Mining Departm't

# THE
# COAL MINERS' POCKETBOOK

# COAL MINERS' POCKETBOOK

FORMERLY THE
**COAL AND METAL MINERS' POCKETBOOK**

## PRINCIPLES, RULES,
## FORMULAS AND TABLES

ELEVENTH EDITION
REVISED, ENLARGED, AND ENTIRELY RESET

# PREFACE TO ELEVENTH EDITION

Perhaps the most important change in the present edition of this book, is the omission of matter pertaining strictly to ore mining, in order that the space formerly occupied by it, and a large amount in addition, might be devoted to coal mining. In consequence of this, the publishers have deemed it wise to change the title from "The Coal and Metal Miners' Pocketbook," to "The Coal Miners' Pocketbook." It is believed that the pocketbook will still prove of interest and value to all men engaged in mining, as well as to many in other branches of engineering, but it seemed no longer possible, within the limits of a compact handbook, to cover both fields in full detail.

The present edition represents a thorough revision. Not only have material changes been made throughout the text, but substantial additions to many sections, which bring the work up to date, and, it is hoped, increase its usefulness.

The subjects of Weights and Measures and Mathematics have been entirely rewritten and greatly enlarged. The department on Surveying has been expanded to include Leveling with the aneroid barometer, so much of Railroad Surveying as is necessary at mines, and Determination of the Meridian, both by solar and pole-star observations; and to it have been added some practical notes on Surveying, as ordinarily practised in flat seams. Under the heading of Mechanics, Strength of Materials, etc., will be found very complete tables of specific gravities and weights of various substances per cubic foot. The department on Concrete Construction is entirely new and complete and is of timely interest. Hydrostatics and Hydraulics have been enlarged by the addition of entirely new tables. Pump Machinery now includes centrifugal pumps. The department on Fuels has been practically rewritten and now contains the best formulas for calculating the heating values of coals from their proximate and ultimate analyses, as well as very complete tables of analyses of American and foreign coals. The notes upon Prospecting include the newer types of drilling machinery. The matter on Mine Timbering is practically new and includes notes on the preservation of timber, steel timbers, etc. The subject of Flushing Culm has been brought up to date with numerous examples drawn from recent practice. The subjects of Explosives and Mining Machines have been rewritten on the lines of modern practice. Under Hoisting and Haulage will be found much entirely new matter on gasoline and storage-battery motors, and the subject of Tracklaying has been entirely rewritten. The subject of the Treatment of Injured Persons is now up to date, and the Glossary has been enlarged.

Those who use this book are kindly and earnestly requested to advise us of any errors or omissions they may note and to offer suggestions for the betterment of subsequent editions.

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# The Coal Miners' Pocketbook

## WEIGHTS AND MEASURES

### LINEAR MEASURE

Immediately following each table of weights or of measures is given a table of equivalents showing the relation existing between the different denominations. All figures on the same horizontal line are of equal or equivalent value.

The United States unit of length, of which unit all other denominations are multiples or submultiples, is the *yard*, originally derived from the Imperial yard of Great Britain. Since 1893, the United States Bureau of Standards has been authorized to derive the yard from the meter, using the relationship established by Congress in the act of July 28, 1866, viz., 1 yard $= \frac{3,600}{3,937}$ meter.

| | |
|---|---|
| 12 inches (in.)........ | = 1 foot........ ft. |
| 3 feet.............. | = 1 yard........ yd. |
| 5.5 yards.......... | = 1 rod........ rd. |
| 40 rods........... | = 1 furlong..... fur. |
| 8 furlongs....... | = 1 mile........ mi. |

| in. | ft. | yd. | rd. | fur. | mi. |
|---|---|---|---|---|---|
| 1 = .083333 | = .027778 | = .005051 | = .000126 | = .000016 | |
| 12 = | 1 = .333333 | = .060606 | = .001515 | = .000189 | |
| 36 = | 3 = | 1 = .181818 | = .004545 | = .000568 | |
| 198 = | 16.5 = | 5.5 = | 1 = .025000 | = .003125 | |
| 7,920 = | 660 = | 220 = | 40 = | 1 = .125000 | |
| 63,360 = | 5,280 = | 1,760 = | 320 = | 8 = | 1 |

The rod of 16.5 ft., and variously known as the *perch* or *pole*, is the same as in surveyor's measure. The furlong is now no longer used.

The *land league* of 3 statute mi. is 15,840 ft.; the *nautical*, or *marine*, *league* of 3 geographical mi. is 18,240 ft.

The nautical, marine, or geographical mile is the $\frac{1}{60}$ part of 1° of a great circle of a sphere whose surface is equal to the surface of the earth. This is commonly taken as 6,080 ft., but is more accurately 6,080.26 ft., and is equivalent to 1.1516 stat. mi. One statute mile equals .8684 naut. mi.

The *fathom* of 6 ft. is used at sea in measuring depths of water, and some times (England) in giving depths of mine shafts.

The *pace* is commonly 3 ft. The U. S. military pace is 30 in.

### SURVEYOR'S LINEAR MEASURE

The surveyor's linear measure is no longer in common use but its denominations are found in descriptions of the boundaries of farms taken from old deeds. Lengths of land lines are now measured and recorded in feet and decimal parts thereof.

| | |
|---|---|
| 7.92 inches (in.)...... | = 1 link.............li. |
| 25 links........... | = 1 rod (16.5 ft.).....rd. |
| 4 rods........... | = 1 chain (66 ft.).....ch. |
| 80 chains.......... | = 1 mile.............mi. |

| in. | li. | rd. | ch. | mi. |
|---|---|---|---|---|
| 1 = | .126263 = | .005051 = | .001263 = | .000016 |
| 7.92 = | 1 = | .040000 = | .010000 = | .000125 |
| 198 = | 25 = | 1 = | .250000 = | .003125 |
| 792 = | 100 = | 4 = | 1 = | .012500 |
| 63,360 = | 8,000 = | 320 = | 80 = | 1 |

Surveyors commonly use the engineer's chain of 50 or 100 ft., the feet being divided into tenths and hundredths.

The annexed scale shows on one side, proportionately reduced, a scale of tenths. On the other, a scale of twelfths, corresponding to inches. To reduce inches to decimal parts of a foot, find the number of inches and fractional parts thereof on the side marked "inches." Opposite, on the scale of



TENTHS OF A FOOT / INCHES
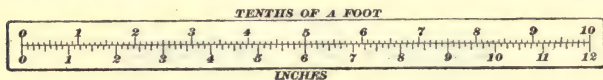
tenths, will be found the decimal part of a foot. Thus, if it is wanted to find the decimal part of a foot represented by 7½ in., find the mark corresponding to 7½ in. on the side marked "inches." Opposite this mark may be read 6 tenths, 2 hundredths, and 5 thousandths; or, expressed decimally, .625. This scale may be laid out, full size, upon stiff cardboard and will be found very useful in figuring lengths in construction work.

## DECIMALS OF AN INCH AND MILLIMETERS FOR EACH 1-64TH IN.

| 64ths of an Inch | Decimal Parts of 1 In. | Millimeters | 64ths of an Inch | Decimal Parts of 1 In. | Millimeters |
|---|---|---|---|---|---|
| 1/64 | .015625 | .397 | 33/64 | .515625 | 13.097 |
| 1/32 | .031250 | .794 | 17/32 | .531250 | 13.494 |
| 3/64 | .046875 | 1.191 | 35/64 | .546875 | 13.891 |
| 1/16 | .062500 | 1.588 | 9/16 | .562500 | 14.288 |
| 5/64 | .078125 | 1.984 | 37/64 | .578125 | 14.684 |
| 3/32 | .093750 | 2.381 | 19/32 | .593750 | 15.081 |
| 7/64 | .109375 | 2.778 | 39/64 | .609375 | 15.478 |
| 1/8 | .125000 | 3.175 | 5/8 | .625000 | 15.875 |
| 9/64 | .140625 | 3.572 | 41/64 | .640625 | 16.272 |
| 5/32 | .156250 | 3.969 | 21/32 | .656250 | 16.669 |
| 11/64 | .171875 | 4.366 | 43/64 | .671875 | 17.066 |
| 3/16 | .187500 | 4.763 | 11/16 | .687500 | 17.463 |
| 13/64 | .203125 | 5.159 | 45/64 | .703125 | 17.859 |
| 7/32 | .218750 | 5.556 | 23/32 | .718750 | 18.256 |
| 15/64 | .234375 | 5.953 | 47/64 | .734375 | 18.653 |
| 1/4 | .250000 | 6.350 | 3/4 | .750000 | 19.050 |
| 17/64 | .265625 | 6.747 | 49/64 | .765625 | 19.447 |
| 9/32 | .281250 | 7.144 | 25/32 | .781250 | 19.844 |
| 19/64 | .296875 | 7.541 | 51/64 | .796875 | 20.241 |
| 5/16 | .312500 | 7.938 | 13/16 | .812500 | 20.638 |
| 21/64 | .328125 | 8.334 | 53/64 | .828125 | 21.034 |
| 11/32 | .343750 | 8.731 | 27/32 | .843750 | 21.431 |
| 23/64 | .359375 | 9.128 | 55/64 | .859375 | 21.828 |
| 3/8 | .375000 | 9.525 | 7/8 | .875000 | 22.225 |
| 25/64 | .390625 | 9.922 | 57/64 | .890625 | 22.622 |
| 13/32 | .406250 | 10.319 | 29/32 | .906250 | 23.019 |
| 27/64 | .421875 | 10.716 | 59/64 | .921875 | 23.416 |
| 7/16 | .437500 | 11.113 | 15/16 | .937500 | 23.813 |
| 29/64 | .453125 | 11.509 | 61/64 | .953125 | 24.209 |
| 15/32 | .468750 | 11.906 | 31/32 | .968750 | 24.606 |
| 31/64 | .484375 | 12.303 | 63/64 | .984375 | 25.003 |
| 1/2 | .500000 | 12.700 | 64/64 | 1.000000 | 25.400 |

## MEASURES OF SURFACE

### SQUARE MEASURE

| 144 square inches (sq. in.) | = 1 square foot | sq. ft. |
| 9 square feet | = 1 square yard | sq. yd. |
| 30.25 square yards | = 1 square rod | sq. rd. |
| 40 square rods | = 1 rood | rood. |
| 4 roods (160 sq. rd.) | = 1 acre | A. or ac. |
| 640 acres = 1 section | = 1 square mile | sq. mi. |

| sq. in. | sq. ft. | sq. yd. | sq. rd. | rood | A. | sq. mi. |
|---|---|---|---|---|---|---|
| 1 = | .006944 = | .000772 = | .000026 | | | |
| 144 = | 1 = | .111111 = | .003673 = | .000092 = | .000023 | |
| 1,296 = | 9 = | 1 = | .033058 = | .000826 = | .000207 | |
| 39,204 = | 272.25 = | 30.25 = | 1 = | .025000 = | .006250 = | .000009 |
| 1,568,160 = | 10,890 = | 1,210 = | 40 = | 1 = | .250000 = | .000391 |
| 6,272,640 = | 43,560 = | 4,840 = | 160 = | 4 = | 1 = | .001563 |
| 4,014,489,600 = | 27,878,400 = | 3,097,600 = | 102,400 = | 2,560 = | 640 = | 1 |

The square rod is also known as the *perch.* The rood, equal to 40 sq. rd., or ¼ A., is obsolete. 640 A. make one section; 320 A., one half-section; 160 A., a quarter section, etc. 36 sections, or 23,040 A., make 1 township (twp.). The areas of small tracts of land, such as city lots, are usually given in square feet and decimals thereof; of larger bodies of land, in acres and decimals of an acre. A square measuring 208.71 ft., or 69.57 yd., on each side contains 1 A.

*Squares* of 100 sq. ft. or of 1 sq. yd. are used in estimating various kinds of work, such as roofing, lathing, plastering, etc. It is advisable to specify the size of the square in all contracts.

### SURVEYOR'S SQUARE MEASURE

The surveyor's square measure is practically obsolete in the United States, although its denominations are commonly found in old deeds in describing the area of lands. As lengths of land lines are now generally measured in feet, tenths, and hundredths, areas are commonly expressed in square feet or in acres and in decimal parts thereof.

| 62.7264 square inches (sq. in.) | = 1 square link | sq. li. |
| 625 square links | = 1 square rod | sq. rd. |
| 16 square rods | = 1 square chain | sq. ch. |
| 10 square chains | = 1 acre | A. or ac. |
| 640 acres = 1 section | = 1 square mile | sq. mi. |
| 36 square miles | = 1 township | tp. or twp. |

| sq. in. | sq. li. | sq. rd. | sq. ch. | A. | sq. mi. |
|---|---|---|---|---|---|
| 1 = | .015942 = | .000026 = | .000002 | | |
| 62.7264 = | 1 = | .001600 = | .000100 = | .000010 | |
| 39,204 = | 625 = | 1 = | .062500 = | .006250 = | .000009 |
| 627,264 = | 10,000 = | 16 = | 1 = | .100000 = | .000156 |
| 6,272,640 = | 100,000 = | 160 = | 10 = | 1 = | .001563 |
| 4,014,489,600 = | 64,000,000 = | 102,400 = | 6,400 = | 640 = | 1 |

1 *township* = 36 sq. mi. = 23,040 A. = 230,400 sq. ch. = 3,686,400 sq. rd. = 2,304,000,000 sq. li. = 144,521,625,600 sq. in.

## MEASURES OF WEIGHT

The United States standard of weight is the troy pound of Great Britain from which the avoirdupois pound is derived in the ratio 1 pound avoirdupois = $\frac{7,000}{5,760}$ pound troy. Since 1893, the United States Bureau of Standards has been authorized to derive the pound avoirdupois from the kilogram, using the

relationship established by Congress in the act of July 28, 1866, viz., 1 pound avoirdupois $= \frac{1}{2.2046}$ kilogram. The weight of the grain in the troy, apothecaries', and avoirdupois pounds, is the same.

### TROY WEIGHT

Troy weight is used in weighing gold and silver.

| | | |
|---|---|---|
| 24 grains (gr.)........ | = 1 pennyweight..... | dwt. |
| 20 pennyweights..... | = 1 ounce.......... | oz. |
| 12 ounces.......... | = 1 pound.......... | lb. |

| gr. | dwt. | oz. | lb. |
|---|---|---|---|
| 1 = | .041667 = | .002083 = | .000174 |
| 24 = | 1 = | .050000 = | .004167 |
| 480 = | 20 = | 1 = | .083333 |
| 5,760 = | 240 = | 12 = | 1 |

1 oz. troy = 1 oz. apothecaries' = 1.09714 oz. avoirdupois
1 lb. troy = 1 lb. apothecaries' = .82286 lb. avoirdupois
1 oz. avoirdupois = .91146 oz. troy or apothecaries'
1 lb. avoirdupois = 1.21528 lb. troy or apothecaries'

### APOTHECARIES' WEIGHT

| | | |
|---|---|---|
| 20 grains (gr.)..... | = 1 scruple..... | sc. |
| 3 scruples........ | = 1 dram....... | dr. |
| 8 drams.......... | = 1 ounce...... | oz. |
| 12 ounces........ | = 1 pound...... | lb. |

### DECIMALS OF A FOOT FOR EACH 1-64TH IN.

| Inch | 0″ | 1″ | 2″ | 3″ | 4″ | 5″ | 6″ | 7″ | 8″ | 9″ | 10″ | 11″ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | .0833 | .1667 | .2500 | .3333 | .4167 | .5000 | .5833 | .6667 | .7500 | .8333 | .9167 |
| 1/64 | .0013 | .0846 | .1680 | .2513 | .3346 | .4180 | .5013 | .5846 | .6680 | .7513 | .8346 | .9180 |
| 1/32 | .0026 | .0859 | .1693 | .2526 | .3359 | .4193 | .5026 | .5859 | .6693 | .7526 | .8359 | .9193 |
| 3/64 | .0039 | .0872 | .1706 | .2539 | .3372 | .4206 | .5039 | .5872 | .6706 | .7539 | .8372 | .9206 |
| 1/16 | .0052 | .0885 | .1719 | .2552 | .3385 | .4219 | .5052 | .5885 | .6719 | .7552 | .8385 | .9219 |
| 5/64 | .0065 | .0898 | .1732 | .2565 | .3398 | .4232 | .5065 | .5898 | .6732 | .7565 | .8398 | .9232 |
| 3/32 | .0078 | .0911 | .1745 | .2578 | .3411 | .4245 | .5078 | .5911 | .6745 | .7578 | .8411 | .9258 |
| 7/64 | .0091 | .0924 | .1758 | .2591 | .3424 | .4258 | .5091 | .5924 | .6758 | .7591 | .8424 | .9258 |
| 1/8 | .0104 | .0937 | .1771 | .2604 | .3437 | .4271 | .5104 | .5937 | .6771 | .7604 | .8437 | .9271 |
| 9/64 | .0117 | .0951 | .1784 | .2617 | .3451 | .4284 | .5117 | .5951 | .6784 | .7617 | .8451 | .9284 |
| 5/32 | .0130 | .0964 | .1797 | .2630 | .3464 | .4297 | .5130 | .5964 | .6797 | .7630 | .8464 | .9297 |
| 11/64 | .0143 | .0977 | .1810 | .2643 | .3477 | .4310 | .5143 | .5977 | .6810 | .7643 | .8477 | .9310 |
| 3/16 | .0156 | .0990 | .1823 | .2656 | .3490 | .4323 | .5156 | .5990 | .6823 | .7656 | .8490 | .9323 |
| 13/64 | .0169 | .1003 | .1836 | .2669 | .3503 | .4336 | .5169 | .6003 | .6836 | .7669 | .8503 | .9336 |
| 7/32 | .0182 | .1016 | .1849 | .2682 | .3516 | .4349 | .5182 | .6016 | .6849 | .7682 | .8516 | .9349 |
| 15/64 | .0195 | .1029 | .1862 | .2695 | .3529 | .4362 | .5195 | .6029 | .6862 | .7695 | .8529 | .9362 |
| 1/4 | .0208 | .1042 | .1875 | .2708 | .3542 | .4375 | .5208 | .6042 | .6875 | .7708 | .8542 | .9375 |
| 17/64 | .0221 | .1055 | .1888 | .2721 | .3555 | .4388 | .5221 | .6055 | .6888 | .7721 | .8555 | .9388 |
| 9/32 | .0234 | .1068 | .1901 | .2734 | .3568 | .4401 | .5234 | .6068 | .6901 | .7734 | .8568 | .9401 |
| 19/64 | .0247 | .1081 | .1914 | .2747 | .3581 | .4414 | .5247 | .6081 | .6914 | .7747 | .8581 | .9414 |
| 5/16 | .0260 | .1094 | .1927 | .2760 | .3594 | .4427 | .5260 | .6094 | .6927 | .7760 | .8594 | .9427 |
| 21/64 | .0273 | .1107 | .1940 | .2773 | .3607 | .4440 | .5273 | .6107 | .6940 | .7773 | .8607 | .9440 |
| 11/32 | .0286 | .1120 | .1953 | .2786 | .3620 | .4453 | .5286 | .6120 | .6953 | .7786 | .8620 | .9453 |
| 23/64 | .0299 | .1133 | .1966 | .2799 | .3633 | .4466 | .5299 | .6133 | .6966 | .7799 | .8633 | .9466 |
| 3/8 | .0312 | .1146 | .1979 | .2812 | .3646 | .4479 | .5312 | .6146 | .6979 | .7812 | .8646 | .9479 |
| 25/64 | .0326 | .1159 | .1992 | .2826 | .3659 | .4492 | .5326 | .6159 | .6992 | .7826 | .8659 | .9492 |
| 13/32 | .0339 | .1172 | .2005 | .2839 | .3672 | .4505 | .5339 | .6172 | .7005 | .7839 | .8672 | .9505 |
| 27/64 | .0352 | .1185 | .2018 | .2852 | .3685 | .4518 | .5352 | .6185 | .7018 | .7852 | .8685 | .9518 |
| 7/16 | .0365 | .1198 | .2031 | .2865 | .3698 | .4531 | .5365 | .6198 | .7031 | .7865 | .8698 | .9531 |
| 29/64 | .0378 | .1211 | .2044 | .2878 | .3711 | .4544 | .5378 | .6211 | .7044 | .7878 | .8711 | .9544 |
| 15/32 | .0391 | .1224 | .2057 | .2891 | .3724 | .4557 | .5391 | .6224 | .7057 | .7891 | .8724 | .9557 |
| 31/64 | .0404 | .1237 | .2070 | .2904 | .3737 | .4570 | .5404 | .6237 | .7070 | .7904 | .8737 | .9570 |
| 1/2 | .0417 | .1250 | .2083 | .2917 | .3750 | .4583 | .5417 | .6250 | .7083 | .7917 | .8750 | .9583 |

| gr. | sc. | dr. | oz. | lb. |
|---|---|---|---|---|
| 1 = | .050 = | .016667 = | .002083 = | .000174 |
| 20 = | 1 = | .333333 = | .041667 = | .003472 |
| 60 = | 3 = | 1 = | .125000 = | .010417 |
| 480 = | 24 = | 8 = | 1 = | .083333 |
| 5,760 = | 288 = | 96 = | 12 = | 1 |

For equivalents in troy and apothecaries' weights, see under the former.

## AVOIRDUPOIS WEIGHT
### Short Ton

| | | |
|---|---|---|
| 27.34375 grains (gr.) | = 1 dram | dr. |
| 16 drams | = 1 ounce | oz. |
| 16 ounces | = 1 pound | lb. |
| 100 pounds | = 1 hundredweight | cwt. |
| 20 hundredweight }<br>2,000 pounds } | = 1 ton | T. |

| gr. | dr. | oz. | lb. | cwt. | T. |
|---|---|---|---|---|---|
| 1 = | .036571 = | .002286 = | .000143 = | .000001 | |
| 27.34375 = | 1 = | .062500 = | .003906 = | .000039 = | .000002 |
| 437.5 = | 16 = | 1 = | .062500 = | .000625 = | .000031 |
| 7,000 = | 256 = | 16 = | 1 = | .010000 = | .000500 |
| 700,000 = | 25,600 = | 1,600 = | 100 = | 1 = | .050000 |
| 14,000,000 = | 512,000 = | 32,000 = | 2,000 = | 20 = | 1 |

The ton of 2,000 lb. is the trade standard of the United States, except in transactions involving anthracite (in Pennsylvania) and certain iron and steel products in bulk. A hundredweight is sometimes known as a quintal and is not uncommonly used among fishermen. The dram is practically obsolete.

## DECIMALS OF A FOOT FOR EACH 1-64TH IN.

| Inch | 0″ | 1″ | 2″ | 3″ | 4″ | 5″ | 6″ | 7″ | 8″ | 9″ | 10″ | 11″ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1/2 | .0417 | .1250 | .2083 | .2917 | .3750 | .4583 | .5417 | .6250 | .7083 | .7917 | .8750 | .9583 |
| 33/64 | .0430 | .1263 | .2096 | .2930 | .3763 | .4596 | .5430 | .6263 | .7096 | .7930 | .8763 | .9596 |
| 17/32 | .0443 | .1276 | .2109 | .2943 | .3776 | .4609 | .5443 | .6276 | .7109 | .7943 | .8776 | .9609 |
| 35/64 | .0456 | .1289 | .2122 | .2956 | .3789 | .4622 | .5456 | .6289 | .7122 | .7956 | .8789 | .9622 |
| 9/16 | .0469 | .1302 | .2135 | .2969 | .3802 | .4635 | .5469 | .6302 | .7135 | .7969 | .8802 | .9635 |
| 37/64 | .0482 | .1315 | .2148 | .2982 | .3815 | .4648 | .5482 | .6315 | .7148 | .7982 | .8815 | .9648 |
| 19/32 | .0495 | .1328 | .2161 | .2995 | .3828 | .4661 | .5495 | .6328 | .7161 | .7995 | .8828 | .9661 |
| 39/64 | .0508 | .1341 | .2174 | .3008 | .3841 | .4674 | .5508 | .6341 | .7174 | .8008 | .8841 | .9674 |
| 5/8 | .0521 | .1354 | .2188 | .3021 | .3854 | .4688 | .5521 | .6354 | .7188 | .8021 | .8854 | .9688 |
| 41/64 | .0534 | .1367 | .2201 | .3034 | .3867 | .4701 | .5534 | .6367 | .7201 | .8034 | .8867 | .9701 |
| 21/32 | .0547 | .1380 | .2214 | .3047 | .3880 | .4714 | .5547 | .6380 | .7214 | .8047 | .8880 | .9714 |
| 43/64 | .0560 | .1393 | .2227 | .3060 | .3893 | .4727 | .5560 | .6393 | .7227 | .8060 | .8893 | .9727 |
| 11/16 | .0573 | .1406 | .2240 | .3073 | .3906 | .4740 | .5573 | .6406 | .7240 | .8073 | .8906 | .9740 |
| 45/64 | .0586 | .1419 | .2253 | .3086 | .3919 | .4753 | .5586 | .6419 | .7253 | .8086 | .8919 | .9753 |
| 23/32 | .0599 | .1432 | .2266 | .3099 | .3932 | .4766 | .5599 | .6432 | .7266 | .8099 | .8932 | .9766 |
| 47/64 | .0612 | .1445 | .2279 | .3112 | .3945 | .4779 | .5612 | .6445 | .7279 | .8112 | .8945 | .9779 |
| 3/4 | .0625 | .1458 | .2292 | .3125 | .3958 | .4792 | .5625 | .6458 | .7292 | .8125 | .8958 | .9792 |
| 49/64 | .0638 | .1471 | .2305 | .3138 | .3971 | .4805 | .5638 | .6471 | .7305 | .8138 | .8971 | .9805 |
| 25/32 | .0651 | .1484 | .2318 | .3151 | .3984 | .4818 | .5651 | .6484 | .7318 | .8151 | .8984 | .9818 |
| 51/64 | .0664 | .1497 | .2331 | .3164 | .3997 | .4831 | .5664 | .6497 | .7331 | .8164 | .8997 | .9831 |
| 13/16 | .0677 | .1510 | .2344 | .3177 | .4010 | .4844 | .5677 | .6510 | .7344 | .8177 | .9010 | .9844 |
| 53/64 | .0690 | .1523 | .2357 | .3190 | .4023 | .4857 | .5690 | .6523 | .7357 | .8190 | .9023 | .9857 |
| 27/32 | .0703 | .1536 | .2370 | .3203 | .4036 | .4870 | .5703 | .6536 | .7370 | .8203 | .9036 | .9870 |
| 55/64 | .0716 | .1549 | .2383 | .3216 | .4049 | .4883 | .5716 | .6549 | .7383 | .8216 | .9049 | .9883 |
| 7/8 | .0729 | .1562 | .2396 | .3229 | .4062 | .4896 | .5729 | .6562 | .7396 | .8229 | .9062 | .9896 |
| 57/64 | .0742 | .1576 | .2409 | .3242 | .4076 | .4909 | .5742 | .6576 | .7409 | .8242 | .9076 | .9909 |
| 29/32 | .0755 | .1589 | .2422 | .3255 | .4089 | .4922 | .5755 | .6589 | .7422 | .8255 | .9089 | .9922 |
| 59/64 | .0768 | .1602 | .2435 | .3268 | .4102 | .4935 | .5768 | .6602 | .7435 | .8268 | .9102 | .9935 |
| 15/16 | .0781 | .1615 | .2448 | .3281 | .4115 | .4948 | .5781 | .6615 | .7448 | .8281 | .9115 | .9948 |
| 61/64 | .0794 | .1628 | .2461 | .3294 | .4128 | .4961 | .5794 | .6628 | .7461 | .8294 | .9128 | .9961 |
| 31/32 | .0807 | .1641 | .2474 | .3307 | .4141 | .4974 | .5807 | .6641 | .7474 | .8307 | .9141 | .9974 |
| 63/64 | .0820 | .1654 | .2487 | .3320 | .4154 | .4987 | .5820 | .6654 | .7487 | .8320 | .9154 | .9987 |
| 1 | | | | | | | | | | | | 1.0000 |

## Long Ton

The grains, drams, ounces, and pounds are the same as in the short ton.

| | |
|---|---|
| 16 ounces..............| = 1 pound............lb. |
| 14 pounds............| = 1 stone..............st. |
| 2 stones............| = 1 quarter............qr. |
| 4 quarters............| = 1 hundredweight.....cwt. |
| 20 hundredweight.....| = 1 ton (2,240 lb.)......T. |

| oz. | lb. | st. | qr. | cwt. | T. |
|---|---|---|---|---|---|
| 1 = | .062500 = | .004464 = | .002232 = | .000558 = | .000028 |
| 16 = | 1 = | .071429 = | .035714 = | .008929 = | .000446 |
| 224 = | 14 = | 1 = | .500000 = | .125000 = | .006250 |
| 448 = | 28 = | 2 = | 1 = | .250000 = | .012500 |
| 1,792 = | 112 = | 8 = | 4 = | 1 = | .050000 |
| 35,840 = | 2,240 = | 160 = | 80 = | 20 = | 1 |

Short tons multiplied by 1.12 equal long tons. Long tons multiplied by .892857 equal short tons. The long ton is the standard in Great Britain and colonies, except Canada, but its use in the United States is limited. The long ton is used in estimating custom duties.

---

# MEASURES OF VOLUME

| | |
|---|---|
| 1,728 cubic inches (cu. in.).....| = 1 cubic foot .....cu. ft. |
| 27 cubic feet .............| = 1 cubic yard.....cu. yd. |

| cu. in. | cu. ft. | cu. yd. |
|---|---|---|
| 1 = | .000579 = | .000021 |
| 1,728 = | 1 = | .037037 |
| 46,656 = | 27 = | 1 |

A *cord of wood* is 128 cu. ft., or a pile 8 ft. long and 4 ft. high when cut in 4-ft. lengths. It is used in estimating amounts of fire and pulp wood, tanbark, etc.

A ton (2,240 lb.) of Pennsylvania anthracite, when broken for domestic use, occupies about 42 cu. ft. of space; bituminous coal about 46 cu. ft.; and coke, about 88 cu. ft.

A bushel of coal is 80 lb. in Kentucky, Illinois, and Missouri; 76 lb. in Pennsylvania and Montana; and 70 lb. in Indiana.

**Masonry.**—A perch of masonry is 24.75 cu. ft., or is a section of wall 16.5 ft. (1 rd. or perch) long, 1.5 ft. thick, and 1 ft. high. It is very frequently taken as 25 cu. ft. Methods and customs of estimating masonry vary locally and it is highly advisable, when preparing contract specifications, to insert in the agreement, upon what basis the measurements are to be made; that is, if by the perch, the number of cubic feet therein. Owing to the confusion in the dimensions of the perch, the term is falling into disuse, and contracts specify measurements either in cubic feet or cubic yards.

Masonry is measured solid, no deductions being made for corners, which are counted twice, or for openings under 3 ft. in width. This is the custom of the trade and holds in law unless the contract specifies differently. Thus, a foundation wall 1 ft. thick, 8 ft. high, and with outside dimensions of 10 ft. by 12 ft., and with one door opening 2 ft. wide and 8 ft. high, actually contains $[(12 \times 2) + (10 - 1 - 1) \times 2] \times 8 - (2 \times 8) = 304$ cu. ft. On the trade basis, the door opening is neglected and the four side walls are counted at their total length (2 of 12 ft., and 2 of 10 ft.), and the wall contains $[(12 \times 2) + (10 \times 2)] \times 8 = 352$ cu. ft.

**Brickwork.**—Brickwork is generally estimated by the thousand bricks laid in the wall, but measurements by the cubic foot and the perch are also used. When making calculations of the volume of walls, etc., to allow for mortar, it is customary to add ¼ in. to the length and thickness of each brick. The following data will be useful in calculating the number of bricks in a wall. For each superficial foot of wall 4 in. in thickness (the width of one brick), allow 7½ bricks; for a 9-in. wall (the width of two bricks), allow 15 bricks; and so on, estimating 7½ bricks for each additional 4 in. in thickness of wall. If brickwork is estimated by the cubic yard, allow 500 bricks to 1 cu. yd. This figure is based on the use of a 8¼ in. by 4 in. by 2¼ in. brick, with mortar joints not over ⅜ in. thick. If the joints are ¼ in. thick, as in face brickwork, 1 cu. yd. will require about 575 bricks. An allowance of 5% should be made for waste in breakage, etc.

**Shipping.**—The *gross tonnage* of a ship is its entire internal capacity, calculated according to certain rather complicated rules laid down in the Revised Statutes of the United States. The *net tonnage* of a ship is obtained by deducting from the gross tonnage the space given over to engines, coal, quarters for the crew, etc.; that is, it is the net space available for cargo or paying load. *Registered tonnage* is the entire internal cubic contents of the vessel divided by 100, or 100 cu. ft. equals 1 registered ton. The term gives no idea of the dimensions of the vessel, and is merely an arbitrary way of forming some conception of its relative size.

*Displacement* is the weight of the volume of water displaced by the hull of a vessel and is often confused with some one of the meanings of tonnage just given. The displacement of a ship naturally varies, depending on the weight of cargo, stores, fuel, etc., aboard. Thus, the displacement of a transatlantic liner will be markedly less on arriving at New York than when leaving England. Displacement is frequently calculated at a *normal*, or standard, depth of water, to which draft the ship is usually loaded, or for which it was designed; from this is derived the expression, say, "displacement 18,500 T. on 23 ft. draft."

For the purpose of calculating vessel freights, estimating stowage capacity, etc., 1 U. S. *shipping ton* equals 40 cu. ft. and is equivalent to 32.143 U. S. bu., or 31.16 imp. bu. of England. The British shipping ton is 42 cu. ft. and is equal to 32.719 imp. bu., or 33.75 U. S. bu.

For weights of various materials see under the heading Specific Gravity.

## LIQUID MEASURE

By act of Congress the standard of liquid measure is the *gallon* of 231 cu. in.

| | | | |
|---|---|---|---|
| 1 gill | gi. | 7.21875 cu. in. |
| 4 gills | = 1 pint | pt. | 28.875 cu. in. |
| 2 pints | = 1 quart | qt. | 57.750 cu. in. |
| 4 quarts | = 1 gallon | gal. | 231.000 cu. in. |
| 31.5 gallons | = 1 barrel | bbl. | 4.211 cu. ft. |
| 2 barrels | = 1 hogshead | hhd. | 8.422 cu. ft. |

| *gi.* | *pt.* | *qt.* | *gal.* | *bbl.* | *hhd.* |
|---|---|---|---|---|---|
| 1 | = .250 | = .125 | = .03125 | = .000992 | = .000496 |
| 4 = | 1 | = .500 | = .12500 | = .003968 | = .001984 |
| 8 = | 2 = | 1 | = .25000 | = .007936 | = .003968 |
| 32 = | 8 = | 4 = | 1 | = .031746 | = .015873 |
| 1,008 = | 252 = | 126 = | 31.5 = | 1 | = .500000 |
| 2,016 = | 504 = | 252 = | 63 = | 2 = | 1 |

The U. S. liquid pint, quart, and gallon are equal, respectively, to .85937 U. S. dry pint, quart, and gallon. The U. S. dry pint, quart, and gallon are equal, respectively, to 1.16365 U. S. liquid pint, quart, and gallon.

A box 19⅜ in. long on each edge contains 1 bbl.

In approximate calculations, 1 cu. ft. of water may be considered equal to 7½ gal., and 1 gal. as weighing 8⅓ lb.

The capacity of a cylinder in U. S. liquid gallons = square of the diameter, in inches × height, in inches × .0034 (accurate within 1 part in 100,000).

The following cylinders contain the given measures very closely:

| | Diam. Inches | Height Inches | | Diam. Inches | Height Inches |
|---|---|---|---|---|---|
| Gill | 1½ | 3 | Gallon | 7 | 6 |
| Pint | 3½ | 3 | 8 gal. | 14 | 12 |
| Quart | 3½ | 6 | 10 gal. | 14 | 15 |

## DRY MEASURE

By act of Congress the standard of dry measure is the bushel defined as a cylinder 18½ in. in diameter, 8 in. deep, and containing 2,150.42 cu. in.

| | | | |
|---|---|---|---|
| 1 pint | pt. | 33.6003125 cu. in. |
| 2 pints | = 1 quart | qt. | 67.200625 cu. in. |
| 4 quarts | = 1 gallon | gal. | 268.8025 cu. in. |
| 2 gallons | = 1 peck | pk. | 537.605 cu. in. |
| 4 pecks | = 1 bushel | bu. | 2,150.42 cu. in. |

|  | pt. | qt. | gal. | pk. | bu. |
|--|--|--|--|--|--|
| 1 = | .500 = | .125 = | .0625 = | .015625 |
| 2 = | | 1 = | .250 = | .1250 = | .031250 |
| 8 = | | 4 = | 1 = | .5000 = | .125000 |
| 16 = | | 8 = | 2 = | 1 = | .250000 |
| 64 = | | 32 = | 8 = | 4 = | 1 |

The standard bushel is a *struck.*, or level full, *bushel.* The *heaped bushel* is approximately equal to 1¼ struck bu., the *cone*, or heap, being not less than 6 in. in height.

The standard bushel is equal to 1.24445, or approximately 1¼, cu. ft. 1 cu. ft. is equal to .80356, or approximately ⅘, bu.

A cube 3.227 in. on an edge contains 1 pt.; one 4.066 in. on an edge, 1 qt.; one 6.454 in. on an edge, 1 gal.; one 8.131 in. on an edge, 1 pk.; and one 12.908 in. on an edge, 1 bu.

The capacity of a cylinder in U. S. bushels = square of diameter in inches ×height in inches×.0003652.

There appears to be no standard barrel, dry measure, although one of 3 struck bu. is frequently recognized.

The relation between the dimensions of the units of dry and liquid measure will be found under the former.

## RELATION BETWEEN VOLUMES AND WEIGHTS OF WATER, U. S. LIQUID MEASURE

The mass of a given volume of water, such as 1 cu. ft. or 1 gal., depends on the conditions under which it is weighed, being less if weighed in air than in a vacuum, at the equator than at the poles, at sea level than at any elevation above, and at higher than at lower readings of either the thermometer or barometer. Reduced from the French measurements made to determine the relations existing between the units of the metric system, the weight *in vacuuo* of 1 cu. ft. of pure distilled water, free from air, at the temperature of its maximum density (4° C., or 39.3° F.) and under a barometric pressure of 760 millimeters (29.92 in.) of mercury, at sea level and at the latitude of Paris (48° 50′ N), is 62.42664 lb. This is the weight commonly used in pocketbooks and is often given as 62.427 lb., 62.43 lb., and even as 62.5 lb., depending on the degree of accuracy required. From this, the weight of 1 cu. in. of water under the given conditions may be taken as .036126 lb., .036 lb., or even .04 lb. At other temperatures commonly used in calculations the weight of 1 cu. ft. of water is: at 32° F., 62.418 lbs.; at 62° F., 62.355 lbs.; and at 212° F., 59.846 lbs.

The following table gives the weight *in air* of 1 gal. and of 1 cu. ft. of distilled water. These weights are reduced from the French measurements and are referred to sea level at the latitude of Paris. They represent the customary or ordinary manner of weighing.

| Temperature | Pressure | Weight of 1 Gal. Pounds | Weight of Cu. Ft. Pounds |
|--|--|--|--|
| 4° C. (39.3° F.)..760 mm. (29.92 in.)... | .8.33586..... | 62.35656 |
| 62° F. (16.7° C.)..760 mm. (29.92 in.)... | .8.32675..... | 62.28844 |
| 62° F. (16.7° C.).. 30 in. (762.0 mm.)... | .8.32673..... | 62.28827 |

The following table of equivalents is based upon the weight of 1 cu. ft. of water *weighed in air* at 39.3° F., and 29.92 in. of mercury, viz.: 62.356562 lb. avoirdupois.

### EQUIVALENT WEIGHTS AND VOLUMES OF WATER

| Gills | Pints | Quarts | Gallons | Cubic Inches | Cubic Foot | Weight of Water Pounds |
|--|--|--|--|--|--|--|
| 1 | .250000 | .125000 | .031250 | 7.218750 | .004178 | .260496 |
| 4 | 1 | .500000 | .125000 | 28.875000 | .016710 | 1.041983 |
| 8 | 2 | 1 | .250000 | 57.750000 | .033420 | 2.083965 |
| 32 | 4 | | 1 | 231 | .133681 | 8.335860 |
| .138528 | .034632 | .017316 | .004329 | 1 | .000579 | .036086 |
| 239.376624 | 59.844156 | 29.922078 | 7.480520 | 1,728 | 1 | 62.356562 |
| 3.838836 | .959709 | .479855 | .119964 | 27.711598 | .016037 | 1 |

## ANGULAR, OR CIRCULAR, MEASURE

60 seconds (sec. or ″) ..... = 1 minute............min. or ′
60 minutes..... ......... = 1 degree...........deg. or °
360 degrees............... = 1 circumference .... cir.

| sec. | min. | deg. | cir. |
|------|------|------|------|
| 1 = | .016667 = | .000278 = | .000008 |
| 60 = | 1 = | .016667 = | .000463 |
| 3,600 = | 60 = | 1 = | .002778 |
| 1,296,000 = | 21,600 = | 360 = | 1 |

For tables of grades, grade angles, etc., see under the head of Tracklaying.

**Laying Off Right Angles.**—Right angles may conveniently be laid off in the field by using the relation existing between the hypotenuse and sides of a right-angled triangle, viz.: The square of the hypotenuse is equal to the sum of the squares of the sides. The commonly used ratio is 3 for the horizontal side, 4 for the vertical side, and 5 for the hypotenuse, or slope. There are many other ratios, such as 8, 15, 17, and the like, a number of which are given in the following table. Likewise, any multiple or submultiple of these ratios may be used. Thus, instead of 3, 4, and 5, $\frac{1}{2}$, $1\frac{1}{2}$, or 2 times the proportion may be used, as 1.5, 2, 2.5, or 4.5, 6, 7.5, or 6, 8, and 10, respectively.
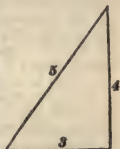
### RATIO OF SIDES OF RIGHT-ANGLED TRIANGLE

| Hori-zontal | Verti-cal | Slope | Hori-zontal | Verti-cal | Slope | Hori-zontal | Verti-cal | Slope |
|---|---|---|---|---|---|---|---|---|
| 3 | 4 | 5 | 12 | 16 | 20 | 18 | 24 | 30 |
| 4 | 3 | 5 | 12 | 9 | 15 | 20 | 15 | 25 |
| 6 | 8 | 10 | 14 | 48 | 50 | 20 | 48 | 52 |
| 7 | 24 | 25 | 15 | 8 | 17 | 24 | 32 | 40 |
| 8 | 6 | 10 | 15 | 20 | 25 | 24 | 18 | 30 |
| 8 | 15 | 17 | 15 | 36 | 39 | 24 | 45 | 51 |
| 9 | 12 | 15 | 16 | 12 | 20 | 24 | 7 | 25 |
| 10 | 24 | 26 | 16 | 30 | 40 | 25 | 60 | 65 |

**Laying Off an Angle With a Tape.**—It is frequently necessary to lay off other than right angles as, for example, when a cut or two must be taken from



Base Line 10 Feet long

Station from which Angle is turned
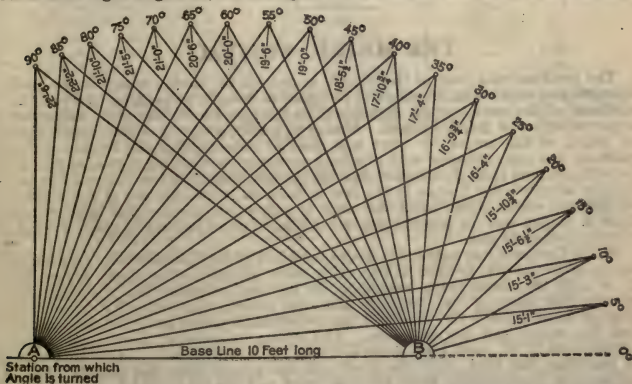
the rib of an entry at a point where a branch entry is to be started, in order that there may be room to place the sights. Suppose it is desired to turn an angle

of 50° to the left.   Fasten the 0 end of an ordinary tape at *A*, then stretch the tape to *B* a distance of 10 ft., and fasten the 10-ft. mark at *B*.   If the angle is to be turned from an entry, the points *A* and *B* can be lined in from the entry sights.   Then fasten the 30-ft. mark of the tape at *A*.   This will leave 20 ft. of slack tape between *A* and *B*.   Find the 19-ft. mark on the tape, and draw the tape tight from both *A* and *B*.   The tape line from *A* to the 19-ft. mark will make an angle of 50° at *A* with the line *A B*.   If the angle is to the right, use the same measurements to the right of the base line.   For other angles than 50° hold the tape at the mark given, on the accompanying diagram, below the required number of degrees.   Thus, for 75°, hold the tape at the 21′ 5″ mark; for 30°, hold it at the 16′ 9¾″ mark, and similarly for other angles.

---

## MEASURE OF TIME

| | | | |
|---|---|---|---|
| 60 seconds (s.) . . . . . | = 1 minute | . . . . min. or $^m$ |
| 60 minutes . . . . . . . . | = 1 hour | . . . . . . . . hr. or $^h$ |
| 24 hours . . . . . . . . . . | = 1 day | . . . . . . . . . da. or $^d$ |

|   sec.   |   min.   |   hr.   |   da.   |
|---|---|---|---|
| 1 = | .016667 = | .000278 = | .000012 |
| 60 = | 1 = | .016667 = | .000694 |
| 3,600 = | 60 = | 1 = | .041667 |
| 86,400 = | 1,440 = | 24 = | 1 |

Solar, astronomical, civil, and standard time are discussed in the chapter on Surveying under the heading Determination of the Meridian.

### LONGITUDE AND TIME

As the earth makes a complete revolution of 360° upon its axis in 24 hr., degrees, minutes, and seconds of arc, or longitude, may be expressed in hours, minutes, and seconds of time.

| *Arc = Time* | *Arc = Time* |
|---|---|
| 360 degrees = 24 hours | 1 minute = 4 seconds |
| 15 degrees = 1 hour | 15 seconds = 1 second |
| 1 degree = 4 minutes | 1 second = .0666+ second |
| 15 minutes = 1 minute | |

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## THE METRIC SYSTEM

The fundamental unit of the metric system is the *meter*, which is the unit of length and which was intended to be equal in length to $\frac{1}{10000000}$ part of the quadrant of the earth, or the distance from the equator to the pole, measured on the meridian of Paris, France.   Owing to imperfection in instruments, etc., the length of the arc of 10° that was used in determining the length of the quadrant was not measured with absolute accuracy, so that the meter, as well as the units derived from it, is not of exactly the dimensions it was intended to be. Since its adoption by France in the law of June 22, 1799, the use of the system has become world wide, until the only countries of commercial importance where it is not now in general use are China, Great Britain, Japan, Russia, and the United States.

While Congress has authority, under Article I of the Constitution, to compel the use of the metric system, it has never done so, contenting itself, under the act of July 28, 1866, with legalizing its use and establishing the relations between the units of the ordinary and the metric systems.   By Executive Order of April 5, 1893, the yard and the avoirdupois pound may now be derived from the meter and the kilogram.   The ratios established by Congress are 1 meter = 39.37 in., and 1 kilogram = 2.2046 lb. avoirdupois.   As these values are less than the true ones, the yard and the pound thus derived are longer and heavier, respectively, than the true yard and the true pound.

The names applied to the various denominations of the system are not always those used in France, but are frequently adapted from the names of

the measures that the metric system has displaced. In almost all of these countries the old and replaced systems are still in use, the extent of this usage being, apparently, in inverse ratio to the commercial importance of the country and to the length of time during which the metric system has been in force.

The original standards of the metric system were a meter and a kilogram of platinum deposited in the Palais des Archives, Paris, at the time of the formal adoption of this system in France. At the second meeting of what is now the International Bureau of Weights and Measures, held in Paris, Sept. 24, 1872, the representatives of thirty nations present decided that the standards of the system should be made of an alloy of 90% platinum and 10% iridium, because of its hardness, fine grain, and ability to withstand the action of acids. A number of meters and kilograms were made and the ones corresponding to the original platinum standards are deposited in the permanent headquarters of the bureau, near Paris. Each nation subscribing to the expenses of the bureau received two copies each of the meter and the kilogram, which are known, respectively, as the *national prototype meter and kilogram*. These are of the same materials and accuracy as the originals. The United States prototypes were received by the President on Jan. 2, 1890, and one of each was selected as a reference standard, while the other is used for working purposes.

The unit of weight of the metric system is the *kilogram*, originally intended to be the weight, in vacuuo, of 1 cubic decimeter of pure, distilled water at the temperature of its maximum density, 4° C. (39.3° F.), with the barometer at 760 millimeters (29.92 in.) of mercury. As with the meter, this relation is not exact, owing to errors in the original standard.

The unit of capacity is the *liter*, which was designed to be the volume of 1 kilogram of pure water under the conditions just named.

The unit of area for land measure is the *are*, a square 10 meters on the side. Among engineers, superficial dimensions are usually expressed in square centimeters, square meters, and square kilometers.

Cubic dimensions are expressed in cubic centimeters, cubic meters, etc.

Multiples of the various units of the metric system are obtained by prefixing to the names of the units (meter, gram, and liter) the Greek words, *deka* or *deca* (10), *hekto* or *hecto* (100), *kilo* (1,000), and *myria* (10,000). The submultiples or divisions are obtained by prefixing to the names of the units, the Latin words *deci* ($\frac{1}{10}$), *centi* ($\frac{1}{100}$), and *milli* ($\frac{1}{1000}$).

### METRIC MEASURES OF LENGTH

| | | | |
|---|---|---|---|
| 1 millimeter | | mm. | .001 |
| 10 millimeters | = 1 centimeter | cm. | .01 |
| 10 centimeters | = 1 decimeter | dm. | .1 |
| 10 decimeters | = 1 meter | m. | 1 |
| 10 meters | = 1 dekameter | Dm. | 10 |
| 10 dekameters | = 1 hektometer | Hm. | 100 |
| 10 hektometers | = 1 kilometer | Km. | 1,000 |
| 10 kilometers | = 1 myriameter | Mm. | 10,000 |

Of these denominations the ones commonly employed are the millimeter, centimeter, meter, and kilometer. The United States Coast and Geodetic Survey, since 1884, has used the value 1 m. = 39.370432 in.; the legal equivalent by Act of Congress is 1 m. = 39.37 in.

### METRIC MEASURES OF SURFACE

| | | | |
|---|---|---|---|
| 1 sq. millimeter | | sq. mm., or mm.² | .000001 |
| 100 sq. millimeters | = 1 sq. centimeter | sq. cm., or cm.² | .0001 |
| 100 sq. centimeters | = 1 sq. decimeter | sq. dm., or dm.² | .01 |
| 100 sq. decimeters | = 1 sq. meter | sq. m., or m.² | 1 |
| 100 sq. meters | = 1 sq. dekameter | sq. Dm., or Dm.² | 100 |
| 100 sq. dekameters | = 1 sq. hektometer | Hm., or Hm.² | 10,000 |
| 100 sq. hektometers | = 1 sq. kilometer | sq. Km., or Km.² | 1,000,000 |
| 100 sq. kilometers | = 1 sq. myriameter | sq. Mm., or Mm.² | 100,000,000 |

The square meter is sometimes known as the *centare;* the square dekameter as the *are;* and the square hektometer as the *hectare.* Farm measurements are generally given in hectares. Engineers use the square millimeter, square centimeter, square meter, and square kilometer. The other denominations are but little used. The square meter, or centare, is a little larger than 1 sq. yd.; the hektometer, or hectare, is about 2½ A.

### METRIC MEASURES OF WEIGHT

| | | | |
|---|---|---|---:|
| 1 milligram | | mg. | .001 |
| 10 milligrams | =1 centigram | cg. | .01 |
| 10 centigrams | =1 decigram | dg. | .1 |
| 10 decigrams | =1 gram | g. | 1 |
| 10 grams | =1 dekagram | Dg. | 10 |
| 10 dekagrams | =1 hektogram | Hg. | 100 |
| 10 hektograms | =1 kilogram | Kg. | 1,000 |
| 10 kilograms | =1 myriagram | Mg. | 10,000 |
| 10 myriagrams | =1 quintal | Q. | 100,000 |
| 10 quintals | =1 tonne | T. | 1,000,000 |

The denominations in common use are the milligram, centigram, gram, kilogram (commonly called kilo), and tonne. Commercially, the kilogram is divided into halves, quarters, etc. As near as may be determined, 1 Kg. =2.2046223 lb. avoirdupois. The ratio legalized by Congress is 1 Kg.=2.2046 lb. avoirdupois.

### METRIC MEASURES OF VOLUME

| | | | |
|---|---|---|---:|
| 1 cubic millimeter | | cu. mm., or mm.$^3$ | .000000001 |
| 1,000 cubic millimeters | =1 cubic centimeter | c.c., cc.$^3$, or cu. cm. | .000001 |
| 1,000 cubic centimeters | =1 cubic decimeter | cu. dm., or dm.$^3$ | .001 |
| 1,000 cubic decimeters | =1 cubic meter | cu. m., or m.$^3$ | 1 |

The cubic millimeter and cubic decimeter are rarely used. The cubic centimeter is a common unit with chemists, and engineers use the cubic meter in the same way as the cubic yard.

### METRIC MEASURES OF CAPACITY

| | | | |
|---|---|---|---:|
| 1 milliliter | | ml. | .001 |
| 10 milliliters | =1 centiliter | cl. | .01 |
| 10 centiliters | =1 deciliter | dl. | .1 |
| 10 deciliters | =1 liter | l. | 1 |
| 10 liters | =1 dekaliter, or | Dl. | 10 |
| | 1 centistere | cs. | |
| 10 dekaliters | =1 hektoliter, or | Hl. | 100 |
| | 1 decistere | ds. | |
| 10 hektoliters | =1 kiloliter, or | Kl. | 1,000 |
| | 1 stere | s. | |
| 10 kiloliters | =1 myrialiter, or | Ml. | 10,000 |
| | 1 dekastere | Ds. | |

The liter and the milliliter (but in its equivalent form the cubic centimeter) are in use among chemists. Engineers use the liter and kiloliter, the latter being called by its equivalent name, cubic meter. Groceries and the like are purchased by the liter, half liter, and quarter liter, instead of by the decimal parts of a liter. Grain is measured by the stere and by the decistere, the latter being called hektoliter.

Congress has not established a ratio between the units of the measures of capacity of the metric and ordinary systems.

### EQUIVALENTS OF VOLUME, WEIGHT OF WATER, AND CAPACITY

Distilled water at 4° C. (39.3° F.) and 760 mm. (29.92 in.) pressure.

| *Volume* | *Weight* | *Capacity* | *Ratio* |
|---|---|---|---:|
| 1 cubic centimeter | =1 gram | =1 milliliter | 1 |
| 10 cubic centimeters | =1 decagram | =1 centiliter | 10 |
| 100 cubic centimeters | =1 hektogram | =1 deciliter | 100 |
| 1 cubic decimeter | =1 kilogram | =1 liter | 1,000 |
| 10 cubic decimeters | =1 myriagram | =1 dekaliter | 10,000 |
| 100 cubic decimeters | =1 quintal | =1 hektoliter | 100,000 |
| 1 cubic meter | =1 tonne | =1 stere | 1,000,000 |

# CONVERSION FACTORS
## METRIC TO UNITED STATES

### LIQUID MEASURE
ml. × .008454 = gi.
l. × 1.056717 = qt.
l. × .264179 = gal.
Hl. × 26.417916 = gal.
Hl. × .838664 = bbl.
s. × 264.179164 = gal.
s. × 8.386640 = bbl.

### MEASURES OF LENGTH
mm. × .039370 = in.
cm. × .393704 = in.
cm. × .032809 = ft.
m. × 39.370432 = in.
m. × 3.280869 = ft.
m. × 1.093623 = yd.
m. × .000621 = mi.
Km. × 3,280.869300 = ft.
Km. × 1,093.623100 = yd.
Km. × .621375 = mi.

### MEASURES OF WEIGHT
mg. × .015432 = gr.
cg. × .154324 = gr.
cg. × .000353 = oz.
g. × 15.432356 = gr.
g. × .035274 = oz.
g. × .002205 = lb.
Kg. × 35.273957 = oz.
Kg. × 2.204622 = lb.
Kg. × .001102 = short T.
tonnes × 1.102311 = short T.
tonnes × .984206 = long T.

### RELATION OF WEIGHT AND VOLUME OF WATER
cu. cm. × 15.432356 = gr.
l. × 2.204622 = lb. (avoir.)
cu. m. × 2,204.622341 = lb.
cu. m. × 264.179164 = gal.
cu. m. × 8.386640 = bbl.
cu. m. × 1.102311 = short T.
cu. m. × .984206 = long T.
Kg. × 1.056717 = qt.

### RELATION OF WEIGHT AND VOLUME OF WATER—*Continued*
Kg. × .264179 = gal.
Kg. × 61.025387 = cu. in.
Kg. × .035316 = cu. ft.

### DRY MEASURE
l. × .908107 = qt.
l. × .028378 = bu.
Hl. × 22.702686 = gal.
Hl. × 2.837836 = bu.
s. × 227.026857 = gal.
s. × 28.378357 = bu.

### MEASURES OF VOLUME
ml. × .061025 = cu. in.
cl. × .610254 = cu. in.
dl. × 6.102539 = cu. in.
l. × 61.025387 = cu. in.
l. × .035316 = cu. ft.
Hl. ds. } × 3.531562 = cu. ft.
Kl. s. } × 35.315617 = cu. ft.
Kl. s. } × 1.307986 = cu. yd.

### SQUARE MEASURE
sq. mm. × .001550 = sq. in.
sq. cm. × .155003 = sq. in.
sq. m. × 10.764104 = sq. ft.
sq. m. × 1.196012 = sq. yd.
sq. m. × .000247 = A.
hectares × 2.471098 = A.
hectares × .003861 = sq. mi.
sq. Km. × 247.109816 = A.
sq. Km. × .386109 = sq. mi.

### CUBIC MEASURE
cu. cm. × .061025 = cu. in.
cu. dm. × 61.025387 = cu. in.
cu. dm. × .035316 = cu. ft.
cu. m. × 35.315617 = cu. ft.
cu. m. × 1.307986 = cu. yd.

## MISCELLANEOUS CONVERSION FACTORS

cm. per sec. × 1.968522 = ft. per min.
cu. cm. per sec. × .015851 = gal. per min.
cm. per m. × .12 = in. per ft.
m. per Km. × .10 = ft. per 100 ft.
m. per Km. × 5.28 = ft. per mi.
m. (depth) per hectare × 1.327697 = A.-ft.
Kg. per m. × .671963 = lb. per ft.
Kg. per sq. cm. × 14.223084 = lb. per sq. in.
Kg. per sq. cm. × .967557 = atmospheres (14.7 lb.) per sq. in.
Kg. per sq. m. × .204812 = lb. per sq. ft.
g. per cu. cm. × .036126 = lb. per cu. in.
Kg. per l. × 8.345217 = lb. per gal. (liquid)
Kg. per cu. m. × .062426 = lb. per cu. ft.

Kg. per cu. m. × .008345 = lb. per gal. (liquid)
francs (fr.) per m. × .176386 = dollars (dol.) per yd.
fr. per Km. × .310441 = dol. per mi.
fr. per hectare × .078063 = dol. per A.
fr. per Kg. × .087498 = dol. per lb.
fr. per tonne × .174996 = dol. per T. (short)
fr. per l. × .182547 = dol. per qt. (liquid)
fr. per l. × .730187 = dol. per gal. (liquid)
fr. per l. × .212419 = dol. per qt. (dry)
fr. per Hl. × .067974 = dol. per bu.
fr. per cu. m. × .147478 = dol. per cu. yd.
marks (mk.) per m. × .217717 = dol. per yd.
mk. per Km. × .383182 = dol. per mi.

mk. per hectare × .096354 = dol. per A.
mk. per Kg. × .108000 = dol. per lb.
mk. per tonne × .216001 = dol. per T. (short)
mk. per l. × .225321 = dol. per qt. (liquid)
mk. per l. × .901282 = dol. per gal. (liquid)
mk. per l. × .262194 = dol. per qt. (dry)
mk. per Hl. × .083902 = dol. per bu.

mk. per cu. m. × .182036 = dol. per cu. yd.
m.-lb. × 7.233077 = ft.-lb.
m.-Kg. × .009297 = B. T. U.
joules × .737308 = ft.-lb.
Kw. × 1.341113 = H. P.
cheval-vapeur × .986329 = H. P.
Poncelet × 1.315105 = H. P.
cal. × 3.968320 = B. T. U.
cal. × 3,087.353112 = ft.-lb.
Gravity (Paris) = 980.90 cm. per sec.

## UNITED STATES TO METRIC

### LIQUID MEASURE

gi. × 118.290925 = ml.
qt. × .946327 = l.
gal. × 3.785310 = l.
gal. × .037853 = Hl.
gal. × .003785 = s.
bbl. × 1.192373 = Hl.
bbl. × .119237 = s.

### RELATION OF WEIGHT AND VOLUME OF WATER

gr. × .064799 = cu. cm.
lb. (avoir.) × .453592 = l.
lb. × .000454 = cu. m.
gal. × .003785 = cu. m.
bbl. × .119237 = cu. m.
short T. × .907185 = cu. m.
long T. × 1.016047 = cu. m.
qt. × .946327 = Kg.
gal. × 3.785310 = Kg.
cu. in. × .016387 = Kg.
cu. ft. × 28.316094 = Kg.

### MEASURES OF LENGTH

in. × 25.399780 = mm.
in. × 2.539978 = cm.
in. × .025400 = m.
ft. × 30.4797260 = cm.
ft. × .304797 = m.
ft. × .000305 = Km.
yd. × .914391792 = m.
yd. × .000914 = Km.
mi. × .001609 = m.
mi. × 1.609330 = Km.

### MEASURES OF WEIGHT

gr. × 64.798918 = mg.
gr. × 6.479892 = cg.
gr. × .064800 = g.
oz. × 2,834.952670 = cg.
oz. × 28.349527 = g.
oz. × .028350 = Kg.
lb. × 453.592428 = g.

### MEASURES OF WEIGHT—*Continued*

lb. × .453592 = Kg.
short T. × 907.184856 = Kg.
short T. × .907185 = tonnes
long T. × 1.016047 = tonnes

### DRY MEASURE

qt. × 1.101191 = l.
gal. × .044048 = Hl.
gal. × .004405 = s.
bu. × 35.238122 = l.
bu. × .352381 = Hl.
bu. × .035238 = s.

### MEASURES OF VOLUME

cu. in. × 16.386623 = ml.
cu. in. × 1.638662 = cl.
cu. in. × .163866 = dl.
cu. in. × .016387 = l.
cu. ft. × 28.316094 = l.
cu. ft. × .283161 = $\begin{cases} \text{Hl.} \\ \text{ds.} \end{cases}$
cu. ft. × .028316 = $\begin{cases} \text{Kl.} \\ \text{s.} \end{cases}$
cu. yd. × .764535 = $\begin{cases} \text{Kl.} \\ \text{s.} \end{cases}$

### SQUARE MEASURE

sq. in. × 645.148422 = sq. mm.
sq. in. × 6.451484 = sq. cm.
sq. ft. × .092901 = sq. m.
sq. yd. × .836112 = sq. m.
A. × 4,046.787846 = sq. m.
A. × .404679 = hectares
sq. mi. × 258.994161 = hectares
A. × .004047 = sq. Km.
sq. mi. × 2.589942 = sq. Km.

### CUBIC MEASURE

cu. in. × 16.386623 = cu. cm.
cu. in. × .016387 = cu. dm.
cu. ft. × 28.316094 = cu. dm.
cu. ft. × .028316 = cu. m.
cu. yd. × .764534 = cu. m.

### MISCELLANEOUS CONVERSION FACTORS

ft. per min. × .507996 = cm. per sec.
gal. per min. × 63.088498 = cu. cm. per sec.
in. per ft. × 8.333333 = cm. per m.
ft. per 100 ft. × 10.00 = m. per Km.
ft. per mi. × .189394 = m. per Km.
A.-ft. × .753183 = m. (depth) per hectare
lb. per ft. × 1.488177 = Kg. per m.
lb. per sq. in. × .070309 = Kg. per sq. cm.

atmospheres (14.7 lb.) × 1.033539 = Kg. per sq. cm.
lb. per sq. ft. × 4.882535 = Kg. per sq. m.
ib. per cu. in. × 27.680653 = g. per cu. cm.
lb. per gal. (liquid) × .119829 = Kg. per l.
lb. per cu. ft. × 16.018897 = Kg. per cu. m.

#### Miscellaneous Conversion Factors—*Continued*

lb. per gal. (liquid) × 119.829666 = Kg. per cu. m.

dol. per yd. × 5.669377 = fr. per m.

dol. per mi. × 3.221224 = fr. per Km.

dol. per A. × 12.810239 = fr. per hectare

dol. per lb. × 11.428835 = fr. per Kg.

dol. per T. × 5.714417 = fr. per tonne

dol. per qt. (liquid) × 5.478052 = fr. per l.

dol. per gal. (liquid) × 1.369513 = fr. per l.

dol. per qt. (dry) × 4.707659 = fr. per l.

dol. per bu. × 14.711434 = fr. per Hl.

dol. per cu. yd. × 6.780771 = fr. per cu. m.

dol. per yd. × 4.593124 = mk. per m.

dol. per mi. × 2.609677 = mk. per Km.

dol. per A. × 10.378394 = mk. per hectare

dol. per lb. × 9.259228 = mk. per Kg.

dol. per T × 4.629614 = mk. per tonne

dol. per qt. (liquid) × 4.438113 = mk. per l.

dol. per gal. (liquid) × 1.109530 = mk. per l.

dol. per qt. (dry) × 3.813588 = mk. per l.

dol. per bu. × 11.918671 = mk. per Hl.

dol. per cu. yd. × 5.493413 = mk. per cu. m.

ft.-lb. × .138254 = m.-Kg.

ft.-lb. × 1.356284 = joules

ft.-lb. × .000324 = cal.

H. P. × .745649 = Kw.

H. P. × 1.013861 = cheval-vapeur

H. P. × .760396 = Poncelet.

B. T. U. × 107.561415 = m.-Kg.

B. T. U. × .251995 = cal.

One dm. = 3.937043 in.; 1 Dk. = 32.808693 ft. or 10.936231 yd.; 1 Hm. = 109.362310 yd.; 1 Mm. = 6.213750 mi. The U. S. 5c. piece, or nickel, is slightly over 2 cm. across.

One sq. dm. = 15.500309 sq. in.; 1 sq. Dm. = 119.601151 sq. yd.; 1 sq. Mm. = 38.610909 sq. mi., or a little more than 1 township.

One Q. = 220.462230 lb.; 1 Mg. = 22.046223 lb.; 1 Hg. = .220462 lb.; the Dg. = .35273957 oz.

The United States 50c. silver coin weighs 12.5 g.; the 25c. coin, 6.25 g.; and the 10c. coin, 2.5 g. These weights have been assigned by congressional enactment.

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# WEIGHTS AND MEASURES OF GREAT BRITAIN AND COLONIES

The measures of length, of surface, of weight, and of volume, while not absolutely identical, are, for all practical purposes, the same as those of the United States. It should be noted, however, that the United States long ton of 2,240 lb. is used in Great Britain and all of her colonies, except Canada, where the ton has been fixed by statute at 2,000 lb. The quarter of 28 lb. and the stone of the 14 lb. are used to some extent.

The chief difference between the measures of Great Britain and the United States is to be found in the sizes of the units of dry and liquid measure. The United States liquid gallon is, by Act of Congress, equal to 231 cu. in., and the United States bushel is, by similar enactment, equal to 2,150.42 cu. in., no reference being made to the weight of water contained in either of the measures. They were founded upon the former British wine gallon and the Winchester *struck* (level full) *bushel*, which have not been current in Great Britain since 1825, when they were replaced by the imperial gallon and bushel. By Act of Parliament, the imperial gallon is the volume of 10 lb. of pure water at 62° F., weighed against brass weights in air at the same temperature and at a barometric pressure of 30 in. of mercury. The imperial bushel is the volume of 80 lb. of pure water under the preceding conditions. From this, it is apparent that the imperial gill, pint, quart, and gallon, are the same in both the liquid and dry measure.

Using the value for the weight of 1 cu. ft. of water, 62.28827 lb., the volume of the imperial gallon is found to be 277.4203 cu. in., and of the imperial bushel, 2,219.3613 cu. in.

### IMPERIAL MEASURE, BOTH LIQUID AND DRY

| | | | Volume Cubic Inches | Weight of Water Pounds |
|---|---|---|---|---|
| 1 gi. | | = | 8.8694 | = .3125 |
| 4 gi. | = 1 pt. | = | 34.6775 | = 1.2500 |
| 2 pt. | = 1 qt. | = | 69.3551 | = 2.5000 |
| 4 qt. | = 1 gal. | = | 277.4203 | = 10.0000 |
| 2 gal. | = 1 pk. | = | 554.8403 | = 20.0000 |
| 4 pk. | = 1 bu. | = | 2,219.3613 | = 80.0000 |

One imp. gal. equals 1.20095 U. S. liquid gal., and 1 imp. bu. equals 1.03206 U. S. bu. Likewise, 1 U. S. liquid gal. equals .83267 imp. gal., and 1 U. S. bu. equals .96894 imp. bu.

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## MONEY

### UNITED STATES CURRENCY

| Denominations | | Mill | Cent | Dime | Dol. | Eagle |
|---|---|---|---|---|---|---|
| 10 mills | =1 cent | 1= | .1= | .01= | .001= | .0001 |
| 10 cents | =1 dime | 10= | 1 = | .1 = | .010= | .0010 |
| 10 dimes | =1 dollar | 100= | 10 = | 1.0 = | .100= | .0100 |
| 10 dollars | =1 eagle | 1,000= | 100 = | 10.0 = | 1.000= | .1000 |
| | | 10,000=1,000 | =100.0 | =10.000 | =1.0000 | |

### STANDARD UNITED STATES COINS

| Gold | | | Silver | | |
|---|---|---|---|---|---|
| Denomination | Value | Weight Grains | Denomination | Value | Weight Grains |
| *Dollar............. | $1.00 | 25.8 | *Trade dollar....... | $1.00 | 420.0 |
| Quarter-eagle....... | 2.50 | 64.5 | Standard silver dollar............... | 1.00 | 412.5 |
| *Three-dollar piece.. | 3.00 | 77.4 | Half-dollar........ | .50 | 192.9 |
| Half-eagle.......... | 5.00 | 129.0 | Quarter-dollar..... | .25 | 96.45 |
| Eagle.............. | 10.00 | 258.0 | Dime............. | .10 | 38.58 |
| Double eagle....... | 20.00 | 516.0 | | | |

Fineness expresses the proportion of pure metal in 1,000 parts; thus, "900 fine" means that 900 of every 1,000 parts are pure metal. Fineness of U. S. coins=900 pure metal, 100 alloy; alloy of gold coin is copper or copper and silver, but in no case shall silver exceed one-tenth of total alloy. Alloy of silver coin is copper.

| Piece | Weight Grains | Contents |
|---|---|---|
| 5-cent (nickel)....... | 77.16 | .75% copper, 25% nickel |
| *3-cent................ | 30 | .75% copper, 25% nickel |
| *2-cent................ | 66 | .95% copper, 5% tin and zinc |
| 1-cent (copper).. ........ | 48 | .95% copper, 5% tin and zinc |

### CURRENCY OF GREAT BRITAIN

| Denominations | | far. | d. | s. | £ |
|---|---|---|---|---|---|
| 4 farthings (far.)..... =1 penny | ......d....... | 1= | .25= | .0208= | .0010 |
| 12 pence.............. =1 shilling | ......s....... | 4= | 1.00= | .0833= | .0042 |
| 20 shillings............ =1 pound | ......£......48= | 12.00= | 1.0000= | .0500 | |
| 21 shillings............ =1 guinea | ......960=240.00= | 20.0000= | 1.0000 | | |

The unit is the *pound sterling*, valued at $4.8665. English silver is .925 fine; gold, .916⅔. The larger silver coins are the shilling, the florin or 2s., the crown or 5s., and the half-crown or 2s. 6d. The gold coins are the sovereign or pound, and the half-pound, or 10s.

| Great Britain | United States | | United States | Great Britain |
|---|---|---|---|---|
| 1 pound ......... | =$4.8665 | | 1 cent ......... | =.49312 pence |
| 1 shilling........ | = .243325 | | 1 dollar......... | =4 sh. 1.312 pence |
| 1 penny.......... | = .0202771 | | 1 dollar....... | =4.109333 shillings |
| | | | 1 dollar....... | =.205466 pound |

*No longer coined.

# FOREIGN MONETARY SYSTEMS AND EQUIVALENTS IN UNITED STATES GOLD

| | | |
|---|---|---|
| Argentine | 100 centavos = 1 peso | $ .9647 |
| Austria | 100 heller = 1 krone | .2026 |
| Belgium | 100 centimes = 1 franc | .1929 |
| Bolivia | 100 centavos = 1 boliviano | .3893 |
| Bosnia | 100 heller = 1 krone | .2026 |
| Brazil[1] | 1,000 reis = 1 milreis | .5463 |
| Bulgaria | 100 stotinki = 1 leva | .1929 |
| Canada | 100 cents = 1 dollar | 1.0000 |
| Chile[1] | 100 centavos = 1 peso | .3649 |
| China[2] | 100 parts = 1 yuan | .4772 |
| Colombia | 100 centavos = 1 peso | 1.0000 |
| Costa Rica | 100 centavos = 1 colon | 1.0000 |
| Cuba[3] | 100 centavos = 1 peso | 1.0000 |
| Denmark | 100 ore = 1 krona | .2679 |
| Ecuador | 100 centavos = 1 sucre | .4866 |
| Egypt[4] | 40 paras = 1 piastre | .0494 |
| France | 100 centimes = 1 franc | .1929 |
| Germany | 100 pfennige = 1 mark | .2381 |
| Great Britain | 20 shillings = 1 pound | 4.8665 |
| Greece | 100 lepta = 1 drachma | .1929 |
| Guatemala[1] | 100 centavos = 1 peso | .3998 |
| Holland | 100 cents = 1 gulden | .4020 |
| Honduras[1] | 100 centavos = 1 peso | .3998 |
| India[5] | 16 annas = 1 rupee | .3244 |
| Italy | 100 centesimi = 1 lira | .1929 |
| Japan | 100 sen = 1 yen | .4984 |
| Mexico | 100 centavos = 1 peso | .4984 |
| Montenegro | 100 heller = 1 krone | .2026 |
| Netherlands | 100 cents = 1 gulden | .4020 |
| Nicaragua | 100 centavos = 1 peso | .3998 |
| Norway | 100 ore = 1 krona | .2679 |
| Panama | 100 centavos = 1 balboa | 1.0000 |
| Paraguay | 100 centavos = 1 peso | .9646 |
| Persia[6] | 20 chahi = 1 kran | .1704 |
| Peru | 100 centavos = 1 sol | .4866 |
| Philippine Islands | 100 centavos = 1 peso | .5000 |
| Porto Rico | 100 centavos = 1 dollar | 1.0000 |
| Portugal[1] | 1,000 reis = 1 milreis | 1.0804 |
| Roumania | 100 bani = 1 leu | .1929 |
| Russia | 100 kopecs = 1 rouble | .5145 |
| Salvador[1] | 100 centavos = 1 peso | .3998 |
| Servia | 100 paras = 1 dinar | .1929 |
| Spain[6] | 100 centavos = 1 peseta | .1929 |
| Sweden | 100 ore = 1 krona | .2679 |
| Switzerland | 100 centimes = 1 franc | .1929 |
| Turkey | 40 paras = 1 piastre | .0439 |
| Uruguay[7] | 100 centesimos = 1 peso | .9647 |
| Venezuela | 100 centimos = 1 bolivar | .1929 |

[1] The actual currency is either depreciated or inconvertible paper of fluctuating value. The values given are for the standard gold coin.

[2] This is the new coinage. The former *tael* is still largely used. Its value differs from town to town and ranges between $.599 at Shanghai to $.660 at Takau. The Haikwan, or tael in which customs are payable, is valued at $.667. The Hong Kong and British dollar, valued at $.431, and the Mexican dollar, valued at $.434, circulate widely.

[3] Cuba has no national currency, gold, silver, or paper, relying upon the money of the United States, Great Britain, and Spain for its needs.

[4] The actual standard is the pound sterling of Great Britain.

[5] Fifteen rupees equal 1 pound sterling.

[6] The values given are for the gold kran and peseta respectively. The actual currency is silver circulating above its metallic value.

[7] The value given is for the silver peso. While under the gold standard, Uruguay has never coined this metal. The standard value of the gold peso is $1.0342.

## VALUES OF FOREIGN COINS

| | | | |
|---|---|---|---|
| Alexander, Bulgaria........ | $ 3.8589 | Groschen, Prussian Poland. | $ .0240 |
| Argentine, Argentine....... | 4.8236 | Gulden, Baden............ | .4000 |
| Boliviano, Bolivia ........ | .9700 | Imperial, Russia........... | 7.7183 |
| Condor, Chile............. | 7.2995 | Jirmilik, Turkey ......... | .8000 |
| Condor, Colombia and | | Kreutzer, Bavaria......... | .0067 |
| Ecuador................. | 9.6470 | Libra, Colombia........... | 4.8665 |
| Crown, Germany.......... | 2.3819 | Libra, Ecuador............ | 4.9429 |
| Crown, Portugal.......... | 10.8043 | Libra, Peru.............. | 4.8665 |
| Crown, Sicily............ | .9600 | Lira, Turkey............. | 4.3960 |
| Crown, Spain............ | 1.9500 | Maria Theresa Dollar, Ab- | |
| Dos, Spain.............. | 3.7084 | yssinia............... | .4139 |
| Doubloon, Central America. | 14.5000 | Maxmillian, Bavaria....... | 3.3000 |
| Doubloon, Chile.......... | 3.6497 | Menelik Dollar, Abyssinia.. | .4139 |
| Doubloon, New Grenada... | 15.3400 | Mohur, India............ | 7.1050 |
| Doubloon, Spain, Mexico .. | 15.6500 | Napoleon, France.......... | 3.8400 |
| Ducat, Austria, Bohemia, | | Peseta, Peru............ | .0973 |
| Hamburg, Hanover...... | 2.2800 | Peso fuerte, Peru......... | .4866 |
| Ducat, Holland........... | 2.2826 | Pistole, Rome............ | 3.3700 |
| Ducat, Denmark.......... | 1.1100 | Pistole, Spain............ | 3.9000 |
| Ducat, Sweden........... | 2.2000 | Pound, Egypt............ | 4.9429 |
| Escudo, Chile............ | 1.8248 | Real, Peru............... | .0486 |
| Florin, Austria-Hungary... | 1.9290 | Rial (Azizi), Morocco...... | .4000 |
| Florin, Hanover (gold).... | 1.6600 | Rial (Hassani), Morocco ... | .4663 |
| Florin, Hanover (silver).... | .5600 | Rupee, Persian........... | .3750 |
| Florin, Holland........... | .4019 | Sou, France............. | .0100 |
| Florin, Prussia........... | .5500 | Sovereign, England........ | 4.8665 |
| Florin, Silesia............ | .4800 | Toman, Persia............ | 1.7046 |
| Fuange, Siam ............ | .0600 | | |

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# MATHEMATICS

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## MATHEMATICAL AND OTHER COMMONLY USED SIGNS AND ABBREVIATIONS

| | |
|---|---|
| $+$ | = plus, or addition |
| $-$ | = minus, or subtraction |
| $\pm$ | = plus or minus |
| $\times$ | = multiplication |
| $\div$ | = division |
| $:$ | = ratio |
| $: :: \atop : = :$ } | = { proportion $2 : 3 :: 4 : 6$ or $2 : 3 = 4 : 6$ shows that *2 is to 3 as 4 is to 6* |
| $=$ | = equality |
| $\sqrt{\phantom{x}}$ | = square root |
| $\sqrt[3]{\phantom{x}}$ | = cube root, etc. |
| $\sqrt{3}$ | = square root of 3 |
| $\sqrt[3]{5}$ | = cube root of 5 |
| $7^2$ | = 7 squared |
| $8^3$ | = 8 cubed |
| $\dfrac{a}{b}$ | = $a/b$ or $a \div b$ |
| $\therefore$ | = therefore, or hence |
| $8 > 7$ | = 8 is greater than 7 |
| $3 < 5$ | = 3 is less than 5 |
| $\square$ | = square |
| $\square'$ | = square feet |
| $\square''$ | = square inches |

| | |
|---|---|
| O | = round |
| ( ) [ ] { } | } signs of aggregation and denote that the numbers enclosed are to be taken together; as $(a+b) \times c$ $= 4 + 3 \times 5 = 35$ |
| ° | = degrees, arc or thermometer |
| ' | = minutes or feet |
| " | = seconds or inches, 30° 40' 4'' is *30 degrees 40 minutes 4 seconds; 4' 6'' is 4 feet 6 inches* |
| ' '' ''' | = accents to distinguish letters as $a'$, $a''$, $a'''$ and read *a prime, a second, a third* |
| $a_1$, $a_2$, $a_b$ | } are read *a sub 1, a sub 2,* and *a sub b* |
| $a^{\frac{2}{3}}$ | = $a$ to the two-thirds power, or the cube root of $a$ squared |
| $a^{\frac{3}{2}}$ | = $a$ to the three halves power, or square root of $a$ cubed |

## MATHEMATICAL SIGNS AND ABBREVIATIONS—*Continued*

| | | | |
|---|---|---|---|
| ∠ | = angle | H. P. | = horsepower |
| ∟ | = right angle | G. C. D. | = greatest common divisor |
| ⊥ | = perpendicular to | B. M. | = board measure |
| log | = logarithm | c. o. d. | = cash on delivery |
| log sin | = logarithmic sine, loga- | ∞ | = infinity |
| log cos | rithmic cosine | Σ | = summation, or the sum |
| sin | = sine | | of a series of terms |
| cos | = cosine | I. H. P. | = indicated horsepower |
| tan | = tangent | B. H. P. | = brake horsepower |
| cotan | = cotangent | A. W. G. | = American wire gauge, or |
| sec | = secant | | Brown & Sharpe |
| cosec | = cosecant | B. W. G. | = Birmingham wire gauge |
| versin | = versed sine | r. p. m. | = revolutions per minute |
| coversin | = coversed sine | c. p. | = candlepower, or chem- |
| π | = *pi*, or ratio of circum- | | ically pure |
| | ference of circle to diam- | B. T. U. | = British thermal units |
| | eter, 3.14159265 | cal | = calories |
| g | = acceleration due to grav- | kw. or | = kilowatts |
| | ity = 32.16 ft. per sec. | K. W. | |
| h, m, s | = hours, minutes, seconds, | f. o. b. | = free on board |
| | 6ʰ 5ᵐ 4ˢ is *6 hours, 5 min-* | c. i. f. | = cost, insurance, freight; |
| | *utes, 4 seconds* | | i. e., included in cost |
| R, or r | = radius | D. C. | = direct current |
| W, or w | = weight | A. C. | = alternating current |

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# ARITHMETIC
## COMMON FRACTIONS

**To Add Fractions.**—If the fractions are of the same denominator, add together the numerators only. Thus, $\frac{1}{8} + \frac{3}{8} + \frac{5}{8} = \frac{9}{8} = 1\frac{1}{8}$.

If they have different denominators, change them to fractions with common denominators, and proceed as before. Thus, $\frac{1}{2} + \frac{1}{4} + \frac{2}{5} = \frac{20}{40} + \frac{10}{40} + \frac{24}{40} = \frac{53}{40}$.

**To Subtract Fractions.**—If the fractions are of the same denominator, subtract the lesser numerator from the greater. Thus, $\frac{7}{16} - \frac{3}{16} = \frac{4}{16} = \frac{1}{4}$.

If they have different denominators, change them to fractions with common denominators, and proceed as before. Thus, $\frac{7}{8} - \frac{2}{3} = \frac{21}{24} - \frac{16}{24} = \frac{5}{24}$.

**To Multiply Fractions.**—Multiply the numerators together for the numerator, and the denominators for the denominator. Thus, $\frac{3}{4} \times \frac{7}{16} \times \frac{2}{3} = \frac{42}{192} = \frac{7}{32}$.

**To Divide Fractions.**—Invert the divisor and multiply. Thus, to divide $\frac{9}{64}$ by $\frac{2}{3}$ the $\frac{2}{3}$ is inverted, and $\frac{9}{64} \div \frac{2}{3} = \frac{9}{64} \times \frac{3}{2} = \frac{27}{128}$.

**To Reduce Compound Fractions to Simple Fractions.**—Multiply the integer by the denominator of the fraction, add the numerator for the new numerator, and place it over the denominator.

EXAMPLE.—Reduce $5\frac{2}{3}$ to a simple fraction.

SOLUTION.— $(5 \times 3) + 2 = 17$, the numerator, the fraction is therefore $\frac{17}{3}$.

**To Reduce Simple Fractions to Compound Fractions.**—Divide the numerator by the denominator, and use the remainder as the numerator of the remaining fraction.

EXAMPLE.—Reduce $\frac{64}{9}$ to a compound fraction.

SOLUTION.—   9 ) 6 4 ( 7
             6 3    Compound fraction is $7\frac{1}{9}$
          —
          1

**To Reduce Common Fractions to Decimal Fractions.**—Annex ciphers to the numerator, and divide by the denominator, and point off as many decimal places in the quotient as there are ciphers annexed.

EXAMPLE.—Reduce $\frac{9}{16}$ to a decimal fraction.

SOLUTION.—               1 6 ) 9.0 0 0 0 (.5 6 2 5
                     8 0
                   —
               1 0 0
                9 6
               —
              4 0
              3 2
             —
              8 0
              8 0

NOTE.—Ciphers annexed to a decimal do not increase its value. 1.13 is the same as 1.1300. Every cipher placed between the first figure of a decimal and the point divides the decimal by 10. Thus, .13 ÷ 10 = .013.

## DECIMALS

*Decimals* are fractions that have for their denominators 10 or a power of 10, but the denominator is usually omitted. Thus, $.1 = \frac{1}{10}$; $.01 = \frac{1}{100}$; $.001 = \frac{1}{1000}$, etc.

**To Add Decimals.**—Place whole numbers under whole numbers, tenths under tenths, hundredths under hundredths, etc., and add, placing the decimal point in the sum directly under the points above. Thus,

```
        .0 0 7 5
        .6 3
       1.0 6
      17.9 3 4 2
      ──────────
      19.6 3 1 7
```

**To Subtract Decimals.**—Arrange the figures as in addition, and proceed as in simple subtraction. Thus,

```
       5.9 6 9 7 8
       3.2 8 6 9 4
       ──────────
       2.6 8 2 8 4
```

**To Multiply Decimals.**—Proceed as in simple multiplication, pointing off as many decimal places in the result as there are decimal places in both multiplicand and multiplier. Thus,

```
      4.6 7 5 3 1   (5 decimal places)
        .0 5 3       (3 decimal places)
      ───────────
      1 4 0 2 5 9 3
    2 3 3 7 6 5 5
    ─────────────────
    .2 4 7 7 9 1 4 3   (8 decimal places)
```

**To Divide Decimals.**—Proceed as in simple division, and point off as many decimal places in the quotient as the number of decimal places in the dividend exceeds those in the divisor.

EXAMPLE 1.—Divide 4.756 by 3.3.

SOLUTION.—

```
      3.3 ) 4.7 5 6 0 0 ( 1.4 4 1 2
            3 3
            ───
            1 4 5
            1 3 2
            ─────
              1 3 6
              1 3 2
              ─────
                4 0
                3 3
                ───
                7 0
                6 6
                ───
                4
```

EXAMPLE 2.—Divide .006 by 20.

SOLUTION.—

```
      2 0 ) .0 0 6 0 ( .0 0 0 3
              6 0
```

**To Reduce Decimals to Common Fractions.**—Omit the decimal point and use the figures thus obtained for the numerator. The denominator will be 1 with as many ciphers attached as there are places after the decimal point and will always be 10 or some multiple thereof. Reduce the fraction thus obtained to its lowest terms. Thus, in the form of a common fraction, the decimal .025 = $\frac{025}{1000} = \frac{25}{1000} = \frac{1}{40}$.

## FORMULAS

The term *formula*, as used in mathematics and in technical books, may be defined as a rule in which symbols are used instead of words; in fact, a formula may be regarded as a shorthand method of expressing a rule.

The well-known rule for finding the indicated horsepower of a steam engine may be stated as follows:

*Divide the continued product of the mean effective pressure, in pounds per square inch, the length of the stroke, in feet, the area of the piston, in square inches, and the number of strokes per minute by 33,000; the result will be the horsepower.*

An examination of the rule will show that four quantities (viz., the mean effective pressure, the length of the stroke, the area of the piston, and the number of strokes) are multiplied together, and the result is divided by 33,000. Hence, the rule might be expressed as follows:

$$\text{Horsepower} = \frac{\text{mean effective pressure}}{\text{(in pounds per square inch)}} \times \frac{\text{stroke}}{\text{(in feet)}}$$
$$\times \frac{\text{area of piston}}{\text{(in square inches)}} \times \frac{\text{number of strokes}}{\text{(per minute)}} \div 33,000$$

This expression may be shortened by representing each quantity by a single letter, thus: representing horsepower by the letter $H$, the mean effective pressure in pounds per square inch by $P$, the length of the stroke in feet by $L$, the area of the piston in square inches by $A$, the number of strokes per minute by $N$, and substituting these letters for the quantities that they represent, the foregoing expression will reduce to

$$H = \frac{P \times L \times A \times N}{33,000},$$

a much simpler and shorter expression. This last expression is called a formula.

It is customary, however, to omit the sign of multiplication between two or more quantities when they are to be multiplied together, or between a number and a letter representing a quantity, it being always understood that when two letters are adjacent with no sign between them, the quantities represented by these letters are to be multiplied. Bearing this fact in mind, the formula just given can be further simplified to

$$H = \frac{P\,L\,A\,N}{33,000}$$

## PROPORTION, OR CAUSE AND EFFECT

*Ratio* is the relation of one number to another as obtained by dividing one by the other. The ratio of 8 to 4 is $8 \div 4 = 2$.

*Simple proportion* is the expression of equality between equal ratios; thus, the ratio of 10 to 5 is 2 and the ratio of 4 to 2 is, also, 2. The relations between these are expressed thus, $10 : 5 :: 4 : 2$, or as $10 : 5 = 4 : 2$. The equality sign ($=$) being easier to write than the double colon ($::$) is the form commonly used.

Proportions may also be expressed as fractions. The preceding ratio, $10 : 5 = 4 : 2$, may be written $\frac{10}{5} = \frac{4}{2}$.

There are four terms in every proportion. The first and last terms are the *extremes*, and the second and third terms are the *means*.

In the ratio $10 : 5$ the first term, 10, is the *antecedent*, and the second term, 5, is the *consequent*. In the question "If 10 men earn \$20, how much will 30 men earn?" which may be expressed in the form $10 : 20 = 30 : y$ ($y$ representing the unknown amount the 30 men will earn), the number of men, 10 and 30, are the *antecedents*, or *causes*, and the sums earned, \$20 and $y$ dollars, are the *consequents* or *effects*.

Quantities are in proportion by *alternation* when antecedent is compared with antecedent and consequent with consequent. Thus, if $10 : 5 = 4 : 2$, then $10 : 4 = 5 : 2$.

Quantities are in proportion by *inversion* when the antecedents are made consequents and the consequents antecedents. Thus, if $10 : 5 = 4 : 2$, then $5 : 10 = 2 : 4$.

In any proportion, the product of the means will equal the product of the extremes. Thus, if $10 : 5 = 4 : 2$, then $5 \times 4 = 10 \times 2$.

A mean proportional between two quantities equals the square root of their product. Thus, a mean proportional between 12 and 3 = the square root of $12 \times 3$, and the proportion is expressed thus, $12 : 6 = 6 : 3$.

If the two means and one extreme of a proportion are given, the other extreme may be found by dividing the product of the means by the given extreme. Thus, $10 : 5 = 4 : (\ )$, then $(4 \times 5) \div 10 = 2$, and the proportion is $10 : 5 = 4 : 2$. This may also be expressed algebraically. If $y =$ the unknown quantity, the proportion is written $10 : 5 = 4 : y$. Then $10 \times y = 5 \times 4$, or $10y = 20$ and $y = 20 \div 10 = 2$, as before.

If the two extremes and one mean are given, the other mean may be found by dividing the product of the extremes by the given mean. Thus, $10 : (\ ) = 4 : 2$, then $(10 \times 2) \div 4 = 5$, and the proportion is $10 : 5 = 4 : 2$. This may be expressed algebraically. Thus, $10 : y = 4 : 2$, or $10 \times y = y \times 4$, or $4y = 20$, and $y = 20 \div 4 = 5$, as before.

EXAMPLE.—If 6 men load 30 cars in 1 da., how many cars will 10 men load?

SOLUTION.—The antecedents, or causes are the number of men and the consequents, or effects, the number of cars loaded by them. If $y$ represents the unknown number of cars loaded by the 30 men, the proportion may be expressed
$$\overset{cause}{6 \text{ (men)}} : \overset{effect}{30 \text{ (cars)}} = \overset{cause}{10 \text{ (men)}} : \overset{effect}{y \text{ (cars)}}.$$
From this, $6 \times y = 30 \times 10$, or $6y = 300$, and $y = 300 \div 6 = 50$, the number of cars loaded by 10 men.

A *compound proportion* is one in which one or both of the ratios contains more than one term. The governing principles are as follows:

1. The product of the simple ratios of the first couplet equals the product of the simple ratios of the second couplet. Thus, $\begin{Bmatrix} 4 : 12 \\ 7 : 14 \end{Bmatrix} = \begin{Bmatrix} 5 : 10 \\ 6 : 18 \end{Bmatrix}$ $=\dfrac{4}{12} \times \dfrac{7}{14} = \dfrac{5}{10} \times \dfrac{6}{18}$.

2. The product of all the terms in the extremes equals the product of all the terms in the means. Thus, in $\begin{Bmatrix} 4 : 12 \\ 7 : 14 \end{Bmatrix} = \begin{Bmatrix} 5 : 10 \\ 6 : 18 \end{Bmatrix}$, $4 \times 7 \times 10 \times 18 = 12 \times 14 \times 5 \times 6$.

3. Any term in either extreme equals the product of the means divided by the product of the other terms in the extremes. Thus, in the same proportion, $4 = \dfrac{5 \times 6 \times 12 \times 14}{7 \times 10 \times 18}$.

4. Any term in either mean equals the product of the extremes divided by the product of the other terms in the means. Thus, in $\begin{Bmatrix} 4 : 12 \\ 7 : 14 \end{Bmatrix} = \begin{Bmatrix} 5 : 10 \\ 6 : 18 \end{Bmatrix}$, $5 = (4 \times 7 \times 10 \times 18) \div (6 \times 12 \times 14)$.

EXAMPLE 1.—If 4 men in 7 da. earn \$24, how much can 14 men earn in 12 da.?

SOLUTION.—Here the antecedents, or causes, are the men and the number of days they worked and the consequents, or effects, are the sums of money they earned. If $y$ is considered equal to the second effect, that is, the unknown number of dollars earned by 14 men in 12 da., the proportion is $\begin{matrix} 4 (\text{men}) \\ 7 (\text{da.}) \end{matrix}$ : 24 (dollars) $= \begin{matrix} 14 (\text{men}) \\ 12 (\text{da.}) \end{matrix}$ : $y$ (dollars). From this, $y \times 4 \times 7 = 24 \times 14 \times 12$, or $28y = 4{,}032$, or $y = \$144$.

EXAMPLE 2.—If 12 men in 35 da. build a wall 140 rd. long, 6 ft. high, how many men can, in 40 da., build a wall of the same thickness 144 rd. long, 5 ft. high?

SOLUTION.—Here, the causes are the men and the number of days worked and the effects are the length and height of wall built by them. It is to be noted that the thickness of the wall is not considered as it is the same for each set of men. If $y$ is made equal to the unknown number of men, $\begin{matrix} 12 (\text{men}) \\ 35 (\text{da.}) \end{matrix}$ : $\begin{matrix} 140 (\text{rd.}) \\ 6 (\text{ft.}) \end{matrix} = \begin{matrix} y (\text{men}) \\ 40 (\text{da.}) \end{matrix}$ : $\begin{matrix} 144 (\text{rd.}) \\ 5 (\text{ft.}) \end{matrix}$. From this, $12 \times 35 \times 144 \times 5 = 140 \times 6 \times y \times 40$, or $33{,}600y = 302{,}400$, or $y = 9$.

## PERCENTAGE

*Percentage* is a process of computation in which the basis of comparison is a hundred.

*Per cent.* means *by*, or *on, the hundred.* 6 per cent. (also written 6%, or .06) of a quantity means 6 of every hundred in the quantity.

The *base* is the amount upon which the percentage is computed. In the case of money at interest, the base is known as the *principal.*

The *rate*, or *rate per cent.*, is the number of hundredths of the base that are to be taken. In monetary transactions, the rate is commonly called *interest.*

The *percentage* is the sum obtained by multiplying the base (or principal) by the rate. In finance, the percentage is always known as the *interest.*

The *amount* is the sum of the base and percentage or, what is the same thing, the sum of the principal and interest.

**To Find the Percentage, Having the Rate and the Base.**—Multiply the base by the rate expressed in hundredths. Thus 6% of 1,930 is found as follows: $1{,}930 \times .06 = 115.80$.

**To Find the Amount, Having the Base and the Rate.**—Multiply the base by 1 plus the rate. Thus, the amount of \$1,930 for 1 yr. at 6% is $\$1{,}930 \times 1.06 = \$2{,}045.80$.

**To Find the Base, Having the Rate and the Percentage.**—Divide the percentage by the rate. Thus, if the rate is 6% and the percentage is 115.80, the base $= 115.80 \div .06 = 1{,}930$.

**To Find the Rate, Having the Percentage and the Base.**—Divide the percentage by the base. Thus, if the percentage is 115.80 and the base 1,930, the rate equals $115.80 \div 1{,}930 = .06$, or 6%.

**To Find the Rate, Having the Amount and Base.**—Subtract the base from the amount; this will give the percentage. Divide the percentage by the base to find the rate. Thus, if the amount is \$2,045.80 and the base is \$1,930, the

percentage (or interest) is $2,045.80 - $1,930 = $115.80. The rate is then $115.80 ÷ 1,930 = .06, or 6%.

**Interest.**—Interest is money paid for the use of money, and may be likened to rent paid for the use of a house by a tenant to his landlord. Interest is figured as a certain per cent. of the money lent; 6% is the prevailing rate in the United States.

In banks interest is generally figured on the basis of there being 360 da., or 12 mo. of 30 da. each, in the year. The following are short rules for calculating 6% interest when 360 da. are taken as 1 yr.:

**Rule I.**—*Multiply the principal by the number of days and divide by 6,000.*

**Rule II.**—*Multiply the principal by the number of months and divide by 200.*

At 6% a year, the interest on $1 for 1 mo. is $\frac{1}{2}$%.

A *note* is a written promise to pay a certain sum of money at a certain time and at a certain rate of interest and, usually, at a certain place. The amount of money to be paid is the principal and is often called the *face* of the note. The *discount* is the interest on the money for the given time and at the given rate, and is so called because it is deducted (or discounted) in advance. The *proceeds* is the net amount received; that is, it is the face of the note less the discount (or interest) paid in advance.

Banks charge interest, as already explained, on the basis of there being 360 da. in the year, and for the exact number of days elapsed. Notes are commonly made for 1, 2, or 3 mo., or for 30, 60, or 90 da. Thus, if on June 15 three sums are borrowed for 1, 2, and 3 mo. time, the notes will be due on the 15th day of July, August, and September, respectively. But interest will not be charged for $\frac{1}{12}$, $\frac{1}{6}$, and $\frac{1}{4}$ yr. in the individual cases, but for 30, 61, and 92 da., and these days are $\frac{30}{360}$, $\frac{61}{360}$, and $\frac{92}{360}$ yr., respectively. On the other hand, should three other sums of money be borrowed on the same day for 30, 60, and 90 da., the notes will fall due on July 15, August 14, and September 13, and interest would be charged for the 30, 60, and 90 da.

**EXAMPLE 1.**—What is the date of payment, discount, and proceeds of a 90-da. note, for $150, dated July 27?

**SOLUTION.**—The date of payment will be 90 da. from July 27, or on October 25, as there are 31 da. each in the months of July and August. The discount will be the interest for 90 da. at 6%, which by rule I is $(150 \times 90) ÷ 6,000 = $2.25. The proceeds will be $150 - $2.25 = $147.75.

**EXAMPLE 2.**—What is the date of payment, discount, and proceeds of a 3-mo. note for $150, dated July 27?

**SOLUTION.**—The date of payment is 3 mo. from July 27, or October 27. As the number of days between these dates is 92, by rule I the discount will be $(150 \times 92) ÷ 6,000 = $2.30. The proceeds will be $150 - $2.30 = $147.70.

**Trade Discount.**—A discount is an abatement from the price of an article for some consideration, frequently the payment of cash upon the receipt of the goods or material purchased. Discounts are generally expressed as a certain per cent. of the purchase price; therefore, the *net*, or real, *cost* of an article is found by multiplying the first cost by 1.00 minus the discount. Thus, if a carload of corn is billed at $438, with a discount of 5% for cash, the price for immediate payment will be $438 \times (1.00 - .05) = 438 \times .95 = $416.10.

Discounts are frequently compound or continuous; that is, there are two or more discounts upon the price of an article. Thus, a discount may be quoted as "ten, ten, and five." This does not mean that the total discount is the sum of the three single discounts, or 25%; each discount and the resultant net price are figured separately. In the case in queston, if the first cost of the article was $100, the first net cost will be $100 \times (1.00 - .10 = .90) = $90; the second net cost will be $90 \times (1.00 - .10 = .90) = $81; and the third net, or final cost will be $81 \times (1.00 - .05 = .95) = $76.95. The total discount is, therefore, $100.00 - $76.95 = $23.05, or 23.05% and not 25%.

## RECIPROCALS

The *reciprocal* of a number is unity, or 1, divided by the number. Thus, the reciprocal of 5 is $1 ÷ 5 = .2$. Reciprocals are always expressed decimally. The reciprocal of a whole number is entirely decimal, while the reciprocal of a number less than unity is either a whole number or a whole number followed by a decimal. Thus, the reciprocal of .5 is $1 ÷ .5 = 2$, and the reciprocal of .6 is $1 ÷ .6 = 1.6666+$. Reciprocals are used to avoid the labor of division. Thus, the operation represented by $100 ÷ 621$ may be performed in the ordinary way by long division, or 100 may be multiplied by the reciprocal of 621. Reciprocals of numbers from 1 to 1,000 are given in connection with the table of powers,

etc., near the end of this volume. From this table, the reciprocal of 621 is found to be .001610306, and $100 \div 621 = 100 \times .001610306 = .1610306$.

Reciprocals of numbers that are multiples or submultiples of those in the table may be obtained directly therefrom by shifting the decimal point. Thus, the reciprocal of 62,100 is $\frac{1}{100}$ of the reciprocal of 621, or .00001610306. The reciprocal of 6.21 is 100 times the reciprocal of 621, or .1613036, and the reciprocal of .0621 is 10,000 times that of 621, or 16.10306.

Reciprocals of numbers intermediate between those in the table may be found therefrom by interpolation. Thus, the reciprocal of 621.25 is .001609579, and by shifting the decimal point the reciprocal of 6.2125 is .16096579.

## ARITHMETICAL PROGRESSION

Quantities are said to be in *arithmetical progression* when they increase or decrease by a common difference. The following is an increasing series in arithmetical progression: 1, 3, 5, 7, 9, 11, 13; if the figures are read backwards, 13, 11, 9, etc., it becomes a decreasing series. In the first series, the *first term* is 1; the *last term* 13; the *number of terms* 7; the *common difference* 2; and the *sum* of the terms 49. In any arithmetical progression,

Let
$$f = \text{first term};$$
$$l = \text{last, or } n\text{th term};$$
$$d = \text{common difference};$$
$$n = \text{number of terms};$$
$$s = \text{their sum}.$$

$$l = f + (n-1)d \qquad (1)$$

$$l = \frac{2s}{n} - f \qquad (2)$$

$$l = \frac{s}{n} + \frac{(n-1)d}{2} \qquad (3)$$

$$l = -\frac{d}{2} \pm \sqrt{2ds + \left(f - \frac{d}{2}\right)^2} \qquad (4)$$

$$f = \frac{2s}{n} - l \qquad (5)$$

$$f = l \pm (n-1)d \qquad (6)$$

$$f = \frac{s}{n} - \frac{(n-1)d}{2} \qquad (7)$$

$$f = \frac{d}{2} \pm \sqrt{\left(l + \frac{d}{2}\right)^2 - 2ds} \qquad (8)$$

$$n = \frac{l-f}{d} + 1 \qquad (9)$$

$$n = \frac{2s}{f+l} \qquad (10)$$

$$n = \frac{d - 2f \pm \sqrt{(2f-d)^2 + 8ds}}{2d} \qquad (11)$$

$$n = \frac{2l + d \mp \sqrt{(2l+d)^2 - 8ds}}{2d} \qquad (12)$$

$$d = \frac{l-f}{n-1} \qquad (13)$$

$$d = \frac{2(s-fn)}{n(n-1)} \qquad (14)$$

$$d = \frac{(l-f)(l+f)}{2s - (l+f)} \qquad (15)$$

$$d = \frac{2(nl-s)}{n(n-1)} \qquad (16)$$

$$s = \frac{n}{2}(f+l) \qquad (17)$$

$$s = \frac{n}{2}[2f + (n-1)d] \qquad (18)$$

$$s = \frac{l+f}{2} + \frac{(l+f)(l-f)}{2d} \qquad (19)$$

$$s = \frac{n}{2}[2l - (n-1)d] \qquad (20)$$

EXAMPLE 1.—During the last month of the year, the mules at a certain mine used 40 bales of hay. The consumption was 2 bales less than this during November, and similarly less for each month until the first of the year. What was the consumption of hay during January, and how many bales were used during the year?

SOLUTION.—Here $n = 12$ (mo.), $d = 2$ (bales), and $l = 40$ (bales during December). It is required to find $f$ and $s$. To find $f$, substituting the various values in formula 6 gives $f = 40 - (12-1) \times 2 = 40 - 22 = 18 =$ number of bales used in January. To find $s$, substituting in formula 20 gives $s = \frac{12}{2} \times [2 \times 40 - (12-1) \times 2] = 6 \times (80-22) = 6 \times 58 = 348$ bales during the year. It will be noted that $f$ has been found in the first part of the solution and it may be used to find $s$ in formula 17 which is more simple than formula 20.

EXAMPLE 2.—A contractor agrees to put down a bore hole for \$1 a ft. for the first 100 ft.; \$3 a ft. for the second 100 ft.; and \$2 a ft. additional for each successive 100 ft. Upon completion of the work he was paid \$6,400. How deep was the hole and what was the cost per foot of the last 100 ft.?

SOLUTION.—Here $f = 100$ (dollars), $s = \$6,400$, and $d = 200$ (dollars increase in cost per 100 ft.). Using formula 11,

$$n = \frac{200 - (2 \times 100) + \sqrt{[(2 \times 100) - 200]^2 + (8 \times 200 \times 6,400)}}{2 \times 200}$$

$$= \frac{200 - 200 + \sqrt{(200 - 200)^2 + 10,240,000}}{400} = \frac{0 + \sqrt{0^2 + 10,240,000}}{400}$$

$$= \frac{\sqrt{10,240,000}}{400} = \frac{3,200}{400} = 8$$

That is, there are 8 sections each 100 ft. deep, or the total depth of the hole is $8 \times 100 = 800$ ft. By using the value of $n$, just found, the last term be determined by means of the simple formula 1. By substitution, the last term $= 100 + (8-1) \times 200 = 100 + 1,400 = 1,500$. That is, the last 100 ft. cost $1,500, or $15 a ft.

## GEOMETRICAL PROGRESSION

A series of quantities, in which each is derived from that which precedes it, by multiplication by a constant quantity, is called a *geometrical progression*. If the multiplier is a whole number, the progression is styled *increasing*; if it is a fraction, the progression is styled *decreasing*. The series 1, 2, 4, 8, 16, 32 has 2 for a multiplier, and is an increasing progression. The series 32, 16, 8, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ has $\frac{1}{2}$ for a multiplier, and is a decreasing progression.

The common multiplier in a geometrical progression is called the *common ratio*; or, briefly, the *ratio*.

Let  $f = $ first term;
$l = $ last term, whose number from $f$ is $n$;
$n = $ number of terms;
$r = $ ratio;
$s = $ sum of terms.

$$l = fr^{n-1} \quad (1)$$

$$l = \frac{f + s(r-1)}{r} \quad (2) \qquad n = \frac{\log\left[\frac{s(r-1)}{f} + 1\right]}{\log r} \quad (6)$$

$$f = \frac{l}{r^{n-1}} \quad (3) \qquad \log r = \frac{\log \frac{l}{f}}{n-1} \quad (7)$$

$$f = s - r(s-l) \quad (4) \qquad r = \frac{s-f}{s-l} \quad (8)$$

$$n = \frac{\log \frac{l}{f}}{\log r} + 1 \quad (5) \qquad s = \frac{f(r^n - 1)}{r - 1} \quad (9)$$

$$s = \frac{lr - f}{r - 1} \quad (10)$$

EXAMPLE 1.—How much will it cost to sink a shaft 1,500 ft. deep at the rate of $\frac{1}{16}$c. for the first 50 ft.; $\frac{1}{8}$c. for the second 50 ft.; $\frac{1}{4}$c. for the third 50 ft.; and so on at the same rate?

SOLUTION.—In this case, $f = \frac{1}{16}$; $n = 30$; and $r = 2$. Substituting in formula 1, $l = \frac{1}{16} \times (2^{29}) = 33,554,432$, and from formula 10

$$s = \frac{33,554,432 \times 2 - \frac{1}{16}}{2 - 1} = \$671,088.63\frac{15}{16}$$

EXAMPLE 2.—At the end of a certain period a mine had produced 126,000 T. of coal. The production for the first 6 mo. was 2,000 T., which was doubled each 6 mo. How long had the mine been in operation?

SOLUTION.—In this case, $s = 126,000$; $f = 2,000$; and $r = 2$. Using formula 6 to find $n$. $n = \dfrac{\log\left[\dfrac{126,000 \times (2-1)}{2,000} + 1\right]}{\log 2} = \dfrac{\log 64}{\log 2} = \dfrac{1.80618}{.30103} = 6$. Hence, the mine had been in operation for six periods of 6 mo. each, or for 3 yr.

## INVOLUTION

*Involution* is the process of finding any power of a number. The power of a number is the product arising from multiplying the number by itself as many times as is indicated by another number known as the exponent. Thus, $4^2$ is read *four squared*, or *four to the second power*, and is equal to $4 \times 4 = 16$. Similarly $5^3$ is read *five cubed*, or *five to the third power*, and is equal to $5 \times 5 \times 5 = 125$. Likewise, $2^5$ is read *two to the fifth power*, or *the fifth power of two*, and is equal to $2 \times 2 \times 2 \times 2 \times 2 = 32$. The figures 2, 3, and 5, written to the right and above the numbers are the *exponents*.

## FIRST NINE POWERS OF FIRST NINE NUMBERS

| Number | Square | Cube | Fourth Power | Fifth Power | Sixth Power | Seventh Power | Eighth Power | Ninth Power |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 4 | 8 | 16 | 32 | 64 | 128 | 256 | 512 |
| 3 | 9 | 27 | 81 | 243 | 729 | 2,187 | 6,561 | 19,683 |
| 4 | 16 | 64 | 256 | 1,024 | 4,096 | 16,384 | 65,536 | 262,144 |
| 5 | 25 | 125 | 625 | 3,125 | 15,625 | 78,125 | 390,625 | 1,953,125 |
| 6 | 36 | 216 | 1,296 | 7,776 | 46,656 | 279,936 | 1,679,616 | 10,077,696 |
| 7 | 49 | 343 | 2,401 | 16,807 | 117,649 | 823,543 | 5,764,801 | 40,353,607 |
| 8 | 64 | 512 | 4,096 | 32,768 | 262,144 | 2,097,152 | 16,777,216 | 134,217,728 |
| 9 | 81 | 729 | 6,561 | 59,049 | 531,441 | 4,782,969 | 43,046,721 | 387,420,489 |

The power of a number may be obtained by multiplying together any two or more lower powers, the sum of whose exponents is equal to the exponent of the required power. Thus, if $n =$ any number, $n^9 = n^4 \times n^5 = n \times n^2 \times n^6 = n^2 \times n^3 \times n^4$, etc. Similarly, $n^7 = n^2 \times n^2 \times n^3 = n^2 \times n^5$, etc.

A table of squares and cubes is given at the end of this volume. The powers of numbers not in the table may be found by interpolation with sufficient accuracy for most purposes. Logarithms afford a rapid method of determining powers.

## EVOLUTION

The *root* of a number is one of the equal factors of a number. The number of equal factors in any number is indicated by a number known as the *index* of the root. This index, which is written to the left and a little above the sign √, shows how many factors compose the number, or how many times the root must be multiplied by itself to produce the number.

**To Find the Square Root of a Number:**

**Rule.—I.** *Separate the given number into periods of two figures each, beginning at the units place.*

**II.** *Find the greatest number whose square is contained in the period on the left; this will be the first figure in the root. Subtract the square of this figure from the period on the left, and to the remainder annex the next period to form a dividend.*

**III.** *Divide this dividend, omitting the figure on the right, by double the part of the root already found, and annex the quotient to that part, and also to the divisor; then, multiply the divisor thus completed, by the figure of the root last obtained, and subtract the product from the dividend.*

**IV.** *Add the root last found to the last trial divisor to form a new trial divisor. Divide the dividend by this new trial divisor and the quotient will be the next figure of the root, which quotient is to be annexed to the trial divisor to form a new complete divisor. Multiply this last complete divisor by the figure of the root last obtained and subtract from the dividend.*

**V.** *Bring down the next period to form a new dividend and continue as before until all the periods have been used.*

**VI.** *If it is desired to carry the root farther, annex periods of two ciphers each, and proceed as before.*

Example.—Find the square root: (a) of 874.225; (b) of .00874225.

Solution.—

(a)
```
 2      8'7 4.2 2'5 0 ( 2 9.5 6 7 +
 2      4
 4 9    4 7 4
   9    4 4 1
 5 8 5    3 3 2 2
     5    2 9 2 5
 5 9 0 6    3 9 7 5 0
       6    3 5 4 3 6
 5 9 1 2 7    4 3 1 4 0 0
              4 1 3 8 8 9
```

(b)
```
 0      .0 0'8 7'4 2'2 5 (.0 9 3 5
 0      0
 9      8 7
 9      8 1
 1 8 3    6 4 2
     3    5 4 9
 1 8 6 5    9 3 2 5
            9 3 2 5
```

**To Find the Cube Root of a Number:**

**Rule.—I.** *Separate the given number into periods of three figures, each beginning at the units place.*

**II.** *Find the greatest number whose cube is contained in the period on the left; this will be the first figure in the root. Subtract the cube of this figure from the period on the left, and to the remainder annex the next period to form a dividend.*

**III.** *Divide this dividend by the partial divisor, which is three times the square of the root already found considered as tens; the quotient is the second figure of the root.*

**IV.** *To the partial divisor add 3 times the product of the second figure of the root by the first, considered as tens, also the square of the second figure; the result will be the complete divisor.*

**V.** *Multiply the complete divisor by the second figure of the root, and subtract the product from the dividend.*

**VI.** *If there are more periods to be brought down, proceed as before, using the part of the root already found, the same as the first figure in the previous process.*

**VII.** *If it is desired to carry the root farther, annex periods of three ciphers each, and proceed as above.*

**EXAMPLE.**—Find the cube root of 12,813,904.

SOLUTION.—

```
                                          1 2'8 1 3'9 0 4 ( 2 3 4
                                  2³       8
First partial divisor, 3×20²   =  1 2 0 0      4 8 1 3
               3×20×3 =            1 8 0
                   3² =               9
First complete divisor         =  1 3 8 9      4 1 6 7
Second partial divisor, 3×23²=1 5 8 7 0 0      6 4 5 9 0 4
               3×230×4 =        2 7 6 0
                   4² =           1 6
Second complete divisor    = 1 6 1 4 7 6       6 4 5 9 0 4
```

A table of squares and cubes is given at the end of this volume by means of which square roots and cube roots may be readily extracted. The roots of numbers whose powers are within the limits of the table, 1,000,000 for squares and 1,000,000,000 for cubes, may be obtained directly, by finding in the proper column, the number whose root is to be extracted and in the column headed Number the root will be found. Thus, if it is desired to extract the cube root of 825,293,672, the number will be found in the column headed Cube, and on the same line in the column headed Number are the figures 938, which is the cube root of the required number.

Often, by shifting and replacing the decimal point, the roots of numbers not in the table may conveniently be found. The decimal point must be shifted the number of figures there are in a period, two in the case of squares and three in that of cubes. Thus, if it is desired to extract the cube root of 14.706125, the decimal point may be shifted either three or six figures (one or two periods) to the right and the cube root of 14,706.125 or of 14,706,125 extracted. After the root is extracted, the decimal point must be restored. In the case just given, by shifting the decimal point six places, or two periods, the cube root of 14,706,125 is found to be 245. As there is but one period before the decimal point in the number, there can be but one figure before the decimal point in the root; hence, the root is 2.45. Similarly, in extracting the cube root of .000000100544625 the decimal point and the six ciphers (two periods) may be dropped and the cube root of 100,544,625 extracted. This root is found to be 465. As there are two periods (of three ciphers each) after the decimal point in the number, there must be two ciphers (one for each period) after the decimal point in the root. Hence, the cube root of .000000100544625 is .00465.

The square or cube roots of numbers not in the table may be found, approximately, by interpolation. When finding the square root of 874.225, the decimal point may be shifted one period, or two figures, to the right, and the square root of 87'422.50 extracted. This is found to be between the squares of 295 and 296. The interpolation is made as follows:

| Number | Root | | Number | Root |
|---|---|---|---|---|
| 87,616 | = 296² | | 87,422.50 | = ? |
| 87,025 | = 295² | | 87,025.00 | = 295² |
| 591 | = first difference | | 397.50 | = second difference |

Then, second difference ÷ first difference = 397.50 ÷ 591 = .656. This is to be added to 295 and the square root of 87,422.50 is found to be 295.656. As the decimal point has been shifted *one period* to the right it must now be shifted *one figure* to the left, and the square root of 874.225 is 29.5656. As the true root, 29.5673, is within .0017 of the root found by interpolating in the table, this method is accurate enough for all practical purposes.

**Finding the Fourth and the Fifth Root of a Number.**—*Fourth roots* may be found by taking the square root of the square root, that is, by extracting the square root twice.

*Fifth roots* are rarely required, and when needed, occur in formulas involving coefficients of friction ($k$, in problems relating to ventilation, etc.) whose values are uncertain within 50% or more. Under such circumstances, it is apparent that even a very large percentage of error in the fifth root is allowable. In the vast majority of cases, no error of importance will be introduced by using for the true root the nearest value thereto, taken directly from the table.

**TABLE OF FIFTH POWERS**

| No. | Power | No. | Power | No. | Power | No. | Power |
|---|---|---|---|---|---|---|---|
| 1.0 | 1.00000 | 3.3 | 391.35393 | 5.6 | 5,507.31776 | 7.9 | 30,770.56399 |
| 1.1 | 1.61051 | 3.4 | 454.35424 | 5.7 | 6,016.92057 | 8.0 | 32,768.00000 |
| 1.2 | 2.48832 | 3.5 | 525.21875 | 5.8 | 6,563.56768 | 8.1 | 34,867.84401 |
| 1.3 | 3.71293 | 3.6 | 604.66176 | 5.9 | 7,149.24299 | 8.2 | 37,073.98432 |
| 1.4 | 5.37824 | 3.7 | 693.43957 | 6.0 | 7,776.00000 | 8.3 | 39,390.40643 |
| 1.5 | 7.59375 | 3.8 | 792.35168 | 6.1 | 8,445.96301 | 8.4 | 41,821.19424 |
| 1.6 | 10.48576 | 3.9 | 902.24199 | 6.2 | 9,161.32832 | 8.5 | 44,370.53125 |
| 1.7 | 14.19857 | 4.0 | 1,024.00000 | 6.3 | 9,924.36543 | 8.6 | 47,042.70176 |
| 1.8 | 18.89568 | 4.1 | 1,158.56201 | 6.4 | 10,737.41824 | 8.7 | 49,842.09207 |
| 1.9 | 24.76099 | 4.2 | 1,306.91232 | 6.5 | 11,602.90625 | 8.8 | 52,773.19168 |
| 2.0 | 32.00000 | 4.3 | 1,470.08443 | 6.6 | 12,523.32576 | 8.9 | 55,840.59449 |
| 2.1 | 40.84101 | 4.4 | 1,649.16224 | 6.7 | 13,501.25107 | 9.0 | 59,049.00000 |
| 2.2 | 51.53632 | 4.5 | 1,845.28125 | 6.8 | 14,539.33568 | 9.1 | 62,403.21451 |
| 2.3 | 64.36343 | 4.6 | 2,059.62976 | 6.9 | 15,640.31349 | 9.2 | 65,908.15232 |
| 2.4 | 79.62624 | 4.7 | 2,293.45007 | 7.0 | 16,807.00000 | 9.3 | 69,568.83693 |
| 2.5 | 97.65625 | 4.8 | 2,548.03968 | 7.1 | 18,042.29351 | 9.4 | 73,390.40224 |
| 2.6 | 118.81376 | 4.9 | 2,824.75249 | 7.2 | 19,349.17632 | 9.5 | 77,378.09375 |
| 2.7 | 143.48907 | 5.0 | 3,125.00000 | 7.3 | 20,730.71593 | 9.6 | 81,537.26976 |
| 2.8 | 172.10368 | 5.1 | 3,450.25251 | 7.4 | 22,190.06624 | 9.7 | 85,873.40257 |
| 2.9 | 205.11149 | 5.2 | 3,802.04032 | 7.5 | 23,730.46875 | 9.8 | 90,392.07968 |
| 3.0 | 243.00000 | 5.3 | 4,181.95493 | 7.6 | 25,355.25376 | 9.9 | 95,099.00499 |
| 3.1 | 286.29151 | 5.4 | 4,591.65024 | 7.7 | 27,067.84157 | 10.0 | 100,000.00000 |
| 3.2 | 335.54432 | 5.5 | 5,032.84375 | 7.8 | 28,871.74368 | | |

This table may be used in the same way as the table of squares and cubes, to find the fifth roots of intermediate values. It must be remembered that the periods are composed of five figures and that there must be one figure in the root for each such period in the number.

EXAMPLE.—Find the fifth root of 3,827,963,000.

SOLUTION.—By pointing off and inserting a decimal point, the fifth root of 38,279.63 is to be extracted. This is seen to lie between 8.2 and 8.3 and an interpolation, which may be made mentally, shows it to be about midway between these or is 8.25. By restoring the decimal point, the fifth root of the given number is found to be 82.5. As the root, by seven-place logarithms, is found to be 82.5265, this simple approximation is accurate enough for use in those formulas in which fifth roots occur. Should greater accuracy be demanded it is possible to interpolate as follows:

| *Number* | *Root* | *Number* | *Root* |
|---|---|---|---|
| 39,390.40463 | = 8.3⁵ | 38,279.63000 | = ? |
| 37,073.98432 | = 8.2⁵ | 37,073.98432 | = 8.2⁵ |
| 2,316.42031 | = first difference | 1,205.64568 | = second difference |

Second difference ÷ first difference = 1,205.64568 ÷ 2,316.42031 = .5205, which is to be annexed to 8.2, making the root 8.25205. By restoring the decimal point the fifth root of 3,827,963,000 is 82.5205.

**Simple Method of Extracting Roots.**—A simple method of extracting roots where tables are not available and the rules have been forgotten, is based upon Sir Isaac Newton's Method of Approximating the Roots of Higher Equations. The method is based on the fact that any number is composed of as many equal factors as are indicated by the index of the root. Thus, the fifth root is one of the five equal factors of the number and the cube root is one of the three equal factors, and similarly.

If the number whose root is required is placed equal to $n$ in the case of the cube root, it may be considered that $a \times b \times c = n$. When these factors are equal, that is, when $a = b = c$, the cube root is found. From the given equation, $a \times b \times c = n$, $c = \dfrac{n}{a \times b}$. If $a$ is made equal to $b$, and a value is assumed for them as near the cube root as possible, the average of the values of $a$, $b$, and $c$, will be nearer the true root than either $a$ or $b$. This value of $c$ is known as the first approximation, and may be placed equal to $a$ and $b$ to find a new value for $c$, the average of the three values being the second approximation. These approximations may be carried indefinitely, but with a little practice it will be found that the first approximation answers all practical purposes.

EXAMPLE 1.—Find the cube root of 987,654,321.

SOLUTION.—Pointing off and inserting a decimal point, it is necessary to extract the cube root of 987.654321. If a table of cubes is not available, one should be prepared. The number will be found between $729 = 9^3$ and $1,000 = 10^3$; that is, the cube root of the number is between 9 and 10. In this case $a \times b \times c = 987.654321$, and when $a = b = c$, the root is found. An interpolation between the cube roots of 729 and 1,000 shows that the cube root of the number is about 9.92. Making $a = b = 9.92$ and substituting gives $9.92 \times 9.92 \times c = 987.654321$, from which $c = 10.036$. The mean of these values is $(9.92 + 9.92 + 10.036) \div 3 = 9.95867$. This is the first approximate root, which may be placed equal to $a$ and $b$ to find a second approximate root. In the case of the first root, by restoring the decimal point, the cube root of 987,654,321 is found to be 995.867, which is identical with that extracted by means of seven-place logarithms. Such coincidence is unusual, but indicates that by carefully selecting the trial factors, a first approximation answers for all but abstract problems.

EXAMPLE 2.—Find the fifth root of 3,827,963,000.

SOLUTION.—Pointing off and placing a decimal point between the periods gives $a \times b \times c \times d \times e = 38,279.63$. From a table of powers, the fifth root of this number is found to lie between 8 and 9, and an interpolation shows it to be about 8.21. Then $8.21 \times 8.21 \times 8.21 \times 8.21 \times e = 38,279.63$. From this $e = 8.425$, and the mean of the five factors is 8.253. Restoring the decimal point shows that the first approximate fifth root of 3,827,963,000 is 82.53. This may be used for a second approximation, giving 82.5261, but the first should answer any purpose. This method may be compared with that based on the use of tables.

# LOGARITHMS

## EXPONENTS

By the use of *logarithms*, the processes of multiplication, division, involution, and evolution are greatly shortened, and some operations may be performed that would be impossible without them. Ordinary logarithms cannot be applied to addition and subtraction. A *logarithm* is the exponent of the power to which a fixed number, called the *base*, must be raised to produce a given number.

Although any positive number except 1 can be used as a base and a table of logarithms calculated, but two numbers have ever been employed. For all arithmetical operations (except addition and subtraction) the logarithms used are called the *Briggs*, or *common*, *logarithms*, and the base used is 10. In abstract mathematical analysis, the logarithms used are variously called *hyperbolic*, *Napierian*, or *natural* logarithms, and the base is 2.718281828+. The common logarithm of any number may be converted into a Napierian logarithm by multiplying the common logarithm by 2.30258509+, which is usually expressed as 2.3026, and sometimes as 2.3. Only the common system of logarithms will be considered here.

As in the common system the base is 10, all numbers are to be regarded as powers of 10; therefore, as $10^1 = 10$, $10^2 = 100$, $10^3 = 1,000$, etc., the logarithms (exponents) of 10, 100, 1,000, etc., are 1, 2, 3, etc., respectively. Similarly,

as $10^{-1} = \frac{1}{10} = .1$, $10^{-2} = \frac{1}{100} = .01$, $10^{-3} = \frac{1}{1000} = .001$, etc., the logarithms (exponents) of .1, .01, .001, etc., are $-1$, $-2$, $-3$, etc., respectively.

From the foregoing, it is seen that while the logarithms of exact powers of 10 and of decimals like, .1, .01, .001, etc., are whole numbers, the logarithms of all other numbers are wholly or in part fractional, the fractional part being expressed decimally. Thus, to produce 20, 10 must have an exponent of approximately 1.30103, or $10^{1.30103} = 20$, very nearly, the degree of exactness depending on the number of decimal places used. Hence, log $20 = 1.30103$. A logarithm, therefore, usually consists of two parts; a whole number, called the *characteristic*, and a fraction, called the *mantissa*. While mantissas are always to be regarded as positive, characteristics may be either positive or negative. From the foregoing, it is apparent that the characteristics of the logarithms of all numbers less than unity are negative, while for numbers greater than unity, they are positive. Negative characteristics are expressed by the sign, $-$, placed above the figures; thus, log $.20 = \bar{1}.30103$.

**Rule for Characteristics.**—*The characteristic of the logarithm of a number equal to or greater than unity is 1 less than the number of digits in the number. In the case of numbers less than unity, the characteristic is determined by the position, with respect to the decimal point, of the first digit in the number. If the first digit is found in the tenths column, the characteristic is $\bar{1}$; if in the hundredths column, it is $\bar{2}$; and similarly; or the characteristic is 1 more than the number of ciphers following the decimal point.*

| | | | |
|---|---|---|---|
| Log .0005 | $= \bar{4}.69897$ | Log 5 | $= .69897$ |
| Log .005 | $= \bar{3}.69897$ | Log 50 | $= 1.69897$ |
| Log .05 | $= \bar{2}.69897$ | Log 500 | $= 2.69897$ |
| Log .5 | $= \bar{1}.69897$ | Log 5,000 | $= 3.69897$ |
| | | Log 50,000 | $= 4.69897$ |

## FINDING THE LOGARITHM OF A NUMBER

A table of logarithms, containing the mantissas of the logarithms from 1 to 9,999 to five places of decimals, is given at the end of this volume. The mantissas of logarithms of larger numbers can be found by interpolation. This table depends on the principle that all numbers having the same figures in the same order have the same mantissa, without regard to the position of the decimal point, which affects the characteristic only. This is apparent from an inspection of the table giving the logarithm of 5 and its multiples and submultiples by 10.

The logarithm of a number having not more than four figures may be found by the following rule:

**Rule.**—*Find the first three significant figures of the number whose logarithm is desired, in the left-hand column; find the fourth figure in the column at the top (or bottom) of the page; and in the column under (or above) this figure, and opposite the first three figures previously found, will be the mantissa or decimal part of the logarithm. The characteristic being found, as previously described, write it at the left of the mantissa, and the resulting expression will be the logarithm of the required number.*

**EXAMPLE.**—Find the logarithm: (*a*) of 6; (*b*) of 48; (*c*) of 300; (*d*) of 3,717; (*e*) of .006195.

**SOLUTION.**—(*a*) The mantissa of the logarithm of 6 is the same as the mantissa of the logarithm of 600. The mantissa is found in the column headed L. 0 and opposite 600 in the column headed N. (number). The first two figures in the mantissa are not repeated for each number, but are found (in this instance) opposite the number 589, and are 77. The last three numbers are found opposite the figures 600. The complete mantissa is 77815. The characteristic is positive and since the number (6) consists of but one digit, the characteristic is $1 - 1 = 0$, therefore, log $6 = .77815$.

(*b*) The mantissa of the logarithm of 48 is the same as the mantissa of the logarithm of 480. As before, this is found in the column headed L. 0 and opposite 480 in the column headed N. The mantissa is 68124. As the number is composed of two figures, the characteristic is $2 - 1 = 1$, therefore, log $48 = 1.68124$.

(*c*) The mantissa of 300 may be taken directly from the table, being found in the column headed L. 0 opposite 300 in the column headed N. The mantissa is 47712. The characteristic, as the number is composed of three figures, is $3 - 1 = 2$, therefore, log $300 = 2.47712$.

(*d*) First find 371 in the column headed N. On the same horizontal line and in the column headed 7, the last three figures of the mantissa are found

to be *019. This star means that the first two figures of the mantissa are to be found below the horizontal line in which 019 is found and not above, and are 57 and not 56. The entire mantissa becomes 57019. As the number is composed of four figures, the characteristic is $4 - 1 = 3$; therefore, log 3,717 = 3.57019.

(e)  The mantissa of 6,195 is found opposite 619. The first two figures, 79, are found in the column headed L. 0, and the last three, 204, in the column headed 5. The entire mantissa is 79204. As the first digit in the number is found in the third decimal place, the characteristic is $\overline{3}$, therefore, log .006195 = 3.79204.

The logarithm of a number consisting of five or more figures may be found by the following rule:

**Rule.—I.** *If the number consists of more than five figures and the sixth figure is 5 or greater, increase the fifth figure by 1 and write ciphers in place of the sixth and remaining figures.*

**II.** *Find the mantissa corresponding to the logarithm of the first four figures, and subtract this mantissa from the next greater mantissa in the table; the remainder is the difference.*

**III.** *Find in the secondary table, headed P. P., a column headed by the same number as that just found for the difference, and in this column, opposite the number corresponding to the fifth figure (or fifth figure increased by 1) of the given number (this figure is always situated at the left of the dividing line of the column), will be found the P. P. (proportional part) for that number. The P. P. thus found is to be added to the mantissa found in II, as in the preceding examples, and the result is the mantissa of the logarithm of the given number, as nearly as may be found with five-place tables.*

To take out the logarithm of a number consisting of more than four figures, it is inexpedient to use more than five figures of the number when using five-place logarithms (the logarithms given at the end of this volume are five-place). Hence, if the number consists of more than five figures and the sixth figure is less than 5, replace all figures after the fifth with ciphers; if the sixth figure is 5 or greater, increase the fifth figure by 1 and replace the remaining figures with ciphers. Thus, if the number is 31,415,926, find the logarithm of 31,416,000; if 31,415,426, find the logarithm of 31,415,000.

EXAMPLE.—Find log 31,416.

SOLUTION.—Find the mantissa of the logarithm of the first four figures, as already explained. This is, in the present case, .49707. Now, subtract the number in the column headed 1, opposite 314 (the first three figures of the given number), from the next greater consecutive number, in this case 721, in the column headed 2. $721 - 707 = 14$; this number is called the *difference*. At the extreme right of the page will be found a secondary table headed P. P., and at the top of one of these columns, in this table, in bold-face type, will be found the difference. It will be noticed that each column is divided into two parts by a vertical line, and that the figures on the left of this line run in sequence from 1 to 9. Considering the difference column headed 14, opposite the number 6 (6 is the last or fifth figure of the number whose logarithm we are taking out) is the number 8.4, which, added to the mantissa just found, disregarding the decimal point in the mantissa, gives $49,707 + 8.4 = 49,715.4$. Now, as 4 is less than 5, it is rejected, giving for the complete mantissa .49715. As the characteristic of the logarithm of 31,416 is $5 - 1 = 4$, log 31,416 = 4.49715.

## TO FIND A NUMBER WHOSE LOGARITHM IS GIVEN

**Rule.—I.** *Consider the mantissa first. Glance along the different columns of the table that are headed 0, until the first two figures of the mantissa are found. Then, glance down the same column until the third figure is found (or 1 less than the third figure). Having found the first three figures, glance to the right along the row in which they are situated until the last three figures of the mantissa are found. Then, the number that heads the column in which the last three figures of the mantissa are found is the fourth figure of the required number, and the first three figures lie in the column headed N, and in the same row in which lie the last three figures of the mantissa.*

**II.** *If the mantissa cannot be found in the table, find the mantissa that is nearest to, but less than, the given mantissa, and which call the next less mantissa. Subtract the next less mantissa from the next greater mantissa in the table to obtain the difference. Also, subtract the next less mantissa from the mantissa of the given logarithm, and call the remainder the P. P. Looking in the secondary table headed P. P. for the column headed by the difference just found, find the number opposite the P. P. just found (or the P. P. corresponding most nearly to that just*

*found); this number is the fifth figure of the required number; the fourth figure will be found at the top of the column containing the next less mantissa, and the first three figures in the column headed N and in the same row that contains the next less mantissa.*

**III.** *Having found the figures of the number as directed, locate the decimal point by the rules for the characteristic, annexing ciphers to bring the number up to the required number of figures if the characteristic is greater than 1.*

EXAMPLE.—Find the number corresponding: (*a*) to the logarithm 3.56867; (*b*) to the logarithm $\bar{2}.05753$.

SOLUTION.—(*a*) The first two figures of the mantissa are 56; glancing down the column, the third figure, 8 (in connection with 820 )is found opposite 370 in the N column. Glancing to the right along the row containing 820, the last three figures of the mantissa, 867, are found in the column headed 4; hence, the fourth figure of the required number is 4, and the first three figures are 370, making the figures of the required number 3,704. As the characteristic is 3, there must be $3+1=4$ figures to the left of the decimal point. Hence, the number is 3,704.

(*b*) The mantissa 05753 is not found in the table. The next less mantissa is found in the column headed 1, opposite the figures 114 in the column headed N; hence, the first four figures are 1,141. The mantissa of log 1141 $=05729$, and of log 1142 $=05707$. The difference is 38. The P. P. (proportional part) is the given logarithm—the lesser tabular logarithm, or 05753 $-05729=24$. Under the head of 38 in the P. P. section, 24 is found between 22.8 (opposite 6) and 26.6 (opposite 7). As 24 is nearer the smaller number, the fifth figure of the number is 6, and the entire number is 11,416. As the characteristic is $\bar{2}$, the number is a decimal and there is $2-1=1$ cipher after the decimal point; hence, the number is .011416.

## MULTIPLICATION BY LOGARITHMS

The principle on which the process of multiplication by means of logarithms is based is that log $ab=$ log $a+$ log $b$. To multiply two or more numbers by using logarithms apply the following rule:

**Rule.**—*Add the logarithms of the several numbers, and the sum will be the logarithm of the product. Find the number corresponding to this logarithm, and the result will be the number sought.*

EXAMPLE 1.—Multiply 4.38, 5.217, and 83 together.

SOLUTION.—    Log  4.38 = .64147
             Log 5.217 = .71742
             Log   83 = 1.91908

Adding,                3.27797 = log (4.38×5.217×83)

Number corresponding to 3.27797 is 1,896.6. Hence, $4.38×5.217×83 =1,896.6$,nearly. By actual multiplication, the product is 1,896.58818, showing that the result obtained by using logarithms was correct to five figures.

When adding logarithms, the algebraic sum is always to be found. Hence, if some of the numbers multiplied together are wholly decimal, the algebraic sum of the characteristics will be the characteristic of the product. It must be remembered that the mantissas are always positive.

EXAMPLE 2.—Multiply 49.82, .00243, 17, and .97 together.

SOLUTION.—

             Log  49.82 = 1.69740
             Log .00243 = $\bar{3}$.38561
             Log     17 = 1.23045
             Log    .97 = $\bar{1}$.98677

Adding,                0.30023 = log (49.82×.00243×17×.97)

Number corresponding to .30023 is 1.9963. Hence, $49.82×.00243×17 ×.97=1.9963$.

In this case the sum of the mantissas was 2.30023. The integral 2 added to the positive characteristics makes their sum $=2+1+1=4$; sum of negative characteristics $=\bar{3}+\bar{1}=\bar{4}$, whence $4+(-4)=0$. If, instead of 17, the number had been .17 in this example, the logarithm of .17 would have been $\bar{1}$.23045, and the sum of the logarithm would have been $\bar{2}$.30023; the product would then have been .019963.

## DIVISION BY LOGARITHMS

The principle upon which the process of division by means of logarithms is based is that $\log \frac{a}{b} = \log a - \log b$.

**Rule I.**—*Subtract the logarithm of the divisor from the logarithm of the dividend, and the result will be the logarithm of the quotient.*

EXAMPLE 1.—Divide 6,784.2 by 27.42.

SOLUTION.—
Log 6,784.2 = 3.83150
Log    27.42 = 1.43807

$difference = \overline{2.39343} = \log (6,784.2 \div 27.42)$

Number corresponding to 2.39343 is 247.42. Hence, $6,784.2 \div 27.42 = 247.42$.

When subtracting logarithms, their algebraic difference is to be found. The operation may sometimes be confusing, because the mantissa is always positive, and the characteristic may be either positive or negative.

**Rule II.**—*When the logarithm to be subtracted is greater than the logarithm from which it is to be taken, or when negative characteristics appear, subtract the mantissa first, and then the characteristic, by changing its sign and adding.*

EXAMPLE 2.—Divide 274.2 by 6,784.2.

SOLUTION.—
Log    274.2 = 2.43807
Log 6,784.2 = 3.83150

$\overline{2}.60657$

First subtracting the mantissa .83150 gives .60657 for the mantissa of the quotient. In subtracting, 1 had to be taken from the characteristic of the minuend, leaving a characteristic of 1. Subtract the characteristic 3 from this, by changing its sign and adding $1 - 3 = \overline{2}$, the characteristic of the quotient. The number corresponding to $\overline{2}.60657$ is .040417. Hence, $274.2 \div 6,784.2 = .040417$.

EXAMPLE 3.—Divide .067842 by .002742.

SOLUTION.—
Log .067842 = $\overline{2}$.83150
Log .002742 = $\overline{3}$.43807

$difference = 1.39343$

As $.83150 - .43807 = .39343$ and $-2 + 3 = 1$, number corresponding to 1.39343 is 24.742. Hence, $.067842 \div .002742 = 24.742$.

The only case that is likely to cause trouble in subtracting is that in which the logarithm of the minuend has a negative characteristic, or none at all, and a mantissa less than the mantissa of the subtrahend. For example, let it be required to subtract the logarithm 3.74036 from the logarithm $\overline{3}$.55145. The logarithm $\overline{3}$.55145 is equivalent to $-3 + .55145$. Now, if both $+1$ and $-1$ are added to this logarithm, it will not change its value. Hence, $\overline{3}.55145 = -3 - 1 + 1 + .55145 = \overline{4} + 1.55145$. Therefore, $\overline{3}.55145 - 3.74036 =$

$\overline{4} + 1.55145$
$3 + .74036$

$difference = \overline{7} + .81109 = \overline{7}.81109$

Had the characteristic of the logarithm been 0 instead of $\overline{3}$, the process would have been exactly the same. Thus, $.55145 = \overline{1} + 1.55145$; hence,

$\overline{1} + 1.55145$
$3 + .74036$

$difference = \overline{4} + .81109 = \overline{4}.81109$

EXAMPLE 4.—Divide .02742 by 67.842.

SOLUTION.—
Log .02742 = $\overline{2}$.43807 = $\overline{3}$ + 1.43807
Log 67.842 = 1.83150 = 1 + .83150

$difference = \overline{4} + .60657 = \overline{4}.60657$

Number corresponding to $\overline{4}$.60657 is .00040417. Hence, $.02742 \div 67.842 = .00040417$.

EXAMPLE 5.—What is the reciprocal of 3.1416?

SOLUTION.—Reciprocal of $3.1416 = \frac{1}{3.1416}$, and $\log \frac{1}{3.1416} = \log 1 - \log 3.1416 = 0 - .49715$. Since $0 = -1 + 1$,

$\overline{1} + 1.00000$
.49715

$difference = \overline{1} + .50285 = \overline{1}.50285$

Number whose logarithm is $\overline{1}$.50285 is .31831.

## INVOLUTION BY LOGARITHMS

The process of involution by means of logarithms is based on the principle that log $a^n = n$ log $a$.

**Rule I.**—*Multiply the logarithm of the number by the exponent that denotes the power to which the number is to be raised; the result will be the logarithm of the required power.*

EXAMPLE 1.—What is: (a) the square of 7.92? (b) the cube of 94.7? (c) the 1.6 power of 512, that is, the value of $512^{1.6}$?

SOLUTION.—(a) Log 7.92 = .89873; exponent of power = 2. Hence, .89873 × 2 = 1.79746 = log $7.92^2$. Number corresponding to 1.79746 is 62.727. Hence, $7.92^2 = 62.727$, nearly.

(b) Log 94.7 = 1.97635; 1.97635 × 3 = 5.92905 = log $94.7^3$. Number corresponding to 5.92905 is 849,280, nearly. Hence, $94.7^3 = 849,280$, nearly.

(c) Log $512^{1.6} = 1.6 \times$ log 512 = 1.6 × 2.70927 = 4.334832, or 4.33483 (when using five-place logarithms) = log 21,619. Hence, $512^{1.6} = 21,619$, nearly.

**Rule II.**—*If the number is wholly decimal, so that the characteristic is negative, multiply the two parts of the logarithm separately by the exponent of the number. If, after multiplying the mantissa, the product has a characteristic, add it, algebraically, to the negative characteristic multiplied by the exponent, and the result will be the negative characteristic of the required power.*

EXAMPLE 1.—Raise .0751 to the fourth power.

SOLUTION.—Log $.0751^4 = 4 \times$ log .0751 = 4 × 2.87564. Multiplying the parts separately, $4 \times \bar{2} = \bar{8}$ and 4 × .87564 = 3.50256. Adding the 3 and 8, 3 + (−8) = −5; therefore, log $.0751^4 = \bar{5}.50256$. Number corresponding to this is .00003181. Hence, $.0751^4 = .00003181$.

A decimal may be raised to a power whose exponent contains a decimal as follows:

EXAMPLE 2.—Raise .8 to the 1.21 power.

SOLUTION.—Log $.8^{1.21} = 1.21 \times \bar{1}.90309$. There are several ways of performing the multiplication.

*First Method.*—Adding the characteristic and mantissa algebraically, the result is −.09691. Multiplying this by 1.21 gives −.1172611, or −.11726, when using five-place logarithms. To obtain a positive mantissa, add +1 and −1; whence, log $.8^{1.21} = −1 + 1 − .11726 = \bar{1}.88274$.

*Second Method.*—Multiplying the characteristic and mantissa separately gives −1.21 + 1.09274. Adding characteristic and mantissa algebraically, gives −.11726; then, adding +1 and −1, log $.8^{1.21} = \bar{1}.88274$.

*Third Method.*—Multiplying the characteristic and mantissa separately gives −1.21 + 1.09274. Adding the decimal part of the characteristic to the mantissa gives −1 + (−.21 + 1.09274) = $\bar{1}.88274$ = log $.8^{1.21}$. The number corresponding to the logarithm $\bar{1}.88274$ = .76338.

Any one of these methods may be used, but the first or the third is recommended. The third saves figures but requires the exercise of more caution than does the first method. Below will be found the entire work of multiplication for both $.8^{1.21}$ and $.8^{.21}$.

| $\bar{1}.90309$ | $\bar{1}.90309$ |
|---|---|
| 1.21 | .21 |
| 90309 | 90309 |
| 180618 | 180618 |
| 90309 | +1.1896489 |
| 1.0927389 | −1 −.21 |
| −1.21 | $\bar{1}.9796489$, or $\bar{1}.97965$ |
| $\bar{1}.8827389$, or $\bar{1}.88274$ | |

In the second case, the negative decimal obtained by multiplying −1 and .21 was greater than the positive decimal obtained by multiplying .90309 and .21; hence, +1 and −1 were added, as shown.

## EVOLUTION BY LOGARITHMS

The process of evolution by logarithms is based on the principle that log $\sqrt[n]{a} = \dfrac{\log a}{n}$.

**Rule.**—*Divide the logarithm of the number by the index of the root; the result will be the logarithm of the root.*

EXAMPLE.—Extract: (a) the square root of 77,851; (b) the cube root of 698,970; (c) the 2.4 root of 8,964,300.

SOLUTION.—(a) Log 77,851 = 4.89127; index of root is 2; hence, log $\sqrt{77,851}$ = 4.89127 ÷ 2 = 2.44564; number corresponding to this is 279.02. Hence, $\sqrt{77,851}$ = 279.02, nearly.

(b) Log $\sqrt[3]{698,970}$ = 5.84446 ÷ 3 = 1.94815 = log 88.746; or, $\sqrt[3]{698,970}$ = 88.746.

(c) Log $\sqrt[2.4]{8,946,300}$ = 6.95251 ÷ 2.4 = 2.89688 = log 788.64; or, $\sqrt[2.4]{8,964,300}$ = 788.64, nearly.

If it is required to extract a root of a number wholly decimal, and the negative characteristic will not exactly contain the index of the root, without a remainder, the following rule may be used:

**Rule.**—*Separate the two parts of the logarithm; add as many units (or parts of a unit) to the negative characteristic as will make it exactly contain the index of the root. Add the same number to the mantissa, and divide both parts by the index. The result will be the characteristic and mantissa of the root.*

EXAMPLE 1.—Extract the cube root of .0003181.

SOLUTION.—Log $\sqrt[3]{.0003181}$ = $\dfrac{\log .0003181}{3}$ = $\dfrac{\overline{4}.50256}{3}$.

$$(\overline{4}+\overline{2}=\overline{6}) + (2 + .50256 = 2.50256)$$
$$(\overline{6} \div 3 = \overline{2}) + (2.50256 \div 3 = .83419)$$

or, log $\sqrt[3]{.0003181}$ = $\overline{2}.83419$ = log .068263

Hence, $\sqrt[3]{.0003181}$ = .068263

EXAMPLE 2.—Find the value of $\sqrt[1.41]{.0003181}$.

SOLUTION.—Log $\sqrt[1.41]{.0003181}$ = $\dfrac{\log. .0003181}{1.41}$ = $\dfrac{\overline{4}.50256}{1.41}$.

If −.23 is added to the characteristic, it will contain 1.41 exactly three times. Hence,

$$[-4 + (-.23) = -4.23] + (.23 + .50256 = .73256)$$
$$(-4.23 \div 1.41 = \overline{3}) + (.73256 \div 1.41 = .51955)$$

or, log $\sqrt[1.41]{.0003181}$ = $\overline{3}.51955$ = log .0033079

Hence, $\sqrt[1.41]{.0003181}$ = .0033079

EXAMPLE 3.—Solve, by logarithms, $\dfrac{497 \times .0181 \times 762}{3,300 \times .6517}$ = ?

SOLUTION.—

| | |
|---|---|
| Log | 497 = 2.69636 |
| Log | .0181 = $\overline{2}$.25768 |
| Log | 762 = 2.88195 |

Log product = 3.83599
Log    3,300 = 3.51851
Log    .6517 = $\overline{1}$.81405

Log product = 3.33256

3.83599 − 3.33256 = .50343 = log 3.1874

Hence, $\dfrac{497 \times .0181 \times 762}{3,300 \times .6517}$ = 3.1874

EXAMPLE 4.—Solve $\sqrt[3]{\dfrac{504,203 \times 507}{1.75 \times 71.4 \times 87}}$ by logarithms.

SOLUTION.—

Log 504,203 = 5.70260
Log     507 = 2.70501

Log product = 8.40761
Log    1.75 = .24304
Log    71.4 = 1.85370
Log     87 = 1.93952

Log product = 4.03626

$\dfrac{8.40761 - 4.03626}{3}$ = 1.45712 = log 28.65

Hence, $\sqrt[3]{\dfrac{504,203 \times 507}{1.75 \times 71.4 \times 87}}$ = 28.65

## SOLUTION OF EQUATIONS BY LOGARITHMS

Logarithms can often be applied to the solution of equations.

**EXAMPLE 1.**—Solve the equation $2.43x^5 = \sqrt[6]{.0648}$.

**SOLUTION.**—Dividing by 2.43, $x^5 = \dfrac{\sqrt[6]{.0648}}{2.43}$. Taking the logs of both numbers,

$5 \log x = \dfrac{\log .0648}{6} - \log 2.43$; $5 \log x = \dfrac{\overline{2}.81158}{6} - .38561 = \overline{1}.80193 - .38561$

$= \overline{1}.41632$. Dividing by 5, $\log x = \overline{1}.88326$, whence $x = .7643$.

**EXAMPLE 2.**—Solve the equation $4.5^x = 8$.

**SOLUTION.**—Taking the logarithms of both numbers, $x \log 4.5 = \log 8$,

whence, $x = \dfrac{\log 8}{\log 4.5} = \dfrac{.90309}{.65321}$. Taking logarithms again, $\log x = \log .90309 - \log$

$.65321 = \overline{1}.95573 - \overline{1}.81505 = .14068$, and $x = 1.3825$.

**REMARKS.**—Logarithms are particularly useful in those cases when the unknown quantity is an exponent, as in the last example, or when the exponent contains a decimal, as in several instances in the examples already given. Such examples can be solved without the use of logarithms, but the process is very long and somewhat involved, and the arithmetical work required is enormous. To solve the example last given without using the logarithmic table and obtain the value of $x$ correct to five figures will require, perhaps, 100 times as many figures as are used in the solution given, and the resulting liability to error will be correspondingly increased; indeed, to confine the work to this number of figures will also require a good knowledge of short-cut methods in multiplication and division, and judgment and skill on the part of the calculator, which can only be acquired by practice and experience.

Formulas containing quantities affected with decimal exponents are generally of an empiric nature; that is, the constants or exponents or both are given such values as will make the results obtained by the formulas agree with those obtained by experiment. Such formulas occur frequently in works treating on thermodynamics, strength of materials, machine design, etc.

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# GEOMETRY

## PRINCIPLES OF GEOMETRY

1. The sum of all the angles formed on one side of a straight line equals two right angles, or 180°.

2. The sum of all the angles formed around a point equals four right angles, or 360°.

3. When two straight lines intersect each other, the opposite or vertical angles are equal.

4. If two angles have their sides parallel, they are equal.

5. If two triangles have two sides, and the included angle of the one equal to two sides and the included angle of the other, they are equal in all their parts.

6. If two triangles have two angles, and the included side of the one equal to two angles and the included side of the other, they are equal in all their parts.

7. In any triangle, the greater side is opposite the greater angle, and the greater angle is opposite the greater side.

8. The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

9. In an isosceles triangle, the angles opposite the equal sides are equal.

10. In any triangle, the sum of the three angles is equal to two right angles, or 180°.

11. If two angles of a triangle are given, the third may be found by subtracting their sum from two right angles, or 180°.

12. A triangle must have at least two acute angles, and can have but one obtuse or one right angle.

13. In any triangle, a perpendicular let fall from the apex to the base is shorter than either of the two other sides.

14. If a triangle is equilateral, it is equiangular, and vice versa.

15. If a straight line from the vertex of an isosceles triangle bisects the base, it bisects the vertical angle and is perpendicular to the base.

16. If one side of a triangle is extended, the exterior angle thus formed, is equal to the sum of the two interior and opposite angles.

17. If two triangles are mutually equiangular, they are similar and their corresponding sides are proportional.

18. Triangles that have an angle in each equal, are to one another as the products of the sides including those equal angles.

19. Similar triangles are to one another as the squares of their corresponding sides.

20. In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

21. If a triangle is inscribed in a semicircle, one side being a diameter, it is right-angled.

22. In any parallelogram, the opposite sides are equal; the opposite angles are equal; it is bisected by its diagonals into two equal triangles, and its diagonals bisect each other.

23. If the sides of a polygon are produced in regular order, the sum of the exterior angles thus formed is equal to 360°.

24. The sum of the interior angles of a polygon is equal to twice as many right angles as the polygon has sides, less four right angles. For example, the sum of the interior angles of a pentagon is $(2 \times 5) - 4 = 6$ right angles, or 540°; of an octagon, $(2 \times 8) - 4 = 12$ right angles, or 1,080°, etc.

25. The diagonals joining the vertices of a regular polygon intersect at the center of the inscribed and circumscribed circles.

26. The angle at the center subtended by the side of a regular polygon is equal to 360° divided by the number of sides.

27. Plane figures are similar when they are bounded by the same number of similar sides and their correspondingly situated angles are equal each to each.

28. The perimeters of similar polygons are to one another as any two corresponding sides; and their areas are to one another as the squares of those sides.

29. The circle is a polygon of an infinite number of sides.

30. A circle may be described about or inscribed within any regular polygon.

31. Through three points not in the same straight line a circle may be made to pass and but one.

32. The diameter of a circle is greater than any chord.

33. Any radius that is perpendicular to a chord, bisects the chord and the arc subtended by it.

34. Arcs and chords of the same circle are proportional to the angles at the center of the circle subtended by them.

35. Similar arcs are proportional to the radii of their circles.

36. A tangent to a circle meets it at one point only, and is perpendicular to the radius at that point.

37. If from a point without a circle tangents are drawn to touch the circle, there are but two such tangents; they are equal, and they make equal angles with the chord joining the points of tangency.

38. The angle between a tangent and a chord is equal to one-half the angle at the center subtended by the chord.

39. The perimeters of circles are to one another as any two corresponding dimensions, and their areas are to one another as the squares of such dimensions.

40. Only five regular polyhedrons are possible; the tetrahedron with four triangular faces; the cube with six square faces; the octahedron with eight triangular faces; the dodecahedron with twelve pentagonal faces; and the icosahedron with twenty triangular faces.

41. The sum of all the angles of the faces of any polyhedron is equal to four right angles taken as many times as the polyhedron has vertices less two.

42. The center of any regular polyhedron and of its circumscribed and inscribed spheres is at the point of intersection of the diagonals joining its opposite vertices.

43. Solids are similar which are bounded by the same number of similar faces similarly placed, and which have their corresponding polyhedral angles equal.

44. The areas of the surfaces of similar solids are to one another as the squares of their similar dimensions, and the volumes of similar solids are to one another as the cubes of like dimensions.

45. The sphere is a regular polyhedron of an infinite number of sides.

46. A sphere may be described about any regular polyhedron, and its radius is equal to the distance from any vertex to the center; and a sphere may be inscribed within any regular polyhedron, and its radius is equal to the perpendicular distance from the center of any face to the center of the figure.

47. Through four points not in the same plane a spherical surface may be made to pass, and but one.

48. From a point without a sphere two tangents may be drawn to a great circle of the sphere, and but two.

49. Through a line without a sphere two tangent planes to the surface of the sphere may be drawn, and but two.

## PROBLEMS IN GEOMETRICAL CONSTRUCTION

1. **To Bisect a Given Straight Line or the Chord or the Arc of a Circle.** Let *AB*, Fig. 1, be the given line or chord and *ACB* the arc of the circle. With *A* and *B* as centers and with a radius greater than one-half the line *AB* or the arc *ACB*, describe arcs intersecting at *E* and *F*. The line *EF* will bisect the line, chord, or arc.
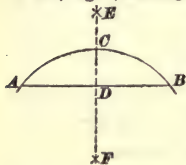
COROLLARY.—The line *EF* will also be perpendicular to the line *AB* and, when prolonged, will pass through the center of the circle of which *ACB* is the arc.

2. **From a Given Point *C*, Without a Straight Line *AB*, to Draw a Perpendicular to the Line.**—From *C*, Fig. 2, as a center, with a radius greater than the distance from *C* to *AB*, describe an arc cutting the line *AB* at *A* and *B*. From *A* and *B* as centers and with a radius greater than one-half of *AB*, describe arcs intersecting at *D*, and draw the line *CD*.
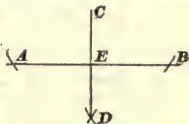
COR.—The line *CD* will bisect that portion of the line included between the points *A* and *B*.

3. **At a Given Point *C* in a Straight Line *AB*, to Erect a Perpendicular to That Line.**—Lay off the points *A* and *B*, Fig. 3, equidistant from *C*, and with *A* and *B* as centers and, with a radius greater than one-half *AB*, describe arcs intersecting at *D*. The line *DC* will be perpendicular to *AB*.

4. **To Erect a Perpendicular at the End *A* of a Given Line *AB*.— *First Method*.**—From any point *C*, Fig. 4, above the line *AB* and with a radius *AC* describe the arc of the circle *AD*, which also cuts the line *AB* at *B*. Connect *B* and *C* and prolong the line to intersect the circle at *D*. The line *AD* will be perpendicular to the line *AB*.
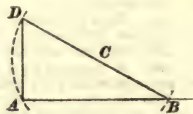
*Second Method*.—From the given point *A*, Fig. 5, set off a distance *AB* equal to three parts by any scale. From *A* and *B* as centers and with radii equal, respectively, to four parts and five parts, draw arcs intersecting at *C*. The line *AC* will be the perpendicular required.

NOTE.—This is one of the methods employed for laying off the coordinates on mine maps.

5. **Through a Given Point *A*, to Draw a Straight Line Parallel to a Given Straight Line *CD*.**—With *A*, Fig. 6, as a center and with a radius greater than the shortest distance from *A* to the line *CD*, describe an indefinite arc *DB*. With *D* as center and with the same radius *DA*, describe the arc *AC*. With *D* as a center and with the radius *AC* describe an arc cutting the arc *DB* at *B*. The line *AB* will be parallel to the line *CD*.
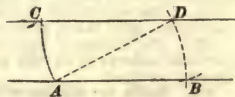
6. **To Draw a Straight Line Parallel to a Given Line and at a Given Distance From It.** *First Method*.—Select any two points *A* and *B*, Fig. 7, on the given line. With these as centers and with radii equal to the distance apart of the lines describe the arcs *C* and *D*. Draw the line *CD* touching the arcs.


FIG. 1


FIG. 2


FIG. 3


FIG. 4


FIG. 5


FIG. 6
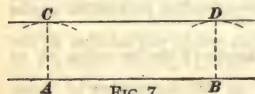

FIG. 7

*Second Method.*—At any two selected points *A* and *B*, Fig. 8, in the given line, erect perpendiculars. With *A* and *B* as centers and with a radius equal to the distance apart of the lines, draw arcs of circles cutting the perpendiculars at *C* and *D*. The line *CD* joining these points of intersection will be parallel to the line *AB*.
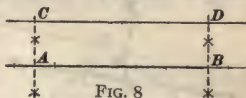
NOTE.—These methods are employed to lay off the coordinates of mine maps.

FIG. 8

7. **To Divide a Straight Line Into Any Number of Equal Parts.**—Let *AB*, Fig. 9, be a straight line that is to be divided into, say, seven equal parts. Draw the parallel lines *AC* and *BD* making any angle with *AB*, and measure off upon each of them seven equal spaces. Connect the points, *1* and *1*, *2* and *2*, etc., by lines. These lines will divide the line *AB* into seven equal parts.
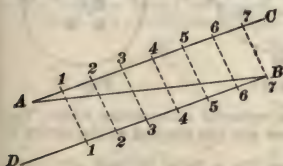
FIG. 9

Or, but one line *AC* need be drawn. By connecting *7* and *B*, the parallel lines drawn through *6*, *5*, *4*, etc., will divide the line as required.

NOTE.—This method is available for dividing a line into any odd number of parts when no scale of such parts is to be had. Thus, to divide a line, 2⅓ in. long into sevenths, seven ¼ or ½ in., may be laid off along the line *AC*. The operation will, if properly carried out, divide the 2⅓ in. into seven equal parts of .3171 in.

A line may be divided into proportional parts by this method. Thus to divide the line *AB* in the ratio of *3* to *4*. Lay off the distance *A–3* equal to three parts, and the distance *4–7* equal to four parts. The line *3–3* will divide the line *AB* in the required ratio.

8. **At a Point A on a Given Straight Line** *AB*, **to Make an Angle Equal to a Given Angle** *EFG*.—From *F*, Fig. 10, as a center and with any radius *FG* describe the arc *EG*. From *A* as a center and with the same radius, describe the arc *CB*; then with a radius equal to the chord *EG*, describe an arc from *B* as a center, cutting *CB* at *D*, and draw the line *AD*. The angle *BAD* will be equal to the angle *EFG*.
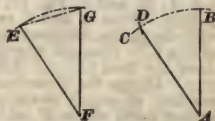
FIG. 10

9. **To Draw Angles of 60° and 30°.**—From any point *A*, Fig. 11, on the line *AB* and with any radius *AB* describe the arc *BD*. With *B* as a center and with the same radius *AB*, describe an arc cutting *BD* at *D*. The line *AD* will form an angle *DAB* with *AB* equal to 60°. The perpendicular *DC* to the base will form the angle *ADC* of 30°.

FIG. 11
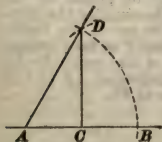
10. **To Draw an Angle of 45°.**—Lay off any distance *AB*, Fig. 12, and at *B* erect a perpendicular to *AB*. With *B* as a center and with a radius equal to *BA* describe an arc cutting the perpendicular at *C*. The line *AC* will form with the line *AB* an angle *CAB* equal to 45°.

Or, the second method under problem 4 may be used.

FIG. 12

11. **To Bisect an Angle** *ABC*.—With any radius and with *B*, Fig. 13, as a center, describe arcs cutting the sides at *A* and *C*. With *A* and *C* as centers, describe arcs of equal radius intersecting at *D*. The line *BD* is the bisector, and the angle *ABD* = angle *DBC*.
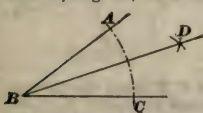
FIG. 13

12. **To Bisect an Open Angle** (*Method by* L. L. LOGAN).—Let *AB* and *CD*, Fig. 14, be the sides of an open angle. With any point *O* as a center, describe a circle cutting the sides at *e*, *f*, *g*, and *h*, and with *e* and *f*, and *g* and *h* as centers and any radius, describe arcs intersecting at *k* and *l*, respectively. Draw *Ok* and *Ol* and *mn*. With *p* and *q* as centers, and any radius, describe arcs intersecting at *R* and *S*. The line drawn through *RS* is the required bisector.
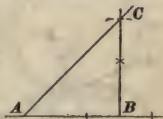
**13. To Find the Center of a Given Circumference or Arc.**—*First Method.*
Take any three points $A$, $B$, and $C$, Fig. 15, on the circumference and unite them by lines $AB$ and $BC$. Bisect these chords by the perpendiculars $DO$ and $EO$; their intersection is the center of the circle.

*Second Method.*—Take any three points $A$, $B$, and $C$, Fig. 16, on the circumference as far apart as convenient. With these three points
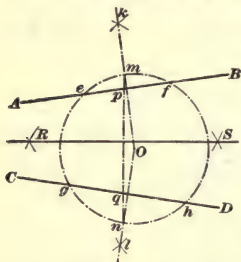
Fig. 14
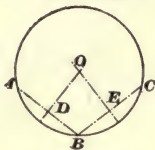
Fig. 15
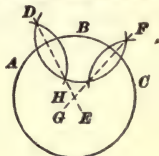
Fig. 16

as centers and with the same radius, draw a series of intersecting arcs. The lines $GF$ and $DE$ through these intersections cut one another at the center $H$ of the circle.

NOTE.—This method is employed to describe a circle through any three points not in the same straight line.

**14. To Describe an Arc of a Circle Passing Through Three Given Points When the Center Is Not Available.**
Let $A$, $B$, and $C$, Fig. 17, be the three points. From $A$ and $C$ as centers and with the radius $AC$ describe the arcs $CY$ and $AX$. Through the third point $B$ draw the lines $CD$ and $AE$ cutting the arcs. Divide the distance $AD$ into any number of equal parts and lay off similar parts above $D$ on the arc $AX$.

Fig. 17

Also lay off like parts above and below $E$ on the arc $CY$. Draw lines $CF$, $CG$, etc., and $AH$, $AL$, etc. Their intersection, $1$, and $2$, will be points on the required circle. The curve may be drawn by splines. The smaller the divisions of the arcs, the more points will there be given in the arc of the circle.

**15. To Draw a Tangent to a Circle Through a Given Point $P$ in the Circumference.**—Find the center $C$, Fig. 18, of the circle by any of the methods described and draw the radial line $CD$. At $P$ erect a perpendicular $AB$ to this line $CD$; the perpendicular $AB$ will be tangent to the circle at $P$.
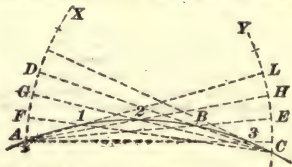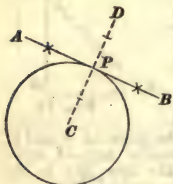
Fig. 18
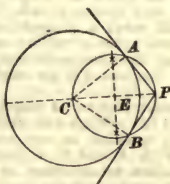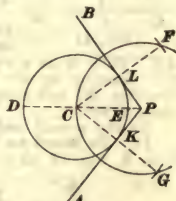
Fig. 19

Fig. 20

**16. To Draw Tangents to a Circle From a Point $P$ Without the Circumference.**—*First Method.*—If the center of the circle is not given find it by any method, and draw the line $PC$, Fig. 19. Bisect this line $PC$ at $E$, and with the radius $EC = PE$, describe a circle cutting the given circle at $A$ and $B$. Connect $A$ and $P$ and $B$ and $P$; the lines $AP$ and $BP$ will be tangent to the circle.

*Second Method.*—As before, let $P$, Fig. 20, be the point. Find the center $C$ of the circle and draw the line $PCD$. With $P$ as a center and with a radius $PC$,

describe the arc FCG. With C as a center and with a radius equal to the diameter DE of the circle, cut this arc at F and G. Draw the lines FC and GC, cutting the circle at L and K. Draw the lines PB and PA through L and K respectively; they will be tangents to the circle.

**17. To Draw an Arc of a Circle Tangent to Two Lines Inclined to Each Other, One Point of Tangency B Being Given.**—Produce the given lines, AB and CD, Fig. 21, until they intersect at E. Bisect the angle AEC by the line EF. Draw a perpendicular to the line AB from the point B. Its intersection G with the line EF, the bisector of the angle, is the center of the required arc. The other point of tangency may be found by dropping a perpendicular GH upon the line CD.
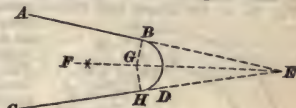
FIG. 21

NOTE.—If an intersection cannot be reached, use the method described in problem 12, for bisecting the angle.

**18. To Construct a Triangle, the Sides Being Given.**—Let AB, Fig. 22, be one of the sides. With A as a center and a radius equal to one of the remaining sides AC, and with B as a center and a radius equal to the third side BC describe the arc AC cutting the arc AC at C. Draw the lines AC and BC; then ABC will be the required triangle.
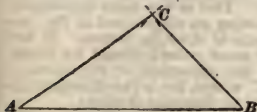
FIG. 22

**19. To Describe a Circle About a Triangle.**—Let ABC, Fig. 23, be the triangle. Bisect any two sides as AB and AC at D and E and at these points erect perpendiculars to the sides intersecting at F. With F as a center and with a radius equal to FA = FB = FC, describe the circle ABC.
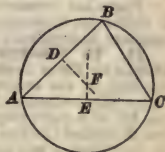
**20. To Inscribe a Circle in a Triangle.**—Let ABC, Fig. 24, be the given triangle. Bisect any two angles, such as A and C, by lines intersecting at D. Drop a perpendicular from D upon any side as DE upon the side AC, and with D as a center and a radius DE, inscribe the circle.
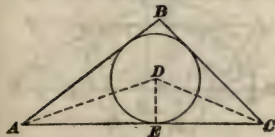
FIG. 23

FIG. 24

**21. To Construct a Hexagon Upon a Given Straight Line.**—Let FE, Fig. 25, be the given line. From its extremities F and E as centers and with the radius FE describe arcs of circles intersecting at G. With a radius GE, draw the circumscribing circle EDCBAF. With the same radius GE = FE set off upon the circumference of this circle the chords ED, DC, CB, and BA. The points so found, when joined, will form the required hexagon.

NOTE.—The side of any hexagon is equal to the radius of its circumscribed circle. As the exterior angles of a hexagon are each equal to 60°, this polygon is readily drawn with a straightedge and a 60°-30° triangle.

FIG. 25

**22. To Describe an Octagon Upon a Given Straight Line.**—Let AH, Fig. 26, be the given line. Produce it in both directions and at A and H erect the perpendiculars AD and HE. Bisect the angles DAa and EHh with the lines AB and HG. With A and H as centers and with a radius AH equal to the length of the given side describe arcs cutting the bisectors at B and G respectively; AB and HG will be sides of the octagon. Draw BC and GF parallel to DA and EH. Make them equal to AH by cutting them (using B and G as centers) with the arcs of a circle whose radius is equal to AH. From C and F as centers and with a radius equal to AH, describe arcs of circles intersecting the perpendiculars at D and E. The lines, CD, DE, and EF, joining the points thus determined, complete the octagon.
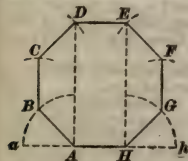
FIG. 26

NOTE.—As the exterior angles of an octagon are each 45°, this polygon is readily drawn with a straightedge and a 45° triangle.

**23. To Construct an Ellipse, the Axes Being Given.**—Let $AB$ and $CD$, Fig. 27, be the major and minor axes, respectively, intersecting, bisecting, and



FIG. 27

perpendicular to each other at $P$. Using $P$ as a center draw two circles with radii equal, respectively, to one-half the axes, or $PA$ and $PC$. From $P$, draw any number of random lines, as $Pa$, $Pb$, . . . $Pf$ to the circumference of the larger circle and drop perpendiculars from the extremities, $a$, $b$, . . . $f$. From the points of intersection, $1$, $2$, . . . $6$, of the lines $Pa$, $Pb$, etc., with the smaller circle draw lines parallel to the major axis $AB$. The points of intersection of these parallels with the verticals previously drawn will be points on the ellipse. This is the most convenient method of drawing an ellipse and is the one used very largely in drafting rooms. It will be noted that more points should be determined where the direction of the curve is changing rapidly, as at and near $B$, than at $C$ where, for a considerable distance on either side of the minor axis, the change in direction is slight.
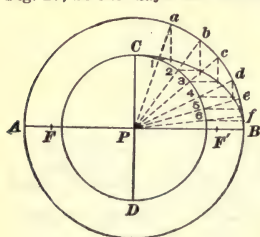
The foci of the ellipse may be found by drawing, with $C$ or $D$ as a center and with $PA = PB$ as a radius, arcs of circles cutting the major axis at $F$ and $F'$.

There are numerous complicated and inaccurate methods of drawing what is called an approximate ellipse, three of which are given, but they do not compare in simplicity with the exact method given.

**24. To Construct an Approximate Ellipse, the Axes Being Given.** *Method by Three Centers.*—Let $a$, Fig. 28, be the center, $bc$ the major, and $ae$ one-half the minor axis of an ellipse.



FIG. 28

Draw the rectangle $bfgc$, and the diagonal line $be$; at a right angle to the line $be$, draw line $fh$ cutting the line $BB$ at $i$. With radius $ae$, and from $a$ as a center, draw the dotted arc $ej$, giving the point $j$ on the line $BB$. From $k$, which is central between $b$ and $j$, draw the semicircle $bmj$, cutting the line $AA$ at $l$. Draw the radius of the semicircle $bmj$, cutting $fg$ at $n$. With radius $mn$, mark on the line $AA$, from $a$ as a center, the point $o$. With radius $ho$, and from center $h$, draw the arc $poq$. With radius $al$, and from $b$ and $c$ as centers, draw arcs cutting the arc $poq$ at the points $p$ and $q$. Draw the lines $hpr$ and $hqs$, and also the lines $pit$ and $qvw$. From $h$ as a center, draw that part of the ellipse lying between $r$ and $s$ with radius $hr$. From $p$ as a center draw that part of the ellipse lying between $r$ and $t$ with the radius $pr$. From $q$, draw the ellipse from $s$ to $w$. With radius $it$, from $i$ as a center, draw the ellipse from $t$ to $b$ with radius $it$, and from $v$ as a center, draw the ellipse from $w$ to $c$, and one-half the ellipse will be drawn. It will be noted that the whole construction has been performed to find the centers $h$, $p$, $q$, $i$, and $v$, and that while $v$ and $i$ may be used to carry the curve around the other side or half of the ellipse, new centers must be provided for $h$, $p$, and $q$; these new centers correspond in position to $h$, $p$, $q$.
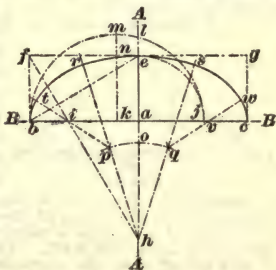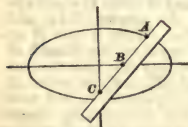
NOTE.—This method is the one commonly employed to lay off concrete, or masonry arches, etc.

*Method by Straightedge.*—On a straightedge, lay off $AB$, Fig. 29, equal to one-half the shorter axis and $AC$ equal to one-half the longer axis. Determine points in the ellipse by marking positions of $A$ as the point $B$ is moved along the major axis, at the same time the point $C$ being kept in the minor axis.



FIG. 29

*Method by Cord.*—Lay off the axes and find the foci as described in problem 23. Stick pins at the foci $F$ and $F'$, Fig. 27. To these pins attach a string making its length equal to $FC + CF'$. The point of a

pencil placed inside the string may be made to describe an ellipse, if the string is kept tightly and uniformly stretched while the pencil is in motion.

NOTE.—This is the method commonly employed to lay off an ellipse upon the ground, as when making garden beds, etc.

The cord and straightedge methods are theoretically capable of describing a true ellipse. That is, the methods are founded on the mathematical principles governing the ellipse, but it is not generally possible to manipulate either the string or straightedge so that perfect results may be obtained.

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# MENSURATION OF SURFACES

## TRIANGLES

A *triangle* is a plane surface bounded by three straight lines. Some of the different kinds of triangles are shown here.
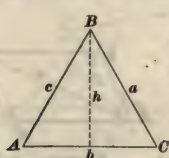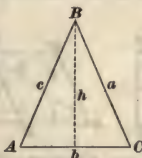


Equilateral          Isosceles          Right-angled
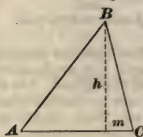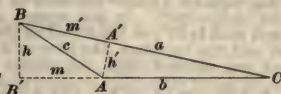


*Acute*                    *Obtuse*
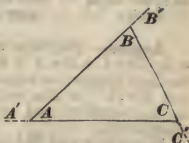
Let  $A$, $B$, $C$ = interior angles
$A'$, $B'$, $C'$ = exterior angles
$b$ = side called base
$A$ = area
$m$ = distance from foot of
$h$ to nearest vertex

$a$, $b$, $c$ = sides opposite angles $A$, $B$, and $C$
$P$ = perimeter
$h$ = perpendicular upon base from vertex of angle opposite

**Angles.**—$A + B + C = 180°$. $A = 180° - (B + C)$, and similarly for $B$ and $C$. $A + A' = 180°$, and similarly for $B + B'$ and $C + C'$. $A' = 180° - A$, and similarly for $B'$ and $C'$. $A' + B' + C' = 360°$. $A' = 360° - (B' + C')$, and similarly for $B'$ and $C'$. $A' = B + C$; $B' = A + C$; $C' = A + B$.

**Perimeter and Sides.**—$P = a + b + c$. In all acute-angled triangles, including equilateral, isosceles, and right-angled triangles, any side $a = \sqrt{b^2 + c^2 - 2bm}$. In an obtuse-angled triangle, the side opposite the obtuse angle $a = \sqrt{b^2 + c^2 + 2bm}$.

**Altitude.**—In any triangle, the *altitude*, or perpendicular distance from the base to the vertex of the opposite angle, is $h = \sqrt{c^2 - m^2}$.

**Area.**—In any triangle, the area is equal to the product of the base by one-half the altitude, or $A = \dfrac{bh}{2}$. Any side may be selected as the base. Thus, in the obtuse-angled triangle, if the base is the side $b$, $A = \dfrac{bh}{2}$; if the base is the side $a$, $A = \dfrac{ah'}{2}$. If the length of the three sides is given, let $p$ = one-half the sum of the three sides, or $p = \dfrac{a + b + c}{2}$, and $A = \sqrt{p(p - a)(p - b)(p - c)}$; that is, *the area is equal to the square root of one-half the sum of the sides multiplied by this one-half sum less each one of the sides, respectively.*

**Special Cases.**—*Equilateral Triangle.*—Angle $A = B = C = 60°$. Side $a = b = c$. Altitude $h = \text{side} \times .866025$. Side $= h \div .866025 = h \times 1.154701$. Side $= \sqrt{\text{area}} \times 1.51967$. Area $= \text{side}^2 \times .443013$. Length of side of square having same area as an equilateral triangle $= \text{side of triangle} \times .658037$. Diameter of circle of same area as an equilateral triangle $= \text{side of triangle} \div 1.34677$. The perpendicular $h$ bisects the angle $B$ and the side $b$; and similarly for the other angles and sides. This triangle is also known as the equiangular or 60° triangle.

*Isosceles Triangle.*—Angle $A = C$. $B = 90° - A$ or $C$. Side $a = c$. The perpendicular $h$ bisects the angle $B$ and the side $b$.

*Right-Angled Triangle.*—Angle $A = 90°$. Angles $B + C = 90°$. Side $a^2 = b^2 + c^2$, and $a = \sqrt{b^2 + c^2}$, $b = \sqrt{a^2 - c^2}$, $c = \sqrt{a^2 - b^2}$.

For other properties and methods of solving triangles, see under Trigonometry.

## PARALLELOGRAMS

A *parallelogram* is a plane figure bounded by four straight lines which are parallel, two and two. Some of the different kinds of parallelograms are shown here.
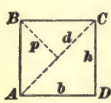


*Square.*—Four
equal sides
and four right
angles
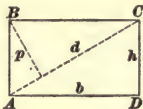
*Rectangle.*—Four
right angles
and opposite
sides equal
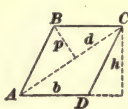
*Rhombus.*—Four
equal sides
and
oblique angles
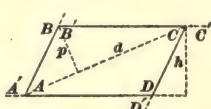
*Rhomboid.*—Four
oblique angles
and
opposite sides equal

**Angles.**—The sum of the exterior angles = the sum of the interior angles $= A + B + C + D = A' + B' + C' + D' = 360°$. In the square and the rectangle, the four angles are equal and each is 90°; in the rhombus and rhomboid, $A = C$ and $B = D$. In the square and the rhombus, the diagonals are perpendicular to one another and bisect one another and the angles at their opposite extremities.

**Perimeter and Sides.**—Let the sides be $a$, $b$, $c$, and $d$, respectively, then the perimeter, $P = a + b + c + d$. In the square and rectangle, any side = area ÷ by an adjacent side. In the rhombus and rhomboid, a side = area ÷ by the altitude $h$. In all four cases, the diagonal $d$ = area ÷ by perpendicular $p$.

**Area.**—In all cases, the area $A = bh = dp$.

**Square.**—Diagonal $d = \text{side} \times 1.41421$. Side = diagonal $\times .707107$. The side of a square equal in area to a given circle = diameter of circle $\times .886277$. The area of the largest square that may be inscribed in a circle $= 2 \times \text{radius of circle}$.[2]

It should be noted that all problems relating to parallelograms, as well as to trapezoids, trapeziums, and regular and irregular polygons, may be solved by resolving these figures into triangles.

## TRAPEZOIDS

A *trapezoid* is a plane figure bounded by four straight lines, only two of which are parallel one to the other. One is shown in the accompanying figure.



**Angles.**—The sum of the interior angles $= 360°$, or $A + B + C + D = A' + B' + C' + D' = 360°$. $A = B'$; $B = A'$; $C = D'$; $D = C'$. $A = 180° - A'$; $B = 180° - B'$; and similarly for $C$ and $D$. $A + B = C' + D'$; $A + C = B' + D'$; $A + D = B' + C'$; $B + C = A' + D'$; and similarly for other combinations of $A$, $B$, $C$, and $D$.

**Perimeter.**—The sides being $a$, $b$, $c$, and $d$, the perimeter $P = a + b + c + d$.

**Diagonal.**—The diagonal $d = 2 \times \text{area} \div (p + p')$.

**Area.**—*Case I.*—Given the two parallel sides $a$ and $c$ and the perpendicular distance between them $h$, $A = \dfrac{h}{2}(a + c)$.

*Case II.*—Given the diagonal $d$ and the perpendiculars upon it $p$ and $p'$.
$$A = \frac{d}{2}(p + p').$$

*Case III.*—Given the two parallel sides $a$ and $c$ and the angles adjacent to one of them, $A$ and $D$, the area,
$$A = \frac{c^2 - a^2}{2(\cot A + \cot B)} = \frac{(c-a)(c+a)\sin A \sin B}{2\sin(A+B)}$$

*Case IV.*—Given all four sides $a$, $b$, $c$, and $d$. Let $c - a = f$, and $\frac{1}{2}(b + d + f) = s$, then area $A = \frac{(a+c)}{f}\sqrt{s(s-d)(s-b)(s-f)}$.

## TRAPEZIUMS

A *trapezium* is a plane figure bounded by four straight lines, no two of which are parallel.
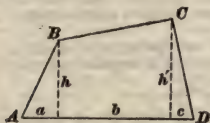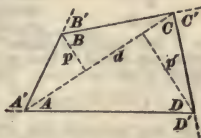


FIG. 1        FIG. 2

**Angles.**—As with trapezoids, the sum of the interior angles = the sum of the exterior angles = 360°. Also, $A = 180° - A'$, and similarly for $B$, $C$, and $D$.

**Perimeter and Diagonal.**—The same relations prevail for the perimeter and the diagonal as for trapezoids.

**Area.**—In the trapezium shown in Fig. 1, $A = \frac{1}{2}[b(h + h') + ah + ch']$.

In the trapezium shown in Fig. 2, $A = \frac{d}{2}(p + p')$.

## POLYGONS

A *polygon* is a plane figure bounded by three or more straight lines. Some of the more common forms are shown in Fig. 1.



*Pentagon*        *Hexagon*        *Heptagon*        *Octagon*

FIG. 1

If all the sides and angles are equal, each to each, the figure is a regular polygon; otherwise it is not. Of the figures previously discussed, the equilateral triangle and the square are regular polygons; the others are irregular.

In any regular polygon, Fig. 2, let the central angle $(AOB = BOC,$ etc.) $= C$; let the interior angle $(ABC = BCD,$ etc.) $= I$; let the exterior angle $(A'AB = B'BC,$ etc.) $= E$. Also let $R = \frac{D}{2} =$ radius of circumscribed circle, and let $r = \frac{d}{2} =$ radius of inscribed circle = apothem. Likewise, let $S =$ length of a side as $AB$, $BC$, etc., and let $N =$ number of sides.

**Central Angle.**—The central angle is equal to the exterior angle and is equal to 360° divided by the number of sides in the polygon, or $C = E = \frac{360°}{N}$. The sum of either the central or the exterior angles of any polygon is 360°.

**Interior Angle.**—The interior angle is equal to 180° minus either the central or the exterior angle, or $I = 180° - C = 180° - E$. The sum of all the interior angles of any polygon is equal to twice as many right angles as the polygon has sides, less four right angles, or $\Sigma I = (2 \times N \times 90°) - 360°$.



FIG. 2

**Diagonals.**—The diagonals $AD$, $BE$, etc., of a regular polygon bisect the interior angles $I$, bisect one another, intersect at the center of the inscribed and circumscribed circles, divide the polygon into as many isosceles triangles as it has sides; also, they are the diameters $D$ of the circumscribed circle.

**Apothems.**—The apothems $LO$, etc., of a regular polygon are perpendicular to the sides. They bisect the sides, the central angles, and one another; divide the fundamental isosceles triangles of the polygon into two equal right angled triangles $BLO + CLO = BOC$; and are the radii $r$ of the inscribed circle.

**Perimeter and Sides.**—The perimeter of any polygon is equal to the sum of the lengths of all its sides. The perimeter of a regular polygon, $P = NS$. Any side, $S = 2\sqrt{R^2 - r^2} = 2R \sin \dfrac{C}{2} = 2r \tan \dfrac{C}{2} = \dfrac{P}{N}$.

**Area.**—In any polygon, the area is equal to the sum of the areas of the triangles into which it is divided by its diagonals. The area of a regular polygon is equal to the area of one of the fundamental triangles, as $AOB$, multiplied by the number of sides. Likewise,

$$A = Nr^2 \tan \frac{C}{2} = \tfrac{1}{2}NR^2 \sin C = \tfrac{1}{2}NSr = \tfrac{1}{2}Pr.$$

The accompanying table gives, for the more important regular polygons, the number of sides; the name; the central angle, which equals the exterior angle; the interior angle; the length of the side, in terms of the radius of both the circumscribed and inscribed circles $R$ and $r$; and the area, in terms of the side $S$ and in terms of the radius of the circumscribed and inscribed circles $R$ and $r$, respectively.

## NAMES AND RELATIONS OF REGULAR POLYGONS

| Number of Sides | Name | Angle | | Side | | Area | | |
|---|---|---|---|---|---|---|---|---|
| | | $C = E$ | $I$ | $R = 1$ | $r = 1$ | $S = 1$ | $R = 1$ | $r = 1$ |
| 3 | Equilateral Triangle | 120° | 60° | 1.73205 | 3.46410 | .43301 | 1.29904 | 5.19615 |
| 4 | Square | 90° | 90° | 1.41421 | 2.00000 | 1.00000 | 2.00000 | 4.00000 |
| 5 | Pentagon | 72° | 108° | 1.17557 | 1.45308 | 1.72048 | 2.37765 | 3.63271 |
| 6 | Hexagon | 60° | 120° | 1.00000 | 1.15470 | 2.59808 | 2.59808 | 3.46608 |
| 7 | Heptagon | 51°25.7' | 128°34.3' | .86776 | .96315 | 3.63391 | 2.73641 | 3.37106 |
| 8 | Octagon | 45° | 135° | .76536 | .82842 | 4.82842 | 2.82840 | 3.31368 |
| 9 | Nonagon | 40° | 140° | .68404 | .72794 | 6.18183 | 2.89253 | 3.27573 |
| 10 | Decagon | 36° | 144° | .61804 | .64984 | 7.69421 | 2.93895 | 3.24921 |
| 11 | Undecagon | 32°43.6' | 147°16.4' | .56346 | .58724 | 9.36581 | 2.97341 | 3.22972 |
| 12 | Dodecagon | 30° | 150° | .51764 | .53590 | 11.19615 | 3.00000 | 3.21538 |

EXAMPLE 1.—What is the length of the side of a triangle inscribed in a circle of 2 in. radius?

SOLUTION.—Here $R = 2$ and the side $S = 2 \times 1.73205 = 3.46410$.

EXAMPLE 2.—What is the length of the side of a pentagon circumscribed about a circle of 4 in. radius?

SOLUTION.—Here $r = 4$ and side $S = 4 \times 1.45308 = 5.81232$ in.

EXAMPLE 3.—What is the area of a hexagon whose side is 2 in. long?

SOLUTION.—Here $S = 2$ and area $= S^2$ or $4 \times 2.59808 = 10.39232$ sq. in.

EXAMPLE 4.—What is the area of a dodecagon that may be inscribed in a circle of 3 in. radius?

SOLUTION.—In this case, $R = 3$ and area $= R^2$ or $9 \times 3.000000 = 27.000000$ sq. in.

EXAMPLE 5.—What is the area of a decagon that may be circumscribed about a circle of 4 in. radius?

SOLUTION.—In this case $r = 4$ and area $= r^2$ or $16 \times 3.24921 = 51.9874$ sq. in.

EXAMPLE 6.—What is the radius of the circle that may be circumscribed about a square whose side is 2 in.?

SOLUTION.—Here $R = 2$ and it is required to find $S$. $S = 2 \div 1.41421 = 1.41421$.

**Area of Irregular Polygons.**—If the figure is bounded by straight lines, to find its area divide it into triangles; the sum of the areas of these will be equal to that of the irregular polygon. This method is commonly used by engineers as a useful check upon the accuracy of the areas obtained by calculation. In many cases the results thus obtained answer every purpose.

If one or more of the boundaries is an irregular line, as would be that of the bank of a river, the area may be found by one of several methods.

*First Method.*—Let $A$ and $E$, Fig. 3, be two corners on the river or on any other irregular boundary. Draw the lines $AF$ and $FE$ to include as much water on the land side as they exclude land on the water side. If the map has been platted by means of coordinates, those of $F$ may be found by measurement, and the area can then be calculated by double latitudes. Or the entire figure may be divided into a series of triangles as shown and the sum of their areas taken as that of the property.
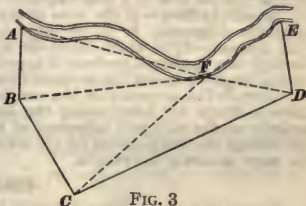


FIG. 3

*Second Method: By Selected Ordinates.*—Draw perpendiculars on $AB$, Fig. 4, from the points of the curve at which its direction changes appreciably, and consider the portion of the curve between two consecutive perpendiculars to be a straight line. The figure is then treated as if divided into a number of trapezoids, whose areas can be computed by the rules already given.
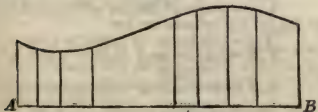


FIG. 4
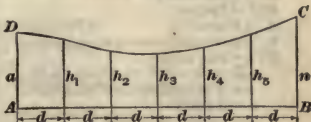


FIG. 5

*Third Method: Trapezoidal Rule.*—The ordinates are measured at regular intervals $d$ along the line, as shown in Fig. 5. If the end ordinates are $a$ and $n$ respectively, the area is $A = \left(\dfrac{a+n}{2} + \Sigma h\right)d$ in which $\Sigma h$ is the sum of all the intermediate ordinates.

EXAMPLE.—If the ordinates from the straight line $AB$ to the curved boundary $DC$, are 19, 18, 14, 12, 13, 17, and 23 li., respectively, and are at equal distances of 50 li., what is the area included between the curved boundary and the straight line?



FIG. 6

SOLUTION.—Area $ABCD = \left(\dfrac{19+23}{2} + 18 + 14 + 12 + 13 + 17\right) \times 50 = 4,750$ sq. li.
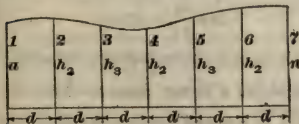
*Fourth Method: Simpson's Rule.* The base line must be divided into an even number of equal parts, as shown in Fig. 6. The area is then equal to $A = (a + n + 4\Sigma h_2 + 2\Sigma h_3)\dfrac{d}{3}$, in which $a + n$ is the sum of the end ordinates; $4\Sigma h_2$ is four times the sum of all intermediate even-numbered ordinates; and $2\Sigma h_3$ is

twice the sum of all intermediate odd-numbered ordinates.   This rule is more accurate than the trapezoidal rule.

EXAMPLE.—What is the area *ABCD* of the polygon, in the example of the Third Method, according to Simpson's rule?

SOLUTION.—$A = [19+23+4\times(18+12+17)+2\times(14+13)]\times\frac{5.0}{3} = 4,733$ sq. li.

*Fifth Method.*—Prepare two drawings upon paper of the same weight and quality, one of the tract of irregular outline and another of about the same size but of a tract whose area is known, both drawings being to the same scale. Cut each out carefully along the boundary and weigh in a chemist's balance sensitive to milligrams (1 mg. = .015 gr., about).   The area of the irregular figure may be calculated from the proportion:   *The unknown area : the known area = the weight of the drawing of the unknown area : the weight of the drawing of the known area.*

*Sixth Method.*—Draw the figure upon cross-section paper.   If each of the small squares represents a certain area to the scale of the map, the number of whole squares and fractions thereof within the boundary, when multiplied by the relative area of a single square, will give the area of the figure.

*Seventh Method.*—If the area of many irregular figures (such as a series of indicator diagrams) is to be determined, a planimeter should be secured. This instrument, of which there are several types, and which may be secured through any dealer in engineer's supplies, affords the most rapid and accurate means for determining the areas of irregular figures.

NOTE.—The accuracy of the results obtained by the use of these approximate methods for determining the areas of irregular figures depends almost entirely on the skill and judgment of the engineer.   Other things being equal, the smaller the subdivisions (in the method involving ordinates or small squares) the more accurate the results.

## CIRCLES

A *circle* is a plane figure bounded by a curved line every point of which is equidistant from an interior point called the *center*.

In the formulas relating to circles the letters have the following meanings: capital letters refer to the larger and lower-case letters to the smaller of two circles concerned in the same formula.

$D, d$ = diameter
$R, r$ = radius
$A, a$ = area
$P, p$ = circumference = perimeter
$H, h$ = rise of arc
$\pi$ = ratio of $P$ to $D$ = 3.1416
$\pi$ = 3.141592653589793238462
$\pi^2$ = 9.86965
$K, k$ = length of chord

$L, l$ = length of arc
$C, c$ = angle at center subtended by chord or arc
$T, t$ = thickness of circular ring
$A, B$ = major and minor axes of ellipse
$a, b$ = semi-major and semi-minor axes
$\sqrt{\pi}$ = 1.772453



$p = \pi d = 3.1416d$
$p = 2\pi r = 6.2832r$
$p = 2\sqrt{\pi A} = 3.5449\sqrt{A}$
$p = \dfrac{2A}{r} = \dfrac{4A}{d}$
$d = \dfrac{p}{\pi} = \dfrac{p}{3.1416} = .3183p$
$d = 2\sqrt{\dfrac{A}{\pi}} = 1.1284\sqrt{A}$

$r = \dfrac{p}{2\pi} = \dfrac{p}{6.2832} = .1592p$
$r = \sqrt{\dfrac{A}{\pi}} = .5642\sqrt{A}$
$A = \dfrac{\pi d^2}{4} = .7854d^2$
$A = \pi r^2 = 3.1416r^2$
$A = \dfrac{pr}{2} = \dfrac{pd}{4}$
$A = \dfrac{p^2}{4\pi} = .0796p^2$
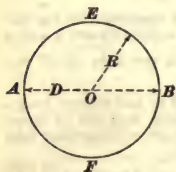
**To Find Diameter of a Circle Equal in Area to a Given Square.**—Multiply one side of the square by 1.12838.

**To Find Radius of a Circle to Circumscribe a Given Square.**—Multiply one side by .7071; or take one-half the diagonal.

**To Find Side of a Square Equal in Area to a Given Circle.**—Multiply the diameter by .88623.

**To Find Side of Greatest Square in a Given Circle.**—Multiply the diameter by .7071.

**To Find Area of Greatest Square in a Given Circle.**—Square the radius and multiply by 2.

### To Find Side of an Equilateral Triangle Equal in Area to a Given Circle.

Multiply the diameter by 1.3468.

Circumferences and areas of circles from 1 to 1,000 units in diameter will be found in connection with the table of squares, cubes, etc. A similar table, but for diameters from $\frac{1}{16}$ to 100, increasing by $\frac{1}{8}$, is also given at the end of the volume. Circumferences and areas of circles whose diameters are not exactly given in the tables, but are within its limits, may be found by interpolation. If the diameters are greater than those given in the table, the circumferences and areas may be found by recalling that the former are proportional to the diameters and the latter to the squares of the diameters. Thus, the circumference and area of a circle 9,380 units in diameter are respectively 10 and 380 times those of a circle 938 units in diameter.

## RINGS

Let $S$ = area and $T$ = thickness of a ring.

$$T = R - r = \frac{D-d}{2} = \frac{P-p}{2\pi} = .159155(P-p)$$

$$S = A - a = \frac{P^2 - p^2}{4\pi} = .079578(P^2 - p^2)$$

$$S = \pi(R^2 - r^2) = \frac{\pi}{4}(D^2 - d^2) = .785398(D^2 - d^2)$$

## SECTORS

A *sector* is a portion of the surface of a circle included between an arc $AEB$ and two radii $OA$ and $OB$.

$$A = \tfrac{1}{2} lr = \frac{90\,l^2}{\pi C} = \frac{\pi}{360} r^2 C = 28.647823 \frac{l^2}{C} = .00872666 r^2 C$$

$$R = 2\frac{A}{l} = \frac{180}{\pi} \frac{l}{C} = \sqrt{\frac{360}{\pi} \frac{A}{C}} = 57.295646 \frac{l}{C}$$

$$= 10.704732 \sqrt{\frac{A}{C}}$$
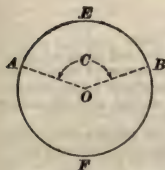
The central angle $C$ must be reduced to degrees and decimal parts thereof.

## CIRCULAR SEGMENTS

A *segment* is a portion of the surface of a circle included between an arc $NEM$ and its chord $NM$.

Area segment $NEM$ = area sector $ONEM$ − area triangle $NOM$.

The area of the sector is found from some one of the formulas already given and that of the triangle is found from

$$A' = \frac{k}{2}(r-h) = \tfrac{1}{2} r^2 \sin C = \tfrac{1}{4} k \cot \frac{C}{2} = (k-h)^2 \tan \frac{C}{2}$$

Other formulas are $k = 2\sqrt{h(2r-h)} = 2r \sin \frac{C}{2}$

$$r = \frac{k^2 + 4h^2}{8h} = \frac{k}{2 \sin \frac{C}{2}} \qquad h = r - \sqrt{r^2 - \left(\frac{k}{2}\right)^2} = r\left(1 - \cos \frac{C}{2}\right)$$

$$l = \frac{2\pi}{360} rC = .017453 rC \qquad \sin \frac{C}{2} = \frac{k}{2r}$$

To determine the area of a heading, the upper part of which is the arc of a circle, the width thereof $k$ and the rise of the arc $h$ being given. Find $r$ from the formula $r = \frac{k^2 + 4h^2}{8h}$. Then find $\frac{1}{2}C$ from $\sin \frac{C}{2} = \frac{k}{2r}$. The angle found is $\frac{1}{2}C$ and must be multiplied by 2. $C$ is to be expressed in degrees and decimals thereof. The area of the sector $ONEM$ is now found from the formula $A = .00872664 r^2 C$. The area of the triangle $NMO$ is found from the formula $A' = \frac{k}{2}(r-h)$. Then $A - A'$ = area segment $NEM$, which is to be added to that of the lower rectangular portion of the heading.

## ELLIPSE

If $A$ and $B$ are the axes of an ellipse and $a$ and $b$ are the semi-, or half, axes, the area $A = \pi ab = 3.141593 \ ab = \frac{\pi}{4}AB = .785398AB$.

No simple formula has been developed for finding the perimeter of an ellipse. In terms of the semi-axes, the formula for the perimeter involves the summation of an expanding series of an infinite number of terms:

$$P = \pi(a+b)\left[1 + \frac{1}{4}\left(\frac{(a-b)}{(a+b)}\right)^2 + \frac{1}{64}\left(\frac{(a-b)}{(a+b)}\right)^4 + \frac{1}{256}\left(\frac{(a-b)}{(a+b)}\right)^6 + \text{etc.}\right]$$

In an ellipse whose semi-axes are 4 and 3, the values within the brackets become $1 + .005102041 + .000006507 + .000000033$. It is apparent, then, except in cases involving great accuracy, that the terms beyond the second may be dropped and the formula for the perimeter of an ellipse may be written, $P = \pi(a+b)\left[1 + \frac{1}{4}\left(\frac{(a-b)}{(a+b)}\right)^2\right]$. This, the correct formula, is fully as simple and is, naturally, far more accurate than any of the so-called shorter approximations.

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# MENSURATION OF SOLIDS

## VALUES USED IN FORMULAS

$V$ = volume of solid
$S$ = area of convex surface
$A$ = area of main base
$a$ = area of second base, or top
$T$ = area of entire surface = $S + A + a$
$h$ = altitude, or perpendicular distance from base to top

$P$ = perimeter of base
$p$ = perimeter of second base or top
$R, r, D, d$ = radius and diameter of main base and of secondary base, respectively; or, radius and diameter of a sphere
$l$ = slant height, or length from base to base measured on surface

## PRISMOID AND PRISMOIDAL FORMULA

A *prismoid* is any solid having two parallel ends or faces of any shape similar or dissimilar, regular or irregular, provided these ends are united by



surfaces, whether plane or curved, on which and through every point of which, a straight line may be drawn from one of the parallel ends to the other. It embraces all polyhedrons, parallelopipeds, prisms, cylinders, cones, pyramids, etc., and even the sphere, which may be regarded as a regular polyhedron of an infinite number of faces.
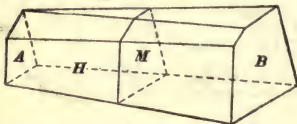
This is true whether the solids are regular or irregular, right or oblique, and applies to their frustums when cut parallel to the base.

The formula for the volume of a prismoid is

$$V = \frac{H(A + 4M + B)}{6}$$

in which $A$ and $B$ = areas of two ends, respectively;
$M$ = area of section taken midway between ends;
$H$ = perpendicular distance between parallel ends.

This is known as the *prismoidal formula*, and from it, by making the proper substitutions for $A$, $M$, and $B$, an equation for the volume of any solid may be deduced, provided this form is included in the definition of a trapezoid. Unless the parallel ends are similar polygons, the mid-section $M$, is not the mean of the sections $A$ and $B$. This formula is extensively used in calculating excavations in railroad work.

## REGULAR POLYHEDRONS

A *polyhedron* is a solid contained within any number of plane sides. A regular polyhedron is one whose bounding planes (faces) are regular polygons of the same shape and area, and whose solid (polyhedral) angles are equal each to each. Unless the sphere is considered a regular polyhedron of an

infinite number of sides, only five regular polyhedrons are possible; these are shown in the accompanying figure.



*Tetrahedron*  *Cube*  *Octahedron*  *Dodecahedron*  *Icosahedron*
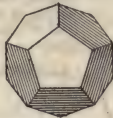
## REGULAR POLYHEDRONS WHOSE EDGES ARE UNITY

| Name | Bounding Polygons | Surface | Volume |
|---|---|---|---|
| Tetrahedron | 4 equilateral triangles | 1.732051 | .117851 |
| Hexahedron or cube | 6 squares | 6.000000 | 1.000000 |
| Octahedron | 8 equilateral triangles | 3.464102 | .471405 |
| Dodecahedron | 12 pentagons | 20.645729 | 7.663119 |
| Icosahedron | 20 equilateral triangles | 8.660254 | 2.181695 |

The surface of any regular polyhedron is equal to the number of faces multiplied by the area of a single face. It maybe found from the accompanying table by squaring the length of the edge of a face and multiplying this by the number, in the column headed Surface, opposite the name of the polyhedron. Thus, the surface area of an octahedron whose edge is 2 in. long, is $2^2$, or $4 \times 3.464102 = 14.856408$ sq. in.

The volume of any polyhedron may be obtained by taking the sum of the volumes of the pyramids into which it may be divided. There will be as many pyramids as there are faces in the polyhedron and the bases of these pyramids will be the several faces of the polyhedron. The volumes of the regular polyhedrons may be obtained from the table by multiplying the cube of the length of the edge by the number, in the column headed Volume, opposite the name of the polyhedron concerned. Thus, the volume of a tetrahedron whose edge is 2 in. long, is $2^3$, or $8 \times .117851 = .942808$ cu. in.

## THE SPHERE

The *sphere* may be defined as a regular polyhedron of an infinite number of sides; or as a solid generated by the revolution of a semicircle about its diameter; or as a solid, every point of whose surface is equidistant from a fixed interior point called the center.

A *great circle* of a sphere is the line formed by the intersection of a plane through the center with the surface of the sphere, as $ABCD$ or $AbCc$. Its radius and diameter $r$ and $d$ are the same as those of the sphere; its area $\pi R^2$ may be placed equal to $Z$.
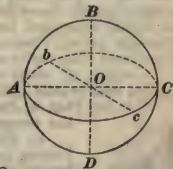


$$R = \sqrt[3]{\frac{3}{4\pi} V} = .62035 \sqrt[3]{V} = \sqrt{\frac{S}{4\pi}} = .2821 \sqrt{S}$$

$$P = \sqrt[3]{6\pi^2 V} = 3.8978 \sqrt[3]{V} = \sqrt{\pi S} = 1.7725 \sqrt{S} = \frac{S}{d} = \pi d = 2\pi r$$

$$S = 4\pi r^2 = 12.5664 r^2 = \pi d^2 = 3.1416 d^2 = \frac{P^2}{\pi} = .3183 P^2 = Pd = 4Z = \frac{6V}{d}$$

$$V = \frac{4}{3}\pi r^3 = 4.1888 r^3 = \frac{\pi}{6} d^3 = .5236 d^3 = \frac{1}{6\pi^2} P^3 = .01689 P^3 = \frac{1}{6} dS = \frac{2}{3} dZ$$

## SPHERICAL SEGMENTS

A *spherical segment* is that portion of a sphere that is included between its surface and a plane cutting it, as *EABCD*. If the plane passes through the center, the sphere is divided into two equal parts, each of which is a hemisphere. Let $EF = h =$ height of segment, and $r'$ and $d'$ be the radius and diameter of the basal plane, and $R$ be the radius of the sphere.
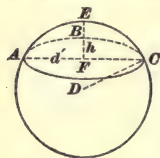
$$S = 2\pi Rh = 6.2832 Rh = \frac{\pi}{4}(d'^2 + 4h^2) = .7854(d'^2 + 4h^2) = Ph$$

To this must be added the area of the base, $\frac{\pi}{4}d'^2$, if the entire surface is required.

$$V = \pi h^2\left(R - \frac{h}{3}\right) = \pi h^2\left(\frac{d'^2 + 4h^2}{8h} - \frac{h}{3}\right)$$

## SPHERICAL ZONES

A *spherical zone* is that portion of a sphere that is included between two parallel intersecting planes.

Let $d$ be the diameter of the sphere and $d'$ and $d''$ be the diameters $AC$ and $ac$ of the two planes, whose distance apart is $EF = h$.
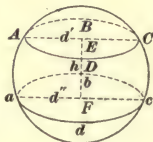
$$S = PZ = \pi dh = \text{surface of sphere} \times \frac{h}{d}$$

$$V = \frac{\pi}{2} h\left(\frac{d'^2 + d''^2}{4} + \frac{h^2}{3}\right)$$
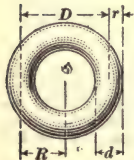
## CYLINDRICAL RINGS

A *cylindrical ring* is produced by bending a cylinder upon itself. Using the notation in the accompanying figure

$$S = 4\pi^2 Rr = 39.4786 Rr = \pi^2 Dd = 9.8697 Dd. \quad V = 2\pi^2 Rr^2$$

$$= 19.7393 Rr^2 = \frac{\pi^2}{4} Dd^2 = 2.4674 Dd^2.$$

## PARALLELOPIPEDS
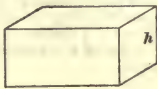


Cube    Rectangular Prism    Rhombohedron    Rhombic Prism
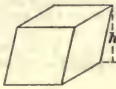
A *parallelopiped* is a solid bounded by six faces, all of which are parallelograms; opposite faces being parallel. In the cube, there are six equal faces and eight equal solid angles; in the rectangular prism, there are three pairs of equal opposite faces and eight equal solid angles; in the rhombohedron, there are six equal faces and four pairs of equal and diagonally opposite angles; in the rhombic prism, there are three pairs of equal opposite faces and four pairs of equal and diagonally opposite angles.

$S =$ sum of areas of six faces. In the cube and rhombohedron $S = 6A$. In the two prisms (calling the equal faces $A$, $B$, and $C$) $S = 2A + 2B + 2C$.

$V = Ah$; that is, the volume is equal to the area of any face multiplied by the perpendicular distance to the opposite face.

In the cube, $V =$ cube of length of one edge. The diagonal joining opposite vertices $=$ an edge $\times 1.732051$. The radius of the inscribed sphere $=$ the edge $\times .5$. The radius of the circumscribed sphere $=$ one-half the diagonal joining opposite vertices $=$ the edge $\times .866026$.

**Frustum of Prism.**—If a section perpendicular to the edges is a triangle, square, parallelogram, or regular polygon,

$$V = \frac{\text{sum of lengths of edges}}{\text{number of edges}} \times \text{area of right section}$$

## CYLINDERS

A *cylinder of revolution*, which is the common form, is a solid that may be considered to have been generated by the revolution of a parallelogram about one edge.

$$S = 2\pi r h = \pi d h = Ph. \quad T = S + 2 \times \text{area of base} = S + 2A$$

$$A = \pi r^2 = \frac{\pi}{4} d^2 = \frac{1}{2} Pr = \frac{Pd}{4}$$

$V$ = area of base × perpendicular height $h$, or $V = Ah = \pi r^2 h$ $= \frac{\pi}{4} d^2 h = \frac{P^2}{4\pi} h = 3.1416 r^2 h = .7854 d^2 h = .0796 P^2 h.$

**Frustum of Cylinder.—**

Let $h$ = one-half sum of greatest and least heights

$$S = Ph = \pi d h$$

$$T = \pi d h + \frac{\pi}{4} d^2 + \text{area of elliptic top.} \quad V = Ah = \frac{\pi}{4} d^2 h$$

**Hollow Cylinders.—**The volumes of hollow cylinders, as the quantity of metal in a pipe, Fig., is found by multiplying the area of the ring made by its cross-section by the length of the cylinder. The area of the cross-section may be determined by the formulas given under Rings.
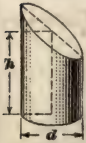
## THE PYRAMID

A *pyramid*, Fig. 1, is a solid having for its base a plane figure of any number of sides, and for its sides, plane triangles terminating in a common point, called the *apex*.

$S = \frac{1}{2} Pl$, and $T = \frac{1}{2} Pl + A$, in which $P$ is the perimeter of the base, $A$ its area, and $l$ the slant height. Note that $l$, Fig. 1, is not measured on an edge, but from the center of one side of the base to the vertex. These formulas apply to the right regular pyramid in which the base is a regular polygon and the axis is perpendicular thereto. If the base is not a regular polygon and the pyramid is oblique (the axis is inclined to the base), each triangular side has different dimensions. To find $S$, the sum of the areas of the different triangular sides must be taken, to which must be added the area of the irregular base if $T$ is desired.

FIG. 1

The volume of any pyramid is equal to the area of the base multiplied by one-third the altitude, or $V = \frac{1}{3} Ah$.

**Frustum of Regular Pyramid.—**If $A$ and $a$ and $P$ and $p$ are the area and perimeter of the lower and upper bases, respectively, of a frustum of a regular pyramid, Fig. 2,

$$S = \frac{1}{2} l (P + p) \text{ and } T = \frac{1}{2} l (P + p) + A + a$$

FIG. 2

If the cutting plane is inclined to the base or the frustum is that of an irregular or oblique pyramid, the surface area $S$ is the sum of the areas of the trapeziums forming the sides, to which must be added the areas of the irregular bases if $T$ is wanted.

The volume of the frustum of any pyramid is $V = \frac{1}{3} h (A + a + \sqrt{Aa})$.

## THE WEDGE

The *wedge*, whether with a blunt edge as $ABCD\text{-}abcd$, or with a sharp edge as $ABCD\text{-}a'b'$, is a special form of the trapezoid, as the ends $abcd$, or $a'b'$ are parallel to the base $ABCD$. In the blunt wedge, $V = \frac{1}{2} fh(m+n)$ and in the sharp wedge $V = \frac{1}{2} fnh$, as $m = 0$. In the latter case, $h = Bb'$.
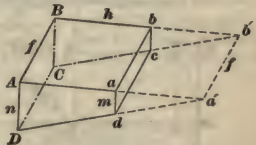
## THE CONE

A *cone*, Fig. 1, is a solid generated by a straight line, one end of which passes through a fixed point, called the *apex* while the other end is free to move around the perimeter of a closed curve, known as the *base*. Cones are regular when the base is a circle and are right or oblique according as the axis is at right angles to or inclined to the base.

FIG. 1

The common form of cone, as shown in accompanying figure, is the *right regular cone of revolution*, which may be
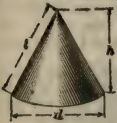
considered to have been generated by the revolution of a right-angled triangle about one side. Its base is a circle and its axis is perpendicular to the center of the base.



FIG. 2

The surface of the cone is $S = \pi r l = \frac{1}{2}\pi dl = 3.1416 rl = 1.5708 dl$.

The total surface, $T = \pi r l + \pi r^2 = \pi r \sqrt{r^2 + h^2} + \pi r^2 = \pi r (l + r)$ $= \pi r (\sqrt{r^2 + h^2} + r)$

The volume,

$$V = \frac{1}{3}A h = \frac{\pi}{12}d^2 h = \frac{P^2}{12\pi}h = \frac{\pi}{3}r^2 h = .2618 d^2 h = .0265 P^2 h = 1.0472 r^2 h.$$

**Frustum of Cone.**—When, as in Fig. 2, the cutting plane is parallel to the base of the cone

$$S = \frac{1}{2}l(P + p) = \frac{1}{2}\pi l(D + d), \text{ and } T = \frac{1}{2}\pi[l(D + d) + \frac{\pi}{4}\frac{1}{2}(D^2 + d^2)]$$

$$V = \frac{1}{4}\pi(D^2 + d^2 + Dd)\frac{h}{3} = .2618 h(D^2 + d^2 + Dd)$$

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# PLANE TRIGONOMETRY

## DEFINITIONS

*Plane trigonometry* treats of the solution of plane triangles. In every triangle there are six parts—three sides and three angles. These parts are so related that when three of the parts are given, one being a side, the other parts may be found.

An angle is measured by the arc included between its sides, the center of the circumference being at the vertex of the angle.

For measuring angles, the circumference is divided into 360 equal parts, called *degrees;* each degree is divided into 60 equal parts called *minutes;* and each minute is divided into 60 equal parts called *seconds.* Divisions smaller than a second are expressed in decimal parts of that unit; thus, 24.56″.

A *quadrant* is one-fourth the circumference of a circle, or 90°.

The *complement* of an arc is 90° minus the arc; the arc *DC*, Fig. 1, is the complement of the arc *BC*, and the angle *DOC* is the complement of the angle *BOC*.

The *supplement* of an arc is 180° minus the arc; the arc *AE* is the supplement of the arc *BDE*, and the angle *AOE* is the supplement of the angle *BOE*.



FIG. 1

In trigonometry, instead of comparing the angles of triangles or the arcs that measure them, the trigonometric functions, known as the sine, cosine, tangent, cotangent, secant, and cosecant, are compared.

The *sine* of an arc is the perpendicular let fall from one extremity of the arc on the diameter that passes through the other extremity. Thus, *CD*, Fig. 2, is the sine of the arc *AC*.



FIG. 2

The *cosine* of an arc is the sine of its complement; or it is the distance from the foot of the sine to the center of the circle. Thus, *CE* or *OD* equals the cosine of arc *AC*.

The *tangent* of an arc is a line that is perpendicular to the radius at one extremity of an arc and limited by a line passing through the center of the circle and the other extremity. Thus, *AT* is the tangent of *AC*.

The *cotangent* of an arc is equal to the tangent of the complement of the arc. Thus, *BT′* is the cotangent of *AC*.

The *secant* of an arc is a line drawn from the center of the circle through one extremity of the arc, and limited by a tangent at the other extremity. Thus, *OT* is the secant of *AC*.
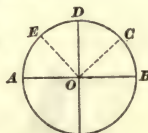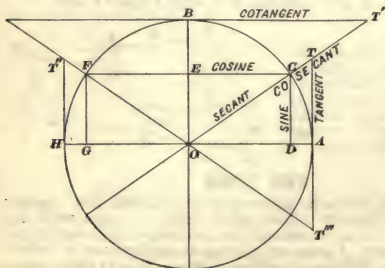
The *cosecant* of an arc is the secant of the complement of the arc. Thus, $OT'$ is the cosecant of $AC$.

The *versed sine* of an arc is that part of the diameter included between the extremity of the arc and the foot of the sine. $DA$ is the versed sine of $AC$.

The *coversed sine* is the versed sine of the complement of the arc. Thus, $BE$ is the coversed sine of $AC$.

## FUNDAMENTAL RELATIONS

If $x$ is any angle, the fundamental relations that its trigonometric functions sustain to one another are:

$$\sin^2 x + \cos^2 x = 1$$
$$\tan x = \frac{\sin x}{\cos x}$$
$$\cot x = \frac{\cos x}{\sin x}$$
$$\cot x = \frac{1}{\tan x}$$

$$\sec x = \frac{1}{\cos x}$$
$$\operatorname{cosec} x = \frac{1}{\sin x}$$
$$\sec^2 x = 1 + \tan^2 x$$
$$\operatorname{cosec}^2 x = 1 + \cot^2 x$$
$$\operatorname{vers} x = 1 - \cos x$$
$$\operatorname{covers} x = 1 - \sin x$$

The tangent and cotangent of the same angles are reciprocals of each other; so also are the secant and cosine; and the cosecant and sine.

The value of the sine and cosine cannot be greater than 1. Tangents and cotangents may have any value from 0 to ∞. Secants and cosecants may have any value between 1 and ∞. Versed sines and coversed sines may have any value between 0 and 2.

## SIGNS OF TRIGONOMETRIC FUNCTIONS

The various trigonometric functions have signs; that is, they are + or −, depending on the magnitude of the angle.

Sines and cosecants of angles between 0° and 180° are +; and those of angles between 180° and 360° are −.

Cosines and secants of angles between 0° and 90° and between 270° and 360° are +; and those of angles between 90° and 270° are −.

Tangents and cotangents of angles between 0° and 90° and between 180° and 270° are +; and those of angles between 90° and 180° and between 270° and 360° are −.

Versed sines and coversed sines are always + regardless of the magnitude of the angle.

## FUNCTIONS OF ANGLES BETWEEN 90° AND 180°

In the solution of obtuse-angled triangles, it is commonly necessary to have to find the functions of an angle of more than 90°. This may readily be done if it is recalled that the functions of an angle of more than 90°. This may readily be function of its supplement. Thus

| | |
|---|---|
| sine 110° = sine of 70° | cosine 110° = cosine 70° |
| tangent 110° = tangent of 70° | cotangent 110° = cotangent of 70° |
| secant 110° = secant of 70° | cosecant 110° = cosecant of 70° |

Thus, if it is desired to find the sine of an angle of 120° 30′, look for the sine of 180° − 120° 30′, or 59° 30′, and similarly for the other functions. In dealing with angles of more than 90° attention should be paid to the sign of the function.

## FUNCTIONS OF 90° + A

| | |
|---|---|
| $\sin (90° + A) = \cos A$ | $\cot (90° + A) = -\tan A$ |
| $\tan (90° + A) = -\cot A$ | $\sec (90° + A) = -\csc A$ |
| $\cos (90° + A) = -\sin A$ | $\csc (90° + A) = \sec A$ |

## FUNCTIONS OF 180° − A AND OF 180° + A

| | |
|---|---|
| $\sin (180° - A) = \sin A$ | $\sin (180° + A) = -\sin A$ |
| $\tan (180° - A) = -\tan A$ | $\tan (180° + A) = \tan A$ |
| $\cos (180° - A) = -\cos A$ | $\cos (180° + A) = -\cos A$ |
| $\cot (180° - A) = -\cot A$ | $\cot (180° + A) = \cot A$ |
| $\sec (180° - A) = -\sec A$ | $\sec (180° + A) = -\sec A$ |
| $\csc (180° - A) = \csc A$ | $\csc (180° + A) = -\csc A$ |

## FUNCTIONS OF (A+B) AND OF (A−B)

$$\sin (A+B) = \sin A \cos B + \cos A \sin B$$
$$\sin (A-B) = \sin A \cos B - \cos A \sin B$$
$$\cos (A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos (A-B) = \cos A \cos B + \sin A \sin B$$
$$\tan (A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$\tan (A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## FUNCTIONS OF 2A AND OF ½A

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin \tfrac{1}{2}A = \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2}A = \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2}A = \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\tan \tfrac{1}{2}A = \frac{1 - \cos A}{\sin A}$$

## SUMS AND DIFFERENCES OF FUNCTIONS

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B)$$
$$\sin A - \sin B = 2 \sin \tfrac{1}{2}(A-B) \cos \tfrac{1}{2}(A+B)$$
$$\cos A + \cos B = 2 \cos \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B)$$
$$\cos A - \cos B = 2 \sin \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(B-A)$$
$$\tan A + \tan B = \frac{\sin (A+B)}{\cos A \cos B}$$
$$\tan A - \tan B = \frac{\sin (A-B)}{\cos A \cos B}$$
$$\sin^2 A - \sin^2 B = \sin (A+B) \sin (A-B)$$
$$\cos^2 A - \cos^2 B = \sin (A+B) \sin (B-A)$$
$$\cos^2 A - \sin^2 B = \cos (A+B) \cos (A-B)$$

## SOLUTION OF RIGHT-ANGLED TRIANGLES
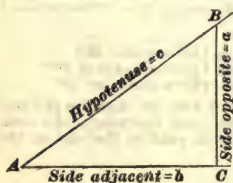


There are six parts in every triangle and if three of them are known the other three may be determined by calculation, provided one of the three known parts is a side. In the case of a right-angled triangle, one of the angles is 90°, but two other parts, one of them a side, are necessary for its solution. Three angles do not determine a triangle, because all triangles whose sides are parallel each to have the same angles but sides of different length.

In right-angled triangles, the following relations between the angles and sides prevail:

$$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{a}{c} \qquad \cot A = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{b}{a}$$

$$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{b}{c} \qquad \sec A = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{b}$$

$$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{a}{b} \qquad \text{cosec } A = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{a}$$

$$\text{vers } A = 1 - \frac{b}{c} \qquad\qquad \text{covers } A = 1 - \frac{a}{c}$$

## RELATIONS BETWEEN ANGLES AND SIDES OF RIGHT-ANGLED TRIANGLES

| Given | Required | Formula | Given | Required | Formula |
|-------|----------|---------|-------|----------|---------|
| $a, A$ | $B, b, c$ | $\begin{cases} B = 90° - A \\ b = a \cot A \\ c = \dfrac{a}{\sin A} = a \csc A \end{cases}$ | | | $\begin{cases} \tan A = \dfrac{a}{b} \\ \tan B = \dfrac{b}{a}, \\ \text{or } B = 90° - A \\ c = \sqrt{a^2 + b^2} \\ c = \dfrac{a}{\sin A} = a \csc A \end{cases}$ |
| $a, B$ | $A, b, c$ | $\begin{cases} A = 90° - B \\ b = a \tan B \\ c = \dfrac{a}{\cos B} = a \sec B \end{cases}$ | $a, b$ | $A, B, c$ | |
| $c, A$ | $B, a, b$ | $\begin{cases} B = 90° - A \\ a = c \sin A \\ b = c \cos A \end{cases}$ | | | $\begin{cases} \sin A = \dfrac{a}{c} \\ \cos B = \dfrac{a}{c} \\ \text{or } B = 90° - A \\ b = \sqrt{c^2 - a^2} \\ \quad = \sqrt{(c+a)(c-a)} \\ b = a \cot A \end{cases}$ |
| | | | $a, c$ | $A, B, b$ | |

**Area.**—The area of a right-angled triangle is equal to one-half the product of the base by the altitude, or area = $\frac{1}{2} ab$.

The area is also equal to one-half the product of any two sides into the sine of the angle between them. Thus, if the angle $A$ and the sides $c$ and $b$ are given, area = $\frac{1}{2} cb \sin A$.

## SOLUTION OF OBLIQUE-ANGLED TRIANGLES

The following relations between the sides and angles apply to all triangles, but are of particular service in solving those with oblique angles:
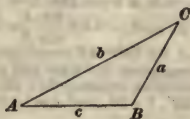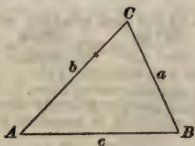


1. *The sides of any plane triangle are proportional to the sines of the angles opposite.*

$$a : b = \sin A : \sin B$$
$$a : c = \sin A : \sin C$$
$$b : c = \sin B : \sin C$$

2. *Any side of a plane triangle equals the sum of the products of each of the other sides into the cosine of the angle that it makes with the first side.*

$$a = b \cos C + c \cos B$$
$$b = a \cos C + c \cos A$$
$$c = a \cos B + b \cos A$$

3. *The square of any side of a plane triangle equals the sum of the squares of the other two, minus twice their product into the cosine of their included angle.*

$$a^2 = c^2 + b^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

4. *The sum of any two sides of a plane triangle is to their difference, as the tangent of one-half the sum of the angles opposite them is to the tangent of one-half their difference.*

$$a + b : a - b = \tan \tfrac{1}{2}(A + B) : \tan \tfrac{1}{2}(A - B)$$
$$a + c : a - c = \tan \tfrac{1}{2}(A + C) : \tan \tfrac{1}{2}(A - C)$$
$$b + c : b - c = \tan \tfrac{1}{2}(B + C) : \tan \tfrac{1}{2}(B - C)$$

5. *The cosine of any angle of a plane triangle is equal to the sum of the squares of the adjacent sides minus the square of the side opposite, the whole divided by twice the product of the adjacent sides.*

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

6. *The area of any plane triangle is equal to one-half the product of any two sides into the sine of the included angle.*

$$\text{Area} = \tfrac{1}{2}cb \sin A = \tfrac{1}{2}ac \sin B = \tfrac{1}{2}ab \sin C$$

7. *The area of any plane triangle is equal to the square root of the continued product of one-half its perimeter into one-half its perimeter minus each side separately.*

If the perimeter, $a + b + c = p$, then,

$$\text{Area} = \sqrt{\tfrac{1}{2}p(\tfrac{1}{2}p - a)(\tfrac{1}{2}p - b)(\tfrac{1}{2}p - c)}$$

## PRACTICAL EXAMPLES

1. *Having given two sides and the included angle, to find the other side and remaining angles.*—Let $b = 30$, $c = 20$, and $A = 38° 20'$; required $a$, $B$, and $C$. Find the angle $B$ from the third formula of the fourth relation, which may be transposed to $\tan \frac{1}{2}(B-C) = \tan \frac{1}{2}(B+C) \times \frac{b-c}{b+c}$. In this $B + C = 180° - A = 180° - 38° 20' = 141° 40'$, and $\frac{1}{2}(B+C) = 70° 50'$; $B - C$ is unknown; $b + c = 30 + 20 = 50$, and $b - c = 30 - 20 = 10$. By substitution, $\tan \frac{1}{2}(B-C) = 2.87700 \times \frac{10}{50} = .57540$. From this $\frac{1}{2}(B-C) = 29° 55'$ (very nearly), and $B - C = 29° 55' \times 2 = 59° 50'$.

$$B + C = 141° 40'$$
$$B - C = 59° 50'$$

By addition, $2B = 201° 30'$
By division, $B = 100° 45'$

From this $A + B = 38° 20' + 100° 45' = 139° 5'$, and $C = 180° - (A+B) = 180° - 139° 5' = 40° 55'$.

Find the side $a$ from the first formula of the first relation, which may be transposed to read, $a = b \times \frac{\sin A}{\sin B} = 30 \times \frac{\sin 38° 20'}{\sin 100° 45'} = 30 \times \frac{.62024}{.98245} = 18.94$. Remember that $\sin 100° 45' = \sin (180° - 100° 45') = \sin 79° 15'$.

2. *Having given two sides and the angle opposite one of them, to find the other side and remaining angles.*—Let the given parts of the triangle shown in Fig. 1, be $A = 38° 20'$, $b = 30$, and $a = 18.94$; from which it is required to find $c$, $B$, and $C$.

Find the angle $B$ from the first formula of the first relation, which transposed is $\sin B = \frac{b}{a} \times \sin A = \frac{30.00}{18.94} \times \sin 38° 20' = \frac{30.00}{18.94} \times .62024 = .98245$; whence $B = 79° 15'$ or $100° 45'$. Unless the shape of the triangle is actually known it is impossible to tell which of these values of $B$ should be taken. In fact, both of them are correct, as a study of the accompanying figure will show. As only $A$, $b$, and $a$, are fixed, it is apparent that $a$ may occupy either position $CB$ or $CB'$ and yet have the same value, 18.94. Such being the case, the angle at $B$ may be (for the position $CB = a' = 18.94$) $CBA = 79° 15'$, or (for the position $CB' = a = 18.94$) $CB'A = 100° 45'$. Hence, angle $C = 180° - (A+B) = 180° - (38° 20' + 79° 15') = 180° - 117° 35' = 62° 25'$, or $C = 180° - (38° 20' + 100° 45') = 180° - 139° 5' = 40° 55'$.


Fig. 1

The side $c$ may now be found from the second formula of the first relation, which may be transposed to read $c = a \frac{\sin C}{\sin A}$, and taking the two values of the angle $C$, $c = 18.94 \times \frac{\sin 62° 25'}{\sin 38° 20'} = 18.94 \times \frac{.88634}{.62024} = 27.07$
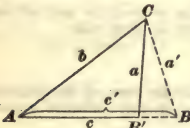
or
$$c = 18.94 \times \frac{\sin 40° \, 55'}{\sin 38° \, 20'} = 18.94 \times \frac{.65496}{.62024} = 20$$

Thus two solutions of this triangle are possible; in the first case, $B = 79° \, 15'$, $C = 62° \, 25'$, $c = 27.07$, and in the second case, $B = 100° \, 45'$, $C = 40° \, 55'$, $c = 20$.

3. *Having given two angles and any side, to find the other angle and the other two sides.*—Let $A = 38° \, 20'$, $B = 100° \, 45'$, $a = 18.94$; to find the remaining angle $C$ and the other sides $b$ and $c$.

Find $C$ from the relation $C = 180° - (A + B) = 180° - (38° \, 20' + 100° \, 45')$ $= 180° - 139° \, 5' = 40° \, 55'$.

The sides may now be found from the first and second formulas given in the first relation after these have been transposed.
$$b = a \frac{\sin B}{\sin A} = 18.94 \times \frac{\sin 100° \, 45'}{\sin 38° \, 20'} = 18.94 \times \frac{.98245}{.62024} = 30$$
$$c = a \frac{\sin C}{\sin A} = 18.94 \times \frac{\sin 40° \, 55'}{\sin 38° \, 20'} = 18.94 \times \frac{.65496}{.62024} = 20$$

4. *Having given the three sides to find the three angles.*—Let $a = 18.94$, $b = 30$, and $c = 20$; required the angles $A$, $B$, and $C$. Using the formulas given in the fifth relation, then $C = 180° - (A + B)$.
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{30^2 + 20^2 - 18.94^2}{2 \times 30 \times 20} = .78440, \text{ and } A = 38° \, 20'$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{18.94^2 + 20^2 - 30^2}{2 \times 18.94 \times 20} = -.18648, \text{ and } B = 100° \, 45'$$
$$C = 180° - (A + B) = 180° - (38° \, 20' + 100° \, 45') = 40° \, 55'$$

Note that the angle corresponding to the cosine .18648 is either $79° \, 15'$ or $100° \, 45'$. By referring to the section Signs of Trigonometric Functions, it will be seen that when the cosine is minus, as it is in this case $(-.18648)$, the angle is between $90°$ and $270°$; hence, the value $B = 100° \, 45'$ is taken.

This example is readily solved by the solution of two right-angled triangles, as shown in Fig. 2. Let fall a perpendicular $CD$ from the opposite vertex $C$ upon the longest side $AB$ dividing it into two segments $AD = m$ and $DB = n$. From geometry, $m + n : b + a = b - a$
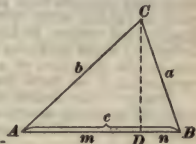

Fig. 2

$: m - n$, and as $m + n = c$, $m - n = \frac{(b + a)(b - a)}{c}$. Combining the value of $m - n$ thus obtained with that of $m + n = c$, the values of $m$ and $n$ may be found. In the right-angled triangles $ACD$ and $BCD$, $b$ and $m$ and $a$ and $n$, respectively, are given, from which the angles $A$ and $B$ may be calculated; angle $C$ found by subtracting the sum of angles $A$ and $B$ from $180°$.

Using the values for the sides, $a = 18.94$, $b = 30$, and $c = 27.066$,
$$m - n = \frac{(30 + 18.94) \times (30 - 18.94)}{27.066} = \frac{48.94 \times 11.06}{27.066} = 19.998.$$

Then

|  |  |  |  |
|---|---|---|---|
| | $m + n = 27.066$ | | $m + n = 27.066$ |
| | $m - n = 19.998$ | | $m - n = 19.998$ |
| By addition | $2m = 47.064$ | By subtraction | $2n = \; 7.068$ |
| | $m = 23.532$ | | $n = \; 3.534$ |

In the triangle $ACD$, $\cos A = \frac{m}{b} = \frac{23.532}{30} = .78440$. Whence $A = 38° \, 20'$.

In the triangle $DCB$, $\cos B = \frac{n}{a} = \frac{3.534}{18.94} = .18659$. Whence $B = 79° \, 15'$.
$$C = 180° - (A + B) = 180° - 117° \, 35' = 62° \, 25'$$

Tables of natural and logarithmic trigonometric functions will be found at the end of the volume; each table is preceded by the necessary explanations for its use.

# SURVEYING

## THE COMPASS

### GENERAL DESCRIPTION

*Surveying* is an extension of mensuration, and, as ordinarily practiced, may be divided into surface work, or *ordinary surveying*, and underground work, or *mine surveying*. With slight modifications, the instruments employed in both are the same, and consist of a compass—if the work is of little importance, and accuracy is not required—a transit, level, transit and level rods, steel tape or chain, and measuring pins, and sometimes certain accessory instruments, as clinometers or slope levels, dipping needles, etc., as will be described later.

The *compass* may be either a pocket compass, or a surveyor's compass, and may be used while held in the hand, or upon a tripod. The Jacob's staff, convenient for use on the surface, is useless in the mine. As the compass cannot be sighted accurately on an object, cannot be read closer than 30′, except by guess, and may be deflected from its true course as much as 2° or 3° by the iron in the rails or water pipes or by electric currents, it is obvious that bearings and angles determined through its use cannot be relied on as being within 15′ of the truth and they may be very much more in error. As present day surveying requires that any angle be known within 1 min. and in special cases, such as tunnel work, within 30″ or even 20″ or 15″, the compass is now no longer used except, in emergencies, when a transit is not available. However, in driving room necks far enough for the permanent sights, in obtaining a rough idea of the direction of a heading, and, on the surface, in connection with the rerunning of old land lines, the compass has its uses.

Owing to the length of time taken by the needle to settle so that it can be read, an accurate transit survey can commonly be made in less time than an inaccurate one with the compass.

### COMPASS ADJUSTMENTS

When adjusting the *levels*, first bring the bubbles into the center by the pressure of the hand on different parts of the plate, and then turn the compass half way around. Should the bubbles run to the ends of the tubes, those ends are the higher; these should then be lowered by tightening the screws immediately under, and loosening those under the lower ends until, by estimation, the error is half removed. The plate should again be leveled and the first operation repeated until the bubbles will remain in the center during an entire revolution of the compass.

The *sights* may next be tested by observing, through the slits, a fine hair or thread, made exactly vertical by a plumb. Should the hair appear on one side of the slit, the sight must be adjusted by filing off its under surface on the side that seems the higher.

The *needle* is adjusted in the following manner: Having the eye nearly in the same plane with the graduated rim of the compass circle, with a small splinter of wood, or a slender iron wire, bring one end of the needle in line with any prominent division of the circle, as the 0 or 90° mark, and notice if the other end corresponds with the degree on the opposite side. If it does, the needle is said to cut opposite degrees; if not, bend the center pin by applying a small brass wrench, furnished with most compasses, about ⅛ in. below the point of the pin, until the ends of the needle are brought into line with the opposite degrees. Then, holding the needle in the same position, turn the compass half way around, and note whether it now cuts opposite degrees; if not, correct half the error by bending the needle, and the remainder by bending the center pin. The operation must be repeated until perfect reversion is secured in the first position. This being obtained, it may be tried on another quarter of the circle; if any error is there manifested, the correction must be made in the center pin only, the needle being already straightened by the previous operation. When again made to cut, it should be tried on the other quarters of the circle, and corrections made in the same manner until the error is entirely removed, and the needle will reverse in every point of the divided circle.

## USING THE COMPASS

When using the compass, the surveyor should keep the south end toward his person, and read the bearings from the north end of the needle. In the surveyor's compass the position of the E and W letters on the face of the compass are reversed from their natural position, in order that the direction of the sight may be correctly read.

The compass circle being graduated to $\frac{1}{2}°$, a little practice will enable the surveyor to read the bearings to quarters—estimating with his eye the space bisected by the point of the needle.

The compass is divided into quadrants, and 0 is placed at the north and south ends; 90° is placed at the E and W marks, and the graduations run right and left from the 0 to 90°. When reading the bearing, the surveyor will notice that if the sights are pointed in a NW direction, the north end of the needle, which always points approximately north, is to the right of the front sight or front end of the telescope, and, as the number of degrees is read from it, the letters marking the cardinal points of the compass read correctly. If the E, or east, mark were on the right side of the circle, a NW course would read NE. This same remark applies to all four quadrants. The compass should always be in a level position.

If all the corners of a field can be seen from a central point, the survey can be made by setting up at that point, and with one corner as a backsight, taking all the other corners as foresights, and by measuring from this point to all of the corners; or the compass can be set up at any corner and a line of survey run around the field. This latter method is called *meandering*. Both methods will give the same result when plotted; but the first is much quicker, as the boundaries of a tract are frequently overgrown with bushes that must be cleared to allow a sight; while a central point can frequently be found that will allow a free sight to all the corners, and the distance can be measured by tape, or stadia. As the central point is nearer the corners than they are to one another, a shorter distance must be chained or cut in the case of a central set-up.

## MAGNETIC VARIATION

*Magnetic declination*, or *variation*, of the needle is the angle made by the magnetic meridian with the true meridian or true north and south line. It is east or west according as the north end of the needle lies east or west of the true meridian. It is not constant, but changes from year to year, and, for this reason, in rerunning the lines of a tract of land, from field notes of some years' standing, the surveyor makes an allowance in the bearing of every line by means of a vernier.

The declination, where a knowledge of it is necessary, should always be determined for the particular place and at the particular time where and when it is needed. Quite a number of the States in cooperation with the United States Coast and Geodetic Survey have established a true meridian by astronomical observations at each county seat. Information as to the location, etc., of the monuments marking the meridian may be obtained from the county surveyor, the recorder of deeds, or some one else in authority at the county court house. However, the variation thus obtained is only available for use a comparatively short distance either east or west of the county seat (assuming that the highest accuracy is desired), because on the average, there is in the United States, a change in the value of the declination of 1′ per mi. in the foregoing directions. From this, it is apparent that the declination at a place 30 mi. east or west of the county seat will probably vary 30′ from that at the monuments referred to. This difference of 30′ is within the limits between which the compass is ordinarily read. In proceeding north or south from the county seat, the change in declination is very much less than in an east or west direction. If the declination cannot be determined, a note should be made of the date of the survey, with a statement to the effect that the bearings are referred to the magnetic meridian, and these notes should appear on the map and should be incorporated in the deed if the survey was made preliminary to a transfer of property.

The United States Coast and Geodetic Survey, Washington, District of Columbia, issues from time to time tables and charts showing the declination at many points in the United States and outlying possessions, together with formulas by means of which the declination may be calculated with a high degree of accuracy at future times. These may be obtained from the Superintendent of the Survey.

**Reading the Vernier.**—The compass vernier, shown in the accompanying illustration, is usually so graduated that 30 spaces on it equal 31 on the limb of the instrument and, commonly, there are 15 spaces on each side of the 0 mark. It is read as follows: Note the degrees and half degrees on the limb of the instrument. If the space passed beyond the degree or half-degree mark by the zero mark on the vernier is less than one-half the space of $\frac{1}{2}°$ on the limb, the number of minutes is, of course, less than 15, and must be read from the lower row of figures. If the space passed is greater than one-half the spacing on the limb, the upper row of figures must be read. The line on the vernier that exactly coincides with a line on the limb is the mark that denotes the number of minutes. If the index is moved to the right, the minutes are read from the left half of the vernier; if moved to the left, they are read from the right side of the vernier.
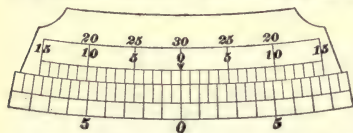
**Turning Off the Variation.**—Moving the vernier to either side, and with it, of course, the compass circle attached, set the compass to any variation by placing the instrument on some well-defined line of the old survey, and by turning the tangent screw (slow-motion screw) until the needle of the compass indicates the same bearing as that given in the old field notes of the original survey. Then screw up the clamping nut underneath the vernier and run all the other lines from the old field notes without further alteration. The reading of the vernier on the limb gives the amount of variation since the original survey was made.

## FIELD NOTES FOR AN OUTSIDE COMPASS SURVEY

Call place of beginning Station 1.

| Stations | Bearings | Distances |
|---|---|---|
| 1–2 | N 35° E | 270.0 |

At 1+ 37 ft. crossed small stream 3 ft. wide.
At 1+116 ft. = first side of road.
At 1+131 ft. = second side of road.
At 1+137 ft. = blazed and painted pine tree, 3 ft. left, marked for a go-by.
Station 2 is a stake at foot of white-oak tree, blazed and painted on four sides for corner.

| | | |
|---|---|---|
| 2–3 | N 83½° E | 129.0 |

Station 3 is a stake-and-stones corner.

| | | |
|---|---|---|
| 3–4 | S 57° E | 222.0 |

3+ 64 ft. = center of small stream 2 ft. wide.
3+196 ft. = white oak go-by, 2 ft. right.
Station 4, cut stone corner.

| | | |
|---|---|---|
| 4–5 | S 34½° W | 355.0 |

4+174 ft. = ledge of sandstone 10 ft. thick, dipping 27° south.

| | | |
|---|---|---|
| 5–1 | N 56½° W | 323.0 |

5+274 ft. = ledge of sandstone 10 ft. thick, dipping 25° south (evidently continuation of same ledge as at 4+174).
Station 1 = place of beginning.

# THE TRANSIT

## GENERAL DESCRIPTION

The *transit* is the only instrument that should be used for measuring angles in any survey where accuracy is desired. The advantages of a transit over a vernier compass are mainly due to the use of a telescope. By its use, angles can be measured either vertically or horizontally, and, as the vernier is used throughout, extreme accuracy is secured.

Fig. 1 shows the interior construction of the sockets of a transit having two verniers to the limb, the manner in which it is detached from its spindle, and how it can be taken apart when desired. The limb *b* is attached to the main socket *c*, which is carefully fitted to the conical spindle *h*, and held in place by the spring catch *s*.

The upper plate *a*, carrying the compass circle, standards, etc., is fastened to the flanges of the socket *k*, which is fitted to the upper conical surface of the

main socket $c$. The weight of all the parts is supported on the small bearings of the end of the socket, as shown, so as to make as little friction as possible where such parts are being turned as a whole.

A small conical center, in which a strong screw is inserted from below, is brought down firmly on the upper end of the main socket $c$, thus holding the two plates of the instrument securely together, and, at the same time, allowing them to move freely around each other. The steel center pin on which the needle rests is held by the small disk fastened to the upper plate by two small screws above the conical center. The clamp to limb $df$, with clamp screw, is attached to the main socket. The instrument
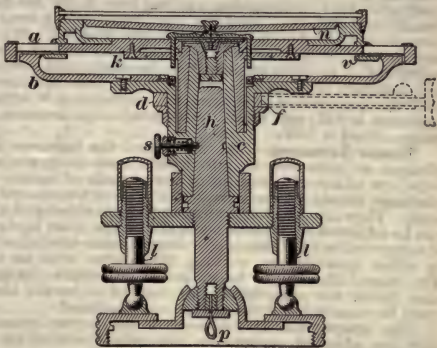


Fig. 1

is leveled by means of the leveling screws $l$ and placed exactly over a point by means of the shifting center. The plummet is attached to the loop $p$.

**Transit Verniers.**—In transits, the limb or plate has two sets of concentric graduations, as shown in part in Fig. 2. The style of marking these graduations may be varied to suit the ideas of the surveyor, but the arrangement shown is a common and a good one. The same $0°$ point is used for both sets of the graduations and is placed near or under the eye end of the telescope. One set of graduations is continuous from this $0°$ to $360°$ toward the left. The other set begins at the same $0°$ point and increases to $90°$ at the left, decreases to $0°$ directly opposite the starting point, increases to $90°$ at the right of this point, and decreases to $0°$ at the starting point. The first described set of graduations is numbered continuously every $10°$ from $0°$ to $360°$. The inner row is numbered, in advancing order, every $10°$ from $0°$ to $90°$; thence it decreases by $10°$ to the $0°$ point; then it again increases by $10°$ to the $90°$ mark at the right; and then decreases by $10°$ to the starting point. This last set of graduations, known as *quadrant graduations*, is marked with an N at the $0°$ point, with E at the $90°$ point; and similarly with S and W at the second $0°$ and second $90°$ points, respectively. Further, as shown, at each marking, the letters N E, S E, S W, or N W are stamped on the plate at its proper quadrant. Thus the $0°$ of the continuous graduation is the same as the N on the quadrant graduation, the $90°$ of the continuous graduation is the same as the E of the quadrant, the $180°$ of the continuous graduation is the S of the quadrant, and the $270°$ of the continuous graduation is the W of the quadrant.
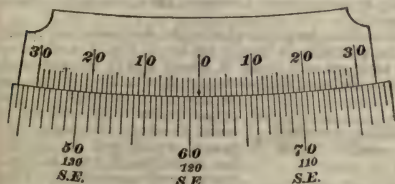
There are two transit verniers; one, known as *vernier A*, is placed as near as possible under the eye end of the telescope, and the other, known as *vernier B*, is placed directly opposite. These verniers are double, that is, they read both ways from the 0 mark so that angles deflected either to the right or left may be read. The vernier is commonly divided so that 30



Fig. 2

spaces on it are equal to 29 spaces on the limb of the transit. Each division of the vernier is, therefore, $\frac{1}{30}$, or, in other words, 1′ shorter than the $\frac{1}{2}°$ graduations on the limb. In Fig. 2, the reading is S $60°30'$ E (from the limb) $+13'$ (from the vernier) $=$ S $60°43'$ E. This is the quadrant reading from the inner row of graduations. The continuous vernier (outer row) reading is $119°$ (from the limb)

+17′ (from the vernier) = 119° 17′. It will be noted that the sum of the two readings is 60° 43′+119° 17′ = 180°. This summation proves and checks the readings. Had the quadrant reading been N 33° 18′ E, the continuous vernier should read 33° 18′. Had the quadrant reading been S 56° 39′ W, the continuous vernier should read 180°+56° 39′ = 236° 39′; and had the quadrant reading been N 76° 29′ W, the continuous vernier should read 360°−76° 19′ = 283° 31′. In other words, when the continuous vernier reads from 0° to 90°, the quadrant reading will be NE; when the continuous is between 90° and 180° the quadrant is SE; when the continuous is between 180° and 270° the quadrant is SW; and when the continuous is between 270° and 360° (or 0°), the quadrant is NW.

**Transit Telescope.**—The interior of the telescope is fitted up with a diaphragm or cross-wire ring to which cross-wires are attached. These cross-wires are either of platinum or as strands of spider web.· For inside work, platinum should be used, as spider web is translucent and cannot readily be seen. They are set at right angles to each other and are so arranged that one can be adjusted so as to be vertical and the other horizontal. This diaphragm is suspended in the telescope by four capstan-headed screws, and can be moved in either direction by working the screws with an ordinary adjusting pin. The transit should not be subjected to sudden changes in temperature that may break the cross-hairs. In case of a break, the cross-hair diaphragm must be removed and the broken wire replaced.

The intersection of the wires forms a very minute point, which, when they are adjusted, determines the optical axis of the telescope, and enables the surveyor to fix it upon an object with the greatest precision.

The imaginary line passing through the optical axis of the telescope is termed the *line of collimation*, and the operation of bringing the intersection of the wires into the optical axis is called the *adjustment of the line of collimation*.

All screws and movable parts should be covered, so that acid water and dust will be kept out. If this is not done, the mine work will destroy a transit. The vertical circle on the transit may be a full circle or a segment. The former is to be preferred, as it is always ready without intermediate clamp screws.

## TRANSIT ADJUSTMENTS

The use of a transit tends to disarrange some of its parts, which detracts from the accuracy of its work, but in no way injures the instrument itself. Correcting this disarrangement of parts is called *adjusting the transit*.

1. *To make the level tubes parallel to the vernier plate.*—Plant the feet of the tripod firmly in the ground. Turn the instrument until one of the levels is parallel to a pair of opposite leveling screws; the other level will be parallel to the other pair. Bring the bubble in each tube to the middle with the pair of leveling screws to which the tube is parallel. Next turn the vernier plate half way around; that is, revolve it through an angle of 180°. If the bubbles have remained in the middle of the tubes, the levels are in proper adjustment. If they have not remained so, but have moved toward either end, bring them half way back to the middle of the tubes by means of the capstan-headed screws attached to the tubes, and the rest of the way back by the leveling screws. Again turn the vernier plate through 180°, and if the bubbles do not remain at the middle of the tubes, repeat the correction. Sometimes the adjustment is made by one trial, but usually it is necessary to repeat the operation. Each level must be adjusted separately.

2. *To make the line of collimation perpendicular to the horizontal axis that supports the telescope.*—With the instrument firmly set at A, Fig. 1, and carefully leveled, sight to a pin or tack set at a point B, about 400 ft. distant, and on level, or nearly level, ground. Reverse the telescope; that is, turn it over on



its axis until it points in the opposite direction, and set a point at about the same distance, which will be at D, for example, if this adjustment needs correction. Un-
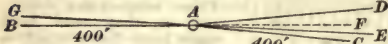
FIG. 1

clamp the vernier plate, and, without touching the telescope, revolve the instrument about its vertical axis sufficiently far to take another sight upon the point B. Then turn the telescope on its axis and locate a third point, as at C. Measure the distance CD, and at E, one-fourth of the distance from C to D, set the pin or tack. Move the cross-hairs, by means of the capstan-headed screws, until the vertical hair exactly covers the pin at E, being careful to move it in the opposite direction from that in which it appears it should be

moved. Having done this, and then having reversed the telescope, the line of sight will not be at the point $B$, but at $G$, a distance from $B$ equal to $CE$. Again sight to $B$, then reverse, and the pin will be at $F$ in the same straight line with $AB$. It may be necessary to repeat the operation to secure an exact adjustment.

3. *To make the horizontal axis of the telescope parallel to the vernier plate, so that the line of collimation will revolve in a vertical plane.*—Sight to some point $A$, Fig. 2, at the top of a building, so that the telescope will be elevated at a large angle. Depress the telescope and set a pin on the ground below at a point $B$. Loosen the clamp, turn over the telescope, and turn the plate around sufficiently far to take an approximately accurate sight upon the point $A$. Then clamp the instrument and again take an exact sight to the point $A$. Next depress the telescope, and set another pin on the ground, which will come at $C$. The distance $BC$ is double the error of adjustment. Correct the error by raising or lowering one end of the telescope axis by means of a small screw placed in the standard for that purpose. The amount the screw must be turned is determined only by repeated trials.



FIG. 2

4. *To make the axis of the attached level of the telescope parallel to the line of collimation.*—Drive two stakes at equal distances from the instrument and in exactly opposite directions. Level the plate carefully, and clamp the telescope in a horizontal position, or as nearly so as possible. Sight to a rod placed alternately upon each stake, and have the stakes driven down until the rod reading is the same on both stakes. When this condition is reached, the heads of the stakes are at the same level. Then move the instrument beyond one stake and set it up so that it will be in line with both stakes. Level the plate again and elevate or depress the telescope so that, when a sight is taken to the rod held on first one stake and then on the other, the reading will be alike on both. In this position, the line of collimation is level, and the bubble in the level attached to the telescope should stand in the center of the bubble tube. If it does not, bring it to the center by turning the nuts at the ends of the tube, being careful at the same time to keep the telescope in the position that gives equal rod readings on both stakes.

# CHAIN, STEEL TAPE, AND PINS

The *chain* is probably the earliest form of distance-measuring instrument. The original surveyor's or Gunters chain, was 66 ft. or 4 rd. in length and was composed of 100 li., each 7.92 in. long. This form of chain is no longer used, but is useful in preliminary work in locating old corners from descriptions in early deeds where distances are expressed in rods (poles, perches, or chains). The engineer's chain composed of 100 li. each 1 ft. long is also falling into disuse except for railroad work. Any chain is so liable to abrasion at the numerous joints and to bending, that distances measured with it cannot be relied on to be accurate within the limits demanded by modern engineering. A 100-ft. chain has 800 wearing surfaces, and should each one of these be worn but $\frac{1}{160}$ in. after several season's work, the chain will be 100 ft. 8 in. (100.67 ft.) in length, and each full 100 ft. measured with it will be recorded as but 99.34 ft. Similarly, bends or kinks shorten the chain so that the distances measured with it are recorded as being too great.

When used at all, the chain should be made of annealed steel wire, each link exactly 1 ft. in length. The links should be so made as to reduce the liability to kink to a minimum. All joints should be brazed, and handles at each end of **D** shape, or modifications of **D** shape, should be provided. These handles should be attached to short links at each end, and the combined length of each of these short links and one handle should be exactly 1 ft. The handles should be attached to the short link in such a manner that the chain may be slightly lengthened or shortened by screwing up a nut at the handle. It should be divided every 10 ft. with a brass tag, on which either the number of points represents the number of tens from the front end, or the number of tens may be designated by figures stamped on the tags.

When a chain is purchased, one that has been warranted as "Correct, U. S. Standard," should be selected, and, before using it, it should be stretched on a level surface, care being taken that it is straight, and no kinks in it, and the extremities marked by some permanent mark. These marks can be used in the future to test the chain. It should be tested frequently, and the length kept to the standard as marked when it was new.

Ordinarily, the chain should be held horizontally, and if either end is held above the ground, a plumb-bob and line should be used to mark the end of the chain on the ground. If used on a regular slope, the chain may be stretched along the ground, and, by having the amount of inclination, the horizontal and vertical distances may either be calculated or found in the Traverse Table.

The *steel tape*, which has superseded the chain, is simply a ribbon of steel not so high in carbon as to be brittle and liable to snap on a short bend, nor of so soft a steel that it will stretch when strongly pulled. Tapes are made of a standard or exact length at a given temperature, say 60° F., and under a certain tension, say, 15 lb. At higher temperatures or under greater pull the tape expands, and distances measured with it are less than the true ones. At lower temperatures or under less tension, the tape contracts and distances measured with it are too long. When a tape is hung unsupported between two points it forms a curve, called a *catenary*, and measures a greater distance between the points than if the tape formed a straight line. However, the apparent shortening of the distance between two points, owing to the tape being subject to more than the normal tension, may be offset by the lengthening of the distance due to sag in the unsupported tape. For every span, there is a corresponding tension where the errors balance, and this tension should be ascertained and used in practice. The errors due to expansion and contraction, arising from changes in temperature, cannot be compensated and must be corrected for in all accurate work. It should be noted that the temperature of the average mine is about 65° and being essentially the same as that at which the tape was graduated, no allowance for expansion or contraction is generally necessary in mine work.

Steel tapes are of two general kinds, and are commonly named from their length, as a 100-ft. tape, a 400-ft. tape, etc. The 50-ft. and 100-ft. tapes are about ⅜ in. wide, coil or wind up in a leather case or upon a small single-handed reel, and, for surveyor's use, are divided throughout their length into feet, tenths, and hundredths.

The steel tape, proper, is a narrow band of metal about one-third the width of the 100-ft. tape and considerably thicker, which is wound upon a wooden or iron reel like a spool. These tapes may be made in any length, but those 400 ft. long appear to be in most general use. The tape is commonly graduated every 5 ft. on brass sleeves soldered upon it. Distances are read to the nearest 5 ft. from the tape and intermediate distances measured with a pocket tape graduated in hundredths of a foot. Sometimes before the 0 mark there is an extra set of divisions into feet, the first foot being further divided into tenths. In this case, the scale for determining the hundredths need be very short, say a tenth or two in length. In use, a handle with a swivel joint is fixed in an eye at the 0 end and the tape unwound from the reel for the desired or required distance, but is not removed from the reel unless a distance equal to that of the tape is to be measured many times. In this case, a second handle may be fixed in the eye at the outer end of the tape.

In order to repair breaks that may occur in the field, clamps are made to hold the broken ends together. Brass sleeves that may be brazed around broken ends by the surveyor or by a competent gunsmith may be purchased. To keep a mark upon the tape for frequent reference, a *clip* (made by bending sharply upon itself a piece of steel ⅛ in.×3 in.) is slipped upon the tape, where it will remain unless subject to considerable force.

What are known as *metallic tapes*, are made in lengths of 25, 50, and 100 ft., and are graduated in hundredths of a foot for surveyors and into inches and eighths for mine foremen. These are similar to the 100-ft. steel tape but are made of linen, with threads of copper woven in to overcome the tendency to stretch. One of these tapes, 100 ft. long, is part of every surveying outfit and it should be used, wherever possible, to save the more costly 100-ft. steel tape. The metallic tape answers every purpose in measuring the dimensions of buildings that must appear on the map, the width of small streams, roads, etc., and the distance to a property corner, if it is a nearby tree or other not sharply defined point.

*Pins* are now but little used except in those classes of work where the use of the chain is permissible. Pins should be from 15 to 18 in. long, made of tempered-steel wire, and should be pointed at one end, and turned with a ring for a handle. When using a 50-ft. chain, a set of pins should consist of eleven, one of which should be distinguished by some peculiar mark. This should be the last pin stuck by the front chainman. When all eleven pins have been stuck, the front chainman calls "Out!" and the back chainman comes forwards and delivers him the ten pins that he has picked up, and he notes the out.

When giving the distance to the transitman, he counts his outs, each of which consists of 500 ft., and adds to their sum the number of fifties as denoted by the pins in his possession, and the odd number of feet and fractional parts of a foot from the last pin to the front end of the chain. Pins cannot be used in underground work as they cannot be stuck in the floor. If the distance to be measured is longer than the tape, a tack is placed in a tie between the stations and a measurement taken to it from each station with the dip of the sights. In outside work, a stake with a tack is used for the same purpose, and is lined in with the transit.

The *clinometer*, or *slope level*, is a valuable instrument for side-note work; but it is not accurate enough for a survey, and its place is taken by the vertical circle on the transit. There are two styles of clinometer, with a bubble and with a pendulum. The latter is the old-fashioned and more accurate German *gradbogen* that is used by some old corps. The bubble variety is much more easily rendered worthless by the breaking of the bubble tube, and, in general, is not so accurate as the other style, which consists of a semicircular protractor cut out of thin brass and furnished with hooks at each end, that it can be hung on a stretched string so that the string will pass through the 0° and 180° points. The dip is read by a pendulum swung from the center of the circle. If made sufficiently large, it will readily read to quarter degrees. By inclining the string parallel to the surface and hanging the clinometer, the dip will be obtained. A pocket instrument combining a compass and clinometer can be obtained from any dealer in surveying instruments.

# TRANSIT SURVEYING

## READING ANGLES

The angle read may be included or deflected. If the transit is set up at 0, and a backsight taken on *B* and a foresight taken on *C*, it will be noted that there are two angles made by the line *CO* with the line *BOA*, namely the included angle *BOC*, and the deflected angle *AOC*. It will be further noted that *AOC* = 180° − *BOC*, and vice versa.

**Reading the Included Angle.**—To read the included angle, set the zeros of the vernier and the limb as near together as possible by the eye, clamp the upper plate and bring the zeros into exact coincidence by means of the upper tangent screw. Set the vertical hair approximately upon the backsight, clamp the lower motion, and by the lower tangent screw, make the setting exact. Loosen the upper clamp, set approximately on the foresight, tighten the upper clamp, and by means of the upper tangent screw make the setting exactly. The vernier will read, say 45°, which is the included angle *BOC*.

**Reading the Deflected Angle.**—After arranging the verniers as just explained, to read the deflected angle, invert the telescope so that the level bubble is above, and set upon the backsight as before. Turn the telescope back to its normal position (this is called *plunging* the telescope) and sight to the foresight as explained. The vernier will read a right angle of 135°. As noted, the sum of the included and deflected angles must always be 180°, and in the case given 45° +135° = 180°.

## MAKING A SURVEY WITH A TRANSIT

**Meridians, or Base Lines.**—Every survey must start from some fixed point and the angles measured in the course thereof must be referred to some line as a base. The nature of the base line depends on the use to which the survey of the property is to be put. When surveying small tracts of land, such as city or town lots, the starting point may be a stake driven into the ground or a corner of the property itself and the base line may be one of the sides of the tract. When surveying farms, it is customary to make one of the property corners the starting point and to use as a base line the magnetic meridian, noting the date on which the survey is made, so that at any subsequent time an allowance may be made for the variation of the needle. When surveying large tracts of land, or even comparatively small ones, upon which mines are to be opened, the starting point is commonly some firmly planted artificial object, and the base line is the true meridian, as determined by astronomical observations on the North Star (Polaris), or on the sun, by methods given under the

head of Latitude and Longitude. The true meridian is the only invariable base line that can always be determined at any time or place by means of the engineer's transit.

**Monuments.**—When establishing a reference meridian or base line for any large coal property, it is not customary to place the monuments marking its extremities exactly in the true north-and-south line; in fact, it is not generally possible to do so. The monuments should be placed where they will not become disturbed by mining operations; where the line of sight between them will not become obstructed by subsequent building operations; and where they will be convenient for use. Hence, monuments should be placed outside the crop line if this comes upon the property so that pillar drawing, followed by settling of the surface, will not throw them out of line. If the property is opened by a shaft, one monument may be placed upon that portion of the surface that will be sustained by the shaft pillar, and the other monument or monuments may be placed upon reservations from under which the coal will not be mined, or may be placed entirely outside the boundaries of the property. If no place that will remain undisturbed during the life of the mine can be found for the second monument, sights may be taken to a number of prominent buildings or natural objects within or without the boundaries of the property. The angles made by the lines joining these objects with the monument as a vertex should be repeatedly read, and the mean of the readings taken as the true angle. Reference points so selected will determine the direction of the meridian at any later date, as it is improbable that all of the five or ten reference points will be disturbed or destroyed.

The cheapest monuments may be made from mine-car axles or from old railroad iron of, say, 60 to 70 lb. to the yd., cut into lengths of from 4 to 8 ft., depending on the nature of the soil. One end should then be sharpened and the axle or rail driven into the ground until about 6 in. project above the surface. A hole is then deeply marked in the top by a center punch and its distance from three or four nearby points is measured. These distances, or *references*, will enable the monument to be found in event of its being subsequently covered with dirt.

Monuments made of rails, unless the ends are driven well below the frost-line, are apt to be moved out of line through the alternate freezing and thawing of the soil; therefore, a better way is to place the monuments in solid, outcropping rock. In this case, a hole some 12 in. deep is drilled in the rock and a bolt of 1-in. or $1\frac{1}{2}$-in. round iron is leaded into it, the head of the bolt being allowed to project 1 or 2 in. above the rock. The bolt is center-punched the same as would be a rail. Sometimes the projecting end of the bolt is threaded so that a cap may be screwed upon it to protect the center. This is a good plan in damp climates if the monument is intended to last many years.

Excellent monuments may be made of dressed stone in which a center-punched bolt is set to mark the exact point. The upper foot, in length, of the stone, about 6 in. of which projects above the surface, is dressed square with a side of 6 to 8 in. The lower portion beneath the ground should be as large, and consequently as heavy, as possible. The length should be 4 ft. or more, so that the bottom is set well below the frost line. Concrete monuments are cheaper than stone and may be constructed of any size. They are, of course, built up in a pit of good depth, the center bolt being placed before the cement has had time to set.

The boundary of a large property may be 5, 10, or more mi. in length. As it is impossible to tell, in advance, when and where new property will be acquired, necessitating an extension of the original survey, it is a most excellent plan to set a pair of monuments at intervals of about 1 mi. along the line of the survey. The wooden pegs used as stations in the original survey, will disappear in 12 to 18 mo. or will have been so displaced by the action of frost as to be useless. If two consecutive stations are made of rails placed in the fashion of monuments, they will serve at any future time as a base for the extension of the survey. These *permanent stations*, as they are frequently called, should be carefully witnessed and referenced.

## OUTSIDE SURVEYS

**Preliminary Work.**—Before the survey of a property is undertaken, it is highly advisable that the surveyor should go over the ground and familiarize himself with the location of all the corners, roads, streams, houses, outcrops, reservations, and other features that are to appear upon the map. While in some cases a map of the property is furnished the surveyor, it is usually necessary for him to prepare one from the deeds to its component tracts as recorded

at the county seat. A large property is usually made up of from 10 to 100 or more tracts varying in size from a fraction of 1 A. to 100 A. or more. As the surveys found in the deeds were made at widely different dates, the bearing of a line common to two or more properties is commonly different in each deed in which it is mentioned. In such cases, the surveyor should mark on his tracing the different bearings given, as well as the different lengths for each line. With this map, a good pocket compass, and a 100-ft. tape, together with what information may be picked up from residents along the line, the surveyor and his assistant can locate and mark the various corners.

**Angular Measurements.**— The angular measurements in a survey may be made by one of two methods. In one, the angle at any station is read but once, the method commonly used being known as the *continuous vernier;* in the other method, the angle at each station is read twice. By the first method, no check on the accuracy of the work is afforded until the initial station of the survey is occupied with the transit and the azimuth of the first line redetermined, affording what is known as a *close.* As the bulk of the time in the field is employed in setting up the transit, it would seem but ordinary good sense to repeat or check the angles at each station that the accuracy of the work may be certain as the survey proceeds. In no case should the results of a single-angle survey be accepted as correct until such a close has been made.

When making a survey by single angles, the procedure is as follows: Set the transit over the monument marking one end of the base line, which is called Sta. 0. Assuming that the base line makes an angle of 48° 21′ to the right of the meridian, this angle is the azimuth of the base line; and as it is to right of the meridian, its bearing is N 48° 21′ E. Set off this azimuth on the limb of the transit and focus the vertical hair on the second mounment; the line of collimation of the transit will now be in a line directed N 48° 21′ E, and if the upper plate is loosened and the instrument set on any distant object, the azimuth and bearing of that point from Sta. 0, may be read from the graduated limb. Suppose the sight is taken to Sta. 1 of the survey. If the reading on one set of graduations is 326° 48′, which is the azimuth of the line 0–1, the reading on the other set will be N 33° 12′ W, which is the bearing of the line 0–1.

Setting up the instrument over Sta. 1, the vernier is read, to see that it still reads 326° 48′. Then invert the telescope so that the level tube is on top, and by means of the lower motion, take a backsight on Sta. 1. When the telescope is plunged into its normal position (level tube below), the line of sight is in the line 0–1 produced with an azimuth of 326° 48′. Loosen the upper motion and sight on Sta. 2 of the survey. If the azimuth from one set of graduations is, say, 266° 10′, the bearing from the other set will be S 86° 10′ W. Continue this work from station to station until the transit finally reoccupies Sta. 0, the monument at one end of the base line. If, now, the azimuth of the base line as determined by reference to the last line of the survey is found to be 48° 21′ as determined astronomically, all of the angles of the survey have been measured correctly, and the survey is said to *close in angle.*

When making a survey by double angles at each station, assume that the azimuths of the lines are the same as those just used for illustration and that the transit is set up at Sta. 1. Set the vernier at 0° and take a backsight on Sta. 0. If the upper motion is loosened and the telescope revolved 180° around its vertical axis, the line of sight will be in the line 0–1 produced. Continue revolving the telescope until it is set upon Sta. 2, when it will have been turned to the left of the line 1–0 produced and the angle made by the line 1–2 with this line (0–1 produced) is a deflection angle to the left, or a *left angle* commonly called. This will be found to be (in the assumed case) 60° 38′. Next, set the vernier on the azimuth of the line 0–1, viz.: 326° 48′ (bearing N 33° 12′ W), and with the telescope inverted take a backsight upon Sta. 0. Plunge the telescope and by the upper motion set on Sta. 2. The azimuth will be found to be 266° 10′ (bearing S 86° 10′ W). As the deflection angle subtracted from the azimuth of the line 0–1 gives the azimuth of the line 1–2, the angles have been correctly read. Thus 326° 48′ (azimuth) − 60° 38′ (deflection angle) = 266° 10′ (azimuth line 1–2). Similarly N 33° 12′ W (bearing line 0–1) + 60° 38′ (left deflection angle) = N 93° 50′ W = S 86° 10′ W (bearing of line 1–2).

It is customary to read and record the deflection angle first, and then to read and record the azimuth or bearing. After the main angles by which the survey is continued have been noted, before moving to the next station, all the corners, houses, roads, streams, etc., that can conveniently be reached from the instrument should be located and entered in the notebook.

Whether the direction of a line of a survey shall be recorded and described by its azimuth or by its bearing is a matter of choice. Few of those for whom the map is chiefly made are familiar with the former term, and the statement that a certain property line or heading has an azimuth of, say, 286° 10′ does not give them any idea as to its direction; whereas, every layman understands the meaning of the equivalent bearing, S 86° 10′ W. For this reason it seems better to give the bearings of lines instead of their azimuths.

**Distance Measurements.**—A general rule that should not be broken is that all operations necessary to carry on the main line of the survey must be done before anything else is attempted. Therefore, after reading and recording the angle between the lines of the survey, and checking it by the method just explained, the distance to the next, or foresight, station must be read. The method of doing this will depend on whether a 100-ft. tape or chain is used, or, as is the better practice, a 400-ft. tape is employed.

The method of using a chain on level ground has been described. On ground sloping, say, down hill from the instrument, one end of the chain is held at the tack in the stake marking the station, and as much of the tape as can be held horizontal is stretched out in the line to the next station. The end of the horizontal portion of the chain is marked on the ground by dropping a plumb-line from it. The length of this level portion is noted on a piece of paper; the end of the tape held at the plumb-bob in the ground and another length of level tape stretched out and its end marked, and length noted as before. This is kept up until the distance between the stations has been covered, when the sum of the single measurements, is equal to the entire measurement. This is a slow, laborious, and generally inaccurate method that has, by mine surveyors, at least, given way to the use of the long steel tape.

If the distance between stations is less than the length of the tape and the ground is level or uniformly sloping, the 0 end of the tape is held at the tack in the stake at the instrument and the distance to the tack in the foresight stake read, as previously explained. If the ground slopes either up or down hill, the angle of elevation or depression of the slope must be taken and recorded as a plus (+) angle if one of elevation, or as a minus (−) angle, if one of depression. If the tape has been stretched along a plane surface, as explained, the line of sight when the so-called vertical angle is read must be parallel to the tape. To do this, a sight must be taken at a point on the foresight rod as far above the ground as is the center of the telescope axis at the instrument station. If the ground is not uniformly sloping, the 0 end of the tape is held at the hole marking the end of the horizontal axis of the telescope, and the measurement is made to the tack in the foresight stake. In this case the vertical angle is measured directly, exactly as the horizontal cross-hair cuts the station tack.

Distance on slope × cos vertical angle = horizontal distance          (1)

Distance on slope × sin vertical angle = difference in elevation      (2)

The reduction of the slope distances to horizontal ones is made in the office, not in the field. If the elevation of the first survey station is known, that of all the other stations is obtained by adding continuously to it the difference in elevation, as obtained from formula 2. While elevations thus obtained are not so accurate as those secured through the use of leveling instruments, they answer every purpose as a basis for a topographical survey made with the stadia.

If the distance is greater than the length of the tape, but less than twice as great, say 790 ft., a stake with a tack is placed about half way between the stations and in the line joining them. The distance and vertical angle to the stake are read after making the foresight, and again from the next station, after taking the backsight. The sum of these distances, after reduction to the horizontal, is the total distance between the two stations.

If the distance is greater than two tape lengths, say, 1,000 ft., two stakes must be set. The first stake *a* may be placed, say, 350 ft. from the instrument, and the second stake *b*, 350 ft. beyond that. Before moving the transit the distance and vertical angle to *a* must be read and the transit set up roughly over *a* and the distance and vertical angle to *b* read. At the next station and on the backsight, the distance and vertical angle to *b* must be read. The sum of these three distances reduced to the horizontal is the distance between the stations.

**Locating Corners, Etc.**—After the main-line angle is read and checked and the distance to the next station measured (or such part of it as is possible without moving the transit) a sight or sights should be taken to any nearby property corner or corners and the azimuth or bearing, as well as the distance and the vertical angle thereto recorded. It should be noted that the deflection

angle between the lines joining stations should be read first, as the instrument must be properly oriented before corners, etc., can be located, and this orientation is secured at each station by using as a backsight for the second reading, the azimuth of the line joining the backsight and instrument stations as determined from the previous set-up. After the corners have been located, stadia sights are taken to any houses, streams, roads, topographic features, etc., that should appear upon the map.

**Keeping Notes.**—The various ways of keeping the main-line and side notes of an outside survey arrange themselves into four groups: (1) The side notes of each sight follow the transit notes of that sight, and on the same page. (2) They are entered in the same book on opposite pages. (3) The transit notes of the whole survey come first, and are followed by the side notes in the same book. (4) Each set of notes has a separate book.

Of these notes, the second and third are in common use. For a survey made by the methods just explained, the accompanying form of transit notes is the usual one. The columns are headed for station (Sta.), Bearing, Angle (deflection angle) which may be either R (right) or L (left), distance (Dist.), and Slope (vertical angle, or pitch).

TRANSIT NOTES

| Sta. | Bearing | Angle | | Dist. | Slope |
|------|---------|-------|------|-------|-------|
| | | R | L | | |
| $M_1-M_2$ | N 48.21 E | (Bea | ring of base | line) | |
| $M_1-1$ | S 26.30 W | | 21.51 | 262.83 | −4.16 |
| $1-2$ | S 67.49 W | 41.19 | | 387.62 | −2.18 |
| $2-3$ | N 86.11 W | 26.00 | | 316.99 | +0.16 |
| $3-4$ | S 55.28 W | | 38.21 | 365.34 | +1.56 |

The bearing of the base line, or that joining the two monuments, $M_1$ and $M_2$, is N 48.21 E. It will be noted that the signs ° and ′ are not used, a period serving to separate the degrees and minutes. This saves time. The notes are simple and self-explanatory. The transit is always assumed to be at the station whose number or letter is given first in the station column. Thus, the instrument is at Sta. $M_1$, 1, 2, and 3. The foresight follows the instrument station in the same horizontal line. Thus, the foresights to Sta. 1, 2, 3, and 4, are made from Sta. $M_1$, 1, 2, and 3, respectively. The backsight at Sta. 1 is, of course, $M_1$, at Sta. 2, it is 1, and similarly. It should be noted at Sta. $M_1$, when the vernier is set for the purpose of taking the azimuth or bearing of the line 1–2, that the setting S 48.21 W and not N 48.21 E is used. This is because the line $M_2-M_1$ runs in exactly the opposite direction from the line $M_1-M_2$, and the survey is moving forwards in the former line.

As stated, the side notes are entered in the same book as the main-line notes, but usually in the back. It is well to head a certain page, say, Sta. 1, and then to follow with all the side notes taken at that station, and similarly for the succeeding stations. The condition of all property corners should be carefully described, as their present state is often very different from what it was at the time the original deeds were made. Thus, a corner described as a hemlock sapling in a deed dated 1790, may now be a tree 2 or 3 ft. in diameter, may be a stump, may have entirely rotted away so that only an expert can tell from the decayed remains that a hemlock once marked the corner, or a good-sized oak, sugar maple, or hickory may have replaced it, the hemlock having been destroyed but a few years after the deed was made and a tree of another species grown in its place.

Frequently there are reservations around farm houses from under which the coal may not be mined. Sometimes the corners of these *reserves* (as they are often called) can be reached in a single sight; if not, a branch line must be run to them from the line of the main survey.

When locating houses by stadia sights, two men are necessary. One can hold the stadia rod at the opposite corners of the house, but a second is required to hold the tape by which the dimensions of the buildings are secured. Roads are located by the fences on either side, the road legally occupying all the space between the fences, even if part of it is grown up in weeds and the wagons

have made but a single line of ruts. Roads are commonly either 1 or 2 rd., that is, 16.5 or 33 ft. in width, and should be so mapped.

Small brooks are located by taking a stadia sight to their center at each important bend. Larger creeks should have one bank located with the stadia, the stadia rodman estimating the width of the stream, which should be entered in the notes. Large rivers should have a regular traverse run along the bank, the opposite side being located by stadia sights. In the case of very large rivers, a transit line must be run along each bank and the shore line located by stadia sights. Stadia sights should be taken to points along the bottoms of all dry gulleys, to the summits of all ridges, and to any marked change in the degree of slope of a hill.

If the vertical angles have been taken when the distances between the stations were measured and the stadia sights taken as just explained, there will be gathered in the course of the survey enough data to make a very complete topographic, or contour, map of the property. This will particularly be true, if, as is customary, the boundaries of the individual farms making up the entire property are surveyed. The reason for surveying the single farms is that very frequently the operating coal company has not the same rights in all of them, so that the method of mining is affected by what the company can and cannot do.

The kind, dip, and strike of all outcropping ledges or other exposures of rock should be noted and mapped. The information thus obtained, combined with the elevations obtained with the stadia, etc., furnishes the data from which geological cross-sections may be made.

The first page of the notebook should give the name of the company for whom the work is done, the location of the property, name of the engineer in charge of the work, the names of the helpers, and each day's work should be dated; and if the members of the corps have changed, this, too, should be a matter of record.

If more than one property is being surveyed from the same office, a separate set of field books should be devoted to each. It is an excellent plan at night to copy at least the main-line notes in a permanent notebook, which is left at the office, or at the farmhouse if the corps is in the field. Sometimes the side notes are also copied. These copies are made so that all the records may not be destroyed, necessitating an entirely new survey, should anything serious happen to the field book. When not in use, all field books, both originals and copies, should be kept in a fireproof vault.

**Closing Surveys.**—To diminish the chance of error, even if double angles have been read, the survey must be closed upon itself or some part of a former closed survey. That is, the transit must a second time occupy the initial station, so that the first azimuth read may be referred back to the last line of the survey. If the azimuth (or bearing) as read the second time agrees with that obtained at the first reading, the angular readings are proved to have been correctly made. If the error in closure is not more than 1' to 3' in a line 4 or 5 mi. in length, most surveyors will balance the survey, but in important work the error must (or should) be located so that the survey will close exactly *in angle.* Usually one or more stations will be selected as the most probable ones at which the error (all or in part) was made. These stations will be those at which good sights were not to be had owing, say, to smoke obscuring the point of the plumb-bob underground or, in the case of surface surveys, to the station stake on either backsight or foresight being beyond a roll in the ground so that the tack or the point of the rod was not visible. These doubtful stations may be reoccupied and the angles remeasured with special care. If not located at the probable places of error, the survey, so far as the angle measurements are concerned, may be rerun in its entirety; but this is rarely necessary. If the angles cannot be made to close upon resurvey, the failure to do so is probably caused by cumulative errors, a few seconds at each station. These errors are, singly, too small to measure, but in a survey of 40 or 50 stations may amount to 1' or more. Errors in linear measurement are far more common than errors in angular measurement, as there is no field check upon the work with the tape unless the distances are read twice, on the foresight from the one station and on the backsight from the next. A wrong reading of the vertical angle will, of course, affect the horizontal distance. Errors in measurement are often of 5 or 10 ft. due to incorrectly reading the graduations on the tape. Perfect linear measurements are far more difficult to make than perfect angular measurements. This is because the correct length of a line is very materially affected by variations in the length of the tape due to the effects of sag, tension, changes in temperature, etc. In highly accurate work, corrections should be made for all of these.

The allowable error in closure depends on many things, the chief of which is financial. If it will cost more to locate and correct the error than the value of the land saved, it will not pay to do so. In other words, far more time (consequently, money) may be spent upon a survey of coal land worth $2,500 an A. than upon land costing but $5 an A. In ordinary rolling country, such as prevails in the coal fields of the eastern states, with instruments in good adjustment and using ordinary precautions, the error in closure should not be greater than 1 ft. in 3,000 ft. to 1 ft. in 5,000 ft., and trained corps will do better. In bituminous mines, which are commonly in flat coal, underground surveys may easily be closed within 1 ft. in 10,000 ft. to 1 ft. in 20,000 ft. The higher accuracy obtainable underground is due chiefly to the fact that mine temperatures are extremely uniform so that corrections for expansion or contraction of the tape are unnecessary. Likewise, the tape is stretched on the ground and errors due to sag are thus eliminated.

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# LEVELING

## DESCRIPTION OF INSTRUMENTS

In *leveling*, but two instruments are used, the level and a leveling rod. The *level* consists of a telescope to which is fitted, on the under side, a long level tube. The telescope rests in a **Y** at each end of a revolving bar, which is attached to a tripod head very similar to that used for a transit. The telescope is similar to the telescope of a transit.

The *leveling rod* is merely a straight bar of wood, 6 ft. or more in length, divided into feet and tenths of a foot. A target divided into four equal parts by two lines, one parallel with the staff, and the other at right angles to it, and painted red and white, so as to make it prominent at a distance, slides on the rod and is provided with a clamp screw. The center of the target is cut out and a vernier, graduated decimally, is set in, which enables the rodman to read as close as $\frac{1}{1000}$ ft. If a long rod is required, it is made of two sliding bars, which, when closed, are similar to a single rod, as described above. When used at points where it is necessary to shove the target to a greater height than 6 or $6\frac{1}{2}$ ft., the target is clamped at the highest graduation on the front of the rod, and the rod is extended by pushing up the back part, which carries the target with it. The readings, in this case, are made either from the vernier on a graduated side, or a vernier on the back. The rodman must always hold his rod perfectly plumb or perpendicular.

## LEVEL ADJUSTMENTS

The proper care and adjustment of the level is of great importance. A very slight error in adjustment will completely destroy the utility of any work done.

1. *To Adjust the Line of Collimation.*—Set the tripod firmly, remove the **Y** pins from the clips, so as to allow the telescope to turn freely, clamp the instrument to the tripod head, and, by the leveling and tangent screws, bring either of the wires upon a clearly marked edge of some object, distant from 100 ft. to 500 ft. Then with the hand, carefully turn the telescope half way around, so that the same wire is compared with the object assumed. Should it be found above or below, bring it half way back by moving the capstan-headed screws at right angles to it, remembering, always, the inverting property of the eyepiece; now bring the wire again upon the object, and repeat the first operation until it will reverse correctly. Proceed in the same manner with the other wire until the adjustment is completed. Should both wires be much out, it will be well to bring them nearly correct before either is separately adjusted.

2. *To Adjust the Level Bubble.*—Clamp the instrument over either pair of leveling screws, and bring the bubble into the center of the tube. Now turn the telescope in the wyes, so as to bring the level tube on either side of the center of the bar. Should the bubble run to the end, it shows that the vertical plane, passing through the center of the bubble, is not parallel to that drawn through the axis of the telescope rings. To rectify the error, bring it by estimation half way back, with the capstan-headed screws, which are set in either side of the level holder, placed usually at the object end of the tube. Again bring the level tube over the center of the bar, and adjust the bubble in the center, turn the level to either side, and, if necessary, repeat the correction until the bubble will keep its position, when the tube is turned $\frac{1}{2}$ in. or more to either side of the center of the bar. The necessity for this operation arises from the fact that when the telescope is reversed, end for end, in the wyes in the other and principal adjustment of the bubble, it is not easy to place the

level tube in the same vertical plane, and, therefore, it is almost impossible to effect the adjustment without a lateral correction.

Having now, in a great measure, removed the preparatory difficulties, it is possible to proceed to make the level tube parallel with the bearings of the Y rings. To do this, bring the bubble into the center with the leveling screws, and then, without jarring the instrument, take the telescope out of the wyes and reverse it end for end. Should the bubble run to either end, lower that end, or what is equivalent, raise the other by turning the small adjusting nuts, on one end of the level, until, by estimation, half the correction is made; again bring the bubble into the center and repeat the whole operation, until the reversion can be made without causing any change in the bubble. It is well to test the lateral adjustment, and make such correction as may be necessary in that, before the horizontal adjustment is entirely completed.

3. To Adjust the Wyes.—To adjust the wyes, or, more precisely, to bring the level into a position at right angles to the vertical axis, so that the bubble will remain in the center during an entire revolution of the instrument, bring the level tube directly over the center of the bar, and clamp the telescope firmly in the wyes. Place it, as before, over two of the leveling screws, unclamp the socket, level the bubble, and turn the instrument half way around, so that the level bar may occupy the same position with respect to the leveling screws beneath. Should the bubble run to either end, bring it half way back by the Y nuts on either end of the bar; now move the telescope over the other set of leveling screws, bring the bubble again into the center, and proceed precisely as just described, changing to each pair of screws, successively, until the adjustment is very nearly perfected, when it may be completed over a single pair.

The object of this approximate adjustment is to bring the upper parallel plate of the tripod head into a position as nearly horizontal as possible, in order that no essential error may arise, in case the level, when reversed, is not brought precisely to its former situation. When the level has been thus completely adjusted, if the instrument is properly made and the sockets are well fitted to one another and the tripod head, the bubble will reverse over each pair of screws in any position. Should the engineer be unable to make it perform correctly, he should examine the outside socket carefully, to see that it sets securely in the main socket, and also notice that the clamp does not bear upon the ring that it encircles. When these are correct, and the error is still manifested, it will probably be in the imperfection of the interior spindle.

After the adjustments of the level have been effected and the bubble remains in the center in any position of the socket, the engineer should carefully turn the telescope in the wyes, and sighting upon the end of the level, which has the horizontal adjustment along each side of the wye, make the tube as nearly vertical as possible. When this has been secured, he may observe, through the telescope, the vertical edge of a building, noticing if the vertical hair is parallel to it; if not, he should loosen two of the cross-wire screws at right angles to each other, and with the hand on these, turn the ring inside, until the hair is made vertical; the line of collimation must then be corrected again, and the adjustments of the level will be complete.

## USING THE LEVEL

When the instrument is being used, its legs must be set firmly into the ground, and neither the hands nor person of the operator be allowed to touch them. The bubble should then be brought over each pair of leveling screws successively, and the instrument leveled in each position, any correction being made in the adjustments that may appear necessary. Care should be taken to bring the wires precisely in focus, and the object distinctly in view, so that all errors of parallax may be avoided.

An error of parallax is seen when the eye of an observer is moved to either side of the center of the eyepiece of a telescope, in which the foci of the object and eyeglasses are not brought precisely upon the cross-wires and object; in such a case, the wires will appear to move over the surface and the observation will be liable to inaccuracy. In all instances, the wires and object should be brought into view so perfectly that the spider lines will appear to be fastened to the surface, and will remain in that position however the eye is moved.

If the socket of the instrument becomes so firmly set in the tripod head as to be difficult of removal in the ordinary way, the engineer should place the palm of the hand under the Y nuts at each end of the bar, and give a sudden upward shock to the bar, taking care, also, to hold his hands so as to grasp it the moment it is free.

## FIELD WORK

If the survey has been carefully made and vertical angles taken at every sight, leveling will be necessary only in cases where extreme accuracy in regard to vertical heights is necessary. In most cases of practical work at collieries, particularly in determining thickness of strata, general rise or fall of an inside road, etc., the elevations calculated by the use of the vertical angle will be close enough, but there are frequently instances when leveling must be done, to insure success in certain work. In this connection, it is well to state that if the transit telescope is supplied with a long level tube, and it is, as a whole, in first-class adjustment, levels can be successfully run with it if the transitman uses due care. Having his instrument in proper adjustment and his notebook ruled, the levelman is ready to proceed with the work.

The rodman holds the rod on the starting point, the elevation of which is either known or assumed. The levelman sets up his instrument somewhere in the direction in which he is going, but not necessarily, or usually, in the precise line. He then sights to the rod and notes the reading as a backsight or + (plus) sight, entering it in the proper column of his notebook, and adding it to the elevation of the starting point as the "height of instrument." The rodman then goes ahead about the same distance, sets his rod on some well-defined and solid point, and the levelman sights again to the target, which the rodman moves up or down the rod until it is exactly bisected by the horizontal cross-hair in the telescope, as he did when giving the backsight. This reading is noted as a foresight or − (minus) sight. The foresight subtracted from the height of instrument gives the elevation of the second station. The rodman holds this latter point, and the levelman goes ahead any convenient distance, backsights to the rod, and proceeds as before. In this case, it is assumed that levels are only being taken between regular stations or two extreme points.

If a number of points in close proximity to each other are to be taken, the rodman, after giving the backsight, holds his rod at each point desired. The readings of any number in convenient sighting distance are taken and recorded as foresights, and any descriptive notes are made in the column of remarks. These are each subtracted from the height of instrument, and the elevation found is noted in column headed Elevation. After all the intermediate points are taken, the rodman goes ahead to some well-defined point, which is called a turning point (T. P.) in the notes. The elevation of this is found and recorded. The rodman remains at this point until the levelman goes ahead, sets up and takes a backsight. This backsight reading, added to the elevation of the turning point, gives a new height of instrument from which to subtract new foresights, and thus obtain the elevation of the next set of points sighted to.

When running levels over a long line, the levelman should set frequent *bench marks* (B. M.). These are any permanent well-defined marks that can be readily found and identified at any future time. By leveling to them he has secured the elevation of points from which to start any subsequent levels that may be necessary. A good bench mark can always be made on the side or root of a large tree or stump by chopping it away so as to leave a wedge-shaped projection with the point up. A nail should be driven in the highest point of this, to mark where the rod was held, and the tree or stump blazed

LEVEL NOTES

| Station | B. S. | F. S. | H. Inst. | Elev. | Remarks |
|---|---|---|---|---|---|
| 1 | 3.412 | | 103.412 | 100. | Assumed elevation of Sta. 1. |
| 2 | | 4.082 | | 99.33 | Sta. 2 of survey. See page—— Vol.—— |
| | | 6.791 | | 96.621 | Sight taken to ground at N. E. cor. John Smith's house. |
| 3 = T. P. | 11.698 | 4.862 | 110.248 | 98.55 | Sta. 3 of survey noted above. |
| 4 | | 9.817 | | 100.431 | Sta. 4 of survey noted above. |
| B. M. 1 | | 6.311 | | 103.937 | B. M. 1 is on north side of large white oak. |
| 5 | | 6.427 | | 103.821 | Sta. 5 of survey noted above. |

above the bench mark. In this blaze, the number of the bench mark, which should, of course, correspond with the number in the notebook, should be cut or painted. In the mines, prominent frogs or castings in the main roads, if permanent, make good bench marks.

In underground leveling, extreme care must be observed to record the algebraic signs of the readings, which show whether the level rod was held in its usual position, indicated by a + sign or the absence of any sign, or upside down, indicated by the − sign.

**Proof of Calculations.**— The calculations are proved by adding together the backsights and also the foresights taken to turning points and last station. Their difference equals the difference of level between the starting point and last station. Thus:

| Foresights | Backsights |
|---|---|
| 4.862 | 3.412 |
| 6.427 | 11.698 |
| 11.289 | 15.110 |
| | 11.289 |
| | $3.821 = 103.821 - 100.0$ or $3.821$ |

## TRIGONOMETRIC LEVELING

*Trigonometric leveling* determines the difference in elevation between two points from the measurement of the distance between the points, and from the vertical angle between them. Although generally less accurate than leveling with a **Y** level, it is much more rapid and is especially adapted for pre-



FIG. 1

liminary work in a hilly country, or for the leveling of mine slopes or pitching rooms where the **Y** level cannot be used with any advantage or accuracy. By reading the angles and by checking the measurements, a very high degree of accuracy can be obtained in trigonometric leveling.
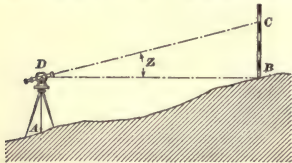
**Case 1.**—Assume the elevation of $A$, Fig. 1, to be 100 ft. above tide. With the transit set up over $A$ and properly leveled, sight to a point $C$ on a rod so that $BC$ equals $AD$. Measure the vertical angle $Z$ and the inclined distance $DC$, then the difference in the elevation between $A$ and $B$ equals $BC = CD \times \sin Z$, and the elevation of $B$ equals $100 + BC$.

**Case 2.**—Assume the elevation of station $A$, Fig. 2, in the roof of a mine to be 100 ft. above tide. Then, with the transit set up directly under $A$ and properly leveled sight to a point $C$ upon the plumb-line suspended from the station $B$, measure the vertical angle $X$, inclined distance $DC$, and roof distance $BC$. From this, the distance $CY = DC \times \sin X$. The elevation of $B$ is then found as follows: The elevation of $B$ = elevation of $A - AD + (DC \times \sin X) + BC$.
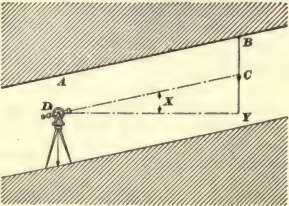
There are many modifications of this simple method, but from these diagrams the most complex modifications can be worked out.



FIG. 2

### TRIGONOMETRIC LEVEL NOTES

| Station | Vertical Angle Degrees | Inclined Distance | Vertical Distance | Height of Instrument Feet | Roof Distance Feet | Elevation Feet |
|---|---|---|---|---|---|---|
| $A$ | | | | | | +100.00 |
| $A - B$ | +5 | 100 | +8.72 | 2 | 3 | 109.72 |
| $B - C$ | +2 | 100 | +3.49 | 3 | 2 | 112.21 |
| $C - D$ | −3 | 100 | −5.23 | 4 | 3 | 105.98 |
| $D - E$ | −4 | 100 | −6.98 | 2 | 1 | 98.00 |

# CONNECTING OUTSIDE AND INSIDE WORK THROUGH SHAFTS AND SLOPES

## SURVEYING SHAFTS

As the dip of the bed increases, it becomes more difficult to make a connection and the chances of accuracy diminish. In the survey of a pitching plane, one station is located, with respect to the adjacent ones, by multiplying the distance by the cosine of the vertical angle. The greatest angular accuracy for a given distance is where the vertical angle is 0°. As the pitch or vertical angle of sight increases the cosine diminishes until, at a vertical sight, distance × cos. vertical angle = 0°.

In the case of an adit level, or a slope of less than 45°, there is no difficulty beyond the want of absolute rigidity in setting up the transit, and the danger of moving it in going about it. The difficulty increases more rapidly than does the pitch, and as the distance × cos. vertical angle diminishes, though the distance is fixed, the chances of error increase. When the slope reaches 60°, there is an impracticability in running a line down a slope, as the line of collimation of the telescope strikes the graduated limb of the instrument. A person can use a prismatic eyepiece and see up the slope; but cannot look down. As it is assumed that it is unnecessary to use an additional telescope, the line must be run by intermediates. To do this, the transit should be set up at the bottom of the slope where the longest sight up the same can be secured and a backsight taken on a station of the underground work; or a backsight should be set for the occasion (both stations will afterwards be connected with the work below). With the prismatic eyepiece, a sight should be taken up the slope on a line that will give the longest sight and, at the same time, afford a good intermediate place to set up the transit, as, on a pitch of 60° or more, it is absolutely necessary that the legs of the transit should be set solidly (in holes in the floor, or between the sills of the track) so that they will not be moved by subsequent walking about it. By this method, all the sights will be taken from one side alone, and the tripod legs can be shortened to make the sight possible without building a standing place—if the man is short.

Call this station $A$; at the foot of the slope locate $B$, where the transit can be readily set up, and as far up the slope as possible (this distance must be at least 100 ft.), and in a continuation of $AB$, locate $C$. Set up at $B$ and take foresight to $C$; locate $D$ under the same conditions that governed the placing of $B$, and, in a continuation of the line $BD$, place $E$. Set up at $D$ with foresight at $E$, and locate $F$ and $G$ as before. The survey is carried by the intermediates $B$, $D$, $F$, etc., to the top, by a series of foresights to $C$, $E$, $G$, etc.

The term *shaft* in American coal-mining practice is applied only to vertical openings, though in metal mining, both in the United States and abroad, it is also applied to highly inclined slopes. For such shafts, most of the methods given in the textbooks are worthless, as they are for transit work and the distance × cos. vertical angle in rare cases may be as great as 20 ft., while the distance varies from 100 to 1,500 ft. Again, to sight down a shaft necessitates the erection of a temporary (and therefore more or less unsteady) support for the tripod of the transit, and the chances of variation in its position as the different sights are made are so great that it is difficult to say when a movement has not taken place that will vitiate the work.

In sighting up a shaft of greater depth than 100 ft., there is annoyance—if not danger—from dripping water or the fall of more solid substances. In a wet shaft the object glass is instantly covered with water, and a sight is impossible. Also, it is necessary to stand upon a platform, and it is hard to tell when this is perfectly rigid. From all these considerations the methods with a transit are never used by engineers in the anthracite regions, and the connections are made as follows:

When the bottom of the shaft can be reached by an adit or a slope in a roundabout route of such length as to render errors in measurement of distance of great importance, the angles are carried by a transit with as long sights as possible, and no distances are measured, from a point on the surface in the shaft to a point vertically below it in the mine. Sometimes the guide of the cage is taken when it has been recently set, as the guides are plumbed into position; but the better way is to suspend an iron plummet by a copper wire: sink the former in a barrel of water or bucket of oil so as to lessen the tendency to swing on account of the pull upon the bob and wires from the air-currents, or falling drops in a wet shaft. The top of the barrel should be

covered with two pieces of plank with a semicircular groove of 3 in. radius cut out of the middle for the passage of the wire, to catch the substances whose fall upon the water would cause waves. The heavier the plummet and the lighter the wire, the less the tendency to swing. This wire can be sighted at by parties above and below at the same time, and the swing can be bisected to get the position of the wire. A number of sights that agree can be taken as accurate.

When the shaft is the only way to get below from above, it must be plumbed with two or more wires suspended as just described. When two wires are used, the wires should be so hung that an instrument can be set up below in a line passing through them produced, and at a sufficient distance from them to insure an accurate sight. When more than two wires are used, the underground station can be located at any point at which all the wires can be seen from the instrument.

**Case 1.**—Two wires are used, which are located as far apart as possible. Two pieces of scantling *cd* and *ef*, Fig. 1, are spiked across the opposite corners of two compartments of a shaft to allow the cages to pass up and down without interference. The station *X* is (roughly) located in a line through the corners *x*, and is connected with the outside survey. From this station locate in the line *Xxx* two spads for holding the wires of the plumb-bobs. These are driven up to the head in the scantlings in such a way that the line of sight passes through the center of the holes in their heads. When the distances *Xa* and *ab* are measured, the work of the survey above ground is com-



Fig. 1

pleted. The light copper wire is rolled upon a reel, and one end is fastened to a light plumb-bob to keep it free from coils or kinks in descending. It can thus be readily lowered without accident. When at the bottom, the upper end is fastened in the spad and the heavy bob applied to the bottom and placed in the empty barrel. The cages are then run slowly up and down, with an observer on each, to see that the wires hang free from top to bottom. By this time the wire will have stretched so that it will be straight, all slack is taken up, the barrel filled with water, and the top boards put in place. As a last check, the distance between the wires below is measured, to see if it agrees with the distance above.

Lining in below a point *Y* on the line *ab*, make a hole in the roof 2 in. in diameter, and drive in a broad plug. Setting up the transit under *Y*, sight at the wires *a* and *b* alternately. A number of methods for illuminating the wires have been used. But the most satisfactory method is to place a large white target behind both wires and illuminate them by a large lamp with a reflector behind it. The wire stands out black against the target, and can be followed across it. As there is considerable distance between the wires, and as the transit is comparatively near them, there is little chance of getting a sight of one, when the telescope is focused upon the other, and so the focus has to be set between them. This gives a hazy sight at each; but both are shown against the white background in strong relief. After the transit head is shifted so that the line of sight coincides approximately with both, they should be focused upon alternately so as to see if the line bisects the swing of each. If so, the work is done; if not, the shifting of the transit head must follow until the end is attained. It frequently requires 2 hr. or more of steady observation to complete the work, and, when it seems as if the proper point were secured, one of the wires will show by its swaying that it has been deflected from the vertical by a peculiar slant of wind, and the result obtained must be checked again. When through, there is no absolute certainty that the point marked is in the accurate extension of the line *ab* at the surface. Having decided on the proper place, a spad must be driven into the plug overhead and a plumb-bob hung to it to see if it is over the axis of the transit, as shown by the screw on the telescope. If not, the spad must be driven so that the point of the bob does so hang, and the point *Y* is said to be in the line *ab*. The distance *Ya* and the angles to any station of the underground survey must then be measured and when the line *ab* is connected with the surveys at daylight and below, the plumb-bobs may be removed.

The disadvantages of this method are that there is no absolute certainty that the point *Y* is in the line *ab* prolonged, and this want of certainty should not exist in so important a measurement.

The work must be performed by daylight, and the length of time necessary to complete it makes it impossible to work the shaft for at least ½ da., and may cause annoyance to the operators, or, if you are working for a lessee, lead them to refuse to let you have the use of the shaft at the time most suitable for your purpose.

Underground it may be hard to obtain a long sight on any line running through the larger axis of the shaft. Any shorter line will give too short a base line and will increase the chances of error.

**Case 2.**—Fig. 2 shows the top set of timbers in a shaft of two hoisting compartments, down which it is desired to carry a known course or meridian on the surface to the entry below. It is necessary first, to find out which side of the shaft is best adapted for setting up the transit, as the point to be marked in the mines will be vertically under the point on the surface; consequently, the side with the widest opening leading from the foot of the shaft should be selected.

Having carried the meridian to a convenient point near the top of the shaft, and having found that the south side of the shaft is the most accessible, determine, with an ordinary string, the location of the point A, from which the hangers for the plumb-lines will be exactly located, by means of the transit. Now mark with chalk on the timbers where the strings cross. These marks, though not accurate, serve as guides in setting the hangers. Make a permanent station at the point A and carry the meridian to it.

The hangers can be made of strap iron, ½ in. thick by 2 in. wide, and at least 16 in. long. In one end of the iron, have a jaw with a fine cut at the apex, or a drill hole just large enough to contain the wire to be used for plumbing. There should be two or three countersunk holes in the hanger, through which to fasten it to the timbers by means of heavy wire nails. A top view of the hanger is shown in Fig. 2. In most shafts there is a space from 2 to 4 in. wide between the ends of the cage and the sides of the timbers. In order to hoist and lower the



Fig. 2

cage to see that the wires are hanging freely, it is best to set the hangers in such a position on the timbers that the wires will hang in the middle of the space. The hangers should be permanently fastened over the chalk marks previously made on the north side of the shaft, with the jaws pointing toward A, and on the south side of the shaft the outer end of the hanger may be fastened temporarily.

Now, set the transit over the station at A, take the backsight, foresight on the wire hole of the hanger C and set the wire hole of the hanger B on the same line. Record this course and foresight on the wire hole of the hanger E, fixing as before the wire hole of the hanger D in the same line. Record this course, and then the meridian is to be carried into the workings below is established. Measure carefully and record the distances A to B, A to C, B to C, A to D, A to E, D to E, B to D, and C to E, in order to establish a point at the bottom of the shaft vertically below A, and check the work in the office.

The transit party can now descend to the bottom of the shaft, taking with it four buckets of oil, the weights or plumb-bobs to be attached to the wire, and all the surveying instruments, leaving a responsible person on the surface to handle the wires. Having arrived at the bottom of the shaft, the transit-man should have the cage hoisted 3 ft. above the landing, throw several planks across the timbers on which to set the buckets of oil, signal to the man on top to lower a wire and fasten it securely, passing it through the wire hole of the hanger, attach the plumb-bob and adjust the wire to such length that, when sustaining the full weight of the plumb-bob, the latter will not touch the bottom of the bucket. He should then place the weight in the oil, using care not to let the full weight come upon the wire with a jerk, but should let the weight down slowly, so that the wire will receive the full strain gradually. The three remaining wires should be set in a similar way.
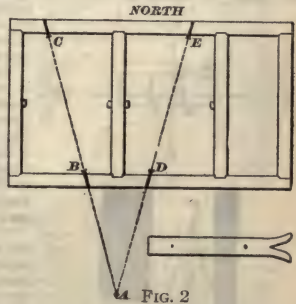
After the wires have been hanging a few minutes with the weights attached, the latter may move from one side to the other of the buckets. They should be watched carefully and the buckets moved until all the weights hang perfectly free, then everything should be let alone until the wires become steady. The cages can now be hoisted and lowered for the purpose of examining the wires to see that they hang free and plumb, care being taken that the cages are not brought so close to the landings as to disturb the hangers at the top, or the buckets at the bottom.

To find a point vertically below *A*, a string may be stretched along the wires *B* and *C*, care being taken not to touch them; another may be stretched along the wires *D* and *E*; then, with a plumb-line, a point on the bottom vertically below the intersection of the strings may be determined. The distances *AB*, *AD*, *BD*, and *CE* should then be measured and compared with the corresponding distances at the top of the shaft. If these distances compare favorably, the wires are, in all probability, steady, and the work of determining the desired course with the transit may be begun.

Set the transit up over the point of intersection just found; backsight on the wires *B* and *C*; foresight on the wires *D* and *E*, and compare the included angle and the distances with the corresponding angle and distances at the surface. If these do not correspond, move the transit in the direction necessary to increase or decrease the angle or distances, as the case may be. Repeat this operation until the exact point vertically below *A* is determined.



A simple device that is of great advantage is to have three links from an ordinary trace chain placed in the wires on the side toward the transit, and a few feet above the buckets. This not only enables the wires to turn freely, but also enables the transitman to sight through one of the links to the wire beyond, whereby he can place the transit in exact line with the wires more easily than if the links were not there.

**Case 3.**— Two, three, or four wires may be used, being secured and hung as before. They are located in the angles of the compartments *x*, Fig. 3. These are connected with four stations *A*, *B*, *C*, and *D*, the lines *AB* and *CD* being at right angles to each other for convenience in the subsequent calculation, and are connected with the outside survey. From *A* and *B*, taking *AB* as a base line, the points *x* are located. The same is repeated from *C* and *D*, taking *CD* as a base line. Thus are obtained four locations of each wire; these are tabulated, and any variations in a reading must be followed by a repetition of the same. The mean of the readings gives the location. (Subsequently, the subject of calculating work will be taken up.) It can be briefly stated that the bearings of each wire to each of the others, as referred to the base line of the survey, are then calculated and the distance between the wires accurately measured. This finishes the work at daylight.

There may be two general types of arrangements of the bottom of the shaft, and both arrangements have been sketched and lettered similarly. The first is a case when the shaft is arranged across the dip of the bed, and the second is parallel to the same. In both cases *0* and *7* are taken as far apart as possible, and all the wires *x* are located from each station with reference to the other. The

Fig. 3

distances between the wires above and below are also accurately measured as a check. There will be four locations of *0* and *7* from the four wires,
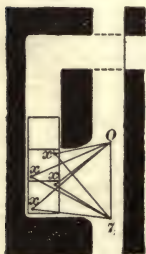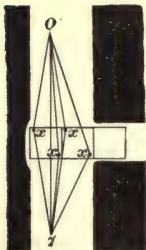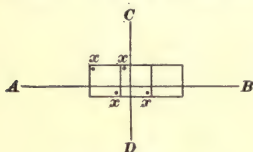
and the mean of these is taken as the correct one. In every case of angle measurement, a series of readings of each angle is taken on different parts of the graduated limb, to avoid instrumental errors, and the mean of these is taken as the true reading. From the locations of *0* and *7*, the course between them, as referred to the mean base line, is calculated, and *07* is the base line for the underground work. The angle readings above and below can be made at the same time with different instruments, and, in taking the readings below, it is not necessary to wait for absolute quiet in the wires, as that is seldom found. A small swing can be bisected by the cross-hair, and the readings are duplicated until a constant result is secured. By this method a greater accuracy and speed is obtained, and the angles below can be accurately measured, no matter how the shaft may be arranged.

**The T-Square Method.**—The T-square method of taking the line underground is especially valuable in shafts with several small compartments or

in cramped places where one cannot line in with the wires. The wires are placed in separate compartments and as far apart as possible. The apparatus is made by the carpenter, and consists of a straightedge and T squares. The former is merely a planed pine board about 8 in. $\times \frac{3}{4}$ in. and 1 ft. longer than the distance between the wires. It rests approximately horizontally on slats tacked across the shaft for supports. It is brought to about $\frac{1}{8}$ or $\frac{1}{16}$ in. from each wire and then nailed to the slats sufficiently to prevent slipping. One man should be at each wire. The T squares are most serviceable if made with a movable head clamped by a thumbscrew, and of planed pine, about $2\frac{1}{2}$ in. $\times \frac{1}{2}$ in. Except in cramped quarters, the T squares will be set at right angles, and should be placed together in clamping, to insure that each set is at precisely the same angle.
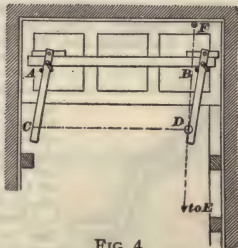


FIG. 4

Fig. 4 shows a cramped position, similar to what sometimes arises, in which the movable head gives more latitude in working. After clamping, the T squares are slid along the straightedge until close to the wire, but not touching it, and are there clamped by a G clamp, both men working at one T square. The ends of the T squares *C* and *D* must be supported on blocks so that the T squares lie approximately in the same horizontal plane. Everything up to the next step need be only approximately and quickly placed, but the greatest care must be exercised in measuring out equal distances *AC* and *BD* from the wires. If the wire vibrates, the middle of its swing must be determined by a pencil or pin. When holding a foot-mark (not the end of the tape) opposite the wire on the T square, an even number of feet should be measured, the point marked with a sharp pencil, and a pin inserted. When this has been done for both wires the parallelogram *ABDC* is obtained, in which the only essential is that *CD* shall be exactly parallel to *AB*, the line of the wires. The transit should now be set over the most convenient of the two points, as *D*. To get the azimuth of *DC* and, consequently, the line of the wires, sight must
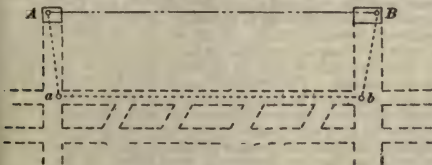


FIG. 5

be taken on a known point *E* for a backsight, and the angle *EDC* measured. Another point *F* must be established on the line of the backsight, in order that its course may be preserved after the instrument and T square are removed.

By this means the closing angle at *E* may be read after the wires are removed from the shaft. Underground, the method is the same, except that *DC* is the known course, and is used as the backsight. To give the coordinates of the instrument at *D* with the greatest precision, the angle *ADC* and the distance *AD* should be measured.

**Check Methods.**—The results obtained by the use of the methods just described should be checked as soon as an underground connection between

the hoisting shaft and air-shaft has been driven. Single wires are hung, one in each shaft, and the distance between them on the surface is measured. The latitude and departure of both *A* and *B*, Fig. 5, are known as they are tied in to the outside survey. In the mine, the stations *a* and *b* are occupied by the transit and the angles *A ab* and *abB* repeatedly measured. The underground distances *Aa*, *ab*, and *bB* are carefully measured. With these data, it is easily possible to calculate the latitude and departure of the mine stations *a* and *b*, and consequently the azimuth of the line *ab*, which serves as a base for all future extensions of the mine survey. It should be noted that Fig. 5 shows the simplest arrangement of the sights that is possible, but two; ordinarily there will be anywhere from four to ten or more stations required to carry the line from one shaft to another. As the surface and mine temperatures may be very different, particularly during the winter and summer months, a correction should be made for the expansion or contraction of the tape.

## SURVEYING SLOPES OR INCLINED SHAFTS

Where a single sight reaches from top to bottom of the slope, the problem is simple enough. A station can be established on the inside of the foot-wall plate at the collar and others in similar positions at each level. The instrument set up over any station can command the whole slope and the level opposite.
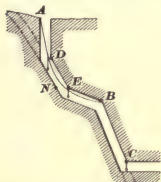


Where the slope is sunk on several dips, the survey is a much more difficult matter. Fig. 1 illustrates cases of common occurrence. The slope may be divided into sections like *ADEB*, which are convex downwards, and others such as *EBC*, which are concave downwards. As a rule a set-up can be avoided at the convex knuckles if desired, and need only be made at those that are concave.

**Bent Plumb-Line Method.**—Point *A* may be invisible from point *B*, but the survey may be carried from one point to the other by the *bent plumb-line method.*

Fig. 1

In this case, establish a station at *A*, the foot-wall side of the collar, the center point being a small nail head projecting horizontally. Attach a long plumb-line to this and carry the other end to *B*. Here it will probably be necessary to use a small screw-eye, with its head turned into the vertical plane of the slope for the center point. Pass the plumb-line through this and draw it fairly tight. Now attach a plumb-bob at an intermediate point and regulate the tautness so that the line is clear at all points. The curves in the slope may be such that two plumb-bobs may have to be hung, as at *D* and *E*, and even a third may become necessary.

As the plumb-line, perhaps 100 ft. long, is apt to be disturbed by the air-currents, it is often better to mark a point on a convenient timber near *D*, and another near *E*, so close to the string that there is no doubt of the points lying in exactly the same vertical plane as the plumb-line. If these points are once established, the string and weights can be taken out of the slope, leaving four points in the same vertical plane, and whose horizontal projections lie in the same course.

Now set up the transit at *A* and measure the azimuth angle from the backsight to *D*, thereby giving the bearing from *A* to *B*. If *D* should be invisible from *B*, depress the telescope after sighting on *D* and locate the point *N* in the same vertical plane, and so situated that it is visible from both *A* and *B*. Measure the vertical angle and distance to *N*. Now set up the transit at *B*, use the course *BE* for a backsight, and foresight to *C*. Measure the vertical angle and distance *BN*. The line *BN* might have been used as a backsight, and *E* serve as an additional check. The point *N* is really an intermediate station, but as it lies in the course *AB*, a set-up here is unnecessary. In simple cases, the bent plumb-line method is a very convenient one for carrying a survey from the surface to the first level, and a longer horizontal projection of the sight *AD* can be secured than if a set-up were made in the shaft at *D*; but in complicated cases, such as the one shown, it may often be quicker to make the extra set-up than to use the plumb-line. In all sights for determining azimuth, the vertical angles must be kept as low as possible, and the horizontal projection of the course long.

**Method by a Single Wire in the Slope.**—When surveying a slope by using a single wire, stretch a rather fine wire, free from kinks, down the slope, as shown in Fig. 2, being careful that it touches nowhere in the slope. Take two plumb-bobs provided with fine round strings and suspend one from *A* and the other

from *B* so that they nearly touch the same side of the wire *MN*. In order to have the plumb-lines as far apart as possible, the line at *B* must be quite long and a can of water provided to keep the bob from swinging. The plumb-line is fastened to a nail *B* nearly in the proper position. A bar of wood, with a block fastened to it, should be placed to one side and a little below *B*. The block must have a hole through which a small screw bolt can easily screw. On this bolt is placed a spool in which a groove is turned and which is sandpapered and greased so that the string will slip easily as the bolt is turned. Now, place the transit in line with the two plumb-bobs, as in an ordinary case of shaft plumbing. Repeat this operation below. The plumb-bobs in both cases hang in the same vertical plane and thus the true bearings are found underground. Even the plumb-lines may be dispensed with, but the method will not then be so accurate. The instrument will be set nearly in the vertical plane passing through the wire, leveled and sighted at *M*. The telescope should be dipped until the lowest point on the wire is visible and the amount by which the cross-hair and wire fail to coincide noted and the instrument shifted accordingly.



FIG. 2

But if this method is tried, the two points sighted at will not be nearly so far apart horizontally as the plumb-lines, and any error in leveling will also vitiate the result. This method of the single wire, however, provides no way of obtaining the coordinates.

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# UNDERGROUND, OR MINE, SURVEYING

## INTRODUCTION

The same instruments and the same methods of measuring and recording angles and distances are used on the surface and in the mine. The chief difference between surface and underground work is one of detail only. In the mine, lamps are needed to give light, sights are taken to a string or to the point of a plumb-bob, and stations are placed in the roof and not in the ground. The main-line notes are kept in the same way above and below ground, but the method of taking and recording side notes is materially different.

Mine surveying in flat coal seams differs in minor details from that used in pitching seams, and this difference in detail has been greatly exaggerated until it has become a prevailing belief that anthracite (pitching) and bituminous (flat, usually) mine surveying methods are absolutely distinct. There is, in all underground work, a difference in the organization of the corps and consequently in some minor details in carrying on the work, depending on whether the survey of an entire and extensive mine is to be undertaken or whether, as is commonly the case, the survey is made merely to connect recent developments with those previously surveyed.

## FLAT WORK

Because it is necessary for the gangways, or entries, in pitching work to follow the foldings and twistings of the seam, the physical difficulties in the way of the surveyor are greater in that kind of work than in flat seams. In pitching seams, short sights and many of them are the rule; in flat work, long sights and few are general. In fact, in flat seams the headings or entries are commonly *driven on points;* that is, they are driven upon some predetermined course for long distances, even 2 or 3 mi. in the case of large properties. At regular intervals, pairs of cross-, or butt, entries are driven at right angles to the main entries; from these, in turn, the rooms are driven. Hence, the surveyor in the average bituminous mine (bituminous mines are commonly flat) is chiefly concerned with prolonging straight lines for several thousand feet in the case of butt entries, and up to 2, 3, or 4 mi. and more in the case of long main entries. The angles measured are chiefly right angles at the points where a butt entry intersects the main entry, or at the mouths of rooms driven from a butt entry.

The bituminous-mine surveyor is very rarely called on to make a complete survey of underground workings. He usually *moves up the points* (sights) at more or less regular intervals of time or of distance driven, measures the distance from the old set of points to the new, takes the necessary side notes, and at once plots them; the mine map, so far as the main headings and cross-entries are concerned, is thus up to date within a month or so at the most.

In fact, whether the sights are moved up with regularity or not, the mine foreman is always able to determine the position of the working face by measuring back from it to the last survey station appearing on the tracing or blueprint furnished him, and adding this distance thereto, laying it off in the direction the heading is being driven.

**Stations.**—An essential part of the equipment of the mine surveyor is a carpenter's brace and from three to six bits, or drills, which can be made from files thrown away by the miner. With these, a hole from ¾ to 1 in. in diameter and from 1 to 1½ in. deep may be drilled in the roof where a station is necessary. From ten to fifty plugs should also be carried by the corps. These plugs are pieces of poplar or other softwood about ⅝ to ¾ in. longer than the drill hole. They are square in section, the side being a little longer than the diameter of the hole in which they are to be used; the lower end may be roughly rounded. After the hole is drilled, one of these plugs is fitted in and driven home by blows from the hatchet until from ½ to ¾ in. projects below the roof. A *spad*, *spud*, or as sometimes called a *nail*, is then driven into the plug. The spads or nails are made from mule-shoe nails, through the flattened heads of which ⅛-in. holes have been drilled or punched. These spads may also be purchased from dealers in surveyors' supplies, but are commonly made by the blacksmith's helper at odd times.

Sometimes stations are made by driving a tack or nail in a tie, which has been notched or flattened with a hatchet to receive it, or by driving a spad in the collar or cap-piece of a set of timbers. Neither of these methods is to be recommended, as the position of a tie, and with it that of the station tack, is practically sure to be changed in a few months from the pounding it receives from passing cars, mules, and men, or from alining the track. Stations in collars are less liable to be shifted than those in the track, but as the pressure due to mining operations comes upon the timbers they, too, are apt to be shifted out of line. Sometimes stations that must be set at an exact distance from a previous station (as when a butt entry is to be started, say, 225.25 ft. from the main entry station) are of necessity made in a tie if the roof at the exact spot is not in condition to hold a plug (or has been shot down for an overcast). In such a case the station is placed in the tie and as soon as the butt entry or other place has advanced sufficiently to have sight plugs set in, these are placed, and the use of the tack abandoned.

In longwall work, much difficulty is experienced at times through the stations being shifted out of line through the settling of the overlying rocks. This settling may extend over a period of years or may stop after a few months. While the settling is in progress, the timbers must frequently be renewed, so that they and the constantly disturbed roof and floor afford a poor place for permanent stations. Hence, during the period of settlement at least, stations in longwall mines must be placed in the floor. For the foregoing reasons, combined with heavings of the floor as the weight comes unevenly upon the pillars, these stations are almost certain to be disturbed, and should not be used to extend a previous survey until one or more older stations have been reoccupied by the transit and the angles and distances remeasured to be absolutely certain that no such disturbance has taken place. If displacement has occurred, it will be necessary to go back along the line until three stations have been found in their original position. From these a new set of lines must be run, using the old stations if solid enough, remeasuring the angles and distances, but not necessarily retaking the side notes.

Stations are commonly marked by enclosing them in a circle, square, or triangle, or by a + mark of white paint. White lead, or Dutch white, thinned with linseed oil is ordinarily used, and is applied with an ordinary sash tool. If the paint has to be kept a number of days, it should be covered with water, which should be poured off before the paint is used.

In addition to marking the station itself so that it may readily be found, some system of lettering or numbering must be devised so that the stations may be distinguished one from the other. Stations on the main entry frequently have the letter M prefixed and are numbered in regular order from the drift mouth, thus, M–1, M–2, etc. Or the stations on the main entry may use the letter A, those on its air-course the letter B, those on the manway the letter C, and similarly for other parallel entries. As butt entries are usually known by a number corresponding to the order in which they were turned off from the main entry, the stations on such entries, while numbered consecutively, have prefixed to them the letter R or L to indicate whether the entry is turned to the right or the left of the main entry, as well as the serial number proper to the entry. Thus, L1–12 or 1L–12 indicates the twelfth station

on the first left-hand cross-entry, or, as commonly said, *on the first left.* Similarly R5–6, R–56, or 5R–6, is the sixth station on the fifth cross-entry to the right. If entries are driven from one side only of the main entry, the prefixed letters R and L may be omitted. In this case, the foregoing stations will be numbered 1–12 and 5–6, respectively. Air-courses are not usually run on sights. If they are, the stations in them may be numbered in the same way as those on the heading proper with a letter, such as A, to denote the fact that they are on the air-course. Stations coming between regular stations on an entry commonly receive the number of the preceding station with some letter added. Thus 56 D is a station on the fifth cross-entry at a point between regular stations 6 and 7.

Some corps number the stations in a heading as are stakes in a railroad survey. The station at the mouth of the heading on the main entry is called 0, and the first station in the heading will be, say, 2+56.29, the second 4+81.96, etc. This means that these stations are 256.29 and 481.96 ft., respectively, from the mouth of the entry. The distance between the stations is, of course, 481.96−256.29 = 225.67 ft. While this system of numbering shows at a glance the exact distance an entry has been driven, such large numbers may be a source of trouble and of possible error when used to describe the backsight, instrument, and foresight stations as entered in the field notes.

Rarely are stations numbered in the order in which they chance to be placed. It may happen that station 56 will be on the main entry, 57 on the fifth right, and 58 on the tenth left and follow, say, stations numbered 36, 48, and 51. This system, or want of system, is not commended, as it leads to confusion.

While the numbers of stations are frequently painted on the roof alongside the station mark, it is better practice to paint them on the rib a short distance before the station is reached. In this way the foresight man can see them without having to walk in a stooping posture.

**Sighting.**—Sights in underground work are commonly taken either to the point of a plumb-bob suspended by a cord passed through the hole in the head of the spad marking the station, or to the cord itself. In the first case, a lamp is held a short distance back of the point of the bob to render it visible against a background of flame. In the latter case, the cord is best illuminated by placing a white paper or cardboard behind it and holding the lamp in front and to one side. The string shows as a dark line against a white ground, but care must be taken not to confuse the string with the shadow it casts upon the paper.

What are known as *plumb-lamps,* or *plummet lamps,* are a great convenience. These are very heavy plumb-bobs, suspended in gimbals to ensure verticality, which are hollowed to form an oil chamber, and are provided with a wick. The bisection of the lamp flame, if the wick is turned low, affords a sufficiently accurate setting of the transit when placing room sights, surveying rooms, and even in moving up sights in a butt entry, provided the backsight is not too close to the instrument and the flame is not greatly disturbed by the ventilating current. It is obvious that the use of the plummet lamp, which will burn for several hours and is always ready for backsighting on, saves the labor of one man. In accurate sighting, the point or the cord of the plumb-lamp may be used as just explained. Where the velocity of the air is great enough to cause the plumb-bob to oscillate, long narrow bobs must be used, and these must be shielded from the direct current by the backsight man. In drafty places, a hole may be dug out in the ballast between the ties and the bob let into this shelter, the sight being taken to the string. Cords for suspending plumb-bobs should be braided to avoid twisting under the weight of the bob, should be well-oiled, and should be hung with a slip knot so that the point may be raised or lowered.

Various means are employed to illuminate the cross-hairs in the telescope. Commonly this is done by holding a lamp a little beyond, above, and to one side of the object glass. Sometimes a reflector is provided. This consists of a piece of silvered metal, inclined at an angle of about 30°, that is soldered on one edge to a metal ring that fits around the object end of the telescope. The reflector has an oval hole through it to permit the passage of the line of sight. The reflection of the light carried in the surveyor's cap sufficiently illuminates the hairs to permit their being centered upon an object. In some rare cases, transits have the horizontal axis pierced (the opening being closed by a piece of glass), through which hole a lamp held at the end of the telescope axis will throw enough light to render the cross-hairs visible.

**Centers.**—If stations are made in the ties, the transit may be set up over them exactly as in outside work. However, stations are commonly made in the roof. The transit may be set under a station if a center mark is punched in the collar surrounding the telescope and to which the transverse axis is attached. When the telescope is level, this mark is immediately above the point of a plumb-bob suspended from beneath the instrument. By slightly loosening the leveling screws, the plate may be shifted until the center of the instrument is exactly below the point of a bob suspended from the station spad. However, care must be taken in tightening the leveling screws, that the instrument is not thrown out of line. Commonly, the station in the roof is transferred to what is called a *center* placed on the floor, and over this center the instrument is set in the ordinary way.

The center may be made from a large square or hexagonal nut some 2 in. across. A wire nail or spad is set in the center of the hole in the nut, which is filled with melted lead or Babbitt metal. The point of the nail is cut off about $\frac{1}{2}$ in. above the top of the nut and sharpened with a file. Or a very satisfactory center may be made by boring a hole $1\frac{1}{4}$ in. in diameter and 1 in. deep in a thick plank, setting a brad in the center with its head down, and filling in with melted lead, at the same time holding the brad in a vertical position with its end projecting slightly above the top of the plank. After cooling, the brad may be sharpened as before.

A plumb-bob is hung from the station and the string adjusted until the point of the bob just clears the point of the center. By sighting across the two points at an angle of 90°, the point in the center may be shifted into coincidence with the point of the bob. If the center has been lost or forgotten, a plank may be laid across the rails under the station and the position of the point of the plumb-bob marked upon it with a pencil. After removing the plumb-bob and its cord, the transit may be set over the center or pencil mark in the usual way.

**Placing Stations on Line.**—Two cases of placing stations on line may arise: (1) The station may be on the prolongation of the heading line and at any indefinite distance from the instrument. (2) The station may (or may not) be on the line of the heading, but is at a fixed, definite distance from the instrument.

The first is the common case when the last station on a straight heading is, say, 200 to 250 ft. back from the face, and a new station must be set ahead and on line so that sight plugs may be placed from it. The transit should be set up at the station nearest the face, a backsight taken on an outer station on the line, and the telescope plunged. Where the roof is solid and from 25 to 50 ft. back from the face, the foresight man should hold his lamp against the roof so that its flame may be lined in by the surveyor, a smear with the lampwick or an X with a piece of chalk should then be made on the roof when the line is secured. The point of the drill may be placed on this mark and the lamp held behind it. If not in line, the drill may be shifted to the right or left until it is. The hole should then be started by pressing upwards and turning the brace. As the point of the drill is apt to slip on smooth rock, as soon as the hole has a grip, another sight should be taken to make sure that the drill is still in line. If it is not in line, a new sight should be taken and a new hole started a few inches away. The hole should be drilled to the required depth and the plug driven firmly home. The point of a spad should then be stuck in the plug near its center with the head hanging vertically and the eye facing the transit. Holding the flame of the lamp behind the spad, it must be moved to the right or left until its point is exactly in line. Still holding the lamp behind it, the spad should be driven firmly home, the blows of the hatchet being inclined so that the head of the spad may be driven to the right or the left as the transitman, who is following the work, may direct. When the spad is driven home, the lamp should be held behind its eye and, with gentle taps of the hatchet, it should be knocked to the right or left until the vertical hair exactly bisects the hole. For many purposes, the alinement is now sufficiently accurate. For important work, however, the plumb-bob should be hung from the spad and the final and exact alinement made by sighting to its point or to the cord. The distance between the old and new stations should be measured and if, in addition, the side notes are taken the work may be placed upon the mine map at once.

The second case arises when a branch or cross-entry is driven from the main entry, either to meet a similar place being driven toward it or to intersect another place at a fixed point. The instrument is set up on the main entry at the point nearest to that from which the branch road is to be driven,

a backsight is taken upon an outer station, and the telescope plunged. The foresight must now be placed not only on the entry line, but at an exact distance, say, 186.27 ft., from the transit. The tape must be stretched in line, and at the exact distance from the station, a pencil mark 3 or 4 in. long made at right angles to the line of sight. This mark may be made on the face of a tie that has been cleaned off, if one is at the right distance; or on a piece of plank made to span the space between two ties. The point of a tack or a brad should be placed in this line and a lamp held behind it. The tack must then be shifted until it is in line and driven home. If the distance the branch road is to be driven is short and the station at its mouth will not be needed again, the instrument may be set over the tack, and after setting the vernier at 0 or at the back azimuth or bearing of the entry line, the deflection angle, azimuth, or bearing on which the new place is to be driven may be set off on the upper plate and upon this line the sight plugs may be placed. The surveyor is recommended to always use the system of reading and recording angles with which he is most familiar, except, of course, when the system is inaccurate.

If the station must be preserved (which is commonly the case), the position of the tack just placed in the tie must be transferred to a permanent station in the roof. To do this, leave the transit as it was after setting the tack. Suspend a plumb-line from the roof, holding the string in such a way between the thumb and forefinger that when the helper says the point is directly over the tack, a smear of chalk (previously applied to the thumb) may easily be made on the roof. Hold the drill on the mark and have it lined in from the transit. After the bit grips the rock, again line in from the transit and check the distance measurement by again dropping a plumb-line upon the tack. Drill the hole and drive home a plug, which may be of somewhat larger surface area than the ordinary station plug. Draw a pencil line across the face of the plug by joining the holes left by the points of two brads set near the inner and outer edges thereof and which have been lined in from the transit. (These brads need not be driven up, as they are used only for giving the ends of the line.) Next suspend the plumb-bob over the edge of a knife blade (the thumb is too blunt), so that the cord is on the pencil line, and when the helper, by sighting across the point and the tack, announces that they are directly over one another (that is, the distance is exact) make a mark, with a pencil, on the plug. A spad may now be placed in this mark and should be on the entry line and at the proper distance from the instrument. However, before driving it home, check the alinement. After driving home, perfect the alinement by sighting to the point of the bob or to its cord. Some prefer, after the plug is driven up, to suspend the plumb-bob from a spad, which is shifted by the transitman to the right or left until in the line of the heading, and by the foresight man's helper inbye and outbye along the heading line until at the exact distance, after which it is slowly driven up, being checked during the process. By the first method, there is no trouble in placing spads within less than $\frac{1}{100}$ ft. of the exact spot and any desired degree of accuracy may be obtained by taking time.

**Placing Sights.**—The entries always and the rooms usually, in flat seams, are *driven on sights*. These sights are a pair of plugs set on the pre-determined line about 2 ft. apart in which are driven spads that are set exactly on line. Pieces of coal, iron nuts, or other weights are hung from the spads and before an undercut is made the foreman, miner, or machine runner sights across the strings and marks the center line on the face with a piece of chalk. Under ordinary conditions, sights should be moved up about every 200 ft. of advance in the working face; although this distance may be increased where the ventilation is good and lessened where it is poor. While a pair of sights may be placed near the face from an instrument station 200 ft. or more back therefrom, the better practice is to place a station ahead as just explained. The transit is then moved to this station, a backsight taken as before, the telescope plunged and the two sight plugs placed some 10 to 15 ft. ahead of the instrument, the final alinement being effected by bisecting the two eyes. This makes the survey station independent of the sight plugs and the spad in it is not apt to be pulled out of line, as will be the case if the station is one of the sight plugs. The practical certainty of finding the station in good condition more than repays for the labor of setting the extra plug and making the extra set-up.

While entry sights are commonly and naturally placed in the center of the opening, many consider it better to place them to one side and over, say, the right-hand rail. By so doing, a conscientious track foreman can use them to

keep his own work in line and, being proud of a fine piece of work, will urge the miners to drive the headings straight. With heavy trains and motors and the high speed required for large outputs, the importance of a straight track is apparent.

When rooms are driven on sights, usually each room is given a pair of sight plugs, but in some cases only every other room is so provided, the intermediate room being kept in line, as well as possible, by leaving a pillar of constant thickness on each side, which thickness is determined by measuring through the cross-cuts. As the direction of rooms is rarely of prime importance, sights in them are not moved up, unless it is absolutely impossible to see the face from them by reason of smoke, roof falls, etc. Room sights are commonly placed in the necks anywhere from 8 to 20 ft. from the entry line. Surveyors prefer to wait until the necks have been turned for a number of rooms before placing the sights in any of them. The instrument is then set up at any convenient station and the line of sight made to coincide with that of the entry. A series of tacks is placed in line the proper distance apart (when rooms are turned at 90°, distance = width of room + width of pillar) and are driven down into the tie or into a plank laid across the rails, one tack for each room. The instrument is set up over each tack in succession and a right angle to the heading line is turned and two sight plugs are placed in the room neck as far from the entry as possible. The distance of each tack from the entry station is noted and the distance from the line of the entry to the outbye room sight plug is also measured and recorded. Room sights may be set by bisecting the eye of the spad and are commonly placed 18 in. to 2 ft. from the rib. Where rooms are inclined to the entry, the distance between their centers measured along the line of the entry is found from the formula:

$$\text{Distance} = \frac{\text{width of room} + \text{thickness of pillar}}{\sin \text{angle of inclination}}$$

**Surveying and Note Keeping.**—The laws of most states require that the mine workings be surveyed and mapped at least once every 6 mo. If the necessary measurements and side notes have been taken at the times the entry sights have been moved up and have been mapped, the mine, so far as the main roads are concerned, is always within a few weeks of being up to date. If this has not been done, the procedure will depend on whether lines of sight are carried up one or both entries of a pair. If each entry has its sights, the 0 end of the tape should be held at the last station appearing on the map and the tape stretched out to the next station. The surveyor may then walk along the tape and, when opposite a break-through, note the distance to both sides of the opening as say +256 to +267 (the opening being 11 ft. wide). The assistant should carry the 0 end of a tape (usually a 50-ft. metallic tape) to the rib at each side of the cross-cut, and the surveyor should measure the distance to the nearest ½-ft. mark to these points from the entry line and note whether the break-through has been driven to the right or left from the entry. Room necks may be located in the same way.

It is advisable, where the entry is crooked, to note the places where the entry comes nearest and farthest from the rib. Many surveyors do not take offsets to the corners of the pillars made by the various openings, merely noting these *plusses*, or distance measured along the tape opposite which these openings come. When so mapped, the entries appear perfectly straight, which makes an attractive but inaccurate map. When the exact distance between stations as well as the necessary vertical angles have been taken at the time the stations were moved up, the use of a transit is unnecessary when making the entry surveys, but it is highly advisable to remeasure these distances as a valuable, in fact as the only possible, check on the original distance measurements until a close is made and the survey calculated.

After taking the notes between the first pair of stations, those between the next pair should be taken, and similarly until the foresight man with the tape reel is at the last station. This may be anywhere from 25 to 200 ft. from the face. To get the entry line, the foresight man, carrying the reel, should be sent ahead; and when he is at the face, the tape may be brought in line by sighting over the entry sight plugs to a lamp held on the reel. After completing the side notes on one entry, those on the parallel entry or entries are taken. These notes are entered in tabular form as taken, beginning at the bottom of the page and working toward the top. The *plusses*, or distances from the instrument, are in one column and in other columns are the offsets to the corners of the openings, each placed in the horizontal line of the proper *plus*.

Each surveyor will have his own set of abbreviations: Common ones are Bt for break-through (or cc for cross-cut), Rm for room, r or rb for rib. Thus +256, 5.5 Btr, means that at 256 ft. from the station and 5.5 from the line is the corner of a break-through, which is driven to the right. Sometimes



Fig. 1

these notes are amplified and illustrated by sketches as shown in Figs. 1 and 2. In most mines, particularly in the case of butt entries, only the room entry is driven on points, the air-course being kept as nearly as possible to a parallel line by maintaining a constant thickness of pillar between it and the room entry. In this case, the distance from the tape or the offset is measured not only to the edge of the cross-cut, but also to the corresponding edge of it on the air-course and to the far side of the air-course as well. These three measurements might be recorded as follows: +254, 5—26—37 and +263, 6—26—36. The plusses are at 254 and 263 ft. from the station and show that the break through is 9 ft. wide. At the first plus, the distance to the corner of the break-through is 5 ft., the distance to the corresponding corner on the air-course is 26 ft. (the pillar being 26−5=21 ft. thick), and 37 ft. to the rib of the air-course, which is 37 −26=11 ft. wide. At the inner side of the cross-cut the pillar is 26−6=20 ft. thick, and the air-course 36−26=10 ft. wide. The advantage in measuring continuously out from the tape is twofold: The surveyor does not have to leave the entry, and consequently has time to enter his notes in a concise and cleanly way as the foresight man does the running around and, above all, by standing on the entry he is able to keep the small tape with which the side notes are taken, exactly at right angles to the large tape; further, notes thus kept are easier to map, requiring but one setting of the scale, regardless of the number of offsets taken from any plus.
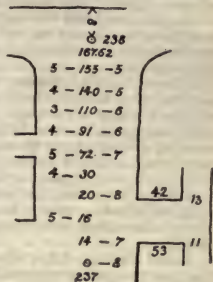
It is usual to make sketches as the side notes are taken, in order to illustrate and make plain any points that might be obscure to the office man that plots the notes. Two forms of such notes are shown in Figs. 1 and 2; both are of the same entry, the air-course parallel to which is located by measurements through the break-throughs. Such notes are begun at the bottom of a page and sketched upwards in the order in which they are taken. The form shown in Fig. 2 is preferable as being the clearer. It will be noticed that in both forms



Fig. 2

the air-course is located not by continuous offsets, but by single measurements through the pillar, the disadvantages of which method have been explained. The ends of pillars are so rarely square that it is commonly difficult to decide at just what point a break-through begins. This is illustrated at *a* and *b*, Fig. 2. The proper way to locate such a rounded pillar is to take a plus and offset at the point where the pillar begins to round (+155, *a*) and a second plus and offset where a sight tangent to the end of the pillar may be had (+157, *b*). The distance between the stations is always noted on the sketch. In the case illustrated, the distance from the last station to the face is shown (8 ft.) beyond the figures for the length of the line.

The survey of a series of rooms in which sights have been placed is a comparatively simple matter. The sight strings are lowered (the miner generally keeps them hung up against the rib), and the helper goes to the face, unwinding the 400-ft. tape as he goes. The 0 end of the tape is held at the first or outer sight plug, the distance of which from the line of the entry was measured when the sights were placed. The transitman, by means of the room sights, places the man at the face in line and takes the side notes in the manner explained. If the rooms have not been driven on sights, it is customary to place a tack in a tie at the mouth of each room, the tacks all being on the entry line, but at irregular distances apart as they are placed so that the instrumentman may see the face. After the tacks are lined in, their respective distances from the instrument are measured and recorded.
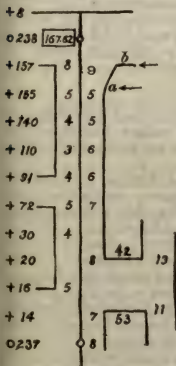
The instrument is set over the tack at the first room, the vernier is set at the azimuth or bearing of the entry, and a backsight is taken upon some station. The foresight man unwinds the tape on his way to the face, where he holds the reel on an **X** he has marked on the coal in chalk, and to which the transitman takes a sight. The 0 of the tape being at the instrument, a line is established to the face, and the side notes may be taken as before explained. In many instances a line is run up every other room only (thus, up rooms 1, 3, 5, 7, etc.) the intermediate rooms being located with sufficient accuracy by offsets through the cross-cuts.

It sometimes happens that practically all the rooms on an entry are so blocked with falls of slate that it is impossible to see to the face and thus survey each room separately. In such a case, a line is run from the entry up some room not blocked by falls and a survey made of the faces of the rooms by running a line along them through the break-throughs. If desired, stations may be established in ties in each room near the face, and sights taken down the room until stopped by the falls; or a line may be run down every other room. Usually, offsets from the line along the face will locate the pillars and face line with sufficient accuracy for all practical purposes, particularly if a good portion of the rooms have been mapped from surveys made before the fall of roof took place.

**Level Notes.**—All level notes are kept as in outside work, with the exception that, as the rod is reversed in getting the elevation of a station in the roof, the record of the reading is prefixed with a minus sign. A record of such a reversed rod, when the target is 3.78 ft. below the station, is recorded $-3.78$.

A shaft is measured (if deep) by a fine steel wire running about an accurately graduated wheel (a sufficient number of turns being laid to prevent slipping) and noting the number of turns before the bottom is reached. The wire may be measured before and after the operation, to insure against stretching. An aneroid mining barometer, if in good condition, will give quite accurate results if a number of trips are made between top and bottom, to give an average. In this case the barometer must be left quiet 10 or 15 min., to be sure that it has expanded or contracted to the proper degree. For rough measurements, the length of the winding rope between top and bottom is taken.

By one of these methods a bench mark should be located below, connected with the outside work, and referred to tide water. The rod must be reversed to get the elevation of all stations in the roof, and all such readings are noted with the minus sign, as $-4.32'$ (read 4.32 ft. below station). Roof stations are almost certain to settle, from the pressure of the superincumbent rocks. To check such settling, the distance from roof to floor must be accurately measured. Some measure from floor to rail of track; this is inaccurate, as the track may be shifted or the grade changed in making repairs, or to take out a sag. Whenever a level survey is begun the distance between roof and floor should be measured to see if it agrees with the notes. If it differs, the fact should be stated under the original notes, as a check for future work.

## PITCHING WORK

The survey of workings in highly inclined coal seams does not differ in methods from those employed in surveying mines in flat seams, but there are sundry minor modifications in detail varying from mine to mine, as peculiar or local difficulties have to be overcome.

**Stations.**—The seams are usually folded along the line of strike so that the entries, or gangways, that are driven approximately upon a water-level, are curved and crooked to the same extent as the seam. For this reason, gangways cannot be driven upon sights, and stations are established as needed as the survey proceeds. As these stations are placed solely with a view to obtaining as long or as many sights from one point as possible and as the spads used do not have to be set exactly on line, much smaller drills and plugs may be used in pitching than in flat work. In some cases, the holes are only $\frac{1}{8}$ to $\frac{1}{2}$ in. in diameter and but $\frac{1}{2}$ in. to 1 in. long as maximum. Various devices for quickly establishing these more or less temporary stations have been adopted in the anthracite regions of Pennsylvania, some of which are here given.

1. The simplest top station is a shallow conical hole, made with the point of the foresight man's hatchet, which is dug into the top rock and rotated, and is called by some a *jigger* station. Corps using these entirely have a jigger consisting of a steel-pointed extension rod, with an offset holding a paint brush. The rod is long enough to allow the point to be driven into the roof at any height, and its rotation marks a circle with the brush, which is also used to mark the number beside it. Centers are set under such

stations and sights are given by another tool—also called a jigger. This is an extension rod, beyond the upper end of which projects a piece of sheet iron shaped like an isoceles triangle, with the upper and smaller angle cut off so as to form an end $\frac{1}{4}$ in. broad, and in this end is cut a **U**-shaped groove.

The sights are given and the centers set by putting the plummet cord in this groove, and placing the end in the jigger hole in the roof. The cord must be more than twice the length of the section of the place, as it must be held in the hand, run over the jigger notch, and hung vertically to the plummet, which must come to the floor when the stations are set. The rod and cord are held in the left hand, and the right is free to steady the bob, give sight, or set the center. The advantages are the quickness with which the centers are set and the sights given, and the ease with which the highest stations are reached. The disadvantages are the impossibility of making the jigger hole perfectly conical, so that the jigger can be set in the same place on two successive sights, and the plummet cord will hang exactly in the same place.

2. A twist drill $\frac{3}{8}$ in. in diameter is used to make a hole in the roof; a piece of cord—or, better, a copper wire—is placed across this, and a hardwood shoe peg is driven into the hole and binds the cord tight. The plummet is tied to the lower end. A cord will soon rot, and, if in the gangway, is pulled out by the drivers for whip lashes, while the wire is more permanent; but even this will be pulled out by catching in the topping of a car in a low place.

3. The use of spads is dispensed with, and all the stations put in rock roof where possible. A $\frac{1}{2}$-in. twist drill makes a vertical hole 1 in. deep. Into this, when a sight is to be taken, the foresight man puts a steel clip with serrated edges. This is made by bending upon itself a thin piece of steel $\frac{3}{16}$ in. wide. When the ends are pressed together it will go into the hole, and the spring of the sides and the serrated edges hold the clip in the hole so that it is hard to pull out. The cord passes through a hole in the center of the bend and is, therefore, in the center of the hole—no matter how the clip is inserted. It is removed by pressing together the ends of the clip. This is the easiest and quickest way of working, as there is no eyehole to be freed from dirt and no knot to be tied and untied. The hanging of the plummet takes a fraction of a second, and the station will remain as long as the roof keeps up. The disadvantages are the putting of the holes inclined to the vertical by a careless man, and the many roofs that are unfit for piercing with a twist drill.

Stations are generally marked upon some regular system, as in flat work, although in some mines the objectionable practice of numbering stations at random as they happen to be placed still prevails. In the case of leased properties two surveys will commonly be made, one by the operator and another by the land owner. When this happens, each corps will have its distinctive mark as, for example, the one a circle and the other a cross (+), with possibly a distinguishing letter selected from the name of the corps as a further means of identification. If both corps use the same station each will place about it its distinguishing sign and number, and the notes will state "Sta. 617 = Sta. 432 of (          ) Corps."

**Surveying Methods.**—The method of surveying gangways and keeping notes does not differ from that employed in flat seams, except from the fact that three consecutive stations not being in line, a deflection angle and bearing or azimuth must be read at each set up. As the grade between stations may be, in fact commonly is, pronounced, particular attention must be paid to reading the vertical angle. Parallel entries (room entry and its air-course) are commonly at such a distance above or below one another that it is not usually possible to locate the one by measurements made through the crosscuts from the other and a separate line must be run in each. In case it is possible to locate the air-course by means of offsets from the main gangway, a clinometer, frequently a brass protractor with a plummet attached, must be hung from the stretched tape to give its angle of inclination. All such inclined offset sights must be reduced to the horizontal before being mapped.

If the seam pitches more than 30°, the rooms are worked with batteries; the heavy timbers forming these usually preclude the possibility of sighting from the gangway to the face. Work of this kind is surveyed by lines out the gangway and back through the faces of the rooms, which are generally clear of timber. The line along the face should be tied into the gangway line as soon as opportunity offers.

If the seam makes much gas, sights must be taken to safety lamps unless the portable battery hat lamps are used. The latter afford a very satisfactory light and, being absolutely clean, are preferable not only to the ordinary safety lamp in gaseous mines, but to the oil lamp in any mine.

The angle of dip of the seam should be taken at each station and at intermediate points if it changes radically. The thickness and quality of the coal should be observed frequently and changes of importance noted on the map.

**Locating Pillars for Surface Support.**—It is customary to leave unmined pillars of coal to support important buildings, reservoirs, etc., on the surface. The usual method of locating these pillars is to extend vertical planes through the boundary lines of such objects, and leave untouched all parts of the superincumbent beds embraced by those planes. This is accurate only when the strata are horizontal or vertical, as beds settle normally to the planes of the strata and not in a vertical line in case the open spaces are stowed. If the spaces are left open, they are first filled by falls, and then the settling goes on according to the above rule. No cut is necessary to show the method of settling, and the place where the bed is to be left untouched may be found as follows: *Draw a vertical section through the point to be supported, and also the underlying bed on the line of the dip of the bed—the section being accurately drawn to any scale. Draw through the extremities of the object to be supported, lines to the bed, which will make right angles with it. The space included will give the dimension of the pillar measured along the dip of the bed, and the dimensions of the object taken at right angles to the first plane will give the other dimension of the pillar.*

## MINE CORPS

The number of men required in making a mine survey and the nature of their duties depend on the nature of the work to be performed. If sights are to be moved up two men, the transitman and foresight man, can do the work; but if distances are to be measured, a third man is advisable to assist with the tape if time is an object. The third man is essential if stations are to be set at exact distances from the instrument. In all ordinary survey work where offsets are to be taken, four men are essential and five are advisable. There must be two men to hold the long tape between stations on the entry, and two to hold the tape with which the offsets are taken, one of whom may be the transitman, but it is better to have a special crew for taking offsets, leaving the surveyor free to record the notes, determine the position of the stations, etc. Much time will be saved if one of the four men can set up the transit and read the angles.

When making a complete survey of an extensive property, particularly in pitching work where short sights are the rule and branch gangways on divers grades are common, it is a material help to place the survey stations before an attempt is made to measure the angles or distances. To do this,
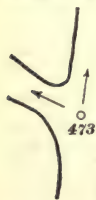


the transitman will require two assistants and the services of a mine foreman or other official familiar with the workings and who will, in emergencies, hold a lamp where needed. The backsight man remains at the station from which the survey is to start and the party goes ahead to the most distant point therefrom that the lamp is visible. At this point, a station is established in the roof with the drill, plug, and spad, and its proper number painted on the rib or roof. The backsight man comes up to the new station and the party goes ahead locating a second and succeeding station or stations as may be needed. Very frequently several sights must be taken from one station, a common case being that shown in the accompanying figure, where the road forks. Here a helper is sent up each branch, the mine foreman holding his lamp at the backsight, and the transitman shifts his position until he finds a point from which the three lamps are visible and there establishes a station. For this work the transit is not necessary, only a bucket holding the brace, and drills, plugs, spads, and a hatchet, and possibly a 100-ft. tape, being taken into the mine.

## CARE OF INSTRUMENTS

The transit should be removed from the tripod and placed in the instrument box with its plates unclamped when not in use. When going to and from work, the transit should not be carried on the transit head, or the spindle will become sprung. Nor should it be carried with the arm crooked under the telescope, as the weight comes on the axis, and that soon gets sprung so that all the adjusting in the world will not make it work right. When carried in the hand, it should be reversed and the hand slipped under the compass plate and brought over so as to clamp both plates. In this way there will be no strain on any part. The person carrying the transit should be the first to

ascend a slope or any pitching place and the last to descend, so that loose stones or dirt that may be dislodged may not affect or endanger the instrument or trip the carrier. He must be sure that the tripod head is screwed firmly on the tripod. The possible slip of the instrument through not observing this caution may be a source of trouble in the failure to agree of the duplicate angles read at each station. As soon as the corps comes back from the mine, the tape must be stretched, tested, wiped, and oiled. It can be inspected to see if marks are too much worn, or it stands in need of mending, the marking pot is cleared of muck, and fresh white paint is mixed, if the corps is going out in 24 hr.; the plummets will have their strings overhauled and freed from knots; hatchets will be sharpened, and axes ground, pouches overhauled, and a supply of tacks or spads taken. The transit is set up and wiped with a cloth wet with alcohol, so as to remove dirt, oil, and paint. If water has gotten between the graduated limb and compass box, the verniers must be uncovered and the whole wiped dry. If sulphureted hydrogen from the powder smoke has tarnished the silver surfaces of any of the graduated circles, it must be removed with whiting. Alcohol should be always used instead of water, as it will quickly evaporate and leave the parts dry. The telescope glasses are then wiped with soft chamois leather, and the instrument is tested for want of adjustment before it is put away in its box.

How often the transit will require adjusting depends on the quality of the instrument and the care it receives when in use. When moving up sights by backsighting and plunging the telescope, the adjustment of the vertical hair must be perfect or the foresight will be to the right or to the left of the prolongation of the line joining the backsight and instrument stations; this adjustment is, thence, of prime importance to the surveyor in flat work, who is chiefly occupied in moving up sights as the workings advance. This adjustment is also of importance when reading deflection angles by the methods explained. Sights may be set without regarding the adjustment of the vertical hair, by setting the vernier at 0°, backsighting, and turning off an angle of 180°, but this involves two accurate readings and settings of the vernier. Deflection angles may be determined in the same way, by subtracting the included angle from 180°, and with the same objections to the method. A method sometimes employed to move up sights, which is independent of the cross-hair adjustment and does not require the reading of an angle is as follows: Assume that stations numbered, say, 200 and 201 are those nearest the face and that Sta. 202 is to be placed on the line 200–201 prolonged. Set up at Sta. 200, and foresight upon Sta. 201, remove the plumb-bob and cord from Sta. 201 and set Sta. 202 at the proper distance ahead on the line thus prolonged.

For plumbing wet shafts, kerosene resists the extinguishing power of water better than fish oil, and is less readily blown out by a strong ventilating current. It makes more smoke, and, in tight headings, or mines with poor ventilation, with a large party, fouls the air much more readily than fish oil. Sometimes a mixture of the two is burnt in very drafty places, where it is hard to maintain a light. Kerosene is burned in the plummet lamp unless it is used with the safety attachment. Sweet oil, or any oil burning without smoke, must then be used. Smoke clogs the openings in the gauze, restricts the entry and escape of gases, and, especially if the gauze is damp with oil, may ignite and communicate the flame from within to the outside body of gas.

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# TRAVERSING AND MAPPING

## TRAVERSING

The *latitude* of a point is its distance north or south of some *parallel of latitude*, or line running east and west. The *departure* of a point is its distance east or west of some *meridian*, or line running north and south; it is the same as the *longitude* of the point. Latitudes are measured in a direction at right angles to the departures. The distance that one end of a line is due north or south of the other end is the difference of latitude of the two ends of the line, and is called the *northing* or *southing*, or simply the latitude of the end considered. The distance that one end of a line is due east or west of the other end is the difference in longitude of the two ends of the line, and is called the *easting* or *westing*, or simply the departure.

The process of calculating and tabulating the latitudes and departures of the courses of a survey is known as *traversing the survey*. To do this, all distances

must either be measured horizontally or be reduced to horizontal distances by means of the vertical angle. The horizontal angles must either be read as quadrant courses, or must be reduced from azimuth to quadrant courses.

$$\text{Latitude} = \text{distance} \times \cos \text{ of bearing}$$
$$\text{Departure} = \text{distance} \times \sin \text{ of bearing}$$

Below is given, in tabular form, the calculated notes of a closed compass survey. All the work shown should be kept in ink in the permanent office record books. The notes in the first three columns, headed Station, Bearing, Distance, are the same as the corresponding columns of the field notes. If the field notes show that the distances were measured along the

TRAVERSED SURVEY NOTES

| Station | Bearing | Distance | Latitude | | Departure | | Total Latitude | | Total Departure | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | North | South | East | West | North | South | East | West |
| 1–2 | N 35° E | 270.00 | 221 | | 155 | | 221 | | 155 | |
| 2–3 | N 83° 30′ E | 129.00 | 15 | | 128 | | 236 | | 283 | |
| 3–4 | S 57° E | 222.00 | | 121 | 186 | | 115 | | 469 | |
| 4–5 | S 34° 15′ W | 355.00 | | 293 | | 200 | | 178 | 269 | |
| 5–1 | N 56° 30′ W | 322.56 | 178 | | | 269 | 0 | 0 | 0 | 0 |
| | | | 414 | 414 | 469 | 469 | | | | |

slope, as would be the case in an ordinary transit survey, two extra columns should be provided, one for the measured distances and another for the vertical angles. If the elevations are to be deduced from the vertical angles, something that is necessary if a topographic map of the property is to be made, two additional columns will be needed, in one of which should be placed the differences in elevation of consecutive stations, and in the other, the total elevation of each station above sea level.

From the latitudes and departures of the individual stations, it is customary to determine the latitude and departure of each station with reference to the first station of the survey. These are commonly called the *total latitudes* and *total departures*, or *total northings, southings, eastings, or westings*, as may be.

The latitudes and departures of the individual stations are calculated by the formulas given. The total latitudes and departures are obtained by adding continuously and algebraically to the assumed latitude and departure of the first station, the latitudes and departures of the individual stations. The first station is frequently called the origin of coordinates, and its northing, southing, easting, and westing are commonly taken as zero (0). As a check on entering the latitudes and departures in the right columns, it should be noted that when the bearing is less than 45°, the departure is less than the latitude; and when the bearing is greater than 45°, the departure is greater than the latitude.

**Errors in Closure.**—If the survey is a continuous one around a tract, and ending at the place of beginning, the sum of the northings should equal the sum of the southings, and the sum of the eastings should equal the sum of the westings. Or, in other words, the sum of all the latitudes north, should equal the sum of all the latitudes south; and the sum of all the departures east, should equal the sum of all the departures west. It is evident that by coming back to the place of beginning the surveyor has traveled the same distance north as he has south, and the same distance east as he has west. However, in practice, as has been intimated under the heading Closing Surveys, no such agreement is possible. In fact, should a survey actually balance or close, it should be assumed that the closure is apparent and not real; the sum of the errors in one direction being exactly offset by the sum of the errors in the opposite direction.

The *error in closure* of a survey is the ratio that the length of the line joining the initial and final stations (as determined by the survey) bears to the

entire distance run. The length of this line is that of the hypotenuse of a right-angled triangle of which the errors in latitude and departure are the two sides. Thus, if the coordinates of the starting point are 0, and after running around a tract of land a distance, by survey of, say, 25,000 ft., it is found that the total eastings exceed the total westings by 4.25 ft., and that the total northings exceed the total southings by 1.5 ft., the survey will have failed to close by $\sqrt{4.25^2 + 1.50^2} = 4.51$ ft. The error in closure will be $25,000 \div 4.51 = 1$ ft. in 5,543 ft. (about). The bearing of the line of error (as it may be called) may be found from the formula:

$$\text{Tan bearing} = \frac{\text{error in departure}}{\text{error in latitude}} = \frac{4.25}{1.5} = 2.83333,$$

from which the bearing is N 70° 34′ E. That is to say, owing to errors in measurement, the final point instead of coinciding with the initial point, is found to be N 70° 34′ E, 4.51 ft. from it.

**Balancing Surveys.**—In surveys made with the compass and chain, it may be safely assumed that the failure to close is as much due to errors in angular measurement as in chaining. In this case, the latitudes and departures may each be corrected by certain amounts, some being increased and other being decreased, until a perfect balance is secured between the northings and southings on the one hand and between the eastings and the westings on the other by means of the following rule:

**Rule I.**—*The correction to be applied to any particular latitude or departure is to the total error in latitude or departure as the corresponding distance is to the entire distance covered by the survey.*

Each correction is to be applied in such a way as to diminish the whole error at the particular station.

In the case of surveys made with the transit, the angular measurements are highly accurate and it is very probable that errors in closing are due almost entirely to incorrect chaining. This is particularly so if the sum of the deflection angles is 360° (in which case the survey closes exactly in angle) or is not more than 1′ different for each mile or two (averaging, say, fourteen stations per mile) surveyed. In this case the rule for determining the corrections to be applied to each individual latitude or departure is:

**Rule II.**—*The correction to be applied to any particular latitude or departure is to the whole error in latitude or departure as the corresponding latitude or departure is to the arithmetical sum of all the latitudes or departures.*

As before, each correction should be so applied as to diminish the whole error at each station.

**Locating Special Work.**—The rules given for finding the error in closure of a survey, as well as its bearing, are applied to determine the length and bearing of a line (as that of a tunnel or entry) required to connect two points whose latitudes and departures are known. Thus, suppose that it is required to connect Sta. 57, whose latitude is 2,046.25 N and departure 18.76 E, with Sta. 49 whose latitude is 1,625.75 N and departure 159.26 E. It is apparent that Sta. 49 is $2,046.25 - 1,625.75 = 420.50$ ft. south, and $159.26 - 18.76 = 140.50$ ft. east of Sta. 57. The distance between the two stations is $\sqrt{420.50^2 + 140.50^2} = 443.35$ ft. Again,

$$\text{Tan bearing} = \frac{\text{difference in departures}}{\text{difference in latitudes}} = \frac{140.5}{420.5} = .33413$$

whence the angle is 18° 29′. As Sta. 49 is south and east of Sta. 57, the bearing and length of the line joining Sta. 57 and Sta. 49 is S 18° 29′ E, 443.35 ft. It must be noted that the exact tangent of 18° 29′ is .33427, or .00014 more than the calculated one. In the distance between the stations, 443.35 ft., a line run on a bearing of S 18° 29′ E will miss Sta. 49 by .06 ft. Hence, an exact closure cannot generally be obtained with an instrument graduated to minutes only. The distance between the stations may be found without having to extract the square root, the bearing having been obtained, by the formula

$$\text{Distance} = \frac{\text{difference in latitudes}}{\cos \text{ of bearing}} = \frac{420.5}{.94842} = 443.36 \text{ ft.}$$

# MAPPING

**Laying Off a Map.**—It is very commonly the case that a mining property has its greatest linear dimension in any other direction than an east-and-west line. Thus a property containing, say, 2,000 A., might have approximate dimensions of $2\frac{1}{2}$ mi. in a general northeast and southwest direction, and of

$1\frac{1}{2}$ mi. at right angles thereto. Mine maps are required by the laws of most states to be on a scale of 100 ft. to 1 in., although 200 ft. to 1 in. is permissible in some cases. On the larger scale, the property just described would have a northeast and southwest length of 128 in. (10 ft. 8 in.) and a length at right angles thereto of 79.2 in. (6 ft. 7.2 in.). It is apparent that if such a property is mapped with its meridian at right angles to the length of the paper (that is like ordinary maps in an atlas with the north toward the top), a goodly portion of the survey will extend both above and below the top and bottom edges of any paper now made for draftsmen's use. Such a property must be laid down with its longest dimension parallel to the longest dimension of the paper, regardless of the direction of the meridian.

To determine the best way to lay off the map on the paper, it is customary to make a skeleton map of the property on a small scale, say, on one of 1,000 ft. to 1 in. (in the foregoing case the dimensions would be 12.8 in.×7.92 in.) and lay this upon a sheet of paper that represents, on the same scale, the paper to be used for the finished map. By shifting one upon another, a position will eventually be found where the property may be drawn upon the sheet. By pricking through with a needle point, the stations may be transferred from the skeleton map to the sheet representing the drawing paper, and the connecting lines drawn. A border should be drawn around this minature map at one-tenth the distance from the edge that the border will be from the edge of the large map. There will now be available for laying off the paper, a minature reproduction of the outlines of the finished map. To draw the coordinate lines, lay off upon the large sheet, using their location on the small map as a guide, the most easterly and most westerly corners of the property. Any other two corners will do, provided they are separated by as long a distance as is conveniently possible. Connect the selected corners by a line and calculate the bearing thereof. If it is assumed that this line has a bearing of N 58° 30′ E, all lines making an angle of 58° 30′ to the left of this base will be north-and-south lines, or meridians.

When mapping extensive surveys, it is a slow and usually an inaccurate process to measure from the initial station the total latitude and departure by which every other station is located, as many of the distances will be very long, from 5,000 to 10,000 ft. or more (from 50 to 100 in. on the scale of the map); therefore, mine maps are laid off in a series of squares 1,000 ft. (10 in.) on edge with their sides in the meridian. To locate a station whose coordinates are, say, latitude 8,250 N and departure 6,500 E, measure along the meridian marked 6,000 a distance 250 ft. north of its intersection with the parallel marked 8,000. At this point erect a perpendicular to the meridian (or draw a line parallel to the latitude) and lay off along it a distance of 500 ft. to the east. The point thus plotted will have the coordinates in question.

To draw these squares, place a meridian, determined by the method explained, upon the map somewhere near the middle. Upon this, mark a series of points exactly 10 in. apart. Through the extreme points draw perpendiculars to the meridian by any of the methods of geometry. Upon these parallels lay off spaces of 10 in. both east and west from the meridian and through these points draw the remaining meridians. On the most eastern and western meridian thus established, lay off further spaces of 10 in., the points marking which may be connected with those on the first meridian laid down upon the map, thus completing the work. These squares should be laid off in pencil and lightly inked in with the utmost accuracy. The work should be done during a single day when conditions of temperature and humidity are as nearly constant as possible, as atmospheric changes will cause paper to expand or contract and thus change the size of the squares. In the case of large properties, the proper placing of the meridians and parallels so that all the corners, etc. will come on one sheet of paper is a matter of painstaking work. Often the shifting of the meridians or parallels 1 or 2 in. either way will accomplish this much to be desired result; and this can only be done by cut-and-try methods combined with more or less calculation and recalculation of the coordinates of the extreme points as the meridians and parallels are shifted. If the map will extend over upon a second sheet, this should be laid off in squares in a similar manner to the first, and should have laid down upon it enough of the workings, etc., appearing last upon the first sheet, that it may be used independently of it. In other words, the second sheet of the map should overlap for 2 or 3 in. on that of the first sheet.

The question of numbering the meridians and parallels or, what is in effect the same thing, determining the location of the zero of coordinates, is a matter of importance. In most maps, some one meridian will be marked 0,

those to the right of it will be designated as 1 E, 2 E, 3 E, etc., and those to the left, 1 W, 2 W, 3 W, etc. Similarly, some one parallel will be marked 0, and those above and below it will be respectively 1 N, 2 N, 3 N, etc., and 1 S, 2 S, 3 S, etc. A better plan is to call the most westerly meridian and most southerly parallel 0. In this way all the latitudes will be north and all the departures will be east, all additions made to determine the total latitudes and departures will be algebraic without shifting from one column to another and there will be but two columns for the total latitudes and departures instead of four. Under this plan, there is much less liability to error when making calculations involving differences in latitudes and departures, for these differences will always be obtained by subtraction and never by addition, as is frequently the case when the first system of numbering the meridians and parallels is used. For example, under the first system two points having latitudes of 200 N and 300 S, respectively, will differ in latitude by $200+300 = 500$ ft.; under the second system, these same points will have latitudes of, say, 800 N and 300 N, the difference being $800-300 = 500$, as before.

**Mapping the Field Notes.**—The stations made to determine the boundaries of the property are first placed upon the map, using the total latitude and departure of each for this purpose; the method having been described. After two consecutive stations have been plotted, as a check on the work, the distance between them should be measured. This should agree with the horizontal distance reduced from the field measurements. After the survey stations are plotted, the property corners should be mapped in the same way, checking up the plotted distance between them and the survey station from which they were determined. By joining these corners, the outer boundaries of the property will now appear upon the map. Preferably by means of a protractor reading to minutes and using any convenient meridian as a base, the side shots to buildings, runs, etc., as determined from each station, should be laid off. These directions should be transferred to the proper station and the distance measurements laid off thereupon; this gives all the points taken from that station. After all the side shots are taken, the map may be inked in, provided all possible checks prove that the penciled work is correct. If a topographic map is desired, the stadia sights taken to determine the topography may be located and contours, 10, 20, 25, or 50 ft. apart drawn in. The flatter the country, the smaller should be the contour interval. In ordinary rolling country, where the contours merely serve as a guide to determine the width of pillar in the mine, a contour interval of 25 ft. suffices; in mountainous country, 50 ft. is close enough.

The mine workings are mapped in exactly the same way as the surface features so far as survey lines are concerned, but there is a difference in mapping, as there is in taking, the side shots. On the surface, points are determined by noting their bearing and distance from some station; underground, points are located by offsets at right angles to the line of sight and must be so mapped.

Property corners are marked by a small circle in black and property lines are reasonably heavy ones joining adjacent circles, but not passing within the circumference, the exact corner being a pin point at the center of the circle. On the map should appear the bearing and length of all property lines. A description of the corner should be given as W. O. (for white oak), stone, etc. If the original corner is gone and something else is in place the description should say "W. O., now stone," or "Stone, orig. W. O.," or "Stone (W. O.)," the original corner being placed in parentheses.

The boundaries of all the individual properties making up the entire tract should appear on the map, as well as the name of the owner of each and the acreage. All reservations from under which the coal cannot be mined or to which the company's rights are unusual or peculiar, must be carefully mapped. The names of the owners of adjacent properties should appear. The outcrop of all workable seams should be given in brown, as well as the location of all test openings thereon and the thickness and character of the coal. The position of all oil and gas wells should be accurately determined and mapped, with memoranda as to their operation, production (if any) etc. Abandoned and improperly plugged wells are a constant source of danger in some parts of the country and too much time cannot be spent in accurately locating them. The base-line monuments, together with the azimuth or bearing of the line joining them should be given, as well as all meridian reference lines connected therewith.

Mine workings should be shown in black, the stations being denoted by small circles in the same color. The numbers of all stations and their elevation above sea level should appear. If more than one seam is worked, the

operations in the separate seams should be given a distinctive color, none of which (except the principal workings which may be shown in black) should be used in mapping any surface feature.

When mapping the operations in a single seam, it is not unusual to outline the work done by different colors to represent the extraction during each semi-annual period. Thus, the workings advanced between January 1 and June 30, 1914, would be shown, say, in blue; those from July 1 to December 31, 1914, in red; those from January 1 to June 30, 1915, in green; and similarly for each succeeding period of 6 mo. While this serves to show the extent of the operations for any semiannual period, and this is desirable, it makes an ugly map, and many prefer to plat the mine workings in one single color, drawing a dash across the face of the working places after each semiannual posting has been made. The date, as 6–30–1914, placed by a dash indicates the date at which the posting was made. If many seams are worked under the one property and all are platted on the same sheet, it leads to confusion and it is a better plan not to map them this way, but to make a series of property maps, one for the workings of each seam, or at the most, for the workings of two adjacent seams. Then, if it is desired to note the relationship between the workings of all the seams, a tracing may be made upon which the workings in all the seams are given.

**Coloring a Map.**—The survey line by which the corners were determined is frequently placed on the map in red ink. It is not necessary to give the bearings and distances of the lines, but the stations should be numbered and their elevation above sea level given if this has been determined. Contour lines appear in brown, those marking even hundreds of feet above sea level being heavier than intermediate ones. Small brooks appear as a single line of Prussian blue; larger ones are shown by two parallel lines; and creeks and rivers have their banks shown as they actually exist. If a creek is named, this name should appear on the map. Roads are denoted in brown and should appear in their legal width. Houses are commonly outlined in black, as are tipples, coke ovens, etc. Railroads are denoted by fine parallel lines, marked with black dashes about $\frac{1}{8}$ in. in length; black-and-white dashes of the same length alternating.

It is a question whether it is advisable to tint a map with water colors or not. If the map is a final or finished one upon which no further operations will appear it is advisable to tint it properly; but in the case of a working map, there will be so many erasures as the workings advance, as pillars are drawn, as new lines of railroad are constructed, etc., that the effect of the tinting is soon spoiled. When tinting is used, the inner edge of the property lines should receive a wash in India ink (which will appear in dark gray-black) about $\frac{1}{8}$ in. wide. Crop lines should receive a similar but narrower band in brown. Roads should have a light wash in yellow ocher, and narrow streams, those appearing less than 1 in. in width (100 ft. wide in nature) a light wash in Prussian blue. Large ponds and wide streams if tinted for their full width should be colored with indigo, as the Prussian blue is rather too vivid. Frequently, streams and lakes are not colored for their full width with a flat tint; instead the color is applied with the maximum intensity at the shore line, being gradually drawn out to nothing 1 in. or less therefrom. The projections of houses, barns, tipples, etc., should receive a light wash of crimson lake. Theoretically, unworked areas of coal should be given a flat tint of India ink, the tint being removed to correspond to the mining operations. This is, of course, not practicable, so it is customary to leave the unworked coal, white, and to color the excavations made in mining. As stated, this makes an attractive map when first completed, but as the workings advance and pillars are drawn, the scratching out of previously applied tints, produces an unpleasant effect.

The paper upon which a mine map is made should be the very best eggshell, linen mounted, obtainable. When not in use it, with all other permanent records, should be kept in a fireproof vault.

# DETERMINATION OF MERIDIAN

## LATITUDE AND LONGITUDE

If a meridian, that is, a circle passing through the axis of the earth, is passed through a given point of the earth's surface, the angular distance of the point from the equator, measured on the meridian, is the *latitude* of that point. A plane parallel to the equator cuts the earth's surface in a circle called a *parallel of latitude.* All the points on a parallel of latitude have the same latitude. The *longitude* of a place is the angle that the plane of the meridian of the place makes with the plane of a reference meridian (usually the meridian of Greenwich). This angle may be measured on the equatorial circle or on the parallel of latitude of the given place. Longitude is counted from the reference meridian toward the west.

## CELESTIAL SPHERE

The *celestial sphere* is an imaginary sphere enclosing all the heavenly bodies. It is of such enormous dimensions that, in comparison with it, the earth may be considered as a mere dot. The earth's axis produced indefinitely is called the *axis of the celestial sphere.* This axis intersects the celestial sphere in two points, called the *north pole* and the *south pole* of the heavens. All the great circles of the celestial sphere passing through this axis are called *hour circles.* The circle in which the plane of the equator intersects the celestial sphere is called the *celestial equator.* The point on the equator that the sun in its apparent motion over the celestial sphere crosses on March 21, as it passes from the southern to the northern hemisphere, is called the *vernal equinox.*

## REFERENCE CIRCLES

The accompanying illustration, which represents the celestial hemisphere, shows all the reference circles that are used for determining the position of a heavenly body. *O* is the position of the earth; *OP*, one-half of the axis of the celestial sphere, *P* being the north pole; *VQV'L*, part of the celestial equator; *X*, the vernal equinox; and *YXC*, part of the sun's path. *PX* is the hour circle passing through *X*, called the *equinoctial colure.* *S* is any star, and *PSA* is the hour circle passing through it. *XA* is the *right ascension* of the star,
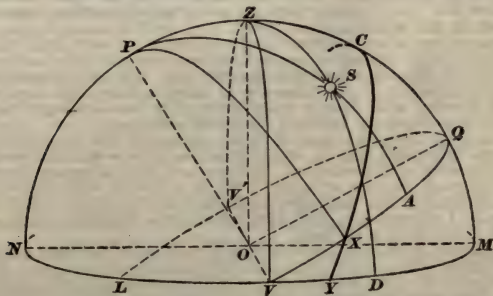


which is the arc on the equator measured eastwards from the vernal equinox to the hour circle passing through the star. *AS* is the *declination* of the star; that is, its angular distance from the equator. The declination is considered positive when the star is north and negative when south of the equator. The complement angle of the declination, *SP*, is called the *polar distance* of the star.

The *zenith* of a point on the earth's surface is the point *Z* in which the line passing through the center of the earth and the given point intersects the celestial sphere above the given point. The *horizon* is the plane *NYM* passing through the given point and perpendicular to this line.

The *celestial meridian* of a given point is a great circle passing through the zenith of the point and the poles. The celestial meridian cuts the horizon in two points *N* and *M*, called, respectively, the *north point* and the *south point.*

A *vertical circle* is one that passes through the zenith and is perpendicular to the horizon.

The *prime vertical* is the vertical circle at right angles to the meridian; it intersects the horizon in two points $V$ and $V'$, called the *west* and the *east point*, respectively.

The *altitude of a heavenly body* is its angular distance $DS$ from the horizon, measured along the vertical circle passing through the body. The *zenith distance*, is the angular distance $SZ$ of the star from the zenith, measured along the same circle. The zenith distance is the complement of the altitude.

The *azimuth of a star* is the angle in the plane of the horizon intercepted by the planes of the meridian and the vertical circle passing through the star. It is measured from the north point toward the east or from the south point toward the west. $NMD$ is the azimuth of $S$, measured from the north toward the east, and $MD$ is the azimuth of $S$ when measured from the south toward the west.

The *hour angle* of a star is the arc $QA$ intercepted on the equator between the meridian and the foot of the hour circle passing through the star; it is measured from the meridian toward the west.

## TIME

The passing of a heavenly body across the meridian of a place is called its *culmination*, or *transit*. It is upper or lower culmination, according as it is then occupying the highest or the lowest position with regard to the horizon.

The interval of time that elapses between two successive upper or lower transits of a star over the same meridian is called a *sidereal day*. This day begins, for any place, when the vernal equinox crosses the meridian above the pole; this instant is called *sidereal noon*. Sidereal hours, minutes, and seconds are reckoned from 0 to 24 hr., starting from sidereal noon. Time expressed in sidereal days and fractions (hours, minutes, seconds) is called *sidereal time*.

From this, it follows that sidereal time is the hour angle of the vernal equinox; also, that the right ascension of a star is equal to the sidereal time of its transit, or culmination. For any other position of the star, the sidereal time equals the algebraic sum of the right ascension and the hour angle of the star.

The interval between two successive upper transits of the sun is called a *true solar day*, or an *apparent day*. Owing to the fact that the motion of the sun is not uniform and that the solar days are not of equal duration, apparent time is not used for the ordinary affairs of life.

The *mean sun* is an imaginary body supposed to start from the vernal equinox at the same time as the true sun, and to move uniformly on the equator, returning to the vernal equinox with the true sun. The time between two successive upper transits of the mean sun is called a *mean solar day*, and time expressed in mean solar days is called *mean solar time*, or simply *mean time*. This is the time shown by ordinary clocks and watches.

A mean solar day is the mean of the duration of all the true solar days in a year (a year being the time in which either the true or the mean sun makes a complete circuit of the heavens). As there are 365.2422 true solar days and 366.2422 sidereal days in a year. 1 mean solar da. $= 366.2422 \div 365.2422$ $= 1.0027379$ sidereal day $= 24^h \ 3^m \ 56.55^s$, sidereal time.

Likewise, 1 sidereal day $= 365.2422 \div 366.2422 = .99726957$ mean solar day $= 23^h \ 56^m \ 4.09^s$, mean solar time.

The *equation of time* is a certain quantity that must be added algebraically to the apparent solar time to obtain the corresponding mean time. The value of this quantity for each day of the year is given in the American Ephemeris.*

**Civil Time and Astronomical Time.**—By *civil time* is meant the time that is usually reckoned in ordinary life. For astronomical purposes, the day is considered to begin at noon, and hours counted from 0 to 24. When time is reckoned in this manner it is called *astronomical time*. The civil day begins at 12 o'clock at night, and the astronomical day begins 12 hr. later. For instance, the date October 17, $7^h \ 14^m \ 3^s$, astronomical time, means $7^h \ 14^m \ 3^s$ after noon of the civil date October 17, and is in civil time, $7^h \ 14^m \ 3^s$ P. M. The astronomical date February 20, $18^h \ 6^m \ 12^s$ means $18^h \ 6^m \ 12^s$ after noon

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*The American Ephemeris and Nautical Almanac may be obtained from the Director of the Nautical Almanac, Naval Observatory, Washington, D. C. Remittance must be made in cash or a post-office money order for $1.25. Stamps and checks are not taken.

of the civil date February 20, or 6h 6m 12s after midnight of February 20; that is, February 21, 6h 6m 12s A. M.

**Longitude and Time.**—The mean sun describes a complete circle in 24 mean solar hours. In 1 hr. it moves over $360° \div 24 = 15°$ of arc; in 1 min. of time, over 15' of arc; and in 1 sec. of time, 15" of arc.

**Relation Between Time and Longitude.**—Let $A$ and $B$ be two places on the earth's surface, $B$ being west of $A$. Let their respective longitudes be $h_a$ and $h_b$, and let the difference between $h_a$ and $h_b$, expressed in measure of time, be $d_g$, Let, also, $T_a$ be the time at $A$ when the time at $B$ is $T_b$. Then,

$$T_a = T_b + d_g \qquad (1)$$

and

$$T_b = T_a - d_g \qquad (2)$$

EXAMPLE 1.—The longitude of Washington, west of Greenwich, is 5h 8m 1s; that of San Francisco, 8h 9m 47s. What is the time at: (a) Washington when it is 9h 3m at San Francisco? (b) San Francisco when it is 19h 54m 30s at Washington?

SOLUTION.—(a) Here $A$, the eastern locality, is Washington and $B$ is San Francisco; also, $d_g = $ 8h 9m 47s − 5h 8m 1s = 3h 1m 46s. Therefore, applying formula 1, $T_a = $ 9h 3m + 3h 1m 46s = 12h 4m 46s

(b) Applying formula 2,

$$T_b = 19h\ 54m\ 30s - 3h\ 1m\ 46s = 16h\ 52m\ 44s.$$

**Standard Time.**—Time referred to the meridian of a given place is called the *local time* of that place. To obviate complications in comparing local times of different localities, for use in ordinary affairs of life *standard times* have been adapted for regions between certain longitudes. The United States is divided into four zones, or sections of standard time. The time in each zone is referred to the meridian passing through its center. These central meridians are 15° or 1h distant from one another and are, respectively, 75°, 90°, 105°, and 120° west of Greenwich; or, in hours, 5h, 6h, 7h, and 8h west of Greenwich. Each of these meridians controls the watch time of all places within 7½° on either side. This is shown as follows:

Time referred to the 75° meridian is called *eastern time;* to the 90° meridian, *central time;* to the 105° meridian, *mountain time;* and to the 120° meridian, *Pacific time.*

| | | 8h 30m | 8h | 7h 30m | 7h | 6h 30m | 6h | 5h 30m | 5h | 4h 30m |
|---|---|---|---|---|---|---|---|---|---|---|
| *Pacific* | | *Mountain* | | *Central* | | *Eastern* | | | | |
| 127 30' | 120° | 112 30' | 105° | 97 30' | 90° | 82 30' | 75° | 67 30' | | |

**To Change Standard Time Into Local Time and Vice Versa.**—Standard time can be changed into local time or local time can be changed into standard time by applying formula 1 or formula 2, according as the given place is east or west of the reference meridian of the zone in which the place is located.

EXAMPLE.—The standard time, by a watch, at a place whose longitude is 81° 37', is 9h 37m 45s A. M.; what is the local time?

SOLUTION.—As the longitude is 81° 37', the place lies within the zone of the 75° meridian; and being west of the latter, formula 2 must be applied. In this case, $T_a = $ 9h 37m 45s and $d_g = $ 81° 37' − 75° = 6° 37' = 26m 28s. Therefore, $T_b = $ 9h 37m 45s − 26m 28s = 9h 11m 17s A. M.

## DETERMINATION BY OBSERVING POLARIS AT CULMINATION

The position of Polaris, or the north star, can easily be ascertained by means of the group of stars called the Dipper, or the Great Bear. As shown in the accompanying illustration, a straight line joining the stars, α and β, called the *pointers*, nearly intersects Polaris. There are two times during the day when the star crosses the meridian. It is then said to be at its *upper* or *lower culmination*, as the star is then occupying either the highest or the lowest position with reference to the horizon. When the star is in either of these positions, the vertical plane passing through it and the observer's station is the meridian of the place, and its intersection with the horizon is therefore a true north-and-south line.

**Field Work.**—Select a date on which Polaris is at either lower or upper culmination during the night (preferably during the early part of the evening). Determine, by means of the accompanying table, the exact time of culmination, being careful to reduce the tabular values to standard civil time. It is safer, in order to avoid confusion, for the observer to set his watch to show local time. About 15m before the time of culmination, set the transit in such a position that an unobstructed view toward the north may be obtained for a distance of between 300 and 500 ft. Drive a stake, and mark by a tack the exact point

## LOCAL MEAN ASTRONOMICAL TIME OF UPPER CULMINATION OF POLARIS

| | 1913 | | 1914 | | 1915 | | 1916 | | 1917 | | 1918 | | 1919 | | 1920 | | 1921 | | 1922 | | Diff. for 1 Day. Minutes |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | h. | m. | h. | m. | h. | m. | h. | m. | h. | m. | h. | m. | h. | m. | h. | m. | h. | m. | h. | m. | |
| Jan. 1 | 6 | 43.8 | 6 | 45.2 | 6 | 46.7 | 6 | 48.1 | 6 | 45.6 | 6 | 47.0 | 6 | 48.4 | 6 | 49.9 | 6 | 47.4 | 6 | 48.8 | 3.95 |
| Jan. 15 | 5 | 48.5 | 5 | 49.9 | 5 | 51.1 | 5 | 52.8 | 5 | 50.3 | 5 | 51.7 | 5 | 53.1 | 5 | 54.6 | 5 | 52.1 | 5 | 53.5 | 3.95 |
| Feb. 1 | 4 | 41.4 | 4 | 42.8 | 4 | 44.3 | 4 | 45.7 | 4 | 43.2 | 4 | 44.6 | 4 | 46.0 | 4 | 47.5 | 4 | 45.0 | 4 | 46.4 | 3.95 |
| Feb. 15 | 3 | 46.1 | 3 | 47.5 | 3 | 49.0 | 3 | 50.4 | 3 | 47.9 | 3 | 49.3 | 3 | 50.7 | 3 | 52.2 | 3 | 49.7 | 3 | 51.1 | 3.94 |
| Mar. 1 | 2 | 50.9 | 2 | 52.5 | 2 | 53.8 | 2 | 55.2 | 2 | 52.6 | 2 | 54.1 | 2 | 55.5 | 2 | 53.0 | 2 | 54.4 | 2 | 55.9 | 3.94 |
| Mar. 15 | 1 | 55.7 | 1 | 57.3 | 1 | 58.6 | 1 | 59.9 | 1 | 57.4 | 1 | 58.9 | 0 | 0.3 | 1 | 57.7 | 1 | 59.2 | 0 | 0.7 | 3.93 |
| Apr. 1 | 0 | 48.9 | 0 | 50.3 | 0 | 51.8 | 0 | 49.3 | 0 | 50.6 | 0 | 52.1 | 0 | 53.5 | 0 | 50.9 | 0 | 52.4 | 0 | 53.9 | 3.93 |
| Apr. 15 | 23 | 49.9 | 23 | 51.3 | 23 | 52.8 | 23 | 50.3 | 23 | 51.6 | 23 | 53.1 | 23 | 54.6 | 23 | 52.0 | 23 | 53.4 | 23 | 54.9 | 3.92 |
| May 1 | 22 | 47.2 | 22 | 46.9 | 22 | 50.0 | 22 | 47.5 | 22 | 48.9 | 22 | 50.4 | 22 | 51.8 | 22 | 49.3 | 22 | 50.7 | 22 | 52.2 | 3.92 |
| May 15 | 21 | 52.3 | 21 | 53.7 | 21 | 55.1 | 21 | 52.6 | 21 | 54.0 | 21 | 55.4 | 21 | 56.9 | 21 | 54.4 | 21 | 55.8 | 21 | 57.3 | 3.92 |
| Jun. 1 | 20 | 55.7 | 20 | 52.1 | 20 | 58.1 | 20 | 55.1 | 20 | 47.2 | 20 | 48.6 | 20 | 50.0 | 20 | 47.6 | 20 | 49.0 | 20 | 50.5 | 3.91 |
| Jun. 15 | 19 | 50.7 | 19 | 49.5 | 19 | 53.5 | 19 | 51.0 | 19 | 52.4 | 19 | 53.8 | 19 | 55.3 | 19 | 52.8 | 19 | 54.2 | 19 | 55.7 | 3.91 |
| Jul. 1 | 18 | 48.1 | 18 | 54.8 | 18 | 50.9 | 18 | 48.7 | 18 | 49.8 | 18 | 51.2 | 18 | 52.7 | 18 | 49.0 | 18 | 51.6 | 18 | 53.1 | 3.92 |
| Jul. 15 | 17 | 53.4 | 17 | 54.1 | 17 | 56.2 | 17 | 54.3 | 17 | 55.1 | 17 | 56.5 | 17 | 58.0 | 17 | 55.5 | 17 | 56.9 | 17 | 58.4 | 3.92 |
| Aug. 1 | 16 | 46.8 | 16 | 46.7 | 16 | 49.6 | 16 | 47.1 | 16 | 48.5 | 16 | 49.9 | 16 | 51.4 | 16 | 48.9 | 16 | 50.3 | 16 | 51.8 | 3.92 |
| Aug. 15 | 15 | 52.0 | 15 | 53.8 | 15 | 55.1 | 15 | 53.6 | 15 | 53.7 | 15 | 55.1 | 15 | 56.6 | 15 | 54.1 | 15 | 56.5 | 15 | 57.0 | 3.92 |
| Sept. 1 | 14 | 45.3 | 14 | 49.0 | 14 | 48.1 | 14 | 45.6 | 14 | 47.0 | 14 | 48.5 | 14 | 49.9 | 14 | 47.4 | 14 | 48.9 | 14 | 50.3 | 3.93 |
| Sept. 15 | 13 | 50.4 | 13 | 54.4 | 13 | 53.7 | 13 | 50.4 | 13 | 52.5 | 13 | 53.6 | 13 | 55.0 | 13 | 54.0 | 13 | 54.0 | 13 | 55.3 | 3.93 |
| Oct. 1 | 12 | 47.6 | 12 | 47.2 | 12 | 50.4 | 12 | 47.9 | 12 | 49.3 | 12 | 50.8 | 12 | 52.2 | 12 | 49.7 | 12 | 51.2 | 12 | 52.6 | 3.93 |
| Oct. 15 | 11 | 52.6 | 11 | 52.1 | 11 | 55.4 | 11 | 52.9 | 11 | 54.4 | 11 | 55.8 | 11 | 57.2 | 11 | 54.4 | 11 | 56.2 | 11 | 57.6 | 3.94 |
| Nov. 1 | 10 | 45.8 | 10 | 54.1 | 10 | 48.6 | 10 | 46.1 | 10 | 47.5 | 10 | 49.0 | 10 | 47.9 | 10 | 47.9 | 10 | 49.4 | 10 | 50.8 | 3.94 |
| Nov. 15 | 9 | 50.7 | 9 | 52.1 | 9 | 53.5 | 9 | 51.0 | 9 | 52.4 | 9 | 53.9 | 9 | 52.8 | 9 | 52.8 | 9 | 54.3 | 9 | 55.7 | 3.94 |
| Dec. 1 | 8 | 47.8 | 8 | 49.6 | 8 | 50.3 | 8 | 48.2 | 8 | 49.2 | 8 | 50.7 | 8 | 52.1 | 8 | 49.6 | 8 | 51.1 | 8 | 52.5 | 3.94 |
| Dec. 15 | 7 | 52.3 | 7 | 53.7 | 7 | 55.1 | 7 | 52.6 | 7 | 54.0 | 7 | 55.5 | 7 | 56.9 | 7 | 54.4 | 7 | 55.9 | 7 | 57.3 | 3.94 |

occupied by the instrument. About 5ᵐ before the time of culmination, direct the telescope to the star, holding a lamp in front and a little toward one side of the objective glass to illuminate the cross-hairs. Set both clamps, and with either tangent screw set the vertical cross-hair exactly on the star. The star will appear to be moving toward the left or toward the right according as it is approaching upper or lower culmination. Follow it in its motion by turning the tangent screw until the exact time of culmination (which, preferably, should be called out by an assistant). This completes the observation of the star. Now depress the telescope, direct it to a point on the ground about 400 or 500 ft. from the instrument, and have an assistant drive a tack in the top of a stake in line with the line of sight; this completes the operation. The line between the two stakes is a true north-and-south line, or true meridian.

**Time of Culmination of Polaris.**—The accompanying table contains the times of upper culmination of Polaris for the dates given. The lower culmination occurs nearly 11ʰ 58ᵐ before and after the upper culmination, and can be determined from the latter. In the table the extreme right-hand column contains the difference between the times of culmination for any two succeeding days. Each difference applies to any day between the date horizontally opposite that difference in the left-hand column, and the following date. Thus, the difference 3.95ᵐ, which is horizontally opposite January 1, indicates that, between January 1 and January 15, the time of culmination decreases by 3.95ᵐ per da. For instance, the time of culmination on January 8 is obtained by subtracting from the time of culmination for January 1 the product 3.95ᵐ×7 =27.65ᵐ, the number of days elapsed from January 1 to January 8 being seven.

It should be borne in mind that the times given in the table are mean local times counted in the astronomical way; that is, from 0ʰ to 24ʰ, beginning at noon.

*Pole* • *Polaris*



*Dipper or Great Bear*

EXAMPLE.—Find the time of upper culmination of Polaris on September 6, 1913.

SOLUTION.—Referring to the table,

Upper culmination, Sept. 1, 1913 . . . . . . . . . . . . . . . . . . = 14ʰ45.3ᵐ
Difference for 1 da . . . . . . . . . . . . . . . . . . . . . . =3.92ᵐ
Correction for 5 da . . . . . . . . . . . . . . . . . . . . =3.92ᵐ×5=   19.6ᵐ
                                                                          _____
Time of culmination on Sept. 6 . . . . . . . . . . . . . . . . . . =14ʰ25.7ᵐ

This means that upper culmination will occur when 14ʰ 25.7ᵐ have elapsed since local noon Sept. 6; that is, at 2ʰ 25.7ᵐ A. M., Sept. 7.

## DETERMINATION BY OBSERVING POLARIS AT ELONGATION

When a star is at its extreme westerly or easterly position, it is said to be at *western* or *eastern elongation.* This position with reference to the meridian of the place is determined by the angle that a vertical plane passing through the star and the point of observation is making with the meridian. This angle is called the *azimuth of the star,* and its values for Polaris, for the years 1913 to 1922 and latitudes 5° to 74°, are given in the accompanying table.

Polaris is at eastern elongation about 5ʰ 55ᵐ before it reaches its upper culmination; and at western elongation, 5ʰ 55ᵐ after upper culmination. The times of elongation can, therefore, be readily determined from those of culmination taken from the table.

EXAMPLE.—Find the time of western elongation of Polaris on March 1, 1914.

SOLUTION.—On referring to the table, it is found that the upper culmination is at 2ʰ 52.5ᵐ, local astronomical time, or 2ʰ 52.5ᵐ, P. M., local civil time. Polaris is at western elongation 5ʰ 55ᵐ later or at 8ʰ 47.5ᵐ, P. M., local civil time.

**Making the Observation and Marking the Meridian.**—Determine the approximate time of elongation as just explained. About 20ᵐ before that time, set the transit over a point properly marked, and level it carefully. Set the vernier at 0. Direct the telescope to the star, and, with both clamps set, follow the star by means of the lower tangent screw. If the star is approaching
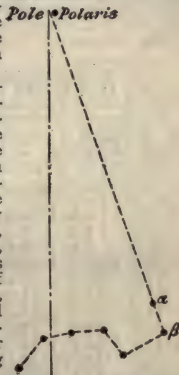
eastern elongation it will be moving to the right; if western, to the left. About the time of elongation, it will be noticed that the star ceases to move horizontally, and that its image appears to follow the vertical cross-hair of the instrument. The star has then reached its elongation and the observation is completed. Take the azimuth from the table. Depress the telescope, and turn it through an angle equal to the azimuth, to the west or to the east, according as the star was at eastern or western elongation. The line of sight will then

### AZIMUTHS OF POLARIS AT ELONGATION

| Lat. Deg. | Year | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1913 | | 1914 | | 1915 | | 1916 | | 1917 | | 1918 | | 1919 | | 1920 | | 1921 | | 1922 | |
| | Deg. | Min. | Deg. | Min. | Deg. | Min. | Deg. | Min. | Deg. | Min. | Deg. | Min. | Deg. | Min. | Deg. | Min. | Deg. | Min. | Deg. | Min. |
| 5 | 1 | 9.8 | 1 | 9.5 | 1 | 9.2 | 1 | 8.8 | 1 | 8.5 | 1 | 8.2 | 1 | 7.9 | 1 | 7.6 | 1 | 7.3 | 1 | 7.0 |
| 6 | | 9.9 | | 9.6 | | 9.3 | | 9.0 | | 8.6 | | 8.3 | | 8.0 | | 7.7 | | 7.4 | | 7.1 |
| 8 | | 10.2 | | 9.8 | | 9.5 | | 9.2 | | 8.9 | | 8.6 | | 8.3 | | 7.9 | | 7.7 | | 7.4 |
| 10 | 1 | 10.6 | 1 | 10.3 | 1 | 10.0 | 1 | 9.7 | 1 | 9.3 | 1 | 9.0 | 1 | 8.7 | 1 | 8.4 | 1 | 8.1 | | 7.8 |
| 12 | | 11.0 | | 10.7 | | 10.4 | | 10.1 | | 9.7 | | 9.4 | | 9.1 | | 8.8 | | 8.5 | | 8.2 |
| 14 | | 11.6 | | 11.3 | | 11.0 | | 10.6 | | 10.3 | | 10.0 | | 9.7 | | 9.4 | | 9.0 | | 8.7 |
| 16 | | 12.3 | | 12.0 | | 11.7 | | 11.3 | | 11.0 | | 10.7 | | 10.4 | | 10.1 | | 9.7 | | 9.4 |
| 18 | | 13.1 | | 12.8 | | 12.4 | | 12.1 | | 11.8 | | 11.5 | | 11.1 | | 10.8 | | 10.5 | | 10.2 |
| 20 | 1 | 14.0 | 1 | 13.7 | 1 | 13.4 | 1 | 13.0 | 1 | 12.6 | 1 | 12.3 | 1 | 11.9 | 1 | 11.6 | 1 | 11.3 | 1 | 10.9 |
| 22 | | 15.0 | | 14.7 | | 14.4 | | 14.1 | | 13.8 | | 13.4 | | 13.1 | | 12.7 | | 12.4 | | 12.1 |
| 24 | | 16.0 | | 15.7 | | 15.4 | | 15.1 | | 14.8 | | 14.5 | | 14.1 | | 13.8 | | 13.5 | | 13.2 |
| 26 | | 17.4 | | 17.0 | | 16.7 | | 16.3 | | 16.0 | | 15.7 | | 15.3 | | 14.9 | | 14.6 | | 14.3 |
| 28 | | 18.8 | | 18.5 | | 18.1 | | 17.8 | | 17.4 | | 17.0 | | 16.7 | | 16.3 | | 16.0 | | 15.7 |
| 30 | 1 | 20.3 | 1 | 19.9 | 1 | 19.6 | 1 | 19.2 | 1 | 18.9 | 1 | 18.5 | 1 | 18.2 | 1 | 17.9 | 1 | 17.5 | 1 | 17.1 |
| 32 | | 22.0 | | 21.6 | | 21.2 | | 20.9 | | 20.5 | | 20.1 | | 19.8 | | 19.4 | | 19.0 | | 18.7 |
| 34 | | 23.8 | | 23.5 | | 23.1 | | 22.8 | | 22.4 | | 22.1 | | 21.7 | | 21.3 | | 21.0 | | 20.6 |
| 36 | | 26.0 | | 25.6 | | 25.2 | | 24.9 | | 24.5 | | 24.1 | | 23.8 | | 23.4 | | 23.0 | | 22.6 |
| 38 | | 28.2 | | 27.8 | | 27.5 | | 27.1 | | 26.8 | | 26.4 | | 26.0 | | 25.6 | | 25.2 | | 24.8 |
| 40 | 1 | 30.7 | 1 | 30.3 | 1 | 30.0 | 1 | 29.6 | 1 | 29.2 | 1 | 28.8 | 1 | 28.4 | 1 | 28.0 | 1 | 27.6 | 1 | 27.2 |
| 42 | | 33.6 | | 33.2 | | 32.8 | | 32.4 | | 32.0 | | 31.6 | | 31.1 | | 30.7 | | 30.3 | | 29.9 |
| 44 | | 36.7 | | 36.3 | | 35.8 | | 35.4 | | 35.0 | | 34.6 | | 34.1 | | 33.6 | | 33.2 | | 32.8 |
| 46 | | 40.1 | | 39.7 | | 39.2 | | 38.8 | | 38.4 | | 37.9 | | 37.5 | | 37.1 | | 36.6 | | 36.2 |
| 48 | | 43.9 | | 43.4 | | 43.0 | | 42.5 | | 42.2 | | 41.8 | | 41.3 | | 40.8 | | 40.3 | | 39.9 |
| 50 | 1 | 48.1 | 1 | 47.7 | 1 | 47.2 | 1 | 46.8 | 1 | 46.3 | 1 | 45.9 | 1 | 45.4 | 1 | 44.9 | 1 | 44.4 | 1 | 43.9 |
| 52 | | 52.9 | | 52.4 | | 52.0 | | 51.5 | | 51.0 | | 50.5 | | 50.0 | | 49.5 | | 49.0 | | 48.5 |
| 54 | | 58.3 | | 57.8 | | 57.3 | | 56.8 | | 56.3 | | 55.8 | | 55.2 | | 54.7 | | 54.2 | | 53.7 |
| 56 | | 4.4 | | 3.8 | 2 | 3.3 | 2 | 2.7 | 2 | 2.2 | 2 | 1.7 | 2 | 1.1 | 2 | 0.5 | 2 | 0.0 | 1 | 59.4 |
| 58 | | 11.3 | | 10.7 | | 10.1 | | 9.6 | | 9.0 | | 8.4 | | 7.8 | | 7.2 | | 6.6 | | 6.0 |
| 60 | 2 | 19.0 | 2 | 18.4 | 2 | 17.8 | 2 | 17.2 | 2 | 16.6 | 2 | 16.0 | 2 | 15.3 | 2 | 14.7 | 2 | 14.0 | 2 | 13.4 |
| 62 | | 28.1 | | 27.4 | | 26.7 | | 26.0 | | 25.4 | | 24.7 | | 24.0 | | 23.4 | | 22.7 | | 22.0 |
| 64 | | 38.7 | | 38.0 | | 37.3 | | 36.5 | | 35.9 | | 35.2 | | 34.5 | | 33.8 | | 33.0 | | 32.3 |
| 66 | | 50.9 | | 50.1 | | 49.4 | | 48.6 | | 47.8 | | 47.0 | | 46.2 | | 45.5 | | 44.7 | | 43.9 |
| 68 | 3 | 5.7 | 3 | 4.8 | 3 | 4.0 | 3 | 3.1 | 3 | 2.2 | 3 | 1.3 | 3 | 0.4 | | 59.6 | | 58.7 | | 57.7 |
| 70 | 3 | 22.8 | 3 | 21.8 | 3 | 20.8 | 3 | 19.9 | 3 | 18.9 | 3 | 17.9 | 3 | 16.9 | 3 | 15.9 | 3 | 15.0 | 3 | 14.0 |
| 72 | | 45.2 | | 44.2 | | 43.1 | | 42.0 | | 41.0 | | 40.0 | | 38.9 | | 37.8 | | 36.8 | | 35.7 |
| 74 | 4 | 12.1 | 4 | 11.0 | 4 | 9.8 | 4 | 8.7 | 4 | 7.5 | 4 | 6.4 | 4 | 5.2 | 4 | 4.1 | 4 | 3.0 | 4 | 1.8 |

be directed along the true meridian, and by marking another point 400 or 500 ft. from that occupied by the instrument, the direction of the true meridian will be established.

This is the most accurate method of determining the true meridian, and, where possible, should be used in preference to others.

As the marking monuments are not commonly set in the meridian, some little change in the method of making the observations from that described is necessary. Having the cross-hair on Polaris at the point of greatest elongation,

the telescope is brought down and the angle between the star and the monument is read. The telescope is inverted and again set on Polaris and the angle to the monument read. This angle may be read four or six times, even more, as the change in position of the pole star for 15ᵐ before and after elongation is not measurable by an ordinary transit. The mean of the two, four, or six readings of the angle is taken as the true angle. By making a sketch of the position of Polaris with reference to the meridian and of the position of the monuments with reference to Polaris, it will be apparent whether the azimuth of the star is to be added to or subtracted from the angle between it and the monuments to give the azimuth of the line joining said reference points. As the determination of the meridian is of great importance it is well, unless the engineer has had experience in the work, to repeat the observations on a second night.

## DETERMINATION BY SOLAR OBSERVATION

**Formula for Azimuth of the Sun.**—One of the most convenient methods of determining the meridian is to measure the altitude of the sun at any hour angle with a transit. At the same time that the altitude is measured, determine, also, the horizontal angle between the sun and a fixed object, or reference mark. Then, the azimuth of the sun is calculated by the formula that follows. The azimuth of the reference mark is then equal to the algebraic sum of the azimuth of the sun and the measured angle between the sun and the mark. Finally, the true north-and-south line may be located from the azimuth of the reference mark.

Let $a$ = required azimuth counted from north toward east;
$z$ = zenith distance of sun, which is equal to 90° minus altitude;
$\delta$ = declination of sun; and
$\phi$ = latitude of observer; then

$$\sin \frac{a}{2} = \sqrt{\frac{\cos \frac{1}{2}(z+\phi+\delta) \sin \frac{1}{2}(z+\phi-\delta)}{\sin z \cos \phi}}$$

Two values of $\frac{a}{2}$ will correspond to the computed $\sin \frac{a}{2}$; one angle will be acute and the other obtuse. The acute angle should be used for morning observations and the obtuse for afternoon observations.

**Values of $\delta$ and $\phi$.**—The method just described requires that the declination of the sun at the time of observation, and the latitude of the place be known. The declination of the sun for every day of the year at the instant of Washington noon, together with the hourly change, is given in the Ephemeris, and has to be reduced to the time of observation as follows:

**Rule.**—*Change the local time to Washington time by adding algebraically to the former the longitude of the place counted from Washington. Take from the Ephemeris the declination corresponding to the preceding Washington noon and add algebraically the product of the hourly change by the time elapsed since Washington noon.*

**Example.**—Find the true declination of the sun for 9 A. M. January 5, 1903, at Philadelphia.

**Solution.**—Jan. 5, 9 A. M., civil time = Jan. 4, 21ʰ, astronomical time. The longitude of Philadelphia is $-7^m 37^s = -.127^h$. The Washington time corresponding to 9 A. M. is $21^h - .127^h = 20.873^h$. From the Ephemeris, the declination at Washington at noon Jan. 4 is $-22° 47' 43''$, and the hourly change is 15.06″. The algebraic increase is, therefore, $15.06 \times 20.873 = 5' 14''$; thus, the declination at 9 A. M. is $-22° 47' 43'' + 5' 14'' = -22° 42' 29''$.

## DETERMINATION OF LATITUDE, AND CORRECTIONS FOR ALTITUDE

**Approximate Determination of Latitude From Polaris.**—In nearly all methods of determining the true meridian, the latitude of the place of observation must be known, at least approximately. In the majority of cases the latitude can be taken from a map or book of reference. In case this cannot be done, a sufficiently close value may be obtained by measuring, with a transit, the altitude of Polaris, which is very nearly (within about 1°) equal to the latitude of the place.

This method of determining latitude is founded on the following very simple and useful principle:

**Principle.**—*The latitude of any place on the earth's surface is equal to the altitude of the pole with respect to the horizon of that place.*

For more accurate work, the tables given in the Ephemeris, entitled, **For Finding the Latitude by Polaris**, may be used. The simple directions for using them are there given in full.

**Latitude by Solar Observation.**—Latitude may be determined by measuring the sun's altitude, with the sextant or transit, at the instant of its passage across the meridian; that is, at apparent noon. The time of apparent noon may be determined by adding algebraically the equation of time to the noon of local mean time, as previously explained. Then begin the observations about 15ᵐ before apparent noon and repeat them every minute or two. At first the altitude will be increasing; then, it will be decreasing. The maximum altitude obtained will be the apparent meridian altitude. To this the corrections that follow must be applied, giving the true altitude. The true altitude is then subtracted from 90°, and the remainder is the zenith distance. The latitude is then equal to the algebraic sum of the zenith distance and the declination of the sun at the instant of apparent noon.

**Corrections for Altitude.**—The observed altitude of a heavenly body must be corrected for: (1) refraction, (2) parallax, and (3) semi-diameter.

1. *Refraction* is the change of direction of the rays of light when they pass from one medium into another of different density. Its amount for different altitudes is given in the table on page 107. It is subtractive. When the altitude is less than about 8° to 10°, the refraction becomes so uncertain that the measurement is of no value for accurate work.

2. *Parallax* is the difference in direction of a heavenly body as actually observed and the direction it would have if seen from the earth's center. This correction is necessary when the sun is observed; its values for different altitudes are given in the accompanying table. It is additive.

## SUN'S PARALLAX IN ALTITUDE TO BE APPLIED TO ALL MEASURED ALTITUDES OF THE SUN
### (Additive to observed altitude)

| Altitude Degrees | Parallax Seconds | Altitude Degrees | Parallax Seconds | Altitude Degrees | Parallax Seconds |
|---|---|---|---|---|---|
| 0 | 9 | 40 | 7 | 69 | 3 |
| 6 | 9 | 45 | 6 | 72 | 3 |
| 12 | 9 | 48 | 6 | 75 | 2 |
| 16 | 8 | 51 | 5 | 78 | 2 |
| 20 | 8 | 54 | 5 | 81 | 1 |
| 25 | 8 | 57 | 5 | 84 | 1 |
| 30 | 8 | 60 | 4 | 87 | 0 |
| 34 | 7 | 63 | 4 | 90 | 0 |
| 36 | 7 | 66 | 3 | | |

3. The correction for *semi-diameter* is also necessary when the sun is observed, owing to the fact that either the upper or the lower edge of the disk, instead of the center, is observed. This correction may be taken from the Ephemeris in the same manner as the sun's declination. For the purpose of ordinary calculations, however, this may be taken from the following table:

| Time of year (approx.)... | Jan. 1, | Apr. 1, | July 1, | Oct. 1 |
|---|---|---|---|---|
| Sun's semi-diameter ..... | 16′ 18″ | 16′ 2″ | 15′ 45″ | 16′ 2″ |

It is additive when the lower limb is observed, and subtractive when the upper one is observed.

**Corrections for Observation of the Sun for Azimuth.**—When the sun is observed for azimuth, a correction for semi-diameter must also be applied to the reading of the horizontal circle; this may be found by dividing the correction for altitude by the cosine of the sun's altitude. This correction is to be added to the reading of the horizontal circle if the hair is placed tangent to the left edge of the sun, and subtracted from the reading of the horizontal circle if the hair is placed tangent to the right edge of the sun.

When making observations of the sun for azimuth, the errors of adjustment, the index error, and the correction for semi-diameter may be eliminated by the following method, which assumes that the vertical circle of the transit is complete:

The instrument is set up with the horizontal plate reading 0° when sighting at the azimuth mark. For forenoon work, the sun should be so sighted that

## MEAN REFRACTION TO BE APPLIED TO ALL MEASURED ALTITUDES
### (Subtractive from apparent altitude)

| App. Altitude | Refraction | App. Altitude | Refraction | App. Altitude | Refraction | App. Altitude | Refraction | App. Altitude | Refraction |
|---|---|---|---|---|---|---|---|---|---|
| ° ′ | ′ ° ″ | ° ′ | ′ ″ | ° ′ | ′ ″ | ° ′ | ′ ″ | ° ′ | ′ ″ |
| 0 0 | 33 0 | 5 0 | 9 54 | 10 0 | 5 15 | 20 0 | 2 35 | 34 0 | 1 24 |
|  |  |  |  | 10 10 | 5 10 | 20 10 | 2 34 | 34 30 | 1 23 |
|  |  |  |  | 10 20 | 5 5 | 20 20 | 2 32 | 35 0 | 1 21 |
|  |  |  |  | 10 30 | 5 0 | 20 30 | 2 31 | 35 30 | 1 20 |
|  |  | 5 20 | 9 23 | 10 40 | 4 56 | 20 40 | 2 29 | 36 0 | 1 18 |
|  |  |  |  | 10 50 | 4 51 | 20 50 | 2 28 | 36 30 | 1 17 |
|  |  |  |  | 11 0 | 4 47 | 21 0 | 2 27 | 37 0 | 1 16 |
|  |  |  |  | 11 10 | 4 43 | 21 10 | 2 26 | 37 30 | 1 14 |
|  |  | 5 40 | 8 54 | 11 20 | 4 39 | 21 20 | 2 25 | 38 0 | 1 13 |
|  |  |  |  | 11 30 | 4 34 | 21 30 | 2 24 | 38 30 | 1 11 |
|  |  |  |  | 11 40 | 4 31 | 21 40 | 2 23 | 39 0 | 1 10 |
|  |  |  |  | 11 50 | 4 27 | 21 50 | 2 21 | 39 30 | 1 9 |
| 1 0 | 24 29 | 6 0 | 8 28 | 12 0 | 4 23 | 22 0 | 2 20 | 40 0 | 1 8 |
|  |  |  |  | 12 10 | 4 20 | 22 10 | 2 19 | 41 0 | 1 5 |
|  |  |  |  | 12 20 | 4 16 | 22 20 | 2 18 | 42 0 | 1 3 |
|  |  |  |  | 12 30 | 4 13 | 22 30 | 2 17 | 43 0 | 1 1 |
|  |  | 6 20 | 8 3 | 12 40 | 4 9 | 22 40 | 2 16 | 44 0 | 0 59 |
|  |  |  |  | 12 50 | 4 6 | 22 50 | 2 15 | 45 0 | 0 57 |
|  |  |  |  | 13 0 | 4 3 | 23 0 | 2 14 | 46 0 | 0 55 |
|  |  |  |  | 13 10 | 4 0 | 23 10 | 2 13 | 47 0 | 0 53 |
|  |  | 6 40 | 7 40 | 13 20 | 3 57 | 23 20 | 2 12 | 48 0 | 0 51 |
|  |  |  |  | 13 30 | 3 54 | 23 30 | 2 11 | 49 0 | 0 49 |
|  |  |  |  | 13 40 | 3 51 | 23 40 | 2 10 | 50 0 | 0 48 |
|  |  |  |  | 13 50 | 3 48 | 23 50 | 2 9 | 51 0 | 0 46 |
| 2 0 | 18 35 | 7 0 | 7 20 | 14 0 | 3 45 | 24 0 | 2 8 | 52 0 | 0 44 |
|  |  |  |  | 14 10 | 3 43 | 24 10 | 2 7 | 53 0 | 0 43 |
|  |  |  |  | 14 20 | 3 40 | 24 20 | 2 6 | 54 0 | 0 41 |
|  |  |  |  | 14 30 | 3 38 | 24 30 | 2 5 | 55 0 | 0 40 |
|  |  | 7 20 | 7 2 | 14 40 | 3 35 | 24 40 | 2 4 | 56 0 | 0 38 |
|  |  |  |  | 14 50 | 3 33 | 24 50 | 2 3 | 57 0 | 0 37 |
|  |  |  |  | 15 0 | 3 30 | 25 0 | 2 2 | 58 0 | 0 35 |
|  |  |  |  | 15 10 | 3 28 | 25 10 | 2 1 | 59 0 | 0 34 |
|  |  | 7 40 | 6 45 | 15 20 | 3 26 | 25 20 | 2 0 | 60 0 | 0 33 |
|  |  |  |  | 15 30 | 3 24 | 25 30 | 1 59 | 61 0 | 0 32 |
|  |  |  |  | 15 40 | 3 21 | 25 40 | 1 58 | 62 0 | 0 30 |
|  |  |  |  | 15 50 | 3 19 | 25 50 | 1 57 | 63 0 | 0 29 |
| 3 0 | 14 36 | 8 0 | 6 29 | 16 0 | 3 17 | 26 0 | 1 56 | 64 0 | 0 28 |
|  |  |  |  | 16 10 | 3 15 | 26 10 | 1 55 | 65 0 | 0 26 |
|  |  | 8 10 | 6 22 | 16 20 | 3 12 | 26 20 | 1 55 | 66 0 | 0 25 |
|  |  |  |  | 16 30 | 3 10 | 26 30 | 1 54 | 67 0 | 0 24 |
|  |  | 8 20 | 6 15 | 16 40 | 3 8 | 26 40 | 1 53 | 68 0 | 0 23 |
|  |  |  |  | 16 50 | 3 6 | 26 50 | 1 52 | 69 0 | 0 22 |
| 3 30 | 13 6 | 8 30 | 6 8 | 17 0 | 3 4 | 27 0 | 1 51 | 70 0 | 0 21 |
|  |  |  |  | 17 10 | 3 3 | 27 15 | 1 50 | 71 0 | 0 19 |
|  |  | 8 40 | 6 1 | 17 20 | 3 1 | 27 30 | 1 49 | 72 0 | 0 18 |
|  |  |  |  | 17 30 | 2 59 | 27 45 | 1 48 | 73 0 | 0 17 |
|  |  | 8 50 | 5 55 | 17 40 | 2 57 | 28 0 | 1 47 | 74 0 | 0 16 |
|  |  |  |  | 17 50 | 2 55 | 28 15 | 1 46 | 75 0 | 0 15 |
| 4 0 | 11 51 | 9 0 | 5 48 | 18 0 | 2 54 | 28 30 | 1 45 | 76 0 | 0 14 |
|  |  |  |  | 18 10 | 2 52 | 28 45 | 1 44 | 77 0 | 0 13 |
|  |  | 9 10 | 5 42 | 18 20 | 2 51 | 29 0 | 1 42 | 78 0 | 0 12 |
|  |  |  |  | 18 30 | 2 49 | 29 30 | 1 40 | 79 0 | 0 11 |
|  |  | 9 20 | 5 36 | 18 40 | 2 47 | 30 0 | 1 38 | 80 0 | 0 10 |
|  |  |  |  | 18 50 | 2 46 | 30 30 | 1 37 | 81 0 | 0 9 |
| 4 30 | 10 48 | 9 30 | 5 31 | 19 0 | 2 44 | 31 0 | 1 35 | 82 0 | 0 8 |
|  |  |  |  | 19 10 | 2 43 | 31 30 | 1 33 | 83 0 | 0 7 |
|  |  | 9 40 | 5 25 | 19 20 | 2 41 | 32 0 | 1 31 | 84 0 | 0 6 |
|  |  |  |  | 19 30 | 2 40 | 32 30 | 1 30 | 86 0 | 0 4 |
|  |  | 9 50 | 5 20 | 19 40 | 2 38 | 33 0 | 1 29 | 88 0 | 0 2 |
|  |  |  |  | 19 50 | 2 37 | 33 30 | 1 26 | 90 0 | 0 0 |

it occupies position *1*, Fig. 1, with reference to the cross-hairs; the time, vertical angle, and horizontal angle are noted. Then the upper plate is loosened, the instrument turned 180° in azimuth, the telescope inverted, and the sun sighted again, as in position *2*. In position *1*, the sun is moving toward both hairs; in position *2*, the telescope should be set approximately as shown by the dotted circle, so that the sun will clear both hairs at the same instant. For
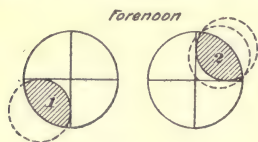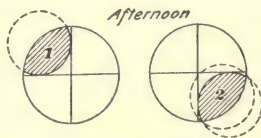


FIG. 1　　　　　　　　　　FIG. 2

afternoon work, the positions shown in Fig. 2 should be used. The observations are taken in *pairs;* if the second observation of a pair cannot be obtained promptly after the first one (owing to a passing cloud, or some other cause), the first must be ignored and considered as useless.

It should be noted that the reversal of the transit between the observations eliminates the index error of the vertical circle, the error of level in the horizontal axis of the telescope, and the error of collimation of the telescope. By sighting in diagonal corners of the field of view and taking the mean of the observations, the corrections (both horizontal and vertical) due to the semi-diameter of the sun are eliminated. To simplify the notes, 180° should be added to (or subtracted from) the horizontal plate reading when the instrument is inverted.

EXAMPLE.—The following measurements were taken in the manner just described. The four means of the circle readings were formed in the field. The declination of the sun was −9° 30′ 5″, and the approximate latitude +39° 57′. Find the azimuth of the reference mark.

| Telescope | Time P. M. | Vertical Circle | | | Horizontal Circle | | |
|---|---|---|---|---|---|---|---|
| Direct | 3:27 | 19° | 39′ | 00″ | 99° | 52′ | 00″ |
| Inverted | 3:29 | 19 | 52 | 00 | 99 | 49 | 00 |
| Mean | 3:28 | 19 | 45 | 30 | 99 | 50 | 30 |
| Direct | 3:32 | 18 | 46 | 00 | 100 | 55 | 30 |
| Inverted | 3:34 | 19 | 3 | 00 | 100 | 49 | 00 |
| Mean | 3:33 | 18 | 54 | 30 | 100 | 52 | 15 |
| Direct | 3:36 | 18 | 4 | 30 | 101 | 46 | 00 |
| Inverted | 3:38 | 18 | 23 | 30 | 101 | 35 | 00 |
| Mean | 3:37 | 18 | 14 | 00 | 101 | 40 | 30 |
| Direct | 3:40 | 17 | 26 | 30 | 102 | 29 | 30 |
| Inverted | 3:42 | 17 | 43 | 00 | 102 | 21 | 00 |
| Mean | 3:41 | 17 | 34 | 45 | 102 | 25 | 15 |

SOLUTION.—

| Mean of the four vertical circle readings | 18° | 37′ | 11″ |
|---|---|---|---|
| Refraction | | −2 | 48 |
| Parallax | | | +8 |
| True altitude of center | 18° | 34′ | 31″ |
| Zenith distance = 90° − true altitude | 71° | 25′ | 29″ |

To find the azimuth of the sun: $z = 71° 25′ 29″$; $\phi = 39° 57′ 0″$; $\delta = -9° 30′ 5″$; $\frac{1}{2}(z + \phi + \delta) = 50° 56′ 12″$; $\frac{1}{2}(z + \phi - \delta) = 60° 26′ 17″$. Substituting these values in the formula for the azimuth of the sun.

$$\sin \tfrac{1}{2}a = \sqrt{\frac{\cos 50° 56′ 12″ \sin 60° 26′ 17″}{\sin 71° 25′ 29″ \cos 39° 57′}}$$

The two values of $\frac{1}{2}a$ are 60° 17′ 15″ and 119° 42′ 45″ ($= 180° - 60° 17′ 15″$). As the observations were made in the afternoon, the obtuse angle should be used. This gives $a = 2 \times 119° 42′ 45″ = 239° 25′ 30″$. The mean of the four horizontal readings is 101° 12′ 8″. Subtracting this from the azimuth of the sun, the azimuth of the reference mark is found to be 239° 25′ 30″ − 101° 12′ 8′ = 138° 13′ 22″.

# RAILROAD SURVEYING

## DEFINITIONS OF CIRCULAR CURVES

The line of a railroad consists of a series of straight lines connected by curves. Each two adjacent lines are united by a curve having the radius
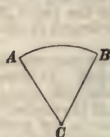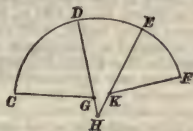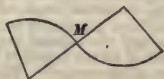


FIG. 1    FIG 2    FIG. 3

best adapted to the conditions of the surface. The straight lines are called *tangents*, because they are tangent to the curves that unite them.

Railroad curves are usually circular and are divided into three general classes, namely, simple, compound, and reverse curves.

A *simple curve* is a curve having but one radius, as the curve $AB$, Fig. 1, whose radius is $AC$.

A *compound curve* is a continuous curve composed of two or more arcs of different radii, as the curve $CDEF$, Fig. 2, which is composed of the arcs $CD$, $DE$, and $EF$, whose respective radii are $GC$, $HD$, and $KE$. In the general class of compound curves may be included what are known as *easement curves*, *transition curves*, and *spiral curves*, now used very generally on the more important railroads.

A *reverse curve* is a continuous curve composed of the arcs of two circles of the same or different radii, the centers of which lie on opposite sides of the curve, as in Fig. 3. The two arcs composing the curve meet at a common point or point of reversal $M$, at which point they are tangent to a common line perpendicular to the line joining their centers. Reverse curves are becoming less common on railroads of standard gauge.

## GEOMETRY OF CIRCULAR CURVES

The following principles of geometry are of special importance as relating to curves:

1. A tangent to a circle is perpendicular to the radius at its tangent point. Thus, in Fig. 4, $AF$ is perpendicular to $BO$ at its tangent point $B$, and $ED$ is perpendicular to $CO$ at $C$.

2. Two tangents to a circle from any point without the circle are equal in length, and make equal angles with the chord joining their points of tangency. Thus, $BE$ and $CE$ are equal, and the angles $EBC$ and $ECB$ are equal.

3. An angle not exceeding 90° formed by a chord and the tangent at one of its extremities, is equal to one-half the central angle subtended by the chord. Thus, the angle $EBC = ECB = \frac{1}{2} BOC$.

4. An angle not exceeding 90° having its vertex in the circumference of a circle and subtended by a chord of the circle, is equal to one-half the central angle subtended by the chord. Thus, the angle $GBH$, whose vertex $B$ is in the
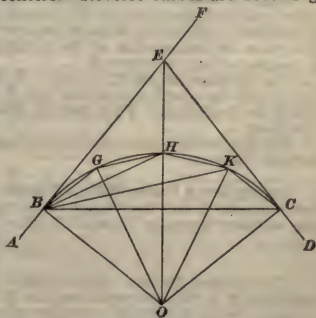


FIG. 4

circumference, is subtended by the chord *GH* and is equal to one-half the central angle *GOH*, subtended by the same chord *GH*.

5. Equal chords of a circle subtend equal angles at its center and also in its circumference, if the angles lie in corresponding segments of the circle. Thus, if *BG*, *GH*, *HK*, and *KC* are equal, *BOG = GOH*, *GBH = HBK*, etc.

6. The *angle of intersection FEC* of two tangents of a circle is equal to the central angle subtended by the chord joining the two points of tangency. Thus, the angle *CEF = BOC*.

7. A radius that bisects any chord of a circle is perpendicular to the chord.

8. A chord subtending an arc of 1° in a circle having a radius = 100 ft. is very closely equal to 1.745 ft.

## ELEMENTS AND METHODS OF LAYING OUT A CIRCULAR CURVE

The *degree of curvature* of a curve is the central angle subtending a chord of 100 ft. Thus, if, in Fig. 4, the chord *BG* is 100 ft. long and the angle *BOG* is 1°, the curve is called a *one-degree curve;* but if, with the same length of chord, the angle *BOG* is 4°, the curve is called a *four-degree curve.*

The *deflection angle* of a chord is the angle formed between any chord of a curve and a tangent to the curve at one extremity of the chord. It is equal to one-half the central angle subtended by the chord. The deflection angle for a chord of 100 ft. is called the *regular deflection angle*, and is equal to one-half the degree of curvature. The deflection angle for a *subchord*—that is, for a chord less than 100 ft.—is equal to one-half the degree of curvature multiplied by the length of the subchord expressed in chords of 100 ft. The length *c* of a subchord or of any chord is given by the formula

in which
$$c = 2R \sin D$$
$R$ = radius;
$D$ = deflection angle of that chord.

**Relation Between Radius and Deflection Angle.**—From the formula just given,

$$R = \frac{c}{2 \sin D}$$

If $D_{100}$ is the deflection angle for a chord of 100 ft., then

$$R = \frac{50}{\sin D_{100}}$$

For a 1° curve, $D_{100} = 30'$ and $R = 5,730$, nearly. For curves less than 10°, the radius may be taken as $\frac{5,730}{D_c}$, in which $D_c$ is the degree of curvature. The accompanying table gives the length of the radius, in feet, for degrees of curvature ranging by intervals of 5' and 10' from 0' to 20°.

**Tangent Distance.**—The point where a curve begins is called the *point of curve*, and is designated by the letters P. C.; and the point where the curve terminates is called the *point of tangency*, and is designated by the letters P. T. The point of intersection of the tangents is called the *point of intersection;* it is designated by the letters P. I.

The distance of the P. C. or P. T. from the P. I. is called the *tangent distance*, and the chord connecting the P. C. and P. T. of a curve is commonly called its *long chord*. This term is also applied to chords more than one station long.

If *I* denotes the angle of intersection and *R* the radius of the curve, then the tangent distance

$$T = R \tan \tfrac{1}{2}I$$

**Laying Out a Curve With a Transit.**—When the angle of intersection *I* has been measured and the degree of curve decided upon, the radius of the curve can be taken from the Table of Radii and Deflections or it can be figured by the formula

$$R = \frac{5,730}{D_c}$$

The tangent distance is then computed and measured back on each tangent from the P. I., thus determining the P. C. and P. T. Subtracting the tangent distance from the station number of the P. I. will give the station number of the P. C. Ordinarily, this will not be an even or full station. The length of the curve is then computed by dividing the angle *I* by the degree of curve, the quotient giving the length of the curve in stations of 100 ft. and decimals thereof. After having found the length of the curve, compute the deflection angles for the chords joining the P. C. with all the station points; set the transit at the P. C.; set the vernier at 0, sight to the intersection point, and turn off

## TABLE OF RADII AND DEFLECTIONS

| Degree | Radii | Chord Deflection | Tangent Deflection |
|---|---|---|---|
| 0° 05′ | 68,754.94 | .145 | .073 |
| 0° 10′ | 34,377.48 | .291 | .145 |
| 0° 15′ | 22,918.31 | .436 | .218 |
| 0° 20′ | 17,188.76 | .582 | .291 |
| 0° 25′ | 13,751.02 | .727 | .364 |
| 0° 30′ | 11,459.19 | .873 | .436 |
| 0° 35′ | 9,822.18 | 1.018 | .509 |
| 0° 40′ | 8,594.49 | 1.164 | .582 |
| 0° 45′ | 7,639.49 | 1.309 | .654 |
| 0° 50′ | 6,875.55 | 1.454 | .727 |
| 0° 55′ | 6,250.51 | 1.600 | .800 |
| 1° 00′ | 5,729.65 | 1.745 | .873 |
| 1° 05′ | 5,288.92 | 1.891 | .945 |
| 1° 10′ | 4,911.15 | 2.036 | 1.018 |
| 1° 15′ | 4,583.75 | 2.182 | 1.091 |
| 1° 20′ | 4,297.28 | 2.327 | 1.164 |
| 1° 25′ | 4,044.51 | 2.472 | 1.236 |
| 1° 30′ | 3,819.83 | 2.618 | 1.309 |
| 1° 35′ | 3,619.13 | 2.763 | 1.382 |
| 1° 40′ | 3,437.87 | 2.909 | 1.454 |
| 1° 45′ | 3,274.17 | 3.054 | 1.527 |
| 1° 50′ | 3,125.36 | 3.200 | 1.600 |
| 1° 55′ | 2,989.48 | 3.345 | 1.673 |
| 2° 00′ | 2,864.93 | 3.490 | 1.745 |
| 2° 05′ | 2,750.35 | 3.636 | 1.818 |
| 2° 10′ | 2,644.63 | 3.781 | 1.891 |
| 2° 15′ | 2,546.64 | 3.927 | 1.963 |
| 2° 20′ | 2,455.70 | 4.072 | 2.036 |
| 2° 25′ | 2,371.04 | 4.218 | 2.109 |
| 2° 30′ | 2,292.01 | 4.363 | 2.181 |
| 2° 35′ | 2,218.09 | 4.508 | 2.254 |
| 2° 40′ | 2,148.79 | 4.654 | 2.327 |
| 2° 45′ | 2,083.68 | 4.799 | 2.400 |
| 2° 50′ | 2,022.41 | 4.945 | 2.472 |
| 2° 55′ | 1,964.64 | 5.090 | 2.545 |

| Degree | Radii | Chord Deflection | Tangent Deflection |
|---|---|---|---|
| 3° 00′ | 1,910.08 | 5.235 | 2.618 |
| 3° 05′ | 1,858.47 | 5.381 | 2.690 |
| 3° 10′ | 1,809.57 | 5.526 | 2.763 |
| 3° 15′ | 1,763.11 | 5.672 | 2.836 |
| 3° 20′ | 1,719.12 | 5.817 | 2.908 |
| 3° 25′ | 1,677.28 | 5.962 | 2.981 |
| 3° 30′ | 1,637.28 | 6.108 | 3.054 |
| 3° 35′ | 1,599.21 | 6.253 | 3.127 |
| 3° 40′ | 1,562.88 | 6.398 | 3.199 |
| 3° 45′ | 1,528.16 | 6.544 | 3.272 |
| 3° 50′ | 1,494.95 | 6.689 | 3.345 |
| 3° 55′ | 1,463.16 | 6.835 | 3.417 |
| 4° 00′ | 1,432.69 | 6.980 | 3.490 |
| 4° 05′ | 1,403.46 | 7.125 | 3.563 |
| 4° 10′ | 1,375.45 | 7.271 | 3.635 |
| 4° 15′ | 1,348.45 | 7.416 | 3.708 |
| 4° 20′ | 1,322.53 | 7.561 | 3.781 |
| 4° 25′ | 1,297.58 | 7.707 | 3.853 |
| 4° 30′ | 1,273.57 | 7.852 | 3.926 |
| 4° 35′ | 1,250.42 | 7.997 | 3.999 |
| 4° 40′ | 1,228.11 | 8.143 | 4.071 |
| 4° 45′ | 1,206.57 | 8.288 | 4.144 |
| 4° 50′ | 1,185.78 | 8.433 | 4.217 |
| 4° 55′ | 1,165.70 | 8.579 | 4.289 |
| 5° 00′ | 1,146.28 | 8.724 | 4.362 |
| 5° 05′ | 1,127.50 | 8.869 | 4.435 |
| 5° 10′ | 1,109.33 | 9.014 | 4.507 |
| 5° 15′ | 1,091.72 | 9.160 | 4.580 |
| 5° 20′ | 1,074.68 | 9.305 | 4.653 |
| 5° 25′ | 1,058.16 | 9.450 | 4.725 |
| 5° 30′ | 1,042.14 | 9.596 | 4.798 |
| 5° 35′ | 1,026.60 | 9.741 | 4.870 |
| 5° 40′ | 1,011.51 | 9.886 | 4.943 |
| 5° 45′ | 996.87 | 10.031 | 5.016 |
| 5° 50′ | 982.64 | 10.177 | 5.088 |
| 5° 55′ | 968.81 | 10.322 | 5.161 |

| Degree | Radii | Chord Deflection | Tangent Deflection |
|---|---|---|---|
| 6° 00′ | 955.37 | 10.467 | 5.234 |
| 6° 05′ | 942.29 | 10.612 | 5.306 |
| 6° 10′ | 929.57 | 10.758 | 5.379 |
| 6° 15′ | 917.19 | 10.903 | 5.451 |
| 6° 20′ | 905.13 | 11.048 | 5.524 |
| 6° 25′ | 893.39 | 11.193 | 5.597 |
| 6° 30′ | 881.95 | 11.339 | 5.669 |
| 6° 35′ | 870.79 | 11.484 | 5.742 |
| 6° 40′ | 859.92 | 11.629 | 5.814 |
| 6° 45′ | 849.32 | 11.774 | 5.887 |
| 6° 50′ | 838.97 | 11.919 | 5.960 |
| 6° 55′ | 828.88 | 12.065 | 6.032 |
| 7° 00′ | 819.02 | 12.210 | 6.105 |
| 7° 05′ | 809.40 | 12.355 | 6.177 |
| 7° 10′ | 800.00 | 12.500 | 6.250 |
| 7° 15′ | 790.81 | 12.645 | 6.323 |
| 7° 20′ | 781.84 | 12.790 | 6.395 |
| 7° 25′ | 773.07 | 12.936 | 6.468 |
| 7° 30′ | 764.49 | 13.081 | 6.540 |
| 7° 35′ | 756.10 | 13.226 | 6.613 |
| 7° 40′ | 747.89 | 13.371 | 6.685 |
| 7° 45′ | 739.86 | 13.516 | 6.758 |
| 7° 50′ | 732.01 | 13.661 | 6.831 |
| 7° 55′ | 724.31 | 13.806 | 6.903 |
| 8° 00′ | 716.78 | 13.951 | 6.976 |
| 8° 05′ | 709.40 | 14.096 | 7.048 |
| 8° 10′ | 702.18 | 14.241 | 7.121 |
| 8° 15′ | 695.09 | 14.387 | 7.193 |
| 8° 20′ | 688.15 | 14.532 | 7.266 |
| 8° 25′ | 681.35 | 14.677 | 7.338 |
| 8° 30′ | 674.69 | 14.822 | 7.411 |
| 8° 35′ | 668.16 | 14.967 | 7.483 |
| 8° 40′ | 661.74 | 15.112 | 7.556 |
| 8° 45′ | 655.45 | 15.257 | 7.628 |
| 8° 50′ | 649.28 | 15.402 | 7.701 |
| 8° 55′ | 643.22 | 15.547 | 7.773 |

| Degree | Radii | Chord Deflection | Tangent Deflection |
|---|---|---|---|
| 9° 00′ | 637.27 | 15.692 | 7.846 |
| 9° 05′ | 631.44 | 15.837 | 7.918 |
| 9° 10′ | 625.71 | 15.982 | 7.991 |
| 9° 15′ | 620.06 | 16.127 | 8.063 |
| 9° 20′ | 614.56 | 16.272 | 8.136 |
| 9° 25′ | 609.14 | 16.417 | 8.208 |
| 9° 30′ | 603.80 | 16.562 | 8.281 |
| 9° 35′ | 598.57 | 16.707 | 8.353 |
| 9° 40′ | 593.42 | 16.852 | 8.426 |
| 9° 45′ | 588.36 | 16.996 | 8.498 |
| 9° 50′ | 583.38 | 17.141 | 8.571 |
| 9° 55′ | 578.49 | 17.286 | 8.643 |
| 10° 00′ | 573.69 | 17.431 | 8.716 |
| 10° 10′ | 564.31 | 17.576 | 8.788 |
| 10° 20′ | 555.23 | 17.721 | 8.860 |
| 10° 30′ | 546.44 | 17.866 | 8.933 |
| 10° 40′ | 537.92 | 18.011 | 9.005 |
| 10° 50′ | 529.67 | 18.156 | 9.078 |
| 11° 00′ | 521.67 | 18.300 | 9.150 |
| 11° 10′ | 513.91 | 18.445 | 9.223 |
| 11° 20′ | 506.38 | 18.590 | 9.295 |
| 11° 30′ | 499.05 | 18.735 | 9.367 |
| 11° 40′ | 491.96 | 18.880 | 9.440 |
| 11° 50′ | 485.05 | 19.024 | 9.512 |
| 12° 00′ | 478.34 | 19.169 | 9.585 |
| 12° 10′ | 471.81 | 19.314 | 9.657 |
| 12° 20′ | 465.46 | 19.459 | 9.729 |
| 12° 30′ | 459.28 | 19.603 | 9.802 |
| 12° 40′ | 453.26 | 19.748 | 9.874 |
| 12° 50′ | 447.40 | 19.893 | 9.946 |
| 13° 00′ | 441.68 | 20.038 | 10.019 |
| 13° 10′ | 436.12 | 20.182 | 10.091 |
| 13° 20′ | 430.69 | 20.327 | 10.164 |
| 13° 30′ | 425.40 | 20.472 | 10.236 |
| 13° 40′ | 420.23 | 20.616 | 10.308 |
| 13° 50′ | 415.19 | 20.761 | 10.381 |

| Degree | Radii | Chord Deflection | Tangent Deflection |
|---|---|---|---|
| 14° 00′ | 410.28 | 24.374 | 12.187 |
| 14° 10′ | 405.47 | 24.663 | 12.331 |
| 14° 20′ | 400.78 | 24.951 | 12.476 |
| 14° 30′ | 396.20 | 25.240 | 12.620 |
| 14° 40′ | 391.72 | 25.528 | 12.764 |
| 14° 50′ | 387.34 | 25.817 | 12.908 |
| 15° 00′ | 383.06 | 26.106 | 13.053 |
| 15° 10′ | 378.88 | 26.394 | 13.197 |
| 15° 20′ | 374.79 | 26.682 | 13.341 |
| 15° 30′ | 370.78 | 26.970 | 13.485 |
| 15° 40′ | 366.86 | 27.259 | 13.629 |
| 15° 50′ | 363.00 | 27.547 | 13.773 |
| 16° 00′ | 359.26 | 27.835 | 13.917 |
| 16° 10′ | 355.59 | 28.123 | 14.061 |
| 16° 20′ | 351.98 | 28.411 | 14.205 |
| 16° 30′ | 348.45 | 28.699 | 14.349 |
| 16° 40′ | 344.99 | 28.986 | 14.493 |
| 16° 50′ | 341.60 | 29.274 | 14.637 |
| 17° 00′ | 338.27 | 29.562 | 14.781 |
| 17° 10′ | 335.01 | 29.850 | 14.925 |
| 17° 20′ | 331.82 | 30.137 | 15.069 |
| 17° 30′ | 328.68 | 30.425 | 15.212 |
| 17° 40′ | 325.60 | 30.712 | 15.356 |
| 17° 50′ | 322.59 | 31.000 | 15.500 |
| 18° 00′ | 319.62 | 31.287 | 15.643 |
| 18° 10′ | 316.71 | 31.574 | 15.787 |
| 18° 20′ | 313.86 | 31.861 | 15.931 |
| 18° 30′ | 311.06 | 32.148 | 16.074 |
| 18° 40′ | 308.30 | 32.436 | 16.218 |
| 18° 50′ | 305.60 | 32.723 | 16.361 |
| 19° 00′ | 302.94 | 33.010 | 16.505 |
| 19° 10′ | 300.33 | 33.296 | 16.648 |
| 19° 20′ | 297.77 | 33.583 | 16.792 |
| 19° 30′ | 295.25 | 33.870 | 16.935 |
| 19° 40′ | 292.77 | 34.157 | 17.078 |
| 19° 50′ | 290.33 | 34.443 | 17.222 |

successively the deflection angles, at the same time measuring the chords and marking the stations. The station of the P. T. is found by adding the length of curve in chords of 100 ft. to the station of the P. C.



FIG. 1

If the entire curve cannot be run from the P. C. on account of obstructions to the view, run the curve as far as the stations are visible from the P. C. and run the remainder of the curve from the last station that can be seen. Suppose that in the 10° curve shown in Fig. 1 the station at $H$, 200 ft. from the P. C., is at $B$, is the last point on the curve that can be set from the P. C. A plug is driven at $H$ and centered carefully by a tack driven at the point. The transit is now moved forward and set up at $H$. As the deflection angle $EBH$ is 10° to the right, an angle of 10° is turned to the left from 0 and the vernier clamped. The instrument is then sighted to a flag at $B$, the lower clamp set, and by means of the lower tangent screw the cross-hairs are made to bisect the flag exactly. The vernier clamp is then loosened, the vernier set at 0, and the telescope plunged. The line of sight will then be on the tangent $IP$, and the deflection angles to $K$ and $C$ can be turned off from this tangent, and the stations at $K$ and $C$ located in the same manner that the stations at $G$ and $H$ were located from $B$, because the angle at $IHB$ between the tangent $IH$ and the chord $BH$ is equal to the angle $EBH$ between the tangent $EB$ and the same chord.

This method of setting the vernier for the backsight when the instrument is moved forwards to a new instrument point on the curve is sometimes called the *method by zero tangent*. The essential principle of the method is that the vernier always reads zero when the instrument is sighted on the tangent to the curve at the point where the instrument is set, and the deflection angles are made to read from the tangent to the curve at this point in the same manner as if this point were the P. C. of the curve.

**Tangent and Chord Deflections.**—Let $AB$, Fig. 2, be a tangent joining the curve $BCEH$ at $B$. If the tangent $AB$ is prolonged to $D$, the perpendicular distance $DC$ from the tangent to the curve is called a *tangent deflection*. If the chord $BC$ is prolonged to the point $G$, so that $CG = CE$, the distance $GE$ is called a *chord deflection*. If the radius $R$ of the curve and the length of the chord $c$ are known, the tangent deflection $f$ can be determined by the formula

$$f = \frac{c^2}{2R}$$

This formula can be used for any length of chord or radius. If $CE = BC$, the chord deflection $= 2f = \frac{c^2}{R}$. For this condition the Table of Radii and Deflections gives the chord deflection and tangent deflection for 100-ft. chords and for degrees of curvature varying by intervals of 5′ and 10′ from 5′ to 20°.



FIG. 2

When the two chords preceding the station considered are of unequal lengths, the chord deflection $= \frac{c_1(c_1 + c_2)}{2R}$, where $c_1$ is the length of the first chord
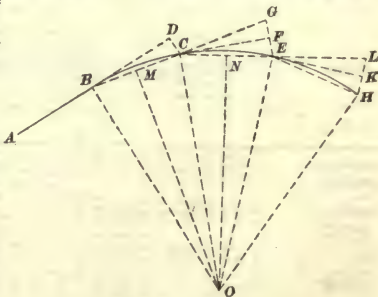
and $c_2$ the length of the second chord preceding the station considered. When the tangent deflection $f$ is known, the chord deflection

$$d_0 = f\left(1 + \frac{c_2}{c_1}\right)$$

**Special Values of Chord and Tangent Deflection.**—For a chord of 100 ft. preceded by one of the same length the chord deflection for a 1° curve is 1.745; for a 2° curve, it is twice that amount, or 3.49; and so on. The tangent deflection, being half the chord deflection, will be .873 ft. for a 1° curve, 1.745 for a 2° curve, etc. The tangent deflection for a chord of any length equals the tangent deflection for a chord of 100 ft. multiplied by the square of the given chord expressed as the decimal part of a chord of 100 ft.

**Application of Chord and Tangent Deflection.**—Let it be required to restore center stakes on the 4° curve, Fig. 3, at each full station. The points $A$ and $B$ determine the direction of the tangent, the point $B$ being the P. C., which is at Sta. 8+25. For a 4° curve the regular chord deflection for 100 ft. is $4 \times 1.745 = 6.98$ ft., and the tangent deflection is 3.49 ft. The distance from P. C. to the next full station is 75 ft.; hence, the tangent deflection $CF = .75^2 \times 3.49 = 1.96$ ft. The point $F$ is found by first measuring 75 ft. from $B$, thus locating the point $C$ in the line $AB$ prolonged, then from $C$ measuring $CF = 1.96$ ft., at right angles to $BC$; the point $F$ thus determined will be Sta. 9. Next, the chord $BF$ is prolonged 100 ft. to $D$; $BF$ is only 75 ft.; $DG$ is computed from the preceding formula; thus, $d_0 = 3.49 (1 + \frac{75}{100}) = 6.11$. This distance is measured at right angles to $BD$; the point $G$ thus determined will be Sta. 10. The point $H$, which is Sta. 11, and the P. T. of the curve, is determined in the same manner, except that, as the chords $FG$ and $GH$ are each 100 ft. long, the regular



Fig. 3

chord deflection of 6.98 ft. is used for $EH$. A stake is driven at each station thus located. Although a chord deflection is not at right angles to the chord theoretically, yet the deflection is so small, as compared with the length of the chord, that for curves of ordinary degree it is usually measured at right angles.

**Middle Ordinate.**—The *middle ordinate* of a chord is the ordinate to the curve at the middle point of the chord. The following formulas give the relation between the length of the chord $c$, the radius of the curve $R$, and the middle ordinate $m$.

$$m = R - \sqrt{R^2 - \frac{c^2}{4}}$$

$$c = 2\sqrt{2Rm - m^2}$$

$$R = \frac{c^2}{8m} + \frac{m}{2}$$

**To Determine Degree of Curve From Middle Ordinate.**—It is sometimes necessary to determine the radius or the degree of a curve in an existing track when no transit is available. By measuring the middle ordinate of any convenient chord, the degree of the curve can be calculated from the relative values of the ordinate and chord. As the track is likely not to be in perfect alinement, it is well to measure the middle ordinate of different chords in different parts of the curve; also, as the middle ordinate of a chord measured to the inner rail will somewhat exceed the middle ordinate of the same chord measured to the outer rail, the ordinate of each chord should be measured to both rails and the average of the two taken as the value of the ordinate. Having measured the middle ordinate of one or more chords, the degree of curve $D_c$ can be found by the formula

$$D_c = \frac{45,840 \, m}{c^2}$$

The following rule is sometimes applied in determining the degree of curve:
**Rule.**—*Measure the middle ordinate to a chord of 67.71 ft.; express it in feet and decimals of a foot, and multiply by 10; the result will be the degree of the curve.*

**Rules for Measuring the Radius of a Curve.**—Stretch a string, say 20 ft. long, or longer if the curve is not a sharp one, across the curve corresponding



FIG. 4

to the line from $A$ to $C$, in Fig. 4. Then measure from $B$ the center of the line $AC$, and at right angles with it, to the rail at $D$. Multiply the distance $A$ to $B$, or one-half the length of the string, in inches, by itself; measure the distance $D$ to $B$ in inches, and multiply it by itself. Add these two products, and divide the sum by twice the distance from $B$ to $D$, measured exactly in inches and fractional parts of inches. This will give the radius of the curve in inches.
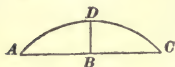
It may be more convenient to use a straightedge instead of a string. Care must be taken to have the ends of the string or straightedge touch the same part of the rail as is taken in measuring the distance from the center. If the string touches the bottom of the rail flange at each end, and the center measurement is made to the rail head, the result will not be correct.

In practice, it will be found best to make trials on different parts of the curve, to allow for irregularities.

ILLUSTRATION.—Let $AC$ be a 20-ft. string; half the distance, or $AB$, is then 10 ft., or 120 in. Suppose $BD$ is found on measurement to be 3 in. Then $120 \times 120 = 14,400$, and $3 \times 3 = 9$; $14,400 + 9 = 14,409$, which, divided by $2 \times 3 = 6$, equals $2,401\frac{1}{2}$ in., or 200 ft. $1\frac{1}{2}$ in., which is the radius of the curve. The formula $\dfrac{AB^2 + BD^2}{2\,BD} = R$, applied to the example is

$$\frac{120^2 + 9}{2 \times 3} = 2,401\tfrac{1}{2} \text{ in.} = 200 \text{ ft. } 1\tfrac{1}{2} \text{ in.}$$

**Other Ordinates.**—Any ordinate $y$ to the curve at a distance $a$ from the middle point of a chord may be determined by means of the formula:

$$y = \sqrt{R^2 - a^2} - R + m$$

By using long chords, a curve may be laid out or obstacles passed by means of ordinates. Suppose that it is required to run out the curve $AEH$, Fig. 5, with several obstacles in the direct line of the curve, as shown, Sta. 3 being the P. C., and the regular stations on the curve being in the positions indicated by the numbers 4, 7, 8, etc. The positions of Stas. 5 and 6 are indicated by the letters $C$ and $D$. The stations are to be located in their proper positions on the curve, between the obstructions, wherever it is possible to do so. In addition to this, it is customary to mark with a tack or otherwise the point where the line of the curve intersects each obstruction.

Beginning at the point of curve $A$, which is at Sta. 3, the curve can be run in as far as the first obstruction, which is the building $P$, setting the stakes on the curve at Stas. 4 and 5, and a tack in the side of the building $P$ at the point where the line of curve intersects, according to the deflection angle as determined by its distance from Sta. 5. It is not possible to proceed further in the regular manner, however, because Sta. 6 cannot be seen from the P. C. Therefore, it is necessary to locate Sta. 7 by deflection angle $V'BE$, from $B$ or Sta. 4, to determine the chord 4–7, which, in this case, is a long



FIG. 5

chord of three stations, and to calculate the ordinates $D'D$ and $C'C$ by substituting for $a$ in the preceding formula the value of $MC' = MD' =$ half a station or 50 ft.

Fig. 5 shows also another method of passing a building $S$, namely, by running an equilateral triangle $FLG$. In this method, the instrument is set up at Sta. 8 and sighted back to the P. C. Then, the telescope is reversed and the deflection angle for Sta. 9 is turned off the same as if no obstruction existed.
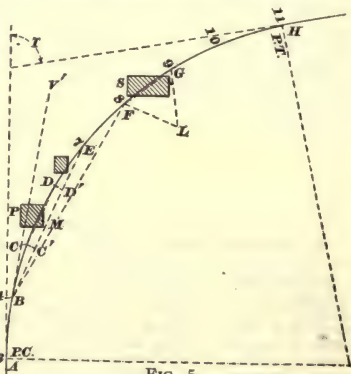
The telescope will then be sighted on the line $FG$, although the point $G$ will not be visible. The angle $GFL$, equal to 60°, is then turned, and the point $L$ is located so that $FL = FG = 100$ ft. The instrument is next moved to $L$, and the line $LG$ is run, making 60° with $FL$. On this line the distance $LG = 100$ ft. is measured, giving the point $G$, which is Sta. 9. The transit is then set up at this point and sighted to $L$, and an angle of 60° is turned off to the right, giving the direction of the line 9–8, the intersection of which with $S$ is marked. The remainder of the curve may be run in the following manner: Set the vernier at an angle equal to the deflection angle of the chord 9–8 to the left from the 0; clamp the upper plate, sight at the point set in the line 9–8; then clamp the lower plate and set vernier at 0. The line of sight will then be in the tangent at point 9, and by plunging the telescope the remainder of the curve can be run as if the point 9 were the P. C.

## FIELD NOTES FOR CURVES

Various styles of field notebooks are published, in which the pages are ruled to suit the different kinds and methods of field work. The accompanying, which are the field notes of a portion of a line containing a curve, represent a good form for recording the field notes of a curve that is run in by the method of zero tangent.

In the first column are recorded the station numbers; in the second column, the deflections with the abbreviations P. C. and P. T., together with the degree of curve and the abbreviation R or L, according as the line curves to the right or left. At each transit point on the curve, the total or central angle from the P. C. to that point is calculated and recorded in the third column. This total angle is double the deflection angle between the P. C. and the transit point. In

| Station | Deflection | Tot.Angle | Mag.Bearing | Ded.Bearing | Remarks | June 20.1912 |
|---------|-----------|-----------|-------------|-------------|---------|---------|
| 9 | | | | | | |
| 8 | - | | | | | |
| 7 | | | | | | |
| 6+95 | 4°54′P.T. | 15°00′ | N35°20′E. | N35°15′E. | | |
| 6+50 | 4°00′ | | | | | |
| 6 | 3°00′ | | | | | Centerline of highway |
| 5+50 | 2°00′ | | | | 5+80 | |
| 5 | 1°00′ | | | | 5+60 | |
| 4+50 | 2°36′ | 5°12′ | | | | |
| 4 | 1°36′ | | | | Int.Angle=15°00′ | 4°Curve R |
| 3+50 | 0°36′ | | | | T=188.61 ft. | Def.Angle for 50 ft.=1°00′ |
| 3+20 | P.C.4°R. | | | | P.C.=3+20 | Def.Angle for 1 ft.=1.2′ |
| 3 | | | | | Length of Curve=375 ft. | |
| 2 | | | | | P.T.=6+95 | |
| 1 | | | | | | |
| 0 | | | N20°15′E. | N20°15′E. | | |

these notes, there is but one intermediate transit point between the P. C. and the P. T. The deflection from the P. C. at Sta. 3+20 to the intermediate transit point at Sta. 4+50 is 2° 36′. The total angle is double this deflection, or 5° 12′, which is recorded on the same line in the third column. The record of total angles at once indicates the stations at which transit points are placed. The total angle at the P. T. will be the same as the angle of intersection, provided the work is correct. When the curve is finished, the transit is set up at the P. T., and the bearing of the forward tangent taken, which affords an additional check upon the previous calculations. The magnetic bearing is recorded in the fourth column, and the deduced, or calculated, bearing is recorded in the fifth column.

## EARTHWORK

**Cuts and Fills.**—When building a railroad, cuts and fills are introduced to equalize the irregularities of the natural soil. Figs. 1 and 2 show a typical fill and cut in ordinary firm earth or gravel.

**Slope Ratio.**—In cuts in the hardest rock, the average slope is made usually $\frac{1}{4}$:1; that is, $\frac{1}{4}$ horizontal to 1 vertical. As the soil becomes less firm, the slope must be flattened until, for a soil of firm earth or gravel, a slope of 1 to 1 may be permissible, although a slope of $1\frac{1}{2}$:1 is commonly adopted. In very soft soil, the slope ratio is sometimes cut down even as far as 4 horizontal to 1 vertical.

The standard practice in a fill is 1½ horizontal to 1 vertical. When a fill is made of the material from a rock cut, it is possible to make a stable embankment with a slope ratio of 1:1. On side-hill work, where a slope ratio of 1¼:1 or even 1:1 might require a very long slope, it is often advisable to make a rough dry wall of the stones from a rock cut that will have a slope ratio of ¾:1, or it may even be steeper.
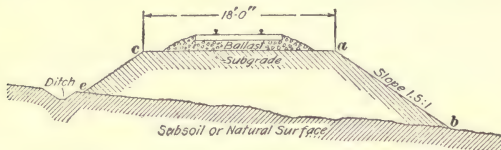


FIG. 1

**Width of Excavations and Embankments.**—The width required for a standard-gauge single-track roadbed may be estimated as follows (see Figs. 1 and 2): The tie will be between 8 and 9 ft. long, usually 8 ft. 6 in., and at the ends the ballast will slope down to subgrade. The extra width required for this will be about 1 or 2 ft. at each end of the tie. Usually, the embankment is widened for about 2 ft. beyond the ballast on each side. The absolute minimum for the width of subgrade for a fill is, therefore, 8½ ft. + 2 × (1 + 2) ft. = 14½ ft. This width would be used only for light-traffic, cheaply constructed roads; 16 to 18 ft. is far more common, while 20 ft. and even more is frequently used, as the danger of accident due to a washing out of the embankment is materially reduced by widening the roadbed.

In cuts, the proper width for two ditches should be added. Unless the soil is especially firm, the ditches should have a side slope of 1½:1. If the ditch is 12 in. wide at the base and 12 in. deep, with side slopes of 1½:1, each ditch will require a total width of 4 ft. This will add 8 ft. to the width of the cut at the elevation of subgrade. The usual distance between track centers for double track is 13 ft. Therefore, whatever rate of side slopes and width of ditches is required for single-track work, the width for double-track work must be 13 ft. greater. When excavation is made through rock, the side slopes of the ditches may properly be made much steeper; the danger of scouring during heavy rain storms being eliminated, the total required width may be very materially reduced from the figures just given. The heavy expense of excavating through solid rock requires that such economy shall be used if possible.

**Grade Profile.**—For the purpose of constructing a road as well as for calculating the earthwork, a grade profile is prepared by setting stakes on the center line at every full station and also at all intermediate points at which the inclination of the natural surface of the ground changes abruptly; then, by leveling, the elevation of the natural surface at each stake is determined and plotted, as explained under Leveling. The established grade is then drawn in.
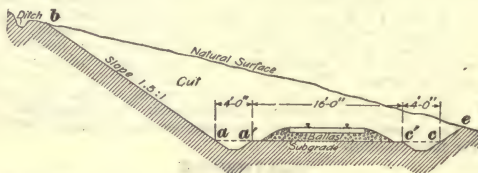


FIG. 2

It consists of a series of straight lines, the elevations of the ends of which are clearly indicated. These elevations are those of the subgrade *ac*, Figs. 1 and 2.

A short portion of a profile is shown in Fig. 3. The horizontal line *XX'* represents a reference plane, and the broken line *AGH* shows the position of the established grade. The station numbers are written along the line *XX'*, and the elevations of the corresponding points of the established grade are

written along the grade line. Thus, in Fig. 3, the elevation of subgrade at Sta. 90, or *A*, is 100 ft.; at Sta. 93, or *G*, it is 102.28 ft.; and at Sta. 94, or *H*, it is 101.78 ft.

The *gradient* of the established grade is the per cent. of rise or fall of grade; that is, the number of feet by which the elevation increases or decreases in 100 ft. It is usually marked on the grade line in the manner shown in Fig. 3.

The *depth of center stake* is the difference between the elevation of the
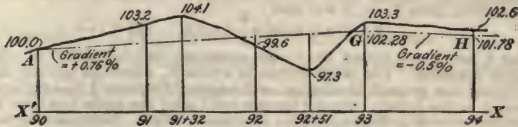


FIG. 3

natural surface at any stake and the elevation of the subgrade. The elevation of the natural surface is found in the level notes, while the elevations of the subgrade are computed from the gradients and also entered in the level notes. The difference for each stake is then figured and entered in a column headed Depth of Center Stake, being preceded by the letter *C* or *F* to indicate cut or fill.

EXAMPLE.—Stakes are set at the stations indicated in the first column of the accompanying field notes. The gradient is +.76% from Sta. 90 to Sta. 93, and −.50% beyond Sta. 93. The elevation of the established grade at Sta. 90 is 100 ft.; the elevation of the natural surface at each stake is given in the third column. Find the center depth at each stake. (See Fig. 3.)

| Station | Subgrade | Elevation | Depth of Center Stake |
|---|---|---|---|
| 94 | 101.8 | 102.6 | C .8 |
| 93 | 102.3 | 103.3 | C 1.0 |
| 92+51 | 101.9 | 97.3 | F 4.6 |
| 92 | 101.5 | 99.6 | F 1.9 |
| 91+32 | 101.0 | 104.1 | C 3.1 |
| 91 | 100.8 | 103.2 | C 2.4 |
| 90 | 100.0 | 100.0 | 0 |

SOLUTION.—The elevations of the subgrade at the station stakes are determined as follows:

| Station | Elevation |
|---|---|
| 91 | $100.00 + 1.00 \times .76 = 100.8$ |
| 91+32 | $100.00 + 1.32 \times .76 = 101.0$ |
| 92 | $100.00 + 2.00 \times .76 = 101.5$ |
| 92+51 | $100.00 + 2.51 \times .76 = 101.9$ |
| 93 | $100.00 + 3.00 \times .76 = 102.3$ |
| 94 | $102.28 + 1.00 \times -.50 = 101.8$ |

The center depth is the difference between the corresponding numbers in the second and third columns. This is a fill if the subgrade is higher than the natural surface; otherwise, it is a cut.

**Slope Stakes.**—In addition to center stakes, slope stakes are used to mark the points where the side slopes of a cut or a fill intersect the natural surface of the ground. In Fig. 4, *c* is the center stake and *m* and *m'* are the slope stakes. The method of locating slope stakes is as follows, all letters referring to Fig. 4:

Let *b* be the width *l l'* of the roadbed; *d*, the depth *ce* of the center stake; and *s* the slope ratio $= lk \div mk = l'k' \div m'k'$. For the upper stake at *m*, let *x* be the distance *mq* from the slope stake to the center line; $y + d$, the elevation of *m* above the subgrade $= qc + ce = mk$. Similarly for the lower stake at *m'*,

let $x'$ be the horizontal distance $m'q'$ from $m'$ to the center line, and let $d-y'$ $=m'k'$, the elevation of $m'$ above the subgrade.

Then,
$$x=\frac{b}{2}+s\times d+s\times y \qquad (1)$$

and
$$x'=\frac{b}{2}+s\times d-s\times y' \qquad (2)$$

If the natural surface $mcm'$ is a level line, so that $q$, $c$, and $q'$ are at the same elevation, then $y=o$, $y'=o$, and
$$x=x'=ca=ca'=\frac{b}{2}+s\times d \qquad (3)$$

Formulas 1 and 2 are called *slope-stake equations* and formula 3 is called the *level-section equation.* The last formula is available when the ground is nearly level. When the ground is sloping or irregular, formula 1 is employed, but not directly, as the value of $y$ is not known until after the stake has been located. The distance $x$ or $x'$ is determined by successive trials. Suppose, for example, that, in Fig. 4, $d=6.3$ and the rod reading on the point $c$ be 5.9. Suppose, also, that $s=1.5:1$. Then, if the ground is level, by formula 3,

$$ac=\frac{20}{2}+1.5\times6.3=19.5 \text{ ft.}$$

To find the location of $m$, the rodman will hold the rod at some point more than 19.5 ft. from $cr$. Suppose that he holds it at $n$, 20 ft from $cr$, and that the



reading on the rod in this position is 2.8. Then, the height of this point above $c$ equals the reading on $c$ minus the reading on $n$, or 5.9 − 2.8 = 3.1 ft. The computed distance from the rod to $cr$ is, by formula 1, $\frac{20}{2}$ + 1.5 × 6.3 + 1.5 × 3.1 =24.2 ft. As the measured distance (20 ft.) is much smaller than this, the rod must be moved much farther out.

FIG. 4

Suppose that the rod is carried out 7 ft. so that the measured distance to $cr$ is 27 ft., and suppose that the reading on the rod in this position is .8 ft. The elevation of this trial point above $c$ will be 5.9−.8=5.1 ft., and by formula 1, the computed distance $x$ is $\frac{20}{2}$+1.5×6.3+1.5×5.1=27.2 ft. This agrees so closely with the measured distance that the slope stake may be driven at this point.

The lower slope stake at $m'$ is set in the same manner as the upper, except that the distance of each trial point below $c$ is measured, and formula 2 is used in computing the corresponding value of $x'$. The distance of the trial point from $cr$ will in this case be taken less than the distance $ca'$ computed by formula 3. As in the preceding case, if the measured distance from $cr$ to the trial point is less than the computed distance, the point should be moved out; if greater, it should be moved in.

**Form of Notes in Cross-Section Work.**—When each slope stake has been set as just explained, its distance from the center line and the elevation of the stake above or below subgrade are entered in the field book in the form of a fraction. The numerator of this fraction is the distance of the stake above or below subgrade, and the denominator is the distance of the stake from the center line. Thus, if the slope stakes in the preceding example are set at Sta. 131, the complete entry in the notebook will be as follows:
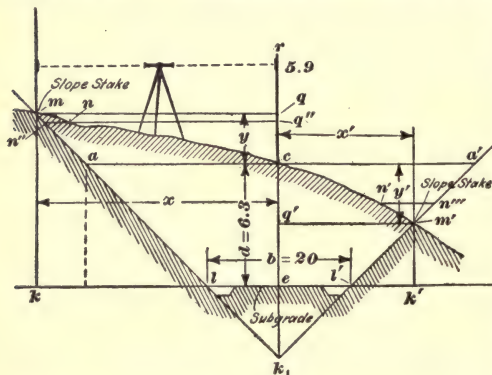
| Station | Subgrade | Elevation | Center Depth | Left | Right |
|---------|----------|-----------|--------------|------|-------|
| 132 | 149.80 | 159.7 | C 9.9 | C 11.4 | C 2.3 |
| 131 | 148.80 | 155.1 | C 6.3 | 27.2 | 13.5 |
| 130 | 147.80 | 147.2 | F .6 | | |

The fraction $\dfrac{C11.4}{27.2}$ indicates that the left slope stake at *m*, Fig. 4, is 27.2 ft. from the center line of the roadbed and 11.4 ft. above subgrade. Similarly, the fraction $\dfrac{C2.3}{13.5}$ indicates that the right slope stake *m'* is 13.5 ft. to the right of the center line and 2.3 ft. above subgrade. These expressions are called *slope-stake fractions*.

When the ground between the slope stakes and the center stake is irregular, the elevations and distances from the center of the intermediate points where the ground changes abruptly are determined and also entered in the notebook in the form of fractions.

## RAILROAD LOCATION

The preliminary survey is made by the methods given, a random line being run along the proposed route and a map of the region made covering several hundred feet on each side of the traverse. The map should show both banks of any streams and enough levels should be taken to show the contours within the width covered by the map. This width will depend on the nature of the ground, being less in hilly than in flat country. In general, the map should cover the ground from the toe of one side hill to the toe of the other and should extend a distance up each hill to an elevation beyond which it would not be practicable to make a cut. After this preliminary line has been mapped, a preliminary estimate of the cost of the proposed work may be made.

**Preliminary Estimate.**—When making a preliminary estimate, great accuracy is not necessary, and no time should be wasted in useless refinements of calculation. The estimate should be high enough to cover all probable cost, and a liberal allowance should be made to cover unforeseen contingencies that may develop during construction. Most experienced engineers make it a rule to add 10% to a preliminary estimate in order to provide for contingencies.

When estimating for earthwork, the cross-sections may be considered as level cuttings; that is, the cross-section surface may be considered as level, unless its slope angle exceeds 10°, in which case a suitable allowance must be made for the slope. The preliminary estimate, which also includes approximate figures for material and labor required for culverts, bridges, trestles, piers, and abutments is then classified and summarized. A sample of a good form of a preliminary estimate of the cost of a proposed railroad follows:

### Estimate of Cost—A & B Railroad

| | |
|---|---:|
| Clearing 625 A. at $20 per A. | $ 12,500 |
| Earth excavation: 900,000 cu. yd. at 17c. | 153,000 |
| Loose-rock excavation: 300,000 cu. yd. at 40c. | 120,000 |
| Solid-rock excavation: 200,000 cu. yd. at 80c. | 160,000 |
| Overhaul exceeding 600 ft.: 300,000 cu. yd. at 1c. | 3,000 |
| Borrowed embankment: 80,000 cu. yd. at 17c. | 13,600 |
| Piling: 12,000 lin. ft. at 25c. | 3,000 |
| Framed trestles: 300,000 ft. B. M. at $35 per M. | 10,500 |
| First-class masonry: 2,800 cu. yd. at $12 | 33,600 |
| Second-class masonry: 4,200 cu. yd. at $8. | 33,600 |
| Box culvert masonry: 2,300 cu. yd. at $5 | 11,500 |
| Dry-rubble masonry: 2,600 cu. yd. at $4. | 10,400 |
| Concrete masonry: 3,000 cu. yd. at $6. | 18,000 |
| Riprap: 2,000 sq. yd. at $1.50. | 3,000 |
| Cast-iron pipe culverts: 40,000 lb. at 3c. | 1,200 |
| Vitrified pipe culverts: 1,800 lin. ft. at $1.50. | 2,700 |
| Total, exclusive of bridges and track. | $589,600 |
| Add 10 per cent. | 58,960 |
| Total cost for grading and trestles. | $648,560 |

**Location.**—The location is the operation of fitting the line to the ground in such a manner as to secure the best adjustment of the alinement and grade, consistent with an economical cost of construction. It is then best projected on the map, and it is called a *paper location*.



FIG. 1

An example of such location is illustrated in Fig. 1. Here, the line follows the valley of Bear River, and the gradient is determined by the slope of the stream. The gradient adopted is .5%, or .5 ft. per station. The preliminary line is shown dotted, and the located line is drawn full.

Let the grade elevation for Sta. 16 be 155 ft.; the grade elevation for Sta. 17 will, therefore, be 155 ft. + .5 ft. = 155.5 ft. The grade elevation for Sta. 18 will be 155.5 + .5 = 156 ft. By the same process, the grade elevation is found for each station shown in the plat; and by means of interpolation between two contour curves, points having the required elevation are located opposite the corresponding stations of the preliminary survey. Each point is marked by a small dot enclosed in a circle. A line joining the points thus designated will be the *grade contour*, or the line where the required gradient meets the surface of the ground. The tangents *AB* and *CD* are then projected so as to conform as closely as practicable to the grade contour, and a suitable curve is inserted for the intersection angle *EFD*. This is most conveniently done by means of a *curved protractor*, an illustration of which is shown in Fig. 2. This instrument, which is made of transparent material, is shifted until there is found a curve that will fit the topography and will close the angle between the tangents, as required.
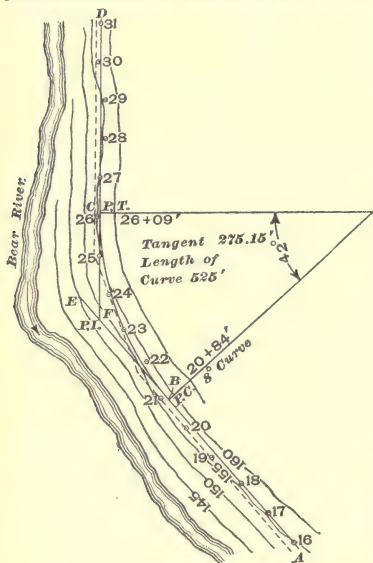
**Curvature.**—There is no fixed rule for limiting curvature, but for a permanent track it is desirable to have the curvature as easy as possible. For all ordinary traffic conditions, it is good practice to use such curves as will best conform to existing topographical conditions. Any curve up to 10° will be no obstacle to a speed of 35 mi. per hr., the average speed of passenger trains. This practice will afford a range in
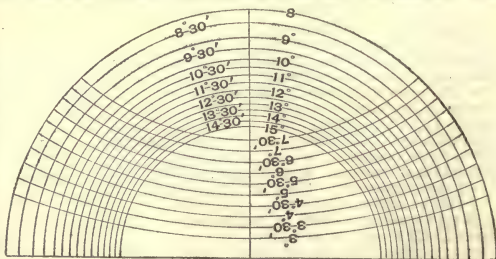


FIG. 2

curvature that will meet the requirements of almost any locality.

**Compensation for Curvature.**—The effect of curvature on a railroad line is to cause a resistance to the movement of trains. When a curve occurs on a

gradient, the effect of the curve resistance on ascending trains is practically the same as increasing the gradient. It is customary, when fixing the final grades, to lighten the grade on a curve by an amount sufficient to offset the resistance due to the curvature. This operation is called *compensating for curvature.* The usual rate of compensation for curvature is .03 to .05 ft. per 100 ft. per degree of curvature. For example, where the maximum gradient on tangents is 1%, the maximum gradient on a 6° curve, allowing a compensation of .03 ft. per degree, would be $1-(.03\times6)=.82\%$. If a compensation of .05 ft. per degree were made, the grade on a 6° curve would be $1-(.05\times6)=.70\%$.

**Final Grade Lines.**—The establishing of final grade lines is illustrated in Fig. 3, where the uncompensated grade is 1.3%, and the compensation for curvature, as shown on the final grade line, is .03 ft. per degree. The location notes for this line are as given on page 122.

The elevation of the grade at Sta. 27 is fixed at 120 ft., and at Sta. 52, at 152.5 ft., giving between these stations an actual rise of 32.5 ft. and an uncompensated grade of 1.3%. These grade points are marked on the profile with small circles. The total curvature between Sta. 27 and Sta. 52 is 108½°. The resistance due to each degree of curvature being taken as equivalent to an increase of .03 ft. in grade, the total resistance due to 108.5° is equivalent to $.03\times108.5=3.255$ ft. additional rise between Sta. 27 and Sta. 52. Hence, the total theoretical grade between these stations is the sum of 32.5 ft., the actual rise, and 3.255 ft. due to curvature, or 35.755 ft. Dividing 35.755 by 25, the number of stations in the given distance, there results $35.755\div25=+1.4302$ ft., as the grade for tangents on this line. The starting point of this grade is at Sta. 27. The P. C. of the first curve is at Sta. 29, giving a tangent of 200 ft. which is equal to two stations. As the grade for tangents is $+1.4302$ ft. per station, the rise in grade between Sta. 27 and Sta. 29 is $1.4302\times2=2.8604$ ft. The elevation of grade at Sta. 27 is 120 ft., and the elevation of grade at Sta. 29 is $120+2.8604=122.8604$ ft. which is recorded on the profile as shown in the diagram, with the rate of grade, namely, $+1.4302$, written above the grade line. The first curve is 8°, and, as the compensation per degree is .03 ft., then, for 8°, and a full station, the compensation is $.03\times8=.24$ ft. The grade on the curve will therefore be the tangent grade minus the compensation, or $1.4302-.24=+1.1902$ ft. per station. The P. C. of this curve is at Sta. 29, the P. T. at Sta. 33, making the total length of the curve 400 ft. or four stations. The grade on this curve is



FIG. 3

$+1.1902$ ft. per station and the total rise on the curve is $1.1902\times4=4.7608$ ft. The elevation of the grade at the P. C. at Sta. 29 is 122.8604; hence, the elevation of grade at the P. T. at Sta. 33 is $122.8604+4.7608=127.6212$ ft., which is recorded on the profile together with the grade, namely, $+1.4302$, written above the grade line. The P. C. of the next curve is at Sta. 37+50, giving an intermediate tangent of 450 ft., or four and one-half stations. The grade for tangents is $+1.4302$ ft. per station; hence, the total rise on the tangent is $1.4302\times4.5=6.4359$ ft. Adding 6.4359 ft., to 127.6212 ft., the elevation
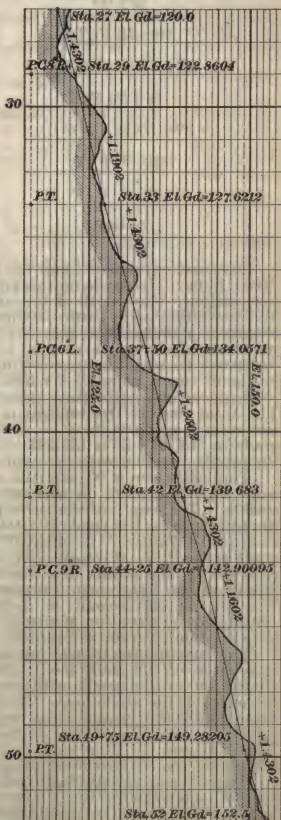
of grade at Sta. 37+50 is found to be 134.0571 ft., which is recorded on the profile, together with the rate of grade for tangents.

## LOCATION NOTES

| Stations | Intersection Angles |
|---|---|
| 52+00 | End of Grade |
| 49+75 P. T. | |
| 44+25 P. C. 9° R. | 49° 30' |
| 42+00 P. T. | |
| 37+50 P. C. 6° L. | 27° 00' |
| 33+00 P. T. | |
| 29+00 P. C. 8° R. | 32° 00' |
| 27+00 | Beginning of grade |

The next curve is 6°, and the compensation in grade per station is .03 ft. ×6 = .18 ft. The grade on this curve will therefore be 1.4302 − .18 = 1.2502 ft. per station. The length of the curve is 450 ft., or four and one-half stations, and the total rise in grade on this curve is +1.2502 ft. ×4.5 = 5.6259 ft. The elevation of the grade at Sta. 37+50, the P. C. of the curve, is 134.0571. The elevation of the grade at Sta. 42, the P. T., is therefore 134.0571+5.6259 = 139.683 ft., which is recorded on the profile, together with the rate of grade on the 6° curve, namely, +1.2502. The P. C. of the next curve is at Sta. 44 +25, giving an intermediate tangent of 225 ft., or two and one-fourth stations. The total rise on the tangent is, therefore, 1.4302×2.25 = 3.21795 ft. The elevation of grade at the P. T. at Sta. 42 is 139.683; therefore, the elevation of grade at Sta. 44+25 is 139.683+3.21795 = 142.90095 ft., which is recorded on the profile, together with the grade +1.4302.

The last curve is 9°, and the compensation in grade per station is .03×9 = .27 ft. The grade on this curve is, therefore, 1.4302 − .27 = 1.1602 ft. per station. The length of the curve is 550 ft., or five and one-half stations, and the total rise on the curve is 1.1602×5.5 = 6.3811 ft. The elevation of grade at Sta. 44+25, the P. C. of the 9° curve, is 142.90095; hence, the elevation of grade at the P. T., at Sta. 49+75, is 142.90095+6.3811 = 149.28205 ft., which is recorded on the profile, together with the grade, +1.1602. The end of the line is at Sta. 52, giving a tangent of 225 ft., or two and one-fourth stations. The rise on this tangent is 1.4302×2.25 = 3.21795 ft., which is added to 149.28205, the elevation of the P. T. at Sta. 49+75. The sum, 152.5 ft., is the elevation of grade at Sta. 52.

The sum of the partial grades should equal the total rise between the extremities of the grade line. The points where the changes of grade occur are marked on the profile with small circles, which are connected by fine lines representing the grade line. These points of change are projected on a horizontal line at the bottom of the profile. The portions of this line that represent curves are dotted, and the portions that represent tangents are drawn full. The P. C. and P. T. of each curve are marked with small circles on this horizontal line, and are lettered as shown in the diagram.

Where the grades are light and the curves have large radii, there will be no need of compensation for curvature. Where the grades exceed .5% and the curves 5°, compensation should be made.

## VERTICAL CURVES

If the grade of the center line of track changes at any point, the two grade lines that intersect at this point form with each other an angle more or less abrupt. If this angle points upwards, it is called a *spur;* if it points downwards, it is called a *sag.* The angles *CVD* in Fig. 1 (*a*) and (*b*) are spurs; the angles *CVD* in Fig. 2 (*a*) and (*b*) are sags.

**Vertical Curve at a Spur.**—If *AV* and *BV*, Fig. 3, are two grade lines meeting at *V*, a vertical curve *CMD* must be introduced to join these lines.

Between $C$ and $D$, the actual grade is established along the vertical curve $CMD$, instead of along $CV$ and $VD$. The projections $RT$ and $TS$ of the distances $VC$ and $VD$ from the vertex to the points at which the vertical curve begins and
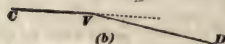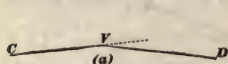


FIG. 1
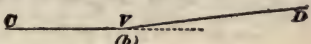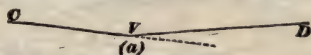


FIG. 2

ends are always chosen equal. If $K$ is the middle point of the straight line $CD$, the vertical curve is always so chosen that it will bisect $VK$; that is, so that $VM = MK$.

Let $E$ be the elevation of $C$, Fig. 3, $E'$ that of $D$, and $H$ that of $V$, so that $E = RC$, $E' = SD$, and $H = VT$. Then,

$$VM = \tfrac{1}{2}\left(H - \frac{E + E'}{2}\right)$$

The distance $VM$ is called the *correction in grade* at the point $V$.

Vertical curves are always made parabolic. It is a property of the parabola that the correction in grade $am$ at any point $a$ is given by the equation,

$$am = VM\left(\frac{Ca}{CV}\right)^2$$

The distance $CV = VD$ is always made a whole number of stations; and, to simplify the work, the grade stakes $a$, $b$, $c$, etc., are so set that they divide the distance $CV$ into a number of equal parts. The corrections in grade at points $a'$, $b'$, and $c'$ along $DV$ are equal to those for the corresponding points along $CV$. That is, if $Ca = Da'$, then $am = a'm'$; if $Cb = Db'$, then $bm = b'm'$, etc.

EXAMPLE.—A



FIG. 3

$+.4\%$ grade meets a $-.5\%$ grade at Sta. 190, the elevation of which is 161.3 ft. If a vertical curve 400 ft. long is inserted, what is the correction in grade and the corrected grade elevation at each station and half station?

SOLUTION.—In this example, $VC = VD = 200$ ft. The elevation of $C$ is 161.3 $-2 \times .4 = 160.5$ ft., $= E$; that of $D$ is 161.3 $-2 \times .5 = 160.3$ ft., $= E'$; that of $K$ is $\tfrac{1}{2}(E' + E) = \tfrac{1}{2} \times (160.5 + 160.3) = 160.4$ ft.; and that of $V$ is $H = 161.3$ ft. Substituting these values in the formula for $VM$,

$$VM = \tfrac{1}{2} \times (161.3 - 160.4) = .45 \text{ ft.}$$

Since, for the first stake, $Ca = 50$ ft. and $CV = 200$ ft., the formula for $am$ gives

$$am = (\tfrac{50}{200})^2 \times VM = \tfrac{1}{16} \times .45 = .03 \text{ ft.} = a'm'$$

Similarly,

$$bm = (\tfrac{100}{200})^2 \times VM = \tfrac{1}{4} \times .45 = .11 = b'm'$$
$$cm = (\tfrac{150}{200})^2 \times VM = \tfrac{9}{16} \times .45 = .25 = c'm'$$

The original and corrected grade elevations are as follows:

| Station | Original Elevation | Correction | Corrected Elevation |
|---|---|---|---|
| 188 | 160.50 | .00 | 160.50 |
| +50 | 160.70 | .03 | 160.67 |
| 189 | 160.90 | .11 | 160.79 |
| +50 | 161.10 | .25 | 160.85 |
| 190 | 161.30 | .45 | 160.85 |
| +50 | 161.05 | .25 | 160.80 |
| 191 | 160.80 | .11 | 160.69 |
| +50 | 160.55 | .03 | 160.52 |
| 192 | 160.30 | .00 | 160.30 |
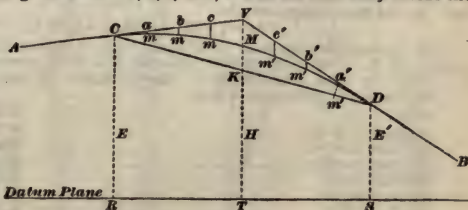
**Vertical Curve at a Sag.**—If two grade lines, $AV$ and $VB$, Fig. 4, meet so as to form a sag, the vertical curve will evidently be wholly above both grade lines. Using the same notation as before, the correction in grade at point $V$ will be
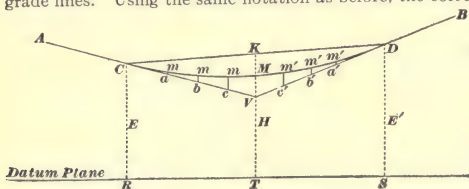


FIG. 4

$$VM = \tfrac{1}{2}\left(\frac{E+E'}{2}-H\right)$$

The correction in the grade at any point $a$ will be given by the preceding formula for $am$, as before. This correction, however, is now to be added to the old elevation of the grade at $a$ to obtain the corrected elevation.

EXAMPLE.—The grade of $CV$, Fig. 4, is $-1.2\%$, that of $VD$ is $+.6\%$, and the elevation of $V$ is $+49.2$ ft. Find the corrections in grade and the corrected elevations at stakes 100 ft. apart, if the length of the vertical curve is 600 ft.

SOLUTION.—The uncorrected grade elevations are as follows:

| *Along CV* | | *Along VD* | |
|---|---|---|---|
| At first stake............ | 52.8 | At fifth stake.......... | 49.8 |
| At second stake......... | 51.6 | At sixth stake......... | 50.4 |
| At third stake.......... | 50.4 | At seventh stake, $D$..... | 51.0 |
| At fourth stake, $V$ ......... | 49.2 | | |

Therefore,     $\tfrac{1}{2}(E+E') = \tfrac{1}{2}\times(52.8+51.0)=51.9$;

and, by the preceding formula,

$$VM = \tfrac{1}{2}\times(51.9-49.2)=1.35 \text{ ft.}$$

The formula for $am$ may now be applied. Correction in grade at second stake, 100 ft. from $C$, is $(\tfrac{100}{300})^2 \times 1.35 = \tfrac{1}{9}\times 1.35 = .15$, correction at sixth stake. Correction at third stake, $(\tfrac{200}{300})^2\times 1.35 = \tfrac{4}{9}\times 1.35 = .60$, correction at fifth stake.

The corrected elevations will be

| | | |
|---|---|---|
| At $C$.................................... | $52.80 + \ .00 =$ | $52.80$ |
| At second stake............................ | $51.60 + \ .15 =$ | $51.75$ |
| At third stake............................. | $50.40 + \ .60 =$ | $51.00$ |
| At fourth stake............................ | $49.20 + 1.35 =$ | $50.55$ |
| At fifth stake............................. | $49.80 + \ .60 =$ | $50.40$ |
| At sixth stake............................. | $50.40 + \ .15 =$ | $50.55$ |
| At $D$..................................... | $51.00 + \ .00 =$ | $51.00$ |

## CURVED TRACK

The difference in length between the inner and the outer rail of a standard-gauge curve may be found by either of the following rules:

**Rule I.**—*Multiply the degree of the curve by the length, in stations of 100 ft., and this product by $1\tfrac{1}{12}$; the result will be the difference in length between the inner and the outer rail, in inches.*

**Rule II.**—*Multiply the distance between the center lines of the rails by the length of the curve, in feet, and divide the product by the radius of the track curve; the quotient will be the required difference in length, expressed in feet.*

For light curves laid to exact gauge, the first rule is the simpler one, but for short curves where the gauge is widened, the second rule should be used.

**Curving Rails.**—When laying track on curves, in order to have a smooth line, the rails themselves must conform to the curve of the center line. To accomplish this, the rails must be curved. The curving should be done with a rail bender with a lever, preferably with the former. To guide those in charge of this work, a table of middle and quarter ordinates for a 30-ft. rail for all degrees of curve should be prepared. The middle ordinates in the following table are calculated by the formula

$$m = \frac{c^2}{8R},$$

in which $m =$ middle ordinate;

$c =$ length of chord, assumed to be of same length as rail;

$R =$ radius of curve.

This formula is not theoretically correct; yet the error is so small that it may be ignored in practical work.

In curving rails, the ordinate is measured by stretching a cord from end to end of the rail against the gauge side, as shown in the accompanying illustration. Suppose the rail $AB$ is 30 ft. in length, and the curve 8°, then the middle ordinate at $a$ should be

$$m = \frac{30^2}{8 \times 716.78} = .157 \text{ ft.} = 1\frac{7}{8} \text{ in.}$$
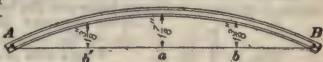
To insure a uniform curve to the rails, the ordinates at the quarter points $b$ and $b'$ should be tested. In all cases the quarter ordinates should be three-quarters of the middle ordinate. In the illustration, if the rail has been properly curved, the quarter ordinates at $b'$ and $b$ will be $\frac{3}{4} \times 1\frac{7}{8}$ in. $= 1\frac{13}{32}$, say $1\frac{3}{8}$.

## MIDDLE ORDINATES FOR CURVING RAILS

| Degree of Curve | Length of Rail, in Feet | | | | | |
|---|---|---|---|---|---|---|
| | 30 | 28 | 26 | 24 | 22 | 20 |
| | Middle Ordinates, in Inches | | | | | |
| 1 | $\frac{1}{4}$ | $\frac{3}{16}$ | $\frac{3}{16}$ | $\frac{3}{16}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| 2 | $\frac{7}{16}$ | $\frac{7}{16}$ | $\frac{3}{8}$ | $\frac{5}{16}$ | $\frac{1}{4}$ | $\frac{3}{16}$ |
| 3 | $\frac{11}{16}$ | $\frac{5}{8}$ | $\frac{9}{16}$ | $\frac{1}{2}$ | $\frac{7}{16}$ | $\frac{5}{16}$ |
| 4 | $\frac{15}{16}$ | $\frac{13}{16}$ | $\frac{11}{16}$ | $\frac{11}{16}$ | $\frac{9}{16}$ | $\frac{7}{16}$ |
| 5 | $1\frac{1}{16}$ | $1\frac{1}{16}$ | $\frac{7}{8}$ | $\frac{13}{16}$ | $\frac{11}{16}$ | $\frac{9}{16}$ |
| 6 | $1\frac{1}{16}$ | $1\frac{1}{4}$ | $1\frac{1}{16}$ | $\frac{15}{16}$ | $\frac{3}{4}$ | $\frac{5}{8}$ |
| 7 | $1\frac{3}{8}$ | $1\frac{1}{16}$ | $1\frac{1}{4}$ | $1\frac{1}{16}$ | $\frac{7}{8}$ | $\frac{11}{16}$ |
| 8 | $1\frac{7}{8}$ | $1\frac{5}{8}$ | $1\frac{5}{16}$ | $1\frac{3}{16}$ | $\frac{7}{8}$ | $\frac{3}{4}$ |
| 9 | $2\frac{1}{8}$ | $1\frac{7}{8}$ | $1\frac{5}{8}$ | $1\frac{1}{2}$ | $1$ | $\frac{15}{16}$ |
| 10 | $2\frac{3}{8}$ | $2\frac{1}{16}$ | $1\frac{3}{4}$ | $1\frac{1}{2}$ | $1\frac{1}{8}$ | $1\frac{1}{16}$ |
| 11 | $2\frac{5}{8}$ | $2\frac{1}{4}$ | $1\frac{15}{16}$ | $1\frac{11}{16}$ | $1\frac{3}{8}$ | $1\frac{1}{8}$ |
| 12 | $2\frac{13}{16}$ | $2\frac{5}{8}$ | $2\frac{1}{8}$ | $2\frac{1}{8}$ | $1\frac{9}{16}$ | $1\frac{1}{4}$ |
| 13 | $3\frac{1}{16}$ | $2\frac{11}{16}$ | $2\frac{5}{16}$ | $2\frac{5}{16}$ | $1\frac{5}{8}$ | $1\frac{3}{8}$ |
| 14 | $3\frac{5}{16}$ | $2\frac{7}{8}$ | $2\frac{1}{2}$ | $2\frac{1}{4}$ | $1\frac{3}{4}$ | $1\frac{1}{2}$ |
| 15 | $3\frac{3}{16}$ | $3\frac{3}{16}$ | $3\frac{1}{16}$ | $2\frac{11}{16}$ | $1\frac{11}{16}$ | $1\frac{9}{16}$ |
| 16 | $3\frac{1}{4}$ | $3\frac{1}{4}$ | $2\frac{13}{16}$ | $2\frac{5}{8}$ | $2\frac{5}{16}$ | $1\frac{11}{16}$ |
| 17 | $4$ | $3\frac{1}{2}$ | $3$ | $2\frac{9}{16}$ | $2\frac{1}{4}$ | $1\frac{3}{4}$ |
| 18 | $4\frac{1}{16}$ | $3\frac{11}{16}$ | $3\frac{3}{16}$ | $2\frac{11}{16}$ | $2\frac{5}{16}$ | $1\frac{7}{8}$ |
| 19 | $4\frac{5}{16}$ | $3\frac{7}{8}$ | $3\frac{3}{8}$ | $2\frac{5}{8}$ | $2\frac{1}{16}$ | $2$ |
| 20 | $4\frac{11}{16}$ | $4\frac{1}{2}$ | $3\frac{1}{16}$ | $3$ | $2\frac{3}{16}$ | $2\frac{1}{8}$ |

## TURNOUTS

A *turnout* is a contrivance for passing from one track to another. The principal parts are the switch, the frog, and two guard-rails. The *switch*, which is the movable part of the turnout, consists of two switch rails $BA$ and $CD$, Fig. 1. The fixed ends $B$ and $C$ of the switch rails are called the *heels* of the switch, and the movable ends $A$ and $D$, the *toes* of the switch. The cross-tie that supports the toes of the switch is called the *head-block*, and the tie-rod at the toes, the *head-rod*. The distance $AA'$ or $DD'$ through which the toes move is called the *throw* of the switch. A frog is shown at $K$ and two guard-rails at $R$ and $R'$.

**Switches.**—There are two kinds of switches, which differ in the arrangement and form of switch rails, namely, the *stub switch* and the *point switch*. In the stub switch, Fig. 1, a part of each main-track rail is bent over to connect with the side track. In the point switch, Fig. 2, the outer rail $DV$ of the main track is spiked rigidly to the ties; the opposite rail $EA'U$, lying partly in the main track and partly in the side track, is also firmly spiked. These two rails are immovable. The two switch rails $BA$ and $CD$ are planed to thin edges at $A$ and $D$. The ends $B$ and $C$ of these rails are the fixed ends or heels; the thin edges at $A$ and $D$ are the toes. The head-block is at $H$, and the head-rod at $g$.

The point of the center line at which the turnout begins is called the *point of switch*. In Figs. 1 and 2, $W$ is the point of switch. In stub switches, the point of switch is midway between the heels; in point switches, it is midway between the toes and above the head-block.
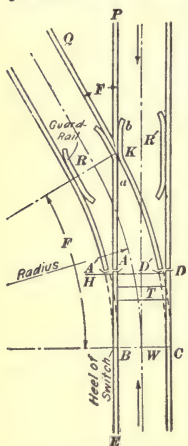
**Frogs and Guard-Rails.**—A *frog* is a combination of rails so arranged that the broad tread of the wheel will always have a surface on which to roll, and that the flange of the wheel will have a channel through which to pass. A frog is shown in position on the track at $K$, Fig. 1, and a larger plan of the part at $ab$, Figs. 1 and 2, is shown in Fig. 3.

The wedge-shaped part $akb$ of the frog is called the *tongue* of the frog, and its point $k$ is called the *actual point of frog*. The actual point of frog is somewhat shortened and rounded. The intersection $c$ of the outside edges $ac$ and $bc$ of the tongue is called the *theoretical point of frog*. When the point of frog is referred to, the theoretical point is usually meant. The bent rails $wr$ are called *wing rails;* the narrowest part $mp$ of the frog is called the *throat*. The throat of the frog must be wide enough to allow the flanges of the wheels to pass through; it is usually made about 2 in. wide.

**Frog Angle and Frog Number.**—The angle $acb$, Fig. 3, between the outside edges of the tongue of the frog is called the *frog angle*. This is also equal to the angle $dce$ between the outside edges of the tongue produced beyond $c$. The frog angle which is represented by $F$ is also equal to the angle between the two tracks.
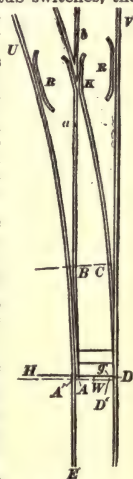
The distance $ab$ between the gauge lines at the end of the tongue is called the *heel width;* the distance $de$, the *mouth width*. If $sch$ is the bisector of the angle $F$, the distance $ch$ is called the *length of frog*.

The ratio of the length to the heel width is called the *frog number*, and is usually denoted by $n$; that is,

$$n = ch \div ab$$

The relation between $n$ and $F$ is expressed by the formulas

$$n = \tfrac{1}{2} \cot \tfrac{1}{2} F$$

and

$$\cot \tfrac{1}{2} F = 2n$$



FIG. 1



FIG. 2



FIG. 3

Frogs are usually designated by their numbers; thus, a No. 8 frog is one in which $n = 8$.

If the distance $sh$ and the widths $ab$ and $de$, Fig. 3, are measured on a frog, the frog number $n$ can be determined by the formula

$$n = \frac{sh}{ab + de}$$

**Guard-Rails.**—Guard-rails, which are usually from 10 to 15 ft. long, are placed opposite the frog on the main track and the switch track, as at $R$ and $R'$ in Figs. 1 and 2. The clear space between the head of the guard-rail and the head of the main or the switch rail should be about 2 in.

**Radius and Lead of a Turnout for Stub Switches.**—Let $RN$, Fig. 4, be the main track and $QP$ the turnout. Let $Q$ be the point of switch and $K$ the point of frog. If a stub switch is employed, the main-track rails will be securely spiked along $YB$ and $LD$; the parts $BG$ and $DV$ of these rails will be movable, so that they may be bent outwards to meet the turnout rails $W$ and $Z$. Here, then, the ends $B$ and $D$ are the heels of the switch, and $G$ and $V$ are the toes. The head-block is underneath $G$ and $V$.

In order to lay out a turnout when the frog angle is given, it is necessary to find the radius $r$, in terms of the frog angle, and the distance $KB$ from the point of frog to the heel of switch, which distance is called the *lead* and is designated by $L$.



FIG. 4

The formulas for $r$ and $L$ are:

$$r = \tfrac{1}{2}g \cot^2 \tfrac{1}{2}F = 2gn^2$$

and

$$L = g \cot \tfrac{1}{2}F = 2gn$$

In these formulas $g$ denotes the gauge. The *standard gauge* of track is 4 ft. $8\frac{1}{2}$ in. = 4.708 ft.

The accompanying table, some parts of which are calculated from the foregoing formulas, can be used in laying out a turnout with a stub switch. The
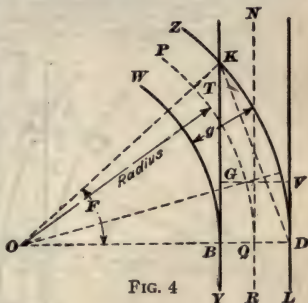
## DIMENSIONS OF STUB-SWITCH TURNOUTS

Track Circular From Heel of Switch to Point of Frog. Throw = 5½ In.

| Frog Number $n$ | Frog Angle $F$ | | | Lead $L$ | Chord $(QT)$ | Radius | Degree of Curve $d$ | Length of Switch Rails | Distance $Ka$, Fig.1 |
|---|---|---|---|---|---|---|---|---|---|
| 4.0 | 14° | 15′ | 00″ | 37.67 | 37.38 | 150.67 | 38° 46′ | 11.73 | 1.50 |
| 4.5 | 12 | 40 | 59 | 42.37 | 42.12 | 190.69 | 30 24 | 13.19 | 1.69 |
| 5.0 | 11 | 25 | 16 | 47.08 | 46.85 | 235.42 | 24 32 | 14.65 | 1.87 |
| 5.5 | 10 | 23 | 20 | 51.79 | 51.58 | 284.85 | 20 13 | 16.15 | 2.06 |
| 6.0 | 9 | 31 | 38 | 56.50 | 56.30 | 339.00 | 16 58 | 17.64 | 2.25 |
| 6.5 | 8 | 47 | 51 | 61.21 | 61.03 | 397.85 | 14 26 | 19.09 | 2.44 |
| 7.0 | 8 | 10 | 16 | 65.92 | 65.75 | 461.42 | 12 26 | 20.53 | 2.62 |
| 7.5 | 7 | 37 | 41 | 70.62 | 70.47 | 529.69 | 10 50 | 22.03 | 2.81 |
| 8.0 | 7 | 9 | 10 | 75.33 | 75.19 | 602.67 | 9 31 | 23.48 | 3.00 |
| 8.5 | 6 | 43 | 59 | 80.04 | 79.90 | 680.36 | 8 26 | 24.93 | 3.19 |
| 9.0 | 6 | 21 | 35 | 84.75 | 84.62 | 762.75 | 7 31 | 26.43 | 3.37 |
| 9.5 | 6 | 1 | 32 | 89.46 | 89.33 | 849.85 | 6 45 | 27.97 | 3.56 |
| 10.0 | 5 | 43 | 29 | 94.17 | 94.05 | 941.67 | 6 05 | 29.37 | 3.75 |
| 10.5 | 5 | 27 | 9 | 98.87 | 98.76 | 1,038.19 | 5 32 | 30.85 | 3.94 |
| 11.0 | 5 | 12 | 18 | 103.58 | 103.47 | 1,139.42 | 5 02 | 32.31 | 4.12 |
| 11.5 | 4 | 58 | 45 | 108.29 | 108.19 | 1,245.36 | 4 36 | 33.78 | 4.31 |
| 12.0 | 4 | 46 | 19 | 113.00 | 112.90 | 1,356.00 | 4 14 | 35.17 | 4.50 |

frog number, which is usually given, is stated in the first column; the corresponding frog angle in the second column; and the lead, or $BK$, Fig. 4, in the third column. Then follow columns containing the chord $QT$, Fig. 4, which is

equal to $2r \sin \frac{1}{2}F$; the radius of the turnout; the corresponding degree of curve
which is equal to $\dfrac{5{,}730}{r}$; the length $l$ of switch rails $AB$, Fig. 1, obtained by the

formula $l = \sqrt{t(2r - t)}$; and the distance $Ka$, Fig. 1, or $cw$, Fig. 3, With different forms of frogs, this distance varies; the engineer should therefore measure



FIG. 5

it for the different frogs he uses, as it is necessary in determining the length of spiked rail $Aa$, Fig. 1.
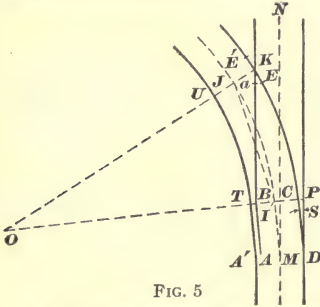
**Turnout Dimensions for Point Switches.**—Let $MN$, Fig. 5, be the center line of the main track and $MJ$ that of the turnout. Let $BA$ and $CD$ be the two switch rails whose fixed ends, or heels, are at $B$ and $C$, and whose toes are at $A$ and $D$. These rails are usually of a uniform length of 15 ft., except for the sharpest curves.

The center line $MIJ$ will, when a point switch is used, have a somewhat different position from that which it has when a stub switch is employed. In the stub-switch turnout, the rails $A'TU$ and $DCK$ are bent to a uniform curve between $M$

and $J$; in a point switch, the outer rail is made up of a straight part $DC$, which is the switch rail, and a curved part $CE$, which is tangent to $DC$ at $C$. On this account, the lead $A'K$ is less with a point switch than with a

## DIMENSIONS OF POINT-SWITCH TURNOUTS

Turnouts With Straight Point Rails and Straight Frog Rails;
Gauge 4 Ft. 8½ In.

| Frog Number $n$ | Frog Angle $F$ | | | Switch Angle $S$ (CDP) | | | Lead $L$ (A'K) | | | Radius $r$ | Degree of Curve $d$ | | | Chord (JI) | Length of Switch Rails (CD) | Length of Straight Frog Rail (KE) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4.0 | 14° | 15′ | 00″ | 3° | 40′ | | 32.20 | | | 125.21 | 47° | 05′ | | 23.09 | 7.5 | 1.50 |
| 4.5 | 12 | 40 | 49 | 3 | 40 | | 34.29 | | | 159.25 | 36 | 36 | | 25.03 | 7.5 | 1.69 |
| 5.0 | 11 | 25 | 16 | 2 | 45 | | 41.85 | | | 197.65 | 29 | 22 | | 29.88 | 10.0 | 1.87 |
| 5.5 | 10 | 23 | 20 | 2 | 45 | | 44.16 | | | 240.44 | 24 | 00 | | 32.03 | 10.0 | 2.06 |
| 6.0 | 9 | 31 | 39 | 1 | 50 | | 56.00 | | | 288.09 | 19 | 59 | | 38.66 | 15.0 | 2.25 |
| 6.5 | 8 | 47 | 51 | 1 | 50 | | 58.84 | | | 340.19 | 16 | 54 | | 41.34 | 15.0 | 2.44 |
| 7.0 | 8 | 10 | 16 | 1 | 50 | | 61.65 | | | 397.65 | 14 | 27 | | 43.98 | 15.0 | 2.62 |
| 7.5 | 7 | 37 | 41 | 1 | 50 | | 64.36 | | | 460.00 | 12 | 29 | | 46.50 | 15.0 | 2.81 |
| 8.0 | 7 | 9 | 10 | 1 | 50 | | 67.04 | | | 527.91 | 10 | 52 | | 48.99 | 15.0 | 3.00 |
| 8.5 | 6 | 43 | 59 | 1 | 50 | | 69.60 | | | 600.94 | 9 | 33 | | 51.38 | 15.0 | 3.19 |
| 9.0 | 6 | 21 | 35 | 1 | 50 | | 72.20 | | | 681.16 | 8 | 25 | | 53.80 | 15.0 | 3.37 |
| 9.5 | 6 | 1 | 32 | 1 | 50 | | 74.70 | | | 767.11 | 7 | 29 | | 56.11 | 15.0 | 3.56 |
| 10.0 | 5 | 43 | 29 | 1 | 50 | | 77.04 | | | 858.14 | 6 | 41 | | 58.28 | 15.0 | 3.75 |
| 10.5 | 5 | 27 | 9 | 1 | 50 | | 79.51 | | | 959.00 | 5 | 59 | | 60.57 | 15.0 | 3.94 |
| 11.0 | 5 | 12 | 18 | 1 | 50 | | 81.82 | | | 1,065.52 | 5 | 23 | | 62.69 | 15.0 | 4.12 |
| 11.5 | 4 | 58 | 45 | 1 | 50 | | 84.09 | | | 1,180.16 | 4 | 51 | | 64.78 | 15.0 | 4.31 |
| 12.0 | 4 | 46 | 19 | 1 | 50 | | 86.16 | | | 1,299.93 | 4 | 24 | | 66.67 | 15.0 | 4.50 |

stub switch. As point switches are used on the main line where very accurate work is required, it is necessary to take account of the fact that the short frog rails are not curved, the part $EE'$ of the rail being straight.

In computing the dimensions of a point-switch turnout, the usual data are the length $AB = DC$ of the switch rail, the angle $CDP$ between the outer switch rail and the main rail. This angle is called the *switch angle*, and will be represented by $S$. The frog number or the frog angle must also be known, as well
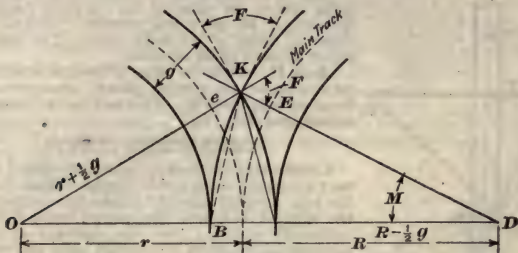


Fig. 6

as the length of the straight part $EE'$. It is then required to determine the radius $OI$ of the center line of a turnout whose outer rail shall be tangent to the switch rail $DC$ at $C$ and to the frog rail $EE'$ at $E$, and to find the lead $A'K$ of this turnout.

The formulas for computing these quantities are so complicated that, in practice, tables giving the various dimensions of point switches are always employed.

The accompanying table contains all the dimensions necessary for laying out a point switch when the frog number is known. It contains the frog angle, the switch angle $CDP$, Fig. 5, the lead $A'K$, the radius $OI$ of the center line of the turnout, the degree of curve of this center line, the chord $JI$, the length $AB = CD$ of the switch rails, and the length $KE = Ka$ of the straight frog rail.

**Turnouts From the Outer Side of a Curved Track.**—A turnout from the outer side of a curved track is shown in Fig. 6. The radius $DE = R$ of the main track, the frog angle $F$, or frog number $n$, and the gauge $g$ are usually known; from these the lead $BK = L$, and the radius $Oe = r$ of the center line of the turnout must be computed. The angle $M$, must first be found by the formula

$$\tan \tfrac{1}{2}M = \frac{g}{2R} \cot \tfrac{1}{2}F = \frac{gn}{R}$$

Then, the lead must be determined by the formula

$$L = 2(R + \tfrac{1}{2}g) \sin \tfrac{1}{2}M$$

Finally, $r$ is given by the formula

$$r + \tfrac{1}{2}g = \frac{R + \tfrac{1}{2}g}{\sin (F - M)} \sin M$$

When $r$ has been found, the degree of curve is given by the formula

$$d = \frac{5,730}{r}$$

If the main-track curve is not very sharp, this value of $d$ may be obtained by subtracting the degree of curve of



Fig. 7

the main track from that obtained from the sixth column of the table for stub switches. The lead $L$ may also be taken from the table.

If the curvature of the main track is very sharp, or if the frog angle is very small, the turnout may curve in the same direction as the main track; in which case, the lead $L$ taken from the stub-switch table will be less than the degree of curve of the main track. The difference between the two degrees of curve will still be equal to the degree of curve of the turnout. If the degrees of curve are equal, the turnout rails will be straight.
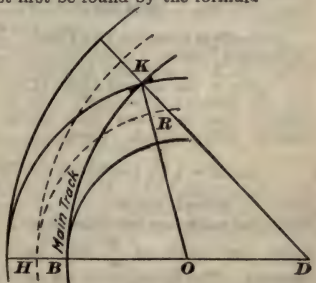
**Turnout from the Inner Side of a Curved Track.**—A turnout from the inner side of a curved track is shown in Fig. 7. The radius $OR$ of the turnout is always less than the radius $DH$ of the main track. The degree of curve of the center line of the turnout and the lead $BK$ are found as follows:



**Rule I.**—*Take from the table for a stub or for a point switch, the value of the degree of curve corresponding to the given frog number. Add this to the degree of curve of the main track. The sum is the degree of curve of the turnout.*

**Rule II.**—*Take the value of the lead from the table for a stub or for a point switch, corresponding to the given frog number. This will be the value of the desired lead BK, Fig. 7.*

## CONNECTING CURVES

A *connecting curve* is a curve introduced to connect a turnout with a side track. Thus, in Fig. 8, the two parallel straight tracks are connected by the turnout $ME$ and the curved track $ED$. The values of $n$ and $g$, and the distance $a$, usually taken as 13 ft., must be known; then the radius $r' = O_1D = O_1E$, and distance $KT$ may be computed by the formulas

$$r' = 2(a-g) n^2 + \tfrac{1}{2}a$$

and

$$KT = \frac{a-g}{g} \times L$$

FIG. 8

$L$ is the lead $PK$ of the turnout, and, in such cases as this, is always to be taken from the table for a stub switch, even when the point switch is inserted, because in deriving the formula for $KT$, $QK$ and $ME$ are assumed to be circular arcs.

## CROSS-OVERS

A *cross-over* is a stretch of track that connects two parallel tracks, and enables a train to pass from one track to the other. Thus, in Fig. 9, if $UV$ and $U'V'$ are two parallel tracks, the track $RZR'$ is a cross-over. This cross-over consists of two equal turnouts $Rm$ and $R'm'$, whose frog angles at $K$ and $K'$ are equal, and a reversed curve $mZm'$ connecting the ends of these turnouts, $Z$ being the point of reversal.

**Cross-Over Between Two Parallel Straight Tracks.**—To lay out the cross-over, it is necessary to know the radius $r$, the central angle $M$, and the distance $BE = B'E'$. The radius $r$ may be taken from the table for stub switches. Then,

$$\sin M = \sqrt{\frac{a}{r}\left(1 - \frac{a}{4r}\right)}$$

and

$$BE = 2r \sin M$$

When the tracks are less than 30 ft. apart, the value of $\frac{a}{4r}$ may be dropped. The formulas for $\sin M$ and $BE$ then become, respectively,

$$\sin M = \sqrt{\frac{a}{r}}$$

and

$$BE = 2\sqrt{ar}$$



FIG. 9

The preceding formulas apply only to stub switches; to apply them to point switches, proceed as follows: Having located one frog point $K$ of the point-switch turnout, measure back from $K$ the lead $KB$ for a stub-switch turnout taken from the table, and from the point $R$ of the center line opposite $B$ run
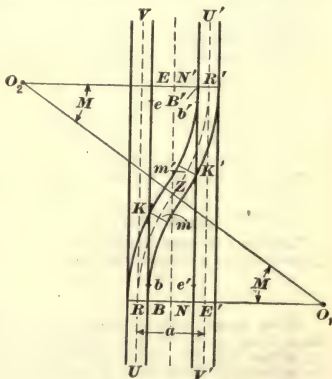
in the curve $RmZ$ to the point of reversal. Then measure off the distance $BE = 2\sqrt{ar}$, and from the point $B'$ opposite to $E$ lay off the stub-switch lead $B'K'$ to locate the second point of frog $K'$. Then run in the center-line curve $R'Z$. The two frog points and the reversed curve $mZm'$ are thus located. Finally, measure back from $K$ and $K'$ the distances $Kb = K'b'$ equal to the lead for point switches, to locate the toes of the point switches at $b$ and $b'$, and complete the location of these switches as explained under Laying Out Turnouts.

It is evident that the whole length of the cross-over when point switches are employed is $be = b'e' = BE - 2 \times Bb = 2\sqrt{ar} - 2 \times Bb$. Therefore,

$$be = b'e' = 2\sqrt{ar} - 2 \times \text{(lead of stub switch} - \text{lead of point switch)}$$

A stake is usually driven at $Z$, midway between the inner rails and midway between the points $N$ and $N'$, and the turnout curves are continued to this point. This is more accurate than to attempt to determine the point of reversal by the use of the central angle $M$.

**Another Form of Cross-Over Between Two Parallel Straight Tracks.**—A second form of cross-over is shown in Fig. 10. In this form, the ends of the two equal turnouts are connected by a straight track $KTK'T'$. The cross-over with a reversed curve, Fig. 9, is much shorter than this straight-track cross-over, and thus requires less length of track and occupies less room. The straight-track form is, however, to be preferred; it is less wearing on the rolling stock because it gives the wheel trucks a better opportunity to adjust themselves to the reversion of curvature.

In order to lay out a straight-track cross-over, it is only necessary to compute the distance $BE = B'E'$, Fig. 10, in addition to the usual dimensions of the two turnouts, which may be done by taking the lead $L$ from the stubswitch table and applying the formula:



FIG. 10

$$BE = 2L - \frac{a}{4n} + (a - 2g)n$$

The turnout $Rm$ having been put in place, the distance $BE$ is laid off and the heels $B'$ and $H'$ of the second turnout are located opposite the point $E$. This turnout is then laid out as far as $m'$, and finally the straight rails $KT'$ and $K'T$ are laid adjoining the ends of the two turnouts.

The only modification of the work for a point switch arises from the fact that the lead $Kb = K'b'$ of the point switch is less than that of the stub switch. The whole length of cross-over is, for a point switch,

$$be = 2L' - \frac{a}{4n} + (a - 2g)n$$

Here $L'$ is the lead taken from the table for point switches.

## LAYING OUT TURNOUTS

**To Lay Out a Stub Switch.**—Having decided on the position of the end $b$, Fig. 11, of the frog rail, measure the total length of the frog and deduct from the length of the rail to be cut, marking with red chalk on the flange of the rail the point at which the rail is to be cut. From Fig. 3,

$$n = \frac{ch}{ab}$$

and
$$ch = n \times ab$$

To calculate the distance from the heel to the theoretical point of frog, the width of the frog at the heel is measured and multiplied by the frog number. For example, if the width of the frog at the heel is $8\frac{1}{2}$ in., and a No. 8 frog is to be used, the theoretical distance from the heel to the point of frog is $8.5 \times 8 = 68$ in. $= 5$ ft. 8 in. Measure off this distance from the point marking the heel of the frog; this will locate the point of frog, which should be distinctly marked with red chalk on the flange of the rail. It is a common practice to make a distinct mark on the web of the main-track rail, directly opposite to the point

of frog. This point, being under the head of the rail, is protected from wear and the weather. The heel of the turnout is then located by measuring back the lead from the point of frog. Next, make a chalk-mark on both main-track rails on a line marking the center of the head-block; a more permanent mark is made with a center punch. Stretch a cord touching these marks, and drive a stake on each side of the track, with a tack in each; this line should be at right angles to the center line of the track, and the stakes should be sufficiently far from the tracks not to be disturbed when putting in switch ties. Next, cut the switch ties to proper length; draw the spikes from the track ties, three or four at a time, and remove the ties from the track, replacing them with switch ties, and tamping the latter securely in place. When all the long ties are tamped, cut the main-track rail for the frog, being careful that the amount cut off is just equal to the length of the frog. If, by increasing or decreasing the length of the lead 5%, the cutting of a rail can be avoided, this should be done, especially for frogs above No. 8.

Full-length rails (30 ft.) should be used for moving or switch rails, and care should be taken to leave a joint of proper width at the head chair. The head-chairs should be spiked to the head-block so that the main-track rails will be in perfect line. From 8 to 10 ft. of the switch rails should be spiked to the ties. The tie-rods are placed between the switch ties, which should not be more than 15 in. from center to center of tie. The connection-rod should be attached to the head-rod and switch stand. With these connections made the switch stand is easily placed to give the proper throw of the switch.

It is common practice to fasten the switch stand to the head-block with track spikes, but a better fastening is made with bolts. The stand is first properly placed, the holes are marked and bored, and the bolts passed through from the under side of the head-block. This obviates all danger of movement of the switch stand in fastening, which is liable to occur when spikes are used, and insures a perfect throw.

The use of track spikes is admissible when holes are bored to receive them, in which case a $\frac{3}{8}$-in. auger should be used for standard track spikes. The switch stand should, when possible, be placed facing the switch, so as to be seen from the engineer's side of the engine — the right-hand side.



FIG. 11



FIG. 12

To find the position of the chord of the arc of the outer rail of the turnout curve, stretch a cord from the heel $a$, Fig. 11, to the point $b$, of the frog. Mark the middle point $c$ and the quarter points $d$ and $e$, and at these points lay off the offsets $dd'$, $cc'$, and $ee'$. Add to these offsets the distance from the gauge line to the outside of the rail flange, and mark the points on the switch ties. Spike the rail to these marks and place the other at easy track gauge from it. Spike the rails of the turnout, as far as the point of frog, to exact gauge, unless the gauge has been widened owing to the sharpness of the curve. Beyond the point of frog, the curve may be allowed to vary a little in gauge to prevent a kink from showing opposite the frog. In case the gauge is widened at the frog, increase the guard-rail distance an equal amount. For a gauge 4 ft. 8$\frac{1}{2}$ in., place the side of the guard-rail that comes in contact with the car wheels at 4 ft. 6$\frac{5}{8}$ in. from the gauge line of the frog. This gives a space of 1$\frac{7}{8}$ in. between the main rail and the guard-rail. In case the gauge is widened $\frac{1}{4}$ or $\frac{1}{2}$ in., increase the guard-rail distance an equal amount.

When the turnout curve is very sharp, it will be necessary to curve the switch rails, to avoid an angle at the head-block. The rails should be carefully curved before being laid, and great pains should be taken to secure a perfect line.

**To Lay Out a Point Switch.**—The frog point $K$, Fig. 12, having been located exactly as for a stub switch, the lead $KB$ is next laid off from $K$ to the toe of switch $B$, and the positions of $B$ and $D$ are marked on the main-track rails. From $D$, the length $DN$ of the switch rail, which is usually 15 ft., is then

measured forwards to $N$, and the position of $N$ is marked on the web or flange of the rail. The heel $M$ is usually $5\frac{3}{4}$ in. from the point $N$. The point $I$ is located on a line perpendicular to $MD$ and at a distance $\frac{1}{2}g$ from $M$. The point $J$ is similarly located from the point $H$. As a check on the work, the length of the chord $JI$ should have the value given in the table for point switches.

Switch ties of the requisite number and length should be prepared and placed in the track in proper order. As in the case of stub switches, all long switch ties should be in place before the rail is cut for placing the frog; also, the ends $M$ and $L$ of the rails, with which the switch points connect, should be exactly even; otherwise the tie-rods will be skewed, and the switch will not work or fit well. The tie-rods should next be fastened in position, care being taken to place them in their proper order, the head-rod being numbered 1. Each rod is marked with a center punch, the number of punch marks corresponding to the number of the rod.

The switch rails are now coupled with the rails $LK$ and $MK$, and the sliding plates are then placed in position and securely spiked to the ties. The head-rod is then connected with the switch stand, and the switch is closed, giving a clear main track. The stand is then adjusted for this position of the switch, and bolted fast to the head-block. Next, rail $BR$ is crowded against the switch point so as to insure a close fit, and secured in place with a rail brace at each tie. The laying of the rails of the turnout is then continued.

The rail $MH$ is to be bent and spiked in place by laying off offsets from the chord $MH$ exactly as explained for stub switches. The rail between $M$ and $H$ usually consists of two pieces of plain rail bent to the proper curve. The outer rail of the main track is not disturbed.

**Switch Timbers.**—Every first-class railroad has its own standards for switches, which include the necessary switch timbers. The number of ties and their lengths may be determined by the following rules:

**Rule I.**—*To find the number of ties required for any switch lead, reduce to inches the distance from the head-block to the last long tie behind the frog, and divide this distance by the number of inches from center to center of ties; the quotient will be the number of ties required.*



$(a)$     FIG. 13     $(b)$

**Rule II.**—*Measure the length of the tie next the head-block and the length of the last long tie behind the frog. Find the difference, in inches, between them. Divide this difference by the number of ties in the switch lead; the quotient will be the increase in length per tie from the head-block toward the frog to have the ends of the tie in proper line on both sides of the track.*

**Practical Method of Laying Out Sharp Curves in a Mine.**—Curves in a mine are usually so sharp that they are designated as curves of so many feet radius, instead of as curves of so many degrees. For example, suppose that it is required to connect the two headings $A$ and $B$, Fig. 13 $(a)$, which are perpendicular to each other, with a curve of 60 ft. radius. Prepare the device shown in $(b)$, by taking three small wires or inelastic strings $fg$, $gh$, and $gk$, each 10 ft. long, and connecting one end of each to a small ring, and the other end of two to the ends of a piece of wood $1\frac{2}{3}$ ft. long. Form a neat loop at end $f$ of string $gf$.

To use this device, lay off on the center line of the heading $B$, $cd$ and $de$ equal to 60 ft. and 10 ft., respectively. Place the loop $f$ of the device described over a small wire peg driven in at $e$, and the ring $g$ over a similar peg at $d$. Take hold of the stick $hk$, pull the strings $gh$ and $gk$ taut, and place the center mark on $hk$ on the center line of the heading $B$. Drive a small peg in at $m$, located by the point $k$, which is on the curve. Move the device forwards, place the loop $f$ over the peg at $d$, the ring $g$ over the peg at $m$, and take hold of the stick $hk$ and pull until the strings $gh$ and $gk$ are taut, and the strings $fg$ and $gh$ are in a straight line. The point $k$ will fall on the curve at $n$, which mark by driving in a peg. To locate other points, proceed exactly as in the last step. The distance $cd$ in any case is found by the formula

$$cd = R \tan \tfrac{1}{2}I,$$
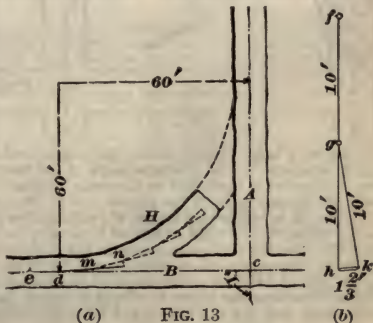
in which        $R$ = radius of curve;
                $I$ = intersection angle of center lines of headings.

Fig. 14 shows a graphic method of laying out curves, when it is desired to connect a main entry $a$ with a cross-entry $b$ by a curve of a given radius. Here, $c$ is the point of curve on the main-entry center line and $d$ is the point of tangency on the center line of the cross-entry, which is assumed to be turned off at right angles from the main entry. From $c$ and $d$, arcs with a radius equal to
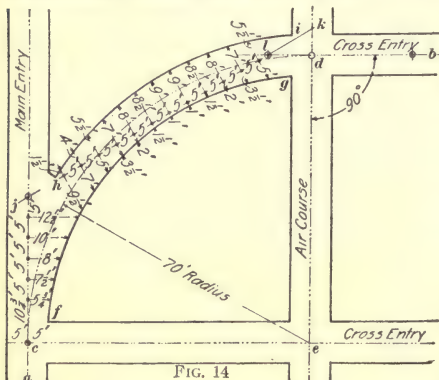


FIG. 14

the desired radius of c u r v a t u r e are described, thus locating the center $e$ of the c u r v e. From this point, with the same radius, the curve $cd$, which is the center of the proposed passageway, is described. The curves $fg$ and $hi$ are then drawn, making the curved entry any desired width. Line $jk$, so l o c a t e d that it will not cut either rib of the curved roadway, should then be drawn and on it points 5 feet apart should be laid off to the given scale and the right and left distances to the rib scaled, as shown in the figure. If a plat of this kind is

drawn on a large enough scale, the angles and distances may be scaled sufficiently accurate for the practical work of the mine foreman. A blueprint or tracing of the plat is furnished to the mine foreman, and from it he lays off the right and left distances from the center line of the main entry and from the line $jk$. The line $jk$ is located by means of two stations placed with the transit at $b$ and a point either to the right or left of $b$, as may be most convenient, and along the line $bl$. For the distances given in the illustration and the radius of curvature of 70 ft., the angle $cjk$ as taken from the drawing is about 120° 15'; the angle at $jld$, about 149° 45'; the distance $cj$, 35 ft. 9 in.; $jl$, 58 ft.; and $ld$, 11 ft. 6 in. These distances are given as about so much, to show that they are scaled from a drawing and are not calculated by trigonometry.

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# STADIA SURVEYING

*Stadia surveying* is the process of measuring distances and elevations by observing through a telescope the distance intercepted on a rod between two horizontal cross-hairs. These hairs are carried on the same ring as the regular horizontal cross-hair, and are equidistant from it. The rod used may be an ordinary level rod, but, if much stadia work is to be done, it is advisable to select some one of the special rods designed for the work; a description of these may be found in any instrument maker's catalog.

The distance intercepted on the rod between the cross-hairs of the telescope bears a certain relation to the distance of the rod from the instrument. As the transit is provided with a vertical circle or arc, the angle of inclination between the line of sight and the horizon may be determined in case of inclined sights. The space intercepted between the cross-hairs on the rod, taken in connection with the vertical angle, enable the distance and elevation of the rod to be calculated. When the line of sight is horizontal, the distance $d$ of the rod from the instrument can be determined from the formula

$$d = sR + i,$$

in which    $R$ = stadia reading or distance intercepted between the stadia wires;
            $s$ = stadia constant;
            $i$ = instrument constant.

The values of the stadia and instrument constants are usually determined by the instrument maker. The *instrument constant* varies from about .75 ft. to 1.33 ft. in different transits, according to the size and power of their telescopes. Its value is usually marked on a card attached to the inside of the instrument box. It is a general rule that unless definitely stated to the contrary, the *stadia constant* is made equal to 100; that is, in a horizontal line of sight, the stadia wires will intercept a distance of 1 ft. on a rod whose distance from the instrument is 100 ft. plus the instrument constant. Thus, if the stadia constant is 100 and the instrument constant is 1.33 ft., if the stadia wires intercept a distance of 8.37 ft. on the rod, the distance from the rod to the transit is $8.37 \times 100 + 1.33 = 838.33$ ft.; the line of sight being horizontal.

Stadia wires are either fixed or adjustable. The former are firmly placed by the instrument maker so that the stadia constant is 100. They cannot get out of adjustment and are much to be preferred to the other and older method of mounting, in which the distance between the wires may be varied to intercept any distance on the rod. In the event of the constants not being known, they may be determined as follows: Select a piece of level ground and set tacks in stakes at exactly 50 ft. apart for a distance of 400 to 800 ft. Let $R_2$ and $R_1$ be two stadia readings taken at the respective distances $d_2$ and $d_1$; then,

$$s = \frac{d_2 - d_1}{R_2 - R_1}$$

and

$$i = \frac{d_1 R_2 - d_2 R_1}{R_2 - R_1}$$

Several pairs of readings and their corresponding distances are substituted in these formulas, and the mean of all the resulting values of $s$ and $i$ calculated.

EXAMPLE.—Determine the stadia and the instrument constant from the following data:

| Distance Measured Feet | Rod Reading Feet |
|---|---|
| 50 | .488 |
| 100 | .988 |
| 200 | 1.988 |
| 300 | 2.991 |
| 400 | 3.986 |

SOLUTION.—Take 50 ft. for the value of $d_1$ and 100 ft., 200 ft., etc. successively for the values of $d_2$, and apply the preceding formulas for $s$ and $i$. For the first pair of observations:

$$s = \frac{100 - 50}{.988 - .488} = 100$$

and

$$i = \frac{50 \times .988 - .488 \times 100}{.988 - .488} = 1.200 \text{ ft.}$$

The other values are figured in a similar manner and the whole is tabulated as follows:

| $s$ | $i$ |
|---|---|
| 100.000 | 1.200 |
| 100.000 | 1.200 |
| 99.880 | 1.258 |
| 100.057 | 1.172 |
| 4)399.937 | 4)4.830 |

means      $99.984 = s$      $1.208 = i$

**Reduction of Inclined Sights.**—For the purpose of determining distances and differences in elevation when the rod is above or below the level of the instrument what are known as stadia reduction tables have been prepared. Their use is best explained by an illustration.

EXAMPLE.—Using a transit with fixed cross-hairs, the stadia constant being 100 and the instrumental constant being 1, the distance intercepted on the rod was 4.25 ft. and the vertical angle was $18° 23'$. What was the horizontal and the vertical distance from the rod to the instrument?

SOLUTION.—Referring to the table in the column headed $18°$ and opposite $22'$ in the column headed M, for $18° 22'$ a horizontal distance of 90.07 is found; and opposite $24'$, for $18° 24'$, a horizontal distance of 90.04 is found. For $18° 23'$, therefore, the horizontal distance will be the mean, or 90.055. The first column of the three lowest lines of the table contains the symbols $c = .75$,

$c = 1.00$, and $c = 1.25$. In this $c$ is the instrument constant, the value of which is given by the maker. In the case in question $c = 1.00$. Following horizontally along the line of $c = 1.00$, the figures .95 are found in the column headed "18° Hor. Dist." This .95 is the proportion of the instrument constant to be added to the reduced distance when the vertical angle is between 18° and 19°. The horizontal distance is then,

$$\text{Distance} = \text{rod reading} \times \text{tabular horizontal distance} + c;$$

From this

$$\text{distance} = 4.25 \times 90.055 + .95 = 382.73 + .95 = 383.68 \text{ ft.}$$

Referring again to the table but using the second of the two columns under 18°, that headed Diff. Elev. and by interpolating between the values of 22' and 24', the Diff. Elev. for 18° 23' is found to be 29.925. The proportion of the instrumental constant is found, in the line beginning $c = 1.00$ in the column headed 18° but under the subcolumn, Diff. Elev., to be .32 for all values of the vertical angle between 18° and 19°. The difference in elevation is then

$$\text{Diff. Elev.} = \text{rod reading} \times \text{tabular Diff. Elev.} + c$$

From this

$$\text{Diff. Elev.} = 4.25 \times 29.925 + .32 = 127.18 + .32 = 127.50 \text{ ft.}$$

**Use of Stadia.**—The stadia is not used as much as it should be by the mining engineer, particularly in securing data for making a contour map of the property of which he is in charge. In rolling and hilly country, some points upon the surface may be from 500 to 1,500 ft. higher than others and this difference in the thickness of the rocks, and consequent pressure upon the coal, will have a great bearing on the thickness of pillars, method of drawing stumps, etc., underground. If the traverse made to determine the boundaries of the property has been made with a 400-ft. tape, the elevations of the stations are known from the distances and vertical angles. By setting up at the various stations, stadia sights may be taken to points at any distances within which the rod may be read. This will be 1,200 or 1,400 ft. for a 12-ft. or 14-ft. rod. Sights of double this distance may be taken if the upper hair and the regular horizontal hair are used instead of the two stadia hairs; that is, if the two upper hairs are used in place of the first and third. Distances so determined must be multiplied by 2, as they are half distances. In this case the engineer must be certain that the horizontal hair is exactly midway between the stadia hairs. This may be ascertained by reading a distance with the upper and middle hairs and then with the middle and lower hairs. If these distances are the same, the hairs are evenly spaced; if the distances do not agree, their mean must be taken.

Cloudy days when the air is free from vibrations are best for stadia work. An ordinary erecting transit will give excellent results up to 1,200 and 1,800 ft., depending on its power, and a high class inverting telescope with large objective may be used for distances up to 3,000 ft. A good rodman is essential, and if long sights are being taken and the slopes are regular, one observer can keep two rodmen busy. The rod should be held at all points where there is any marked change in the degree of slope. Small inequalities in the surface may be neglected, it being apparent that an error of 20 to 30 ft. in determining the thickness of the rocks overlying the seam is of no importance.

When the sights are too long to permit of the rod being plainly read, the rodman should drive a stake and place a tack in it to mark the station. The edge of the rod should then be held on the tack, the rodman running his hand up and down the side of the rod that is to be sighted to. The transitman should then move up and take further stadia sights from the station just established. In traversing valleys or swales cutting across a property or in determining the line of a ridge all the work may be done with the stadia with a degree of accuracy well within reasonable requirements and far more easily, rapidly, and cheaply, than with a transit, 400 ft. tape, and leveling instrument. In locating isolated and distant houses, the opposite side of a wide river from that on which the traverse is being run, in fact in all work where an error of 1 : 500 or 1 : 1,000 is allowable, the stadia is to be preferred to the tape.

Stadia Reduction Table

| M. | 0° Hor. Dist. | 0° Diff. Elev. | 1° Hor. Dist. | 1° Diff. Elev. | 2° Hor. Dist. | 2° Diff. Elev. | 3° Hor. Dist. | 3° Diff. Elev. | 4° Hor. Dist. | 4° Diff. Elev. | 5° Hor. Dist. | 5° Diff. Elev. | 6° Hor. Dist. | 6° Diff. Elev. | 7° Hor. Dist. | 7° Diff. Elev. | 8° Hor. Dist. | 8° Diff. Elev. | 9° Hor. Dist. | 9° Diff. Elev. | 10° Hor. Dist. | 10° Diff. Elev. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 100.00 | .00 | 99.97 | 1.74 | 99.88 | 3.49 | 99.73 | 5.23 | 99.51 | 6.96 | 99.24 | 8.68 | 98.91 | 10.40 | 98.51 | 12.10 | 98.06 | 13.78 | 97.55 | 15.45 | 96.98 | 17.10 |
| 2 | 100.00 | .06 | 99.97 | 1.80 | 99.87 | 3.55 | 99.72 | 5.28 | 99.51 | 7.02 | 99.23 | 8.74 | 98.90 | 10.45 | 98.50 | 12.15 | 98.05 | 13.84 | 97.53 | 15.51 | 96.96 | 17.16 |
| 4 | 100.00 | .12 | 99.97 | 1.86 | 99.87 | 3.60 | 99.72 | 5.34 | 99.50 | 7.07 | 99.22 | 8.80 | 98.88 | 10.51 | 98.48 | 12.21 | 98.03 | 13.89 | 97.52 | 15.56 | 96.94 | 17.21 |
| 6 | 100.00 | .17 | 99.97 | 1.92 | 99.87 | 3.66 | 99.71 | 5.40 | 99.49 | 7.13 | 99.21 | 8.85 | 98.87 | 10.57 | 98.47 | 12.26 | 98.01 | 13.95 | 97.50 | 15.62 | 96.92 | 17.26 |
| 8 | 100.00 | .23 | 99.96 | 1.98 | 99.86 | 3.72 | 99.70 | 5.46 | 99.48 | 7.19 | 99.20 | 8.91 | 98.86 | 10.62 | 98.46 | 12.32 | 98.00 | 14.01 | 97.48 | 15.67 | 96.90 | 17.32 |
| 10 | 100.00 | .29 | 99.96 | 2.04 | 99.86 | 3.78 | 99.70 | 5.52 | 99.47 | 7.25 | 99.19 | 8.97 | 98.85 | 10.68 | 98.44 | 12.38 | 97.98 | 14.06 | 97.46 | 15.73 | 96.88 | 17.37 |
| 12 | 100.00 | .35 | 99.96 | 2.09 | 99.86 | 3.84 | 99.69 | 5.57 | 99.46 | 7.30 | 99.18 | 9.03 | 98.83 | 10.74 | 98.43 | 12.43 | 97.97 | 14.12 | 97.44 | 15.78 | 96.86 | 17.43 |
| 14 | 100.00 | .41 | 99.96 | 2.15 | 99.85 | 3.89 | 99.69 | 5.63 | 99.46 | 7.36 | 99.17 | 9.08 | 98.82 | 10.79 | 98.41 | 12.49 | 97.95 | 14.17 | 97.43 | 15.84 | 96.84 | 17.48 |
| 16 | 100.00 | .47 | 99.96 | 2.21 | 99.85 | 3.95 | 99.68 | 5.69 | 99.45 | 7.42 | 99.16 | 9.14 | 98.81 | 10.85 | 98.40 | 12.55 | 97.93 | 14.23 | 97.41 | 15.89 | 96.82 | 17.54 |
| 18 | 100.00 | .52 | 99.95 | 2.27 | 99.84 | 4.01 | 99.68 | 5.75 | 99.44 | 7.48 | 99.15 | 9.20 | 98.80 | 10.91 | 98.39 | 12.60 | 97.92 | 14.28 | 97.39 | 15.95 | 96.80 | 17.59 |
| 20 | 100.00 | .58 | 99.95 | 2.33 | 99.84 | 4.07 | 99.67 | 5.80 | 99.43 | 7.53 | 99.14 | 9.25 | 98.78 | 10.96 | 98.37 | 12.66 | 97.90 | 14.34 | 97.37 | 16.00 | 96.78 | 17.65 |
| 22 | 99.99 | .64 | 99.95 | 2.38 | 99.83 | 4.13 | 99.66 | 5.86 | 99.42 | 7.59 | 99.13 | 9.31 | 98.77 | 11.02 | 98.36 | 12.72 | 97.88 | 14.40 | 97.35 | 16.06 | 96.76 | 17.70 |
| 24 | 99.99 | .70 | 99.95 | 2.44 | 99.83 | 4.18 | 99.66 | 5.92 | 99.41 | 7.65 | 99.11 | 9.37 | 98.76 | 11.08 | 98.34 | 12.77 | 97.87 | 14.45 | 97.33 | 16.11 | 96.74 | 17.76 |
| 26 | 99.99 | .76 | 99.94 | 2.50 | 99.82 | 4.24 | 99.65 | 5.98 | 99.40 | 7.71 | 99.10 | 9.43 | 98.74 | 11.13 | 98.33 | 12.83 | 97.85 | 14.51 | 97.31 | 16.17 | 96.72 | 17.81 |
| 28 | 99.99 | .81 | 99.94 | 2.56 | 99.82 | 4.30 | 99.64 | 6.04 | 99.38 | 7.76 | 99.09 | 9.48 | 98.73 | 11.19 | 98.31 | 12.88 | 97.83 | 14.56 | 97.29 | 16.22 | 96.70 | 17.86 |
| 30 | 99.99 | .87 | 99.94 | 2.62 | 99.81 | 4.36 | 99.63 | 6.09 | 99.38 | 7.82 | 99.08 | 9.54 | 98.72 | 11.25 | 98.29 | 12.94 | 97.82 | 14.62 | 97.28 | 16.28 | 96.68 | 17.92 |
| 32 | 99.99 | .93 | 99.93 | 2.67 | 99.81 | 4.42 | 99.63 | 6.15 | 99.37 | 7.88 | 99.07 | 9.60 | 98.71 | 11.30 | 98.28 | 13.00 | 97.80 | 14.67 | 97.26 | 16.33 | 96.66 | 17.97 |
| 34 | 99.99 | .99 | 99.93 | 2.73 | 99.80 | 4.48 | 99.62 | 6.21 | 99.36 | 7.94 | 99.05 | 9.65 | 98.69 | 11.36 | 98.27 | 13.05 | 97.78 | 14.73 | 97.24 | 16.39 | 96.64 | 18.03 |
| 36 | 99.99 | 1.05 | 99.93 | 2.79 | 99.80 | 4.53 | 99.61 | 6.27 | 99.35 | 7.99 | 99.04 | 9.71 | 98.68 | 11.42 | 98.25 | 13.11 | 97.76 | 14.79 | 97.22 | 16.44 | 96.62 | 18.08 |
| 38 | 99.99 | 1.11 | 99.92 | 2.85 | 99.79 | 4.59 | 99.60 | 6.33 | 99.34 | 8.05 | 99.03 | 9.77 | 98.67 | 11.47 | 98.24 | 13.17 | 97.75 | 14.84 | 97.20 | 16.50 | 96.60 | 18.14 |
| 40 | 99.98 | 1.16 | 99.92 | 2.91 | 99.79 | 4.65 | 99.59 | 6.38 | 99.33 | 8.11 | 99.01 | 9.83 | 98.65 | 11.53 | 98.22 | 13.22 | 97.73 | 14.90 | 97.18 | 16.55 | 96.57 | 18.19 |
| 42 | 99.98 | 1.22 | 99.91 | 2.97 | 99.78 | 4.71 | 99.59 | 6.44 | 99.32 | 8.17 | 99.00 | 9.88 | 98.64 | 11.59 | 98.20 | 13.28 | 97.71 | 14.95 | 97.16 | 16.61 | 96.55 | 18.24 |
| 44 | 99.98 | 1.28 | 99.91 | 3.02 | 99.77 | 4.76 | 99.58 | 6.50 | 99.31 | 8.23 | 98.99 | 9.94 | 98.63 | 11.64 | 98.19 | 13.33 | 97.69 | 15.01 | 97.14 | 16.66 | 96.53 | 18.30 |
| 46 | 99.98 | 1.34 | 99.90 | 3.08 | 99.77 | 4.82 | 99.57 | 6.56 | 99.30 | 8.28 | 98.97 | 10.00 | 98.61 | 11.70 | 98.17 | 13.39 | 97.68 | 15.06 | 97.12 | 16.72 | 96.51 | 18.35 |
| 48 | 99.98 | 1.40 | 99.90 | 3.14 | 99.76 | 4.88 | 99.56 | 6.61 | 99.29 | 8.34 | 98.96 | 10.05 | 98.60 | 11.76 | 98.16 | 13.45 | 97.66 | 15.12 | 97.10 | 16.77 | 96.49 | 18.41 |
| 50 | 99.98 | 1.45 | 99.90 | 3.20 | 99.76 | 4.94 | 99.56 | 6.67 | 99.28 | 8.40 | 98.94 | 10.11 | 98.58 | 11.81 | 98.14 | 13.50 | 97.64 | 15.17 | 97.08 | 16.83 | 96.47 | 18.46 |
| 52 | 99.98 | 1.51 | 99.89 | 3.26 | 99.75 | 4.99 | 99.55 | 6.73 | 99.27 | 8.45 | 98.93 | 10.17 | 98.57 | 11.87 | 98.13 | 13.56 | 97.62 | 15.23 | 97.06 | 16.88 | 96.45 | 18.51 |
| 54 | 99.97 | 1.57 | 99.89 | 3.31 | 99.74 | 5.05 | 99.54 | 6.78 | 99.26 | 8.51 | 98.92 | 10.22 | 98.56 | 11.93 | 98.11 | 13.61 | 97.61 | 15.28 | 97.04 | 16.94 | 96.42 | 18.57 |
| 56 | 99.97 | 1.63 | 99.89 | 3.37 | 99.74 | 5.11 | 99.53 | 6.84 | 99.25 | 8.57 | 98.92 | 10.28 | 98.54 | 11.98 | 98.10 | 13.67 | 97.59 | 15.34 | 97.02 | 16.99 | 96.40 | 18.62 |
| 58 | 99.97 | 1.69 | 99.88 | 3.43 | 99.73 | 5.17 | 99.52 | 6.90 | 99.25 | 8.63 | 98.91 | 10.34 | 98.53 | 12.04 | 98.08 | 13.73 | 97.57 | 15.40 | 97.00 | 17.05 | 96.38 | 18.68 |
| 60 | 99.97 | 1.74 | 99.88 | 3.49 | 99.73 | 5.23 | 99.51 | 6.96 | 99.24 | 8.68 | 98.91 | 10.40 | 98.51 | 12.10 | 98.06 | 13.78 | 97.55 | 15.45 | 96.98 | 17.10 | 96.36 | 18.73 |
| c = .75 | .75 | .01 | .75 | .02 | .75 | .03 | .75 | .05 | .75 | .06 | .75 | .07 | .75 | .08 | .75 | .10 | .74 | .11 | .74 | .12 | .74 | .14 |
| c = 1.00 | 1.00 | .01 | 1.00 | .03 | 1.00 | .04 | 1.00 | .06 | 1.00 | .08 | .99 | .09 | .99 | .11 | .99 | .13 | .99 | .15 | .99 | .16 | .98 | .18 |
| c = 1.25 | 1.25 | .02 | 1.25 | .03 | 1.25 | .05 | 1.25 | .08 | 1.25 | .10 | 1.24 | .11 | 1.24 | .14 | 1.24 | .16 | 1.23 | .18 | 1.23 | .21 | 1.23 | .23 |

| M. | 20° Hor. Dist. | 20° Diff. Elev. | 19° Hor. Dist. | 19° Diff. Elev. | 18° Hor. Dist. | 18° Diff. Elev. | 17° Hor. Dist. | 17° Diff. Elev. | 16° Hor. Dist. | 16° Diff. Elev. | 15° Hor. Dist. | 15° Diff. Elev. | 14° Hor. Dist. | 14° Diff. Elev. | 13° Hor. Dist. | 13° Diff. Elev. | 12° Hor. Dist. | 12° Diff. Elev. | 11° Hor. Dist. | 11° Diff. Elev. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 88.30 | 32.14 | 89.40 | 30.78 | 90.45 | 29.39 | 91.45 | 27.96 | 92.40 | 26.50 | 93.30 | 25.00 | 94.15 | 23.47 | 94.94 | 21.92 | 95.68 | 20.34 | 96.36 | 18.73 |
| 2 | 88.26 | 32.18 | 89.36 | 30.83 | 90.42 | 29.44 | 91.42 | 28.01 | 92.37 | 26.55 | 93.27 | 25.05 | 94.12 | 23.52 | 94.91 | 21.97 | 95.65 | 20.39 | 96.34 | 18.78 |
| 4 | 88.23 | 32.23 | 89.33 | 30.87 | 90.38 | 29.48 | 91.39 | 28.06 | 92.34 | 26.59 | 93.24 | 25.10 | 94.09 | 23.58 | 94.89 | 22.02 | 95.63 | 20.44 | 96.32 | 18.84 |
| 6 | 88.19 | 32.27 | 89.29 | 30.92 | 90.35 | 29.53 | 91.35 | 28.10 | 92.31 | 26.64 | 93.21 | 25.15 | 94.07 | 23.63 | 94.86 | 22.08 | 95.61 | 20.50 | 96.29 | 18.89 |
| 8 | 88.15 | 32.32 | 89.26 | 30.97 | 90.31 | 29.58 | 91.32 | 28.15 | 92.28 | 26.69 | 93.18 | 25.20 | 94.04 | 23.68 | 94.84 | 22.13 | 95.58 | 20.55 | 96.27 | 18.95 |
| 10 | 88.11 | 32.36 | 89.22 | 31.01 | 90.28 | 29.62 | 91.29 | 28.20 | 92.25 | 26.74 | 93.16 | 25.25 | 94.01 | 23.73 | 94.81 | 22.18 | 95.56 | 20.60 | 96.25 | 19.00 |
| 12 | 88.08 | 32.41 | 89.18 | 31.06 | 90.24 | 29.67 | 91.26 | 28.25 | 92.22 | 26.79 | 93.13 | 25.30 | 93.98 | 23.78 | 94.79 | 22.23 | 95.53 | 20.66 | 96.23 | 19.05 |
| 14 | 88.04 | 32.45 | 89.15 | 31.10 | 90.21 | 29.72 | 91.22 | 28.30 | 92.19 | 26.84 | 93.10 | 25.35 | 93.95 | 23.83 | 94.76 | 22.28 | 95.51 | 20.71 | 96.21 | 19.11 |
| 16 | 88.00 | 32.49 | 89.11 | 31.15 | 90.18 | 29.76 | 91.19 | 28.34 | 92.15 | 26.89 | 93.07 | 25.40 | 93.93 | 23.88 | 94.73 | 22.34 | 95.49 | 20.76 | 96.18 | 19.16 |
| 18 | 87.96 | 32.54 | 89.08 | 31.19 | 90.14 | 29.81 | 91.16 | 28.39 | 92.12 | 26.94 | 93.04 | 25.45 | 93.90 | 23.93 | 94.71 | 22.39 | 95.46 | 20.81 | 96.16 | 19.21 |
| 20 | 87.93 | 32.58 | 89.04 | 31.24 | 90.11 | 29.86 | 91.12 | 28.44 | 92.09 | 26.99 | 93.01 | 25.50 | 93.87 | 23.99 | 94.68 | 22.44 | 95.44 | 20.87 | 96.14 | 19.27 |
| 22 | 87.89 | 32.63 | 89.00 | 31.28 | 90.07 | 29.90 | 91.09 | 28.49 | 92.06 | 27.04 | 92.98 | 25.55 | 93.84 | 24.04 | 94.66 | 22.49 | 95.41 | 20.92 | 96.12 | 19.32 |
| 24 | 87.85 | 32.67 | 88.96 | 31.33 | 90.04 | 29.95 | 91.06 | 28.54 | 92.03 | 27.09 | 92.95 | 25.60 | 93.81 | 24.09 | 94.63 | 22.54 | 95.39 | 20.97 | 96.09 | 19.38 |
| 26 | 87.81 | 32.72 | 88.93 | 31.38 | 90.00 | 30.00 | 91.02 | 28.58 | 92.00 | 27.13 | 92.92 | 25.65 | 93.79 | 24.14 | 94.60 | 22.60 | 95.36 | 21.03 | 96.07 | 19.43 |
| 28 | 87.77 | 32.76 | 88.89 | 31.42 | 89.97 | 30.04 | 90.99 | 28.63 | 91.97 | 27.18 | 92.89 | 25.70 | 93.76 | 24.19 | 94.58 | 22.65 | 95.34 | 21.08 | 96.05 | 19.48 |
| 30 | 87.74 | 32.80 | 88.86 | 31.47 | 89.93 | 30.09 | 90.96 | 28.68 | 91.93 | 27.23 | 92.86 | 25.75 | 93.73 | 24.24 | 94.55 | 22.70 | 95.32 | 21.13 | 96.03 | 19.54 |
| 32 | 87.70 | 32.85 | 88.82 | 31.51 | 89.90 | 30.14 | 90.92 | 28.73 | 91.90 | 27.28 | 92.83 | 25.80 | 93.70 | 24.29 | 94.52 | 22.75 | 95.29 | 21.18 | 96.00 | 19.59 |
| 34 | 87.66 | 32.89 | 88.78 | 31.56 | 89.86 | 30.19 | 90.89 | 28.77 | 91.87 | 27.33 | 92.80 | 25.85 | 93.67 | 24.34 | 94.50 | 22.80 | 95.27 | 21.24 | 95.98 | 19.64 |
| 36 | 87.62 | 32.93 | 88.75 | 31.60 | 89.83 | 30.23 | 90.86 | 28.82 | 91.84 | 27.38 | 92.77 | 25.90 | 93.65 | 24.39 | 94.47 | 22.85 | 95.24 | 21.29 | 95.96 | 19.70 |
| 38 | 87.58 | 32.98 | 88.71 | 31.65 | 89.79 | 30.28 | 90.82 | 28.87 | 91.81 | 27.43 | 92.74 | 25.95 | 93.62 | 24.44 | 94.44 | 22.91 | 95.22 | 21.34 | 95.93 | 19.75 |
| 40 | 87.54 | 33.02 | 88.67 | 31.69 | 89.76 | 30.32 | 90.79 | 28.92 | 91.77 | 27.48 | 92.71 | 26.00 | 93.59 | 24.49 | 94.42 | 22.96 | 95.19 | 21.39 | 95.91 | 19.80 |
| 42 | 87.51 | 33.07 | 88.64 | 31.74 | 89.72 | 30.37 | 90.76 | 28.96 | 91.74 | 27.52 | 92.68 | 26.05 | 93.56 | 24.55 | 94.39 | 23.01 | 95.17 | 21.45 | 95.89 | 19.86 |
| 44 | 87.47 | 33.11 | 88.60 | 31.78 | 89.69 | 30.41 | 90.72 | 29.01 | 91.71 | 27.57 | 92.65 | 26.10 | 93.53 | 24.60 | 94.36 | 23.06 | 95.14 | 21.50 | 95.86 | 19.91 |
| 46 | 87.43 | 33.15 | 88.56 | 31.83 | 89.65 | 30.46 | 90.69 | 29.06 | 91.68 | 27.62 | 92.62 | 26.15 | 93.50 | 24.65 | 94.33 | 23.11 | 95.12 | 21.55 | 95.84 | 19.96 |
| 48 | 87.39 | 33.20 | 88.53 | 31.87 | 89.61 | 30.51 | 90.66 | 29.11 | 91.65 | 27.67 | 92.59 | 26.20 | 93.47 | 24.70 | 94.31 | 23.16 | 95.09 | 21.60 | 95.82 | 20.02 |
| 50 | 87.35 | 33.24 | 88.49 | 31.92 | 89.58 | 30.55 | 90.62 | 29.15 | 91.61 | 27.72 | 92.56 | 26.25 | 93.45 | 24.75 | 94.28 | 23.22 | 95.07 | 21.66 | 95.79 | 20.07 |
| 52 | 87.31 | 33.28 | 88.45 | 31.96 | 89.54 | 30.60 | 90.59 | 29.20 | 91.58 | 27.77 | 92.53 | 26.30 | 93.42 | 24.80 | 94.25 | 23.27 | 95.04 | 21.71 | 95.77 | 20.12 |
| 54 | 87.27 | 33.33 | 88.41 | 32.01 | 89.51 | 30.65 | 90.55 | 29.25 | 91.55 | 27.81 | 92.49 | 26.35 | 93.39 | 24.85 | 94.23 | 23.32 | 95.02 | 21.76 | 95.75 | 20.18 |
| 56 | 87.24 | 33.37 | 88.38 | 32.05 | 89.47 | 30.69 | 90.52 | 29.30 | 91.52 | 27.86 | 92.46 | 26.40 | 93.36 | 24.90 | 94.20 | 23.37 | 94.99 | 21.81 | 95.72 | 20.23 |
| 58 | 87.20 | 33.41 | 88.34 | 32.09 | 89.44 | 30.74 | 90.48 | 29.34 | 91.48 | 27.91 | 92.43 | 26.45 | 93.33 | 24.95 | 94.17 | 23.42 | 94.97 | 21.87 | 95.70 | 20.28 |
| 60 | 87.16 | 33.46 | 88.30 | 32.14 | 89.40 | 30.78 | 90.45 | 29.39 | 91.45 | 27.96 | 92.40 | 26.50 | 93.30 | 25.00 | 94.15 | 23.47 | 94.94 | 21.92 | 95.68 | 20.34 |
| c = .75 | .70 | .26 | .71 | .25 | .71 | .24 | .72 | .23 | .72 | .21 | .72 | .20 | .73 | .19 | .73 | .17 | .73 | .16 | .73 | .15 |
| c = 1.00 | .94 | .35 | .94 | .33 | .95 | .32 | .95 | .30 | .96 | .28 | .96 | .27 | .97 | .25 | .97 | .23 | .98 | .22 | .98 | .20 |
| c = 1.25 | 1.17 | .44 | 1.18 | .42 | 1.19 | .40 | 1.19 | .38 | 1.20 | .35 | 1.20 | .34 | 1.21 | .31 | 1.21 | .29 | 1.22 | .27 | 1.22 | .25 |

| M | 21° Hor. Dist. | 21° Diff. Elev. | 22° Hor. Dist. | 22° Diff. Elev. | 23° Hor. Dist. | 23° Diff. Elev. | 24° Hor. Dist. | 24° Diff. Elev. | 25° Hor. Dist. | 25° Diff. Elev. | 26° Hor. Dist. | 26° Diff. Elev. | 27° Hor. Dist. | 27° Diff. Elev. | 28° Hor. Dist. | 28° Diff. Elev. | 29° Hor. Dist. | 29° Diff. Elev. | 30° Hor. Dist. | 30° Diff. Elev. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 87.16 | 33.46 | 85.97 | 34.73 | 84.73 | 35.97 | 83.46 | 37.16 | 82.14 | 38.30 | 80.78 | 39.40 | 79.39 | 40.45 | 77.96 | 41.45 | 76.50 | 42.40 | 75.00 | 43.30 |
| 2 | 87.12 | 33.51 | 85.93 | 34.78 | 84.69 | 36.01 | 83.41 | 37.20 | 82.09 | 38.34 | 80.74 | 39.44 | 79.34 | 40.49 | 77.91 | 41.48 | 76.45 | 42.43 | 74.95 | 43.33 |
| 4 | 87.08 | 33.54 | 85.89 | 34.82 | 84.65 | 36.05 | 83.37 | 37.23 | 82.05 | 38.38 | 80.69 | 39.47 | 79.30 | 40.52 | 77.86 | 41.52 | 76.40 | 42.46 | 74.90 | 43.36 |
| 6 | 87.04 | 33.59 | 85.85 | 34.86 | 84.61 | 36.09 | 83.33 | 37.27 | 82.00 | 38.41 | 80.65 | 39.51 | 79.25 | 40.55 | 77.81 | 41.55 | 76.35 | 42.49 | 74.85 | 43.39 |
| 8 | 87.00 | 33.63 | 85.80 | 34.90 | 84.57 | 36.13 | 83.28 | 37.31 | 81.96 | 38.45 | 80.60 | 39.54 | 79.20 | 40.59 | 77.77 | 41.58 | 76.30 | 42.53 | 74.80 | 43.42 |
| 10 | 86.96 | 33.67 | 85.76 | 34.94 | 84.52 | 36.17 | 83.24 | 37.35 | 81.92 | 38.49 | 80.55 | 39.58 | 79.15 | 40.62 | 77.72 | 41.61 | 76.25 | 42.56 | 74.75 | 43.45 |
| 12 | 86.92 | 33.72 | 85.72 | 34.98 | 84.48 | 36.21 | 83.20 | 37.39 | 81.87 | 38.53 | 80.51 | 39.61 | 79.11 | 40.66 | 77.67 | 41.65 | 76.20 | 42.59 | 74.70 | 43.47 |
| 14 | 86.88 | 33.76 | 85.68 | 35.02 | 84.44 | 36.25 | 83.15 | 37.43 | 81.83 | 38.56 | 80.46 | 39.65 | 79.06 | 40.69 | 77.62 | 41.68 | 76.15 | 42.62 | 74.65 | 43.50 |
| 16 | 86.84 | 33.80 | 85.64 | 35.07 | 84.40 | 36.29 | 83.11 | 37.47 | 81.78 | 38.60 | 80.41 | 39.69 | 79.01 | 40.72 | 77.57 | 41.71 | 76.10 | 42.65 | 74.60 | 43.53 |
| 18 | 86.80 | 33.84 | 85.60 | 35.11 | 84.35 | 36.33 | 83.07 | 37.51 | 81.74 | 38.64 | 80.37 | 39.72 | 78.96 | 40.76 | 77.52 | 41.74 | 76.05 | 42.68 | 74.55 | 43.56 |
| 20 | 86.77 | 33.89 | 85.56 | 35.15 | 84.31 | 36.37 | 83.02 | 37.54 | 81.69 | 38.67 | 80.32 | 39.76 | 78.92 | 40.79 | 77.48 | 41.77 | 76.00 | 42.71 | 74.49 | 43.59 |
| 22 | 86.73 | 33.93 | 85.52 | 35.19 | 84.27 | 36.41 | 82.98 | 37.58 | 81.65 | 38.71 | 80.28 | 39.79 | 78.87 | 40.82 | 77.42 | 41.81 | 75.95 | 42.74 | 74.44 | 43.62 |
| 24 | 86.69 | 33.97 | 85.48 | 35.23 | 84.23 | 36.45 | 82.93 | 37.62 | 81.60 | 38.75 | 80.23 | 39.83 | 78.82 | 40.86 | 77.38 | 41.84 | 75.90 | 42.77 | 74.39 | 43.65 |
| 26 | 86.65 | 34.01 | 85.44 | 35.27 | 84.18 | 36.49 | 82.89 | 37.66 | 81.56 | 38.78 | 80.18 | 39.86 | 78.77 | 40.89 | 77.33 | 41.87 | 75.85 | 42.80 | 74.34 | 43.67 |
| 28 | 86.61 | 34.06 | 85.40 | 35.31 | 84.14 | 36.53 | 82.85 | 37.70 | 81.51 | 38.82 | 80.14 | 39.90 | 78.73 | 40.92 | 77.28 | 41.90 | 75.80 | 42.83 | 74.29 | 43.70 |
| 30 | 86.57 | 34.10 | 85.36 | 35.35 | 84.10 | 36.57 | 82.80 | 37.74 | 81.47 | 38.86 | 80.09 | 39.93 | 78.68 | 40.96 | 77.23 | 41.93 | 75.75 | 42.86 | 74.24 | 43.73 |
| 32 | 86.53 | 34.14 | 85.31 | 35.40 | 84.06 | 36.61 | 82.76 | 37.77 | 81.42 | 38.89 | 80.04 | 39.97 | 78.63 | 40.99 | 77.18 | 41.97 | 75.70 | 42.89 | 74.19 | 43.76 |
| 34 | 86.49 | 34.18 | 85.27 | 35.44 | 84.01 | 36.65 | 82.72 | 37.81 | 81.38 | 38.93 | 80.00 | 40.00 | 78.58 | 41.02 | 77.13 | 42.00 | 75.65 | 42.92 | 74.14 | 43.79 |
| 36 | 86.45 | 34.23 | 85.23 | 35.48 | 83.97 | 36.69 | 82.67 | 37.85 | 81.33 | 38.97 | 79.95 | 40.04 | 78.54 | 41.06 | 77.09 | 42.03 | 75.60 | 42.95 | 74.09 | 43.82 |
| 38 | 86.41 | 34.27 | 85.19 | 35.52 | 83.93 | 36.73 | 82.63 | 37.89 | 81.28 | 39.00 | 79.90 | 40.07 | 78.49 | 41.09 | 77.04 | 42.06 | 75.55 | 42.98 | 74.04 | 43.84 |
| 40 | 86.37 | 34.31 | 85.15 | 35.56 | 83.89 | 36.77 | 82.58 | 37.93 | 81.24 | 39.04 | 79.86 | 40.11 | 78.44 | 41.12 | 76.99 | 42.09 | 75.50 | 43.01 | 73.99 | 43.87 |
| 42 | 86.33 | 34.35 | 85.11 | 35.60 | 83.84 | 36.80 | 82.54 | 37.96 | 81.19 | 39.08 | 79.81 | 40.14 | 78.39 | 41.16 | 76.94 | 42.12 | 75.45 | 43.04 | 73.93 | 43.90 |
| 44 | 86.29 | 34.40 | 85.07 | 35.64 | 83.80 | 36.84 | 82.49 | 38.00 | 81.15 | 39.11 | 79.76 | 40.18 | 78.34 | 41.19 | 76.89 | 42.15 | 75.40 | 43.07 | 73.88 | 43.93 |
| 46 | 86.25 | 34.44 | 85.02 | 35.68 | 83.72 | 36.88 | 82.45 | 38.04 | 81.10 | 39.15 | 79.72 | 40.21 | 78.30 | 41.22 | 76.84 | 42.19 | 75.35 | 43.10 | 73.83 | 43.95 |
| 48 | 86.21 | 34.48 | 84.98 | 35.72 | 83.67 | 36.92 | 82.41 | 38.08 | 81.06 | 39.18 | 79.67 | 40.24 | 78.25 | 41.26 | 76.79 | 42.22 | 75.30 | 43.13 | 73.78 | 43.98 |
| 50 | 86.17 | 34.52 | 84.94 | 35.76 | 83.63 | 36.96 | 82.36 | 38.11 | 81.01 | 39.22 | 79.62 | 40.28 | 78.20 | 41.29 | 76.74 | 42.25 | 75.25 | 43.16 | 73.73 | 44.01 |
| 52 | 86.13 | 34.57 | 84.90 | 35.80 | 83.58 | 37.00 | 82.32 | 38.15 | 80.97 | 39.26 | 79.58 | 40.31 | 78.15 | 41.32 | 76.69 | 42.28 | 75.20 | 43.18 | 73.68 | 44.04 |
| 54 | 86.09 | 34.61 | 84.86 | 35.85 | 83.54 | 37.04 | 82.27 | 38.19 | 80.92 | 39.29 | 79.53 | 40.35 | 78.10 | 41.35 | 76.64 | 42.31 | 75.15 | 43.21 | 73.63 | 44.07 |
| 56 | 86.05 | 34.65 | 84.82 | 35.89 | 83.50 | 37.08 | 82.23 | 38.23 | 80.87 | 39.33 | 79.48 | 40.38 | 78.06 | 41.39 | 76.59 | 42.34 | 75.10 | 43.24 | 73.58 | 44.09 |
| 58 | 86.01 | 34.69 | 84.77 | 35.93 | 83.46 | 37.12 | 82.18 | 38.26 | 80.83 | 39.36 | 79.44 | 40.42 | 78.01 | 41.42 | 76.55 | 42.37 | 75.05 | 43.27 | 73.52 | 44.12 |
| 60 | 85.97 | 34.73 | 84.73 | 35.97 | 83.46 | 37.16 | 82.14 | 38.30 | 80.78 | 39.40 | 79.39 | 40.45 | 77.96 | 41.45 | 76.50 | 42.40 | 75.00 | 43.30 | 73.47 | 44.15 |
| c = .75 | .70 | .27 | .69 | .29 | .69 | .30 | .68 | .31 | .68 | .32 | .67 | .33 | .66 | .35 | .66 | .36 | .65 | .37 | .65 | .38 |
| c = 1.00 | .98 | .37 | .92 | .38 | .92 | .40 | .91 | .41 | .90 | .43 | .89 | .45 | .89 | .46 | .88 | .48 | .87 | .49 | .86 | .51 |
| c = 1.25 | 1.16 | .46 | 1.15 | .48 | 1.15 | .50 | 1.14 | .52 | 1.18 | .54 | 1.12 | .56 | 1.11 | .58 | 1.10 | .60 | 1.09 | .62 | 1.08 | .64 |

## BAROMETRIC LEVELING

Of the two types of barometer in general use, the *mercurial barometer* is better adapted for work at permanent stations, and the *aneroid barometer*, shown in the accompanying figure, by reason of its portability, is better suited for use in the field. In the barometer, the difference in the pressure of the air at two different stations not in the same horizontal plane, is made the basis for measuring the difference in their elevation. The aneroid barometer consists of a circular metallic air-tight box, either of brass or aluminum (because of its lightness). One side is covered with a thin corrugated plate and only enough air is left within the box to compensate for the diminished stiffness of the cover at higher temperatures. This cover rises or falls as the outer pressure changes, and this motion is greatly magnified by a series of levers and is transmitted to a pointer moving over a scale on the circumference of the outer face. This scale is commonly doubly graduated. The inner scale is graduated by inches, tenths, and hundredths, to correspond with the graduations of a standard mercurial barometer. The outer scale is graduated in thousands, hundreds, and tens of feet, indicating elevations above sea level corresponding to the atmospheric pressures recorded by the inner scale. The
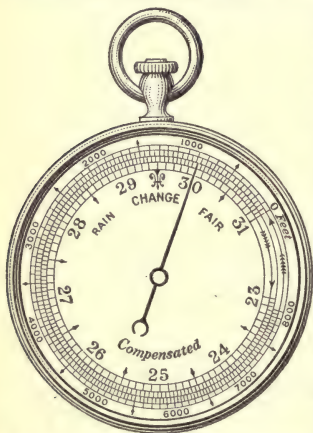


fineness of these outer graduations varies, the ordinary aneroid reading direct to 10 ft. and by estimation to, say, 2 ft., while very large forms of the instrument are provided with a vernier, magnifying glass, etc., by which they may be read to single feet. Such refinement in reading is unnecessary, as instrument and weather irregularities introduce much greater uncertainties than those caused by too coarse graduating.

In some aneroids the two sets of graduations are fixed, the zero of the altitude scale being opposite the 31-in. mark of the mercurial scale, and this relation cannot be changed, but in most instruments of this kind the zero may be shifted either by turning an arrangement like the stem of a watch or by turning the milled edge of the aneroid itself. The zero of the altitude scale may be made to coincide with either the 30- or 31-in. mark of the mercurial scale, but whichever mark is decided on by the instrument maker must always be used. That this is the case will be obvious from the table of Barometric

Elevations, which shows that a difference of 1 in. in the length of the mercurial column between 29 and 30 inches corresponds to a difference of 924 ft. in elevation, whereas a difference of 1 in. of mercury between 13 and 14 in. corresponds to a difference of 2,020 ft. in elevation. Hence, as the length of the altitude scale corresponding to a difference in elevation of 1,000 ft. is not uniform, any shifting of the zeros of the two scales will bring inharmonious graduations opposite one another.

Aneroids vary in size from $1\frac{3}{4}$ to $2\frac{1}{2}$ to 3 in. in diameter for the standard forms up to 5 in. in diameter for the larger ones provided with a vernier, etc. They are commonly graduated from 31 in. down to 27, 25, 21, 19, 17, and 14 in. of mercury, corresponding to approximate elevations of from 1,000 ft. below sea level to 3,000, 5,000, 10,000, 12,000, 16,000, and 20,000 ft. above sea level. The larger barometers are no more accurate than the smaller ones and nothing is gained by having them graduated to record smaller pressures (greater elevations) than those prevailing where the instrument is to be used. Thus, east of the Mississippi River, a 2-in. or $2\frac{1}{2}$-in. aneroid graduated from 27 to 32 in. (corresponding to elevations of 3,000 ft. above to 2,000 ft. below sea level), will answer for all purposes of the coal-mining engineer, except for exploratory work in the highest mountains of the Carolinas, etc. A barometer reading to 17 in. (about 16,000 ft.) will answer for all parts of the continental

United States, as the highest peaks of the Rocky Mountains are but little over 14,000 ft. high. It must be remembered that the less the pressure (the greater the altitude) the barometer will record, the finer are the graduations on both scales and, consequently, the more difficult is precise reading.

A small screw in the center of the back permits the index pointer to be accurately adjusted to correspond with the reading of a standard mercurial barometer. This adjustment is originally made by the instrument maker, but the aneroid should be compared from time to time with a standard barometer at some station of the United States Weather Bureau, or elsewhere.

The word *compensated* stamped on the face of an aneroid barometer does not mean that in determining elevations differences in temperature are not to be considered, but only that the instrument reads correctly at all temperatures and that no allowance need be made for the effect of changes in temperature upon the instrument itself.

Too much reliance must not be placed on the accuracy of elevations as determined by the aneroid. All this instrument does is to measure the pressure of the air at a given place at a certain time. As this pressure must and does vary at the same place as the weather changes, it must be apparent that the difference in elevation between two distant points can be determined with even approximate accuracy only when two barometers are used and which are read simultaneously at the two points in question. At sea level, 1 in. of the mercurial column corresponds to a difference in elevation of about 900 ft. Changes of $\frac{1}{10}$ in. frequently take place in 1 hr., $\frac{2}{10}$ in. in 1 da., and in event of storms, ranges of 1 in. are not unusual. Thus, as $\frac{1}{10}$ in. represents about 90 ft. of elevation, a single reading of the barometer may give an elevation for a place 900 ft. greater or less than the true one under unfavorable atmospheric conditions, and one of as much as 100 ft. under the best. On the other hand, if the barometer is read two or three times daily for a period of 1 yr. or more, the temperature being noted at the same time and the proper corrections made, a very fair idea may be obtained as to the difference in elevation between the station in question and that of any other station at which simultaneous observations have been made. Thus, the daily readings may be made by the engineer at some isolated station and their mean for a period of time compared with those made at any station of the United States Weather Bureau any number of miles distant.

**Barometric Formulas.**[*]—The general formula for obtaining the difference of elevation between two points is,

$$z = 60,520[1 + .001017(t + T - 64)] \log \frac{H}{h}$$

in which $z$ = difference of elevation of the two points;

$h$ and $t$ = reading of barometer and thermometer at upper station;
$H$ and $T$ = reading of barometer and thermometer at lower station.

This equation may be referred to an approximate sea level (height of mercurial column 30 in. instead of 29.92 in.) and to a mean station temperature of 50° F., that is $t + T$ is made equal to 100° F., in which $t$ and $T$ may have any values as long as their sum equals 100° F. Making the substitutions

$$z = 62,737 \log \frac{30}{h} - 62,737 \log \frac{30}{H}$$

The accompanying table of Barometric Elevations, from the United States Coast and Geodetic Survey, contains values for $62,737 \log \frac{30}{h}$ for all readings of the aneroid from 13 to 31 in. for use in connection with the foregoing formula, no allowance being made for the amount of aqueous vapor in the air. At other temperatures and for an assumed average humidity a correction obtained from the table of Corrections for Temperature and Humidity must be applied to the difference in elevation as obtained from the first table. Thus, if $A$ is the difference in elevation obtained from the table of Barometric Elevations, and $C$ the correction for temperature and humidity from the second table,

$$z = A (1 + C)$$

**EXAMPLE.**—The means of the readings of the barometer and the thermometer at the summit and base of a mountain were: Summit, barometer 17.92 in., thermometer 26° F.; base, barometer 24.15, thermometer 64° F. If the

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[*] Adapted from The Theory and Practice of Surveying, by J. B. Johnson, published by John Wiley & Sons, New York City.

## BAROMETRIC ELEVATIONS*

| h | A | Difference for .01 | h | A | Difference for .01 | h | A | Difference for .01 |
|---|---|---|---|---|---|---|---|---|
| Inches | Feet | Feet | Inches | Feet | Feet | Inches | Feet | Feet |
| 13.0 | 22,785 | −20.9 | 19.0 | 12,445 | −14.3 | 25.0 | 4,968 | −10.9 |
| 13.1 | 22,576 | 20.8 | 19.1 | 12,302 | 14.2 | 25.1 | 4,859 | 10.8 |
| 13.2 | 22,368 | 20.6 | 19.2 | 12,160 | 14.2 | 25.2 | 4,751 | 10.8 |
| 13.3 | 22,162 | 20.4 | 19.3 | 12,018 | 14.1 | 25.3 | 4,643 | 10.8 |
| 13.4 | 21,958 | 20.1 | 19.4 | 11,877 | 14.0 | 25.4 | 4,535 | 10.7 |
| 13.5 | 21,757 | 20.0 | 19.5 | 11,737 | 13.9 | 25.5 | 4,428 | 10.7 |
| 13.6 | 21,557 | 19.9 | 19.6 | 11,598 | 13.9 | 25.6 | 4,321 | 10.6 |
| 13.7 | 21,358 | 19.8 | 19.7 | 11,459 | 13.8 | 25.7 | 4,215 | 10.6 |
| 13.8 | 21,160 | 19.8 | 19.8 | 11,321 | 13.8 | 25.8 | 4,109 | 10.5 |
| 13.9 | 20,962 | 19.7 | 19.9 | 11,184 | 13.7 | 25.9 | 4,004 | 10.5 |
| 14.0 | 20,765 | 19.5 | 20.0 | 11,047 | 13.6 | 26.0 | 3,899 | 10.5 |
| 14.1 | 20,570 | 19.3 | 20.1 | 10,911 | 13.5 | 26.1 | 3,794 | 1.04 |
| 14.2 | 20,377 | 19.1 | 20.2 | 10,776 | 13.4 | 26.2 | 3,690 | 10.4 |
| 14.3 | 20,186 | 18.9 | 20.3 | 10,642 | 13.4 | 26.3 | 3,586 | 10.3 |
| 14.4 | 19,997 | 18.8 | 20.4 | 10,508 | 13.3 | 26.4 | 3,483 | 10.3 |
| 14.5 | 19,809 | 18.6 | 20.5 | 10,375 | 13.3 | 26.5 | 3,380 | 10.3 |
| 14.6 | 19,623 | 18.6 | 20.6 | 10,242 | 13.2 | 26.6 | 3,277 | 10.2 |
| 14.7 | 19,437 | 18.5 | 20.7 | 10,110 | 13.1 | 26.7 | 3,175 | 10.2 |
| 14.8 | 19,252 | 18.4 | 20.8 | 9,979 | 13.1 | 26.8 | 3,073 | 10.1 |
| 14.9 | 19,068 | 18.2 | 20.9 | 9,848 | 13.0 | 26.9 | 2,972 | 10.1 |
| 15.0 | 18,886 | 18.1 | 21.0 | 9,718 | 12.9 | 27.0 | 2,871 | 10.1 |
| 15.1 | 18,705 | 18.0 | 21.1 | 9,859 | 12.9 | 27.1 | 2,770 | 10.0 |
| 15.2 | 18,525 | 17.9 | 21.2 | 9,460 | 12.8 | 27.2 | 2,670 | 10.0 |
| 15.3 | 18,346 | 17.8 | 21.3 | 9,332 | 12.8 | 27.3 | 2,570 | 10.0 |
| 15.4 | 18,168 | 17.6 | 21.4 | 9,204 | 12.7 | 27.4 | 2,470 | 9.9 |
| 15.5 | 17,992 | 17.5 | 21.5 | 9,077 | 12.6 | 27.5 | 2,371 | 9.9 |
| 15.6 | 17,817 | 17.4 | 21.6 | 8,951 | 12.6 | 27.6 | 2,272 | 9.9 |
| 15.7 | 17,643 | 17.3 | 21.7 | 8,825 | 12.5 | 27.7 | 2,173 | 9.8 |
| 15.8 | 17,470 | 17.2 | 21.8 | 8,700 | 12.5 | 27.8 | 2,075 | 9.8 |
| 15.9 | 17,298 | 17.1 | 21.9 | 8,575 | 12.4 | 27.9 | 1,977 | 9.7 |
| 16.0 | 17,127 | 16.9 | 22.0 | 8,451 | 12.4 | 28.0 | 1,880 | 9.7 |
| 16.1 | 16,958 | 16.9 | 22.1 | 8,327 | 12.3 | 28.1 | 1,783 | 9.7 |
| 16.2 | 16,789 | 16.8 | 22.2 | 8,204 | 12.2 | 28.2 | 1,686 | 9.6 |
| 16.3 | 16,621 | 16.7 | 22.3 | 8,082 | 12.2 | 28.3 | 1,589 | 9.6 |
| 16.4 | 16,454 | 16.6 | 22.4 | 7,960 | 12.2 | 28.4 | 1,493 | 9.6 |
| 16.5 | 16,288 | 16.4 | 22.5 | 7,838 | 12.1 | 28.5 | 1,397 | 9.5 |
| 16.6 | 16,124 | 16.3 | 22.6 | 7,717 | 12.0 | 28.6 | 1,302 | 9.5 |
| 16.7 | 15,961 | 16.3 | 22.7 | 7,597 | 12.0 | 28.7 | 1,207 | 9.5 |
| 16.8 | 15,798 | 16.2 | 22.8 | 7,477 | 11.9 | 28.8 | 1,112 | 9.4 |
| 16.9 | 15,636 | 16.0 | 22.9 | 7,358 | 11.9 | 28.9 | 1,018 | 9.4 |
| 17.0 | 15,746 | 16.0 | 23.0 | 7,239 | 11.8 | 29.0 | 924 | 9.4 |
| 17.1 | 15,316 | 15.9 | 23.1 | 7,121 | 11.7 | 29.1 | 830 | 9.4 |
| 17.2 | 15,157 | 15.8 | 23.2 | 7,004 | 11.7 | 29.2 | 736 | 9.3 |
| 17.3 | 14,999 | 15.7 | 23.3 | 6,887 | 11.7 | 29.3 | 643 | 9.3 |
| 17.4 | 14,842 | 15.6 | 23.4 | 6,770 | 11.6 | 29.4 | 550 | 9.2 |
| 17.5 | 14,686 | 15.5 | 23.5 | 6,654 | 11.6 | 29.5 | 458 | 9.2 |
| 17.6 | 14,531 | 15.5 | 23.6 | 6,538 | 11.5 | 29.6 | 366 | 9.2 |
| 17.7 | 14,377 | 15.4 | 23.7 | 6,423 | 11.5 | 29.7 | 274 | 9.2 |
| 17.8 | 14,223 | 15.4 | 23.8 | 6,308 | 11.4 | 29.8 | 182 | 9.1 |
| 17.9 | 14,070 | 15.3 | 23.9 | 6,194 | 11.4 | 29.9 | 91 | 9.1 |
| 18.0 | 13,918 | 15.2 | 24.0 | 6,080 | 11.3 | 30.0 | 00 | 9.1 |
| 18.1 | 13,767 | 15.1 | 24.1 | 5,967 | 11.3 | 30.1 | − 91 | 9.0 |
| 18.2 | 13,617 | 15.0 | 24.2 | 5,854 | 11.3 | 30.2 | 181 | 9.0 |
| 18.3 | 13,468 | 14.9 | 24.3 | 5,741 | 11.2 | 30.3 | 271 | 9.0 |
| 18.4 | 13,319 | 14.9 | 24.4 | 5,629 | 11.1 | 30.4 | 361 | 9.0 |
| 18.5 | 13,172 | 14.7 | 24.5 | 5,518 | 11.1 | 30.5 | 451 | 8.9 |
| 18.6 | 13,025 | 14.7 | 24.6 | 5,407 | 11.1 | 30.6 | 540 | 8.9 |
| 18.7 | 12,879 | 14.6 | 24.7 | 5,296 | 11.0 | 30.7 | 629 | 8.8 |
| 18.8 | 12,733 | 14.6 | 24.8 | 5,186 | 10.9 | 30.8 | 717 | 8.8 |
| 18.9 | 12,589 | 14.4 | 24.9 | 5,077 | −10.9 | 30.9 | 805 | − 8.8 |
| | | −14.4 | | | | | | |

*Calculated for barometer at sea level = 30 in. and a mean temperature of 50° F.

## CORRECTIONS FOR TEMPERATURE AND HUMIDITY

| $T+t$ F.° | $C$ | $T+t$ F.° | $C$ | $T+t$ F.° | $C$ |
|---|---|---|---|---|---|
| 0  | $-.1025$ | 60  | $-.0380$ | 120 | $+.0262$ |
| 5  | .0970 | 65  | .0326 | 125 | .0315 |
| 10 | .0915 | 70  | .0273 | 130 | .0368 |
| 15 | .0860 | 75  | .0220 | 135 | .0420 |
| 20 | .0860 | 80  | .0166 | 140 | .0472 |
| 25 | .0752 | 85  | .0112 | 145 | .0524 |
| 30 | .0698 | 90  | .0058 | 150 | .0575 |
| 35 | .0645 | 95  | $-.0004$ | 155 | .0626 |
| 40 | .0592 | 100 | $+.0049$ | 160 | .0677 |
| 45 | .0539 | 105 | .0102 | 165 | .0728 |
| 50 | .0486 | 110 | .0156 | 170 | .0779 |
| 55 | .0433 | 115 | .0209 | 175 | .0829 |
| 60 | $-.0380$ | 120 | $+.0262$ | 180 | $+.0879$ |

elevation of the base was 6,025 ft. above sea level, what was the elevation of the summit?

SOLUTION.—From the first table,

$$\text{Height for 17.92 in.} = 14,039.6 \text{ ft.}$$
$$\text{Height for 24.15 in.} = 5,910.5 \text{ ft.}$$

$$\text{Approx. difference in elevation} = 8,129.1 \text{ ft.}$$

As $T+t=90°$, from the second table, $C=-.0058$, and

$$z = 8,129.1 \times (1-.0058) = 8,082 \text{ ft.}$$
$$\text{Elevation of base} = 6,025 \text{ ft.}$$

$$\text{Elevation, top of mountain} = 14,010 \text{ ft.}$$

**Use of Barometer.**—The mining engineer ordinarily has but one aneroid and this is not commonly provided with a thermometer. With a single instrument reliable results are difficult to obtain, and depend as much on good and uniform weather conditions as on the skill and carefulness of the engineer. In fact, the results obtained during storms, whether of wind or rain, are not to be relied on at all. The single aneroid, then, should be used only when weather conditions are of the best. In the morning, before starting out, its reading should be noted and recorded. If a thermometer is available, it, too, should be read. As the aneroid is usually employed in exploratory work in which more or less time is required to examine various coal openings, it should be read a few minutes after reaching an opening and again on leaving, the times and temperatures being noted as well. After the examination of one opening is completed, the engineer should hasten as rapidly as possible to the next; that is, he should move rapidly from place to place but should remain a sufficient time at each to estimate the changes in pressure (and consequently in apparent elevation) that are taking place. By taking two observations at each opening at intervals of, say, ½ hr., a correction curve showing the changes in pressure may be worked out by means of which an allowance may be made for these changes while the barometer is being carried from place to place. Thus, a reading at Sta. A at 9.15 A. M., showed an elevation of 810 ft., and a reading at Sta. B at 9.30 A. M. showed one of 860 ft., and a second at B at 10 A. M. indicated 875 ft. This last reading shows a change in apparent elevation of 15 ft. in 30 min., or at the rate of .5 ft. per min. The apparent difference in elevation between A and B (first readings) is 50 ft., but a part of this difference is due to a change in the atmospheric pressure. The two readings at Sta. B show that the elevations are apparently increasing at the rate of .5 ft. per min.; therefore, as it took 15 min. to go from A to B, the elevation of B over A apparently increased 7.5 ft. Hence, the actual difference in elevation between A and B is not 50 ft. but $50-7.5=42.5$ ft. Very satisfactory results may be obtained in this way; and if the time between stations is short, corrections for changes in temperature, etc., need not be made, provided the difference in

elevation is not great. By taking double readings at each station, a continuous curve can be worked out and applied to correcting the day's observations. If possible, the aneroid should be reread at the various openings on the way home, and the mean of the afternoon and morning readings taken as the true reading. The instrument should be read upon reaching the starting point at night and again when leaving the next morning.

When two barometers are available, their readings are compared at starting, one being carried into the field and the other retained at headquarters where it is read by an assistant throughout the day at intervals of 10 or 15 min., a record being kept of the time, temperature, and pressure. The field man, then, need take but one reading at a station, preferably just before leaving, and should likewise note the temperature and time. These double readings, unless the field-man is so far from the base that weather conditions are markedly different, afford a complete check on fluctuations in pressure due to changed atmospheric conditions.

**Care of the Barometer.**—The aneroid should not be removed from its case; should not be subjected to violent jars; nor exposed to or read while affected by artificial heat. It should be read in a horizontal position, and on sunny days should be allowed to remain in the shade for, say, 5 min. before being read, that all its parts may have time to assume the temperature of the air at the station.

## PRACTICAL PROBLEMS IN SURVEYING

**1. To Prolong a Straight Line.**—Let $AB$, Fig. 1, be a straight line whose position on the ground is fixed by stakes set at $A$ and $B$, and let it be required to prolong the line to $C$. This can be done in two ways; namely, by foresight only, or by backsight and foresight, the
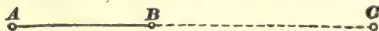


Fig. 1

latter method being commonly called backsight.

*By Foresight.*—The transit is set over the point at $A$, and the line of sight directed to a flag held at $B$; if the point $C$ is to be set at a given distance from $B$, the chainmen measure the required distance, the head chainman being kept in line by the transitman. When the required distance has been measured, the point $C$, which evidently lies in the prolongation of $AB$, is marked by a stake or otherwise.

*By Backsight.*—The transit is set over the point at $B$ and a sight taken on a flag held at $A$. The telescope is then plunged so that it is directed along the prolongation of $AB$. Any required distance $BC$ may then be measured from $B$ in the direction indicated by the line of sight.

**2. To Run a Line Over a Hill When the Ends of the Line Are Invisible From Each Other.**—The points $A$ and $B$, Fig. 2, are supposed to be on the opposite sides of a hill, and to be invisible from each other. It is desired to run a line between them, or to locate some intermediate points.
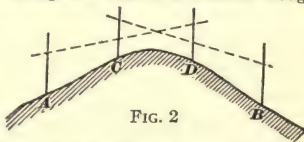


Fig. 2

Having set two poles at $A$ and $B$, two flagmen with poles station themselves at $C$ and $D$, approximately in line with $A$ and $B$, and in such positions that the poles at $B$ and $D$ are visible from $C$, and those at $C$ and $A$ are visible from $D$. The flagman at $C$ lines in the pole at $D$ between $C$ and $B$, and then the flagman at $D$ lines in that at $C$ between $D$ and $A$. Then the flagman at $C$ again lines in that at $D$, and so on, until $C$ is in the line between $D$ and $A$ at the same time that $D$ is in line between $C$ and $B$. The points $C$ and $D$ will then be in line with $A$ and $B$.
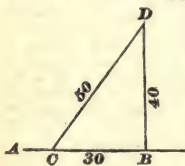


Fig. 3

**3. To Erect a Perpendicular to a Line at a Given Point.**—Let it be required to erect a perpendicular to the line $AB$ at the point $B$, Fig. 3. A triangle whose sides are in the proportion of 3, 4, and 5 is a right triangle, the longest side being the hypotenuse; for $5^2 = 4^2 + 3^2$. The following method is based on this principle: Lay off on $BA$ a distance $BC$ of 30 ft. (or li.). Fix one end of the chain at one of the extremities as $C$, and the end of the nineties link at the other extremity $B$. Hold the end of the fiftieth link

and draw the chain until both parts are taut. The point $D$ where the end of the fiftieth link is held will then be a point in the perpendicular, and the direction of the latter will therefore be $BD$.

The distance $BC$ may be any other convenient multiple of 3. In general, if $BC$ is denoted by $3a$, $BD$ must be $4a$, and $CD$ must be $5a$. Thus, $BC$ may be made equal to 21 $(=3\times7)$ li.; in which case $BD$ must be $4\times7=28$, and $CD$ must be $5\times7=35$ li. As $35+28=63$, one end of the chain must be fixed at one of the extremities of $BC$, the end of the sixty-third link at the other extremity, and the chain pulled from the end of the thirty-fifth link until both parts are taut.



FIG. 4

**4. To Determine the Angle Between Two Lines.**—Let $AD$ and $AE$, Fig. 4, be two lines on the ground. To determine the angle $DAE$, measure off from $A$ on $AD$ and $AE$ equal distances $AB$ and $AC$. Measure the distance $BC$. Then the angle $DAE$ is calculated from the relation
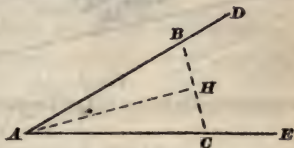
$$\sin \tfrac{1}{2}DAE = \frac{\tfrac{1}{2}BC}{AB}$$

EXAMPLE.—If $AB$ and $AC$ are each 100 ft. and $BC$ is 57.6 ft., what is the value of the angle $DAE$?

SOLUTION.—Substituting the values of $BC$ and $AB$ in the formula,

$$\sin \tfrac{1}{2}DAE = \frac{\tfrac{1}{2}\times57.6}{100} = .28800;$$

whence, $\tfrac{1}{2}DAE = 16°\ 44'$, nearly; and, therefore, $DAE = 16°\ 44' \times 2 = 33°\ 28'$.

**5. To Find the Distance of an Inaccessible Point.**—*Case I.*—Let it be required to determine the distance from the point $B$ to an inaccessible point $P$,



FIG. 5

Fig. 5. Measure $BC$ in any convenient direction and run a line $A'D'$ parallel to $BC$. Measure $AD$, the distance between the points where the lines $PB$ and $PC$ intersect $A'D'$. Measure also $AB$. Then,

$$BP = \frac{AB \times BC}{AD - BC}$$

EXAMPLE.—If, in Fig. 5, $BC = 100$ ft., $AB = 52.4$ ft., and $AD = 124.2$ ft., what is the distance $BP$?

SOLUTION.—Substituting these values in the formula,

$$BP = \frac{52.4 \times 100}{124.2 - 100} = 216.5 \text{ ft.}$$

*Case II.*—Measure a horizontal base line $AB$, Fig. 6, and take the angles formed by the lines $BAC$ and $ABC$, which gives two angles and the included side. Assuming the angle $A$ to be $60°$; the angle $B$ $50°$; and the side $AB = 500$ ft., angle $C = 180° - (60° + 50°) = 70°$.

Then, $\sin 70° : AB = \sin A : BC$, and $\sin 70° : AB = \sin B : AC$; or, $.939693 : 500 = .866025 : BC$, or $460.8+$, and $.939693 : 500 = .766044 : AC$, or $407.6+$.

By logarithms:



FIG. 6

$$\begin{array}{ll}
\text{Log } 500 = & 2.698970 \\
\text{Log sin } 60° = & 9.937531 \\
\hline
& 12.636501 \\
\text{Log sin } 70° = & 9.972986 \\
\hline
& 2.663515 = \text{log of } 460.8+.
\end{array}$$

$$\begin{array}{ll}
\text{Log } 500 = & 2.698970 \\
\text{Log sin } 50° = & 9.884254 \\
\hline
& 12.583224 \\
\text{Log sin } 70° = & 9.972986 \\
\hline
& 2.610238 = \text{log of } 407.6+.
\end{array}$$
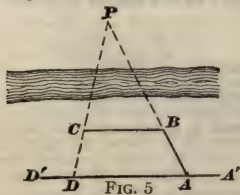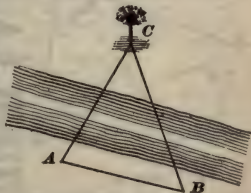
**6. To Determine the Distance Between Two Points Invisible From Each Other or Separated by an Impassable Barrier.**—*Case I.*—Let it be required to find the distance between two points $A$ and $B$, Fig. 7, that are invisible from each other. First run a random line $AD'$ in such a manner that it will pass as near $B$ as can be estimated. From $B$ drop a perpendicular $BD$ on $AD'$ and compute the required distance $AB$ by the formula
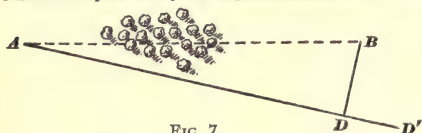
$$AB = \sqrt{AD^2 + BD^2}$$

EXAMPLE.—If, in Fig. 7, the distance $AD$ is 206.1 ft. and the distance $BD$ is 35.1 ft., what is the distance from $A$ to $B$?

SOLUTION.—Here $AD = 206.1$ and $BD = 35.1$; therefore, substituting in the formula, $AB = \sqrt{206.1^2 + 35.1^2} = 209.1$ ft.

*Case II.*—Select any convenient station, as $C$, Fig. 8, measure the lines $CA$ and $CB$, and the angle included between these sides, so as to obtain two sides and the included angle. Assuming the angle $C$ to be 60°, the side $CA$, 600 ft., and the side $CB$, 500 ft., the following formula is obtained:

$$CA + CB : CA - CB = \tan \frac{A+B}{2} : \tan \frac{B-A}{2}.$$

Then, $\dfrac{A+B}{2} = \dfrac{180° - 60°}{2}$, or 60°.

Then, $1,100 : 100 = \tan 60° : \tan \dfrac{B-A}{2}$; or, $1,100 : 100 = 1.732050 : .157459$,

or tangent of $\dfrac{B-A}{2}$, or 8° 57'.

Then, $\qquad 60° + 8° 57' = 68° 57'$, or angle $B$,
and $\qquad 60° - 8° 57' = 51° 03'$, or angle $A$,

Having found the angles, find the third side by the same method as case II, of problem 5.

The foregoing formula, worked out by logarithms, is as follows:

$$\begin{aligned}
\text{Log } 100 &= 2.000000 \\
\text{Log tan } 60° &= 10.238561 \\
&\overline{\phantom{0}12.238561} \\
\text{Log } 1,100 &= 3.041393 \\
&\overline{\phantom{0}9.197168} = \text{log tan of } \frac{B-A}{2}, \text{ or 8° 57'.}
\end{aligned}$$

Then,
$\qquad 60° + 8° 57' = 68° 57'$, or angle $B$,
and $\qquad 60° - 8° 57' = 51° 03'$, or angle $A$

NOTE.—The greater angle is always opposite the greater side.

**7. To Find the Distance Between Two Inaccessible Objects When Points Can Be Found From Which Both Objects May Be Seen.**—Let $AB$, Fig. 9, be the line, the ends $A$ and $B$ of which are inaccessible. Select two points $P$ and $Q$ from which both ends of the line can be seen, and at a distance from each other of about 300 or 400 ft. Measure the line $PQ$, and the angles $K$, $L$, $M$, and $N$.
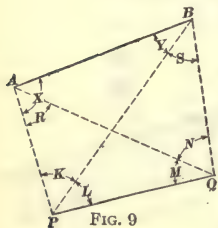
Then, from triangle $APQ$,

$$AP = \frac{PQ \sin M}{\sin R}$$

in which $R = 180° - (K+L) - M$.
From triangle $BPQ$,

$$BP = \frac{PQ \sin (M+N)}{\sin S}$$

in which $S = 180° - L - (M+N)$.

FIG. 7

FIG. 8

FIG. 9

Then, from triangle $ABP$,

$$\tan \tfrac{1}{2}(X-Y) = \frac{BP-AP}{BP+AP} \cot \tfrac{1}{2}K$$

and

$$AB = \frac{(BP-AP)\cos \tfrac{1}{2}K}{\sin \tfrac{1}{2}(X-Y)}$$

EXAMPLE.—If, in Fig. 9, the distance $PQ$ is 400 ft., and the angles, as measured, are $K = 37°\ 10'$, $L = 36°\ 30'$, $M = 52°\ 15'$, $N = 32°\ 55'$, what is the distance $AB$?

SOLUTION.—In the triangle $APQ$, $R = 180° - (37°\ 10' + 36°\ 30' + 52°\ 15')$ $= 54°\ 05'$, and

$$AP = \frac{400 \sin 52°\ 15'}{\sin 54°\ 05'} = 390.53 \text{ ft.}$$

In the triangle $BPQ$, $S = 180° - (36°\ 30' + 52°\ 15' + 32°\ 55') = 58°\ 20'$, $M+N = 52°\ 15' + 32°\ 55' = 85°\ 10'$, and

$$BP = \frac{400 \sin 85°\ 10'}{\sin 58°\ 20'} = 468.30 \text{ ft.}$$

Also, $K = 37°\ 10'$, $\tfrac{1}{2}K = 18°\ 35'$, and

$$\tan \tfrac{1}{2}(X-Y) = \frac{(468.30-390.53)}{468.30+390.53} \cot 18°\ 35'$$

whence, $\tfrac{1}{2}(X-Y) = 15°\ 04'$, and therefore

$$AB = \frac{(468.30-390.53)\cos 18°\ 35'}{\sin 15°\ 04'} = 283.58 \text{ ft.}$$

**8. To Determine the Angle Between Two Lines AB and CD, Whose Point of Intersection P is Inaccessible, Also, the Distances BP and DP.**—This problem is of frequent occurrence in railroad work, the two given lines being the center lines of two straight tracks that are to be connected by a curve Measure the distance $BD$, Fig. 10, and the angles $K$ and $L$. Then, $M = 180° - (K+L)$, $I = K+L$, $BP = \dfrac{BD \sin L}{\sin M}$, and $DP = \dfrac{BD \sin K}{\sin M}$.

FIG. 10

**9. Survey of a Closed Field.**—If a closed field is to be surveyed without the aid of an angle-measuring instrument, the area may be divided into triangles by means of diagonals, which are measured on the ground. The area of each triangle may then be determined by the formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ in which $a$, $b$, and $c$ represent the three sides and $s$ represents one-half of their sum, or $\dfrac{a+b+c}{2}$.
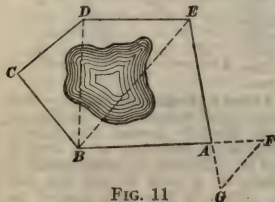
FIG. 11

When obstacles make it impossible to measure directly the diagonals of a field, as, for instance, the diagonal $BE$, Fig. 11, a tie line $FG$ parallel to $BE$ is run and measured. Then,

$$BE = \frac{GF \times AB}{AF}$$

To run the line $FG$, produce $BA$ and select any convenient point $F$ and measure $AF$. Then produce $EA$ and locate $G$ from the relation

$$AG = \frac{AF \times AE}{AB}$$

EXAMPLE.—In Fig. 11, let the lengths of the sides be as follows: $AB = 320$ ft., $BC = 217$ ft., $CD = 196$ ft., $DE = 285$ ft., and $EA = 304$ ft. It is required to calculate the length of the diagonal $BE$ by means of a tie-line.

SOLUTION.—Let the line $BA$ be prolonged 100 ft. beyond $A$; that is, make $AF = 100$ ft. Then $AG$ must be equal to $\dfrac{AF \times AE}{AB} = \dfrac{100 \times 304}{320} = 95$ ft. Let the length of $GF$, as found by measurement, be 125 ft. Then,

$$BE = \frac{GF \times AB}{AF} = \frac{125 \times 320}{100} = 400 \text{ ft}$$
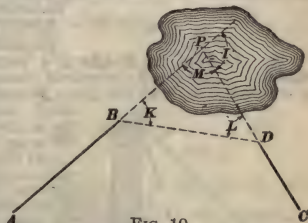
**10. To Determine the Height of a Vertical Object Standing on a Horizontal Plane.**—Measure from the foot of the object any convenient horizontal distance $AB$, Fig. 12; at the point $A$, take the angle of elevation $BAC$. Then, as $B$ is known to be a right angle, two angles and the included side of a triangle are obtained. Assuming that the line $AB$ is 300 ft. and the angle $BAC = 40°$, the angle $C = 180° - (90° + 40°) = 50°$. Then, sin $C : AB = $ sin $A : BC$, or $.766044 : 300 = .642788 : BC$, or $BC = 251.73+$ ft. Or, by logarithms:
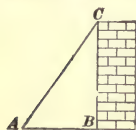
$$\text{Log } 300 = 2.477121$$
$$\text{Log sin } 40° = 9.808067$$
$$\overline{12.285188}$$
$$\text{Log sin } 50° = 9.884254$$
$$\overline{2.400934 \text{ or log of } 251.73+ \text{ ft.}}$$

**11. To Find the Distance of a Vertical Object Whose Height is Known.** At a point $A$, Fig. 13, take the angle of elevation to the top of the object. Knowing that the angle $B$ is a right angle, the angles $B$ and $A$ and the side $BC$ are known. Assuming that the side $BC = 200$ ft. and the angle $A = 30°$, a triangle is formed as follows: Angle $A = 30°$, $B = 90°$, $C = 60°$, and the side $BC = 200$ ft. Then sin $A : BC = $ sin $C : AB$, or $.5 : 200 = .866025 : AB$, or $AB = 346.41$ ft. By logarithms:

$$\text{Log } 200 = 2.301030$$
$$\text{Log sin } 60° = 9.937531$$
$$\overline{12.238561}$$
$$\text{Log sin } 30° = 9.698970$$
$$\overline{2.539591 \text{ or log of } 346.41 \text{ ft.}}$$
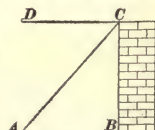
**12. To Find the Height of a Vertical Object Standing Upon an Inclined Plane.**—Measure any convenient distance $DC$, Fig. 14, on a line from the foot of the object, and, at the point $D$, measure the angles of elevation $EDA$ and $EDB$, and the angle $CDB$; also at $C$ measure the angle $BCD$. In the triangle $BDC$, the side $BD$ may be calculated for the angles at $D$ and $C$ and the side $CD$ are known. Then, in the right-angled triangle $BED$ the sides $BE$ and $ED$ may be calculated, as the side $BD$ and the angle at $D$ are known. Next, in the right-angled triangle $AED$, the side $AE$ may be calculated, for the side $ED$ and the angle at $D$ are known. Finally, the height of the object may be found by subtracting the length $AE$ from the length $BE$.

**13. To Find the Height of an Inaccessible Object Above a Horizontal Plane.**—*First Method.*—Measure any convenient horizontal line $AB$, Fig. 15, directly toward the object, and take the angles of elevation at $A$ and $B$. In the triangle $ABC$, the side $AC$ may be calculated as the angles at $A$ and $B$ and the side $AB$ are known. Then, in the right-angled triangle $CDA$, the side $CD$, which is the height of the object, may be calculated as the angle at $A$ and the side $AC$ are known.

*Second Method.*—If it is not convenient to measure a horizontal base line toward the object, measure any line $AB$, Fig. 16, and also measure the horizontal angles $BAD$, $ABD$, and the angle of elevation $DBC$. Then, by means of the two triangles $ABD$ and $CBD$, the height $CD$ can be found. Then, with the line $AB$ and the angles $BAD$ and $ABD$ known, the third angle is readily found, and the side $BD$ can be found. Then, in the triangle $BDC$, the angle $B$ is known, by measurement, $D = 90°$, and the side $BD$ is known. Then, the side $CD$, or the vertical height, can be found by preceding methods.


Fig. 12


Fig. 13


Fig. 14


Fig. 15


Fig. 16
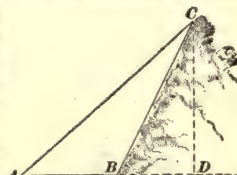
# MECHANICS

## ELEMENTS OF MECHANICS

### GENERAL LAW

All machinery, however complicated, is merely a combination of six elementary forms, viz.: the *lever*, the *wheel and axle*, the *pulley*, the *inclined plane* the *wedge*, and the *screw*; and these six can be still further reduced to the lever and the inclined plane. They are termed *mechanical powers*, but they do not produce force; they are only methods of applying and directing it. The law of all mechanics is:

Law.—*The power multiplied by the distance through which it moves is equal to the weight multiplied by the distance through which it moves.*

Thus, 20 lb. of power moving through 5 ft. = 100 lb. of weight moving through 1 ft. In the following discussion friction is not considered.

### LEVERS

There are three classes of levers: (1) power at one end, weight at the other, and fulcrum between, as shown in Fig. 1; (2) power at one end, fulcrum at
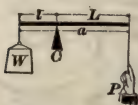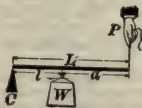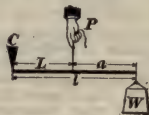


FIG. 1         FIG. 2         FIG. 3

the other, and weight between, as shown in Fig. 2; (3) weight at one end, fulcrum at the other, and power between, as shown in Fig. 3.

The handle of a blacksmith's bellows is a lever of the first class; the hand is the power and the bellows the weight, with the pivot between as the fulcrum. A crowbar used for prying down top rock is a lever of the second class; the hand is the power, the rock to be barred down the weight, and the point in the roof against which the bar presses is the fulcrum. The treadle of a grindstone is a lever of the third class; the foot is the power, the hinge at the back of the foot is the fulcrum, and the moving of the machinery is the weight. A lever is in equilibrium when the arms balance each other. The distances through which the power and the weight move depend on the comparative length of the arms.

Let

$P$ = power;
$W$ = weight;
$L$ = power's distance from fulcrum $C$;
$l$ = weight's distance from fulcrum $C$;
$a$ = distance between power and weight;

Arranging these terms according to the law of mechanics,

$$PL = Wl, \text{ or } P : W = l : L$$

$$P = \frac{Wl}{L} \qquad W = \frac{PL}{l}$$

In levers of the first class, $a = L + l$;

whence $\qquad L = \dfrac{Wa}{W+P} \qquad\qquad l = \dfrac{Pa}{W+P}$

In levers of the second class, $a = L - l$;

whence $\qquad L = \dfrac{Wa}{W-P} \qquad\qquad l = \dfrac{Pa}{W-P}$

In levers of the third class, $a = l - L$;

whence $\qquad L = \dfrac{Wa}{P-W} \qquad\qquad l = \dfrac{Pa}{P-W}$

In first- and second-class levers, as ordinarily used, power is gained and time is lost; in the third class, power is lost and time is gained.

EXAMPLE.—Having a weight of 2,000 lb. to lift with a lever, the short end of which is 2 ft. from the fulcrum and the long end 10 ft., how much power will be required?

SOLUTION.—Applying the formula $L : l = W : P$, $10 : 2 = 2,000 : P$; whence $P = (2,000 \times 2) \div 10 = 400$ lb.

The *compound lever*, Fig. 4, consists of several levers so constructed that the short arm of the first acts on the long arm of the second, and so on to the last. If the distance from $A$ to the fulcrum is four times the distance from the
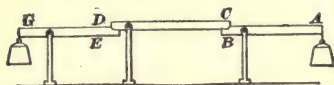
FIG. 4

fulcrum to $B$, then a power of 5 lb. at $A$ will lift 20 lb. at $B$. If the arms of the second lever are of the same comparative length, the 20-lb. power obtained at $B$ will exert a pressure of 80 lb. on $E$; and if the third lever has the same comparative lengths, this 80 lb. at $E$ will lift 320 lb. at $G$. Thus, a power of 5 lb. at $A$ will balance a weight of 320 lb. at $G$. But, in order to raise the weight 1 ft., the power must pass through $320 \div 5 = 64$ ft.

## WHEEL AND AXLE

The *wheel and axle*, Fig. 5, is a modification of the lever. The ordinary windlass is a common form. The power is applied to the handle, the bucket is the weight, and the axis of the windlass is the fulcrum. The long arm of the lever is the handle, and the short arm is the radius of the axle. Thus, $F$ is the fulcrum, $Fc$ the long arm, and $Fb$ the short arm. The wheel and axle has the advantage that it is a kind of perpetual lever. It is not necessary to prop up the weight and readjust the lever, but both arms work continuously.

FIG. 5

By turning the handle or wheel around once, the rope will be wound once around the axle, and the weight will be lifted that distance. Applying the law of mechanics, power × circumference of wheel = weight × circumference of axle; or, as the circumferences of circles are proportional to their radii, $P : W = r : R$; whence $PR = Wr$. Therefore,

$$P = \frac{Wr}{R} \qquad R = \frac{Wr}{P}$$

$$W = \frac{RP}{r} \qquad r = \frac{RP}{W}$$
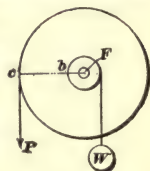
A *train*, Fig. 6, consists of a series of wheels and axles that act on one another on the principle of a compound lever. The *driver* is the wheel to which power is applied. The *driven wheel* or *follower*, is the one that receives motion from the *driver*. The *pinion* is the small gear-wheel on the axle.
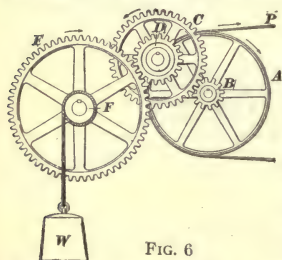
FIG. 6

If the diameter of the wheel $A$ is 16 in., and of the pinion $B$ 4 in., a pull of 1 lb. applied at $P$ will exert a force of 4 lb. on the wheel $C$; if the diameter of $C$ is 6 in., and of $D$ 3 in., a force of 4 lb. on $C$ will exert a force of 8 lb. on $E$. If $E$ is 16 in. in diameter, and $F$ 4 in., a force of 8 lb. on $E$ will raise a weight of 32 lb. on $F$. In order, however, to lift this amount, according to the principle already named, the weight will only pass through one thirty-second of the distance of
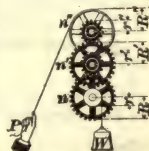
FIG. 7

the power. Thus, power is gained and speed lost. To reverse this, power may be applied to the axle, and, with a correspondingly heavy power, speed gained. Referring to Fig. 7, applying the law of mechanics,

$$P = \frac{Wrr'r''}{RR'R''} \qquad W = \frac{PRR'R''}{rr'r''}$$

$$n : n'' = r'r'' : RR'$$
$$v : v' = rr'r'' : RR'R''$$

in which      $n, n', n''$ = number of revolutions;
     $v, v'$ = velocity or speed of rotation;
     $r, r', r''$, etc. = radii of pinions;
     $R, R', R''$, etc. = radii of wheels.

## INCLINED PLANE

In Fig. 8, the power must descend a distance equal to $AC$ in order to elevate the weight to the height $BC$; hence, $P \times$ length of inclined plane $= W \times$ height of inclined plane, or $P : W =$ height of inclined plane : length of inclined plane; or,

$$P = \frac{Wh}{l} \qquad W = \frac{Pl}{h} = \frac{P}{\sin a}$$



FIG. 8

**To Find Weight Required to Balance Any Weight on Any Inclined Plane.**—Multiply the given weight by the sine of the angle of inclination. Thus, to find the weight required to balance loaded car weighing 2,000 lb. on a plane pitching 18°, multiply 2,000 by the sine of 18°, or $2,000 \times .309017 = 618.034$ lb.

Or, if the length of the plane and the vertical height are given, multiply the load by the quotient of the vertical height divided by the length. Thus, if a plane between two levels is 300 ft. long and rises 92.7 ft., and the load is 2,000 lb., the balancing weight is found as follows: $2,000 \times \frac{92.7}{300} = 618$ lb.

*Case I.*—To find the horsepower required to hoist a given load up an inclined plane in a given time, use the formula

$$\frac{\text{(Load, in lb.} + \text{weight of hoisting rope, in lb.)} \times \text{vertical height load is raised, in ft.}}{33,000 \times \text{time of hoisting, in minutes}}$$

EXAMPLE.—Find the horsepower required to raise, in 3 min., a car weighing 1 T. and containing 1 T. of material up an inclined plane 1,000 ft. long and pitching 30°, if the rope weighs 1,500 lb.

SOLUTION.—The total load equals car + contents + rope = 2,000 + 2,000 + 1,500 = 5,500 lb. The vertical height through which the load is hoisted equals $1,000 \times \sin 30° = 1,000 \times .5 = 500$ ft.; therefore, H. P. = $\frac{5,500 \times 500}{33,000 \times 3} = 27.7$.

*Case II.*—When the power acts parallel to the base, use the formula
$$W \times \text{height of inclined plane} = P \times \text{length of base.}$$

These rules are theoretically correct, but in practice an allowance of about 30% must be made for friction and contingencies.

## SCREW

The *screw* consists of an inclined plane wound around a cylinder. The inclined plane forms the thread, and the cylinder, the body. It works in a nut that is fitted with reverse threads to move on the thread of the screw. The nut may run on the screw, or the screw in the nut. The power may be applied to either, as desired, by means of a wrench or a lever.

When the power is applied at the end of a lever, it describes a circle of which the lever is the radius $r$. The distance through which the power passes is the circumference of the circle; and the height to which the weight is lifted at each revolution of the screw is the distance between two of the threads, called the *pitch*, $p$. Therefore, $P \times$ circumference of circle $= W \times$ pitch; or $P : W = p : 2\pi r$; whence,

$$P = \frac{Wp}{2\pi r} \qquad W = \frac{2\pi r P}{p}$$

The power of the screw may be increased by lengthening the lever or by diminishing the distance between the threads.

EXAMPLE.—How great a weight can be raised by a force of 40 lb. applied at the end of a wrench 14 in. long, using a screw with 5 threads per in.?

SOLUTION.—Substituting in the formula $W \times \frac{1}{5} = 40 \times 28 \times 3.1416$; whence $W = 17,593$ lb.

## WEDGE

The *wedge* usually consists of two inclined planes placed back to back, as shown in Fig. 9. In theory, the same formula applies to the wedge as to the inclined plane, Case II.

$$P : W = \text{thickness of wedge : length of wedge.}$$
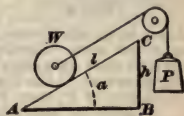
Friction, in the other mechanical powers, materially diminishes their efficiency; in this it is essential, since, without it, after each blow the wedge would fly back and the whole effect be lost. Again, in the others the power is applied as a steady force; in this it is a sudden blow, and is equal to the momentum of the hammer.



FIG. 9

## PULLEY
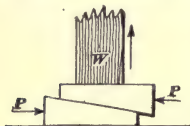
The *pulley* is simply another form of the lever that turns about a fixed axis or fulcrum. With a single fixed pulley, shown in Fig. 10, there can be no gain of power or speed, as the force $P$ must pull down as much as the weight $W$, and both move with the same velocity. It is simply a lever of the first class with equal arms, and is used to change the direction of the force. If $v$ = velocity of $W$; $v'$ = velocity of $P$; then $P = W$ and $v = v'$.



FIG. 10      FIG. 11

A form of the single pulley, where it moves with the weight, is shown in Fig. 11. In this, one-half of the weight is sustained by the hook, and the other half by the power. As the power is only one-half the weight, it must move through twice the space; in other words, by taking twice the time, twice as much can be raised. Here power is gained and time lost; therefore, $P = \frac{1}{2} W$ and $v' = 2v$.

**Combinations of Pulleys.**—(1) In Fig. 12, the weight $W$ is sustained by three cords, each of which is stretched by a tension equal to the $P$; hence,
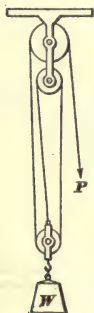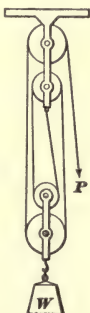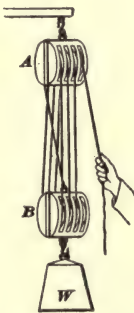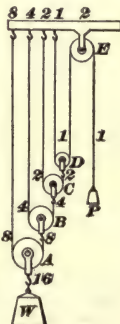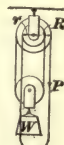


FIG. 12      FIG. 13      FIG. 14      FIG. 15      FIG. 16

1 lb. of power will balance 3 lb. of weight. (2) In Fig. 13, a power of 1 lb. will in the same manner sustain a weight of 4 lb., and must descend 4 in. to raise the weight 1 in. (3) Fig. 14 represents the ordinary tackle block used by mechanics, which can be calculated by the following general rule:

**Rule.**—*In any combination of pulleys where one continuous rope is used, a load on the free end will balance a weight on the movable block as many times as great as the load on the free end as there are parts of the rope supporting the load, not counting the free end.*

(4) In the cord marked *1*, Fig. 15, each part has a tension equal to the power $P$; and in the cord marked *2*, each part has a tension equal to *2 P*, and so on with the other cords. The sum of the tensions acting on the weight $W$ is 16; hence, $W = 16\,P$. If $n$ = number of pulleys,

$$P = \frac{W}{2^n} \qquad W = 2^n P$$

**Differential Pulley.**—In the differential pulley, shown in Fig. 16, $W = \dfrac{2\,P\,R}{R-r}$.

In all pulley combinations, nearly one-half the effective force is lost by friction.

## FALLING BODIES

When the center of gravity of a moving body passes over equal distances in equal intervals of time, the body has a *uniform motion;* otherwise, the motion is *variable.* The *velocity* in a uniform motion is constant and is equal to the distance traversed by the center of gravity of the body in a unit of time, as feet per second, miles per hour, etc. When, in a variable motion, the velocity increases or decreases uniformly with the time, the motion is designated, respectively, as *uniformly accelerated* or *uniformly retarded,* and, the rate of increase or decrease is called *acceleration* or *retardation,* being equal to the amount that the velocity increases or decreases in a unit of time. A body falling under the action of gravity is a case of uniformly accelerated motion, the acceleration being equal to 32.16 ft. per sec. and being usually denoted by $g$.

Let $\quad\quad\quad t$ = number of seconds that body falls;

$\quad\quad\quad\quad\quad v$ = velocity, in feet per second, at end of time $t$;

$\quad\quad\quad\quad\quad h$ = distance that body falls during time $t$.

Then, $\quad\quad\quad v = gt = \dfrac{2h}{t} = \sqrt{2gh} = 8.02\sqrt{h}$

$$h = \frac{vt}{2} = \frac{gt^2}{2} = \frac{v^2}{2g} = .015547v^2$$

$$t = \frac{v}{g} = \frac{2h}{v} = \sqrt{\frac{2h}{g}} = .24938\sqrt{h}$$

## WORK

*Work* is the overcoming of resistance through a distance. The unit of work is the *foot-pound;* that is, it equals 1 lb. raised vertically 1 ft. The amount of work done is equal to the resistance, in pounds, multiplied by the distance, in feet, through which it is overcome. If a body is lifted, the resistance is the weight, or the overcoming of the attraction of gravity, the work done being the weight $W$, in pounds, multiplied by the height $h$ of the lift, in feet, or $Wh$ ft.-lb.

*Power* is the amount of work performed in a unit of time. One horsepower is 550 ft.-lb. of work in 1 sec., 33,000 ft.-lb. in 1 min., 1,980,000 ft.-lb. in 1 hr. In the metric system, 1 H. P. is 75 meter-kilograms per second, usually written 75 m. Kg.-sec.

*Kinetic energy* is the capacity of a moving body to perform work. If the moving body has a weight $W$ and a velocity $v$, the work that it is capable of doing in being brought to rest is $\dfrac{Wv^2}{2g}$. A body falling through a height of $h$ ft. acquires during its fall a velocity of $v = \sqrt{2gh}$; its kinetic energy is therefore,

$$\frac{W(\sqrt{2gh})^2}{2g} = Wh$$

EXAMPLE 1.—What is the horsepower of a stream of water discharging 12 cu. ft. per sec. through a height of 125 ft.?

SOLUTION.—The kinetic energy per second, is $62.5 \times 12 \times 125$ ft.-lb., 62.5 being the weight of 1 cu. ft. of water. The horsepower is, therefore,

$$\frac{62.5 \times 12 \times 125}{550} = 170.5$$

EXAMPLE 2.—What is the kinetic energy per second of a jet of water whose area of cross-section is .1 sq. ft. and whose velocity is 10 ft. per sec.?

SOLUTION.—In this case, $W = 62.5 \times .1 \times 10 = 62.5$ lb. The kinetic energy is therefore, $\quad\quad \dfrac{62.5 \times 10^2}{2g} = \dfrac{6,250}{64.32} = 97.2$ ft.-lb. per sec.

## COMPOSITION AND RESOLUTION OF FORCES

The *resultant* of two or several forces acting on a body is the single force that, if acting alone, would produce the same effect as the several forces combined. The latter forces are called *components* with respect to the resultant.

*Composition of forces* is the process of finding the resultant when the components are known, and the converse process of finding the components when the resultant is given, is called *resolution of forces.*

**Parallelogram of Forces.**—If two forces, as $F_1$ and $F_2$, Fig. 1, are represented in magnitude and direction by two lines, as $OA$ and $OB$, their resultant $R$



FIG. 1

will be represented in magnitude and direction by the diagonal $OC$ of the parallelogram $OACB$ which is constructed by drawing $BC$ and $AC$ parallel to $OA$ and $OB$, respectively, and joining the intersection $C$ with $O$.
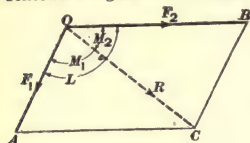
The resultant $R$ can also be determined analytically: its magnitude by the formula $R = \sqrt{F_1{}^2 + F_2{}^2 - 2F_1F_2 \cos L}$, and the angles $M_1$ and $M_2$ that $R$ makes with $F_1$ and $F_2$, respectively, may be found by the formulas,

$$\sin M_1 = \frac{F_2 \sin L}{R}$$

and

$$\sin M_2 = \frac{F_1 \sin L}{R}$$

For *rectangular components*, $L = 90°$. The formulas then become:

$$R = \sqrt{F_1^2 + F_2^2}$$

$$\sin M_1 = \frac{F_2}{R}$$

$$\sin M_2 = \frac{F_1}{R}$$



FIG. 2

**Resolution of Forces.**—A given force may have an innumerable number of combinations of components. The problem is, however, determinate when the directions of the components are given. Let $OC$, Fig. 2, represent in magnitude and direction the force $R$ acting at $O$, and let it be required to find its components in the directions $OX_2$ and $OX_1$. Draw from $C$, lines parallel to these directions meeting $OX_1$ at $A$ and $OX_2$ at $B$. Then, $OA$ and $OB$ are the required components $F_1$ and $F_2$. They may also be determined analytically by the formulas,

$$F_1 = \frac{R \sin M_2}{\sin (M_1 + M_2)}$$

and

$$F_2 = \frac{R \sin M_1}{\sin (M_1 + M_2)}$$

When $F_1$ and $F_2$ are perpendicular to each other, $M_1 + M_2 = 90°$ and

$$F_1 = R \sin M_2$$
$$F_2 = R \sin M_1$$

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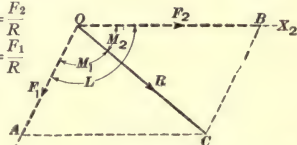
# MOMENTS OF FORCES



The *moment* of a force about a point is the product obtained by multiplying the magnitude of the force by the perpendicular distance from the point to the line of action of the force. In the accompanying figure, the moment of $F$ about the point $C$ is $Fp$; and about the point $C_1$ it is $Fp_1$.

The point to which a moment is referred, or about which a moment is taken is called the *center of moments*, or *origin of moments*. The perpendicular $p$ or $p_1$ from the origin of moments on the line of action of the force is called the *lever arm* or simply the *arm*, of the force with respect to the origin.

A moment is expressed in foot-pounds, inch-tons, etc., according to the units to which the force and its arm are referred.

The moment is either positive or negative, depending on the direction in which the force tends to cause rotation. It is positive for clockwise motion, and negative for counter-clockwise motion. Thus, the moment of $F$ about $C$ is positive and the moment about $C_1$ is negative, because, if the arms $p$ and $p_1$ were bars, the force would tend to rotate $p$ in a clockwise direction, and $p_1$ in a counter-clockwise direction.

# CENTER OF GRAVITY

The *center of gravity* of a figure or a body is that point upon which the figure or the body will balance no matter in what position it may be placed, provided it is acted upon by no other force than gravity.

If a plane figure is alike, or symmetrical, on both sides of a center line, the latter line is termed an *axis of symmetry*, and the center of gravity lies in this line. If the figure is symmetrical about any other axis, the intersection of the two axes will be the center of gravity of the section; thus, the center of gravity of a *parallelogram* is at the intersection of the diagonals and that of a *circle* or an *ellipse* is at the geometrical center of the figure. The center of gravity of a *triangle* lies on a line drawn from a vertex to the middle point of the opposite side, and at a distance from that side equal to one-third the length of the line; or it is at the intersection of lines drawn from the vertexes to the middle points of the opposite sides.



FIG. 1

To find the center of gravity of a *trapezoid*, Fig. 1, lay off $BF = DC$ and $DE = AB$; the center of gravity is at the intersection of $EF$ with $M_1 M_2$, the line joining the middle points of the parallel sides. $GM_1$ can also be determined by the formula

$$GM_1 = \frac{m(b_1 + 2b_2)}{3(b_1 + b_2)}$$
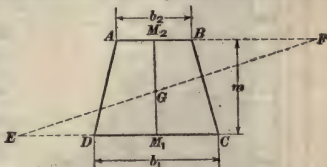
The center of gravity of any *quadrilateral* may be determined as follows: First divide it, with a diagonal, into two triangles, and join with a straight line the centers of gravity of the two triangles; then, with the second diagonal, divide the figure into two other triangles and join the centers of gravity of these triangles with a straight line. The center of gravity of the quadrilateral is at the intersection of the lines joining the centers of gravity of the two sets of triangles.

For an *arc of a circle*, the center of gravity lies on the radius drawn to the middle point of the arc (an axis of symmetry) and at a distance from the center equal to the length of the chord multiplied by the radius and divided by the length of the arc.

For a *semicircle*, the distance from the center $= \frac{2r}{\pi} = .6366\,r$, when $r =$ radius.

For the area included in a *half circle*, the distance of the center of gravity from the center is

$$\frac{4r}{3\pi} = .4244\,r$$

For a *circular sector*, the distance of the center of gravity from the center equals two-thirds of the length of the chord multiplied by the radius and divided by the length of the arc.

For a *circular segment*, let $A$ be its area and $C$ the length of its chord; then the distance of the center of gravity from the center of the circle is equal to $\frac{C^3}{12A}$.

The center of gravity of any *irregular plane figure* can be determined by applying the following principle: The static moment of any plane figure with regard to a line in its plane—that is, the product of its area $A$ by the distance $D$ of its center of gravity from that line—is equal to the algebraic sum of the static moments of the separate parts into which the figure may be divided, with regard to the same axis, or

$$AD = a_1d_1 + a_2d_2, \text{ etc.},$$

in which, $a_1$, $a_2$, etc., denote the areas of the subdivided parts of the figure, and $d_1$, $d_2$, etc. are the distances of their respective centers of gravity from the reference line. Solving this equation for the value of $D$,

$$D = \frac{a_1d_1 + a_2d_2 + \text{etc.}}{A}$$

The figure whose center of gravity is required is divided into separate parts whose centers of gravity are easily ascertained, usually into rectangles or triangles. A suitable axis is then assumed with reference to which the expressions $a_1d_1$, $a_2d_2$, etc. are found, and their sum is divided by $A = a_1 + a_2 + \text{etc.}$, the quotient giving $D$. The center of gravity of the whole figure lies, therefore, on a line parallel to the assumed axis and distant $D$ from it. In a similar manner,

another line containing the center of gravity is obtained, the intersection of the two lines giving its exact position.

**EXAMPLE 1.**—Find the center of gravity of the cross-section of the dam shown in Fig. 2.
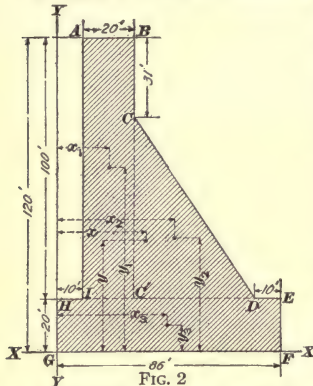


FIG. 2

SOLUTION.—Divide the section into the rectangles $ABC'I$ and $HEFG$ and the triangle $CDC'$, and assume the lines $X-X$ and $Y-Y$ as reference lines. Then,

$$y = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

and

$$x = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

From the illustration, $a_1 = 100 \times 20$ $= 2,000$, $y_1 = 70$, $x_1 = 20$; $a_2 = \dfrac{69 \times 46}{2}$ $= 1,587$, $y_2 = 43$, $x_2 = 45.33$; $a_3 = 86 \times 20$ $= 1,720$, $y_3 = 10$, and $x_3 = 43$. Substituting these values,

$$y = \frac{2,000 \times 70 + 1,587 \times 43 + 1,720 \times 10}{2,000 + 1,587 + 1,720}$$
$$= 42.48$$
$$x = \frac{2,000 \times 20 + 1,587 \times 45.33 + 1,720 \times 43}{5,307}$$
$$= 35.03$$

**EXAMPLE 2.**—Find the center of gravity of the bridge chord section shown in Fig. 3.

SOLUTION.—The center of gravity is on the line $YY$, which is an axis of symmetry. To find the distance $y$, divide the section into angles and plates and take moments about $XX$. The areas and centers of gravity of the angles might be located by the preceding principles or taken from a manufacturer's handbook. They are: For the $4'' \times 4'' \times \frac{1}{2}''$ angle, area $= 3.75$ sq. in. and distance from center of gravity to back of angle $= 1.18$ in.; for the $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ angle, area $= 3.25$ sq. in. and distance from center of gravity to back of angle $= 1.06$ in. Hence, the moment of the bottom angles is $2 \times 3.75 \times 1.18 = 8.85$ and that of the top angles is $2 \times 3.25 \times (15 - 1.06) = 90.61$. The moment of the two web-plates is $2 \times 15 \times \frac{1}{2} \times 7.5 = 112.5$, and that of the cover-plate, $24 \times \frac{1}{2} \times 15.25 = 183.00$. The sum of the moments is, $8.85 + 90.61 + 112.5 + 183.00 = 394.96$. The sum of the areas is $2 \times 3.25 + 2 \times 3.75 + 24 \times \frac{1}{2} + 2 \times 15 \times \frac{1}{2} = 41$ sq. in. Then, $y = 394.96 \div 41 = 9.63$ in.



FIG. 3

**Center of Gravity of Solids.**—For a solid having three axes of symmetry, all perpendicular to each other, like a sphere, cube, right parallelopiped, etc., the point of intersection of these axes is the center of gravity.

For a cone or pyramid, draw a line from the apex to the center of gravity of the base; the required center of gravity is one-fourth the length of this line from the base, measured on the line.

For two bodies, the larger weighing $W$ lb., and the smaller $P$ lb., the center of gravity will lie on the line joining the centers of gravity of the two bodies and at a distance from the larger body equal to $\dfrac{Pa}{P+W}$, where $a$ is the distance between the centers of gravity of the two bodies.

For any number of bodies, first find the center of gravity of two of them, and consider them as one weight whose center of gravity is at the point just found. Find the center of gravity of this combined weight and a third body. So continue for the rest of the bodies, and the last center of gravity will be the center of gravity of the whole system of bodies.

To find the center of gravity mechanically, suspend the object from a point near its edge and mark on it the direction of a plumb-line from that point; then suspend it from another point and again mark the direction of a plumb-line. The intersection of these two lines will be directly over the center of gravity.
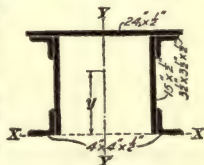
## MOMENT OF INERTIA

The *moment of inertia* of a plane surface about a given axis is the sum of the products obtained by multiplying each of the elementary areas, into which the surface may be conceived to be divided, by the square of its distance from the axis.

### MOMENTS OF INERTIA, ETC.

| Name of Section | | $I$ | $\dfrac{I}{c}$ | $r^2$ |
|---|---|---|---|---|
| Solid circular | | $\dfrac{\pi d^4}{64}$ | $\dfrac{\pi d^3}{32}$ | $\dfrac{d^2}{16}$ |
| Hollow circular | | $\dfrac{\pi(d^4-d_1^4)}{64}$ | $\dfrac{\pi(d^4-d_1^4)}{32d}$ | $\dfrac{d^2+d_1^2}{16}$ |
| Solid square | | $\dfrac{d^4}{12}$ | $\dfrac{d^3}{6}$ | $\dfrac{d^2}{12}$ |
| Hollow square | | $\dfrac{d^4-d_1^4}{12}$ | $\dfrac{d^4-d_1^4}{6d}$ | $\dfrac{d^2+d_1^2}{12}$ |
| Solid rectangular | | $\dfrac{bd^3}{12}$ | $\dfrac{bd^2}{6}$ | $\dfrac{b^2}{12}$ |
| Hollow rectangular | | $\dfrac{bd^3-b_1d_1^3}{12}$ | $\dfrac{bd^3-b_1d_1^3}{6d}$ | $\dfrac{b^3d-b_1^3d_1}{12(bd-b_1d_1)}$ |
| Solid triangular | | $\dfrac{bd^3}{36}$ | $\dfrac{bd^2}{24}$ | $\dfrac{d^2}{18}$ |
| Solid elliptic | | $\dfrac{\pi bd^3}{64}$ | $\dfrac{\pi bd^2}{32}$ | $\dfrac{b^2}{16}$ |
| Hollow elliptic | | $\dfrac{\pi}{64}(bd^3-b_1d_1^3)$ | $\dfrac{\pi(bd^3-b_1d_1^3)}{32d}$ | $\dfrac{b^3d-b_1^3d_1}{16(bd-b_1d_1)}$ |
| I-beam | | $\dfrac{bd^3-b_1d_1^3}{12}$ | $\dfrac{bd^3-b_1d_1^3}{6d}$ | $\dfrac{b^3d-b_1^3d_1}{12(bd-b_1d_1)}$ |
| Cross with equal arms (approximate) | | | | $\dfrac{d^2}{22.5}$ |
| Angle with equal arms (approximate) | | | | $\dfrac{d^2}{25}$ |

The moment of inertia is usually designated by the letter $I$. The value of the moment of inertia used in calculating the strength of beams and columns is usually taken about the neutral axis of the figure, which, with the exception of reinforced-concrete sections, passes through the center of gravity of the figure.

Formulas for the values of $I$ about an axis passing through the center of gravity of the section are given for various forms of sections in the

accompanying table. For any other section, it can be computed by means of the following principles:

**Principle I.**—*The moment of inertia of a section about any axis is equal to the algebraic sum of the moments of inertia about the same axis, of the separate parts of which the figure may be conceived to consist.*
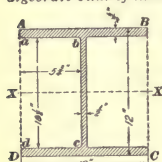


FIG. 1

**Principle II.**—*The moment of inertia of any figure about an axis not passing through the center of gravity, is equal to the moment of inertia about a parallel axis through the center of gravity, plus the product of the entire area of the section by the square of the distance between the two axes.*

EXAMPLE 1.—Find the moment of inertia about the neutral axis



FIG. 2

$XX$ of the Bethlehem **I** column section having dimensions as shown in Fig. 1.

SOLUTION.—Conceive the section to consist of the square $ABCD$ minus twice the rectangle $abcd$. Then, by applying principle I and the formulas of the table for moments of inertia.

$$I = \frac{12^4}{12} - \frac{2 \times 5.75 \times 10.5^3}{12} = 618.6$$

NOTE.—This result can be obtained directly by the **I**-beam formula, given in the same table.

EXAMPLE 2.—Find the moment of inertia of the section shown in Fig. 2 about the neutral axis parallel to the cover-plate.

SOLUTION.—The neutral axis passes through the center of gravity, which has been found to be 9.63 in. from the back of the bottom angles. The distances of the centers of gravity of the subdivisions of this section from the axis $XX$, Fig. 2, are:

For the cover-plate $15.25 - 9.63$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . $= 5.62$
For the web-plates $9.63 - 7.50$ . . . . . . . . . . . . . . . . . . . . . . . . . . . $= 2.13$
For the $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ **L**'s, $15.00 - 1.06 - 9.63$ . . . . . . . . . . . $= 4.31$
For the $4'' \times 4'' \times \frac{1}{2}''$ **L**'s, $9.63 - 1.18$ . . . . . . . . . . . . . . . . . $= 8.45$

The moments of inertia of the respective parts about their own neutral axes parallel to $XX$ are:

For the cover-plate . . . . . . . . . . . . . . . . . . . . . . . $\dfrac{24 \times (\frac{1}{2})^3}{12} =$    $.25$

For the web-plates . . . . . . . . . . . . . . . . . . . . . . . $\dfrac{2 \times \frac{1}{2} \times 15^3}{12} = 281.25$

From a steel manufacturer's handbook, the value of $I$ for a $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ **L** is found to be 3.64; and for a $4'' \times 4'' \times \frac{1}{2}''$ **L** it is 5.56. Applying principle II, the moment of inertia of the entire section is, $I = .25 + 24 \times \frac{1}{2} \times 5.62^2 + 281.25 + 2 \times 15 \times \frac{1}{2} \times 2.13^2 + 2 \times 3.64 + 2 \times 3.25 \times 4.31^2 + 2 \times 5.56 + 2 \times 3.75 \times 8.45^2$
$= 1,403.22$.

## RADIUS OF GYRATION

Let $I$ denote the moment of inertia of any section and $a$ its area; then, the relation between $I$ and $a$ is expressed in the formula, $I = ar^2$, in which $r$ is a constant depending on the shape of the section and is called the *radius of gyration* of the section referred to the same axis as $I$. Then,



FIG. 3

$$r = \sqrt{\frac{I}{a}}.$$
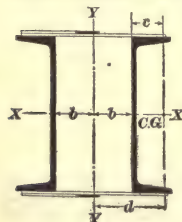
EXAMPLE 1.—What is the radius of gyration of the section shown in Fig. 1 about the axis $XX$?

SOLUTION.—The moment of inertia of this section has been found to be 618.6 and its area is $2 \times 12 \times \frac{3}{4} + 10.5 \times \frac{1}{2} = 23.25$ sq. in. Substituting in the formula,

$$r = \sqrt{\frac{618.6}{23.25}} = 5.16$$

EXAMPLE 2.—Determine the distance $b$ in the strut made up of two latticed channels, as shown in Fig. 3, so that the radii of gyration about the axes $XX$ and $YY$ will be equal.

SOLUTION.—Let $I_x$, $r_x$, $I_y$, $r_y$ be, respectively, the moments of inertia and radii of gyration of a single ⅃ about the axes $XX$ and $YY$; $a$ its area and $CG$, its center of gravity, then, from the figure, $b = d - c$, and $I_x = ar_x^2$; also, $I_y = ar_y^2 + ad^2$. Hence, by the condition of the problem, $ar_x^2 = ar_y^2 + ad^2$, or $r_x^2 = r_y^2 + d^2$. Whence, $d = \sqrt{r_x^2 - r_y^2}$. The values of $r_x$, $r_y$ and $c$ for any ⅃ may be taken from a steel manufacturer's handbook. For instance, for a 15-in. ⅃ of 33 lb. $r_x = 5.62$, $r_y = .912$, and $c = .794$; hence, $d = \sqrt{5.62^2 - .912^2} = 5.546$, and $b = d - c = 5.546 - .794 = 4.752$.

A practical rule giving good approximate results for a channel column or strut is to subtract $r_y$ from $r_x$; the result is $b$. Applying this rule to the 15-in. ⅃ of 33 lb. column or strut, $b = 5.62 - .912 = 4.708$.

## SECTION MODULUS AND MOMENT OF RESISTANCE

The expression $\dfrac{I}{c}$, in which $I$ is the moment of inertia and $c$ the distance of the outermost fiber of the section from the neutral axis, is called the *section modulus*. For a given material, this quantity is a measure of the capacity of the section to resist bending. Multiplied by the unit stress to which the outermost fibers are subjected under given loads, the product gives the amount of bending moment the section is resisting, and is called *moment of resistance*. If $f$ is the unit stress that certain loads develop in the outermost fibers of the section, the moment of resistance is

$$M_r = \frac{I}{c} f$$

EXAMPLE 1.—What is the section modulus of a 20-in. I beam of 75 lb. whose moment of inertia is 1,268.9?

SOLUTION.—As the neutral axis passes through the center of the section, the distance $c$ is in this case equal to one-half the depth; that is $20 \div 2 = 10$. The section modulus is therefore $\dfrac{I}{c} = \dfrac{1,268.9}{10} = 126.9$

EXAMPLE 2.—When subjected to loads perpendicular to the cover-plates the outermost fibers of the section shown in Fig. 2, are stressed to 16,000 lb. per sq. in. What is the resisting moment of the section?

SOLUTION.—The moment of inertia of the section has been found to be 1,403.22, and the outermost fibers are 9.63 in. from the neutral axis; hence, the section modulus is equal to $1,403.22 \div 9.63 = 145.7$; this multiplied by 16,000 gives 2,331,200 in.-lb.

Formulas for obtaining directly the section moduli of sections frequently used are given in the table of Moments of Inertia, etc.

## FRICTION

*Friction* is the resistance that a body meets from the surface on which it moves. It depends on the degree of roughness of the surfaces in contact, and is directly proportional to the perpendicular pressure between the surfaces. It is independent of the extent of the surfaces in contact as long as the normal pressure remains the same. It is generally greater between surfaces of the same material than between those of different materials, and greater between soft bodies than hard ones.

**Coefficient of Friction.**—The ratio between the resistance to the motion of a body due to friction and the perpendicular pressure between the surfaces is called the *coefficient of friction*. When the coefficient of friction between two surfaces is known, the frictional resistance is obtained by multiplying the normal pressure by the coefficient.

EXAMPLE 1.—What is the resistance per linear foot of a retaining wall against sliding when the normal pressure on the foundation is 10,000 lb. per lin. ft. of wall and the coefficient of friction of the masonry on the foundation is .65?

SOLUTION.—The frictional resistance is $10,000 \times .65 = 6,500$ lb.

The coefficient of friction of the wheels of suddenly stopping engines and cars on the rails is usually assumed at .20. The rails on bridges or trestles will transfer to the bridge or trestle tower the frictional forces produced by the brakes in order to stop the cars, causing stresses that must be provided for.

EXAMPLE 2.—What is the longitudinal force on a bridge caused by the sudden stopping of a car weighing 60,000 lb?

SOLUTION.—The longitudinal force will be $60,000 \times .20 = 12,000$ lb.

**Angle of Friction.**—When a body, as $B$ in the accompanying illustration, weighing $W$ lb. is placed on an inclined plane making an angle $a$ with the horizontal, the normal pressure is $N = W \cos a$; and, if the coefficient of friction is denoted by $f$, the frictional resistance against sliding down of the body is
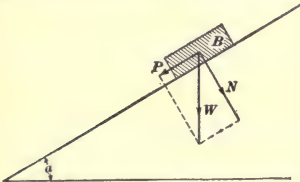


$F = fN = fW \cos a$. This force acts in a direction opposite to that of the force $P = W \sin a$. When the angle $a$ is such that $F$ just balances, or is equal to $P$, so that the slightest force will cause the body to slide, the angle is then called the *angle of friction*. The tangent of that angle is equal to the coefficient of friction, or $f = \tan a$.

**Angle of Repose.**—On a sloping bank of loose material, such as sand, earth, etc., when the angle of slope is such that the particles are on the point of moving, the angle is called the *angle of repose*. It is the same as the angle of friction of the material on itself. The slope is then called the *slope of repose*, or the *natural slope of the material*, for it is the slope that the material will assume when subject to gravity only.

EXAMPLE.—The coefficient of friction of dry sand on itself is .65; what is its angle of repose?

SOLUTION.—The angle of repose is the same as the angle of friction, whose tangent equals the coefficient of friction; consequently, $.65 = \tan a$, and from a table of natural tangents $a = 33°$.

The accompanying tables give coefficients of friction and angles of repose of a number of materials.

**COEFFICIENTS OF FRICTION AND ANGLES OF REPOSE FOR MASONRY MATERIALS**

| Material | Coefficient of Friction | Angle of Repose Degrees |
|---|---|---|
| Fine-cut granite, on same, dry | .60 | 31 |
| Fine-cut granite, on rough-pointed granite, dry | .65 | 33 |
| Rough-pointed granite, on same, dry | .70 | 35 |
| Well-dressed soft limestone, on same, dry | .75 | 37 |
| Concrete blocks, on same, dry | .65 | 33 |
| Concrete blocks, on fine-cut granite, dry | .60 | 31 |
| Common brick, on same, dry | .65 | 33 |
| Common brick, on well-dressed soft limestone, dry | .65 | 33 |
| Common brick, on well-dressed hard limestone, dry | .60 | 31 |
| Common brick, on same, with slightly damp mortar | .75 | 37 |
| Hard brick, on same, with slightly damp mortar | .70 | 35 |
| Hard limestone, on same, with slightly damp mortar | .65 | 33 |
| Common brick, on same, with fresh mortar | .50 | 27 |
| Well-dressed granite, on same, with fresh mortar | .50 | 27 |
| Granite, roughly worked, on dry sand and gravel | .50 to .60 | 27 to 31 |
| Granite, roughly worked, on wet sand | .35 to .45 | 19 to 24 |
| Granite, roughly worked, on dry clay | .50 | 27 |
| Granite, roughly worked, on moist clay | .35 | 19 |

**Rolling Friction.**—The friction between the circumference of a rolling body and the surface upon which it rolls is known as *rolling friction*. It is due to the compressibility of substances, the weight of the rolling body causing a small depression in the supporting surface and a flattening of the roller. Its magnitude depends on the materials of the roller and the supporting surface, and is proportional to the normal pressure exercised by the roller on the rolling surface. It depends also on the diameter of the roller, being less for large rollers

than for small ones. On highways with soft compressible surfaces, the resistance is also affected by the width of the wheel tires, being greater for narrow tires than for wide ones.

## COEFFICIENTS OF FRICTION, ANGLES OF REPOSE, AND WEIGHTS OF EARTHS

| Material | Coefficient of Friction | Angle of Repose Degrees | Weight Pounds per Cubic Foot |
|---|---|---|---|
| Mixed earth, dry | .70 | 35 | 95 |
| Mixed earth, damp | .80 | 39 | 115 |
| Mixed earth, wet | .40 | 22 | 115 |
| Sand, dry | .65 | 33 | 110 |
| Sand, wet | .05 | 3 | 125 |
| Loam, dry | .70 | 35 | 75 to 100 |
| Loam, wet | .50 | 27 | 90 to 120 |
| Clay, dry | 1.00 | 45 | 100 |
| Clay, wet | .30 | 17 | 125 |

## COEFFICIENTS AND ANGLES OF FRICTION FOR MISCELLANEOUS MATERIALS

| Materials | Coefficient of Friction | Angle of Friction Deg. Min. | |
|---|---|---|---|
| Cast iron on cast iron | .15 | 8 | 32 |
| Cast iron on brass | .15 | 8 | 32 |
| Cast iron on oak | .49 | 26 | 6 |
| Wrought iron on wrought iron | .14 | 7 | 58 |
| Wrought iron on cast iron | .19 | 10 | 46 |
| Wrought iron on brass | .17 | 9 | 39 |
| Wrought iron on mahogany | .18 | 10 | 12 |
| Wrought iron on oak | .62 | 31 | 47 |
| Steel on cast iron | .20 | 11 | 19 |
| Steel on brass | .15 | 8 | 32 |
| Steel on ice | .014 | 0 | 48 |
| Yellow copper on cast iron | .19 | 10 | 46 |
| Yellow copper on oak | .62 | 31 | 48 |
| Brass on cast iron | .22 | 12 | 25 |
| Brass on brass | .20 | 11 | 19 |
| Brass on wrought iron | .16 | 9 | 6 |
| Bronze on cast iron | .21 | 11 | 52 |
| Bronze on wrought iron | .16 | 9 | 5 |
| Bronze on bronze | .20 | 11 | 19 |
| Oak on oak | .48 | 25 | 38 |
| Oak on elm | .25 | 14 | 3 |
| Oak on cast iron | .37 | 20 | 19 |
| Leather on oak | .33 | 18 | 16 |
| Leather belt on oak drum | .27 | 15 | 7 |
| Leather belt on cast iron | .56 | 29 | 15 |
| Leather packing | .56 | 29 | 15 |

The first table on page 162 gives the maximum, minimum, and mean values of the coefficient of rolling friction for different roadway surfaces. They are expressed in pounds required to overcome the resistance on a level road of a gross ton (2,240 lb.). The mean value is also expressed as a ratio between the frictional resistance and the load.

The friction of liquids moving in contact with solid bodies is independent of the pressure, because the forcing of the particles of the fluid over the

projections on the surface of the solid body is aided by the pressure of the surrounding particles of the liquid, which tend to occupy the places of those

## ROLLING FRICTION FOR DIFFERENT ROADWAY SURFACES

| Character of Roadway Surface | Rolling Friction | | | |
| --- | --- | --- | --- | --- |
| | In Pounds per Gross Ton | | | Mean In Terms of Load |
| | Maximum | Minimum | Mean | |
| Earth, ordinary............. | 300 | 125 | 200 | $\frac{1}{11}$ |
| Earth, dry and hard......... | 125 | 75 | 100 | $\frac{2}{45}$ |
| Gravel, common.............. | 147 | 140 | 143 | $\frac{1}{16}$ |
| Gravel, hard rolled.......... | • • | | 75 | $\frac{1}{30}$ |
| Macadam, ordinary.......... | 140 | 60 | 90 | $\frac{1}{25}$ |
| Macadam, good.............. | 80 | 41 | 60 | $\frac{1}{37}$ |
| Macadam, best.............. | 64 | 30 | 50 | $\frac{1}{45}$ |
| Cobblestone, ordinary....... | | | 140 | $\frac{1}{16}$ |
| Cobblestone, good.∴........ | | | 75 | $\frac{1}{30}$ |
| Granite block, ordinary...... | | | 90 | $\frac{2}{25}$ |
| Granite block, good.......... | 80 | 45 | 56 | $\frac{1}{40}$ |
| Granite block, best.......... | 40 | 25 | 34 | $\frac{1}{66}$ |
| Belgian block, ordinary...... | | | 56 | $\frac{1}{40}$ |
| Belgian block, good.......... | 50 | 26 | 38 | $\frac{1}{60}$ |
| Plank....................... | 56 | 32 | 44 | $\frac{1}{50}$ |
| Wooden block, in good condition................... | 40 | 20 | 30 | $\frac{1}{75}$ |
| Asphalt..................... | 39 | 15 | 22 | $\frac{1}{100}$ |

forced over. Therefore, the coefficients of friction of liquids over solids do not correspond with those of solids over solids. The resistance is directly as the area of surface or contact.

## COEFFICIENTS OF FRICTION IN AXLES

| Axle | Bearing | Ordinary Lubrication | Lubricated Continuously |
| --- | --- | --- | --- |
| Bell metal......... | Bell metal ....... | .097 | |
| Cast iron.......... | Bell metal ....... | .07 | .049 |
| Wrought iron....... | Bell metal....... | .07 | .05 |
| Wrought iron....... | Cast iron........ | .07 | .05 |
| Cast iron.......... | Cast iron........ | .07 | .05 |
| Cast iron.......... | Lignum vitæ...... | .10 | |
| Wrought iron....... | Lignum vitæ...... | .12 | |

Friction naturally varies with the character of the surfaces, lubrication, and the nature of the lubricant. The best lubricants for the purposes should always be used, and the supply should be regular. When machinery is well lubricated, the lubricant keeps the surfaces apart, and the frictional resistance becomes very small, or about the same as the friction of liquids.

**Frictional Resistance of Shafting.—**

Let    $K$ = coefficient of friction;
       $W$ = work absorbed, in foot-pounds;
       $P$ = weight of shafting and pulleys + resultant stress of belts;
       $H$ = horsepower absorbed;
       $D$ = diameter of journal, in inches;
       $R$ = number of revolutions per minute.

| Then, | ORDINARY OILING | CONTINUOUS OILING |
|---|---|---|
| | $W = .0182PD$ | $.0112PD$ |
| | $H = .000000556PDR$ | $.000000339PDR$ |
| | $K = .066$ | $.044$ |

As a rough approximation, 100 ft. of shafting, 3 in. diameter, making 120 rev. per min., requires 1 H.P.

For friction of air in mines, see Coefficient of Friction, under Ventilation.

**Friction of Mine Cars.**—The friction of mine cars varies so much that it is impossible to give a formula for calculating it in every case. No two mine cars will show the same frictional resistance, when tested with a dynamometer, and, therefore, nothing but an average friction can be dealt with. The construction of the car, the condition of the track, and the lubrication are important factors in determining the amount of friction.

Some of the requisites of good oil box and journal bearings may be stated. Tightness is a prerequisite, and, in dry mines where the dust is very penetrating, this is especially important; the bearings should be sufficiently broad; the oil box large enough to hold sufficient oil to run 1 mo. without renewal, and so constructed that, while it may be quickly and easily opened, it will not open by jarring or by being accidentally struck by a sprag or a lump of coal.

There are a number of patented self-oiling wheels that are improvements on the old-style plain wheels, and each of these has undoubtedly some point of superiority over the old style.

Among the most extensively used of these patented wheels are those with annular oil chambers, and those with patent bushings. Their superiority consists in the fact that, if properly attended to, a well-lubricated bearing is secured with greater regularity and less work than when the old-style wheel is used.

With a view of adopting a standard wheel, the Susquehanna Coal Co., of Wilkes-Barre, Pa., experimented for a number of years with different styles of self-lubricating wheels.

Mr. R. Van A. Norris, E. M., assistant engineer, made a series of 989 tests with old-style wheels, some of which had patent removable bushings, and others annular oil chambers, and the self-oiling wheel. The old wheels were found to be practically alike in regard to friction. All the wheels were of the loose outside type, 16 in in diameter, mounted on 2⅜ in. steel axles, with journals 5¼ in. long. The axles passed loosely through solid cast boxes, bolted to the bottom sills of the cars, and were not expected to revolve.

The table of friction tests shows the results obtained with both old- and new-style wheels, and is of interest to all colliery managers, inasmuch as the figures given for the old-style wheels alone are the most complete in existence and, as stated before, they are good averages.

Tests were made on the starting and running riction of each style of wheel, under the conditions of empty and loaded cars level and grade track, curves, and tangents. The instruments were a Pennsylvania Railroad spring dynamometer, graduated to 3,000 lb., with a sliding recorder, a hydraulic gauge (not recording) reading to 10,000 lb., graduated to 25 lb , and a spring balance, capacity 300 lb., graduated to 3 lb. All these were tested and found correct previous to the experiments.

Most of the observations on single cars were made with the 300-lb. balance. The two types of old-style wheels have been classed together in the table. Each car was carefully oiled before testing, and several of each type were used, the results being averages from the number of trials shown in the table.

In the experiments on the slow start and motion, the cars were started very slowly by a block and tackle, and the reading was taken at the moment of starting. They were then kept just moving along the track for a considerable distance, and the average tractive force was noted, the whole constituting one experiment.

The track selected for these experiments was a perfectly straight and level piece of 42 in. gauge, about 200 ft. long, in rather better condition than the average mine track. The cars were 41¾ in. gauge, 3½ ft. wheel base, 10 ft. long, capacity about 85 cu. ft., with 6-in. topping.

To ascertain the tractive force required at higher speeds, trips of one, four and twenty cars, both empty and loaded, were attached to a mine locomotive and run about 1 mi. for each test, the resistance at various points on the track, where its curve and grade were known, being noted, care also being taken to run at a constant speed. Unfortunately, only four of the new-style cars were available on the tracks where these trials were made.

The remarkably low results for the twenty-car trips are attributed to variations in the condition of the track, and the fact that the whole train

## SUMMARY OF FRICTION TESTS ON OLD-STYLE MINE-CAR WHEELS

DIMENSIONS OF WHEEL

16 in, diameter of tread
2¼ in, diameter of axle
5½ in, length of journal

| | Empty | | | | | | Loaded | | | | | | No. of Tests | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Weight of Car | Tractive Force per Car | Tractive Force Due to Gravity | Tractive Force per Car Due to Friction | Tractive Force per Ton Due to Friction | Percentage of Weight | Weight of Car | Tractive Force per Car | Tractive Force Due to Gravity | Tractive Force per Car Due to Friction | Tractive Force per Ton Due to Friction | Percentage of Weight | Empty | Loaded |
| **LEVEL** | | | | | | | | | | | | | | |
| Average slow start | 2,240 | 100 | | 100 | 100 | 4.46 | 8,500 | 325 | | 325 | 85 | 3.80 | 16 | 12 |
| Average slow start | 2,140 | 83 | | 83 | 86 8/16 | 3.88 | 7,885 | 357 | | 357 | 101 4/16 | 4.53 | 63 | 53 |
| Average motion 50 ft. per min. | 2,140 | 54 | | 54 | 56 4/16 | 2.52 | 7,885 | 205 | | 205 | 58 7/16 | 2.60 | 54 | 60 |
| Average motion 1,000 ft. per min., 1 car. | 2,140 | 62½ | | 62½ | 64 4/16 | 2.89 | 7,885 | 262 | | 262 | 74 4/16 | 3.32 | 17 | 74 |
| Average motion 1,000 ft. per min., 4 cars. | 2,240 | | | | | 2.80 | | | | | | | | |
| Average motion 1,000 ft. per min., 20 cars. | 2,240 | 47 | | 47 | 47 | 2.20 | 9,000 | 117 | | 117 | 29 | 1.30 | 6 | 18 |
| Average starting jerk, 20 cars. | 2,240 | 96 | | 96 | 96 | 4.29 | 9,000 | 175 | | 175 | 44 | 1.95 | 3 | 7 |
| Average starting jerk, 2 cars, rope haul. | 2,240 | 630 | | 630 | 630 | 28.12 | | | | | | | 10 | |
| **GRADE** | | | | | | | | | | | | | | |
| Average slow start 12°. | 2,140 | 550 | 445 | 105 | 110 | 4.90 | 7,885 | 1,950 | 1,639 | 311 | 88 4/16 | 3.94 | 2 | 2 |
| Average motion 50 ft. per min., 12°. | 2,140 | 510 | 445 | 65 | 68 1/16 | 3.18 | 7,885 | 1,800 | 1,639 | 161 | 45 4/16 | 2.00 | 10 | 16 |
| Average motion 1,000 ft. per min., 1⅓°, 1 car. | 2,140 | 125 | 65 | 60 | 62 1/16 | 2.80 | 7,885 | 425 | 205 | 220 | 62 4/16 | 2.79 | 10 | 12 |
| Average motion 1,000 ft. per min., 1⅓° 1 car. | 2,240 | | | | | 2.80 | | | | | | | | |
| Average motion 200 ft. per min., rope haul, 2°. | 2,240 | 140 | 78 | 62 | 62 | 3.80 | *Cars pulled from side* | | | | | | 15 | |
| Average motion 200 ft. per min., rope haul, 2° 30′. | 2,240 | 183 | 98 | 85 | 85 | 5.00 | | | | | | | 15 | |
| Average motion 200 ft. per min., rope haul, 5° 10′. | 2,240 | 315 | 202 | 113 | 113 | 5.00 | | | | | | | 15 | |
| Average motion 200 ft. per min., rope haul, 6° 10′. | 2,240 | 353 | 240 | 113 | 113 | 5.00 | | | | | | | 15 | |
| **CURVE** | | | | | | | | | | | | | | |
| Average slow start, 85 ft. radius. | 2,240 | 125 | | 125 | 125 | 5.58 | 8,500 | 400 | 227 | 400 | 106 | 4.70 | 10 | 10 |
| Average slow start, 11 ft. radius, 1⅓° grade. | 2,240 | | | | | | 8,700 | 819 | | 592 | 152 | 6.80 | 5 | 8 |
| Average 20 cars, 1,000 ft. per min., 350 ft. radius. | 2,240 | 62 | | 62 | 62 | 2.80 | 9,000 | 143 | | 143 | 36 | 1.60 | 5 | 7 |
| Average 20 cars, 1,000 ft. per min., 450 ft. radius. | 2,240 | 50 | | 50 | 50 | 2.23 | 9,000 | 126 | | 126 | 32 | 1.40 | 2 | |
| Average 4 cars, 1,000 ft. per min., 350 ft. radius | 2,240 | 100 | | 100 | 100 | 4.46 | | | | | | | | |

Total number of tests    276   286

## SUMMARY OF FRICTION TESTS ON SELF-OILING MINE-CAR WHEELS

| DIMENSIONS OF WHEELS<br><br>16 in., diameter of tread<br>2¼ in., diameter of axle<br>5¼ in., length of journal | Empty | | | | | | Loaded | | | | | | No. of Tests | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Weight of Car | Tractive Force per Car | Tractive Force Due to Gravity | Tractive Force per Car Due to Friction | Tractive Force per Ton Due to Friction | Percentage of Weight | Weight of Car | Tractive Force per Car | Tractive Force Due to Gravity | Tractive Force per Car Due to Friction | Tractive Force per Ton Due to Friction | Percentage of Weight | Empty | Loaded |
| **LEVEL** | | | | | | | | | | | | | | |
| Average slow start | 2,415 | 66¾ | | 66¾ | 62 | 2.78 | 9,125 | 200 | | 200 | 49 | 2.20 | 6 | 20 |
| Average slow start | 2,415 | 62 | | 62 | 55 1/10 | 2.48 | 8,160 | 193 | | 193 | 53 | 2.36 | 82 | 48 |
| Average motion 50 ft. per min. | 2,415 | 40 | | 40 | 37 1/10 | 1.66 | 8,160 | 133 | | 133 | 36 1/10 | 1.63 | 81 | 72 |
| Average motion 1,000 ft. per min., 1 car | 2,415 | 36¾ | | 36¾ | 33 1/10 | 1.48 | 8,160 | 158 | | 158 | 43 1/10 | 1.93 | 6 | 39 |
| Average motion 1,000 ft. per min., 4 cars | 2,415 | 37¼ | | 37¼ | 34¼ | 1.56 | | | | | | | 6 | |
| **GRADE** | | | | | | | | | | | | | | |
| Average slow start 12° | 2,415 | 600 | 502 | 98 | 90 1/10 | 4.06 | 8,160 | 2,000 | 1,696 | 304 | 83 1/10 | 3.73 | 2 | 2 |
| Average motion 50 ft. per min., 12° | 2,415 | 540 | 502 | 38 | 35 1/10 | 1.56 | 8,160 | 1,825 | 1,696 | 129 | 35 2/5 | 1.58 | 6 | 12 |
| Average motion 1,000 ft. per min., 1¾°, 1 car | 2,415 | 100 | 73 | 27 | 25 | 1.12 | 8,160 | 400 | 249 | 151 | 41 1/10 | 1.85 | 6 | 6 |
| Average motion 1,000 ft. per min., 1½°, 1 car | | | | | | | 8,160 | 350 | 212 | 138 | 37 1/10 | 1.69 | | 16 |
| **CURVE** | | | | | | | | | | | | | | |
| Average slow start, 85 ft. radius | 2,415 | 75 | | 75 | 75 | 3.10 | 9,125 | 275 | | 275 | 67 | 3.00 | 2 | 8 |
| Average slow start, 11 ft. radius, 1¼° grade | | | | | | | 9,125 | 564 | 239 | 325 | 80 | 3.60 | | 7 |
| Average 4 cars, 1,000 ft. per min., 350 ft. radius | | | | | | | | | | | | | | |
| Total number of tests | | | | | | | | | | | | | 197 | 230 |

was seldom pulling directly on the locomotive, the cars moving by jerks, so that correct observations were impracticable. The hydraulic gauge was used for these twenty-car tests, and the needle showed vibrations from 1 to 4 T. and back. The mean was taken as nearly as possible. The gauge was rather too quickly sensitive for the work, and the Pennsylvania Railroad dynamometer was not strong enough to stand the starting jerks and the strain of accelerating speed.

The tests marked Rope Haul were made on an empty-car haulage system, about 500 ft. long, with overhead endless rope running continuously at a speed of 180 ft. per min., the cars being attached to the moving rope by a chain, a ring at the end of which was slipped over a pin on the side of the car. The increase of friction on the heavier grades was due to the rope pulling at a greater angle across the car. Correction was not made for this angularity at the time, and the rope has since been rearranged, so that the correction cannot now be made. There were not enough curve experiments to permit the deduction of any general formula for the resistance of these cars on curves.

The experiments on grade agree fairly well with those on a level, the rather higher values obtained being probably due more to the greater effort required in moving them, and the consequent jerkiness of the motion, than to any real increase in resistance. As the experiments on all styles of wheels were made in an exactly similar manner, the comparative value of the results is believed to be nearly correct, the probable error in each set of experiments, as computed by the method of least squares, varying from about 4% for slow start and motion to 12% for the rapid motion and twenty-car trips.

**Ball and Roller Bearings.**—Some of the leading manufacturers now provide mine-car wheels with either ball or roller bearings. In the former type, a series of steel balls placed within the hub bear upon the axle; and in the latter type, a series of steel rollers. In other features, however, such as the method of lubrication, etc., the improved wheels are essentially the same as the old. Only a limited number of tests have thus far been published upon the savings effected by the use of these improved bearings in power-transmission shafting, mine-car wheels, etc., but these indicate a diminution of from 15 to 75% in the friction over the old type of bearings. In the case of mine cars, those equipped with the new type of bearing require about one-half the drawbar pull formerly demanded either to start them from rest or to continue them in motion.

Quoting from one of the leading manufacturers of spiral roller bearings: "The saving in power varies to a certain extent with conditions. In some cases it has run as high as 60%; in others, it has been as low as 30%; a safe average saving is 40% to 50%. The following series of tests was made in November, 1912. The twenty-five cars used in the test had been in constant use for about 6 mo.; the wheels were 16 in. and the axles $2\frac{1}{4}$ in. in diameter.

| Type of Wheel | Average Starting Pull Pounds | Average Constant Pull Pounds | Gross Weight Pounds | Up-Grade Per Cent. | Constant Tractive Effort Per Ton Pounds | Tractive Effort Per Ton, Corrected For Level Pounds |
|---|---|---|---|---|---|---|
| Plain | 297 | 120 | 5,800 | .25 | 41.3 | 26.8 |
| Spiral } Roller } | 104 | 80 | 5,800 | .25 | 27.5 | 13.0 |

**Lubrication.**—There is probably no factor that has a more direct bearing on the cost of production per ton of coal and ores than the lubrication of mine machinery, and yet it is doubtful if there is another item connected with the operation of a mine less understood by owners, their managers, and engineers in charge.

Steam plants are equipped with boilers of the highest known efficiency; heaters are used that, by utilizing waste steam, will heat the feedwater for boilers to the highest point. Modern engines that will develop a horsepower with the least amount of steam are installed; bends, instead of elbows, are placed in steam and exhaust pipes, so that the friction and back pressure may be reduced to a minimum. In a word, everything is done in the equipment of a plant to secure economy in its operation. After all this is done,

frequently a long step is taken in the opposite direction by the use of an oil unsuited to the existing conditions, and those in charge of the plant are led to believe that the lubrication is all that could be desired, simply because the engines and machinery run quietly and the temperature of the bearings does not become alarmingly high. The office of a lubricant is not merely to secure this result, but, primarily, to reduce friction and wear to a minimum; and an oil that will do this is the best oil to use, no matter what the price per gallon may be.

Few realize the great loss in power due to the friction of wearing parts. One of the greatest living authorities on lubrication writes:

"It may probably be fairly estimated that one-half the power expended in the average case, whether in mill, mine, or workshop, is wasted on lost work, being consumed in overcoming the friction of poorly lubricated surfaces."

He adds that a reduction of 50% in the work lost by friction has often been secured by a change of lubricants.

As one of many instances showing the loss that will occur by the use of inferior lubricants, attention is called to two flour mills located in one of the Middle States. One of the plants was equipped with a condensing engine capable of developing 1 H. P. on 24 lb. of water per hr.; the other plant had a simple engine, taking 30 lb. of water per hr. The plant containing the condensing engine was purchased by the owner of the plant containing the simple engine. The new owner of the plant was surprised to learn that the cost of operation per barrel of flour manufactured was equally as great in the new plant as in the old one. The engines were indicated, and valves found to be properly adjusted and the engine working within the economical range, so far as load was concerned. The loss was then attributed to the boilers, but an evaporative test proved that there was no practical difference here, as the boilers, in both instances, were evaporating a fraction over 8 lb. of water per lb. of coal. At this point, the question of lubrication was taken up, and, on the advice of an expert sent by a prominent manufacturer of lubricants to look over the plant, an entire change was made in the lubricants used, and, as a result, a money saving of over $2.25 per da. (practically $700 per annum — this in a plant of less than 250 H. P.) was effected, notwithstanding the fact that the new lubricants used cost considerably more per gallon than those formerly used. This simply indicates that the price of an oil is of little importance in comparison with its friction-reducing power. Friction costs money, because it means greater cost of operation per unit of output.

Among the expenses chargeable to waste power, due to inferior lubrication, may be included: (1) The cost of power produced in excess of that really required to operate the mine per ton of output. In this calculation should be included the proper proportion of salaries of engineers, and all other items that contribute to the cost of the motive department, as well as the cost of mining the fuel consumed in producing this excess power. (2) Wear and tear of machinery, which is constantly doing more work per ton of coal mined than should be required of it.

There is also an element of danger that ought to receive serious consideration, as, while it is true that cylinder and bearing lubricants of indifferent merit will, under ordinary conditions, keep the cylinders from groaning and the bearings from becoming hot, experiments have proved that, in accomplishing such results, the oils in use were being taxed to their utmost; and there is record of many instances where, as a result of using oils of such limited endurance, accidents of a serious nature have occurred, necessarily causing shut-downs just at the time when the operation of the plant to its fullest capacity was imperative. It is most difficult to do much more than point out the danger due to the use of inferior lubricants, leaving it to the purchaser himself to determine as to the intrinsic worth of the lubricants offered to him. In making his selection he would do well to consult with and heed the advice of some highly responsible manufacturer of lubricants who has given to the question, in all its phases, the most careful study, and who would most probably have the benefit of a wide experience in the application as well as the manufacture of lubricants. Some buyers have, to their ultimate regret, adopted, as a method of determining the merits of lubricants, a schedule of laboratory tests. Such a method is not only useless, but it is misleading to any one other than a manufacturer of lubricants, who makes use of it merely as a means of insuring uniformity in his manufactured products, and not as a measure whereby to judge their practical value. Indeed, many oils can be very properly described by practically the same schedule of tests, and yet are widely apart when their utility for a given service is considered.

As a general guide in purchasing cylinder oil for mine lubrication, it might be said that a dark-colored oil is of greater value, as a rule, than one that has been filtered to a red or light amber color, as the process of filtration necessarily takes from the oil a considerable percentage of its lubricating value, and at the same time the process is an expensive one. In short, if a light-colored oil is insisted upon, a high price must be paid for an inferior lubricant. As a word of caution, however, it would be well to add right here that irresponsible manufacturers frequently take advantage of the fact that the most efficient and best known cylinder oils are dark-colored, and endeavor, with more or less success, to market as "cylinder oil" products absolutely unsuited to the lubrication of steam cylinders, and that would consequently be expensive could they be procured without cost.

For the lubrication of engine bearings, where modern appliances for feeding are used, an engine oil of a free running nature is best, as it more quickly reaches the parts requiring lubrication than an oil of a more sluggish nature. It, of course, must not be an oil susceptible to temperature changes, but must be capable of performing the service required of it under the most severe conditions, where an oil of less backbone would fail. Such an oil would also be suitable for the lubrication of dynamos, and should also give satisfaction where used in lubricating the cylinders of air compressors. Where the machinery is of an old type and loose-jointed, or when the bearings are open and the oil is applied directly to them by means of an oiler, an engine oil of a more sluggish, or viscid, nature is best.

Perhaps of equal importance to the lubrication of power machinery must be considered the lubrication of the axles of mine cars. This is important, first, because of the fact that perhaps three-fourths of the oil used about a coal mine is used for this purpose, and, secondly, because there is really a marked difference in the quality and, therefore, in the efficiency of lubricants used for this purpose. Fully nine-tenths of the prominent railroads of this country are today using car-axle oil, costing perhaps as much per gallon as much of the so-called cylinder oil that is used in coal mines, they having discovered, by exhaustive experiments, that the increased efficiency gained by using an oil of such quality many times offsets the difference in the cost per gallon and enables them to secure a greater mileage without any increase in their power or other fixed charges. This will apply just as forcibly to the lubrication of coal cars, no matter whether the power is derived from mules or electric motors; therefore, this feature of lubrication of mine equipment should receive more careful attention than it does receive, as a rule.

There is considerable waste in the lubrication of mine cars. This waste is hard to avoid, and, naturally, makes the buyer hesitate before adopting the use of a car oil that costs very much per gallon; but even in the face of this waste the increased efficiency secured by the use of a high-grade car oil will warrant its use. Such waste is pretty hard to correct in mines where the old-fashioned style of car axle is still in use, and where the oil is applied through an ordinary spout oil can into the axle box, and allowed to drip off the axles and on to the ground. When axles are equipped in the same manner as those of freight cars, or where cars are equipped with one of the several different styles of patent car wheels and axles that are coming into use quite extensively, it is possible to regulate the feeding of the oil to the axles, so as to reduce the waste to a minimum. One of these patent car wheels, which is perhaps better known than any other, is constructed with a hollow hub that acts as a reservoir for the oil, the oil passing from this reservoir through small holes on to a felt washer, which it must saturate, and by which it is applied to the axles. Such wheels require a limpid oil, as a heavy, sluggish oil will not so readily saturate the felt washer referred to. A tight cap is adjusted to the end of the axle, to prevent waste of oil. These wheels will run quite a length of time without reoiling after the reservoir is once filled. While it costs something to equip mine cars with these patent axles, such an outlay will result in more economical operation, particularly if at the same time the very best quality of car oil obtainable is used.

**Lubricant Tests.**—There are certain simple tests that may readily be made to determine the suitability of certain oils for certain grades of work. When testing oils for use in connection with engines running under constant load and speed, Mr. W. W. Davis, of Boston, recommends that a thermometer be placed in the bearing so that the bulb rests on the shaft, a constant feed of oil being maintained. Another thermometer is hung in the engine room near the bearing and away from drafts of air, so as to show the temperature of the room. Commence the test when the engine is started, note the rise of

temperature at frequent intervals, also that of the room; continue the test until the temperature of the bearing ceases to rise. Every bearing will in the course of a few hours reach a point where heat is radiated as fast as generated. Deducting the temperature of the room from that of the bearing will give the rise in temperature due to friction. If the engine runs during the day only, the bearing will cool off over night and after cleaning thoroughly with gasoline, will be in condition to test another oil the next day.

While it is true that the coefficient of friction often decreases with the rise in temperature, in everyday practice it is safe to assume that of two oils the one that will keep the bearing the cooler is the best lubricant, so in tests of this kind the oil showing the least rise in temperature will be the better lubricant. Such tests can also be made in ring-oiled bearings of motors, dynamos, or shafting.

When testing the value of two or more cylinder oils, Mr. Ward recommends that one oil be fed in at a given rate for a few days, the cylinder head then removed, and the inner surface wiped over with a piece of soft white paper. If there is no stain of oil and a liberal amount has been used, either the steam is very wet or not enough fatty oil has been used in compounding the lubricant. A separator will remove the excess moisture from the steam, when further tests will indicate if there is enough fatty oil present.

The same tests can be used to determine the least amount necessary to maintain good lubrication. By gradually reducing the amount of oil fed and examining the surfaces from time to time the proper amount necessary to maintain good lubrication can be determined. Where tests of this kind are to be made some means must be provided for the easy removal of the cylinder heads.

## BEST LUBRICANTS FOR DIFFERENT PURPOSES (THURSTON)

| | |
|---|---|
| Low temperatures, as in rock drills driven by compressed air.................. | Light mineral lubricating oils. |
| Very great pressures and slow speed...... | Graphite, soapstone, and other solid lubricants. |
| Heavy pressures and slow speed......... | The solid mineral lubricants and lard, tallow, and other greases. |
| Heavy pressures and high speed......... | Sperm oil, castor oil, and heavy mineral oils. |
| Light pressures and high speed......... | Sperm, refined petroleum, olive, rape, cottonseed. |
| Ordinary machinery.................. | Lard oil, tallow oil, heavy mineral oils, and the heavier vegetable oils. |
| Steam cylinders...................... | Heavy mineral oils, lard, tallow. |
| Watches and other delicate mechanism. | Clarified sperm, neat's foot, porpoise, olive, and light mineral lubricating oils. |

For mixture with mineral oils, sperm is best; lard is much used; olive and cottonseed are good.

# STRENGTH OF MATERIALS

## DEFINITIONS

*Stress* is the cohesive force by which the particles of a body resist the external load that tends to produce an alteration in the form of the body. It is always equal to the effective external force acting upon the body; thus, a bar subjected to a direct pulling force of 1,000 lb. endures a stress of 1,000 lb.

*Unit stress* is the stress or load per unit of area, usually taken per square inch of section. For instance, if the bar just mentioned is 1 in. × 2 in. in section, the unit stress of the bar will be $1,000 \div 2$ (sectional area) = 500 lb.

*Tensile stress* is produced when the external forces tend to stretch a body, or pull the particles away from one another. A rope by which a weight is suspended is an example of a body subjected to tensile stress.

*Compressive stress* is produced when the forces tend to compress the body, or push the particles closer together. A post or column of a building is subjected to compresive stress.

## AVERAGE ULTIMATE STRENGTHS OF METALS, IN POUNDS PER SQUARE INCH

| Kind of Metal | Compression | Tension | Elastic Limit | Shearing | Modulus of Rupture | Modulus of Elasticity |
|---|---|---|---|---|---|---|
| *Aluminum:* | | | | | | |
| Aluminum, commercial | 12,000 | 15,000 | 6,500 | 12,000 | | 11,000,000 |
| Aluminum, nickel..... | | 40,000 | 22,000 | | | |
| *Brass, Bronze, and Copper:* | | | | | | |
| Brass, cast.......... | (30,000) | 24,000 | 6,000 | 36,000 | 20,000 | 9,000,000 |
| Brass wire, annealed (softened by reheating)............... | | 50,000 | | | | |
| Brass wire, unannealed | | 80,000 | 16,000 | | | 14,000,000 |
| Bronze, aluminum..... | 120,000 | 75,000 | | | | |
| Bronze, gun metal..... | (20,000) | 32,000 | 10,000 | | 53,000 | 10,000,000 |
| Bronze, manganese.... | 120,000 | 60,000 | 30,000 | | | |
| Bronze, phosphor ..... | | 50,000 | 24,000 | | | 14,000,000 |
| Bronze, Tobin........ | | 66,000 | 40,000 | | | 4,500,000 |
| Copper, bolts........ | 30,000 | 30,000 | | | | |
| Copper, cast......... | (40,000) | 24,000 | 6,000 | 30,000 | 22,000 | 10,000,000 |
| Copper wire, annealed (softened by reheating)............... | | 36,000 | | | | 15,000,000 |
| Copper wire, unannealed | | 60,000 | 10,000 | | | 18,000,000 |
| *Cast and Wrought Iron:* | | | | | | |
| Iron, cast........... | 80,000 | 15,000 | 6,000 | 18,000 | 30,000 | 12,000,000 |
| Iron chains.......... | | 35,000 | | | | |
| Iron, corrugated...... | | | | | 40,000 | |
| Iron wire, annealed (softened by reheating)............... | | 60,000 | | | | 15,000,000 |
| Iron wire, unannealed. | | 80,000 | 27,000 | | | 25,000,000 |
| Iron, wrought, shapes.. | 46,000 | 48,000 | 26,000 | 40,000 | 44,000 | 27,000,000 |
| Iron, wrought, rerolled bars............... | 48,000 | 50,000 | 27,000 | 40,000 | 48,000 | 26,000,000 |
| *Lead:* | | | | | | |
| Lead, cast........... | | 2,000 | 1,000 | | | 1,000,000 |
| Lead pipe........... | | 1,600 | | | | |
| *Cast and Structural Steel:* | | | | | | |
| Steel, castings........ | 70,000 | 70,000 | 40,000 | 60,000 | 70,000 | 30,000,000 |
| Steel, structural, soft.. | 56,000 | 56,000 | 30,000 | 48,000 | 54,000 | 29,000,000 |
| Steel, structural, medium.............. | 64,000 | 64,000 | 33,000 | 50,000 | 60,000 | 20,000,000 |
| Steel wire, annealed (softened by reheating)............... | | 80,000 | 40,000 | | | 29,000,000 |
| Steel wire, unannealed | | 120,000 | 60,000 | | | 30,000,000 |
| Steel wire, crucible.... | | 180,000 | 80,000 | | | 30,000,000 |
| Steel wire, for suspension bridges........ | | 200,000 | 90,000 | | | 30,000,000 |
| Steel wire, special tempered............... | | 300,000 | | | | |
| *Tin and Zinc:* | | | | | | |
| Tin, cast............ | (6,000) | 3,500 | 1,800 | | 4,000 | 4,000,000 |
| Zinc, cast........... | (20,000) | 5,000 | 4,000 | | 7,000 | 13,000,000 |

NOTE.–Compression values enclosed in parentheses indicate loads producing 10% reduction in original lengths.

*Shearing stress* is produced when the forces tend to cause the particles in one section of a body to slide over those of the adjacent section. A steel plate acted on by the knives of a shear, and a beam carrying a load, are subjected to shearing stress.

Tension, compression, and shear are called *simple* or *direct stresses*, to distinguish them from bending and torsion.

The amount of alteration in form of a body produced by a stress is called *deformation*, or *strain*. It may be tensile deformation, compressive deformation, or shearing deformation, according as the stress producing it is tensile, compressive, or shearing. The *rate of deformation*, also called *unit deformation*, is the deformation of a body, subjected to tension or compression, per unit of

## AVERAGE ULTIMATE STRENGTHS OF WOODS, IN POUNDS PER SQUARE INCH

| Kind of Timber | Tension | | Compression | | | Transverse | | Shearing | |
|---|---|---|---|---|---|---|---|---|---|
| | With Grain | Across Grain | With Grain | | Across Grain | Extreme Fiber Stress | Modulus of Elasticity | With Grain | Across Grain |
| | | | End Bearing | Columns Under 10 or 15 Diameters | | | | | |
| White oak..... | 12,000 | 2,000 | 7,000 | 5,000 | 2,000 | 7,000 | 1,500,000 | 800 | 4,000 |
| White pine..... | 7,000 | 500 | 5,500 | 3,500 | 700 | 4,000 | 1,000,000 | 400 | 2,000 |
| Southern long-leaf or Georgia pine.......... | 12,000 | 600 | 7,000 | 5,000 | 1,400 | 7,000 | 1,500,000 | 600 | 5,000 |
| Douglas fir..... | 8,000 | | 5,700 | 4,500 | 800 | 5,000 | 1,400,000 | 500 | |
| Short-leaf yellow pine...... | 9,000 | 500 | 6,000 | 4,500 | 1,000 | 6,000 | 1,200,000 | 400 | 4,000 |
| Red pine (Norway pine)..... | 8,000 | 500 | 5,000 | 4,000 | 800 | 5,000 | 1,130,000 | | |
| Spruce and Eastern fir.... | 8,000 | 500 | 6,000 | 4,000 | 700 | 4,000 | 1,200,000 | 400 | 3,000 |
| Hemlock...... | 6,000 | | | 4,000 | 600 | 3,500 | 900,000 | 350 | 2,500 |
| Cypress....... | 6,000 | | 5,000 | 4,000 | 700 | 5,000 | 900,000 | | |
| Cedar......... | 7,000 | | 5,500 | 3,500 | 700 | 4,000 | 700,000 | 400 | 1,500 |
| Chestnut...... | 8,500 | | | 4,000 | 900 | 5,000 | 1,000,000 | 600 | 2,000 |
| California redwood......... | 7,000 | | | 4,000 | 600 | 4,500 | 700,000 | 400 | |
| California spruce....... | | | | 4,000 | | 5,000 | 1,200,000 | | |
| Factor of safety | 10 | 10 | 5 | 5 | 4 | 6 | 2 | 4 | 4 |

length. If an iron bar 6 ft. long is subjected to a force that elongates it 1 in., the rate of deformation will be 1 in. ÷ 72 (length of the bar, in inches) = .0139 in.

The *modulus* or *coefficient of elasticity* is the ratio between the stresses and corresponding deformations for a given material, which may have a somewhat different modulus of elasticity for tension, compression, and shear. If *l* is the increase per unit of length of a material subjected to tensile stress, and *p* the unit stress producing this elongation, the modulus of elasticity

$$E = \frac{p}{l}$$

For example, a wrought-iron bar, 80 in. long, subjected to a unit tensile stress *p* of 10,000 lb., stretched .029 in. The unit strain *l*, or stretch per inch of length, is .029 in. ÷ 80 in. = .0003625 in. Then,

$$E = \frac{10,000}{.0003625} = 27,586,200 \text{ lb. per sq. in.}$$

The relation $E = p \div l$ is true only when equal additions of stress cause equal increases of strain. Previous to rupture, this condition ceases to exist, and the material is said to be strained beyond the *elastic limit*, which, therefore, is that degree of stress within which the modulus of elasticity is nearly constant and equal to the unit stress divided by the unit strain.

The *ultimate strength* of a given material in tension, compression, or shear is that unit stress which is just sufficient to break it, and is equal to the maximum stress causing rupture divided by the original area of the cross-section. The accompanying tables show the average ultimate strengths, in pounds per square inch, of both metals and woods.

*Working stress* is the maximum unit stress to which the parts of a structure are to be subjected.

*Factor of safety* is the ratio of ultimate strength to working stress. The factor of safety required for a structure depends on the material and on the character of the loads applied—that is, whether the loads are quiescent or such that cause impact and vibrations. For stone and brick, a factor of safety of from 10 to 30 is used; for timber, from 8 to 15; for cast iron, from 6 to 20; for reinforced concrete, from 4 to 6; and for structural steel, from 3 to 6.

It is obvious that structures subjected to loads causing impact should be designed for a higher factor of safety than those having to carry static loads. When a structure, as a bridge, carries both dead load and live loads, the modern practice favors the specifying of one working unit stress for both kinds of loads, and providing for the effect of vibration by increasing the live-load stress or bending moment by an amount $I$ determined from a so-called impact formula. The formula most in use for railroad bridges is

$$I = \frac{300}{L + 300} S$$

in which $S$ = maximum live-load stress or bending moment in member,
$L$ = length, in feet, of single track that must be loaded in order to obtain value $S$.

# SIMPLE, OR DIRECT, STRESS

**Formula for Simple Stress.**—If $P$ is an external force producing tension, compression, or shear uniformly distributed over an area $A$, and $s$ is the unit working stress, the fundamental formula for designing parts of structures subjected to a simple, or direct, stress is

$$P = sA$$

When designing members that are in tension, $A$ must be taken as the net area of the section. This is determined by deducting from the gross section the greatest number of pin, bolt, or rivet holes that can be cut by a plane at right angles to the section. Rivet holes are usually taken $\frac{1}{8}$ in. larger than the diameter of the rivet.

**Important Applications of Formulas for Direct Stress.**—1. Tension members and short compression members of roof or bridge trusses are examples of simple stress, and their sections are determined by the preceding formula.

**EXAMPLE.**—A tension member of a roof truss is made of two $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ angles connected by one line of rivets $\frac{7}{8}$ in. in diameter. What stress will it carry at 16,000 lb. per sq. in.?
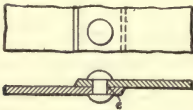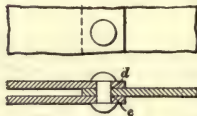


FIG. 1            FIG. 2

SOLUTION.—The gross sectional area of a $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ angle is 3.25 sq. in. The deduction for one rivet hole is $(\frac{7}{8} + \frac{1}{8}) \times \frac{1}{2} = .5$. The net area is $3.25 - .5 = 2.75$. The carrying capacity of the angle is therefore $2.75 \times 16,000 = 44,000$ lb.

2. Riveted joints also are examples of simple stress. In the joint shown in Fig. 1, the rivet is in *single shear*, because there is only one section $e$ of the rivet subjected to a shearing stress. The amount $R$ that one rivet will carry being equal to the area of the cross-section of the rivet multiplied by the unit

## SHEARING AND BEARING VALUES OF RIVETS, IN POUNDS

| Diam. of Rivet Inch | Area of Rivet Square Inch | Shear Values at 6,000 Lb. per Sq. In. | | Bearing Values for Different Thicknesses of Plate, in Inches, at 12,000 Lb. per Sq. In. | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Single Shear | Double Shear | 1/4 | 5/16 | 3/8 | 7/16 | 1/2 | 9/16 | 5/8 | 11/16 | 3/4 | 13/16 | 7/8 |
| 1/2 | .1963 | 1,180 | 2,360 | 1,500 | 1,880 | 2,250 | 2,630 | 3,000 | | | | | | |
| 5/8 | .3068 | 1,840 | 3,680 | 1,880 | 2,340 | 2,810 | 3,280 | 3,750 | 4,220 | 4,690 | | | | |
| 3/4 | .4418 | 2,650 | 5,300 | 2,250 | 2,810 | 3,380 | 3,940 | 4,500 | 5,060 | 5,630 | 6,190 | 6,750 | | |
| 7/8 | .6013 | 3,610 | 7,220 | 2,630 | 3,280 | 3,940 | 4,590 | 5,250 | 5,910 | 6,560 | 7,220 | 7,880 | 8,530 | 9,190 |
| 1 | .7854 | 4,710 | 9,420 | 3,000 | 3,750 | 4,500 | 5,250 | 6,000 | 6,750 | 7,500 | 8,250 | 9,000 | 9,750 | 10,500 |
| | | At 7,500 Lb. | | Bearing Values at 15,000 Lb. per Sq. In. | | | | | | | | | | |
| 1/2 | .1963 | 1,470 | 2,940 | 1,880 | 2,340 | 2,810 | 3,280 | 3,750 | | | | | | |
| 5/8 | .3068 | 2,300 | 4,600 | 2,340 | 2,930 | 3,520 | 4,100 | 4,690 | 5,280 | 5,860 | | | | |
| 3/4 | .4418 | 3,310 | 6,630 | 2,810 | 3,520 | 4,220 | 4,920 | 5,630 | 6,330 | 7,030 | 7,720 | 8,440 | | |
| 7/8 | .6013 | 4,510 | 9,020 | 3,280 | 4,100 | 4,920 | 5,740 | 6,560 | 7,380 | 8,200 | 9,030 | 9,850 | 10,670 | 11,480 |
| 1 | .7854 | 5,890 | 11,780 | 3,750 | 4,690 | 5,630 | 6,560 | 7,500 | 8,440 | 9,380 | 10,310 | 11,250 | 12,190 | 13,130 |
| | | At 9,000 Lb. | | Bearing Values at 18,000 Lb. per Sq. In. | | | | | | | | | | |
| 1/2 | .1963 | 1,770 | 3,530 | 2,250 | 2,810 | 3,380 | 3,940 | 4,500 | | | | | | |
| 5/8 | .3068 | 2,760 | 5,520 | 2,810 | 3,520 | 4,220 | 4,920 | 5,630 | 6,330 | 7,030 | | | | |
| 3/4 | .4418 | 3,980 | 7,950 | 3,380 | 4,220 | 5,060 | 5,910 | 6,750 | 7,590 | 8,440 | 9,280 | 10,130 | | |
| 7/8 | .6013 | 5,410 | 10,820 | 3,940 | 4,920 | 5,910 | 6,890 | 7,880 | 8,860 | 9,840 | 10,830 | 11,810 | 12,800 | 13,780 |
| 1 | .7854 | 7,070 | 14,140 | 4,500 | 5,630 | 6,750 | 7,880 | 9,000 | 10,130 | 11,250 | 12,380 | 13,500 | 14,630 | 15,750 |
| | | At 11,000 Lb. | | Bearing Values at 22,000 Lb. per Sq. In. | | | | | | | | | | |
| 1/2 | .1963 | 2,160 | 4,320 | 2,750 | 3,440 | 4,130 | 4,810 | 5,500 | | | | | | |
| 5/8 | .3068 | 3,370 | 6,750 | 3,440 | 4,300 | 5,160 | 6,020 | 6,880 | 7,740 | 8,600 | | | | |
| 3/4 | .4418 | 4,860 | 9,720 | 4,130 | 5,160 | 6,190 | 7,220 | 8,250 | 9,280 | 10,320 | 11,340 | 12,380 | | |
| 7/8 | .6013 | 6,610 | 13,230 | 4,810 | 6,020 | 7,220 | 8,430 | 9,630 | 10,810 | 12,040 | 13,240 | 14,440 | 15,640 | 16,840 |
| 1 | .7854 | 8,640 | 17,280 | 5,500 | 6,880 | 8,250 | 9,630 | 11,000 | 12,380 | 13,750 | 15,130 | 16,500 | 17,880 | 19,250 |

shearing stress, or $R = sA$, the number $n$ of rivets required to transfer a stress $T$ by single shear is

$$n = \frac{T}{R} = \frac{T}{As}$$

In Fig. 2, the rivet is subjected to shear on two sections, $d$ and $e$, and it is said to be in *double shear*. The amount of stress that one rivet can carry in double shear is twice that of one in single shear, and, using the preceding notation,

$$n = \frac{T}{2R}$$

The *bearing value* of a rivet is the compressive stress induced by the rivet in bearing on the plate, and is also calculated by the simple-stress formula

$$P = sA$$

in which

$P$ = value of rivet in bearing;
$s$ = unit working stress in bearing;
$A$ = bearing area.

It is customary to assume that the bearing area $A$ is the thickness of the plate multiplied by the diameter of the rivet. In calculating the required number of rivets, both the shearing and the bearing value of one rivet are determined and the critical value (the smaller) used.

The accompanying table gives the shearing and bearing values of rivets, in pounds, for different values of the working stress.

3. **Strength of Cylindrical Shells and Pipes With Thin Walls.**—When a cylinder is subjected to internal pressure, the tensile stress developed in the walls or shell of the cylinder is called *circumferential stress*, or *hoop tension*.

Let
$s$ = intensity of this stress;
$d$ = internal diameter of cylinder;
$p$ = intensity of pressure on inner surface of the cylinder;
$t$ = thickness of shell.

$$t = \frac{pd}{2s} \qquad s = \frac{pd}{2t}$$

The first formula serves to compute the thickness when $p$, $d$, and $s$ (working stress) are given; and the second one is used to compute the intensity of stress when the intensity of pressure $p$ and the dimensions of the cylinder are given.

EXAMPLE.—What should be the thickness of walls of a cast-iron water pipe, inside diameter 24 in., to resist a water pressure of 200 lb. per sq. in., using a unit working stress of 2,000 lb.?

SOLUTION.—Here, $d = 24$, $p = 200$, and $s = 2,000$. Substituting in the formula for $t$,

$$t = \frac{200 \times 24}{2 \times 2000} = 1.2 \text{ in.}$$

4. **Temperature Stresses.**—If a bar subjected to change of temperature is constrained so that it can neither expand nor contract, the constraint exerts on it a force sufficient to prevent the deformation. This causes in the bar a corresponding stress called *temperature stress*. It is compressive when the change of temperature is a rise, and tensile when a fall.

Let $T$ be the stress induced in a bar, whose area is $a$, by a rise or fall of $t°$; also let $c$ be the coefficient of expansion and $E$ the modulus of elasticity of the material. Then, $T = ctaE$

The coefficient of expansion for a number of substances is given in the section on Heat, Fuels, Etc.

EXAMPLE.—A wrought-iron bar 1.5 in. square has its ends fastened to firm supports. What is the stress produced in it by a change of 50° in its temperature?

SOLUTION.—Here, $E = 25,000,000$; $a = 1.5 \times 1.5 = 2.25$ sq. in., and $t = 50$; and, $c = .00000686$. Substituting in the formula, $T = .00000686 \times 50 \times 2.25 \times 25,000,000 = 19,294$ lb.

# BEAMS

A *beam* is a body resting upon supports and liable to transverse stress. Beams are designated by the number and location of the supports, and may be simple, cantilever, fixed, or continuous.

A *simple beam* is one that is supported at each end, the distance between its supports being the *span*.

A *cantilever* is a beam that has one or both ends overhanging the support; or a beam that has one end firmly fixed and the other end free.

A *fixed beam* is one that has both ends firmly secured.

A *continuous* beam is one which rests upon more than two supports.

**Reactions.**—The loads acting on a beam are balanced by the reactions or supporting forces; their sum must therefore be equal to the sum of the loads. To find any reaction, as $R_2$, at $B$, Fig. 1, take moments of all the external forces about the other support $A$ and divide their sum by the span. With reference to Fig. 1,

$$R_2 = \frac{W_1 a + W_2 b + W_3 c}{l}$$

The reaction $R_1$ can be found in a similar manner by taking moments about the support $B$. Their sum $R_1 + R_2$ must be equal to the sum of loads $W_1 + W_2 + W_3$.
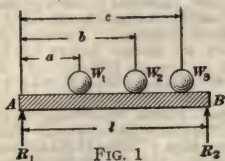


FIG. 1
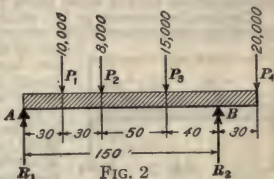


FIG. 2

EXAMPLE.—Find the reactions of a cantilever bridge loaded as shown in Fig. 2.

SOLUTION.—Substituting given values in the formula and noting that the moment of $P_4$ about $B$ is of opposite sign to the moments of the other loads,

$$R_1 = \frac{10,000 \times 120 + 8,000 \times 90 + 15,000 \times 40 - 20,000 \times 30}{150} = 12,800 \text{ lb.}$$

and

$$R_2 = \frac{10,000 \times 30 + 8,000 \times 60 + 15,000 \times 110 + 20,000 \times 180}{150} = 40,200 \text{ lb.}$$

The sum of the loads is $10,000 + 8,000 + 15,000 + 20,000 = 53,000$. The sum of the reactions is $40,200 + 12,800 = 53,000$.

**External Shear and Bending Moment.**—The forces acting on a beam tend, on the one hand, to shear its fibers vertically and, on the other hand, to bend it, producing compressional stresses in the fibers on one side of the neutral axis and tensional on the other side. The tendency to shear the fibers vertically is determined by the external shear, and that of bending by the bending moment. For brevity, external shear is often called simply *shear*, but it must not be confused with shearing stress at the section.

Forces acting upwards are considered positive, and those acting downwards, negative. The *external shear* at any section of a beam is the algebraic sum of all the external forces (loads and reactions) on one side of the section. It is equal to either reaction minus the sum of the loads between that reaction and the section considered. The maximum shear is always equal to the greater reaction. For a simple beam with a uniformly distributed load, the maximum shear is at the supports, and is equal to one-half the load, or to the reaction; the shear changes at every point of the loaded length, the minimum shear being zero at the center of the span. The maximum shear in a simple beam having a single load concentrated at the center is equal to one-half the load, and is uniform throughout the beam. Where a beam supports several concentrated loads, changes in the amount of shear occur only at the points where the loads are applied.

The external shear is resisted by the internal shear, or shearing stress, of the beam, which is numerically equal to the external shear. If the external shear is denoted by $V$, and the area of the cross-section by $A$, the average intensity of shearing stress in the section is $\frac{V}{A}$. This shearing stress is not uniformly distributed, and in beams of rectangular cross-section, the maximum intensity of shearing stress is $\frac{3V}{2A}$. Hence, a rectangular beam must be so designed that this value will not exceed the working shearing strength of the material.

In metallic beams with thin webs (plate girders), the shearing stress may be considered as uniformly distributed over the cross-section of the web. There is, also, at every horizontal or longitudinal section of the beam, a horizontal shearing stress the intensity of which at any point is equal to the intensity of the vertical shearing stress at that point.

Although the maximum intensity of shearing stress, both horizontal and vertical, in wooden beams is usually small, the shearing strength of wood along the grain is also small. As the horizontal external shear usually acts along the grain, the safe load for a wooden beam may depend on its shearing strength and not on its bending strength. For instance, the safe load for a beam 4 in. × 12 in. and 4 ft. long is 16,000 lb., uniformly distributed, when based on a fiber strength of 1,000 lb. per sq. in. Such a load will produce a shearing stress per unit of area equal to $\frac{3 \times 8,000}{2 \times 48} = 250$ lb. per sq. in., which exceeds the working shearing stress for the wood along the grain by about 100 lb. per sq. in.

The *bending moment* at any section of a loaded beam is equal to the algebraic sum of the moments of all the external forces (loads and reactions) to the right or



FIG. 3

left of the section about that section. For example, the bending moments at several points on the beam shown in Fig. 3 are as follows: At $W_1 = R_1a$; at $W_2 = R_1(a+b) - W_1b$; at $W_3 = R_1(a+b+c) - [W_2c + W_1(b+c)]$, or $R_2d$.

The bending moment varies, depending on the shear, and attains a maximum value at the point where the shear changes sign. If the loads are concentrated at several points, the maximum bending moment will be under the load at which the sum of all the loads between one support up to and including the load in question first becomes equal to, or greater than, the reaction at the support. Hence, to find the maximum bending moment in any simple beam:

**Rule.**—*Compute the reactions and determine the point where the shear changes sign. Calculate the moment about this point of either reaction, and of each load between the reaction and the point, and subtract the sum of the latter moments from the former.*

**EXAMPLE.**—What is the maximum bending moment of the beam loaded as shown in Fig. 4?

**SOLUTION.**—The reactions due to the uniform load are equal to one-half of the load; those due to the concentrated loads are computed by the principle given under Reactions. Both added give $R_1 = 18,170$ lb. and $R_2 = 14,330$ lb. Beginning at $R_1$ and sub-



FIG. 4

tracting the loads in succession, it is found that the shear just to the left of the load $d$ is $18,170 - 16,500$; and just to right of the load $d$ it becomes negative. Hence, the shear changes sign under the load $d$ and the bending moment is maximum at that point. It is equal to

$$18,170 \times 13 - 10,000 \times 7 - \frac{13^2 \times 500}{2} = 123,960 \text{ ft.-lb.}$$

Formulas for the maximum bending moments and shears for beams loaded and supported in different ways are given in the following table.

For a beam supporting moving loads, the maximum bending moment occurs:

1. For a single load, when the load is at the middle of the span.

2. For two equal loads, under either load, when the two loads are on opposite sides of the center, and one of the loads is at a distance from the center equal to one-fourth the distance between the loads.

3. For two unequal loads, under the heavier load, when that load and the center of gravity of the two loads are equidistant from the center of gravity of the beam.

**EXAMPLE.**—A beam 24 ft. long supports two moving loads 6 ft. apart. The left-hand load is 8,000 lb., and the right-hand load is 4,000 lb. Find the maximum bending moment.

**SOLUTION.**—The center of gravity of the loads is 2 ft. from the left-hand load. The maximum bending moment occurs under the heavy load, and obtains when the latter is 1 ft. to the left of the center of the beam. The left reaction is, then, $\frac{12,000 \times 11}{24} = 5,500$ lb., and the maximum bending moment is $5,500 \times 11 = 60,500$ ft.-lb.

**Designing of Beams.**—In every section of a carrying beam there is induced an internal moment called the *moment of resistance*, which is equal to the bending
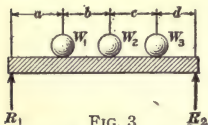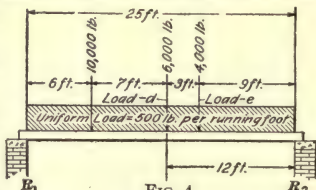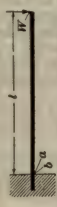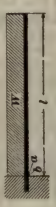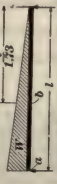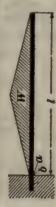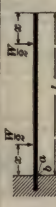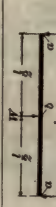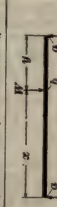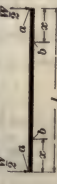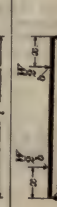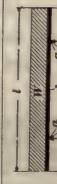
## FORMULAS FOR MAXIMUM SHEAR AND BENDING MOMENTS OF BEAMS

| Case | Method of Loading | Maximum Shear | Maximum Moment | Case | Method of Loading | Maximum Shear | Maximum Moment |
|------|-------------------|---------------|----------------|------|-------------------|---------------|----------------|
| I | (diagram) | $W$ | $Wl$ | X | (diagram) | $\dfrac{W}{2}$ | $\dfrac{Wl}{8}$ |
| II | (diagram) | $W$ | $\dfrac{Wl}{2}$ | XI | (diagram) | $\dfrac{W}{2}$ | $\dfrac{Wl}{6}$ |
| III | (diagram) | $W$ | $\dfrac{Wl}{3}$ | XII | (diagram) | $\dfrac{2W}{3}$ | $\dfrac{52Wl}{405}$ |
| IV | (diagram) | $W$ | $\dfrac{2Wl}{3}$ | XIII | (diagram) | $\tfrac{11}{16}W$ | $\tfrac{3}{16}Wl$ |
| V | (diagram) | $W$ | $\dfrac{Wl}{2}$ | XIV | (diagram) | $\tfrac{5}{8}W$ | $\dfrac{Wl}{8}$ |
| VI | (diagram) | $W$ | $\dfrac{Wl}{4}$ | XV | (diagram) | $\dfrac{W}{2}$ | $\dfrac{Wl}{8}$ |
| VII | (diagram) | $\dfrac{W}{2}$<br>$Wy$ or $Wx\dfrac{l}{l}$ | $\dfrac{Wxy}{l}$ | XVI | (diagram) | $\dfrac{W}{2}$ | $\dfrac{Wl}{12}$ |
| VIII | (diagram) | | | XVII | (diagram) | $\dfrac{Wx}{2}$ | $\dfrac{Wx}{2}$ |
| IX | (diagram) | $\dfrac{W}{2}$ | $\dfrac{Wx}{2}$ | XVIII | (diagram) | $\dfrac{Wx}{l}$ or $W\left(\dfrac{l-2x}{2l}\right)$ | $\dfrac{Wx^2}{2l}$ or $W\left(\dfrac{l}{4}-x\right)$ or $W\dfrac{l}{2}\left(\dfrac{l}{4}-x\right)$ |

moment at that section. As previously explained, the resisting moment is equal to $\frac{I}{c}f$; and, if the maximum bending moment is denoted by $M$, $M = \frac{I}{c}f$; whence,

$$\frac{M}{f} = \frac{I}{c}$$

which is the fundamental formula for the designing of beams; $f$ is the working stress in flexure, which is the modulus of rupture divided by a suitable factor of safety.

The *modulus of rupture*, also called the *ultimate strength of flexure*, is the extreme fiber stress that a material subjected to bending can withstand. Its value is intermediate between the ultimate strength in compression and tension. The accompanying tables give the average values of the modulus of rupture for a number of materials.

When a beam is to be designed to carry certain loads, the maximum bending moment is determined and divided by $f$. The latter is usually given or is found by dividing the modulus of rupture of the material by a suitable factor of safety. The problem then reduces itself to the finding of a section that has a value of $\frac{I}{c}$, the section modulus, equal to $\frac{M}{f}$. For rolled-steel sections, the value of $\frac{I}{c}$ can be taken from a manufacturer's handbook. For a rectangular section,

$$\frac{I}{c} = \frac{bd^2}{6}$$

$b$ being the breadth and $d$ the depth of the section. As the expression contains two unknown quantities $b$ and $d$, a value for either one may be assumed and substituted, and the formula solved for the other. If a built-up beam is used the section has to be found by trial; a suitable section is first assumed and its section modulus is computed by the principles given under the heading Moment of Inertia; if necessary, it is modified until it is equal to $\frac{M}{f}$.

EXAMPLE.—Design both a rolled-steel **I** beam and a solid wooden beam 10 ft long, each to carry a uniform load of 250 lb. per ft. in addition to a central load of 2,000 lb., assuming for wood a working stress of 1,000 lb. per sq. in. and for steel 15,000 lb. per sq. in.

SOLUTION.—The maximum bending moment occurs at the middle of the beam and is equal to the sum of the moments due to the uniform load and the central load; therefore,

$$M = \frac{2,000 \times 120}{4} + \frac{250 \times 10 \times 120}{8} = 97,500 \text{ in.-lb.}$$

For a steel beam, $\frac{M}{f} = \frac{97,500}{15,000} = 6.5$. From a manufacturer's handbook, a 6-in. **I** beam of 12.25 lb. has a section modulus of 7.3 and can therefore be used. For a wooden beam, $\frac{M}{f} = \frac{97,500}{1,000} = 97.5 = \frac{bd^2}{6}$, assuming that $b = 6$ in., $d = \sqrt{97.5} = 10$ in. nearly.
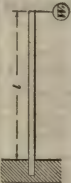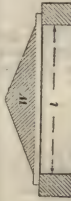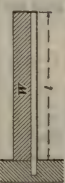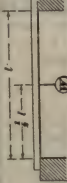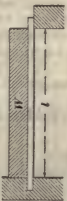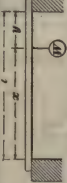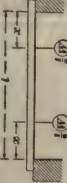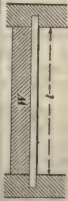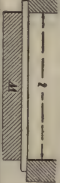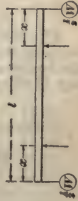
**Stiffness.**—When designing a beam, it is sometimes necessary to ascertain the amount that it will deflect under given loads. This, for instance, is the case when designing supports for machinery parts or joists for plastered ceilings, in which latter case the deflections should not exceed $\frac{1}{360}$ of the span. The following table gives deflection formulas for the most usual cases. In these formulas $l$ is the span, in inches; $W$, the total load acting on the beam; and $I$, the moment of inertia of the cross-section of the beam; and $E$, the modulus of elasticity of the material.

EXAMPLE 1.—A simple timber beam 10 ft. long, and having a width of 4 in. and a depth of 12 in., carries a uniform load of 400 lb. per ft. What is the deflection?

SOLUTION.—According to the table, the deflection for a uniformly distributed load is $\frac{5}{384}\frac{Wl^3}{EI}$. In this case, $l = 10 \times 12 = 120$; $W = 400 \times 10 = 4,000$; $E = 1,500,000$; and $I = \frac{4 \times 12^3}{12} = 576$. Substituting in the formula,

$$\text{Deflection} = \frac{5 \times 4,000 \times 120^3}{384 \times 1,500,000 \times 576} = .1 \text{ in.}$$

## FORMULAS FOR DEFLECTION OF BEAMS

| Case | Method of Loading | Deflection Inches | Case | Method of Loading | Deflection Inches |
|---|---|---|---|---|---|
| I |  | $\dfrac{W l^3}{3\,EI}$ | IX |  | $\dfrac{W l^3}{60\,EI}$ |
| II |  | $\dfrac{W l^3}{8\,EI}$ | X |  | $\dfrac{47\,W l^3}{3{,}600\,EI}$ |
| III |  | $\dfrac{W l^3}{15\,EI}$ | XI |  | $\dfrac{3\,W l^3}{322\,EI}$ |
| IV |  | $\dfrac{W l^3}{48\,EI}$ | XII |  | $\dfrac{5\,W l^3}{926\,EI}$ |
| V |  | $\dfrac{Wxy(2l-x)\sqrt{3x(2l-x)}}{27lEI}$ | XIII |  | $\dfrac{W l^3}{192\,EI}$ |
| VI |  | $\dfrac{Wx}{48\,EI}(3l^2-4x^2)$ | XIV |  | $\dfrac{W l^3}{384\,EI}$ |
| VII |  | $\dfrac{5\,W l^3}{384\,EI}$ | XV |  | For overhang: $\dfrac{Wx}{12\,EI}(3xl-4x)^2$ For part between supports: $\dfrac{Wx}{16\,EI}(l-2x)^2$ |
| VIII |  | $\dfrac{3\,W l^3}{320\,EI}$ | | | |

## COLUMNS

The strength of a compression member depends on the ratio of its length to its least lateral dimension, or, what is the same thing, on the *ratio of slenderness;* that is, the ratio of its length to its radius of gyration.

For compression members whose ratio of slenderness does not exceed 30, the formula $s = \dfrac{P}{A}$, for simple stress, may be used.

When this ratio exceeds 30, but is not more than 150, $s$ should be deduced from *Rankine's formula,*

$$s = \frac{s_u}{1 + \dfrac{k_1 l^2}{r^2}}$$

in which
$\qquad s_u$ = ultimate strength in compression;
$\qquad l$ = length;
$\qquad r$ = radius of gyration.
$\qquad k_1$ = coefficient from table.

The ultimate strength in compression $s_u$ should be divided by a suitable factor of safety. Both $l$ and $r$ are expressed in the same unit. The values of $k_1$, which depend on the material of the column and the condition of its ends—that is whether fixed or round—are given in the following table:

### VALUES OF $k_1$ (RANKINE'S FORMULA)

| Material | Both Ends Flat or Fixed | One End Round | Both Ends Round |
|---|---|---|---|
| Cast iron..... | $\dfrac{1}{5,000}$ | $\dfrac{1.78}{5,000}$ | $\dfrac{4}{5,000}$ |
| Wrought iron. | $\dfrac{1}{36,000}$ | $\dfrac{1.78}{36,000}$ | $\dfrac{4}{36,000}$ |
| Steel......... | $\dfrac{1}{25,000}$ | $\dfrac{1.78}{25,000}$ | $\dfrac{4}{25,000}$ |
| Wood....... | $\dfrac{1}{3,000}$ | $\dfrac{1.78}{3,000}$ | $\dfrac{4}{3,000}$ |

When the value of $\dfrac{l}{r}$ exceeds 150, *Euler's formula,* which is given later, should be used.

The *straight line formula* is more convenient for determining the value of $s$, and is now in extensive use. It is only approximate, giving values of $s$ that differ somewhat from those obtained by Rankine's formula; but the difference is on the side of safety. For the same notation as before, the straight-line formula is
$$s = s_u - k \frac{l}{r}$$

### CONSTANTS FOR THE STRAIGHT-LINE AND EULER'S FORMULAS

| | Medium Steel | | Wrought Iron | | Cast Iron |
|---|---|---|---|---|---|
| | Flat Ends | Pin Ends | Flat Ends | Pin Ends | Flat Ends |
| $s_u$ ......... | 52,500 | 52,500 | 42,000 | 42,000 | 80,000 |
| $k$ ............ | 179 | 220 | 128 | 157 | 438 |
| limit of $\dfrac{l}{r}$ ...... | 195 | 159 | 218 | 178 | 122 |
| $nE\pi^2$ ........ | 666 m | 444 m | 666 m | 444 m | 395 m |

The values of $s_u$ and $k$ are given in the accompanying table, in which will also be found the limit of $\frac{l}{r}$ within which the formula may be used. When $\frac{l}{r}$ exceeds this limit, Euler's formula, which follows, should be used.

EXAMPLE.—What is the ultimate strength per square inch of a medium-steel column 25 ft. long both ends of which are fixed and the radius of gyration of which is 2.5?

## SAFE LOADS FOR HOLLOW, CYLINDRICAL, CAST-IRON COLUMNS
### (*The Carnegie Steel Co., Limited*)

| Outside Diameter Inches | Thickness of Metal Inches | Length of Columns in Feet | | | | | | | | | Sectional Area Inches | Weight of Columns per Foot of Length Pounds |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | | |
| | | Safe Load, in Tons of 2,000 Lb. | | | | | | | | | | |
| 6 | ½ | 26.2 | 23.0 | 20.1 | 17.5 | 15.2 | 13.2 | 11.5 | | | 8.6 | 26.95 |
| 6 | ¾ | 37.5 | 33.0 | 28.8 | 25.0 | 21.7 | 18.9 | 16.5 | | | 12.4 | 38.59 |
| 6 | ⅞ | 42.7 | 37.6 | 32.8 | 28.5 | 24.7 | 21.5 | 18.8 | | | 14.1 | 43.96 |
| 6 | 1 | 47.6 | 41.9 | 36.5 | 31.8 | 27.6 | 24.0 | 21.0 | | | 15.7 | 49.01 |
| 6 | 1⅛ | 52.2 | 46.0 | 40.1 | 34.8 | 30.2 | 26.3 | 23.0 | | | 17.2 | 53.76 |
| 7 | ¾ | 47.7 | 43.1 | 38.5 | 34.3 | 30.4 | 26.9 | 23.9 | 21.2 | 18.9 | 14.7 | 45.96 |
| 7 | 1 | 61.1 | 55.2 | 49.3 | 43.8 | 38.9 | 34.4 | 30.6 | 27.1 | 24.2 | 18.9 | 58.90 |
| 7 | 1⅛ | 67.2 | 60.8 | 54.3 | 48.3 | 42.8 | 37.9 | 33.7 | 29.9 | 26.7 | 20.8 | 64.77 |
| 8 | ¾ | 57.9 | 53.3 | 48.6 | 44.1 | 39.7 | 35.8 | 32.2 | 28.9 | 26.1 | 17.1 | 53.29 |
| 8 | 1 | 74.6 | 68.7 | 62.5 | 56.7 | 51.1 | 46.0 | 41.4 | 37.3 | 33.6 | 22.0 | 68.64 |
| 8 | 1¼ | 89.9 | 82.8 | 75.5 | 68.4 | 61.7 | 55.5 | 49.9 | 44.9 | 40.5 | 26.5 | 82.71 |
| 9 | ¾ | 68.1 | 63.6 | 58.9 | 54.2 | 49.6 | 45.2 | 41.2 | 37.5 | 34.1 | 19.4 | 60.65 |
| 9 | 1 | 88.0 | 82.3 | 76.2 | 70.0 | 64.1 | 58.4 | 53.2 | 48.4 | 44.1 | 25.1 | 78.40 |
| 9 | 1¼ | 106.6 | 99.6 | 92.2 | 84.8 | 77.6 | 70.8 | 64.4 | 58.7 | 53.4 | 30.4 | 94.94 |
| 9 | 1½ | 123.8 | 115.7 | 107.1 | 98.5 | 90.1 | 82.2 | 74.8 | 68.1 | 62.0 | 35.3 | 110.26 |
| 9 | 1¾ | 139.6 | 130.5 | 120.8 | 111.1 | 101.6 | 92.7 | 84.4 | 76.8 | 69.9 | 39.9 | 124.36 |
| 10 | 1 | 101.4 | 95.9 | 89.8 | 83.6 | 77.4 | 71.5 | 65.8 | 60.5 | 55.5 | 28.3 | 88.23 |
| 10 | 1¼ | 123.3 | 116.5 | 109.1 | 101.6 | 94.1 | 86.8 | 79.9 | 73.4 | 67.5 | 34.4 | 107.23 |
| 10 | 1½ | 143.7 | 135.8 | 127.3 | 118.5 | 109.7 | 101.2 | 93.2 | 85.6 | 78.7 | 40.1 | 124.99 |
| 10 | 1¾ | 162.7 | 153.8 | 144.1 | 134.1 | 124.2 | 114.6 | 105.5 | 97.0 | 89.1 | 45.4 | 141.65 |
| 11 | 1 | 114.8 | 109.4 | 103.5 | 97.3 | 91.0 | 84.8 | 80.2 | 73.1 | 67.7 | 31.4 | 98.03 |
| 11 | 1¼ | 139.9 | 133.3 | 126.1 | 118.6 | 110.9 | 103.3 | 97.8 | 89.4 | 82.5 | 38.3 | 119.46 |
| 11 | 1½ | 163.5 | 155.9 | 147.5 | 138.6 | 128.7 | 120.8 | 114.3 | 104.1 | 96.4 | 44.8 | 139.68 |
| 11 | 1¾ | 185.7 | 177.1 | 167.5 | 157.4 | 147.3 | 137.2 | 128.8 | 118.3 | 109.5 | 50.9 | 158.68 |
| 11 | 2 | 206.6 | 196.9 | 186.3 | 175.1 | 163.8 | 152.6 | 144.4 | 131.5 | 121.8 | 56.6 | 176.44 |
| 12 | 1 | 128.0 | 122.9 | 117.2 | 111.0 | 104.7 | 98.4 | 92.2 | 86.1 | 80.4 | 34.6 | 107.51 |
| 12 | 1¼ | 156.4 | 150.1 | 143.1 | 135.7 | 127.9 | 120.2 | 112.6 | 105.2 | 98.2 | 42.2 | 131.41 |
| 12 | 1½ | 183.3 | 175.9 | 167.7 | 159.9 | 149.9 | 140.9 | 132.0 | 123.3 | 115.1 | 49.5 | 154.00 |
| 12 | 1¾ | 208.7 | 200.4 | 191.0 | 181.1 | 170.7 | 160.4 | 150.3 | 140.5 | 131.1 | 56.4 | 175.53 |
| 12 | 2 | 232.7 | 223.4 | 213.0 | 201.9 | 190.4 | 178.9 | 167.6 | 156.6 | 146.1 | 62.8 | 195.75 |
| 13 | 1 | 141.2 | 136.3 | 130.7 | 124.7 | 118.5 | 112.1 | 105.8 | 99.5 | 93.5 | 37.7 | 117.53 |
| 13 | 1¼ | 172.8 | 166.8 | 160.0 | 152.7 | 145.0 | 137.2 | 129.4 | 121.8 | 114.4 | 46.1 | 143.86 |
| 13 | 1½ | 203.0 | 195.9 | 187.9 | 179.3 | 170.3 | 161.1 | 152.0 | 143.1 | 134.3 | 54.2 | 168.98 |
| 13 | 1¾ | 231.6 | 223.6 | 214.5 | 204.7 | 194.4 | 183.9 | 173.5 | 163.3 | 153.3 | 61.9 | 192.88 |
| 13 | 2 | 258.9 | 249.9 | 239.7 | 228.7 | 217.3 | 205.5 | 193.9 | 182.5 | 171.3 | 69.1 | 215.56 |
| 14 | 1 | 154.3 | 149.6 | 144.3 | 138.5 | 132.3 | 125.9 | 119.5 | 113.1 | 106.8 | 40.8 | 127.60 |
| 14 | 1¼ | 189.2 | 183.4 | 176.9 | 169.7 | 162.2 | 154.4 | 146.5 | 138.6 | 131.0 | 50.1 | 156.31 |
| 14 | 1½ | 222.6 | 215.8 | 208.1 | 199.7 | 190.8 | 181.7 | 172.3 | 163.1 | 154.1 | 58.9 | 183.67 |
| 14 | 1¾ | 254.4 | 246.7 | 237.9 | 228.3 | 218.1 | 207.6 | 197.0 | 186.5 | 176.2 | 67.4 | 210.00 |
| 14 | 2 | 284.8 | 276.2 | 266.4 | 255.6 | 244.2 | 232.4 | 220.6 | 208.8 | 197.2 | 75.4 | 235.12 |
| 15 | 1 | 167.4 | 162.9 | 157.8 | 152.1 | 146.0 | 139.7 | 133.3 | 126.8 | 120.4 | 44.0 | 137.28 |
| 15 | 1¼ | 205.5 | 200.0 | 193.7 | 186.7 | 179.3 | 171.5 | 163.6 | 155.7 | 147.9 | 54.0 | 168.48 |
| 15 | 1½ | 242.1 | 235.7 | 228.2 | 220.0 | 211.2 | 202.1 | 192.8 | 183.5 | 174.2 | 63.6 | 198.74 |
| 15 | 1¾ | 277.2 | 269.8 | 261.3 | 251.9 | 241.9 | 231.4 | 220.7 | 210.1 | 199.5 | 72.9 | 227.45 |
| 15 | 2 | 310.8 | 302.5 | 293.0 | 282.5 | 271.2 | 259.5 | 247.5 | 235.5 | 223.6 | 81.7 | 254.90 |

Solution.—By the straight-line formula,

$$s = 60,000 - 179 \times \frac{25 \times 12}{2.5} = 38,520 \text{ lb. per sq. in.}$$

Using Rankine's formula,

$$S = \frac{60,000}{1 + \frac{(25 \times 12)^2}{25,000 \times 2.5^2}} = 38,070 \text{ lb. per sq. in.}$$

**Euler's Formula.**—Structural members in compression whose ratio of slenderness exceeds 150 should preferably not be used. Sometimes, however, long columns cannot be avoided, and when $\frac{l}{r}$ exceeds the limits for which the preceding formulas may be applied, Euler's formula should be used. This formula is as follows:

$$\frac{P}{A} = \frac{n\pi^2 E}{\left(\frac{l}{r}\right)^2}$$

in which       $E$ = modulus of elasticity of material;

                $n$ = constant.

The value of the constant $n$ depends on the end condition; it has the value of 1 for columns with both ends pivoted and 4 for columns with both ends fixed. The table of constants on page 180 gives the values of $n\pi^2 E$, expressed in millions of pounds.

**Formula for Wooden Columns.**—The formula for determining the strength of wooden columns having flat or square ends was deduced from exhaustive tests of full-size specimens, made at the Watertown Arsenal, Mass., and may be expressed as follows:

$$S = U - \frac{Ul}{100d}$$

in which   $S$ = ultimate strength of column, per square inch of section;

           $U$ = ultimate compressive strength of material, per square inch;

           $l$ = length of column, in inches;

           $d$ = dimension of least side of column, in inches.

This formula may be applied to all wooden columns, the length or height of which is not under 10 times nor over 45 times the dimension of the least side. In other words, $\frac{l}{d}$ should not be less than 10 nor more than 45. If the length is less than 10 times the least side, the direct compressive strength of the material per square inch, multiplied by the sectional area of the column in square inches, will give the strength of the column. If the length is over 45 times the least side, Rankine's formula should be used.

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# COMBINED STRESSES

**Bending Combined With Compression or Tension.**—Assume that $P$ is the axial force acting on the beam; $M$, the maximum bending moment to which the beam is subjected; $A$, the cross-sectional area of the beam; $I$, its moment of inertia; and $c$, the distance from the neutral axis of the most distant fiber, having the same kind of stress (tension or compression) as that caused by $P$. Then, the working stress should not exceed

$$s = \frac{P}{A} + \frac{Mc}{I}$$

In case of compression, $s$ should, in addition, be deduced from one of the compression formulas previously given.

The preceding formula for $s$ is the one commonly used in practice, but it is only approximate. When more accurate results are required, the following formula should be used,

$$s = \frac{P}{A} + \frac{Mc}{I \pm k\frac{Pl^2}{E}}$$

Here, $l$ is the span; $E$, the modulus of elasticity, and $k$, a constant having the following values:

*Value of k*

For a cantilever loaded at end.............................. $\frac{1}{8}$
For a cantilever loaded uniformly........................... $\frac{1}{4}$
For a beam supported at both ends and loaded at center..... $\frac{1}{12}$
For a beam supported at both ends and loaded uniformly...... $\frac{5}{48}$
For a beam fixed at both ends and loaded at center.......... $\frac{1}{16}$
For a fixed beam uniformly loaded.......................... $\frac{1}{32}$

The minus sign before $k$ is for the case when the direct stress is compressive, and the plus sign, when it is tensile.

## STRENGTH OF HEMP AND MANILA ROPES AND OF CHAINS

**Ropes.**—If $C$ is the circumference of a rope, in inches, and $P$ the working load, in pounds, then, for hemp and manila rope,

$$P = 10C^2$$

This formula gives a factor of safety of from $7\frac{1}{2}$ for manila or tarred hemp rope to about 11 for the best three-strand hemp rope.

For iron-wire rope of seven strands, nineteen wires to a strand,

$$P = 600C^2$$

and for the best steel-wire rope of seven strands, nineteen wires to the strand,

$$P = 1,000C^2$$

The last two formulas are based on a factor of safety of 6.

**Chains.**—If $P$ is the safe load, in pounds, and $d$ the diameter of link, in inches, then, for open-link chains made from a good quality of wrought iron,

$$P = 12,000d^2$$

and for stud-link chains, $P = 18,000d^2$

**Chain Cables.**—The strength of a chain link is less than twice that of a straight bar of a sectional area equal to that of one side of the link. A weld exists at one end and a bend at the other, each requiring at least one heat, which produces a decrease in the strength. The report of the committee of the U. S. Testing Board, on tests of wrought-iron and chain cables, contains the following conclusions:

"That, beyond doubt, when made of American bar iron, with cast-iron studs, the studded link is inferior in strength to the unstudded one.

"That when proper care is exercised in the selection of material, the strength of chain cables will vary by about 5% to 17% of the resistance of the strongest. Without this care the variation may rise to 25%.

"That with proper material and construction the ultimate resistance of the chain may be expected to vary from 155% to 170% of that of the bar used in making the links, and show an average of about 163%.

"That the proof test of a chain cable should be about 50% of the ultimate resistance of the weakest link."

From a great number of tests of bars and unfinished cables, the committee considered that the average ultimate resistance and proof tests of chain cables made of the bars, whose diameters are given, should be such as are shown in the accompanying table.

### ULTIMATE RESISTANCE AND PROOF TESTS OF CHAIN CABLES

| Diameter of Bar Inches | Average Resistance = 163% of Bar Pounds | Proof Test Pounds | Diameter of Bar Inches | Average Resistance = 163% of Bar Pounds | Proof Test Pounds |
|---|---|---|---|---|---|
| 1 | 71,172 | 33,840 | $1\frac{9}{16}$ | 162,283 | 77,159 |
| $1\frac{1}{16}$ | 79,544 | 37,820 | $1\frac{5}{8}$ | 174,475 | 82,956 |
| $1\frac{1}{8}$ | 88,445 | 42,053 | $1\frac{11}{16}$ | 187,075 | 88,947 |
| $1\frac{3}{16}$ | 97,731 | 46,468 | $1\frac{3}{4}$ | 200,074 | 95,128 |
| $1\frac{1}{4}$ | 107,440 | 51,084 | $1\frac{13}{16}$ | 213,475 | 101,499 |
| $1\frac{5}{16}$ | 117,577 | 55,903 | $1\frac{7}{8}$ | 227,271 | 108,058 |
| $1\frac{3}{8}$ | 128,129 | 60,920 | $1\frac{15}{16}$ | 241,463 | 114,806 |
| $1\frac{7}{16}$ | 139,103 | 66,138 | 2 | 256,040 | 121,737 |
| $1\frac{1}{2}$ | 150,485 | 71,550 | | | |

# PRACTICAL PROBLEMS IN THE STRENGTH OF BEAMS AND PROPS

**To Find the Quiescent Breaking Load of a Horizontal Square or Rectangular Beam Supported at Both Ends and Loaded at the Middle.**—Multiply the breadth, in inches, by the square of depth, in inches; divide the product by distance in feet, between supports, and multiply the quotient by the constant given in the Table of Constants for Seasoned Timber. Take as the safe working load one-third of the breaking load.

**To Find the Quiescent Breaking Load of a Horizontal Cylindrical Beam.** Divide the cube of the diameter, in inches, by the distance between the supports, in feet, and multiply the quotient by the constant.

When the load is uniformly distributed on the beam, the results obtained by the foregoing rules should be doubled.

EXAMPLE 1.—Find the quiescent breaking load and safe working load of a yellow-pine collar 8 in. square, 12 ft. between legs.

SOLUTION.—Breaking load $= \dfrac{8 \times 8^2}{12} \times 500 = 21,333$ lb. for seasoned, and 10,666 lb. for green timber.

Safe working load $= 7,111$ lb. for seasoned, and 3,556 lb. for green timber.

EXAMPLE 2.—Find the quiescent breaking load, and the safe working load of a hemlock collar 10 in. diameter, 7 ft. between legs.

SOLUTION.—Breaking load $= \dfrac{10^3}{7} \times 236 = 33,714$ lb. for seasoned timber, and $33,714 \div 2 = 16,857$ lb. for green timber.

Safe working load $= 33,714 \div 3 = 11,238$ lb. for seasoned, and $33,714 \div 6$ or $11,238 \div 2 = 5,619$ lb. for green timber.

**To Find the Load a Rectangular Collar Will Support When Its Depth Is Increased.**—When the length and width remain constant, the load varies as the square of the depth.

EXAMPLE.—A rectangular collar 10 in. deep supports 15,000 lb. What will it support if its depth is increased to 12 in.?

SOLUTION.—Applying the rule just given $10^2 : 12^2 = 15,000 : 21,600.$

**Having the Length and Diameter of a Collar, to Find the Diameter of a Longer Collar to Support the Same Weight.**—For the same load, the strength of collars varies as the cubes of their diameters, and inversely as their lengths.

EXAMPLE.—If a collar 6 ft. long and 8 in. diameter supports a certain weight, what must be the diameter of a collar 12 ft. long to support the same weight?

## TABLE OF CONSTANTS FOR SEASONED TIMBER

| Woods | Constant | | Woods | Constant | |
|---|---|---|---|---|---|
|  | Square or Rectangular | Round |  | Square or Rectangular | Round |
| Ash, white......... | 650 | 383 | Locust........... | 600 | 353 |
| Ash, swamp....... | 400 | 236 | Lignum vitæ..... | 650 | 383 |
| Ash, black........ | 300 | 177 | Larch........... | 400 | 236 |
| Balsam, Canada... | 350 | 206 | Maple........... | 550 | 324 |
| Beech, white...... | 450 | 265 | Oak, red or black | 550 | 324 |
| Beech, red........ | 550 | 324 | Oak, white....... | 600 | 353 |
| Birch, black...... | 450 | 265 | Oak, live......... | 600 | 353 |
| Birch, yellow...... | 450 | 266 | Pine, white....... | 450 | 265 |
| Cedar, white...... | 250 | 147 | Pine, yellow...... | 500 | 295 |
| Chestnut.......... | 450 | 265 | Pine, pitch....... | 550 | 324 |
| Elm............... | 350 | 206 | Poplar........... | 550 | 324 |
| Elm, rock......... | 600 | 353 | Spruce........... | 450 | 265 |
| Hemlock.......... | 400 | 236 | Sycamore........ | 500 | 295 |
| Hickory.......... | 650 | 383 | Willow.......... | 350 | 206 |
| Ironwood ........ | 600 | 353 |  |  |  |

SOLUTION.—Applying the rule just given $\sqrt[3]{12}:\sqrt[3]{6}\ =8$ in. : **6.35 in.**

**Having the Loads of Two Beams of Equal Length and the Diameter of One. to Find the Diameter of the Other.**—When the lengths are equal, the diameters vary as the cube roots of the loads, or the cubes of the diameters vary as the loads.

EXAMPLE 1.—A beam 11 in. in diameter supports a load of 32,160 lb. What will be the diameter of another beam the same length, to support a load of 19,440 lb.?

SOLUTION.—Applying the rule just given

$$\sqrt[3]{32,160}:\sqrt[3]{19,440}=11:9.3.$$

EXAMPLE 2.—A beam 8 in. in diameter will support a load of 10,240 lb. What load will a beam the same length and 7 in. in diameter support?

SOLUTION.—Applying the rule just given $8^3:7^3=10,240:6,860.$

The preceding Table of Constants has been calculated for seasoned timber; for green timber, take one-half of these constants. The safe working load is one-third of the breaking load.

**To Find the Diameter of a Collar When the Weight Increases in Proportion to the Length.**—Find the required diameter to support the same weight as the short collar. Then the length of the short collar is to the length of the long one as the diameter found to support the original weight is to the required diameter.

EXAMPLE.—If a collar 6 ft. long, 8 in. in diameter, supports a certain weight, what must be the diameter of a collar 12 ft. long to support twice the weight?

SOLUTION.—From the rule just given, $1:2=\dfrac{8^3}{6}:\dfrac{(\quad)^3}{12}$, or $1:2=2\times8^3:(\quad)^3$,

or $\sqrt[3]{1}:\sqrt[3]{2}::8\sqrt[3]{2}:12.7.$

**To Find the Breaking Load of Either Square or Rectangular Wooden Pillars or Props.**—Call one side of the square or the least side of the rectangle the breadth. Divide the square of the length, in inches, by the square of the breadth, in inches, multiply the quotient by .004, add 1 to the product and divide the crushing load by the result. Then multiply this quotient by the number of square inches in the end of the prop

Or, breaking load in lb. $=\dfrac{\text{Crushing load}}{\left(\dfrac{l^2}{b^2}\times.004\right)+1}\times bd,$

in which
$l=$ length, in inches;
$b=$ breadth, in inches;
$d=$ depth, in inches

## CRUSHING LOADS OF WELL-SEASONED AMERICAN WOODS

| Wood | Crushing Load Pounds per Square Inch | Wood | Crushing Load Pounds per Square Inch |
|---|---|---|---|
| Ash.................. | 6,800 | Maple, sugar, black.... | 8,000 |
| Beech................ | 7,000 | Maple, white, red...... | 6,800 |
| Birch................ | 8,000 | Oak, white, red, black.. | 7,000 |
| Cedar, red........... | 6,000 | Oak, scrub, basket..... | 6,000 |
| Cedar, white......... | 4,400 | Oak, chestnut, live..... | 7,500 |
| Chestnut............. | 5,300 | Oak, pin.............. | 6,500 |
| Hemlock............. | 5,300 | Pine, white........... | 5,400 |
| Hickory............. | 8,000 | Pine, pitch........... | 5,000 |
| Linden.............. | 5,000 | Pine, Georgia......... | 8,500 |
| Locust, black, yellow... | 9,800 | Poplar............... | 5,000 |
| Locust, honey........ | 7,000 | Spruce, black......... | 5,700 |
| Maple, broad-leafed, Oregon............. | 5,300 | Spruce, white......... | 4,500 |
|  |  | Willow............... | 4,400 |

For green timber, take one-half of the crushing strength given in the foregoing table. The safe working load equals one-third of crushing load.

## SAFE LOADS UNIFORMLY DISTRIBUTED FOR STANDARD AND SPECIAL I BEAMS
(Tons of 2,000 Pounds.)

| Distance Between Supports, in Feet | 3" I 5.5 Lb. | Add for Every Pound Increase in Weight | 4" I 7.5 Lb. | Add for Every Pound Increase in Weight | 5" I 9.75 Lb. | Add for Every Pound Increase in Weight | 6" I 12.25 Lb. | Add for Every Pound Increase in Weight | 7" I 15 Lb. | Add for Every Pound Increase in Weight | 8" I 18 Lb. | Add for Every Pound Increase in Weight |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 1.76 | .16 | 3.18 | .21 | 5.16 | .26 | 7.75 | .31 | 11.04 | .36 | 15.17 | .42 |
| 6 | 1.47 | .13 | 2.65 | .18 | 4.30 | .22 | 6.46 | .26 | 9.20 | .30 | 12.64 | .35 |
| 7 | **1.26** | .11 | 2.27 | .15 | 3.69 | .19 | 5.54 | .22 | 7.89 | .26 | 10.84 | .30 |
| 8 | 1.10 | .10 | **1.99** | .13 | 3.23 | .16 | 4.84 | .19 | 6.90 | .23 | 9.48 | .26 |
| 9 | 0.98 | .09 | 1.77 | .12 | **2.87** | .14 | 4.31 | .17 | 6.13 | .20 | 8.43 | .23 |
| 10 | 0.88 | .08 | 1.59 | .11 | 2.58 | .13 | 3.88 | .16 | 5.52 | .18 | 7.59 | .21 |
| 11 | 0.80 | .08 | 1.45 | .11 | 2.35 | .12 | **3.52** | .14 | 5.02 | .16 | 6.90 | .19 |
| 12 | 0.73 | .07 | 1.33 | .10 | 2.15 | .11 | 3.23 | .13 | 4.60 | .15 | 6.32 | .18 |
| 13 | 0.68 | .07 | 1.22 | .09 | 1.98 | .10 | 2.98 | .12 | **4.25** | .14 | 5.83 | .16 |
| 14 | 0.63 | .06 | 1.14 | .08 | 1.84 | .09 | 2.77 | .11 | 3.94 | .13 | 5.42 | .15 |
| 15 | 0.59 | .06 | 1.06 | .08 | 1.72 | .08 | 2.58 | .10 | 3.68 | .12 | 5.06 | .14 |
| 16 | 0.55 | .05 | 0.99 | .07 | 1.61 | .08 | 2.42 | .10 | 3.45 | .11 | 4.74 | .13 |
| 17 | 0.52 | .05 | 0.94 | .07 | 1.52 | .08 | 2.28 | .09 | 3.25 | .11 | 4.46 | .12 |
| 18 | 0.49 | .05 | 0.88 | .06 | 1.43 | .07 | 2.15 | .09 | 3.07 | .10 | 4.21 | .12 |
| 19 | 0.46 | .04 | 0.84 | .06 | 1.36 | .07 | 2.04 | .08 | 2.91 | .09 | 3.99 | .11 |
| 20 | 0.44 | .04 | 0.80 | .06 | 1.29 | .07 | 1.94 | .08 | 2.76 | .09 | 3.79 | .11 |
| 21 | 0.42 | .04 | 0.76 | .05 | 1.23 | .06 | 1.85 | .07 | 2.63 | .09 | 3.61 | .10 |

| Distance Between Supports, in Feet | 9" I 21 Lb. | Add for Every Pound Increase in Weight | 10" I 25 Lb. | Add for Every Pound Increase in Weight | 12" I 31.5 Lb. | Add for Every Pound Increase in Weight | 12" I 40 Lb. Special |
|---|---|---|---|---|---|---|---|
| 12 | 8.39 | .20 | 10.85 | .22 | 15.99 | .26 | 19.92 |
| 13 | 7.74 | .18 | 10.02 | .20 | 14.76 | .24 | 18.39 |
| 14 | 7.19 | .17 | 9.30 | .19 | 13.70 | .23 | 17.08 |
| 15 | 6.71 | .16 | 8.68 | .17 | 12.79 | .21 | 15.94 |
| 16 | **6.29** | .15 | **8.14** | .16 | 11.99 | .20 | 14.94 |
| 17 | 5.92 | .14 | 7.66 | .15 | 11.29 | .19 | 14.06 |
| 18 | 5.59 | .13 | 7.24 | .14 | 10.66 | .18 | 13.28 |
| 19 | 5.30 | .12 | 6.86 | .14 | 10.10 | .17 | 12.58 |
| 20 | 5.03 | .12 | 6.51 | .13 | 9.59 | .16 | 11.95 |
| 21 | 4.79 | .11 | 6.20 | .12 | 9.14 | .15 | 11.38 |
| 22 | 4.58 | .11 | 5.92 | .12 | 8.72 | .14 | 10.87 |
| 23 | 4.38 | .10 | 5.66 | .11 | 8.34 | .14 | 10.39 |
| 24 | 4.19 | .10 | 5.43 | .11 | 7.99 | .13 | 9.96 |
| 25 | 4.03 | .09 | 5.21 | .10 | 7.67 | .13 | 9.56 |
| 26 | 3.87 | .09 | 5.01 | .10 | 7.38 | .12 | 9.19 |
| 27 | 3.73 | .09 | 4.82 | .10 | 7.11 | .12 | 8.85 |
| 28 | 3.59 | .08 | 4.65 | .09 | 6.85 | .11 | 8.54 |
| 29 | 3.48 | .08 | 4.49 | .09 | 6.62 | .11 | 8.24 |
| 30 | 3.36 |  | 4.34 |  | 6.40 | .11 | 7.97 |

Safe loads given include weight of beam. Maximum fiber stress 16,000 lb. per sq. in. For spacings below the heavy lines, the deflections will be greater than the allowable limit for plastered ceilings, equaling $\frac{1}{360}$ of span.

EXAMPLE.—What is the breaking load of a well-seasoned hemlock post 10 in. $\times$ 8 in. and 12 ft. long?

SOLUTION.—Applying the rule just given, $5,300 \div \left[1 + \left(\frac{144^2}{8^2} \times .004\right)\right]$ $= 2,308.4$ lb. per sq. in. of area. $2,308.4 \times 80 = 184,672$ lb. Ans.

**To Find the Breaking Load of a Cylindrical Wooden Prop.**—Find the breaking load of a square prop whose ends are equal in area to those of the cylindrical one, and proceed according to foregoing rule.

EXAMPLE.—What is the safe working load for a hemlock mine prop 10 in. diameter, 10 ft. long?

SOLUTION.—The area of the end of the prop $= 78.54$ sq. in. A square of equal area will have sides equal to $\sqrt{78.54} = 8.86 +$ in. Then,

$$5,300 \div \left[1 + \left(\frac{120^2}{8.86^2} \times .004\right)\right] = 3,058.3 \text{ lb. per each sq. in. of area.}$$

And $3,058.3 \times 78.54 = 240,198$ lb. This is the crushing strength of a similar prop of seasoned timber, but, as mine timber is used in its green state, take one-half of 240,198 lb., or 120,099 lb., as the crushing load of the prop in question. Then, the safe working load is one-third of this, or 40,033 lb.

The strength of similar props varies as the cubes of their diameters, and inversely as their lengths.

### IRON AND STEEL BEAMS

Constants for use in calculating the strength of iron and steel beams are: Cast iron, 2,000; wrought iron, 2,200; steel, 5,000. Hard steel will break the same as cast iron; soft steel will bend like wrought iron. The elastic limit of wrought iron is reached at about 2,200 lb. As it does not break, the limit of elasticity should be used.

**To Find the Quiescent Breaking Load of a Horizontal Square or Rectangular Iron or Steel Beam Supported at Both Ends and Loaded at the Middle.**—Multiply the square of its depth, in inches, by its breadth, in inches; multiply this result by the constant for the material used, and divide by the length, in feet, between the supports. For the net load, subtract one-half of the weight of the beam.

**To Find the Quiescent Breaking Load of a Cylindrical Iron or Steel Beam.** Find the breaking load of a square beam the sides of which are equal to the diameter of the round one, and multiply by .6.

The *safe working load* in each of the preceding cases is one-third of the breaking load. If the load is equally distributed over the beam, it will be twice as great.

# CONCRETE

## CEMENTING MATERIALS

### DEFINITIONS

Any substance that becomes plastic under certain treatment and subsequently reverts to a tenacious and inelastic condition may, in a broad sense, be termed a *cement*. However, nearly all the cementing materials employed in building construction are obtained by the heating, or *calcination*, as it is called, of minerals composed wholly or in part of lime. The different composition of these minerals, as well as the properties of the calcined products, enables the various resulting substances to be classified as *limes*, *hydraulic cements*, *plasters*, and *miscellaneous cements*. Although all these materials have cementing properties, the term *cement* is commonly used to apply only to the group made up of hydraulic cements, *hydraulic* meaning that these substances possess the ability to *set*, or become hard, under water.

Limes and hydraulic cements (commonly called simply *cements*) are composed essentially of oxide of calcium, or lime, generally called *quicklime*, with which may be combined certain argillaceous, or clayey, elements, notably silica and alumina, it being to these elements that the hydraulic properties of certain of these materials are due. The quantity of silica and alumina present in these substances enables them to be classified as *common limes*, *hydraulic limes*, and *cements*.

The ratio of the quantity of silica and alumina present in these materials to the quantity of lime is called the *hydraulic index*. In common limes, this index is less than $\frac{1}{100}$; in hydraulic limes, it lies between $\frac{1}{100}$ and $\frac{40}{100}$; and in cements, it exceeds $\frac{40}{100}$. These limes merge into one another so gradually, however, that it is often difficult to distinguish the dividing line between them.

## LIMES

The commercial varieties of lime may be classified as common, hydrated, and hydraulic. The common limes, also called quicklimes, may be subdivided into rich, or fat, lime and meager, or poor, lime.

**Common Limes.**—The grade of common lime known as *fat*, or *rich*, *lime* is almost pure oxide of calcium, $CaO$, and contains only about 5% of impurities. It has a specific gravity of about 2.3 and a great affinity for water, of which it absorbs about one-quarter of its weight. This absorption is accompanied by a great rise in temperature, by the lime bursting, and by the giving off of vapor. The lime finally crumbles into a powder. This powder occupies from two and one-half to three and one-half times as much volume as the original lime, the exact amount depending on its initial purity. When the lime is in this plastic state, it is said to be *slaked*. It is then unctuous and soft to the touch, and from this peculiarity it derives the name of *fat* or *rich*.

*Meager*, or *poor*, *lime* consists of from 60 to 90% of pure lime, the remainder being impurities, such as sand or other foreign matter. These impurities have no chemical action on the lime, but simply act as adulterants. Compared with fat lime, poor lime slakes more slowly and evolves less heat. The resulting paste is also thinner and not so smooth, greatly resembling fat slaked lime mixed with sand. Poor lime is not so good for building purposes as fat lime, nor has it such extensive use.

**Hydrated Lime.**—The class of lime called *hydrated lime* (calcium hydrate) is merely thoroughly slaked fat lime dried in the form of a fine powder, $Ca(OH)_2$. It is used extensively in conjunction with cement for making mortar, and also in the sand-lime brick industry.

**Hydraulic Limes.**—Limes that contain enough quicklime to slake when water is added, and enough clay or sand to form a chemical combination when wet, thus giving them the property of setting under water, are called *hydraulic limes*. The slaking qualities vary, the time of setting under water also varies, but these limes usually become as hard as stone in 3 or 4 da. The use of hydraulic lime has rapidly decreased in this country. Large quantities were formerly imported from Europe.

## CEMENTS

Cement may be divided into four general classes: Portland, natural, puzzolan (also called pozzuolana), and mixed. The relative importance of each cement is indicated by the order in which it is named.

*Portland cement* may be defined as the product resulting from the process of grinding an intimate mixture of calcareous (containing lime) and argillaceous (containing clay) materials, calcining (heating) the mixture until it starts to fuse, or melt, and grinding the resulting clinker to a fine powder. It must contain not less than 1.7 times as much lime by weight as it does of those materials that give the lime its hydraulic properties, and must contain no materials added after calcination, except small quantities of certain substances used to regulate the activity or the time of setting.

*Natural cement* is the product resulting from the burning and subsequent pulverization of an argillaceous limestone or other suitable rock in its natural condition, the heat of burning being insufficient to cause the material to start to melt.

*Puzzolan cement* is a material resulting from grinding together, without subsequent calcination, an intimate mixture of slaked lime and a puzzolanic substance, such as blast-furnace slag or volcanic scoria. That made from slag is know as *slag cement*.

*Mixed cements* cover a wide range of products obtained by mixing, or blending, the foregoing cements with one another or with other inert substances. *Sand cements*, *improved cements*, and many second-grade cements belong to this class. Mixed cements, however, are of comparatively little importance.

**Properties of Cements.**—The hydraulic cements differ from the limes in that they do not slake after calcination and that they set, or harden, under water. They can be formed into a paste with water without any sensible increase in volume and with little, if any, disengagement of heat. They do

not shrink appreciably in hardening, so that the sand and broken stone with which they are mixed are employed merely through motives of economy and not, as with limes, of necessity.

The color of the different grades of cement is variable, but in certain cases it is distinctive. Portland cement is a dark bluish or greenish gray; if it is a light yellow, it may indicate underburning. Natural cement ranges in color from a light straw, through the grays, to a chocolate brown. Slag cement is gray with usually a tinge of lilac. In general, however, the color of cement is no criterion of its quality.

Cement is packed either in wooden barrels or in cloth or paper bags, the latter being the form of package most commonly employed. A barrel of Portland or of slag cement contains the equivalent of 4 bags, while but 3 bags of natural cement equal a barrel. The average weights of the various cements are given in the accompanying table.

In proportioning mortar or concrete by volume, a common assumption is that a bag of Portland cement occupies .9 cu. ft. This practice, however, is, not entirely uniform.

### AVERAGE WEIGHTS OF HYDRAULIC CEMENTS

| Kind of Cement | Net Weight of Bag Pounds | Net Weight of Barrel Pounds | Weight per Cubic Foot Pounds | |
|---|---|---|---|---|
| | | | Packed | Loose |
| Portland........... | 94 | 376 | 100 to 120 | 70 to 90 |
| Natural........... | 94 | 282 | 75 to 95 | 45 to 65 |
| Slag.............. | $82\frac{1}{2}$ | 330 | 80 to 100 | 55 to 75 |

Portland cement may be distinguished by its dark color, heavy weight, slow rate of setting, and greater strength. Natural cement is characterized by lighter color, lighter weight, quicker set, and lower strength. Slag cement is somewhat similar to Portland, but may be distinguished from it by its lilac-bluish color, by its lighter weight, and by the greater fineness to which it is ground.

Portland cement is adaptable to any class of mortar or concrete construction, and is unquestionably the best material for all such purposes. Natural and slag cements, however, are cheaper, and, under certain conditions, may be substituted for the more expensive Portland cement. All heavy construction, especially if exposed, all reinforced-concrete work, sidewalks, concrete blocks, foundations of buildings, piers, walls, abutments, etc. should be made with Portland cement. In second-class work, as in rubble masonry, brick sewers, unimportant work in damp or wet situations, or in heavy work in which the working loads will not be applied until long after completion, natural cement may be employed to advantage. Slag cement is best adapted to heavy foundation work that is immersed in water or is at least continually damp. This kind of cement should never be exposed directly to dry air, nor should it be subjected either to attrition or impact.

### SAND AND ITS MIXTURES

*Sand* is an aggregation of loose grains of crystalline structure, derived from the disintegration of rocks. It is called *silicious, argillaceous,* or *calcareous,* according to the character of the rock from which it is derived. Sand is obtained from the seashore, from the banks and beds of rivers, and from land deposits.

The first class, called *sea sand,* contains alkaline salts that attract and retain moisture and cause efflorescence in brick masonry. This efflorescence is not at first apparent but becomes more marked as time goes on. It can be removed temporarily, at least, by washing the stonework in very dilute hydrochloric acid.

The second, termed *river sand,* is generally composed of rounded particles, and may or may not contain clay or other impurities.

The third, called *pit sand,* is usually composed of grains that are more angular; it often contains clay and organic matter. When washed and screened it is a good sand for general purposes.

Sand is used in making mortar because it prevents excessive shrinkage and reduces the quantity of lime or cement required. Lime adheres better to the particles of sand than it does to its own particles; hence, it is considered that sand adds strength to lime mortar. On cement mortar, on the contrary, sand has a weakening effect.

**Properties of Sand.**—The weight of sand is determined by merely filling a cubic-foot measure with dried sand and obtaining its weight. Dry sand weighs from 80 to 120 lb. per cu. ft.; moist sand, however, occupies more space and weighs less per cubic foot. The weight of sand is more or less dependent on its specific gravity and on the size and shape of the sand grains, but, other things being equal, the heaviest sand makes the best mortar.

The specific gravity of sand ranges from 2.55 to 2.80. For all practical purposes the specific gravity may be assumed to be 2.65 with little danger of error.

By *percentage of voids* is meant the amount of air space in the sand. Structurally, it is one of the most important properties of sand. The greater these voids, the more cement paste will be required to fill them in order to give a dense mortar. The percentage of voids may be determined by observing the quantity of water that can be introduced into a vessel filled with sand, but it is best computed as follows:

$$\text{percentage of voids} = 100 - \frac{100 \times \text{weight per cubic foot}}{62.5 \times \text{specific gravity}}$$

EXAMPLE.—What is the percentage of voids in a sand having a specific gravity of 2.65 and weighing 105 lb. per cu. ft.?

SOLUTION.—Substituting in the formula, the percentage of voids is

$$100 - \frac{100 \times 105}{62.5 \times 2.65} = 100 - 63.4 = 36.6$$

The percentage of voids depends principally on the size and shape of the sand grains and the gradation of its fineness, and hence will vary from 25 to 50%. Sand containing over 45% of voids should not be used to make mortars.

The *shape* of the grains of sand is of chief importance in the influence that the sand exerts on the percentage of voids. Obviously, a sand with rounded grains will compact into a more dense mass than one whose grains are angular or flat like particles of mica. Therefore, the more nearly the grains approach the spherical in shape, the more dense and strong will be the mortar for the same amount of cement. This fact is contrary to the common opinion on the subject.

The fineness of sand is determined by passing a dried sample through a series of sieves having 10, 20, 30, 40, 50, 74, 100, and 200 meshes, respectively, to the linear inch. The result of this test, expressed in the amount of sand passing each sieve, is known as the *granulometric composition* of the sand. Material that does not pass a ¼-in. screen is not considered to be sand, and should be separated by screening. Sand that is practically all retained on a No. 30 sieve is called *coarse*, while 80 or 90% of sand known as *fine* will pass through this sieve. Fine sand produces a weaker mortar than coarse sand, but a mixture of fine and coarse sand will surpass either one in those cases, at least, where there is not enough cement to fill voids using either sand.

The *purity*, or cleanness, of sand may be roughly ascertained by rubbing it between the fingers and observing how much dirt remains. To determine the percentage of the impurities more accurately, a small dried and weighed sample is placed in a vessel and stirred up with water. The sand is allowed to settle, the dirty water poured off, and the process repeated until the water pours off clear. The sand is then dried and weighed. The loss in weight gives the quantity of impurities contained in the sand. The presence of dirt, organic loam, mica, etc. is decidedly injurious and tends to weaken the resulting mortar. Clay or fine mineral matter in small proportions may actually result in increased strength, but excessive quantities of these materials may be a possible source of weakness. The best modern practice limits the quantity of impurities found by this washing test to 5%.

Attention is called to the fact that the sand found and used around many collieries is inferior. It is apt to be dirty and to consist of fine uniform grains. Such sand is sometimes suitable for building brattices or small foundations where a certain amount of air-tightness or weight, but not strength, is required. In all important work or in reinforced concrete, however, good, carefully selected sand should be used. The sand should always be tested to see whether it will make a mortar or a concrete of the desired qualities.

**Preparation of Sand.**—Sand is prepared for use by (1) screening to remove the pebbles and coarser grains, the fineness of the meshes of the screen depending on the kind of work in which the sand is to be used; (2) washing, to remove salt, clay, and other foreign matter; and (3) drying if necessary. When dry sand is required, it is obtained by evaporating the moisture either in a machine, called a *sand dryer*, or in large, shallow, iron pans supported on stones, with a wood fire placed underneath.

## LIME AND CEMENT MORTARS

*Mortars* are composed of lime or cement and sand mixed to the proper consistency with water. The proportions of the ingredients depend on the character of the work in which the mortar is to be used.

In proportioning mortar, the quantities of the separate ingredients are usually designated by a ratio, such as 1–1, 1–2, 1–3, etc. Thus, 1–2 signifies that 1 part of lime or cement is used to 2 parts of sand, etc. For great accuracy these measurements should be made by weight, but they are usually specified to be measured by volume.

**Lime Mortars.**—In lime mortar, besides effecting an economy, the presence of sand is necessary to prevent the shrinkage that would otherwise occur during the hardening of the paste.

When a mortar is made of lime and sand, enough lime should be present to just cover completely each grain of sand. An excess of lime over this quantity will cause the mortar to shrink excessively on drying, while a deficiency of lime will produce a weak and crumbly mortar. The correct quantity of lime depends on the character of the ingredients, the method of treatment and, to some extent on the judgment of the builder. The mixtures employed vary from 1–2½ to 1–5. Building laws in many municipalities require the use of a 1–3 mixture, and for most materials this proportion will be found satisfactory, although for rich, fat limes a 1–3½ or a 1–4 mixture is sometimes preferable.

In mixing lime mortar, a bed of sand is made in a mortar box, and the lime distributed as evenly as possible over it, first measuring both the lime and the sand in order that the proportions specified may be obtained. The lime is then slaked by pouring on water, after which it should be covered with a layer of sand, or, preferably, a tarpaulin, to retain the vapor given off while the lime is undergoing the chemical reaction of slaking. Additional sand is then used, if necessary, until the mortar attains the proper proportions.

Care should be taken to add just the proper quantity of water to slake the lime completely to a paste. If too much water is used, the mortar will never attain its proper strength, while if too little is used at first, and more is added during the process of slaking, the lime will have a tendency to chill, thereby injuring its setting and hardening properties. Rather than make up small batches, it is considered better practice to make lime mortar in large quantities and to keep it standing in bulk so that it can be used as needed.

Lime mortar is employed chiefly for brickwork of the second class, and its use is continually decreasing as that of cement increases. It is absolutely unsuitable for any important construction, because it possesses neither strength nor the property of resisting water. It cannot be used in damp or wet situations, nor should it ever be laid in cold weather, as it is very susceptible to the action of frost, being much injured thereby. Moreover, since it hardens by the action of dry air, only the exterior of lime mortar ever becomes fully hardened, so that anything like a concrete with lime as a matrix is impossible. However, for second-class brickwork, such as is commonly used in the walls of smaller buildings, lime mortars are economical and sufficiently good.

The strength of lime mortars is extremely variable, depending on the ingredients themselves and on their treatment, environment, etc. It is unsafe to figure a lime-mortar joint as possessing much strength, since only a part of the joint is hardened and capable of developing any strength at all. The tensile strength of thoroughly hardened 1–3 lime mortars averages from 40 to 70 lb. per sq. in., and the compressive strength from 150 to 300 lb.

**Cement Mortars.**—The sand for all mortars should be clean, of suitable size and granulometric composition. For structures designed to withstand heavy unit stresses, or for those intended to resist either the penetration of moisture or the actual pressure of water, the selection of the sand should be most carefully made. A simple method of determining the best sand for cement mortar is to prepare mixtures of the cement, sand, and water, using the same quantities in each case, and then to place each mixture in a measure; that mixture giving the least volume of mortar may be considered to contain the most desirable sand for use.

Limestone screenings, brick dust, crushed cinders, etc., are sometimes substituted for sand in making mortars, and, if care is taken in their selection, they may prove economical and entirely suitable for certain purposes.

The theory of the composition of a correctly proportioned mortar is that the cement paste will just a little more than fill all the voids between the particles of sand, thus giving an absolutely dense mortar at the least expense. The correct proportion of cement to sand, therefore, is more or less variable, depending on the granulometric composition of the sand. Since, however, Portland-cement paste that has set weighs nearly as much as sand, and since the average sand contains about 30 to 40% of voids, it is evident that 1–3 mixtures most nearly approach the best and most economical proportion.

Mortars, however, are made in proportions varying from 1–1 to 1–8. The richer mixtures are used for facing, pointing, waterproofing, granolithic mixtures, etc., the 1–2 mixture being usually made for such purposes. The leaner mixtures are used for rough work, filling, backing, etc., but should never be employed where either much strength or much density is desired. Natural-cement mortars are commonly made 1 part of sand less than Portland-cement mortars intended for the same purpose; that is, where a 1–3 Portland-cement mortar would be used, a 1–2 natural mortar would be required, although natural-cement mortars should be decreased by about 2 parts of sand to equal the strength of Portland. In other words, a 1–4 Portland mortar closely equals the strength of a 1–2 natural mortar. Puzzolan cements are usually proportioned the same as Portlands.

Cements are commonly proportioned by volume, the unit volume of the cement barrel being assumed. If a 1–3 mortar is desired, a box having a capacity of 10.8 cu. ft. is filled with sand and mixed with 4 bags or 1 bbl. of cement. A box 3 ft. $3\frac{7}{16}$ in. square and 1 ft. deep will have a capacity of very nearly 10.8 cu. ft. and, besides, makes a convenient size of box for actual work.

For general purposes, the mortar should be of a plastic consistency—firm enough to stand at a considerable angle yet soft enough to work easily. Wet mortars are easiest to work and are the strongest. However, they are subject to greater shrinkage, are slower setting, and are more easily attacked by frost. Dry mortars, on the other hand, are often friable and porous.

In the accompanying table are given the quantities of materials required to produce 1 cu. yd. of compacted mortar. The proportions are by volume, a cement barrel being assumed to contain 3.6 cu. ft. Of course, the exact values vary with the variety of sand, etc., but the table will serve as an approximation.

## MATERIALS REQUIRED PER CUBIC YARD OF MORTAR

| Kind of Mixture | Portland Cement Barrels | Loose Sand Cubic Yards |
|---|---|---|
| 1–1 | 4.95 | .65 |
| 1–2 | 3.28 | .88 |
| 1–3 | 2.42 | 1.01 |
| 1–4 | 1.99 | 1.06 |
| 1–5 | 1.62 | 1.11 |
| 1–6 | 1.34 | 1.15 |
| 1–7 | 1.18 | 1.17 |
| 1–8 | 1.05 | 1.18 |

EXAMPLE.—How much cement and sand will be required to obtain 8.5 cu. yd. of 1–3 Portland-cement mortar?

SOLUTION.—According to the table, 1 cu. yd. of a 1–3 Portland-cement mortar requires 2.42 bbl. of cement; therefore, 8.5 cu. yd. will require 8.5 × 2.42 = 20.57 bbl. of cement. Also, since 1 cu. yd. of a mixture of this kind requires 1.01 cu. yd. of sand, the quantity of sand required will be 8.5 × 1.01 = 8.59 cu. yd.

Mortar that is to be mixed by hand is prepared on a platform or in a mortar box. The sand is first measured by means of a bottomless box with handles on the sides. After filling the box, the sand is struck off level, the box lifted up, and the sand spread in a low, flat pile. The required number of bags of cement are then emptied on the sand and spread evenly over it. The pile is then mixed with shovels, working through it not less than four times. After

this operation, the dry mixture is formed into a ring, or crater, and the water intended to be used is poured into the center. The material from the sides of the basin is then shoveled into the center until the water is entirely absorbed, after which the pile is worked again with shovels and hoes until the mixture is uniform and in a plastic condition.

Another method of mixing, where a mortar box is used, is to gather the mixed dry materials at one end of the box and pour in the water at the other end, drawing the mixture into the water with a hoe, a little at a time, and hoeing until a plastic consistency is obtained.

**Properties and Uses of Cement Mortars.**—The strength of a mortar is measured by its resistance to tensile, compressive, cross-breaking, and shearing stresses, and also by determinations of its adhesion to inert surfaces, its resistance to impact, abrasion, etc. There is no definitely fixed ratio between the strength of mortar subjected to these different stresses, but there is nevertheless, a close relation between them, so that, practically, it may be assumed that if a mortar shows either abnormally high or low values in any one test, the same relation will develop when tested under other stresses. In practice, therefore, the strength of mortar is commonly determined through its resistance to tensile stresses, and its resistance to other forms of stress is computed from these results.

### TENSILE STRENGTH OF CEMENT MORTARS

| Proportions | | Age of Mortar | | | | | |
|---|---|---|---|---|---|---|---|
| | | 7 da. | 28 da. | 3 mo. | 7 da. | 28 da. | 3 mo. |
| Cement Parts | Sand Parts | Tensile Strength, in Pounds per Square Inch | | | | | |
| | | Portland Cement | | | Natural Cement | | |
| 1 | 1 | 450 | 600 | 610 | 160 | 245 | 280 |
| 1 | 2 | 280 | 380 | 395 | 115 | 175 | 215 |
| 1 | 3 | 170 | 245 | 280 | 85 | 130 | 165 |
| 1 | 4 | 125 | 180 | 220 | 60 | 100 | 135 |
| 1 | 5 | 80 | 140 | 175 | 40 | 75 | 110 |
| 1 | 6 | 50 | 115 | 145 | 25 | 60 | 90 |
| 1 | 7 | 30 | 95 | 120 | 15 | 50 | 75 |
| 1 | 8 | 20 | 70 | 100 | 10 | 45 | 65 |

The tensile strength of mortar has been shown to vary with the character of its ingredients, with its consistency, its age, and with many other factors. In the above table is given a fair average of the tensile strength that may be expected from mortars of Portland and natural cements that are made in the field and with a sand of fair quality but not especially prepared.

The strength of Portland-cement mortar increases up to about 3 mo.; after that period, it remains practically constant for an indefinite time. Natural-cement mortar, on the other hand, continues to increase in strength for 2 or 3 yr., its ultimate strength being about 25% in excess of that attained in 3 mo. The strength of slag-cement mortar averages about three-quarters of that of Portland-cement mortar.

The compressive strength of cement mortars is usually given in textbooks as being from eight to ten times the tensile strength. This value is rather high for the average mortar, a ratio of from 6 to 8 being one more nearly realized in practice. The ratio increases with the age and richness of the mortar, and varies considerably with the quality of the sand. Portland-cement mortars of 1–3 mixture that are 3 mo. old develop, on an average, a compressive strength of about 1,800 lb. per sq. in., while 1–2 natural-cement mortars average about 1,600 lb.

The strength of mortars in cross-breaking and shear may be taken at about one and one-half to two times the tensile strength, with a fair amount of accuracy.

The adhesion of mortars to inert materials varies both with the character of the mortar and with the roughness and porosity of the surfaces with which they are in contact. The adhesion of 1-2 Portland-cement mortar, 28 da. old, to sandstone averages about 100 lb. per sq. in.; to limestone, 75 lb.; to brick, 60 lb.; to glass, 50 lb.; and to iron or steel, 75 to 125 lb. Natural-cement mortars have nearly the same adhesive strength as those made of Portland cement.

In bricklaying and in other places in which mortar is employed it is frequently desired to use a material that is more plastic or smooth than pure cement mortar. This quality is usually obtained by adding from 10 to 25% of lime to the mortar. This addition of lime not only renders the mortar more plastic, and hence easier to work, but also increases both its adhesive strength and its density, which assists in making the mortar waterproof. Hydrated lime is to be preferred for use in cement mortar, because its complete slaking is assured. Hydrated lime may also be readily handled and measured on the work.

Occasionally, small quantities of cement are added to lime mortars so as to make them set quicker and to increase their strength. Such mixtures, however, are not especially economical nor are they convenient in practice.

**Retempering of Mortar.**—Mortar composed of cement, sand, and water soon begins to set and finally becomes hard. When it is desired to use this material, more water has to be added and the mixture worked until it again becomes plastic. This process is called *retempering*. Laboratory tests generally show that retempering slightly increases the strength of mortar, but the reworking is more thorough as a rule in the laboratory than would be the case in actual work. Any part of the hardened mortar that is not retempered is a source of weakness when incorporated in the building. The adhesive strength of cement, moreover, is greatly diminished by this process. For these reasons, it is generally inadvisable to permit the use of retempered mortars; but if they are allowed, great care should be taken to see that the second working is thorough and complete.

**Laying Mortar in Freezing Weather.**—Frost, or even cold, has a tendency to retard greatly the set of cement mortars. When the temperature, moreover, is so low that the water with which the mortar is mixed freezes before it combines with the cement, it may, if care is not exercised, result in complete destruction of the work. A single freezing is not particularly harmful, because when thawing occurs, the arrested chemical action continues. A succession of alternate freezings and thawings, however, is extremely injurious. Nevertheless, Portland-cement mortars may be laid even under the worst conditions if certain precautions are observed, but mortars of natural cement should never be used in extremely cold weather, as they are generally completely ruined by freezing.

The bad results that arise during mild frosts may be successfully guarded against by heating the sand and water and by using a quick-setting cement mixed rich and as dry as possible. In extremely cold weather, salt must be added to the water, so as to convert it into a brine that requires a temperature lower than 32° F. to freeze it. The common rule for adding salt is to use a quantity equal to 1% of the weight of the water for each degree of temperature that is expected below 32° F. Thus, at 32° F., a 1% solution would be used, while at 25°, an 8% solution would be required. Solutions greater than 12% should not be employed, and if a temperature below 20° F. is expected, heat must be used in addition to the salt. The finished work should also be protected with canvas or straw. Manure should not be used for this purpose, because the acids it contains tend to rot the cement. Unless the conditions are such as to make it imperative, it is not advisable to lay mortars during freezing weather.

**Shrinkage of Mortars.**—Cement mixtures exposed to the air shrink during the process of hardening, while those immersed in water tend to expand. The shrinkage of ordinary cement mortars is slight, and when they are used as a bonding material it need not be considered. When used as a monolith, as in sidewalks, shrinkage is guarded against by keeping the mortar wet during setting. This can be done by covering with moist straw or by sprinkling the mixture with water.

**Grouting.**—By grouting is meant the process of filling spaces in masonry with a thin, semifluid mixture known as *grout*. This mixture consists of cement, 1 or 2 parts of sand, and an excess of water. Grout can be used for filling the voids in walls of rubble masonry for backing arches and tunnels, for filling the joints between paving brick, and in all places where mortar cannot be laid in the ordinary manner. When hardened, grout is weak, friable, and porous.

# CEMENT TESTING

## FIELD INSPECTION AND SAMPLING

When *cement* is mentioned anywhere in the following pages, *Portland cement*, is meant, as this is by far the most important cement.

In order to determine correctly the structural value of a shipment of cement, an examination in the field is very necessary. A number of packages of cement should be weighed at intervals, and the average weight should never be permitted to fall below 94 lb. per bag, as mortar and concrete are usually proportioned on the assumption of this weight. Each package should also be plainly marked with the brand and name of the manufacturer; those not branded should be discarded, and, if possible, a mixture of different brands should be avoided.

A possible indication of inferiority is the presence of lumps throughout the bulk of the material. On standing, cement gradually absorbs moisture from the air. At first this moisture is present in merely a minute and harmless state, but eventually it combines chemically with the cement; that is, in the same manner as when cement and water are actually mixed together in practice. In the first condition, lumps usually appear, but they are so soft that they may be readily crushed with the fingers, and of course would be entirely broken up when mixed into mortar. When, however, the cement contains lumps that are hard and pebble-like and that can be crushed only with considerable effort, it indicates that chemical action has begun. Cement containing any appreciable amount of these hardened lumps is generally of decidedly inferior quality, and it should never be permitted to enter any important part of a structure.

Storing cement too long will tend to weaken it. Cement from 2 to 6 mo. old is usually the safest and will produce the best results.

The color of Portland cement, ranging from bluish to yellowish gray, affords no indication of quality except in cases where different shipments or different parts of the same shipment show a variation in color, thus pointing to a lack of uniformity.

**Sampling.**—When securing a sample for testing, the essential point is to get one that will fairly represent the entire shipment whose qualities are to be determined. The common practice is to take a small portion of material from every tenth barrel or, what is the same thing, from every fortieth bag. When tests are to be made, however, on a shipment of only a few barrels, more packages than one in ten should be opened; and when the shipment is large, say over 150 bbl., it should be subdivided and each portion tested separately. The bags selected should be taken at random and from different layers and not all from one part of the pile.

The cement, moreover, should be taken not only from the top of the packages, but from the center and sides as well. When the cement is contained in barrels, a sampling auger is used to extract the sample, a hole being bored in the staves midway between the heads. After the samples of cement have been taken from the packages they are thoroughly mixed in a can or basin, and this mixed sample is used for the various tests. To make a complete series of tests, the sample should contain from 6 to 8 lb. The cement, after sampling and before testing, must be well protected, as exposure to heat, cold, dampness, or any other abnormal condition may seriously affect the results.

**Purpose and Classification of Tests.**—In order that a mortar or a concrete made with cement shall give good results in actual construction, it must possess two important properties, namely, *strength* and *durability*. The primary purpose of cement testing, therefore, is to determine whether any particular shipment of cement possesses sufficient strength and durability to admit of its use in construction.

A determination of the quality of cement necessitates the employment of several tests, which may be classified as *primary tests* and *secondary tests*. The former tests, which include tests for *soundness* and tensile strength, are made to give directly a measure of the essential qualities of strength and durability. Unfortunately, neither of these tests is capable of being made with precision. Therefore, the secondary tests, which include tests to determine the time of setting, the fineness, the specific gravity, and the chemical analysis, are made to obtain additional information in regard to the character of the material. However, with the possible exception of the test of time of setting, the secondary tests have but little importance and only indicate by their results indirectly the properties of the material.

## PRIMARY TESTS

**Tests for Soundness.**—The property of cement that tends to withstand any forces that may operate to destroy or disintegrate it is known as *soundness*. This property, which is sometimes called *constancy of volume*, is the most important requisite of a good cement.

The most common cause of unsoundness in Portland cement is an excess of free or uncombined lime, which crystallizes with great increase of volume,



(a)        (b)        (c)

(d)        (e)        (f)
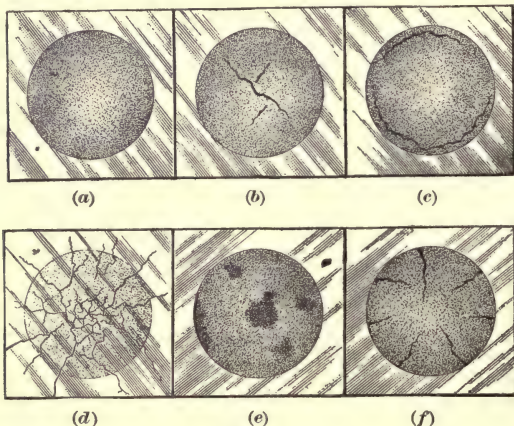
Fig. 1

and thus breaks and destroys the bond of the cement. This excess of lime may be due to incorrect proportioning or to insufficient grinding of the raw materials, to underburning, or to lack of sufficient storing before use, called *seasoning*. A certain amount of seasoning is usually necessary, because almost every cement, no matter how well proportioned or burned it may be, will contain a small amount of this excess of lime, which, on standing, will absorb moisture from the air, slake, and become inert.

Excess of magnesia or the alkalies may also cause unsoundness, but the ordinary cement rarely contains a sufficient amount of these ingredients to be harmful. Sulphate of lime is occasionally responsible for unsoundness, but this ingredient usually acts in the opposite direction, tending to make sound a cement that otherwise might disintegrate.



(a)

(b)

(c)

Fig. 2

The property of soundness is determined in one or more of three ways: by measurements of expansion, by normal tests, and by accelerated tests.

*Measurements of expansion* are made by forming specimens of cement, usually in the shape of prisms, and measuring the change in volume by means of a micrometer screw. At the present time, however, it is believed that expansion is not a sure index of unsoundness, so that this test is seldom employed.

*Normal tests* consist in making specimens of cement mixed with water, preserving them in air or in water under normal conditions, and observing their behavior. The common practice is to make on glass plates about 4 in. square, from a paste of neat or pure cement, two circular pats about 3 in. in diameter, $\frac{1}{2}$ in. thick at the center, and tapering to a thin edge. These pats are kept in moist air for 24 hr.; then one of them is placed in fresh water of ordinary temperature and the other
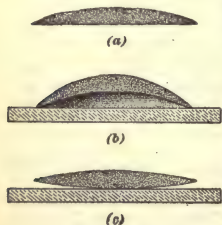
is preserved in air. The condition of the pats is observed 7 da. and 28 da. from the date of making, and thereafter at such times as may be desired.

The most characteristic forms of failure are illustrated in Figs. 1 and 2. Fig. 1 (a) shows a pat in good condition. View (b) illustrates shrinkage cracks that are due, not to inferior cement, but to the fact that the pat has been allowed to dry out too quickly after being made. Pats must be kept in a moist atmosphere while hardening, or these cracks, indicative merely of careless manipulation, will develop. View (c) shows cracks that are due to the expansion of the cement; this condition is common in the air pats, and is not indicative of injurious properties. Pats kept in water, however, should not show these cracks. View (d) shows cracking of the glass plate to which the pat is attached; this cracking is caused by expansion or contraction of the cement, combined with strong adhesion to the glass; it rarely indicates injurious properties. View (e) illustrates blotching of the pats, the cause of which should always be investigated by chemical analysis or otherwise, which may or may not warrant the rejection of the material. Slag cements or cements adulterated with slag invariably show this blotching. View (f) shows the radial cracks that mark the first stages of disintegration; such cracks should never occur with good material. They are signs of real failure, and cement showing them should never be used.

Fig. 2 shows three pats that, for different reasons, have left the glass plate on which they were made. The disk shown in (a) left the plate because of lack of adhesion; the one in (b), through contraction; and the one in (c), through expansion. The condition illustrated in (a) is never dangerous in either air or water; that in (c) is only dangerous when existing in a marked degree; and that in (b) hardly ever occurs in water, but in air it often indicates dangerous properties. Air pats that develop the curvature shown in (b) generally disintegrate later. A curvature of about $\frac{3}{8}$ in. in a 3-in. pat can be considered to be about the limit of safety.

The normal pat tests are the only absolutely fair and accurate methods of testing cements for soundness, but the serious objection to them lies in the fact that frequently several months or even years elapse before failure in the cement so tested becomes apparent. To overcome this difficulty the *accelerated tests* have been devised. These tests are intended to produce in a few hours results that require months in the normal tests.

Many forms of accelerated tests have been devised. At present, however, the only tests employed commercially are the boiling test and the steam test.

The *boiling test* is made by forming specimens of neat-

Fig. 3

cement paste into pats, such as are employed for the normal tests, or preferably into balls about $1\frac{1}{2}$ in. in diameter. The specimens are allowed to remain in moist air for 24 hr. and are then tested.
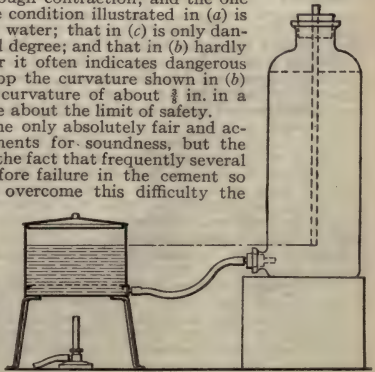
The form of the apparatus used for the boiling test is shown in Fig. 3. It consists of a copper tank that is heated by a Bunsen burner and is filled with water. The water in the tank is kept at a uniform height by means of a constant-level bottle. A wire screen placed an inch from the bottom of the tank prevents the specimens from coming into contact with the heated bottom. The test pieces, which are 24 hr. old, are placed in the apparatus, which is filled with water of a normal temperature, and heat is applied at a rate such that the water will come to boiling in about $\frac{1}{2}$ hr. Quiet boiling is continued for 3 hr., after which the specimens are removed and examined. Care must be taken that the water employed is clean and fresh, because impure water may seriously affect the results. The same water, also, should never be used for more than one test. A good cement will not be affected by this treatment, and the ball will remain firm and hard. Inferior cement will fail by checking, cracking, or entirely disintegrating.

The *steam test* is made in the same way as the boiling test, except that instead of immersing the specimens in water, they are kept in the steam above the water. The apparatus employed is the same as that used for the boiling

test.  The wire screen, however, is raised so that it is an inch above the surface of the water; also, there must be provided a cover that is close enough to retain the steam without creating pressure.  The steam test is less severe than the boiling test and is somewhat less accurate.

The result of the normal tests, if properly made and interpreted, may be considered reliable guides to the soundness of the material, and cement failing in these tests should always be rejected.  The accelerated tests, on the other hand, merely furnish indications, and are by no means infallible.  A cement passing the boiling test can generally be assumed sound and safe for use, but, if failure occurs, it simply means that other tests should be performed with greater care and watchfulness.  It often is advisable to hold for a few weeks cement that fails in boiling, so that the expansive elements may have an opportunity to hydrate and become inert; but if the material fulfils all the conditions except the boiling test, and is sound in the normal tests up to 28 da., it is generally safe for use.  All things being equal, however, a cement that will pass the boiling test is to be preferred.

**Tests for Tensile Strength.**—The tensile-strength test is for the purpose of ascertaining a measure of the ability of the material to withstand the loads that the structure must carry.  This test is made by forming specimens, called *briquets*, of cement and cement mortar, and determining the force necessary to rupture them in tension at the expiration of fixed intervals of time.  Cement constructions are rarely called on to withstand tensile stresses, but if the tensile strength is known, the resistance to other forms of stress may be computed with a fair degree of accuracy.  The tensile-strength test is the most convenient for laboratory determinations, on account of the small size of the specimens and the comparatively low stress required to cause rupture.

Cement is tested both neat or pure and in a mortar commonly composed of 1 part of cement and 3 parts of sand.  The periods at which the briquets are broken have been fixed by usage at 7 da. and 28 da. after making, although tests covering much longer periods of time are necessary in research or in investigative work.

The strength of cement and cement mortars varies considerably with the amount of water employed in making the briquets.  Dry mixtures ordinarily give the higher results for short-time tests, and wet mixtures show stronger with a greater lapse of time.  For testing purposes, therefore, it is essential that all cements be mixed, not with the same amount of water, but with the amount that will bring all the cements to the same physical condition, or to what is called *normal consistency*.  Different cements require different percentages of water because of their varying chemical composition, degree of burning, age, fineness, etc,

The normal consistency of neat-cement pastes may be determined by either of the methods that follow.  In these tests, the quantities are given in grams.

The first method is taken from that part of the final report of the Special Committee on Uniform Tests of Cement of the American Society of Civil Engineers which may be found on pp. 679 to 684, inclusive, of Vol. LXXV, Dec., 1912, of the Transactions of that Society.  In this method that quantity of cement that it is proposed to use subsequently in each batch for making test pieces should be weighed, but in no case should less than 500 grams be taken.  The amount is placed on a non-absorbent mixing slab in the form of a crater, and a definite amount of water poured into the center.  The cement is turned over from the sides into the center with a trowel until the water is absorbed.  It is then kneaded vigorously for 1 min. and quickly formed into a ball and tossed six times from one hand to the other maintained 6 in. apart.  During the operation of kneading and making the ball the hands should be protected by rubber gloves.  The ball of cement on the palm of the hand is then pressed into the large end of the hard-rubber ring of the Vicat apparatus in such a way that the ring is completely filled with paste.  A Vicat apparatus working on the same principle as the one illustrated in the Society's report and with the same weight of movable rod and the same principal dimensions is described in connection with Tests for Times of Setting.  The excess paste at the large end of the ring is removed with one movement of the palm of the hand, the ring is placed large end down on the plate under the movable rod, and the excess paste on the top is sliced off with a trowel held at an angle.  The paste must not be compressed.  The plunger is brought into contact with the surface of the material, quickly released, and its penetration noted.  The penetration should be exactly 10 millimeters in ½ min.; if the test shows a greater or less amount, other trials must be made, using more or less water, until the correct consistency is obtained.

The other method is to form of the paste a ball about 2 in. in diameter and to drop this ball on a table from a height of about 2 ft. If the cement is of the correct consistency, the ball will not crack nor will it flatten to less than one-half its original thickness. The percentage of water required will vary from 16 to 25, depending on the characteristics of the material, the average cement taking about 20%.

The consistency of sand mortars, however, cannot be obtained by either of the foregoing methods, because the sand grains do not permit of the measurement of the consistency by penetration, and the mixture is too incoherent for use of the ball method.

The accompanying table gives the amount of water required to make mortar of normal consistency and is given in the report of the committee just referred to above. It is for a mortar consisting of 1 part of cement and 3 parts of Standard Ottawa Sand. The amount of water is given as a percentage of the combined weight of the cement and sand. Each percentage corresponds with the water required for a normal consistency with the neat cement formed by the method recommended by the Committee.

### PERCENTAGE OF WATER FOR STANDARD SAND MORTAR

| Water in Neat Cement Per Cent. | Water in Standard Mortar Per Cent. | Water in Neat Cement Per Cent. | Water in Standard Mortar Per Cent. | Water in Neat Cement Per Cent. | Water in Standard Mortar Per Cent. |
|---|---|---|---|---|---|
| 15 | 8.0 | 23 | 9.3 | 31 | 10.7 |
| 16 | 8.2 | 24 | 9.5 | 32 | 10.8 |
| 17 | 8.3 | 25 | 9.7 | 33 | 11.0 |
| 18 | 8.5 | 26 | 9.8 | 34 | 11.2 |
| 19 | 8.7 | 27 | 10.0 | 35 | 11.3 |
| 20 | 8.8 | 28 | 10.2 | 36 | 11.5 |
| 21 | 9.0 | 29 | 10.3 | 37 | 11.7 |
| 22 | 9.2 | 30 | 10.5 | 38 | 11.8 |

The following formula has been devised to give the normal consistency to any mortar with any sand. It will be noted that the values derived by the formula differ somewhat from those given in the table.

Let $x$ = per cent. of water required for sand mixture;
$N$ = per cent. of water required to bring neat cement to normal consistency;
$n$ = parts of sand to one of cement;
$S$ = constant depending on character of sand.

Then,
$$x = \frac{3N + Sn + 1}{4(n+1)}$$

For crushed-quartz sand, the constant $S$ is 30; for Ottawa sand, it becomes 25; and for the bar and bank sands used in construction, it varies from 25 to 35, and must be determined for each particular sand.

EXAMPLE.—How much water is required in a mixture of 1 part of cement and 3 parts of crushed-quartz sand? The neat cement requires 19% of water to give normal consistency.

SOLUTION.—Here, $N = 19$, $S = 30$, and $n = 3$. Substituting these values in the formula,
$$x = \frac{3 \times 19 + 30 \times 3 + 1}{4 \times (3+1)} = 9.3\%$$

**Sand for Mortar Tests.**—The size, gradation, and shape of the particles of sand with which cement mortars are made have great influence on the resulting strength. There are two varieties of standard sand for cement testing, one an artificial sand of crushed quartz, the particles of which are angular in shape, and the other a natural sand from Ottawa, Illinois, the particles of which are almost spherical. Both sands are sifted to a size that will pass a sieve of 20 meshes to the inch and be retained on a sieve of 30 meshes, the diameters of the sieve wires being .0165 and .0112 in., respectively. The Ottawa sand will develop strengths in 1-3 mortars about 20 to 30% greater than those obtained with crushed quartz, and it is theoretically the better sand for testing. On most important works, tests for purposes of comparison are also made of the actual sand entering the construction.

**Briquets.**—The form of tensile briquet, adopted as standard in the United States, is shown in the accompanying figure; its cross-section is exactly 1 sq. in. These briquets are made in molds that come either single or in gangs of three, four, or five. The gang molds are preferable, as they tend to produce greater uniformity in the results. Molds should be made of brass or of some other non-corrodible material; those made of cast iron soon rust and become unfit for use.

When making the briquets, 1,000 g. of cement is carefully weighed and placed on the mixing table in the form of a crater, and into the center of this is poured the amount of water that is necessary to give the correct normal consistency. Cement from the sides of the crater is then turned into the center, by means of a trowel, until all the water is absorbed, after which the mass is vigorously worked with the hands, as dough is kneaded, for 1½ min. When sand mixtures are being tested, 250 g. of cement and 750 g. of sand are first weighed and thoroughly mixed dry until the color of the pile is uniform; then the water is added and the operation is completed by vigorous kneading.
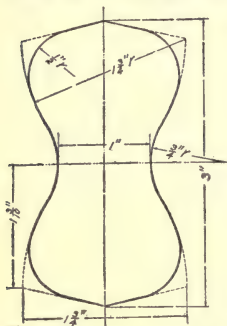
After kneading, the material is immediately placed into the molds, which should have been wiped with oil to prevent the cement from sticking to them. The entire mold is filled with material at once—not compacted in layers—and pressed in firmly with the fingers without any ramming or pounding. An excess of material is then placed on the mold and a trowel drawn over it under moderate pressure, at each stroke cutting off more and more of the excess material, until the surface of the briquets is smooth and even. The mold is then turned over, and more material placed in it and smoothed, as before. The mixing and molding should be performed on a surface of slate, glass, or some other smooth, non-absorbent material. During the mixing the operator should wear rubber gloves, so as to protect his hands from the action of the lime in the cement.

For 24 hr. after making, the briquets are stored in a damp closet so that the cement can harden in a moist atmosphere. The damp closet is simply a tight box of soapstone with doors of wood lined with zinc, or some similar arrangement, with a receptacle for water at the bottom and racks for holding the briquets. The briquets remain in the molds while in the damp closet, but at the expiration of 24 h, they are removed, marked, and placed in clean water near 70° F. until broken.

**Testing Machines.**—There are many styles of testing machines on the market; that shown is called a *shot machine* and is made by the Fairbanks Company. It is constructed on the cast-iron frame *a*, and is operated as follows: The cup *f* is hung on the end of the beam *d*, the poise *r* placed at the zero mark, and the beam balanced by turning the weight *l*. The hopper *b* is then filled with fine shot, and the briquet to be tested is placed in the clips *h*. The hand-wheel *p* is now tightened sufficiently to cause the graduated beam *d* to rise until the indicators at *k* are in line. In the case is a tension arrangement containing a worm and worm-gear that is connected to an axis that is threaded and passes up through the hand wheel *p* and into a block connected to the lower clip. A knob in the case engages this gear when it is required. After the hand-wheel *p* has been tightened until the indicators are in line, the worm is engaged and the automatic valve *j* is opened so as to allow the shot to run into the cup *f*. The flow of the shot can be regulated by means of a small valve located where the spout joins the reservoir. As the briquet stretches, the beam is kept stationary by applying tension to the briquet by means of the hand wheel *s*. When the briquet breaks, the beam *d* drops and by means of the lever *t* automatically closes the valve *j*. After the specimen has broken, the cup with its contents is removed, and the counterpoise *g* is hung in its place. The cup *f* is then hung on the hook under the large ball *e*, and the shot weighed. The weighing is done by using the poise *r* on the graduated beam *d* and the weights *n* on the counterpoise *g*. The result will show the number of pounds required to break the specimen. A mold for a single briquet is shown at *c*.

The load should be applied in all tests at the uniform rate of 600 lb. per min. The briquets should be broken as soon as they are removed from the storage tanks and while they are still wet, because drying out tends to lower their strength. The average of from three to five briquets should be taken as the result of a test.

Results of Tensile-Strength Tests.—The tensile strength of briquets tested in the preceding manner should increase with age up to about 3 mo., and should then remain practically stationary for longer periods. The average results of tests of Portland cement made in the Philadelphia laboratories, covering a period of several years and based on over 200,000 briquets, are given in the accompanying table.

Specifications for strength commonly stipulate minimum values for the 7- and 28-da. tests, the customary requirements for Portland cement being 500 lb. at 7 da. and 600 lb. at 28 da., when tested neat, and 170 lb. at 7 da. and 240 lb. at 28 da., when tested in a mortar consisting of 1 part of cement and 3 parts of crushed-quartz sand. When Ottawa sand is used, the requirements for mortar should be raised to 200 and 280 lb., respectively. Retrogression in strength in the periods specified is not often allowed. Although this retrogression in neat briquets between 7 and 28 da. is not necessarily indicative of undesirable
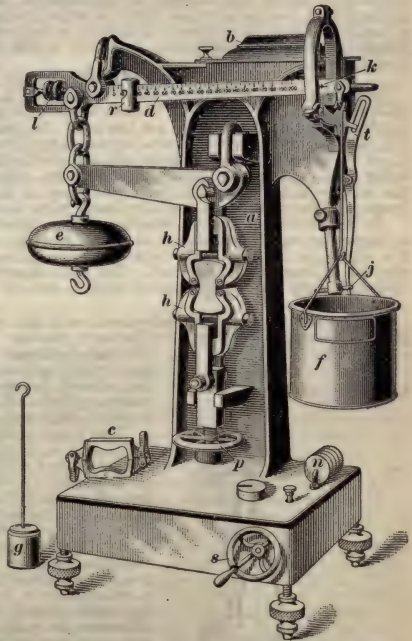
## TENSILE STRENGTH OF CEMENT BRIQUETS

| Mixture | 1 Hr. in Air 23 Hrs. in Water Pounds per Square Inch | 1 Da. in Air 6 Da. in Water Pounds per Square Inch | 1 Da. in Air 27 Da. in Water Pounds per Square Inch | 1 Da. in Air 89 Da. in Water Pounds per Square Inch |
|---|---|---|---|---|
| Neat........................ | 420 | 710 | 770 | 775 |
| 1 cement, 1 crushed-quartz sand............ | 360 | 590 | 695 | 700 |
| 1 cement, 2 crushed-quartz sand............ | 210 | 370 | 455 | 465 |
| 1 cement, 3 crushed-quartz sand............ | 105 | 210 | 300 | 310 |
| 1 cement, 4 crushed-quartz sand............ | 60 | 130 | 210 | 230 |
| 1 cement, 5 crushed-quartz sand............ | 35 | 80 | 155 | 195 |

properties according to some authorities, yet if the mortar briquets show retrogression, the cement should be condemned. Abnormally high strength in the 7-da. test of neat cement, say over 900 lb., may generally be taken as an indication of weakness rather than of superiority, because such a condition is usually created by an excess of lime or of sulphates, either of which may be injurious. Neat cement testing from 600 to 800 lb., at 7 da. is generally the most desirable.

## SECONDARY TESTS

**Tests for Time of Setting.**—The time-of-setting test is made to determine whether or not the cement will become hard at the time most desirable in actual construction. If it begins to set too soon, the crystallization of the particles will have begun before the mortar or concrete is thoroughly tamped into place. If, on the other hand, the cement sets too slowly the material is more likely to suffer from exposure to heat, cold, dampness, and frost; also, the progress of the work will be much delayed on account of the greater interval required between different sections, and the longer time the forms must be left up.

In the setting of cements, two stages are recognized: When the paste begins to harden or to offer resistance to change of form, called *initial set;* and when the setting is complete, or when the mass cannot be appreciably distorted without rupture, called *hard set.* The time-of-setting test consists, therefore, in determining the time required for the cement to reach these two critical points.

The test is made by mixing cement with the amount of water required to produce normal consistency, in the same manner as for neat tensile briquets,



**FIG. 1**

forming specimens, placing them under one of the forms of apparatus, and observing the time that elapses between the moment the mixing water is added and the moments when the paste acquires initial set and hard set.

The *Vicat needle,* shown in Fig. 1, consists of a frame $k$, holding a movable rod $l$, which carries a cap $d$ at the upper end and a needle $h$ at the lower. A screw $f$ holds the rod in any desired place. The position of the needle is shown by a pointer moving over a graduated scale. The rod with needle and cap weighs exactly 300 g. and the needle is 1 mm. in diameter with the end cut off square. When making tests of normal consistency, the plunger $b$, which is 1 cm. in diameter, is substituted for the needle $h$, and the cap $a$ for the cap $d$, the difference in weight between the needle and plunger being compensated by the difference in the weight of the caps. The mold $i$ for holding the cement paste is in the form of a truncated cone. It has an upper diameter of 6 cm., a lower diameter of 7 cm., and a height of 4 cm., and rests on a $4'' \times 4'' \times \frac{1}{4}''$ glass plate $j$.

After the cement paste is mixed, the mold is filled by forcing the cement through the large end; then, after turning it over and smoothing the top, it is placed on the glass plate under the needle. The needle is lowered until it is exactly in contact with the surface of the paste, then quickly released and the depth to which it penetrates is read from the graduated scale. Initial set is said to have taken place when the needle ceases to penetrate to within 5 mm. of the bottom of the specimen; and hard set takes place when the same needle ceases to make an impression on the surface. Trials of penetration are made every 5 or 10 min. until these points are reached.

Time of setting varies considerably with the amount of mixing water employed, so that it is essential that every sample tested be brought exactly to normal consistency; otherwise, the results may be in decided error. Variations in temperature in both environment and in the mixing water, also influence the results. Standard practice requires that both the materials and the room in which the tests are made be at a temperature of as nearly 70° F. as practicable.

When specifying results to be obtained in testing the time of setting, it is obvious that a minimum value should be stipulated for initial set and a maximum, as well as a minimum, for hard set. It must also be remembered that a cement mixed with an aggregate and with an excess of water in the
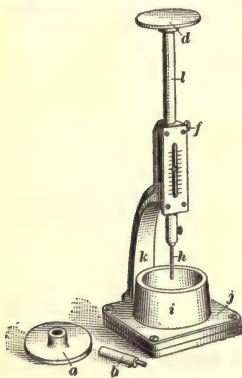
field, will require from two to four times as long to set as the neat-cement paste mixed with little water in the laboratory. Cement, therefore, showing an initial set at the expiration of 20 min. with the Vicat needle, will rarely begin to set on the actual work in less than ¾ hr., which gives ample time for mixing and placing the materials, and cement setting in less than 10 hr., will usually have hardened completely in the work in 24 or at least in 36 hr. Specifications usually stipulate that Portland cement shall show initial set in not less than 20 minutes and shall develop hard set in not less than 1 hr. nor more than 10 hr. Cement reaching initial set in less than 12 or 15 min. should never be used for any work.

**Tests for Fineness.**—The fineness of cement is important, because it affects both the strength and the soundness of the product. The fineness of cement is determined by passing it through a series of sieves of different mesh and then measuring the amount retained on each. Three sieves are commonly employed, namely, those having 50, 100, and 200 wires to the linear inch. Sieves for cement testing should never be used until they have been carefully examined and found to conform to the following standard specifications:

1. Cloth for cement sieves shall be of woven brass wire of the following diameters: No. 50, .0090 in.; No. 100, .0045 in.; and No. 200, .00235 in.

2. Mesh to count on any part of the sieve as follows: No. 50, not less than 48 nor more than 50 per lin. in.; No. 100, not less than 96 nor more than 100 per lin. in.; and No. 200, not less than 188 nor more than 200 per lin. in.

3. Cloth to be mounted squarely and to show no irregularities of spacing.

The method of using the sieves in the fineness test is to weigh out 50 g. of cement on a scale sensible at least to $\frac{1}{20}$ g. and to place it on the No. 200 sieve, on which it is shaken until not more than $\frac{1}{20}$ g. passes the sieve at the end of 1 min. of shaking. The arrival of this stage of completion can be watched either by using a pan under the sieve or by shaking over a piece of paper. The residue remaining on the sieve is weighed, placed on the No. 100 sieve and the operation repeated, again weighing the residue. The amount remaining on the No. 50 sieve is then determined similarly. The process of sifting can be accelerated by placing a small quantity of coarse shot or pebbles on the sieves with the cement during the shaking. These may be separated from the cement by passing the residue with the shot through a coarse sieve, such as the No. 20.

Portland cement should be ground to such a fineness that it will leave a residue of not more than 25%, by weight, on the No. 200 sieve, and not more than 8% on the No. 100 sieve. Of these two requirements, the first is the more important, because it is only that part of the cement passing the finest sieve that is active in the setting and hardening of the material. The amount remaining on the No. 100 sieve is also important, because this part is most liable to cause unsoundness in the cement, and although specifications do not call for tests with the No. 50 sieve, it is usually employed for the same reason as the No. 100 sieve. Any appreciable residue on this sieve indicates that the material is much more liable to unsoundness. Any cement failing to pass the fineness test should be watched more carefully in the soundness and strength tests, but if these tests show good results up to 28 da., the cement can, as a rule, be used safely. It must be remembered, however, that only that part passing the No. 200 sieve is really cement, so that a coarsely ground shipment is practically equivalent to one adulterated with weak and unsound material.

**Tests for Specific Gravity.**—The object of the specific gravity test is to furnish indications of the degree of burning, the presence or absence of adulteration, and the amount of seasoning that the cement has received. When Portland cement is burned, the separate ingredients are in close contact and gradually combine by a process of diffusion. The greater the amount of this burning the more thoroughly are the elements combined. Thus, by their contraction they give, in volume, a higher density or specific gravity. Since, to secure good cement the burning must have been made within definite limits, it follows that the specific gravity must also lie within fixed limits if the cement has been properly manufactured.

The common adulterants of Portland cement, namely, limestone, natural cement, sand, slag, cinder, etc., all have specific gravities ranging from 2.6 to 2.75, while the specific gravity of Portland cement averages about 3.15. An appreciable amount of adulteration, therefore, is at once indicated in the results of the test.

Seasoning is indicated because the cement on standing gradually absorbs water and carbonic acid from the air. These ultimately combine with it and thus lower the specific gravity.

Of the many forms of apparatus employed for the specific-gravity test, the *Le Chatelier flask*, shown in Fig. 2, is the one most commonly used. It is also the one adopted by the technical societies as standard. It consists of a glass flask about 30 cm. high. The lower part up to mark *a* contains 120 cc., and the bulb between the marks *a* and *b* contains exactly 20 cc. The neck of the flask above the mark *b* is graduated into $\frac{1}{10}$ cc. The funnel *c* inserted in the neck is to facilitate the introduction of the cement.
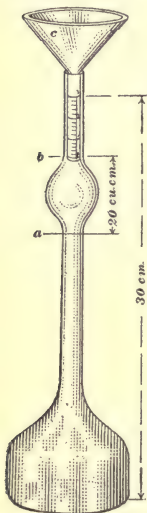


Fig. 2

The method of conducting the specific-gravity test is as follows: 64 g. of cement is carefully weighed on scales that should have a sensibility of at least .005 g. The flask is filled to the lower mark *a* with benzine or kerosene, which has no action on the cement, and is carefully adjusted precisely to the mark by adding the liquid a drop at a time. The funnel is then placed in the neck of the flask and the weighed cement introduced slowly through it, the last traces of the cement being brushed through with a camel's-hair brush. The funnel is then removed and the height of the benzine read from the graduations, estimating to .01 cc. The displaced volume is then 20 plus the reading, in cubic centimeters, and the specific gravity of the cement is 64 divided by that quantity. For example, suppose that the reading on the flask is .54, then the displaced volume will be $20 + .54 = 20.54$ and the specific gravity will be $64 \div 20.54 = 3.116$.

The apparatus must be protected from changes in temperature while in use; even touching the flask with the fingers will change the volume of the liquid noticeably. The flask is sometimes immersed in water during the tests to prevent these changes of temperature, but this precaution is unnecesary if proper care is exercised.

The specific gravity of well-burned Portland cement averages about 3.15 and should not fall below 3.1. If it falls below 3.1, tests should also be made on dried and on ignited samples to ascertain whether or not this condition has been produced by reason of excessive seasoning. As a rule, low specific gravity merely indicates well-seasoned cement, and if sound and sufficiently strong, such cement is the best sort of material for use, as its durability is scarcely open to question.

**Tests of Natural and Slag Cements.**—The methods of conducting tests of *natural* and *slag cements* are, in all important particulars, identical with those employed for Portland cement, although the results obtained and the interpretation to be put on them are often radically different. In the testing, the only essential difference is in the amount of water required by these cements to produce normal consistency; natural cement requires from 23 to 35% and slag cement takes about 18%, or an average of 2 or 3% less than Portland. Tests of natural cement for tensile strength are also frequently made on 1–1 and 1–2 mortars, but recent practice is to test mortars of all kinds of cement in 1–3 mixtures. For these cements, moreover, the specific-gravity test has practically no significance, except in determining the uniformity with which the different brands are made.

# CEMENT SPECIFICATIONS

A good example of a complete modern specification for Portland cement is here given.

## SPECIFICATIONS FOR PORTLAND CEMENT

**Kind.**—All cement shall be Portland of the best quality, dry and free from lumps. By Portland cement is meant the finely pulverized product resulting from the calcination to incipient fusion of an intimate mixture of properly proportioned argillaceous and calcareous materials to which no addition greater than 3% has been made subsequent to calcination.

**Packages.**—Cement shall be packed in strong cloth or canvas bags, or in sound barrels lined with paper, which shall be plainly marked with the brand

and the name of the manufacturer. Bags shall contain 94 lb. net and barrels shall contain 376 lb. net.

**Inspection.**—All cement must be inspected, and may be reinspected at any time. The contractor must submit the cement, and afford every facility for inspection and testing, at least 12 da. before desiring to use it. The chief engineer shall be notified at once on receipt of each shipment at the work. No cement will be inspected or allowed to be used unless delivered in suitable packages properly branded. Rejected cement must be immediately removed from the work.

**Protection.**—The cement must be protected in a suitable building having a wooden floor raised from the ground, or on a wooden platform properly protected with canvas. It shall be stored so that each shipment will be separate, and each lot shall be tagged with identifying number and date of receipt.

**Quality.**—The acceptance or rejection of a cement to be used will be based on the following requirements:

*Specific gravity:* Not less than 3.1.
*Ultimate tensile strength per square inch:* POUNDS
   7 da. (1 da. in air, 6 da. in water)........................... 500
  28 da. (1 da. in air, 27 da. in water)........................... 600
   7 da. (1 da. in air, 6 da. in water), 1 part cement to 3 parts of standard quartz sand.......................................... 170
  28 da. (1 da. in air, 27 da. in water), 1 part of cement to 3 parts of standard quartz sand.......................................... 240

*Fineness:* Residue on No. 100 sieve not over 8%, by weight; residue on No. 200 sieve not over 25%, by weight.

*Set:* It shall require at least 20 min. to develop initial set, and shall develop hard set in not less than 1 hr. nor more than 10 hr. These requirements may be modified where the conditions of use make it desirable.

*Constancy of Volume:* Pats of cement 3 in. in diameter, ½ in. thick at center, tapering to thin edge, immersed in water after 24 hr. in moist air, shall show no signs of cracking, distortion, or disintegration. Similar pats in air shall also remain sound and hard. The cement shall pass such accelerated tests as the chief engineer may determine.

*Analysis:* Sulphuric anhydride, $SO_3$, not more than 1.75%; magnesia, $MgO$, not more than 4%.

The common requirements for high-grade Portland, natural, and slag cements are given in the following table.

## REQUIREMENTS FOR HIGH-GRADE CEMENTS

| Requirements | Portland Cement | Natural Cement | Slag Cement |
|---|---|---|---|
| *Specific gravity:* | | | |
|   Not less than....................... | 3.1 | 2.8 | 2.7 |
| *Fineness:* | | | |
|   Residue on No. 100 sieve, not over..... | 8% | 10% | 3% |
|   Residue on No. 200 sieve, not over..... | 25% | 30% | 10% |
| *Time of Setting:* | | | |
|   Initial, not less than................. | 20 min. | 10 min. | 20 min. |
|   Hard, not less than................... | 1 hr. | 30 min. | 1 hr. |
|   Hard, not more than.................. | 10 hr. | 3 hr. | 10 hr. |
| *Tensile strength per sq. in.* | | | |
|   7 da., neat, not less than............. | 500 lb. | 125 lb. | 350 lb. |
|   28 da., neat, not less than............ | 600 lb. | 225 lb. | 450 lb. |
|   7 da., 1–3 quartz, not less than....... | 170 lb. | 50 lb. | 125 lb. |
|   28 da., 1–3 quartz, not less than...... | 240 lb. | 110 lb. | 200 lb. |
| *Soundness:* | | | |
|   Normal pats in air and water for 28 da. to be ........................... | sound and hard | sound and hard | sound and hard |
|   Boiling test to be ................. | sound and hard | | sound and hard |
| *Analysis:* | | | |
|   Magnesia, $MgO$, not over............ | 4% | | 4% |
|   Anhydrous sulphuric acid, $SO_3$, not over | 1.75% | | |
|   Sulphur, $S$, not over................. | | | 1.3% |

# PLAIN CONCRETE

## DEFINITIONS AND TERMS

*Concrete* is usually made of cement, sand, and broken stone. The cement in a plastic state, either by itself or with the sand that is generally mixed with it, is called the *matrix*, and the broken stone, gravel, or other material used as a filler is called the *aggregate*. The sand is correctly classed as a part of the aggregate, although some engineers mix it with the matrix. The aggregate is used to cheapen concrete. Pure, or neat, cement, when wet with water, would in a way fulfil all the physical requirements of concrete, but it would be too expensive.

In the concrete of today, hydraulic cement is used almost exclusively. For this reason, the term concrete, as commonly used, refers only to that variety. In specifying any other kind of concrete, the usual custom is to mention it by its full name, as *bituminous concrete, lime concrete*, etc. Such varieties, however, are of comparatively little importance.

The term concrete, besides being restricted to hydraulic-cement concrete, has another restriction: the aggregate must not be sand alone, although it may be partly sand. A mixture of hydraulic cement, sand, and water is called by the special name of *mortar*.

Concrete is usually named from the kind of aggregate used. For example, *stone concrete* embodies the use of broken stone or coarse pebbles, while in *cinder concrete*, the aggregate consists of cinders or broken slag.

The proportion of cement and sand to the broken stone depends on the spaces between the stones, which are known as *voids*. In all instances, there must be sufficient mortar to fill the voids entirely and to cover all surfaces of the separate stones.

## AGGREGATES OTHER THAN SAND

The aggregates or broken stone used in concrete work should possess three qualities: (1) They should be hard and strong, so as to resist crushing and shearing or transverse stresses; (2) they should have surface texture that will permit the cement mortar to adhere to their surfaces; and (3) where the concrete is to be used for building construction, such as in reinforced-concrete work, and for fireproofing, they should possess refractory, or fire-resisting, qualities. Usually, aggregates that break in such a way as to allow the smallest spaces, or interstices, between the particles, will make the strongest concrete for construction purposes because the voids can be most economically filled with cement mortar.

**Size of Aggregates.**—When measuring broken stone, the size of the stone is determined by the size of the ring through which it will pass. For instance, a 2-in. stone is one that will pass through a ring, or hole, that is 2 in. in diameter.

The broken stone used in concrete work varies in size with the nature of the work. For foundation and mass construction, it is the custom to use broken stone of a size that will pass through a 2- or 2½-in. ring. For filling the spandrels of bridges or the spaces between walls, where mere bulk is desired, broken stone of a much larger size is used.

In reinforced-concrete work, the broken stone must be small, owing to the narrow spaces in the forms. For columns and wall work, stone that will pass through a 1- or ¾-in. ring is suitable, while for filling beam and girder forms, where numerous reinforcing rods occur, the broken stone is sometimes so small as to pass through a ½-in. ring.

The latest practice in making concrete is to use stone as it comes from the crusher, without screening it. While such stone, termed the *run of crusher*, contains broken stone of a size specified, it also has smaller particles of stone and such stone dust as is carried along with the broken stone from the crusher. Where the run of crusher is used, the proportion of the cement and sand must be changed, because the stone dust takes the place of some of the sand. In using run of crusher the very finest dust should be washed or screened out as it tends to coat the large pieces and to prevent the cement from adhering to them.

The size of the aggregates has much to do with the quality and strength of the concrete. It can, however, be stated as a general proposition, that the larger the stones the stronger will be the concrete. This fact was well proved in a series of tests made at the Watertown Arsenal in 1898. These tests also showed that the concrete becomes heavier per cubic foot, or, in other words,

more dense, the larger the stone used. All these tests were made with concrete manufactured in the proportion of 1 part of cement, 1 part of sand, and 3 parts of broken stones, or a 1–1–3 (1 to 1 to 3) mixture, as it is usually expressed. The figures on cinder concrete (a 1 to 1 to 3) are added simply to give a comparison of weights, for it will be noted that the cinder concrete is older than the other concretes, and therefore stronger in proportion.

Aggregates that consist of stone of varying sizes are best for making concrete, owing to the fact that they pack closer. It is well, however, to screen all the fine particles, such as ½-in. sizes, and use them with the sand, as otherwise they will not mix properly with the cement.

**Selection of Aggregates.**—Usually the character of the aggregates used in mixing concrete depends on the availability of the supply. Where there is much choice in the selection of the aggregates those that are hardest and break with a cubical fracture will make the best concrete, although rounded pebbles are considered by some engineers to possess great advantages.

Some years ago the American Society of Civil Engineers, American Society for Testing Materials, American Railway Engineering and Maintenance of Way Association, and the Association of American Portland Cement Manufacturers appointed committees to obtain information concerning the practice in and properties of concrete and reinforced concrete and recommend formulas for design, etc. This general committee is commonly known as the Joint Committee and references will be made to its report in the following pages. These references are taken from the Proceedings of the American Society of Civil Engineers, Vol. XXXIX, No. 2, pp. 117 to 168, where the Progress Report of the Special Committee of that Society on concrete and reinforced concrete will be found. It was presented to that society by its committee on Jan. 15, 1913.

The relative merit of various aggregates for concrete cannot of course be defined accurately, because in any one aggregate the quality may vary considerably. The working stresses for concrete have been discussed by the Joint Committee. This committee recommends in its report certain tests for the ultimate strength of concrete. In the absence of such tests there are given certain values for the strength of concrete that should be obtained under certain conditions. The values given vary among other things with the kind of aggregate used. The aggregates are arranged into four groups in so far as they govern the strength of the concrete. These groups are as follows:

First group, granite, trap rock.
Second group, gravel, hard limestone, and hard sandstone.
Third group, soft limestone and sandstone.
Fourth group, cinders.

The difference in quality between any two adjacent groups is not constant. Elsewhere in the report it is stated: "Cinder concrete should not be used for reinforced-concrete structures. It may be allowable in mass for very light loads or for fire-protection purposes. The cinders should be composed of hard, clean, vitreous clinker, free from sulphides, unburned coal, or ashes."

## PROPORTIONING OF INGREDIENTS

**Effect on Strength and Imperviousness.**—The strength of concrete depends on the strength of the cement and the thoroughness with which the cement binds together the various pieces of aggregate. The more completely the voids are filled, the more completely will the aggregate be held together. Therefore, the more solid and condensed the concrete is, the less voids it will have, and the stronger it will be. The same is true with regard to making concrete water-proof; the more dense the concrete is, the more nearly water-proof it is.

A mixture of 1 part of cement, 1½ parts of sand, and 3 parts of stone, which would be considered extravagantly rich for a dry place, is probably as dense a concrete, and as good for waterproofing qualities, as can be made.

When a concrete is made of cement, sand, and stone, and the stone is of such a size that it will pass through a 3-in. ring, but will not pass through a 2½-in. ring, the concrete is weaker and requires more cement than one made with graded stone from 3-in. down. When the stone is graded in size, the stones of smaller size fill the voids between the larger stones and thus reduce the quantity of cement and sand required.

**Proportioning by Weights.**—The ingredients for a sample batch of concrete are weighed out in known proportions and mixed to the desired consistency on a sheet of steel. They are then tamped in a piece of 10-in. pipe capped at one end. The pipe thus partly filled is weighed, and subtracting the weight of the receptacle a check is obtained. The height of the concrete in the pipe is then

## COMPRESSIVE STRENGTH OF CONCRETE MADE OF DIFFERENT-SIZED STONES

| Material | First Group | | | Second Group | | | Third Group | | | Fourth Group | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Age. Days | Weight per Cubic Foot Pounds | Compressive Strength Pounds per Square Inch | Age. Days | Weight per Cubic Foot Pounds | Compressive Strength Pounds per Square Inch | Age. Days | Weight per Cubic Foot Pounds | Compressive Strength Pounds per Square Inch | Age. Days | Weight per Cubic Foot Pounds | Compressive Strength Pounds per Square Inch |
| Trap, ¼ in. | 7 | 145.56 | 1,391 | 19 | 149.00 | 2,220 | 32 | 146.44 | 2,800 | 76 | 153.34 | 5,021 |
| Trap, ½ in. | 8 | 147.01 | 1,900 | 20 | 150.12 | 2,769 | 32 | 148.27 | 3,200 | 73 | 158.54 | 5,272 |
| Trap, 1 in. | 7 | 159.26 | 3,390 | 20 | 160.65 | 4,254 | 34 | 160.88 | 4,917 | | | |
| Trap, 1½ in. | 11 | 157.80 | 3,189 | 26 | 160.56 | 4,006 | 41 | 161.14 | 4,562 | | | |
| Trap, 2½ in. | 7 | 158.37 | 2,400 | 22 | 159.27 | 4,143 | 48 | 157.39 | 2,583 | | | |
| Pebbles, ⅜ in. | 7 | 146.76 | 1,298 | 21 | 150.51 | 1,298 | 32 | 161.44 | 4,140 | | | |
| Pebbles, 1¼ in. | 7 | 149.63 | 2,276 | 22 | 151.75 | 2,276 | 34 | 147.02 | 2,992 | 65 | 161.76 | 4,523 |
| Pebbles, 3 in. | 11 | 151.96 | 2,800 | 26 | 150.63 | 2,800 | 29 | 151.51 | 3,817 | 70 | 148.76 | 3,870 |
| Cinders | 31 | 115.90 | 2,329 | 90 | 114.12 | 2,834 | 41 | 153.60 | 3,400 | 61 | 150.89 | 4,018 |
| | | | | | | | 46 | 153.21 | | | | |

measured and the mixture dumped out before it hardens. After all the apparatus is cleaned another batch of concrete is mixed, using the same weight of water, cement, and total weight of sand and stone as before, but a different ratio of weight of sand to weight of stone. The height of this concrete in the pipe is measured. This operation is repeated. The concrete occupying the least volume is the densest and strongest that can be made with that particular sand and stone as they are and with the given proportion of cement. The volumes corresponding to the weights can be found by trial measurements.

**Usual Proportions of Materials.** — It is not always necessary to use the strongest concrete, as the concrete may be required to withstand only slight stresses and be simply used for its weight. The strongest concrete would then be unnecessarily expensive. Often the foregoing method of proportioning concrete is not employed and the engineer specifies a mixture from his own experience without testing the aggregates in any way, except to see that the stone is under the specified maximum size and properly graded, and that the sand is in large grains, graded down, and free from dirt and loam. A common proportion for unimportant work is 1–3–6. This proportion may be used for foundations below ground, in engine bases, in the foundations for asphalt pavements, and for similar purposes. A richer mixture 1–2–4 is used in piers, in dams, in important reinforced-concrete work, and in other places where great strength is desired.

In regard to the proportioning of ingredients the Joint Committee states as follows:

"*Quality and Proportions.*—The materials to be used in concrete should be carefully selected, of uniform quality, and proportioned with a view to securing as nearly as possible a maximum density.

"*Unit of Measure.*—The unit of measure should be the cubic foot. A bag of cement, containing 94 lb. net should be considered the equivalent of 1 cu. ft. The measurement of the fine and coarse aggregates should be by loose volume.

"*Relation of Fine and Coarse Aggregates.*—The fine and coarse aggregates should be used in such relative proportions as will insure maximum density. In unimportant work, it is sufficient to do this by individual judgment, using correspondingly higher proportions of cement; for important work these proportions should be carefully determined by density experiments, and the sizing of the fine and coarse aggregates should be uniformly maintained or the proportions changed to meet the varying sizes.

"*Relation of Cement and Aggregates.*—For reinforced-concrete construction, 1 part of cement to a total of 6 parts of fine and coarse aggregates measured separately should generally be used. For columns, richer mixtures are generally preferable; and in massive masonry or rubble concrete, a mixture of 1–9 or even 1–12 may be used.

"These proportions should be determined by the strength or the wearing qualities required in the construction at the critical period of its use. Experienced judgment based on individual observation and tests of similar conditions in similar localities is an excellent guide as to the proper proportions for any particular case.

"For all important construction, advance tests should be made of concrete composed of the materials in the proportions, and of the consistency to be used in the work. These tests should be made under laboratory conditions to obtain uniformity in mixing, proportioning, and storage, and in case the results do not conform to the requirements of the work, aggregates of a better quality should be chosen or richer proportions used to obtain the desired results."

**Water for Concrete.**—The wetter the concrete is, the easier it will be put in place, but mixtures that are too wet are not so strong as medium mixtures. The amount of water that will make the best mixture is such that after the concrete has been put in place and rammed it will quake like jelly when struck with a spade, and water will come to the surface. If the concrete is wetter than this, the water will have a slight chemical effect on the cement, and, moreover, the sand and cement will tend to separate from the broken stone.

In cinder concrete, owing to the porosity of the cinders, it is necessary to use a little more water, so that the cement will be liquid enough to fill the little cavities in each cinder. This precaution is indispensable when the concrete is to be used with steel, as otherwise the steel will be rapidly corroded by the action of air reaching it through the pores in the cinders.

## DESTRUCTIVE AGENCIES

Various causes may affect the strength and durability of concrete. The principal causes have been discussed by the Joint Committee already referred to, and as the various effects are often more or less complex it is probably best to quote directly from the Joint Committee.

"*Corrosion of Metal Reinforcement.*—Tests and experience indicate that steel sufficiently embedded in good concrete is well protected against corrosion, no matter whether located above or below water level. It is recommended that such protection be not less than 1 in. in thickness. If the concrete is porous, so as to be readily permeable by water, as when the concrete is laid with a very dry consistency, the metal may corrode on account of the presence of moisture and air.

"*Electrolysis.*—The most recent experimental data available on this subject seem to show that while reinforced-concrete structures may, under certain conditions, be injured by the flow of electric current in either direction between the reinforcing material and the concrete, such injury is generally to be expected only where voltages are considerably higher than those that usually occur in concrete structures in practice. If the iron is positive, trouble may manifest itself by corrosion of the iron accompanied by cracking of the concrete; if the iron is negative, there may be softening of the concrete near the surface of the iron, resulting in a destruction of the bond. The former, or anode effect, decreases much more rapidly than the voltage, and almost, if not quite,

disappears at voltages that are most likely to be encountered in practice. The cathode effect, on the other hand, takes place even at very low voltages, and is, therefore, more important from a practical standpoint than that of the anode.

"Structures containing salt or calcium chloride, even in very small quantities, are very much more susceptible to the effects of electric currents than normal concrete, both the anode and cathode effects progressing much more rapidly in the presence of chlorine.

"There is great weight of evidence to show that normal reinforced-concrete structures free from salt are in very little danger under most practical conditions, while non-reinforced-concrete structures are practically immune from electrolysis troubles. The results of experiments now in progress may yield more conclusive information on this subject.

"*Sea-Water.*—The data available concerning the effect of sea-water on concrete or reinforced concrete are limited and inconclusive. Sea walls out of the range of frost action have been standing for many years without apparent injury; in many harbors where the water is brackish, through rivers discharging into them, serious disintegration has taken place. This has occurred chiefly between low- and high-tide levels, and is due, evidently, in part to frost. Chemical action also appears to be indicated by the softening of the mortar. To effect the best resistance to sea-water, the concrete must be proportioned, mixed, and placed so as to prevent the penetration of sea-water into the mass or through the joints. The cement should be of such chemical composition as will best resist the action of sea-water; the aggregates should be carefully selected, graded, and proportioned with the cement so as to secure the maximum possible density; the concrete should be thoroughly mixed; the joints between old and new work should be made water-tight; and the concrete should be kept from exposure to sea-water until it is thoroughly hard and impervious.

"*Acids.*—Concrete of first-class quality, thoroughly hardened, is affected appreciably only by strong acids that seriously injure other materials. A substance like manure is injurious to green concrete, but after the concrete has hardened thoroughly it satisfactorily resists the action of such acid.

"*Oils.*—When concrete is properly made and the surface is carefully finished and hardened, it resists the action of such mineral oils as petroleum and ordinary engine oils. Oils that contain fatty acids produce injurious effects, forming compounds with the lime that result in a disintegration of the concrete in contact with them.

"*Alkalies.*—The action of alkalies on concrete is problematic. In the reclamation of arid land, where the soil is heavily charged with alkaline salts, it has been found that concrete, stone, brick, iron, and other materials are injured under certain conditions. It would seem that at the level of the ground water, in an extremely dry atmosphere, such structures are disintegrated through the rapid crystallization of the alkaline salts, resulting from the alternate wetting and drying of the surface. Such destructive action can be prevented by the use of a protective coating, and is minimized by securing a dense concrete."

**Effect of Fire on Concrete.**—Concrete is essentially a fire-proof material. All the ingredients of which it is composed are of a highly refractory nature, the aggregates being the elements of the mixture that are most quickly affected by intense heat. This is especially true of granite and limestone aggregates, the former being likely to crack or burst when heated, and the latter to calcine. After cement has set, the chemical union of its particles is liable to destruction by fire, because intense heat robs the cement of the water of crystallization, or *dehydrates* the cement, thus softening the material and making it crumbly. If concrete in a mass is subjected to intense heat, this action of dehydration extends into the concrete for a depth of only $\frac{1}{8}$ to $\frac{1}{2}$ in., and is not likely to penetrate farther.

**Effect of Mine Water on Concrete.**—The water from coal mines contains sulphuric acid, ammonium compounds, and other chemicals decidedly injurious to concrete. The use of concrete about the mines has assumed large proportions only in the last few years, and as yet no cheap method that is always effective has been uniformly adopted to protect the concrete from this water. Since the mine water will not attack silica, sand containing at least 92% silica should be used. The stone employed should also be acid-resisting and at least 90% insoluble in dilute hydrochloric acid. The cement should be properly burned and should be the best obtainable for such work. Some of the coal companies have special specifications for cement. Although the foregoing precautions will not make concrete entirely permanent in some conditions, they will increase its life.

**Expansion and Contraction.**—Considère, a French concrete expert, has found by experiment that a 1–3 mortar will shrink from about .05 to .15% when setting in air, and that the shrinkage will be two to three times as great with neat cement. The shrinkage in concrete will be much less than with neat cement or cement mortar.

The shrinkage of concrete is lessened by embedding in it steel rods or bars, as these, by their tensile resistance, prevent the shrinkage of the material when setting. By the experiments of Considère, it is found that with 1–3 mortar reinforced with steel the shrinkage when setting is about one-fifth that of the same mortar without the steel reinforcement.

**Effect of Thermal Changes on Concrete.**—Nearly all materials expand slightly as they become heated. Concrete and steel also follow this law. The contraction or the expansion of concrete due to changes in temperature is about the same as that of steel. The average coefficient of expansion of a 1–2–4 concrete for each Fahrenheit degree in change of temperature is .0000055. Experiments made on 1–3–6 concrete give a coefficient of expansion of .0000065, which is practically the same as the coefficient of steel.

**Effect of Vibration on Concrete.**—The effect of constant vibration on concrete structures has not been definitely determined. Many buildings and bridges constructed of concrete reinforced with steel rods and bars have withstood heavy and constant vibration, both continuous and intermittent, for an extended period of years with no apparent deterioration in strength. Fresh concrete is always, however, subject to deterioration by vibration, and the strength of concrete subjected to jar or shock when setting is materially reduced, because the process of crystallization between the particles, and the consequent cohesion of the mass, seems to be partly destroyed.

## WORKING STRESSES AND STRENGTH VALUES OF CONCRETE

The ultimate strength of concrete varies so with the proportion of the mixture, manner of working, character of ingredients, and age of material, that it is necessary to assume low unit working stresses for it.

There can be no unit stresses recommended for use for all conditions. It takes experience to make good concrete. Moreover, complete and detailed instructions and directions must be followed; more complete and detailed than there is room for here. The Joint Committee's report recommends for allowable stresses for concrete, certain percentages of the crushing strength of cylinders of the concrete, of certain size, and tested under certain conditions. In the absence of tests to learn this crushing strength, this report states that for cylinders of certain size, age, and method of manufacture, if made of 1–2–4, according to the directions laid down, of gravel, hard limestone, or hard sandstone, the crushing strength should be 2,000 pounds per square inch. The allowable stresses for such a concrete, when made of Portland cement, with static loads are:

|  | *Pounds per Square Inch* |
|---|---|
| *For Bearing* | |
| Axial compression, columns under 12 diameters | 450 |
| Compression in extreme fiber of beam, due to bending | 650 |
| Shear in beams, without web reinforcement | 40 |
| Shear, pounding shear | 120 |
| Bond, plain bars | 80 |
| Bond, drawn wire | 40 |

The strength of concrete varies, of course, with the age and richness of the mixture. To show this variation, the ultimate compressive strength of concrete of various ages and mixtures made from Portland cement, sand, and crushed stone, is given in the accompanying table. These results represent the product of some six hundred tests made by W. Purves Taylor, engineer-in-charge of the municipal testing laboratory of Philadelphia. Of course, to obtain similar figures, the concrete must be made and tested as it was in this experiment. The table, however, shows the increase of strength with age and richness of mixture.

## CONCRETE MIXTURES

**Methods of Measuring Ingredients.**—After deciding what proportions of ingredients will be used for the concrete, the engineer must be able to calculate the quantity of each material that he must order. An ordinary box car holds from 400 to 600 bags of cement. The purchaser is charged for the bags by the manufacturer, unless they are of paper, but he gets a rebate for those that are returned.

## AVERAGE ULTIMATE CRUSHING STRENGTH OF CONCRETE

| Proportion of Ingredients | | | Ultimate Crushing Strength Pounds per Square Inch | | | |
|---|---|---|---|---|---|---|
| Cement | Sand | Stone | 7 da. | 1 mo. | 3 mo. | 6 mo. |
| 1 | 2 0 | 4 | 1,600 | 2,150 | 2,400 | 2,500 |
| 1 | 2.5 | 5 | 1,430 | 1,950 | 2,250 | 2,350 |
| 1 | 3.0 | 6 | 1,250 | 1,800 | 2,100 | 2,200 |
| 1 | 3.5 | 7 | 1,100 | 1,660 | 1,960 | 2,080 |
| 1 | 4.0 | 8 | 980 | 1,520 | 1,820 | 1,950 |
| 1 | 4.5 | 9 | 850 | 1,400 | 1,690 | 1,840 |
| 1 | 5.0 | 10 | 750 | 1,260 | 1,550 | 1,720 |
| 1 | 5.5 | 11 | 650 | 1,120 | 1,420 | 1,600 |
| 1 | 6.0 | 12 | 600 | 1,000 | 1,300 | 1,500 |

Cement is usually measured by the barrel just as it comes from the manufacturer, or as 4 bags to the barrel, while broken stone and sand are measured loose in a barrel. Portland cement, after it is taken out of its original package and stirred up, fills a larger volume than when packed. It is therefore necessary to state just how the cement is to be measured; and, as said before, the custom is to measure it by the barrel, compact. A cement barrel contains about 3.8 cu. ft.

**Fuller's Rule for Quantities.**—A practical rule has been devised by W. B. Fuller whereby, after the proportions of ingredients have been fixed, the quantity of material for a certain work may be obtained with reasonable closeness. It is called *Fuller's rule for quantities*, and may be expressed in mathematical symbols as follows:

Let $c$ = number of parts of cement;

$s$ = number of parts of sand;

$g$ = number of parts of gravel or broken stone;

$C$ = number of barrels of Portland cement required for 1 cu. yd. of concrete;

$S$ = number of cubic yards of sand required for 1 cu. yd. of concrete;

$G$ = number of cubic yards of stone or gravel required for 1 cu. yd. of concrete.

Then
$$C = \frac{11}{c+s+g}, \quad S = \frac{3.8}{27} Cs, \text{ and } G = \frac{3.8}{27} Cg$$

If the broken stone is of uniformly large size, with no smaller stone in it, the voids will be greater than if the stone were graded. Therefore, 5% must be added to each value found by the preceding formulas.

EXAMPLE.—If a 1–2–4 mixture is considered, what will be: (a) the number of barrels of cement? (b) the number of cubic yards of sand? and (c) the number of cubic yards of stone required for 1 cu. yd. of concrete?

SOLUTION.—(a) Here $c = 1$, $s = 2$, and $g = 4$. Substituting these values in the first formula, $C = \frac{11}{1+2+4} = 1.57$.

(b) Substituting the values of $C$ and $s$ in the second formula, $S = \frac{3.8}{27} \times 1.57 \times 2 = .44$.

(c) Substituting the values of $C$ and $g$ in the third formula, $G = \frac{3.8}{27} \times 1.57 \times 4 = .88$.

## WORKING OF CONCRETE

**Mixing of Concrete.**—Concrete may be mixed either by hand or by machine. For small work, the concrete is mixed by hand in small batches, such as would be made up from 1 or 2 bags of cement. When mixing, hand work should be performed on a flat, water-tight platform. The sand, after it has been measured, is spread over the platform in an even layer. Upon the sand is placed the cement, and these two materials are turned over with shovels at least three

times, or until the uniform color of the mixture indicates that they are thoroughly incorporated. The stones, or aggregates, having previously been well wetted, are then placed on the top of the mixture of sand and cement, and these materials are also turned at least three times, water being added after the first turning. The water should always be added in small quantities. If a hose is used for this purpose, it should be fitted with a sprinkling nozzle, as otherwise much of the cement is liable to be washed out of the mixture. The concrete, when ready for placing, should be of uniform consistency, either mealy for a dry mix or mushy for a wet mix. In large work, the mixing should be done by machine.

**Retempering of Concrete.**—If the cement of the concrete has attained its initial set before being placed—that is, if the concrete has commenced to harden—remixing with water, or *retempering the concrete*, as it is called, should not be allowed; and if concrete treated in this manner has been deposited in the forms, it should be taken out and removed from the site of the operation, because concrete cannot be retempered properly, except in small quantities for laboratory tests.

**Concreting at High Temperatures.**—If the weather is extremely warm, the stone and sand are liable to become heated to a high temperature. In such cases, when the materials are being mixed, the water necessary for the crystallization of the cement is rapidly absorbed by the stone and the sand, or else is rapidly evaporated by contact with them. Besides, the extreme heat will hasten the setting of the cement, which gives it a tendency to cake in the mixing machine, producing lumpy and inferior concrete. In order to overcome such difficulties, the stone should be thoroughly wetted with a hose, and the sand and stone should be kept under cover, away from the direct rays of the sun. Likewise, the mixing platform or machine should be roofed over. It is well, also, to wet down the finished concrete work with a hose several times a day in extremely hot weather, and less frequently in moderate temperatures.

**Concreting in Freezing Weather.**—Although it is practicable to mix and place concrete at a temperature as low as 27° F., it is not advisable to lay concrete work when the temperature is below 32°; neither should it be mixed and placed even at this temperature, if there is a possibility that the temperature will fall. If concrete is frozen, its setting is retarded and it is liable to become worthless, never properly setting and obtaining the requisite hardness and strength. There is, however, no certainty of the action of frost on concrete, as frozen concrete will frequently thaw out and set, with apparently little loss of strength.

To prevent the freezing of concrete when the temperature has fallen below 32° F., salt is sometimes used in the mixture.

One rule is 1% of salt to the water for each degree below 32° F., as already stated in the case for mortar. More than 12% salt should never be used. The addition of salt, however, is never advisable if a surface finish is required, as the salt is liable to cause efflorescence, or a white deposit, on the surface causing the work to become very unsightly.

Aggregates that are coated with ice or that have been exposed to severe weather for a long time should be heated or thawed out before being used. Concrete that is exposed to freezing after it is set should always be protected by placing over it a layer of boards and straw, or salt hay, or cement bags; or, where the work is in the nature of a reinforced-concrete floor system, by heating the interior of the structure by means of salamanders or fires.

**Joining of Old Concrete With New.**—New and old concrete can be joined only with difficulty, and the strength of such a connection is always uncertain. The joining of old and new concrete work is best done by thoroughly chipping, or cutting away, the old surface, saturating it with water, and working into it thin coats of a 1–1 Portland-cement mortar, and, then, while the coating is still fresh, placing against it the new concrete.

There are some high-grade, imported cements that, in the form of cement mortar, more readily adhere to old concrete work than the usual Portland cements. These cements are frequently used for patching and piecing out work already in place.

# ELEMENTS OF STEEL REINFORCEMENT

## PRINCIPLES OF CONSTRUCTION

When a beam is subjected to tranverse stress, due to loads, the portion of the beam section above the *neutral axis*, or axis along which there is no stress, is in compression, while in that portion below the neutral axis, tensile stresses are created. Ordinarily, concrete is about ten times as strong in compression as it is in tension. Thus, it can readily be seen that a beam of plain concrete without steel reinforcement will fail primarily from lack of



(a)                              (b)
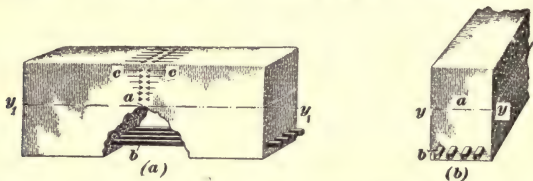
tensile resistance, without realizing its full compressive strength. In order, therefore, to make concrete an economical material to use in construction, its deficiency in tensile resistance must be made up by embedding steel rods, bars, or some other form of metallic reinforcement in that portion of the beam section subjected to tensile stress.

In order to explain more fully this primary principle of reinforced, concrete, reference is made to the reinforced, rectangular concrete beam here shown. The neutral line of the section is shown at $y_1 y_1$ in the side view (a), while the neutral axis is represented by $y y$ in the end view (b). When the concrete beam is under tranverse stress, there is neither tensile nor compressive stress at the neutral axis. Therefore, the point a in the beam which is on the neutral axis, is subjected to zero stress.

Should the concrete be cut away below the neutral plane, leaving the steel reinforcing rods, or bars, exposed at b, the strength of the beam will not be greatly affected, as the necessary tension below the neutral axis is supplied by the reinforcing rods of steel, while the necessary compression above it is furnished by the concrete, as at c. The amount of compression in each square inch of concrete above the neutral axis varies from zero at the axis to a maximum at the extreme upper surface of the beam. The concrete below the neutral axis $y y$ is usually so filled with very fine cracks that all the tension must be carried by the steel alone.

In ordinary reinforced-concrete column construction, merely vertical rods are employed. They are tied together, however, at intervals with wire or other ties.

The principle of *hooped columns* is best explained as follows: It is well known that a column of sand will not resist compression, because it will spread, while a cylinder of very thin metal will sustain only a small load. However, if the cylinder is filled with sand, the tensile strength of the cylinder combined with the compressive resistance of the sand, will produce a column capable of resisting considerable compression. This principle is applied to the reinforcement of concrete columns by binding, or tying, together the concrete with cylindrical hoops, or helical, or spiral, windings of steel a few inches below the surface.

## PARTS OF STEEL REINFORCEMENT

In the accompanying illustration is shown a perspective view (a) of a complete bay of a reinforced-concrete floor system, and a diagrammatic representation (b) of a typical system of reinforcement for a concrete girder and column. In (a), the heavy members A running between columns are commonly known as *girders*, and the lighter members B running between girders, as *beams*. In both (a) and (b), the rods, or bars, a are the *main reinforcing bars*, or *rods*, of the girders. The beams, of course, have similar main reinforcing bars. Of these main reinforcing bars, several are bent up, as at b, to form *trussed bars*. The web reinforcement of the girders is shown at c, and consists of **U**-shaped pieces of iron or steel, called *stirrups*. The rods that

reinforce the slab of the reinforced-concrete floor system, called *slab rods*, are shown at *d*. This slab reinforcement may consist of straight rods, expanded



*(a)*



*(b)*
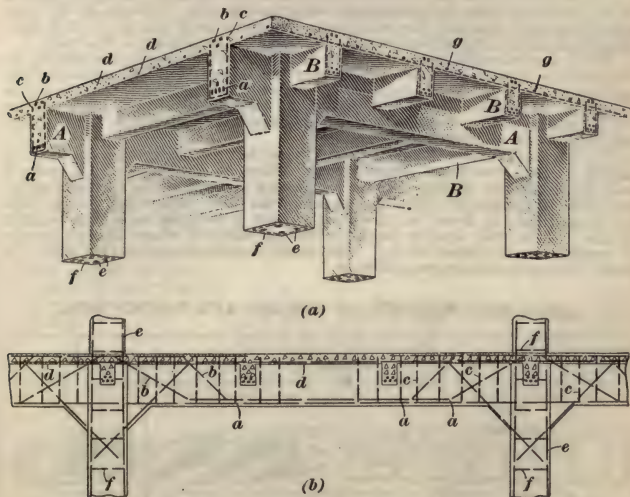
metal, woven-wire lath, or any other metallic reinforcement. The stirrups are bent over at the upper ends or fastened to the slab rods so that they will not pull out. The rods of the columns *e* are called *longitudinal column rods*, and the ties *f, column ties*.

Any rod, or bar, used to resist shearing stresses is designated as a *shear bar*. A rod, or bar, used to resist the shrinkage of the concrete in setting, or to provide against cracks due to thermal changes, is called a *shrinkage rod*. Shrinkage rods are shown at *g* in view (*a*). Sometimes all the slab rods run one way and an occasional shrinkage bar is used at right angles to them as shown, to prevent shrinkage cracks. A rod used to connect abutting beams or girders is called a *tension bar* or a *tie bar*. The short rods used at the splice when longitudinal column rods are butted are called *splice rods*, or *bars*.

**Members to Resist Lines of Failure.**—In the accompanying figure are illustrated a typical beam having the usual type of steel reinforcement and
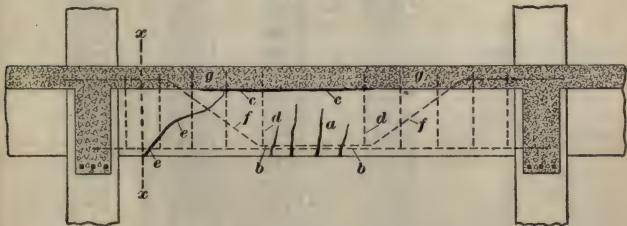


the several methods of failure that might occur. At *a* are shown cracks, or lines of failure, that would be caused by lack of tensile resistance in the main reinforcing rods *b*. These cracks, are usually invisible, and generally extend

from the bottom surface to the neutral axis. They are nearly always present in concrete, but, of course, so long as the steel holds in, the beam will not fail.

If the main reinforcing rods do not extend to the bearings, failure by vertical shear may occur near the abutments, along the line $x\,x$. Failures of this kind seldom happen, because the main rods usually extend across all such lines of vertical shear, and add greatly to the shearing resistance of the beam.

If the slab concrete is not placed at the same time that the concrete of the beam section is poured, failure by shearing usually occurs at the junction of the beam with the slab, as shown at $c$. The shearing resistance at this junction should be increased, however, by extending stirrups $d$ into the slab. If the crack $c$ opens, it usually joins with a crack, like $e$, at each end of the beam, as suggested in the preceding paragraph.

The lines of failure indicated at $e$ are those that usually occur from diagonal tension stresses that cross these lines of failure at right angles. A beam is held against failure in this manner by placing stirrups in the concrete either vertically or obliquely. The bending up of the main reinforcing rods to form the trussed bar, as shown at $ff$, will also assist in resisting such stresses, and, besides, will provide against negative bending moment where tension instead of compression is created at $gg$. The line of fracture shown at $ee$ is typical of nearly all reinforced-concrete failures.

## AREAS AND WEIGHTS OF SQUARE AND ROUND BARS

| Size Inches | Square ■ | | Round ● | |
|---|---|---|---|---|
| | Area Inches | Weight per Foot Pounds | Area Inches | Weight per Foot Pounds |
| 1/16 | .0039 | .013 | .0031 | .010 |
| 1/8 | .0156 | .053 | .0123 | .042 |
| 3/16 | .0352 | .120 | .0276 | .094 |
| 1/4 | .0625 | .213 | .0491 | .167 |
| 5/16 | .0977 | .332 | .0767 | .261 |
| 3/8 | .1406 | .478 | .1104 | .376 |
| 7/16 | .1914 | .651 | .1503 | .511 |
| 1/2 | .2500 | .850 | .1963 | .668 |
| 9/16 | .3164 | 1.076 | .2485 | .845 |
| 5/8 | .3906 | 1.328 | .3068 | 1.043 |
| 11/16 | .4727 | 1.607 | .3712 | 1.262 |
| 3/4 | .5625 | 1.913 | .4418 | 1.502 |
| 13/16 | .6602 | 2.245 | .5185 | 1.763 |
| 7/8 | .7656 | 2.603 | .6013 | 2.044 |
| 15/16 | .8789 | 2.989 | .6903 | 2.347 |
| 1 | 1.0000 | 3.400 | .7854 | 2.670 |
| 1 1/16 | 1.1289 | 3.838 | .8866 | 3.014 |
| 1 1/8 | 1.2656 | 4.303 | .9940 | 3.379 |
| 1 3/16 | 1.4102 | 4.795 | 1.1075 | 3.766 |
| 1 1/4 | 1.5625 | 5.312 | 1.2272 | 4.173 |
| 1 5/16 | 1.7227 | 5.857 | 1.3530 | 4.600 |
| 1 3/8 | 1.8906 | 6.428 | 1.4849 | 5.049 |
| 1 7/16 | 2.0664 | 7.026 | 1.6230 | 5.518 |
| 1 1/2 | 2.2500 | 7.650 | 1.7671 | 6.008 |
| 1 9/16 | 2.4414 | 8.301 | 1.9175 | 6.520 |
| 1 5/8 | 2.6406 | 8.978 | 2.0739 | 7.051 |
| 1 11/16 | 2.8477 | 9.682 | 2.2365 | 7.604 |
| 1 3/4 | 3.0625 | 10.413 | 2.4053 | 8.178 |
| 1 13/16 | 3.2852 | 11.170 | 2.5802 | 8.773 |
| 1 7/8 | 3.5156 | 11.953 | 2.7612 | 9.388 |
| 1 15/16 | 3.7539 | 12.763 | 2.9483 | 10.024 |
| 2 | 4.0000 | 13.600 | 3.1416 | 10.681 |

## REINFORCING MATERIALS

The reinforcement for concrete is almost uniformly steel, but the grade to be used should be determined by one who has made a careful study of this matter. He should both write the specifications and inspect the steel afterwards to make sure that the steel specified has been furnished. Steel used for reinforcement should be free from rust and scale or any coating that will tend to weaken the bond between the metal and the concrete.

**Plain Bar Iron.** The cheapest form of metallic reinforcement for concrete is the *plain, round, rolled bar.* This bar can be obtained in any part of the United States and as its price per pound is lower than



FIG. 1

that of any other form of rolled steel, it is the cheapest and most available material. For slabs, $\frac{3}{8}$- to $\frac{1}{2}$-in. round bars are used, while for beam, girder, and column reinforcement, from $\frac{5}{8}$- to $1\frac{1}{4}$-in. bars are ordinarily employed.

The principal objection to the use of plain, round bars in reinforced-concrete work is that they are not gripped, or held, well by the concrete. Plain square and flat bars are sometimes used for the reinforcement of concrete, though, generally, both of these sections, when so used, are deformed by twisting.



(a)    FIG. 2    (b)
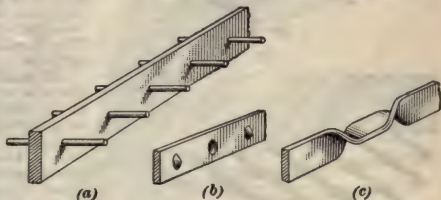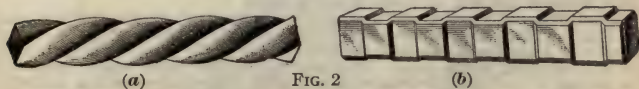
In the nomenclature of reinforced concrete, round, rolled sections are designated as *rods*; square sections as *bars*; and rectangular sections as *flats*, or *flat bars*.

In the preceding table are given the areas and weights of square and round bars from $\frac{1}{16}$- to 2-in. sizes.

**Bars of Special Construction.**—Some early forms of bars used in reinforcement of concrete are shown in Fig. 1. That shown in (a) is known as the *Hyatt bar*. In (b) is shown the *Staff bar;* this consists of a flat bar, through which a countersunk punch



(a)

has been partly driven, thus forcing the metal out on the opposite side so as to form projections. The *De Mann bar* is shown in (c).

There are numerous bars on the market having special mechanical bonds. The complete descriptions of such bars can be obtained from their manufacturers. Only a few will be mentioned here as examples.

**Square-Twisted Bars.**—The square-twisted bar consists of a square bar that is twisted by being given a certain number of turns around its axis, either while it is hot or while it is cold; this bar is often known as the *Ransome bar*. By twisting the bar to the screw shape, as shown in Fig. 2 (a), a form is obtained that has great resistance to pulling from a mass of concrete. If the square bars are twisted cold, their elastic limit and ultimate strength are increased from 8 to 25%. The square-twisted bar can be obtained in various sizes for various purposes.

$\frac{3}{4}'' \times 2\frac{7}{8}''$ Bar-Weight = 2.7 lb. per foot-Area = 0.79 Sq. in.

(b)

FIG. 3

**Corrugated Bar.**—In Fig. 2 (b) is shown a corrugated bar known as the *Johnson bar*, after its inventor, A. L. Johnson. The corrugations on the sides, of course, increase greatly over smooth bars the grip on the concrete.

**Kahn Trussed Bar.**—Fig. 3 (*a*) shows the Kahn Trussed Bar. The section of one of these bars is shown in (*b*).



(*a*)



(*b*)

FIG. 4

The fins are partly sheared across and also in a direction parallel with the axis of the bar, and are bent up, as shown in (*a*), so as to form a grip with the concrete and to provide the stirrups, or web members, necessary to resist diagonal stresses. The Kahn bar is made in various sections.
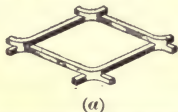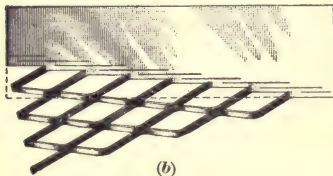
**Expanded Metal.**—One form of metallic reinforcement for concrete is the distorted steel plate known as *expanded metal*, a familiar illustration of which is shown in Fig. 4 (*a*). This form of reinforcement is manufactured by partly shearing a sheet of steel in parallel rows, as shown in (*b*), and then pulling the material sidewise, thus forming a diamond mesh. In this way, the area of a sheet is increased about eight times, with a corresponding decrease in weight per unit area and without any waste of material. The material is made in various sizes. Various forms of metal lath are to be had.

**Woven Wire.**—Various forms of wire cloth, or woven wire, are also on the market. Among them is *Clinton wire cloth*. It is a fabric that is secured at the intersections by a perfect, electric weld, and it has at intervals a double wire that twines in and out, as shown at *a*, Fig. 5.

**Floor Systems.**—A complete floor system constructed of loose rods is shown in Fig. 6. The beam reinforcement consists of three reinforcing rods. Two of these rods *a* run straight through the entire length of the beam, while the third *b* is bent upwards at the ends. This bent member provides tensile resistance at the top of the beams and thus takes care of the negative bending moment, which occurs in all beams fixed at the end. The bend in such rods is usually made at an angle of about 30° with the horizontal. The rods should be straight at the center of the span for at least one-third the distance between the supports.

A tie-rod *c*, that is 4 or 5 ft. in length, and sometimes bent down at the ends, should be placed over the top of the beam juncture.

The girder reinforcement consists of five rods, two of them *c* being bent up to provide against negative bending moment.

In the best work, two short rods *f* are located transversely through the column. These rods tie the adjoining girders together and provide additional rigidity at the junction of the girders with the column.

The slab rods *h* are generally spaced at about 6 in. from center to center. They should bond with the stirrups, or web reinforcement, of the beams, and may be threaded through, interlocked, or wired to them. It is customary to provide shrinkage rods that extend at right angles to the regular slab reinforcement, in order to prevent shrinkage cracks in the concrete. For this purpose, ¼-in. round or square rods *j* are generally spaced about 2 ft. from center to center. In order to bond the concrete over the main girders securely, it is also good practice to provide over these important members rods *d* of about the same size as the slab rods. These rods should run through holes punched in the top of the stirrup, as illustrated at *g*, and should extend at right angles to the axis of the girder. Sometimes, similar rods are used in the slab over beams, as shown at *k*.



FIG. 5

The longitudinal reinforcement of the concrete columns consists of four round rods *l*. It is customary to project them above the concrete of each story about a foot and to splice them by lapping and wiring or by using pipe
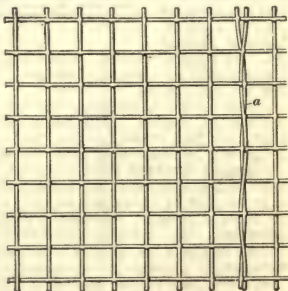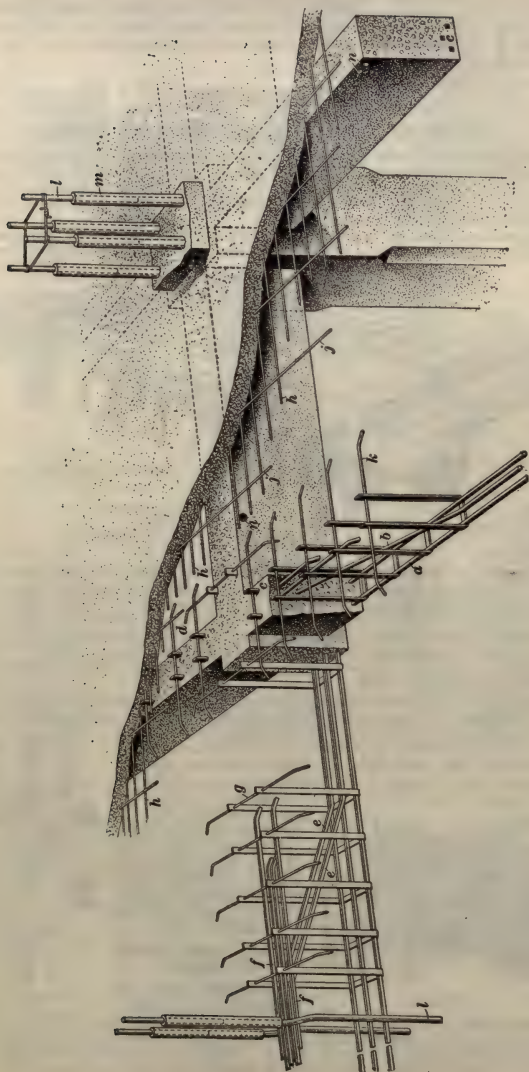
Fig. 6

sockets $m$, as illustrated. Frequently, it is not possible to lay out beforehand the electric-light or power wiring, but if this installation is to be adopted $1\frac{1}{4}$-in. pipes to serve as a passageway should be embedded near the center of the span of all beams and girders close to the under side of the slab construction, as at $n$.

# FORM WORK

## CONSTRUCTION AND FINISH OF FORM WORK

In the erection of reinforced-concrete work, nothing requires more careful consideration than the construction of the *form work*, or molds, necessary to shape and support the concrete until it has thoroughly set and hardened. Throughout the practice of reinforced-concrete construction various methods of form constructions are in use.

The greatest economy is gained by constructing the forms so that they can be used over and over again in the structure. Economy in construction can
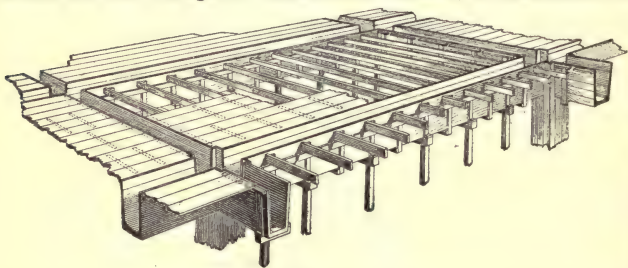


Fig. 1

also be gained by fastening the form work together with a minimum amount of nailing. Every nail that is driven gives trouble when the forms are taken down to be replaced for the upper floors. In many constructions, wedges and clamps are used instead of nails or screws if the forms are to be reused.

In some instances, both wooden and metal forms are coated on the side next to the concrete in order that the forms may be detached more readily. Coating the forms also serves to prevent the marking of the grain of the wooden forms on the finished concrete work.

Dead oil, or crude petroleum, has been used with success for this purpose. It is not unusual to soap wooden forms, and in some cases tallow and bacon fat have been employed. The latter is especially recommended for coating metal forms, as it seems to give the best results with forms of this material. In some instances, wooden forms have been covered on the inside with paper, and even canvas has been used, although it is usually found that the paper adheres to the concrete work and is detached only with difficulty. It is not customary, however, to oil or coat the forms unless they are to be used for fine exterior work.
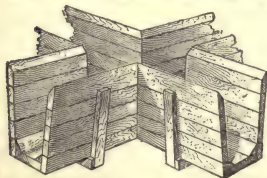


Fig. 2



Fig. 3

## FORMS FOR FLOOR SYSTEMS

**Common Types of Form Work.**—In Fig. 1 is shown a type of form work extensively used for the construction of reinforced-concrete floor systems, and in Fig. 2 is shown a perspective of the forms at the intersection of a beam and girder. This form work is designed so that light $\frac{7}{8}$-in. dressed tongued-and-grooved material may be used extensively in its
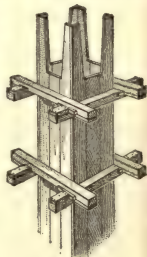
construction. It is so arranged that the sides of the beams and girders, together with the slab form boards, may be removed without the necessity of
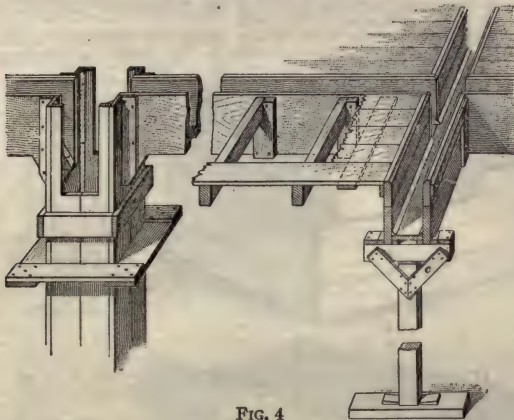


Fig. 4

removing the supports directly underneath the beams and girders. The form for columns in this type of construction is shown in Fig. 3.



Fig. 5

Fig. 6

(a) (b)
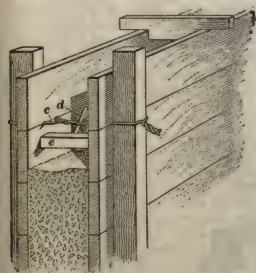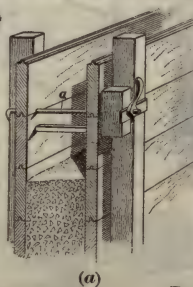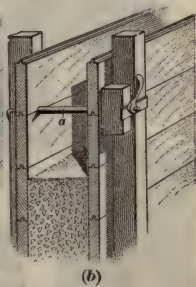
**Forms Constructed of Plank.**—A superior type of form for a reinforced-concrete floor system is shown in Fig. 4. The wooden forms are supported by 3″×4″ studs. As it is important to bring the forms to a true level, a double adjustment wedge is provided at the bottom of the studs. The forms for the columns are made of 1½- or 2-in. material. In the construction of the beam and girder forms 2-in. planks are generally used for the sides. In order to form a chamfer on the lower edges of the beams and girders, triangular fillet pieces are nailed in the forms. It is customary in this type of construction to make the forms for the slabs of ⅞-in. plain boards, frequently using tongued-and-grooved material for the purpose.



Fig. 7

**Wall Forms With Wire Ties.**—A type of wall-form construction that is frequently used is illustrated in Fig. 5. In order to prevent the sides of the

forms from spreading when the concrete is tamped in place, a wire tie *c* is used. This tie is made taut by twisting with a bar, or stick *d*. To keep the form boards the proper distance apart for the thickness of the wall, a block or stick *e*, of wood is sometimes inserted.

**Wall-Form Construction With Clamp Bolts.**—A wall-form construction similar to that shown in Fig. 5 is illustrated in Fig. 6 (*a*). In this form, how-



(*a*)

(*a*)

(*b*)

(*b*)

(*c*)          (*a*)

(*e*)
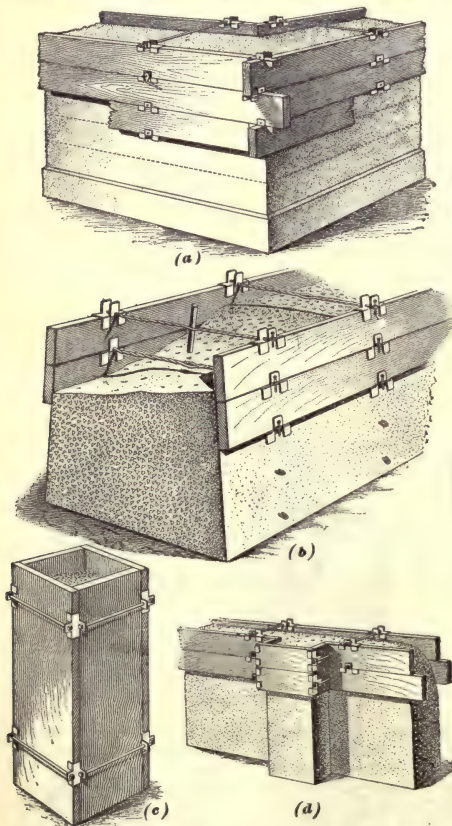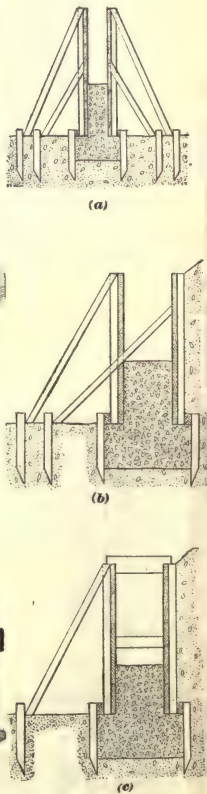
FIG. 8                                    FIG. 9

ever, a clamp bolt *a*, instead of a wire tie, is used to prevent spreading. If a bolt of this character is used, it must be knocked out before the concrete has finally set and when the form boards are to be raised to form the next course of concrete. The bolt is preferable to the wire tie, because it is removed from the concrete. Wire ties are usually cut off close to the concrete work after the form boards have been removed, and as the ends frequently project, they rust and thus stain the wall.

In (b) is shown the construction of a wall form in which a pipe separator *a* is used with the clamp bolt. The pipes may be driven out of the concrete after it has obtained its initial set, or they may be left in place.

**Clamping Devices and Plank Holders for Wall Forms.**—Many devices that aid in the construction of concrete walls have been invented. One of the most useful of these devices is the *Sullivan pressed-steel plank holder*, various forms of which are shown in Fig. 7. These holders are formed from an iron plate by shearing and bending it so as to form clips. The application of this type of plank holder is illustrated in Fig. 8.

**Braces for Wall Forms.**—If a wall is to be constructed in a place where there is no embankment, a double set of forms braced as shown in Fig. 9 (a) must be used. If the soil of an embankment against which a concrete wall is to be built is unstable, it is necessary to sheath and brace it. This may be accomplished in the manner shown in (b) or (c).

## CONCRETE MIXERS

In the construction of a reinforced-concrete structure, the quantity of concrete to be placed decides the amount of equipment and the character of the machinery that is to be employed. The character of the work also influences these two factors. In all instances, the concrete plant should be equipped with machinery suitable for the size of the work and the number of men that will be available in the construction operation. On small work, the concrete is frequently mixed by hand; it is, however, unusual for the concrete in large operations to be so mixed. The successful contractor will employ the mixing machine that is found most efficient and will give careful attention to its erection in the field.

There are many kinds of concrete mixers in commercial use. These mixers are classified according to the principles upon which they operate, and are known as *batch mixers* and *continuous mixers*.

Frequently, the selection of the mixer, especially in work where new equipment is to be used, is left to the superintendent. In making the selection, the superintendent should bear in mind the character of the work; that is, whether it is of more importance to turn out great quantities of concrete than it is to have a uniform mixture, or whether, as in a reinforced-concrete building, the most important consideration is to have concrete delivered from the mixer of uniform consistency. Usually, for heavy mass work the continuous mixer is advantageous, but the batch mixer is now more used for work such as that included in ordinary reinforced-concrete buildings.

The batch mixer often consists of a metal box in the shape of a cube, a cylinder, etc. The materials are put in this box and are mixed by revolving the box on a horizontal axis. There are often deflectors on the inside of the box to help in mixing. The machines are sometimes discharged by tipping them until the contents slide out of an opening or by other suitable means.

The continuous mixer often consists of a trough or cylinder or long box square in section. The materials enter continuously at one end and are discharged mixed at the other. They are usually mixed, in their passage through the machine, by paddles or deflectors, or by revolving the cylinder or box. The ingredients are often moved from the receiving to the discharging end of the machine either by the trough or cylinder being tilted or by paddles that gradually force the mass in one direction.

## CONCRETE STRUCTURES

It is proposed to mention briefly a few of the uses in engineering to which concrete can be put. Complete examples of work with all details cannot be given in such a short space, but the few words and illustrations may suggest to the designer ideas that may be elaborated and changed to suit various conditions. The calculations of stresses and proportioning of parts is beyond the scope of this work. They should be left to a competent designing engineer.

**Tank Tower of Reinforced Concrete.**—The tower shown in Fig. 1 was designed to carry a 200,000-gal. steel tank. The tower consists of eight concrete columns spaced on centers around the circumference of a 26½-ft. circle and surrounding a hollow concrete cylinder with an inside diameter of 8 ft. The columns and cylinders are held together by two intermediate platforms

and a heavy platform, or slab, at the top. The footing is spread over the entire base of the tower, and is in the form of a sixteen-sided polygon, being 6 ft. in thickness and 38 ft. in diameter at the bottom. The central shaft, in addition to carrying some of the load of the tank, acts as a cylinder around which the reinforced-concrete stairs are run and likewise forms a shaft for the pipes leading to and from the tank. The columns of the lower tier are 3 ft. 6 in. square; those of the second tier 3 ft. square; and those of the top tier, 2 ft. 6 in. square. The offset is made at the outside of the column, as shown in Fig. 2.

The details of construction and the method of reinforcing the columns, cylinder, and balcony floor construction are illustrated in Fig. 3, a half plan of the structure being shown in (a), and a section through the column construction, balcony, and cylinder, in (b). As will be observed, the thickness of the cylinder is dropped off 3 in. at each tier; thus, the walls of the cylinder at the bottom are 18 in. in thickness, and at the second and the third tier they are 15 in. and 12 in., respectively. The reinforcing rods are likewise reduced in size in both the columns and the cylinder from the first tier upwards. In the lower tiers, the columns have four 1¾-in. round rods; in the second tier, four 1⅝-in. rods; and in the upper tier, four 1⅜-in. rods. The vertical reinforcing rods are placed near the corners of the columns and are tied in with ¼-in. wire ties placed 12 in. from center to center. These rods were figured in estimating the compressive strength of the columns. Their ends are neatly fitted and are surrounded with a pipe sleeve. The bottom rods project into the reinforced-concrete footing and are slipped into pipe
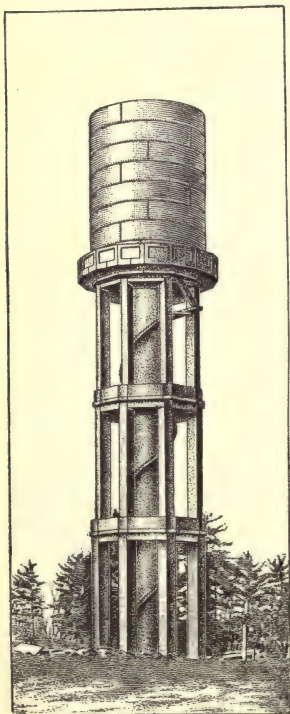


FIG. 1
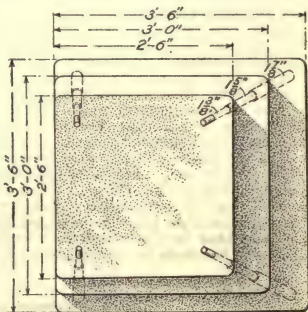


FIG. 2

sleeves provided with a 12-in. circular cast-iron base. In this way, the bearing strength of the bars was well realized. The cylinder is reinforced with vertical rods placed about 12 in. from center to center, and horizontal reinforcement, consisting of 1-in. hoop rods, is also provided in it. The balcony floors are reinforced with rods that radiate from the cylinder and bear upon the concrete lintels spanning the space between the columns, each of these lintels being reinforced with four corrugated bars. The floor of the third balcony, which carries the tank, is about 36½ ft. in diameter and 16 in. in thickness. It is heavily reinforced with radiating 1-in. rods, and is cantilevered about 4 ft. beyond the

column. At each balcony is provided a monolithic railing, or balustrade. This railing is about 3 ft. high and 6 in. in thickness and is reinforced. In the construction of this work, a 1–3–5 mixture was used for the foundation and a 1–2–4 mixture, placed very wet, was employed for the rest of the construction. The aggregates used throughout the construction were sand and good,
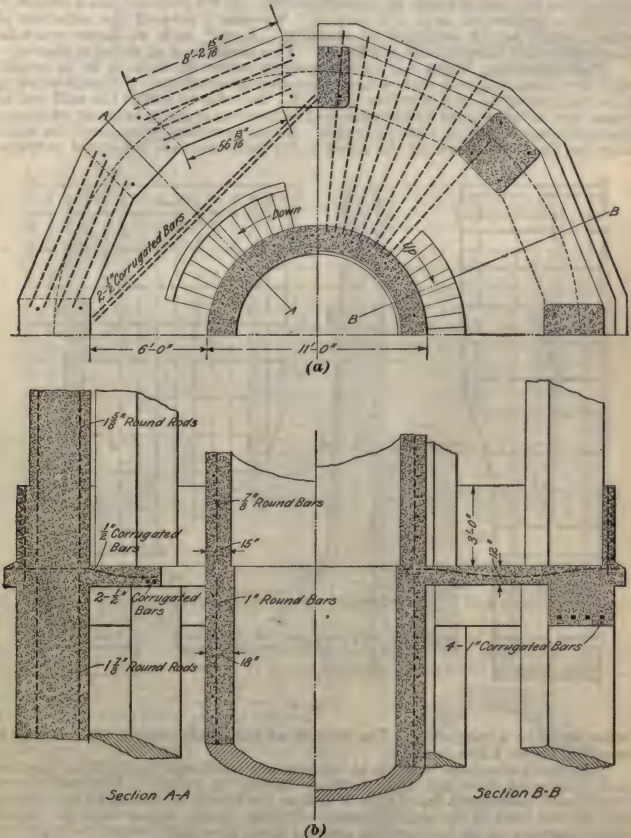


Fig. 3

clean gravel, which, being very coarse, took the place of broken stone. No effort was made to finish the structure after erection, but in placing the concrete a very wet mixture was used. It was thoroughly spaded along the sides of the forms, so as to force the gravel away from the surface and allow the neat cement to mold smooth against the form boards. The work that resulted was smooth and presentable.

**Reinforced-Concrete Retaining Walls.**—Fig. 4 shows a retaining wall of moderate height. The base is spread out at $AB$ so as to make the wall less easily tipped. The earth behind the wall resting on the part $B$ has to be lifted if the wall tips, so this earth also assists in preventing the wall from turning over. The part $A$, called the *toe*, extends out in front of the wall, and by increasing the lever arm of the loads also tends to prevent tipping. The load of earth on the part $B$ tends to prevent sliding. If the wall still has a tendency to slide, a projection, as at $c$, is put on. Weepers $d$ prevent the collection of water, which would endanger its stability. A wall such as shown must be thoroughly and scientifically reinforced. One method of so doing is shown.

**High Retaining Walls.**—If retaining walls are to be over 12 ft. in height, they are often designed with a buttress every 10 or 12 ft. A design suitable for a high retaining wall is shown in Fig. 5. In order to stiffen the wall and to cause it to act merely as a slab subjected to transverse stress between supports, the buttresses shown at $a$ are provided. As the resultant pressure of the earth is in the direction shown by the arrow $b$, the buttresses are subjected to trans-
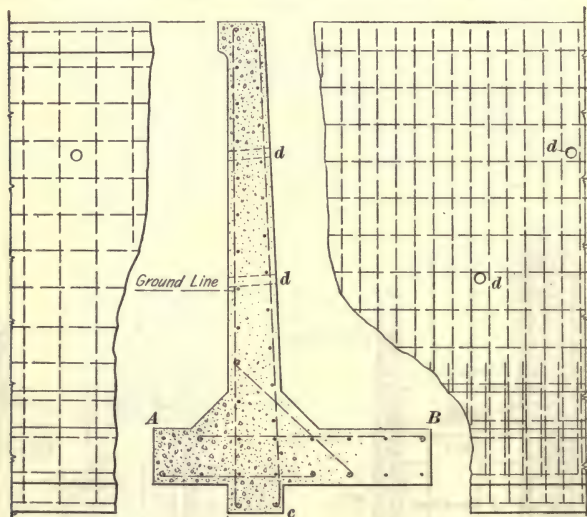


FIG. 4

verse stress as a cantilever. The tension at the inside edge of the buttress is provided for by Kahn bars $c$.

The wall spans from buttress to buttress are reinforced in the same manner as a reinforced floor slab span between beams or girders. The wall is reinforced with horizontal Kahn bars $d$ and as the thrust of the earth per square foot on the back of the wall increases with the depth, these bars are placed closer together at the bottom, the spacing being increased toward the top.

To prevent its failure, the footing is reinforced along the inside edge with the beam $e$. This beam is monolithic with the buttress, so that the footing is logically reinforced with transverse bars $f$. In order that the entire design may be stiffened and strengthened, gussets or fillets are inserted in the junctions between the buttresses and the footing. To reinforce the footing further, longitudinal bars $g$ are provided. Additional reinforcement $h$ running in a vertical direction is provided in the wall.

**Conduits.**—Reinforced concrete is used for such constructions as water conduits, sewers, aqueducts, etc. A type of conduit for carrying water is

shown in Fig. 6. As will be observed, it is reinforced with expanded metal. In nearly all instances, such conduits are constructed with collapsible centering



PLAN

SECTION AT A-B

REAR ELEVATION
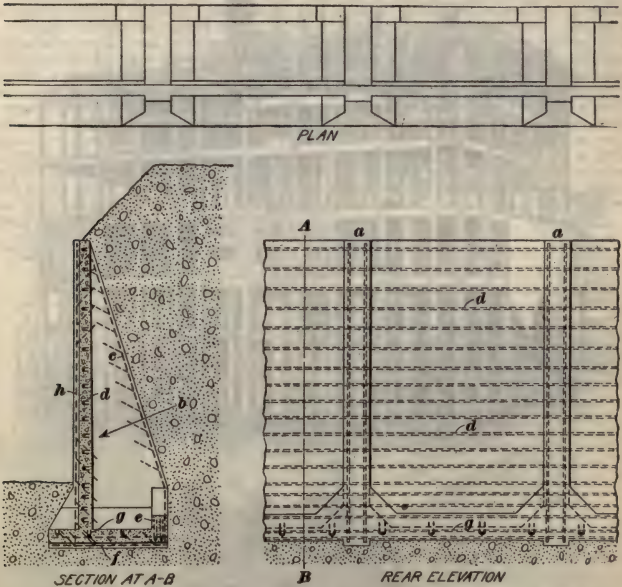
FIG. 5

inside, and with forms and lagging for the outside work. The work is always carried along in sections.

**Coal Breakers in Reinforced Concrete.**—The Taylor coal breaker was the first all reinforced-concrete breaker constructed in the anthracite fields, although the Pine Hill breaker, at Minersville, Pa., was built in 1906, of reinforced concrete from the foundations to the main breaker floor, including the coal pockets, slate pockets, and shaker and jig supports.

Under favorable conditions, the average wooden anthracite breaker has a life of about 20 yr., but mostly conditions are unfavorable to such longevity, and this, coupled with the rapidly advancing price of timber suitable for such structures, has caused engineers to consider the more durable iron and reinforced concrete as building materials. Wherever anthracite is prepared wet, the decay of the timbers is hastened.

The rear of the breaker in the course of construction is shown in Fig. 7. The first two rows of posts in the rear of the breaker are about 2 ft. square and up to the second or pocket floor have a length of 65 ft. Directly under and between the pockets, where the most weight will come, the 2-ft.-square posts are supplemented with 36-in.-square posts. As the posts are
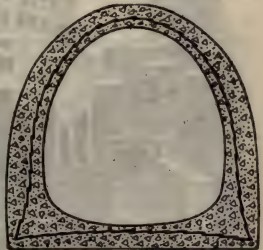


FIG. 6
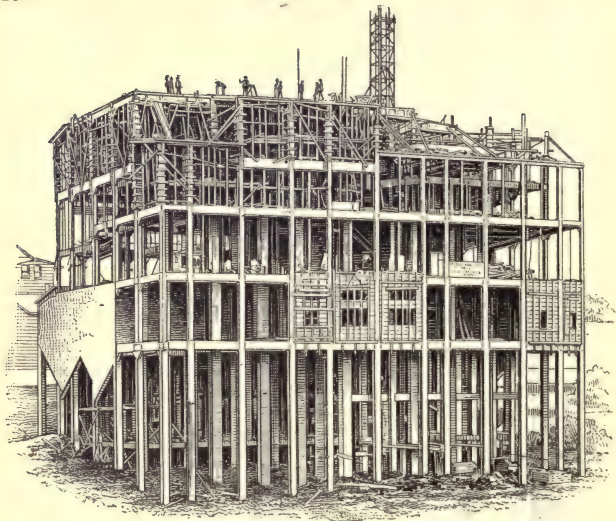
FIG. 7



FIG. 8

carried upwards this size is decreased until at the top floors they are but 12 in. square.

In Fig. 8 the breaker is shown practically finished with the top forms still in place, and as it looks today from the outside, could be taken for an office building. It will be noted that the architect has furnished windows for daylight; also each post is supplied with tubes for wiring for electric lights, thus making this breaker an exceptional one for light and comfort of the employes. In the construction of the posts and beams, corrugated rods $1\frac{1}{2}$ in. in diameter were placed at each corner, then bound with hoop bands and wire so as to form a rectangular cage. They were reinforced by smaller corrugated iron rods from $\frac{5}{8}$-in. diameter up to $1\frac{1}{2}$-in., as the occasion demanded. Inside this cage pipes for the electric wiring were placed and then wooden forms were constructed around these skeleton post frames. Concrete was poured in a little at a time from the top of the forms and tamped with iron rods about the pipes and rods. In this way the posts were built up to the height of a floor, where the rods for the beams were tied with those of the posts. The beams were made the same as the posts.



FIG. 9

It will be noted that the coal pockets are large and provided with two chutes so that two cars can be loaded from the same pocket at the same time. There are fourteen chutes for each track, or twenty-eight for both tracks. To the rear of Fig. 8 is seen the old Taylor breaker, constructed of wood.

It may be of interest to know that while the Taylor breaker is concrete, 500,000 ft. of lumber will be needed inside for machinery bed-plates and other fittings. However, very little if any of this lumber is large sized and there are no sticks such as breaker framing demands.

**Concrete Coal Pockets.**—The Pennsylvania Coal Co. has erected a new breaker at Throop, Pa. In the construction of this breaker both reinforced concrete and steel are used; reinforced concrete for the coal and rock pockets, and part of the washery, and steel for the main part of the building, which will contain the washery.

The capacity of the coal pockets is 3,500 T. The coal cars are to be loaded directly beneath the pockets, while box cars are to pass on the outside of the building and are to be loaded from a chute leading from the center of the bottom slab of the pocket. The slope on the bottom of the coal pockets is 9 in 12 and on the rock pockets 6 in 12. The width of these pockets varies from 10 ft. 8 in. to 16 ft. Where the pocket is not over 12 ft. wide, the floor slab is designed so

as to carry the load without any beam for support. But for the pockets 16 ft. wide a beam is placed beneath the floor slab. Fig. 9 shows a view through the coal pockets of this breaker during construction. The forms, it will be noted, are still in place.

**Concrete Shaft Lining.**—Concrete is now being used to line shafts in mines. As yet there is no uniform method of doing this work and the designs are changed to suit various conditions to be met. The shaft with concrete lining about to be described is the main shaft of the Filbert mine of the H. C. Frick Coke Co.; it is located in Fayette County, Pa.

The shaft, elliptical in plan, measures 13 ft. × 28 ft. in the clear on the center lines of the axes, and has a depth of 550 ft. below the top of the coping to the



Section B-B.　　　Section A-A.
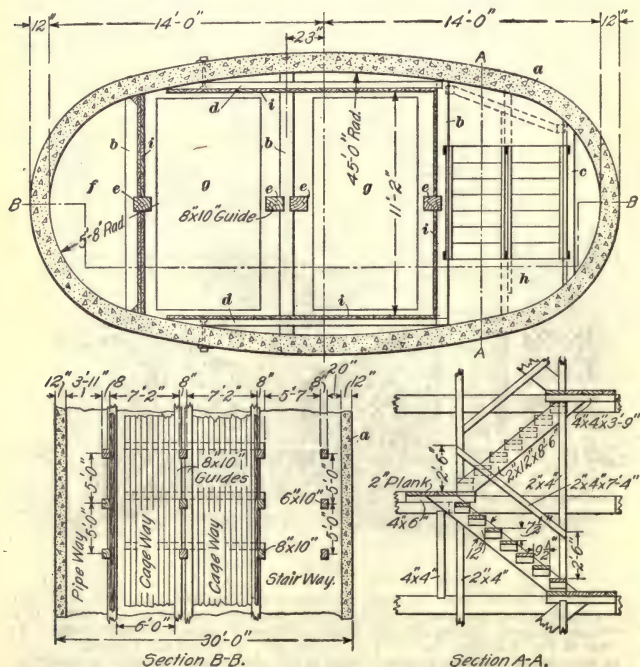
FIG. 10

bottom of the 9-ft. coal bed, and is provided with a sump 15 ft. below the coal. This shaft is divided into four compartments, containing two cageways, a stair-way, and pipeway.

The plan of the shaft is shown in Fig. 10. At $a$ is shown the concrete lining; at $b$, the $8'' \times 10''$ yellow-pine buntons; at $c$, the $6'' \times 10''$ buntons; at $d$, the $4'' \times 8''$ yellow-pine nailing strips; and at $e$, the yellow-pine cage guides. The compartment reserved for the various pipes that go up the shaft is shown at $f$. The two hoisting cages are shown at $g$. At $h$ is a compartment for foot travel-ing fitted with steps and landings. The hoisting part proper is largely lined with 1-in., yellow pine, tongued-and-grooved sheathing.

The circumference of the inside of the concrete lining is 69 ft., with a clear-opening area of 310 sq. ft., comprising 195 sq. ft. for the two cageways, 80 sq. ft.

for the stairway, and 35 sq. ft. for the pipeway. The ends of the shaft conform to a radius of 5 ft. 8 in. and the curvature of the sides to a 45-ft. radius. From the top of the coping to solid rock, a distance of 19 ft., the structure is of heavy concrete construction forming a solid foundation for the structural steel head-frame of 135 ft. superimposed thereon. Fig. 11 shows the irregular-shaped plan and elevation of this portion of the work. This shaft has two water rings constructed therein; one at 78 ft., and the other at 494 ft. below the top of the coping.

All concrete for lining the shafts is composed of 1 part Portland cement; 2 parts clean, sharp, river sand, and 5 parts of stone crushed to pass through 1½-in. ring. About 50% of the stone used for concrete was obtained from the
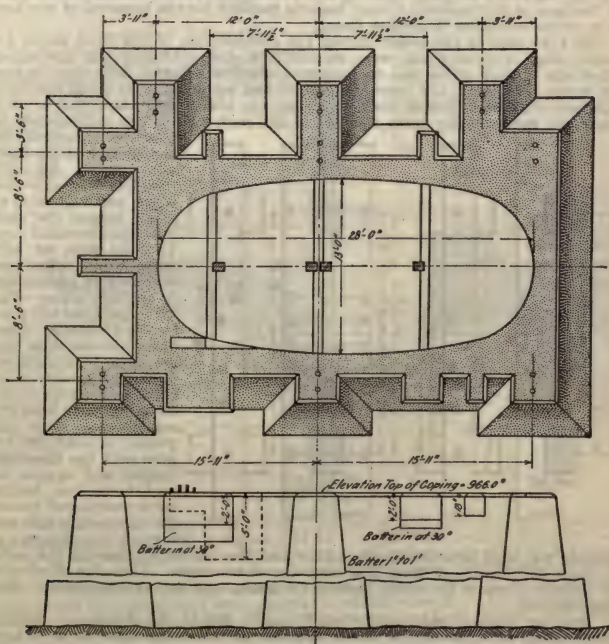


FIG. 11

materials excavated from the shafts. About 30% was shipped in, crushed ready for use, while about 20% was obtained from a quarry on the grounds.

In sinking this shaft, an excavation was first made to a depth of 65 ft. All excavating was done to the full measurements of the outside perimeter of the concrete lining, being kept to a correct line by plumb-bobs suspended from a template placed above the opening. Derricks were used to handle the material excavated. At the depth mentioned, the concreting was begun. Forms were put in near the bottom to the height of 5 ft. and filled. Then 5 ft. more of forms were put on top of the first forms and again filled. Then another section was built and filled and so on until the surface was reached. The shaft was then sunk about 50 ft. deeper and concreted up from the bottom in the same way. Then another section of the shaft was sunk, and so on until the job was completed.

The dividing struts for the compartments of the shafts are 8″ × 10″ buntons, spaced vertically 5 ft. center to center. The buntons support the guide rails for the cages—these are 8 in. × 10 in., surfaced on four sides and fastened to the dividing struts or buntons as shown in Fig. 12. The buntons are set in the concrete shaft lining to a depth of 6 in. on each end, allowing from 6 in. up of concrete beyond their ends, thus insuring a water-tight wall. The minimum thickness of the concrete lining wall is 12 in. In soft strata, the concrete is as much as 33 in. in thickness, as no voids were left between the rock and lining, all such being carefully filled with concrete. Bolts for the guides have a 1-in. stud nut and two wrought-iron washers ⅜ in. thick.

Throughout a portion of the shaft, wedge-shaped blocks with a base 6 in. × 8 in. and 4 ft. long, tapering to a point, were placed on the buntons just at the face of the concrete lining, making pockets for the removal of the buntons, when necessary to replace them.

To take care of the water during the construction of the shaft above the water rings, two methods were successfully used. During the concreting, when any flow of water was encountered, sheet-iron plates were used to turn the water away from the concrete until it had set sufficiently so as not to be damaged by water flowing over it. Then the sheets were deflected so as to cause the water to run down the face of the rock wall of the shaft, where, at
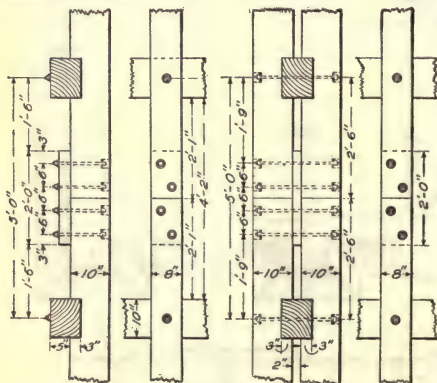


intervals around the circumference of the shaft, were placed 3-in. tile pipe set in loose broken stone and extending down behind the shaft lining to the water ring. The tile pipe, set with open joints, took care of the water, conducting it to the water ring. To hold the tile pipes to their proper place and to insure their non-stoppage from concrete packing in around them, sheet-iron strips of No. 22 gauge were bent into semicircular form and placed around the tile pipe with the open side toward and against the rock face; then broken stone was placed around the pipe within this protection. In this way an

**Fig. 12**

opening was kept between the rock face and the tile pipe, allowing the water to reach the drain and thus flow into the water rings. This also eliminated the danger of water gathering behind the lining wall and exerting undue pressure thereon.

Another method successfully used was to locate the fissure or opening where larger streams entered the excavation, and to enlarge them so as to form a reservoir of several cubic feet capacity; then to tightly close the openings of this reservoir with concrete and sheet iron, having placed a wrought-iron pipe of sufficient capacity to handle the water gathered therein. This pipe conducts the water to a pipe set vertically in the lining-wall concrete and leading to the water rings. This latter method was used only where a large flow of water was encountered.

The water rings in the shaft were constructed behind the shaft-lining wall, where a niche was blasted out of the rock to form an opening 2 ft. wide and 4 ft. high throughout the entire perimeter of the shaft. The bottom of this niche was concrete in gutter shape, with a drop of 3 in. from its highest to its lowest point, located at the pipe-compartment end of the shaft. At the lowest point a wrought-iron pipe, 2½ in. in diameter, set through the lining wall on an angle of 45° and protected by strainer plates, connected with a line in the pipe compartment and led the water to a permanent disposal pump. In the shaft-lining wall, on its inner face and throughout the entire perimeter of the wall, and at a point near the top of the water ring, a groove 4½ in. deep and 5 in. from the face

of the groove to the edge of the lip extending 1½ in. beyond the inner face of the wall was constructed to catch any seepage or water from the surface of the lining. This groove and lip has a fall of 3 in. from its highest to its lowest point, where a 2-in. wrought-iron pipe passes through the lining wall on an angle of 45° leading all water caught in the groove to the water ring. Thus, all the water both behind the lining wall and all water on the inner surface is collected in the water rings and thence led to the pumps and expelled from the shaft.

In the shaft, the approaches leading to the shaft bottom from both the loaded and empty sides are of concrete. These are of 18-ft. span having a minimum thickness of 24 in. for side walls and crowns. Each arch extends
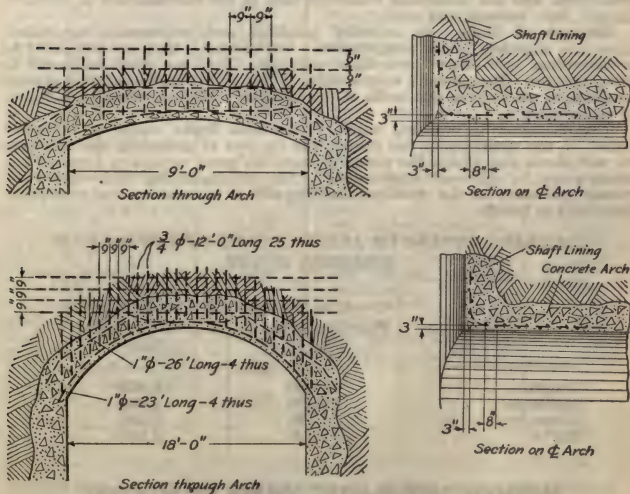


FIG. 13

13 ft. from the face of the shaft lining. Arches of 9-ft. span having a minimum thickness of 18 in. extend a distance of 11 ft. 6 in. from both ends of the face of the shaft lining. These arches connect with the cross-entries and manway around the shaft bottom. An archway of 4-ft. span was placed in the side walls for entrance to the run-around. At the intersection of all arches with the shaft-lining wall, steel bars were placed for reinforcement. The method of this reinforcement is shown in Fig. 13. All centering was built of 2-in. oak plank, spaced 2½ ft. center to center, covered with 2-in. tongued-and-grooved oak lagging. Concreting for the four arches was brought up at one time and also at the same time as the shaft-lining wall was placed.

# MASONRY

## MATERIALS OF CONSTRUCTION

### STONE

The materials employed in the construction of masonry are stone, brick, terra cotta, and the cementing materials used in the manufacture of mortars; namely, lime, cement, and sand.

**Strength of Stone.**—In ordinary buildings and engineering structures, stones are generally under compression. Occasionally, they are subjected to cross-stresses, as in lintels over wide openings. They are never subjected to direct tension. As a general rule, a stone should not be subjected to a greater compressive stress than one-tenth of the ultimate crushing strength, as found by experiment.

The resistance to crushing varies within wide limits, owing to the great variety in the structure of the stones; the method of preparing and finishing the test pieces also affects the results; hence, the great variations found in the values given by different experiments. The accompanying table shows the average resistance of the principal building stones to crushing and to rupture when used as beams.

### CRUSHING STRENGTH AND MODULUS OF RUPTURE OF BUILDING STONE

| Stone | Crushing Strength Pounds per Square Inch | Modulus of Rupture Pounds per Square Inch |
|---|---|---|
| Granite............. | 15,000 | 1,800 |
| Sandstone........... | 10,000 | 1,200 |
| Limestone........... | 13,000 | 1,500 |
| Marble............. | 14,000 | 2,160 |

### MINIMUM SAFE-BEARING VALUES OF MASONRY MATERIALS

| Materials | Safe-Bearing Value Tons per Square Foot |
|---|---|
| Granite, capstone................................ | 50 |
| Squared masonry............................ | 25 |
| Sandstone, capstone............................ | 25 |
| Squared masonry........................... | 12 |
| Rubble, laid in lime mortar................. | 5 |
| Rubble, laid in cement mortar.............. | 10 |
| Limestone, capstone......................... | 36 |
| Squared masonry........................... | 18 |
| Rubble, laid in lime mortar................. | 5 |
| Rubble, laid in cement mortar.............. | 10 |
| Bricks, hard, laid in lime mortar.............. | 7 |
| Hard, laid in Portland cement mortar...... | 14 |
| Hard, laid in Rosendale cement mortar...... | 10 |
| Concrete, 1 Portland, 2 sand, 5 broken stone... | 10 |

## ULTIMATE UNIT CRUSHING STRENGTH OF VARIOUS STONES AND STONE MASONRY PIERS

| Material | Compressive Strength Pounds per Square Inch | Material | Compressive Strength Pounds per Square Inch |
|---|---|---|---|
| Granite, Colo............ | 15,000 | Limestone, Marquette, Mich............... | 8,000 |
| Granite, Conn........... | 14,000 | Limestone, Conshohock- en, Pa.............. | 15,000 |
| Granite, Mass........... | 16,000 | | |
| Granite, Me............ | 15,000 | Marble, Montgomery Co., Pa............. | 11,000 |
| Granite, Minn........... | 25,000 | | |
| Granite, N. Y........... | 16,000 | Marble (dolomite), Lee, Mass............. | 22,800 |
| Granite, N. H........... | 12,000 | | |
| Bluestone............. | 15,000 | Marble (dolomite), Pleasantville, N. Y. ... | 22,000 |
| Sandstone, Middletown, Conn............... | 7,000 | Marble, Italian......... | 12,000 |
| | | Marble, Vt............ | 10,000 |
| Sandstone, Longmeadow, Mass............... | 10,000 | Slate................. | 10,000 |
| | | Piers, ashlar, bluestone.. | 2,100 |
| Sandstone, Hudson River, N. Y.......... | 12,000 | Piers, ashlar, granite .... | 2,100 |
| | | Piers, ashlar, limestone.. | 1,500 |
| Sandstone(brown), Little Falls, N. Y........... | 10,000 | Piers, ashlar, common sandstone.......... | 1,050 |
| Sandstone, Ohio........ | 8,000 | | |
| Sandstone (brown), Hummelstown, Pa... | 12,000 | Piers, rubble, cement mortar.............. | 900 |
| Limestone, Kingston, N. Y.............. | 12,000 | Piers, rubble, lime mor- tar................. | 480 |
| Limestone, Garrison Station, N. Y........ | 18,000 | | |
| Limestone (oolitic), Bed- ford, Ind. ........... | 8,000 | | |

## ULTIMATE CRUSHING STRENGTH OF BRICK MASONRY PIERS
(*Average Age of Brickwork, 6 Mo.*)

| Material | Composition of Mortar | Compressive Strength Pounds per Square Inch |
|---|---|---|
| Wire-cut brick.................. | 1 cement, 5 sand | 3,000 |
| Dry-pressed brick............... | 1 cement, 5 sand | 3,400 |
| Dry-pressed brick............... | 1 cement, 1 lime, 3 sand | 2,300 |
| Repressed brick................. | 1 cement, 5 sand | 1,700 |
| Light-hard, sand-struck brick...... | 1 cement, 5 sand | 1,900 |
| Light-hard, sand-struck brick...... | 1 cement, 7 sand | 853 |
| Hard, sand-struck brick........... | 1 cement, 1 sand | 2,100 |
| Hard, sand-struck brick........... | 1 cement, 1 lime, 3 sand | 1,500 |
| Hard, sand-struck brick........... | 1 cement, 5 sand | 1,200 |
| Sand-lime brick................. | 1 cement, 3 sand | 1,100 |
| Sand-lime brick................. | 1 lime, 3 sand | 450 |
| Sand-lime brick................. | Neat cement | 1,400 |
| Terra-cotta work................ | 1 cement, 3 sand | 2,000 |

**Absorptive Power of Stone.**—The absorptive power of a stone is a very important property, a low absorption generally indicating a good quality. The accompanying table gives the average percentage of water absorbed by stones.

### ABSORPTIVE POWER OF STONE

| Stone | Absorptive Capacity Per Cent. |
|---|---|
| Granites......... | .066 to .155 |
| Sandstones...... | .410 to 5.480 |
| Limestones...... | .200 to 5.000 |
| Marbles......... | .080 to .160 |
| Trap............ | .000 to .019 |

**Durability of Stone.**—The following rough estimate, based on observations made in the city of New York, indicates the number of years a sound stone may be expected to last without being discolored or disintegrated to such an extent as to require repairs:

|  | *Life of Stone* |
|---|---|
| *Name of Stone* | *Years* |
| Coarse brownstone.... | 5 to 15 |
| Compact brownstone.. | 100 to 200 |
| Limestone........... | 20 to 40 |
| Granite.............. | 75 to 200 |
| Marble.............. | 40 to 200 |

## BRICK

**Size and Weight.**—The dimensions of bricks vary considerably. The standard adopted by the National Brickmakers' Association is, for common clay brick, $8\frac{1}{4}$ in. $\times 4$ in. $\times 2\frac{1}{4}$ in., and for face or pressed brick (clay) $8\frac{1}{4}$ in. $\times 4\frac{1}{4}$ in. $\times 2\frac{1}{4}$ in. The weight of a common clay brick is about $4\frac{1}{2}$ lb.; that of a pressed-clay, enameled brick, about 7 lb. Enameled and glazed bricks are made in two sizes: English size, 9 in. $\times 3$ in. $\times 4\frac{1}{2}$ in.; American size, $8\frac{3}{8}$ in. $\times 2\frac{1}{4}$ in. $\times 4\frac{1}{2}$ in. The usual dimensions for firebricks are 9 in. $\times 4\frac{1}{2}$ in. $\times 2\frac{1}{4}$ in.; various sizes and forms are made to suit the required work. The dimensions of the lime-sand bricks are $8\frac{3}{8}$ in. $\times 4\frac{1}{4}$ in. $\times 2\frac{13}{16}$ in. The weight varies between 5 and 6 lb. The accompanying table gives the approximate weight and resistance to crushing of brick.

### WEIGHT AND STRENGTH OF BRICK

| Kind of Brick | Weight Pounds per Cubic Foot | Crushing Strength Pounds per Square Inch |
|---|---|---|
| Best pressed-clay...................... | 150 | 5,000 to 15,000 |
| Common hard-clay..................... | 125 | 5,000 to 8,000 |
| Soft-clay............................. | 100 | 450 to 600 |
| Lime-sand............................ | 120 | 3,600 to 7,600 |
| Firebrick............................. | 120 | 1,000 to 1,500 |

**Requisites for Good Brick.**—Bricks of good quality should be of regular shape, with parallel surfaces, plane faces, and sharp square edges. They should be of uniform texture; burnt hard; and thoroughly sound, free from cracks and flaws. They should emit a clear ringing sound when struck a sharp blow. A hard well-burned brick should not absorb more than one-tenth of its weight of water; it should have a specific gravity of 2 or more. The crushing strength of a brick laid flat should be at least 6,000 lb. per sq. in. The modulus of rupture should be at least 1,000 lb. per sq. in.

# WIRE ROPES*

## GENERAL DESCRIPTION

### WIRE-ROPE MATERIALS

Wire ropes are used about mines chiefly for hoisting from shafts, for haulage and the transmission of power, for the cables of aerial tramways, for the guy ropes of derricks and smokestacks, etc., and, rarely, for the cables of small, short-span, suspension bridges, as where the town or settlement is situated on the opposite side of a narrow stream from the mine. While wire ropes are now almost universally made of steel, manufacturers still make and list iron ropes, which have a limited field of usefulness.

*Swedish*, *Swedes*, or *charcoal-iron* ropes are made of a very pure wrought or puddled iron having a tensile strength of from 50,000 to 100,000 lb. per sq. in. These ropes are soft, tough, and pliable, and are adapted especially for passenger elevators, small hoists, steering gear of vessels, etc., where the loads are intermittently applied and are not too great, or where the speed is high and the bending stresses great. It will be noted from tables given later that the ultimate breaking strength of a $6 \times 19$ Swedish iron hoisting rope 1 in. in diameter, is but 14.5 T., whereas, the breaking strength of a steel rope of the same kind and size is from 30 to 45 T. For general mine use, iron ropes have been almost entirely superseded by steel ropes because of their greater strength and elasticity.

*Steel ropes* are generally made of open-hearth steel having a tensile strength of from 150,000 to 275,000 lb. per sq. in. and in some cases even more, the tensile strength depending on the composition of the metal and the method of its treatment. Steel ropes are in almost every way superior to iron ropes. The principal advantage is that they have more than double the strength of iron ropes of the same size; consequently, for equal strains, they can be made of much less diameter than iron ropes and can, therefore, be used in connection with much smaller and lighter drums, sheaves, or pulleys. Iron-wire ropes are not so elastic as ropes made of steel wire, hence a larger sheave is required for iron than for steel ropes of the same diameter. Iron ropes, however, are usually more flexible than steel ropes, are less brittle though not so strong, and better resist the acid in mine water. A 1-in. Swedish iron rope has about the same strength as a $\frac{5}{8}$-in. ordinary cast-steel rope; weighs 1.58 lb. per ft. as opposed to .62 lb.; costs (list price) 26c. per ft. as opposed to 16.5c.; and requires a sheave or drum 6 ft. in diameter as against one 2.5 ft. for a cast-steel rope.

*Cast-steel*, *crucible-steel*, and *crucible cast-steel ropes* are the trade names given to the ordinary grades of ropes made from wire having an ultimate tensile strength of 160,000 to 210,000 lb. per sq. in. The breaking strength of a $6 \times 19$ standard hoisting rope of this grade and 1 in. in diameter is given, in the manufacturers' tables, as 30 T., more than twice the strength of a similar iron rope. Ropes of this material are those commonly used in and around mines for haulage and hoisting purposes.

*Extra strong cast-steel*, *extra strong crucible-steel*, *special steel*, and *patent steel ropes* are the trade names for the next stronger grades of ropes, intermediate in strength between cast-steel and plow-steel ropes in strength. The wire from which they are made has an ultimate tensile strength of from 190,000 to 230,000 lb. per sq. in. The breaking strength of a $6 \times 19$ hoisting rope of this grade and 1 in. in diameter is given as 34 T., 11.33% more than that of an ordinary cast-steel rope of the same dimensions. These ropes are also standard and are in general use where it is desirable to increase the factor of safety while retaining the same diameter of rope.

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*Plow-*, or *plough-steel ropes* are made of wires having an ultimate tensile strength of 220,000 to 250,000 lb. per sq. in. The breaking strength of a $6 \times 19$ hoisting rope of this grade and 1 in. in diameter is given as 38 T., or 11.18% more than that of an extra strong cast-steel rope of the same dimensions. Ropes of this grade are not generally recommended, except where it is necessary to have the maximum of tensile strength with the least weight of rope, or where it is necessary to employ a rope of much greater strength but of the same diameter. The first necessity will arise when hoisting through extremely deep shafts; the second, where by reason of increased loads it becomes essential to have a stronger rope, but at the same time the diameter of the rope is fixed by the size of existing drums, sheaves, etc. Plow-steel ropes are extensively employed for logging lines, dredge and wrecking ropes, ballast unloading ropes, quarry ropes, etc.

*Extra, special,* or *improved plow-steel ropes* are made of wires having an ultimate tensile strength of from 240,000 to 300,000 lb. per sq. in. A rope of this grade of the size cited before has a breaking strength of 45 T., or about 11.84% more than an ordinary plow-steel rope of the same dimensions. The comments upon standard plow steel ropes apply as well to ropes of this grade.

## CONSTRUCTION OF WIRE ROPES

Wire ropes consist of a number of strands, each composed of the same number of single wires twisted around a hemp or wire core or center to form a single rope. The hemp core adds practically nothing to the strength of the rope but, being saturated with lubricant, tends to prevent rusting of the wires and, being soft, acts as a cushion for the individual strands, thus reducing internal friction and wear. A wire core adds largely to the internal friction and consequent wear of rope as the strands rub upon the wire center; increases the weight; and, while adding about 10% to the strength, reduces the flexibility in a marked degree, at the same time adding 10% to the cost. Manufacturers, whose judgment should be final, recommend ropes with wire cores only for standing lines, such as the guy ropes of derricks, etc., because they lack the flexibility demanded of running ropes (those used for hoisting, haulage, etc.), and because of their much greater internal wear in bending around drums, sheaves, and the like.



FIG. 1

**Lay of Ropes.**—The lay of a rope is the direction of the twist of the strands composing it. Ropes are either right or left lay, the former being the ordinary construction as shown in Fig. 1 where the strands are bent to the right. The left-lay construction is shown in Fig. 2.

The term lay is also used to describe the direction of twist of the individual wires composing the strands in a rope. Thus, in Fig. 1, while the rope is right lay (strands twisted to the right), the strands are left lay, the single wires being twisted to the left. Similarly, in Fig. 2, while the rope is left lay, the strands are right lay.

Finally, the term lay is used to designate the pitch of the rope; that is, the rate at which the strands twist or, what is the same thing, the ratio that the length of strand required for one complete turn bears to the diameter of the rope.

In ordinary rope making, the lay or (better) pitch of the wires varies from 2.5 to 3.5 times the diameter of the rope, and that of the strands from 6.5 to 9 times the diameter of the rope. The lay exerts an important influence upon the life of a rope. For the same kind and size of rope, the shorter the lay or pitch, the greater the flexibility



FIG. 2

and elasticity, but the less the strength. This falling off in the strength, due to the shortening of the pitch, is brought about by the nicking, or cutting, of one wire by another, which is naturally less when the ropes cross one another at a long angle (long pitch) than when they cross at a sharp angle (short pitch).

In practice, ropes are commonly classified as ordinary-lay or regular-lay ropes, and as Lang lay or universal-lay ropes. In the *ordinary lay ropes,* which are shown in Fig. 1 and Fig. 2, the wires in the strands are twisted in the opposite direction from that of the strands in the rope, while in *Lang lay ropes,* shown in Fig. 3, the single wires and the strands are twisted in the same

direction. Lang lay ropes may be either right or left twist, and their price is the same as that of ordinary-lay ropes.

The principal advantages of the Lang lay are that, the wires and strands being twisted in the same direction, the surface of the rope is smoother, the outside wires do not so soon become worn as a much longer surface of each wire is exposed to wear, and, the wires being straighter, these ropes are somewhat more flexible. The disadvantages of Lang lay ropes are a tendency to untwist, rendering them unsuited for hoisting except where guides are used; they can be spliced to ropes of ordinary-lay only with difficulty; and when the wires break, the loose ends are very troublesome, because a much greater length of each wire is exposed



FIG. 3

than in ordinary-lay ropes. Under careful inspection, a regular-lay haulage rope may be used for some time after a few of its wires are broken here and there throughout its length, except when a dangerous risk would be incurred by so doing. The Lang lay ropes are commonly used for haulage, particularly where grips are used to attach the cars to the ropes, and are sometimes used for hoisting, but only where the cage works in guides.

## HOISTING ROPES

### ROUND ROPES

**6 × 19 Ropes.**—Ropes used for hoisting through shafts are round or flat and in either case may be of uniform or tapering section. Round ropes of uniform section, are practically the only ones used in American mines. The standard American hoisting rope, shown in Fig. 1, is composed of 6 strands of 19 wires each (114 wires) wrapped around a hemp center. It is frequently spoken of as a 6 × 19 rope. These ropes are commonly made of cast-steel or, where greater strength is required, extra strong cast-steel. The diameter of sheave recommended for use with a 1-in. rope of this type is variously given at 4 to 4.5 ft.
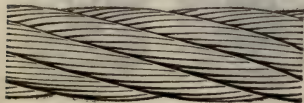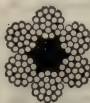


FIG. 1

**8 × 19 Ropes.**—Where extreme flexibility is required, ropes composed of 8 strands of 19 wires each (152 wires) may be employed. From the tables given later, it will be noted that the maximum diameter of rope of this section commonly carried in stock is $1\frac{1}{2}$ in. as against $2\frac{1}{4}$ in. for the 6 × 19. Fig. 2 shows that the core is much larger in proportion to the area of metal than in a 6 × 19 rope of the same size and quality. Consequently, this rope is not so strong as a 6 × 19 rope of the same diameter (24 T. as against 30 T. for 1-in. cast-steel rope) and is more liable to flatten out under heavy pressure. The diameter of sheave suggested for use with a 1-in. rope of this type is variously given as 2.5 to 3.25 ft.;



FIG. 2



FIG. 3

materially less than that required for a 6 × 19 rope. Ropes of this class are recommended for derricks and similar work where small sheaves must be employed, but it should be noted that, so far as the working life of this type is

concerned, the increased flexibility in a very considerable measure offsets its decreased strength.

**6×37 Ropes.**—A form of very flexible rope that is not infrequently used in preference to the 8×19, is shown in Fig. 3; in this, the rope is composed of 6 strands of 37 wires each. As there is a much greater area of metal in proportion to the hemp core than in an 8×19, the breaking strength of a 1-in. cast-steel rope of this type is given as 29 T., only 1 T. less than that of the standard 6×19 rope and 3 T. more than the 8×19. The diameter of sheave suggested for this rope is the same as that for the 8×19;
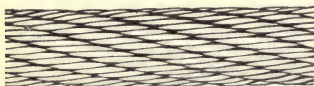


FIG. 4

viz., 2.5 to 3.25 ft. As the wires in this rope are, of necessity, smaller than those in a 6×19 rope of the same diameter, it is apparent that this type of rope is not so well adapted to withstand abrasion as those containing larger wires. This rope is employed where the bending strains are very great, as in logging operations, for use with electric cranes, etc. When galvanized, this rope is largely used for hawsers in towing, etc.

**Non-Spinning Ropes.**—Non-spinning hoisting rope, made by one of the leading manufacturers, is shown in Fig. 4. It is composed of 18 strands of 7 wires each (126 wires), 12 of the strands being laid in a reversed direction about 6 which, in turn, are laid about a hemp core. Because of the reversed directions in which the inner and outer sets of strands are laid, there is no tendency to twist and the rope is, thence, adapted to hoisting where the load is not raised between guides but hangs freely as a bucket in shaft-sinking. The rope is slightly more flexible than the standard 6×19 rope and slightly stronger. However, it cannot be spliced.

**Flattened-Strand Ropes.**—In order to present a larger and smoother wearing surface, and thus to increase the life of the rope, flattened-strand wire ropes have been devised. In these ropes the strands have an elliptic, or triangular, cross-section, depending on the shape of the metal



(*a*)       (*b*)

FIG. 5

center of the strand, and the rope has either a hemp or a wire core. They are made either 5×28 (5 strands of 28 wires each, or 140 wires) as shown in Fig. 5 (*a*), or 6×25 (6 strands of 25 wires each, or 150 wires) as shown in (*b*). These ropes are made of Swedes iron, and of cast-steel, extra strong cast-steel, and extra plow-steel. The breaking strength of a cast-steel rope of this make, 5×28, and 1 in. in diameter is given as 30 T., the same as that of the same size and kind round, 6×19 standard rope. A 6×25 cast-steel rope of this type 1 in. in diameter is given as 33 T., which is greater than that of the corresponding 6×19 round rope and nearly as great as that of an extra strong cast-steel rope. There is claimed for this rope greater flexibility, less liability of the wires becoming brittle, and freedom from all tendency to spin or kink; also that they maintain their form better than round ropes.
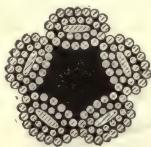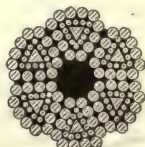
**Seale Ropes.**—A form of rope made by some manufacturers and used for hoisting, but possibly better adapted to haulage purposes, is shown in Fig. 6, as it is of the 6×19 type used in standard hoisting



FIG. 6

ropes. This is known as *Seale rope* or *Seale lay rope*, and consists of 6 strands of 19 wires each, in which 9 large wires are twisted around 9 small ones, which in turn surround one of the larger size. This rope is intermediate in flexibility and ability to stand abrasion between the standard ropes of 7-wire (haulage) and 19-wire (hoisting) strands. This type of rope, on account of the large outside wires, will withstand heavy frictional wear and is used on slopes, planes, and cable

roads where the rope commonly drags, provided, however, that there are no bends of sharp angle to overstrain the outer wires. While there is more metal in the outer wires than in those of the standard rope, there is correspondingly less in the inner wires, and closer inspection of the outer wires is, therefore, necessary to prevent the rope being used too long. The price of these ropes is the same as the standard $6 \times 19$ hoisting rope of the same grade.

## FLAT ROPES

Flat ropes, Fig. 7, are composed of a number of loosely twisted ropes of four wires each and without hemp centers. The ropes, of alternately right and left lay, are placed side by side and are then sewed together with soft iron or annealed steel wire to form a single rope. The sewing wires, which vary in number from 8 to 12 pass through the centers of the individual ropes from side to side and often have to be renewed, as they naturally wear faster than the wires composing the rope proper.

Flat ropes may be made to order of any width to give any desired strength, but the width must, of necessity, be some multiple of the diameter of a single strand. They are made of the same grades of steel as round ropes and, under certain conditions present material advantages over the ordinary type. In very deep shafts



FIG. 7

round ropes have a tendency to twist and untwist, or to spin, something that flat ropes do not do. The width of the reel upon which a flat rope winds is very much less than that of the drum used for round ropes, and as the rope coils upon itself like a ribbon, it tends to equalize the load upon the engine, the effect being approximately the same as that produced by conical drums. Likewise, in hoisting, the rope is always in the same vertical plane, thus avoiding the wear that round ropes are subject to when wound on a drum. Flat ropes are not used in coal mines in the United States, owing to the comparatively shallow depths of the shafts, but are quite extensively used in the metal-mining districts, where vertical lifts of 2,000 ft. and over are common.
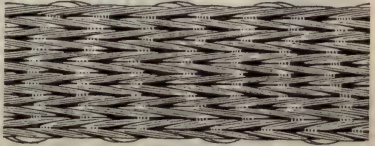
## TAPER ROPES

Taper ropes, both round and flat, have been used in deep hoisting. Such a rope has its diameter or width reduced uniformly throughout its length by dropping a single wire at a time, or by decreasing the size of the wire used at regular intervals, so as to reduce the sectional area of the rope in proportion to the weight to be supported. The reason for using taper rope is as follows: When the load is at the bottom of the shaft, the upper part of the rope sustains both the load to be hoisted and the weight of the rope itself. As the rope is wound up, the load on the rope at the drum gradually decreases and, therefore, the size of the rope may be proportionately decreased. Owing to the difficulties of manufacture, taper ropes cannot be made as perfect as straight ropes and their cost is greater; furthermore, they cannot be used for haulage and other purposes when partly worn, as is the case with straight ropes.

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# HAULAGE ROPES

$6 \times 7$ **Ropes.**—For underground haulage and for the transmission of power, a rope of the section shown in Fig. 1, and either ordinary or Lang lay, is in
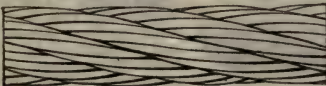


FIG. 1

general use in American mines, to the practical exclusion of any other type. It is composed of 6 strands of 7 wires each ($6 \times 7$) twisted around a hemp center. It is decidedly stiffer

than a $6 \times 19$ standard hoisting rope and requires larger sheaves, as will appear from the tables. Owing to the small number of wires (there are but 42 as against 114, 152, and 222, in the $6 \times 19$, $8 \times 19$, and $6 \times 37$ hoisting ropes) this

rope should be used with a higher factor of safety than is employed with hoisting ropes, as the breaking of one or two wires materially reduces the strength of the rope. These ropes are made of Swedes iron and of the four grades or strengths of steel previously mentioned. As in the case of hoisting ropes, haulage ropes of iron require the use of deceidedly larger sheaves than do those of steel. Manufacturers recommend a sheave from 10.5 to 11 ft. in diameter for a 1-in. iron rope of this type as against a sheave 7 to 8 ft. in diameter for a corresponding steel rope.

**Flattened-Strand Ropes.**—Flattened-strand rope, similar in general construction to the rope of the same name used for hoisting, is also employed for haulage. Ropes of this type are shown in Fig. 2 (a) and (b). The former shows the $5 \times 9$ (45 wires) rope very similar to the $5 \times 28$ hoisting rope of the same type. View (b) shows the $6 \times 8$ haulage rope, which is not unlike the $6 \times 25$ hoisting rope. The ultimate breaking strength of a 1-in., cast-steel, ordinary, haulage rope is given as 31 T., and those of the $5 \times 9$, and $6 \times 8$, flattened-strand rope of the same diameter and material are 31 and 34 T., respectively. The diameter of sheave suggested for the standard $6 \times 7$ 1-in. haulage rope is 7, and 5.75 ft. for either type of the flattened-strand rope. The comments made upon hoisting ropes of this type apply here.



(a)     (b)

FIG. 2

**Seale Ropes.**—Seale lay ropes are used to a certain extent for haulage and those of the Lang lay type are very commonly employed for the same purpose, as explained before.

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# ROPES FOR MISCELLANEOUS PURPOSES

**Ropes for Cableways.**—Many of the ropes described, and particularly the $6 \times 7$ Lang lay, are used for the track or supporting cable of what are variously known as *cableways, wire-rope tramways, aerial tramways,* and the like. In this system of transportation, the materials to be moved are carried in buckets suspended from wheeled trucks, which are hauled
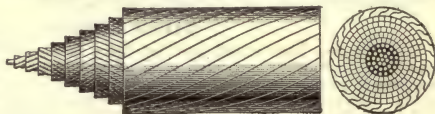


FIG. 1

by a lighter rope upon a fixed rope known as the *cable*, or *track cable*. Such cables are subject to extreme wear and to produce a rope having the maximum of wearing surface, what are known as *locked-wire cables* and *locked-coil cables* have been devised.

The locked-wire construction is shown in Fig. 1. The outside wires are drawn of such a shape as to interlock one with the other, making a smooth cylindrical surface for the carrier wheels to run upon. Ropes of this type have wire cores and are, consequently, stiffer than ordinary ropes of the same size; but as they are proportionally stronger for equal strengths there is probably not much difference in the stiffness of the two forms of construction. The advantages claimed for this rope are lessened wear, both on the part of the rope and on that of the wheels of the traveling carriage; absence of any tendency to twist and turn; and freedom from unraveling should any of the wires break.
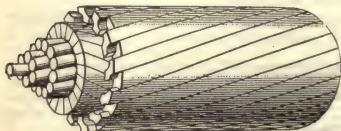


FIG. 2

The locked-coil construction is illustrated in Fig. 2. It differs from the preceding only in the smaller number and larger size of the wires, which makes it stiffer.

Another and cheaper, but very satisfactory, construction for track cables, shown in Fig. 3, is known as the *tramway strand*, or *smooth-coil*. It is merely a heavy strand of very large wires resembling a spirally fluted cylinder in appearance. The large wires give increased durability over smaller wires, owing to the greater surface exposed to wear.

A disadvantage of the track cables here illustrated is that they cannot be spliced. In order to connect them the coupling shown in Fig. 4 must be employed. This consists of two, narrow, tapered sockets, joined at the middle by a plug with right- and left-hand screw



FIG. 3

threads. The ends of the wires are spread apart in the funnel-shaped apertures, and the space between them filled with conical thimbles and narrow wedges made approximately to the shape of the interstices. The sockets are attached to the ends of the cables by a special form of press, after which they are brought nearly together and in line and the proper end of the plug inserted in each. The plug is then turned until the sockets are brought together.

**Ropes for Suspension Bridges.**—Small suspension bridges at mines are frequently built of old and partly worn hoisting or haulage ropes, but a special form of rope, shown in Fig. 5, is not infrequently used for this purpose, particularly if the span is considerable. This is the familiar 6×7 haulage rope, but with a wire instead of a hemp center, which gives increased strength but with lessened flexibility, the latter quality not being of prime importance in bridge construction.

There are many other purposes for which ropes are used in and around mines, such as in running ropes for derricks, aerial tramways, and steam



FIG. 4

shovels, as guy ropes for derricks and smokestacks, as rigging, hawsers, and mooring ropes for vessels engaged in the transportation of coal, etc. The forms of some of these ropes are in Figs. 5, 6, 7, and 8.

**Derrick Ropes.**—Guys for derricks and stacks, shrouds and stays aboard ship, etc., are commonly made of ordinary 6×7 rope with a hemp center; though where greater flexibility is required of a 6 strand rope with 12 instead of 7 wires to the strand, that is a 6×12 rope, is used. The wires are commonly single or double galvanized; that is, they receive a single or double coating of zinc to prevent rusting.

**Hawsers.**—Steel hawsers, mooring lines, and running rigging for vessels, which must be more flexible than ropes used for guys, standing rigging, etc., are very commonly made of the 6×37 ropes. Special rope for these purposes is made of the section shown in Fig. 6, in which 6 strands (each with a hemp
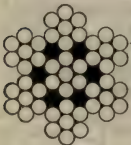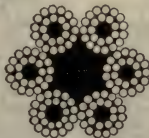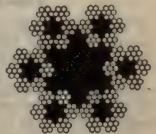


FIG. 5      FIG. 6      FIG. 7      FIG. 8

center) of 12 wires each are wrapped around a common hemp core. Such ropes are very much stronger than those of Manila hemp of the same size and are fully as flexible.

Another form of rope used for the same purposes is shown in Fig. 7. In this case, 6 strands of 24 wires each with a hemp center are wrapped around a common hemp core. This is more flexible than the form shown in Fig. 6 and

is consequently well adapted for mooring lines, which must be wound upon capstans, piles, etc. of comparatively small diameter. The strands vary in their make-up; with some manufacturers, the 24 wires are all of the same size, while with others, the 12 inner wires are considerably smaller than the outer ones, as in Seale lay ropes.

*Tiller rope* is made of six small 6×7 ropes laid around a hemp center as shown in Fig. 8. Containing 252 wires, this is the most flexible wire-rope made. It is used mostly for steering or tiller ropes on steamers, for hand-lines on passenger elevators, and in any place where a smooth and very flexible rope is required. The ultimate strengths of tiller ropes are about one-third less than those of standard 6×19 wire ropes of the same size and grade. The minimum diameter of sheaves recommended for usual loads are: for iron, 30 times the rope diameter; for steel, 25 times the rope diameter. Owing to the small size of the wires, tiller rope should be subject to as little abrasion as possible.

With the exception of tiller rope, and that only when used for passenger elevators, the ropes that have been described are but rarely made of Swedes iron, steel giving much better results. These ropes are almost always galvanized, at a cost of about 10% above that of untreated ropes.

For use in and around oil wells, ropes of 6×7, 6×12, and 6×19 construction are commonly employed. The 6×7 is used for sand lines and, when left-lay, for cleaning out or redrilling wet holes. Casing lines and drilling lines for new holes, the latter left-lay, are made of 6×19 rope.

# ROPE DRUMS AND FASTENINGS

**Fastening Rope to Drum.**—A common method of fastening a rope to a drum is shown in Fig. 1 (*a*), where the rope is passed through a hole in the drum shell and then around the shaft, clamping the end to the rope between the shaft and the shell, as shown. Care should be taken to make the radius of curvature of the hole at *a* as large as possible, so that the wire will not be bent any sharper than is necessary.
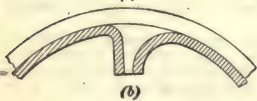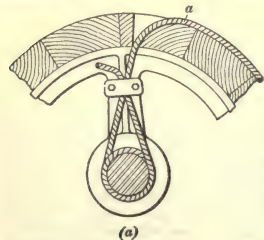


When an iron drum is used, the thickness of the rim does not afford enough depth in which to bend the rope, so it is necessary to build in a pocket for the purpose, as shown in (*b*). In no case should the rope be bent sharply at right angles where it passes through the drum shell.

The securing of the rope to the drum or the drum shaft by several coils around each is unnecessary. With one coil around either the drum or the shaft, a pull of 1 lb. will resist a weight of 9 lb.; if two coils, a pull of 1 lb. will resist 9×9=81 lb.; if three coils, 9×9×9=729 lb., and so on, multiplying the former result by 9 for each additional coil.

**(a)**

**(b)**

FIG. 1

**Rope Sockets.**—The common method of attaching the socket shown in Fig. 2 to a rope is as follows: The rope is pushed through from the small end and is allowed to project any convenient distance. It is then firmly wrapped with wire at a point a little more than twice the depth of the socket from the end. The end of each strand is untwisted, a few of the wires cut away, and the others bent back upon themselves.

This makes the end of the rope conical, in which condition it is drawn back into the socket, and a conical wedge is rammed into the center of the hemp core to spread the wires against the side of the socket. The socket and a small length of the rope next thereto are covered with a layer of moist fireclay, and melted Babbitt metal is



FIG. 2

run into the socket, until it fills the space completely and thus cements the whole into a solid mass. This entire operation must be carefully performed, otherwise all of the wires will not be engaged, and thus an undue strain will

be thrown upon other wires or strands, possibly resulting in the failure of the rope at some distance from the socket.

The following method of attaching the socket is employed by John A. Roebling's Sons Company. As before, the rope is fitted through the socket and allowed to project. Wires are then securely served around the rope, the strands opened for a distance equal to the length of the basket of the socket, and the hemp center cut off the length of the opening. The wires are then well cleaned with kerosene and wiped dry. After the strands are separated into wires, which may be conveniently done with a small piece of pipe, the wires are placed into a solution of equal parts of water and hydrochloric acid or muriatic acid, *HCl*, for 5 min., and then are cleaned off. They are then redipped into the solution, which has been made weaker by the addition of 1 part of water (now 2 parts of water to 1 part of acid). The wires are then bunched and bound together by wire about 1 in. from the top. When the socket has been pushed over the wires until their ends are even with the top of the basket, fireclay is placed around the rope at the bottom of the socket, to serve as a shield, and melted zinc is poured into the basket of the socket. It will be noted that in this method of fastening none of the wires are cut out and none are bent back upon themselves; the rope is merely opened up, untwisted, and the ends of the wires bunched together.

Instead of using a socket, the end of the rope may be bent around a thimble and the end fastened by clamping the parts of the rope with clevices, as shown in Fig. 3 (*a*); or with iron bands that are sometimes held in place by nails, inserted between the strands, as in (*b*), though this practice is not recommended; or by wrapping the rope with wire, as in (*c*). In the last method, the end of the rope is frayed, and the loosened wires are arranged evenly around the main portion of the rope before wrapping is commenced.



(*a*)

(*b*)

(*c*)
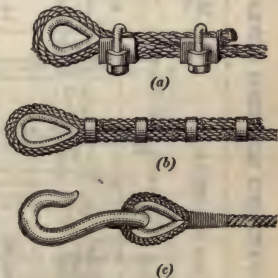
Fig. 3

## WIRE-ROPE TABLES

The accompanying tables (eleven in number) giving the ultimate strengths of wire ropes, proper size of sheaves to be used therewith, etc. are taken from the latest practice of the leading American manufacturers of wire rope. The strengths given are ultimate strengths, the working strains to which the rope is actually subjected being usually one-fifth of these; that is, a factor of safety of 5 is commonly employed, although one of 6 is not unusual, and even 10 is used, particularly in elevator work where passengers are carried. The use of these tables when selecting a hoisting rope for any particular service is best illustrated by means of an example.

EXAMPLE.—A total load of 8 T. is to be hoisted from a shaft 300 ft. deep. What size rope should be employed, allowing a factor of safety of 5?

SOLUTION.—Since the factor of safety is 5, the rope selected must have an ultimate strength of $5 \times 8 = 40$ T. Standard hoisting ropes are $6 \times 19$ and commonly of cast steel. The first table shows that a $1\frac{1}{8}$-in. rope of this kind has a breaking strength of 38 T., and one $1\frac{1}{4}$ in. in diameter, a breaking strength of 56 T. The weight of the smaller rope is 2 lb. per ft., and of the larger 2.45 lb. In the one case the weight of the rope will be $300 \times 2 = 600$ lb. $= .3$ T., and in the other $300 \times 2.45 = 735$ lb. $= .36$ T. If a $1\frac{1}{8}$-in. rope is used, the rope is called upon to sustain a load of 40.3 T.; if a $1\frac{1}{4}$-in. rope is used, the strain is 40.36 T. Therefore, the smaller rope is not quite strong enough and the larger rope is considerably over strength. Probably the $1\frac{1}{8}$-in. rope with a factor of safety a little less than 5 would answer, but better practice demands the selection of the larger rope; the factor of safety being nearly 6. The table also shows that the minimum diameter of sheave or drum for the $1\frac{1}{8}$-in. rope is 5 ft., and for the $1\frac{1}{4}$-in. rope, 6 ft.

## SIZES AND STRENGTHS OF STANDARD HOISTING ROPES

| Diameter of Rope, Inches | Approximate Total Strength of Rope, in Tons of 2,000 Lb. | | | | | | | | | | | | | Minimum Diameter of Sheave or Drum, Feet | | | | Approximate Weight Per Foot, Pounds | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 6 Strands of 19 Wires Each | | | | | 6 Strands of 37 Wires Each | | | | 8 Strands of 19 Wires Each | | | | 6×19 | | 6×37 Steel / 8×19 Steel | 6×19 / 6×37 Steel / 8×19 Steel | 6×19 / 6×37 Steel | 8×19 Steel |
| | Swedish Iron | Cast Steel | Extra Cast Steel | Plow Steel | Extra Plow Steel | Cast Steel | Extra Cast Steel | Plow Steel | Extra Plow Steel | Cast Steel | Extra Cast Steel | Plow Steel | Ex. Pl'w Ste'l | Iron | Steel | | | | |
| 2¾ | 111.0 | 211.00 | 243.00 | 275.00 | 315.00 | 200.00 | 233.00 | 265.00 | 278.00 | | | | | 18 | 14 | | | 11.95 | |
| 2½ | 92.0 | 170.00 | 200.00 | 229.00 | 263.00 | 160.00 | 187.00 | 214.00 | 225.00 | | | | | 16 | 12½ | | | 9.85 | |
| 2¼ | 72.0 | 133.0 | 160.00 | 186.00 | 210.00 | 125.00 | 150.00 | 175.00 | 184.00 | | | | | 14½ | 11 | | | 8.00 | |
| 2 | 55.0 | 106.0 | 123.00 | 166.00 | 166.00 | 105.00 | 117.00 | 130.00 | 137.00 | | | | | 13 | 9¾ | | | 6.30 | |
| 1¾ | 50.0 | 96.0 | 112.00 | 127.00 | 150.00 | | | | | | | | | 12 | 9 | | | 5.55 | |
| 1⅝ | 44.0 | 85.0 | 99.00 | 94.00 | 133.00 | 84.00 | 95.00 | 108.00 | 113.00 | 58.00 | 66.00 | 74.00 | 80.0 | 11 | 8¼ | 9 | 4¾ | 4.85 | 3.19 |
| 1½ | 38.0 | 72.0 | 83.00 | 82.00 | 110.00 | 71.00 | 79.00 | 90.00 | 95.00 | 51.00 | 57.00 | 64.00 | 68.0 | 10 | 7½ | 8¼ | 4½ | 4.15 | 2.70 |
| 1⅜ | 33.0 | 64.0 | 73.00 | 72.00 | 98.00 | 63.00 | 71.00 | 80.00 | 84.00 | 47.00 | 47.00 | 52.00 | 56.0 | 9½ | 7 | 7¾ | 4¼ | 3.55 | 2.20 |
| 1¼ | 28.8 | 56.0 | 64.00 | 58.00 | 84.00 | 55.00 | 61.00 | 68.00 | 71.00 | 42.00 | 42.00 | 46.00 | 46.0 | 8⅞ | 6½ | 6¾ | 4 | 3.00 | 1.80 |
| 1⅛ | 18.6 | 47.0 | 53.00 | 47.00 | 69.00 | 45.00 | 50.00 | 55.00 | 58.00 | 34.00 | 38.00 | 43.00 | 36.0 | 8 | 6 | 5½ | 3 | 2.45 | 1.42 |
| 1 | 14.5 | 38.0 | 43.00 | 38.00 | 56.00 | 34.00 | 32.00 | 44.00 | 46.00 | 26.00 | 29.70 | 33.00 | 26.0 | 7 | 5 | 5 | 3 | 2.00 | 1.08 |
| ⅞ | 11.8 | 34.0 | 34.00 | 29.00 | 45.00 | 29.00 | 25.00 | 35.00 | 37.00 | 20.00 | 17.60 | 26.00 | 22.0 | 6¼ | 4½ | 4½ | 2½ | 1.58 | .80 |
| ¾ | 8.5 | 30.0 | 26.00 | 23.00 | 35.00 | 23.00 | 19.00 | 27.00 | 29.00 | 15.30 | 10.10 | 20.00 | 15.0 | 6 | 4 | 3¾ | 2 | 1.20 | .56 |
| ⅝ | 6.8 | 17.5 | 20.20 | 15.50 | 26.30 | 17.50 | 12.60 | 21.00 | 23.00 | 12.40 | 8.00 | 14.00 | 12.0 | 5 | 3½ | 3¼ | 2 | .89 | .45 |
| | 4.7 | 12.5 | 14.00 | 12.30 | 19.00 | 12.50 | 10.50 | 14.00 | 16.00 | 8.70 | 6.30 | 11.60 | 9.5 | 4½ | 3 | 2¾ | 1½ | .62 | .35 |
| | 3.9 | 10.4 | 11.20 | 8.00 | 14.50 | 9.50 | 8.25 | 11.50 | 12.50 | 5.70 | 4.66 | 8.70 | | 4 | 2½ | 2¼ | 1½ | .50 | .27 |
| | 2.9 | 6.5 | 7.25 | 5.80 | 12.10 | 7.25 | 6.35 | 9.25 | 9.75 | 4.20 | 3.05 | 6.90 | | 3 | 2¼ | 2 | 1 | .39 | .20 |
| | 2.1 | 4.8 | 5.30 | 3.80 | 9.40 | 4.20 | 4.65 | 7.20 | 7.50 | 3.05 | 2.02 | 5.12 | | 2¾ | 2 | 1¾ | 1 | .30 | .13 |
| | 1.5 | 3.1 | 3.55 | 3.05 | 6.75 | | | 5.10 | 5.30 | 1.80 | | 3.35 | | 2¼ | 1½ | 1¼ | 1 | .22 | .09 |
| | 1.1 | 2.2 | 2.43 | 2.65 | 4.50 | | | | | | | 2.25 | | 1¾ | 1¼ | 1 | 1 | .15 | |
| | | | | | 3.15 | | | | | | | | | 1¾ | 1¼ | | | .10 | |

## SIZES AND STRENGTHS OF PATENT, FLATTENED-STRAND, HOISTING ROPES

| Diameter of Rope Inches | Approximate Total Strength of Rope, in Tons of 2,000 Lb. | | | | | | | Minimum Diameter of Sheave or Drum for Either Section of Rope Feet | | | | Approximate Weight per Foot of Iron or Steel Rope Pounds | | Diameter of Rope Inches |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 5 Strands of 28 Wires Each | | | | 6 Strands of 25 Wires Each | | | | | | | | | |
| | Swedish Iron | Cast Steel | Extra Cast Steel | Extra Plow Steel | Cast Steel | Extra Cast Steel | Extra Plow Steel | Swedish Iron | Cast Steel | Extra Cast Steel | Extra Plow Steel | 5 × 28 | 6 × 25 | |
| 2¼ | 72.0 | 133.0 | 160.0 | 210.0 | 146.0 | 176.0 | 231.0 | 11¼ | 8¾ | 8¾ | 12 | 8.00 | 9.20 | 2¼ |
| 2 | 55.0 | 106.0 | 123.0 | 166.0 | 117.0 | 135.0 | 183.0 | 10¾ | 8 | 8 | 11 | 6.30 | 7.25 | 2 |
| 1¾ | 44.0 | 85.0 | 99.0 | 133.0 | 94.0 | 109.0 | 146.0 | 9 | 7¼ | 7¼ | 9½ | 4.85 | 5.60 | 1¾ |
| 1⅝ | 38.0 | 72.0 | 83.0 | 110.0 | 79.0 | 91.0 | 121.0 | 7 | 6¼ | 6¼ | 8 | 4.15 | 4.75 | 1⅝ |
| 1½ | 33.0 | 64.0 | 73.0 | 98.0 | 70.0 | 80.0 | 108.0 | 7 | 6 | 6 | 8 | 3.55 | 4.00 | 1½ |
| 1⅜ | 28.0 | 56.0 | 64.0 | 84.0 | 62.0 | 70.0 | 92.0 | 6¼ | 5¾ | 5¾ | 7½ | 3.00 | 3.45 | 1⅜ |
| 1¼ | 22.8 | 47.0 | 53.0 | 69.0 | 52.0 | 58.0 | 76.0 | 6 | 5½ | 5½ | 7 | 2.45 | 2.80 | 1¼ |
| 1⅛ | 18.6 | 38.0 | 43.0 | 56.0 | 42.0 | 47.0 | 62.0 | 5½ | 5 | 5 | 6 | 2.00 | 2.30 | 1⅛ |
| 1 | 14.5 | 30.0 | 34.0 | 45.0 | 33.0 | 37.0 | 50.0 | 5 | 4½ | 4½ | 5 | 1.58 | 1.80 | 1 |
| ⅞ | 11.8 | 23.0 | 26.0 | 35.0 | 25.0 | 29.2 | 39.0 | 4½ | 4 | 4 | 4½ | 1.20 | 1.80 | ⅞ |
| ¾ | 8.5 | 17.5 | 20.2 | 26.3 | 19.3 | 22.2 | 29.0 | 4 | 3¾ | 3 | 4 | .89 | 1.00 | ¾ |
| ⅝ | 6.7 | 12.5 | 14.2 | 19.0 | 13.8 | 15.4 | 21.0 | 3¾ | 2¾ | 2¼ | 3½ | .62 | .72 | ⅝ |
| ½ | 4.9 | 10.0 | 11.2 | 14.5 | 13.0 | 12.3 | 16.3 | 3 | 1¾ | 1¾ | 3 | .50 | .58 | ½ |
| 7/16 | 3.9 | 8.4 | 9.2 | 12.1 | 9.3 | 10.1 | 13.3 | 2½ | 1¼ | 1½ | 2¾ | .39 | .45 | 7/16 |
| ⅜ | 2.4 | 4.8 | 5.3 | 6.75 | 5.3 | 5.8 | 7.4 | 1 | 1 | 1 | 2 | .22 | .25 | ⅜ |

## SIZES AND STRENGTHS OF FLAT HOISTING ROPES

| Width and Thickness Inches | Weight per Foot Pounds | Approximate Strength, in Tons of 2,000 lb. | | Width and Thickness Inches | Weight per Foot Pounds | Approximate Strength, in Tons of 2,000 Lb. | |
|---|---|---|---|---|---|---|---|
| | | Cast Steel | Plow Steel | | | Cast Steel | Plow Steel |
| ½×7 | 5.90 | 129 | 158 | ⅜×5½ | 3.90 | 76 | 99 |
| ½×6 | 5.20 | 113 | 138 | ⅜×5 | 3.40 | 72 | 93 |
| ½×5½ | 4.82 | 105 | 128 | ⅜×4½ | 3.12 | 63 | 81 |
| ½×5 | 4.27 | 97 | 118 | ⅜×4 | 2.70 | 58 | 76 |
| ½×4½ | 4.00 | 89 | 108 | ⅜×3½ | 2.30 | 49 | 64 |
| ½×4 | 3.30 | 81 | 99 | ⅜×3 | 2.00 | 45 | 58 |
| ½×3½ | 2.97 | 72 | 89 | ⅜×2½ | 1.75 | 36 | 46 |
| ½×3 | 2.38 | 56 | 69 | ⅜×2 | 1.30 | 27 | 35 |

## SIZES AND STRENGTHS OF STANDARD 6×7 HAULAGE ROPES

| Diameter of Rope Inches | Approximate Strength, in Tons of 2,000 lb. | | | | | Minimum Diameter of Sheave or Drum Feet | | Weight per Foot Pounds |
|---|---|---|---|---|---|---|---|---|
| | Swedish Iron | Cast Steel | Extra Cast Steel | Plow Steel | Extra Plow Steel | Iron | Steel | |
| 1½ | 32.0 | 63.0 | 73.00 | 82.0 | 90.00 | 16½ | 13 | 3.55 |
| 1⅜ | 28.0 | 53.0 | 63.00 | 72.0 | 79.00 | 15 | 11½ | 3.00 |
| 1¼ | 23.0 | 46.0 | 54.00 | 60.0 | 67.00 | 13¾ | 10½ | 2.45 |
| 1⅛ | 19.0 | 37.0 | 43.00 | 47.0 | 52.00 | 12¼ | 9¼ | 2.00 |
| 1 | 15.0 | 31.0 | 35.00 | 38.0 | 42.00 | 11 | 8 | 1.58 |
| ⅞ | 12.0 | 24.0 | 28.00 | 31.0 | 33.00 | 9¾ | 7¼ | 1.20 |
| ¾ | 8.8 | 18.6 | 21.00 | 23.0 | 25.00 | 8½ | 5¾ | .89 |
| 11/16 | 7.3 | 15.4 | 16.70 | 18.0 | 20.00 | 7½ | 5¼ | .75 |
| ⅝ | 6.0 | 13.0 | 14.50 | 16.0 | 17.50 | 7 | 4¾ | .62 |
| 9/16 | 4.8 | 10.0 | 11.00 | 12.0 | 13.00 | 6¼ | 4¼ | .50 |
| ½ | 3.7 | 7.7 | 8.85 | 10.0 | 11.00 | 5½ | 3¾ | .39 |
| 7/16 | 2.6 | 5.5 | 6.25 | 7.0 | 7.75 | 4½ | 3¼ | .30 |
| ⅜ | 2.2 | 4.6 | 5.25 | 5.9 | 6.50 | 4 | 2¾ | .22 |
| 5/16 | 1.7 | 3.5 | 3.95 | 4.4 | | 3½ | 2⅜ | .15 |
| ¼ | 1.2 | 2.5 | 2.95 | 3.4 | | 3 | 2 | .125 |

## GALVANIZED STEEL CABLES FOR SUSPENSION BRIDGES

| Diameter of Rope Inches | Weight per Foot Pounds | Ultimate Strength in Tons of 2,000 Lb. | Diameter of Rope Inches | Weight per Foot Pounds | Ultimate Strength in Tons of 2,000 Lb. |
|---|---|---|---|---|---|
| 2⅝ | 12.70 | 310 | 1¾ | 5.90 | 144 |
| 2½ | 11.60 | 283 | 1⅝ | 5.10 | 124 |
| 2⅜ | 10.50 | 256 | 1½ | 4.34 | 106 |
| 2¼ | 9.50 | 232 | 1⅜ | 3.70 | 90 |
| 2⅛ | 8.52 | 208 | 1¼ | 3.10 | 75 |
| 2 | 7.60 | 185 | 1⅛ | 2.57 | 62 |
| 1⅞ | 6.73 | 164 | | | |

## SIZES AND STRENGTHS OF PATENT, FLATTENED-STRAND HAULAGE ROPES

| Diameter of Rope Inches | Approximate Total Strength of Rope, in Tons of 2,000 Lb. | | | | | | | Minimum Diameter of Sheave or Drum for Either Section of Rope Feet | | | Approximate Weight per Foot of Iron or Steel Rope Pounds | | Diameter of Rope Inches |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 5 Strands of 9 Wires Each | | | | 6 Strands of 8 Wires Each | | | | | | | | |
| | Swedish Iron | Cast Steel | Extra Cast Steel | Extra Plow Steel | Cast Steel | Extra Cast Steel | Extra Plow Steel | Swedish Iron | Cast Steel | Extra Plow Steel | 5×9 | 6×8 | |
| 1½ | | 63.0 | 73.00 | | 68.0 | 79.0 | | 8½ | 8 | | 3.65 | 4.00 | 1½ |
| 1⅜ | | 53.0 | 63.00 | | 57.0 | 68.0 | | 8 | 8 | | 3.10 | 3.45 | 1⅜ |
| 1¼ | 23.0 | 46.0 | 54.00 | 67.0 | 50.0 | 58.0 | 73.0 | 9½ | 7¼ | 9¼ | 2.55 | 2.80 | 1¼ |
| 1⅛ | 19.0 | 37.0 | 43.00 | 52.0 | 40.0 | 46.0 | 56.0 | 8½ | 6¼ | 8 | 2.05 | 2.30 | 1⅛ |
| 1 | 15.0 | 31.0 | 35.00 | 42.0 | 34.0 | 38.0 | 46.0 | 7¼ | 5¾ | 6¼ | 1.65 | 1.80 | 1 |
| ⅞ | 12.0 | 24.0 | 28.00 | 33.0 | 26.0 | 30.0 | 36.0 | 6½ | 5 | 6 | 1.24 | 1.38 | ⅞ |
| ¾ | 8.8 | 18.6 | 21.00 | 25.0 | 20.0 | 22.7 | 27.0 | 6 | 4½ | 5½ | .92 | 1.00 | ¾ |
| ⅝ | 6.0 | 13.0 | 14.50 | 17.5 | 14.0 | 15.7 | 19.0 | 4½ | 3½ | 4½ | .64 | .72 | ⅝ |
| ½ | 3.7 | 7.7 | 8.85 | 11.0 | 8.3 | 9.6 | 11.9 | 3½ | 2½ | 3½ | .40 | .45 | ½ |
| ⅜ | 2.2 | 4.6 | 5.25 | 6.5 | 5.0 | 5.7 | 7.0 | 2¼ | 2 | 2¼ | .23 | .25 | ⅜ |

### CAST STEEL LOCKED-WIRE CABLE

| Diameter of Rope Inches | Weight per Foot Pounds | Ultimate Strength, in Tons of 2,000 Lb. | Diameter of Rope Inches | Weight per Foot Pounds | Ultimate Strength in Tons of 2,000 Lb. |
|---|---|---|---|---|---|
| 2½ | 15.60 | 275 | 1⅛ | 3.15 | 55.00 |
| 2¼ | 12.50 | 220 | 1 | 2.50 | 43.00 |
| 2 | 10.00 | 170 | ⅞ | 1.88 | 36.00 |
| 1¾ | 7.65 | 129 | ¾ | 1.30 | 27.00 |
| 1⅝ | 6.60 | 114 | ⅝ | .90 | 19.00 |
| 1½ | 5.70 | 95 | 9/16 | .72 | 13.25 |
| 1⅜ | 4.75 | 80 | ½ | .57 | 11.50 |
| 1¼ | 3.80 | 67 | | | |

### TRAMWAY OR SMOOTH-COIL CABLE

| Diameter of Rope Inches | Weight per Foot Pounds | Ultimate Strength, in Tons of 2,000 Lb. | | Diameter of Rope Inches | Weight per Foot Pounds | Ultimate Strength, in Tons of 2,000 Lb. | |
|---|---|---|---|---|---|---|---|
| | | Cast Steel | Plow Steel | | | Cast Steel | Plow Steel |
| 2½ | 13.10 | 285.0 | 335.0 | 1⅜ | 4.01 | 88.8 | 105.0 |
| 2¼ | 10.36 | 233.0 | 266.0 | 1¼ | 3.23 | 71.8 | 84.6 |
| 2⅛ | 9.35 | 204.0 | 240.0 | 1⅛ | 2.70 | 60.0 | 70.7 |
| 2 | 8.40 | 185.0 | 218.0 | 1 | 2.20 | 49.2 | 58.0 |
| 1⅞ | 7.28 | 161.0 | 189.0 | ⅞ | 1.69 | 37.6 | 44.4 |
| 1¾ | 6.59 | 145.8 | 171.0 | ¾ | 1.24 | 27.6 | 32.5 |
| 1⅝ | 5.63 | 124.0 | 146.0 | ⅝ | .86 | 19.2 | 22.3 |
| 1½ | 4.88 | 108.4 | 127.5 | | | | |

## GALVANIZED IRON AND STEEL RUNNING ROPE

| Diameter of Rope Inches | Weight per Foot Pounds | Ultimate Strength in Tons of 2,000 Lb. | | Diameter of Rope Inches | Weight per Foot Pounds | Ultimate Strength in Tons of 2,000 Lb. | |
|---|---|---|---|---|---|---|---|
| | | Iron | Cast Steel | | | Iron | Cast Steel |
| 1 1/16 | 1.18 | 10.1 | 22.5 | 9/16 | .33 | 2.80 | 6.50 |
| 1 | 1.05 | 8.7 | 19.5 | 1/2 | .26 | 2.20 | 5.00 |
| 7/8 | .80 | 6.9 | 15.5 | 7/16 | .20 | 1.70 | 3.90 |
| 13/16 | .68 | 6.0 | 13.5 | 3/8 | .14 | 1.30 | 2.85 |
| 3/4 | .59 | 5.1 | 11.5 | 5/16 | .10 | .82 | 1.98 |
| 5/8 | .42 | 3.6 | 8.0 | | | | |

## GALVANIZED STEEL HAWSERS

| Diameter of Rope Inches | Approximate Circumference Inches | Weight per Foot Pounds | Ultimate Strength, in Tons of 2,000 Lb. | Size of New Manila Rope of Same Strength Inches | Diameter of Rope Inches | Approximate Circumference Inches | Weight per Foot Pounds | Ultimate Strength, in Tons of 2,000 Lb. | Size of New Manila Rope of Same Strength Inches |
|---|---|---|---|---|---|---|---|---|---|
| 2 1/16 | 6 1/2 | 4.43 | 83 | | 1 1/2 | 4 1/2 | 2.36 | 45 | 12.00 |
| 2 | 6 1/4 | 4.20 | 77 | | 1 7/16 | 4 1/2 | 2.16 | 41 | 11.50 |
| 1 15/16 | 6 | 3.89 | 71 | | 1 3/8 | 4 1/4 | 2.00 | 38 | 11.00 |
| 1 11/16 | 5 3/4 | 3.42 | 66 | | 1 1/4 | 4 | 1.63 | 31 | 10.00 |
| 1 3/4 | 5 1/2 | 3.23 | 61 | 13.5 | 1 3/16 | 3 3/4 | 1.47 | 28 | 9.25 |
| 1 11/16 | 5 1/4 | 2.94 | 57 | 13.0 | 1 1/8 | 3 1/2 | 1.33 | 26 | 8.75 |
| 1 5/8 | 5 | 2.76 | 53 | 12.5 | | | | | |

## GALVANIZED STEEL MOORING LINES

| Diameter of Rope Inches | Weight per Foot Pounds | Ultimate Strength, in Tons of 2,000 Lb. | Diameter of Rope Inches | Weight per Foot Pounds | Ultimate Strength, in Tons of 2,000 Lb. |
|---|---|---|---|---|---|
| 2 1/16 | 5.81 | 113 | 1 3/8 | 2.62 | 50 |
| 2 | 5.51 | 106 | 1 1/4 | 2.15 | 42 |
| 1 13/16 | 5.09 | 98 | 1 3/16 | 1.93 | 38 |
| 1 11/16 | 4.48 | 88 | 1 1/8 | 1.75 | 34 |
| 1 5/8 | 4.24 | 82 | 1 1/16 | 1.54 | 27 |
| 1 11/16 | 3.86 | 76 | 1 | 1.38 | 25 |
| 1 1/2 | 3.63 | 74 | 7/8 | 1.05 | 20 |
| 1 3/8 | 3.10 | 63 | 13/16 | .90 | 17 |
| 1 7/16 | 2.92 | 55 | 3/4 | .78 | 14 |

## WIRE-ROPE CALCULATIONS

The *working load*, also called the proper working load, is the maximum load that a rope should be permitted to support under working conditions. The stress on a rope to which a load is attached, and which bends over a sheave, is made up of the load on the rope, known as the *load stress*, and the bending of the rope about a sheave or drum, known as the *bending stress*. That is, if

$S$ = total safe stress;
$S_b$ = bending stress;
$S_l$ = load stress;
$S = S_b + S_l$ and $S_l = S - S_b$.

The total stress must not equal the elastic limit of the material composing the rope and is usually taken as from one-third to one-fourth the approximate breaking stress. It is only quite recently that account has been taken of this second stress and it is not by any means universal practice to consider it when calculating the size of rope needed for a given purpose.

If a given weight is to be hoisted with a wire rope, the proper size of rope to use may be taken directly from the tables, but these do not take account of the bending stress, except by allowing for it in the factor of safety assumed.

A second method calculates the bending stress. The following formulas and the diagram based on them are given by Mr. E. T. Sederholm, former chief engineer for Fraser & Chalmers, and will be found in the hoisting-engine catalog of the Allis-Chalmers Co. The general formula for the bending stress is

$$S_b = \frac{E\,a\,A}{D};$$

in which
$S_b$ = bending stress;
$E$ = modulus of elasticity;
$a$ = diameter of each wire;
$D$ = diameter of drum or sheave, in inches;
$A$ = total area of wire cross-section, in inches.

For a rope of 19 wires to the strand the diameter of each wire is about one-fifteenth (exactly $\frac{100}{15.52}$) of the diameter of the rope. That is, if $d$ = diameter of rope, $a = \frac{d}{15.52}$; and by substituting this in the formula, $S_b = \frac{E\,A\,d}{15.52\,D}$.

The modulus of elasticity for the different kinds of wire is given different values by different authorities. Mr. Sederholm uses 29,400,000 in his formula and diagram, and Mr. Hewitt 28,500,000, the same modulus being used for the different materials of which ropes are made.

The cross-section of metal $A$ in a wire rope is approximately $.4d^2$, or it may be more accurately calculated by multiplying the cross-section of each wire, as given by a wire table, by the number of wires in the rope.

EXAMPLE.—Find the bending stress in a 19-wire, cast-steel hoisting rope 2 in. in diameter, winding on an 8-ft. drum, if $A = .4d^2$, and $E = 29,400,000$.

SOLUTION.—Applying the formula just given, the bending moment is

$$S_b = \frac{29,400,000 \times 2^3 \times .4}{10 \times 15.52 \times 96 \times 2,000} = 31.51 \text{ T.}$$

The approximate breaking stress for such a rope is 106 T., and if a factor of 3 is assumed $106 \div 3 = 35 +$ T. for the safe working stress, and $35 - 32 = 3$ T., for the safe lifting load under the given conditions.

Mr. Wm. Hewitt, of the Trenton Iron Co., has given a similar but more complicated formula for the bending stress, which is supposed to give somewhat more accurate results, as he has introduced terms that allow for the actual radius of the bend at the outside fiber of the rope, while the Sederholm formula assumes the radius of the bend to be the radius of the sheave.

Mr. Hewitt's formula is as follows:

$$S_b = \frac{EA}{2.06\,\frac{R}{d} + C}$$

in which $S_b$ = bending stress, in pounds;
$E$ = modulus of elasticity (28,500,000);
$A$ = aggregate area of wires, in square inches;
$R$ = radius of drum or sheave, in inches;
$d$ = diameter of individual wires, in inches;
$C$ = a constant depending on number of wires in strands.

The values of $d$ and $C$ are:

| *7-Wire Rope* | *19-Wire Rope* |
|---|---|
| $d = \frac{1}{9}$ diameter of rope | $d = \frac{1}{15}$ diameter of rope |
| $C = 9.27$ | $C = 15.45$ |

For 12-wire and 16-wire ropes the values are intermediate in proportion to the number of wires. In the case of ropes having strands composed of different sizes of wires, take the larger of the outer layer for the value of $d$.

(*By permission of E. T. Sederholm, Chief Engr., Fraser & Chalmers, Chicago.*)
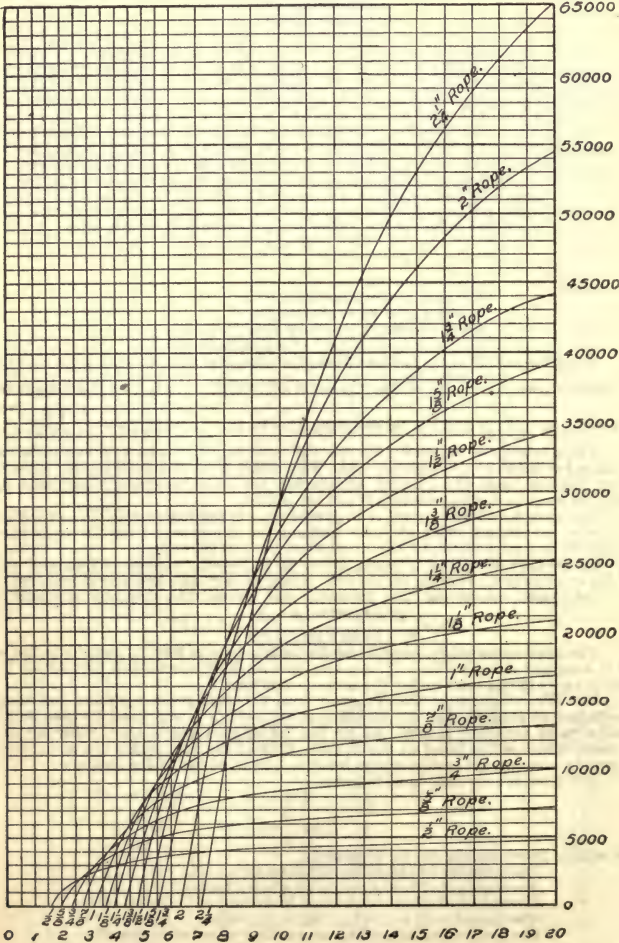
Mr. Hewitt assumes one-third of the approximate breaking stress as the maximum safe stress and uses 28,500,000 for the modulus of elasticity.

If the problem given under the Sederholm formula is worked out by the Hewitt formula the safe working load will be $11\frac{1}{2}$ T., while the table gives $106 \div 5 = 21.2$ T.

The Sederholm diagram gives for a load of 21.2 T. a sheave between 13 and 14 ft. in diameter, the formula gives a sheave of $11\frac{1}{2}$ ft. in diameter, while the table gives $9\frac{3}{4}$ ft. It is evident that there is a wide difference of opinion among the wire-rope authorities and a good opportunity for experimental work along this line.

In using the Sederholm or Hewitt formulas, there are two unknown quantities, the diameter of the rope $d$ and the diameter $D$ or radius $R$ of the drum. $d$ varies inversely as $D$, that is, for a given load, the smaller $d$ is taken the larger $D$ must be to give the same conditions of safety.

If a certain ratio between $S_b$ and $S_l$ could be assumed in the formula $S = S_b + S_l$ the problem could be easily solved, but an examination of this ratio in a number of cases where good results have been obtained from hoisting ropes shows it to vary from $\dfrac{S_l}{S_b} = 1$ to $\dfrac{S_l}{S_b} = \dfrac{2}{5}$. In the transmission of power by wire ropes, Mr. Hewitt assumes $\dfrac{S_l}{S_b} = \dfrac{1}{2}$, but this relation will scarcely hold in a hoisting problem, and the foregoing problem must be solved by the cut-and-try method.

**Proper Working Load.**—For steel hoisting ropes, made with 19 wires to the strand, when used on drums of different diameters, the proper working load may be found from the following formula, in which the total strain on the rope, including bending strain and the strain due to load is assumed at 50,000 lb. per sq. in. of actual steel section.

Let
$d$ = diameter of rope, in inches;
$D$ = diameter of drum, in inches;
$S$ = strain per square inch due to bending;
$L$ = proper working load, in pounds.

$$S = 1,894,000 \times \frac{d}{D}$$

$$L = 20,000d^2 - 757,600 \times \frac{d^3}{D}$$

### STARTING STRESS ON ROPE

| Dynamometer Tests | Starting Stress | |
|---|---|---|
| | Tons | Cwt. |
| *First Test* | | |
| Empty cage lifted gently............................... | 1 | 16 |
| Empty cage with $2\frac{1}{2}$ in. slack chain...................... | 2 | 10 |
| Empty cage with 6 in. slack chain...................... | 4 | 0 |
| Empty cage with 12 in. slack chain..................... | 5 | 10 |
| *Second Test* | | |
| Cage and four empty cars weighed by machine........... | 2 | 17 |
| Cage and four empty cars lifted gently................. | 3 | 0 |
| Cage and four empty cars with 3 in. slack chain........ | 5 | 0 |
| Cage and four empty cars with 6 in. slack chain........ | 5 | 10 |
| Cage and four empty cars with 12 in. slack chain....... | 7 | 0 |
| *Third Test* | | |
| Cage and full cars weighed by machine.................. | 5 | 1 |
| No. 1, lifted gently................................... | 5 | 1 |
| No. 2, lifted gently.................................. | 5 | 3 |
| No. 1, with 3 in. slack chain......................... | 8 | 10 |
| No. 2, with 3 in. slack chain......................... | 8 | 10 |
| No. 1, with 6 in. slack chain......................... | 10 | 10 |
| No. 2, with 6 in. slack chain......................... | 11 | 10 |
| No. 1, with 9 in. slack chain......................... | 12 | 10 |
| No. 2, with 9 in. slack chain......................... | 11 | 10 |

**Starting Stress on Hoisting Rope.**—When selecting a hoisting rope, due allowance must be made for the shock and extra stress imposed on the rope when the load is started from rest. Experiments made by placing a dynamometer between the rope and the cage have shown that the starting stress may be from two to three times the actual load. The experiments referred to were made in England and are here given; the tons are those of 2,240 lb., and the hundredweights (cwt.), 112 lb.

## STRESS OF ROPE ON PLANES

| Rise per 100 Ft. Horizontal Feet | Angle of Inclination | Stress per Ton of 2,000 Lb. Pounds | Rise per 100 Ft. Horizontal Feet | Angle of Inclination | Stress per Ton of 2,000 Lb. Pounds |
|---|---|---|---|---|---|
| 5 | 2° 52′ | 140 | 105 | 46° 24′ | 1,484 |
| 10 | 5° 43′ | 240 | 110 | 47° 44′ | 1,516 |
| 15 | 8° 32′ | 336 | 115 | 49° 00′ | 1,535 |
| 20 | 11° 10′ | 432 | 120 | 50° 12′ | 1,573 |
| 25 | 14° 03′ | 527 | 125 | 51° 21′ | 1,597 |
| 30 | 16° 42′ | 613 | 130 | 52° 26′ | 1,620 |
| 35 | 19° 18′ | 700 | 135 | 53° 29′ | 1,642 |
| 40 | 21° 49′ | 782 | 140 | 54° 28′ | 1,663 |
| 45 | 24° 14′ | 860 | 145 | 55° 25′ | 1,682 |
| 50 | 26° 34′ | 933 | 150 | 56° 19′ | 1,699 |
| 55 | 28° 49′ | 1,003 | 155 | 57° 11′ | 1,715 |
| 60 | 30° 58′ | 1,067 | 160 | 58° 00′ | 1,730 |
| 65 | 33° 02′ | 1,128 | 165 | 58° 47′ | 1,744 |
| 70 | 35° 00′ | 1,185 | 170 | 59° 33′ | 1,758 |
| 75 | 36° 53′ | 1,238 | 175 | 60° 16′ | 1,771 |
| 80 | 38° 40′ | 1,287 | 180 | 60° 57′ | 1,782 |
| 85 | 40° 22′ | 1,332 | 185 | 61° 37′ | 1,794 |
| 90 | 42° 00′ | 1,375 | 190 | 62° 15′ | 1,804 |
| 95 | 43° 32′ | 1,415 | 195 | 62° 52′ | 1,813 |
| 100 | 45° 00′ | 1,450 | 200 | 63° 27′ | 1,822 |

**Stress in Hoisting Ropes on Inclined Planes of Various Degrees.**—The preceding table is based on an allowance of 40 lb. per T. for rolling friction, but there will be an additional stress due to that portion of the weight of the rope which acts vertically.

**Relative Effect of Various Sized Sheaves or Drums on Life of Wire Ropes.** Mine officials and other users of wire ropes have often felt the want of a table or set of tables that would enable them to determine at a glance what effect the use of various sized sheaves would have on various sized ropes. The following tables have been specially prepared for the Coal and Metal Miner's Pocketbook by Mr. Thomas E. Hughes, of Pittsburg, Pennsylvania. The cast-steel ropes for inclines are made of 6 strands of 7 wires each, laid around a hemp core, the cast-steel hoisting ropes are made of 6 strands of 19 wires each, laid a hemp core; and the iron hoisting ropes, of 6 strands of 19 wires each, laid around a hemp core.

## CAST-STEEL ROPES FOR INCLINES

| Diameter of Rope. Inches | Percentages of Life for Various Diameters | | | | | | |
|---|---|---|---|---|---|---|---|
| | 100 | 90 | 80 | 75 | 60 | 50 | 25 |
| | Diameters of Sheaves or Drums in Feet | | | | | | |
| 1½ | 16.00 | 14.00 | 12.00 | 11.00 | 9.00 | 7.00 | 4.75 |
| 1⅜ | 14.00 | 12.00 | 10.00 | 8.50 | 7.00 | 6.00 | 4.50 |
| 1¼ | 12.00 | 10.00 | 8.00 | 7.25 | 6.00 | 5.50 | 4.25 |
| 1⅛ | 10.00 | 8.50 | 7.75 | 7.00 | 6.00 | 5.00 | 4.00 |
| 1 | 8.50 | 7.75 | 6.75 | 6.00 | 5.00 | 4.50 | 3.75 |
| ⅞ | 7.75 | 7.00 | 6.25 | 5.75 | 4.50 | 3.75 | 3.25 |
| ¾ | 7.00 | 5.50 | 5.50 | 5.00 | 4.25 | 3.50 | 2.75 |
| ⅝ | 6.00 | 5.25 | 4.50 | 4.00 | 3.25 | 3.00 | 2.50 |
| ½ | 5.00 | 4.50 | 4.00 | 3.50 | 2.75 | 2.25 | 1.75 |

## CAST-STEEL HOISTING ROPES

| Diameter of Rope. Inches | Percentages of Life for Various Diameters | | | | | | |
|---|---|---|---|---|---|---|---|
| | 100 | 90 | 80 | 75 | 60 | 50 | 25 |
| | Diameters of Sheaves or Drums in Feet | | | | | | |
| $1\frac{1}{2}$ | 14.00 | 12.00 | 10.00 | 8.50 | 7.00 | 6.00 | 4.50 |
| $1\frac{3}{8}$ | 12.00 | 10.00 | 8.00 | 7.00 | 6.00 | 5.25 | 4.25 |
| $1\frac{1}{4}$ | 10.00 | 8.50 | 7.50 | 6.75 | 5.50 | 5.00 | 4.00 |
| $1\frac{1}{8}$ | 9.00 | 7.50 | 6.50 | 5.50 | 5.00 | 4.50 | 3.75 |
| 1 | 8.00 | 7.00 | 6.00 | 5.50 | 4.50 | 4.00 | 3.50 |
| $\frac{7}{8}$ | 7.50 | 6.75 | 5.75 | 5.00 | 4.25 | 3.50 | 3.00 |
| $\frac{3}{4}$ | 5.50 | 4.50 | 4.00 | 3.75 | 3.25 | 3.00 | 2.25 |
| $\frac{5}{8}$ | 4.50 | 4.00 | 3.75 | 3.25 | 3.00 | 2.50 | 2.00 |
| $\frac{1}{2}$ | 4.00 | 3.00 | 3.00 | 2.75 | 2.25 | 2.00 | 1.50 |
| $\frac{3}{8}$ | 3.00 | | | 2.00 | | 1.50 | |

## IRON HOISTING ROPES

| Diameter of Rope. Inches | Percentages of Life for Various Diameters | | | | | | |
|---|---|---|---|---|---|---|---|
| | 100 | 90 | 80 | 75 | 60 | 50 | 25 |
| | Diameters of Sheaves or Drums in Feet | | | | | | |
| $1\frac{1}{2}$ | 12.00 | 11.00 | 9.00 | 7.50 | 6.00 | 5.00 | 4.25 |
| $1\frac{3}{8}$ | 10.00 | 9.00 | 7.50 | 7.00 | 5.25 | 4.75 | 4.00 |
| $1\frac{1}{4}$ | 9.00 | 7.75 | 6.50 | 5.75 | 4.50 | 4.00 | 3.50 |
| $1\frac{1}{8}$ | 8.00 | 6.75 | 5.50 | 5.00 | 4.25 | 3.50 | 3.00 |
| 1 | 6.75 | 6.00 | 5.00 | 4.75 | 4.00 | 3.25 | 2.75 |
| $\frac{7}{8}$ | 6.75 | 6.00 | 5.00 | 4.50 | 4.00 | 3.00 | 2.50 |
| $\frac{3}{4}$ | 5.00 | 4.75 | 4.00 | 3.75 | 3.00 | 2.75 | 2.00 |
| $\frac{5}{8}$ | 4.50 | 3.75 | 3.25 | 3.00 | 2.75 | 2.25 | 1.75 |
| $\frac{1}{2}$ | 3.50 | 3.25 | 3.00 | 2.75 | 2.00 | 1.50 | 1.00 |
| $\frac{3}{8}$ | 3.00 | | 2.00 | | | 1.25 | 1.00 |

## CARE OF WIRE ROPES

**Ordinary Method of Splicing Wire Rope.**—The tools required for splicing wire ropes by the ordinary method are a pair of iron nippers for cutting off strands; two marline spikes, one round and one oval, for opening strands; one knife to cut hemp center; two clamps to untwist rope to insert ends of strands or, in place of them, two short hemp-rope slings, with a stick for each as a lever; a wooden mallet and some rope twine. Also, a bench and vise are handy.

The length of the splice depends on the size of the rope. The larger ropes require the longer splices. The splice of ropes from $\frac{3}{8}$ in. to $\frac{3}{4}$ in. in diameter should not be less than 20 ft.; from $\frac{3}{4}$ in. to $1\frac{1}{8}$ in., 30 ft.; and from $1\frac{1}{8}$ in. up, 40 ft.

To splice a rope, tie each end with a piece of cord at a distance equal to one-half the length of the splice, or 10 ft. back from the end for a $\frac{3}{8}$-in. rope, after which unlay each end as far as the cord. Then cut out the hemp center, and bring the two ends together as close as possible, placing the strands of the one end between those of the other, as shown in Fig. 1 (*a*). Now remove the cord *k* from the end *M* of the rope, and unlay any strand, as *a*, and follow it up with the strand of the other end *M'* of the rope that corresponds to it, as *a'*. About 6 in. of *a* is left out, and *a'* is cut off about 6 in. from the rope, thus leaving two short ends, as shown at *P* in (*b*), which must be tied for the present by cords as shown. Wind the cord *k* again around the end *M* of the rope, to prevent the unraveling of the strands; after which remove the cord *k'* on the other or *M'* end of the rope, and unlay the strand *b*; follow it up, as before, with the strand *b'*, leaving the ends out, and tying them down for the present, as described in the case of strands *a* and *a'*, see *O*; also, replace the cord *k'*. Again remove the cord *k* and unlay the next strand, as *c*, and follow it up with *c'*, stopping, however, this time within 4 ft. of the first set. Con-

tinue this operation with the remaining six strands, stopping 4 ft. short of the preceding set each time. The strands are now in their proper places, with the ends passing each other at intervals of 4 ft., as shown in (c). To dispose of the loose ends, clamp the rope in a vise at the left of the strands *a* and *a'*,
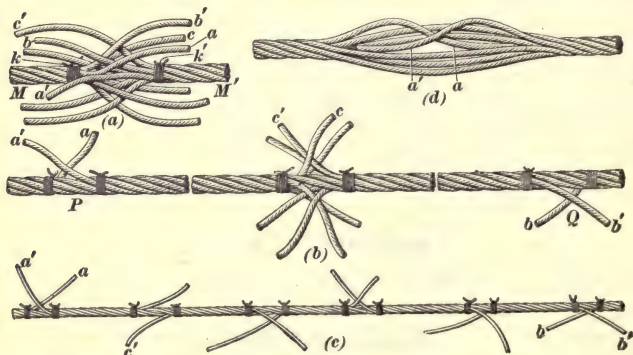


FIG. 1

view (c), and fasten a clamp to the rope at the right of these strands; then remove the cords tied around the rope that hold these two strands down; after which turn the clamp in the opposite direction to which the rope is twisted, thereby untwisting the rope, as shown in (d). The rope should be untwisted enough to allow its hemp core to be pulled out with a pair of nippers. Cut off 24 in. of the hemp cores, 12 in. at each side from the point of intersection of the strands *a* and *a'*, and push the ends of the strands in their place as shown in (d). Then allow the rope to twist up to its natural shape, and remove the clamps. After the rope has been allowed to twist up, the strands tucked in generally bulge out somewhat. This bulging may be reduced by lightly tapping the bulged part of the strands with a wooden
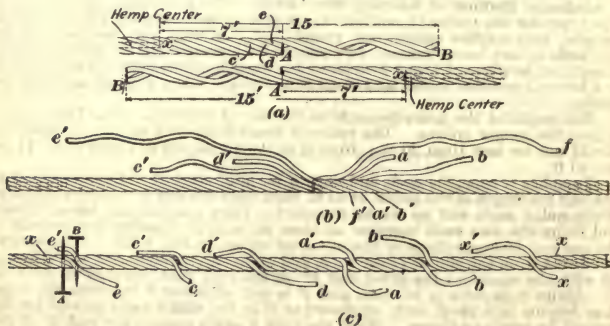


FIG. 2

mallet, which will force their ends farther into the rope. Proceed in the same manner to tuck in the other ends of the strands.

**Rapid Method of Splicing Wire Rope.**—The only tools needed in the rapid method of splicing wire ropes are a cold chisel and hammer for cutting

and trimming the strands, and two needles 12 in. long, made of good steel and tapered ovally to a point. Cut off the ends of the ropes to be spliced and unlay three adjacent strands of each back 15 ft.; cut out the hemp center to this point and relay the strands for 7 ft. and cut them off. Pull the ropes by each other until they have the position shown in Fig. 2, (a), cut off a and d', b and c', view (b), making their lengths approximately 10 and 12½ ft., respectively, measured from the point where the hemp centers were cut. Place the ropes together, unlay e, d, c, Fig. 2, view (a), keeping the strands together, and follow with e', d', c', (b). Similarly, unlay f', a', b', and follow with f, a, b, until the rope appears as in (c). Next run the strands into the middle of the rope. To do this, cut off the end of the strand e', so that when it is put in place it will just reach to the end x of the hemp core, and then push the needle A, through the rope from the under side, leaving two strands at the front of the needle, as shown. Push the needle B through from the upper side and as close to the needle A as possible, leaving the strands e and e' between them; place the needle A on the knee and turn the needle B around with the coil of the rope, and force the strand e' into the center of the rope. Repeat this operation with the other ends and cut them off so that the ends coming together in the center of the rope will butt against each other as nearly as possible.

**Wear of Wire Ropes.**—The deterioration of wire ropes may be either external or internal, and may be due (1) to abrasion, due to the rubbing of the outside surface of the rope against other objects, or to the internal chafing of the wires composing the strands against one another; (2) to injury from overloading, to shock due to sudden starting of the load, or to repeated bendings about too sharp angles or over sheaves or rollers of too small a diameter for the size of the rope; (3) to rust or corrosion of the wire from acid waters, or to decay of the hemp cores.

As a result of abrasion, the wires in a rope are either flattened or torn apart. With properly designed drums and head-frame and properly placed sheaves, a hoisting rope is but slightly abraded, and the wear is due chiefly to bending or to overloading. A haulage rope is subjected to constant abrasion in passing over rollers and sheaves and from dragging along the bottom and sides of the haulage ways and from the grips. It is also often subject to severe shocks and abrasion from the lashing or vibration when the winding engine starts.

The wear and tear on a rope increases as its velocity is increased; hence, conditions permitting, it is better to increase the output by increasing the load within allowable limits rather than by increasing the velocity of the rope.

**Inspection of Ropes.**—The life of a hoisting rope depends not only on its quality, but also on the conditions under which it is used and on the carefulness of the engineer in handling the load. A rope should be inspected often and at regular intervals; at some mines, the hoisting ropes are inspected every morning before lowering the men. The cage is slowly lowered and then raised, each rope being carefully examined by an inspector to detect any broken wires. Particular attention should be given to the part of the rope where it is attached to the socket at the cage, as this part is more subject to corrosion and sharp bending than any other. When the core fails at any point the rope should be discarded at once as the wires are likely to kink and break internally as the rope passes over the sheave. At some mines, hoisting ropes are discarded at regular intervals, whether they show wear or not. Haulage ropes do not require as frequent examination and are not discarded as quickly as hoisting ropes, as much less in the way of life and property is dependent on them. If a new piece of loose hemp rope is given one turn around the haulage rope and each end held firmly while the rope is run the presence of loose wire ends will be shown.

**Lubrication of Wire Ropes.**—Mine water has a very corrosive action on wire ropes, and a rope will soon be destroyed unless the water is prevented from coming in contact with the metal. For this reason, black oil or some lubricating preparation is applied to the rope, but any lubricant used must be free from acids or other substances that will corrode the wire.

For hoisting ropes, 1 bu. of freshly slaked lime to 1 bbl. of pine or coal tar makes a good lubricant; with pine tar, which contains no acid, tallow may be used instead of lime. Another mixture contains tar, summer oil, axle grease, and a little pulverized mica, mixed to such a consistency that it will penetrate thoroughly between the wires and will not dry or strip off. The lubricant should not be thick enough to render difficult the thorough inspection of the rope, and all lubricants of this nature should be used sparingly after the first application, as the rope should be kept clean and free from grit. Graphite is also used for the purpose.

Lubricants may be applied by running the engine slowly and allowing the rope to pass through a bunch of waste saturated with lubricant; by rubbing the lubricant into the rope by means of a brush; or by pouring the oil into the groove of the sheave as the rope is run slowly back and forth. A new hoisting rope should be passed through a bath of hot lubricant and thus be thoroughly lubricated.

Haulage ropes are not usually lubricated as thoroughly as hoisting ropes on account of the grease causing slipping of grips and gathering of dirt and dust, but they can be treated with raw linseed oil thickened with lamp-black boiled with an equal portion of pine tar, and the mixture applied while hot. Ordinary black oil, such as is used to oil mine cars and hoisting ropes, can be used on haulage ropes where no friction grips are employed. These mixtures, if fluid, can be poured on the rope as it is run over the sheave, or applied from a leather-lined box filled with oil. Patent lubricants known as *cable shields* or *rope fillers*, which fill the interstices between the strands, are often used on tail and main ropes.

**General Precautions.**—Wire rope is as pliable as new hemp rope of the same strength; the former will therefore run on the same sized sheaves and pulleys as the latter. But the greater the diameter of the sheaves, pulleys, and drums, the longer wire rope will last. In the construction of machinery for wire rope, it will be found good economy to make the drums and sheaves as large as possible.

The tables of wire-rope manufacturers give proper diameters of drum or sheave at from 50 to 65 times the rope diameter; but the expression should more properly be the minimum admissible diameter. For ordinary service, by using sheaves and drums from 75 to 100 times the diameter of the rope, the average life of hoisting ropes will be materially lengthened. For rapid hoisting, during which abnormal strains are most likely to occur, or where a low factor of safety is employed, a sheave diameter of 150 times that of the rope is to be recommended.

Experience has demonstrated that the wear increases with the speed; it is therefore better to increase the load than the speed. Wire rope is manufactured either with a wire or a hemp center; the latter is more pliable than the former, and will wear better where there is short bending.

Wire rope must not be coiled or uncoiled like hemp rope. When mounted on a reel, the reel should be mounted on a spindle or flat turntable to pay off the rope. When shipped in a small coil, without reel, the coil should be rolled over the ground like a wheel, and the rope run off in that way. All untwisting or kinking must be avoided.

A rope should not be changed from a large drum to a small one, for it will not work so well, neither will it last as long. This is also true, but in a lesser degree, of ropes changed from a small drum to a large one. After having been used for some time on a drum, the rope adapts itself to that diameter and resents a change. Rope sheaves should be made to fit the rope, and should be filled in with well-seasoned blocks of oak or other hardwood, set on end; this will save the rope and increase adhesion.

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# CABLEWAYS AND TRAMWAYS

**Cableways.**—A suspension cableway is a hoisting and conveying device using a suspended cable for a trackway. There are two types: the inclined, or semi-gravity, Fig. 1, and the horizontal, Fig. 2.

The *inclined cableway* consists of a cable inclined 20° to 22° to the horizontal, and passing over a cast-iron saddle *B* on top of a tower or frame *A*. It is anchored by logs *D* buried about 5 ft. underground, or from iron plugs secured in the rock, when the rock is near the surface. The trolley carriage *G* runs down the incline of the cableway by gravity until it reaches a stop. A hoisting rope *E* operated by a winding drum *F* leads over a sheave pulley *e*, thence to a pulley in the carriage *G*, thence to a fall block *M*, upwards again to a second pulley in the carriage, and downwards again to the fall block. Winding in the rope hoists the fall block to the carriage, the carriage remaining at the lower stop. When the fall block collides with the carriage, both the carriage and the fall block are pulled up the incline cable, and when the carriage arrives at the head-tower, a gate, or hook, *O* is lowered to hold it in place. The fall block is then lowered and the load discharged. The engine *F* has usually a $10'' \times 12''$ double cylinder and a single friction drum 37 in. in diameter.

FIG. 1

FIG. 2
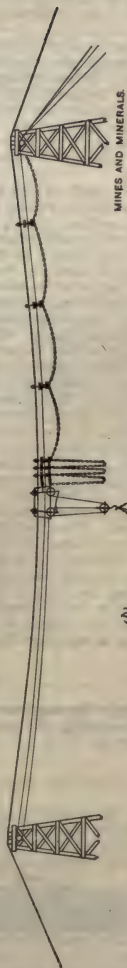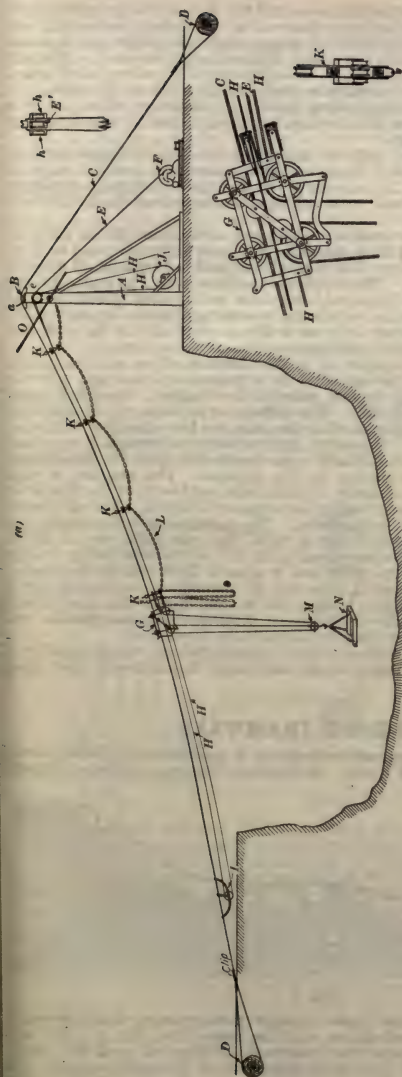
MINES AND MINERALS.

An endless rope $H$ takes several turns around the sheave $J$ to prevent it from slipping, and both ends are passed over sheaves at the top of the derrick, one end being secured to the front of the carriage, while the other end is taken through the carriage and around the return sheave $I$ and fastened to the rear end of the carriage $G$. The endless-rope wheel $J$ is provided with a band brake, which, when applied, holds the carriage securely at any point on the cable.

All ropes pass through the supporting trolleys $K$, which are connected by a chain $L$. These trolleys follow the carriage by gravity, and the chain may or may not be fast to the carriage.

Instead of chain-connected trolleys, patent button-stop, fall-rope carriers, which are lighter, may be used. These are spaced along an auxiliary rope on which buttons are screwed. The carriers are picked up by a horn on the front of the carriage. These are said to be cheaper for operation than the chain trolleys.

The length of the span for inclined cableways varies from 200 to 1,200 ft. The main rope is from $1\frac{1}{2}$ to $2\frac{1}{4}$ in. in diameter, the hoisting rope from $\frac{5}{8}$ to $\frac{3}{4}$ in., and the endless rope $\frac{7}{16}$ and $\frac{1}{2}$ in. The rope mostly used is 6 strands, 19 wires to the strand, crucible cast steel. The hoisting rope lasts from 1 to 2 yr., and the main cable from 5 to 10 yr. These cableways are widely used about the slate quarries in Eastern Pennsylvania, where the operating expense for each cable, where two or more are connected with one boiler, is about $5 per da. of 10 hr.; this includes the engineer, steam, and maintenance of the cableway.

The *horizontal cableway* requires a double friction drum and reversible link-motion engine. It may be operated at any inclination of the carrying cable and either from the high or low point of the support, though, if possible, it should be from the higher end. The endless, or traction, rope is attached to one of the drums of the engine so that the engineer has complete control of the carriage; hence, because of its greater applicability, this system is supplanting the inclined, although the inclined costs one-fourth less for installation. The method of operation is similar to the inclined. The amount of rope required is the same in each system. A horizontal cableway of the Hamilton Coal Co., near Tarentum, Pennsylvania, has a span of 2,200 ft. The stationary rope is 2,500 ft. long and $2\frac{1}{4}$ in. in diameter. The hoisting rope is 4,500 ft. long and $\frac{5}{8}$ in. in diameter. The head-tower is 80 ft. high, the tail-tower is 100 ft. and the rope deflects 80 ft. The skip used holds 3 T. of coal and makes 10 trips per hour. Five men operate the plant and it takes 2 T. of coal for the engine. Based on a capacity of 100 T. per da., the cost of carrying the coal is 13c. per T. For a cableway of average length, 1,000–1,500 ft., the cost of operation should not be one-half the above cost. A cableway, 2,140 ft. long was used in constructing the dam at the power plant at Glens Falls, New York.

One or both towers of a cableway may be mounted on wheels capable of moving on a track at right angles to the cable and the cableway then made to cover a wide territory.

## WIRE-ROPE TRAMWAYS

**Single Tramways.**—A tramway, in America, is a cableway of the horizontal type consisting of a number of spans. In England, the term cableway includes tramways.
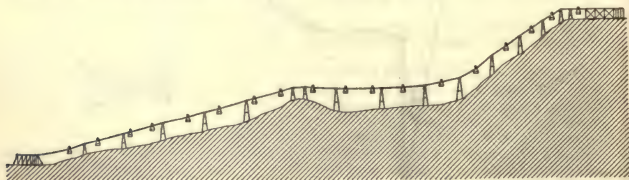


FIG. 1

Single wire-rope tramways have a single moving rope, which serves to support and advance the load at one and the same time, Fig. 1. This rope passes over suitable sheaves at the intermediate supports, and the load is carried in buckets suspended from it by gooseneck or straight hangers. The hangers are usually attached to the cable by means of a clip, which is either

inserted in the center of the cable or strapped to it. The carriers are often loaded and unloaded while in motion, the loading being accomplished by a traveling mechanical hopper and the unloading by a drop bottom to the bucket. If the line is level, or the grade light, the hangers are provided with box heads filled with wood or leather and rubber, which rest on the rope; the rubber or wood providing sufficient friction to prevent the hangers' slipping. With this system, long spans are evidently out of the question, because with a long span the angle of the rope in the vertical plane, at the supports, becomes so great that the friction will not hold the box head. For all practical purposes, grades exceeding 1 : 4 are to be avoided; and for steeper grades, to prevent slipping, a clamp or a clip inserted in the rope is used to fasten the hanger to the rope.

The single-moving rope tramways carry loads not exceeding 200 lb. The speed of the rope for the variety in which the loads are fastened to the rope may be as high as 450 ft. per min., and for one in which the hanger is loose, 200 ft. per min. The single moving-rope tramway has a capacity up to 200 T. per da., and may be built, say, $1\frac{1}{2}$ to 2 mi. long.

**Double Tramways.**—The more satisfactory and substantial kind of wire-rope tramway has one or more fixed ropes, which constitute the permanent way, and an endless traction rope. The loaded carrier travels outwards on one fixed cable and returns by a parallel one suspended from the opposite side of the same supports. The terminals have suitable appliances for loading and unloading the buckets, either by hand or automatically.

The intermediate supports are built of wood or steel framing, with saddles of cast iron $a$, Fig. 2 in which the fixed cables $b$ rest. The traction rope $c$ is supported (in the absence of a bucket) by the rollers $d$, set conveniently on the supports. The load is carried in buckets $e$, or other contrivances suitable for the purpose, which are suspended from a trolly $f$, which runs on the fixed cables, the wheels of which are large enough to pass over the rope couplings, and
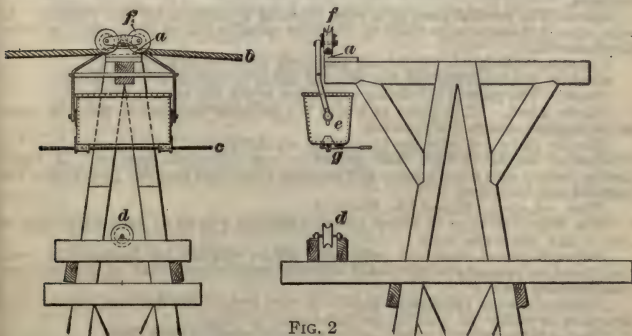


Fig. 2

also to clear the saddles. Grips $g$ attach the carriers to the traction rope. These grips may be operated by hand or automatically.

This kind of tramway is capable of carrying individual loads up to 1,400 or 1,500 lb., not including the weights of the bucket and hanger itself. The speed of traction rope may be from 150 to 350 ft. per min. The capacity is from 200 to 1,000 T. per da. of 10 hr. These figures represent good, safe, practice, but they are not, of course, inflexible.

The maximum length of line that may be built in one section varies largely with conditions of load, spacing of supports, contour of ground, etc. Wire-rope tramways work under great difficulties, and probably $2\frac{1}{2}$ to 4 mi. is the economical limit. This has been exceeded, but for a much greater distance the friction becomes too great for economical working of the traction rope. This does not, however, limit the length of tramway which may be built, as the power station may be located at a convenient intermediate point, dividing the line into sections. Several intermediate power stations may be used, and the length of the line greatly increased above the limit given. A tramway at Grand Encampment, Wyoming, is 16 mi. long and carries 40 T. of ore each hour.

# GLOSSARY OF ROPE TERMS

*Annealed Wire Rope.*—A wire rope made from wires that have been softened by annealing and the tensile strength thereby lowered.

*Bending Stress.*—The stress produced in the outer fibers of a rope by bending over a sheave or drum.

*Breaking Strain, Breaking Strength, Breaking Stress.*—The least load that will break a rope. These terms are used indiscriminately to mean the load that will break a rope. The stress on a rope at the moment of breaking is the *breaking stress,* and the strain or deformation produced in the material by this stress is the breaking strain.

*Bright Rope.*—Rope of any construction, whose wires have not been galvanized, tinned, or otherwise coated.

*Cable-Laid Rope.*—Wire cables made of several ropes twisted together, each rope being composed of strands twisted together without limitation as to the number of strands or direction of twist. A fiber cable-laid rope is a rope having three strands of hawser-laid rope, twisted right-handed.

*Cable.*—Same as cable-laid rope; a fiber cable consists of three hawsers laid up left-handed.

*Cast Steel.*—Steel that has been melted, cast into ingots, and rolled out into bars.

*Clamp.*—A device for holding two pieces or parts of rope together by pressure.

*Clip.*—A device similar to a clamp but smaller and for the same purpose.

*Coir.*—Cocoanut-husk fiber.

*Compound.*—A lubricant applied to the inside and outside of ropes preventing corrosion and lessening abrasion of the rope when in contact with hard surfaces.

*Core.*—The central part of a rope forming a cushion for the strands. In wire ropes it is sometimes made of wire, but usually it is of hemp, jute, or some like material.

*Coupling.*—A device for joining two rope ends without splicing.

*Crucible Steel.*—A fine quality of steel made by the crucible process.

*Drum.*—The part of a hoisting engine on which the rope is wound.

*Elastic Limit.*—That point at which the deformations in the material cease to be proportional to the stresses.

*Elevator Rope.*—A rope use to operate an elevator.

*Endless Rope.*—A rope that moves in one direction, one part of which carries loaded cars from a mine at the same time that another part brings the empties into the mine.

*Fiber.*—A single thread-like filament.

*Flat Rope.*—A rope in which the strands are woven or sewed together to form a flat, braid-like rope.

*Flattened-Strand Rope.*—A wire rope whose strands are flattened or oval. and therefore presents an increased wearing surface over that of the ordinary round-strand rope.

*Flattened-Strand Triangular Rope.*—A wire rope of the flattened-strand construction in which the strands are triangular in shape.

*Fleet.*—Movement of a rope sidewise when winding on a drum.

*Fleet Wheel.*—A grooved wheel or sheave that serves as a drum and about which one or more coils of a haulage rope pass.

*Galvanized Rope.*—Rope made of wires that have been galvanized or coated with zinc to protect them from corrosion.

*Grip Wheel.*—A wheel, the periphery of which is fitted with a series of togglejointed, cast-steel jaws that grip the rope automatically.

*Guy.*—A strand or rope used to support a pole, structure, derrick, or chimney, etc.

*Haulage Rope.*—A rope used for haulage purposes.

*Hawser.*—Any wire rope used for towing on lake or sea. A fiber hawser consists of three strands laid up right-handed.

*Hawser-Laid Rope* has three strands of yarn twisted left-handed, the yarns being laid up right-handed. Synonymous with cable-laid rope as applied to wire ropes.

*Hawser Wire Rope.*—Galvanized rope of iron or steel, usually composed of 6 strands, 12 wires each, principally used in marine work for towing purposes.

*Hemp.*—A tough, strong fiber obtained from the hemp plant.

*Hoisting Rope.*—A rope composed of a sufficient number of wires and strands to insure flexibility. Such ropes are used in shafts, elevators, quarries, etc.

*Idler.*—A sheave or pulley running loose on a shaft to guide or support a rope.

*Jute.*—A fiber obtained from the inner bark of two Asiatic herbs: *Corchorus capsularis* and *C. olitorius.*

*Lang Lay Rope.*—A rope in which the wires in each strand are twisted in the same direction as the strands in the rope.

*Lay.*—The direction, or length, of twist of the wires and strands in a rope.

*Live Load.*—A load that is variable in distinction from a constant load.

*Load Stress.*—The stress produced by the load.

*Locked-Wire Rope.*—A rope with a smooth cylindrical surface, the outer wires of which are drawn to such shape that each one interlocks with the other and the wires are disposed in concentric layers about a wire core instead of in strands. Particularly adapted for haulage and rope-transmission purposes.

*Manila.*—The fiber of *Musa textilis;* Manila hemp.

*Modulus of Elasticity.*—The ratio between the amount of extension or compression of a material and the load producing this same extension or compression.

*Plow Steel.*—A select grade of steel of high tensile strength; first used in rope for plowing fields.

*Proper Working Load.*—The maximum load that a rope should be permitted to support under working conditions. (See *working load.*)

*Regular-Lay Rope.*—A rope in which the wires in each strand are twisted in opposite direction to the strands in the rope.

*Round-Strand Rope.*—A rope made of round twisted strands.

*Running Rope.*—A flexible rope that will pass through blocks and used for hoisting on shipboard. The term is also often used for any moving rope.

*Sheave.*—A wheel or pulley around or over which a rope passes.

*Shroud Laid,* or *Four-Strand, Rope* has four strands laid around a core.

*Sisal.*—A hemp; the fiber of the *Agave Sisalona.*

*Socket.*—A device fastened to the end of a rope by means of which the rope may be attached to its load; the socket may be opened or closed.

*Splice.*—The joining of two ends of rope by interweaving the strands.

*Step Socket.*—A special form of socket for use on locked-wire rope.

*Stirrup.*—An adjustable bale of a socket.

*Stone Wire.*—Wire smaller than No. 14 put up in 12-lb. coils, which are about 8 in. inside diameter.

*Strand.*—A varying number of wires or fibers twisted together; the strands in turn are twisted together, forming a rope.

*Stress.*—A force or combination of forces tending to change the shape of a body.

*Strain.*—A change of shape produced in a body. (Stress and strain are often used incorrectly as synonymous terms.)

*Surging.*—The flapping of a moving rope.

*Swedish Iron.*—A soft and comparatively pure iron.

*Switch Rope.*—A short length of rope fitted with a hook on one end and a link on the other, used for the switching of freight cars.

*Tail-Rope.*—(1) The rope that is used to draw the empties back into a mine in a tail-rope haulage system. (2) A rope attached beneath the cage when the cages are hoisted in balance.

*Taper Rope.*—A rope that has a gradually diminishing diameter from the upper to the lower end. The diameter of the rope is decreased by dropping one wire at a time at regular intervals. Both round and flat ropes may be made tapered, and such ropes are intended for deep-shaft hoisting with a view to proportioning the diameter of the rope to the load to be sustained at different depths.

*Tensile Strength.*—The stress required to break a rope by pulling it in two.

*Thimble.*—An oval iron ring around which a rope end is bent and fastened to form an eye.

*Tiller Rope.*—A very flexible wire rope composed of six small ropes, usually of seven-wire strands laid about a hemp core.

*Tinned Rope.*—Rope made of wires that have been coated with tin to protect them from corrosion.

*Torsion.*—The process of twisting a wire, thereby showing its ductility.

*Traction Rope.*—A rope used for transmitting the power in a wire-rope tramway and to which the buckets are attached.

*Transmission Rope.*—A rope used for transmitting power.

*Traveler.*—A truck rolling along a suspended rope for supporting a load to be transported.

*Turnbuckle.*—A form of coupling so threaded or swiveled that by turning it the tension of a rope or rod may be regulated.

*Ultimate Tensile Strength.*—Same as tensile strength.

*Universal Lay.*—Another term for lang lay.

*Whipping.*—The flopping of a moving rope.

*Wire Gauge.*—Standard sizes or diameters for wire.

*Wire Rope.*—A rope whose strands are made of wires, twisted or woven together.

*Working Load.*—The maximum load that a rope can carry under the conditions of working without danger of straining. (Same as *proper working load.*)

*Wrought Iron.*—A comparatively pure and malleable iron.

*Yarn.*—Twisted fiber of which rope strands are made.

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# POWER TRANSMISSION

## TRANSMISSION BY WIRE ROPES

The term transmission, as here used, applies simply to the modification of belt driving, using grooved wheels or sheaves at each end of the line. The power is applied to one sheave and taken off from the other. The friction between rope and sheaves depends directly on the weight and tension of the rope and on the nature of the surfaces in contact. This pressure is better obtained by using a large, heavy rope at a low tension than by using a smaller rope at a high tension. The deflection, or sag, of the rope, between the sheaves, is the same for both upper and lower parts of the rope when the transmission is not running and should be, according to John A. Roebling's Sons Co., equal to about $\frac{1}{30}$ of the span. The deflection may be calculated by the formula from the Trenton Iron Co.:

$$h = \frac{ws^2}{8t},$$

in which
$h$ = deflection, in feet;
$w$ = weight of rope per foot, in pounds;
$s$ = span, in feet;
$t$ = tension, in pounds.

When driving from the under side, this part of the rope will be tightened and its deflection decreased, while the upper part of the rope becomes slackened and its deflection increased. Under proper conditions, the deflection of the lower rope should be about $\frac{1}{40}$, and that of the upper about $\frac{1}{24}$ of the span. The difference in the tensions of the two parts of the rope is the effective pull of the driving sheaves, enabling power to be transmitted.

Transmission ropes are subject to three stresses: (1) The direct tension, due to the power transmitted, plus the friction and weight of the rope; (2) the bending stress, due to the bending of the rope around the sheaves; (3) the centrifugal tension, due to the centrifugal force in the rapidly running rope.

The following data on stresses in transmission ropes are given by Mr. Wm. Hewitt, of the Trenton Iron Co.: When transmitting power by wire rope, working tension should not exceed the difference between the maximum safe stress and the bending stress. It may be greater, therefore, as the bending stress is less, but to avoid slipping, a certain ratio must exist between the tensions in taut and slack portions of the rope when running, which is determined by the formula $T = Se^{fn\pi}$;

in which $T$ = tension in taut portion of rope;
$S$ = tension in slack portion;
$e$ = base of Naperian system of logarithms = 2.7182818;
$n$ = number of half laps of rope about sheaves or drums at either end of line;
$\pi$ = 3.1416;
$f$ = coefficient of friction depending on kind of filling in grooves of sheaves, or character of material on which rope tracks.

The useful effort of transmitting force is the difference between the tension of the taut and slack portions of the rope, $T - S = S(e^{fn\pi} - 1)$, and to obtain this, the initial tension, or tension when the rope is at rest, must be one-half the sum of the two tensions.

$$T + S = S(e^{fn\pi} + 1) = \frac{e^{fn\pi} + 1}{e^{fn\pi} - 1}(T - S)$$

The following are some of the values of $f$:

| | |
|---|---|
| Dry rope on a grooved iron drum | .120 |
| Wet rope on a grooved iron drum | .085 |
| Greasy rope on a grooved iron drum | .070 |
| Dry rope on wood-filled sheaves | .235 |
| Wet rope on wood-filled sheaves | .170 |
| Greasy rope on wood-filled sheaves | .140 |
| Dry rope on rubber and leather filling | .495 |
| Wet rope on rubber and leather filling | .400 |
| Greasy rope on rubber and leather filling | .205 |

The values of the coefficients corresponding to the foregoing values of $f$, for one up to six half laps of the rope, are given in the accompanying table.

## VALUE OF COEFFICIENTS

| $f=$ | $n=$ Number of Half Laps About Sheaves or Drums at Either End of Line | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| | Values of $e^{fn\pi}$ | | | | | |
| .070 | 1.246 | 1.552 | 1.934 | 2.410 | 3.003 | 3.741 |
| .085 | 1.306 | 1.706 | 2.228 | 2.910 | 3.801 | 4.964 |
| .100 | 1.369 | 1.875 | 2.566 | 3.514 | 4.810 | 6.586 |
| .120 | 1.458 | 2.125 | 3.099 | 4.518 | 6.586 | 9.602 |
| .130 | 1.504 | 2.263 | 3.405 | 5.122 | 7.706 | 11.593 |
| .140 | 1.552 | 2.410 | 3.741 | 5.808 | 9.017 | 13.998 |
| .150 | 1.602 | 2.566 | 4.111 | 6.586 | 10.551 | 16.902 |
| .170 | 1.706 | 2.910 | 4.964 | 8.467 | 14.445 | 24.641 |
| .200 | 1.875 | 3.514 | 6.586 | 12.346 | 23.140 | 43.376 |
| .205 | 1.904 | 3.626 | 6.904 | 13.146 | 25.031 | 47.663 |
| .235 | 2.092 | 4.378 | 9.160 | 19.166 | 40.100 | 83.902 |
| .250 | 2.193 | 4.810 | 10.551 | 23.140 | 50.637 | 111.318 |
| .265 | 2.299 | 5.286 | 12.153 | 27.941 | 64.239 | 147.693 |
| .300 | 2.566 | 6.586 | 16.902 | 43.376 | 111.318 | 285.680 |
| .350 | 3.001 | 9.017 | 27.077 | 81.307 | 244.152 | 733.145 |
| .400 | 3.514 | 12.346 | 43.376 | 152.405 | 535.488 | 1,849.140 |
| .410 | 3.626 | 13.146 | 47.663 | 172.814 | 626.577 | 2,271.775 |
| .450 | 4.111 | 16.902 | 69.487 | 285.680 | 1,174.480 | 4,828.510 |
| .495 | 4.716 | 22.425 | 106.194 | 502.881 | 2,381.400 | |
| .500 | 4.810 | 23.140 | 111.318 | 535.488 | 2,575.940 | |
| | Values of $\dfrac{e^{fn\pi}+1}{e^{fn\pi}-1}$ | | | | | |
| .070 | 9.130 | 4.623 | 3.141 | 2.418 | 1.999 | 1.729 |
| .085 | 7.536 | 3.833 | 2.629 | 2.047 | 1.714 | 1.505 |
| .100 | 6.420 | 3.287 | 2.280 | 1.795 | 1.525 | 1.358 |
| .120 | 5.345 | 2.777 | 1.953 | 1.570 | 1.358 | 1.232 |
| .130 | 4.968 | 2.584 | 1.832 | 1.485 | 1.298 | 1.189 |
| .140 | 4.623 | 2.418 | 1.729 | 1.416 | 1.249 | 1.154 |
| .150 | 4.322 | 2.280 | 1.643 | 1.358 | 1.209 | 1.126 |
| .170 | 3.833 | 2.047 | 1.505 | 1.268 | 1.149 | 1.085 |
| .200 | 3.287 | 1.795 | 1.358 | 1.176 | 1.090 | 1.047 |
| .205 | 3.212 | 1.762 | 1.338 | 1.165 | 1.083 | 1.043 |
| .235 | 2.831 | 1.592 | 1.245 | 1.110 | 1.051 | 1.024 |
| .250 | 2.676 | 1.525 | 1.209 | 1.090 | 1.040 | 1.018 |
| .265 | 2.539 | 1.467 | 1.179 | 1.072 | 1.032 | 1.014 |
| .300 | 2.280 | 1.358 | 1.126 | 1.047 | 1.018 | 1.007 |
| .350 | 2.000 | 1.249 | 1.077 | 1.025 | 1.008 | 1.003 |
| .400 | 1.795 | 1.176 | 1.047 | 1.013 | 1.004 | 1.001 |
| .410 | 1.765 | 1.164 | 1.043 | 1.012 | 1.003 | 1.001 |
| .450 | 1.643 | 1.126 | 1.029 | 1.007 | 1.002 | 1.000 |
| .495 | 1.538 | 1.093 | 1.019 | 1.004 | 1.001 | 1.000 |
| .500 | 1.525 | 1.090 | 1.018 | 1.004 | 1.001 | 1.000 |

For a given diameter of sheave, and a variable diameter of wire, a ratio exists between these diameters corresponding to a maximum working tension. This ratio results, approximately, in a working tension of one-third and a bending stress of two-thirds of the maximum safe tension, which is from one-third to two-fifths of the ultimate stress, and practically determines the minimum diameter of sheave for any rope. The ratio for any size of wire varies slightly, according to the number of wires composing the rope, and in terms of rope diameter is,

|  | *Steel* | *Iron* |
|---|---|---|
| For  7-wire rope..................... | 79.6 | 160.5 |
| For 12-wire rope..................... | 59.3 | 120.0 |
| For 19-wire rope..................... | 47.2 | 95.8 |

from which the following table is derived.

### MINIMUM DIAMETERS OF SHEAVES

| Diameter of Rope Inch | Steel Rope | | | Iron Rope | | |
|---|---|---|---|---|---|---|
| | 7-Wire | 12-Wire | 19-Wire | 7-Wire | 12-Wire | 19-Wire |
| | Diameter of Sheaves, in Inches | | | | | |
| $\frac{1}{4}$ | 20 | 15 | 12 | 40 | 30 | 24 |
| $\frac{5}{16}$ | 25 | 19 | 15 | 50 | 38 | 30 |
| $\frac{3}{8}$ | 30 | 22 | 18 | 60 | 45 | 36 |
| $\frac{7}{16}$ | 35 | 26 | 21 | 70 | 53 | 42 |
| $\frac{1}{2}$ | 40 | 30 | 24 | 80 | 60 | 48 |
| $\frac{9}{16}$ | 45 | 33 | 27 | 90 | 68 | 54 |
| $\frac{5}{8}$ | 50 | 37 | 30 | 100 | 75 | 60 |
| $1\frac{1}{8}$ | 55 | 41 | 32 | 110 | 83 | 66 |
| $\frac{3}{4}$ | 60 | 44 | 35 | 120 | 90 | 72 |
| $\frac{7}{8}$ | 70 | 52 | 41 | 140 | 105 | 84 |
| 1 | 80 | 59 | 47 | 160 | 120 | 96 |

**Sheaves.**—To decrease the bending stresses, the sheaves for wire-rope transmissions are generally of as large diameter as is practicable to give the required speed to the rope. Large sheaves are also advantageous because with them the rope is run at a high velocity allowing of a lower tension, and permitting a rope of smaller diameter to be used than would be possible with smaller sheaves, provided, of course, that the span is of sufficient length to give the necessary weight.

Sheaves are generally made of cast iron when not exceeding 12 ft. in diameter; when larger than this, they are usually built up with wrought-iron arms. Sheaves, upon which the rope is to make but a single half-turn, are made with **V**-shaped grooves in their circumference. The bottom part of the groove is widened to receive the filling, which consists of some substance to give a bed for the rope to run on and protect it from wear, and to increase the friction so that the rope will not slip. This filling is made of blocks of wood, rubber, leather, or other material. Rubber and leather have been used separately, but blocks of rubber separated by pieces of leather have been found to give the best results.

**Power Transmitted.**—The horsepower transmitted is equal to the resistance overcome (the effective pull), in pounds, multiplied by the speed of the rope, in feet per minute, and divided by 33,000 that is (formula from John A. Roebling's Sons Co.),

$$H = \frac{TV}{33,000}$$

in which $H$ = horsepower transmitted;

$T$ = difference in tension between driving and driven sides of rope;

$V$ = speed of rope, in feet per minute.

When applying this formula, $V$ is either given or assumed. $T$ is equal to the weight of the rope suspended between the sheaves multiplied by 3 (for the proportion of deflection stated).

To transmit a given horsepower, the speed of the rope may be increased and the tension (effective pull) correspondingly decreased, and a smaller rope may be used provided other considerations will allow it.

For determining the horsepower that can be transmitted over a given transmission, the following formula is given by the Trenton Iron Co.:

$$H = [cd^2 - .000006\,(W + g' + g'')]s$$

in which $H$ = horsepower that can be transmitted;

 $c$ = constant, depending on material of rope, filling in grooves of sheaves, and number of half laps about sheaves or drums at either end of line;

 $d$ = diameter of rope, in inches;

 $W$ = weight of rope, in pounds;

 $g'$ = weight of terminal sheaves and shafts;

 $g''$ = weight of intermediate sheaves and shafts.

The accompanying table gives the value of $c$ for ropes on different materials.

### TABLE OF CONSTANTS FOR ROPES ON DIFFERENT MATERIALS

| $c$ = for Steel Rope on | Number of Half Laps About Sheaves or Drums at Either End of Line | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| | Value of $c$ | | | | | |
| Iron............... | 5.61 | 8.81 | 10.62 | 11.65 | 12.16 | 12.56 |
| Wood............. | 6.70 | 9.93 | 11.51 | 12.26 | 12.66 | 12.83 |
| Rubber and leather | 9.29 | 11.95 | 12.70 | 12.91 | 12.97 | 13.00 |

The values of $c$ for iron rope are one-half of those given. It is evident from these figures that when more than three laps are made it is immaterial what the surface is on which the rope tracks, as far as frictional adhesion is concerned.

From the foregoing formula, assuming the sheaves to be of equal diameter, and of a size not less than the minimum diameter given in the table, it is possible to find the horsepower that may be transmitted by a steel rope, as is shown in the accompanying table.

### HORSEPOWER THAT MAY BE TRANSMITTED BY A STEEL ROPE MAKING A SINGLE LAP ON WOOD-FILLED SHEAVES

| Diameter of Rope Inch | Velocity of Rope, in Feet per Second | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| | Horsepower That May Be Transmitted | | | | | | | | | |
| $\frac{1}{2}$ | 4 | 8 | 13 | 17 | 21 | 25 | 28 | 32 | 37 | 40 |
| $\frac{9}{16}$ | 7 | 13 | 20 | 26 | 33 | 40 | 44 | 51 | 57 | 62 |
| $\frac{5}{8}$ | 10 | 19 | 28 | 38 | 47 | 56 | 64 | 73 | 80 | 89 |
| $\frac{11}{16}$ | 13 | 26 | 38 | 51 | 63 | 75 | 88 | 99 | 109 | 121 |
| $\frac{3}{4}$ | 17 | 34 | 51 | 67 | 83 | 99 | 115 | 130 | 144 | 159 |
| $\frac{13}{16}$ | 22 | 43 | 65 | 86 | 106 | 128 | 147 | 167 | 184 | 203 |
| $\frac{7}{8}$ | 27 | 53 | 79 | 104 | 130 | 155 | 179 | 203 | 225 | 247 |
| $\frac{15}{16}$ | 32 | 63 | 95 | 126 | 157 | 186 | 217 | 245 | | |
| $\frac{1}{1}$ | 38 | 76 | 103 | 150 | 186 | 223 | | | | |
| $\frac{7}{8}$ | 52 | 104 | 156 | 206 | | | | | | |
| 1 | 68 | 135 | 202 | | | | | | | |

The horsepower that may be transmitted by iron ropes is one-half of the above.

The table gives the maximum amount of power capable of being transmitted under the conditions stated, so that when using wood-lined sheaves, it is well to make some allowance for the stretching of the rope, and to advocate somewhat heavier equipments than the table would give; that is, if it is desired to transmit 20 H. P., for instance, to put in a plant that would transmit 25 to 30 H.P., thus avoiding the necessity of having to take up a comparatively small amount of stretch. On rubber and leather filling, however, the amount of power capable of being transmitted is considerably greater than on wood, so that this filling is generally used; and in this case no allowance need be made for stretch, as such sheaves will likely transmit the power given by the table, under all possible deflections of the rope.

The transmission of more than 250 H. P. is impracticable with filled sheaves, because the tension is so great that the filling will quickly cut out, and the frictional adhesion on a metallic surface is insufficient where the rope makes but a single lap, or a half lap at either end of the line.

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## TRANSMISSION BY HEMP ROPE

There is a growing tendency toward the substitution of hemp and cotton ropes for belting and line shafting as a means of transmitting power in large factories and shops. The advantages claimed for the rope-driving system are: (1) Economy; for a rope system is cheaper to install than either leather belting or shafting. (2) In the rope system, there is less loss of power by slipping. (3) Flexibility; that is, the ease with which the power is transmitted to any distance and in any direction.

In the United States, a single rope is carried round the pulley as many times as is necessary to produce the required power, and the necessary tension is obtained by passing the rope round a tension pulley weighted to give the desired tension. The ropes used in rope transmission are either of hemp, manila, or cotton; manila ropes are mostly used in the United States. They are of three strands, hawser laid, and may be from $\frac{1}{2}$ in. to 2 in. in diameter.

The weight of ordinary manila or cotton rope is about .3 $D^2$ lb. per ft. of length, where $D$ = diameter of rope, in inches. Letting $w$ = weight per foot of length, $w$ = .3 $D^2$. The breaking strength of the rope varies from 7,000 to 12,000 lb. per sq. in. of cross-section. The average value may be taken as 7,000 $D^2$, when $D$ is the diameter of rope.

For a continuous transmission, it has been determined by experiment that the best results are obtained when the tension in the driving side of the rope is about $\frac{1}{35}$ of the breaking strength. That is,

$$T_1 = \text{tension in tight side} = \frac{7,000\ D^2}{35} = 200\ D^2$$

The ropes run in **V**-shaped grooves, and the coefficient of friction is, of course, greater than on a smooth surface. The coefficient for grooves with sides at an angle of 45° may be taken at from .25 to .33.

The horsepower that can be transmitted by a single rope running under favorable conditions is given by the formula

$$H = \frac{v\ D^2}{825}\left(200 - \frac{v^2}{107.2}\right),$$

in which

$H$ = horsepower transmitted;
$D$ = diameter of rope, in inches;
$v$ = velocity of rope, in feet per second.

The maximum power is obtained at a speed of about 84 ft. per sec. For higher velocities, the centrifugal force becomes so great that the power is decreased, and when the speed reaches 145 ft. per sec. the centrifugal force just balances the tension, so that no power at all is transmitted. Consequently, a rope should not run faster than about 5,000 ft. per min., and it is preferable on the score of durability to limit the velocity to 3,500 ft. per min.

EXAMPLE.—A rope flywheel is 26 ft. in diameter, and makes 55 rev. per min. The wheel is grooved for 35 turns of $1\frac{1}{2}$ in. rope. What horsepower may be transmitted?

SOLUTION.—Velocity, in feet per second is

$$v = \frac{26 \times \pi \times 55}{60} = \frac{4,492}{60} = 74.9 \text{ ft.}$$

Applying the formula $H = \dfrac{v \, D^2}{825}\left(200 - \dfrac{v^2}{107.2}\right)$, the horsepower transmitted by one rope or turn is

$$\frac{74.9 \times (1\frac{1}{2})^2}{825} \times \left[200 - \frac{(74.9)^2}{107.2}\right] = 30.16.$$

Then, $30.16 \times 35 = 1,055.6$ H. P. transmitted by the 35 ropes.

EXAMPLE.—How many times should a 1-in. rope be wrapped around a grooved wheel in order to transmit 200 H. P., the speed being 3,500 ft. per min.?

SOLUTION.—3,500 ft. per min. = $3,500 \div 60 = 58\frac{1}{3}$ ft. per sec. Applying the formula, the horsepower transmitted with one turn is,

$$H = \frac{58\frac{1}{3} \times 1^2}{825} \times \left[200 - \frac{(58\frac{1}{3})^2}{107.2}\right] = 11.9$$

Hence, $200 \div 11.9 = 16.8$, say 17 turns.

Rope pulleys differ from belt pulleys only in their rims. The inclination of the sides of the grooves may vary from 30° to 60°. The more acute the angle, the greater the coefficient and, consequently, the wear on the rope.

The long radius $R$ is determined by drawing a line through the center of the rope at an angle of $22\frac{1}{2}°$ with the horizontal, and producing it until it intersects a line drawn through the tops of the ribs dividing the grooves, then, with this point of intersection as a center, drawing the curve forming the side of the groove tangent to the circumference of the rope. The advantage claimed for this groove is that the rope will turn more freely in it, thus presenting new sets of fibers to the sides of the grooves and increasing the life of the rope.

The diameter of a rope pulley should be at least 30 times the diameter of the rope. Good results are obtained when the diameters of pulleys and idlers on the driving side are 40 times, and those on the driven side 30 times, the rope diameter. Idlers used simply to support a long span may have diameters as small as 18 rope diameters, without injuring the rope.

When possible, the lower side of the rope should be the driving side, for in that case the rope embraces a greater portion of the circumference of the pulley, and increases the arc of contact.

When the continuous system of rope transmission is used, the tension pulley should act on as large an amount of rope as possible. It is good practice to use a tension pulley and carriage for every 1,200 ft. of rope, and have at least 10% of the rope subjected directly to the tension.

Aside from the grooved rim, rope pulleys are constructed the same as other pulleys. They may be cast solid, in halves, or in sections. The pulley grooves must be turned to exactly the same diameter; otherwise, the rope will be severely strained.

## HORSEPOWER OF MANILA ROPES

*(Link-Belt Engineering Co.)*

| Diam. of Rope Inches | Weight per Foot Pounds | Breaking Strain Pounds | Working Strain Pounds | 1,000 Ft. per Min. | | 2,000 Ft. per Min. | | 3,000 Ft. per Min. | | 4,000 Ft. per Min. | | 5,000 Ft. per Min. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | H.P. | Tens. Wt. | H.P. | Tens. Wt. | H.P. | Tens. Wt. | H.P. | Tens. Wt. | H.P. | Tens. Wt. |
| ⅝ | .15 | 4,000 | 121 | 2¼ | 90 | 4½ | 90 | 6¼ | 80 | 7½ | 80 | 8½ | 70 |
| ¾ | .18 | 5,000 | 151 | 2¾ | 110 | 5½ | 110 | 7¼ | 100 | 9½ | 100 | 10½ | 90 |
| ⅞ | .27 | 7,500 | 227 | 4¼ | 170 | 8½ | 170 | 11¾ | 160 | 14½ | 150 | 16 | 130 |
| 1 | .33 | 9,000 | 272 | 5 | 200 | 10 | 200 | 14 | 180 | 17½ | 170 | 19 | 150 |
| 1⅛ | .45 | 12,250 | 371 | 7 | 280 | 13½ | 270 | 19 | 250 | 23½ | 230 | 26 | 210 |
| 1¼ | .50 | 14,000 | 424 | 8 | 320 | 15½ | 310 | 22 | 290 | 27 | 270 | 29½ | 240 |
| 1⅜ | .65 | 18,062 | 547 | 10¼ | 410 | 20 | 400 | 28¼ | 370 | 34¼ | 350 | 38½ | 310 |
| 1½ | .73 | 20,250 | 613 | 11½ | 460 | 22 | 440 | 31½ | 420 | 39 | 390 | 43½ | 350 |
| 1⅝ | .82 | 25,000 | 760 | 14½ | 570 | 27½ | 550 | 39½ | 520 | 49 | 490 | 55½ | 448 |
| 1¾ | 1.08 | 30,250 | 916 | 17 | 680 | 33½ | 660 | 47¼ | 630 | 58½ | 580 | 64½ | 520 |
| 2 | 1.27 | 36,000 | 1,000 | 20½ | 810 | 40 | 790 | 56½ | 740 | 69½ | 670 | 77½ | 620 |

## LINE SHAFTING

*Shafting* is usually made cylindrically true, either by a special rolling process, when it is known as *cold-rolled shafting*, or it is turned up in a machine called a *lathe*. In the latter case, it is called *bright shafting*. What is known as *black shafting* is simply bar iron rolled by the ordinary process and turned where it receives the couplings, pulleys, bearings, etc.

Bright turned shafting varies in diameter by $\frac{1}{4}$ in. up to about $3\frac{1}{2}$ in.; above this the diameter of the shafting varies by $\frac{1}{2}$ in. The actual diameter of a bright shaft is $\frac{1}{16}$ in. less than the commercial diameter, it being designated from the diameter of the ordinary round bar iron from which it is turned. Thus, a length of what is called 3-in. bright shafting is only $2\frac{15}{16}$ in. in diameter. Cold-rolled shafting is designated by its commercial diameter; thus, a length of what is called 3-in. shafting is 3 in. in diameter.

### CONSTANTS FOR LINE SHAFTING

| Material of Shaft | No Pulleys Between Bearings | With Pulleys Between Bearings |
|---|---|---|
| Steel or cold-rolled iron. | 65 | 85 |
| Wrought iron............ | 70 | 95 |
| Cast iron.............. | 90 | 120 |

Cold-rolled iron is considerably stronger than ordinary turned wrought iron, the increased strength being due to the process of rolling, which seems to compress the metal and so make it denser—not merely skin deep, but practically throughout the whole diameter.

Let

$D$ = diameter of shaft;
$R$ = revolutions per minute;
$H$ = horsepower transmitted;
$C$ = constant given in table.

In the above table the bearings are supposed to be spaced so as to relieve the shaft of excessive bending; also, in the third vertical column, an average number and weight of pulleys and power given off is assumed.

When determining these constants allowance was made to insure the stiffness as well as strength of the shaft.

$$H = \frac{D^3R}{C} \qquad D = \sqrt[3]{\frac{CH}{R}} \qquad R = \frac{CH}{D^3}$$

Shafts are subject to forces that produce stresses of two kinds—transverse and torsional. When the machines to be driven are below the shaft, there is a transverse stress on the shaft, due to the weight of the shaft itself, of the pulley and the tension of the belt. Sometimes, the power is taken off horizontally on one side, in which case the tension of the belt produces a horizontal transverse stress, while the weight of the pulley acts with the weight

### MAXIMUM DISTANCE BETWEEN BEARINGS

| Diameter of Shaft Inches | Distance Between Bearings Feet | |
|---|---|---|
| | Wrought-Iron Shaft | Steel Shaft |
| 2 | 11 | 11.50 |
| 3 | 13 | 13.75 |
| 4 | 15 | 15.75 |
| 5 | 17 | 18.25 |
| 6 | 19 | 20.00 |
| 7 | 21 | 22.25 |
| 8 | 23 | 24.00 |
| 9 | 25 | 26.00 |

of the shaft to produce a vertical transverse stress. When the machinery to be driven is placed on the floor above the shaft, the tension of the belt produces a transverse stress in an opposite direction to that due to the weight of the shaft and pulley.

The torsional strength of shafts, or their resistance to breaking by twisting, is proportional to the cube of their diameter. Their stiffness or resistance to bending is proportional to the fourth power of their diameters, and inversely

proportional to the cube of the lengths of their spans. No simple general formula can be given that will safely apply to engine and other shafting that is subjected to the bending stresses produced by overhung cranks, the weight of heavy flywheels, the pull of large belts, or to severe shocks produced by the intermittent action of the power or load. The calculations for such shafts should always be based on the special conditions involved.

In the preceding table are given the maximum distances between the bearings of some continuous shafts that are used for the transmission of power. Pulleys from which considerable power is to be taken should always be placed as close to a bearing as possible.

The diameters of the different lengths of shafts composing a line of shafting may be proportional to the quantity of power delivered by each respective length. In this connection, the positions of the various pulleys depend on the distance between the pulley and the bearing, and on the amount of power given off by the pulleys. Suppose, for example, that a piece of shafting delivers a certain amount of power; then, the shaft will deflect or bend less if the pulley transmitting that power is placed close to a hanger or bearing, than if it is placed midway between the two hangers or bearings. It is impossible to give any rule for the proper distance of bearings that could be used universally, as in some cases the requirements demand that the bearings be nearer together than in others. If the work done by a line of shafting is distributed quite equally along its entire length, and the power can be applied near the middle, the strength of the shaft need be only one-half as great as when the power is applied at one end of the shaft.

### HORSEPOWER SHAFTING WILL TRANSMIT

| Diameter of Shaft Inches | Revolutions per Minute | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 100 | 125 | 150 | 175 | 200 | 225 | 250 | 300 | 350 | 400 |
| | Horsepower Transmitted | | | | | | | | | |
| 1 1/16 | 1.2 | 1.4 | 1.7 | 2.1 | 2.4 | 2.6 | 3.1 | 3.6 | 4.3 | 5.0 |
| 1 3/16 | 2.4 | 3.1 | 3.7 | 4.3 | 4.9 | 5.5 | 6.1 | 7.3 | 8.5 | 9.7 |
| 1 7/16 | 4.3 | 5.3 | 6.4 | 7.4 | 8.5 | 9.5 | 10.5 | 12.7 | 14.8 | 16.9 |
| 1 11/16 | 6.7 | 8.4 | 10.1 | 11.7 | 13.4 | 15.1 | 16.7 | 20.1 | 23.4 | 26.8 |
| 1 15/16 | 10.0 | 12.5 | 15.0 | 17.5 | 20.0 | 22.5 | 25.0 | 30.0 | 35.0 | 40.0 |
| 2 3/16 | 14.3 | 17.8 | 21.4 | 24.9 | 28.5 | 32.1 | 35.6 | 42.7 | 49.8 | 57.0 |
| 2 7/16 | 19.5 | 24.4 | 29.3 | 34.1 | 39.0 | 44.1 | 48.7 | 58.5 | 68.2 | 78.0 |
| 2 11/16 | 26.0 | 32.5 | 39.0 | 43.5 | 52.0 | 58.5 | 65.0 | 78.0 | 87.0 | 104.0 |
| 2 15/16 | 33.8 | 42.2 | 50.6 | 59.1 | 67.5 | 75.9 | 84.4 | 101.3 | 118.2 | 135.0 |
| 3 3/16 | 43.0 | 53.6 | 64.4 | 75.1 | 85.8 | 95.6 | 107.3 | 128.7 | 150.3 | 171.6 |
| 3 7/16 | 53.6 | 67.0 | 79.4 | 93.8 | 107.2 | 120.1 | 134.0 | 158.8 | 187.6 | 214.4 |
| 3 11/16 | 65.9 | 82.4 | 97.9 | 115.4 | 121.8 | 148.3 | 164.8 | 195.7 | 230.7 | 243.6 |
| 3 15/16 | 80.0 | 100.0 | 120.0 | 140.0 | 160.0 | 180.0 | 200.0 | 240.0 | 280.0 | 320.0 |
| 4 7/16 | 113.9 | 142.4 | 170.8 | 199.3 | 227.8 | 256.2 | 284.7 | 341.7 | 398.6 | 455.6 |
| 4 15/16 | 156.3 | 195.3 | 234.4 | 273.4 | 312.5 | 351.5 | 390.6 | 468.7 | 546.8 | 625.0 |

### BELT PULLEYS

**Solid and Split Pulleys.**—Besides being used with ropes or chains for the hoisting of loads, pulleys are extensively employed with belts for transmitting power. Belt pulleys may be divided into two classes, namely, solid pulleys and split pulleys. A *solid pulley* is one in which the arms, hub, and rim are cast in one solid piece. A *split pulley* is one that is cast in halves that are afterwards bolted together; the latter style of pulley is more readily placed on or removed from a shaft than is the solid pulley. Pulleys are generally cast in halves or parts when they are more than 6 ft. in diameter. This is done on account of the shrinkage strains in large pulley castings, which render the pulleys liable to crack as a result of unequal cooling of the metal.

**Wooden Pulleys.**—Although most belt pulleys are made of cast iron, wrought iron, and steel, wooden pulleys have come into extensive use. These are built of segments of wood, usually maple, securely glued together. It is possible to procure wooden split pulleys that are fitted with removable bushings, thus allowing the same pulley to be adapted readily to shafting of different diameters. Wooden pulleys are somewhat lighter than cast-iron pulleys for the same service.

**Driving and Driven Pulleys.**—The pulley that imparts motion to the belt is called the *driving pulley*, or the *driver*, and the one that receives motion from the belt is called the *driven pulley*, or simply the *driven*. When two pulleys are connected by a belt, the speeds at which the pulleys run are inversely proportional to their diameters. Thus, if two pulleys have diameters of 12 in. and 24 in., the speed of the smaller is to the speed of the larger as 24 to 12, or as 2 to 1. The speed of a pulley or of a shaft is usually stated in revolutions per minute, abbreviated rev. per min. or R. P. M.

**Diameter and Speed of Driver.**—It often becomes necessary to calculate the size or the speed of a pulley that drives or is being driven by a machine.

Let $D$ = diameter of driving pulley, in inches;
$d$ = diameter of driven pulley, in inches;
$N$ = number of rev. per min. of driving pulley;
$n$ = number of rev. per min. of driven pulley.

Then, if the diameter of the driven and the required speeds of both pulleys are given, the diameter of the driver may be found by the formula

$$D = \frac{dn}{N} \quad (1)$$

If the speed of the driver is to be found, it is necessary to use the formula

$$N = \frac{dn}{D} \quad (2)$$

**Example.**—A 12-in. pulley on a certain machine is to run at 160 rev. per min. and is to be driven by belt from a pulley on the shaft of an engine that make 96 rev. per min. What must be the diameter of the pulley on the engine shaft?

**Solution.**—Substituting in formula 1,

$$D = \frac{12 \times 160}{96} = 20 \text{ in.}$$

**Diameter and Speed of Driven.**—If the diameter of the driving pulley and the desired speeds of both pulleys are known, the required diameter of the driven pulley may be found by the formula

$$d = \frac{DN}{n} \quad (1)$$

If the speed of the driven pulley is to be found, it is necessary to use the formula

$$n = \frac{DN}{d} \quad (2)$$

**Example 1.**—A 30-in. pulley on a line shaft running at 120 rev. per min. is to drive a pulley on a machine at 300 rev. per min. What must be the diameter of the pulley on the machine?

**Solution.**—Substituting in formula 1,

$$d = \frac{30 \times 120}{300} = 12 \text{ in.}$$

**Example 2.**—A driving pulley 48 in. in diameter makes 175 rev. per min. and is connected by belt to a driven pulley 14 in. in diameter. What is the speed of the driven pulley?

**Solution.**—Substituting in formula 2,

$$n = \frac{48 \times 175}{14} = 600 \text{ rev. per min.}$$

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# BELTING

A *belt* is a flexible band by which motion is transmitted from one pulley to another. The materials most commonly used for belts are leather, cotton, and rubber, although thin, flat bands of steel are coming into use. Leather belts are usually made either single or double. A *single belt* is one composed of a single thickness of leather, and a *double belt* is one composed of two thicknesses

of leather cemented and riveted together throughout the whole length of the belt. Still heavier belts, consisting of three or four thicknesses of leather, and known as *triple* or *quadruple belts*, are sometimes made for heavy drives. Cotton belts are made up of a number of layers, or plies, sewed together and treated with a water-proofing substance. They are termed *two-ply, three-ply*, etc., according to the number of plies they contain. Four-ply cotton belting is usually considered equal to single leather belting. Rubber belts are particularly adapted for use in damp or wet places. They withstand changes of temperature without injury, are durable, and are said to be less liable to slip than are leather belts.

**Sag of Belts.**—The distance between pulley centers depends on the size of the pulleys and of the belt; it should be great enough so that the belt will run with a slight sag and a gently undulating motion, but not great enough to cause excessive sag and an unsteady flapping motion of the belt. In general, the centers of small pulleys carrying light narrow belts should be about 15 ft. apart and the belt sag $1\frac{1}{2}$ to 2 in.; for large pulleys and heavy belts the distance should be 20 to 30 ft. and the sag $2\frac{1}{2}$ to 5 in. Loose-running belts will last much longer than tight ones, and will be less likely to cause heating and wear of bearings.

**Speed of Belts.**—The higher the speed of a belt, the less may be its width to transmit a given horsepower; consequently, a belt should be run at as high a speed as conditions will permit. The greatest allowable speed for a belt joined by lacing is about 3,500 ft., per min. for ordinary single and double leather belts. For belts joined by cementing, when the joint has about the same strength as the solid belt, the velocity may be as high as 5,000 ft. per min. Higher speeds than these have been used, but there is little to be gained by exceeding about 4,800 ft. per min. In choosing a proper belt speed, due regard must be paid to commercial conditions. Although a high speed of the belt means a narrow and cheaper belt, the increased cost of the larger pulleys that may be required may offset the gain due to the high speed of the belt, at least so far as the first cost is concerned. The speed of a belt, in feet per minute, may be found by multiplying the number of revolutions per minute of the pulley by 3.1416 times the diameter of the pulley, in inches, and dividing the product by 12.

**Horsepower of Belts.**—The pull on a belt is greatest on the tight, or driving, side, and least on the slack side. The difference between the tensions, or pulls, in these two sides is called the *effective pull*. The effective pull that may be allowed per inch of width for single leather belts with different arcs of contact is given in the accompanying table. The *arc of contact* is the portion of the circumference of the smaller pulley that is covered by the belt. The horsepower that can be transmitted by a single leather belt may be found by the formula

$$H = \frac{CWV}{33,000}. \quad (1)$$

in which
$H$ = horsepower of belt;
$C$ = effective pull, taken from table;
$W$ = width of belt, in inches;
$V$ = speed of belt, in feet per minute.

If it is desired to find the width of single belt required to transmit a given horsepower, the formula becomes

$$W = \frac{33,000H}{CV} \quad (2)$$

**EXAMPLE 1.**—What horsepower can be transmitted by a single leather belt 4 in. wide running at a speed of 2,500 ft. per min., if the belt covers one-third of the circumference of the small pulley?

**SOLUTION.**—The fraction of the circumference covered by the belt is $\frac{1}{3} = .333$. From the table, the allowable effective pull corresponding to this value is 28.8. Substituting in formula 1,

$$H = \frac{28.8 \times 4 \times 2,500}{33,000} = 8.7 \text{ H. P.}$$

**EXAMPLE 2.**—A single leather belt is to run at a speed of 3,000 ft., per min. and is to transmit 18 H. P. Find the width of the belt, if the arc of contact is 150°.

**SOLUTION.**—The effective pull corresponding to an arc of contact of 150°, from the table, is 33.8. Substituting in formula 2,

$$W = \frac{33,000 \times 18}{33.8 \times 3,000} = 5.9 \text{ in.}$$

A 6-in. belt would be used.

## ALLOWABLE EFFECTIVE PULL

| Arc of Contact | | Allowable Effective Pull |
| --- | --- | --- |
| Degrees | Fraction of Circumference | Pounds per Inch of Width |
| 90 | .250 | 23.0 |
| 112½ | .312 | 27.4 |
| 120 | .333 | 28.8 |
| 135 | .375 | 31.3 |
| 150 | .417 | 33.8 |
| 157½ | .437 | 34.9 |
| 180 or over | .500 or over | 38.1 |

The horsepower of a double leather belt may be taken as 1⅔ times that of a single leather belt of the same width running under the same conditions. Accordingly, the width of a double leather belt required for any service is only $\frac{7}{10}$ that of a single belt for the same service.

**Lacing of Belts.**—A very satisfactory way of lacing belts less than 3 in. wide is shown in Fig. 1, in which *A* is the outside of the belt and *B* is the side that runs against the face of the pulley. The ends of the belt are first cut square, and then holes are punched in the ends, in corresponding positions opposite one another. The number of holes in each row should always be odd, in the style of lacing shown, using three holes in belts up to 2 in. wide and five holes in belts between 2 and 3 in. wide. The lacing is first drawn through one of the middle holes from the under side, or pulley side, at *1*. Then it is drawn across the upper side and is passed down through *2*, across under the belt to *3*, up through *3*, across and down again through *2*, back under the belt and up through *3* again,



FIG. 1

then across and down through *4* and finally up through *5*, where a barb is cut in the edge of the lacing to prevent it from pulling out. This completes the lacing of one half. The other end of the lacing is then carried through the holes in the other half, in the same order.
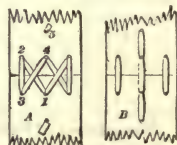
For belts wider than 3 in., the lacing shown in Fig. 2 may be used. In this case, there are two rows of holes in each end of the belt to be joined. The row nearer the end of the belt should have one more hole than the row farther away. For belts up to 4½ in. wide use three holes in the first row and two holes in the second row. For belts up to 6 in. wide, use four and three holes, respectively. For wider belts, make the total number of holes in both rows
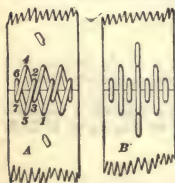


FIG. 2

either one or two more than the number of inches of width of the belt, the object being to get an odd total number of holes. For example, a 10-in. belt should have eleven holes, and a 13-in. belt should have fifteen holes. The outside holes of the first row should not be nearer the side edges of the belt than ¾ in. and not nearer the joint edge than ⅞ in. The second row should be at least 1½ in. from the joint edge. In Fig. 2, *A* is the outside face and *B* the face next the pulley. The lacing is first drawn up through *1* from the pulley side, and then is carried through *2*, *3*, *4*, *5*, *6*, *7*, *6*, *7*, *4*, *5*, *2*, *3*, *8*, and out at *9* to be fastened. The other end of the lacing is used on the other half of the belt in the same way.

**Care and Use of Belts.**—It is a much disputed question as to which side of a leather belt should be run next to the pulley. The more common practice, it is believed, is to run the belt with the hair, or grain, side nearest the pulley. This side is harder and more liable to crack than the flesh side; by running it on the inside, the tendency is to cramp or compress it as it passes over the pulley, while, if it ran on the outside, the tendency would be for it to stretch

and crack. The flesh side is the tougher side, but for the reason just given the life of the belt will be longer if the wear comes on the grain side. The lower side of the belt should be the driving side, the slack side running from the top of the driving pulley. The sag of the belt will then cause it to encompass a greater length of the circumference. Long belts, running in any other direction than vertical, work better than short ones, as their weight holds them more firmly to their work.

It is bad practice to use rosin to prevent slipping. Rosin gums the belt, causes it to crack, and prevents slipping for only a short time. If a belt in good condition persists in slipping, a wider belt should be used. Sometimes, larger pulleys on the driving and driven shafts are of advantage, as they increase the belt speed and reduce the stress on the belt. Belts may be kept soft and pliable by being oiled once a month with castor oil or neat's-foot oil.

When rubber belts are used, animal oils or animal grease should never be used on them. If the belt should slip, it may be lightly moistened on the side nearest the pulley with boiled linseed oil.

**Flapping of Belts.**—One of the most annoying troubles experienced with belting of all kinds is the violent flapping of the slack side. Flapping may be due to one or both of the pulleys running out of true, causing the belt to be alternately stretched and released. This will usually cause a belt to flap when running at a high speed. If the belt is rather slack, tightening it may lessen or stop the flapping. Another frequent cause of the flapping of a belt is the want of alinement of the pulleys. To remedy this, the pulleys should be brought in line; should this fail, the belt should be tightened if it is rather loose. If no improvement is noticed and it is not possible to turn the pulleys, the belt speed should be reduced a little, either by the substitution of smaller pulleys or by changing the speed of the driving shaft, according to circumstances.

With belts running at speeds above 4,000 ft. per min., flapping may occur when the pulleys are perfectly true and in line with each other, even when the belt has the proper tension. This is believed to be due to air becoming entrapped between the face of the pulley and the belt; in this case the trouble may be prevented by perforating the belt with a series of small holes. Perforated belts may now be bought in the market.

Another cause of flapping is that the distance between the pulleys may be too great. In general, the distance between the pulleys should not exceed 15 ft. for belts up to 4 in. wide; 20 ft. for belts from 4 to 12 in.; 25 ft., from 12 to 18 in.; and 30 ft., for wider belts. A belt that is not joined square will flap, especially when running at a rather high speed.

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# SPECIFIC GRAVITY, WEIGHT, AND OTHER PROPERTIES OF MATERIALS

## DEFINITIONS

The specific gravity of a body is the ratio of its weight to the weight of an equal bulk of pure water, at a standard temperature (62° F. = 16.670° C.). Some experimenters have used 60° F. as the standard temperature, others 32° and still others, 39.1°. To reduce a specific gravity, referred to water at 39.1° F., to the standard of water at 62° F., multiply by 1.00112.

**Rule I.**—*Given the specific gravity referred to water at 62° F., multiply by 62.355 to find the weight of 1 cu. ft. of the substance.*

**Rule II.**—*Given weight per cubic foot, to find specific gravity, multiply by .016037.*

**Rule III.**—*Given specific gravity, to find the weight per cubic inch, multiply by .036085.*

**To Find the Specific Gravity of a Solid Heavier Than Water.**—Weigh the body both in air and in water, and divide the weight in air by the difference of the weights in air and water.

EXAMPLE.—If a piece of coal weighs 480 gr. in air and weighs 82 gr. in water, what is its specific gravity?

SOLUTION.—As 480 − 82 = 398, or loss of weight in water. Then 480 ÷ 398 = 1.206, the specific gravity of coal.

**To Find the Specific Gravity of a Solid Lighter Than Water.**—Attach to it another body heavy enough to sink it; weigh severally the compound mass and the heavier body in water, divide the weight of the body in air by the weight of the body in air plus the weight of the sinker in water minus the combined weight of the sinker and body in water.

**To Find the Specific Gravity of a Fluid.**—Weigh both in and out of the fluid a solid (insoluble) of known specific gravity, and divide the product of the weight lost in the fluid and the specific gravity of the solid by the weight of the solid.

The weight of 1 cu. ft. of water at a temperature of 62° is about 1,000 oz. avoir., and the specific gravity of a body, multiplied by 1,000, shows the weight of 1 cu. ft. of that body in ounces avoirdupois. Then, if the magnitude of the body is known, its weight can be computed; or, if its weight is known, its magnitude can be calculated, provided its specific gravity is known; or, of the magnitude, weight, and specific gravity, any two being known, the third may be found.

**To Find the Weight of a Body, in Ounces, From Its Magnitude.**—Multiply the magnitude of the body in cu. ft., by the specific gravity of the substance multiplied by 1,000.

**To Find the Magnitude of a Body, in Cubic Feet, From Its Weight.**—Divide the weight of the body in ounces by 1,000 times the specific gravity of the body.

NOTE.—The specific gravity of any substance is equal to its weight in grams per cubic centimeter.

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# SPECIFIC GRAVITY OF COMMON SUBSTANCES
## SPECIFIC GRAVITY OF MINERALS AND EARTHS

| Material | Specific Gravity | Material | Specific Gravity |
|---|---|---|---|
| Alabaster, gypseous ... | 2.31 | Lime, quick........... | 1.50 |
| Alabaster, calcareous .. | 2.76 | Limestone............ | 2.70 |
| Alum................ | 1.72 | Magnetic iron ore..... | 5.09 |
| Amethyst............ | 3.92 | Malachite............ | 4.01 |
| Asbestos, starry....... | 3.07 | Marble, African....... | 2.71 |
| Asphaltum........... | 1.40 | Marble, common...... | 2.67 |
| Barytes (heavy spar).. | 3.05 to 3.38 | Marble, Egyptian..... | 2.67 |
| Bluestone............ | 2.45 to 3.00 | Marble, Parian....... | 2.84 |
| Borax............... | 1.71 | Marble, Italian, white. | 2.71 |
| Bort (black diamond).. | 3.50 | Marl................ | 1.60 to 2.34 |
| Brick............... | 2.00 | Mica................ | 2.80 |
| Chalk............... | 2.78 | Niter................ | 1.90 |
| Clay................ | 1.90 | Opal................ | 2.09 |
| Coral, red........... | 2.70 | Phosphorus.......... | 1.77 |
| Diamond............ | 3.50 to 3.53 | Plaster of Paris....... | 1.87 to 2.47 |
| Earth, loose......... | 1.36 | Potash.............. | 2.10 |
| Emerald............. | 3.95 | Quartz.............. | 2.66 |
| Emerald, aquamarine.. | 2.73 | Rotten stone......... | 1.98 |
| Emery.............. | 4.00 | Ruby............... | 3.95 |
| Feldspar............ | 2.60 | Salt, common........ | 2.13 |
| Flint, black......... | 2.58 | Saltpeter............ | 2.09 |
| Flint, white......... | 2.59 | Sand................ | 2.65 |
| Garnet............. | 3.60 to 4.20 | Sandstone........... | 2.08 to 2.52 |
| Glass, bottle........ | 2.73 | Sapphire............ | 3.98 |
| Glass, flint......... | 3.50 | Serpentine.......... | 2.81 |
| Glass, green........ | 2.64 | Shale............... | 2.60 |
| Glass, white........ | 2.90 | Slate............... | 2.80 |
| Granite, Patapsco..... | 2.64 | Soil, common........ | 1.98 |
| Granite, Quincy...... | 2.65 | Specular iron ore..... | 5.25 |
| Granite, Scotch...... | 2.62 | Stone, mill.......... | 2.48 |
| Granite, Susquehanna. | 2.70 | Sulphur, native....... | 2.03 |
| Graphite............ | 2.20 | Talc................ | 2.90 |
| Grindstone.......... | 2.14 | Topaz............... | 3.50 |
| Gypsum, opaque...... | 2.17 | Tourmaline.......... | 2.07 |
| Jasper.............. | 2.65 | Turquoise........... | 2.84 |
| Lapis lazuli......... | 2.96 | Zircon.............. | 4.50 |

## SPECIFIC GRAVITY OF METALS

| Metal | Specific Gravity | Metal | Specific Gravity |
|---|---|---|---|
| Aluminum, wrought | 2.660 | Magnesium | 1.740 |
| Antimony | 6.712 | Manganese | 8.000 |
| Bismuth | 9.746 | Mercury, solid, at −40° F | 15.632 |
| Brass, common | 8.500 | Mercury, liquid, at 32° F | 13.619 |
| Brass, wire | 8.548 | Mercury, liquid, at 60° F | 13.580 |
| Bronze, bell-metal | 8.060 | Mercury, liquid, at 212° F | 13.375 |
| Bronze, gun-metal | 8.500 | Molybdenum | 8.620 |
| Calcium | 1.580 | Nickel, cast | 8.280 |
| Chromium | 6.000 | Platinum, hammered | 20.337 |
| Cobalt | 8.500 | Platinum, rolled | 21.042 |
| Copper, cast | 8.700 | Platinum, wire | 22.009 |
| Copper, pure | 8.788 | Potassium | .860 |
| Copper, wire and rolled | 8.878 | Silver, pure | 10.474 |
| Gold, hammered | 19.361 | Sodium | .970 |
| Gold, pure, cast | 19.258 | Steel, cast | 7.919 |
| Gold, 22 carat fine | 17.486 | Steel, common, soft | 7.833 |
| Iridium | 18.680 | Steel, hard and tempered | 7.818 |
| Iron, cast | 7.207 | Tin, Bohemian | 7.312 |
| Iron, hammered | 7.780 | Tin, English | 7.201 |
| Iron, pure | 7.768 | Titanium | 5.300 |
| Iron, wrought and rolled | 7.780 | Tungsten | 17.600 |
| Lead, hammered | 11.388 | Zinc, cast | 6.860 |
| Lead, pure | 11.330 | Zinc, rolled | 7.101 |

## SPECIFIC GRAVITY OF LIQUIDS

| Liquid | Specific Gravity | Liquid | Specific Gravity |
|---|---|---|---|
| Acid, acetic | 1.062 | Cider | 1.018 |
| Acid, hydrochloric | 1.200 | Egg | 1.090 |
| Acid, nitric | 1.217 | Ether, sulphuric | .739 |
| Acid, phosphoric | 1.558 | Honey | 1.450 |
| Acid, sulphuric | 1.841 | Human blood | 1.054 |
| Alcohol, commercial | .833 | Milk | 1.032 |
| Alcohol, proof-spirit | .925 | Oil, linseed | .940 |
| Alcohol, pure | .792 | Oil, olive | .915 |
| Alcohol, wood | .800 | Oil, turpentine | .870 |
| Beer, lager | 1.034 | Oil, whale | .932 |
| Bordeaux, wine | .992 | Petroleum | .878 |
| Bromine | 2.996 | Water, distilled | 1.000 |
| Champagne | .997 | Water, sea | 1.030 |
| Chloroform | 1.530 | Wine | .992 |

## SPECIFIC GRAVITY OF GASES AND VAPORS

| Gas or Vapor | Specific Gravity | Gas or Vapor | Specific Gravity |
|---|---|---|---|
| Air, 32° F. and 1 atmos. | 1.000 | Methane | .553 |
| Alcohol, vapor | 1.589 | Nitrogen | .917 |
| Ammonia | .589 | Olefiant gas | .967 |
| Bromine vapor | 4.540 | Oxygen | 1.106 |
| Carbon dioxide | 1.520 | Smoke, bituminous coal | .102 |
| Carbon monoxide | .967 | Smoke, wood | .900 |
| Chlorine | 2.450 | Steam at 212° F | .488 |
| Chloroform, vapor | 5.300 | Sulphureted hydrogen | 1.1912 |
| Hydrogen | .069 | Turpentine vapor | 4.700 |
| Mercury vapor (ideal) | 6.974 | Water vapor | .623 |

## SPECIFIC GRAVITY OF DRY WOODS

| Wood | Specific Gravity | Wood | Specific Gravity |
|---|---|---|---|
| Alder | .80 | Hazel | .60 |
| Apple | .89 | Lemon | .70 |
| Ash | .86 | Lignum vitae | 1.33 |
| Bay | .82 | Linden | .60 |
| Beech | .85 | Logwood | .91 |
| Box, French | .96 | Mahogany, Honduras | .56 |
| Box, Dutch | 1.33 | Maple | .79 |
| Box, Brazilian, red | 1.03 | Mulberry | .90 |
| Cedar, wild | .60 | Oak | .95 |
| Cedar, Palestine | .61 | Orange | .71 |
| Cedar, American | .56 | Pear | .66 |
| Cherry | .67 | Pine, southern | .72 |
| Cork | .25 | Pine, white | .40 |
| Ebony, American | 1.22 | Poplar | .38 |
| Elder | .70 | Poplar, white Spanish | .53 |
| Elm | .56 | Sassafras | .48 |
| Filbert | .60 | Spruce | .50 |
| Fir, male | .55 | Spruce, old | .46 |
| Fir, female | .50 | Walnut | .61 |

## SPECIFIC GRAVITY OF MISCELLANEOUS SUBSTANCES

| Substance | Specific Gravity | Substance | Specific Gravity |
|---|---|---|---|
| Amber | 1.085 | India rubber | .935 |
| Air | .0012 | Ivory | 1.822 |
| Beeswax | .965 | Lard | .947 |
| Bone | 1.8 to 2.0 | Pearl, oriental | 2.650 |
| Butter | .942 | Spermaceti | .943 |
| Cotton | 1.950 | Sugar | 1.605 |
| Fat | .923 | Tallow, sheep and ox | .923 |
| Gunpowder, loose | .900 | Tallow, calf | .934 |
| Gunpowder, shaken | 1.000 | Tar | 1.000 |
| Gum Arabic | 1.452 | Wool | 1.610 |

# AVERAGE WEIGHT OF VARIOUS SUBSTANCES
## WEIGHT OF 1 CU. FT. OF VARIOUS METALS

| Metal | Weight Pounds | Metal | Weight Pounds |
|---|---|---|---|
| Aluminum | 167 | Magnesium | 109 |
| Antimony | 418 | Manganese | 499 |
| Bismuth | 613 | Mercury, at 32° F. | 849 |
| Brass, cast | 504 | Mercury, at 60° F. | 846 |
| Brass, rolled | 524 | Mercury, at 212° F. | 836 |
| Bronze | 546 | Molybdenum | 538 |
| Chromium | 456 | Nickel | 548 |
| Cobalt | 560 | Platinum, hammered | 1,270 |
| Copper, cast | 552 | Platinum, rolled | 1,313 |
| Copper, rolled | 555 | Platinum, wire | 1,372 |
| Gold, cast, 24 carat | 1,204 | Silver, hammered | 657 |
| Gold, pure, hammered | 1,217 | Silver, pure | 653 |
| Gun-metal bronze | 529 | Sulphur | 125 |
| Iridium | 1,400 | Steel | 490 |
| Iron, cast | 450 | Tin | 458 |
| Iron, wrought | 480 | Tungsten | 1,097 |
| Lead, commercial, cast | 712 | Zinc | 437 |

## WEIGHT OF 1 CU. FT. OF VARIOUS WOODS, WHEN DRY*

| Wood | Weight Pounds | Wood | Weight Pounds |
|---|---|---|---|
| | | Cypress, bald | 29 |
| Alder | 42 | Cypress, Spanish | 40 |
| Apple | 47 | Dagame | 56 |
| Arbor vitae | 19 | Dogwood | 50 |
| Ash, black | 39 | Ebony | 76 |
| Ash, blue | 34 | Elder tree | 43 |
| Ash, green | 39 | Elm, cork | 45 |
| Ash, Oregon | 35 | Elm, slippery | 43 |
| Ash, red | 38 | Elm, white | 34 |
| Ash, white | 39 | Elm, wing | 46 |
| Aspen | 27 | Filbert tree | 38 |
| Bamboo | 22 | Fir, balsam | 23 |
| Basswood | 28 | Fir, great silver | 22 |
| Bay tree | 51 | Fir, red, or California | 29 |
| Beech | 42 | Fir, red, or noble | 28 |
| Bethabara | 76 | Fir, white | 22 |
| Birch, paper, or white | 37 | Greenheart | 72 |
| Birch, red | 35 | Gum, cotton | 32 |
| Birch, sweet | 47 | Gum, sour | 39 |
| Birch, yellow | 40 | Gum, sweet | 37 |
| Blue beech (ironwood) | 45 | Hackmatack (American | |
| Blue gum (fever tree) | 43 to 69 | larch) | 38 |
| Box elder | 26 | Hawthorn | 57 |
| Boxwood, Brazilian, red | 64 | Hazel | 38 |
| Boxwood, Dutch | 83 | Hemlock | 26 |
| Boxwood, French | 57 | Hemlock, western | 28 |
| Buckeye, Ohio | 28 | Hickory, mocker nut | 53 |
| Buckeye, sweet | 27 | Hickory, pecan | 49 |
| Bullet tree | 65 | Hickory, pignut | 56 |
| Butternut | 25 | Hickory, shagbark, or shell- | |
| Buttonwood (sycamore) | 35 | bark | 51 |
| Cabacalli | 56 | Holly | 36 |
| Catalpa, or Indian bean | 27 | Hornbeam | 47 |
| Catalpa, hardy | 25 | Ironwood, or blue beech | 45 |
| Cedar, California, white | 25 | Ironwood, or hop hornbeam | 51 |
| Cedar, canoe | 23 | Jasmine, Spanish | 48 |
| Cedar, incense | 25 | Joshua tree | 23 |
| Cedar, Indian | 82 | Juneberry | 54 |
| Cedar, juniper | 35 | Karri | 63 |
| Cedar, Palestine | 38 | Kauri | 37 |
| Cedar, Port Oxford | 28 | Laburnum | 57 |
| Cedar, red | 30 | Lancewood | 53 |
| Cedar, white, or post | 23 | Larch | 38 |
| Cedar, white (arbor vitæ) | 19 | Larch, tamarack | 46 |
| Cedar, wild | 37 | Laurel, California | 40 |
| Cedar, yellow | 29 | Laurel, Madroña | 43 |
| Cherry, wild black | 36 | Lemon | 45 |
| Chestnut | 28 | Lignum vitæ | 83 |
| Chinkapin | 36 | Linden | 38 |
| Citron | 45 | Locust, black, or yellow | 45 |
| Cocoa wood | 65 | Locust, honey | 42 |
| Cocobolo | 55 | Logwood | 58 |
| Cottonwood | 24 | Madroña | 43 |
| Cottonwood, black | 23 | Mahoe | 41 |
| Cucumber tree | 29 | Mahogany | 45 |

*The weight of wood depends largely on the amount of moisture it contains. The weights in this and the following tables are for very dry wood and not for green wood. Separate tables are given for Philippine, Indian, and Australian woods. The weight of moisture contained in the wood must be added to the values given in these tables.

TABLE—(*Continued*)

| Wood | Weight Pounds | Wood | Weight Pounds |
|------|---------------|------|---------------|
| Mahogany, Mexican....... | 32 | Pine, white, (Pacific States and British Columbia) ... | 24 |
| Mahogany, Spanish........ | 53 | | |
| Mahogany, white.......... | 33 | Pine, white, (Rocky Mountains) ................. | 27 |
| Maple, Oregon............ | 30 | | |
| Maple, red............... | 38 | Pingow................... | 47 |
| Maple, silver, or soft....... | 32 | Plum tree............... | 49 |
| Maple, sugar, or hard...... | 43 | Pockwood................ | 81 |
| Mastic tree............... | 53 | Pomegranate tree.......... | 85 |
| Medlar................... | 59 | Poon.................... | 36 |
| Mesquit.................. | 47 | Poplar, or large-tooth aspen | 28 |
| Missel tree............... | 59 | Poplar, yellow, or tulip tree | 26 |
| Mora..................... | 57 | Quebracho................ | 82 |
| Mulberry, red or black..... | 36 | Quince tree.............. | 44 |
| Oak, black............... | 45 | Redwood................. | 26 |
| Oak, burr................. | 46 | Roller wood.............. | 52 |
| Oak, chestnut............. | 46 | Rosewood................ | 68 |
| Oak, cow................. | 46 | Sal..................... | 60 |
| Oak, English.............. | 51 | Sassafras................. | 31 |
| Oak, live, California....... | 51 | Shadblow................. | 54 |
| Oak, live, Southern United States.................. | 59 | Shadbush................. | 54 |
| | | Spruce, black............. | 28 |
| Oak, pin.................. | 43 | Spruce, Douglas........... | 32 |
| Oak, post................. | 50 | Spruce, Norway.......... | 29 |
| Oak, red................. | 45 | Spruce, single (balsam fir).. | 23 |
| Oak, Spanish.............. | 43 | Spruce, Sitka............ | 26 |
| Oak, white (North-central and Eastern United States)............... | 50 | Spruce, white, (Northern United States)......... | 25 |
| Oak, white, (Pacific Coast from British Columbia to California) ............ | 46 | Spruce, white (Rocky Mountains and British Columbia) ............... | 21 |
| Orange, osage............ | 48 | Sycamore, or buttonwood.. | 35 |
| Orange tree.............. | 44 | Sycamore, California...... | 30 |
| Paddlewood.............. | 52 | Tamarack................ | 38 |
| Palm, Washington........ | 32 | Teak.................... | 50 |
| Palmetto, cabbage........ | 27 | Tonka................... | 64 |
| Pear..................... | 41 | Tooart.................. | 67 |
| Persimmon............... | 49 | Tulip tree............... | 26 |
| Pine, bull................ | 29 | Tulip wood.............. | 61 |
| Pine, Cuban.............. | 39 | Vine tree................ | 83 |
| Pine, Kauri.............. | 60 | Walnut, black............ | 38 |
| Pine, loblolly............ | 33 | Walnut, Circassian........ | 35 |
| Pine, long-leaf, or Georgia.. | 38 | Walnut, English.......... | 36 |
| Pine, northern............ | 34 | Walnut, Italian........... | 42 |
| Pine, Norway............ | 31 | Walnut, Persian.......... | 36 |
| Pine, Oregon............. | 32 | Walnut, white........... | 25 |
| Pine, pitch............... | 32 | Wasahba................ | 76 |
| Pine, short-leaf, or Carolina. | 32 | Whitewood.............. | 26 |
| Pine, sugar............... | 22 | Willow, black............ | 27 |
| Pine, white, (North-Central and Northeastern States). | 24 | Yarura.................. | 52 |
| | | Yew, Dutch.............. | 49 |
| | | Yew, Spanish............ | 50 |
| | | Yucca, or Joshua tree...... | 23 |

## WEIGHT OF 1 CU. FT. OF PHILIPPINE WOODS, WHEN DRY

| Wood | Weight Pounds | Wood | Weight Pounds |
|------|------|------|------|
| Acle | 37 | Liusin | 44 |
| Amuguis | 43 | Lumbayao | 35 |
| Apitong | 41 | Macaasin | 44 |
| Aranga | 54 | Malasantol | 40 |
| Balacat | 33 | Malugay | 40 |
| Balacbacan | 34 | Mayapis | 25 |
| Bansalaguin | 53 | Molave | 49 |
| Banuyo | 33 | Narra | 36 |
| Batitinan | 49 | Palo Maria | 39 |
| Betis | 49 | Sacat | 37 |
| Calantas | 27 | Sasalit | 55 |
| Dungon | 49 | Supa | 45 |
| Guijo | 43 | Tanguile | 30 |
| Ipii | 47 | Tindalo | 48 |
| Lauan | 29 | Yacal | 52 |

## WEIGHT OF 1 CU. FT. OF AUSTRALIAN WOODS, WHEN DRY*

| Wood | Weight Pounds |
|------|------|
| Acacia dealbata (silver wattle) | 57 |
| Acacia decurrens (common wattle) | 47 |
| Acacia implexa | 44 |
| Acacia melanoxylon (blackwood; lightwood) | 47 |
| Acacia mollissima (silver wattle) | 50 |
| Acacia pycnantha (golden wattle) | 52 |
| Acacia salicina | 48 |
| Araucaria cunninghamii (pine) | 45 |
| Aster argophyllus (musk tree) | 40 |
| Banksia integrifolia (coast honeysuckle tree) | 50 |
| Banksia marginata (common honeysuckle tree) | 38 |
| Banksia serrata (heath honeysuckle tree) | 50 |
| Callitris verrucosa (dessert sandarac pine, or cypress) | 43 |
| Castanaspermum australe (black bean) | 57 |
| Casuarina torulusa (forest oak) | 66 |
| Casuarina quadravalvis (drooping she oak) | 61 |
| Cedrela australis (cedar) | 28 |
| Ceratopetalum apetalum (coachwood) | 42 |
| Dacrydium cupressinum (rimu) | 38 |
| Dissiliaria baloghioides (teak) | 60 |
| Dysoxylon muelleri (red bean) | 46 |
| Eucalyptus amygdalina regnans (mountain ash or peppermint tree) | 60 |
| Eucalyptus botryoides (blue gum, Gippsland mahogany, or bastard mahogany) | 60 |
| Eucalyptus corymbosa (bloodwood) | 58 |
| Eucalyptus corynocalyx (sugar gum) | 69 |
| Eucalyptus diversicolor (karri) | 61 |
| Eucalyptus globulus (blue gum) | 57 |
| Eucalyptus gomphocephala (tuart) | 66 |
| Eucalyptus goniocalyx (bastard box, spotted gum) | 72 |
| Eucalyptus haewastowa (spotted gum) | 69 |
| Eucalyptus hemiphloia (canary wood, white box, or gray box) | 48 |

* On account of the unsettled nomenclature of Australian woods, this table gives botanical names, with common names, so far as possible, in parenthesis after the botanical name. Also see note at foot of table, Weight of Various Woods, page 279.

TABLE—(*Continued*)

| Wood | Weight Pounds |
|---|---|
| Eucalyptus largiflorens (slaty gum) | 77 |
| Eucalyptus leucoxylon (iron bark, red flowering, or black iron bark) | 70 |
| Eucalyptus longifolia (wollybutt tree) | 69 |
| Eucalyptus maculata (spotted gum) | 63 |
| Eucalyptus marginata (jarrah) | 54 |
| Eucalyptus melliodora (yellow box) | 69 |
| Eucalyptus microcorys (tallow wood) | 59 |
| Eucalyptus obliqua (messmate, stringy bark) | 57 |
| Eucalyptus pilularis (blackbutt, or flintwood) | 53 |
| Eucalyptus piperita (blackbutt, white stringy bark tree) | 69 |
| Eucalyptus resinifera (mahogany) | 68 |
| Eucalyptus robusta (swamp mahogany) | 67 |
| Eucalyptus rostrata (red gum tree) | 56 |
| Eucalyptus saligna (gray gum) | 61 |
| Eucalyptus siderophloia (iron bark) | 68 |
| Eucalyptus sieberiana (iron bark, gumtop stringy bark, mountain ashes) | 56 |
| Eucalyptus tereticarnis (flooded gum) | 68 |
| Eucalyptus viminalis (manna gum tree, drooping gum, or white gum tree) | 43 |
| Eugenia smithii (myrtle) | 57 |
| Exocarpus cupressiformis (native cherry tree) | 50 |
| Fagus cunninghamii (evergreen beech or native myrtle) | 45 |
| Hakea leucoptera (water tree) | 51 |
| Heterodendron oleifolium | 53 |
| Lomatia fraseri | 42 |
| Melalenca decussata | 59 |
| Melalenca parviflora | 62 |
| Myporum insulare | 51 |
| Myrsine variabilis | 45 |
| Panax murrayi (palm panax) | 22 |
| Pimelea microcephala | 55 |
| Pittosporum bicolor (white wood) | 48 |
| Pomaderris apetala (hazel) | 48 |
| Prostanthera lasianthas (mint tree) | 51 |
| Santalum acuminatum (native peach or quandong) | 52 |
| Santalum persicarium (native sandalwood) | 47 |
| Senecio bedfordii (native dogwood) | 56 |
| Syncarpia laurifolia (turpentine) | 63 |
| Tristania conferta (brush or white box) | 67 |
| Tristania nerifolia (water gum) | 63 |
| Viminaria denudata | 39 |

## WEIGHT OF 1 CU. FT. OF INDIAN WOODS

(*Berkley-Clark*)

| Wood | Weight Pounds | Wood | Weight Pounds |
|---|---|---|---|
| Bibla | 56 | Poon | 39 |
| Blackwood | 56 | Red eyne | 68 |
| Erroul | 63 | Teak, Jungle | 41 |
| Hedoo | 39 | Teak, Northern | 55 |
| Khair | 73 | Teak, Southern | 48 |
| Kullum | 41 | | |

## WEIGHT OF AMERICAN TIMBERS

| Wood | Specific Gravity | Weight per Cubic Foot Pounds | Weight per Foot Board Measure Pounds | Remarks |
|---|---|---|---|---|
| California redwood .... | .39 | 24.16 | 2.01 | The weights given are the averages of a large number of determinations for commercially dry lumber containing less than 15% of moisture. The weights of unseasoned green lumber will be from 20 to 40 % greater. Green oak is commonly taken to weigh 5 lb. per running foot board measure. |
| California spruce....... | .40 | 24.97 | 2.08 | |
| Cedar................ | .37 | 23.10 | 1.93 | |
| Chestnut............. | .66 | 41.20 | 3.43 | |
| Cypress.............. | .46 | 28.72 | 2.39 | |
| Douglas fir........... | .51 | 31.84 | 2.65 | |
| Hemlock............. | .40 | 24.97 | 2.08 | |
| Red pine (Norway pine) | .50 | 31.31 | 2.60 | |
| Short-leaf yellow pine... | .51 | 31.84 | 2.65 | |
| Southern, long-leaf, or Georgia yellow pine.... | .61 | 38.08 | 3.17 | |
| Spruce and eastern fir.. | .40 | 24.97 | 2.08 | |
| White oak............. | .80 | 49.94 | 4.16 | |
| White pine............ | .38 | 23.72 | 1.98 | |

## WEIGHT OF 1 SQ. FT. OF BUILDING MATERIALS

| Name of Material | | Average Weight Pounds | Name of Material | | Average Weight Pounds |
|---|---|---|---|---|---|
| Corrugated (2½ -in.) galvanized iron | No. 16.. | 2.91 | Slate, single thickness. | ¼ in. thick.... | 1.81 |
| | No. 18.. | 2.36 | | ⅟₁₆ in. thick .. | 2.71 |
| | No. 20.. | 1.82 | | ½ in. thick... | 3.62 |
| | No. 22.. | 1.54 | | ⅜ in. thick. ... | 5.43 |
| | No. 24.. | 1.27 | | ½ in. thick... | 7.25 |
| | No. 26.. | .99 | | ⅝ in. thick... | 9.06 |
| | No. 27.. | .93 | | ½ in. thick. .: | 10.87 |
| | No. 28.. | .86 | Slag roof, four-ply........ | | 4.00 |
| Corrugated galvanized iron, No. 20, average amount of side lap, unboarded...... | | 2.25 | Steel roofing, standing seam | | 1.00 |
| | | | Tiles, Spanish, 14½ in.×10½ in., 7½ in. to weather..... | | 8.50 |
| Copper roofing, 16-oz., standing seam.......... | | 1.25 | Tiles, plain, 10½ in.×6½ in. ×⅜ in., 5½ in. to weather. | | 18.00 |
| Felt and pitch, without sheathing.......... | | 3.00 | White-pine sheathing, 1 in. thick.................. | | 2.00 |
| Glass, ¼ in. thick.......... | | 1.75 | Yellow-pine sheathing, 1 in. thick.................. | | 3.00 |
| Hemlock sheathing, 1 in. thick................ | | 2.00 | Gravel roof and four-ply felt | | 5.50 |
| Lead, about ⅛ in. thick.... | | 6 to 8 | Gravel roof and five-ply felt | | 6.00 |
| Lath-and-plaster ceiling (ordinary)............. | | 6 to 8 | Roofing, three-ply ready (asphalt, rubberoid, etc.) | | 6 to 10 |
| Mackite, 1 in. thick, with plaster................ | | 10.00 | Purlins, wooden, with 12- to 16-ft. span.......... | | 2.00 |
| Neponset roofing felt, two layers............... | | .50 | Chestnut or maple sheathing, 1 in. thick.......... | | 4.00 |
| Spruce sheathing, 1 in. thick | | 2.00 | Ash, hickory, or oak sheathing, 1 in. thick......... | | 5.00 |
| Shingles, common, 6 in.×18 in., 5 in. to weather...... | | 2.00 | Sheet iron, ⅟₁₆ in. thick ..... | | 3.00 |
| Skylight of glass, ⅟₁₆ to ¼ in., including frame........ | | 4 to 10 | Thatch................... | | 6.50 |

## WEIGHT OF 1 CU. FT. OF BUILDING MATERIALS

| Material | Weight Pounds | Material | Weight Pounds |
|---|---|---|---|
| Asphalt-pavement composition | 130 | Earth, soft, flowing mud | 108 |
| Bluestone | 160 | Earth, dense mud | 125 |
| Brick, best pressed | 150 | Firebrick | 150 |
| Brick, common and hard | 125 | Granite | 165 to 170 |
| Brick, paving | 150 | Gravel | 117 to 125 |
| Brick, soft, inferior | 100 | Iron, cast* | 450 |
| Brickwork, in lime mortar, average | 120 | Iron, wrought† | 480 |
| | | Limestone | 146 to 168 |
| Brickwork, in cement mortar, average | 130 | Marble | 168 |
| | | Masonry, squared granite or limestone | 165 |
| Brickwork, pressed brick, thin joints | 140 | Masonry, granite or limestone rubble | 150 |
| Cement, Portland, packed | 100 to 120 | Masonry, granite or limestone dry rubble | 138 |
| Cement, Portland, loose | 70 to 90 | Masonry, sandstone | 145 |
| Cement, natural packed | 75 to 95 | Mineral wool | 12 |
| | | Mortar, hardened | 90 to 100 |
| Cement, natural, loose | 45 to 65 | Quicklime, ground, loose, or small lumps | 53 |
| Cement, slag, packed | 80 to 100 | Quicklime, ground, thoroughly shaken | 75 |
| Cement, slag, loose | 55 to 75 | Sand, pure quartz, dry | 90 to 106 |
| Concrete, cinder | 105 | Sandstone, building, dry | 130 to 151 |
| Concrete, gravel | 140 | Slate | 160 to 180 |
| Concrete, slag | 135 | Snow, fresh fallen | 5 to 12 |
| Concrete, stone | 140 | Steel, structural‡ | 489.6 |
| Concrete, reinforced, average | 150 | Terra cotta | 110 |
| Earth, loam, dry and loose | 72 to 80 | Terra-cotta masonry work | 112 |
| Earth, loam, shaken | 82 to 92 | Tile | 110 to 120 |
| Earth, loam, moderately rammed | 90 to 100 | | |

## WEIGHT OF 1 CU. FT. OF MISCELLANEOUS MATERIALS

| Material | Weight Pounds | Material | Weight Pounds |
|---|---|---|---|
| Acid, acetic | 66 | Ashes | 40 |
| Acid, fluoric | 94 | Asphalt, pure | 80 |
| Acid, hydrochloric | 75 | Borax | 107 |
| Acid, nitric | 76 | Bran | 16 |
| Acid, phosphoric | 97 | Chalk | 156 |
| Acid, sulphuric | 115 | Charcoal, birch | 34 |
| Alabaster, white | 171 | Charcoal, fir | 28 |
| Alabaster, yellow | 169 | Charcoal, oak | 21 |
| Alcohol, commercial | 52 | Charcoal, pine | 18 |
| Alcohol, grain | 49.6 | Chrome ore dust, well shaken | 160 |
| Alcohol, wood | 49.9 | Clay, ordinary | 120 to 150 |
| Ammonia, 28% | 56 | Clay, potter's, dry | 119 |
| Antimony | 418 | Clinker | 85 |
| Apples | 38 | Coal, anthracite, broken | 54 |
| Asbestos, starry | 192 | | |

*Weight per cubic inch, .260 lb.     †Weight per cubic inch, .277 lb.
‡Weight per cubic inch, .283 lb.

TABLE—(Continued)

| Material | Weight Pounds | Material | Weight Pounds |
|---|---|---|---|
| Coal, anthracite, moderately shaken. | 58 | Magnesite, calcined.... | 110 |
| Coal, anthracite, solid*............ | 93 | Mica................. | 183 |
| Coal, bituminous, broken, loose...... | 50 | Millstone............. | 155 |
| Coal, bituminous, slacked... | 52 | Naphtha ............. | 53 |
| Coal, bituminous, solid†............. | 84 | Niter................ | 119 |
| Coal, cannel, solid..... | 79 | Oats................ | 26 |
| Coke, loose‡......... | 23 to 32 | Oil, linseed.......... | 59 |
| Coral, red............ | 169 | Oil, olive............ | 57 |
| Cork................ | 15 | Oil, turpentine.....÷.. | 54 |
| Corn on cob, husked.. | 56 | Oil, whale†........... | 58 |
| Corn on cob, unhusked | 58 | Ore, iron, magnetite... | 312 |
| Corn, shelled......... | 45 | Ore, iron, hematite.... | 306 |
| Corn meal, bolted..... | 37 | Paper, calendered book | 50 |
| Corn meal, unbolted... | 38 | Paper, leather-board... | 59 |
| Corundum............ | 244 | Paper, manila........ | 37 |
| Cotton yarn, skeins.... | 11 | Paper, news.......... | 38 |
| Crowd of men........ | 134 to 157 | Paper, strawboard..... | 33 |
| Emery............... | 250 | Paper, supercalendered book............... | 69 |
| Ether................ | 45 | Paper, wrapping....... | 10 |
| Feldspar............. | 166 | Paper, writing........ | 64 |
| Flint................ | 162 | Paving stone.......... | 150 |
| Glass, common....... | 156 to 172 | Pearl, oriental........ | 165 |
| Glass, flint.......... | 180 to 196 | Peat, dry, compressed. | 20 to 30 |
| Gneiss, common...... | 168 | Petroleum........... | 55 |
| Gneiss, in loose piles.... | 96 | Pitch............... | 72 |
| Grindstone........... | 134 | Plaster of Paris, cast.. | 80 |
| Gun-metal............ | 528 | Plumbago............ | 140 |
| Gunpowder, loose...... | 56 | Potatoes, sweet........ | 41 |
| Gunpowder, shaken.... | 63 | Potatoes, white....... | 48 |
| Gunpowder, solid...... | 105 | Pumice stone......... | 57 |
| Gutta percha......... | 61 | Quartz, common, pure. | 165 |
| Gypsum.............. | 143 | Rosin............... | 69 |
| Hay, alfalfa, baled.... | 12.5 to 14.3 | Rope, manila ......... | 42 |
| Hay, alfalfa, double compressed bales.... | 25.53 | Rottenstone.......... | 124 |
| Hay, alfalfa, cylindrical, double compressed bales........ | 36.36 | Salt, coarse........... | 45 |
| | | Salt, West-Indies, well-dried............... | 74 |
| Hay, clover, baled..... | 14 | Saltpeter............. | 131 |
| Hay, clover, compressed............ | 24 | Shales............... | 162 |
| Hay, clover, in mow... | 4.6 | Soil, common......... | 124 |
| Hematite iron ore..... | 306 | Soapstone............ | 170 |
| Ice................. | 57 | Spelter (zinc)......... | 437 |
| India rubber.......... | 58 | Straw............... | 19 |
| Ivory................ | 114 | Sugar............... | 100 |
| Land plaster.......... | 80 | Sulphur............. | 125 |
| Leather, sole, in lbs... | 17 | Talc, block........... | 181 |
| Magnesia, carbonate... | 150 | Tallow, sheep or ox.... | 58 |
| | | Tar................. | 63 |
| | | Trap rock............ | 170 |
| | | Turf, or peat......... | 20 to 30 |
| | | Turnips.............. | 44 |
| | | Vinegar............. | 68 |
| | | Wheat............... | 48 |

*Anthracite increases about 75% in bulk when broken to any market size; 1 T. loose, averages from 38 to 46 cu. ft.

†A heaped bushel, loose, weighs about 74 lb., and 1 T. occupies from 43 to 48 cu. ft. Bituminous coal, when broken, occupies 75% more space than in the solid.

‡A heaped bushel, loose, weighs from 35 to 42 lb.; 1 T. occupies 80 to 97 cu. ft.

## PROPERTIES OF COAL

### SPECIFIC GRAVITY OF AMERICAN COALS

(*U. S. Bureau of Mines*)

| State | County | Locality | Seam | Selected Lumps | Average Lumps |
|---|---|---|---|---|---|
| Alabama | Walker | Carbon Hill | Jagger | 1.32 | 1.37 |
| Alabama | Bibb | Garnsey | Underwood | 1.30 | 1.38 |
| Alabama | Bibb | Belle Ellen | Youngblood | 1.28 | 1.32 |
| Arkansas | Sebastian | Midland | Hartshorne | 1.32 | 1.44 |
| Illinois | Williamson | Bush | No. 6 | 1.31 | 1.33 |
| Illinois | Madison | Donkville | No. 6 | 1.22 | 1.26 |
| Illinois | Logan | Lincoln | No. 5 | 1.22 | 1.31 |
| Illinois | Sangamon | Auburn | No. 6 | 1.24 | 1.28 |
| Indiana | Sullivan | Hymera | No. 5 | 1.28 | 1.42 |
| Indiana | Sullivan | Hymera | No. 4 | 1.25 | 1.39 |
| Indiana | Pike | Littles | No. 5 | 1.27 | 1.40 |
| Indiana | Vigo | Terre Haute | No. 7 | 1.29 | 1.30 |
| Indiana | Vigo | Macksville | No. 7 | 1.25 | 1.36 |
| Indiana | Park | Rosedale | No. 6 | 1.24 | 1.29 |
| Indiana | Sullivan | Dugger | No. 4 | 1.26 | 1.30 |
| Indiana | Pike | Hartwell | No. 5 | 1.26 | 1.32 |
| Kansas | Linn | Jewett | Weir-Pittsburg | 1.23 | 1.34 |
| Kentucky | Bell | Straight Creek | Straight Creek | 1.27 | 1.40 |
| Kentucky | | Big Black Mt. | High Splint | 1.29 | 1.30 |
| Kentucky | Johnson | Paintsville | No. 1 | 1.27 | 1.28 |
| Kentucky | Muhlenburg | Central City | No. 9 | 1.31 | 1.44 |
| Maryland | Garrett | Westernport | Lw. Kittaning | 1.36 | 1.41 |
| Missouri | Randolph | Huntsville | | 1.21 | 1.36 |
| New Mexico | Colfax | Van Houten | Raton | 1.29 | 1.37 |
| New Mexico | Colfax | Brilliant | Raton | 1.30 | 1.39 |
| New Mexico | Colfax | Blossburg | Raton | 1.31 | 1.35 |
| North Dakota | Stark | Lehigh | Lignite | 1.25 | 1.44 |
| North Dakota | McLean | Wilton | Lignite | 1.22 | 1.23 |
| Ohio | Jackson | Wellston | No. 4 | 1.29 | 1.35 |
| Ohio | Jackson | Wellston | No. 5 | 1.31 | 1.36 |
| Ohio | Perry | Shawnee | No. 6 | 1.30 | 1.33 |
| Ohio | Jefferson | Bradley | No. 8 | 1.30 | 1.39 |
| Ohio | Jefferson | Rush Run | No. 8 | 1.29 | 1.33 |
| Ohio | Guernsey | Danford | No. 7 | 1.30 | 1.34 |
| Ohio | Perry | Dixie | No. 6 Hocking | 1.30 | 1.42 |
| Ohio | Vinton | Clarion | No. 4 | 1.30 | 1.36 |
| Pennsylvania | Westmoreland | Greensburg | Pittsburg | 1.30 | 1.35 |
| Pennsylvania | Washington | Ellsworth | Pittsburg | 1.30 | 1.31 |
| Pennsylvania | Washington | Ellsworth | Pittsburg | 1.28 | 1.33 |
| Pennsylvania | Westmoreland | East Millsboro | Pittsburg | 1.30 | 1.33 |
| Pennsylvania | Westmoreland | Ligonier | Pittsburg | 1.33 | 1.41 |
| Pennsylvania | Cambria | Ehrenfeld | Lw. Kittanning | 1.31 | 1.36 |
| Pennsylvania | Somerset | Kimmelton | Lw. Kittanning | 1.35 | 1.39 |
| Pennsylvania | Allegheny | Bruce | Pittsburg | 1.30 | 1.36 |
| Tennessee | Campbell | Gatliff | Log Mountain | 1.28 | 1.33 |
| Tennessee | Campbell | Gatliff | Regal Block | 1.29 | 1.32 |
| Tennessee | Roane | Oliver Springs | Wind Rock | 1.29 | 1.37 |
| Tennessee | Cumberland | Waldensia | Lower Sewanee | 1.29 | 1.31 |
| Tennessee | Fentress | Wilder | Wilder | 1.34 | 1.39 |

TABLE—(Continued)

| State | County | Locality | Seam | Specific Gravity | |
|---|---|---|---|---|---|
| | | | | Selected Lumps | Average Lumps |
| Tennessee | White | Clifty | 1st. Abv. Sewanee | 1.34 | 1.37 |
| Tennessee | Marion | Orme | Battle Creek | 1.35 | |
| Texas | Milan | Olsen | Lignite | | 1.25 |
| Texas | Wood | Hoyte | Lignite | | 1.26 |
| Virginia | Lee | Crab Orchard | Wilson | 1.27 | 1.32 |
| Virginia | Lee | Crab Orchard | McConnell | 1.28 | 1.37 |
| Virginia | Wise | Tom's Creek | Upper Banner | 1.27 | 1.28 |
| Virginia | Lee | Darby | Darby | 1.28 | 1.28 |
| Washington | King | Renton | Black Lignite | 1.28 | 1.33 |
| Washington | Kittitas | Roslyn | | 1.32 | 1.39 |
| West Virginia | Preston | Bretz | Upper Freeport | 1.31 | 1.35 |
| West Virginia | Fayette | Page | Ansted | 1.27 | 1.30 |
| West Virginia | Fayette | Page | Eagle | 1.27 | 1.28 |
| West Virginia | Harrison | Clarksburgh | Pittsburg | 1.28 | 1.31 |
| West Virginia | Marion | Monongah | Pittsburg | 1.29 | 1.35 |
| West Virginia | Preston | Bretz | Bakerstown | 1.28 | 1.41 |
| West Virginia | Mingo | Glen Alum | Glen Alum | 1.30 | 1.34 |
| West Virginia | Fayette | McDonald | Sewell | 1.26 | 1.38 |
| West Virginia | Kanawha | Acme | Keystone | 1.27 | 1.34 |
| West Virginia | Kanawha | Winifrede | Winifrede | 1.28 | 1.34 |
| Wyoming | Weston | Cambria | Cambria Fuel Co. | 1.31 | 1.37 |
| Wyoming | Crook | Aladdin | Stillwell Coal Co. | 1.28 | 1.40 |
| Wyoming | Carbon | Hanna | | 1.33 | 1.35 |
| Wyoming | Sweetwater | Rock Springs | | 1.26 | 1.30 |
| Wyoming | Uinta | Kemmerer | | 1.28 | |
| | | Average of 70 American Coals | | 1.286 | 1.348 |

## WEIGHTS AND MEASUREMENTS OF COAL

(Coxe Bros. & Co., Chicago, Ill.)

| Coal | Weight per Cubic Foot Pounds | Cubic Feet per Ton, 2,000 Lb. | Coal | Weight per Cubic Foot Pounds | Cubic Feet per Ton, 2,000 Lb. |
|---|---|---|---|---|---|
| Lehigh lump........ | 55.26 | 36.19 | Free-burning egg.... | 56.07 | 35.67 |
| Lehigh cupola...... | 55.52 | 36.02 | Free-burning stove.. | 56.33 | 35.50 |
| Lehigh broken....... | 56.85 | 35.18 | Free-burning nut... | 56.88 | 35.16 |
| Lehigh egg......... | 57.74 | 34.63 | Pittsburg.......... | 46.48 | 43.03 |
| Lehigh stove........ | 58.15 | 34.39 | Illinois............ | 47.22 | 42.35 |
| Lehigh nut......... | 58.26 | 34.32 | Connellsville coke... | 26.30 | 76.04 |
| Lehigh pea......... | 53.18 | 37.60 | Hocking............ | 49.30 | 40.56 |
| Lehigh buckwheat... | 54.04 | 37.01 | Indiana block...... | 43.85 | 45.61 |
| Lehigh dust........ | 57.25 | 34.93 | Erie............... | 48.07 | 41.61 |
| | | | Ohio cannel........ | 49.18 | 40.66 |

## AVERAGE WEIGHT AND BULK OF AMERICAN COALS
### (W. R. Johnson)

| Coal | Specific Gravity | Weight of 1 Cu. Ft. Solid Pounds | Weight of 1 Cu. Ft. Heaped Pounds | Bulk of 1 T., Heaped Cubic Feet |
|---|---|---|---|---|
| 1. Anthracites............... | 1.500 | 93.78 | 53.05 | 42.35 |
| 2. Bituminous, free-burning.. | 1.358 | 84.93 | 52.84 | 42.42 |
| 3. Bituminous, coking....... | 1.342 | 83.90 | 49.28 | 45.71 |
| Average of 1, 2, and 3.... | 1.400 | 87.54 | 51.72 | 43.49 |
| Foreign and Western...... | 1.318 | 82.39 | 49.31 | 45.51 |
| Cokes................. | .... | .... | 32.13 | 69.76 |

## SPECIFIC GRAVITIES OF VARIOUS COALS

| Name of Coal | Specific Gravity | Weight of 1 Cu. Ft. Pounds | Weight of 1 Cu. Yd. Tons |
|---|---|---|---|
| Newcastle Hartley, England....... | 1.29 | 80.6 | .972 |
| Wigan, 4 ft., England............ | 1.20 | 75.0 | .914 |
| Portland, England............... | 1.30 | 81.2 | .978 |
| Anthracite, Wales.............. | 1.39 | 86.9 | 1.047 |
| Eglington, Scotland............ | 1.25 | 78.1 | .941 |
| Anthracite, Irish............... | 1.59 | 99.4 | 1.193 |
| Anthracite, Pennsylvania....... | 1.55 | 96.9 | 1.167 |
| Bituminous, Pennsylvania....... | 1.40 | 87.5 | 1.054 |
| Block coal, Indiana............. | 1.27 | 79.4 | .956 |

To Mr. Irving A. Stearns, mining engineer and former general superintendent of the Pennsylvania Railroad Co.'s Coal Department, we are indebted for the following summary of tests made by the mining engineers of this company. Although these tests were made 25 yr. ago, they are still of value as

## WEIGHT OF SUSQUEHANNA COAL CO.'S WHITE ASH ANTHRACITE

| Size | Size of Mesh, in Inches | | Weight per Cubic Foot Pounds | Cubic Feet From 1 Cubic Foot of Solid |
|---|---|---|---|---|
| | Over | Through | | |
| Lump.................. | 4½ to 9 | | 57 | 1.614 |
| Broken................ | 2½ to 2⅞ | 3¼ to 4½ | 53 | 1.755 |
| Egg.................. | 1⅞ to 2¼ | 2½ to 2⅞ | 52 | 1.769 |
| Large stove............ | 1¼ to 1⅞ | 1⅞ to 2¼ | 51½ | 1.787 |
| Small stove............ | 1 to 1¼ | 1¼ to 1⅞ | 51½ | 1.795 |
| Chestnut.............. | ⅝ to ⅞ | 1 to 1¼ | 51 | 1.804 |
| Pea.................. | ⅜ to ⅝ | ⅝ to ⅞ | 50¾ | 1.813 |
| No. 1 buckwheat....... | 3/16 to ⅜ | ⅜ to ⅝ | 50¾ | 1.813 |
| No. 2 buckwheat....... | | 1/16 to ⅜ | 50¾ | 1.813 |

giving the weight of anthracite, but the sizes corresponding to the different grades are not now in use. Sizes of anthracite should be taken from the table Sizes of Prepared Anthracite. These determinations showed an average value for the specific gravity of the coal from the various seams upon this

company's property of 1.4784, together with an average weight per cubic foot of coal in the solid of 92.5 lb.

## CONTENTS OF HORIZONTAL COAL SEAMS*

| Thickness of Seam | | Lignite† | Bituminous† | Anthracite† |
|---|---|---|---|---|
| Feet | Inches | Tons per Acre | Tons per Acre | Tons per Acre |
| | 1 | 141.32 | 152.62 | 151.41 |
| | 2 | 282.63 | 305.24 | 302.82 |
| | 3 | 423.95 | 457.87 | 454.23 |
| | 4 | 565.27 | 610.49 | 605.64 |
| | 5 | 706.58 | 763.11 | 757.06 |
| | 6 | 847.90 | 915.73 | 908.47 |
| | 7 | 989.22 | 1,068.35 | 1,059.88 |
| | 8 | 1,130.54 | 1,220.98 | 1,211.29 |
| | 9 | 1,271.85 | 1,373.60 | 1,362.70 |
| | 10 | 1,413.17 | 1,526.22 | 1,514.11 |
| | 11 | 1,554.49 | 1,678.84 | 1,665.63 |
| 1 | 0 | 1,695.80 | 1,831.47 | 1,816.93 |
| 2 | 0 | 3,391.61 | 3,662.93 | 3,633.86 |
| 3 | 0 | 5,087.41 | 5,494.40 | 5,450.79 |
| 4 | 0 | 6,783.21 | 7,325.87 | 7,267.73 |
| 5 | 0 | 8,479.01 | 9,157.34 | 9,084.66 |
| 6 | 0 | 10,174.82 | 10,988.80 | 10,901.59 |
| 7 | 0 | 11,870.62 | 12,820.27 | 12,718.52 |
| 8 | 0 | 13,566.42 | 14,651.74 | 14,535.45 |
| 9 | 0 | 15,262.23 | 16,483.20 | 16,352.38 |
| 10 | 0 | 16,958.03 | 18,314.67 | 18,169.32 |

Mr. Robert A. Quin, Manager Susquehanna Coal Co., has very kindly furnished the two following tables. The first shows the present (1914) sizes of anthracite, and the second shows the number of cubic feet per ton occupied by each of these sizes, according to the region in which it is mined.

## SIZES OF PREPARED ANTHRACITE

| Size | Through | | Over | |
|---|---|---|---|---|
| | Square Inches | Round Inches | Square Inches | Round Inches |
| Lump...................... | | | 5¾ | 6½ |
| Steamboat................. | 5¾ | 6½ | 4 | 4½ |
| Broken.................... | 4 | 4½ | 2¾ | 3¼ |
| Egg....................... | 2¾ | 3¼ | 2 | 2¼ |
| Stove..................... | 2 | 2¼ | 1¾ | 1⅞ |
| Chestnut.................. | 1¾ | 1⅞ | ⅞ | 1 |
| Pea....................... | ⅞ | 1 | 9/16 | ⅝ |
| No. 1 Buckwheat........... | 9/16 | ⅝ | ¼ | ⅜ |
| No. 2 Buckwheat........... | ¼ | ⅜ | ⅛ | 3/16 |
| No. 3 Buckwheat........... | ⅛ | 3/16 | 1/16 | 3/32 |

* This table is based on the average specific gravity of American coals.
† The contents of the bituminous coal and lignite is given in tons of 2,000 lb.; the contents of anthracite seams, in tons of 2,240 lb.

## CUBIC FT. IN 1 T. OF ANTHRACITE BROKEN IN TRADE SIZES

| Size | Wyoming Region Cubic Feet | Shamokin Region Cubic Feet | Schuylkill Region Cubic Feet | Lykens Region Cubic Feet |
|------|------|------|------|------|
| Lump................... | 37.713 | | 38.00 | |
| Steamboat................ | 38.55 | | | |
| Broken.................. | 39.03 | 40.138 | 38.60 | 43.00 |
| Egg..................... | 38.66 | 42.499 | 39.40 | 43.41 |
| Stove................... | 37.72 | 41.559 | 40.30 | 43.92 |
| Nut..................... | 40.43 | 42.308 | 41.00 | 44.78 |
| Pea..................... | 41.63 | 42.692 | 41.70 | 45.16 |
| No. 1 Buckwheat......... | 41.96 | 45.373 | 42.30 | 45.71 |
| No. 2 Buckwheat......... | 42.65 | 43.282 | 43.10 | 46.09 |
| No. 3 Buckwheat......... | 43.36 | 43.686 | 43.90 | |

## WEIGHTS OF ENGLISH AND FRENCH COALS

[ *Delabeche & Playfair (D. K. Clark)* ]

| Name of Coal | Specific Gravity | Weight of 1 Cu. Ft. Solid | Heaped | Cubic Feet in 1 T. Heaped |
|------|------|------|------|------|
| *Wales.*  Anthracite............ | 1.370 | 85.4 | 58.3 | 38.4 |
| Port Mawr (highest)............ | 1.390 | 86.7 | 53.3 | 42.0 |
| Llynvi (one of the lowest)....... | 1.280 | 80.3 | 53.1 | 42.0 |
| Average of 37 samples.......... | 1.315 | 82.3 | 53.1 | 42.7 |
| *Newcastle.*  Hedley's Hartley (highest) | 1.310 | 81.8 | 52.0 | 43.1 |
| Original Hartley (one of the lowest) | 1.250 | 78.0 | 49.1 | 45.6 |
| Average of 18 samples.......... | 1.256 | 78.3 | 49.8 | 45.3 |
| *Derbyshire and Yorkshire.*  Elsecar.... | 1.296 | 80.8 | 47.2 | 47.4 |
| Butterley...................... | 1.280 | 79.8 | 47.4 | 47.3 |
| Stavely........................ | 1.270 | 79.8 | 49.9 | 44.9 |
| Loscoe, soft.................... | 1.285 | 79.6 | 45.9 | 48.8 |
| Average of 8 samples........... | 1.292 | 79.6 | 45.9 | 47.4 |
| *Lancashire.*  Laffack  Bushy  Park (highest)................... | 1.350 | 84.1 | 52.6 | 42.6 |
| Cannel, Wigan (lowest)......... | 1.230 | 76.8 | 48.3 | 46.4 |
| Average of 28 samples.......... | 1.273 | 79.4 | 49.7 | 45.2 |
| *Scotland.*  Grangemouth (highest)..... | 1.290 | 80.5 | 54.3 | 40.1 |
| Wallsend, Elgin................ | 1.200 | 74.8 | 54.6 | 41.0 |
| Average of 8 samples........... | 1.259 | 78.6 | 50.0 | 42.0 |
| *Ireland.*  Slievardagh, Anthracite.... | 1.590 | 99.6 | 62.8 | 35.7 |
| *France.*  Labarthe................. | | | 55.0 | 40.8 |
| Auvergne and Blanzy............ | | | 54.3 | 41.2 |
| Combelle...................... | | | 53.7 | 41.7 |
| Lataupe....................... | | | 53.1 | 42.2 |
| Saint Etienne.................. | | | 52.5 | 42.7 |
| Decise........................ | | | 51.8 | 43.2 |
| Mons......................... | | | 50.0 | 44.8 |
| Creusot....................... | | | 49.3 | 45.4 |
| Average of French bituminous coals. | | | 52.5 | 42.75 |
| Anthracite..................... | | | 56.2 | 40.0 |

# WIRE AND SHEET-METAL GAUGES
## TABLE OF WIRE AND SHEET-METAL GAUGES

| Number of Gauge | U. S. Standard Gauge for Sheet and Plate Iron and Steel. Inch* | British Imperial Standard Wire Gauge. Millim.† | Birmingham Gauge Inch | American or Brown & Sharpe Gauge Inch | Roebling's Gauge Inch | Trenton Iron Co.'s Wire Gauge Inch | English Legal Standard Inch |
|---|---|---|---|---|---|---|---|
| 0000000 | .5 | 12.7 | | | .49 | | .500 |
| 000000 | .469 | 11.78 | | | .46 | | .464 |
| 00000 | .438 | 10.97 | | | .43 | .45 | .432 |
| 0000 | .406 | 10.16 | .454 | .46 | .393 | .40 | .4 |
| 000 | .375 | 9.45 | .425 | .40964 | .362 | .36 | .372 |
| 00 | .344 | 8.84 | .38 | .3648 | .331 | .33 | .348 |
| 0 | .313 | 8.23 | .34 | .32486 | .307 | .305 | .324 |
| 1 | .281 | 7.62 | .3 | .2893 | .283 | .285 | .3 |
| 2 | .266 | 7.01 | .284 | .25763 | .263 | .265 | .276 |
| 3 | .25 | 6.4 | .259 | .22942 | .244 | .245 | .252 |
| 4 | .234 | 5.89 | .238 | .20431 | .225 | .225 | .232 |
| 5 | .219 | 5.38 | .22 | .18194 | .207 | .205 | .212 |
| 6 | .203 | 4.88 | .203 | .16202 | .192 | .19 | .192 |
| 7 | .188 | 4.47 | .18 | .14428 | .177 | .175 | .176 |
| 8 | .172 | 4.06 | .165 | .12849 | .162 | .16 | .16 |
| 9 | .156 | 3.66 | .148 | .11443 | .148 | .145 | .144 |
| 10 | .141 | 3.26 | .134 | .10189 | .135 | .13 | .128 |
| 11 | .125 | 2.95 | .12 | .09074 | .12 | .1175 | .116 |
| 12 | .109 | 2.64 | .109 | .08081 | .105 | .105 | .104 |
| 13 | .094 | 2.34 | .095 | .07196 | .092 | .0925 | .092 |
| 14 | .078 | 2.03 | .083 | .06408 | .08 | .08 | .08 |
| 15 | .07 | 1.83 | .072 | .05707 | .072 | .07 | .072 |
| 16 | .0625 | 1.63 | .065 | .05082 | .063 | .061 | .064 |
| 17 | .0563 | 1.42 | .058 | .04526 | .054 | .0525 | .056 |
| 18 | .05 | 1.22 | .049 | .0403 | .047 | .045 | .048 |
| 19 | .0438 | 1.01 | .042 | .03589 | .041 | .04 | .04 |
| 20 | .0375 | .91 | .035 | .03196 | .035 | .035 | .036 |
| 21 | .0344 | .81 | .032 | .02846 | .032 | .031 | .032 |
| 22 | .0313 | .71 | .028 | .02535 | .028 | .028 | .028 |
| 23 | .0281 | .61 | .025 | .02257 | .025 | .025 | .024 |
| 24 | .025 | .56 | .022 | .0201 | .023 | .0225 | .022 |
| 25 | .0219 | .51 | .02 | .0179 | .02 | .02 | .02 |
| 26 | .0188 | .45 | .018 | .01594 | .018 | .018 | .018 |
| 27 | .0172 | .42 | .016 | .01419 | .017 | .017 | .0164 |
| 28 | .0156 | .38 | .014 | .01264 | .016 | .016 | .0148 |
| 29 | .0141 | .35 | .013 | .01126 | .015 | .015 | .0136 |
| 30 | .0125 | .31 | .012 | .01002 | .014 | .014 | .0124 |
| 31 | .0109 | .29 | .01 | .00893 | .0135 | .013 | .0116 |
| 32 | .0101 | .27 | .009 | .00795 | .013 | .012 | .0108 |
| 33 | .0094 | .25 | .008 | .00708 | .011 | .011 | .01 |
| 34 | .0086 | .23 | .007 | .0063 | .01 | .01 | .0092 |
| 35 | .0078 | .21 | .005 | .00561 | .0095 | .0095 | .0084 |
| 36 | .007 | .19 | .004 | .005 | .009 | .009 | .0076 |
| 37 | .0066 | .17 | | .00445 | .0085 | .0085 | .0068 |
| 38 | .0063 | .15 | | .00396 | .008 | .008 | .006 |
| 39 | | .13 | | .00353 | .0075 | .0075 | .0052 |
| 40 | | .12 | | .00314 | .007 | .007 | .0048 |
| 41 | | .11 | | | | | .0044 |
| 42 | | .10 | | | | | .004 |
| 43 | | .09 | | | | | .0036 |
| 44 | | .08 | | | | | .0032 |
| 45 | | .07 | | | | | .0028 |
| 46 | | .06 | | | | | .0024 |
| 47 | | .05 | | | | | .002 |
| 48 | | .04 | | | | | .0016 |
| 49 | | .03 | | | | | .0012 |
| 50 | | .025 | | | | | .001 |

*Legal standard      †Legal standard in Great Britain

## STANDARD DECIMAL GAUGE*

| Thickness | | | Weight of Square Foot, in Pounds | | Thickness | | | Weight of Square Foot, in Pounds | |
|---|---|---|---|---|---|---|---|---|---|
| Decimal Inch | Fraction Inch | Millimeters | Iron | Steel | Decimal Inch | Fraction Inch | Millimeters | Iron | Steel |
| .002 | $\frac{1}{500}$ | .0508 | .08 | .082 | .060 | $\frac{3}{50}$ | 1.5240 | 2.40 | 2.448 |
| .004 | $\frac{1}{250}$ | .1016 | .16 | .163 | .065 | $\frac{13}{200}$ | 1.6510 | 2.60 | 2.652 |
| .006 | $\frac{3}{500}$ | .1524 | .24 | .245 | .070 | $\frac{7}{100}$ | 1.7780 | 2.80 | 2.856 |
| .008 | $\frac{1}{125}$ | .2032 | .32 | .326 | .075 | $\frac{3}{40}$ | 1.9050 | 3.00 | 3.060 |
| .010 | $\frac{1}{100}$ | .2540 | .40 | .408 | .080 | $\frac{2}{25}$ | 2.0320 | 3.20 | 3.264 |
| .012 | $\frac{3}{250}$ | .3048 | .48 | .490 | .085 | $\frac{17}{200}$ | 2.1590 | 3.40 | 3.468 |
| .014 | $\frac{7}{500}$ | .3556 | .56 | .571 | .090 | $\frac{9}{100}$ | 2.2860 | 3.60 | 3.672 |
| .016 | $\frac{2}{125}$ | .4064 | .64 | .653 | .095 | $\frac{19}{200}$ | 2.4130 | 3.80 | 3.876 |
| .018 | $\frac{9}{500}$ | .4572 | .72 | .734 | .100 | $\frac{1}{10}$ | 2.5400 | 4.00 | 4.080 |
| .020 | $\frac{1}{50}$ | .5080 | .80 | .816 | .110 | $\frac{11}{100}$ | 2.7940 | 4.40 | 4.488 |
| .022 | $\frac{11}{500}$ | .5588 | .88 | .898 | .125 | $\frac{1}{8}$ | 3.1750 | 5.00 | 5.100 |
| .025 | $\frac{1}{40}$ | .6350 | 1.00 | 1.020 | .135 | $\frac{27}{200}$ | 3.4290 | 5.40 | 5.508 |
| .028 | $\frac{7}{250}$ | .7112 | 1.12 | 1.142 | .150 | $\frac{3}{20}$ | 3.8100 | 6.00 | 6.120 |
| .032 | $\frac{4}{125}$ | .8128 | 1.28 | 1.306 | .165 | $\frac{33}{200}$ | 4.1910 | 6.60 | 6.732 |
| .036 | $\frac{9}{250}$ | .9144 | 1.44 | 1.469 | .180 | $\frac{9}{50}$ | 4.5720 | 7.20 | 7.344 |
| .040 | $\frac{1}{25}$ | 1.0160 | 1.60 | 1.632 | .200 | $\frac{1}{5}$ | 5.0800 | 8.00 | 8.160 |
| .045 | $\frac{9}{200}$ | 1.1430 | 1.80 | 1.836 | .220 | $\frac{11}{50}$ | 5.5880 | 8.80 | 8.976 |
| .050 | $\frac{1}{20}$ | 1.2700 | 2.00 | 2.040 | .240 | $\frac{6}{25}$ | 6.0960 | 9.60 | 9.792 |
| .055 | $\frac{11}{200}$ | 1.3970 | 2.20 | 2.244 | .250 | $\frac{1}{4}$ | 6.3500 | 10.00 | 10.200 |

* The weights per square foot of sheet metal given in this table are based on a weight of 480 lb. per cu. ft., for iron and one of 489.6 lb. per cu. ft. for steel.

# MISCELLANEOUS TABLES

## WEIGHT OF WROUGHT-IRON BOLTHEADS, NUTS, AND WASHERS

| Diameter of Bolt Inches | Hexagon Heads and Nuts Per Pair | Square Heads and Nuts Per Pair | Round Washers Per Pair |
|---|---|---|---|
| $\frac{1}{4}$ | 20 to 1 lb. | 16 to 1 lb. | 20 to 1 lb. |
| $\frac{3}{8}$ | 10 to 1 lb. | $8\frac{1}{2}$ to 1 lb. | 10 to 1 lb. |
| $\frac{1}{2}$ | 5 to 1 lb. | $4\frac{1}{2}$ to 1 lb. | 5 to 1 lb. |
| $\frac{5}{8}$ | $2\frac{1}{4}$ to 1 lb. | $2\frac{1}{2}$ to 1 lb. | 3 to 1 lb. |
| $\frac{3}{4}$ | 2 to 1 lb. | .56 lb. | .63 lb. |
| $\frac{7}{8}$ | .77 lb. | .88 lb. | .77 lb. |
| 1 | 1.25 lb. | 1.31 lb. | 1.25 lb. |
| $1\frac{1}{8}$ | 1.75 lb. | 2.10 lb. | 1.75 lb. |
| $1\frac{1}{4}$ | 2.13 lb. | 2.56 lb. | 2.25 lb. |
| $1\frac{3}{8}$ | 3.00 lb. | 3.60 lb. | 3.25 lb. |
| $1\frac{1}{2}$ | 3.75 lb. | 4.42 lb. | 4.25 lb. |
| $1\frac{5}{8}$ | 4.75 lb. | 5.70 lb. | 5.25 lb. |
| $1\frac{3}{4}$ | 5.75 lb. | 7.00 lb. | 6.50 lb. |
| $1\frac{7}{8}$ | 7.27 lb. | 8.72 lb. | 8.00 lb. |
| 2 | 8.75 lb. | 10.50 lb. | 9.60 lb. |

## WEIGHTS OF SHEETS AND PLATES OF STEEL, WROUGHT IRON, COPPER, AND BRASS

(*Cambria Steel Co*)

| American, or Brown & Sharpe, Gauge Number | Thickness Inch | Weight Per Square Foot, in Pounds | | | |
|---|---|---|---|---|---|
| | | Steel | Iron | Copper | Brass |
| 0000 | .460000 | 18.7680 | 18.4000 | 20.8380 | 19.6880 |
| 000 | .409642 | 16.7134 | 16.3857 | 18.5568 | 17.5327 |
| 00 | .364796 | 14.8837 | 14.5918 | 16.5253 | 15.6133 |
| 0 | .324861 | 13.2543 | 12.9944 | 14.7162 | 13.9041 |
| 1 | .289297 | 11.8033 | 11.5719 | 13.1052 | 12.3819 |
| 2 | .257627 | 10.5112 | 10.3051 | 11.6705 | 11.0264 |
| 3 | .229423 | 9.3605 | 9.1769 | 10.3929 | 9.8193 |
| 4 | .204307 | 8.3357 | 8.1723 | 9.2551 | 8.7443 |
| 5 | .181940 | 7.4232 | 7.2776 | 8.2419 | 7.7870 |
| 6 | .162023 | 6.6105 | 6.4809 | 7.3396 | 6.9346 |
| 7 | .144285 | 5.8868 | 5.7714 | 6.5361 | 6.1754 |
| 8 | .128490 | 5.2424 | 5.1396 | 5.8206 | 5.4994 |
| 9 | .114423 | 4.6685 | 4.5769 | 5.1834 | 4.8973 |
| 10 | .101897 | 4.1574 | 4.0759 | 4.6159 | 4.3612 |
| 11 | .090742 | 3.7023 | 3.6297 | 4.1106 | 3.8838 |
| 12 | .080808 | 3.2970 | 3.2323 | 3.6606 | 3.4586 |
| 13 | .071962 | 2.9360 | 2.8785 | 3.2599 | 3.0800 |
| 14 | .064084 | 2.6146 | 2.5634 | 2.9030 | 2.7428 |
| 15 | .057068 | 2.3284 | 2.2827 | 2.5852 | 2.4425 |
| 16 | .050821 | 2.0735 | 2.0328 | 2.3022 | 2.1751 |
| 17 | .045257 | 1.8465 | 1.8103 | 2.0501 | 1.9370 |
| 18 | .040303 | 1.6444 | 1.6121 | 1.8257 | 1.7250 |
| 19 | .035890 | 1.4643 | 1.4356 | 1.6258 | 1.5361 |
| 20 | .031961 | 1.3040 | 1.2784 | 1.4478 | 1.3679 |
| 21 | .028462 | 1.1612 | 1.1385 | 1.2893 | 1.2182 |
| 22 | .025346 | 1.0341 | 1.0138 | 1.1482 | 1.0848 |
| 23 | .022572 | .92094 | .90288 | 1.0225 | .96608 |
| 24 | .020101 | .82012 | .80404 | .91058 | .86032 |
| 25 | .017900 | .73032 | .71600 | .81087 | .76612 |
| 26 | .015941 | .65039 | .63764 | .72213 | .68227 |
| 27 | .014195 | .57916 | .56780 | .64303 | .60755 |
| 28 | .012641 | .51575 | .50564 | .57264 | .54103 |
| 29 | .011257 | .45929 | .45028 | .50994 | .48180 |
| 30 | .010025 | .40902 | .40100 | .45413 | .42907 |
| 31 | .008928 | .36426 | .35712 | .40444 | .38212 |
| 32 | .007950 | .32436 | .31800 | .36014 | .34026 |
| 33 | .007080 | .28886 | .28320 | .32072 | .30302 |
| 34 | .006305 | .25724 | .25220 | .28562 | .26985 |
| 35 | .005615 | .22909 | .22460 | .25436 | .24032 |
| 36 | .005000 | .20400 | .20000 | .22650 | .21400 |
| 37 | .004453 | .18168 | .17812 | .20172 | .19059 |
| 38 | .003965 | .16177 | .15860 | .17961 | .16970 |
| 39 | .003531 | .14406 | .14124 | .15995 | .15113 |
| 40 | .003144 | .12828 | .12576 | .14242 | .13456 |

## WEIGHT OF CAST-IRON PIPE PER FT., IN POUNDS*

| Diameter of Pipe Inches | Thickness of Pipe, in Inches | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ¼ | ⅜ | ½ | ⅝ | ¾ | ⅞ | 1 | 1⅛ | 1¼ | 1⅜ | 1½ | 1¾ | 2 |
| | Weight of Pipe, in Pounds | | | | | | | | | | | | |
| 1 | 3.07 | 5.07 | 7.38 | | | | | | | | | | |
| 1¼ | 3.69 | 6.00 | 8.61 | | | | | | | | | | |
| 1½ | 4.30 | 6.92 | 9.84 | | | | | | | | | | |
| 1¾ | 4.92 | 7.84 | 11.10 | | | | | | | | | | |
| 2 | 5.53 | 8.76 | 12.30 | 16.2 | | | | | | | | | |
| 2¼ | 6.15 | 9.69 | 13.50 | 17.7 | | | | | | | | | |
| 2½ | 6.76 | 10.60 | 14.80 | 19.2 | 24.0 | | | | | | | | |
| 2¾ | 7.37 | 11.50 | 16.00 | 20.8 | 25.9 | | | | | | | | |
| 3 | 7.98 | 12.50 | 17.20 | 22.3 | 27.7 | 33.4 | | | | | | | |
| 3½ | 9.21 | 14.30 | 19.70 | 25.4 | 31.4 | 37.7 | | | | | | | |
| 4 | 10.30 | 16.10 | 22.20 | 28.5 | 35.1 | 42.0 | | | | | | | |
| 4½ | 11.70 | 18.00 | 24.60 | 31.5 | 38.8 | 46.3 | | | | | | | |
| 5 | 12.90 | 19.80 | 27.10 | 34.6 | 42.5 | 50.6 | | | | | | | |
| 5½ | 14.20 | 21.70 | 29.50 | 37.7 | 46.1 | 54.9 | | | | | | | |
| 6 | 15.40 | 23.50 | 32.00 | 40.8 | 49.8 | 59.2 | 68.9 | | | | | | |
| 6½ | 16.60 | 25.40 | 34.50 | 43.8 | 53.5 | 63.5 | 73.8 | 84.4 | | | | | |
| 7 | 17.80 | 27.20 | 36.90 | 46.9 | 57.2 | 67.8 | 78.7 | 89.4 | | | | | |
| 7½ | 19.10 | 29.10 | 39.40 | 50.0 | 60.9 | 72.1 | 83.7 | 95.5 | 108 | | | | |
| 8 | 20.30 | 30.90 | 41.80 | 53.1 | 64.6 | 76.4 | 88.6 | 101.0 | 114 | 127 | | | |
| 8½ | 21.50 | 32.80 | 44.30 | 56.1 | 68.3 | 80.7 | 93.5 | 107.0 | 120 | 134 | 148 | | |
| 9 | 22.80 | 34.60 | 46.80 | 59.2 | 72.0 | 85.1 | 98.4 | 112.0 | 126 | 140 | 155 | | |
| 9½ | 24.00 | 36.40 | 49.20 | 62.3 | 75.7 | 89.3 | 103.0 | 118.0 | 132 | 147 | 163 | | |
| 10 | 25.10 | 38.30 | 51.70 | 65.3 | 79.4 | 93.6 | 108.0 | 123.0 | 138 | 154 | 170 | 202 | |
| 11 | 27.60 | 42.00 | 56.60 | 71.5 | 86.7 | 102.0 | 118.0 | 134.0 | 151 | 168 | 185 | 220 | |
| 12 | 30.00 | 45.70 | 61.50 | 77.7 | 94.1 | 111.0 | 128.0 | 145.0 | 163 | 181 | 199 | 237 | 275 |
| 13 | 32.50 | 49.40 | 66.40 | 83.8 | 102.0 | 120.0 | 138.0 | 156.0 | 175 | 195 | 214 | 254 | 294 |
| 14 | 35.00 | 53.10 | 71.40 | 89.4 | 109.0 | 128.0 | 148.0 | 168.0 | 188 | 208 | 229 | 271 | 314 |
| 15 | 37.40 | 56.70 | 76.30 | 96.1 | 116.0 | 137.0 | 158.0 | 179.0 | 200 | 222 | 244 | 289 | 334 |
| 16 | 39.10 | 60.40 | 81.20 | 102.0 | 124.0 | 145.0 | 167.0 | 190.0 | 212 | 235 | 258 | 306 | 353 |
| 17 | 42.30 | 64.10 | 86.10 | 108.0 | 131.0 | 154.0 | 177.0 | 201.0 | 225 | 249 | 273 | 323 | 373 |
| 18 | 44.80 | 67.80 | 91.00 | 115.0 | 139.0 | 163.0 | 187.0 | 212.0 | 237 | 262 | 288 | 340 | 393 |
| 19 | 47.30 | 71.50 | 96.00 | 121.0 | 146.0 | 171.0 | 197.0 | 223.0 | 249 | 276 | 303 | 357 | 412 |
| 20 | 49.70 | 75.20 | 101.00 | 127.0 | 153.0 | 180.0 | 207.0 | 234.0 | 261 | 289 | 317 | 375 | 432 |
| 21 | 52.20 | 78.90 | 106.00 | 133.0 | 161.0 | 188.0 | 217.0 | 245.0 | 274 | 303 | 332 | 392 | 452 |
| 22 | 54.60 | 82.60 | 111.00 | 139.0 | 168.0 | 196.0 | 227.0 | 256.0 | 286 | 316 | 347 | 409 | 471 |
| 23 | 57.10 | 86.30 | 116.00 | 145.0 | 175.0 | 206.0 | 236.0 | 267.0 | 298 | 330 | 362 | 426 | 491 |
| 24 | 59.60 | 89.90 | 121.00 | 152.0 | 183.0 | 214.0 | 246.0 | 278.0 | 311 | 343 | 375 | 444 | 511 |
| 25 | 62.00 | 93.60 | 126.00 | 158.0 | 190.0 | 223.0 | 256.0 | 289.0 | 323 | 357 | 391 | 461 | 531 |
| 26 | 64.50 | 97.30 | 131.00 | 164.0 | 198.0 | 231.0 | 266.0 | 300.0 | 335 | 370 | 406 | 478 | 550 |
| 27 | 66.90 | 101.00 | 135.00 | 170.0 | 205.0 | 240.0 | 276.0 | 311.0 | 348 | 384 | 421 | 495 | 570 |
| 28 | 69.40 | 105.00 | 140.00 | 176.0 | 212.0 | 249.0 | 286.0 | 323.0 | 360 | 397 | 436 | 512 | 590 |
| 29 | 71.80 | 109.00 | 145.00 | 182.0 | 220.0 | 257.0 | 295.0 | 334.0 | 372 | 411 | 450 | 530 | 609 |
| 30 | 74.20 | 112.00 | 150.00 | 188.0 | 227.0 | 266.0 | 305.0 | 345.0 | 384 | 424 | 465 | 547 | 629 |

* These weights are for plain pipe. For hautboy pipe, add 8 in. in length for each joint. For copper, add ¼; for lead, ⅔; for welded iron, 1/16.

# CONTENTS OF CYLINDERS OR PIPES FOR 1 FT. IN LENGTH*

### DIAMETERS IN INCHES

| Diameter Inches | Diameter in Decimals of a Foot | Gallons of 231 Cu. In. (U. S. Standard) | Weight of Water in Pounds, in 1 Ft. of Length | Diameter Inches | Diameter in Decimals of a Foot | Gallons of 231 Cu. In. (U. S. Standard.) | Weight of Water in Pounds, in 1 Ft. of Length |
|---|---|---|---|---|---|---|---|
| ⅛ | .0208 | .0025 | .02122 | 5 | .4167 | 1.020 | 8.488 |
| ¼ | .0417 | .0102 | .08488 | 5½ | .4583 | 1.234 | 10.270 |
| ⅜ | .0625 | .0230 | .19098 | 6 | .5000 | 1.469 | 12.223 |
| 1 | .0833 | .0408 | .33952 | 6½ | .5417 | 1.724 | 14.345 |
| 1¼ | .1042 | .0638 | .53050 | 7 | .5833 | 1.999 | 16.636 |
| 1½ | .1250 | .0918 | .76392 | 7½ | .6250 | 2.295 | 19.098 |
| 1¾ | .1458 | .1249 | 1.0398 | 8 | .6667 | 2.611 | 21.729 |
| 2 | .1667 | .1632 | 1.3581 | 8½ | .7083 | 2.948 | 24.530 |
| 2¼ | .1875 | .2066 | 1.7188 | 9 | .7500 | 3.305 | 27.501 |
| 2½ | .2083 | .2550 | 2.1220 | 9½ | .7917 | 3.682 | 30.641 |
| 2¾ | .2292 | .3085 | 2.5676 | 10 | .8333 | 4.080 | 33.952 |
| 3 | .2500 | .3672 | 3.0557 | 10½ | .8750 | 4.498 | 37.432 |
| 3½ | .2917 | .4998 | 4.1591 | 11 | .9167 | 4.937 | 41.082 |
| 4 | .3333 | .6528 | 5.4323 | 11½ | .9583 | 5.396 | 44.901 |
| 4½ | .3750 | .8263 | 6.8750 | 12 | 1.0000 | 5.875 | 48.891 |

### DIAMETERS IN FEET

| Diameter Inches | Diameter in Decimals of a Foot | Gallons of 231 Cu. In. (U. S. Standard) | Weight of Water in Pounds, in 1 Ft. of Length | Diameter Inches | Diameter in Decimals of a Foot | Gallons of 231 Cu. In. (U. S. Standard.) | Weight of Water in Pounds, in 1 Ft. of Length |
|---|---|---|---|---|---|---|---|
| 1¼ | 1.25 | 9.18 | 76.392 | 10 | 10.00 | 587.6 | 4,889.12 |
| 1½ | 1.50 | 13.22 | 110.000 | 10½ | 10.50 | 647.7 | 5,404.24 |
| 1¾ | 1.75 | 17.99 | 149.73 | 11 | 11.00 | 710.9 | 5,915.84 |
| 2 | 2.00 | 23.50 | 195.56 | 11½ | 11.50 | 777.0 | 6,485.72 |
| 2¼ | 2.25 | 29.74 | 247.51 | 12 | | 846.1 | 7,040.00 |
| 2½ | 2.50 | 36.72 | 305.57 | 13 | | 992.8 | 8,710.00 |
| 2¾ | 2.75 | 44.43 | 369.74 | 14 | | 1,152.0 | 10,096.00 |
| 3 | 3.00 | 52.88 | 440.00 | 15 | | 1,322.0 | 11,000.50 |
| 3¼ | 3.25 | 65.28 | 544.37 | 16 | | 1,504.0 | 12,516.00 |
| 3½ | 3.50 | 71.97 | 631.00 | 17 | | 1,698.0 | 14,166.00 |
| 3¾ | 3.75 | 82.62 | 687.53 | 18 | | 1,904.0 | 15,841.00 |
| 4 | 4.00 | 94.0 | 782.24 | 19 | | 2,121.0 | 17,691.00 |
| 4¼ | 4.25 | 106.1 | 885.40 | 20 | | 2,350.0 | 19,556.50 |
| 4½ | 4.50 | 119.0 | 990.04 | 21 | | 2,591.0 | 21,617.00 |
| 4¾ | 4.75 | 132.5 | 1,105.71 | 22 | | 2,844.0 | 23,663.00 |
| 5 | 5.00 | 146.9 | 1,222.28 | 23 | | 3,108.0 | 25,943.00 |
| 5¼ | 5.25 | 161.9 | 1,351.06 | 24 | | 3,384.0 | 28,160.00 |
| 5½ | 5.50 | 177.7 | 1,478.96 | 25 | | 3,672.0 | 30,557.00 |
| 5¾ | 5.75 | 194.3 | 1,621.43 | 26 | | 3,971.0 | 34,840.00 |
| 6 | 6.00 | 211.5 | 1,760.00 | 27 | | 4,283.0 | 35,641.00 |
| 6¼ | 6.25 | 229.5 | 1,915.18 | 28 | | 4,606.0 | 40,384.00 |
| 6½ | 6.50 | 248.2 | 2,177.48 | 29 | | 4,941.0 | 41,117.00 |
| 6¾ | 6.75 | 267.7 | 2,233.96 | 30 | | 5,288.0 | 44,002.00 |
| 7 | 7.00 | 287.9 | 2,524.00 | 31 | | 5,646.0 | 46,984.00 |
| 7½ | 7.50 | 330.5 | 2,750.12 | 32 | | 6,017.0 | 50,064.00 |
| 8 | 8.00 | 376.0 | 3,128.96 | 33 | | 6,398.0 | 53,242.00 |
| 8½ | 8.50 | 424.5 | 3,541.60 | 34 | | 6,792.0 | 56,664.00 |
| 9 | 9.00 | 475.9 | 3,960.16 | 35 | | 7,197.0 | 59,891.50 |
| 9½ | 9.50 | 530.2 | 4,422.84 | 36 | | 7,614.0 | 63,364.00 |

\* The contents of pipes or cylinders in gallons or pounds are to each other as the squares of their diameters. Thus, a pipe 9 ft. in diameter will contain 9 times as much as a 3-ft. pipe, or 4 times as much as a 4½-ft. pipe.

## STANDARD DIMENSIONS OF WROUGHT-IRON WELDED PIPES

| Nominal Diameter Inches | External Diameter Inches | Thickness Inch | Internal Diameter Inches | Internal Circumference Inches | External Circumference Inches | Length of Pipe per Square Foot of Internal Surface, in Feet | Length of Pipe per Square Foot of External Surface, in Feet | Internal Area Inches | Length of Pipe Containing 1 Cu. Ft. | Gallons in 1 Ft. in Length | Weight per Foot Pounds | Number of Threads per Inch of Screw |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1/8 | .40 | .068 | .27 | .85 | 1.27 | 14.15 | 9.440 | .057 | 2,513.0 | .002 | .24 | 27 |
| 1/4 | .54 | .088 | .36 | 1.14 | 1.70 | 10.50 | 7.075 | .104 | 1,383.3 | .002 | .42 | 18 |
| 3/8 | .67 | .091 | .49 | 1.55 | 2.12 | 7.67 | 5.657 | .192 | 751.2 | .005 | .56 | 18 |
| 1/2 | .84 | .109 | .62 | 1.96 | 2.65 | 6.13 | 4.502 | .305 | 472.4 | .010 | .84 | 14 |
| 3/4 | 1.05 | .113 | .82 | 2.59 | 3.30 | 4.64 | 3.637 | .533 | 270.0 | .023 | 1.13 | 14 |
| 1 | 1.31 | .134 | 1.05 | 3.29 | 4.13 | 3.66 | 2.903 | .863 | 166.9 | .040 | 1.67 | 11½ |
| 1¼ | 1.66 | .140 | 1.38 | 4.33 | 5.21 | 2.77 | 2.301 | 1.496 | 96.25 | .063 | 2.26 | 11½ |
| 1½ | 1.90 | .145 | 1.61 | 5.06 | 5.97 | 2.37 | 2.010 | 2.038 | 70.66 | .091 | 2.69 | 11½ |
| 2 | 2.37 | .154 | 2.07 | 6.49 | 7.46 | 1.85 | 1.611 | 3.355 | 42.91 | .163 | 3.67 | 11½ |
| 2½ | 2.87 | .204 | 2.47 | 7.75 | 9.03 | 1.55 | 1.328 | 4.783 | 30.10 | .255 | 5.77 | 8 |
| 3 | 3.50 | .217 | 3.07 | 9.64 | 11.00 | 1.24 | 1.091 | 7.388 | 19.50 | .367 | 7.55 | 8 |
| 3½ | 4.00 | .226 | 3.55 | 11.15 | 12.57 | 1.08 | 0.955 | 9.887 | 14.57 | .500 | 9.05 | 8 |
| 4 | 4.50 | .237 | 4.03 | 12.65 | 14.14 | .95 | 0.849 | 12.730 | 11.31 | .652 | 10.73 | 8 |
| 4½ | 5.00 | .247 | 4.51 | 14.15 | 15.71 | .85 | 0.765 | 15.939 | 9.02 | .826 | 12.49 | 8 |
| 5 | 5.56 | .259 | 5.04 | 15.85 | 17.47 | .78 | 0.629 | 19.990 | 7.20 | 1.02 | 14.56 | 8 |
| 6 | 6.62 | .280 | 6.06 | 19.05 | 20.81 | .63 | 0.577 | 28.889 | 4.98 | 1.46 | 18.77 | 8 |
| 7 | 7.62 | .301 | 7.02 | 22.06 | 23.95 | .54 | 0.505 | 38.737 | 3.72 | 2.00 | 23.41 | 8 |
| 8 | 8.62 | .322 | 7.98 | 25.08 | 27.10 | .48 | 0.444 | 50.039 | 2.88 | 2.61 | 28.35 | 8 |
| 9 | 9.69 | .344 | 9.00 | 28.28 | 30.43 | .42 | 0.394 | 63.633 | 2.29 | 3.30 | 34.08 | 8 |
| 10 | 10.75 | .366 | 10.02 | 31.47 | 33.77 | .38 | 0.355 | 78.838 | 1.82 | 4.08 | 40.64 | 8 |

## STRENGTH OF METALS PER SQUARE INCH

| Material | Ultimate Tensile Pounds | Ultimate Compression Pounds | Ultimate Shearing Pounds | Modulus of Rupture Pounds | Modulus of Elasticity Millions |
|---|---|---|---|---|---|
| Wrought iron | 50,000 | 44,000 | 44,000 | 48,000 | 27 |
| Shape iron | 48,000 | | | | 26 |
| Structural steel { | 60,000 | | | | |
| | 65,000 | 52,000 | 52,000 | 60,000 | 29 |
| Cast iron | 18,000 | 81,000 | 25,000 | 45,000 | 12 |
| Steel, castings | 70,000 | 70,000 | 60,000 | 70,000 | 30 |
| Brass, cast | 24,000 | *30,000 | 36,000 | 20,000 | 9 |
| Bronze, phosphor | 50,000 | | | | 14 |
| Bronze, aluminum | 75,000 | 120,000 | | | |
| Aluminum, commercial | 15,000 | 12,000 | 12,000 | | 11 |

*Unit stress producing 10% reduction in original length.

## STANDARD AND EXTRA-GAUGE STEEL BOILER TUBES

| Outside Diameter Inches | Standard Thickness | | Nominal Weight per Foot, in Pounds | | | | |
|---|---|---|---|---|---|---|---|
| | Nearest Birm. Wire Gauge | Inch | Standard Thickness | One Extra Wire Gauge | Two Extra Wire Gauges | Three Extra Wire Gauges | Four Extra Wire Gauges |
| 1 | 13 | .095 | .90 | 1.04 | 1.13 | 1.24 | 1.35 |
| 1¼ | 13 | .095 | 1.15 | 1.33 | 1.45 | 1.60 | 1.74 |
| 1½ | 13 | .095 | 1.40 | 1.62 | 1.77 | 1.96 | 2.14 |
| 1¾ | 13 | .095 | 1.66 | 1.91 | 2.09 | 2.31 | 2.53 |
| 2 | 13 | .095 | 1.91 | 2.20 | 2.41 | 2.67 | 2.93 |
| 2¼ | 13 | .095 | 2.16 | 2.49 | 2.73 | 3.03 | 3.32 |
| 2½ | 12 | .109 | 2.75 | 3.05 | 3.39 | 3.72 | 4.12 |
| 2¾ | 12 | .109 | 3.04 | 3.37 | 3.74 | 4.11 | 4.56 |
| 3 | 12 | .109 | 3.33 | 3.69 | 4.10 | 4.51 | 5.00 |
| 3¼ | 11 | .120 | 3.96 | 4.46 | 4.90 | 5.44 | 5.90 |
| 3½ | 11 | .120 | 4.28 | 4.82 | 5.30 | 5.88 | 6.38 |
| 3¾ | 11 | .120 | 4.60 | 5.18 | 5.69 | 6.32 | 6.86 |
| 4 | 10 | .134 | 5.47 | 6.09 | 6.76 | 7.34 | 8.23 |
| 4½ | 10 | .134 | 6.17 | 6.88 | 7.64 | 8.31 | 9.32 |
| 5 | 9 | .148 | 7.58 | 8.52 | 9.27 | 10.40 | 11.23 |
| 6 | 8 | .165 | 10.16 | 11.19 | 12.57 | 13.58 | 14.65 |
| 7 | 8 | .165 | 11.90 | 13.11 | 14.74 | 15.93 | 17.19 |
| 8 | 8 | .165 | 13.65 | 15.04 | 16.91 | 18.28 | 19.73 |
| 9 | 7 | .180 | 16.76 | 19.07 | 20.63 | 22.27 | 24.18 |
| 10 | 6 | .203 | 21.00 | 22.98 | 24.82 | 26.95 | 29.47 |
| 11 | 5 | .220 | 25.00 | 27.36 | 29.71 | 32.51 | 34.29 |
| 12 | 4½ | .229 | 28.50 | 31.19 | 34.01 | 36.52 | 39.92 |
| 13 | 4 | .238 | 32.06 | 35.25 | 38.57 | 40.70 | 45.98 |

## STANDARD LAP-WELDED CHARCOAL-IRON BOILER TUBES

| Diameter Inches | | Thickness Inch | Circumference Inches | | Transverse Area Square Inches | | Length of Tube in Feet per Square Foot of Surface | | Weight per Foot Pounds |
|---|---|---|---|---|---|---|---|---|---|
| Outside | Inside | | Outside | Inside | Outside | Inside | Outside | Inside | |
| 1 | .810 | .095 | 3.142 | 2.545 | .785 | .515 | 3.820 | 4.479 | .90 |
| 1¼ | 1.060 | .095 | 3.927 | 3.330 | 1.227 | .882 | 3.056 | 3.604 | 1.15 |
| 1½ | 1.310 | .095 | 4.712 | 4.115 | 1.767 | 1.348 | 2.547 | 2.916 | 1.40 |
| 1¾ | 1.560 | .095 | 5.498 | 4.901 | 2.405 | 1.911 | 2.183 | 2.448 | 1.65 |
| 2 | 1.810 | .095 | 6.283 | 5.686 | 3.142 | 2.573 | 1.910 | 2.110 | 1.91 |
| 2¼ | 2.060 | .095 | 7.069 | 6.472 | 3.976 | 3.333 | 1.698 | 1.854 | 2.16 |
| 2½ | 2.282 | .109 | 7.854 | 7.169 | 4.909 | 4.090 | 1.528 | 1.674 | 2.75 |
| 2¾ | 2.532 | .109 | 8.639 | 7.955 | 5.940 | 5.035 | 1.389 | 1.508 | 3.04 |
| 3 | 2.782 | .109 | 9.425 | 8.740 | 7.069 | 6.079 | 1.273 | 1.373 | 3.33 |
| 3¼ | 3.010 | .120 | 10.210 | 9.456 | 8.296 | 7.116 | 1.175 | 1.269 | 3.96 |
| 3½ | 3.260 | .120 | 10.996 | 10.242 | 9.621 | 8.347 | 1.091 | 1.172 | 4.28 |
| 3¾ | 3.510 | .120 | 11.781 | 11.027 | 11.045 | 9.676 | 1.019 | 1.088 | 4.60 |
| 4 | 3.732 | .134 | 12.566 | 11.724 | 12.566 | 10.939 | .955 | 1.024 | 5.47 |
| 4½ | 4.232 | .134 | 14.137 | 13.295 | 15.904 | 14.066 | .849 | .903 | 6.17 |
| 5 | 4.704 | .148 | 15.708 | 14.778 | 19.635 | 17.379 | .764 | .812 | 7.58 |
| 6 | 5.670 | .165 | 18.850 | 17.813 | 28.274 | 25.250 | .637 | .674 | 10.16 |
| 7 | 6.670 | .165 | 21.991 | 20.954 | 38.485 | 34.942 | .546 | .573 | 11.90 |
| 8 | 7.670 | .165 | 25.133 | 24.096 | 50.266 | 46.204 | .477 | .498 | 13.65 |
| 9 | 8.640 | .180 | 28.274 | 27.143 | 63.617 | 58.630 | .424 | .442 | 16.76 |
| 10 | 9.594 | .203 | 31.416 | 30.141 | 78.540 | 72.292 | .382 | .398 | 21.00 |

*PROPERTIES OF MATERIALS*

## WEIGHT OF WROUGHT IRON*

| Thickness or Diameter Inches | Weight of 1 Sq. Ft. Pounds | Weight of Square Bar 1 Ft. Long Pounds | Weight of Round Bar 1 Ft. Long Pounds | Thickness or Diameter Inches | Weight of 1 Sq. Ft. Pounds | Weight of Square Bar 1 Ft. Long Pounds | Weight of Round Bar 1 Ft. Long Pounds |
|---|---|---|---|---|---|---|---|
| 1/8 | 5.052 | .0526 | .0414 | 4 3/8 | 176.8 | 64.47 | 50.63 |
| 1/4 | 10.10 | .2105 | .1653 | 4 1/2 | 181.9 | 68.20 | 53.57 |
| 3/8 | 15.16 | .4736 | .3720 | 4 5/8 | 186.9 | 72.05 | 56.59 |
| 1/2 | 20.21 | .8420 | .6613 | 4 3/4 | 192.0 | 75.99 | 59.69 |
| 5/8 | 25.26 | 1.316 | 1.033 | 4 7/8 | 197.0 | 80.05 | 62.87 |
| 3/4 | 30.31 | 1.895 | 1.488 | 5 | 202.1 | 84.20 | 66.13 |
| 7/8 | 35.37 | 2.579 | 2.025 | 5 1/4 | 212.2 | 92.83 | 72.91 |
| 1 | 40.42 | 3.368 | 2.645 | 5 1/2 | 222.3 | 101.9 | 80.02 |
| 1 1/8 | 45.47 | 4.263 | 3.348 | 5 3/4 | 232.4 | 111.4 | 87.46 |
| 1 1/4 | 50.52 | 5.263 | 4.133 | 6 | 242.5 | 121.3 | 95.23 |
| 1 3/8 | 55.57 | 6.368 | 5.001 | 6 1/4 | 252.6 | 131.6 | 103.3 |
| 1 1/2 | 60.63 | 7.578 | 5.952 | 6 1/2 | 262.7 | 142.3 | 111.8 |
| 1 5/8 | 65.68 | 8.893 | 6.985 | 6 3/4 | 272.8 | 153.5 | 120.5 |
| 1 3/4 | 70.73 | 10.31 | 8.101 | 7 | 282.9 | 165.0 | 129.6 |
| 1 7/8 | 75.78 | 11.84 | 9.300 | 7 1/4 | 293.0 | 177.0 | 139.0 |
| 2 | 80.83 | 13.47 | 10.58 | 7 1/2 | 303.1 | 189.5 | 148.8 |
| 2 1/8 | 85.89 | 15.21 | 11.95 | 7 3/4 | 313.2 | 202.3 | 158.9 |
| 2 1/4 | 90.94 | 17.05 | 13.39 | 8 | 323.3 | 215.6 | 169.3 |
| 2 3/8 | 95.99 | 19.00 | 14.92 | 8 1/4 | 333.4 | 229.3 | 180.1 |
| 2 1/2 | 101.0 | 21.05 | 16.53 | 8 1/2 | 343.5 | 243.4 | 191.1 |
| 2 5/8 | 106.1 | 23.21 | 18.23 | 8 3/4 | 353.6 | 247.9 | 202.5 |
| 2 3/4 | 111.2 | 25.47 | 20.01 | 9 | 363.8 | 272.8 | 214.3 |
| 2 7/8 | 116.2 | 27.84 | 21.87 | 9 1/4 | 373.9 | 288.2 | 226.3 |
| 3 | 121.3 | 30.31 | 23.81 | 9 1/2 | 384.0 | 304.0 | 238.7 |
| 3 1/8 | 126.3 | 32.89 | 25.83 | 9 3/4 | 394.1 | 320.2 | 251.5 |
| 3 1/4 | 131.4 | 35.57 | 27.94 | 10 | 404.2 | 336.8 | 264.5 |
| 3 3/8 | 136.4 | 38.37 | 30.13 | 10 1/4 | 414.3 | 353.9 | 277.9 |
| 3 1/2 | 141.5 | 41.26 | 32.41 | 10 1/2 | 424.4 | 371.3 | 291.6 |
| 3 5/8 | 146.5 | 44.26 | 34.76 | 10 3/4 | 434.5 | 389.2 | 305.7 |
| 3 3/4 | 151.6 | 47.37 | 37.20 | 11 | 444.6 | 407.5 | 320.1 |
| 3 7/8 | 156.6 | 50.57 | 39.72 | 11 1/4 | 454.7 | 426.3 | 334.8 |
| 4 | 161.7 | 53.89 | 42.33 | 11 1/2 | 464.8 | 445.4 | 349.8 |
| 4 1/8 | 166.7 | 57.31 | 45.01 | 11 3/4 | 474.9 | 465.0 | 365.2 |
| 4 1/4 | 171.8 | 60.84 | 47.78 | 12 | 485.0 | 485.0 | 380.9 |

*This table is for wrought iron. Multiply by .95 for weight of cast iron; by 1.02 for steel; by 1.16 for copper; by 1.09 for brass; by 1.48 for lead.

## DIAMETER AND NUMBER OF WOOD SCREWS

| Formulas for Wood Screws | No. | Diameter | No. | Diameter | No. | Diameter |
|---|---|---|---|---|---|---|
| $N$ = number | 0 | .056 | 11 | .201 | 22 | .347 |
| $D$ = diameter | 1 | .069 | 12 | .215 | 23 | .361 |
| $D = (N \times .01325) + .056$ | 2 | .082 | 13 | .228 | 24 | .374 |
| $N = \dfrac{D - .056}{.01325}$ | 3 | .096 | 14 | .241 | 25 | .387 |
| | 4 | .109 | 15 | .255 | 26 | .401 |
| | 5 | .122 | 16 | .268 | 27 | .414 |
| | 6 | .135 | 17 | .281 | 28 | .427 |
| | 7 | .149 | 18 | .293 | 29 | .440 |
| | 8 | .162 | 19 | .308 | 30 | .453 |
| | 9 | .175 | 20 | .321 | | |
| | 10 | .188 | 21 | .334 | | |

## SPIKES AND NAILS

Columns below are grouped as: **Standard Steel-Wire Nails** (Common, Finishing) · **Common Iron Nails** · **Steel-Wire Spikes**

| Sizes | Length Inches | Common Diameter Inch | Common Number per Pound | Finishing Diameter Inch | Finishing Number per Pound | Sizes | Length Inches | Number per Pound | Length Inches | Diameter Inch | Number per Pound |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2d | 1 | .0524 | 1,060 | .0453 | 1,558 | 2d | 1 | 800 | 3 | .1620 | 41 |
| 3d | 1¼ | .0588 | 640 | .0508 | 913 | 3d | 1¼ | 400 | 3½ | .1819 | 30 |
| 4d | 1½ | .0720 | 380 | .0508 | 761 | 4d | 1½ | 300 | 4 | .2043 | 23 |
| 5d | 1¾ | .0764 | 275 | .0571 | 500 | 5d | 1¾ | 200 | 4½ | .2294 | 17 |
| 6d | 2 | .0808 | 210 | .0641 | 350 | 6d | 2 | 150 | 5 | .2576 | 13 |
| 7d | 2¼ | .0858 | 160 | .0641 | 315 | 7d | 2¼ | 120 | 5½ | .2893 | 11 |
| 8d | 2½ | .0935 | 115 | .0720 | 214 | 8d | 2½ | 85 | 6 | .2893 | 10 |
| 9d | 2¾ | .0963 | 93 | .0720 | 195 | 9d | 2¾ | 75 | 6½ | .2249 | 7½ |
| 10d | 3 | .1082 | 77 | .0808 | 137 | 10d | 3 | 60 | 7 | .2249 | 7 |
| 12d | 3¼ | .1144 | 60 | .0808 | 127 | 12d | 3¼ | 50 | 8 | .3648 | 5 |
| 16d | 3½ | .1285 | 48 | .0907 | 90 | 16d | 3½ | 40 | 9 | .3648 | 4½ |
| 20d | 4 | .1620 | 31 | .1019 | 62 | 20d | 4 | 20 | | | |
| 30d | 4½ | .1819 | 22 | | | 30d | 4½ | 16 | | | |
| 40d | 5 | .2043 | 17 | | | 40d | 5 | 14 | | | |
| 50d | 5½ | .2294 | 13 | | | 50d | 5½ | 11 | | | |
| 60d | 6 | .2576 | 11 | | | 60d | 6 | 8 | | | |

## WEIGHT OF 100 BOLTS WITH SQUARE HEADS AND NUTS
*(The Carnegie Steel Co., Limited)*

| Length Under Head to Point Inches | Diameter of Bolts — ¼ In. | ⁵⁄₁₆ In. | ⅜ In. | ⁷⁄₁₆ In. | ½ In. | ⅝ In. | ¾ In. | ⅞ In. | 1 In. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Weight, in Pounds | | | | |
| 1½ | 4.0 | 7.0 | 10.5 | 15.2 | 22.5 | 39.5 | 63.0 | | |
| 1¾ | 4.4 | 7.5 | 11.3 | 16.3 | 23.8 | 41.6 | 66.0 | | |
| 2 | 4.8 | 8.0 | 12.0 | 17.4 | 25.2 | 43.8 | 69.0 | 109.0 | 163 |
| 2¼ | 5.2 | 8.5 | 12.8 | 18.5 | 26.5 | 45.8 | 72.0 | 113.3 | 169 |
| 2½ | 5.5 | 9.0 | 13.5 | 19.6 | 27.8 | 48.0 | 75.0 | 117.5 | 174 |
| 2¾ | 5.8 | 9.5 | 14.3 | 20.7 | 29.1 | 50.1 | 78.0 | 121.8 | 180 |
| 3 | 6.3 | 10.0 | 15.0 | 21.8 | 30.5 | 52.3 | 81.0 | 126.0 | 185 |
| 3½ | 7.0 | 11.0 | 16.5 | 24.0 | 33.1 | 56.5 | 87.0 | 134.3 | 196 |
| 4 | 7.8 | 12.0 | 18.0 | 26.2 | 35.8 | 60.8 | 93.1 | 142.5 | 207 |
| 4½ | 8.5 | 13.0 | 19.5 | 28.4 | 38.4 | 65.0 | 99.1 | 151.0 | 218 |
| 5 | 9.3 | 14.0 | 21.0 | 30.6 | 41.1 | 69.3 | 105.2 | 159.6 | 229 |
| 5½ | 10.0 | 15.0 | 22.5 | 32.8 | 43.7 | 73.5 | 111.3 | 168.0 | 240 |
| 6 | 10.8 | 16.0 | 24.0 | 35.0 | 46.4 | 77.8 | 117.3 | 176.6 | 251 |
| 6½ | | | 25.5 | 37.2 | 49.0 | 82.0 | 123.4 | 185.0 | 262 |
| 7 | | | 27.0 | 39.4 | 51.7 | 86.3 | 129.4 | 193.7 | 273 |
| 7½ | | | 28.5 | 41.6 | 54.3 | 90.5 | 135.0 | 202.0 | 284 |
| 8 | | | 30.0 | 43.8 | 59.6 | 94.8 | 141.5 | 210.7 | 295 |
| 9 | | | | 46.0 | 64.9 | 103.3 | 153.6 | 227.8 | 317 |
| 10 | | | | 48.2 | 70.2 | 111.8 | 165.7 | 244.8 | 339 |
| 11 | | | | 50.4 | 75.5 | 120.3 | 177.8 | 261.9 | 360 |
| 12 | | | | 52.6 | 80.8 | 128.8 | 189.9 | 278.9 | 382 |
| 13 | | | | | 86.1 | 137.3 | 202.0 | 296.0 | 404 |
| 14 | | | | | 91.4 | 145.8 | 214.1 | 313.0 | 426 |
| 15 | | | | | 96.7 | 154.3 | 226.2 | 330.1 | 448 |
| 16 | | | | | 102.0 | 162.8 | 238.3 | 347.1 | 470 |
| 17 | | | | | 107.3 | 171.0 | 250.4 | 364.2 | 492 |
| 18 | | | | | 112.6 | 179.5 | 262.6 | 381.2 | 514 |
| 19 | | | | | 117.9 | 188.0 | 274.7 | 398.3 | 536 |
| 20 | | | | | 123.2 | 196.5 | 286.8 | 415.3 | 558 |
| Per Inch Additional | 1.4 | 2.1 | 3.1 | 4.2 | 5.5 | 8.5 | 12.3 | 16.7 | 21.8 |

## PROPORTIONS OF THE UNITED STATES STANDARD SCREW THREADS, NUTS, AND BOLT HEADS

| Diam. of Screw Inches | Threads per Inch | Diam. of Core Inch | Width of Flat Inch | Inside Diam. Inch | Outside Diam. Inch | Diagonal Inch | Height of Head Inches |
|---|---|---|---|---|---|---|---|
| 1/4 | 20 | .185 | .0062 | 1/2 | 37/64 | 45/64 | 1/4 |
| 5/16 | 18 | .240 | .0070 | 19/32 | 11/16 | 27/32 | 19/64 |
| 3/8 | 16 | .294 | .0078 | 11/16 | 51/64 | 31/32 | 11/32 |
| 7/16 | 14 | .344 | .0089 | 25/32 | 29/32 | 1 7/64 | 25/64 |
| 1/2 | 13 | .400 | .0096 | 7/8 | 1 1/64 | 1 15/64 | 7/16 |
| 9/16 | 12 | .454 | .0104 | 31/32 | 1 1/8 | 1 3/8 | 31/64 |
| 5/8 | 11 | .507 | .0113 | 1 1/16 | 1 15/64 | 1 1/2 | 17/32 |
| 3/4 | 10 | .620 | .0125 | 1 1/4 | 1 7/16 | 1 49/64 | 5/8 |
| 7/8 | 9 | .731 | .0140 | 1 7/16 | 1 21/32 | 2 1/32 | 23/32 |
| 1 | 8 | .837 | .0156 | 1 5/8 | 1 7/8 | 2 19/64 | 13/16 |
| 1 1/8 | 7 | .940 | .0180 | 1 13/16 | 2 3/32 | 2 9/16 | 29/32 |
| 1 1/4 | 7 | 1.065 | .0180 | 2 | 2 5/16 | 2 53/64 | 1 |
| 1 3/8 | 6 | 1.160 | .0210 | 2 3/16 | 2 17/32 | 3 3/32 | 1 3/32 |
| 1 1/2 | 6 | 1.284 | .0210 | 2 3/8 | 2 3/4 | 3 23/64 | 1 3/16 |
| 1 5/8 | 5 1/2 | 1.389 | .0227 | 2 9/16 | 2 61/64 | 3 5/8 | 1 9/32 |
| 1 3/4 | 5 | 1.490 | .0250 | 2 3/4 | 3 11/64 | 3 57/64 | 1 3/8 |
| 1 7/8 | 5 | 1.615 | .0250 | 2 15/16 | 3 25/64 | 4 5/32 | 1 15/32 |
| 2 | 4 1/2 | 1.712 | .0280 | 3 1/8 | 3 39/64 | 4 27/64 | 1 9/16 |
| 2 1/4 | 4 1/2 | 1.962 | .0280 | 3 1/2 | 4 3/64 | 4 61/64 | 1 3/4 |
| 2 1/2 | 4 | 2.175 | .0310 | 3 7/8 | 4 15/32 | 5 31/64 | 1 15/16 |
| 2 3/4 | 4 | 2.425 | .0310 | 4 1/4 | 4 29/32 | 6 1/64 | 2 1/8 |
| 3 | 3 1/2 | 2.628 | .0357 | 4 5/8 | 5 11/32 | 6 35/64 | 2 5/16 |
| 3 1/4 | 3 1/2 | 2.878 | .0357 | 5 | 5 25/32 | 7 1/16 | 2 1/2 |
| 3 1/2 | 3 1/4 | 3.100 | .0384 | 5 3/8 | 6 13/64 | 7 19/32 | 2 11/16 |
| 3 3/4 | 3 | 3.317 | .0410 | 5 3/4 | 6 41/64 | 8 1/8 | 2 7/8 |
| 4 | 3 | 3.566 | .0410 | 6 1/8 | 7 5/64 | 8 21/32 | 3 1/16 |
| 4 1/4 | 2 7/8 | 3.798 | .0435 | 6 1/2 | 7 1/2 | 9 3/16 | 3 1/4 |
| 4 1/2 | 2 3/4 | 4.027 | .0460 | 6 7/8 | 7 15/16 | 9 23/32 | 3 7/16 |
| 4 3/4 | 2 5/8 | 4.255 | .0480 | 7 1/4 | 8 3/8 | 10 1/4 | 3 5/8 |
| 5 | 2 1/2 | 4.480 | .0500 | 7 5/8 | 8 13/16 | 10 25/32 | 3 13/16 |
| 5 1/4 | 2 1/2 | 4.730 | .0500 | 8 | 9 1/4 | 11 5/16 | 4 |
| 5 1/2 | 2 3/8 | 4.953 | .0526 | 8 3/8 | 9 43/64 | 11 27/32 | 4 3/16 |
| 5 3/4 | 2 1/4 | 5.203 | .0526 | 8 3/4 | 10 7/64 | 12 3/8 | 4 3/8 |
| 6 | 2 1/4 | 5.423 | .0555 | 9 1/8 | 10 35/64 | 12 29/32 | 4 9/16 |

The threads have an angle of 60°, with flat tops and bottoms, and are of the following proportions:

*Notation of Letters, All Dimensions in Inches*

$D$ = outside diameter of screw;
$d$ = diameter of root of thread, or of hole in nut;
$p$ = pitch of screw;
$t$ = number of threads per inch;
$f$ = flat top and bottom;
$o$ = outside diameter of hexagon nut or bolt head;

$i$ = inside diameter of hexagon, or side of square nut or bolt head;
$s$ = diagonal of square nut or bolt head;
$h$ = height of rough or unfinished bolt head.

The height of a finished nut or bolt head is made equal to the diameter $D$ of the screw.

$$p = \frac{\sqrt{16D+10}-2.909}{16.64}, \qquad t = \frac{1}{p} \qquad s = 1.414i$$

$$d = D - \frac{1.299}{t} \qquad i = \frac{3D}{2} + \frac{1}{8} \qquad o = 1.155i \qquad f = \frac{p}{8}$$

## WEIGHT OF 1 LIN. FT. OF FLAT WROUGHT IRON*

| Size Inches | Weight Pounds | Size Inches | Weight Pounds | Size Inches | Weight Pounds |
|---|---|---|---|---|---|
| 1 × ¼ | .85 | 5¼ × ⅜ | 6.65 | 4 × ⅝ | 8.45 |
| 1¼ × ¼ | 1.06 | 5½ × ⅜ | 6.97 | 4¼ × ⅝ | 8.98 |
| 1½ × ¼ | 1.27 | 5¾ × ⅜ | 7.29 | 4½ × ⅝ | 9.51 |
| 1¾ × ¼ | 1.48 | 6 × ⅜ | 7.60 | 4¾ × ⅝ | 10.03 |
| 2 × ¼ | 1.69 | | | 5 × ⅝ | 10.56 |
| 2¼ × ¼ | 1.90 | 1 × ½ | 1.69 | 5¼ × ⅝ | 11.09 |
| 2½ × ¼ | 2.11 | 1¼ × ½ | 2.11 | 5½ × ⅝ | 11.62 |
| 2¾ × ¼ | 2.32 | 1½ × ½ | 2.53 | 5¾ × ⅝ | 12.15 |
| 3 × ¼ | 2.53 | 1¾ × ½ | 2.96 | 6 × ⅝ | 12.67 |
| 3¼ × ¼ | 2.75 | 2 × ½ | 3.38 | | |
| 3½ × ¼ | 2.96 | 2¼ × ½ | 3.80 | 1 × ¾ | 2.53 |
| 3¾ × ¼ | 3.17 | 2½ × ½ | 4.22 | 1¼ × ¾ | 3.17 |
| 4 × ¼ | 3.38 | 2¾ × ½ | 4.65 | 1½ × ¾ | 3.80 |
| 4¼ × ¼ | 3.59 | 3 × ½ | 5.07 | 1¾ × ¾ | 4.44 |
| 4½ × ¼ | 3.80 | 3¼ × ½ | 5.49 | 2 × ¾ | 5.07 |
| 4¾ × ¼ | 4.01 | 3½ × ½ | 5.92 | 2¼ × ¾ | 5.70 |
| 5 × ¼ | 4.22 | 3¾ × ½ | 6.33 | 2½ × ¾ | 6.33 |
| 5¼ × ¼ | 4.44 | 4 × ½ | 6.76 | 2¾ × ¾ | 6.97 |
| 5½ × ¼ | 4.65 | 4¼ × ½ | 7.18 | 3 × ¾ | 7.60 |
| 6 × ¼ | 5.07 | 4½ × ½ | 7.60 | 3¼ × ¾ | 8.24 |
| | | 4¾ × ½ | 8.03 | 3½ × ¾ | 8.87 |
| 1 × ⅜ | 1.27 | 5 × ½ | 8.45 | 3¾ × ¾ | 9.51 |
| 1¼ × ⅜ | 1.58 | 5¼ × ½ | 8.87 | 4 × ¾ | 10.14 |
| 1½ × ⅜ | 1.90 | 5½ × ½ | 9.30 | 4¼ × ¾ | 10.77 |
| 1¾ × ⅜ | 2.22 | 5¾ × ½ | 9.72 | 4½ × ¾ | 11.41 |
| 2 × ⅜ | 2.53 | 6 × ½ | 10.14 | 4¾ × ¾ | 12.04 |
| 2¼ × ⅜ | 2.85 | | | 5 × ¾ | 12.67 |
| 2½ × ⅜ | 3.17 | 1 × ⅝ | 2.11 | 5¼ × ¾ | 13.31 |
| 2¾ × ⅜ | 3.49 | 1¼ × ⅝ | 2.64 | 5½ × ¾ | 13.94 |
| 3 × ⅜ | 3.80 | 1½ × ⅝ | 3.17 | 5¾ × ¾ | 14.57 |
| 3¼ × ⅜ | 4.12 | 1¾ × ⅝ | 3.70 | 6 × ¾ | 15.21 |
| 3½ × ⅜ | 4.44 | 2 × ⅝ | 4.22 | | |
| 3¾ × ⅜ | 4.75 | 2¼ × ⅝ | 4.75 | 1½ × 1 | 5.07 |
| 4 × ⅜ | 5.07 | 2½ × ⅝ | 5.28 | 2 × 1 | 6.76 |
| 4¼ × ⅜ | 5.39 | 2¾ × ⅝ | 5.81 | 3 × 1 | 10.14 |
| 4½ × ⅜ | 5.70 | 3 × ⅝ | 6.33 | 4 × 1 | 13.52 |
| 4¾ × ⅜ | 6.02 | 3¼ × ⅝ | 6.87 | 5 × 1 | 16.90 |
| 5 × ⅜ | 6.33 | 3½ × ⅝ | 7.39 | 6 × 1 | 20.28 |
| | | 3¾ × ⅝ | 7.92 | 7 × 1 | 23.66 |

# TIMBER AND BOARD MEASURE

## TIMBER MEASURE

**Volume of Round Timber.**—The cubic contents of a round log are those of a cylinder of the same dimensions. As logs usually taper throughout their length, the diameter taken is the mean of the two end diameters. The cubic contents of a cylinder are expressed by the formula, $\frac{\pi d^2}{4} l = \frac{\pi d}{4} dl$. From this it is possible to deduce, for field use, the following:

**Rule.**—*The number of cubic feet in a round log is equal to one-quarter of the mean girth* $\left(\text{circumference or } \frac{\pi d}{4}\right)$ *multiplied by the mean diameter multiplied by the length. If, as is usually the case, the girth and diameter are given in inches, the result obtained must be divided by 144.*

*Multiply by .95 for weight of cast iron; by 1.02 for weight of steel; by 1.16 for copper; by 1.09 for brass; by 1.48 for lead.

When the accompanying table of quarter girths is used, it is only necessary to measure the circumference of each end of the log and divide the sum by 2, for the mean girth. Opposite one-quarter of this mean girth, find the area in feet. This multiplied by the length of the log, gives the cubic contents in feet.

EXAMPLE.—The circumferences of a 20-ft. log are 48 and 60 in., respectively. What is the cubic contents of the log?

SOLUTION.— $(48 + 60) \div 2 = 54 =$ mean girth. $54 \div 4 = 13\frac{1}{2} =$ one-quarter girth. The area corresponding to a quarter girth of $13\frac{1}{2}$ in. is 1.26 sq. ft. Hence, $1.26 \times 20 = 25.20$, number of cubic feet in the log.

## TABLE OF QUARTER GIRTHS

| Quarter Girths Inches | Area Feet | Quarter Girths Inches | Area Feet | Quarter Girths Inches | Area Feet |
|---|---|---|---|---|---|
| 6 | .250 | $12\frac{1}{4}$ | 1.04 | 19 | 2.50 |
| $6\frac{1}{4}$ | .272 | $12\frac{1}{2}$ | 1.08 | $19\frac{1}{2}$ | 2.64 |
| $6\frac{1}{2}$ | .294 | $12\frac{3}{4}$ | 1.12 | 20 | 2.77 |
| $6\frac{3}{4}$ | .317 | 13 | 1.17 | $20\frac{1}{2}$ | 2.91 |
| 7 | .340 | $13\frac{1}{4}$ | 1.21 | 21 | 3.06 |
| $7\frac{1}{4}$ | .364 | $13\frac{1}{2}$ | 1.26 | $21\frac{1}{2}$ | 3.20 |
| $7\frac{1}{2}$ | .390 | $13\frac{3}{4}$ | 1.31 | 22 | 3.36 |
| $7\frac{3}{4}$ | .417 | 14 | 1.36 | $22\frac{1}{2}$ | 3.51 |
| 8 | .444 | $14\frac{1}{4}$ | 1.41 | 23 | 3.67 |
| $8\frac{1}{4}$ | .472 | $14\frac{1}{2}$ | 1.46 | $23\frac{1}{2}$ | 3.83 |
| $8\frac{1}{2}$ | .501 | $14\frac{3}{4}$ | 1.51 | 24 | 4.00 |
| $8\frac{3}{4}$ | .531 | 15 | 1.56 | $24\frac{1}{2}$ | 4.16 |
| 9 | .562 | $15\frac{1}{4}$ | 1.61 | 25 | 4.34 |
| $9\frac{1}{4}$ | .594 | $15\frac{1}{2}$ | 1.66 | $25\frac{1}{4}$ | 4.51 |
| $9\frac{1}{2}$ | .626 | $15\frac{3}{4}$ | 1.72 | 26 | 4.69 |
| $9\frac{3}{4}$ | .659 | 16 | 1.77 | $26\frac{1}{2}$ | 4.87 |
| 10 | .694 | $16\frac{1}{4}$ | 1.83 | 27 | 5.06 |
| $10\frac{1}{4}$ | .730 | $16\frac{1}{2}$ | 1.89 | $27\frac{1}{2}$ | 5.25 |
| $10\frac{1}{2}$ | .766 | $16\frac{3}{4}$ | 1.94 | 28 | 5.44 |
| $10\frac{3}{4}$ | .803 | 17 | 2.00 | $28\frac{1}{2}$ | 5.64 |
| 11 | .840 | $17\frac{1}{4}$ | 2.09 | 29 | 5.84 |
| $11\frac{1}{4}$ | .878 | $17\frac{1}{2}$ | 2.12 | $29\frac{1}{2}$ | 6.04 |
| $11\frac{1}{2}$ | .918 | $17\frac{3}{4}$ | 2.18 | 30 | 6.25 |
| $11\frac{3}{4}$ | .959 | 18 | 2.25 | | |
| 12 | 1.000 | $18\frac{1}{2}$ | 2.37 | | |

## BOARD MEASURE

The unit of board measure (B. M.) is the *board foot*, which is the contents of a board 1 ft. square and 1 in. thick. When calculating board feet all thicknesses under 1 in. are counted as if 1 in., while all over 1 in. are counted at their exact value. Thus, a $\frac{3}{4}$-in. plank is counted as a 1-in. plank, and a $1\frac{1}{2}$-in. plank as $1\frac{1}{2}$ in. When estimating the number of board feet that should be cut from a log, numerous rules have been devised, but the one in most general use is known as *Doyle's, Connecticut River,* or *Scribner's rule*; it is as follows:

**Rule.**—*The diameter of the small end of the log, inside the bark, is first measured. From this diameter, 4 in. are deducted for slabbing and squaring up. The square of one-fourth of this remainder, multiplied by the length, in feet, will give the resultant board feet.*

**Table of Board Feet.**—The number of feet board measure for 1 ft. in length of planks of all thicknesses, increasing by single inches, up to 12 in., and thence, by each 2 in. up to 24 in., are given in the accompanying table. In the rare cases where the thickness is not an even inch, the value for the board feet may be found by interpolation. For timbers larger than $12 \times 24$, it is necessary to take the board feet of two pieces each one-half the size of the stick under consideration. Thus, the number of board feet in one piece $16 \times 24$ is twice the number of board feet in a piece $8 \times 24$ or in one $16 \times 12$.

To economize space, the values have been carried to the first repeating decimal only, which is marked with a (*). Unless the extension indicated is

made, the final figure should be made 1 greater than in the table; thus, if the figures in the table are 1.6*, this should be used as 1.7; or, better, as 1.67, or 1⅔, because the correct value is 1.66666, repeating indefinitely.

### TABLE OF BOARD FEET

| Depth, Inches | Width of Plank, in Inches | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| | Size of Plank, in Board Feet | | | | | | | | | | | |
| 1 | .083* | .16* | .25 | .3* | .416* | .5 | .583* | .6* | .75 | .83* | .916* | 1 |
| 2 | .166* | .33* | .50 | .6* | .833* | 1.0 | 1.166* | 1.3* | 1.50 | 1.66* | 1.833* | 2 |
| 3 | .250 | .50 | .75 | 1.0 | 1.250 | 1.5 | 1.750 | 2.0 | 2.25 | 2.50 | 2.750 | 3 |
| 4 | .333* | .66* | 1.00 | 1.3* | 1.666* | 2.0 | 2.333* | 2.6* | 3.00 | 3.33* | 3.666* | 4 |
| 5 | .416* | .83* | 1.25 | 1.6* | 2.083* | 2.5 | 2.916* | 3.3* | 3.75 | 4.16* | 4.583* | 5 |
| 6 | .500 | 1.00 | 1.50 | 2.0 | 2.500 | 3.0 | 3.500 | 4.0 | 4.50 | 5.00 | 5.500 | 6 |
| 7 | .583* | 1.16* | 1.75 | 2.3* | 2.916* | 3.5 | 4.083* | 4.6* | 5.25 | 5.83* | 6.416* | 7 |
| 8 | .666* | 1.33* | 2.00 | 2.6* | 3.333* | 4.0 | 4.666* | 5.3* | 6.00 | 6.66* | 7.333* | 8 |
| 9 | .750 | 1.50 | 2.25 | 3.0 | 3.750 | 4.5 | 5.250 | 6.0 | 6.75 | 7.50 | 8.250 | 9 |
| 10 | .833* | 1.66* | 2.50 | 3.3* | 4.166* | 5.0 | 5.833* | 6.6* | 7.50 | 8.33* | 9.166* | 10 |
| 11 | .916* | 1.83* | 2.75 | 3.6* | 4.583* | 5.5 | 6.416* | 7.3* | 8.25 | 9.16* | 10.833* | 11 |
| 12 | 1.000 | 2.00 | 3.00 | 4.0 | 5.000 | 6.0 | 7.000 | 8.0 | 9.00 | 10.00 | 11.000 | 12 |
| 14 | 1.166* | 2.33* | 3.50 | 4.6* | 5.833* | 7.0 | 8.166* | 9.3* | 10.50 | 11.66* | 12.833* | 14 |
| 16 | 1.333* | 2.66* | 4.00 | 5.3* | 6.666* | 8.0 | 9.333* | 10.6* | 12.00 | 13.33* | 14.666* | 16 |
| 18 | 1.500 | 3.00 | 4.50 | 6.0 | 7.500 | 9.0 | 10.500 | 12.0 | 13.50 | 15.00 | 16.500 | 18 |
| 20 | 1.666* | 3.33* | 5.00 | 6.6* | 8.333* | 10.0 | 11.666* | 13.3* | 15.00 | 16.66* | 18.333* | 20 |
| 22 | 1.833* | 3.66* | 5.50 | 7.3* | 9.166* | 11.0 | 12.833* | 14.6* | 16.50 | 18.33* | 20.166* | 22 |
| 24 | 2.000 | 4.00 | 6.00 | 8.0 | 10.000 | 12.0 | 14.000 | 16.0 | 18.00 | 20.00 | 22.000 | 24 |

# HYDROSTATICS

*Hydrostatics* treats of the equilibrium of liquids and of their pressures on the walls of vessels containing them; the science depends on the way in which the molecules of a liquid form a mass under the action of gravity and molecular attraction, the latter of which is so modified in liquids as to give them their state of liquidity. While the particles of a liquid cohere, they are free to slide upon one another without the least apparent friction; and it is this perfect mobility that gives them the mechanical properties considered in hydrostatics.

The fundamental property of hydrostatics is a physical axiom, and on it are based nearly all the principles of hydrostatics; it is as follows:

*When a pressure is exerted on any part of the mass of a liquid, that pressure is transmitted undiminished to all parts of the mass, and in all directions.*

**Equilibrium of Liquids.**—The equilibrium of liquids is a property that can be easily demonstrated, and examples are frequently seen. Thus, if two barrels are connected at the bottom with a pipe, and water is poured in one until it reaches within 1 ft. of the top, the water in the other will be found to have attained the same height.

**Pressure of Liquids on Surfaces.**—The general law governing the pressure of liquids on surfaces is as follows:

**Law.**—*The pressure of a liquid on any surface immersed in it is equal to the weight of a column of the liquid whose base is the surface pressed, and whose height is the perpendicular depth of the center of gravity of the surface below the surface of the liquid.*

The pressure exerted by liquids is independent of the shape or size of the vessel or cavity containing the liquid. The pressure of a liquid against any point of any surface, either curved or plane, is always perpendicular to the surface at that point. At any given depth, the pressure of a liquid is equal in every direction, and is in direct proportion to the vertical depth below the surface.

The weight of 1 cu. ft. of fresh water, at ordinary temperature of the atmosphere, that is, from 32° F. to 90° F., is usually assumed at 62.5 lb. This is a trifle more than the actual weight, but is sufficiently close for most practical purposes.

**To Find Pressure Exerted by Quiet Water Against Side of Gangway or Heading.**—Multiply the area of the side, in square feet, by the perpendicular distance from the surface of the water to a point equidistant between the top and bottom of the submerged heading or gangway, and multiply the product by 62.5. The result will be the pressure, in pounds, exclusive of atmospheric pressure; this latter need not be considered in ordinary mining work.

EXAMPLE.—If an abandoned colliery, opened by a slope on a pitch of 25° and 100 yd. long, is allowed to fill with water, what is the average pressure exerted on each square foot of the lower rib of the gangway, assuming that the gangways were driven dead level, and that the length of the slope was measured to a point on the lower rib equidistant between top and bottom of gangway?

SOLUTION.—Perpendicular height of water is $300 \times \sin 25° = 126.78$ ft. Then, the pressure on each square foot of the lower rib of gangway is $126.78 \times 62.5$ lb., $= 7,923.75$ lb., or over $3\frac{1}{2}$ gross tons. The total pressure exerted along the gangway may readily be found by multiplying 7,923.75 lb. by the number of square feet in the lower rib against which it rests.

**Pressure Against Dams, Etc.**—To find the total pressure of quiet water against and perpendicular to any surface whatever, as a dam, embankment, or the bottom, side, or top of any containing vessel, water pipe, etc., no matter whether said surface is vertical, horizontal, or inclined; or whether it is flat or curved; or whether it reaches to the surface of the water or is entirely below it, apply the following rule:

**Rule.**—*Multiply the area, in square feet, of the surface pressed by the vertical depth, in feet, of its center of gravity below the surface of the water, and this product by 62.5; the result will be the pressure in pounds.*

Thus, assume that in Figs. 1, 2, and 3 the depth of water in each dam is 12 ft., and the wall or embankment is 50 ft. long. In Fig. 1 the total pressure will equal $12 \times 50 \times 6 \times 62.5 = 225,000$ lb. In Figs. 2 and 3, the walls or embankments, being inclined, expose a greater surface to pressure, say, 18 ft. from $A$ to $B$. Then the total pressure equals $18 \times 50 \times 6 \times 62.5 = 337,500$ lb.
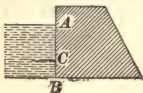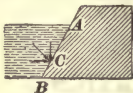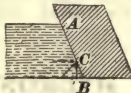


The results obtained are the total pressures without regard to direction. In Fig. 1 the total pressure calculated, or 225,000 lb., is horizontal, tending either to overturn the wall or make it slide on its base. The center of pressure is at $C$, or one-third of the vertical depth from the bottom. In Fig. 2, the pressure is exerted in two directions; one pressure, acting horizontally, tends to overthrow or slide the wall, while the other, acting vertically, tends to hold it in place. In Fig. 3, the pressure is also exerted in two directions; one pressure, acting horizontally, tends to overthrow or slide the wall, while the other tends to lift. So long as the vertical depth of water remains the same, the horizontal pressure remains the same, no matter what inclination is given the wall. Thus, in Figs. 2 and 3, the horizontal pressure is the same as in Fig. 1, or 225,000 lb.

**Distribution of Pressure.**—The total pressure of the water is distributed over the entire depth of the submerged part of the back of the wall, and is least at the top, gradually increasing toward the bottom. But so far as regards the united action of every portion of it, in tending to overthrow the wall, considered as a single mass of masonry incapable of being bent or broken, it may all be assumed to be applied at $C$, which is one-third of the vertical depth from the bottom in Fig. 1, or, what is the same thing, one-third of the slope distance from the bottom in Figs. 2 and 3.

No matter how much water is in a dam or vessel, the pressure remains the same, so long as the area pressed and the vertical depth of its center of gravity below the level surface of the water remains unchanged. Thus, if the water in dam shown in Fig. 1 extended back 1 m., it would exert no more pressure against the wall than if it extended back only 1 ft.

In any two vessels having the same base, and containing the same depth of water, no matter what quantity, the pressures on the bases are equal. Thus, if Figs. 4 and 5 have the same base and are filled with water to the same depth, the pressure on the bases will be equal. This fact, that the pressure on a given

surface, at a given depth, is independent of the quantity of water, is called the *hydrostatic paradox.*

As the pressure of water against any point is at right angles to the surface at that point, props or other strengthening material for the bracing of such structures as a sloping dam, should be so placed as to offer the greatest resistance in a line at right angles to the sloping surface, and these supports should be strongest and closest together at the bottom. For the same reason, the hoops on a circular tank should be strongest and closest at the bottom.

**Transmission of Pressure Through Water.**—Water, in common with other liquids, possesses the important property of transmitting pressure equally in all directions. Thus, if a vessel is constructed



FIG. 4    FIG. 5



FIG. 6

with two cylinders, the area of one being 10 sq. in., and that of the other 100 sq. in., and the vessel is filled with water, as in Fig. 6, and pistons are fitted to the cylinders, a pressure of 100 lb. applied at the smaller will balance 1,000 lb. at the larger. This is the principle of the hydrostatic press. Air and other gaseous fluids transmit pressure equally in all directions, like liquids, but not as rapidly.

**To Find Pressure on Plane Surface at Any Given Depth of Water.**—For pounds per square inch, multiply depth in feet by .434. For pounds per square foot, multiply depth in feet by 62.5. For tons per square foot, multiply

**PRESSURE AT DIFFERENT VERTICAL DEPTHS ALSO TOTAL PRESSURE AGAINST A PLANE 1 FT. WIDE EXTENDING VERTICALLY FROM SURFACE OF WATER TO SAME DEPTHS**

| Depth Feet | Pressure Pounds per Sq. Ft. | Total Pressure Pounds | Depth Feet | Pressure Pounds per Sq. Ft. | Total Pressure Pounds | Depth Feet | Pressure Pounds per Sq. Ft. | Total Pressure Pounds |
|---|---|---|---|---|---|---|---|---|
| 1 | 62.5 | 31 | 27 | 1,687 | 22,781 | 65 | 4,062 | 132,031 |
| 2 | 125 | 125 | 28 | 1,750 | 24,500 | 70 | 4,375 | 153,125 |
| 3 | 187 | 281 | 29 | 1,812 | 26,281 | 75 | 4,687 | 175,781 |
| 4 | 250 | 500 | 30 | 1,875 | 28,125 | 80 | 5,000 | 200,000 |
| 5 | 312 | 781 | 31 | 1,937 | 30,031 | 85 | 5,312 | 225,781 |
| 6 | 375 | 1,125 | 32 | 2,000 | 32,000 | 90 | 5,625 | 253,125 |
| 7 | 437 | 1,531 | 33 | 2,062 | 34,031 | 95 | 5,937 | 282,031 |
| 8 | 500 | 2,000 | 34 | 2,125 | 36,125 | 100 | 6,250 | 312,500 |
| 9 | 562 | 2,531 | 35 | 2,187 | 38,281 | 110 | 6,875 | 378,125 |
| 10 | 625 | 3,125 | 36 | 2,250 | 40,500 | 120 | 7,500 | 450,000 |
| 11 | 687 | 3,781 | 37 | 2,312 | 42,781 | 130 | 8,125 | 528,100 |
| 12 | 750 | 4,500 | 38 | 2,375 | 45,125 | 140 | 8,750 | 612,500 |
| 13 | 812 | 5,281 | 39 | 2,437 | 47,531 | 150 | 9,375 | 703,125 |
| 14 | 875 | 6,125 | 40 | 2,500 | 50,000 | 160 | 10,000 | 800,000 |
| 15 | 937 | 7,031 | 41 | 2,562 | 52,531 | 170 | 10,625 | 903,125 |
| 16 | 1,000 | 8,000 | 42 | 2,625 | 55,125 | 180 | 11,250 | 1,012,500 |
| 17 | 1,062 | 9,031 | 43 | 2,687 | 57,781 | 190 | 11,875 | 1,128,125 |
| 18 | 1,125 | 10,125 | 44 | 2,750 | 60,500 | 200 | 12,500 | 1,250,000 |
| 19 | 1,187 | 11,281 | 45 | 2,812 | 63,281 | 225 | 14,062 | 1,582,031 |
| 20 | 1,250 | 12,500 | 46 | 2,875 | 66,125 | 250 | 15,625 | 1,953,125 |
| 21 | 1,312 | 13,781 | 47 | 2,937 | 69,031 | 275 | 17,187 | 2,363,281 |
| 22 | 1,375 | 15,125 | 48 | 3,000 | 72,000 | 300 | 18,750 | 2,812,500 |
| 23 | 1,437 | 16,531 | 49 | 3,062 | 75,031 | 350 | 21,875 | 3,828,125 |
| 24 | 1,500 | 18,000 | 50 | 3,125 | 78,125 | 400 | 25,000 | 5,000,000 |
| 25 | 1,562 | 19,531 | 55 | 3,437 | 94,531 | 450 | 28,125 | 6,328,125 |
| 26 | 1,625 | 21,125 | 60 | 3,750 | 112,500 | 500 | 31,250 | 7,812,500 |

depth in feet by .0279. The pressure per square foot at different depths increases directly as the depths. The total pressure against a plane 1 ft. wide at different depths increases as the square of the depths.

**Pressure of Water in Pipes.**—As water exerts a pressure equally in all directions, it is important that, in pipe lines, the pipe should be sufficiently thick to assure strength enough to resist a bursting pressure. In ordinary practice, the thickness of cast-iron water pipes of different bores is calculated by Mr. J. T. Fanning's formula, given in his Hydraulic Engineering, which is as follows, the thickness and bore being given in inches and the pressure in pounds per square inch:

$$\text{Thickness} = \frac{(\text{pressure} + 100) \times \text{bore}}{.4 \times \text{ultimate tensile strength}} + .333 \times \left(1 - \frac{\text{bore}}{100}\right)$$

This formula, worked out for different heads and different sizes of bore, yields the results given in the accompanying table. In this table, the ultimate tensile strength of cast iron is taken at 18,000 lb. per sq. in. The addition of 100 lb. to the pressure is made to allow for water ram. The valves of water pipes should be closed slowly, and especially as the pipes increase in diameter. If this rule is not observed, the momentum of the water which is suddenly stopped creates a great pressure against the pipes in all directions and throughout the entire length of the line above the valve, even if it is many miles long, and there is danger of the pipes bursting at any point. For this reason, stop-gates are shut by screws, because they prevent any very sudden closing; but in pipes of large diameters, even the screws must be worked very slowly to prevent the pipes bursting.

## THICKNESS OF PIPE FOR DIFFERENT HEADS AND PRESSURES

| Head, in Feet .... | 50 | 100 | 200 | 300 | 500 | 1,000 |
|---|---|---|---|---|---|---|
| Pressure, in Pounds per Square Inch | 21.7 | 43.4 | 86.8 | 130 | 217 | 434 |
| Bore Inches | Thickness of Pipe, in Inches | | | | | |
| 2 | .36 | .37 | .38 | .39 | .42 | .48 |
| 3 | .37 | .38 | .40 | .42 | .45 | .54 |
| 4 | .39 | .40 | .42 | .45 | .50 | .61 |
| 6 | .41 | .43 | .47 | .50 | .57 | .75 |
| 8 | .45 | .47 | .52 | .57 | .66 | .90 |
| 10 | .47 | .50 | .56 | .62 | .74 | 1.04 |
| 12 | .49 | .53 | .60 | .67 | .82 | 1.18 |
| 16 | .55 | .60 | .70 | .79 | .98 | 1.46 |
| 18 | .57 | .63 | .74 | .85 | 1.06 | 1.60 |
| 20 | .61 | .67 | .79 | .91 | 1.15 | 1.75 |
| 24 | .66 | .73 | .87 | 1.02 | 1.30 | 2.03 |
| 30 | .74 | .83 | 1.01 | 1.19 | 1.55 | 2.46 |
| 36 | .82 | .93 | 1.15 | 1.36 | 1.80 | 2.88 |
| 48 | .98 | 1.13 | 1.42 | 1.70 | 2.28 | 3.73 |

**Wooden Pipe.**—The sizes of wooden pipes up to 6 in., and sometimes up to 12 in. in diameter are commonly bored from a single log. The sizes up to 48 in. in diameter are made from a series of staves placed side by side and wrapped with wire, steel bands, etc., and coated with tar or some similar preservative. The sizes given in the accompanying table are made in the factories and shipped to destination. Larger sizes 54-, 60-, 66-, 72-, 78-, 84-, 90-, 96-, 108-, and 120-in. pipes are made on the ground where used from staves of the proper cross-section. As built, the staves break joint at least 24 in. so that they interlock longitudinally, forming a continuous pipe. While being placed, the completed pipe is wrapped with steel wire or bands. For heads up to 400 ft. (173 lb. per sq. in.) wooden pipe has been used for many years and possesses numerous advantages over cast- or wrought-iron pipe. Owing to the smoothness of its interior, its carrying capacity for water is much greater

## STANDARD SIZES OF WOOD PIPE

| Inside Diameter of Pipe Inches | Outside Diameter of Pipe Inches | Weight per Foot 80-lb. Pressure Pounds | Number of Feet a 40-Ft. Car Will Hold |
|---|---|---|---|
| 1½ | 3¾ | 2 | 17,000 |
| 2 | 4¾ | 4 | 11,000 |
| 3 | 6¼ | 6½ | 6,000 |
| 4 | 7¼ | 7½ | 4,500 |
| 4 | 8¼ | 12 | 4,000 |
| 5 | 9¼ | 15 | 3,000 |
| 6 | 10¼ | 18 | 2,700 |
| 6 | 10½ | 13 | 2,700 |
| 8 | 12⅜ | 16 | 2,000 |
| 10 | 14¼ | 21 | 1,600 |
| 12 | 16½ | 25 | 1,200 |
| 14 | 18⅜ | 27 | 1,000 |
| 16 | 20⅜ | 29 | 800 |
| 18 | 22⅜ | 31 | 700 |
| 20 | 24⅜ | 35 | 600 |
| 24 | 28½ | 50 | 425 |
| 30 | 36½ | 90 | 200 |
| 36 | 42½ | 110 | 175 |
| 48 | 54⅜ | 160 | 65 |

than is that of cast or wrought pipe. This difference is estimated to be about 10% in favor of the wooden pipe when it and the iron pipe are new and as much as 30 or 40% when both are old. Wooden pipe is not affected by acid mine water or by electrolysis as is iron pipe, and for these reasons is in much favor in many mines where the head is not too great. The standard thicknesses are made to resist pressures due to 100 ft., 200 ft., 300 ft., and 400 ft. head, or 43 lb., 87 lb., 130 lb., and 173 lb. per sq. in. pressure. In exceptional cases, wooden pipe has been built to work under 700-ft. head, say, under a pressure of 300 lb.

**Compressibility of Liquids.**—Liquids are not entirely incompressible, but they are so nearly so, that for most purposes they may be considered as incompressible. The bulk of water is diminished about $\frac{1}{1000}$ by a pressure of 324 lb. per sq. in., or 22 atmospheres; varying slightly with its temperature. It is perfectly elastic, regaining its original bulk when the pressure is removed.

# HYDRAULICS

## DEFINITIONS

*Hydraulics* treats of liquids in motion, and in this, as in hydrostatics, water is taken as the type. In theory, its principles are the same as those of falling bodies, but in practice they are so modified by various causes that they cannot be relied on except as verified by experiment. The discrepancy arises from changes of temperature that vary the fluidity of the liquid, from friction, the shape of the orifice, etc. When dealing with water only, the first cause need not be considered. In theory the velocity of a jet is the same as that of a body falling from the surface of the water.

**To Find Theoretical Velocity of Jet of Water.**—Let $v$ = velocity, $g$ = acceleration of gravity (32.16 ft.), and $d$ = distance of orifice below surface of water. Then,
$$v = \sqrt{2gd}.$$

EXAMPLE.—The depth of water above the orifice is 64 ft.; what is the velocity?

SOLUTION.—Substituting in the formula 64 for $d$ and 32.16 for $g$,

$$v = \sqrt{2 \times 32.16 \times 64} = 64.16.$$

**To Find Theoretical Quantity of Water Discharged in Given Time.**—Multiply the area of the orifice by the velocity of the water, and that product by the number of seconds.

EXAMPLE.—What quantity of water will be discharged in 5 sec. from an orifice having an area of 2 sq. ft., at a depth of 16 ft.?

SOLUTION.— $2 \times \sqrt{2 \times 32.16 \times 16} = 64.16$ cu. ft., or the amount discharged in 1 sec., and in 5 sec. the amount will be $5 \times 64.16 = 320.8$ cu. ft.

The foregoing rules are only theoretical, and are only useful as foundations on which to build practical formulas.

**Flow of Water Through Orifices.**—The standard orifice, or an orifice so arranged that the water when flowing from it touches only a line, as is the case when flowing through a hole in a very thin plate, is used for the measurement of water. The contraction of the jet, which always occurs when water issues from a standard orifice, is due to the circumstance that the particles of water as they approach the orifice move in converging directions, and that these directions continue to converge for a short distance beyond the plane of the orifice. This contraction causes only the inner corner of the orifice to be touched by the escaping water, and takes place in orifices of any shape, its cross-section being similar to the orifice until the place of greatest contraction is passed. Owing to this contraction, the actual discharge from an orifice is always less than the theoretical discharge.

**Coefficient of Contraction.**—The coefficient of contraction is the number by which the area of the orifice is to be multiplied in order to find the area of the least cross-section of the jet. By experiment, this coefficient has been found to be about .62 (Merriman's "Hydraulics"); or, in other words, the minimum cross-section of the jet is 62% of the cross-section of the orifice.

**Coefficient of Velocity.**—The coefficient of velocity is the number by which the theoretical velocity of flow from the orifice is to be multiplied in order to find the actual velocity at the least cross-section of the jet. This may be taken for practical work as .98; or, in other words, the actual flow at the contracted section is 98% of the theoretical velocity.

**Coefficient of Discharge.**—The coefficient of discharge is the number by which the theoretical discharge is to be multiplied in order to obtain the actual discharge. This has been found by thousands of experiments to be equal to the product of the coefficients of contraction and velocity, and for practical work it may be taken as .61; or, the actual discharge from standard orifices is 61% of the theoretical discharge.

NOTE.—While the coefficients for standard orifices with sharp edges have been determined fairly close, those for the more complicated cases of weirs, and especially for the flow of water through long pipes, are simply the nearest approximation to the truth that it has been possible to obtain. In all cases, the coefficient should be one that has been determined under conditions similar to those in the problem in hand. For instance, it is not practicable to use the coefficient for small pipes in solving problems relating to large ones, or for short pipes in solving problems relating to long ones.

**Suppression of Contraction.**—When a vertical orifice has its lower edge at the bottom of a reservoir, the particles of water flowing through its lower portion move in lines nearly perpendicular to the plane of the orifice, and the contraction of the jet does not form on the lower side. The same thing occurs in a lesser degree when the lower edge of the orifice is within a distance of three times its least diameter from the bottom. The suppression of contraction will occur on the side if it is placed within a distance of three times its least diameter from the side of a reservoir, the suppression of contraction being the greater the nearer the orifice is to the side. By rounding the edge of the orifice sufficiently, the contraction can be completely suppressed, and the discharge can be increased. As stated before, the value of the coefficient of contraction for a standard square-edged orifice is .62, but with a rounded orifice it may have any value between .62 and 1, depending on the degree of rounding. The coefficient of discharge for square-edged orifices has a mean value of .61; this is increased with rounded edges and may have any value between .61 and 1, although it is not probable that values greater than .95 can be obtained except by the most careful adjustment of the rounded edges to the exact curve of a completely contracted jet. A rounded interior orifice is therefore always a source of error when the object of the orifice is the measurement of the discharge.

## GAUGING WATER

Water is sold by two methods; i. e., the flowing unit and the capacity unit. The flowing unit is 1 cu. ft. per sec. In the western part of North America the *miners' inch* has come into use quite largely, while in Australia and New Zealand the cubic foot per second is the common measure; 1 cu. ft. per sec. being 1 *head*, 10 heads of water is 10 cu. ft. per sec., regardless of the actual hydrostatic head under which the water is delivered. Water is sometimes sold for irrigation by the capacity unit, that is, so much land covered to a certain depth, as, for instance, the *acre-foot*, which means that 1 A. has been covered to a depth of 1 ft., or that 43,560 cu. ft. of water has been furnished.

**Miners' Inch.**—The miners' inch may be roughly defined as the quantity of water that will flow in 1 min. through a vertical standard orifice having a section of 1 sq. in. and a head of $6\frac{1}{2}$ in. above the center of the orifice. This quantity equals 1.53 cu. ft., and the mean quantity may be taken at, approximately, 1.5 cu. ft. per min. The laws or customs defining the miners' inch in different districts vary so that the amount of water actually delivered varies from 1.2 to 1.76 cu. ft. per min. The principal reasons for these variations are the methods adopted for measuring the water where large quantities are used; as, for instance, at Smartsville, California, an opening 4 in. deep, 250 in. long, with a head of 7 in. above the top edge, is said to furnish 1,000 miners' inches, while it actually furnishes considerably over 1,000. In other places, the size of the opening for measuring the amounts is restricted, and may actually furnish less than the rated amount. In Montana the common method of measurement was formerly through a vertical rectangle 1 in. high, with a head on the center

### DUTY OF MINERS' INCH

(*Risdon Iron Works, Evans's Elevator Catalogue*)

North Bloomfield Mine

| Years | Amount of Gravel Washed Cubic Yards | Miners' 24-Hour Inches | Grades | Amount Washed per Miners' Inch Cubic Yards | Water Used per Cubic Foot of Gravel Moved Cubic Feet | Height of Bank Feet |
|---|---|---|---|---|---|---|
| 1870-74 | 3,250,000 | 710,987 | $6\frac{1}{2}$ in. to 12 ft. | 4.60 | 18 | 100 |
| 1875 | 1,858,000 | 386,972 | $6\frac{1}{2}$ in. to 12 ft. | 4.80 | 17 | 100 |
| 1876 | 2,919,700 | 700,000 | $6\frac{1}{2}$ in. to 12 ft. | 4.17 | 20 | 200 |
| 1877 | 2,993,930 | 595,000 | $6\frac{1}{2}$ in. to 12 ft. | 3.86 | 21 | 265 |
| Totals | 11,021,630 | 2,392,959 | | 4.60 | 18 | |

La Grange Mine

| Years | Amount of Gravel Washed Cubic Yards | Miners' 24-Hour Inches | Grades | Amount Washed per Miners' Inch Cubic Yards | Water Used per Cubic Foot of Gravel Moved Cubic Feet | Height of Bank Feet |
|---|---|---|---|---|---|---|
| 1874-76 | 676,968 | 624,745 | 4 in. to 16 ft. | 1.08 | 74.0 | 10 to 48 |
| 1875-76 | 683,244 | 375,155 | 4 in. to 16 ft. | 1.82 | 43.9 | 6 |
| 1874-76 | 284,932 | 207,010 | 4 in. to 16 ft. | 1.37 | 58.0 | 50 to 80 |
| 1875-78 | 459,570 | 302,960 | 4 in. to 16 ft. | 1.52 | 52.0 | 40 to 50 |
| 1880-81 | 329,120 | 203,325 | 4 in. to 16 tt. | 1.57 | 50.0 | 10 to 80 |
| Totals | 2,433,834 | 1,713,195 | | 1.42 | 56.0 | |

of the orifice of 4 in. The number of miners' inches discharged was considered to be the same as the number of linear inches in the length of the orifice; thus, under the given head, an orifice 1 in. deep and 60 in. long could discharge 60 miners' inches.

The State Legislature of Montana has now passed a law defining the miners' inch as the number of gallons of water discharged in a given time, regardless of the character of the openings or methods of measurement. The statement is as follows: "Where water rights, expressed in miners' inches, have been granted, 100 miners' inches shall be considered equivalent to a flow of $2\frac{1}{2}$ cu. ft. per sec. (18.7 gal.), and this proportion shall be observed in determining the equivalent flow represented by any number of miners' inches." If this amount is reduced to cubic feet per minute, it will be found to be equal to a flow of 1.5 cu. ft. per min., which is the value given for the miners' inch.

**Duty or Work Performed by a Miners' Inch of Water.**—Few tests have been made in regard to the duty of a miners' inch of water, but the North Bloomfield mine and the La Grange mine, in California, have carried on a series of experiments extending over several years. At the La Grange mine, the observations were carried on simultaneously upon several different claims, hence parallel dates appear. The following table gives the results of these experi-

| Number of Sluice Heads | Dimensions of Aperture, in Inches | | Depth of Pressure Board Above Top of Aperture Inches |
|---|---|---|---|
| | Width | Depth | |
| 1 | 16 | 1 | $5\frac{1}{2}$ |
| 2 | 16 | 2 | $4\frac{3}{4}$ |
| 3 | 16 | $2\frac{1}{4}$ | $9\frac{1}{16}$ |
| 4 | 16 | $2\frac{3}{8}$ | $10\frac{5}{8}$ |
| 5 | 16 | $3\frac{1}{2}$ | $9\frac{1}{4}$ |
| 6 | 32 | $2\frac{1}{4}$ | $8\frac{5}{8}$ |
| 7 | 32 | $2\frac{1}{2}$ | $9\frac{1}{2}$ |
| 8 | 32 | $2\frac{3}{4}$ | 10 |
| 9 | 32 | 3 | $10\frac{1}{2}$ |
| 10 | 32 | $3\frac{1}{2}$ | $8\frac{3}{4}$ |

ments. In general the duty is governed by the size, capacity, character of pavement, and grade of sluices, together with the supply of water. A heavy grade will compensate for a limited supply. With an abundant supply of water and material, the capacity of the sluices will depend on the character of the material washed, the size and minimum grade of the sluices, and the character of the riffles used. In the North Bloomfield mine the sluices were 6 ft. wide, and 32 in. deep. The riffles were principally blocks (wood), but rock riffles were used in the tail sluices. The larger portion of the material moved was top gravel. In the La Grange mine the sluices were 4 ft. wide and 30 in. deep, and were paved with blocks.

**Sluice Head.**—In Tasmania and the Australian Provinces, water is sold by the sluice head. For the following information relating to Tasmania, we are indebted to Roy Bamford, Esq., South Mt. Cameron, Tasmania.

"The sluice head as leased by mining companies from the Government at a rental of £1 per yr. for each head, amounts to 24 cu. ft. per min. and is measured as follows:

"A Government gauge box for measuring water is 12 ft. long, open on top,



and set truly horizontal. The outlet end of the box is closed with a board 1 in. in thickness with the exception of an aperture, which is always the full width of the gauge box. In all cases, the lower end of the aperture is 2 in. above the bottom of the gauge box. When measuring the discharge
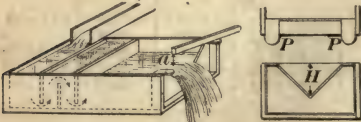
FIG. 1

through the aperture, the water must stand on a level with the top of the

board. When more than ten sluice heads of water are required to be measured, two or more of such gauge boxes shall be used.''

**Gauging by V Notch.**—The right-angled V notch is frequently used for gauging the flow of comparatively small streams. The notch is usually fitted into a box provided with baffle boards, Fig. 1, or, where this is not practicable, the water should be so impounded above the notch as to remove all possibility of surface currents producing a perceptible velocity of approach. The distance $a$ of the surface of the water below the top of the box is taken at a point some distance back from the notch (at least 18 to 20 in.), where the sur-

## DISCHARGE OF WATER THROUGH A RIGHT-ANGLED V NOTCH

| Head Inches. | Quantity Cubic Feet per Minute | Head Inches | Quantity Cubic Feet per Minute | Head Inches | Quantity Cubic Feet per Minute | Head Inches | Quantity Cubic Feet per Minute | Head Inches | Quantity Cubic Feet per Minute |
|---|---|---|---|---|---|---|---|---|---|
| $h$ | $Q*$ | $h$ | $Q$ | $h$ | $Q$ | $h$ | $Q$ | $h$ | $Q$ |
| 1.05 | .3457 | 3.25 | 5.827 | 5.45 | 21.22 | 7.65 | 49.53 | 9.85 | 93.18 |
| 1.10 | .3884 | 3.30 | 6.054 | 5.50 | 21.71 | 7.70 | 50.34 | 9.90 | 94.37 |
| 1.15 | .4340 | 3.35 | 6.285 | 5.55 | 22.20 | 7.75 | 51.16 | 9.95 | 95.56 |
| 1.20 | .4827 | 3.40 | 6.523 | 5.60 | 22.70 | 7.80 | 51.99 | 10.00 | 96.77 |
| 1.25 | .5345 | 3.45 | 6.765 | 5.65 | 23.22 | 7.85 | 52.83 | 10.05 | 97.98 |
| 1.30 | .5896 | 3.50 | 7.012 | 5.70 | 23.74 | 7.90 | 53.67 | 10.10 | 99.20 |
| 1.35 | .6480 | 3.55 | 7.266 | 5.75 | 24.26 | 7.95 | 54.53 | 10.15 | 100.43 |
| 1.40 | .7096 | 3.60 | 7.524 | 5.80 | 24.79 | 8.00 | 55.39 | 10.20 | 101.67 |
| 1.45 | .7747 | 3.65 | 7.788 | 5.85 | 25.33 | 8.05 | 56.26 | 10.25 | 102.92 |
| 1.50 | .8432 | 3.70 | 8.058 | 5.90 | 25.87 | 8.10 | 57.14 | 10.30 | 104.18 |
| 1.55 | .9153 | 3.75 | 8.332 | 5.95 | 26.42 | 8.15 | 58.03 | 10.35 | 105.45 |
| 1.60 | .9909 | 3.80 | 8.613 | 6.00 | 26.98 | 8.20 | 58.92 | 10.40 | 106.73 |
| 1.65 | 1.0700 | 3.85 | 8.899 | 6.05 | 27.55 | 8.25 | 59.82 | 10.45 | 108.02 |
| 1.70 | 1.1530 | 3.90 | 9.191 | 6.10 | 28.12 | 8.30 | 60.73 | 10.50 | 109.31 |
| 1.75 | 1.2400 | 3.95 | 9.489 | 6.15 | 28.70 | 8.35 | 61.65 | 10.55 | 110.62 |
| 1.80 | 1.3300 | 4.00 | 9.792 | 6.20 | 29.28 | 8.40 | 62.58 | 10.60 | 111.94 |
| 1.85 | 1.4240 | 4.05 | 10.100 | 6.25 | 29.88 | 8.45 | 63.51 | 10.65 | 113.26 |
| 1.90 | 1.5220 | 4.10 | 10.410 | 6.30 | 30.48 | 8.50 | 64.45 | 10.70 | 114.60 |
| 1.95 | 1.6250 | 4.15 | 10.730 | 6.35 | 31.09 | 8.55 | 65.41 | 10.75 | 115.94 |
| 2.00 | 1.7310 | 4.20 | 11.060 | 6.40 | 31.71 | 8.60 | 66.37 | 10.80 | 117.29 |
| 2.05 | 1.8410 | 4.25 | 11.390 | 6.45 | 32.33 | 8.65 | 67.34 | 10.85 | 118.65 |
| 2.10 | 1.9550 | 4.30 | 11.730 | 6.50 | 32.96 | 8.70 | 68.32 | 10.90 | 120.02 |
| 2.15 | 2.0740 | 4.35 | 12.070 | 6.55 | 33.60 | 8.75 | 69.30 | 10.95 | 121.41 |
| 2.20 | 2.1960 | 4.40 | 12.420 | 6.60 | 34.24 | 8.80 | 70.30 | 11.00 | 122.81 |
| 2.25 | 2.3230 | 4.45 | 12.780 | 6.65 | 34.89 | 8.85 | 71.30 | 11.05 | 124.21 |
| 2.30 | 2.4550 | 4.50 | 13.140 | 6.70 | 35.56 | 8.90 | 72.31 | 11.10 | 125.61 |
| 2.35 | 2.5900 | 4.55 | 13.510 | 6.75 | 36.23 | 8.95 | 73.33 | 11.15 | 127.03 |
| 2.40 | 2.7300 | 4.60 | 13.890 | 6.80 | 36.89 | 9.00 | 74.36 | 11.20 | 128.45 |
| 2.45 | 2.8750 | 4.65 | 14.270 | 6.85 | 37.58 | 9.05 | 75.40 | 11.25 | 129.90 |
| 2.50 | 3.0240 | 4.70 | 14.650 | 6.90 | 38.27 | 9.10 | 76.44 | 11.30 | 131.35 |
| 2.55 | 3.1770 | 4.75 | 15.040 | 6.95 | 38.96 | 9.15 | 77.49 | 11.35 | 132.81 |
| 2.60 | 3.3350 | 4.80 | 15.440 | 7.00 | 39.67 | 9.20 | 78.55 | 11.40 | 134.27 |
| 2.65 | 3.4980 | 4.85 | 15.850 | 7.05 | 40.38 | 9.25 | 79.63 | 11.45 | 135.75 |
| 2.70 | 3.6660 | 4.90 | 16.260 | 7.10 | 41.10 | 9.30 | 80.71 | 11.50 | 137.23 |
| 2.75 | 3.8380 | 4.95 | 16.680 | 7.15 | 41.83 | 9.35 | 81.80 | 11.55 | 138.73 |
| 2.80 | 4.0140 | 5.00 | 17.110 | 7.20 | 42.56 | 9.40 | 82.90 | 11.60 | 140.23 |
| 2.85 | 4.1960 | 5.05 | 17.540 | 7.25 | 43.30 | 9.45 | 84.01 | 11.65 | 141.75 |
| 2.90 | 4.3820 | 5.10 | 17.970 | 7.30 | 44.06 | 9.50 | 85.12 | 11.70 | 143.28 |
| 2.95 | 4.5740 | 5.15 | 18.420 | 7.35 | 44.82 | 9.55 | 86.24 | 11.75 | 144.82 |
| 3.00 | 4.7700 | 5.20 | 18.870 | 7.40 | 45.58 | 9.60 | 87.37 | 11.80 | 146.36 |
| 3.05 | 4.9710 | 5.25 | 19.320 | 7.45 | 46.36 | 9.65 | 88.52 | 11.85 | 147.91 |
| 3.10 | 5.1780 | 5.30 | 19.790 | 7.50 | 47.14 | 9.70 | 89.67 | 11.90 | 149.48 |
| 3.15 | 5.3880 | 5.35 | 20.260 | 7.55 | 47.92 | 9.75 | 90.83 | 11.95 | 151.05 |
| 3.20 | 5.6050 | 5.40 | 20.730 | 7.60 | 48.72 | 9.80 | 92.00 | 12.00 | 152.64 |

*1 cu. ft. contains 7.48 U. S. gal.; 1 U. S. gal. weighs 8.34 lb.

face of the water is unaffected by the flow through the notch. The distance $a$, subtracted from the total depth of the notch $H$, gives the head $h$ of the water passing over the notch. The discharge, in cubic feet per second, may be found by the formula

$$Q = .306\sqrt{h^5} = .306h^2\sqrt{h},$$

in which $Q$ = quantity, in cubic feet per minute;
$h$ = head, in inches.

The accompanying table gives the discharge, in cubic feet per minute, through a right-angled **V** notch, as shown in Fig. 1, for heads varying from 1.05 in. to 12 in.
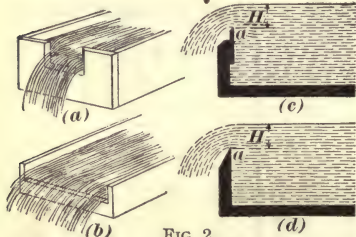


FIG. 2

**Gauging by Weirs.**—A weir is an obstruction placed across a stream for the purpose of diverting the water so as to make it flow through a desired channel, which may be a notch or opening in the weir itself. The term usually applies to rectangular notches in which the water touches the bottom and ends, the opening being a notch without any upper edge. Weirs are of two general classes: *weirs with end contractions*, Fig. 2 (a), and *weirs without end contractions*, as in (b). The crest and edges of the weir with end contractions should be sharp, as shown in (c) and (d). The head of water $H$ producing the flow over the weir should be measured at a sufficient distance from the crest to avoid the effects of the curve of the surface as it flows over the crest. The water above the weir should be motionless, or if it has any perceptible current toward the weir, this should be determined and taken into account in the formula. Fig. 3 illustrates a weir constructed across a small stream for measuring its flow. The head is measured from the stake $E$ some distance back of the weir, the top of the stake being level with the crest of the weir $B$. The discharge over the weir may be calculated from the following formula:



FIG. 3

Let $l$ = length of weir, in feet;
$H$ = head, in feet;
$v$ = velocity with which water approaches weir, in feet;
$h$ = head equivalent to velocity with which water approaches weir;
$c$ = coefficient of discharge;
$Q$ = theoretic discharge, in cubic feet per second;
$Q'$ = actual discharge, in cubic feet per second.

For weirs with end contractions and a velocity of approach, the actual discharge is

$$Q' = 5.347\ cl\sqrt{(H + 1.4h)^3}$$

Where the water has no velocity of approach,

$$Q' = 5.347\ cl\sqrt{H^3}$$

For weirs without end contractions, but with a velocity of approach, the actual discharge is

$$Q' = 5.347\ cl\sqrt{(H + \tfrac{4}{3}h)^3}$$

Where the water has no velocity of approach,

$$Q' = 5.347\ cl\sqrt{H^3}$$

The velocity with which the water approaches the weir may be found by determining the approximate discharge from the stream without any allowance for velocity of approach, and then dividing this discharge, in cubic feet per

second, by the area of the stream, in square feet, where it approaches the weir, which will give the velocity of approach, in feet per second. Having obtained the value of $v$, the equivalent head $h$ may be found by the formula

$$h = .01555v^2$$

As $v$ is small in a properly constructed weir, it is usually neglected unless great accuracy is required.

The values of coefficients of discharge, as determined from experiments, for weirs with end and for weirs without end contractions are given in the accompanying tables. In the first two tables, the values of the coefficients are given in feet and tenths. When only a close approximation is required, it is desired to take all of the measurement in feet and inches. The third table should not be used unless the length of the crest is at least three or four times the depth of water passing over the weir, for if this is not the case, there will be serious errors caused by end contractions.

## COEFFICIENT OF DISCHARGE FOR WEIRS WITH END CONTRACTIONS

| Effective Head Feet | Length of Weir, in Feet | | | | | | |
|---|---|---|---|---|---|---|---|
| | .66 | 1 | 2 | 3 | 5 | 10 | 19 |
| | Value of Coefficient | | | | | | |
| .10 | .632 | .639 | .646 | .652 | .653 | .655 | .656 |
| .15 | .619 | .625 | .634 | .638 | .640 | .641 | .642 |
| .20 | .611 | .618 | .626 | .630 | .631 | .633 | .634 |
| .25 | .605 | .612 | .621 | .624 | .626 | .628 | .629 |
| .30 | .601 | .608 | .616 | .619 | .621 | .624 | .625 |
| .40 | .595 | .601 | .609 | .613 | .615 | .618 | .620 |
| .50 | .590 | .596 | .605 | .608 | .611 | .615 | .617 |
| .60 | .587 | .593 | .601 | .605 | .608 | .613 | .615 |
| .70 | | .590 | .598 | .603 | .606 | .612 | .614 |
| .80 | | | .595 | .600 | .604 | .611 | .613 |
| .90 | | | .592 | .598 | .603 | .609 | .612 |
| 1.00 | | | .590 | .595 | .601 | .608 | .611 |
| 1.20 | | | .585 | .591 | .597 | .605 | .610 |
| 1.40 | | | .580 | .587 | .594 | .602 | .609 |
| 1.60 | | | | .582 | .591 | .600 | .607 |

## COEFFICIENT OF DISCHARGE FOR WEIRS WITHOUT END CONTRACTIONS

| Effective Head Feet | Length of Weir, in Feet | | | | | | |
|---|---|---|---|---|---|---|---|
| | 19 | 10 | 7 | 5 | 4 | 3 | 2 |
| | Value of Coefficient | | | | | | |
| .10 | .657 | .658 | .658 | .659 | | | |
| .15 | .643 | .644 | .645 | .645 | .647 | .649 | .652 |
| .20 | .635 | .637 | .637 | .638 | .641 | .642 | .645 |
| .25 | .630 | .632 | .633 | .634 | .636 | .638 | .641 |
| .30 | .626 | .628 | .629 | .631 | .633 | .636 | .639 |
| .40 | .621 | .623 | .625 | .628 | .630 | .633 | .636 |
| .50 | .619 | .621 | .624 | .627 | .630 | .633 | .637 |
| .60 | .618 | .620 | .623 | .627 | .630 | .634 | .638 |
| .70 | .618 | .620 | .624 | .628 | .631 | .635 | .640 |
| .80 | .618 | .621 | .625 | .629 | .633 | .637 | .643 |
| .90 | .619 | .622 | .627 | .631 | .635 | .639 | 645 |
| 1.00 | .619 | .624 | .628 | .633 | .637 | .641 | 648 |
| 1.20 | .620 | .626 | .632 | .636 | .641 | .646 | |
| 1.40 | .622 | .629 | .634 | .640 | .644 | | |
| 1.60 | .623 | .631 | .637 | .642 | .647 | | |

## DISCHARGE PER MINUTE FOR EACH INCH IN LENGTH OF WEIR FOR DEPTHS FROM 1-8 IN. TO 25 IN.

| | Depth of Water, in Inches | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0 | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{2}$ | $\frac{5}{8}$ | $\frac{3}{4}$ | $\frac{7}{8}$ |
| | Discharge per Inch of Length per Minute, in Cubic Feet | | | | | | | |
| 0 | | .01 | .05 | .09 | .14 | .20 | .26 | .33 |
| 1 | .40 | .47 | .55 | .65 | .74 | .83 | .93 | 1.03 |
| 2 | 1.14 | 1.24 | 1.36 | 1.47 | 1.59 | 1.71 | 1.83 | 1.96 |
| 3 | 2.09 | 2.23 | 2.36 | 2.50 | 2.63 | 2.78 | 2.92 | 3.07 |
| 4 | 3.22 | 3.37 | 3.52 | 3.68 | 3.83 | 3.99 | 4.16 | 4.32 |
| 5 | 4.50 | 4.67 | 4.84 | 5.01 | 5.18 | 5.36 | 5.54 | 5.72 |
| 6 | 5.90 | 6.09 | 6.28 | 6.47 | 6.65 | 6.85 | 7.05 | 7.25 |
| 7 | 7.44 | 7.64 | 7.84 | 8.05 | 8.25 | 8.45 | 8.66 | 8.86 |
| 8 | 9.10 | 9.31 | 9.52 | 9.74 | 9.96 | 10.18 | 10.40 | 10.62 |
| 9 | 10.86 | 11.08 | 11.31 | 11.54 | 11.77 | 12.00 | 12.23 | 12.47 |
| 10 | 12.71 | 13.95 | 13.19 | 13.43 | 13.67 | 13.93 | 14.16 | 14.42 |
| 11 | 14.67 | 14.92 | 15.18 | 15.43 | 15.67 | 15.96 | 16.20 | 16.46 |
| 12 | 16.73 | 16.99 | 17.26 | 17.52 | 17.78 | 18.05 | 18.32 | 18.58 |
| 13 | 18.87 | 19.14 | 19.42 | 19.69 | 19.97 | 20.24 | 20.52 | 20.80 |
| 14 | 21.09 | 21.37 | 21.65 | 21.94 | 22.22 | 22.51 | 22.79 | 23.08 |
| 15 | 23.38 | 23.67 | 23.97 | 24.26 | 24.56 | 24.86 | 25.16 | 25.46 |
| 16 | 25.76 | 26.06 | 26.36 | 26.66 | 26.97 | 27.27 | 27.58 | 27.89 |
| 17 | 28.20 | 28.51 | 28.82 | 29.14 | 29.45 | 29.76 | 30.08 | 30.39 |
| 18 | 30.70 | 31.02 | 31.34 | 31.66 | 31.98 | 32.31 | 32.63 | 32.96 |
| 19 | 33.29 | 33.61 | 33.94 | 34.27 | 34.60 | 34.94 | 35.27 | 35.60 |
| 20 | 35.94 | 36.27 | 36.60 | 36.94 | 37.28 | 37.62 | 37.96 | 38.31 |
| 21 | 38.65 | 39.00 | 39.34 | 39.69 | 40.04 | 40.39 | 40.73 | 41.09 |
| 22 | 41.43 | 41.78 | 42.13 | 42.49 | 42.84 | 43.20 | 43.56 | 43.92 |
| 23 | 44.28 | 44.64 | 45.00 | 45.38 | 45.71 | 46.08 | 46.43 | 46.81 |
| 24 | 47.18 | 47.55 | 47.91 | 48.28 | 48.65 | 49.02 | 49.39 | 49.76 |

## CONVERSION FACTORS

**Cubic Feet Into Gallons.**—1 cu. ft. = 1,728 cu. in. = 1,728 ÷ 231 gal. = 7.4805194 gal.

**Gallons Into Cubic Feet.**—1 U. S. liq. gal. = 231 cu. in. = 231 ÷ 1,728 cu. ft. = .133680555 cu. ft

**Feet per Second Into Miles per Hour.**—1 ft. per sec. = 3,600 ft. per hr. = $\frac{3600}{5280}$, or $\frac{15}{22}$ mi. per hr.

**Miles per Hour Into Feet per Second.**—1 mi. per hr. = 5,280 ft. per hr. = $\frac{5280}{3600}$, or $\frac{22}{15}$ feet per sec.

**Second-Feet per Day Into Gallons.**—1 sec.-ft., or 7.4805194 gal. per sec. for 1 da., or 86,400 sec. = 646,316.87616 gal.

**Millions of Gallons Into Second-Feet per Day.**—1,000,000 gal. per 24 hr. = $\frac{231,000,000}{1,728 \times 86,400}$ cu. ft. per sec., or 1.5472286 sec. ft.

**Second-Feet per Day Into Acre-Feet.**—1 sec.-ft. flow for 1 da. = 86,400 cu. ft. = 86,400 ÷ 43,560 = 1.983473 A.-ft.

**Acre-Feet Into Second-Feet Flow for 24 Hours.**—1 A.-ft. each 24 hr. = 43,560 cu. ft. each 86,400 sec. = 43,560 ÷ 86,400, or $\frac{11}{22}$ sec.-ft. flow for 24 hr.

**Acre-Feet Into Gallons.**—1 A.-ft. = 43,560 cu. ft. (43,560 × 1,728) ÷ 231 = 325,851.428 gal.

**Millions of Gallons Into Acre-Feet.**—1,000,000 U. S. liq. gal., or 231,000,000 cu. in. = 133,680.555 cu. ft. = 133,680.555 ÷ 43,560 = 3.0688832 A.-ft.

**Second-Feet Into Minute Gallons.**—1 cu. ft. contains 1,728 cu. in.; 1 gal. has a capacity of 231 cu. in.; 1 sec.-ft. equals $[(1,728 \div 231) \times 60]$ gal. per min. $= 448.831164$ min.-gal.

**Minute-Gallons Into Second-Feet.**—1 gal. contains 231 cu. in.; 1 cu. ft. contains 1,728 cu. in.; 1 gal. per min. equals $[(231 \div 1,728) \div 60]$ sec.-ft., $= .0022280092$ sec.-ft.

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# FLOW OF WATER IN OPEN CHANNELS

**Ditches.**—In the case of hydraulic mining and irrigation, water is usually conveyed through ditches. The ditch line should be carefully surveyed and all brush and trees removed, and the underbrush cut away and burned, before beginning to excavate the ditch. The form of ditch and its grade will depend largely on the amount of water to be conveyed and the character of the soil in the section under consideration. As a general rule, the average flow of water in a ditch should not be less than 2 ft. per sec., and under most circumstances should not exceed 4 ft., though in rare cases where the formation is suitable, mean velocities of 5 ft. per sec. are employed. Sand will deposit from a current flowing at the rate of 1½ ft. per sec., and if the current does not have a velocity of at least 2 ft. per sec., vegetation is liable to block the ditch line.

The following letters will be used in the formulas for determining the various factors relating to ditches:

$h$ = difference in level between ends of canal or ditch, or between two points under consideration;

$l$ = horizontal length of portion of canal or ditch under consideration;

$s$ = slope = ratio $\dfrac{h}{l}$ = sine of slope;

$a$ = area of water cross-section in square feet;

$p$ = wet perimeter = portion of outline of cross-section of stream in contact with channel, in feet;

$r$ = hydraulic radius, or hydraulic mean depth = ratio $\dfrac{a}{p}$;

$c'$ = coefficient, depending on nature of surface of ditch;

$c$ = coefficient depending on nature of surface of ditch, as determined by Kutter's formula;

$v$ = mean velocity of flow, in feet per second;

$v'$ = surface velocity of a stream;

$v_b$ = bottom velocity of a stream;

$x$ = bottom or one side of a section the form of which is one-half a regular hexagon, in feet;

$Q$ = quantity of water flowing, in cubic feet per second;

$n$ = coefficient of roughness in Kutter's formula.

**Safe Bottom Velocity.**—The bottom velocity of a stream may be obtained from the average velocity by the formula

$$v_b = v - 10.87\sqrt{rs}$$

The accompanying table gives values of safe bottom and mean velocities, corresponding with certain materials, as given by Ganguillet and Kutter:

### SAFE BOTTOM AND MEAN VELOCITIES OF STREAMS

| Material of Channel | Safe Bottom Velocity $v_b$ Feet per Second | Mean Velocity $v$ Feet per Second |
|---|---|---|
| Soft brown earth | .249 | .328 |
| Soft loam | .499 | .656 |
| Sand | 1.000 | 1.312 |
| Gravel | 1.998 | 2.625 |
| Pebbles | 2.999 | 3.938 |
| Broken stone, flint | 4.003 | 5.579 |
| Conglomerate, soft slate | 4.988 | 6.564 |
| Stratified rock | 6.006 | 8.204 |
| Hard rock | 10.009 | 13.127 |

**Resistance of Soils to Erosion by Water.**—The following resistances of various soils to erosion by water have been selected from the experiments of W. A. Burr:

| Soil | Feet per Second |
|------|------|
| Pure sand resists erosion by flow of | 1.10 |
| Sandy soil, 15% clay, resists erosion by flow of | 1.20 |
| Sandy loam, 40% clay, resists erosion by flow of | 1.80 |
| Loamy soil, 65% clay, resists erosion by flow of | 3.00 |
| Clay loam, 85% clay, resists erosion by flow of | 4.80 |
| Agricultural clay, 95% clay, resists erosion by flow of | 6.20 |
| Clay, resists erosion by flow of | 7.35 |

**Carrying Capacity of Ditches.**—Ditches should never be run full, but should be constructed large enough so that they will carry the desired amount of water when from three-fourths to seven-eighths full. For any given cross-section, the greatest flow will be attained when the hydraulic radius or hydraulic mean depth is equal to one-half of the actual depth of the channel. The cross-section of a ditch or conduit that has the greatest possible carrying capacity is a half circle, and the nearest practical approach to this is a half hexagon. Knowing the cross-section of a ditch, its dimensions may be found by the formula:

$$x = \sqrt{\frac{2a}{2.598}}$$

As the obtuse angle between the side and bottom of the ditch is 120°, the form can be easily laid off. The carrying capacity of ditches generally increases after they have been in use some time, as the ditch becomes lined with a fine scum that closes the pores in the soil and prevents leakage; this may increase the amount by as much as 10%.

**Grade.**—The grade of the ditch must be sufficient to create the desired velocity of flow, and depends largely on the character of the material composing the surface of the ditch. If the surface is smooth, as, for instance, where the ditch is cut through clay or is lined with masonry, the grade can be considerably less than where the surface is rough, or when cut through coarse gravel or when lined with rough stone. In mountainous countries, where the ground is hard, deep narrow ditches with steep grades are generally preferred to larger channels with gentle slopes, as the cost of excavation is considerably less; but steep grades and narrow ditches are suitable only when the banks can resist the rapid flow. In California, grades of from 16 to 20 ft. per mi. are used, and 10 ft. per mi. is quite common. Water channels of a uniform cross-section should have a uniform grade; otherwise, the flow will be checked in places, which will result in deposits of sand or silt in some portions of the ditch, which are liable to cause the banks to be overflowed and the ditch to be ultimately destroyed. When designing any given ditch, the grade is generally assumed to correspond to the formation of the country and the velocity figured from the grade. In case $v$ is found to be so great that it will cut the banks, it will be necessary either to reduce the grade or to change the form of the ditch so as to reduce the velocity.

*Ditch Banks,* when possible, should be composed of solid material, but when necessary to use excavated material care must be taken to see that the material is so placed as to avoid settling and cracking as much as possible. All stumps, roots, etc. should be separated from the material to be used for embankments. If artificial banks are necessary, they should be built of masonry if the expense is not too great; or, the water may be carried across depressions in pipes or flumes. When the character of the material through which the ditch is constructed is not sufficiently firm to resist the desired current velocity, it is necessary to line the ditch. In some locations the ditches are simply smoothed on the inside and lined with from ¾ in. to 1 in. of cement mortar, made up of Portland cement and sharp sand. In other cases, they are lined with dry stonework laid up in order and carefully bonded together. Sometimes the stonework is pointed with cement or mortar on the inside, so as to present a more uniform surface to the flow. In other cases, the sides are simply revetted with stone.

**Influence of Depth on Ditch.**—The depth of the flow in a ditch has considerable influence on the scouring or eroding of the bottom and the banks, owing to the fact that a much greater average velocity can be attained in a deep stream than in a shallow stream, without causing an excessive velocity of the water in contact with the wet perimeter. For this reason, in cases where banks will stand it, it is best to use narrow deep ditches rather than wide flat

ditches, though each location has to be treated in accordance with its own special conditions, and no general rule can be laid down.

**Measuring the Flow of Water in Channels.**—The laws for the resistance to the flow may be expressed by the relation $ha = c'lpv^2$; or

$$v = \sqrt{\frac{h}{c'l} \times \frac{a}{p}} = \sqrt{\frac{l1}{c'} \times s \times r}$$

If $c = \sqrt{\frac{l1}{c'}}$, the formula becomes

$$v = c\sqrt{rs}$$

The coefficient $c$ is usually found by means of Kutter's formula, one form of which is as follows:

$$c = \frac{23 + \frac{1}{n} + \frac{.00155}{s}}{.5521 + \left(23 + \frac{.00155}{s}\right)\frac{n}{\sqrt{r}}}$$

The values for $n$, the coefficient of roughness under various conditions, are given in the accompanying table:

## COEFFICIENT OF ROUGHNESS UNDER VARIOUS CONDITIONS

| Character of Channel | Value of $n$ |
|---|---|
| Clean, well-planed timber | .009 |
| Clean, smooth, glazed iron, and stoneware pipes | .010 |
| Masonry, smoothly plastered with cement, and for very clean, smooth, cast-iron pipe | .011 |
| Unplaned timber, ordinary cast-iron pipe, and selected pipe sewers, well laid and thoroughly flushed | .012 |
| Rough iron pipes and ordinary sewer pipes, laid under usual conditions | .013 |
| Dressed masonry and well-laid brickwork | .015 |
| Good rubble masonry and ordinary rough or fouled brickwork | .017 |
| Coarse rubble masonry and firm, compact gravel | .020 |
| Well-made earth canals in good alinement | .0225 |
| Rivers and canals in moderately good order and perfectly free from stones and weeds | .025 |
| Rivers and canals in rather bad condition and somewhat obstructed by stones and weeds | .030 |
| Rivers and canals in bad condition, overgrown with vegetation and strewn with stones and other detritus, according to condition | .035 to .05 |

As it is quite difficult to obtain the value of $c$ by Kutter's formula, the following three approximate formulas for $v$ are given:

For canals with earthen banks,

$$v = \sqrt{\frac{100,000r^2s}{9r + 35}}$$

If the ditch is lined with dry stonework,

$$v = \sqrt{\frac{100,000r^2s}{8r + 15}}$$

If the ditch is lined with rubble masonry,

$$v = \sqrt{\frac{100,000r^2s}{7.3r + 6}}$$

To find the quantity $Q$ of water flowing through any channel in a given time, multiply the velocity by the area, or $Q = av$.

**Flow in Brooks and Rivers.**—When a stream is so large that it becomes impracticable to employ a weir for measuring its flow, fairly accurate results may be arrived at by determining the velocity of the current at various points

in a carefully surveyed cross-section of the stream, thus determining both $v$ and $a$. The greatest velocity of current occurs at a point some distance below the surface, in the deepest part of the channel. When determining the current velocities in the different portions of a stream, it is frequently advantageous to divide the stream into divisions. This may be accomplished by stretching a wire across and tying strings or rags about the wire at various points. The mean velocity of the current between these points can be determined by current meters, or by floats. The points for observation should be chosen where the channel is comparatively straight and the current uniform. Surface floats may be used, in which case the mean velocity of the point where the float is used may be found as follows: If $v'$ equals the observed velocity, then the mean velocity will be $v = .9v'$.

By taking observations of the velocity of the current in each section of a stream, the amount of water flowing may be determined for each separate section. The total amount of water flowing in the stream will be the sum of the amounts in each section. The average velocity of the entire stream may be found by dividing the total amount of water flowing by the total area of the cross-section of the stream. The correction necessary to reduce surface velocity to mean velocity may be made as follows: Measure off nine-tenths of the ordinary distance, and figure the time as though for the full distance. For instance, if only 90 ft. is employed, the time will be taken and the problem figured as though it were 100 ft., because the mean velocity is only nine-tenths of the surface velocity.

## FLUMES

*Flumes* are used for conveying water when a ditch line would be abnormally long, or when the material to be excavated is very hard. They may be constructed of timber or of metal, but metal flumes are comparatively rare, as piping can be used instead. The line of the proposed flume should be carefully cleared of all standing timber, and the brush burned for at least 20 ft. each side of the flume line to prevent danger from fire. The life of an ordinary flume, which is supported on or constructed of timber, is always short, varying, as a rule, from 10 to 20 yr., depending on whether the flume is allowed to run dry a portion of the year or is always full of water, the care with which it was originally constructed, and the attention paid to repairs.

**Grade and Form of Flumes.**—Flumes are usually set on a much steeper grade than is possible in ditches, the grade frequently being as much as 25 to 30 ft. per mi., and in special cases even more. The result of this is that the carrying capacity of flumes is much greater than that of ditches of the same size.

The form of flume depends largely on the material of which it is constructed. Metal flumes may have a semicircular form, while wooden flumes are either rectangular or **V**-shaped. The former is used almost exclusively for conveying water, and the latter quite extensively for fluming timber or cord wood from the mountains to the shipping point in the valley.

*Timber flumes* should be so constructed that the water will meet with but small resistance, and the bottom and side should be enclosed in a frame of timbers so braced or secured that there is no possible chance of the sides spreading or lifting from the bottom, and thus causing leakage. As a rule, all mortised-and-tenoned joints should be avoided in flume construction. Fig. 1 shows a timber flume in which no joints are cut; the bottoms of the posts are kept in place by stringers spiked on the sills and the tops are tied together by pieces bolted on. Fig. 2 shows a construction in which the posts are let into the sills and
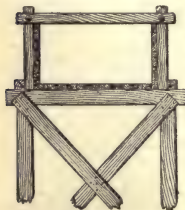


FIG. 1
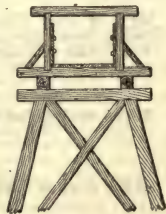


FIG. 2

supported by diagonal braces. The ties across the top of the posts are also notched to receive the upper ends of the posts. As a rule, these ties are only placed on every third or fourth frame, the diagonal braces being depended on to hold the other posts in place. The joints between the planking may be battened on the inside with strips of ½-in. lumber, 4 or 5 in. wide, or the edges

of the planking may be dressed and painted before they are put together, so as to form a tight joint.

**Connection With Ditches.**—Where flumes connect with ditches or dams, the posts for several boxes should be made longer, so that they may receive another sideboard to prevent the water from splashing over the sides. The flume should also be widened out or flared, both at its entry and discharge ends. Where the flume passes through a bank of earth, an outer siding may be nailed on the outside of the posts, to protect the flume from rotting.

**Trestles.**—Where flumes are carried on trestles, the individual frames supporting the flume are usually placed on heavy stringers, which in turn are supported upon trestle bents from 12 to 16 ft. apart, the frames supporting the flume being placed about 4 ft. apart.

**Curves.**—Where flumes are laid around curves, the outer edge of the flume should be elevated so as to prevent splashing and to cause the flowing water to have a uniform depth across the width of the flume. It is impossible to give any definite rule as to the amount that the outer edge of the flume should be raised, but this is usually accomplished by judging the amount when the flume is first constructed, and correcting this by wedging up after the water is flowing. The individual boxes of the flume may have to be cut into two or three portions on curves, and at times the side planks are sawed partly through, so as to enable them to be bent to the desired curve.

**Waste Gates.**—Waste gates should be placed every $\frac{1}{2}$ mi., to empty the flume for repairs, or in case of accident. They are also useful for flushing snow out of a flume. In snowy regions, flumes are frequently protected by sheds over their exposed portions.

**Flow of Water Through Flumes.**—As smooth wooden surfaces offer considerably less resistance to the flow of water than earth or stone canals, the coefficients must necessarily be somewhat reduced, and the following formula is useful in giving the flow of water through flumes:

$$v = \sqrt{\frac{100,000r^2s}{6.6r + 0.46}}$$

That flumes may have their full carrying capacity, they have to be of sufficient length to get the water in motion, or, as it is technically expressed, "to put the water in train." It is largely on this account that flumes have to be made of a larger cross-section at both the entrance and the exit.

In cold countries it may be best to construct the flume narrower than it is deep, as in cold weather the ice in the narrow flume freezes a crust entirely across the surface, thus protecting the water from further action of the elements and frequently prolonging the flow through the flume for several weeks, while wide shallow flumes will not freeze on the surface so quickly, but will freeze in from the bottom and sides until they are practically a solid mass of ice.

When a flume is laid on the ground along a bank, it should be laid as close to the bank as possible, so as to protect it from snow or landslides, and so that in the winter the snow will drift in under and behind it, thus preventing the circulation of the air about the flume. This will protect the flume, and may prolong the flow for some time after cold weather sets in.

# TUNNELS

*Tunnels* are sometimes used for conveying water, in connection with flume or ditch lines. Where a tunnel is unlined, it is best to give the roof the shape of the Gothic arch, because this stands better and resists scaling to a greater extent than the round arch, which usually scales off until it has the form of the Gothic arch. If tunnels are to be used as water conduits, without lining, care should be taken to make the inside of the tunnel as smooth as possible. In some cases, in order to increase the carrying capacity, the tunnel has been lined with wooden-stave pipe, backed with concrete, the pipe requiring no metal bands, but depending on the concrete to keep it in place. When such linings are employed, it is not practicable to have them exposed to the alternate action of the water and the atmosphere; hence, the tunnel should be kept continually full of water. To accomplish this, the tunnel may be dropped below the grade of the ditch or flume line, so that it is always under a slight hydrostatic pressure, and even if the water is turned off from the line, the tunnel will remain full of water, the same as an inverted siphon. Sometimes tunnels are lined with cement, being given either a circular or oval form, or they may have a flat bottom, with flat sides and an arched roof. The cement

may be placed directly on the country rock composing the walls of the tunnel or the tunnel may be lined with brick or stone, and then cemented on the inside.

**Flow Through Tunnels.**—The flow of water through tunnels, when they are only partly filled, is calculated by the formulas for flow in open channels, while in the case of lined tunnels that are run full, the flow is calculated by formulas for calculating the flow through pipes.

# FLOW THROUGH PIPES

**Hydraulic Gradient.**—If a pipe of uniform cross-section is connected with a reservoir, and water is allowed to discharge through its open end, the pressure on the pipe at any point is equal to the vertical distance from the center of the pipe at that point to an imaginary line, called the *hydraulic gradient* or *hydraulic grade line*. This is a line drawn from a point slightly below the surface of the
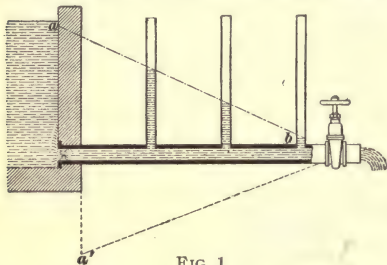


FIG. 1

water in the reservoir to the outlet of the pipe, as *ab*, Fig. 1. The distance from the surface of the water to the point *a* is equal to the head lost in overcoming the friction at the entrance to the pipe, and is rarely over 1 ft. A pipe laid along the line *ab* will carry exactly the same amount of water as when laid horizontally, as shown, but there will be practically no pressure tending to burst the pipe at any point along this line; while if it is laid along the line from the point *a'* (the reservoir being made deeper), it will still deliver exactly the same amount of water, but the pressure tending to burst the pipe will be greatly increased. In order that a pipe may have a maximum discharge, no point in the line must rise above the hydraulic gradient; it makes no difference in the discharge how far below the gradient it may fall.

In Fig. 2, the pipe rises above the hydraulic gradient *ac*; in this case a new hydraulic gradient *ab* must be established and the flow calculated for this head. The pipe *bc* simply acts to carry off the water delivered to it at *b*. If the upper side of the pipe is open at the point *b*, the water will have no tendency to escape, but, on the contrary, air will probably enter and the pipe flow only partly full from *b* to *c*.

**Flow in Pipes.**—Darcy, a French engineer, made a series of experiments on different diameters of cast-iron pipe, with different degrees of internal roughness, from which he calculated a series of formulas. The following are some of these formulas, as arranged by the late E. Sherman Gould, C. E., E. M. Darcy found that the character of the inside surface of the pipe played a very important part in its discharge, and he deduced a formula and determined a series of coefficients for it, but
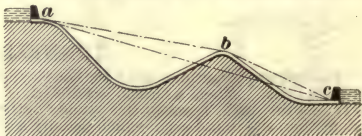


FIG. 2

Mr. Gould calls attention to the fact that the coefficients for pipes from 8 to 48 in. in diameter practically cancel the numerical factor employed in Darcy's formula, and that a slightly different factor applies to pipes from 3 to 8 in., so that the following simple formulas, in which the factors given apply, may be obtained:

$$Q = \text{discharge, in cubic feet per second;}$$
$$q = \text{discharge, in U. S. gallons per minute;}$$
$$D = \text{diameter of pipe, in feet;}$$
$$d = \text{diameter of pipe, in inches;}$$
$$H = \text{total head, in feet;}$$
$$h = \text{head per 1,000 ft.;}$$
$$V = \text{velocity, in feet per second.}$$

*Pipes Between 3 and 8 In. in Diameter*

Rough inside surface, $Q = .89\sqrt{D^5h} = .89D^2\sqrt{Dh}$; $V = 1.13\sqrt{Dh}$

Smooth inside surface, $Q = .89\sqrt{2D^5h} = 1.25D^2\sqrt{Dh}$; $V = 1.6\sqrt{Dh}$

*Pipes Above 8 In. in Diameter*

Rough inside surface, $Q = \sqrt{D^5h} = D^2\sqrt{Dh}$; $V = 1.27\sqrt{Dh}$

Rough inside surface, d in inches, $Q = \dfrac{d^2}{288}\sqrt{\dfrac{dh}{3}}$

Smooth inside surface, $Q = 1.4\sqrt{D^5h} = 1.4D^2\sqrt{Dh}$; $V = 1.78\sqrt{Dh}$

As a rule, it is best to calculate any pipe line by the formula for pipes having a rough internal surface, for if this is not done the results are liable to be disappointing, as all pipes become more or less rough with use.

**Eytelwein's Formula for Delivery of Water in Pipes:**

$D$ = diameter of pipe, in inches;
$H$ = head of water, in feet;
$L$ = length of pipe, in feet;
$W$ = water discharged per minute, in cubic feet.

$$W = 4.71\sqrt{\frac{D^5H}{L}} \qquad D = .538\sqrt[5]{\frac{L \times W^2}{H}}$$

**Hawksley's Formula:**

$G$ = number of gallons delivered per hour;
$L$ = length of pipe, in yards;
$H$ = head of water, in feet;
$D$ = diameter of pipe, in inches.

$$D = \tfrac{1}{15}\sqrt[5]{\frac{G^2L}{H}} \qquad G = \sqrt{\frac{(15D)^5H}{L}}$$

**Neville's General Formula:**

$v$ = velocity, in feet per second;
$r$ = hydraulic mean depth, in feet;
$s$ = sine of inclination, or total fall divided by total length

$$v = 140\sqrt{rs} - 11\sqrt[3]{rs}$$

In cylindrical pipes, $v \times 47.124d^2$ = discharge per minute, in cubic feet, or $v \times 293.7286d^2$ = discharge per minute in gallons, $d$ being the diameter of the pipe in feet.

**Comparison of Formulas.**—The various formulas for velocity are as follows:

$R$ = mean hydraulic depth, in feet = area ÷ wet perimeter = $\dfrac{d}{4}$ for circular section of pipe;

$S$ = sine of slope = $\dfrac{H}{L}$;

$v$ = velocity, in feet per second;
$d$ = diameter of pipe, in feet;
$L$ = length of pipe, in feet;
$H$ = head of water, in feet.

Prony, $\qquad v = 97.05\sqrt{RS} - .08 \qquad v = 99.88\sqrt{RS} - .154$

Eytelwein, $\qquad v = 50\sqrt{\dfrac{dH}{L + 50d}}$

Eytelwein, $\qquad v = 108\sqrt{RS} - .13$

Hawksley, $\qquad v = 48\sqrt{\dfrac{dH}{L + 54d}}$

Neville, $\qquad v = 140\sqrt{RS} - 11\sqrt[3]{RS}$

Darcy, $\qquad v = C\sqrt{RS}$

For the value of $C$ see the following table. The maximum value of $C$ for very large pipes is 113.3.

Kutter, $\qquad v = C\sqrt{RS}$

in which $\qquad C = \dfrac{181 + \dfrac{.00281}{S}}{1 + \dfrac{.026}{\sqrt{d}}\left(41.6 + \dfrac{.00281}{S}\right)}$

Weisbach, $$h = \frac{L}{r}\left(.0036 + \frac{.0043}{\sqrt{v}}\right)\frac{v^2}{2g},$$

in which $h$ = head necessary to overcome friction in the pipe;
$r$ = mean radius of pipe, in feet;
$g$ = gravity = 32.2.

Darcy, $$h = \frac{.02L}{d}\left(1 + \frac{1}{12d}\right)\frac{v^2}{2g}$$

### VALUE OF C IN DARCY'S FORMULA

Diameter of Pipes, in Inches

| $\frac{1}{2}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

Value of $C$.

| 65 | 80 | 93 | 99 | 102 | 103 | 105 | 106 | 107 | 108 | 109 | 109.5 | 110 | 110.5 | 110.7 | 111 | 111.5 | 111.5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

**Loss of Head in Pipe by Friction.**—In each 100 ft. in length the loss of head, by friction, in pipes of different diameters, when discharging various quantities of water per minute is given in the accompanying table, which has been prepared by the Pelton Water Wheel Co.
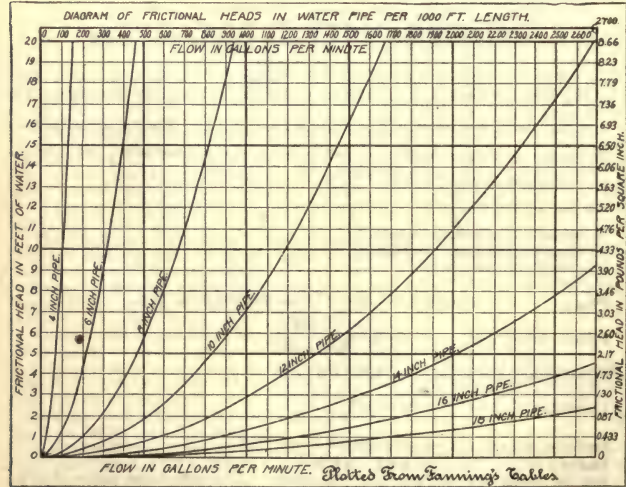


FIG. 3

EXAMPLE.—Having 200-ft. head and 600 ft. of 11-in. pipe, carrying 119 cu. ft. of water per minute, what is the effective head?

SOLUTION.—In right-hand column, under 11-in. pipe, find 119 cu. ft. Opposite this will be found the coefficient of friction for this amount of water, which is .444. Multiplying this by the number of hundred feet of pipe, which

is 6, gives 2.66 ft., which is the loss of head. Therefore, the effective head is $200 - 2.66 = 197.34$.

The following formula, deduced by William Cox, gives practically the same results as the accompanying table, and will be found useful in many instances:

$$F = \frac{L}{1,000D}(4V^2 + 5V - 2),$$

in which
$F =$ friction head;
$L =$ length of pipe, in feet;
$D =$ diameter of pipe, in inches;
$V =$ velocity, in feet per second.

The diagram shown in Fig. 3 gives the frictional heads in 1,000 ft. of water pipe. It shows the flow, in gallons per minute, of the common sizes and gives the frictional heads in both feet of water and pounds per square inch.

**Friction of Knees and Bends.**—To obtain the friction of knees and bends, the following formulas may be taken as giving close approximate results. It is well to bear in mind that right angles should be avoided whenever possible, and that bends should be made with as large a radius as circumstances will allow. The position of the angle $A$ is shown in Fig. 4.

Let $A =$ angle of bend or knee with forward
line of direction;
$V =$ velocity of water, in feet per second;
$R =$ radius of center line of bend;
$r =$ radius of bore of pipe (or $\frac{1}{2}$ diameter);
$K =$ coefficient for angles of knees;
$L =$ coefficient for curvature of bends;
$H =$ head of water, in feet, necessary to
overcome friction of bends, or knees.
$H = .0155V^2K.$
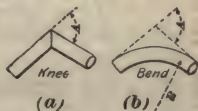


FIG. 4.

The value of $K$ is as follows for different angles:

| $A°$ | $20°$ | $40°$ | $60°$ | $80°$ | $90°$ | $100°$ | $120°$ |
|---|---|---|---|---|---|---|---|
| $K$ | .046 | .139 | .364 | .74 | .98 | 1.26 | 1.86 |

For bends,

$$H = .0155V^2\left(\frac{A}{180}L\right)$$

Values of $L$ for various ratios of the radius of bend to radius of bore:

| When $\frac{r}{R} =$ | .1 | .2 | .3 | .4 | .5 | .6 | .7 | .8 | .9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| Circular section $L$ | .131 | .138 | .158 | .206 | .294 | .44 | .66 | .98 | 1.4 | 2.0 |
| Rectangular section $L$ | .124 | .135 | .18 | .25 | .4 | .64 | 1.01 | 1.55 | 2.3 | 3.2 |

## RELATIVE QUANTITIES OF WATER DELIVERED IN 24 HOURS, IN 1 HOUR, AND IN 1 MINUTE

| Gal. in 24 Hr. | Gal. in 1 Hr. | Gal. in 1 Min. | Gal. in 24 Hr. | Gal. in 1 Hr. | Gal. in 1 Min. | Gal. in 24 Hr. | Gal. in 1 Hr. | Gal. in 1 Min. |
|---|---|---|---|---|---|---|---|---|
| 2,500,000 | 104,166.6 | 1,736.0 | 650,000 | 27,083.3 | 451.3 | 150,000 | 6,250.0 | 104.1 |
| 2,000,000 | 83,333.3 | 1,388.8 | 600,000 | 25,000.0 | 416.7 | 100,000 | 4,166.6 | 69.4 |
| 1,500,000 | 62,500.0 | 1,041.7 | 550,000 | 22,916.6 | 381.9 | 75,000 | 3,125.0 | 52.1 |
| 1,000,000 | 41,666.6 | 694.4 | 500,000 | 20,833.3 | 347.2 | 60,000 | 2,500.0 | 41.6 |
| 950,000 | 39,583.3 | 659.7 | 450,000 | 18,750.0 | 312.5 | 50,000 | 2,083.3 | 34.7 |
| 900,000 | 37,500.0 | 625.0 | 400,000 | 16,666.6 | 277.7 | 25,000 | 1,041.6 | 17.3 |
| 850,000 | 35,416.6 | 590.2 | 350,000 | 14,583.3 | 243.0 | 20,000 | 833.3 | 13.8 |
| 800,000 | 33,333.3 | 555.5 | 300,000 | 12,500.0 | 208.3 | 15,000 | 625.0 | 10.4 |
| 750,000 | 31,250.0 | 520.8 | 250,000 | 10,416.7 | 173.6 | 10,000 | 416.6 | 6.9 |
| 700,000 | 29,166.6 | 486.1 | 200,000 | 8,333.3 | 138.8 | 5,000 | 208.3 | 3.4 |

# ACTUAL AMOUNT, OR 80% OF THE THEORETICAL FLOW, IN PIPES FROM 1 IN. TO 30, IN. DIAMETER*

Inside Diameter of Pipes, in Inches

Discharge of Pipes, in Gallons per Minute

| Fall or Slope | 1 | 1¼ | 1½ | 1¾ | 2 | 2½ | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 15 | 18 | 24 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 in 1,000 | .7 | 1.2 | 2.0 | 3.0 | 4.2 | 7.6 | 12.3 | 26 | 46 | 74 | 111 | 156 | 212 | 279 | 356 | 446 | 793 | 1,267 | 2,652 | 4,698 |
| 2 in 1,000 | 1.0 | 1.9 | 3.0 | 4.6 | 6.5 | 11.6 | 18.6 | 39 | 69 | 111 | 166 | 234 | 316 | 415 | 530 | 663 | 1,174 | 1,873 | 3,909 | 6,911 |
| 3 in 1,000 | 1.3 | 2.4 | 3.9 | 5.8 | 8.2 | 14.7 | 23.6 | 49 | 88 | 140 | 209 | 294 | 398 | 522 | 666 | 832 | 1,473 | 2,347 | 4,891 | 8,638 |
| 4 in 1,000 | 1.6 | 2.8 | 4.6 | 6.8 | 9.8 | 17.4 | 27.9 | 58 | 103 | 165 | 248 | 346 | 468 | 613 | 782 | 977 | 1,727 | 2,751 | 5,727 | 10,109 |
| 5 in 1,000 | 1.8 | 3.3 | 5.4 | 7.9 | 11.1 | 19.8 | 31.7 | 66 | 117 | 187 | 278 | 392 | 530 | 694 | 885 | 1,105 | 1,956 | 3,110 | 6,470 | 11,415 |
| 6 in 1,000 | 2.0 | 3.6 | 5.9 | 8.7 | 12.4 | 22.0 | 35.2 | 73 | 130 | 208 | 308 | 434 | 586 | 767 | 979 | 1,222 | 2,159 | 3,426 | 7,145 | 12,601 |
| 7 in 1,000 | 2.2 | 4.0 | 6.4 | 9.6 | 13.5 | 23.3 | 38.4 | 80 | 142 | 226 | 335 | 473 | 639 | 833 | 1,066 | 1,331 | 2,349 | 3,737 | 7,769 | 13,772 |
| 8 in 1,000 | 2.4 | 4.3 | 6.9 | 10.3 | 14.6 | 25.9 | 41.4 | 86 | 153 | 244 | 362 | 509 | 687 | 899 | 1,147 | 1,432 | 2,527 | 4,019 | 8,351 | 14,720 |
| 9 in 1,000 | 2.6 | 4.6 | 7.4 | 11.1 | 15.6 | 27.7 | 44.2 | 92 | 163 | 260 | 385 | 543 | 733 | 959 | 1,223 | 1,522 | 2,694 | 4,284 | 8,900 | 15,684 |
| 10 in 1,000 | 2.7 | 4.9 | 7.9 | 11.7 | 16.5 | 29.4 | 46.9 | 98 | 173 | 277 | 408 | 575 | 777 | 1,016 | 1,298 | 1,617 | 2,853 | 4,536 | 9,421 | 16,599 |
| 1 in 100 | 4.1 | 7.3 | 11.7 | 17.4 | 24.5 | 43.6 | 69.1 | 144 | 254 | 404 | 598 | 840 | 1,134 | 1,482 | 1,888 | 2,355 | 4,149 | 6,653 | 13,664 | 23,993 |
| 2 in 100 | 5.7 | 10.2 | 16.2 | 24.5 | 34.6 | 54.3 | 86.3 | 179 | 317 | 504 | 749 | 1,046 | 1,411 | 1,844 | 2,348 | 2,928 | 5,157 | 8,184 | 16,958 | 29,828 |
| 3 in 100 | 6.1 | 11.0 | 17.2 | 25.5 | 35.9 | 63.5 | 101.1 | 210 | 370 | 588 | 870 | 1,222 | 1,647 | 2,152 | 2,740 | 3,416 | 5,998 | 9,539 | 19,754 | 34,732 |
| 4 in 100 | 6.9 | 12.2 | 19.5 | 28.9 | 40.6 | 71.7 | 114.1 | 237 | 418 | 663 | 981 | 1,377 | 1,856 | 2,424 | 3,082 | 3,838 | 6,769 | 10,738 | 22,228 | 39,073 |
| 5 in 100 | 7.6 | 13.7 | 21.6 | 31.9 | 44.9 | 79.1 | 126.0 | 261 | 461 | 731 | 1,082 | 1,518 | 2,046 | 2,665 | 3,401 | 4,239 | 7,457 | 11,826 | 24,474 | 43,011 |
| 6 in 100 | 8.3 | 14.7 | 23.5 | 34.7 | 48.8 | 85.1 | 137.7 | 284 | 500 | 793 | 1,175 | 1,648 | 2,221 | 2,900 | 3,691 | 4,601 | 8,091 | 12,830 | 26,642 | 46,642 |
| 7 in 100 | 9.0 | 15.8 | 25.2 | 37.4 | 52.5 | 92.6 | 147.2 | 305 | 537 | 854 | 1,262 | 1,765 | 2,384 | 3,113 | 3,962 | 4,938 | 8,683 | 13,766 | 28,477 | 50,029 |
| 8 in 100 | 9.6 | 16.9 | 26.9 | 39.8 | 56.0 | 98.7 | 156.8 | 325 | 572 | 910 | 1,343 | 1,874 | 2,538 | 3,314 | 4,217 | 5,286 | 9,240 | 14,648 | 30,296 | 53,218 |
| 9 in 100 | 10.1 | 17.9 | 28.5 | 42.2 | 59.2 | 104.5 | 165.9 | 344 | 606 | 959 | 1,421 | 1,992 | 2,684 | 3,504 | 4,459 | 5,557 | 9,768 | 15,484 | 32,018 | 56,238 |
| 10 in 100 | 10.7 | 18.7 | 29.8 | 44.2 | 62.4 | 109.6 | 175.5 | 363 | 639 | 1,012 | 1,499 | 2,102 | 2,832 | 3,697 | 4,704 | 5,863 | 10,305 | 16,336 | 34,016 | 59,331 |
| 1 in 10 | 14.8 | 26.7 | 42.3 | 62.2 | 86.0 | 154.6 | 239.9 | 498 | 876 | 1,389 | 2,051 | 2,884 | 3,871 | 5,051 | 6,425 | 8,004 | 14,059 | 22,273 | 46,016 | 80,770 |
| 2 in 10 | 18.4 | 32.4 | 51.4 | 76.2 | 106.6 | 188.4 | 298.2 | 617 | 1,085 | 1,720 | 2,539 | 3,537 | 4,789 | 6,246 | 7,947 | 9,899 | 17,380 | 27,524 | 56,839 | 99,731 |
| 3 in 10 | 21.5 | 37.8 | 59.9 | 88.8 | 124.5 | 219.0 | 347.3 | 718 | 1,262 | 2,001 | 2,953 | 4,134 | 5,568 | 7,262 | 9,234 | 11,504 | 20,192 | 31,972 | 66,005 | 115,785 |
| 4 in 10 | 24.2 | 42.6 | 67.7 | 99.9 | 140.1 | 246.4 | 390.7 | 808 | 1,419 | 2,248 | 3,319 | 4,647 | 6,266 | 8,161 | 10,365 | 12,923 | 22,679 | 35,903 | 74,102 | 129,971 |
| 5 in 10 | 27.0 | 47.0 | 74.5 | 110.1 | 154.3 | 271.3 | 430.1 | 824 | 1,561 | 2,474 | 3,651 | 5,113 | 6,886 | 8,974 | 11,411 | 14,209 | 24,932 | 39,465 | 81,442 | 142,825 |
| 6 in 10 | 29.0 | 51.0 | 80.9 | 119.4 | 167.4 | 294.2 | 466.4 | 964 | 1,692 | 2,682 | 3,957 | 5,407 | 7,456 | 9,724 | 12,364 | 15,391 | 27,009 | 42,682 | 88,264 | 154,068 |
| 7 in 10 | 31.1 | 54.7 | 86.8 | 128.2 | 179.6 | 315.7 | 500.0 | 1,033 | 1,815 | 2,876 | 4,242 | 5,940 | 7,993 | 10,423 | 13,252 | 16,501 | 28,946 | 45,811 | 94,508 | 165,708 |
| 8 in 10 | 33.1 | 58.2 | 92.5 | 136.4 | 191.1 | 335.8 | 532.0 | 1,099 | 1,931 | 3,058 | 4,510 | 6,315 | 8,499 | 11,081 | 14,088 | 17,540 | 30,767 | 48,690 | 100,438 | 176,090 |
| 9 in 10 | 35.0 | 61.6 | 97.6 | 144.2 | 202.0 | 354.9 | 562.9 | 1,162 | 2,040 | 3,230 | 4,764 | 6,671 | 8,975 | 11,704 | 14,880 | 18,525 | 32,492 | 51,416 | 106,052 | 185,902 |

*To obtain the flow in cubic feet, divide the quantity given by 7.5.

# LOSS OF HEAD BY FRICTION

### Inside Diameter of Pipe, in Inches

| Velocity Ft. per Sec. | 1 Loss of Head, Ft. | 1 Cu. Ft. per Min. | 2 Loss of Head, Ft. | 2 Cu. Ft. per Min. | 3 Loss of Head, Ft. | 3 Cu. Ft. per Min. | 4 Loss of Head, Ft. | 4 Cu. Ft. per Min. | 5 Loss of Head, Ft. | 5 Cu. Ft. per Min. | 6 Loss of Head, Ft. | 6 Cu. Ft. per Min. | 7 Loss of Head, Ft. | 7 Cu. Ft. per Min. | 8 Loss of Head, Ft. | 8 Cu. Ft. per Min. | 9 Loss of Head, Ft. | 9 Cu. Ft. per Min. | 10 Loss of Head, Ft. | 10 Cu. Ft. per Min. | 11 Loss of Head, Ft. | 11 Cu. Ft. per Min. | 12 Loss of Head, Ft. | 12 Cu. Ft. per Min. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.0 | 2.37 | .65 | 1.185 | 2.62 | .791 | 5.89 | .593 | 10.4 | .474 | 16.3 | .395 | 23.5 | .338 | 32.0 | .296 | 41.9 | .264 | 53.0 | .237 | 65.4 | .216 | 79.2 | .198 | 94.2 |
| 2.2 | 2.80 | .73 | 1.404 | 2.88 | .936 | 6.48 | .702 | 11.5 | .561 | 18.0 | .468 | 25.9 | .401 | 35.3 | .351 | 46.1 | .312 | 58.3 | .281 | 72.0 | .255 | 87.1 | .234 | 103.0 |
| 2.4 | 3.27 | .79 | 1.639 | 3.14 | 1.093 | 7.07 | .819 | 12.5 | .650 | 19.6 | .547 | 28.2 | .468 | 38.5 | .410 | 50.3 | .365 | 63.6 | .327 | 78.5 | .297 | 95.0 | .273 | 113.0 |
| 2.6 | 3.78 | .86 | 1.891 | 3.40 | 1.260 | 7.65 | .945 | 13.6 | .757 | 21.3 | .631 | 30.6 | .540 | 41.7 | .473 | 54.4 | .420 | 68.9 | .378 | 85.1 | .344 | 103.0 | .315 | 122.0 |
| 2.8 | 4.32 | .92 | 2.160 | 3.66 | 1.440 | 8.24 | 1.080 | 14.6 | .864 | 22.9 | .720 | 32.9 | .617 | 44.9 | .540 | 58.6 | .480 | 74.2 | .432 | 91.6 | .392 | 111.0 | .360 | 132.0 |
| 3.0 | 4.89 | .99 | 2.440 | 3.92 | 1.620 | 8.83 | 1.220 | 15.7 | .978 | 24.5 | .815 | 35.3 | .698 | 48.1 | .611 | 62.8 | .542 | 79.5 | .488 | 98.2 | .444 | 119.0 | .407 | 141.0 |
| 3.2 | 5.47 | 1.06 | 2.730 | 4.18 | 1.820 | 9.42 | 1.370 | 16.7 | 1.098 | 26.2 | .915 | 37.7 | .785 | 51.3 | .686 | 67.0 | .609 | 84.8 | .549 | 105.0 | .499 | 127.0 | .457 | 151.0 |
| 3.4 | 6.09 | 1.12 | 3.050 | 4.45 | 2.040 | 10.00 | 1.520 | 17.8 | 1.220 | 27.8 | 1.021 | 40.0 | .875 | 54.5 | .765 | 71.2 | .679 | 90.1 | .612 | 111.0 | .557 | 134.0 | .510 | 160.0 |
| 3.6 | 6.76 | 1.19 | 3.380 | 4.71 | 2.260 | 10.60 | 1.690 | 18.8 | 1.350 | 29.4 | 1.131 | 42.4 | .970 | 57.7 | .848 | 75.4 | .755 | 95.4 | .679 | 118.0 | .617 | 142.0 | .566 | 169.0 |
| 3.8 | 7.48 | 1.26 | 3.740 | 4.97 | 2.490 | 11.20 | 1.860 | 19.9 | 1.490 | 31.0 | 1.250 | 44.7 | 1.070 | 60.9 | .936 | 79.6 | .831 | 101.0 | .749 | 124.0 | .680 | 150.0 | .624 | 178.0 |
| 4.0 | 8.20 | 1.32 | 4.100 | 5.23 | 2.730 | 11.80 | 2.050 | 20.9 | 1.640 | 32.7 | 1.370 | 47.1 | 1.175 | 64.1 | 1.027 | 83.7 | .913 | 106.0 | .822 | 131.0 | .747 | 158.0 | .683 | 188.0 |
| 4.2 | 8.97 | 1.39 | 4.490 | 5.49 | 2.980 | 12.30 | 2.240 | 22.0 | 1.790 | 34.3 | 1.490 | 49.5 | 1.280 | 67.3 | 1.122 | 87.9 | .998 | 111.0 | .897 | 137.0 | .816 | 166.0 | .749 | 198.0 |
| 4.4 | 9.77 | 1.45 | 4.890 | 5.75 | 3.250 | 13.50 | 2.430 | 23.0 | 1.950 | 36.0 | 1.620 | 51.8 | 1.390 | 70.5 | 1.220 | 92.1 | 1.086 | 116.0 | .977 | 144.0 | .888 | 174.0 | .815 | 207.0 |
| 4.6 | 10.60 | 1.52 | 5.300 | 6.03 | 3.510 | 14.10 | 2.640 | 24.0 | 2.110 | 37.6 | 1.760 | 54.1 | 1.510 | 73.7 | 1.320 | 96.3 | 1.177 | 122.0 | 1.059 | 150.0 | .963 | 182.0 | .883 | 217.0 |
| 4.8 | 11.45 | 1.58 | 5.720 | 6.28 | 3.810 | 14.70 | 2.850 | 25.1 | 2.280 | 39.2 | 1.900 | 56.5 | 1.630 | 76.9 | 1.430 | 100.0 | 1.270 | 127.0 | 1.145 | 157.0 | 1.040 | 190.0 | .954 | 226.0 |
| 5.0 | 12.31 | 1.65 | 6.170 | 6.54 | 4.110 | 15.90 | 3.080 | 26.2 | 2.460 | 40.9 | 2.050 | 58.9 | 1.760 | 80.2 | 1.540 | 105.0 | 1.370 | 132.0 | 1.230 | 163.0 | 1.122 | 198.0 | 1.028 | 235.0 |
| 5.2 | 13.24 | 1.72 | 6.620 | 6.80 | 4.410 | 16.50 | 3.300 | 27.2 | 2.640 | 42.5 | 2.210 | 61.2 | 1.890 | 83.3 | 1.650 | 109.0 | 1.470 | 138.0 | 1.320 | 170.0 | 1.200 | 206.0 | 1.104 | 244.0 |
| 5.4 | 14.20 | 1.78 | 7.100 | 7.06 | 4.730 | 17.70 | 3.550 | 28.2 | 2.840 | 44.2 | 2.370 | 63.6 | 2.030 | 86.6 | 1.770 | 113.0 | 1.570 | 143.0 | 1.410 | 177.0 | 1.280 | 214.0 | 1.183 | 254.0 |
| 5.6 | 15.16 | 1.85 | 7.580 | 7.32 | 5.060 | 17.70 | 3.790 | 29.3 | 3.030 | 45.8 | 2.530 | 65.9 | 2.170 | 89.8 | 1.890 | 117.0 | 1.680 | 148.0 | 1.510 | 183.0 | 1.370 | 222.0 | 1.260 | 264.0 |
| 5.8 | 16.17 | 1.91 | 8.090 | 7.58 | 5.400 | 17.70 | 4.040 | 30.3 | 3.250 | 47.4 | 2.700 | 68.3 | 2.320 | 93.0 | 2.010 | 121.0 | 1.800 | 154.0 | 1.610 | 190.0 | 1.460 | 229.0 | 1.340 | 274.0 |
| 6.0 | 17.23 | 1.98 | 8.610 | 7.85 | 5.740 | 20.60 | 4.310 | 31.4 | 3.450 | 49.1 | 2.870 | 70.7 | 2.460 | 96.2 | 2.150 | 125.0 | 1.920 | 159.0 | 1.710 | 196.0 | 1.560 | 237.0 | 1.430 | 283.0 |
| 7.0 | 22.89 | 2.31 | 11.450 | 9.16 | 7.620 | 20.60 | 5.720 | 36.6 | 4.570 | 57.2 | 3.810 | 82.4 | 3.260 | 112.0 | 2.850 | 146.0 | 2.520 | 185.0 | 2.280 | 229.0 | 2.070 | 277.0 | 1.910 | 330.0 |

Inside Diameter of Pipe, Inches

| Velocity Ft. per Sec. | 13 Loss of Head, Ft. | 13 Cu. Ft. per Min. | 14 Loss of Head, Ft. | 14 Cu. Ft. per Min. | 15 Loss of Head, Ft. | 15 Cu. Ft. per Min. | 16 Loss of Head, Ft. | 16 Cu. Ft. per Min. | 18 Loss of Head, Ft. | 18 Cu. Ft. per Min. | 20 Loss of Head, Ft. | 20 Cu. Ft. per Min. | 22 Loss of Head, Ft. | 22 Cu. Ft. per Min. | 24 Loss of Head, Ft. | 24 Cu. Ft. per Min. | 26 Loss of Head, Ft. | 26 Cu. Ft. per Min. | 28 Loss of Head, Ft. | 28 Cu. Ft. per Min. | 30 Loss of Head, Ft. | 30 Cu. Ft. per Min. | 36 Loss of Head, Ft. | 36 Cu. Ft. per Min. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.0 | .183 | 110 | .169 | 128 | .158 | 147 | .147 | 167 | .132 | 212 | .119 | 262 | .108 | 316 | .098 | 377 | .091 | 442 | .084 | 513 | .079 | 589 | .066 | 848 |
| 2.2 | .216 | 121 | .200 | 141 | .187 | 162 | .175 | 184 | .156 | 233 | .140 | 288 | .127 | 348 | .116 | 414 | .108 | 486 | .099 | 564 | .093 | 648 | .078 | 933 |
| 2.4 | .252 | 133 | .234 | 154 | .218 | 176 | .205 | 201 | .182 | 254 | .164 | 314 | .149 | 380 | .136 | 452 | .126 | 531 | .116 | 616 | .109 | 707 | .091 | 1,018 |
| 2.6 | .290 | 144 | .270 | 167 | .252 | 191 | .236 | 218 | .210 | 276 | .189 | 340 | .171 | 412 | .157 | 490 | .145 | 575 | .134 | 667 | .126 | 766 | .104 | 1,100 |
| 2.8 | .332 | 156 | .308 | 179 | .288 | 206 | .270 | 234 | .240 | 297 | .216 | 366 | .195 | 443 | .180 | 528 | .165 | 619 | .153 | 718 | .144 | 824 | .119 | 1,188 |
| 3.0 | .375 | 167 | .349 | 192 | .325 | 221 | .306 | 251 | .271 | 318 | .245 | 393 | .222 | 475 | .204 | 565 | .188 | 663 | .174 | 770 | .163 | 883 | .135 | 1,273 |
| 3.2 | .422 | 177 | .392 | 205 | .366 | 235 | .343 | 268 | .305 | 339 | .275 | 419 | .249 | 507 | .229 | 603 | .211 | 708 | .195 | 821 | .182 | 942 | .152 | 1,357 |
| 3.4 | .471 | 188 | .438 | 218 | .408 | 250 | .383 | 284 | .339 | 360 | .306 | 445 | .278 | 538 | .255 | 641 | .235 | 752 | .218 | 872 | .204 | 1,001 | .169 | 1,442 |
| 3.6 | .522 | 199 | .485 | 231 | .452 | 265 | .425 | 301 | .377 | 382 | .339 | 471 | .308 | 570 | .283 | 678 | .261 | 796 | .242 | 923 | .226 | 1,060 | .188 | 1,527 |
| 3.8 | .576 | 210 | .535 | 243 | .499 | 280 | .468 | 318 | .416 | 403 | .374 | 497 | .340 | 601 | .312 | 716 | .288 | 840 | .267 | 974 | .249 | 1,119 | .207 | 1,612 |
| 4.0 | .632 | 221 | .587 | 256 | .548 | 294 | .513 | 335 | .456 | 424 | .410 | 523 | .373 | 633 | .342 | 754 | .315 | 885 | .293 | 1,026 | .273 | 1,178 | .228 | 1,697 |
| 4.2 | .691 | 232 | .641 | 269 | .598 | 309 | .561 | 352 | .499 | 445 | .449 | 550 | .408 | 665 | .374 | 791 | .345 | 929 | .320 | 1,077 | .299 | 1,237 | .249 | 1,782 |
| 4.4 | .751 | 243 | .698 | 282 | .651 | 324 | .611 | 368 | .542 | 466 | .488 | 576 | .444 | 697 | .407 | 829 | .375 | 973 | .348 | 1,129 | .325 | 1,296 | .271 | 1,866 |
| 4.6 | .815 | 254 | .757 | 295 | .707 | 339 | .662 | 385 | .588 | 488 | .529 | 602 | .482 | 728 | .442 | 867 | .407 | 1,017 | .378 | 1,180 | .353 | 1,355 | .294 | 1,951 |
| 4.8 | .881 | 265 | .818 | 308 | .763 | 353 | .715 | 402 | .636 | 509 | .572 | 628 | .521 | 760 | .476 | 905 | .440 | 1,062 | .409 | 1,231 | .381 | 1,414 | .318 | 2,036 |
| 5.0 | .949 | 276 | .881 | 321 | .822 | 368 | .770 | 419 | .685 | 530 | .617 | 654 | .561 | 792 | .513 | 942 | .474 | 1,106 | .440 | 1,283 | .411 | 1,472 | .342 | 2,121 |
| 5.2 | 1.020 | 287 | .947 | 333 | .883 | 383 | .828 | 435 | .736 | 551 | .662 | 680 | .602 | 823 | .552 | 980 | .510 | 1,150 | .473 | 1,334 | .441 | 1,531 | .368 | 2,206 |
| 5.4 | 1.092 | 298 | 1.014 | 346 | .947 | 397 | .889 | 452 | .788 | 572 | .710 | 707 | .645 | 855 | .591 | 1,018 | .546 | 1,194 | .507 | 1,385 | .473 | 1,590 | .394 | 2,291 |
| 5.6 | 1.167 | 309 | 1.083 | 359 | 1.011 | 412 | .949 | 469 | .843 | 594 | .758 | 733 | .690 | 887 | .632 | 1,055 | .583 | 1,239 | .542 | 1,437 | .506 | 1,649 | .421 | 2,376 |
| 5.8 | 1.245 | 321 | 1.155 | 372 | 1.078 | 427 | 1.011 | 486 | .899 | 615 | .809 | 759 | .735 | 918 | .674 | 1,093 | .622 | 1,283 | .578 | 1,488 | .540 | 1,708 | .450 | 2,460 |
| 6.0 | 1.325 | 332 | 1.229 | 385 | 1.148 | 442 | 1.076 | 502 | .957 | 636 | .861 | 785 | .782 | 950 | .717 | 1,131 | .662 | 1,327 | .615 | 1,539 | .574 | 1,767 | .479 | 2,545 |
| 7.0 | 1.750 | 387 | 1.630 | 449 | 1.520 | 515 | 1.430 | 586 | 1.270 | 742 | 1.143 | 916 | 1.040 | 1,109 | .953 | 1,319 | .879 | 1,548 | .817 | 1,796 | .762 | 2,061 | .636 | 2,968 |

## RESERVOIRS

When selecting a site for a reservoir, the following points should be observed:

1. A proper elevation above the point at which the water is required.

2. The total supply available, including observations as to the rainfall and snowfall.

3. The formation and character of the ground, with reference to the amount of absorption and evaporation.

The most desirable formation of ground for a reservoir site is one of compact rock, like granite, gneiss, or slate; porous rocks, like sandstones and limestones, are not so desirable. Steep bare slopes are best for the country surrounding a reservoir, as the water escapes from them quickly. The presence of vegetation above the reservoir causes a considerable amount of absorption; but, at the same time, the rainfall is usually greater in a region covered with vegetation than in a barren region, hence the streams have a more uniform flow. A reservoir must be made large enough to hold a supply capable of meeting the maximum demand. The area of a reservoir should be determined, and a table made showing its contents for every foot in depth, so that the amount of water available can always be known.

## MINE DAMS

Dams may be constructed in mines to isolate a portion of the workings so that they can be flooded to extinguish fires, or, where an extremely wet formation has been penetrated, they may be constructed to prevent the water flowing into the workings. Mine dams should be of sufficient strength to resist any column of water that will be likely to come against them. The dam should be arched toward the direction from which the pressure comes, and should be given a good firm bearing in both walls and in the floor and roof. Fig. 1 illustrates a brick dam that was constructed in Kehley's Run Colliery, at Shenandoah, Pennsylvania, to isolate a portion of the seam so that it might be flooded to extinguish a mine fire. This is one of the largest mine dams that has ever been constructed. It is composed of three brick arches, each having a thickness of 5 ft., placed one against the other so that they act as one solid structure. The gangway at this point is about 20 ft. wide, and the distance to the next upper level is about 119 ft. It was intended that this should be the maximum head of water that the dams would ever have to resist, though they were made sufficiently strong to resist a head of water reaching to the surface. The separate walls were constructed one at a time, and the cement allowed to set before the next wall was placed. The back wall was carried to a greater depth and height than the others, so as to make sure of the fact that all slips or partings had been closed. The total pressure upon the dam when the water was in the mine was about 70,000 lb. per sq. ft.



FIG. 1

Dams constructed to permit the flooding of a mine usually require no passages through them, but where dams are constructed to confine the water to certain parts of the workings, and so reduce pumping charges, it may be necessary to provide both manways and drain pipes through the dams. Fig. 2 illustrates a plan and cross-section of a dam in the Curry Mine, at Norway, Michigan, constructed to keep out of the mine workings the water that came from some exploring drifts. As originally constructed, it was a sandstone dam 10 ft. thick and arched on the back face with a radius of 6 ft. A piece of 20-in. pipe provided a manway through the masonry and was held in place
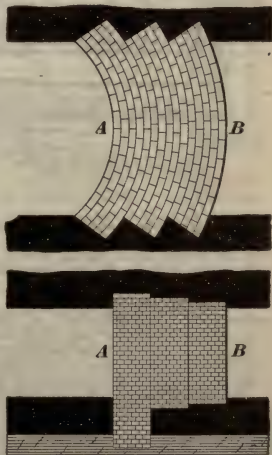
by three sets of clamps and bolts passing through the stonework. A 5-in. drain pipe was also carried through the dam and secured by clamps. When the pressure came upon the dam it was found to leak, the water was drained off and a 22-in. brick wall built 2 ft. 4 in. back of the dam, the space between being filled with concrete, and the manway and drain pipe extended through the brick wall. Before closing the drain pipe, horse manure was fastened against the face of the brick wall by means of a plank partition. After this



the manway and drain pipe were closed, and when the pressure came on, the dam was found to leak a small amount, but this soon practically ceased, showing that the manure had closed the leaks. A gauge in the head of the manway on this dam showed a pressure of 211 lb., which corresponded to a static head of 640 ft. of water. The total pressure against the dam was something over 800 T., which it successfully resisted.

Dams are now successfully made of concrete, which may be used alone or as a support to and surrounding an inner framing of wooden or steel beams, steel rods, etc. The roof and floor as well as the two ribs should be notched deeply so that the dam on all four sides may have ample bearing in order that it may not be bodily pushed forwards by the weight of the water behind it.

## OUTSIDE DAMS

Dams are used for retaining water in reservoirs, for diverting streams in mining, and for storing débris coming from coal washeries in cañons or narrow valleys.

Foundations for dams must be solid to prevent settling, and water-



**Fig. 2**

tight to prevent leakage under the base of the dam. Whenever possible, the foundation should be solid rock. Gravel is better than earth, but when gravel is employed it will be necessary to drive sheet piling under the upper toe of the dam, to prevent water from seeping through the formation under the dam. Vegetable soil should be avoided, and all porous material, such as sand, gravel, etc. should be stripped off until hard pan or solid rock is reached. In case springs occur in the area covered by the foundation of the dam, they must be traced; and if they originate on the upper side their flow must be confined to that side of the dam, so that they will have no tendency to become passageways for water from the upper to the lower face of the dam, thus providing holes that may ultimately destroy the entire foundation of the structure.
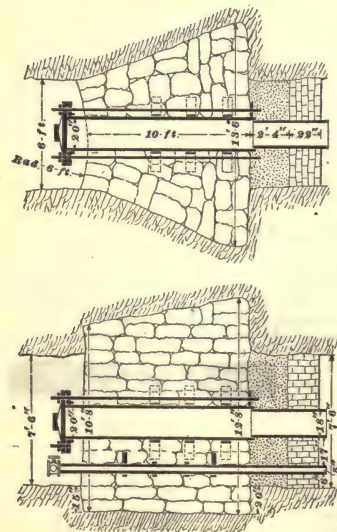
**Wooden Dams.**—Wooden dams are constructed of round, sawed, or hewn logs. The timbers are usually at least 1 ft. square, or, if round, from 18 to 24 in. in diameter. A series of cribs from 8 to 10 ft. square are constructed by building up the logs in log-house fashion and securing them together with tree-nails. The individual cribs are secured to one another with treenails or by means of bolts. The cribs are usually filled with loose rock to keep them in place, and in many cases are secured to the foundation by means of bolts. A layer of planking on the upper face of the dam makes it watertight, and when the spillway is over the crest of the dam the top of the cribs must be planked. In most cases, it is also necessary to provide an apron for the water to fall on. The apron may be set on small cribs, or on timbers projecting from the cribs of the dam itself.

**Abutments and Discharge Gates.**—Abutments are structures at the ends of a dam. They may be constructed from timber, masonry, or dry stonework.

If possible, abutments should have a curved outline, and should be so placed that there is no possibility of the water overflowing them, or getting behind them during floods. If the regular discharge from a dam takes place from the main face, the gates may be arranged in connection with one of the abutments, or by means of a tunnel and culvert through the dam. In either case, some structure should be constructed above the outlet so as to prevent driftwood, brush, and other material from stopping the discharge gates. When the discharge gates are placed at one side of the dam, they are usually arranged outside of the regular abutment, between it and a special abutment, the discharge being through a series of gates into a flume, ditch, or pipe.

**Spillways, or Waste Ways.**—Spillways, or waste ways are openings provided in a dam for the discharge of water during floods or freshets, or for the discharge of a portion not being used at any time. The spillway may be over the crest of the dam; though, where the topography favors such a construction, the main dam may be of sufficient height to prevent water from passing its crest and the spillway arranged at another outlet over a lower dam. Waste ways, proper, are openings through the dam, and are intended for the discharge of the large quantities of water that come down during freshets or floods. In the case of timber dams, the waste ways are usually surrounded by heavy cribs, and have an area of from 40 to 50 sq. ft. each.

There are two general forms of construction employed for waste ways. One consists of a comparatively narrow opening in the dam, extending to a considerable depth (8 or 10 ft.). Water is allowed to discharge through this during flood time, but when it is desired to stop the flow, planks are placed across the up-stream face of the opening in such a manner as to close it. The opening, which is usually not over 3 or 4 ft. wide, is provided with guides on the upper face of the dam; between these the planks are slid down, the individual pieces of planking being at least 1 ft. longer than the opening that they are to cover. The other device frequently used consists in providing the waste way, at one side of the regular spillway, with a crest 2 or 3 ft. lower than the regular spillway. The crest of this waste way is composed of heavy timber, and 4 or 5 ft. above there is placed a parallel timber; the space between the two is then closed by *flash boards*, which are pieces of 2- in. or 3-in. plank, 6 or 8 in. wide, and long enough to extend from 1 to 2 ft. above the upper timber. The planks are placed against both timbers so as to close the space. Through the upper end of each plank is bored a hole through which a piece of rope is passed and a knot tied in the end of the rope; these ropes are secured by staples to the upper timber. When it becomes necessary to open the waste way, men go under with peevies, cant hooks, or pinch bars, and pry up the planks in such a way as to draw the longer end out of contact with the lower timber, when the force of the water will immediately carry the plank down the stream as far as the rope will allow. After the first plank has been loosened, the succeeding ones can be pulled up with comparative ease, and two men can open a 25-ft. or 30-ft. section of waste way in a very few minutes. The ropes keep the plank from being lost, and the opening can be closed again by passing the planks down into the water to one side of the opening and moving them into the current. Some skill is required both in opening and closing the waste ways.

**Stone Dams.**—Where cement or lime is expensive, but where suitable rubble stone can be obtained, dams are frequently constructed without the use of mortar. In such cases, the upper and lower faces of the dam should be of hammer-dressed stone, carefully bonded, and sometimes the stones in the lower face of the dam are anchored by means of bolts. The dam can be made watertight by placing a skin of planking on the upper face. In case water should ever pass over the crest of such a dam, much of it will settle through the openings in the stone into the interior of the dam, and subject the stones in the lower portion of the face to a hydrostatic pressure, provided an opening has not been made for the escape of such water. For this reason, culverts or openings should be made through the lower portion of the dam, to discharge any such water. When such dams as this are constructed, the regular spillway is not placed over the face of the dam, but at some other point, and usually over a timber dam.

**Earth Dams.**—Earth dams are used for reservoirs of moderate height. They should be at least 10 ft. wide on top, and a height of more than 60 ft. is unusual. When the material of which the dam is composed is not water-tight, as for instance, gravel, sand, etc., it is sometimes necessary to construct a puddle wall of clay in the center of the regular dam. This consists of a narrow dam of clay mixed with a certain proportion of sand. The puddle wall

should not be less than from 6 to 8 ft. thick at the top of the dam, and should be given a slight batter on each side. It is constructed during the building of the dam, and should be protected from contact with the water by a considerable thickness of earth on the upper face. The upper face of an earthen dam is frequently protected by means of plank or a pavement of stone. The lower face is frequently protected by means of sod, or sod and willow trees. Sometimes earth dams are provided with a masonry core in place of the puddle
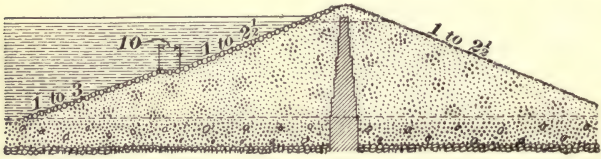


wall, to render them water-tight. This consists of a masonry wall carried to an impervious stratum, and up through the center of the dam. The masonry core should never be less than 2 or 3 ft. thick at the top, and should be given a batter of at least 10% on each side. At the regular water level, earthen dams are liable to have a small bench or shelf formed, and on this account, during the construction, such a bench or shelf is sometimes built into the earth dam. The accompanying figure shows a dam with a masonry core, with the upper face covered with rubble and the lower face covered with grass.

## IRRIGATION QUANTITY TABLES

| Amount of Water Required to Cover 1 A. to Given Depths | | | Second-Feet Reduced to Gallons and Acre-Feet* | | | | Gallons Required to Cover a Given Number of Acres to Depth of 1 Ft. | |
|---|---|---|---|---|---|---|---|---|
| Depth | | Cubic Feet, or Second Feet, Contained in 1 A. | Gallons | Second-Feet | Gallons per Minute | Gallons per Pumping Day of 12 Hr. | Acre-Feet per Pumping Day of 12 Hr. | Acres, or Number of Acre-Feet | Gallons |
| Ft. | In. | | | | | | | | |
| | 1 | 3,630 | 27,154 | ¼ | 112.2 | 80,790 | .2479 | 1 | 325,851 |
| | 2 | 7,260 | 54,309 | ½ | 224.4 | 161,579 | .4959 | 2 | 651,703 |
| | 3 | 10,890 | 81,463 | ¾ | 336.6 | 242,369 | .7438 | 3 | 977,554 |
| | 4 | 14,520 | 108,617 | 1 | 448.8 | 323,158 | .9917 | 4 | 1,303,406 |
| | 5 | 18,150 | 135,771 | 1¼ | 561.0 | 403,948 | 1.2397 | 5 | 1,629,257 |
| | 6 | 21,780 | 162,926 | 1½ | 673.2 | 484,738 | 1.4876 | 6 | 1,955,109 |
| | 7 | 25,410 | 190,080 | 1¾ | 785.5 | 565,527 | 1.7355 | 7 | 2,280,960 |
| | 8 | 29,040 | 217,234 | 2 | 897.7 | 646,317 | 1.9835 | 8 | 2,606,812 |
| | 9 | 32,670 | 244,389 | 2½ | 1122.1 | 807,896 | 2.4793 | 9 | 2,932,663 |
| | 10 | 36,300 | 271,542 | 3 | 1346.5 | 969,475 | 2.9752 | 10 | 3,258,515 |
| | 11 | 39,930 | 298,697 | 4 | 1795.3 | 1,292,634 | 3.9669 | 15 | 4,887,772 |
| 1 | 00 | 43,560 | 325,851 | 5 | 2244.2 | 1,615,792 | 4.9586 | 20 | 6,517,029 |
| 1 | 2 | 50,820 | 380,160 | 6 | 2693.0 | 1,938,951 | 5.9503 | 25 | 8,146,286 |
| 1 | 4 | 58,080 | 434,469 | 7 | 3141.8 | 2,262,109 | 6.9421 | 30 | 9,775,544 |
| 1 | 6 | 65,340 | 488,777 | 8 | 3590.6 | 2,585,268 | 7.9338 | 40 | 13,034,058 |
| 1 | 8 | 72,600 | 543,086 | 9 | 4039.5 | 2,908,426 | 8.9255 | 60 | 19,551,087 |
| 1 | 10 | 79,860 | 597,394 | 10 | 4488.3 | 3,231,585 | 9.9173 | 80 | 26,068,116 |
| 2 | 00 | 87,120 | 651,703 | 20 | 8976.6 | 6,463,170 | 19.8345 | 160 | 52,136,232 |

*One cubic foot of water per second (exact 7.48052 gal.) constant flow is known as the *second-foot*.
   The *acre-foot* is the quantity of water required to cover 1 A. to a depth of 1 ft.

**Refuse Dams.**—Refuse dams are placed across the bed of streams to hold back refuse from mines and washeries, and to prevent damage to the valleys below. They are made of stone, timber, or brush. No attempt is made to render the refuse dam water-tight, the only object being that it should retard the flow of the stream and give it a greater breadth of discharge, so that the water naturally drops and deposits the sediment that it is carrying. The sediment soon silts or fills up against the face of the dam, the area above the dam becoming a flat expanse or plain over which the water finds its way to the dam. When these dams are constructed of stone, the individual stones on the lower face and crest of the dam should be so large that the current will be unable to displace them, while the upper face and core of the dam may be composed of finer material. In case a breach should occur in the refuse dam, it will not necessarily endanger the region farther down the stream, as is the case when a break occurs in a water dam. The reason for this is that the refuse dam is not made watertight, and hence there is never much pressure against it, or a large volume of water held back that can rush suddenly down the stream should a break occur. The only result of the break would be that more or less of the gravel, sand, slate, etc., behind the dam would be washed through the breach.

**Wing Dams.**—Wing dams are used for turning streams from their courses, so as to expose all or a portion of the bed for placer mining or other purposes. They are usually of a temporary nature, and are constructed of brush and stones, light cribs filled with stones, and of large stones, or timber. Sometimes the course of a stream is turned by an obstruction made of sand bags and a wing dam constructed behind this of frames of timber, the intervening space being filled with gravel or earth; in some cases, the timber is covered with stone and the surface riprapped so that if the flow ever comes over the top of the structure it will not destroy it.

**Masonry and Concrete Dams.**—When high masonry or concrete dams are to be employed they should be designed by a competent hydraulic engineer. Masonry or concrete dams are not, as a rule, used around coal mines, owing to the fact that the length of time during which the dam is required rarely warrants such expensive construction.

# WATER-POWER

**Theoretical Efficiency of Water-Power.**—The gross power of a fall of water is the product of the weight of water discharged in a unit of time, and the total head or difference in elevation of the surface of the water, above and below the fall. The term head, used in connection with waterwheels, is the difference in height between the surface of water in the penstock and that in the tailrace, when the wheel is running.

If $Q$ = cubic feet of water discharged per minute;
$W$ = weight of 1 cu. ft. of water = 62.5 lb.;
$H$ = total head, in feet;
$WQH$ = gross power, in foot-pounds per minute;
$\dfrac{WQH}{33,000}$ = horsepower.

Substituting the value for $W$, gives

$$\frac{QH \times 62.5}{33,000} = .00189QH, \text{ as horsepower of a fall.}$$

The total power can never be utilized by any form of motor, because there is a loss of head, both at the entrance to, and exit from, the wheel, and there are also, losses of energy due to friction of the water in passing through the wheel. The ratio of the power developed by the wheel to the gross power of the fall, is the efficiency of the wheel. A head of water can be made use of in any one of the following ways:

1. By its weight, as in the water balance, or overshot wheel.
2. By its pressure, as in the hydraulic engine, hydraulic presses, cranes, etc., or in a turbine water wheel.
3. By its impulse, as in the undershot and impulse wheels, such as Peltons, etc.
4. By a combination of these.

**Horsepower of a Running Stream.**—The gross horsepower is

$$\text{H. P.} = \frac{QH \times 62.5}{33,000} = .00189QH$$

in which $Q$ = quantity actually impinging on float or bucket, in cubic feet per minute;

$H$ = theoretical head added to velocity of stream.

or
$$H = \frac{v^2}{2g} = \frac{v^2}{64.4},$$

in which $v$ = velocity, in feet per second.

For example, if the floats of an undershot waterwheel are 2 ft.×10 ft., and the stream has a velocity of 3 ft, per sec., i. e., $v=3$, $H = 9 \div 64.4 = .139$, and $Q = 2 \times 10 \times 3 \times 60 = 3,600$ cu. ft. per min.

From this, H. P. $= 3,600 \times .139 \times .00189 = .945$ H. P., or a gross horsepower for practically .05 sq. ft. of wheel surface; but, under ordinary circumstances, it is impossible to attain more than 40% of this, or practically .02 H. P. per sq. ft. of surface, which requires 50 sq. ft. of float surface to each horsepower furnished.

**Current Motors.**—A current motor fully utilizes the energy of a stream only when it is so arranged that it can take all the velocity out of the water; that is, when the water leaves the floats or vanes with no velocity. In practice, it is impossible to obtain even a close approximation to these results, and hence only a small fraction of the energy of a running stream can be utilized by the current motor.

Current motors are frequently used to obtain small amounts of power from a large stream, as, for instance, for pumping a limited amount of water for irrigation. For this work, an ordinary undershot wheel having radial paddles is usually employed. At one end of the wheel a series of small buckets is placed, and so arranged that each bucket will dip up water at the bottom of the wheel and discharge it into the launder, near the top of the wheel. The shape of the buckets should be such that only the amount of water that the bucket is capable of carrying to the launder will be dipped up, for, if the bucket is constantly slopping or pouring water as it ascends, a large amount of useless work is performed in raising this extra water and then pouring it out again, as only the portion that reaches the launder can be of any service. Current motors are not practicable for furnishing large amounts of power.

**Utilizing Power of Waterfall.**—The power of a waterfall may be utilized by a number of different styles of motors, but each has certain advantages.

When the head is low (not over 5 or 6 ft.), *breast or undershot wheels* are frequently employed. If these are properly proportioned, it is possible to realize from 25% to 50% of the theoretical power of the fall, but the wheels are large and cumbersome compared with the duty they perform, and are not often installed at present, especially near manufacturing centers.

For falls up to 40 or 50 ft., *overshot wheels* are very commonly employed, and they have been used for even greater heads than this. The overshot wheel derives its power both from the impulse of the water entering the buckets, and from the weight of the water as it descends on one side of the wheel in the buckets; the latter factor is by far the more important of the two. When properly proportioned, overshot wheels may realize from 70% to 90% of the power of the waterfall, but they are large and cumbersome compared with the power that they give, and are not often installed except in isolated regions, where they are made from timber by local mechanics.

For heads varying from 50 ft. up, *impulse wheels* are very largely used. These are also sometimes called *hurdy gurdies*, and are usually of the Pelton type, consisting of a wheel provided with buckets, so arranged about its periphery that they receive an impinging jet of water and turn it back upon itself, discharging it with practically no velocity, and converting practically all the energy into useful work. The efficiency of these wheels varies from 85% to 90% under favorable circumstances. This style of wheel is especially adapted for very high heads and comparatively small amounts of water. There are a number of instances where wheels are operating under a head of as much as 2,000 ft. This style of impulse wheel is an American development; in Europe, a style of impulse turbine has been used to some extent, but has not found very much favor in the United States.

*Turbines*, or *reaction wheels*, are very largely employed, especially for moderate heads. When properly designed to fit the working conditions, they can be used for heads varying from 4 to 5 ft. up to considerably over 100 ft., and when properly placed are capable of utilizing the entire head, a factor that gives them a decided advantage over any other style of waterwheel. Turbines are capable of returning 85% to 90% of the theoretical energy as useful power, and are largely used, especially where a considerable volume of water at a low head, or a smaller volume at a moderate head, can be obtained.

## PUMP MACHINERY

### CLASSIFICATION OF PUMPS

*Pumps* are employed for unwatering mines, handling water at placer mines, irrigation, water-supply systems, boiler feeds, etc. For unwatering mines, two general systems of pumping are employed: (1) The pump is placed in the mine and is operated by a motor on the surface, the power being transmitted through a line of moving rods. (2) Both the motor and pump are placed in the mine, the motor being an engine driven by steam, compressed air, hydraulic motor, or an electric motor.

**Cornish Pumps.**—Any method of operating pumps by rods is commonly called a *Cornish system*. Formerly, the motor in the Cornish system consisted of a steam engine placed over the shaft head, which operated the pump by a direct line of rods. With this arrangement, there is great danger of accident to the engine from the settling of the ground around the shaft, or from fire in the shaft; also, the position of the motor renders access to the shaft difficult. To overcome these objections, the engine is frequently placed at one side of the shaft, and the rods operated by a bob; this has become the common practice, and is generally called the *Cornish rig*. The engine employed in the most modern plants is generally of the Corliss type, and is provided with a governor to guard against the possibility of the engine running away, in case the rods should break.

This system requires no steam line down the shaft, and is independent of the depth of water in the mine, so that the pump is not stopped by the drowning of a mine, but the moving rods are a great inconvenience in the shaft, and they absorb a great amount of power by friction.

**Simple and Duplex Pumps.**—In the simple pumps a steam cylinder is connected directly to a water cylinder, and the steam valves are operated by tappets. Such a pump is more or less dependent on inertia at certain points of the stroke to insure the motion of the valves, hence will not start from any place, but is liable to become stalled at times.

In the duplex pump, two steam cylinders and two water cylinders are arranged side by side, and the valves so placed that when one piston is at mid-stroke it throws the steam valve for the other cylinder, etc. With this arrangement, the pump will start from any point, and can never be stalled for lack of steam, due to the position of the valves. Ordinarily, duplex pumps are to be preferred for mine work.

The *packing* for the water piston of a pump may be either inside or outside. Any form of packing that is inside the cylinder, either upon a moving piston or surrounding the ram, and so situated that any wear will allow communication between the opposite ends of the cylinder, is called *inside* packing. It may consist simply of piston rings about the piston, as in the case of an ordinary steam-engine piston *G*, Fig. 1, or stationary rings may be employed about the outside of a moving ram, or long piston *P*. In either case,



FIG. 1

the cylinder heads have to be removed before the condition of the packing can be inspected, and any leak does not make itself visible.

When outside packing is employed, separate rams are used in opposite ends of the cylinder, there being no internal communication between the chambers in which the rams work. The rams are packed by ordinary outside stuffingboxes and glands. The arrangement consists practically of two single-acting pumps arranged to work alternately, so that one is forcing water while the other is drawing water. Fig. 2 shows a horizontal section of a cylinder so arranged, together with the yoke rods that operate the ram at the farther end of the cylinder.

As a rule, inside-packed pumps should be avoided in mines, because acid or gritty waters are liable to cut the packing, and make the pumps leak in a
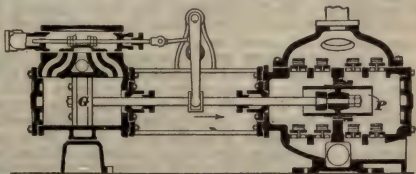
very short time. For dipping work in single stopes or entries, small single or duplex outside-packed pumps may be employed. It is generally best to operate such pumps by compressed air, for the exhaust will then be beneficial



FIG. 2

to the mine air. If steam is employed, it is frequently necessary to introduce a trap and remove entrailed water from the steam before it enters the pump, and to dispose of the exhaust by piping it out or condensing it. Such isolated steam pumps are about the most wasteful form of steam-driven motor in existence.
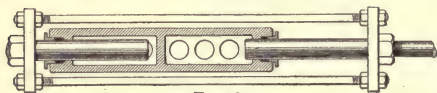
For sinking, center-packed single or duplex pumps are usually employed, the duplex style being the better. For station work, where much water is to be handled, large compound, or triple-expansion, condensing, duplex pumping engines are employed. They may, or may not, be provided with cranks and a flywheel. Engineers differ greatly upon this point, and, as a rule, for very high lifts and great pressures, the flywheel is employed.

The main points in consideration are the first cost of the pump, and the amount that will be saved by using the more expensive engine. The large flywheel pumping engines are several times as expensive as the direct-acting steam pumps, and the question is as to whether their greater efficiency will more than counterbalance the increased outlay. Most engineers favor flywheel pumps for handling large volumes of water where the work is approximately constant, and direct-acting pumps, without flywheels or cranks, for handling small amounts of water, or for very irregular service, owing to the fact that if the flywheel pump is driven below its normal speed it does not govern properly, nor work economically. Until recently, water was removed from mines in lifts of about 300 to 350 ft., pumps being placed at stations along the shaft.

While a series of station pumps are still employed in some cases, they are generally intended to take care of water coming into the shaft, or workings at or near their level, and are not employed for handling water in successive stages or lifts. For handling the bulk of the water from the bottom of the shaft, large pumping engines are employed that frequently force the water to the surface from depths of over 1,000 ft. These high-duty pumping plants, when near the shaft and operated by steam with a condenser, frequently show a very high efficiency. When air is employed to operate such a plant, a much higher efficiency can be obtained if the compressed air is heated before it is used in the high-pressure cylinder and during its passage from the high-pressure to the low-pressure cylinder. This has been very successfully accomplished by means of a steam reheater, the small amount of steam necessary being conveyed to the station in the small pipe, and entirely condensed in the reheater, from which it is trapped as water.

The duty of steam pumps is approximately as follows: For small-sized steam pumps, the steam consumption is from 130 to 200 lb. per H. P. per hr., when operating in the workings of a mine at some distance from the boiler. For larger sizes of simple steam pumps, the consumption runs from 80 to 130 lb. per H. P. per hr. Compound-condensing pumps, such as are commonly used as station pumps, consume from 40 to 70 lb. per H. P. per hr. Triple-expansion, condensing, high-class pumping engines consume from 24 to 26 lb. per H. P. per hr. The Cornish pump consumes varied amounts of steam in proportion to the water delivered, depending largely on the friction of the gearing, bobs, rods, etc., but its efficiency is usually considerably below the best class of pumping engines.

**Speed of Water Through Valves, Pipes, and Pump Passages.**—The speed of water through the valves and passages of a pump should not exceed 250 ft. per min., and care should be taken to see that the passages are not too abruptly deflected. The flow of water through the discharge pipe should not exceed 500 ft. per min., and for single-cylindered pumps it is usually figured at between 250 and 400 ft. per min. In the case of very large pumps, greater velocities may be allowed. The suction pipe for the pump should be larger than the discharge pipe. Ordinarily the suction pipe for a pump should not exceed 250 ft. in length, and should not contain more than two elbows. The following formula gives the diameter of the suction and discharge pipes of a pump:

$G$ = United States gallons per minute;
$d'$ = diameter of suction pipe, in inches;

$d''$ = diameter of discharge pipe, in inches;
$v'$ = velocity of water, in feet per minute, in suction pipe = from $.50v''$ to $.75v''$
$v''$ = velocity of water, in feet per minute, in discharge pipe.

$$d' = 4.95 \sqrt{\frac{G}{v'}} \qquad\qquad d'' = 4.95 \sqrt{\frac{G}{v''}}$$

## RATIO OF STEAM AND WATER CYLINDERS IN A DIRECT-ACTING PUMP

Let $A$ = area of steam cylinder;
$D$ = diameter of steam cylinder;
$P$ = steam pressure, in pounds per square inch;
$H$ = head of water = $2.309p$;
$a$ = area of pump cylinder;
$d$ = diameter of pump cylinder;
$p$ = pressure per square inch, corresponding to head $H = .433H$;
$E$ = efficiency of pump = $\dfrac{\text{work done in pump cylinder}}{\text{work done in steam cylinder}}$;

$$A = \frac{ap}{EP} \qquad d = D\sqrt{\frac{EP}{p}} \qquad \frac{A}{a} = \frac{p}{EP} = \frac{.433H}{EP}$$

$$a = \frac{EAP}{p}; \qquad P = \frac{ap}{EA}; $$

$$D = d\sqrt{\frac{p}{EP}} \qquad p = \frac{EAP}{a}; \qquad H = 2.309EP \times \frac{A}{a}.$$

If $E = 75\%$, then $H = 1.732P \times \dfrac{A}{a}$.

$E$ is commonly taken at from .7 to .8 for ordinary direct-acting pumps. For the highest class of pumping engines it may amount to .9. The steam pressure $P$ is the mean effective pressure, according to the indicator diagram; the pressure $p$ is the mean total pressure acting on the pump plunger or piston, including the suction, as would be shown by the indicator diagram of the water cylinder. The pressure on the pump cylinder is frequently much greater than that due to the height of the lift, on account of the friction in the valves and passages, which increases rapidly with the velocity of the flow.

**Piston Speed of Pumps.**—For small pumps, it is customary to assume a speed of 100 ft. per min., but, in the case of very small short-stroke pumps, this is too high, owing to the fact that the rapid reverses make the flow through the valves and change in the direction of the current too frequent. When the stroke of the pump is somewhat longer (18 in. or more), higher speeds can be employed, and in the case of large pumping engines having long strokes, speeds of as much as 200 to 250 ft. per min. are successfully used without jar or hammer.

### STROKES FOR PISTON SPEED OF 100 FT. PER MIN.

| Length of Stroke Inches | Number of Strokes | Length of Stroke Inches | Number of Strokes | Length of Stroke Inches | Number of Strokes |
|---|---|---|---|---|---|
| 4 | 300 | 12 | 100 | 24 | 50 |
| 5 | 240 | 14 | 86 | 26 | 46 |
| 6 | 200 | 16 | 75 | 28 | 43 |
| 7 | 172 | 18 | 67 | 30 | 40 |
| 8 | 150 | 20 | 60 | 36 | 33 |
| 10 | 120 | 22 | 55 | 40 | 30 |

**Boiler Feed-Pumps.**—In practice, it has been shown that a piston speed greater than 100 ft. per min. results in excessive wear and tear on a boiler feed-pump, especially when the water is warm. This is because vapor forms in the cylinders, and results in a water hammer. In determining the proper size of a pump for feeding a steam boiler, not only the steam employed in running the engine, but that necessary for the pumps, heating system, etc. must be taken into consideration.

## THEORETICAL CAPACITY OF PUMPS AND HORSEPOWER REQUIRED TO RAISE WATER

Let $Q$ = cubic feet of water per minute;
$G$ = United States gallons per minute;
$G'$ = United States gallons per hour;
$d$ = diameter of cylinder, in inches;
$l$ = stroke of piston, in inches;
$N$ = number of single strokes per minute;
$v$ = speed of piston, in feet per minute;

### RATIOS OF AREAS TO DIAMETERS

| Diameter of Water Cylinders Inches | Diameter of Steam Cylinder, in Inches | | | | | |
|---|---|---|---|---|---|---|
| | 5 | 6 | 8 | 10 | 12 | 14 |
| | Ratios of Areas | | | | | |
| 5/8 | 64.00 | 92.16 | 163.84 | 256.00 | 368.64 | 501.76 |
| 3/4 | 44.44 | 64.00 | 113.77 | 177.77 | 256.00 | 348.44 |
| 7/8 | 32.65 | 47.02 | 83.59 | 130.61 | 188.10 | 256.00 |
| 1 | 25.00 | 36.00 | 64.00 | 100.00 | 144.00 | 196.00 |
| 1 1/8 | 19.75 | 28.44 | 50.56 | 79.01 | 113.78 | 154.86 |
| 1 1/4 | 16.00 | 23.04 | 40.96 | 64.00 | 92.16 | 125.44 |
| 1 3/8 | 13.22 | 19.04 | 33.85 | 52.89 | 76.17 | 103.66 |
| 1 1/2 | 11.11 | 16.00 | 28.44 | 44.44 | 64.00 | 87.11 |
| 1 5/8 | 9.46 | 13.63 | 24.23 | 37.87 | 54.53 | 74.22 |
| 1 3/4 | 8.16 | 11.75 | 20.90 | 32.65 | 47.02 | 64.00 |
| 1 7/8 | 7.11 | 10.24 | 18.20 | 28.44 | 40.96 | 55.75 |
| 2 | 6.25 | 9.00 | 16.00 | 25.00 | 36.00 | 49.00 |
| 2 1/4 | 4.93 | 7.11 | 12.64 | 19.75 | 28.44 | 38.71 |
| 2 1/2 | 4.00 | 5.76 | 10.24 | 16.00 | 23.04 | 31.36 |
| 2 3/4 | 3.30 | 4.76 | 8.46 | 13.22 | 19.04 | 25.91 |
| 3 | 2.77 | 4.00 | 7.11 | 11.11 | 16.00 | 21.77 |
| 3 1/4 | 2.37 | 3.40 | 6.06 | 9.46 | 13.63 | 18.56 |
| 3 1/2 | 2.04 | 2.93 | 5.22 | 8.16 | 11.75 | 16.00 |
| 3 3/4 | 1.77 | 2.56 | 4.55 | 7.11 | 10.24 | 13.93 |
| 4 | 1.56 | 2.25 | 4.00 | 6.25 | 9.00 | 12.25 |
| 4 1/4 | 1.38 | 1.99 | 3.54 | 5.53 | 7.97 | 10.85 |
| 4 1/2 | 1.23 | 1.77 | 3.15 | 4.93 | 7.11 | 9.67 |
| 4 3/4 | 1.10 | 1.59 | 2.83 | 4.43 | 6.38 | 8.68 |
| 5 | 1.00 | 1.44 | 2.56 | 4.00 | 5.76 | 7.84 |
| 5 1/2 | .82 | 1.19 | 2.11 | 3.30 | 4.76 | 6.47 |
| 6 | .69 | 1.00 | 1.77 | 2.77 | 4.00 | 5.44 |
| 6 1/2 | .59 | .85 | 1.51 | 2.37 | 3.40 | 4.63 |
| 7 | .51 | .73 | 1.30 | 2.04 | 2.93 | 4.00 |
| 7 1/2 | .44 | .64 | 1.13 | 1.77 | 2.56 | 3.48 |
| 8 | .39 | .56 | 1.00 | 1.56 | 2.25 | 3.06 |
| 8 1/2 | .34 | .49 | .88 | 1.38 | 1.99 | 2.71 |
| 9 | .30 | .44 | .79 | 1.23 | 1.77 | 2.42 |
| 9 1/2 | .27 | .39 | .70 | 1.11 | 1.59 | 2.17 |
| 10 | | .36 | .64 | 1.00 | 1.44 | 1.96 |
| 11 | | .29 | .52 | .82 | 1.19 | 1.62 |
| 12 | | .25 | .44 | .69 | 1.00 | 1.36 |
| 13 | | | .37 | .59 | .85 | 1.16 |
| 14 | | | .33 | .51 | .73 | 1.00 |
| 15 | | | .28 | .44 | .64 | .87 |
| 16 | | | .25 | .39 | .56 | .76 |
| 17 | | | | .34 | .49 | .67 |
| 18 | | | | .30 | .44 | .60 |

$W$ = weight moved, in pounds per minute;
$P$ = pressure, in pounds per square foot = $62.5H$;
$p$ = pressure, in pounds per square inch = $.433H$;
$H$ = height of lift, in feet;
H. P. = horsepower.

Then,

$$Q = \frac{\pi}{4} \times \frac{d^2}{144} \times \frac{Nl}{12} = .0004545 N d^2 l$$

$$G = \frac{\pi}{4} \times \frac{N d^2 l}{231} = .0034 d^2 l \qquad\qquad G' = .204 N d^2 l$$

## OF STEAM AND WATER CYLINDERS

| Diameter of Steam Cylinders, in Inches | | | | | | | |
|---|---|---|---|---|---|---|---|
| 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
| Ratios of Areas | | | | | | | |
| 455.11 | | | | | | | |
| 334.37 | | | | | | | |
| 256.00 | 324.00 | 400.00 | | | | | |
| 202.27 | 256.00 | 316.05 | | | | | |
| 163.84 | 207.36 | 256.00 | 309.76 | | | | |
| 135.41 | 171.37 | 211.57 | 256.00 | | | | |
| 113.77 | 144.00 | 177.77 | 215.11 | 256.00 | | | |
| 96.95 | 122.70 | 151.48 | 183.29 | 218.11 | | | |
| 83.59 | 105.79 | 130.61 | 158.05 | 188.10 | 220.73 | | |
| 72.82 | 92.16 | 113.78 | 137.67 | 163.84 | 192.29 | | |
| 64.00 | 81.00 | 100.00 | 121.00 | 144.00 | 169.00 | 196.00 | 225.00 |
| 50.56 | 64.00 | 79.01 | 95.60 | 113.78 | 133.53 | 154.86 | 177.77 |
| 40.96 | 51.84 | 64.00 | 77.44 | 92.16 | 108.16 | 125.44 | 144.00 |
| 33.81 | 42.84 | 52.89 | 64.00 | 76.17 | 89.39 | 103.66 | 119.01 |
| 28.44 | 36.00 | 44.44 | 53.77 | 64.00 | 75.11 | 87.11 | 100.00 |
| 24.23 | 30.67 | 37.87 | 45.82 | 54.53 | 64.00 | 74.22 | 85.21 |
| 20.90 | 26.44 | 32.65 | 39.51 | 47.02 | 55.18 | 64.00 | 73.47 |
| 18.20 | 23.04 | 28.44 | 34.42 | 40.96 | 48.07 | 55.75 | 64.00 |
| 16.00 | 20.25 | 25.00 | 30.25 | 36.00 | 42.25 | 49.00 | 56.25 |
| 14.22 | 17.93 | 22.14 | 26.79 | 31.89 | 37.43 | 43.41 | 49.83 |
| 12.64 | 16.00 | 19.75 | 23.90 | 28.44 | 33.38 | 38.71 | 44.44 |
| 11.34 | 14.36 | 17.73 | 21.45 | 25.53 | 29.96 | 34.75 | 39.89 |
| 10.24 | 12.96 | 16.00 | 19.36 | 23.04 | 27.04 | 31.36 | 36.00 |
| 8.46 | 10.71 | 13.22 | 16.00 | 19.04 | 22.35 | 25.01 | 29.75 |
| 7.11 | 9.00 | 11.11 | 13.44 | 16.00 | 18.77 | 21.77 | 25.00 |
| 6.06 | 7.66 | 9.46 | 11.45 | 13.63 | 16.00 | 18.56 | 21.30 |
| 5.22 | 6.61 | 8.16 | 9.87 | 11.75 | 13.79 | 16.00 | 18.37 |
| 4.55 | 5.76 | 7.11 | 8.60 | 10.24 | 12.00 | 13.93 | 16.00 |
| 4.00 | 5.06 | 6.25 | 7.56 | 9.00 | 10.56 | 12.25 | 14.06 |
| 3.54 | 4.48 | 5.53 | 6.69 | 7.97 | 9.35 | 10.85 | 12.45 |
| 3.15 | 4.00 | 4.93 | 5.98 | 7.11 | 8.34 | 9.67 | 11.11 |
| 2.83 | 3.59 | 4.43 | 5.36 | 6.38 | 7.49 | 8.68 | 9.97 |
| 2.56 | 3.24 | 4.00 | 4.84 | 5.76 | 6.76 | 7.84 | 9.00 |
| 2.11 | 2.67 | 3.30 | 4.00 | 4.76 | 5.58 | 6.47 | 7.43 |
| 1.77 | 2.25 | 2.77 | 3.36 | 4.00 | 4.69 | 5.44 | 6.25 |
| 1.51 | 1.91 | 2.37 | 2.86 | 3.40 | 4.00 | 4.63 | 5.32 |
| 1.30 | 1.65 | 2.04 | 2.46 | 2.93 | 3.44 | 4.00 | 4.59 |
| 1.13 | 1.44 | 1.77 | 2.15 | 2.56 | 3.00 | 3.48 | 4.00 |
| 1.00 | 1.26 | 1.56 | 1.89 | 2.25 | 2.64 | 3.06 | 3.51 |
| .88 | 1.12 | 1.38 | 1.67 | 1.99 | 2.34 | 2.71 | 3.11 |
| .79 | 1.00 | 1.23 | 1.49 | 1.77 | 2.08 | 2.41 | 2.77 |

The diameter of piston required for a given capacity per minute will be

$$d = 46.9 \sqrt{\frac{Q}{Nl}} = 17.15 \sqrt{\frac{G}{Nl}}, \text{ or } d = 13.54 \sqrt{\frac{Q}{v}} = 4.95 \sqrt{\frac{G}{v}}$$

The actual capacity of a pump will vary from 60% to 95% of the theoretical capacity, depending on the tightness of the piston, valves, suction pipe, etc.

$$\text{H. P.} = \frac{QP}{33,000} = \frac{QH \times 144 \times .433}{33,000} = \frac{QH}{529.2} = \frac{Gp}{1,714.5}$$

The actual horsepower required will be considerably greater than the theoretical, on account of the friction in the pump; hence, at least 20% should be added to the power for friction and usually about 50% more is added to cover leaks, etc., so that the actual horsepower required by the pump is about 70% more than the theoretical.

EXAMPLE 1.—What size of pump will throw 30 gal. of water per min. up 125 ft., from the bottom of a pit or prospect shaft to the station pump at the main shaft?

SOLUTION.—An allowance of probably 25% should be made with a small pump of this character, to overcome slippage or leaking through the valves, past the piston, etc., and hence the total amount of water to be handled is 40 gal. per min. The formula for the diameter of piston is $d = 4.95 \sqrt{\frac{G}{v}}$; therefore, assuming that $v = 100$ ft. per min., $d = 4.95 \sqrt{.4} = 4.95 \times .63 = 3.12$. In practice, a $3\frac{1}{2}$-in. pump will probably be employed.

EXAMPLE 2.—Find the approximate horsepower necessary to lift 30 gal. per min. in Example 1.

SOLUTION.—

$$\text{H. P.} = \frac{Gp}{1,714.5} = \frac{30 \times .433 \times 125}{1,714.5} = .95, \text{ or practically 1 H. P.}$$

In order to cover leakage through valves, friction, etc., an addition of at least 75% should be made to a very small pump like this, and so $1\frac{3}{4}$ H. P. would be counted on.

The table on page 339 gives theoretical horsepowers only. Approximately the actual horsepower for a 100-ft. lift may be found by multiplying the tabular figures by 1.7; for a 200-ft. lift, by 1.45; and for a 300-ft. lift, by 1.3, for triplex pumps.

**Depth of Suction.**—Theoretically, a perfect pump will raise water to a height of nearly 34 ft. at the sea level; but, owing to the fact that a perfect vacuum can never be attained with the pump, that the water always contains more or less air, and that more or less watery vapor will form below the piston, it is never possible to reach this theoretical limit, and, in practice, it is not possible to draw water much, if any, over 30 ft. at the sea level, even when the water is cold. Warm water cannot be lifted as high as cold water because a larger amount of watery vapor forms. With boiler feed-pumps handling hot water, for pumps should flow to the pumps by gravity.

For pumps and connections in the best possible condition, it is generally figured that the suction lift will be three-fourths of that theoretically possible. However, pumps are very commonly out of order to a certain degree so that the lifts given in the following table agree very well with actual practice.

## SUCTION LIFT OF PUMPS AT DIFFERENT ALTITUDES

| Altitude Above Sea Level | | Atmospheric Pressure at Altitude Pounds per Square Inch | Theoretical Lift Feet | Practical Lift Feet |
|---|---|---|---|---|
| Miles | Feet | | | |
| $\frac{1}{4}$ | 1,320 | 14.70 | 33.95 | 22 |
| $\frac{1}{4}$ | 2,640 | 14.02 | 32.38 | 21 |
| $\frac{3}{4}$ | 3,960 | 13.33 | 30.79 | 20 |
| 1 | 5,280 | 12.66 | 29.24 | 18 |
| $1\frac{1}{4}$ | 6,600 | 12.02 | 27.76 | 17 |
| $1\frac{1}{2}$ | 7,920 | 11.42 | 26.38 | 16 |
| 2 | 10,560 | 10.88 | 25.13 | 15 |
| | | 9.88 | 22.82 | 14 |

## THEORETICAL HORSEPOWER REQUIRED TO RAISE WATER TO DIFFERENT HEIGHTS

Height Water is Raised, in Feet — Horsepower Required

| Water Raised Gallons per Minute | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 60 | 75 | 90 | 100 | 125 | 150 | 175 | 200 | 250 | 300 | 350 | 400 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | .006 | .012 | .019 | .025 | .031 | .037 | .044 | .05 | .06 | .06 | .07 | .09 | .11 | .12 | .16 | .19 | .22 | .25 | .31 | .37 | .44 | .50 |
| 10 | .012 | .025 | .037 | .050 | .062 | .075 | .087 | .10 | .11 | .12 | .15 | .19 | .22 | .25 | .31 | .37 | .44 | .50 | .62 | .75 | .87 | 1.00 |
| 15 | .019 | .037 | .056 | .075 | .094 | .112 | .131 | .15 | .17 | .19 | .22 | .28 | .34 | .37 | .47 | .56 | .66 | .75 | .94 | 1.12 | 1.31 | 1.50 |
| 20 | .025 | .050 | .075 | .100 | .125 | .150 | .175 | .20 | .22 | .25 | .30 | .37 | .45 | .50 | .62 | .75 | .87 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 |
| 25 | .031 | .062 | .093 | .125 | .156 | .187 | .219 | .25 | .28 | .31 | .37 | .47 | .56 | .62 | .78 | .94 | 1.09 | 1.25 | 1.56 | 1.87 | 2.19 | 2.50 |
| 30 | .037 | .075 | .112 | .150 | .187 | .225 | .262 | .30 | .33 | .37 | .45 | .56 | .67 | .75 | .94 | 1.12 | 1.31 | 1.50 | 1.87 | 2.25 | 2.62 | 3.00 |
| 35 | .043 | .087 | .131 | .175 | .219 | .262 | .306 | .35 | .39 | .44 | .52 | .66 | .79 | .87 | 1.08 | 1.31 | 1.53 | 1.75 | 2.19 | 2.62 | 3.06 | 3.50 |
| 40 | .050 | .100 | .150 | .200 | .250 | .300 | .350 | .40 | .45 | .50 | .60 | .75 | .90 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | 2.50 | 3.00 | 3.50 | 4.00 |
| 45 | .056 | .112 | .168 | .225 | .281 | .337 | .394 | .45 | .51 | .56 | .67 | .84 | 1.01 | 1.12 | 1.41 | 1.69 | 1.97 | 2.25 | 2.81 | 3.37 | 3.94 | 4.50 |
| 50 | .062 | .125 | .187 | .250 | .312 | .375 | .437 | .50 | .56 | .62 | .75 | .94 | 1.12 | 1.25 | 1.56 | 1.87 | 2.19 | 2.50 | 3.12 | 3.75 | 4.37 | 5.00 |
| 60 | .075 | .150 | .225 | .300 | .375 | .450 | .525 | .60 | .67 | .75 | .90 | 1.12 | 1.35 | 1.50 | 1.87 | 2.25 | 2.62 | 3.00 | 3.75 | 4.50 | 5.25 | 6.00 |
| 75 | .093 | .187 | .281 | .375 | .469 | .562 | .656 | .75 | .84 | .94 | 1.12 | 1.41 | 1.69 | 1.87 | 2.34 | 2.81 | 3.28 | 3.75 | 4.69 | 5.62 | 6.56 | 7.50 |
| 90 | .112 | .225 | .337 | .450 | .562 | .675 | .787 | .90 | 1.01 | 1.12 | 1.35 | 1.68 | 2.02 | 2.25 | 2.81 | 3.37 | 3.94 | 4.50 | 5.62 | 6.75 | 7.87 | 9.00 |
| 100 | .125 | .250 | .375 | .500 | .625 | .750 | .875 | 1.00 | 1.12 | 1.25 | 1.50 | 1.87 | 2.25 | 2.50 | 3.12 | 3.75 | 4.37 | 5.00 | 6.25 | 7.50 | 8.75 | 10.00 |
| 125 | .156 | .312 | .469 | .625 | .781 | .937 | 1.094 | 1.25 | 1.41 | 1.56 | 1.87 | 2.34 | 2.81 | 3.12 | 3.91 | 4.69 | 5.47 | 6.25 | 7.81 | 9.37 | 10.94 | 12.50 |
| 150 | .187 | .375 | .562 | .750 | .937 | 1.125 | 1.312 | 1.50 | 1.69 | 1.87 | 2.25 | 2.81 | 3.37 | 3.75 | 4.69 | 5.62 | 6.56 | 7.50 | 9.37 | 11.25 | 13.12 | 15.00 |
| 175 | .219 | .437 | .656 | .875 | 1.093 | 1.312 | 1.531 | 1.75 | 1.97 | 2.19 | 2.62 | 3.28 | 3.94 | 4.37 | 5.47 | 6.56 | 7.66 | 8.75 | 10.94 | 13.12 | 15.31 | 17.50 |
| 200 | .250 | .500 | .750 | 1.000 | 1.250 | 1.500 | 1.750 | 2.00 | 2.25 | 2.50 | 3.00 | 3.75 | 4.50 | 5.00 | 6.25 | 7.50 | 8.75 | 10.00 | 12.50 | 15.00 | 17.50 | 20.00 |
| 250 | .312 | .625 | .937 | 1.250 | 1.562 | 1.875 | 2.187 | 2.50 | 2.81 | 3.12 | 3.75 | 4.69 | 5.62 | 6.25 | 7.81 | 9.37 | 10.94 | 12.50 | 15.62 | 18.75 | 21.87 | 25.00 |
| 300 | .375 | .750 | 1.125 | 1.500 | 1.875 | 2.250 | 2.625 | 3.00 | 3.37 | 3.75 | 4.50 | 5.62 | 6.75 | 7.50 | 9.37 | 11.25 | 13.12 | 15.00 | 18.75 | 22.50 | 26.25 | 30.00 |
| 350 | .437 | .875 | 1.312 | 1.750 | 2.187 | 2.625 | 3.062 | 3.50 | 3.94 | 4.37 | 5.25 | 6.56 | 7.87 | 8.75 | 10.94 | 13.12 | 15.31 | 17.50 | 21.87 | 26.25 | 30.62 | 35.00 |
| 400 | .500 | 1.000 | 1.500 | 2.000 | 2.500 | 3.000 | 3.500 | 4.00 | 4.50 | 5.00 | 6.00 | 7.50 | 9.00 | 10.00 | 12.50 | 15.00 | 17.50 | 20.00 | 25.00 | 30.00 | 35.00 | 40.00 |
| 500 | .625 | 1.250 | 1.875 | 2.500 | 3.125 | 3.750 | 4.375 | 5.00 | 5.62 | 6.25 | 7.50 | 9.37 | 11.25 | 12.50 | 15.62 | 18.75 | 21.87 | 25.00 | 31.25 | 37.50 | 43.75 | 50.00 |

**Amount of Water Raised by a Single-Acting Lift Pump.**—In the case of all pumps having a piston or ram, the amount of water lifted is usually considerably less than the piston displacement, owing to the leakage through the valves, etc., but with single-acting lift pumps, having bucket plungers with a clack valve in the plunger, the amount lifted may actually exceed the plunger displacement; that is, the volume of water may actually be greater than the length of the stroke multiplied by the number of strokes, for, during the up-stroke, the water both above and below the piston is set in motion, and during the down-stroke, the inertia of the water actually carries more water through the valve than would pass through it on account of the space passed through. This increases as the speed or number of strokes increases.

**Capacity of Pumps.**—In the accompanying table are given the capacities of pumps; these values are for single strokes; to find the capacity for one revolution the capacity for a stroke must be multiplied by **2**.

## CAPACITY OF PUMPS

| Diameter of Piston or Plunger, Inches | Length of Piston or Plunger Stroke, in Inches | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 12 | 13 |
| | Displacement per Stroke of Pump, in Gallons | | | | | | | |
| 1¼ | .0106 | .0159 | .0212 | .0266 | .0319 | .0372 | .0638 | .0691 |
| 1⅜ | .0129 | .0193 | .0257 | .0321 | .0386 | .0450 | .0771 | .0835 |
| 1½ | .0153 | .0229 | .0306 | .0382 | .0459 | .0535 | .0918 | .0994 |
| 1¾ | .0208 | .0312 | .0416 | .0521 | .0625 | .0729 | .1249 | .1353 |
| 2 | .0272 | .0408 | .0544 | .0680 | .0816 | .0952 | .1632 | .1768 |
| 2¼ | .0344 | .0516 | .0688 | .0860 | .1033 | .1205 | .2065 | .2238 |
| 2½ | .0425 | .0638 | .0850 | .1063 | .1275 | .1488 | .2550 | .2763 |
| 2¾ | .0514 | .0771 | .1029 | .1286 | .1543 | .1800 | .3086 | .3343 |
| 3 | .0612 | .0918 | .1224 | .1530 | .1836 | .2142 | .3672 | .3978 |
| 3¼ | .0718 | .1077 | .1437 | .1796 | .2154 | .2514 | .4310 | .4668 |
| 3½ | .0833 | .1249 | .1666 | .2082 | .2499 | .2915 | .4997 | .5414 |
| 3¾ | .0956 | .1434 | .1913 | .2391 | .2869 | .3347 | .5738 | .6216 |
| 4 | .1088 | .1632 | .2176 | .2720 | .3264 | .3808 | .6530 | .7072 |
| 4¼ | .1228 | .1843 | .2457 | .3071 | .3684 | .4300 | .7370 | .7984 |
| 4½ | .1377 | .2065 | .2754 | .3443 | .4131 | .4819 | .8262 | .8950 |
| 4¾ | .1534 | .2301 | .3068 | .3835 | .4603 | .5370 | .9205 | .9972 |
| 5 | .1700 | .2550 | .3400 | .4250 | .5100 | .5950 | 1.0200 | 1.1050 |
| 5¼ | .1874 | .2811 | .3749 | .4686 | .5623 | .6560 | 1.1245 | 1.2183 |
| 5½ | .2057 | .3086 | .4114 | .5143 | .6171 | .7200 | 1.2342 | 1.3370 |
| 5¾ | .2248 | .3373 | .4497 | .5621 | .6745 | .7870 | 1.3490 | 1.4614 |
| 6 | .2448 | .3672 | .4896 | .6120 | .7344 | .8568 | 1.4690 | 1.5912 |
| 6¼ | .2656 | .3984 | .5312 | .6641 | .7969 | .9297 | 1.5940 | 1.7270 |
| 6½ | .2872 | .4309 | .5745 | .7182 | .8618 | 1.0050 | 1.7240 | 1.8674 |
| 6¾ | .3099 | .4648 | .6197 | .7747 | .9296 | 1.0850 | 1.8590 | 2.0140 |
| 7 | .3332 | .4999 | .6665 | .8331 | .9997 | 1.1660 | 1.9990 | 2.1660 |
| 7½ | .4084 | .6126 | .8168 | 1.0210 | 1.2250 | 1.4290 | 2.4500 | 2.6540 |
| 8 | .4352 | .6529 | .8704 | 1.0890 | 1.3060 | 1.5230 | 2.6110 | 2.8290 |
| 9 | .5508 | .8263 | 1.1020 | 1.3770 | 1.6520 | 1.9280 | 3.3050 | 3.5800 |
| 10 | .6800 | 1.0200 | 1.3600 | 1.7000 | 2.0400 | 2.3800 | 4.0800 | 4.4200 |
| 10½ | .7497 | 1.1250 | 1.4990 | 1.8740 | 2.2490 | 2.6240 | 4.4980 | 4.8730 |
| 11 | .8228 | 1.2340 | 1.6460 | 2.0570 | 2.4680 | 2.8800 | 4.9370 | 5.3480 |
| 12 | .9792 | 1.4690 | 1.9580 | 2.4480 | 2.9380 | 3.4270 | 5.8750 | 6.3650 |
| 13 | 1.1490 | 1.7230 | 2.2970 | 2.8720 | 3.4450 | 4.0220 | 6.8940 | 7.4670 |
| 14 | 1.3320 | 1.9980 | 2.6650 | 3.3310 | 3.9970 | 4.6640 | 7.9940 | 8.6610 |
| 15 | 1.5300 | 2.2950 | 3.0600 | 3.8250 | 4.5900 | 5.3540 | 9.1800 | 9.9430 |
| 16 | 1.7400 | 2.6100 | 3.4800 | 4.3500 | 5.2200 | 6.0900 | 10.4400 | 11.3100 |
| 18 | 2.2030 | 3.3050 | 4.4060 | 5.5080 | 6.6100 | 7.7110 | 13.2200 | 14.3200 |
| 20 | 2.7200 | 4.0800 | 5.4400 | 6.8000 | 8.1600 | 9.5200 | 16.3200 | 17.6800 |
| 22 | 3.2910 | 4.9360 | 6.5820 | 8.2280 | 9.8740 | 11.5200 | 19.7500 | 21.3900 |
| 24 | 3.9160 | 5.8750 | 7.8330 | 9.7920 | 11.7500 | 13.7100 | 23.5000 | 25.4600 |

TABLE—(CONTINUED)

| Diameter of Piston or Plunger, Inches | Length of Piston or Plunger Stroke, in Inches | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 16 | 18 | 20 | 24 | 25 | 33 | 36 | 38 |
| | Displacement per Stroke of Pump, in Gallons | | | | | | | |
| 1¼ | .0850 | .0956 | .1062 | .1275 | .1328 | .1753 | .1912 | .2019 |
| 1⅜ | .1029 | .1157 | .1286 | .1543 | .1607 | .2121 | .2314 | .2442 |
| 1½ | .1224 | .1377 | .1530 | .1836 | .1912 | .2524 | .2754 | .2907 |
| 1¾ | .1666 | .1874 | .2082 | .2499 | .2603 | .3436 | .3748 | .3956 |
| 2 | .2176 | .2448 | .2720 | .3264 | .3400 | .4488 | .4896 | .5168 |
| 2¼ | .2754 | .3098 | .3442 | .4131 | .4303 | .5680 | .6196 | .6541 |
| 2½ | .3400 | .3825 | .4250 | .5100 | .5313 | .7013 | .7650 | .8075 |
| 2¾ | .4114 | .4628 | .5143 | .6171 | .6428 | .8485 | .9257 | .9771 |
| 3 | .4896 | .5508 | .6120 | .7344 | .7650 | 1.0100 | 1.1020 | 1.1630 |
| 3¼ | .5746 | .6464 | .7183 | .8619 | .8978 | 1.1851 | 1.2930 | 1.3647 |
| 3½ | .6664 | .7497 | .8330 | .9996 | 1.0410 | 1.3744 | 1.4994 | 1.5830 |
| 3¾ | .7650 | .8610 | .9562 | 1.1480 | 1.1953 | 1.5778 | 1.7212 | 1.8169 |
| 4 | .8704 | .9792 | 1.0880 | 1.3060 | 1.3600 | 1.7952 | 1.9584 | 2.0672 |
| 4¼ | .9826 | 1.1054 | 1.2282 | 1.4740 | 1.5353 | 2.0270 | 2.2110 | 2.3336 |
| 4½ | 1.1010 | 1.2393 | 1.3770 | 1.6524 | 1.7212 | 2.2720 | 2.4786 | 2.6163 |
| 4¾ | 1.2274 | 1.3780 | 1.5340 | 1.8410 | 1.9180 | 2.5310 | 2.7620 | 2.9150 |
| 5 | 1.3600 | 1.5300 | 1.7000 | 2.0400 | 2.1250 | 2.8050 | 3.0600 | 3.2300 |
| 5¼ | 1.5000 | 1.6870 | 1.8740 | 2.2490 | 2.3430 | 3.0930 | 3.3740 | 3.5610 |
| 5½ | 1.6460 | 1.8510 | 2.0570 | 2.4680 | 2.5710 | 3.3940 | 3.7030 | 3.9080 |
| 5¾ | 1.7990 | 2.0230 | 2.2480 | 2.6980 | 2.8110 | 3.7100 | 4.0470 | 4.2720 |
| 6 | 1.9580 | 2.2030 | 2.4480 | 2.9380 | 3.0600 | 4.0380 | 4.4060 | 4.6500 |
| 6¼ | 2.1250 | 2.3900 | 2.6560 | 3.1880 | 3.3200 | 4.3830 | 4.7810 | 5.0470 |
| 6½ | 2.2980 | 2.5850 | 2.8730 | 3.4470 | 3.5910 | 4.7400 | 5.1710 | 5.4580 |
| 6¾ | 2.4790 | 2.7880 | 3.0990 | 3.7180 | 3.8730 | 5.1130 | 5.5780 | 5.8870 |
| 7 | 2.6660 | 2.9990 | 3.3320 | 3.9990 | 4.1650 | 5.4990 | 5.9980 | 6.3320 |
| 7¾ | 3.2670 | 3.6750 | 4.0840 | 4.9000 | 5.1050 | 6.7390 | 7.3510 | 7.7590 |
| 8 | 3.4820 | 3.9170 | 4.3520 | 5.2230 | 5.4400 | 7.1810 | 7.8340 | 8.2690 |
| 9 | 4.4060 | 4.9570 | 5.5080 | 6.6100 | 6.8850 | 9.0890 | 9.9150 | 10.4600 |
| 10 | 5.4400 | 6.1200 | 6.8000 | 8.1600 | 8.5000 | 11.2200 | 12.2400 | 12.9200 |
| 10½ | 5.9980 | 6.7470 | 7.4970 | 8.9960 | 9.3700 | 12.3700 | 13.4900 | 14.2400 |
| 11 | 6.5820 | 7.4050 | 8.2280 | 9.8730 | 10.2800 | 13.5800 | 14.8100 | 15.6300 |
| 12 | 7.8340 | 8.8130 | 9.7920 | 11.7500 | 12.2400 | 16.1600 | 17.6300 | 18.6000 |
| 13 | 9.1920 | 10.3400 | 11.4900 | 13.7800 | 14.3600 | 18.9600 | 20.6900 | 21.8300 |
| 14 | 10.6600 | 11.9900 | 13.3200 | 15.9800 | 16.6600 | 21.9900 | 23.9900 | 25.3200 |
| 15 | 12.2300 | 13.7700 | 15.2900 | 18.3600 | 19.1200 | 25.2400 | 27.5400 | 29.0700 |
| 16 | 13.9200 | 15.6600 | 17.4000 | 20.8800 | 21.7600 | 28.7200 | 31.3300 | 33.0700 |
| 18 | 17.6200 | 19.8200 | 22.0300 | 26.4400 | 27.5400 | 36.3500 | 39.6600 | 41.8600 |
| 20 | 21.7600 | 24.4800 | 27.2000 | 32.6400 | 34.0000 | 44.8800 | 48.9600 | 51.6800 |
| 22 | 26.3300 | 29.6200 | 32.9100 | 39.4900 | 41.1400 | 54.3000 | 59.2400 | 62.5300 |
| 24 | 31.3300 | 35.2500 | 39.1600 | 47.0000 | 48.9600 | 64.6300 | 70.5000 | 74.4200 |

**Pump Valves.**—As a rule, a large number of small valves having a comparatively small opening are preferable to a small number of large valves with a greater opening, and most modern pumps are built on these lines. A small valve represents a proportionately larger surface of discharge with the same lift than the large valve, hence whatever the total area of the valve-seat opening, its full contents can be discharged with less lift through numerous small valves than through one large valve. Cornish pumps generally have one large metal valve.

## POWER PUMPS

In a power pump the reciprocating motion is transmitted to the water plunger or piston by means of a crank driven by belting or gearing, instead of in a straight line directly from the steam or air piston. Power pumps are not

generally used for pumping against heavy pressures. They may be single, duplex, or triplex, single-acting or double-acting, and either of piston or plunger types, although double-acting and triple-acting plunger pumps are the most common. The triplex pump consists of three single-acting plunger pumps driven by cranks 120° apart on a single shaft. A tight and a loose pulley provide the means for starting and stopping the pump without disturbing the engine on the main shaft. The pulley shaft is geared to the crank-shaft by a pinion and spur wheel. By setting the cranks 120° apart, the strokes follow and overlap one another giving a uniform flow of water and a uniform expenditure of power. In the case of duplex single-acting pumps, the cranks are placed 180° apart and the discharge is the same as from one double-acting pump of the same diameter of plunger and length of stroke. Duplex, double-acting power pumps with their cranks set 90° apart give a steady discharge, for when one plunger is at the end of its stroke and at rest, the other plunger is moving and discharging water at its maximum rate. The crank-shaft of these pumps is sometimes supplied with gears at each end to equalize the strain, particularly when heavy pressures are to be overcome.

**Electrically Driven Power Pumps.**—Where water is to be delivered from isolated workings to the sumps for the large station pumps, electrically driven power pumps are far more efficient than steam pumps. In some cases, it is probably best to equip the entire mine with electric pumps, both in the isolated workings and at the stations, because they can be driven by a high-class compound-condensing engine on the surface, directly connected to a generator, and furnishing electricity through conductors to the various pumps.

The total efficiency of a series of small electric pumps that aggregate a sufficient amount of power to enable this arrangement to be used, is very much higher than the total efficiency of a number of small isolated steam or compressed-air pumps introduced into the workings. With compound-condensing engines upon the surface, operating electric pumps underground, the steam consumption per pump horsepower per hour, for the smaller sizes, would only be about 40 lb. per H. P. per hr.; for medium-sized electric pumps, about 30 lb. per hr., and larger sizes from 20 to 30 lb. per H. P. per hr. These figures show that for pumping from isolated portions of the mine the electric pump is much more efficient than the steam pump, as the current can be frequently obtained from the lines operating the underground haulage system, furnishing light, etc.

An efficiency of at least 50% should be obtained in any well-designed, well-built pump, so that a close approximation to the actual current consumption can be obtained by doubling the theoretical consumption given in the accompanying table.

## THEORETICAL CONSUMPTION OF ELECTRIC CURRENT FOR PUMPING WATER PER 1,000 GAL.

| Total Elevation Feet | Equivalent Pressure per Square Inch | Kilowatts per 1,000 Gal. | Total Elevation Feet | Equivalent Pressure per Square Inch | Kilowatts per 1,000 Gal. |
|---|---|---|---|---|---|
| 10 | 4.33 | .0312 | 160 | 69.29 | .500 |
| 20 | 8.66 | .0624 | 170 | 73.63 | .531 |
| 30 | 12.99 | .0937 | 180 | 77.96 | .562 |
| 40 | 17.32 | .124 | 190 | 82.29 | .593 |
| 50 | 21.65 | .156 | 200 | 86.62 | .624 |
| 60 | 25.99 | .187 | 210 | 91.14 | .655 |
| 70 | 30.32 | .218 | 220 | 95.48 | .686 |
| 80 | 34.65 | .249 | 230 | 99.82 | .717 |
| 90 | 38.95 | .281 | 240 | 104.16 | .748 |
| 100 | 43.31 | .312 | 250 | 108.50 | .779 |
| 110 | 47.64 | .343 | 260 | 112.84 | .813 |
| 120 | 51.97 | .374 | 270 | 116.91 | .841 |
| 130 | 56.30 | .406 | 280 | 121.24 | .875 |
| 140 | 60.63 | .437 | 290 | 125.57 | .903 |
| 150 | 64.96 | .468 | 300 | 129.90 | .934 |

**Precautions Necessary With Electrically Driven Mine Pumps.**—Where electricity is used in mining, the prevailing tendency is to lay the blame for troubles of various kinds to its use, because electricity is the least understood and most mysterious force employed. It is especially important, therefore, that every precaution be taken to minimize the possibility of accident from electric causes, either by shock or by fire. While bare wires are necessarily employed as trolley wires, all wires used as feeders, on the headings, and wires leading to the pump should be well insulated, as even a slight shock received by the attendant working at or near the pump, or trackmen, or timbermen working on the entries, may prove fatal should the man receiving the shock fall against the exposed wire. Whenever possible, a dry, wooden platform should be provided about the pump; or, if this is considered objectionable on account of the fire risk, a good, dry, cement floor should be laid in the pump room. A small stool with insulator pins and glasses for legs forms a safeguard and should be used whenever it is necessary to adjust or change brushes or work on the commutator while running.

It is hardly surprising that a fire should originate in a mine when a small electric pump is placed on a wooden floor in a frame pump house and the attendant has a seat or bunk with straw mattress, where he reclines and smokes; or when oil and waste are thrown about and the fuse boxes and rheostat are fastened to a board placed against an inflammable partition, while a strong draft passing through the place, and no available means at hand of extinguishing a fire adds to the danger.

The following precautions are advised in placing an electric pump: Place the pump, if possible, in a special room excavated for the purpose. Where a break-through or passage between two rooms must be used, it should be closed with brick or stone and not with wooden brattice. Make the place fireproof; allow no wooden boxes or furnishings of inflammable material; enforce strict regulations in regard to oil and waste used about the pump; have a dry, cement floor and keep it clean. Matches must not be left in or around the pump house, or any illuminating oil kept therein, and lubricating oils should be carried to the pump house only in small quantities and in closed cans. Cotton waste when saturated with oil is liable to spontaneous combustion, and should be thrown into a tight can and taken out of the mine each day. The discharge pipe of the pump should be tapped with 1-in. connections, and a length of hose with a nozzle kept in the pump house ready for use in case of fire.

The pump house should be well lighted by incandescent lamps. Where lamps are used in series, at least two circuits should be run, since one burn-out will extinguish an entire series. The transmission wire for the pump should be strung on glass or porcelain insulators and care taken to prevent contact with the pump frame or the mine timbers. The line should be taken as directly as possible to the switch and fuses, and should be protected against a series ground or a short circuit.

The following table gives the gallons per minute delivered from various sized pumps operating at different piston speeds.

**Centrifugal Pumps.**—In mining practice, centrifugal pumps commonly driven by an electric motor placed on the same shaft, have been used for many years in raising water from local dips and swamps into the main sump. The absence of valves makes them well adapted to pumping dirty or gritty water and the fact that the moving parts have a rotatory instead of a reciprocating motion makes them especially suitable where electric power is available. Their small size, and consequent portability, is a commending feature for underground work. Where the water is strongly acid these pumps probably wear out sooner than ordinary reciprocating pumps unless made of special acid-resisting metal.

The original form of single-stage centrifugal pump was designed for handling large volumes of water under small heads, say, from 60 to 100 ft. In modern installations, heads of 500 ft., 750 ft., and even more, are overcome by the use of multistage pumps, so that this type is now frequently used in pumping from the main sump to the surface in one operation.

The pump consists of a series of revolving, turbine-like wheels or impellers, set side by side on the same shaft. The water thrown off by the first wheel is taken up by the second and by it passed on to the third, and similarly, according to the number of stages. If there is but one revolving part or impeller, which throws the water into the discharge pipe, the pump is *single-stage*; if there are two revolving parts, the second discharging the water forced into it by the first, it is a *two-stage pump*, and similarly for each additional revolving

## DISCHARGE OF PUMPS AT VARIOUS PISTON SPEEDS

| Diameter of Cylinder Inches | Area Square Inches | Contents for 1 Ft. Length | | Gallons per Minute Delivered at Indicated Piston Speeds* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Cubic Feet | Gallons | 100 | 125 | 150 | 175 | 200 | 225 | 250 | 275 | 300 | 350 | 400 | 500 |
| 1 | .7854 | .0055 | .0408 | 4.1 | 5.1 | 6.1 | 7.1 | 8.2 | 9.2 | 10.2 | 11.2 | 12.2 | 14.3 | 16.3 | 20.4 |
| 2 | 3.1416 | .0218 | .1632 | 16.3 | 20.4 | 24.5 | 28.6 | 32.6 | 36.7 | 40.8 | 44.9 | 49.0 | 57.1 | 65.3 | 81.6 |
| 3 | 7.0686 | .0491 | .3672 | 36.7 | 45.9 | 55.1 | 64.3 | 73.4 | 82.6 | 91.8 | 101.0 | 110.0 | 129.0 | 147.0 | 183.6 |
| 4 | 12.5660 | .0873 | .6528 | 65.3 | 81.6 | 97.9 | 114.0 | 131.0 | 147.0 | 163.0 | 180.0 | 196.0 | 228.0 | 261.0 | 326.0 |
| 5 | 19.6350 | .1364 | 1.0200 | 102.0 | 128.0 | 153.0 | 179.0 | 204.0 | 230.0 | 255.0 | 281.0 | 306.0 | 357.0 | 408.0 | 510.0 |
| 6 | 28.2740 | .1963 | 1.4690 | 147.0 | 184.0 | 220.0 | 257.0 | 294.0 | 330.0 | 367.0 | 404.0 | 441.0 | 514.0 | 588.0 | 734.0 |
| 6½ | 33.1830 | .2304 | 1.7240 | 172.0 | 216.0 | 259.0 | 302.0 | 345.0 | 388.0 | 431.0 | 474.0 | 517.0 | 603.0 | 690.0 | 862.0 |
| 7 | 38.4850 | .2673 | 1.9990 | 200.0 | 250.0 | 300.0 | 350.0 | 400.0 | 450.0 | 500.0 | 550.0 | 600.0 | 700.0 | 800.0 | 1,000.0 |
| 7½ | 44.1790 | .3068 | 2.2950 | 230.0 | 287.0 | 344.0 | 402.0 | 459.0 | 516.0 | 574.0 | 631.0 | 689.0 | 803.0 | 918.0 | 1,148.0 |
| 8 | 50.2650 | .3491 | 2.6110 | 261.0 | 326.0 | 392.0 | 457.0 | 522.0 | 588.0 | 653.0 | 718.0 | 783.0 | 914.0 | 1,044.0 | 1,306.0 |
| 8½ | 56.7450 | .3941 | 2.9480 | 295.0 | 369.0 | 442.0 | 516.0 | 590.0 | 663.0 | 737.0 | 811.0 | 884.0 | 1,032.0 | 1,179.0 | 1,474.0 |
| 9 | 63.6170 | .4418 | 3.3050 | 331.0 | 413.0 | 496.0 | 578.0 | 661.0 | 744.0 | 826.0 | 909.0 | 992.0 | 1,157.0 | 1,322.0 | 1,652.0 |
| 9½ | 70.8820 | .4922 | 3.6820 | 368.0 | 460.0 | 552.0 | 644.0 | 736.0 | 828.0 | 921.0 | 1,013.0 | 1,105.0 | 1,289.0 | 1,473.0 | 1,841.0 |
| 10 | 78.5400 | .5454 | 4.0800 | 408.0 | 510.0 | 612.0 | 714.0 | 816.0 | 918.0 | 1,020.0 | 1,122.0 | 1,224.0 | 1,428.0 | 1,632.0 | 2,040.0 |
| 10½ | 86.5900 | .6013 | 4.4980 | 450.0 | 562.0 | 675.0 | 787.0 | 900.0 | 1,012.0 | 1,125.0 | 1,237.0 | 1,349.0 | 1,574.0 | 1,799.0 | 2,250.0 |
| 11 | 95.0330 | .6600 | 4.9370 | 494.0 | 617.0 | 741.0 | 864.0 | 987.0 | 1,111.0 | 1,234.0 | 1,358.0 | 1,481.0 | 1,728.0 | 1,975.0 | 2,468.0 |
| 11½ | 103.8700 | .7213 | 5.3960 | 540.0 | 675.0 | 809.0 | 944.0 | 1,079.0 | 1,214.0 | 1,349.0 | 1,484.0 | 1,619.0 | 1,889.0 | 2,158.0 | 2,698.0 |
| 12 | 113.1000 | .7854 | 5.8780 | 588.0 | 734.0 | 881.0 | 1,028.0 | 1,175.0 | 1,322.0 | 1,469.0 | 1,616.0 | 1,763.0 | 2,056.0 | 2,350.0 | 2,938.0 |
| 12½ | 122.7200 | .8522 | 6.3750 | 638.0 | 797.0 | 956.0 | 1,116.0 | 1,275.0 | 1,434.0 | 1,594.0 | 1,753.0 | 1,913.0 | 2,231.0 | 2,550.0 | 3,188.0 |
| 13 | 132.7300 | .9218 | 6.8950 | 690.0 | 862.0 | 1,034.0 | 1,207.0 | 1,379.0 | 1,551.0 | 1,724.0 | 1,896.0 | 2,069.0 | 2,413.0 | 2,758.0 | 3,448.0 |
| 13½ | 143.1400 | .9940 | 7.4360 | 744.0 | 930.0 | 1,115.0 | 1,301.0 | 1,487.0 | 1,673.0 | 1,859.0 | 2,045.0 | 2,231.0 | 2,603.0 | 2,974.0 | 3,716.0 |
| 14 | 153.9400 | 1.0690 | 7.9970 | 800.0 | 1,000.0 | 1,200.0 | 1,399.0 | 1,599.0 | 1,799.0 | 1,999.0 | 2,199.0 | 2,399.0 | 2,799.0 | 3,199.0 | 3,998.0 |
| 14½ | 165.1300 | 1.1470 | 8.5780 | 858.0 | 1,072.0 | 1,287.0 | 1,501.0 | 1,716.0 | 1,930.0 | 2,145.0 | 2,359.0 | 2,573.0 | 3,002.0 | 3,431.0 | 4,290.0 |
| 15 | 176.7100 | 1.2270 | 9.1800 | 918.0 | 1,148.0 | 1,377.0 | 1,607.0 | 1,836.0 | 2,066.0 | 2,295.0 | 2,525.0 | 2,754.0 | 3,213.0 | 3,672.0 | 4,590.0 |
| 15½ | 188.6900 | 1.3100 | 9.8010 | 980.0 | 1,225.0 | 1,470.0 | 1,715.0 | 1,960.0 | 2,205.0 | 2,450.0 | 2,695.0 | 2,940.0 | 3,430.0 | 3,920.0 | 4,900.0 |
| 16 | 201.0600 | 1.3960 | 10.4400 | 1,044.0 | 1,305.0 | 1,566.0 | 1,827.0 | 2,088.0 | 2,349.0 | 2,610.0 | 2,871.0 | 3,132.0 | 3,654.0 | 4,176.0 | 5,220.0 |
| 16½ | 213.8200 | 1.4850 | 11.1100 | 1,111.0 | 1,389.0 | 1,667.0 | 1,944.0 | 2,222.0 | 2,500.0 | 2,778.0 | 3,055.0 | 3,333.0 | 3,889.0 | 4,444.0 | 5,556.0 |
| 17 | 226.9800 | 1.5760 | 11.7900 | 1,179.0 | 1,474.0 | 1,769.0 | 2,063.0 | 2,358.0 | 2,653.0 | 2,948.0 | 3,242.0 | 3,537.0 | 4,127.0 | 4,716.0 | 5,896.0 |
| 17½ | 240.5300 | 1.6700 | 12.4900 | 1,249.0 | 1,561.0 | 1,873.0 | 2,185.0 | 2,498.0 | 2,810.0 | 3,122.0 | 3,434.0 | 3,747.0 | 4,371.0 | 4,996.0 | 6,244.0 |
| 18 | 254.4700 | 1.7670 | 13.2200 | 1,322.0 | 1,653.0 | 1,983.0 | 2,314.0 | 2,644.0 | 2,975.0 | 3,305.0 | 3,636.0 | 3,966.0 | 4,627.0 | 5,288.0 | 6,610.0 |
| 18½ | 268.8000 | 1.8670 | 13.9600 | 1,396.0 | 1,745.0 | 2,094.0 | 2,443.0 | 2,792.0 | 3,141.0 | 3,490.0 | 3,839.0 | 4,188.0 | 4,886.0 | 5,584.0 | 6,980.0 |
| 19 | 283.5300 | 1.9690 | 14.7300 | 1,473.0 | 1,841.0 | 2,210.0 | 2,578.0 | 2,946.0 | 3,314.0 | 3,683.0 | 4,051.0 | 4,419.0 | 5,156.0 | 5,892.0 | 7,366.0 |
| 19½ | 298.6500 | 2.0740 | 15.5100 | 1,551.0 | 1,939.0 | 2,327.0 | 2,714.0 | 3,102.0 | 3,490.0 | 3,878.0 | 4,265.0 | 4,653.0 | 5,429.0 | 6,204.0 | 7,756.0 |
| 20 | 314.1600 | 2.1820 | 16.3200 | 1,632.0 | 2,040.0 | 2,448.0 | 2,856.0 | 3,264.0 | 3,672.0 | 4,080.0 | 4,488.0 | 4,896.0 | 5,712.0 | 6,528.0 | 8,160.0 |
| 24 | 452.3900 | 3.1420 | 23.5000 | 2,350.0 | 2,938.0 | 3,525.0 | 4,113.0 | 4,700.0 | 5,288.0 | 5,875.0 | 6,463.0 | 7,050.0 | 8,225.0 | 9,400.0 | 11,750.0 |

*1 gal. = 231 cu. in. = .13368 cu. ft.; 1 cu. ft. = 1728 cu. in.; 1 gal. of water at 39.2° = 8.33888 lb.; 1 cu. ft. of water = 7.48052 gal., and weighs 62.423 lb.

part. These pumps are designated by the size of their outlet as, for instance, a 2-in. or 4-in. pump, meaning with a 2-in. or 4-in. discharge pipe.

The height of lift depends on the quantity of water to be discharged and the circumferential velocity of the revolving disk and is proportional to the area of the discharge orifices at the circumference of the disk. The circumferential velocity depends on the diameter of the disks as well as on the number of revolutions per minute. As the number of revolutions per minute is commonly limited by those of the electric driving motor, the diameter of the disks, that is, the size of the wheel or pump, is varied to suit the head against which the machine is working. For a given lift, the total efficiency of centrifugal pumps increases with the size of the pump, being about .45 to .50 for the smallest sizes of single stage pumps, and .70 to .75 for the largest, the multistage pumps giving better results.

One of the most important points to investigate when selecting a centrifugal pump is the total head against which it will have to work, as this head determines the speed at which the pump can most economically run and the speed determines the size necessary to throw a given volume of water. The horsepower may be determined by the same formula used for reciprocating pumps,

$$BHP = (GPM \times \text{head in feet}) \div (3,960 \times \text{efficiency of pump}),$$

in which, $BHP$ = brake horsepower;

$GPM$ = number of gallons to be delivered per minute.

In this formula, if the efficiency is made equal to unity, the horsepower determined will be the net horsepower theoretically required to raise the given number of gallons the given height. As stated, the efficiency varies widely, and may be taken at .50 for small wheels operating under low heads and .75 for large multistage wheels operating under high heads. However, the determination of the proper size of pump had better be left to the manufacturer who, on receipt of the proper data, will supply the right kind and size of pump and will guarantee its efficiency. The data required are the quantity of water to be pumped per minute and its kind (clean, acid, muddy, gritty, etc., size of grains of impurity, etc.), the suction lift, or distance from the water level in the sump to the center line of the pump, the discharge lift, or difference in elevation between the center line of the pump and the point of discharge, together with a sketch showing a plan of the discharge line with its length and size and the number, location, and radius of the various bends. The pressure, in pounds per square inch, of steam or compressed air, or voltage of electric current should be given, all estimates being made for the power available at the point of its application.

When bolting the pump to the foundation, care must be taken not to spring the bedplate. Every joint in the suction pipe should be air-tight. The pump should be installed to run in the direction indicated by the arrow, wording, or other instruction stamped or cast on the casing. The stuffingboxes should be properly packed and the water-seal ring should be in the proper position. The bearings should be cleaned and filled with a good grade of engine oil. Long sweep elbows only, and as few of them as possible, should be used in the suction and discharge piping. It is also advisable to use large pipe lines, as this reduces the power necessary to drive the pump and will save money in the long run. To prevent freezing in cold weather, the pump should always be drained when not in use by unscrewing the plug in the bottom of the pump casing.

All centrifugal pumps that do not operate with pressure on the suction side (that is, all pumps drawing water from a sump that is below the level of the center of the pump) must have the casing and suction pipe filled with water before starting. This may be accomplished in various ways. Where steam or compressed air is the power an ejector is used, but this method of priming is not available in mining practice where electricity is used. If a hand primer is used, the air cock on the top of the pump is opened and the primer, which is a small hand pump, is worked until water flows through this cock. Then the pump is started.

If there is a foot-valve on the bottom of the suction pipe, the pump can be primed by running water into it from an overhead tank, or any other source of supply, as the valve will hold the water in the pump. In the event of the column pipe being full of water, as is commonly the case when the pump has been shut down temporarily (as during the night shift), a by-pass may be arranged by which the water in the column pipe may be drawn off in sufficient quantity to prime the pump. After priming, the pump can be started and, when full speed has been attained, the discharge valve opened.

## PUMPS FOR SPECIAL PURPOSES

**Sinking Pumps.**—Sinking pumps may be either single or duplex in their action, and may be inside or outside packed. Outside-packed single-acting pumps are in many ways preferable, owing to the fact that they are less liable to get out of order. One requisite of any sinking pump is that it should have as few exposed parts as possible, and that these parts should be so placed that they will be protected to as great an extent from injury by blasting as possible. Sinking pumps are usually provided with a telescopic section in the suction pipe, and sometimes also in the discharge pipe, so that they can be moved down several feet without having to break the joints of the piping.

**Pumps for Acid Waters.**—Where mine waters are acid in their nature, bronze, bronze-lined, or lead lined pumps are usually employed, and in some cases even wooden pumps have been used, as, for instance, in the Swedish copper mines, though this practice is disappearing in favor of the use of bronze or copper linings. The pipes for such pumps should be of bronze or copper tubing, or should be lined with some substance that will not be affected by the acid of the water. Sometimes wooden linings are employed, placed as shown in Figs. 1 and 2, Fig. 1 being a section of the pipe with the lining complete, and Fig. 2 a cross-section of one of the individual boards used in the lining. These are usually made of pine about ⅜ in. thick, and are grooved on each end as shown.
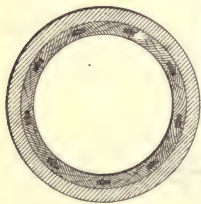
FIG. 1      FIG. 2

They are sprung in so as to complete a circle on the inside of the pipe, and then long, thin, wooden keys driven into the grooves. When the water is allowed to go into the pipes, the linings swell and make all joints perfectly tight. Elbows and other crooked sections are lined with sheet lead beaten in with a mallet. Concrete-lined pumps have been recently introduced in mines where there is acid water.

## PUMP FOUNDATIONS

The *foundation* for pumping machinery depends entirely on the type of pump. Direct-acting duplex pumps probably require the least foundation, for the piston and plunger moving in opposite directions almost balance the machine in line with the plunger motion and the strains due to reversing are contained almost wholly within the machine itself. Small duplex-pump foundations are made of a solid mass of brick or concrete, while large pumps are often set on separate piers, one for the water end and one for the steam end. The foundations must go down to sufficiently hard material to bear the weight of the pump; or, if the substratum consists of loose sand or gravel, the foundation must be spread so that the pressure does not exceed 1 T. per sq. ft. Single pumps require a somewhat heavier foundation than duplex pumps, owing to the greater shocks to which they are subjected.

Foundations should be built of hard brick laid in cement mortar, or of concrete; or, in the case of large pumps, stone may be used. The pump should be secured to foundation bolts anchored to plates underneath the masonry. If solid rock exists at the place where the pump is to be located, the surface of the rock is leveled to suit the bedplate or footings of the pump; holes for the foundation bolts are then drilled to a sufficient depth, and the bolts, which should have a good length and a roughened shank, are placed in the holes and fastened by pouring molten lead around them.

*Pump houses,* or *pump stations* as they are often called, are generally placed near the foot of a shaft or slope in a room excavated for the purpose in the solid rock or coal. If the roof needs support, timbers or steel supports may be used.

## PUMP MANAGEMENT

All pumps, when new, should be run slowly until the parts have become thoroughly adjusted to their bearings, when the speed may be increased. Because a new pump works stiffly is no cause for alarm, for, while a machinist can properly construct the parts, he cannot always forsee the strains caused by the action of the pump, when the parts are assembled and which require certain adjustments to be made after the pump is at work. By running the pump slowly with the parts properly lubricated and making such adjustments as may be necessary, stiffness will gradually disappear and the highest efficiency

of the pump will be attained, provided other matters on which the pump's action depend have received proper attention.

The causes that affect a pump, impair its efficiency, and prevent it from performing its full duty are: wear; the improper adjustment of valves, valve stems, and levers; the improper packing of plungers and stuffingboxes; drawing up the stuffingbox glands too tightly; lost motion due to permitting the working parts to wear and not adjusting them to the new conditions; accumulations of foreign matter under the valves or in the strainer; broken valves and valve springs; leakage in valves; taking air in the suction pipe; clogged or broken discharge pipes; and the use of poor gaskets.

At many mines, the pumps are capable of a larger capacity than is obtained by the low speed at which they are run, but it is important that such pumps be run continuously, as any serious interruption in pumping might cause trouble elsewhere. It is customary, therefore, to keep on hand a supply of duplicate valves, moving parts, and packing, in order that when it becomes necessary to make repairs they may be made without great loss of time.

A common cause of pumps refusing to work properly is due to their taking air below the suction valves. Small leaks will cause the piston to jump, owing to the water not entering through the suction valves soon enough to fill the entire chamber. This trouble may be remedied by seeing that all joints in the suction pipe and between the pipe and the pump are air-tight. Leaks may sometimes be detected by the hearing or by the flame from a candle being drawn toward the hole. If the leaks are small and not at the pipe joints, a coat of asphalt paint may stop them; if large, they should be drilled larger, the hole threaded, and a screw plug inserted. If the leak is at the joint between two pipes, the pipes should be uncoupled and screwed together again, using graphite pipe grease for a lubricant; or, if the joint is a flanged one, a new gasket should be placed between the flanges, and the pipes lined up before the bolts are tightened.

Sometimes, a pump fails to catch the water when started owing to leakage of the valves in the suction chamber. The trouble may be caused by the valve and valve seat being corroded; by chips or gravel getting under the valves and preventing them from seating properly; or by the valves and seats becoming worn so that leakage cannot be prevented without changing the parts.

Many pumps that will not raise water in the suction pipe when empty, owing to the pump having been idle for some time, will continue to draw water after once being started. In such cases, it is necessary to *prime the pump*, by which is meant filling the suction pipe and part of the suction chamber, if there is one, and in some cases, also, the pump barrel, with water, so that the pump may start under conditions similar to those under which it must work. To prime the pump, it is simply necessary to open the cock, or valve, in the priming pipe and allow water from the column pipe to flow down into the suction pipe and the pump. When these are full, the valve is again closed and the pump is ready to start.

Pumps sometimes fail to raise the water when the full head is resting on the valves in the discharge chamber. This may be due to air accumulating between the suction and the discharge decks, which air is compressed and expanded by the motion of the plunger. Air valves should be provided in the water cylinder for the purpose of allowing this confined air to escape. Violent jarring and trembling often take place if the discharge chamber is not provided either with an air chamber where the lift is not above 150 ft., or with an alleviator, for lifts above that distance. This jarring is due to the column of water in the discharge pipe coming to rest suddenly between strokes and having to be again put in motion.

In case the pump column is filled with water and the pump is stopped, the water will run back through the pump if the foot-valve is not tight. To prevent this, a gate valve or a check-valve is placed a short distance from the pump in the column pipe. A gate valve wears less than a check-valve, and presents no obstruction to the flow of water when the valve is open. This valve is useful in the column pipe to keep the pressure off the valves when the pump is not at work, and also for keeping water from running back into the pump chamber when the valves are being repaired.

When starting compound pumps, the steam on the high-pressure-cylinder piston is not always powerful enough to move the plungers against the resistance of the water in the discharge pipe; but, by opening the gate valve in the by-pass piping, the pressure on the plungers is relieved for a sufficient number of strokes to allow the steam to reach the low-pressure piston, when the

combined force of the two pistons will do the work and the by-pass pipe can be closed.

Valves in the steam end sometimes wear unevenly or the valve stems by continual action wear and cause lost motion, thus causing a back pressure and irregular action. Anything wrong in the steam end can usually be determined by the irregular exhaust, but even this may be deceptive in case the water-end valves are leaking. If the steam valves are suspected, the steam-chest cover may be raised for their inspection, but the valves should not be disturbed until it has been determined, by moving the water piston backwards and forwards several times, that they do not open and close properly. The trouble may be in the levers or toggles that throw them, and the adjustments may be properly made without disturbing the valves. In many duplex pumps, there are very slight differences between the two sides, and the amount of lost motion between the valve stem and the valve should be carefully adjusted. Too little lost motion will cause short stroking, while too much will allow the pistons to strike the heads. The adjustment requires skill.

Sometimes, the valve seat or the valve has soft spots that wear faster than the rest of the valve and seat. Through these slight depressions, steam will blow and cut both valve and seat if attention is not given them; back pressure will then seriously interfere with the working of the pump. If the defect is in the valve, a new one can take its place; but the valve seat, if a part of the steam cylinder, will require an entirely new cylinder, and hence it is economy to scrape the seat until the depressions are removed. A try plate made of steel having a perfectly level surface is covered with chalk and carefully rubbed over the valve seat; the elevations will have chalk on them, the depressions will not. The elevations are scraped with a chisel made of the best steel until they are worn down so that chalk sticks to every part of the seat alike. The valve is treated in the same way if it can be done without too much expense. The valve and the valve seat when removable should be sent to the shop to be reground.

The first step after a pump has been erected is to clean out the steam piping. In order that this may be done without carrying foreign matter into the pump, the piping is left disconnected from the pump and steam at full boiler pressure is allowed to blow freely through the piping and valves for a few minutes. Steam is then shut off and the piping is connected to the pump.

The next step is to blow out the steam cylinders. To do this, the cylinder heads should be put on, leaving the pistons and valves out of the cylinders. The stuffingboxes should be closed, which is most conveniently done by placing a piece of board between the stuffingbox and the reversed gland and then setting up the nut on the stuffingbox studs. When the gland is drawn home by a nut outside of it, a circular piece of pine board may be placed between the end of the gland and the inside of the nut in order to close the opening through which the piston rod passes. Steam may now be turned on the main steam pipe leading to the pump; by opening the throttle valve wide at short intervals, the sand and scale in the ports and other passages and spaces of the steam end can be blown out. After the cylinders have been blown out, the heads and covers should be removed and all foreign matter blown into the corners and chambers of the cylinders removed by hand. The pistons, valves, cylinder heads, and other covers can then be put in place. The blowing out of the pipes and cylinders after erection is often neglected or but imperfectly done, with serious consequences to the machine; it cannot be too thoroughly done, particularly in that type of pump where the steam ports and exhaust ports are on top, for in this particular case the sand and grit are deposited in the bottom of the cylinder for the piston to ride on.

The packing of all rods and stems is the next step. If fibrous packing is used, the boxes should be filled full and the glands tightened down very moderately. The tightening of the glands can best be done when steam is on and the machine is in motion, when they should be tightened only sufficiently to stop leakage and no more. When excessive tightening is required to stop leakage, the packing should be completely renewed. Some pumps are fitted with metallic packing; this packing is usually prepared by specialists and fully guaranteed, and their directions for use should be carefully followed. In case of failure or unsatisfactory results, the makers should be consulted.

The oiling of the machinery is the next step and is a very important one. All rubbing surfaces should be provided with suitable oiling devices appropriate to the particular place and service. The quality of oil should be carefully selected to suit the velocity and pressure of the rubbing surfaces on which it is used. For use within the steam cylinder, heavy mineral oil is the only oil

capable of withstanding the high temperature; and when starting up new pumps only the best quality should be used, regardless of price. A liberal use of this oil for the first month will go far toward reducing subsequent oil bills.

The pumping engine must often run continuously or without interruption for a month or even longer. This requires that all oiling devices be so arranged that they can be supplied and adjusted while the machine is in motion. It is a good plan to provide two sets of oiling systems for all the principal journals, so that if one fails the other can be used while the disabled one is being overhauled. All oil holes are generally stopped with wooden plugs or bits of waste twisted into the hole, or are otherwise protected while the machine is being erected. These should now be removed and the oil holes and oil channels thoroughly cleaned. Bearings should be flooded with oil at first to wash out any dust or grit that may have reached the rubbing surfaces.

The steam end is then ready to be warmed up, and it may be mentioned that from now on the method of starting a pump is the same whether the pump is a new one or an old one. To warm up the steam end, the throttle is opened just a little and with the drain cocks opened wide, steam is allowed to blow through the cylinder until no more water passes out of the drain cocks, using the steam by-pass pipes in case of multiple-expansion pumps. If the pump has a valve gear that can be operated by hand, the warming up can be hastened by working the valve back and forth slowly. While the steam end is warming up, the water end should be made ready by opening the stop-valve in the delivery pipe and otherwise seeing to it that the pump has a free delivery. If a stop-valve is fitted to the suction pipe, this should be opened. If the machine is compound or triple expansion, the water by-pass valves must be opened until the machine has made a sufficient number of strokes to bring the intermediate and low-pressure cylinders into action, when the by-pass valves should be closed. If the pump is fitted with dash-relief valves, these should be closed before starting, in order to keep the pistons as far from the heads as possible in starting. Should the pump exhaust into an independent condenser, this should be started and a vacuum obtained before starting the pumps.

To start the pump, open the throttle slowly and let the pistons work back and forth very slowly a few times, gradually increasing the velocity until full speed is attained. After the pump has been running a few minutes, close the drain cocks. If the pump has dash-relief valves, the length of stroke may now be carefully adjusted.

To stop the pump, close the throttle, open the drain cocks, and close the gate valve in the discharge pipe, if one is fitted. Afterwards, shut down the condenser.

## MISCELLANEOUS FORMS OF WATER ELEVATORS

**Jet Pump.**—In the jet pump, the energy of a jet of water is utilized for raising a larger volume through a small distance, or a mixture of water and solid material through a short distance.

**Vacuum Pump.**—The *pulsometer*, which is the most important representative of the vacuum pumps, consists of two chambers in a large casting, with suitable automatic valves arranged at the top and bottom of the chambers. Steam is introduced into one of the chambers, then the valve at the top closed. This steam will condense, forming a vacuum that draws water from the suction into the chamber. When the chamber is filled with water, steam is again introduced and forces the water out through the discharge pipe. The operation is then repeated, more water being drawn in by the condensation of the steam. The two chambers work alternately, one being engaged in drawing water in while the other forces it out. The total steam efficiency of this form of pump is small, though it may actually be above that of small steam pumps employed in isolated portions of a mine. The advantages are that the pump possesses no intricate mechanism nor reciprocating parts, requires no lubrication, and is not injured by gritty or acid materials. On this account it may be employed for pumping water in concentration works, coal-washing plants, and similar places where the water is liable to contain grit or dirt.

**Air-Lift Pumps.**—By introducing compressed air at the bottom of a pipe submerged in any liquid, the air in the pipe rises as bubbles, and so reduces the specific gravity of the fluid in the pipe. This causes the fluid in the pipe to rise above the level of that surrounding the pipe. The difference in specific gravity can never be great, and hence the fluid can never be elevated to any considerable height without having the lower end immersed to a correspondingly great depth. On this account it is frequently necessary to drill a well

considerably below the water-bearing strata, so as to obtain the proper ratio between the submerged portion of the pipe and the height to which the water is to be lifted. Some advantages of this form of pump are that there are no moving parts, no lubrication is required, and gritty material does not interfere with the operation. If the pump is constructed of suitable material, it may be employed for handling acids or solutions in electrolytic or chemical works. This style of pump is also quite extensively employed for pumping water from Artesian wells. It has not been successful as a mine pump, owing to the ratio between the part immersed and the lift.

**Water Buckets.**—Where only a limited amount of water collects in the mine workings, it is frequently removed by means of a special water bucket or water car during the hours that the hoisting engine would otherwise be idle. Where very large amounts of water are to be removed from deep shafts, it has been found economical to do this with special water buckets.

One of the best illustrations of this class of work is the Gilberton water shaft, which has been equipped at the Gilberton Colliery of the Philadelphia and Reading Coal and Iron Co. The collieries draining to this shaft require the removal of 6,000,000 gal. of water per 24 hr. during the wet season, and this has to be lifted from a depth of 1,100 ft. In order to accomplish the work by means of steam pumps, it required a number of pump stations in different parts of the mine, each of which had to be attended by a pumpman, and a large number of steam lines were required in the mine. In order to remove the danger of fire caused by these steam lines, and to dispense with the large amount of labor otherwise necessary, it was decided to hoist the water, and a shaft 22 ft.×26 ft. 8 in. outside of timbers, was sunk. This shaft contains two compartments 7 ft.×7 ft., in which the water buckets are operated, and two compartments 7 ft.×11 ft. 8 in. that are utilized for cages to lower men, timber, and other supplies. The water tanks employed in the special water compartments are 5 ft. 6 in. in diameter, and 14 ft. long. They are provided with special devices, sliding on regular cage guides, and empty themselves automatically at the surface by means of a trip or sliding valve. Two pairs of direct-acting hoisting engines, with 45-in.×60-in. cylinders, operating drums 14 ft. 8 in. in diameter by 15-ft. face, are employed. These operate the water buckets in cages by means of 2-in. crucible steel ropes, at 50 rev. per min., which is equivalent to a piston speed of 500 ft. per min. The drums will hoist two tanks of 2,400 gal. per min. This gives an output of 7,000,000 gal. per 24 hr. By slightly increasing the speed of the engine, this amount can be increased 10%, which is 25% in excess of the calculated maximum demand on the shaft. The cages in the cage compartments are so arranged that they can be disconnected, and water buckets substituted for them. This would be a total output of over 14,000,000 gal. per 24 hr. at the normal speed of the engine. One great advantage of this style of pumping plant is that there is absolutely no fear of drowning the pumps.

The following figures by Mr. F. E. Brackett give the cost of operation and the efficiency of the water hoist at the Coleman shaft, Cambria County, Pennsylvania. The Coleman shaft is about 660 ft. deep and the water hoist consists of two Wellman-Seaver-Morgan boiler-shaped automatic skips of 1,200 gal. capacity each, with the customary valves, etc. "The bailing of the water was carried on intermittently by the main hoisting engine. It was found that bailing from 20 to 30 min. per hr. was sufficient to keep the water down.

"When bailing with one skip, one skip of water was delivered every 75 sec. but by a slight effort a skip could be delivered in 60 sec. When two skips were in use, the time necessary to deliver a skip was from 31 to 38 sec., averaging 34 sec. Of this time, 20 sec. were occupied in hoisting the skip 700 ft. and the remaining 14 sec. were occupied in slowing down and dumping. The actual dumping only occupied about 5 sec. The capacity of the two skip hoists was, therefore, $(1,200 \div 34) \times 60 = 2,120$ gal. per min.

"The amount of coal consumed in hoisting the 800 gal. per min. made by the mine at this time was 23 gross T. per da. of 24 hr. It was estimated that 85% of this, or 19 T., was consumed in hoisting the water. The consumption of steam by the hoisting engine, as computed from its dimensions, was 74 lb. per useful H. P. per hr. As it requires 141 H. P. to hoist 800 gal. of water per min. 700 ft., the amount of steam required was 250,416 lb. per da. Dividing the water by the coal, the efficiency of the boilers on this kind of intermittent work is only 6 lb. of steam per lb. of coal. The duty of the plant, as computed from these data, is about 15,400,000 ft.lb. of work per 100 lb. of coal, which is extremely low."

Mr. Brackett reaches the conclusion of other mining engineers that water hoisting from shallow depths is very uneconomical "as there is very little opportunity to use the steam expansively. Even should an attempt be made to do so, the time occupied in getting up speed, when the steam must be admitted at nearly full stroke, occupies a large percentage, sometimes all, of the total time under steam. In the second place, there is a large amount of power wasted during every winding by the application of brakes to bring the load to rest. Besides these, the intermittent use of steam necessarily interferes with the economical operation of the boilers. Enough steam cannot be raised during the demand for it without wasting fuel during the time there is little or no demand for it. By careful design, especially on windings of greater length, no doubt these losses can be reduced to some extent, but as a general proposition the plan of hoisting water instead of pumping it should not be adopted, unless there exists some other reason that is of greater weight than the economical side of the question."

Some years ago the Hamilton iron mine, in Michigan, was drowned by a sudden inrush of water that drove the pumpmen from the pumps. In order to remove this large volume of water, special bailing buckets were substituted for the ordinary mine skips. These bailing buckets ran on the inclined skip road, and unwatered the mine in a remarkably short time.
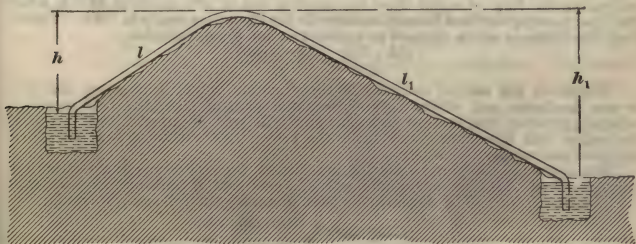


**Siphons.**—The principle on which the siphon works is shown in the accompanying figure. The atmospheric pressure on the surface of the water in the upper basin will force the water up the short leg $l$ to the crown or summit, from whence it will flow by gravity through the long leg $h$ into the lower basin. The vertical height $h$ is called the *lift*, and the vertical fall $h_1$, the *fall* of the siphon. The difference between these heights $(h_1 - h)$ may be called the *siphon head*. The pressure of the atmosphere on the water in each basin acts to keep the pipes $l$, $h$ filled with water.

The conditions required for the successful operation of a siphon may be stated as follows:

1. The height of the summit of the siphon above the surface of the water in the upper basin, or the vertical lift of the siphon, must not exceed a practical limit to which the atmosphere will force the water. This height is theoretically 34 ft. at sea level, but is less at points above the sea level. A safe rule to apply in determining the vertical height to which the atmosphere will force the water in the suction pipe of a pump, or in the short leg of a siphon, is to take .8 of the barometric height, expressed in inches, for the vertical lift of the pump, or siphon, in feet. Thus, if the barometer stands at 30 in., the lift of the pump or siphon, may be taken as $30 \times .8 = 24$ ft. Where the suction pipe is nearly vertical, and consequently of shorter length, this height may be somewhat increased; but where the suction pipe is inclined and its length therefore considerable, the vertical lift should be decreased proportionately.

2. The longer leg, or the discharge pipe of the siphon, must fall through a greater vertical height than the short leg, or draft pipe.

3. When the fall $h_1$ of the siphon exceeds the practical limit to which the atmosphere will force the water, care must be taken to arrange the fall of the siphon relative to its lift, or to increase the length of the long leg, or to use a pipe of a smaller diameter, or to throttle the discharge by means of a valve, so as to prevent the siphon from running empty in a few hours, which it will do whenever the fall is so great that the water in the long leg runs away from the crown faster than the atmospheric pressure forces it up the short leg.

Another important condition requisite to the successful operation of a siphon, is the submerging of both ends of the siphon pipe in their respective basins, in order to prevent air from being drawn into the pipe and reaching the crown, or summit, of the siphon. This takes place more readily in the short leg of the siphon than in the long leg, due to the direction of the flow of the water. The submergence of the discharge end of the siphon, however, is important in order to insure a full flow in the pipe. Air carried into a siphon by the water will gradually accumulate at the highest point of the siphon pipe and will interfere with the working unless an air trap is provided at that point through which the air can be let off from time to time.

In order that a siphon may operate successfully without a throttling valve, its length, diameter, lift, and fall must bear a certain relation to one another, or fulfil certain conditions, without which the pipe has a tendency to empty itself. Referring to the accompanying figure, and calling the head, length, and diameter of pipe, $h$, $l$, and $d$, respectively, on the suction end; and $h_1$, $l_1$, and $d_1$ on the discharge end, as the flow is uniform throughout the pipe,

$$\frac{hd^5}{l} = \frac{h_1 d_1^5}{l_1}$$

But the pressure of the atmosphere acts on both ends; at sea level, this will support a water column of $14.7 \div .434 = 34$ ft., practically. Hence, the head producing the flow of water from the suction end to the crown is $34 - h$, while the head causing the flow from the crown to the discharge end is $h_1 - 34$. Therefore, the formula when applied to siphons at sea level, becomes

$$\frac{(34 - h)d^5}{l} = \frac{(h_1 - 34)d_1^5}{l_1}$$

Whenever the second member of this formula, which represents the flow in the discharge end of the siphon, becomes greater than the first member, which represents the suction end, the pipe will tend to empty itself, because the water will then flow away from the crown faster than the atmospheric pressure can supply the waste.

Whenever the foregoing equation is satisfied, the siphon needs no throttling valve to restrict its flow, although a valve at each end of the siphon is necessary for filling the pipe. The discharge is then given by the following formula,

$$G = 2.83 d^2 \sqrt{\frac{d(h_1 - h)}{f(h_1 + l)}}$$

EXAMPLE.—A siphon pipe 4 in. in diameter and 1,000 ft. long, has a rise of 15 ft. and a fall of 40 ft.; how many gallons of water will it discharge in 1 hr.?

SOLUTION.—Assuming that, for mine work, $f = .01$, and substituting the given values in the formula, $G = 2.83 \times 4^2 \times \sqrt{\frac{4 \times (40 - 15)}{.01 \times 1,000}} = 143+$ gal. per min. The quantity discharged in 1 hr. is then $60 \times 143 = 8,580$ gal.

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# HEAT AND FUELS

*Heat* is a form of energy produced by the rapid vibrations of the molecules of a body. All bodies are assumed to be built up of molecules that are held together by cohesion but yet are in a state of rapid movement in relation to one another. The application of heat to a body causes a more rapid vibration of the molecules, and the withdrawal of heat causes a less rapid vibration thereof; it is to the rate of vibration that the sense of hotness or coldness is due.

**Thermometers.**—Changes in the temperature of a body are commonly measured by a thermometer, although very high temperatures are measured in other ways as by means of a pyrometer, etc. Because of its uniform expansion and its sensitiveness to heat, mercury is commonly used in the construction of thermometers, provided the temperatures to be measured range between, say, $-35°$ F. and $+625°$ F. This is because mercury freezes at about $-38°$ F. and volatilizes at $+675°$ F., beginning to give off some vapor at even a lower temperature. For measuring temperatures below the freezing point of mercury, alcohol is commonly employed although the United States Bureau of Standards prefers toluene.

In all thermometers, the freezing and boiling points of water under mean atmospheric pressure at sea level determine two fixed points, but the division of the scale between these points is made in one of three different ways. In

the Fahrenheit thermometer, which is in universal use in the United States and Great Britain, the boiling point of water is called 212° and the freezing point 32°, the 0° of the scale being 32° below the freezing point and at what was then supposed to be the lowest temperature attainable. In the centigrade thermometer, in use in those countries that employ the metric system and in England and the United States in scientific work, the freezing point of water is called 0° and the boiling point 100°. In the Réaumur thermometer, in use in Russia and in Germany (for domestic purposes), the freezing point of water is called 0° and its boiling point 80°. The following formulas serve to convert the readings of one scale into those of the others:

$$F° = \tfrac{9}{5} C° + 32° = \tfrac{9}{4} R° + 32°$$
$$C° = \tfrac{5}{9}(F° - 32°) = \tfrac{5}{4} R°$$
$$R° = \tfrac{4}{9}(F° - 32°) = \tfrac{4}{5} C°$$

Thus, 1,000° C. is equal to $\tfrac{9}{5} \times 1,000 + 32 = 1,832°$ F., and, smiilarly, 490° F. is equal to $\tfrac{5}{9}(490 - 32) = 254.5°$ C. However, when the relation between a given number of degrees of the scales is desired other formulas must be used. Because between the freezing and boiling points of water there are 100° on the centigrade scale and 180° on the Fahrenheit scale, 1° C. = 1.8° F., and 1° F. = .555° + C. Thus, a range of temperature represented by 1,000° C. is equal to a range of 1,800° F.

### COMPARISON OF THERMOMETER SCALES

|  | Fahrenheit Degrees | Centigrade Degrees | Réaumur Degrees |
|---|---|---|---|
| Absolute zero | −459.64 | −273.13 | −218.51 |
| Zero, Fahrenheit | 0.00 | − 17.78 | − 14.22 |
| Freezing point | 32.00 | 0.00 | 0.00 |
| Maximum density of water | 39.10 | 3.94 | 3.15 |
| Boiling point | 212.00 | 100.00 | 80.00 |

**Absolute Zero.**—At 32° F., a perfect gas expands $\frac{1}{491.64}$ part of its volume if its temperature is increased 1°, the pressure remaining unchanged. Consequently, at $32° + 491.64° = 523.64°$ the gas will occupy double its original volume; and if the temperature is reduced to $491.64° - 32° = -459.64°$, the gas will disappear. Presumably some change in the rate of contraction takes place before the minimum temperature is reached, but the law may conveniently be used within the range of temperature where it is known to hold good. This temperature of −459.64° F. (commonly taken as −460° F.) is known as *absolute zero*. On the centigrade scale, this point is reached at −273.13°, usually taken as −273°. From this, any perfect gas expands $\tfrac{1}{491}$ or $\tfrac{1}{273}$ of its volume for each 1° increase in temperature above that of absolute zero, depending on whether the Fahrenheit or centigrade thermometer is used.

Temperatures reckoned from the absolute zero are known as absolute temperatures. To find the absolute temperature on the Fahrenheit scale, add 460° to the reading of the thermometer. Thus, 62° F. = 460 + 62 = 522° absolute F., and −54° F. = 460 − 54 = 406° absolute F. Similarly, in the centigrade scale, 62° C. = 273 + 62 = 335° absolute C., and −54° C. = 273 − 54 = 219° absolute C.

**British Thermal Unit.**—The quantitative measure of heat in use by English-speaking nations is known as the British thermal unit, commonly written B. T. U. It is the quantity of heat required to raise the temperature of 1 lb. of water 1° at 62° F.; that is, from 62° to 63°. For accurate work, it is necessary to specify the particular degree at which the temperature is measured for the amount of heat required to raise the temperature of 1 lb. of water is not the same for all parts of the thermometric scale. British thermal units were at one time referred to the temperature of the maximum density of water, 39.1° F., but recent practice uses 62° F. as being nearer the mean value.

The heating value of a fuel is commonly expressed in the number of British thermal units per pound of coal, oil, etc., it will yield on burning. Thus, it may be stated that a certain coal has a fuel or heat value of 12,000 B. T. U.

This means that 1 lb. of the fuel when burned under perfect conditions will yield sufficient heat to raise the temperature of 12,000 lb. of water 1° F., or will raise the temperature of 1 lb. of water 12,000° F.

**Calorie.**—The calorie is the equivalent in the metric system of the British thermal unit. It is the amount of heat required to raise the temperature of 1 kilogram of pure water from 15° to 16° C. As in the case of the British thermal unit, the calorie was at one time referred to as the temperature of water at its maximum density, or 3.94° C. The French and English systems are not exactly equivalent or interchangeable because the quantity of heat required to raise the temperature of a given mass of water from 59° F. (15° C.) to 60.8° F. (16° C.) is not exactly the same as that required to raise its temperature from 62° F. to 63° F. This difference, however, is very little, but .03%.

There are two calories in common use. The one just defined, in which the unit weight is the kilogram, is in use commercially and is known as the *large calorie.* Chemists use the *small calorie*, $\frac{1}{1000}$ of the former, being the amount of heat required to raise the temperature of 1 gram of water from 15° C. to 16° C. These are also known as the *kilogram-calorie* and *gram-calorie,* respectively.

**Pound Calorie.**—The pound calorie is the quantity of heat required to raise the temperature of 1 lb. of water from 15° to 16° C. This unit is not infrequently employed in stating the calorific power or heat value of coals and in metallurgical calculations where the weights of the substances involved are given in pounds.

**Equivalence of Heat Units.**—Neglecting the difference in the specific heat of water at different temperatures (as noted before), the relations between the different thermal units may be determined as follows:

$$1 \text{ B. T. U.} = .252 \text{ large cal.} = 252 \text{ small cal.} = .0555 \text{ lb.-cal.}$$
$$1 \text{ large cal.} = 1,000 \text{ small cal.} = 3.968 \text{ B. T. U.} = 2.2046 \text{ lb.-cal.}$$
$$1 \text{ small cal.} = .001 \text{ large cal.} = .003968 \text{ B. T. U.} = .0022046 \text{ lb.-cal.}$$
$$1 \text{ lb.-cal.} = 1.8 \text{ B. T. U.} = .4536 \text{ large cal.} = 453.6 \text{ small cal.}$$

When calories are expressed per kilogram and it is desired to find the equivalent number of British thermal units per pound, the factor for multiplying is 1.8 as will appear from the following fractions, which show the relation between the units of the two systems:

$$\frac{\text{B. T. U. per lb.}}{\text{Cal. per kg.}} = \frac{\text{lb. raised deg. F.}}{\text{kg. raised deg. C.}}$$

As pounds are in each numerator and kilograms are in each denominator they may be canceled, and $\frac{\text{B. T. U.}}{\text{Cal.}} = \frac{\text{Deg. F.}}{\text{Deg. C.}} = \frac{1.8}{1}$, because 1° F. = 1.8° C. From this B. T. U. (per pound) = 1.8 cals. (per kilo).

**Mechanical Equivalent of Heat.**—Heat being a form of energy is capable of performing work. Joule's investigations showed that 1 B. T. U. was equivalent to 772 ft.-lb. of work, but later determinations indicate that the true mechanical equivalent of 1 B. T. U. is 777.52 ft.-lb., which is commonly taken as 778 ft.-lb. Thus, if 1 lb. of Pocahontas coal has a fuel value of 15,000 B. T. U., it is capable of doing 15,000×778 = 11,670,000 ft.-lb. of work, The unit of work in the metric system is the meter-kilogram (often called the kilogram-meter) and is equal to 1 kg. raised through a height of 1 m. One calorie is equal to 426.8028 m.-kg. One B. T. U. = 107.5614 m.-kg., and 1 cal. is equal to 3,087.3531 ft.-lb. The number 778 is called the mechanical equivalent of heat, or, sometimes, Joule's equivalent.

**Expansion by Heat.**—All bodies change in volume as the temperature to which they are subjected is changed; they commonly expand as they are heated and contract as they are cooled. The rate of expansion is commonly expressed as a coefficient, which indicates the relative amount the substance expands in length for an increase of 1° in temperature. The rate of expansion is not the same at all temperatures, as it increases slightly, in the case of metals, as higher temperatures are reached.

In the first table on page 359 are given the coefficients of linear expansion for 1° F. of some of the more common materials. The coefficients of surface expansion are twice the values in the table; and the coefficients of cubic expansion, or expansion in volume, are three times the tabular values. Thus, a bar of wrought iron 60 in. long, if heated from 60° F. to 460° F., or through 400°, will expand 60×.00000677×400 = .16248 in. In a similar way, a sphere of brass measuring 1,000 cu. in. at 32° F., will have its volume increased by 1,000 ×300×(3×.0000104) = 9.36 cu. in. if heated to 332° F.

## EQUIVALENT TEMPERATURES BY THE FAHRENHEIT AND CENTIGRADE THERMOMETERS

| Degrees Fahr. | Degrees Cent. | Degrees Fahr. | Degrees Cent. | Degrees Fahr. | Degrees Cent. | Degrees Fahr. | Degrees Cent. |
|---|---|---|---|---|---|---|---|
| −459.64 | −273.13 | 2 | 16.7 | 62 | 16.7 | 122 | 50.0 |
| 400 | 240.0 | 3 | 16.1 | 63 | 17.2 | 123 | 50.6 |
| 350 | 212.2 | 4 | 15.6 | 64 | 17.8 | 124 | 51.1 |
| 300 | 184.4 | 5 | 15.0 | 65 | 18.3 | 125 | 51.7 |
| 250 | 156.7 | 6 | 14.5 | 66 | 18.9 | 126 | 52.2 |
| 200 | 128.9 | 7 | 13.9 | 67 | 19.5 | 127 | 52.8 |
| 150 | 101.1 | 8 | 13.3 | 68 | 20.0 | 128 | 53.3 |
| 100 | 73.3 | 9 | 12.8 | 69 | 20.6 | 129 | 53.9 |
| 50 | 45.6 | 10 | 12.2 | 70 | 21.1 | 130 | 54.5 |
| 49 | 45.0 | 11 | 11.7 | 71 | 21.7 | 131 | 55.0 |
| 48 | 44.4 | 12 | 11.1 | 72 | 22.2 | 132 | 55.6 |
| 47 | 43.9 | 13 | 10.6 | 73 | 22.8 | 133 | 56.1 |
| 46 | 43.3 | 14 | 10.0 | 74 | 23.3 | 134 | 56.7 |
| 45 | 42.8 | 15 | 9.5 | 75 | 23.9 | 135 | 57.2 |
| 44 | 42.2 | 16 | 8.9 | 76 | 24.5 | 136 | 57.8 |
| 43 | 41.7 | 17 | 8.3 | 77 | 25.0 | 137 | 58.3 |
| 42 | 41.1 | 18 | 7.8 | 78 | 25.6 | 138 | 58.9 |
| 41 | 40.6 | 19 | 7.2 | 79 | 26.1 | 139 | 59.5 |
| 40 | 40.0 | 20 | 6.7 | 80 | 26.7 | 140 | 60.0 |
| 39 | 39.4 | 21 | 6.1 | 81 | 27.2 | 141 | 60.6 |
| 38 | 38.9 | 22 | 5.6 | 82 | 27.8 | 142 | 61.1 |
| 37 | 38.3 | 23 | 5.0 | 83 | 28.3 | 143 | 61.7 |
| 36 | 37.8 | 24 | 4.5 | 84 | 28.9 | 144 | 62.2 |
| 35 | 37.2 | 25 | 3.9 | 85 | 29.5 | 145 | 62.8 |
| 34 | 36.7 | 26 | 3.3 | 86 | 30.0 | 146 | 63.3 |
| 33 | 36.1 | 27 | 2.8 | 87 | 30.6 | 147 | 63.9 |
| 32 | 35.6 | 28 | 2.2 | 88 | 31.1 | 148 | 64.5 |
| 31 | 35.0 | 29 | 1.7 | 89 | 31.7 | 149 | 65.0 |
| 30 | 34.4 | 30 | 1.1 | 90 | 32.2 | 150 | 65.6 |
| 29 | 33.9 | 31 | −.6 | 91 | 32.8 | 151 | 66.1 |
| 28 | 33.3 | 32 | .0 | 92 | 33.3 | 152 | 66.7 |
| 27 | 32.8 | 33 | +.6 | 93 | 33.9 | 153 | 67.2 |
| 26 | 32.2 | 34 | 1.1 | 94 | 34.5 | 154 | 67.8 |
| 25 | 31.7 | 35 | 1.7 | 95 | 35.0 | 155 | 68.3 |
| 24 | 31.1 | 36 | 2.2 | 96 | 35.6 | 156 | 68.9 |
| 23 | 30.6 | 37 | 2.8 | 97 | 36.1 | 157 | 69.5 |
| 22 | 30.0 | 38 | 3.3 | 98 | 36.7 | 158 | 70.0 |
| 21 | 29.4 | 39 | 3.9 | 99 | 37.2 | 159 | 70.6 |
| 20 | 28.9 | 40 | 4.5 | 100 | 37.8 | 160 | 71.1 |
| 19 | 28.3 | 41 | 5.0 | 101 | 38.3 | 161 | 71.7 |
| 18 | 27.8 | 42 | 5.6 | 102 | 38.9 | 162 | 72.2 |
| 17 | 27.2 | 43 | 6.1 | 103 | 39.5 | 163 | 72.8 |
| 16 | 26.7 | 44 | 6.7 | 104 | 40.0 | 164 | 73.3 |
| 15 | 26.1 | 45 | 7.2 | 105 | 40.6 | 165 | 73.9 |
| 14 | 25.6 | 46 | 7.8 | 106 | 41.1 | 166 | 74.5 |
| 13 | 25.0 | 47 | 8.3 | 107 | 41.7 | 167 | 75.0 |
| 12 | 24.4 | 48 | 8.9 | 108 | 42.2 | 168 | 75.6 |
| 11 | 23.9 | 49 | 9.5 | 109 | 42.8 | 169 | 76.1 |
| 10 | 23.3 | 50 | 10.0 | 110 | 43.3 | 170 | 76.7 |
| 9 | 22.8 | 51 | 10.6 | 111 | 43.9 | 171 | 77.2 |
| 8 | 22.2 | 52 | 11.1 | 112 | 44.5 | 172 | 77.8 |
| 7 | 21.7 | 53 | 11.7 | 113 | 45.0 | 173 | 78.3 |
| 6 | 21.1 | 54 | 12.2 | 114 | 45.6 | 174 | 78.9 |
| 5 | 20.6 | 55 | 12.8 | 115 | 46.1 | 175 | 79.5 |
| 4 | 20.0 | 56 | 13.3 | 116 | 46.7 | 176 | 80.0 |
| 3 | 19.4 | 57 | 13.9 | 117 | 47.2 | 177 | 80.6 |
| 2 | 18.9 | 58 | 14.5 | 118 | 47.8 | 178 | 81.1 |
| −1 | 18.3 | 59 | 15.0 | 119 | 48.3 | 179 | 81.7 |
| 0 | 17.78 | 60 | 15.6 | 120 | 48.9 | 180 | 82.2 |
| +1 | 17.2 | 61 | 16.1 | 121 | 49.5 | 181 | 82.8 |

**TABLE**—(*Continued*)

| Degrees Fahr. | Degrees Cent. | Degrees Fahr. | Degrees Cent. | Degrees Fahr. | Degrees Cent. | Degrees Fahr. | Degrees Cent. |
|---|---|---|---|---|---|---|---|
| 182 | 83.3 | 244 | 117.8 | 306 | 152.2 | 368 | 186.7 |
| 183 | 83.9 | 245 | 118.3 | 307 | 152.8 | 369 | 187.2 |
| 184 | 84.5 | 246 | 118.9 | 308 | 153.3 | 370 | 187.8 |
| 185 | 85.0 | 247 | 119.5 | 309 | 153.9 | 371 | 188.3 |
| 186 | 85.6 | 248 | 120.0 | 310 | 154.5 | 372 | 188.9 |
| 187 | 86.1 | 249 | 120.6 | 311 | 155.0 | 373 | 189.5 |
| 188 | 86.7 | 250 | 121.1 | 312 | 155.6 | 374 | 190.0 |
| 189 | 87.2 | 251 | 121.7 | 313 | 156.1 | 375 | 190.6 |
| 190 | 87.8 | 252 | 122.2 | 314 | 156.7 | 376 | 191.1 |
| 191 | 88.3 | 253 | 122.8 | 315 | 157.2 | 377 | 191.7 |
| 192 | 88.9 | 254 | 123.3 | 316 | 157.8 | 378 | 192.2 |
| 193 | 89.5 | 255 | 123.9 | 317 | 158.3 | 379 | 192.8 |
| 194 | 90.0 | 256 | 124.5 | 318 | 158.9 | 380 | 193.3 |
| 195 | 90.6 | 257 | 125.0 | 319 | 159.5 | 381 | 193.9 |
| 196 | 91.1 | 258 | 125.6 | 320 | 160.0 | 382 | 194.5 |
| 197 | 91.7 | 259 | 126.1 | 321 | 160.6 | 383 | 195.0 |
| 198 | 92.2 | 260 | 126.7 | 322 | 161.1 | 384 | 195.6 |
| 199 | 92.8 | 261 | 127.2 | 323 | 161.7 | 385 | 196.1 |
| 200 | 93.3 | 262 | 127.8 | 324 | 162.2 | 386 | 196.7 |
| 201 | 93.9 | 263 | 128.3 | 325 | 162.8 | 387 | 197.2 |
| 202 | 94.5 | 264 | 128.9 | 326 | 163.3 | 388 | 197.8 |
| 203 | 95.0 | 265 | 129.5 | 327 | 163.9 | 389 | 198.3 |
| 204 | 95.6 | 266 | 130.0 | 328 | 164.5 | 390 | 198.9 |
| 205 | 96.1 | 267 | 130.6 | 329 | 165.0 | 391 | 199.5 |
| 206 | 96.7 | 268 | 131.1 | 330 | 165.6 | 392 | 200.0 |
| 207 | 97.2 | 269 | 131.7 | 331 | 166.1 | 393 | 200.6 |
| 208 | 97.8 | 270 | 132.2 | 332 | 166.7 | 394 | 201.1 |
| 209 | 98.3 | 271 | 132.8 | 333 | 167.2 | 395 | 201.7 |
| 210 | 98.9 | 272 | 133.3 | 334 | 167.8 | 396 | 202.2 |
| 211 | 99.5 | 273 | 133.9 | 335 | 168.3 | 397 | 202.8 |
| 212 | 100.0 | 274 | 134.5 | 336 | 168.9 | 398 | 203.3 |
| 213 | 100.6 | 275 | 135.0 | 337 | 169.5 | 399 | 203.9 |
| 214 | 101.1 | 276 | 135.6 | 338 | 170.0 | 400 | 204.5 |
| 215 | 101.7 | 277 | 136.1 | 339 | 170.6 | 401 | 205.0 |
| 216 | 102.2 | 278 | 136.7 | 340 | 171.1 | 402 | 205.6 |
| 217 | 102.8 | 279 | 137.2 | 341 | 171.7 | 403 | 206.1 |
| 218 | 103.3 | 280 | 137.8 | 342 | 172.2 | 404 | 206.7 |
| 219 | 103.9 | 281 | 138.3 | 343 | 172.8 | 405 | 207.2 |
| 220 | 104.5 | 282 | 138.9 | 344 | 173.3 | 406 | 207.8 |
| 221 | 105.0 | 283 | 139.5 | 345 | 173.9 | 407 | 208.3 |
| 222 | 105.6 | 284 | 140.0 | 346 | 174.5 | 408 | 208.9 |
| 223 | 106.1 | 285 | 140.6 | 347 | 175.0 | 409 | 209.5 |
| 224 | 106.7 | 286 | 141.1 | 348 | 175.6 | 410 | 210.0 |
| 225 | 107.2 | 287 | 141.7 | 349 | 176.1 | 411 | 210.6 |
| 226 | 107.8 | 288 | 142.2 | 350 | 176.7 | 412 | 211.1 |
| 227 | 108.3 | 289 | 142.8 | 351 | 177.2 | 413 | 211.7 |
| 228 | 108.9 | 290 | 143.3 | 352 | 177.8 | 414 | 212.2 |
| 229 | 109.5 | 291 | 143.9 | 353 | 178.3 | 415 | 212.8 |
| 230 | 110.0 | 292 | 144.5 | 354 | 178.9 | 416 | 213.3 |
| 231 | 110.6 | 293 | 145.0 | 355 | 179.5 | 417 | 213.9 |
| 232 | 111.1 | 294 | 145.6 | 356 | 180.0 | 418 | 214.5 |
| 233 | 111.7 | 295 | 146.1 | 357 | 180.6 | 419 | 215.0 |
| 234 | 112.2 | 296 | 146.7 | 358 | 181.1 | 420 | 215.6 |
| 235 | 112.8 | 297 | 147.2 | 359 | 181.7 | 421 | 216.1 |
| 236 | 113.3 | 298 | 147.8 | 360 | 182.2 | 422 | 216.7 |
| 237 | 113.9 | 299 | 148.3 | 361 | 182.8 | 423 | 217.2 |
| 238 | 114.4 | 300 | 148.9 | 362 | 183.3 | 424 | 217.8 |
| 239 | 115.0 | 301 | 149.5 | 363 | 183.9 | 425 | 218.3 |
| 240 | 115.6 | 302 | 150.0 | 364 | 184.5 | 426 | 218.9 |
| 241 | 116.1 | 303 | 150.6 | 365 | 185.0 | 427 | 219.5 |
| 242 | 116.7 | 304 | 151.1 | 366 | 185.6 | 428 | 220.0 |
| 243 | 117.2 | 305 | 151.7 | 367 | 186.1 | 429 | 220.6 |

**TABLE**—(*Continued*)

| Degrees Fahr. | Degrees Cent. | Degrees Fahr. | Degrees Cent. | Degrees Fahr. | Degrees Cent. | Degrees Fahr. | Degrees Cent. |
|---|---|---|---|---|---|---|---|
| 430 | 221.1 | 459 | 237.2 | 488 | 253.3 | 925 | 496.1 |
| 431 | 221.7 | 460 | 237.8 | 489 | 253.9 | 950 | 510.0 |
| 432 | 222.2 | 461 | 238.3 | 490 | 254.5 | 975 | 523.9 |
| 433 | 222.8 | 462 | 238.9 | 491 | 255.0 | 1,000 | 537.8 |
| 434 | 223.3 | 463 | 239.5 | 492 | 255.6 | 1,050 | 565.6 |
| 435 | 223.9 | 464 | 240.0 | 493 | 256.1 | 1,100 | 593.3 |
| 436 | 224.5 | 465 | 240.6 | 494 | 256.7 | 1,150 | 621.1 |
| 437 | 225.0 | 466 | 241.1 | 495 | 257.2 | 1,200 | 648.9 |
| 438 | 225.6 | 467 | 241.7 | 496 | 257.8 | 1,250 | 696.7 |
| 439 | 226.1 | 468 | 242.2 | 497 | 258.3 | 1,300 | 704.4 |
| 440 | 226.7 | 469 | 242.8 | 498 | 258.9 | 1,350 | 732.2 |
| 441 | 227.2 | 470 | 243.3 | 499 | 259.5 | 1,400 | 760.0 |
| 442 | 227.8 | 471 | 243.9 | 500 | 260.0 | 1,450 | 787.8 |
| 443 | 228.3 | 472 | 244.4 | 525 | 273.9 | 1,500 | 815.6 |
| 444 | 228.9 | 473 | 245.0 | 550 | 287.8 | 1,600 | 871.1 |
| 445 | 229.5 | 474 | 245.6 | 575 | 301.7 | 1,700 | 926.7 |
| 446 | 230.0 | 475 | 246.1 | 600 | 315.6 | 1,800 | 982.2 |
| 447 | 230.6 | 476 | 246.7 | 625 | 329.4 | 1,900 | 1,036.7 |
| 448 | 231.1 | 477 | 247.2 | 650 | 343.3 | 2,000 | 1,093.3 |
| 449 | 231.7 | 478 | 247.8 | 675 | 357.2 | 2,100 | 1,148.9 |
| 450 | 232.2 | 479 | 248.3 | 700 | 371.1 | 2,200 | 1,204.4 |
| 451 | 232.8 | 480 | 248.9 | 725 | 385.0 | 2,300 | 1,260.0 |
| 452 | 233.3 | 481 | 249.5 | 750 | 398.9 | 2,400 | 1,315.6 |
| 453 | 233.9 | 482 | 250.0 | 775 | 412.8 | 2,500 | 1,371.1 |
| 454 | 234.5 | 483 | 250.6 | 800 | 426.7 | 2,600 | 1,426.7 |
| 455 | 235.0 | 484 | 251.1 | 825 | 440.6 | 2,700 | 1,482.2 |
| 456 | 235.6 | 485 | 251.7 | 850 | 454.4 | 2,800 | 1,537.8 |
| 457 | 236.1 | 486 | 252.2 | 875 | 468.3 | 2,900 | 1,593.3 |
| 458 | 236.7 | 487 | 252.8 | 900 | 482.2 | 3,000 | 1,648.9 |

## EQUIVALENT TEMPERATURES BY THE CENTIGRADE AND FAHRENHEIT THERMOMETERS

| Degrees Cent. | Degrees Fahr. | Degrees Cent. | Degrees Fahr. | Degrees Cent. | Degrees Fahr. | Degrees Cent. | Degrees Fahr. |
|---|---|---|---|---|---|---|---|
| −273.13 | −459.64 | 33 | 27.4 | 7 | 19.4 | 19 | 66.2 |
| 250 | 418.0 | 32 | 25.6 | 6 | 21.2 | 20 | 68.0 |
| 225 | 373.0 | 31 | 23.8 | 5 | 23.0 | 21 | 69.8 |
| 200 | 328.0 | 30 | 22.0 | 4 | 24.8 | 22 | 71.6 |
| 175 | 283.0 | 29 | 20.2 | 3 | 26.6 | 23 | 73.4 |
| 150 | 238.0 | 28 | 18.4 | 2 | 28.4 | 24 | 75.2 |
| 125 | 193.0 | 27 | 16.6 | −1 | 30.2 | 25 | 77.0 |
| 100 | 148.0 | 26 | 14.8 | 0 | 32.0 | 26 | 78.8 |
| 75 | 103.0 | 25 | 13.0 | +1 | 33.8 | 27 | 80.6 |
| 50 | 58.0 | 24 | 11.2 | 2 | 35.6 | 28 | 82.4 |
| 49 | 56.2 | 23 | 9.4 | 3 | 37.4 | 29 | 84.2 |
| 48 | 54.4 | 22 | 7.6 | 4 | 39.2 | 30 | 86.0 |
| 47 | 52.6 | 21 | 5.8 | 5 | 41.0 | 31 | 87.8 |
| 46 | 50.8 | 20 | 4.0 | 6 | 42.8 | 32 | 89.6 |
| 45 | 49.0 | 19 | 2.2 | 7 | 44.6 | 33 | 91.4 |
| 44 | 47.2 | 18 | −.4 | 8 | 46.4 | 34 | 93.2 |
| 43 | 45.4 | 17 | +1.4 | 9 | 48.2 | 35 | 95.0 |
| 42 | 43.6 | 16 | 3.2 | 10 | 50.0 | 36 | 96.8 |
| 41 | 41.8 | 15 | 5.0 | 11 | 51.8 | 37 | 98.6 |
| 40 | 40.0 | 14 | 6.8 | 12 | 53.6 | 38 | 100.4 |
| 39 | 38.2 | 13 | 8.6 | 13 | 55.4 | 39 | 102.2 |
| 38 | 36.4 | 12 | 10.4 | 14 | 57.2 | 40 | 104.0 |
| 37 | 34.6 | 11 | 12.2 | 15 | 59.0 | 41 | 105.8 |
| 36 | 32.8 | 10 | 14.0 | 16 | 60.8 | 42 | 107.6 |
| 35 | 31.0 | 9 | 15.8 | 17 | 62.6 | 43 | 109.4 |
| 34 | 29.2 | 8 | 17.6 | 18 | 64.4 | 44 | 111.2 |

**TABLE**—(*Continued*)

| Degrees Cent. | Degrees Fahr. | Degrees Cent. | Degrees Fahr. | Degrees Cent. | Degrees Fahr. | Degrees Cent. | Degrees Fahr. |
|---|---|---|---|---|---|---|---|
| 45 | 113.0 | 108 | 226.4 | 171 | 339.8 | 234 | 453.2 |
| 46 | 114.8 | 109 | 228.2 | 172 | 341.6 | 235 | 455.0 |
| 47 | 116.6 | 110 | 230.0 | 173 | 343.4 | 236 | 456.8 |
| 48 | 118.4 | 111 | 231.8 | 174 | 345.2 | 237 | 458.6 |
| 49 | 120.2 | 112 | 233.6 | 175 | 347.0 | 238 | 460.4 |
| 50 | 122.0 | 113 | 235.4 | 176 | 348.8 | 239 | 462.2 |
| 51 | 123.8 | 114 | 237.2 | 177 | 350.6 | 240 | 464.0 |
| 52 | 125.6 | 115 | 239.0 | 178 | 352.4 | 241 | 465.8 |
| 53 | 127.4 | 116 | 240.8 | 179 | 354.2 | 242 | 467.6 |
| 54 | 129.2 | 117 | 242.6 | 180 | 356.0 | 243 | 469.4 |
| 55 | 131.0 | 118 | 244.4 | 181 | 357.8 | 244 | 471.2 |
| 56 | 132.8 | 119 | 246.2 | 182 | 359.6 | 245 | 473.0 |
| 57 | 134.6 | 120 | 248.0 | 183 | 361.4 | 246 | 474.8 |
| 58 | 136.4 | 121 | 249.8 | 184 | 363.2 | 247 | 476.6 |
| 59 | 138.2 | 122 | 251.6 | 185 | 365.0 | 248 | 478.4 |
| 60 | 140.0 | 123 | 253.4 | 186 | 366.8 | 249 | 480.2 |
| 61 | 141.8 | 124 | 255.2 | 187 | 368.6 | 250 | 482.0 |
| 62 | 143.6 | 125 | 257.0 | 188 | 370.4 | 251 | 483.8 |
| 63 | 145.4 | 126 | 258.8 | 189 | 372.2 | 252 | 485.6 |
| 64 | 147.2 | 127 | 260.6 | 190 | 374.0 | 253 | 487.4 |
| 65 | 149.0 | 128 | 262.4 | 191 | 375.8 | 254 | 489.2 |
| 66 | 150.8 | 129 | 264.2 | 192 | 377.6 | 255 | 491.0 |
| 67 | 152.6 | 130 | 266.0 | 193 | 379.4 | 256 | 492.8 |
| 68 | 154.4 | 131 | 267.8 | 194 | 381.2 | 257 | 494.6 |
| 69 | 156.2 | 132 | 269.6 | 195 | 383.0 | 258 | 496.4 |
| 70 | 158.0 | 133 | 271.4 | 196 | 384.8 | 259 | 498.2 |
| 71 | 159.8 | 134 | 273.2 | 197 | 386.6 | 260 | 500.0 |
| 72 | 161.6 | 135 | 275.0 | 198 | 388.4 | 275 | 527.0 |
| 73 | 163.4 | 136 | 276.8 | 199 | 390.2 | 300 | 572.0 |
| 74 | 165.2 | 137 | 278.6 | 200 | 392.0 | 325 | 617.0 |
| 75 | 167.0 | 138 | 280.4 | 201 | 393.8 | 350 | 662.0 |
| 76 | 168.8 | 139 | 282.2 | 202 | 395.6 | 375 | 707.0 |
| 77 | 170.6 | 140 | 284.0 | 203 | 397.4 | 400 | 752.0 |
| 78 | 172.4 | 141 | 285.8 | 204 | 399.2 | 425 | 797.0 |
| 79 | 174.2 | 142 | 287.6 | 205 | 401.0 | 450 | 842.0 |
| 80 | 176.0 | 143 | 289.4 | 206 | 402.8 | 475 | 887.0 |
| 81 | 177.8 | 144 | 291.2 | 207 | 404.6 | 500 | 932.0 |
| 82 | 179.6 | 145 | 293.0 | 208 | 406.4 | 550 | 1,022.0 |
| 83 | 181.4 | 146 | 294.8 | 209 | 408.2 | 600 | 1,112.0 |
| 84 | 183.2 | 147 | 296.6 | 210 | 410.0 | 650 | 1,202.0 |
| 85 | 185.0 | 148 | 298.4 | 211 | 411.8 | 700 | 1,292.0 |
| 86 | 186.8 | 149 | 300.2 | 212 | 413.6 | 750 | 1,382.0 |
| 87 | 188.6 | 150 | 302.0 | 213 | 415.4 | 800 | 1,472.0 |
| 88 | 190.4 | 151 | 303.8 | 214 | 417.2 | 850 | 1,562.0 |
| 89 | 192.2 | 152 | 305.6 | 215 | 419.0 | 900 | 1,652.0 |
| 90 | 194.0 | 153 | 307.4 | 216 · | 420.8 | 950 | 1,742.0 |
| 91 | 195.8 | 154 | 309.2 | 217 | 422.6 | 1,000 | 1,832.0 |
| 92 | 197.6 | 155 | 311.0 | 218 | 424.4 | 1,100 | 2,012.0 |
| 93 | 199.4 | 156 | 312.8 | 219 | 426.2 | 1,200 | 2,292.0 |
| 94 | 201.2 | 157 | 314.6 | 220 | 428.0 | 1,300 | 2,372.0 |
| 95 | 203.0 | 158 | 316.4 | 221 | 429.8 | 1,400 | 2,552.0 |
| 96 | 204.8 | 159 | 318.2 | 222 | 431.6 | 1,500 | 2,732.0 |
| 97 | 206.6 | 160 | 320.0 | 223 | 433.4 | 1,600 | 2,912.0 |
| 98 | 208.4 | 161 | 321.8 | 224 | 435.2 | 1,700 | 3,092.0 |
| 99 | 210.2 | 162 | 323.6 | 225 | 437.0 | 1,800 | 3,272.0 |
| 100 | 212.0 | 163 | 325.4 | 226 | 438.8 | 1,900 | 3,452.0 |
| 101 | 213.8 | 164 | 327.2 | 227 | 440.6 | 2,000 | 3,632.0 |
| 102 | 215.6 | 165 | 329.0 | 228 | 442.4 | 2,100 | 3,812.0 |
| 103 | 217.4 | 166 | 330.8 | 229 | 444.2 | 2,200 | 3,992.0 |
| 104 | 219.2 | 167 | 332.6 | 230 | 446.0 | 2,300 | 4,172.0 |
| 105 | 221.0 | 168 | 334.4 | 231 | 447.8 | 2,400 | 4,352.0 |
| 106 | 222.8 | 169 | 336.2 | 232 | 449.6 | 2,500 | 4,532.0 |
| 107 | 224.6 | 170 | 338.0 | 233 | 451.4 | 2,600 | 4,712.0 |

## COEFFICIENTS OF LINEAR EXPANSION PER 1° F.

| Substance | Coefficient | Substance | Coefficient |
|---|---|---|---|
| Aluminum | .00001140 | Marble | .00000400 |
| Brass | .00001040 | Masonry { from | .00000206 |
| Brick | .00000306 | { to | .00000409 |
| Cement, concrete { from | .00000550 | Mercury | .00003334 |
| { to | .00000780 | Platinum | .00000494 |
| Copper | .00000961 | Porcelain | .00000200 |
| Glass { from | .00000399 | Sandstone { from | .00000400 |
| { to | .00000521 | { to | .00000670 |
| Gold | .00000841 | Steel, untempered | .00000599 |
| Granite | .00000460 | Steel, tempered | .00000702 |
| Iron, cast | .00000587 | Tin | .00001160 |
| Iron, wrought | .00000677 | Wood, pine | .00000276 |
| Lead | .00001580 | Zinc | .00001634 |

It should be noted that if the length of a bar that has been heated is determined, and then its length is found after its temperature has been reduced by the amount it has been heated, the value obtained for the contraction in length will not be the same as first obtained for the expansion. Thus, in the case of the wrought iron bar, its final length is 60.16248 in., the expansion from 60° to 460° being .16248 in. If the same bar is cooled from 460° to 60°, the contraction will be 60.16248 × .00000677 × 400 = .16292 in., or .00044 in. more than the expansion from 60° to 460°.

The reason for this difference is purely a mathematical one. If $C$ is the coefficient of expansion and $T_1$, $T_2$, $L_1$, and $L_2$, are, respectively, the initial and final temperatures and the initial and final lengths, the formula becomes,

$$L_2 = \left[\frac{1 + C(T_2 - 32)}{1 + C(T_1 - 32)}\right] L_1$$

**Conduction of Heat.**—The progress of heat from places of higher to places of lower temperature in the same body is called *conduction*. Good conductors are those through which the heat wave moves rapidly; poor conductors are those in which this movement is slow. The relative heat conductivities of some of the metals are given in an accompanying table.

### RELATIVE HEAT CONDUCTIVITIES OF METALS

| Metal | Conductivity | Metal | Conductivity |
|---|---|---|---|
| Silver | 100.0 | Iron | 11.9 |
| Copper | 73.6 | Steel | 11.6 |
| Gold | 53.2 | Lead | 8.5 |
| Aluminum | 31.3 | Platinum | 8.4 |
| Zinc | 28.1 | Bismuth | 1.8 |
| Tin | 15.2 | Mercury | 1.3 |

A non-conductor is a substance that will not conduct heat. No perfectly non-conducting substances are known, although a number of substances are such poor conductors that they are commonly classed as non-conductors. The metals are the best conductors, silver ranking first, while liquids and gases are very poor conductors. Organic substances, as cotton, wool, straw, bran, etc., and rocks and earths, like magnesia, asbestos, etc., are very poor conductors; this quality is taken advantage of in the manufacture of boiler and steam-pipe coverings.

**Radiation of Heat.**—The communication of heat from a hot body to a colder one across an intervening space is called *radiation*. The best example of radiated heat is that received from the sun. The intensity of heat radiation from a given source: Varies as the temperature of the source; varies inversely

as the square of the distance from the source; and grows less as the inclination of the rays to the surface grows less.

The radiating power of heated surfaces depends greatly on the form, shape, and material of which they are composed. Thus, in the case of a cubic vessel, filled with hot water, having its vertical sides coated, respectively, with polished silver, tarnished lead, mica, and lampblack, the radiating power of the sides was experimentally determined to be as 25 : 45 : 80 : 100. From this, bright surfaces radiate less heat than dark ones having the same temperature.

Those bodies or surfaces that reflect heat readily do not absorb it to any extent, and conversely. Thus, lampblack, which reflects few of the heat rays impinging upon it, absorbs nearly all; and polished silver absorbs but about 2.5% of the heat rays, reflecting 97.5%.

**Specific Heat.**—The specific heat of a substance is the ratio between the quantity of heat required to raise or lower the temperature of that substance 1° and the quantity of heat required to raise or lower an equal weight of water 1°. Thus, if the specific heat of lead is .0299, the amount of heat expressed in British thermal units required to raise a certain weight of this metal 1° will raise the same weight of water only .0299 of 1°, or, what is the same thing, as 1 B. T. U. will raise the temperature of 1 lb. of water 1° F., .0299 B. T. U. will raise the temperature of 1 lb. of lead 1° F.

For strict accuracy, the temperature should be noted at which the specific heat is measured, because it has been found that the specific heat is variable for high temperatures. For ordinary temperatures, however, the values given in the accompanying tables may be considered constant. As stated before, the specific heat of water varies with the temperature.

## SPECIFIC HEAT OF WATER AT VARIOUS TEMPERATURES

| Temperature | | Specific Heat | Temperature | | Specific Heat |
|---|---|---|---|---|---|
| Degrees Centigrade | Degrees Fahrenheit | | Degrees Centigrade | Degrees Fahrenheit | |
| 0 | 32 | 1.0094 | 50 | 122 | .9980 |
| 5 | 41 | 1.0053 | 55 | 131 | .9985 |
| 10 | 50 | 1.0023 | 60 | 140 | .9994 |
| 15 | 59 | 1.0003 | 65 | 149 | 1.0004 |
| 16.11 | 61 | 1.0000 | 70 | 158 | 1.0015 |
| 20 | 68 | .9990 | 75 | 167 | 1.0028 |
| 25 | 77 | .9981 | 80 | 176 | 1.0042 |
| 30 | 86 | .9976 | 85 | 185 | 1.0056 |
| 35 | 95 | .9974 | 90 | 194 | 1.0071 |
| 40 | 104 | .9974 | 95 | 203 | 1.0086 |
| 45 | 113 | .9976 | 100 | 212 | 1.0101 |

## SPECIFIC HEATS OF SOLIDS

| Substance | Specific Heat | Substance | Specific Heat |
|---|---|---|---|
| Aluminum | .2143 | Iron, wrought | .1152 |
| Ashes | .2100 | Lead | .0299 |
| Brass | .0883 | Platinum | .0323 |
| Charcoal | .2410 | Steel, soft | .1175 |
| Copper | .0951 | Steel, hard | .1165 |
| Glass | .1937 | Sulphur | .2026 |
| Ice | .5040 | Tin | .0518 |
| Iron, cast | .1189 | Zinc | .0935 |

## SPECIFIC HEATS OF LIQUIDS

| Substance | Specific Heat | Substance | Specific Heat |
|---|---|---|---|
| Alcohol, 32°......... | .5475 | Petroleum........... | .4980 |
| Alcohol, 176°......... | .6794 | Sea water, 64°....... | .9800 |
| Benzene, 50°......... | .4066 | Sulphur, melted...... | .2350 |
| Benzene, 122°........ | .4502 | Sulphuric acid....... | .3363 |
| Glycerine............ | .5760 | Tin, melted......... | .0637 |
| Lead, melted......... | .0410 | Turpentine, Oil of..... | .4110 |
| Mercury, 32°......... | .0335 | | |

In the table of the specific heats of certain gases, it will be noted that two values are given for these. This is because it requires less heat to raise the temperature of a gas when the volume remains constant than when the pressure is constant and the volume varies.

## SPECIFIC HEATS OF GASES

| Substance | Specific Heat at Constant Pressure | Specific Heat at Constant Volume | Substance | Specific Heat at Constant Pressure | Specific Heat at Constant Volume |
|---|---|---|---|---|---|
| Air............... | .2375 | .1690 | Methane.......... | .5929 | .4505 |
| Carbon dioxide..... | .2170 | .1535 | Nitrogen......... | .2438 | .1727 |
| Carbon monoxide.. | .2479 | .1758 | Oxygen.......... | .2175 | .1551 |
| Hydrogen......... | 3.4090 | 2.4123 | Superheated steam | .4805 | .3460 |

EXAMPLE.—Assuming that all the heat of combustion is utilized, how many pounds of cast iron, which has a specific heat of .1189, may be raised from 60° to 260° in temperature, by the burning of 1 lb. of Pocahontas coal with a heating value of 14,500 B. T. U.?

SOLUTION.—The number of British thermal units required to raise the temperature of 1 lb. of cast iron from 60° to 260° is $(260-60) \times .1189 = 23.78$ B. T. U. As 1 lb. of coal yields 14,500 B. T. U., it will raise $14,500 \div 23.78 = 610$ (about) lb. of cast iron from 60° to 260°.

The specific heat of an alloy or of a mixture of gases, etc., is found by multiplying the percentage by weight of each one of the several constituents by its specific heat and dividing the sum of these products by 100.

EXAMPLE 1.—What is the specific heat of an alloy composed of 21% of copper, 40% of tin, and 39% of zinc?

SOLUTION.—

| | | |
|---|---|---|
| Copper | $21 \times .0951 =$ | 1.9971 |
| Tin | $40 \times .0518 =$ | 2.0720 |
| Zinc | $39 \times .0935 =$ | 3.6465 |
| | 100 | 7.7156 |

Hence, the specific heat is $7.7156 \div 100 = .077156$.

EXAMPLE 2.—What is the specific heat of an afterdamp composed of 4.5% $CO_2$, 1.5% $CO$, 80.0% $N$, and 14% $O$, by weight?

SOLUTION.—

| | | |
|---|---|---|
| $N$ | $80.0 \times .2438 =$ | 19.50400 |
| $O$ | $14.0 \times .2175 =$ | 3.04500 |
| $CO_2$ | $4.5 \times .2170 =$ | .97650 |
| $CO$ | $1.5 \times .2479 =$ | .37185 |
| | 100.0 | 23.89735 |

Hence the specific heat is $23.89735 \div 100 = .2389735$, say, 2390.

**Sensible and Latent Heat.**—The heat that serves only to increase the temperature of a body to which it is imparted, and which may be measured by means of a thermometer, is known as *sensible heat*. For illustration, the heat required to raise the temperature of a volume of water from the freezing point at 32° to the boiling point at 212°, or through 180° is sensible heat. In steam engineering, the amount of heat in the water above that at 32° is commonly called the heat of the liquid.

However, heat may be applied to ice at 32° and to water at 212° without increasing the temperature of either. In the one case the ice is changed to water at the same temperature, 32°, and in the other case the water is changed into steam at the same temperature, 212°. In neither case can the heat added be measured by the thermometer until all the ice is converted into water and all the water changed into steam. This heat that is absorbed in changing the state or condition of a body, which does work in overcoming the cohesion of the molecules, is called *latent heat*. This heat is given up when the body resumes its original liquid or solid state.

The heat absorbed in melting a body is called the *latent heat of fusion* and in the case of melting ice into water is equal to 144 B. T. U. The heat absorbed in changing a body from the liquid to the gaseous state is known as the *latent heat of volatilization*. In the case of water this is called the latent heat of *evaporation*, and is commonly taken as being equal to 965.8 B. T. U. Later investigations by Marks and Davis indicate that the true value of the latent heat of evaporation of water is 970.4 B. T. U. Using the former value, the number of British thermal units required to change 1 lb. of ice at 32° F. into steam at 212° F., may be divided as follows:

$$\begin{aligned}
\text{Latent heat of fusion} &= 144.0 \\
\text{Sensible heat from } 32° \text{ to } 212° &= 180.0 \\
\text{Latent heat of evaporation} &= \underline{965.8} \\
&\phantom{=}\ 1,289.8
\end{aligned}$$

Should the ice have been at a temperature below the freezing point there must be added to the 1,289.8 B. T. U. thus obtained the amount of heat required to raise the ice to the melting point. Thus, if the ice is at 0°, the amount of heat required to raise it to 32° will be equal to $(32-0) \times$ specific heat of ice $= 32 \times .504 = 16.13$ B. T. U., and the total heat required to convert 1 lb. of ice at 0° into steam at 212° will be equal to $16.13 + 1,289.8 = 1,305.93$ B. T. U. Should the water have been at, say, 60°, the total heat required to evaporate it into steam at 212° will be:

$$\begin{aligned}
\text{Sensible heat from } 60° \text{ to } 212° &= 152.0 \text{ B. T. U.} \\
\text{Latent heat of evaporation} &= \underline{965.8} \text{ B. T. U.} \\
&\phantom{=}\ 1,117.8 \text{ B. T. U.}
\end{aligned}$$

In those cases where it is necessary to calculate the heat of formation of steam at a higher temperature than 212°, Regnault's formula may be used. This gives the number of heat units required to convert water at 32° into steam at any temperature, $T$.

$$\text{Heat units} = 1,081.4 + .305\,T$$

EXAMPLE 1.—How many British thermal units are required to convert 1 lb. of water at 60° into steam at 400°?

SOLUTION.—From the formula, heat units $= 1,081.4 + (.305 \times 400) = 1,203.4$ B. T. U. But the initial temperature of the water was 60° or $60 - 32 = 28°$ above the freezing point. Inasmuch as a rise of 1° in the temperature of the water requires the expenditure of 1 B. T. U., 28 heat units must be deducted from the total previously obtained, and the required number is $1,203.4 - 28 = 1,175.4$ B. T. U.

EXAMPLE 2.—How many pounds of water at 60° may be evaporated into steam at 400° by the burning of 1 lb. of Pocahontas coal yielding 14,500 B. T. U. per lb.?

SOLUTION.—From example 1, it will require 1,175.4 B. T. U. to convert 1 lb. of water into steam under the assumed conditions. Consequently, 1 lb. of coal will evaporate $14,500 \div 1,175.4 = 12.3$ lb. of water at 60° into steam at 400°.

## MELTING POINTS AND LATENT HEAT OF FUSION OF METALS

| Metal | Fusing Point Degrees F. | Latent Heat Fusion B. T. U. | Metal | Fusing Point Degrees F. | Latent Heat Fusion B. T. U. |
|---|---|---|---|---|---|
| Aluminum | 1,157 | 180.0 | Nickel | 2,642 | 111.6 |
| Copper | 1,985 | 77.9 | Platinum | 3,227 | 49.0 |
| Gold | 1,946 | 29.3 | Silver | 1,764 | 43.8 |
| Iron, wrought | 2,912 | 126.0 | Sulphur | 237 | 16.8 |
| Lead | 619 | 7.2 | Tin | 446 | 24.9 |
| Mercury | −38 | 5.1 | Zinc | 788 | 40.7 |

The boiling point of water decreases as the altitude above sea level increases, as is shown in an accompanying table.

## BOILING POINT OF WATER AT VARIOUS ALTITUDES

| Boiling Point Degrees Fahrenheit | Elevation Above Sea Level Feet | Atmospheric Pressure Pounds per Square Inch | Barometer at 32° F. Inches | Boiling Point Degrees Fahrenheit | Elevation Above Sea Level Feet | Atmospheric Pressure Pounds per Square Inch | Barometer at 32° F. Inches |
|---|---|---|---|---|---|---|---|
| 184 | 15,221 | 8.20 | 16.70 | 199 | 6,843 | 11.29 | 22.99 |
| 185 | 14,649 | 8.38 | 17.06 | 200 | 6,304 | 11.52 | 23.47 |
| 186 | 14,075 | 8.57 | 17.45 | 201 | 5,764 | 11.76 | 23.95 |
| 187 | 13,498 | 8.76 | 17.83 | 202 | 5,225 | 12.01 | 24.45 |
| 188 | 12,934 | 8.95 | 18.22 | 203 | 4,697 | 12.26 | 24.96 |
| 189 | 12,367 | 9.14 | 18.61 | 204 | 4,169 | 12.51 | 25.48 |
| 190 | 11,799 | 9.34 | 19.02 | 205 | 3,642 | 12.77 | 26.00 |
| 191 | 11,243 | 9.54 | 19.43 | 206 | 3,115 | 13.03 | 26.53 |
| 192 | 10,685 | 9.74 | 19.85 | 207 | 2,589 | 13.30 | 27.08 |
| 193 | 10,127 | 9.95 | 20.27 | 208 | 2,063 | 13.57 | 27.63 |
| 194 | 9,579 | 10.17 | 20.71 | 209 | 1,539 | 13.85 | 28.19 |
| 195 | 9,031 | 10.39 | 21.15 | 210 | 1,025 | 14.13 | 28.76 |
| 196 | 8,481 | 10.61 | 21.60 | 211 | 512 | 14.41 | 29.33 |
| 197 | 7,932 | 10.83 | 22.05 | 212 | 0 | 14.70 | 29.92 |
| 198 | 7,381 | 11.06 | 22.57 | | | | |

**Combustion.**—In dealing with the burning of fuels, whether solid, liquid, or gaseous, combustion may be defined as the rapid chemical combination of carbon, hydrogen, and sulphur, or their compounds, with the oxygen of the air, the reaction being accompanied by the production of light and heat. When the combustible or burnable element unites with all the oxygen it is capable of absorbing, the combustion is perfect; otherwise it is imperfect.

The more common chemical reactions in the burning of fuels are here given. The Roman numerals above the symbols employed in expressing the reaction give the relative volumes of the gaseous substances involved, and the Arabic numerals below the reactions are the approximate molecular weights of these substances. The actual weights concerned in the reactions are proportional to the molecular weights.

In the complete combustion of carbon to carbon dioxide, the reaction is

$$\overset{I}{C} + \overset{I}{O_2} = CO_2$$
$$12 + 32 = 44$$
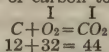
giving 14,544 B. T. U. per lb. of carbon burned.

In the incomplete burning of carbon to carbon monoxide, the reaction is

$$\overset{I}{2C} + \overset{II}{O_2} = 2CO$$
$$24 + 32 = 56$$
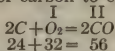
giving 4,450 B. T. U. per lb. of carbon burned.

In the combustion of carbon monoxide to carbon dioxide, the reaction is

$$\overset{II}{2CO} + \overset{I}{O_2} = \overset{II}{2CO_2}$$
$$56 + 32 = 88$$
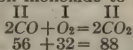
giving 4,325 B. T. U. per lb. and 347 B. T. U. per cu. ft. of $CO$ burned.

In the complete combustion of hydrogen to form water, the reaction is

$$\overset{II}{2H_2} + \overset{I}{O_2} = \overset{II}{2H_2O}$$
$$4 + 32 = 36$$
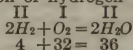
giving 62,028 B. T. U. per lb. and 349 B. T. U. per cu. ft. of hydrogen burned.

In the complete combustion of hydrogen sulphide to carbon dioxide and sulphur dioxide, the reaction is

$$\overset{II}{2H_2S} + \overset{III}{3O_2} = \overset{II}{2H_2O} + \overset{II}{2SO_2}$$
$$68 + 96 = 36 + 128$$
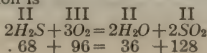
giving by calculation 7,459 B. T. U. per lb. and 709 B. T. U. per cu. ft. of hydrogen sulphide burned.

In the complete combustion of methane to form carbon dioxide and water, the reaction is

$$\overset{I}{CH_4} + \overset{II}{2O_2} = \overset{I}{CO_2} + \overset{II}{2H_2O}$$
$$16 + 64 = 44 + 36$$
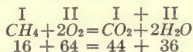
giving 23,513 B. T. U. per lb. and 1,053 B. T. U. per cu. ft. of methane burned.

In the complete combustion of acetylene to form carbon dioxide and water, the reaction is

$$\overset{II}{2C_2H_2} + \overset{V}{5O_2} = \overset{IV}{4CO_2} + \overset{II}{2H_2O}$$
$$52 + 160 = 176 + 36$$
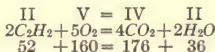
giving 21,465 B. T. U. per lb. and 1,556 B. T. U. per cu. ft. of acetylene burned.

In the complete combustion of olefiant gas to carbon dioxide and water, the reaction is

$$\overset{I}{C_2H_4} + \overset{III}{3O_2} = \overset{II}{2CO_2} + \overset{II}{2H_2O}$$
$$28 + 96 = 88 + 36$$
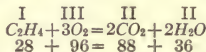
giving 21,344 B. T. U. per lb. and 1,675 B. T. U. per cu. ft. of olefiant gas burned.

In the complete combustion of ethane to carbon dioxide and water, the reaction is

$$\overset{II}{2C_2H_6} + \overset{VII}{7O_2} = \overset{IV}{4CO_2} + \overset{VI}{6H_2O}$$
$$60 + 224 = 176 + 108$$
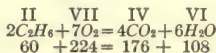
giving 22,230 B. T. U. per lb. and 1,862 B. T. U. per cu. ft. of ethane burned.

In the complete combustion of sulphur to sulphur dioxide, the reaction is

$$\overset{I}{S} + \overset{I}{O_2} = SO_2$$
$$32 + 32 = 64$$
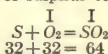
giving 4,050 B. T. U. per lb. of sulphur burned.

The following reactions are important in dealing with fuels, particularly of the gaseous type, as one or all of them are concerned in the manufacture of producer or water gas.

If carbon dioxide is forced through a bed of incandescent coke, it absorbs a certain amount of carbon to form carbon monoxide according to the reaction

$$\overset{I}{CO_2} + C = \overset{II}{2CO}$$
$$44 + 12 = 56$$
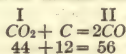
This reaction is not accompanied by the generation of heat but by its absorption at the rate of 10,150 B. T. U. per lb. of carbon burned.

If steam is injected into white-hot coke, the vapor is decomposed into carbon monoxide and hydrogen. The temperature must be very high and the steam supply partial, the reaction being

$$\overset{I}{C} + \overset{I}{H_2O} = \overset{I}{CO} + H_2$$
$$12 + 18 = 28 + 2$$
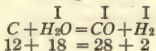
In this case, also, there is an absorption of heat and to the extent of 5,883 B. T. U. per lb. of carbon involved.

If in the last case the steam supply is increased and the temperature lowered, the reaction is

$$\overset{II}{C} + \overset{I}{2H_2O} = \overset{II}{CO_2} + 2H_2$$
$$12 + 36 = 44 + 4$$
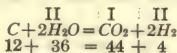
In this case the absorption of heat is 6,066 B. T. U. per lb. of carbon consumed.

When using any of the foregoing equations as a basis for calculating the volumes and weights of the gaseous substances entering into a reaction, it must be remembered that the same is assumed to have taken place at standard temperature and pressure, viz.: 32° F., and 29.92 in. of mercury. Further discussion of the nature and products of combustion will be found near the end of this section and in the section on Mine Ventilation.

# FUELS

## FUELS IN GENERAL

Substances that are burned for the purpose of generating heat for commercial purposes are called *fuels*. As regards their physical state they are divided into solid, liquid, and gaseous fuels. The *solid fuels* include wood, charcoal, coal, peat, coke, sawdust, and other substances of vegetable origin. *Liquid fuels* include petroleum and its derivatives, naphtha, gasoline, and other oils, and grain, wood, and denatured alcohol. *Gaseous fuels* include natural gas and various manufactured gases, such as coal gas, water gas, producer gas, coke-oven gas, blast-furnace gas, etc. The products of combustion, or gases, from beehive coke ovens are frequently used for steam raising, but can hardly be called fuels as they contain no combustible constituents, their value being in their actual, sensible heat.

The chief combustible element in all fuels is carbon, which has a heat value of 14,544 B. T. U. per lb. In most solid fuels, carbon chiefly exists as such, but in the liquid and gaseous fuels and to a less extent in coal, it occurs as a *hydrocarbon*, that is, as a gaseous compound of carbon and hydrogen. The heat value per pound of gas is given under the head of Combustion in the discussion of the combustion of the various hydrocarbon gases.

The second important combustible element is hydrogen, which has a heat value of 62,028 B. T. U. per lb. of gas. It exists in the free, or uncombined, state in natural gas and in certain manufactured fuel gases, such as water gas, but is more commonly present in the form of a hydrocarbon, or combined with oxygen to form water. If both hydrogen and oxygen exist in a fuel, it is assumed that all the oxygen is combined with the hydrogen in the form of water, $H_2O$. The hydrogen thus combined has no fuel value and must therefore be deducted from the total amount of hydrogen present in calculating the heating value of the fuel. The hydrogen left after deducting what is combined with the oxygen is called the *available hydrogen*. As the weight of hydrogen in water is one-eighth the weight of the oxygen, the percentage of available hydrogen is obtained from the formula, $h = H - \dfrac{O}{8}$. in which $h$, $H$, and $O$, are, respectively, the percentages of available hydrogen, total hydrogen, and oxygen, in the fuel.

## WOOD AS FUEL

*Wood* is composed of woody fiber, or cellulose, $C_6H_{10}O_5$, which makes up the chief part of its bulk; the constituents of the sap; and water. The most important of the sap constituents is a soluble gum, *lignine*, amounting, on the average, to 13% of the wood. The cellulose and lignine are both combustible, whereas the water is not only not combustible, but its evaporation absorbs a good portion of the heat generated by the burning of the other constituents. Dry wood is, therefore, a much better fuel than undried wood.

Newly felled wood contains from 25 to 50% of water, the amount varying greatly with different kinds, but averaging about 40%. Exposed to the air at ordinary temperatures, wood loses a large part of its moisture and shrinks, reaching a minimum of about 20% of moisture after about 2 yr. of air drying, but it absorbs water and swells in air highly charged with moisture.

Ordinary air-dried wood may be considered as having the following composition: hygroscopic water, 20%; oxygen and hydrogen in the proportion in which they unite to form water, 40%; and charcoal, including 1% of ash, 40%.

The effective value of all kinds of wood per pound, when dry, is substantially the same, and is commonly estimated at 40% of that of the same weight of average coal. In the accompanying tables are given the weight per cord of air-dried woods arranged in the order of their fuel value per cord, the weight of coal equivalent to 1 cord of air-dried wood, and Gottlieb's values for the composition and calorific value per pound of different varieties of wood.

## WEIGHTS PER CORD OF DRY WOOD ARRANGED ACCORDING TO FUEL VALUES

| Wood | Weight Pounds | Wood | Weight Pounds |
|---|---|---|---|
| Hickory (shell bark).... | 4,469 | Beech................ | 3,126 |
| Hickory (red heart).... | 3,705 | Hard maple........... | 2,878 |
| White oak............ | 3,821 | Southern pine........ | 3,375 |
| Red oak.............. | 3,254 | Virginia pine......... | 2,680 |
| Spruce............... | 2,325 | Yellow pine.......... | 1,904 |
| New Jersey pine....... | 2,137 | White pine........... | 1,868 |

## WEIGHT OF COAL EQUIVALENT TO 1 CORD OF AIR-DRIED WOOD

| Kind of Wood | Weight of 1 Cord Pounds | Weight of Coal Equivalent to 1 Cord of Wood Pounds |
|---|---|---|
| Hickory or hard maple............ | 4,500 | 1,800 to 2,000 |
| White oak........................ | 3,850 | 1,540 to 1,715 |
| Beech, red and black oak.......... | 3,250 | 1,300 to 1,450 |
| Poplar, chestnut, and elm......... | 2,350 | 940 to 1,050 |
| Pine, average.................... | 2,000 | 800 to 925 |

## COMPOSITION AND CALORIFIC VALUE PER POUND OF WOOD
*(Gottlieb)*

| Kind of Wood | Composition | | | | | Calorific Value | |
|---|---|---|---|---|---|---|---|
| | C | H | N | O | Ash | Calories | B. T. U. |
| Oak................ | 50.16 | 6.02 | .09 | 43.36 | .37 | 4,620 | 8,316 |
| Ash................ | 49.18 | 6.27 | .07 | 43.91 | .57 | 4,711 | 8,480 |
| Elm................ | 48.99 | 6.20 | .06 | 44.25 | .50 | 4,728 | 8,510 |
| Beech.............. | 49.06 | 6.11 | .09 | 44.17 | .57 | 4,774 | 8,591 |
| Birch.............. | 48.88 | 6.06 | .10 | 44.67 | .29 | 4,771 | 8,586 |
| Fir................ | 50.36 | 5.92 | .05 | 43.39 | .28 | 5,035 | 9,063 |
| Pine............... | 50.31 | 6.20 | .04 | 43.08 | .37 | 5,085 | 9,153 |

It is safe to assume that from 2.25 to 2.5 lb. of dry wood are equivalent in fuel value to 1 lb. of soft (bituminous) coal of average quality and that, as stated, the fuel value of the same weight of different woods is very nearly the same; that is, 1 lb. of hickory is worth no more for fuel than 1 lb. of pine, assuming both to be dry.

The efficiency of wood fuel depends largely on whether it is wet (as cut) or dry and in a measure as to whether it is fired as cord wood, in 4-ft. lengths, or as sawdust or *hogeed wood*, the latter term being applied to the fine and shredded material produced by running slabs and logs through a macerator or hogging machine. Such refuse may contain as much as 60% of moisture and requires that a large combustion chamber be provided as well as a large area of heated firebrick to radiate heat to the fuel in order to evaporate the water. To secure this extra space, extension furnaces are commonly used, and added room in the firebox may be had by dropping the grate to the level of the boiler-house floor with an ashpit below. When cord wood is fired, extension furnaces are not generally necessary, although the grates should be dropped. Babcock

& Wilcox, in "Steam," state that "with proper draft conditions, 150 lb. of this fuel (sawdust and hogged chips) containing about 30 to 40% of moisture can be burned per square foot of grate surface per hour, and in a properly designed furnace 1 sq. ft. of grate surface can develop from 5 to 6 boiler H. P. Where the wood contains 50% of moisture or over, it is not usually safe to figure on obtaining more than 3 to 4 H. P. per sq. ft. of grate surface."

## PEAT AS FUEL

*Peat*, or as it is sometimes called, *turf*, results from the accumulation, in place, of partly decomposed and disintegrated vegetable matter, chiefly of varieties of moss (sphagnum), where the ordinary decay and decomposition of such material has been more or less suspended, although the form and a considerable part of the structure of the plant organs are more or less destroyed. It is found in bogs and marshes, where periodic overflows or times of saturation by water are favorable to the growth of plant life and the preservation of its remains under water. According to its origin and the conditions under which it has accumulated, peat may vary in color from brown to black. In texture it may vary from light, spongy matter, that is porous, coarse, fibrous, or even woody, and easily falls to pieces when dry, to forms that are nearly or quite devoid of structure, and which, when wet, are as plastic as clay, and when dry form dense, hard masses resembling lignite. In all cases peat is nearly or quite saturated with water, containing, under usual natural conditions, from 80 to 95%.

When dry, peat is generally lighter colored than when freshly dug and will usually float if placed in water, although this is not always true of the dark-colored, plastic kinds that are high in ash and when thoroughly dry are as compact and nearly as hard as coal. Except for such types, raw or untreated peat is easily crumbled to powder when handled, and makes bulky and unsubstantial fuel that does not bear transportation well. The name *muck* is commonly applied to black impure peats of the more completely decomposed types.

It is estimated that the 12,000 sq. mi. of workable peat bogs in the United States contain 13,000,000,000 T. of marketable peat. These deposits are mostly found in the colder and moister sections of the country, in New England and westwards from close to the southern boundary of New York nearly to the ninetieth meridian and thence northwards to Canada. This is supplemented by a narrow strip of bog land extending down the Atlantic coast to Florida, includes all of that state, and reaches westwards, probably across Texas, to the Mexican border. Areas of unknown extent are met along the Pacific coast in California, Oregon, and Washington. The Canadian deposits of peat are estimated to cover 35,000 sq. mi.

Peat is commonly prepared by cutting the material (after the bog has been properly drained) into regular shaped pieces somewhat larger than an ordinary building brick, which are subsequently stacked with air spaces between and dried in the open or under sheds. In some cases, the peat is subjected to a process of grinding or macerating and pressing before being pressed into bricks and is known as machine peat, pressed peat, condensed peat, or machine-formed peat. Peat, after being air-dried, may be ground to a powder, further dried, and pressed into briquettes under a pressure of 18,000 to 30,000 lb. per sq. in. Either charcoal or coke may be made from peat in suitably designed furnaces or retorts. In the producer, air-dried peat yields large volumes of fuel gas of the most excellent quality; this seems the most satisfactory use of using it in manufacturing operations.

According to the United States Geological Survey, freshly dug peat from Bethel, Conn., gave by analysis, moisture, 88.72%; volatile matter, 6.54%; fixed carbon, 3.13%; ash, 1.61%; and sulphur (separately determined), .08%. The calorific value of this freshly dug peat was but 927 B. T. U. per lb.

A consignment of compressed or machine peat from near Orlando, Fla., and tested at the St. Louis Exposition contained by analysis, moisture, 21%; volatile matter, 51.72%; fixed carbon, 22.11%; ash, 5.17%; and sulphur (separately determined), .45%. The ultimate analysis of the same peat was carbon, 46.57%; hydrogen, 6.51%; nitrogen, 2.33%; oxygen, 38.97%; ash, 5.17%; and sulphur, .45%. The calorific value of the peat was 8,127 B. T. U. per lb.

As a general rule ordinary air-dried peat has about one-half the fuel value of bituminous coal, say, from 4,000 to 5,500 B. T. U. per lb.; the calorific value of machined peat being much higher. The cost of machine peat will range between $.75 to $1.50 per T., so that in fuel value it is about equivalent to bituminous coal at $3 per T.

# COAL

## CONSTITUENTS OF COAL

*Coal* consists of the finely comminuted remains of vegetable matter that have been preserved, under water, from complete decay. Whether it has resulted from the accumulation of drift material, as at the mouths of large rivers, or from the growth of trees, shrubs, and mosses in place in bogs, is an undecided question. The accompanying table shows the theoretical change from wood to anthracite.

## CHANGES IN CHEMICAL COMPOSITION FROM WOOD TO ANTHRACITE

| Substance | Carbon Per Cent. | Hydrogen Per Cent. | Oxygen Per Cent. |
|---|---|---|---|
| Woody fiber........................ | 52.65 | 5.25 | 42.10 |
| Peat from Vulcaire.................. | 59.57 | 5.96 | 34.47 |
| Lignite from Cologne................ | 66.04 | 5.27 | 28.69 |
| Earthy brown coal.................. | 73.18 | 5.58 | 21.14 |
| Coal from Belestat, secondary........ | 75.06 | 5.84 | 19.10 |
| Coal from Rive de Gier.............. | 89.29 | 5.05 | 5.66 |
| Anthracite, Mayenne, transition formation | 91.58 | 3.96 | 4.46 |

The chemical elements present in coal are the carbon, hydrogen, oxygen, and nitrogen of the original vegetable matter, together with the ash thereof, and some sulphur and phosphorus. There are other elements and their combinations present, but the ones named are those commonly determined by the chemist. The carbon exists separately and, known as *fixed carbon*, is the chief combustible substance in most coals. A certain amount of the carbon is combined with some of the hydrogen in various gases called hydrocarbons. The nitrogen exists as a gas, and while part of the oxygen may also exist as a gas and a small portion is probably combined with the carbon as carbon dioxide, the bulk of it is combined with hydrogen in the proportions necessary to form water. The water is commonly called *moisture*, and the other gaseous constituents are called *volatile matter, volatile combustible matter,* or *volatile hydrocarbons.*

Chemists report the composition of coal in the form either of a proximate analysis or of an ultimate analysis. In the former, the constituents are reported in the various combinations in which they occur in the coal, as moisture, fixed carbon, volatile matter, and ash, the percentages of which should add up 100. Both the sulphur and phosphorus are *separately determined;* that is, their amounts are not included to make up the 100% of the four chief constituents. In an ultimate analysis, the constituents are determined in their elementary or ultimate form and the percentages of carbon, hydrogen, oxygen, nitrogen, sulphur, phosphorus if determined, and ash should add up 100. Numerous comparisons of proximate and ultimate analyses of the same coals are given in an accompanying table.

*Moisture* in coal consists of two portions, first, *surface moisture,* or that which is on the exterior surface of each lump, and which may be dried off in ordinary dry air; second, the *hygroscopic moisture,* or that which is held by capillary attraction in the pores of the coal and can only be driven out of a lump of coal by heating it considerably above 212° F. The percentage of surface moisture that may be held in a pile of coal depends on the size of the pieces; the smaller the coal, the greater is the amount of moisture that it will hold. Thus, buckwheat anthracite, or slack bituminous coal, after exposure to rain, may hold as much as 8 or 10%.

The amount of hygroscopic moisture depends on the kind of coal; thus, anthracite contains practically none, or less than 1%; semibituminous coal rarely over 1%; bituminous coal from Pennsylvania, between 1 and 2%; from Ohio, about 4%; from Illinois, 8 to 14%; while lignite may contain 20% or more. A sample of Illinois coal originally containing 14% of moisture, and thoroughly dried by heating to from 240° F. to 280° F., reabsorbed the same amount of moisture when exposed to ordinary air for 2 mo.

In addition to representing 20 lb. per T. of worthless material for each 1% present, moisture causes an actual loss of heat from the fact that every pound of water has to be evaporated into steam at 212° and the steam then raised to an uncertain temperature, which is approximately that of the fire and may be from 1,500° to 2,500°. Thus, the water contained in a coal analyzing 10% moisture will be 200 lb. per T. and will require to raise it from, say, 60° to 1,500°, 200×[1,117.8+(1,500−32)×.48]=364,488 B. T. U., which is all the heat generated by the combustion of about 33 lb. of ordinary coal having a calorific value of 11,000 B. T. U. per lb.

*Fixed carbon* is the chief heat-producing constituent of coal, and, the amount of impurities remaining the same, the relative heating values of coals are fairly well determined by a comparison of their content of this substance. Although the fixed carbon of a coal evaporates much less water than an equivalent weight of the volatile hydrocarbons when properly burned, in ordinary practice so much of the latter is lost through careless firing or improper furnace construction, that the relative heating value of a coal may be fairly approximated by assuming that the fixed carbon is the only useful constituent.

*Volatile matter* is that part of the coal that is driven off as a combustible gas when the coal is heated. When a large percentage of volatile matter is present, coals ignite easily and burn with a long yellow flame, and, in ordinary methods of combustion, give off dense smoke. The relative proportions of volatile matter and fixed carbon in a coal, other things being equal, determine its adaptability to any particular purpose, as appears under the Classification of Coals.

The volatile combustible matter in coal consists of carbon, hydrogen, and oxygen in various proportions, differing with the character of the coal. It is found that, with the exception of cannel coal, the larger the percentage of volatile matter in a coal, the greater, usually, is the proportion of oxygen in the volatile matter. It also appears that in the semibituminous coals, after deducting as much of the hydrogen as is needed to form water with the oxygen, that is, one-eighth as much as the oxygen, the remainder, or the available hydrogen, is combined with carbon in about the proportion forming methane, or marsh gas, $CH_4$, or three parts, by weight, of carbon to one part of hydrogen; while in the bituminous coals it is combined in about the proportions of five parts, by weight, of carbon to one part of hydrogen. The low heating values per pound of combustible in coals that are high in volatile matter and in oxygen, are thus accounted for.

The following calculations made on coals having the ultimate analyses given in the table on pages 382 to 385 illustrate the increase in moisture and the decrease in available hydrogen and calorific power of American coals in proceeding westwards from the New River field of West Virginia by way of Pittsburg, Pa., to Illinois.

No. 114, New River, available $H = 4.66 − \dfrac{4.13}{8} = 4.14$; B. T. U. = 14,765

No. 85, Pittsburg, available $H = 4.63 − \dfrac{7.47}{8} = 3.70$; B. T. U. = 13,952

No. 28, Illinois, available $H = 5.44 − \dfrac{18.71}{8} = 3.10$; B. T. U. = 10,719

The *ash* of coal comes partly from that properly belonging to the vegetable matter from which it was formed and partly from sediments washed into the coal swamp during times of flood. Just what proportion of coal ash represents that of the original plant growth will, naturally, depend on the composition of that growth, something impossible to determine, but it would seem that any ash in coal to the extent of more than 2%, is due to extraneous mineral matter. In composition, coal ash approximates that of fireclay, with the addition of ferric oxide, sulphate of lime, magnesia, potash, and phosphoric acid.

White-ash coals are generally freer from sulphur than red-ash coals, which contain iron pyrites, but there are exceptions, as in a certain Peruvian coal, which contains more than 10% of sulphur and yields not a small percentage of white ash.

The fusibility of ash varies according to its composition. It is the more infusible the more nearly its composition approaches fireclay, or silicate of alumina, and becomes more fusible with the addition of other substances, such as iron, lime, etc. Coals high in sulphur usually give a very fusible ash, on account of the iron with which the sulphur is in combination. A fusible ash tends to form a clinker on the grate bars, and therefore is objectionable.

The quantity of ash in different coals as they are sent to market differs greatly according to the quality of the coal itself and the care taken to remove the slate, dirt, etc. that accompany them as they come from the mine. A lump of coal may contain only 5% of ash, while the average of the coal, including slate and dirt as it is mined, may contain 15%. A considerable part of the slate may be removed from the larger sizes by picking after screening, and from the smaller sizes by washing.

In the case of anthracite, the table on page 386 brings out the increase in ash and decrease in calorific power as the sizes grow smaller. The table on page 387 is of value in showing the actual commercial ash present in bituminous coals, many of which, in the case of carefully selected lumps, will yield by analysis less than one-half the ash given in the table.

*Sulphur*, having a calorific value of 4,050 B. T. U. per lb., is always present in coal. While some few coals, notably the Georges Creek-Cumberland semibituminous, occasionally contain but a trace of this element, it is commonly present to the extent of .5 to 5% and even more. Sulphur is generally classed as an impurity in coals because of the corroding action of its fumes upon metals. The terms *low sulphur* and *high sulphur* as applied to coals, are generally relative, although a coal containing less than 1% sulphur would everywhere be placed among the former and one having 5% among the latter. The amount of this element that will render a coal unfit for any particular service is discussed under the Classification of Coals.

Sulphur occurs in coal in at least three forms. It usually occurs in combination with iron as the bisulphide, $FeS_2$, which is known by various names as fool's gold, brasses, etc., and very commonly only as sulphur. In pyrites, sulphur has about one-half the heating value per pound of carbon. The pyrites may occur in minute crystalline grains disseminated through the mass of the coal or segregated in small patches upon the cleavage planes; as sheets or plates, of considerable area in proportion to their thickness, that are parallel to the bedding; as regular layers interstratified with the coal; and as lense-shaped masses, sometimes weighing 100 lb., known as *sulphur balls*. Sulphur balls, when pure, contain 53.33% sulphur and are a source of income at some mines where they are placed in the gob and from time to time gathered up and shipped to the chemical works, where they are employed in the manufacture of sulphuric acid.

Sulphur may be present as a sulphate, usually that of calcium or lime, $CaSO_4$, less commonly as magnesium sulphate, $MgSO_4$, and sometimes as the salt of other metals. The sulphates are commonly found in thin whitish or grayish plates on the vertical joint planes of the coal; in this combination, sulphur has no fuel value.

Sulphur may also be present in combination with the organic constituents of the coal, in which form it is known as *vegetable sulphur* and has a fuel value.

*Phosphorus*, as a natural constituent of the original vegetable matter, is always found in coal. The amount of phosphorus is always very small and does not affect the heating value of the coal, and is only of importance if the coal is destined to make coke for use in furnaces working on Bessemer iron. Phosphorus probably exists as a single or double phosphate of lime and alumina. Mr. Charles Catlett notes, in the Big seam near Columbiana, Ala., the existence of light-colored, resinous grains of evansite, hydrous aluminum phosphate, $Al_6P_2O_{15}.18H_2O$.

Phosphorus is commonly segregated near both the roof and floor of the seam and by rejecting a few inches of the top and bottom coal, the coke made of the remainder may often be brought within the Bessemer limits. Its amount also varies as the workings are extended and analyses of the coal should be made weekly or monthly as the headings advance.

## CLASSIFICATION OF COALS

The most convenient commercial classification of coals is based on the relative amounts of combustible matter therein, both fixed carbon and volatile hydrocarbons, as determined by a proximate analysis; this is shown in the accompanying table.

It must be remembered that this classification is decidedly arbitrary and that the coals of any one group overlap at either end of the scale into those of the other groups. This is particularly true of the lignites, many varieties of which cannot be distinguished from bituminous coal of the better grades if the proximate analysis is made the criterion. Many of the coals of Arkansas are low in volatile matter and may be grouped with either the semianthracite or semibituminous. Similarly, the low-volatile coals of, say, Somerset County

## CLASSIFICATION OF COALS BASED ON THEIR CONTENT OF FIXED CARBON AND VOLATILE MATTER

| Kind of Coal | Fixed Carbon Per Cent. | Volatile Matter Per Cent. |
|---|---|---|
| Anthracite................................... | 97.0 to 92.5 | 3.0 to 7.5 |
| Semianthracite............................... | 92.5 to 87.5 | 7.5 to 12.5 |
| Semibituminous............................... | 87.5 to 75.0 | 12.5 to 25.0 |
| Bituminous, Eastern United States............. | 75.0 to 60.0 | 25.0 to 40.0 |
| Bituminous, Western United States............. | 65.0 to 50.0 | 35.0 to 50.0 |
| Lignite or brown coal........................ | under 50 | over 50 |

and Cambria County, Pa., may be classed as either bituminous or semibituminous.

Owing to this overlapping of the different groups in the scale, a classification based on calorific values is also unsatisfactory. Likewise, geological age fails to furnish a satisfactory basis of classification. As a general rule, the younger the coal geologically, the higher is its content of moisture and volatile matter and, consequently, the lower is it in fixed carbon. Thus, the coals of the Tertiary age are all lignites and commonly of the brown-coal type; the coals of the underlying Cretaceous are commonly black lignites (subbituminous) with some true bituminous coals among them; and the true bituminous, semibituminous, semianthracite, and anthracite coals are almost entirely confined to the true coal measures of the Carboniferous age, in which, the lower volatile coals are found near the base of the series. This rule appears to hold good in regions where the rocks have either remained undisturbed or at best have been but slightly folded, but metamorphism has played an important part in changing the characteristics of seams in mountain districts. The Cretaceous of Colorado is marked by beds of subbituminous lignite and bituminous coals in the level country and foot-hills, which the metamorphism incident to mountain building has altered to anthracite in the midst of the Rockies; therefore, the same seam may present the characteristics of bituminous coal in flat unbroken regions, of semibituminous coal in the foot-hills, and of semianthracite and possibly of true anthracite in the higher and more disturbed mountains.

**Anthracite.**—Anthracite, or *hard coal*, as it is frequently called, is the densest, hardest, and most lustrous of all varieties. It has a conchoidal fracture, frequently displays iridescence, and is characterized both in the lump and in the bed by the absence of cleavage planes. Its specific gravity ranges from 1.3 to 1.8 with an average value of about 1.5. It contains from 3 to 7% of volatile matter, does not coke, is kindled slowly and with difficulty, requires a strong draft through the firebox, and burns with a short almost colorless flame, which is smokeless or essentially so. Practically all the anthracite mined in the United States comes from a small area in northeastern Pennsylvania. A series of analyses of anthracite from that state are given in the table on page 386. These are commercial analyses made on large shipments and show that on account of the uniformly high ash content, the calorific value of anthracite is rather low in comparison with standard bituminous coals. It will also be observed that the smaller sizes (pea, barley, rice, etc.) are much less efficient than the larger broken, egg, and the like. The high content of ash in the smaller sizes is due very largely to imperfect preparation.

An analysis of the so-called anthracite from Cranston, R. I., is No. 92 of the table on pages 382 to 385. The coal is highly graphitic in character and of far more geological interest than commercial value. Its existence has been known for many years, during which time the numerous attempts to exploit it have always ended in failure.

An analysis of anthracite from the Cretaceous seam near Crested Butte, Colo., and from the same formation at Madrid, N. Mex., are given in the table on page 387.

Analyses of anthracite from Alaska will be found in the table on page 390, from New Zealand in the table on page 391, and of the so-called anthracites (more properly semianthracites) from Banff, Alberta, Canada, in the table on page 388.

In the Pocono formation of the Subcarboniferous in the Sleepy Creek and Third Hill Mountains of Berkley and Morgan Counties, W. Va., are found a series of deposits of anthracite that are so irregular in thickness, uncertain in area, and so crushed and otherwise disturbed as to be commercially worthless. Their content of volatile matter is about 10%, which more properly groups them with the semianthracites.

The so-called anthracites of Blacksburg, Montgomery County, W. Va., an analysis of which is No. 105 of the table on pages 382 to 385, are really semi-anthracites and commercially, are of but little more value than those from Sleepy Creek Mountains, W. Va., or Cranston, R. I.

**Semianthracite.**—Semianthracite contains 7.5% to 12.5% of volatile matter and passes by insensible gradations on the one hand into anthracite and on the other hand into semibituminous coal. It is not so hard or dense as true anthracite, is not so lustrous, and, when freshly broken, will leave soot upon the hands, something anthracite will not do. It is somewhat lighter than anthracite, ignites quite readily, and burns more freely than hard coal.

A series of proximate analyses of Pennsylvania semianthracites mined in the northward extension of the anthracite region is given in the table on page 386. The average of eleven analyses of semianthracite from Alaska will be found in the table on page 390, and an analysis of this grade of coal from Banff, Alberta, Canada, is given in the table on page 388. Several proximate analyses of semianthracite from Arkansas, and one from Virginia will be found in the table on page 387, and complete analyses and calorific values of the same in the table on pages 382 to 385.

**Semibituminous Coals.**—The semibituminous coals containing from 12.5 to 25% of volatile matter, and, neglecting the ash and moisture, from 75 to 87.5% of fixed carbon are everywhere the favorites for steam raising. Typical examples are the well-known Pocahontas and New River coals of West Virginia, the Georges Creek-Cumberland coal of Maryland, the Broad Top coals of Pennsylvania, and the Kittanning coals of Cambria, Clearfield, and Somerset Counties in the latter state. All these coals contain about 18% of volatile matter, from 5% to 10% of ash and water and from 70 to 80% of fixed carbon, with sulphur little if any over 1% and have a calorific value of 14,000 B. T. U. per lb., in some cases even more. Some of the coals in this group are fairly hard and blocky, a structure noticeable in those from the Pittsburg seam in the Georges Creek-Cumberland and Broad Top regions, while others are very soft, as typified in the Pocahontas and New River coals. The Kittanning coals of Pennsylvania are intermediate between the Pocahontas and Georges Creek in hardness. Containing more volatile matter than anthracite, these coals kindle more readily and burn more rapidly with a steady fire.

Complete analyses and calorific values of these coals from Arkansas (so-called semianthracite), Pennsylvania, Maryland, West Virginia, and Oklahoma are given in the table on pages 382 to 385. Other, and shorter analyses of these coals from Maryland, Pennsylvania, and West Virginia are given in the table on page 387. Coals of this type from Alaska are noted in the table on page 390, and some foreign and Canadian semibituminous coals are listed in the tables on pages 388 and 389, respectively.

**Bituminous Coals.**—The bituminous coals include about 75% of the output of the mines of the United States. Those produced in the eastern states contain from 25% to a maximum of 40% of volatile matter, or, say, 30 to 32% as an average. Those mined in the central basin and west thereof range as high as 50% in volatile matter, a general average being about 40%. The western bituminous coals are generally characterized by a very much higher content of water than their eastern namesakes, which frequently runs up to 10%, and usually contain more sulphur. These coals vary from hard to soft and from blocky to columnar in structure. Their specific gravity is normally about 1.3. They burn with a yellow flame and much smoke, and, on distillation, yield hydrocarbon oils, tar, etc. According to the use to which they are put or to certain physical and chemical characteristics, they are subdivided into numerous groups.

**Subbituminous Coals.**—The very convenient term subbituminous coals, which originated with Dr. M. R. Campbell, of the United States Geological Survey, is given to that large and valuable group of coals, that possesses some of the undesirable features of the true lignites or brown-coals, together with many of the desirable features of the true bituminous coals. They are sometimes called *black lignites* from their color, which is often highly lustrous and not to be distinguished from that of bituminous coals proper. They have a

brown streak and a specific gravity of 1.22 to 1.25. They burn with a long, bright flame, with considerable smoke like bituminous coals but do not coke. In composition and calorific power, they closely resemble and are, in some cases, even superior to, the true bituminous coals of the Central Basin, as will be seen by comparing the analyses, in the table on pages 382 to 385 of subbituminous coal No. 14, from Lafayette, Colo., or No. 125, from Hanna, Wyo., with analysis No. 28, of bituminous coal from Livingston, Ill., or with No. 35, from Linton, Ind.

The distinction between the two groups lies in their different behavior on weathering. True bituminous coals break down under atmospheric action into smaller and smaller cubes or prisms, the faces of which are more or less parallel to the cleavage planes (butt, face, etc.) of the coal in the bed. As Doctor Campbell remarks, "Exception is to be noted, however, in the case of cannel coal, splint coal, and many forms of block coal, of which the Brazil block of Indiana may be considered the type. Such coals always show cleavage faces on large blocks, but the blocks do not split readily. These coals generally have other characteristics by which they may be identified without recourse to their weathering properties."

On the other hand, subbituminous coal, on weathering, breaks up into irregularly shaped fragments, and, in particular, separates along the bedding planes into plates. This latter peculiarity is the sole distinction between the two groups of coals. To quote further from Doctor Campbell: "In applying these criteria (weathering, etc.) some coals will be classed as bituminous which have a brown streak, are young geologically, and generally have been regarded as lignites or lignitic coals; but they resist the weather, stand shipment well, and have a high calorific value, which makes them to all intents and purposes bituminous coal."

**Lignite.**—Primarily, the distinction between subbituminous coal and lignite is one of color alone; the former is black and the latter brown. As the subbituminous coals have been segregated from the lignites and given a distinctive name, the original term is now confined to the typical lignite, or brown-coal. Lignites are generally inferior as fuels, compared to the subbituminous coals, are usually higher in moisture and volatile matter and lower in fixed carbon, usually show their vegetable origin more plainly, weather more rapidly, and are less well adapted to transportation. It should be noted that many subbituminous coals and lignites, even in the dry climate of the Rocky Mountain region where they are largely mined, will completely disintegrate into slack within 2 to 4 mo.

Bituminous coals, for trade purposes, are subdivided into many groups with distinctive names, depending on the use to which they are put or to which they are best adapted, or depending on some peculiarity of structure or composition. Some of these varieties are here noted.

**Gas Coals.**—The coals suitable for the manufacture of illuminating gas in closed retorts by the destructive distillation of the coal itself without the admission of either air or steam are termed gas coals. These coals yield the original type of gas used for lighting, a type now quite largely superseded by water gas made by forcing steam through incandescent anthracite or coke. Probably the best, and, in any event, the earliest used gas coals in the United States are those mined along the line of the Pennsylvania Railroad near Irwin and along the Baltimore & Ohio Railroad on the Youghiogheny River, in Westmoreland County, Pa. As there produced, these coals commonly contain as much as 37% of volatile matter, from 6 to 8% of ash with considerably less than 1% of sulphur, and being hard and blocky, bear transportation well. Their yield is rather more than 10,000 cu. ft. of gas per T. of coal charged into the retorts, the gas being of 17 to 21 c. p. The residual coke amounts to about 60% of the weight of the original coal and is shiny, fairly strong, and well adapted to domestic purposes or steam raising. Their yield of nitrogenous products, such as ammonia, is also high. These characteristics of Westmoreland coal, particularly the low content of sulphur, are those demanded of standard gas coals.

**Domestic Coals.**—The term domestic, as applied to coal, refers as much to its size as to its composition or other features. In the anthracite consuming sections of the eastern states, the domestic sizes are stove and chestnut adapted to burning in ranges and small heaters, and egg suitable for use in furnaces. In the bituminous regions, there is frequently sold a domestic lump, which may mean a coal specially screened over bars, say, 3 in. apart, or it may refer to coal that will pass over a 3-in. bar screen and through one with bars set, say, 6 in. apart. In any case, coals for domestic use are well screened and

usually over bars much more widely spaced than those used in preparing coal for steam raising. While it is true that domestic coals are classified more by size than anything else, the possession of certain qualities will make one coal more desirable than another for household purposes. The coal that sustains a mild, steady combustion, and remains ignited at a low temperature with a comparatively feeble draft, is the best. A coal burning with a smoky flame is objectionable as producing much soot and dirt, especially for open grates or cooking purposes. For self-feeding stoves, or for base burners, a dry non-coking coal is necessary. A very free and fiercely burning coal is not desirable, particularly in stoves, as the temperature cannot be easily regulated. A sulphurous coal is also bad, as it produces stifling gases with a defective draft, and corrodes the grates and fire-bowls. The difficulty from clinkering is not so great in domestic uses, as the temperature is not generally high enough to fuse the ash. A stony, hard ash that will not pass between the grate bars is bad, and light pulverulent ash is best.

**Blacksmith, or Smithing, Coals.**—A coal suitable for blacksmith purposes should have a high heating power, should contain as much less than $1\%$ of sulphur as possible, should be low in ash, and should coke sufficiently to form a hollow fire, that is, should form an arch on the forge. The semibituminous coals from the New River and Pocahontas regions of West Virginia, that from the Georges Creek-Cumberland district of Maryland, and the Broad Top (Huntingdon County) field of Pennsylvania, make excellent blacksmith coals as mined. The slack of many coking coals, if washed to reduce the content of sulphur (especially) and ash, serve excellently as smithing coal. The best known blacksmith coal of the east and one formerly used for this purpose to the practical exclusion of all others is known by the name of Blossburg from the town of that name in Tioga County, Pa., where it was first mined. An average analysis of this coal (commercial sample) from the Morris Run mines, where the coal is at its best gave moisture, $1.12\%$; volatile matter, $18.57\%$; fixed carbon, $72.10\%$; ash, $7.63\%$; and sulphur, $.583\%$. It will be noted that this is a semibituminous coal essentially the same as those already described.

**Steam Coals**—In the Eastern states and where anthracite is concerned, the term steam refers entirely to the size of the fuel. Thus, if (as noted under that title) the domestic sizes of anthracite are chestnut, stove, and egg, the steam sizes are smaller than these and include pea, buckwheat, barley, rice, etc. The term steam coal is also often used to indicate any coal, anthracite or bituminous, that is too poor to be used for any purpose except steam raising; the idea being that anything that will burn is good enough to find place in the firebox.

For steam making, the superiority of coals high in combustible constituents is admitted, and those with the higher percentage of fixed carbon are the most desirable. But the consideration of the steaming qualities of a coal involves, also, a consideration of the form of furnace and of all the conditions of combustion. The evaporative power of a coal in practice cannot be stated without reference to the conditions of combustion, and every practical test of a coal, to be thorough, should lead to a determination of the best form of furnace for that coal, and should furnish knowledge as to what class of furnaces in actual use such coal is specially adapted. It is not sufficient that in comparative tests of coals the same conditions should exist with each, but there should also be determined the best conditions for each coal.

Of coals high in fixed carbon, the semianthracites and the semibituminous rank as high as the anthracite in meeting the various requirements of a quick and efficient steaming coal.

For railway use, these coals have been found to excel anthracites in evaporating power. The comparative absence, in semibituminous coals, of smoke, which means loss of combustible matter as well as discomfort to the traveler, is sufficient to suggest their superiority over bituminous coals for such use.

Steaming coal should kindle readily and burn quickly but steadily, and should contain only enough volatile matter to insure rapid combustion. It should be low in ash and sulphur, should not clinker, and when it is to be transported should not easily crumble and break.

**Coking Coals.**—Coking coals are those that become pasty or semiviscid in the fire and produce, if the burning or heating is carried on with the partial or entire exclusion of air and the process is not allowed to proceed too far, a hard or porous mass known as *coke*, which consists, essentially, of the fixed carbon and the ash of the original coal, the peculiar structure being due to the escape from the partly melted mass of individual bubbles of gas driven off by the heat.

Commercial coke making is carried on in firebrick structures known as *coke ovens*, which are of two general and very distinct types. The original form of oven and still the chief type used in the United States consists of a hemispherical shell of firebrick into which air sufficient to burn the escaping gases is admitted over the door during the coking process. This is called, from the marked similarity in form, a beehive coke oven. The modern type, called a retort oven, usually consists of a long (18 to 40 ft.) and narrow (18 to 30 in.) rectangular chamber placed like a book on edge, the height being 10, 12, or more ft. In this chamber the coal is heated without access of air, the process being analogous to the production of illuminating gas. Ovens of this type are frequently called *by-product ovens* as they are well adapted to the recovery from the gases of the tar, ammonia, etc., which are commonly wasted in coking in the beehive oven.

Neither anthracite nor semianthracite on one end of the scale will coke, nor will the subbituminous and lignite coals on the other. This limits the possible possession of coking qualities to the semibituminous and bituminous coals, ranging from 12.5 and 50% in volatile matter. In the United States, those semibituminous coals so low in volatile matter as to approximate the semianthracite in composition either will not coke at all or but very indifferently. The same is true at the other end of the scale of the bituminous coals very high in volatile matter, as these coals in the Western states are almost invariably non-coking. The difficulty in making any coke at all, or at best but a very indifferent one, from Western bituminous coals seems to be due to the large amount of water they contain. Probably 90 or 95% of American coke is made from coal containing between 18 and 35% of volatile matter. But all coals falling within these limits will not coke, and the reason why, of two coals having essentially the same analytic composition, one should coke and the other not, is a much disputed question.

Even if a coal will make some kind of a coke, whether this is of value or not depends on the use to which it is to be put. For the manufacture of pig iron in the blast furnace, the coke must be firm and tough and not dense, but with a pronounced cellular structure and a hard cell wall. If the pig iron from the blast furnace is to be used in the manufacture of steel by the Bessemer process, the makers prefer a coke containing from 1 to 3% of volatile matter and moisture combined, 10 to 12% of ash, and 89 to 85% of fixed carbon with sulphur and phosphorus not over 1% and .02%, respectively. This is the composition of the standard so-called Bessemer coke. Unfortunately, very little coke is now obtainable that possesses both this purity and the proper physical qualities. The objection to sulphur in the coke is that it enters the iron and, not being removed in the Bessemer converter, remains in the steel, which it makes *red short* or brittle when hot. Similarly, phosphorus is not removed in this process, but it makes the steel *cold short*, or brittle when cold, a very serious objection in structures subject to shock, as rails. Further, low-phosphorus iron ores are becoming more and more difficult to obtain, so that the lower the content of this impurity in the coke, the higher may it be in the ore. If the pig iron is to be used for making steel by the basic open-hearth process, phosphorus and to a less extent sulphur are not so objectionable in the coke, as they are largely removed in the furnace.

For use in the foundry in the manufacture of castings, the coke should possess the same physical qualities demanded of good blast-furnace coke, but the percentage of both sulphur and phosphorus may be much higher. In fact, many foundry managers prefer a high phosphorus coke, as the presence of this element in the metal makes it very fluid when melted so that it readily fills the smallest openings in the mold. It is generally demanded that coke for foundry purposes shall be capable of melting 10 lb. of iron per pound of fuel. Such a result is not often obtained and in ordinary practice 1 lb. of coke will melt but 8 lb. of iron.

In smelting operations where the metals are recovered as matte (sulphide of iron, etc.) large amounts of sulphur in the coke are not objectionable, as the element is essential to the process.

For domestic use, it is desirable that the coke be hard so that it will bear crushing and screening to the proper sizes and subsequent transportation to market. The amount of sulphur and phosphorus in domestic coke is not of great importance, but the percentage of ash should be as low as is consistent with due strength, and the ash should not clinker.

From the foregoing, it is evident that a coal that will make a coke suitable for one purpose will not make a coke suitable for another. The fusibility of the carbon, the amount of disposable hydrogen, the tenacity with which the

gaseous constituents are held, the amounts of sulphur and phosphorus in the coal, the rapidity and temperature of the coking process, the state of the coal when charged into the oven (whether as run of mine or slack), even the process itself, all affect in one way and another the physical and chemical qualities of the coke and, consequently, the use to which it is best adapted. More or less weathered coal from near the outcrop will not make as good coke as that mined under more cover; many coals that will coke but indifferently as run of mine coke excellently in the fine state; and others, which make a poor showing in the beehive oven, are well adapted to use in retort ovens.

Ordinary analyses do not indicate whether or not a coal is a good coking coal, and they indicate simply by giving the amount of carbon, ash, and sulphur, what will be the probable purity of the coke formed. To produce a standard Bessemer coke in the beehive oven, the typical coal has essentially the analysis of No. 18, of the table on pages 382 to 385, mined from the Pittsburg seam in the Connellsville region of Pennsylvania. Other analyses of this seam show the volatile matter to be as high as 32%, with the fixed carbon, ash, and sulphur as low as 59%, 7%, and .5%, respectively. Coals having analyses that differ materially from this give most excellent cokes that are in every way equal to that from the Connellsville region. As illustrations of this may be cited the cokes made from the semibituminous coals from the New River and Pocahontas fields of West Virginia, which coals contain only about 18% of volatile matter, as well as coke made from the coal mined from the Freeport seam, which often contains over 35% of volatile matter. Coke made from coals containing less than 5 or 6% of ash, while pure, is not generally desirable as blast-furnace fuel, as it lacks the strength to support the burden, and breaking up before burning tends to be blown out the stack by the blast. It must be remembered that coals that will not yield a satisfactory coke from either the chemical or physical view point in the beehive oven, often will do so in the retort oven. Thus, when investigating the coking qualities of the coal from any field carload lots should be shipped to both beehive and retort oven plants for tests under actual working conditions.

**Yield of Coke.**—There are several methods used by field engineers in arriving at an estimate of the quantity and quality of coke that a given coal will produce. It is obvious that if there is no loss of fixed carbon in the process, all this element will be in the coke together with all the ash. In the Connellsville region, it is found that about 40% of the sulphur is volatilized, 60% remaining in the coke. With this understanding, what is called the theoretic coke obtainable from a coal such as No. 85 from Connellsville may be figured as follows:

|  | Original Coal | Remaining in Coke | Analysis of Coke |
|---|---|---|---|
| Moisture | .97 |  |  |
| Volatile matter | 29.09 |  |  |
| Fixed carbon | 60.85 | 60.85 | 87.00 |
| Ash | 9.09 | 9.09 | 13.00 |
|  | 100.00 | 69.94 | 100.00 |
| Sulphur, separate | .90 × .60 | .54 | .77 |

The figures in the third column are arrived at by dividing those in the second column by .6994, which figure is often spoken of as the *theoretic yield* and which means simply, that the coal in question should yield under perfect conditions 69.94% of its weight as coke, which should have the composition given in the third column. These results agree very closely with those obtained in actual practice in the Connellsville region; in fact, there is frequently a greater yield than the theoretic one due to the deposition of carbon in the pores of the coke during the process.

Many engineers assume that under average conditions 1.5 T. of coal are required to make 1 T. of coke, and that there is enough sulphur volatilized to insure that the percentage of this impurity in the coke shall not exceed that in the original coal. Using the foregoing coal, the figures follow:

|  |  | Analysis of Coke Per Cent. |
|---|---|---|
| Ash in coke | 9.09 × 1.5 = | 13.63 |
| Fixed carbon by difference | 100.00 − 13.63 = | 86.37 |
|  |  | 100.00 |
| Sulphur, separate | | .90 |

As the coking process, in the beehive oven at least, is by no means a perfect one, the analysis will be affected by the volatile matter in the coke due to imperfect burning and to the moisture resulting from watering down the charge. The sum of these should not exceed 2% in fairly good practice.

Either of the foregoing methods of estimating the yield and composition of coke give good results if the coal contains enough volatile matter to furnish the necessary heat for the coking process. It fails in the case of the semibituminous coals, as a certain amount of their fixed carbon is burned in supplying the heat for the coking operation. Using the analysis of Pocahontas coal, the theoretic yield of coke is, of course, the sum of the fixed carbon and ash or $73.87 + 5.25 = 79.12\%$, and its composition, using the first method given, should be, fixed carbon, $93.36\%$; ash, $6.64\%$; and sulphur (separately) .48%. But it is found that the yield in the beehive oven from coals of this type is commonly about 63% of the weight of the coal charged. Because all the ash remains in the coke and the fixed carbon alone is consumed, using a ton of 2,000 lb. as a basis, the composition of the coke is as follows:

2,000 lb. of coal yields $2,000 \times .6300 = 1,260$ lb.; fixed carbon + ash
2,000 lb. of coal yields $2,000 \times .0525 = 105$ lb.; ash
2,000 lb. of coal yields $1,260 - 105 = 1,155$ lb.; fixed carbon

Hence, 2,000 lb. of coal will yield 63%, or 1,260 lb., of coke, which contains 1,155 lb., or 91.67%, of fixed carbon and 105 lb., or 8.33%, of ash. The sulphur may be estimated to be from .64 as in the coal to .48% as determined by the first method.

If the coal is coked in a retort oven, the yield of coke will exceed the theoretic, and a fair average may be taken as 78% of the weight of the charge. Using the Connellsville coal mentioned, the yield and composition of coke in a retort oven will be about as follows:

2,000 lb. of coal yields $2,000 \times .7800 = 1,560$ lb.; fixed carbon + ash
2,000 lb. of coal yields $2,000 \times .0909 = 182$ lb.; ash
2,000 lb. of coal yields $1,560 - 182 = 1,378$ lb.; fixed carbon

Hence 2,000 lb. of coal will yield 78%, or 1,560 lb., of coke, which contains 1,378 lb., or 88.33%, of fixed carbon and 182 lb., or 11.67% of ash. It is apparent that coking in a retort oven gives more and better coke than coking in the beehive oven. The increased output is particularly noticeable in coking semibituminous coals.

Of the total coke produced in the beehive oven, between 95 and 96% will be of large size suitable for shipment to any market; 3 to 4% will be fine or small coke (breeze or braise) which may be separated from the ash by screening and sold for domestic use; and about 1% of ashes, which is usually worthless but is sometimes ground and used for foundry facings. The foregoing represents what may be called Connellsville practice; if semibituminous coal is used, the resultant coke is softer and the amount of fine coke and ashes is considerably greater.

**Pishel's Test for Coking Qualities of Coal.**—There has recently been developed, by Mr. Max. A. Pishel, a simple field test for determining the coking or non-coking properties of coals which should have extensive application. As described by Mr. Pishel in the columns of the *Colliery Engineer*, the procedure is: "Pulverize in a mortar a small quantity of the coal to be tested until it will pass through a 100-mesh sieve. Pour out the loose material and note the amount that adheres to the mortar. With some coals, the mortar and pestle will be deeply coated with coal dust that adheres so strongly that it can be removed with difficulty; with other coals, there will be only a thin film of coal dust adhering to the mortar and pestle; while with still others both mortar and pestle will be nearly as clean after the operation is completed as they were before it began. The degree of adhesion depends on the grade of the coal with reference to its coking qualities. If it adheres strongly, the coal will make a good coke; if it adheres only partly, the coal will make an inferior grade of coke; and if it does not adhere, the coal is non-coking." Porcelain, glass, earthenware, iron, or agate mortars may be used, the desideratum being that the material is hard and smooth. The results may be obtained as well with a small mortar that may be carried in the pocket as with a large one.

The structure of the coal in the bed affords some clue to its coking qualities; at least, it has been often noted that where the Pittsburg and Freeport seam coals, as well as those from the Pocahontas and New River districts, make a good coke, the coal shows a distinct columnar structure and tends to break out of the bed in long prisms or fingers.

**Non-Coking Coals.**—The term non-coking coals is applied to those bituminous coals that do not coke even when highly heated but which retain the shape of the original lump until reduced to ashes. There are numerous varieties known by distinctive names, and all are valuable domestic fuels.

**Fat and Dry, or Lean, Coals.**—Fat coals are those that possess a large amount of volatile matter and consequently burn with a long oily flame. Dry, or lean, coals are, obviously, the reverse of fat coals and burn with short flame and little smoke. Cannel coal affords an illustration of the first and Pocahontas of the second of these groups.

**Free-Burning Coal.**—The term free-burning is applied to those coals that burn easily with a light draft. The term is rather loosely used, being applied by some to non-coking coals, by others to semibituminous coals as opposed to anthracite, and by still others to coals containing a larger amount of volatile matter.

**Cannel Coal.**—Cannel coal is a variety of bituminous coal that is very rich in volatile matter, which makes it a very valuable gas coal. It kindles readily and burns with a dense smoky flame. It is compact, with little or no luster and without any appearance of banded structure, breaking with a conchoidal fracture. Its content of volatile is commonly about 50%, its color is dull black to grayish black, and its specific gravity is about 1.23. Certain varieties show what appear to be small concretions about the size of a dime scattered over the surface of a fresh fracture; these are called birds-eye cannel from the fancied resemblance of these structures. The name cannel coal is a corruption of the term *candle coal* given to it because long splints of it readily ignite and burn with a long flame, spitting and sputtering like a candle burning in a draft. It was at one time a popular grate fuel but, although extensive deposits exist, is now little mined.

**Splint Coal.**—Splint coal has a dull black color, and is much harder and less breakable than ordinary bituminous coal. It is readily fissile, like slate, but breaks with difficulty on cross-fracture. It ignites less readily than ordinary bituminous coal, but makes a hot fire, and is a good house coal, although its content of ash is usually high. Both it and cannel coal, while occurring in distinct seams, are very commonly found as a layer or layers interstratified in seams of ordinary coal.

## PROXIMATE ANALYSIS OF COAL

The following is the outline of the method recommended for the proximate analysis of coal by a committee of the American Chemical Society, Messrs. W. F. Hillebrand, C. B. Dudley, W. A. Noyes.

**Sampling.**—At least 5 lb. of coal should be taken for the original sample, with care to secure pieces that represent the average. These should be broken up and quartered down to obtain the smaller sample, which is to be reduced to a fine powder for analysis. The quartering and grinding should be carried out as rapidly as possible, and immediately after the original sample is taken, to prevent gain or loss of moisture. The powdered coal should be kept in a tightly stoppered tube, or bottle, until analyzed. Unless the coal contains less than 2% of moisture, the shipment of large samples in wooden boxes should be avoided.

In boiler tests, shovelfuls of coal should be taken at regular intervals and put in a tight covered barrel, or some air-tight covered receptacle, and the latter should be placed where it is protected from the heat of the furnace.

In sampling a mine, the map of the mine should be carefully examined and points for sampling located in such a manner as to represent fairly the body of the coal. These points should be placed close to the working face. Before sampling, a fresh cut of the face should be made from top to bottom to a depth that will insure the absence of possible changes or of sulphur and smoke from the blasting powders. The floor should be cleaned and a piece of canvas spread to catch the cuttings. Then, with a chisel, a cutting from floor to roof, say 3 in. wide and about 1 in. deep should be made. The shale or other impurities that it is the practice at that mine to reject should not be chiseled out, however. The length of the cutting made should then be measured, but the impurities should not be included in this measurement. With a piece of flat iron and a hammer, all pieces should be broken to ½-in. cubes or less, without removing from the cloth, then quartered, and transferred to a sealed bottle or jar. For the "run-of-mine" sample, samples taken at several points in this manner should be mixed and quartered down. If the vein varies in thickness at different points, the samples taken at each point should correspond in amount to the thickness of the vein. For instance, a

small measure may be filled as many times with the coal of the sample as the vein is feet in thickness. Should there appear differences in the nature of the coal, it will be more satisfactory to take, in addition to the general sample, samples of such portions of the vein as may display these differences.

**Moisture.**—Dry 1 g. of the coal in an open porcelain or platinum crucible at 104° to 107° C. for 1 hr., best in a double-walled bath containing pure toluene. Cool in a desiccator and weigh covered.

**Volatile Combustible Matter.**—Place 1 g. of fresh, undried coal in a platinum crucible weighing 20 to 30 g., and having a tightly fitting cover. Heat over the full flame of a Bunsen burner for 7 min. The crucible should be supported on a platinum triangle with the bottom 6 to 8 cm. above the top of the burner. The flame used should be fully 20 cm. high when burning free, and the determination made in a place free from drafts. The upper surface of the cover should burn clear but the under surface should remain covered with carbon. To find volatile combustible matter, subtract the percentage of moisture from the loss found here.

**Ash.**—Burn the portion of coal used for the determination of moisture at first over a very low flame, with the crucible open and inclined, until free from carbon. If properly treated, this sample can be burned much more quickly than the dense carbon left from the determination of volatile matter.

**Fixed Carbon.**—The fixed carbon is found by subtracting the percentage of ash from the percentage of coke.

**Sulphur (Eschka's Method).**—Mix thoroughly 1 g. of the finely powdered coal with 1 g. of magnesium oxide and $\frac{1}{2}$ g. of dry sodium carbonate, in a thin 75 to 100 c. c. platinum dish or crucible. The magnesium oxide should be light and porous, not a compact, heavy variety. Heat the dish on a triangle over an alcohol lamp, held in the hand at first; gas must not be used, because of the sulphur it contains. Stir the mixture frequently with a platinum wire and raise the heat very slowly, especially with soft coals. Keep the flame in motion and barely touching the dish, at first, until strong glowing has ceased, and then increase gradually until, in 15 min., the bottom of the dish is at a low red heat. When the carbon is burned, transfer the mass to a beaker and rinse the dish, using about 50 c. c. of water. Add 15 c. c. of saturated bromine water and boil for 5 min. Allow to settle, decant through a filter, boil a second and a third time with 30 c. c. of water, and wash until the filtrate gives only a slight opalescence with silver nitrate and nitric acid. The volume of the filtrate should be about 200 c. c. Add 1$\frac{1}{2}$ c. c. of concentrated hydrochloric acid, or a corresponding amount of dilute acid (8. c. c. of an acid of 8%). Boil until the bromine is expelled, and add to the hot solution, drop by drop, especially at first, and with constant stirring, 10 c. c. of a 10% solution of barium chloride. Digest on the water bath, or over a low flame, with occasional stirring until the precipitate settles clear quickly. Filter and wash, using either a Gooch crucible or a paper filter; the latter may be ignited moist in a platinum crucible, using a low flame until the carbon is burned.

In the case of coals containing much pyrites or calcium sulphate, the residue of magnesium oxide should be dissolved in hydrochloric acid and the solution tested for sulphuric acid.

When the sulphur in the coal is in the form of pyrites, that compound is converted almost entirely into ferric oxide in the determination of ash, and, as three atoms of oxygen replace four atoms of sulphur, the weight of the ash is less than the weight of the mineral matter in the coal by five-eighths the weight of the sulphur. While the error from this source is sometimes considerable, a correction for proximate analyses is not recommended. When analyses are to be used as a basis for calculating the heating effect of the coal, a correction should be made.

## FORMS OF REPORTING ANALYSES

The proximate analysis of a coal may be reported in one of several ways.

An analysis designated *as received* refers to the fact that the sample received no preliminary drying before analysis and usually represents the coal exactly as mined or as loaded on the railroad car, etc.

An analysis marked *as fired* refers to the fact that the sample for analysis was taken from the fuel in the boiler room, usually at the time of a test, and represents the coal as fired into the furnace.

An analysis denoted *air dried* refers to the fact that the sample was dried at a uniform temperature, usually the standard one of 62° F., for a number of hours before being analyzed.

In the first two cases, the moisture in the sample is commonly reported as moisture, or water, although the temperature at which the water is driven off is sometimes given. Thus, there are such expressions as "moisture at 212°," or "at 100°," "at 221°," or "at 105°," depending on the temperature and thermometric scale employed.

In the third case, the water is reported in two parts. The first, which represents the difference in the amount of water in the sample as received and after being dried in the laboratory at 62° F. is known as *air drying loss*, or as *loss on drying*. The second part represents the difference between the water in the air-dried sample and that given off on heating at 105° C. This is commonly reported as explained in connection with the first two cases, the temperature usually being stated.

An analysis reported *moisture free*, or as of *dry coal*, is one in which the sum of the percentages of fixed carbon, volatile matter, and ash equal 100. The moisture is reported as a separate item like the sulphur, and usually as, say, "moisture at 105° C."

An analysis reported as *dry and free from ash*, or as *ash and moisture free*, and even, although wrongly, as *pure coal*, is one in which the percentages of fixed carbon and volatile matter equal 100, the moisture and ash being reported separately.

Analyses reported in one form may readily be reduced to another. A proximate analysis made on a sample as received, as fired, or air-dried, may be reduced to a moisture-free basis by dividing each of the constituents, except the moisture, by 100 − moisture. If it is desired to reduce the analysis to a moisture- and ash-free basis, all the constituents except the moisture and ash, which are dropped, are divided by 100 − (moisture+ash). These divisors are applicable to the sulphur and calorific value (British thermal units) as well.

An ultimate analysis may, also, be reduced from the as received to the moisture free, or to the ash- and moisture-free basis by using the same divisors as before, provided the amount of moisture in the coal is known. The ash, sulphur, carbon, and nitrogen are divided in the regular way, but from the hydrogen must be deducted one-ninth of the moisture and from the oxygen, eight-ninths of the moisture.

### COAL 114 FROM SEWELL SEAM, McDONALD, W. VA.

| | | Air Dried | Divisor | Moisture Free | Divisor | Ash and Moisture Free |
|---|---|---|---|---|---|---|
| **Proximate Analysis** | Moisture............... | .68 | | 23.43 | | 24.71 |
| | Volatile matter......... | 23.28 | .9932 | 23.43 | .9418 | 24.71 |
| | Fixed carbon.......... | 70.91 | .9932 | 71.40 | .9418 | 75.29 |
| | Ash................... | 5.14 | .9932 | 5.17 | | |
| | Total............... | 100.00 | | 100.00 | | 100.00 |
| | Sulphur, separate...... | .91 | .9932 | .92 | .9418 | .97 |
| **Ultimate Analysis** | Carbon................ | 83.56 | .9932 | 84.13 | .9418 | 88.71 |
| | Hydrogen (−⅑ moisture)... | 4.66 | .9932 | 4.62 | .9418 | 4.86 |
| | Nitrogen.............. | 1.60 | .9932 | 1.61 | .9418 | 1.70 |
| | Oxygen (−⅛ moisture)..... | 4.13 | .9932 | 3.55 | .9418 | 3.74 |
| | Sulphur............... | .91 | .9932 | .92 | .9418 | .97 |
| | Ash.................. | 5.14 | .9932 | 5.17 | | |
| | Total.............. | 100.00 | | 100.00 | | 100.00 |
| | Calorific power........ | 14,765 | .9932 | 14,866 | .9418 | 15,680 |

The calculation shown in the table on page 380 is made upon a typical coal from the New River field, No. 114 of the table on pages 382 to 385, mined from the Sewell seam, at McDonald, W. Va. The second column contains both the proximate and the ultimate analyses and the calorific value, or power, in British thermal units per pound. The divisor in the third column is obtained by deducting the moisture from 100 and dividing by 100, or divisor $= (100 - .68) \div 100 = .9932$. By dividing all the figures in the second column by this divisor, those in the fourth column are obtained, and these represent the composition of the coal on a moisture-free, or dry-coal basis. The second divisor, that in the fifth column, is obtained by deducting the sum of the moisture and ash from 100 and dividing the result by 100, or divisor $= (100 - .68 - 5.14) \div 100 = .9418$. The figures in the second column (as before) excepting the ash and moisture if divided by this factor will give the figures in the sixth column, which represent the composition of the coal on an ash- and moisture-free, or on the dry-and-free-from-ash basis.

In dealing with the ultimate analysis, before applying either divisor, there must be deducted from the hydrogen and oxygen one-ninth and eight-ninths, respectively, of the moisture (.68) as given in the proximate analysis. The two dividends will be, respectively, $4.66 - (.68 \div 9) = 4.5844$, and $4.13 - (.68 \div 9) \times 8 = 3.5256$.

Should it be desired to give the percentage of moisture on a moisture-free basis, that is, as a separate determination similar to the sulphur, it may be obtained by dividing the moisture in the original sample by the factor in the third column, or moisture, separately determined $= .68 \div .9932 = .684$. If the moisture and ash are desired on the ash- and moisture-free basis, they may be had by dividing the percentage of these constituents in the second column by the divisor in the fifth column, and are, respectively, $.68 \div .9419 = .72$, and $5.14 \div .9419 = 5.45$.

## ANALYSES OF TYPICAL COALS

The table on pages 382 to 385 gives the proximate and the ultimate analyses and the calorific values, in British thermal units per pound, of a number of typical American coals, selected largely from the Reports of the Fuel Testing Plant at the St. Louis Exposition of 1904. Other anaylses are taken from various reports of the United States Geological Survey and the United States Bureau of Mines. Except as noted, all the analyses and heat determinations represent carload lots and were made on samples air-dried at the same temperature. For this reason, the relative values of the different coals are indicated by their calorific power as given in British thermal units. . The exceptions are: The proximate and the ultimate analyses and the heating values of coals 105, 110, and 111 were made upon samples marked "as received"; that is, the preliminary air-drying was omitted, and the analyses indicate more moisture and a lower calorific value in these coals than if they had been air-dried. Also, the ultimate analyses, but not the calorific values, of coals 15, 16, 17, 18, and 19, were made upon samples as received.

In the column headed Kind, A = anthracite; B = bituminous; B. L. = black lignite, the subbituminous coal of the United States Geological Survey; G. A. = graphitic anthracite; L = lignite, the true lignite or brown coal; P = peat; S. A. = semianthracite; S. B. = semibituminous.

In the column headed Grade, L = lump; N = nut; P = pea; R = run of mine; S = slack or culm. L. N. refers to a mixture of lump and nut; S. N. refers to a mixture of slack and nut; and, similarly, for other combinations of the letters.

In the proximate analyses, the sums of the moisture, volatile matter, fixed carbon, and ash should equal 100%. The sulphur is separately determined. In the ultimate analyses, the sums of the carbon, hydrogen, nitrogen, oxygen, sulphur, and ash should equal 100%.

In the table on page 386 is given a series of proximate analyses of Pennsylvania anthracite and semianthracite. Those marked C were made on samples of the coal as received or as fired, and represent a series of monthly or semimonthly shipments, usually extending over a year and in some cases aggregating 15,000 T. in weight. They were made with great care in order to determine the proper price to be paid for the coal, which was sold on analysis. The analyses marked D were made on air-dried samples previously collected in the mine by breaking down a full section of the working face. In all cases, regardless of the origin or preliminary treatment of the sample before analysis, the calorific values were determined on air-dried samples.

In the table on page 387, there is given a series of proximate analyses and calorific values of some important coals not contained in the table on pages

## PROXIMATE AND ULTIMATE ANALYSES AND HEATING VALUES OF AMERICAN COALS
*(United States Geological Survey)*

| No. | State | Nearest Town | Name of Seam | Kind | Grade | Proximate Analysis | | | | Ultimate Analysis | | | | | | B.T.U |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | | | | | | Moisture | V.M. | F.C. | Ash | C | H | N | O | S | Ash | |
| 1 | Ala. | Horse Creek | Horse Creek | B. | L.N. | 1.55 | 32.10 | 53.71 | 12.64 | 72.16 | 4.96 | 1.66 | 7.85 | .73 | 12.64 | 12,958 |
| 2 | Ala. | Carbon Hill | Jagger | B. | L.N.P. | 2.58 | 33.15 | 51.74 | 12.53 | 73.15 | 4.79 | 1.55 | 10.87 | 1.02 | 12.53 | 12,449 |
| 3 | Ala. | Garnsey | Underwood | B. | R. | 2.72 | 20.46 | 53.46 | 14.36 | 69.99 | 4.55 | 1.14 | 9.41 | .55 | 14.36 | 12,461 |
| 4 | Ala. | Belle Ellen | Youngblood | B. | R. | 6.43 | 28.56 | 52.09 | 12.92 | 69.07 | 5.23 | 1.18 | 10.52 | 1.08 | 12.92 | 12,395 |
| 5 | Ala. | Lehigh | Black Creek | B. | R. | 5.51 | 25.05 | 53.28 | 16.08 | 66.58 | 4.70 | 1.27 | 9.92 | 1.40 | 16.08 | 11,932 |
| 6 | Ala. | Dolomite | Pratt | B. | R. | 2.86 | 25.80 | 63.98 | 7.36 | 80.42 | 4.76 | 1.46 | 5.19 | .59 | 7.58 | 14,440 |
| 7 | Ark. | Bonanza | Jenny Lind | S.B. | L. | .74 | 16.26 | 73.66 | 9.34 | 80.03 | 4.13 | 1.40 | 3.52 | 1.90 | 9.34 | 13,961 |
| 8 | Ark. | Denning | Spadra | S.B. | S. | 1.79 | 14.36 | 73.52 | 10.33 | 79.43 | 3.64 | 1.40 | 3.36 | 2.01 | 10.33 | 13,402 |
| 9 | Ark. | Coal Hill | Denning | S.B. | L.S. | 1.28 | 18.82 | 67.69 | 12.21 | 77.29 | 3.74 | 1.39 | 4.70 | 2.01 | 12.21 | 13,406 |
| 10 | Ark. | Huntingdon | Huntingdon | S.B. | L. | 1.17 | 17.83 | 68.12 | 12.88 | 75.68 | 4.00 | 1.47 | 8.07 | 2.02 | 12.88 | 13,410 |
| 11 | Ark. | Midland | Hartshorn | L. | L. | 5.47 | 16.27 | 66.57 | 11.69 | 77.63 | 4.33 | 1.26 | 2.02 | 1.69 | 11.69 | 12,690 |
| 12 | Cal. | Tesla | Tesla | L. | R. | 18.51 | 35.33 | 30.67 | 15.49 | 47.34 | 5.93 | .60 | 27.53 | 3.05 | 15.49 | 8,105 |
| 13 | Cal. | Stone Canyon | Laramie | B.L. | R. | 13.49 | 47.74 | 41.03 | 6.37 | 67.50 | 6.18 | 1.19 | 14.50 | 4.26 | 6.37 | 12,727 |
| 14 | Col. | Lafayette | Sopris | B. | R. | 1.26 | 31.03 | 53.39 | 14.32 | 61.13 | 5.75 | 1.22 | 24.95 | .58 | 6.37 | 10,546 |
| 15 | Col. | Sopris | Walsen | B. | R. | 1.41 | 31.52 | 49.16 | 17.91 | 71.12 | 5.04 | 1.18 | 8.41 | .64 | 14.16 | 12,929 |
| 16 | Col. | Bowen | Cameron | B. | S. | 1.73 | 35.72 | 48.36 | 14.19 | 67.82 | 4.63 | 1.36 | 7.87 | .68 | 17.82 | 12,127 |
| 17 | Col. | Rugby | Berwind | B. | S. | 1.66 | 33.24 | 51.57 | 13.53 | 67.60 | 4.63 | 1.41 | 11.38 | .81 | 13.99 | 12,366 |
| 18 | Col. | Berwind | Yampa | B. | R. | 6.69 | 36.68 | 45.88 | 10.75 | 70.69 | 4.86 | 1.76 | 9.15 | .64 | 13.18 | 13,019 |
| 19 | Col. | Oak Creek | Newcastle | B. | R. | 2.73 | 38.09 | 53.60 | 4.98 | 64.90 | 5.14 | 1.41 | 16.24 | 1.51 | 10.65 | 11,561 |
| 20 | Col. | Newcastle | Little River | B. | R. | 2.73 | 38.09 | 53.60 | 4.98 | 73.45 | 5.05 | 1.76 | 14.22 | .54 | 4.98 | 13,372 |
| 21 | Fla. | Orlando | Peat, air-dried | P. | L. | 21.00 | 51.72 | 22.11 | 5.17 | 36.08 | 5.29 | 2.33 | 38.97 | .45 | 5.17 | 8,127 |
| 22 | Ga. | Menlo | Little River | B. | L.N. | 3.80 | 15.88 | 65.83 | 14.49 | 70.59 | 4.32 | 1.09 | 8.24 | 1.27 | 14.49 | 12,791 |
| 23 | Ill. | O'Fallon | Belleville, No. 6 | B. | R. | 6.28 | 38.94 | 41.08 | 13.72 | 62.01 | 5.09 | 1.27 | 13.86 | 4.25 | 13.72 | 11,448 |
| 24 | Ill. | Marion | Cartersville, No. 7 | B. | R. | 5.96 | 30.29 | 52.16 | 11.59 | 67.30 | 4.92 | 1.43 | 12.99 | 1.77 | 11.59 | 12,103 |
| 25 | Ill. | Coffeen | Pana, No. 5 | B. | R.L. | 5.13 | 32.68 | 47.46 | 14.73 | 60.51 | 4.88 | 1.23 | 17.37 | 4.45 | 14.73 | 11,158 |
| 26 | Ill. | Collinsville | No. 7 | B. | R. | 10.83 | 36.24 | 39.75 | 13.18 | 64.29 | 5.35 | .99 | 15.06 | 4.53 | 13.18 | 10,816 |
| 27 | Ill. | Cartersville | No. 7 | B. | R.L. | 8.86 | 31.25 | 48.23 | 11.66 | 64.29 | 5.24 | 1.29 | 18.71 | 2.46 | 11.66 | 11,902 |
| 28 | Ill. | Livingston | No. 5 | B. | L. | 12.25 | 33.76 | 41.66 | 12.33 | 58.30 | 5.44 | .80 | 18.71 | 1.42 | 12.33 | 10,719 |
| 29 | Ill. | Ziegler | No. 5 | B. | L. | 10.72 | 29.86 | 50.06 | 9.36 | 66.74 | 5.30 | 1.40 | 16.29 | .91 | 9.36 | 11,686 |
| 30 | Ill. | Herrin | No. 7 | B. | L. | 7.78 | 29.85 | 52.39 | 9.98 | 67.45 | 5.06 | 1.21 | 14.98 | 1.32 | 9.98 | 11,959 |

| No. | State | Locality | Bed | | | Moisture | Volatile Matter | Fixed Carbon | Ash | Carbon | Hydrogen | Nitrogen | Oxygen | Sulphur | Ash | B.t.u. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 31 | Ind. | Terre Haute | No. 6 | B.B. | R. | 12.79 | 35.45 | 39.67 | 12.09 | 59.84 | 5.58 | .82 | 18.49 | 3.18 | 12.09 | 10,899 |
| 32 | Ind. | Seelyville | No. 3 | B.B. | R. | 7.88 | 36.85 | 41.07 | 14.20 | 59.75 | 5.22 | .90 | 14.76 | 5.14 | 14.20 | 11,146 |
| 33 | Ind. | Linton | No. 4 | B.B. | R. | 13.58 | 32.07 | 46.20 | 8.15 | 63.53 | 5.65 | 1.42 | 20.34 | .91 | 8.15 | 11,419 |
| 34 | Ind. | Linton | No. 5 | B.B. | R. | 10.30 | 36.31 | 41.64 | 11.75 | 61.00 | 5.38 | 1.06 | 16.58 | 4.23 | 11.75 | 11,218 |
| 35 | Ind. | Brazil | Bottom Bed, Brazil Block | B. | S. N. | 16.91 | 26.85 | 38.87 | 17.37 | 52.97 | 5.48 | 1.01 | 21.28 | 1.89 | 17.37 | 9,524 |
| 36 | Iowa | Maxville | No. 7 | B.B. | L. | 5.24 | 37.83 | 45.73 | 11.20 | 66.48 | 5.19 | 1.12 | 12.46 | 3.56 | 11.20 | 12,085 |
| 37 | Iowa | Laddsdale | Middle | B.B. | L., S. | 5.21 | 31.76 | 46.51 | 16.52 | 61.80 | 4.61 | .97 | 10.90 | 5.20 | 16.52 | 11,392 |
| 38 | Iowa | Marion Co. | Big Vein | B.B. | R. | 4.25 | 37.02 | 41.74 | 16.99 | 60.36 | 4.84 | 1.46 | 11.15 | 5.20 | 16.99 | 11,129 |
| 39 | Iowa | Altoona | Third | B.B. | L. | 4.52 | 40.96 | 38.99 | 15.53 | 60.62 | 4.93 | .93 | 11.16 | 6.83 | 15.53 | 11,356 |
| 40 | Iowa | Centerville | Mystic | B.B. | L. | 10.03 | 37.27 | 41.22 | 11.48 | 61.25 | 5.31 | .94 | 16.56 | 4.46 | 11.48 | 11,227 |
| 41 | Iowa | Chariton | Lower | B.B. | R. | 9.22 | 32.71 | 44.52 | 13.55 | 59.89 | 5.35 | 1.22 | 16.57 | 3.42 | 13.55 | 10,989 |
| 42 | Kans. | Fleming | Lower Weir-Pittsburg | B. | R. | 3.74 | 33.11 | 50.01 | 13.14 | 68.22 | 4.91 | 1.09 | 8.30 | 4.34 | 13.14 | 12,404 |
| 43 | Kans. | Scammon | Weir-Pittsburg | B.B. | R. | 2.50 | 33.80 | 51.25 | 12.45 | 69.07 | 4.91 | 1.20 | 6.69 | 5.68 | 12.45 | 12,900 |
| 44 | Kans. | Jewett | Weir-Pittsburg | B.B. | L. N. | 9.04 | 29.69 | 45.55 | 15.72 | 60.99 | 5.01 | 1.06 | 13.50 | 3.72 | 15.72 | 11,142 |
| 45 | Kans. | West Mineral | Weir-Pittsburg | B.B. | L. | 4.10 | 31.65 | 53.71 | 10.54 | 70.25 | 5.10 | 1.06 | 9.28 | 3.77 | 10.54 | 12,895 |
| 46 | Kans. | Atchison | | B.B. | L. | 3.57 | 37.00 | 46.80 | 12.63 | 65.02 | 5.04 | 1.07 | 13.61 | 2.63 | 12.63 | 12,337 |
| 47 | Ky. | Straight Ck. | Straight Creek | B.B. | L. | 1.92 | 36.56 | 57.08 | 4.44 | 78.31 | 5.36 | 1.85 | 8.80 | 1.24 | 4.44 | 14,319 |
| 48 | Ky. | Earlington | No. 11 | B.B. | L. | 5.36 | 38.99 | 46.27 | 9.38 | 67.16 | 5.43 | 1.23 | 12.68 | 3.72 | 9.38 | 12,539 |
| 49 | Ky. | Barnsley | No. 9 | B.B. | R. | 5.85 | 36.90 | 46.96 | 10.29 | 66.75 | 5.27 | 1.43 | 12.66 | 3.60 | 10.29 | 12,292 |
| 50 | Ky. | Wheatcroft | No. 11 | B.B. | R. | 2.54 | 36.08 | 46.79 | 14.59 | 66.50 | 4.53 | 1.28 | 8.43 | 4.67 | 14.59 | 12,294 |
| 51 | Ky. | Sturges | No. 11 | B.B. | R. | 5.46 | 30.99 | 55.63 | 7.92 | 72.59 | 5.07 | 1.19 | 12.05 | 1.18 | 7.92 | 12,239 |
| 52 | Ky. | McHenry | No. 1B | B.B. | N. | 8.70 | 35.00 | 47.34 | 8.96 | 65.63 | 5.61 | 1.20 | 15.46 | 3.14 | 8.96 | 12,078 |
| 53 | Ky. | Big Black Mt. | High Splint | B.B. | L. | 1.60 | 38.58 | 56.01 | 3.81 | 79.67 | 4.99 | 1.61 | 9.23 | .69 | 3.81 | 14,324 |
| 54 | Ky. | Near Paintsville | No. 1 | B.B. | R. | 5.12 | 36.49 | 55.63 | 2.76 | 77.20 | 5.47 | 1.45 | 12.55 | .57 | 2.76 | 13,743 |
| 55 | Md. | Frostburg | Big Vein | S.B. | R. | .74 | 18.09 | 73.88 | 7.29 | 82.39 | 4.99 | 1.29 | 3.43 | .61 | 7.29 | 14,555 |
| 56 | Md. | Near West'nport | | S.B. | R. | 2.33 | 16.11 | 68.43 | 13.13 | 75.21 | 3.99 | 1.49 | 4.80 | 1.38 | 13.13 | 13,255 |
| 57 | Mo. | Bevier | Bevier | B.B. | R. | 9.14 | 34.53 | 39.02 | 17.31 | 56.25 | 4.96 | .99 | 15.19 | 5.30 | 17.31 | 10,451 |
| 58 | Mo. | Higbee | | B.B. | R. | 12.92 | 33.82 | 39.64 | 13.62 | 57.16 | 5.43 | .75 | 17.86 | 5.18 | 13.62 | 10,548 |
| 59 | Mo. | Novinger | | B.B. | N. | 16.36 | 29.12 | 35.01 | 19.51 | 49.44 | 5.23 | .87 | 21.42 | 3.53 | 19.51 | 9,007 |
| 60 | Mo. | Barnett | | B.B. | R. | 5.39 | 44.91 | 44.47 | 5.23 | 72.67 | 5.55 | .75 | 10.25 | 5.55 | 5.23 | 13,529 |
| 61 | Mo. | Huntsville | | B.B. | L. | 2.49 | 45.44 | 38.79 | 13.28 | 65.71 | 4.74 | 1.08 | 8.85 | 6.34 | 13.28 | 12,213 |
| 62 | Mont. | Bridger | Bridger | L. | S. | 8.56 | 32.36 | 45.69 | 13.39 | 60.39 | 5.06 | 1.06 | 19.56 | .54 | 13.39 | 10,685 |

## PROXIMATE AND ULTIMATE ANALYSES AND HEATING VALUES OF AMERICAN COALS—(Continued)

| No. | State | Nearest Town | Name of Seam | Kind | Grade | Proximate Analysis | | | | Ultimate Analysis | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | | | | | | Moisture | V.M. | F.C. | Ash | C | H | N | O | S | Ash | B.T.U. |
| 63 | Mont. | Red Lodge | No. 1 | L. | S. | 7.34 | 37.92 | 42.17 | 12.57 | 58.20 | 4.97 | 1.26 | 21.90 | 1.10 | 12.57 | 10,269 |
| 64 | Mont. | Bear Creek | No. 2 | L. | S. | 8.21 | 37.98 | 47.67 | 6.14 | 60.86 | 5.30 | 1.43 | 24.80 | 1.47 | 6.14 | 11,422 |
| 65 | N. Mex. | Gibson | Raton | B.L. | R. | 2.72 | 31.85 | 50.86 | 14.57 | 69.96 | 4.88 | 1.17 | 8.73 | .69 | 14.57 | 12,539 |
| 66 | N. Mex. | Van Houten | Weaver | B. | L.S. | 10.86 | 35.14 | 46.90 | 7.10 | 64.34 | 4.73 | 1.05 | 21.14 | .64 | 7.10 | 11,435 |
| 67 | N. Mex. | Williston | Raton | B. | R. | 3.45 | 32.00 | 47.82 | 16.67 | 66.19 | 4.95 | 1.23 | 10.23 | .73 | 16.67 | 11,893 |
| 68 | N. Dak. | Lehigh | Cedar Coulee | L. | R. | 16.70 | 37.10 | 39.49 | 6.71 | 55.16 | 5.61 | .91 | 30.98 | .63 | 6.71 | 9,491 |
| 69 | N. Dak. | Lehigh | Lehigh | L. | R. | 15.42 | 38.73 | 33.61 | 12.24 | 52.66 | 5.22 | .71 | 27.15 | 2.02 | 12.24 | 9,061 |
| 70 | Ohio | Mineral City | No. 5 | B. | L. | 4.49 | 40.55 | 47.43 | 7.53 | 69.64 | 5.68 | 1.29 | 12.93 | 2.93 | 7.53 | 13,291 |
| 71 | Ohio | Bellaire | No. 8 | B. | R. | 7.71 | 39.30 | 47.18 | 9.38 | 69.58 | 5.41 | 1.20 | 10.69 | 3.96 | 9.38 | 13,381 |
| 72 | Ohio | Near Wellston | No. 4 | B. | R. | 4.14 | 39.32 | 42.02 | 11.95 | 62.49 | 5.11 | 1.11 | 14.43 | 4.43 | 11.95 | 13,515 |
| 73 | Ohio | Shawnee | No. 7 | B. | R. | 9.90 | 33.66 | 44.86 | 11.58 | 63.06 | 5.28 | 1.23 | 17.04 | 4.13 | 11.58 | 11,279 |
| 74 | Ohio | Danford | Hocking, No. 6 | B. | L. | 6.65 | 33.94 | 48.86 | 10.55 | 62.38 | 5.38 | 1.20 | 14.83 | 3.13 | 10.55 | 12,179 |
| 75 | Ohio | Dixie | | B. | R. | 7.55 | 38.00 | 46.08 | 8.37 | 67.02 | 5.48 | 1.29 | 15.00 | 2.84 | 8.37 | 12,128 |
| 76 | Ohio | Bradley | No. 8 | B. | L.N. | 3.53 | 37.45 | 49.90 | 9.12 | 65.15 | 5.15 | 1.31 | 15.80 | 3.47 | 9.12 | 13,072 |
| 77 | Okla. | Panama | | S.A. | R. | .64 | 14.29 | 76.66 | 8.41 | 82.06 | 4.35 | 1.68 | 2.26 | 1.24 | 8.41 | 14,083 |
| 78 | Okla. | Henryetta | Henryetta | B. | L. | 3.87 | 35.73 | 50.05 | 10.35 | 69.85 | 5.14 | 1.29 | 11.38 | 1.99 | 10.35 | 12,620 |
| 79 | Okla. | Hartshorne | Hartshorne | B. | R. | 1.70 | 35.19 | 49.79 | 11.32 | 71.49 | 5.00 | 1.72 | 8.91 | 1.56 | 11.32 | 12,969 |
| 80 | Okla. | Edwards | McAlester | B. | R. | 3.45 | 37.45 | 47.82 | 11.28 | 68.18 | 4.85 | 1.50 | 10.52 | 3.67 | 11.28 | 12,469 |
| 81 | Okla. | Lehigh | Lehigh | B. | L.N. | 4.91 | 37.79 | 43.00 | 13.40 | 63.21 | 4.84 | 1.38 | 13.15 | 4.02 | 13.40 | 11,389 |
| 82 | Pa. | Ellsworth | Pittsburg | S.B. | R. | 1.47 | 34.83 | 57.59 | 6.11 | 78.78 | 5.20 | 1.55 | 7.47 | .89 | 6.11 | 14,155 |
| 83 | Pa. | Hastings | D | S.B. | R. | .86 | 22.67 | 68.78 | 8.19 | 79.00 | 4.46 | 1.33 | 5.37 | 1.65 | 8.19 | 14,062 |
| 84 | Pa. | Wehrum | B | B. | R. | .34 | 18.12 | 71.45 | 10.09 | 78.61 | 4.43 | 1.17 | 1.82 | 3.88 | 10.09 | 14,193 |
| 85 | Pa. | Connellsville | Pittsburg | S.B. | R. | .97 | 29.09 | 60.85 | 9.09 | 83.11 | 4.37 | 1.57 | 7.47 | 3.90 | 9.09 | 13,952 |
| 86 | Pa. | Ehrenfeld | Lower Kittanning | B. | R. | .63 | 17.32 | 75.22 | 6.83 | 80.74 | 4.63 | 1.29 | 5.80 | .97 | 6.83 | 14,706 |
| 87 | Pa. | White | Upper Freeport | S.B. | R. | 1.09 | 28.71 | 57.87 | 12.31 | 74.06 | 4.83 | 1.44 | 5.01 | 1.56 | 12.31 | 13,406 |
| 88 | Pa. | Lloydell | Miller | B. | R. | .38 | 16.10 | 74.06 | 10.49 | 75.69 | 4.83 | 1.23 | 6.44 | 1.55 | 10.49 | 14,267 |
| 89 | Pa. | Huff | Pittsburg | S.B. | R. | .91 | 29.03 | 59.57 | 8.83 | 77.42 | 4.77 | 1.58 | 3.58 | 1.03 | 8.83 | 13,738 |
| 90 | Pa. | Kimmelton | Lower Kittanning | B. | R. | .50 | 17.27 | 70.12 | 11.63 | 75.21 | 4.00 | 1.29 | 4.08 | 2.09 | 11.63 | 13,783 |
| 91 | Pa. | Scranton | | A. | S. | 2.08 | 7.27 | 74.24 | 16.33 | 74.21 | 2.81 | 1.80 | 4.08 | .77 | 16.33 | 12,472 |
| 92 | R. I. | Cranston | | G.A. | R. | .42 | 5.02 | 75.11 | 19.45 | 76.63 | .69 | .17 | 2.99 | .07 | 19.45 | 11,212 |

| | State | Mine | Bed | | | | | | | | | | | | | B.t.u. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 93 | Tenn. | Fork Ridge | Mingo | B. | R. | 1.87 | 33.93 | 52.71 | 11.49 | 71.36 | 1.65 | 8.92 | 1.63 | 11.49 | | 12,942 |
| 94 | Tenn. | Brushy Mt. | Petros | B. | R. | 1.35 | 35.13 | 53.32 | 10.20 | 73.23 | 1.69 | 6.78 | 3.38 | 10.20 | | 13,421 |
| 95 | Tenn. | Coalmont | Sewanee | B. | L. | 1.36 | 35.68 | 56.22 | 14.46 | 71.84 | 1.32 | 9.81 | .96 | 14.46 | | 12,843 |
| 96 | Tenn. | Gatliff | Regal Block | B. | R. | 2.25 | 28.43 | 54.79 | 7.28 | 74.80 | 1.78 | 5.91 | 1.02 | 7.28 | | 13,480 |
| 97 | Tenn. | Waldensia | Lower Sewanee | B. | R. | 1.02 | 36.97 | 55.69 | 8.53 | 72.14 | 1.01 | 8.98 | .80 | 8.53 | | 12,886 |
| 98 | Tenn. | Gatliff | Log Mountain | L. | R. | 1.95 | 37.51 | 54.05 | 7.03 | 75.97 | 1.81 | 8.98 | .70 | 7.03 | | 13,734 |
| 99 | Tex. | Hoyt | | L. | R. | 9.76 | 36.17 | 42.77 | 9.96 | 58.43 | .97 | 24.48 | 1.30 | 9.96 | | 10,228 |
| 100 | Tex. | Olsen | | | R. | 9.88 | 42.75 | 43.65 | 10.30 | 58.43 | 1.18 | 23.66 | 1.04 | 10.30 | | 10,287 |
| 101 | Tex. | Crockett | | L. | R. | 13.40 | 43.68 | 29.00 | 14.85 | 52.06 | .95 | 25.53 | .57 | 14.85 | | 9,358 |
| 102 | Utah | Price | No. 3 | S. B. | R. | 2.34 | 48.92 | 48.82 | 5.06 | 83.14 | 1.43 | 12.21 | .75 | 5.06 | | 13,671 |
| 103 | Va. | Pocahontas | Upper Banner | B. | L. | 1.63 | 17.17 | 75.34 | 5.86 | 82.15 | 1.02 | 4.65 | .68 | 5.86 | | 14,672 |
| 104 | Va. | Tom's Creek | Big Seam | A. | P. | .87 | 32.36 | 62.19 | 4.58 | 80.27 | 1.63 | 5.91 | .63 | 4.58 | | 14,796 |
| 105 | Va. | Blacksburg | Darby | B. | L. | 4.80 | 10.12 | 67.05 | 18.03 | 78.56 | .66 | 7.50 | .81 | 18.03 | | 11,961 |
| 106 | Va. | Darby | No. 4 | | L. | 2.40 | 24.28 | 55.54 | 4.42 | 77.21 | 1.35 | 9.73 | 1.27 | 4.42 | | 14,224 |
| 107 | Va. | Richlands | | B. L. | L. | .65 | 36.54 | 64.77 | 10.30 | 63.35 | 1.33 | 5.45 | .80 | 10.30 | | 13,963 |
| 108 | Wash. | Renton | Roslyn | B. | R. | 5.20 | 36.97 | 45.68 | 12.58 | 67.18 | 1.29 | 16.82 | .45 | 12.58 | | 11,291 |
| 109 | Wash. | Roslyn | No. 3 | B. | R. | 1.88 | 48.73 | 48.73 | 12.42 | 67.18 | 1.26 | 10.60 | .69 | 12.42 | | 12,751 |
| 110 | Wash. | Carbonado | No. 4 | B. | R. | 4.66 | 41.37 | 50.31 | 15.96 | 78.31 | 2.11 | 5.38 | .90 | 15.96 | | 12,132 |
| 111 | Wash. | Taylor | | B. | R. | 6.20 | 36.92 | 55.36 | 6.37 | 85.56 | 1.41 | 14.60 | 2.56 | 6.37 | | 10,888 |
| 112 | W. Va. | Kingmont | Pittsburg | S. B. | R. | 1.35 | 37.05 | 52.60 | 8.63 | 83.02 | 1.55 | 7.61 | .91 | 8.63 | | 14,164 |
| 113 | W. Va. | Clarksburg | Pittsburg | B. | R. | 1.12 | 23.28 | 57.16 | 5.14 | 83.63 | 1.44 | 5.13 | 1.35 | 5.14 | | 13,937 |
| 114 | W. Va. | McDonald | Sewell | B. | R. | 1.68 | 21.74 | 56.39 | 4.95 | 82.65 | 1.60 | 4.18 | .66 | 4.95 | | 14,765 |
| 115 | W. Va. | Winifrede | Winifrede | S. B. | R. | 1.50 | 18.05 | 72.53 | 5.09 | 79.35 | 1.52 | 7.83 | .69 | 5.09 | | 14,247 |
| 116 | W. Va. | Rush Run | Fire Creek | S. B. | R. | .64 | 31.87 | 74.38 | 6.95 | 78.95 | 1.70 | 4.23 | .91 | 6.95 | | 14,942 |
| 117 | W. Va. | Big Sandy | No. 8 | S. B. | R. | .62 | 29.53 | 62.95 | 4.01 | 78.21 | 1.34 | 3.03 | .91 | 4.01 | | 14,733 |
| 118 | W. Va. | Page | Ansted | B. | R. | 1.17 | 27.99 | 62.07 | 6.79 | 76.36 | 1.36 | 3.00 | .80 | 6.79 | | 14,821 |
| 119 | W. Va. | Powellton | Powellton | B. | R. | 1.01 | 28.72 | 62.68 | 8.23 | 74.61 | 1.63 | 6.39 | 1.49 | 8.23 | | 14,171 |
| 120 | W. Va. | Bretz | Bakerstown | B. | R. | 1.00 | 35.21 | 61.87 | 8.43 | 72.68 | 1.41 | 6.39 | .90 | 8.43 | | 14,225 |
| 121 | W. Va. | Bretz | Upper Freeport | B. | R. | .98 | 29.20 | 55.50 | 7.71 | 70.30 | 1.50 | 5.13 | 1.24 | 7.71 | | 14,139 |
| 122 | W. Va. | Monarch | Cedar Grove | B. | R. | 1.58 | 36.24 | 59.97 | 10.18 | 76.57 | 1.48 | 6.11 | .99 | 10.18 | | 13,757 |
| 123 | W. Va. | Coalton | Roaring Creek | B. | L. | .65 | 45.27 | 54.42 | 7.39 | 74.61 | 1.43 | 7.90 | .66 | 7.39 | | 10,118 |
| 124 | W. Va. | Spring Hill | Black Band | B. | R. | 1.95 | 36.24 | 54.42 | 7.48 | 72.68 | .90 | 10.89 | .29 | 7.48 | | 13,694 |
| 125 | Wyo. | Hanna | | B. L. | R. | 9.21 | 45.27 | 51.68 | 3.63 | 62.68 | 1.52 | 23.23 | .86 | 3.63 | | 11,009 |
| 126 | Wyo. | Rock Springs | | B. L. | R. | 6.00 | 38.69 | 51.68 | 22.26 | 55.29 | .80 | 18.32 | 4.17 | 22.26 | | 12,519 |
| 127 | Wyo. | Cambria | | | R. | 2.73 | 37.40 | 37.40 | 3.52 | 55.29 | 4.54 | 18.27 | .55 | 3.52 | | 10,364 |
| 128 | Wyo. | Kemmerer | | B. L. | R. | 8.68 | 41.31 | 46.49 | 3.52 | 66.95 | 5.81 | 22.07 | 1.10 | 3.52 | | 11,621 |
| 129 | Wyo. | Monarch | | B. L. | R. | 17.69 | 37.96 | 39.56 | 4.79 | 58.41 | 6.09 | 28.99 | .63 | 4.79 | | 10,355 |

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## PROXIMATE ANALYSES AND HEATING VALUES OF PENNSYLVANIA ANTHRACITES

*(United States Bureau of Mines)*

| | | Moisture | Volatile Matter | Fixed Carbon | Ash | Sulphur | B. T. U. |
|---|---|---|---|---|---|---|---|
| **Anthracites** | Morea, broken............ | C | 3.38 | 4.86 | 82.45 | 9.31 | .60 | 13,057 |
| | Deringer, egg............ | C | 3.67 | 5.29 | 81.42 | 9.62 | .66 | 13,009 |
| | Kingston, grate.......... | C | 3.34 | 4.74 | 81.08 | 10.84 | .65 | 12,717 |
| | Wilkes-Barre, screenings.. | C | 4.17 | 5.95 | 79.02 | 10.86 | .70 | 12,654 |
| | Nanticoke, barley........ | C | 5.45 | 8.43 | 72.12 | 14.00 | .64 | 12,045 |
| | Mid Valley, pea.......... | C | 3.95 | 7.38 | 72.85 | 15.82 | .92 | 11,989 |
| | Deringer, pea ........... | C | 5.11 | 4.99 | 74.87 | 15.03 | .62 | 11,831 |
| | Pittston, pea............ | C | 3.62 | 5.82 | 73.40 | 17.16 | .82 | 11,798 |
| | Mt. Hope, mammoth screenings............. | C | 5.60 | 6.61 | 71.62 | 16.17 | .75 | 11,581 |
| | Newcastle, pea .......... | C | 4.93 | 5.21 | 72.96 | 16.90 | .65 | 11,574 |
| | Schuylkill, rice.......... | C | 5.07 | 4.99 | 71.58 | 18.32 | .75 | 11,382 |
| | Pittston, No. 2 buckwheat | C | 5.93 | 6.72 | 70.35 | 16.80 | .95 | 11,359 |
| | Girard, mammoth, buckwheat................ | C | 6.21 | 5.64 | 70.70 | 17.45 | .56 | 11,141 |
| | Mt. Hope, barley........ | C | 6.55 | 5.26 | 70.83 | 17.36 | .69 | 11,132 |
| | Tower City, Lykens Valley | | 3.33 | 3.27 | 84.28 | 9.12 | .60 | 13,351 |
| | Bernice, egg ............ | C | 2.12 | 8.80 | 73.43 | 15.65 | .58 | 12,575 |
| **Semianthracites** | Bernice, Randall & Shaad mine................ | D | .50 | 9.60 | 77.90 | 12.03 | .83 | 13,510 |
| | Bernice, O'Boyle & Fay mine................ | D | .60 | 9.50 | 77.40 | 12.49 | 1.41 | 13,425 |
| | Bernice, Connell mine .... | D | .70 | 8.65 | 76.75 | 13.90 | .66 | 13,185 |
| | Lopez, Northern Anthracite Company......... | D | .60 | 9.10 | 78.95 | 11.35 | .65 | 13,605 |
| | Loyalsock.............. | | 1.49 | 11.07 | 78.88 | 7.09 | .86 | |

382 to 285. Those analyses marked C are of commercial lots delivered to the various government departments and represent the coal as received or, as fired. The analyses marked D were made from air-dried samples; in some cases they represent carload lots, and in others they are samples taken in the mine. The calorific values were determined upon air-dried samples except those given in *italics*, which were calculated from the analysis by means of Kent's formula. The analyses that are not marked with a C or a D are believed to represent air-dried samples, but those reporting them failed to state the facts in the case.

One of the accompanying tables gives a series of analyses of Alaskan coals taken from the reports of the United States Geological Survey. Among them are three analyses of coals from the Yukon Territory of Canada. The report does not state whether the analyses are of air-dried or as-received samples; judging from the amount of water in some specimens, it is presumed that they, at least, were of samples as received. The sulphur is separately determined.

A series of proximate analyses of foreign coals, many of which are used on the Pacific coast of the United States, is given in the table on page 387. The sample of coal from Argentine and the one from Rio Grande do Sul, Brazil, were tested at the United States Fuel Testing Plant, St. Louis, Missouri. They were treated in the same way as the samples of American coal given in the table on page 387. These two analyses are of air-dried samples and the heating values are 6,320 and 9,058 B. T. U., respectively. The heating value of the Victoria coal from Windmill shaft is given as 12,871 B. T. U. The other analyses are taken from various authorities, who fail to state whether they were made on air-dried or on as-received samples; judging from the small amount of water in most of them, the samples appear to have been air-dried before analysis. The sulphur is usually separately determined.

## PROXIMATE ANALYSES OF MISCELLANEOUS AMERICAN COALS

*(United States Geological Survey)*

| State | Kind of Coal | | Moisture | Volatile Matter | Fixed Carbon | Ash | Sulphur | B. T. U. |
|-------|--------------|---|----------|-----------------|--------------|-----|---------|----------|
| Ala. | Pratt mines, lump......... | C | 1.33 | 30.55 | 58.88 | 9.24 | 2.10 | 13,884 |
| Ark. | Semianthracite, Spadra..... | D | 1.12 | 11.54 | 78.62 | 8.72 | 2.01 | 13,853 |
| Ark. | Semianthracite, Clarksville | D | 1.03 | 10.53 | 80.06 | 8.38 | 2.51 | 13,965 |
| Ark. | Semianthracite, Russelville. | D | .68 | 9.93 | 79.94 | 9.43 | 1.76 | 13,896 |
| Ariz. | Tuba, Black Mesa Field .... | D | 7.80 | 33.80 | 44.50 | 13.90 | 1.30 | 10,650 |
| Ariz. | Oraibi, Black Mesa Field ... | D | 8.10 | 33.30 | 47.80 | 10.84 | 1.14 | 11,020 |
| Ariz. | St. Michael................ | D | 9.89 | 39.07 | 46.95 | 4.05 | .42 | 12,501 |
| Cal. | Stone Cañon............... | D | 4.86 | 47.74 | 41.03 | 6.37 | 4.26 | 12,727 |
| Cal. | Tesla..................... | D | 18.02 | 39.22 | 26.39 | 16.37 | 3.07 | 8,105 |
| Cal. | Trafton................... | D | 12.20 | 35.20 | 41.40 | 11.20 | 4.58 | 10,130 |
| Cal. | Mean of ten analyses....... | | 11.32 | 45.09 | 35.91 | 7.68 | | |
| Col. | Canon City, Chandler...... | D | 6.80 | 36.50 | 50.28 | 6.42 | .44 | 11,833 |
| Col. | Canon City, Nonac........ | D | 6.60 | 35.30 | 52.44 | 5.66 | .98 | 12,001 |
| Col. | Colorado Springs, Cell...... | D | 5.50 | 37.80 | 48.49 | 8.21 | .53 | 10,885 |
| Col. | Crested Butte............. | D | 2.80 | 5.05 | 77.55 | 14.60 | .60 | *12,563* |
| Ill. | Pawnee, washed nut ...... | C | 7.93 | 34.23 | 49.78 | 8.06 | 1.79 | 12,215 |
| Ill. | Pana, washed nut ......... | C | 7.08 | 34.94 | 48.60 | 9.38 | 2.19 | 12,151 |
| Kans. | Cherokee lump............. | C | 3.50 | 33.73 | 52.43 | 10.34 | 4.60 | 12,989 |
| Ky. | Elkhorn Field, Millard ..... | D | 1.62 | 32.68 | 57.39 | 8.31 | 1.07 | *13,664* |
| Ky. | Elkhorn Field, Flatwoods... | D | 1.78 | 34.27 | 55.55 | 8.40 | .83 | *13,357* |
| Ky. | Elkhorn Field, L. Elkhorn.. | D | 1.44 | 31.80 | 56.84 | 9.92 | .56 | *13,473* |
| Ky. | Elkhorn Field, U. Elkhorn.. | D | 1.68 | 33.20 | 61.35 | 3.77 | .60 | *14,228* |
| Md. | Georges Creek, av. 53 anal... | | .70 | 18.81 | 72.96 | 7.26 | 1.01 | *14,526* |
| Mich. | Bay City, Lower Verne .... | D | 8.71 | 38.45 | 41.16 | 11.68 | 2.72 | 12,359 |
| Mich. | Bay City, Upper Verne .... | D | 3.78 | 41.18 | 49.34 | 5.70 | 2.50 | 13,489 |
| Mich. | Saginaw................... | D | 10.67 | 33.59 | 53.80 | 1.94 | 1.01 | 12,868 |
| Mont. | Electric Field.............. | D | 1.00 | 20.00 | 55.19 | 23.81 | .44 | 11,420 |
| Mont. | Electric Field.............. | D | .60 | 30.30 | 57.75 | 11.35 | .87 | 13,610 |
| Mont. | Electric Field.............. | D | 1.10 | 28.20 | 50.57 | 20.13 | 1.37 | 11,790 |
| N. Mex. | Madrid................... | | 5.70 | 2.18 | 86.13 | 5.99 | .69 | 13,268 |
| Ore. | Cöös Bay, average......... | D | 9.60 | 36.30 | 40.90 | 13.20 | | 9,720 |
| Ore. | Cöös Bay, average......... | | 10.41 | 46.15 | 36.85 | 6.59 | 1.02 | |
| Ore. | Rogue Riv. Valley, Medford | D | 9.49 | 23.87 | 32.54 | 34.10 | 1.18 | 7,683 |
| Pa. | Cambria Co., Beech Creek.. | C | 2.42 | 20.31 | 70.13 | 7.14 | 1.10 | 14,087 |
| Pa. | Reynoldsville............. | C | 3.04 | 29.79 | 60.85 | 6.32 | 1.23 | 14,040 |
| Pa. | Somerset County, C' bed ... | C | 2.56 | 16.47 | 71.58 | 9.39 | .93 | 13,748 |
| Utah | Castle Gate............... | D | 3.10 | 40.58 | 48.10 | 8.22 | .54 | *11,956* |
| Utah | Price..................... | D | 4.04 | 43.23 | 47.27 | 5.46 | .58 | *11,729* |
| Utah | Winter Quarters........... | D | 4.37 | 41.84 | 47.77 | 6.02 | .89 | *11,908* |
| Va. | Blacksburg................ | | 2.98 | 10.94 | 64.14 | 21.94 | .68 | 11,669 |
| Va. | Dante, Widow Kennedy.... | D | 1.11 | 31.79 | 61.36 | 5.74 | 1.48 | *14,276* |
| Va. | Dante, Lower Banner...... | D | 1.01 | 32.70 | 60.39 | 5.90 | .85 | *14,203* |
| Va. | Dante, Upper Banner...... | D | 1.07 | 32.83 | 58.68 | 7.42 | .67 | *13,910* |
| W. Va. | Fairmont, average of sixty-three mines............ | | .75 | 38.16 | 54.63 | 6.45 | 2.30 | *13,509* |
| W. Va. | Pocahontas, average of thirty-eight mines......: | | .73 | 17.43 | 77.71 | 4.63 | .62 | *15,032* |
| W. Va. | Piney, Raleigh County..... | C | 2.50 | 20.58 | 71.70 | 5.22 | .74 | 14,533 |
| W. Va. | Kanawha Gas............. | C | 3.18 | 30.57 | 59.40 | 6.85 | .92 | 13,849 |
| W. Va. | Pocahontas, thin vein...... | C | 2.36 | 18.56 | 71.70 | 7.38 | .81 | 14,135 |
| W. Va. | Pocahontas, No. 3, 3-in. lump.................. | C | 3.08 | 17.56 | 74.79 | 4.57 | .66 | 14,557 |
| W. Va. | Pocahontas, No. 3, M.R. ... | C | 2.63 | 18.25 | 73.87 | 5.25 | .64 | 14,528 |

## PROXIMATE ANALYSES AND HEATING VALUES OF CANADIAN COALS
*(Canadian Department of Mines)*

| Field | Town | Company | Mine | Seam | Grade | Moisture | Volatile Matter | Fixed Carbon | Ash | Sulphur | B.T.U. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Alberta** | | | | | | | | | | | |
| Edmonton | Strathcona | S.C.Co. | Strathcona | | L | 22.7 | 41.0 | 47.6 | 11.4 | .4 | 10,728 |
| Edmonton | Edmonton | P.C.Co. | Parkdale | | L | 22.5 | 37.8 | 51.3 | 10.9 | .4 | 10,908 |
| Edmonton | Edmonton | St.C.Co. | Standard | | L | 23.5 | 42.0 | 49.9 | 8.1 | 1.4 | 11,358 |
| Belly River | Taber | C.,W.C.Co. | Taber | | L | 13.0 | 36.0 | 49.9 | 14.1 | 1.4 | 11,034 |
| Belly River | Lethbridge | A.,Ry.&I.Co. | Galt | | L | 8.4 | 37.5 | 51.5 | 11.0 | .8 | 11,719 |
| Lundbreck | Lundbreck | L.,B.C.Co. | Lundbreck | | R | 5.0 | 30.1 | 40.2 | 29.7 | 1.2 | 9,810 |
| Frank-Blairmore | Passburg | Leitch | Leitch | Bellevue No. 1 | R | 1.9 | 27.0 | 55.1 | 17.9 | .6 | 12,240 |
| Frank-Blairmore | Hillcrest | H.C.&C.Co. | Hillcrest | | R | 3.0 | 29.3 | 55.4 | 15.3 | .6 | 12,456 |
| Frank-Blairmore | Lille | W.C.Cols. | Lille | No. 1 | R | .9 | 27.6 | 56.9 | 15.5 | .8 | 12,384 |
| Frank-Blairmore | Bellevue | W.C.Cols. | Bellevue | No. 1 | R | 1.7 | 25.0 | 58.6 | 16.4 | .5 | 12,474 |
| Coleman | Coleman | I.C.&C.Co. | Denison | No. 2 | R | 2.0 | 25.1 | 55.1 | 19.8 | .4 | 11,718 |
| Coleman | Coleman | I.C.&C.Co. | Denison | No. 4 | R | 2.0 | 23.9 | 59.9 | 16.2 | .6 | 12,528 |
| Canmore-Bankhead | Canmore | H.W. McN. Co. | No. 1 | | R | 1.2 | 17.2 | 70.5 | 12.3 | .8 | 13,212 |
| Canmore-Bankhead | Banff | B. Mines | Bankhead | | P | 1.0 | 11.8 | 76.0 | 12.2 | .6 | 13,320 |
| Canmore-Bankhead | Banff | B. Mines | Bankhead | | B | 1.1 | 12.6 | 71.5 | 15.9 | .6 | 12,672 |
| **British Columbia** | | | | | | | | | | | |
| Crowsnest Pass | Michel | C.P.C.Co. | No. 3 | | L | 1.4 | 24.8 | 62.7 | 12.5 | .5 | 13,266 |
| Crowsnest Pass | Michel | C.P.C.Co. | No. 7 | | L | 1.9 | 22.6 | 65.5 | 11.9 | .4 | 13,356 |
| Crowsnest Pass | Michel | C.P.C.Co. | No. 8 | | L | 3.0 | 24.1 | 65.7 | 10.2 | .6 | 13,482 |
| Crowsnest Pass | Hosmer | H.M. | Hosmer | No. 2 | L | 1.7 | 21.3 | 63.4 | 15.3 | .3 | 12,708 |
| Crowsnest Pass | Hosmer | H.M. | Hosmer | No. 6 | R | 2.6 | 25.6 | 62.0 | 12.4 | .5 | 13,086 |
| Crowsnest Pass | Hosmer | H.M. | Hosmer | No. 8 | R | 4.0 | 28.0 | 64.5 | 7.5 | .6 | 13,986 |
| Crowsnest Pass | Fernie | C.P.C.Co. | Coal Creek No. 2 | | R | 2.2 | 26.3 | 64.7 | 9.0 | .6 | 13,822 |
| Crowsnest Pass | Fernie | C.P.C.Co. | Coal Creek, No. 5 | | R | 1.6 | 24.0 | 65.2 | 10.8 | .5 | 13,482 |
| Nicola Valley | Coutlee | N.V.C.C.&C.Co. | Middlesboro, No. 1 | | R | 4.4 | 39.1 | 46.4 | 14.5 | .9 | 11,682 |
| Nicola Valley | Coutlee | N.V.C.C.&C.Co. | Middlesboro, No. 2 | | R | 2.9 | 39.0 | 48.1 | 12.9 | .7 | 12,068 |

| Province | District | Company | Colliery | Seam | | Moisture | Vol. Matter | Fixed Carbon | Ash | Sulphur | B.T.U. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Vancouver Island. | Wellington | W. C. Co. | Extension | Douglas | L | 1.8 | 40.1 | 49.8 | 10.1 | .4 | 13,158 |
| Vancouver Island. | Nanaimo | W. F. Co. | Esplanade No. 1 | Newcastle | L | 2.2 | 41.2 | 48.5 | 10.3 | .9 | 12,834 |
| Vancouver Island. | Nanaimo | W. F. Co. | Esplanade No. 1 | Newcastle | L | 2.4 | 41.5 | 46.6 | 11.9 | 1.3 | 12,474 |
| Vancouver Island. | Cumberland | W. F. Co. | Union, No. 1 | Newcastle | L | | 31.6 | 56.5 | 11.9 | 1.0 | 12,870 |
| Vancouver Island. | Cumberland | W. F. Co. | Union, No. 7 | | L | | 28.0 | 60.1 | 11.9 | .9 | 12,978 |
| Vancouver Island. | Suquash | P C.C. Co | Suquash | | L | | 34.3 | 42.7 | 23.0 | 1.0 | 11,106 |
| **New Brunswick** | | | | | | | | | | | |
| Grand Lake........ | Minto | G. H. King | Kings | | L | 1.3 | 32.2 | 53.4 | 14.4 | 5.8 | 12,888 |
| **Nova Scotia** | | | | | | | | | | | |
| Sydney. | Cape Breton | No. Atl. Cols. Co. | Port Morien | Gowrie | L | 3.5 | 34.7 | 53.0 | 12.3 | 6.4 | 12,708 |
| Sydney. | Glace Bay | Dominion Coal Co. | No. 7 | Hub | L | 2.4 | 36.5 | 57.6 | 5.9 | 2.4 | 13,860 |
| Sydney. | Glace Bay | Dominion Coal Co. | No. 9 | Harbour | L | 3.4 | 38.6 | 55.5 | 5.9 | 3.7 | 14,004 |
| Sydney. | Glace Bay | Dominion Coal Co. | No. 5 | Phalen | L | 4.0 | 35.0 | 59.1 | 5.9 | 1.8 | 14,040 |
| Sydney. | Glace Bay | Dominion Coal Co. | No. 10 | Emery | R | 4.9 | 25.1 | 63.8 | 11.1 | 2.5 | 13,122 |
| Sydney. | Lingan | Dominion Coal Co. | No. 12 | Lingan | L | 3.5 | 35.1 | 60.1 | 4.8 | 1.8 | 13,788 |
| Sydney. | Sidney Mines | N. S. S. & C. Co. | Sidney, No. 1 | Main | L | 5.4 | 37.3 | 55.5 | 7.2 | 4.8 | 13,770 |
| Sydney. | Sidney Mines | N. S. S. & C. Co. | Sidney, No. 3 | Main | L | 9.3 | 37.4 | 55.9 | 6.7 | 2.9 | 13,680 |
| Inverness. | Inverness | I. C. & Ry. Co. | Inverness | | L | 4.0 | 39.0 | 46.4 | 14.6 | 2.5 | 11,772 |
| Inverness. | Port Hood | P. H., R. Ry. & C. Co. | Port Hood | | L | 4.7 | 40.0 | 45.5 | 14.5 | 6.0 | 12,150 |
| Pictou. | Thorburn | Acadia Coal Co. | Vale | 6 Foot | L | 2.1 | 37.1 | 45.6 | 17.3 | 7.9 | 12,024 |
| Pictou. | Stellarton | Acadia Coal Co. | Allan Shaft | Foord | R | 3.6 | 32.1 | 56.6 | 11.3 | 1.0 | 12,230 |
| Pictou. | Stellarton | Acadia Coal Co. | Albion | Third | R | | 33.3 | 52.0 | 14.7 | .6 | 12,852 |
| Pictou. | Stellarton | Acadia Coal Co. | Albion | Cage Pit | R | 3.6 | 31.4 | 58.1 | 10.5 | 1.4 | 13,176 |
| Pictou. | Westville | Acadia Coal Co. | Acadia | Main | L | 1.8 | 26.0 | 64.8 | 9.2 | .9 | 13,860 |
| Pictou. | Westville | Acadia Coal Co. | Drummond | Main | L | 1.4 | 24.7 | 60.8 | 14.5 | .9 | 12,960 |
| Springhill. | Springhill | C. C. & Ry. Co. | No. 2 | | L | 2.8 | 32.3 | 58.5 | 9.2 | 2.5 | 12,394 |
| Springhill. | Springhill | C. C. & Ry. Co. | No. 3 | | L | 2.8 | 33.5 | 55.0 | 11.5 | 1.6 | 12,996 |
| Joggins. | Chignecto | M. Coal Co. | Chignecto | | L | 3.6 | 41.0 | 45.7 | 13.3 | 1.8 | 12,150 |
| Joggins. | River Herbert | M. C. Ry. & P. Co. | Minudie | | L | 3.8 | 35.7 | 48.8 | 15.5 | 6.4 | 12,150 |
| Joggins. | Joggins | M. C. Ry. & P. Co. | Joggins | | L | 1.3 | 36.6 | 44.8 | 18.6 | 6.7 | 11,826 |
| Joggins. | Joggins | M. C. Ry. & P. Co. | Joggins | | L | | 35.7 | 49.8 | 14.5 | 5.4 | 11,592 |
| **Saskatchewan** | | | | | | | | | | | |
| Souris. | Taylortown | W. D. Cols. Co. | Taylortown | | L | 28.6 | 49.0 | 42.9 | 8.1 | .6 | 10,692 |
| Souris. | Estevan | E. C. & B. Co. | Eureka | | R | 30.9 | 40.0 | 43.2 | 16.8 | .5 | 9,648 |
| **Yukon** | | | | | | | | | | | |
| Whitehorse.. | Tantalus | W. P. & Y. Ry. Co. | Tantalus | Upper | Sa. | | 25.0 | 58.0 | 17.0 | .5 | 12,060 |
| Whitehorse.. | Tantalus | W. P. & Y. Ry. Co. | Tantalus | Middle | Sa. | | 26.7 | 54.1 | 19.2 | .5 | 11,358 |
| Whitehorse.. | Tantalus | W. P. & Y. Ry. Co. | Tantalus | Lower | Sa. | | 27.8 | 56.0 | 16.2 | 1.1 | 12,222 |

A series of proximate analyses and heating values of Canadian coals arranged by provinces, coal fields, mines, etc., is given in the table on pages 388 and 389. The analyses and heating-value determinations were made on samples dried at 212° F., consequently the sum of the volatile matter, fixed carbon, and ash equals 100. Both the moisture and sulphur were separately determined and do not enter in this total. By reason of the analyses, etc. being made on dry samples, the values in the table are higher than if made on air-dried samples and cannot be compared with those given in the table on pages 382 to 385. The samples were collected at the mine either at the working face or from coal being shipped and represent several tons in each case. The following abbreviations

## PROXIMATE ANALYSES OF ALASKAN COALS
### (*United States Geological Survey*)

| District and Kind of Coal | Mois-ture | Volatile Matter | Fixed Carbon | Ash | Sul-phur |
|---|---|---|---|---|---|
| Anthracite | | | | | |
| Bering River, average of seven analyses.................... | 7.88 | 6.15 | 78.23 | 7.74 | 1.30 |
| Matanuska River............... | 2.55 | 7.08 | 84.32 | 6.05 | .57 |
| Semianthracite | | | | | |
| Bering River, average of eleven analyses.................... | 5.80 | 8.87 | 76.06 | 9.27 | 1.08 |
| Semibituminous | | | | | |
| Bering River, average of twenty-eight analyses................ | 4.18 | 14.00 | 72.42 | 9.39 | 1.73 |
| Cape Lisburne, average of three analyses.................... | 3.66 | 17.47 | 75.95 | 2.92 | .96 |
| Matanuska River, average of sixteen analyses ................. | 2.71 | 20.23 | 65.39 | 11.60 | .57 |
| Bituminous | | | | | |
| Lower Yukon, average of eleven analyses.................... | 4.68 | 31.14 | 56.62 | 7.56 | .48 |
| Subbituminous, or black lignite | | | | | |
| Matanuska River, average of four analyses.................... | 6.56 | 35.43 | 49.44 | 8.57 | .37 |
| Koyukuk River................. | 4.47 | 34.32 | 48.26 | 12.95 | |
| Nation River.................. | 1.39 | 40.02 | 55.55 | 3.04 | 2.98 |
| Alaska Peninsula, average of five analyses.................... | 2.34 | 38.68 | 49.75 | 9.22 | 1.07 |
| Cape Lisburne, average of eleven analyses.:.................. | 9.35 | 38.01 | 47.19 | 5.45 | .35 |
| Anaktuvuk River............... | 6.85 | 36.39 | 43.38 | 13.38 | .54 |
| Port Graham.................. | 16.87 | 37.48 | 39.12 | 6.53 | .39 |
| Southeast Alaska, average of five analyses.................... | 1.97 | 37.84 | 35.18 | 24.23 | .57 |
| Wainright Inlet................ | 10.65 | 42.99 | 42.94 | 3.42 | .62 |
| Colville River................. | 11.50 | 30.23 | 30.27 | 27.90 | .50 |
| Upper Yukon, Canada, average of thirteen analyses............. | 13.08 | 39.88 | 39.28 | 7.72 | 1.26 |
| Upper Yukon, Circle Province, average of three analyses....... | 10.45 | 41.81 | 40.49 | 7.27 | 1.30 |
| Upper Yukon, Rampart Province, average of six analyses......... | 11.42 | 41.15 | 36.95 | 10.48 | .33 |
| Seward Peninsula............... | 24.92 | 38.15 | 33.58 | 3.35 | .68 |
| Chitistone River............... | 1.65 | 51.50 | 40.75 | 6.10 | |
| Katchemak Bay, average of six analyses.................... | 19.85 | 40.48 | 30.99 | 8.68 | .35 |
| Nenana River.................. | 13.02 | 48.81 | 32.40 | 5.77 | .16 |
| Kodiak Island................. | 12.31 | 51.48 | 33.80 | 2.41 | .17 |
| Unga Island, average of two analyses.................... | 10.92 | 53.36 | 28.25 | 7.47 | 1.36 |
| Tyonck, average of four analyses... | 8.55 | 54.20 | 30.92 | 6.53 | 3.38 |
| Chistochina River............. | 15.91 | 60.35 | 19.46 | 4.28 | |

are used: L, lump; R, run of mine; B, buckwheat (anthracite); P, pea (anthracite); Sa, mine sample; A. Ry. & I. Co., Alberta Railway and Irrigation Co.; B. Mines, Bankhead Mines, Ltd.; C. C. & Ry. Co., Cumberland Coal and Railway Co.; C. P. C. Co., Crowsnest Pass Coal Co.; C. W. C. Co., Canada West Coal Co.; Dom. C. Co., Dominion Coal Co.; E. C. & B. Co., Eureka Coal and Brick Co.; H. C. & C. Co., Hillcrest Coal and Coke Co.; H. M., Hosmer Mines, Ltd.; I. C. & C. Co., International Coal and Coke Co.; I. C.

## PROXIMATE ANALYSES OF FOREIGN COALS
### (*Various Authorities*)

| Coal | Moisture | Volatile Matter | Fixed Carbon | Ash | Sulphur |
|---|---|---|---|---|---|
| Argentine, Province of Mendoza.... | 7.67 | 18.39 | 31.00 | 42.85 | 1.21 |
| Brazil, Rio Grande do Sul........... | 10.96 | 26.78 | 38.82 | 23.44 | 2.94 |
| Brazil, Pernambuco................. | 1.90 | 18.82 | 58.73 | 20.55 | |
| British Columbia, Crowsnest Pass, average...................... | 1.09 | 21.07 | 70.54 | 7.29 | .37 |
| British Columbia, Comox, average... | 1.18 | 28.41 | 62.91 | 7.49 | 1.54 |
| British Columbia, Nanaimo, average. | 2.12 | 34.07 | 55.94 | 7.93 | .64 |
| Chili, Straits of Magellan........... | 1.64 | 24.85 | 69.52 | 3.99 | .97 |
| England, Durham, coking average.... | .91 | 13.12 | 81.54 | 4.43 | .92 |
| England, South Durham, Brockwell.. | 1.12 | 22.05 | 72.17 | 4.66 | 1.22 |
| England, South Durham, Busty seam. | .86 | 25.24 | 68.24 | 5.66 | 1.30 |
| England, Bearspark colliery......... | .85 | 27.06 | 68.97 | 3.12 | .78 |
| India, Umaria..................... | 4.84 | 34.07 | 46.33 | 14.76 | 1.08 |
| India, Warora..................... | 6.00 | 34.00 | 43.04 | 16.96 | 1.20 |
| India, Johilla..................... | 5.76 | 33.03 | 44.22 | 16.99 | .42 |
| Japan, average eight analyses........ | 2.62 | 42.49 | 50.07 | 4.82 | .92 |
| Japan, Yubari mine................. | 1.22 | 42.43 | 51.75 | 4.60 | .47 |
| Japan, Sorachi mine................ | 1.63 | 30.14 | 60.72 | 7.51 | .27 |
| Japan, Poronai mine, Seam 2........ | 2.59 | 36.83 | 56.32 | 4.26 | .60 |
| Japan, Ikushunbetsu mine.......... | 1.77 | 47.96 | 42.62 | 7.65 | .16 |
| Mexico, Sabinas Coal Co., Sabinas ... | .69 | 22.46 | 58.05 | 18.80 | |
| Mexico, Coahuila Coal Co., Coahuila.. | .39 | 19.91 | 64.93 | 14.77 | .86 |
| New South Wales, Southern Field, average...................... | .97 | 23.10 | 56.26 | 10.67 | .46 |
| New South Wales, Western, average.. | 1.87 | 31.49 | 52.61 | 14.03 | .63 |
| New South Wales, Northern, average. | 1.92 | 35.09 | 51.08 | 8.91 | .54 |
| New South Wales, Killingworth mine. | 1.24 | 38.82 | 32.46 | 27.48 | 3.50 |
| New Zealand, Lake Coleridge........ | 1.80 | 1.96 | 84.12 | 12.12 | |
| New Zealand, Millerton Mines....... | 1.16 | 20.50 | 74.83 | 3.51 | |
| New Zealand, Paparoa, Beds 1, 2, 3.. | 1.03 | 16.38 | 78.78 | 3.81 | .29 |
| New Zealand, Paparoa, Beds 4, 5 .... | .78 | 23.55 | 72.77 | 2.90 | .48 |
| New Zealand, Kaitongata mines..... | 20.06 | 28.93 | 44.60 | 6.41 | |
| New Zealand, Westport mines....... | 2.60 | 37.17 | 56.01 | 4.22 | |
| Nova Scotia, Sydney, average....... | 10.60 | 39.20 | 56.70 | 5.10 | 1.20 |
| Philippines, Compostella, Cebu...... | 7.80 | 37.56 | 51.96 | 2.68 | |
| Philippines, Mt. Uling, Cebu........ | 6.30 | 35.30 | 53.55 | 4.45 | .40 |
| Philippines, average nine analyses, Cebu........................... | 14.00 | 31.08 | 50.35 | 4.58 | |
| Philippines, Batan................. | 5.82 | 40.29 | 52.40 | 1.49 | .66 |
| Philippines, Batan................. | 4.53 | 45.89 | 46.96 | 2.62 | .39 |
| Philippines, Batan................. | 5.62 | 38.68 | 54.42 | 1.36 | .14 |
| Philippines, Batan................. | 6.08 | 40.36 | 51.24 | 2.32 | .40 |
| Philippines, Batan, average five analyses....................... | 13.57 | 36.91 | 44.92 | 4.60 | |
| Scotland, Lanarkshire splint........ | 6.86 | 34.42 | 55.53 | 3.19 | .70 |
| Transvaal, Brugspruit............. | .80 | 28.02 | 64.62 | 6.56 | .86 |
| Transvaal, Bethel................. | .30 | 41.23 | 52.16 | 6.31 | Trace |
| Transvaal, Springs................. | .57 | 14.10 | 63.00 | 22.00 | |
| Victoria, Coal Creek, Bed 2......... | 7.00 | 21.60 | 55.66 | 12.35 | |
| Victoria, Windmill shaft............ | 5.94 | 33.67 | 55.23 | 5.16 | |

& Ry. Co., Inverness Coal and Railway Co.; I. C. Co., Intercolonial Coal Co.; L. B. C. Co., Lund Breckenridge Coal Co.; Leitch, Leitch Collieries, Ltd.; M. Coal Co., Minudie Coal Co.; No. Atl. Cols., North Atlantic Collieries, Ltd.; N. S. S. & C. Co., Nova Scotia Steel and Coal Co.; N. V. C. & C. Co., Nicola Valley Coal and Coke Co.; P. C. Co., Parkdale Coal Co.; P. C. C. Co., Pacific Coast Coal Co.; P. H., R. Ry. & C. Co., Port Hood, Richmond, Railway and Coal Co.; S. C. Co., Strathcona Coal Co.; St. C. Co., Standard Coal Co.; W. C. Co., Wellington Colliery Co.; W. C. Cols., West Canadian Collieries; W. D. Cols. Co., Western Dominion Collieries Co.; W. F. Co., Western Fuel Co.; W. P. & Y. Ry. Co., White Pass and Yukon Railway Co., Ltd.

## DETERMINATION OF HEATING VALUE OF COAL FROM A PROXIMATE ANALYSIS

A very good idea of the calorific power of a coal, from a commercial standpoint at least, may be obtained from a proximate analysis by some simple calculations. Of the two methods of doing this, that devised by William Kent and known by his name has a fairly general application. A method devised by Lord and Haas is restricted to a given field, but for the coals within that field it is probably more accurate than Kent's.

**Kent's Method.**—According to Kent, a relation exists between the amount of fixed carbon in the combustible portion of a coal (coal ash-and-moisture free, according to the fifth column in the calculation given under the head Proximate Analysis of Coal *ante*) and its calorific value per pound of combustible, which is shown in the following table.

These figures are correct within 2% for all coals containing more than 63% of fixed carbon in the combustible, but for coals containing less than 60% fixed carbon or more than 40% volatile matter in the combustible they are liable to an error, in either direction, of about 4%. The greater variation in the coals low in fixed carbon and high in volatile matter is due to the fact that they differ considerably in the percentage of oxygen in the volatile matter.

### APPROXIMATE HEATING VALUE OF COALS

| Per Cent. of Fixed Carbon in Coal Dry and Free From Ash | Heating Value per Pound Combustible | |
|---|---|---|
| | B. T. U. | Calories |
| 100 | 14,580 | 8,100 |
| 97 | 14,940 | 8,300 |
| 94 | 15,210 | 8,450 |
| 90 | 15,480 | 8,600 |
| 87 | 15,660 | 8,700 |
| 80 | 15,840 | 8,800 |
| 72 | 15,660 | 8,700 |
| 68 | 15,480 | 8,600 |
| 63 | 15,120 | 8,400 |
| 60 | 14,760 | 8,200 |
| 57 | 14,220 | 7,900 |
| 55 | 13,860 | 7,700 |
| 53 | 13,320 | 7,400 |
| 51 | 12,420 | 6,900 |

**Method of Lord and Haas.**—The following formula is based on extensive experiments made at the Ohio State University by Messrs. Lord and Haas and the results calculated by it have been found to agree very closely with those obtained with the Mahler calorimeter.

$$\text{B. T. U.} = \frac{K(100 - A - S - M) + (S \times 4,050)}{100} \quad (1)$$

in which the ash $A$, sulphur $S$, and moisture $M$ are expressed as percentages, and $K$ is a constant determined from a number of chemical and calorimetric determinations by the following formula, in which the values substituted are the averages of a number of analyses:

$$K = \frac{\text{B. T. U.} - (\text{sulphur} \times 4,050)}{100 - (\text{ash} + \text{sulphur} + \text{moisture})} \quad (2)$$

These gentlemen found that the value of $K$, which depends on the amount of moisture, ash, and sulphur in the coal, is practically constant for the same coal in the same field, regardless of any local variation in the relative proportions of these constituents. The value of $K$ as determined by them for various coals from Ohio, Pennsylvania, and West Virginia is given in the following table.

### VALUE OF $K$ FOR VARIOUS COALS

| Coal | Value of $K$ |
|---|---|
| Upper Freeport, Ohio and Pennsylvania................. | 15,116 |
| Pittsburg, Pennsylvania......................... | 15,183 |
| Middle Kittanning (Darlington coal), Pennsylvania........ | 15,062 |
| Middle Kittanning (Hocking Valley coal), Ohio............ | 14,265 |
| Thacker, West Virginia........................ | 15,410 |
| Pocahontas, West Virginia..................... | 15,829 |
| Fairmount, West Virginia..................... | 15,675 |

ILLUSTRATION 1.—*Lord and Haas' Method.*—Using coal No. 103 from Pocahontas, Va., as the sulphur is separately determined in the usual way, the percentages of ash and moisture must be recalculated, to what they would be if the sulphur were included. This is done by dividing them by $(100+S) \div 100 = 1.0074$, as the coal contains .74% of sulphur. From this the adjusted moisture and ash are, respectively, 1.62% and 5.82%. The value of $K$ to be used, 15,829, is taken from the table. Substituting in formula 1:

$$\text{B. T. U.} = \frac{15,829 \times (100 - 5.82 - .74 - 1.62) + (.74 \times 4,050)}{100} = 14,564$$

As the true calorific value is 14,672, the difference is 108 B. T. U., or .736%.
Kent's Method.—Using the same coal as before, No. 103, from the Pocahontas field in Virginia, the heating value of which has been determined by calorimeter to be 14,672 B. T. U. per lb., the proximate analysis is: Moisture, 1.63%; volatile matter, 17.17%; fixed carbon, 75.34%; and ash, 5.86%; the sum of these four constituents is 100%. The coal dry and free from ash is made up of $75.34 \div (75.34 + 17.17 = 92.51) = 81.44\%$ of fixed carbon, and $17.17 \div (75.34 + 17.17 = 92.51) = 18.56\%$ of volatile matter. By interpolating in the second column of the table, the heating value per pound of combustible of a coal dry and free from ash, and containing 81.44% of fixed carbon, is found to be 15,803 B. T. U. As just shown, the total combustible (the sum of the fixed carbon and volatile matter) in the coal is 92.51. Hence, the calorific value of this coal is $15,803 \times .9251 = 14,619$ B. T. U. This agrees very closely with the calorimetric value, the difference being but 48 B. T. U., or .361%.
ILLUSTRATION 2.—*Kent's Method.*—Using coal No. 78, of the table given on pages 382 to 385, from Henryetta, Okla., the heating value of which is 12,620 B. T. U., and which contains by analysis: Moisture, 3.87%; volatile matter, 35.73%; fixed carbon, 50.05%; and ash, **10.35%**. The fixed carbon in the coal dry and free from ash is 58.35% and the volatile matter is 41.65%. By interpolating in the second column of the table giving the approximate heating value of coals, the calorific value of a coal per pound of combustible and containing 58.35% of fixed carbon on a dry and ash-free basis, is 14,463 B. T. U. But the total combustible (fixed carbon+volatile matter) in this coal is $50.05 + 35.73 = 85.78\%$, hence its heating value is $14,463 \times .8578 = 12,406$ B. T. U. This differs from the true calorific value by 214 B. T. U., or 1.6%. It will be noted, however, that this coal contains less than 60% of fixed carbon, and is, thence, within the group in which the formula does not apply within 4%.
Lord and Haas' Method.—Taking the coal just used and recalculating the analysis to include the sulphur, the analysis is: Moisture, 3.80%; ash, 10.15%; and sulphur, 1.95%. The recalculated volatile matter is 35.03%, and fixed carbon 49.07%. This coal is not included in the table of those for which the value of $K$ has been determined. It is possible, however, for illustrative purposes only, to assume a value for this constant.
In content of volatile matter this coal is not unlike No. 82 from the Pittsburg seam, at Ellsworth, Pa., 35.73% as against 34.83%. Using the value of $K$, 15,183, for the Pittsburg seam, and substituting in formula 1, gives for the calorific value of this coal 12,848 B. T. U. per lb. This is 228 B. T. U., or **1.80%**, greater than the true value, 12,620 B. T. U.

In the amount of fixed carbon, 49.07%, this coal is not unlike No. 75, a Hocking coal from Dixie, Ohio, carrying 46.08% of fixed carbon. Using the value of $K$, 14,265, for Hocking coal, and substituting in the formula, gives the calorific value of coal No. 78 to be 12,076 B. T. U. This is 544 B. T. U., or 4.31% less than the true value.

The illustrations show that while this method gives excellent results with those coals for which the value of $K$ has been experimentally determined, it cannot be relied on to give good results where $K$ is unknown. Nor do Messrs. Lord and Haas make any such claim for it.

## DETERMINATION OF HEATING VALUE OF COAL FROM AN ULTIMATE ANALYSIS

**Dulong's Formula.**—The available method of determining the heating value of coal from an ultimate analysis is based on the formula devised by Dulong and known by his name. The amount of heat obtained in the burning of 1 lb. of coal under theoretically perfect conditions is expressed as follows:

$$\text{Lb. Cal.} = 8,080C + 34,462\left(H - \frac{O}{8}\right) + 2,250S$$

in which $C$, $H$, $O$, and $S$ are the percentages of the elements the symbols represent and the coefficients are the calories evolved in burning 1 lb. of carbon, hydrogen, and sulphur, respectively. The figures within the parenthesis $\left(H - \frac{O}{8}\right)$ represent the *available hydrogen*, i. e., the amount of that element over and above that required to combine with the oxygen to form water. American engineers commonly employ the British thermal unit in place of the pound-calorie. However, if calculations are made in pound-calories, they may readily be reduced to British thermal units by multiplying by 1.8.

In terms of the British thermal unit, the formula becomes,

$$\text{B. T. U.} = 14,544C + 62,032H + 4,050S$$

In this formula $C$, $H$, $O$, and $S$ are the percentages of carbon, hydrogen, oxygen, and sulphur, respectively, as determined by an ultimate analysis, and 14,544, 62,032, and 4,050, are the number of British thermal units evolved in burning 1 lb. of those of the foregoing elements that are combustible.

ILLUSTRATIONS.—Using coal No. 103 from Pocahontas, Va.,

$$\text{B. T. U.} = 14,544 \times .8314 + 62,032 \times \left(.0458 - \frac{.0465}{8}\right) + 4,050 \times .0075 = 14,603$$

This is within 69 B. T. U., or .470% of the calorimeter value.

In the case of coal No. 78, from Henryetta, Okla.,

$$\text{B. T. U.} = 14,544 \times .6985 + 62,028 \times \left(.0514 - \frac{.1138}{8}\right) + 4,050 \times .0199 = 12,547$$

which is 73 B. T. U., or .58%, too low.

Dulong's formula may be relied on to give results within 2% of the true calorimetrically determined value, and the agreement is generally much closer, as has been shown. It should be remembered that if the more accurate values for the calorific powers of the combustible elements are substituted in the formula, the results obtained are lower than those had through the use of the earlier and approximate ones.

A comparison of the results obtained in calculating the heating values of coals No. 103 and No. 78, by the different methods available is here given in tabular form.

| Method Employed | No. 103 B. T. U. | No. 78 B. T. U. |
|---|---|---|
| Actual value by calorimeter | 14,672 | 12,620 |
| Dulong's formula | 14,603 | 12,547 |
| Kent's formula | 14,619 | 12,406 |
| Lord and Haas' formula | 14,564 | |
| Lord and Haas' formula, $K$ for Pittsburg coal, in No. 78 | | 12,848 |
| Lord and Haas' formula, $K$ for Hocking coal, in No. 78 | | 12,076 |

## PETROLEUM AS FUEL

Next to natural gas, petroleum is the ideal fuel, 1 lb. of it having a heating value about 50% greater than 1 lb. of average coal. Of the 209,556,048 bbl. produced in the United States in 1910, 24,586,108 bbl. were used for fuel by the railroads along the Pacific coast and in the Southwest, displacing, say, some 7,000,000 T. of coal. If to the consumption of the railroads is added that of steamships, central-station power plants, and other large industrial concerns, it is probable that the amount of oil used as fuel in the regions named is equivalent to about 20,000,000 T. of coal.

Petroleum is a dark greenish-black to light-brown oil produced by the decomposition of organic matter contained in the rocks, but whether the organic matter is of vegetable or animal origin is undetermined. The oil consists of a series of hydrocarbons that may be distilled off in a series of gradually increasing density as the temperature is increased. The residue, consisting of the least volatile portion, which commonly remains as a solid known as the *base*, affords a means of classifying oils into three groups: paraffin oils, asphalt oils, and olefin oils.

The oils in the first group are those produced in the eastern and middle western states and being limited in production and high in yield of very valuable light oils (gasoline, kerosene, etc.) are too high in cost to be used as fuel. In the second group come the oils of California and Texas, produced in large quantities and at low cost, and furnishing the vast bulk of the fuel oil used in the West and Southwest. In the third group are the oils of Baku, Russia, on the Caspian Sea, and, except in so far as they possibly displace American coal in foreign markets, are of no especial interest to the mining engineer.

**Composition of Crude Petroleum.**—Petroleum in the form in which it issues from the earth is known as *crude oil*. It usually contains from 83 to 87% of carbon; from 10 to 16% of hydrogen; and small amounts of oxygen, nitrogen, and sulphur. Crude oil contains from less than 1% to over 30% of water. The amount of water depends largely on the care with which the oil is pumped from the well. The oil from old producing wells commonly contains more water than that from wells newly drilled. In fact, in many districts, the percentage of water gradually increases during the life of the well, eventually the entire output being salt water. As the amount of water in the crude oil is uncertain and variable and as it separates out if left undisturbed, allowance must be made therefor in providing storage or, as more commonly called, *tankage*. The accompanying table gives the ultimate analyses of oils from various sources.

As stated, the various hydrocarbons composing crude petroleum may be separated by distillation at different temperatures; thus, gasoline is driven off by heating from 140° to 158° F.; a light benzine or naphtha at from 158° to 248° F.; heavier benzines at 248° to 347° F.; kerosene, or ordinary illuminating oil, at 338° F. and upwards; lubricating oils at 482° F. and above; paraffin wax at a higher temperature; leaving a tarry residuum that may be further distilled until nothing but a small quantity of coke remains in the still. If the distillation is stopped after the kerosene has been driven off, the residue may be used for fuel oil.

**Flash Point and Firing Point.**—If a sample of fuel oil or of crude oil is placed in an open cup and heat is applied, the oil will begin to vaporize and inflammable gases will be driven off. If, while the heating proceeds, a lighted match is passed at intervals over the surface of the oil and about ½ in. from it, a point will be reached at which the vapor rising from the oil will ignite and burn with a flicker of blue flame. The temperature of the oil when this flame first becomes apparent is termed the *flash point* of the oil. If the heating of the sample is continued, the vapors will be given off more rapidly and eventually they will ignite and burn continuously at the surface of the oil when the lighted match is brought near. The temperature of the oil when the burning becomes continuous is termed the *firing point* of the oil. The flash point and the firing point of an oil depend on the composition, specific gravity, and source of the oil. As a general rule, the heavier oils have a much higher flash point than the lighter ones and an attempt has been made to use this as a basis for classifying them. A specific gravity of .85 is taken as the basis, oils heavier than this having a flash point above 60° F., and oils lighter than this having a flash point lower than 60° F., although this is very far from always being true. It is obvious that a high flash point is very desirable in a fuel oil in order to avoid danger of explosion.

## ULTIMATE ANALYSES OF CRUDE PETROLEUM

| | Carbon | Hydrogen | Oxygen | Sulphur | Water | Specific Gravity | Flash Point Degrees | Calorific Power per Pound B. T. U. |
|---|---|---|---|---|---|---|---|---|
| United States | | | | | | | | |
| California......... | 85.04 | 11.52 | .99[1] | 2.45 | 1.40 | | | 17,871 |
| California......... | 81.52 | 11.51 | 6.92[1] | 0.55 | | | 230 | 18,667 |
| Kentucky......... | 85.20 | 13.36 | 1.11[2] | | | | | 20,635[3] |
| Ohio............. | 83.40 | 14.70 | 1.30 | 0.60 | | | | 19,580 |
| Ohio............. | 84.20 | 13.10 | .70[2] | | | .887 | | 19,539[3] |
| Ohio, Mecca...... | 86.30 | 13.07 | .63[2] | | | | | 20,660[3] |
| Pennsylvania, Franklin ...... | 84.90 | 14.10 | 1.40 | | | .886 | | 19,210 |
| Pennsylvania, Oil Creek........... | 82.00 | 14.80 | 3.20[2] | | | .816 | | 20,590[3] |
| Texas, Beaumont.. | 84.60 | 10.90 | 2.87 | 1.63 | | .924 | 180 | 19,060 |
| Texas, Beaumont.. | 83.30 | 12.40 | 3.83 | 0.50 | | .926 | 216 | 19,481 |
| Texas, Beaumont.. | 86.10 | 12.30 | .92[1] | 1.75 | | | | 19,060 |
| Texas............ | 87.15 | 12.33 | | 0.32 | | .908 | 370 | 19,338 |
| West Virginia ..... | 84.30 | 14.10 | 1.60[2] | | | .841 | | 21,240 |
| West Virginia ..... | 83.20 | 13.20 | 3.60[2] | | | .857 | | 20,052[3] |
| Foreign Countries | | | | | | | | |
| Austria, Galicia.... | 85.30 | 12.60 | 2.10[2] | | | .855 | | 20,105[3] |
| Austria, Galicia.... | 82.20 | 12.10 | 5.70[2] | | | .870 | | 18,416 |
| Borneo............ | 85.70 | 11.00 | 3.31 | | | | | 19,240 |
| Burmah........... | 83.80 | 12.70 | 3.50[2] | | | .875 | | 19,835[3] |
| Canada, West..... | 81.30 | 13.40 | 2.30[2] | | | .857 | | 19,998[3] |
| Canada, Petrolia... | 84.50 | 13.50 | 2.00[2] | | | .870 | | 20,552[3] |
| China, Fu-li-fu .... | 83.50 | 12.90 | 3.60[2] | | | .860 | | 19,910[3] |
| Germany, Hanover | 80.40 | 12.70 | 6.90[2] | | | .892 | | 19,079[3] |
| Germany, Pechelbronn........... | 85.70 | 12.00 | 2.30[2] | | | .892 | | 19,772[3] |
| Germany, Schwabweiler........... | 86.20 | 13.30 | .50[2] | | | .861 | | 20,794[3] |
| Italy, Parma ...... | 84.00 | 13.40 | 1.80[2] | | | .786 | | 20,436[3] |
| Java, Rembang.... | 87.10 | 12.00 | .90[2] | | | .923 | | 19,707[3] |
| Java, Tjabados.... | 83.60 | 14.00 | 2.40[2] | | | .827 | | 20,700[3] |
| Java, Gagor....... | 85.00 | 11.20 | 2.80[2] | | | .927 | | 19,137[3] |
| Roumania......... | 83.00 | 12.20 | 4.80[2] | | | .901 | | 19,350[3] |
| Russia, Baku...... | 87.40 | 12.60 | .10[2] | | | .882 | | 20,566[3] |
| Russia, Baku...... | 86.60 | 12.30 | 1.10[2] | | | .938 | | 20,187[3] |
| Russia, Baku...... | 85.30 | 11.60 | 3.10[2] | | | .954 | | 19.404[3] |

**Calorific Value of Fuel Oil.**—The combustible elements in oil are the same as those in coal, namely, carbon and hydrogen, and usually some sulphur. The calorific value per pound may be determined by means of Dulong's formula, which is applied exactly as in the case of an ultimate analysis of coal. This formula was used to calculate the greater number of the calorific powers given in the accompanying table. From the results of available tests, it is found that the heat of combustion per pound of fuel oil varies from 17,000 to 21,000 B. T. U., California oil averaging about 18,600 B. T. U., and Texas oil some 1,000 B. T. U. higher. At 18,600 B. T. U. per lb. and assuming an average specific gravity of .885, 1 gal. of oil weighs 7.37 lb., and will yield 137,082 B. T. U., and a barrel of 42 gal. will weigh 310 lb. and will yield 5,766,000 B. T. U. Using the same specific gravity and a calorific value of 19,600 B. T. U. per lb., 1 gal. of oil will develop 144,452 B. T. U., and a barrel 6,076,000 B. T. U. The theoretical comparative fuel values of coals of different heating powers

[1] Includes nitrogen.
[2] Oxygen, by difference.
[3] Calculated by means of Dulong's formula.

and of fuel oil yielding, respectively, 18,600 and 19,600 B. T. U., per lb. are given in the following table.

This table, however, is of more theoretical than practical interest, as it is based on the assumption that combustion is perfect whether oil or coal is used, and that, in consequence, the efficiency of an oil-burning and of a coal-burning boiler is the same. This is far from the case, as the efficiency of a properly designed oil-burning boiler is the greater. To this must be added the advantages outlined, so that only an actual test of the two fuels will determine which is the more economical under a given set of conditions.

## COMPARATIVE VALUE OF COAL AND OIL AS FUEL

| Thermal Value of Different Coals B. T. U. per Pound | At 18,600 B. T. U. per Lb. | | | At 19,600 B. T. U. per Lb. | | |
|---|---|---|---|---|---|---|
| | Pounds of Coal Equivalent to 1 Lb. of Petroleum | Pounds of Coal Equivalent to 1 Bbl. of Petroleum | Barrels of Petroleum Equivalent to 1 T. of Coal | Pounds of Coal Equivalent to 1 Lb. of Petroleum | Pounds of Coal Equivalent to 1 Bbl. of Petroleum | Barrels of Petroleum Equivalent to 1 T. of Coal |
| 10,000 | 1.860 | 577 | 3.47 | 1.960 | 608 | 3.29 |
| 11,000 | 1.691 | 524 | 3.82 | 1.782 | 552 | 3.62 |
| 12,000 | 1.550 | 481 | 4.16 | 1.633 | 506 | 3.95 |
| 13,000 | 1.431 | 444 | 4.51 | 1.508 | 467 | 4.28 |
| 14,000 | 1.329 | 412 | 4.85 | 1.400 | 434 | 4.61 |
| 15,000 | 1.240 | 384 | 5.21 | 1.307 | 405 | 4.95 |

**Advantages and Disadvantages of Oil Fuel.**—Babcock and Wilcox summarize the advantages of fuel oil as follows:

1. The cost of handling is much lower, the oil being fed by simple mechanical means, resulting in

2. A general labor saving throughout the plant in the elimination of stokers, coal passers, ash handlers, etc.

3. For equal heat value, oil occupies very much less space than coal. This storage space may be at any distance from the boiler without detriment.

4. Higher efficiencies and capacities are obtainable with oil than with coal. The combustion is more perfect as the excess air is reduced to a minimum; the furnace temperature may be kept practically constant, as the furnace doors need not be opened for cleaning or working fires; smoke may be eliminated with the consequent increased cleanliness of the heating surfaces.

5. The intensity of the fire can be almost instantaneously regulated to meet load fluctuations.

6. Oil, when stored, does not lose in calorific value as does coal, nor are there any difficulties arising from disintegration, such as may be found where coal is stored.

7. Cleanliness and freedom from dust and ashes in the boiler room with a consequent saving in wear and tear on machinery; little or no damage to surrounding property due to such dust.

The disadvantages of oil are:

1. The necessity that the oil have a reasonably high flash point to minimize the danger of explosions.

2. City or town ordinances may impose burdensome conditions relative to location and isolation of storage tanks, which in the case of a plant situated in a congested portion of the city, might make the use of this fuel prohibitive.

3. Unless the boilers and furnaces are especially adapted for the use of this fuel, the boiler upkeep cost will be higher than if coal is used.

The relative cost of the two fuels per unit of power produced is, of course, the deciding factor in the premises, and this varies so greatly in such short intervals of time, even from day to day, that current quotations of the delivered cost of both fuels must always be used in making calculations of the savings possible when substituting oil for coal, and vice versa.

## GASEOUS FUELS

**Kinds of Gas.**—The different gases used as fuel are the following, arranged in the order of their heating value: (1) *Natural gas*, which is obtained from wells in different parts of the world; (2) *illuminating gas*, or *coal gas*, which is made either by distilling coal in retorts or by enriching water gas with the volatile matter distilled from cannel coal or with vapors distilled from petroleum; (3) *coke-oven gases*, which are mainly those coming from by-product ovens, although occasionally the gases from the beehive ovens are used under boilers; (4) *water gas*, which is made by blowing steam through a bed of glowing anthracite or coke, by the reaction $C + H_2O = CO + 2H$; (5) *producer gas*, which is made by blowing air into burning bituminous coal, in which case the volatile matter, including condensible tarry vapors, is distilled, and the coke is burned to carbon monoxide; producer gas is also made by blowing air into burning anthracite, thus producing carbon monoxide; (6) *combined water gas* and *producer gas*, which is made by blowing air mixed with steam into a producer charged with burning bituminous coal; (7) *blast-furnace gas*, which is the waste gas coming from the top of a blast furnace, and which contains a certain amount of carbon monoxide available as fuel.

The composition and heating value of different gases, as given by H. A. Humphrey in the Proceedings of the Institution of Civil Engineers of Great Britain, are shown in the following table.

### ANALYSES AND HEATING VALUES OF VARIOUS GASES

| Constituent Gases | Pittsburg Natural Gas Volume Per Cent. | Coal Gas (Illuminating) Volume Per Cent. | Solvay Coke-Oven Gas Volume Per Cent. | Lencanchez Producer Gas From Anthracite Volume Per Cent. | Dowson Producer Gas From Anthracite Volume Per Cent. | Siemen's Producer Gas From Bituminous Coal Volume Per Cent. | Mond Producer Gas From Bituminous Coal Volume Per Cent. |
|---|---|---|---|---|---|---|---|
| Hydrogen, $H$ | 22.0 | 48.0 | 56.9 | 20.0 | 18.7 | 8.6 | 24.8 |
| Marsh gas (methane), $CH_4$ | 67.0 | 39.5 | 22.6 | | .3 | 2.4 | 2.3 |
| $C_nH_{2n}$ gases | 6.0 | 3.8 | 3.0 | 4.0 | .3 | | |
| Carbon monoxide, $CO$ | .6 | 7.5 | 8.7 | 21.0 | 25.1 | 24.4 | 13.2 |
| Carbon dioxide, $CO_2$ | .6 | | 3.0 | 5.0 | 6.6 | 5.2 | 12.9 |
| Nitrogen, $N$ | 3.0 | .5 | 5.8 | 49.5 | 49.0 | 59.4 | 46.8 |
| Total volume (approximate) | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Total combustible gases | 95.6 | 98.8 | 91.2 | 45.0 | 44.4 | 35.4 | 40.3 |
| Heating value (B. T. U. per cubic foot at 64° F.) | 892.4 | 686.0 | 511.0 | 207.5 | 160.0 | 134.5 | 154.6 |

Some additional analyses of natural, producer, and coke-oven gases, together with their average heating values, are given in another table.

The heating value per pound and per cubic foot, measured at 32° F., and atmospheric pressure of the several constituents of a mixed fuel are given in the table on page 400. The figures in the first column are those of Favre and Silbermann except in the two cases noted. Other heating values differing slightly from these are given by different authorities, owing to a difference in the experiments by which the values have been determined. In the third column are shown approximate figures, giving the British thermal units per pound of combustible, that are within the limits of error of chemical analysis.

**Blast-Furnace Gases.**—Blast-furnace gas varies greatly in its composition, and in six analyses made in 1 da., the carbon dioxide varied from 6.6 to 7.7%, the carbon monoxide from 20.1 to 31.7%, and the nitrogen and hydrocarbons

## ANALYSES OF NATURAL, PRODUCER, AND COKE-OVEN GASES

| Constituent | Natural Gas | | | | Producer Gas | | | By-Product Coke-Oven Gases | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | No. 1 Per Cent. | No. 2 Per Cent. | No. 3 Per Cent. | No. 4 Per Cent. | Maximum Per Cent. | Minimum Per Cent. | Average Per Cent. | *Otto-Hoffman | | Oven Not Stated |
| | | | | | | | | Rich Gas (Illuminating) | Poor Gas (Fuel Gas) | |
| Carbon dioxide, $CO_2$ | .80 | .60 | .58 | 1.00 | 3.0 | 8.0 | 5.5 | 2.9 | 2.0 | 1.30 |
| Carbon monoxide, $CO$ | 1.00 | .80 | .78 | 2.10 | 18.0 | 25.0 | 23.0 | 6.2 | 5.0 | 6.00 |
| Oxygen, $O$ | 1.10 | .80 | .98 | .80 | | .5 | | .1 | .4 | .45 |
| Ethylene, $C_2H_4$ | .70 | .98 | | | | .5 | | 5.0 } | 2.5 } | 3.40 } |
| Ethane, $C_2H_6$ | 3.60 | 5.50 | 7.92 | 5.20 | | | | | | |
| Marsh gas (methane), $CH_4$ | 72.18 | 65.26 | 60.70 | 57.85 | 1.0 | 4.0 | 3.0 | 37.4 | 29.2 | 33.50 |
| Hydrogen, $H$ | 20.62 | 26.12 | 29.03 | 9.64 | 6.0 | 12.0 | 8.0 | 44.3 | 51.8 | 50.45 |
| Nitrogen, $N$ | | | | 23.41 | 58.0 | 65.0 | 60.5 | 4.1 | 9.1 | 4.90 |
| Average heating value B. T. U. per cubic foot | | 1.007 | | | 120 to 140 | | | 707.8 | 515.0 | |

*From the Everett Plant near Boston, Mass., where the gases coming from the ovens are, separated into a rich portion used for illuminating purposes and a poor portion used for fuel purposes.

## HEATING VALUE OF GASES AT 32° F.

| Substance | Burned in Oxygen | | Approximate Values per Pound B. T. U. |
| --- | --- | --- | --- |
| | Per Pound B. T. U. | Per Cubic Foot B. T. U. | |
| Hydrogen to $H_2O$...................... | 62,032 | 346.75 | 62,000 |
| Carbon to carbon dioxide, $CO_2$.......... | 14,544 | | 14,600 |
| Carbon monoxide, $CO$......... | 4,451 | | 4,450 |
| Carbon monoxide, $CO$, to carbon dioxide $CO_2$, per unit of $CO$................. | 4,325 | 338.00 | 4,300 |
| Carbon monoxide, $CO$, to carbon dioxide, $CO_2$, per unit of carbon $C$............. | | | 10,150 |
| Marsh gas (methane), $CH_4$, to $H_2O$ and $CO_2$................................ | 23,513 | 1,050.00 | |
| Ethylene (olefiant-gas), $C_2H_4$, to $H_2O$ and $CO_2$............................. | 21,344 | 1,568.00 | |
| Benzole gas, $C_6H_6$, to $H_2O$ and $CO_2$ ....... | 17,847 | | |
| Acetylene, $C_2H_2$, to $H_2O$ and $CO_2$......... | 18,196* | 1,351.60 | |
| Sulphur to $SO_2$.......................... | 4,050† | | 4,000 |

from 60.5 to 72.2% by volume. The heating value calculated from the average analyses was 1,175 B. T. U. per lb., and at the temperature of 584° F., at which it entered the steam boiler furnace, it carried 140 B. T. U. per lb. additional heat. The 1,175 B. T. U. per lb. are equal to about 94 B. T. U. per cu. ft., measured at 32° F., or only about one-tenth of the value of 1 cu. ft. of natural gas.

Modern blast furnaces, working on Bessemer iron, will produce about 8,500 lb. of available gas per ton of pig iron made. This gas will have an average composition of about $CO_2$, 10.80%; $CO$, 28.00%; $H$, 2.50%; $CH_4$, .2%; and $N$, 58.50%; and will yield 85 to 100 B. T. U. per cu. ft., or 1,050 to 1,250 B. T. U. per lb., assuming that the weight of 1 cu. ft. thereof is .081 lb. The heat from the waste gases of the blast furnaces in the United States, which produced about 26,700,000 T. of pig iron in 1910, was sufficient, if entirely utilized, to have displaced 10,400,000 T. of coal having a fuel value of 12,000 B. T. U. per lb.

**Natural Gas.**—Natural gas, produced from drilled wells, usually in or near the oil fields, is a valuable fuel in many states. The total production in 1910 was 509,155,309,000,000 cu. ft. The value is placed at $70,756,158, or 13.90 c. per 1,000 cu. ft., varying from 7.16 c. per cu. ft. in West Virginia to 73.77 c. in South Dakota. Assuming a fuel value of 1,000 B. T. U. per cu. ft., this gas displaced 21,000,000 T. of coal rated at 12,000 B. T. U. per lb. The 1,327,722 domestic consumers used 169,823,030,000,000 cu. ft. of gas, for which they paid an average price of 24.4 c. per 1,000 cu. ft., the rate charged varyingf rom 16.9 c. per 1,000 ft. in Oklahoma to $1 in Michigan. The 18,267 industrial establishments used 339,332,279,000,000 cu. ft., for which they paid an average price of 8.63 c. per 1,000 cu. ft., the rate charged varying from 5.1 c. per 1,000 cu. ft. in West Virginia to 69.5 c. in South Dakota.

The great variation in the composition of natural gas from different fields is shown in the table on page 399. Assuming a calorific value of 1,000 B. T. U. per cu. ft., 24,000 cu. ft. of gas will yield as much heat as 1 T. of coal rated at 12,000 B. T. U. per lb. To displace 1 T. of high-grade Pocahontas or New River coal rated at, say, 14,750 B. T. U. per lb., will require 29,500 cu. ft. of gas. The number of cubic feet of gas required to displace 1 T. of coal when the calorific powers of each are known, may be found by dividing twice the calorific power of the coal per pound by the ratio the calorific power of the gas bears to 1,000 B. T. U.

EXAMPLE.—(a) How many thousand cubic feet of gas having a calorific power of 925 B. T. U. per cu. ft., will be required to displace 1 T. of coal rated at 13,965 B. T. U. per lb.? (b) How many cubic feet will be required if the heating value of the gas is 1,155 B. T. U. per lb.?

*Calculated. †N. W. Lord.

SOLUTION.—(a) It is assumed that the efficiency of the boiler is the same regardless of the fuel used. $\frac{13,965 \times 2}{.925} = 30,195$ cu. ft.

(b) $\frac{13,965 \times 2}{1.155} = 24,181$ cu. ft.

It is possible, also, to deduce from this that 1,000 cu. ft. of the first gas $= 2,000 \div 30.195 = 66.23$ lb. of the coal, and 1,000 cu. ft. of the second gas $= 2,000 \div 24.181 = 82.71$ lb. of the coal. Likewise, 1 lb. of the coal is equal to $1,000 \div 66.23 = 15.09$ cu. ft. of the first gas, and is equal to $1,000 \div 82.71 = 12.10$ cu. ft. of the second gas.

In the matter of natural gas in the Pittsburg, Pa., district, a committee of the Western Society of Engineers report that 1 lb. of good coal is equal to $7\frac{1}{2}$ cu. ft. of natural gas. When burned with just enough air, its temperature of combustion is 4.200° F. The Westinghouse Air Brake Company found from experiment that 1 lb. of Youghiogheny coal is equal $12\frac{1}{4}$ cu. ft. of natural gas, or 1,000 cu. ft. of natural gas is equal 81.6 lb. of coal. Indiana natural gas gives 1,000,000 B. T. U. per 1,000 cu. ft., and weighs .045 lb. per cu. ft.

Natural gas, when used for steam raising, is commonly under a pressure of about 8 oz. and is fired through a large number of small burners to prevent what is known as *lancing*, or the issuing of the flame in a long jet similar in appearance and largely in action to that of a blow-pipe.

**By-Product Gas.**—By-product gas, so called, is given off in large amount in connection with the numerous by-product processes of coking. When the coking plant is situated at a steel works, the gas is used for the generation of steam; if situated in or near a city, the gas is enriched and used for illuminating and general fuel purposes, as is done at the plant of the New England Gas and Coke Company at Everett, near Boston, Mass.

In 1910 there were 4,078 by-product ovens in the United States, of which all but 27 were in blast. They consumed 9,529,042 T. of coal and produced 7,138,734 T. of coke, an average yield of 74.9%. A certain amount of the gas given off by the coking coal is required to furnish the heat demanded by the process; the gas not so used, the surplus or available gas, amounted, in 1910, to 27,692,858,000 cu. ft., valued at \$3,017,908, or about 11 c. per 1,000 cu. ft. The production appears to be, from these statistics, at the rate of 2,900 cu. ft. per T. of coal charged, or 3,880 cu. ft. per T. of coke drawn. These figures are approximate and to be used with caution because at a small number of plants no attempt was made to save the gas, and at another, the yield of gas had to be estimated.

The amount of gas available depends on so many factors that it is impossible to predicate just how much and what grade of gas a given coal will yield. Obviously, the amount of gas available from coking 1 T. of high-volatile Connellsville coal is much greater than that to be had from a lean semibituminous coal. The composition of the gas will vary with the composition of the coal and with the process employed in coking. The gas will also depend, in a very great measure, on whether the ovens are designed primarily for coke making, as at a steel or iron plant, or whether intended for the manufacture of illuminating gas and the recovery of by-products (in this case, the coke being a by-product), as at Everett, near Boston.

In regard to the yield of gas per ton of coal charged, the 280 Koppers regenerative ovens at Joliet, Ill., when charged with a mixture of 80% Pocahontas and 20% high-volatile coal, yielded 84% of the coal charged as coke and 10,000 cu. ft. of gas per T., of which rather more than 5,000 cu. ft. was surplus and available as fuel. The same oven, operating in Germany on a coal not dissimilar to the Connellsville, Pa., yields between 5,000 and 6,000 cu. ft. of available gas per T. Mr. F. E. Lucas, superintendent of the coke-oven department of the Dominion Iron and Steel Co., Sydney, N. S., estimates the average yield of surplus gas from average coal in by-product ovens as 5,000 cu. ft. per T. of coal charged, and that this gas has a fuel value of from 450 to 500 B. T. U. per cu. ft. At Mulheim-on-the-Ruhr, Germany, a plant of 50 Koppers ovens supplies that city and Barmen, 40 mi. distant, with gas; these ovens take 8 to 10 T. of coal at a charge. The time of coking is 24 hr. but only the richer portion of the gas, that evolved from the second to the twelfth hour, which is about 50% of the yield, is distributed. During these 10 hr., each oven produces 70,600 cu. ft. of gas of a calorific value well over 600 B. T. U. per cu. ft., with the average composition: $CO_2$, 1.2%; $CO$, 6.8%; $H$, 49.5%; $CH_4$, 38.3%; and $N$, 4.2%. Mr. Edwin A. Moore, estimates that a plant of 100 United-Otto ovens consuming 750 T. of coal will produce 3,472,000 cu. ft.

of available gas, or at the rate of 4,630 cu. ft. of gas per T. of coal charged. Mr. Moore calls attention to the fact that the gases given off during the early portion of the coking process (first 10 hr.) have a fuel value of 650 B. T. U. per cu. ft., and during the latter period, but 525 B. T. U.  It would appear that the average coal will yield about 4,500 cu. ft. of available gas per T., and that the gas has a heating value of some 500 B. T. U. per cu. ft.; on which basis the gas has about 8.5% of the heating value of the coal from which it is made, assuming the latter to be rated at around 13,000 B. T. U. per lb.

On the basis of containing 500 B. T. U. per cu. ft., the 27,692,858,000 cu. ft. of by-product gas marketed in 1910 displaced about 580,000 T. of coal with a fuel value of 12,000 B. T. U. per lb.

As this gas is highly charged with moisture, arrangements must be made for getting rid of the condensed water.  Further, as the gas carries large amounts of tar and heavy hydrocarbons that clog the burners, arrangements must be made whereby they may be cleaned out by blowing steam through them.

**Coke-Oven Gas.**—Beehive coke-oven gases are used at numerous mines for the generation of the necessary steam for operating the power plant connected therewith.  As these gases do not contain any combustible portion, their value lies in their sensible heat.  The total horsepower available may be obtained from the formula, 

$$HP = \frac{W(T-t)s}{33,000}$$

in which

$W$ = weight of gases passing per hour;
$T$ = temperature of gases entering boiler;
$t$ = temperature of gases leaving boiler;
$s$ = specific heat of gases.

As the temperature of the gases entering the boiler is rarely as great as 2,000° F., and as with coal firing the furnace temperature ranges from 2,500° to 3,000°, the heating surface first passed over will not absorb as much heat in the waste-heat boilers, and, consequently, the heating surface per boiler horsepower should be increased.  From 12 to 15 ft. in water-tube boilers is about right, and from 15 to 20 ft. in return tubular and shell boilers.

In a series of tests with a high-class water-tube boiler, the following results were obtained with the waste heat from the coke ovens:  Temperature of the gases entering the boiler, 1,720° F.; temperature of gases leaving the boiler, 650° F.; boiler heating surface, 1,611 sq. ft.; water evaporated per hour from and at 212° F., 6,465 lb.; water evaporated from and at 212° per oven per hour, 294 lb.; water evaporated from and at 212° per lb. of coal coked, 1.7 lb.; water evaporated from and at 212° per sq. ft. of heating surface, 4 lb.

At the Sydney Mines of the Nova Scotia Steel and Coal Company, the waste heat from coke ovens of the Bauer type is used in the generation of steam.  In referring to a test run made upon a single boiler, Mr. Thomas J. Brown, the superintendent, obtained these results:  Average horsepower per hour, 331; maximum horsepower per hour, 436; minimum horsepower per hour, 179; evaporation from and at 212° per lb. of coal charged into the ovens, 1.18 lb. of water; coal charged into the ovens per boiler horsepower, 29.23 lb. Mr. Brown further states, that the boilers have 3,140 sq. ft. of heating surface, with a flue temperature at the rear of the boilers of between 600° and 700°, and that the proportion of about 9.5 sq. ft. of heating surface per horsepower developed seems to be about right for this class of fuel.

At Marianna, Pa., 2,000 H. P. are developed from a battery of 75 beehive ovens at a cost of $2.50 per da. as opposed to a cost of $63.50 if coal were used. At the Stag Cañon Mines, Dawson, N. Mex., 2,400 H. P. are developed from the waste gases of 218 beehive ovens, the gases being delivered under the boilers at temperatures ranging from 1,800° F. to 2,600° F. and leaving the stack at from 600° to 1,150° F.  Mr. R. D. Martin, referring to the waste heat ovens in use at Agujita and Lampacitos, Coahuila, Mex., estimates that each coke oven is capable of developing 12 boiler H. P. from a coal containing, volatile matter, 21.1%; fixed carbon, 67.4%; and ash, 11.5%.  Here the temperature in the firebox, as determined with Saeger cones, was as high as 2,600° F. Mr. Howard N. Eavenson, in connection with the Continental Coke Co., No. 1 plant, estimates that the waste heat from 6 T. of coal charged into a coke oven will yield as many boiler horsepower as 1 T. of coal directly fired.

**Coal Gas.**—Coal gas, frequently called *illuminating gas* from the use to which it is ordinarily put, is made by heating bituminous coal high in volatile matter in fireclay retorts of a semielliptic cross-section.  The retorts are about

15 in. high by 26 in. wide inside, and 9 to 10 ft. long if single-ended, or 18 to 20 ft. long if double-ended. The retort walls are about 4 in. thick and each retort is connected with a pipe that allows the gases to escape as fast as formed. After passing through various devices to remove the ammonia, tar, and sulphur, the gas passes into a gas holder and is ready for distribution.

Analyses of typical gas coals from the Pittsburg, Pa., field are given in the following table.

## ANALYSES OF GAS COALS

| Constituents | Westmoreland Coal Company | | | Pennsylvania Gas Coal Company | | |
|---|---|---|---|---|---|---|
| | South Side Mine | Foster Mine | Larri- mer No. 2 | Irwin No. 1 | Irwin | Sewick- ley |
| Water............. | 1.410 | 1.310 | 1.560 | 1.78 | 1.280 | 1.490 |
| Volatile matter.... | 37.655 | 37.100 | 39.185 | 35.36 | 38.105 | 37.153 |
| Fixed carbon...... | 54.439 | 55.004 | 54.352 | 59.29 | 54.383 | 58.193 |
| Sulphur............ | .636 | .636 | .643 | .68 | .792 | .658 |
| Ash............... | 5.860 | 5.950 | 5.950 | 2.89 | 5.440 | 2.506 |
| Total......... | 100.000 | 100.000 | 100.000 | 100.00 | 100.000 | 100.000 |

Under ordinary conditions 1 T. of such coal should produce about 10,000 cu. ft. of gas of 17 c. p., 1,400 lb. of coke, 12 gal. of tar, and 4 lb. of ammonia.

The following may be considered as the average composition of purified coal gas:

|  | *Per Cent.* |
|---|---|
| Hydrocarbon vapors.............................. | .6 |
| Heavy hydrocarbons............................. | 4.4 |
| Carbon dioxide, $CO_2$.............................. | 3.4 |
| Carbon monoxide, $CO$............................ | 10.1 |
| Marsh gas, $CH_4$................................. | 30.6 |
| Oxygen, $O$..................................... | .3 |
| Hydrogen, $H$................................... | 45.9 |
| Nitrogen, $N$................................... | 4.7 |
| Total......................................... | 100.0 |

The use of illuminating gas as fuel for steam-raising is limited by its cost which, while sometimes as low as 80 c. per 1,000 cu. ft., is usually about $1. A certain amount is used directly in small gas engines in the larger cities, but the larger amount is used for illumination or in cooking ranges, domestic heating stoves, and the like. Coal gas made from gas coals in retorts is being largely displaced by water gas.

**Water Gas.**—Water gas contains the same combustible constituents as coal gas but not in the same proportions. It is made commercially by the contact of steam with incandescent carbon, in the form of anthracite or coke, which decomposes the steam separating the hydrogen from the oxygen. The oxygen takes up carbon from the coal or coke and forms carbon monoxide, along with a small amount of carbon dioxide. The resultant gases therefore are mainly hydrogen and carbon monoxide mechanically mixed together. This is what is called *blue*, or *uncarbureted, water gas*. It burns with a non-luminous flame and is consequently useless for lighting purposes, except in incandescent lamps of the Welsbach type. In actual practice, this water gas is always enriched with oil gas, which furnishes the hydrocarbons necessary to make a luminous flame. The oil gas was made separately in many of the older forms of apparatus, but it is now commonly produced in the same apparatus in which the water gas is made.

The only impurity that must be removed from water gas is hydrogen sulphide, which is formed from the sulphur that is always present in varying amounts in the coal or coke and sometimes in the oil. The hydrogen sulphide

is removed by purification with lime or iron oxide in the same way that the purification of coal gas is accomplished.

Carbon dioxide, which is formed either by imperfect contact of the steam with the incandescent carbon, or because the temperature of the carbon is too low, is not a dangerous impurity, but is merely an inert gas incapable of combustion. It, however, absorbs heat when the gas is burned, and is consequently injurious to the heating and lighting power. It can be removed by purification with lime, but this is not necessary if the generating apparatus is handled properly, as the quantity made will be very small. No ammonia is produced.

The following is a volumetric analysis of purified water gas:

|  | Per Cent. |
|---|---|
| Hydrocarbon vapors | 1.2 |
| Heavy hydrocarbons | 12.6 |
| Carbon dioxide, $CO_2$ | 3.0 |
| Carbon monoxide, $CO$ | 28.0 |
| Marsh gas, $CH_4$ | 20.2 |
| Oxygen, $O$ | .4 |
| Hydrogen, $H$ | 31.4 |
| Nitrogen, $N$ | 3.2 |
| Total | 100.0 |

Water gas requires from 30 to 40 lb. of coal or coke per 1,000 cu. ft. of gas made, and from 4 to 5 gal. of oil, depending on the candlepower required. Usually between 5 and 6 c. p. is obtained from each gallon of oil used. The specific gravity of 24 c. p. water gas is about .625, air being taken as unity.

Pure uncarbureted water gas has no perceptible odor, but the carbureted gas has an odor fully as strong as coal gas. This is mainly due to the hydrocarbons from the oil that is used for enriching. It should be noted that these hydrocarbons are not added if the gas is to be used for heating or in gas engines.

**Producer Gas.**—Producer gas is made in a cylindrical riveted shell of boiler plate, lined with firebrick. A thick bed of fuel is maintained in the bottom of the producer and through this is passed a moderate supply of air, with or without water vapor or steam. By properly regulating the air supply, a partial or incomplete combustion of the fuel is maintained, resulting in the gradual consumption of all the combustible matter. The coke, instead of remaining as a by-product, as in the manufacture of coal gas or in by-product or retort coke ovens, is all consumed in making the gas. When dry air alone is forced through the fire, the resulting gas is known as *air gas* and is the true producer gas; when the air is mixed with steam or water vapor the resulting gas is called *mixed gas*, and is frequently made in producers; and when air is not used at all and steam alone is forced through the fire, the product is called water gas as previously explained.

The quantity of producer gas derivable from 1 T. of fuel will vary according to the fuel used, the type of producer plant, and the method of operating. The United States Geological Survey, at its Fuel Testing Plant at the Louisiana Purchase Exposition, held in St. Louis, in 1904, made an exhaustive series of tests of American coals used in gas producers, and from its reports the following tables, etc., are taken.

### QUANTITY OF GAS PRODUCED PER POUND OF FUEL IN AN UP-DRAFT PRESSURE PRODUCER

| Fuel | Maximum | | Minimum | | Average | |
|---|---|---|---|---|---|---|
| | As Fired Cubic Feet | Dry Cubic Feet | As Fired Cubic Feet | Dry Cubic Feet | As Fired Cubic Feet | Dry Cubic Feet |
| Bituminous coal | 100.8 | 100.3 | 37.0 | 40.9 | 60.5 | 64.7 |
| Lignite | 45.9 | 52.8 | 26.1 | 38.8 | 35.8 | 45.7 |
| Peat | | | | | 30.3 | 38.3 |

The yield of gas, in cubic feet per pound of dry fuel, which may be expected in the up-draft producer from various fuels is, roughly, as follows: Coke or charcoal, 90; anthracite, 75; bituminous coal, 65; lignite, 46; and peat, 38. On the basis of the Survey's tests the yield of gas and the heat value of the gas per ton of fuel as fired are approximately as in the table here given.

**YIELD AND HEAT VALUE OF GAS PER TON OF FUEL AS FIRED IN AN UP-DRAFT PRESSURE PRODUCER**

| Kind of Fuel | Cubic Feet of Gas per Ton of Fuel as Fired | British Thermal Units in Gas, per Cubic Foot | British Thermal Units in Gas, per Ton of Fuel as Fired |
|---|---|---|---|
| Coke or charcoal........... | 170,000 | 140 | 23,800,000 |
| Anthracite................ | 140,000 | 135 | 19,000,000 |
| Bituminous coal........... | 120,000 | 152 | 18,300,000 |
| Lignite................... | 72,000 | 158 | 11,400,000 |
| Peat...................... | 60,000 | 175 | 10,000,000 |

It will be noted from this table that while the inferior fuels yield less gas per ton, as might be expected, the heating value of the gas, in British thermal units per cubic foot, is greater than in gas made from high-class coals.

**Gas Producers.**—There are three general types of gas producer in use. In the *suction* type, the drawing of the air and steam through the fire and, consequently, the generation of the gas, is accomplished by the suction in the engine cylinder in which the gas is used. While the gases are scrubbed, etc., to get rid of the tar that otherwise would clog the cylinders, the absence of storage tanks for the gas and the fact that the suction of the engine causes the operation of the producer, renders absolute separation of the tar difficult if not impossible. Hence, only low volatile coals are adapted to use in this type of producer, and because the price of such coals is always high, suction plants, though numerous, are of comparatively small power, few exceeding 300 H. P. each, and most of them not exceeding 100 H. P.

The *up-draft pressure producer* is the common American type in which the gas is developed under a slight pressure due to the introduction of the air and steam blasts, and the gas is stored in holders until required by the engine. As the generation of the gas is independent of the suction stroke of the engine, tar and other impurities may be removed by suitable devices and hence the use of bituminous coal, lignite, and peat is possible. This form of producer is offered in many types, some of which are without gas holders and are proving eminently satisfactory. If the holder is omitted, automatic devices must be introduced for controlling the pressure and the supply of gas to the engine.

**TYPICAL ANALYSES BY VOLUME OF PRODUCER GAS**

| | From Bituminous Coal Per Cent. | | From Lignite Per Cent. | | From Peat Per Cent. | |
|---|---|---|---|---|---|---|
| | U. D. | D. D. | U. D. | D. D. | U. D. | D. D. |
| Carbon dioxide, $CO_2$....... | 9.84 | 6.22 | 10.55 | 11.87 | 12.40 | 10.94 |
| Oxygen, $O_2$............... | .04 | .13 | .16 | .01 | | .41 |
| Ethylene, $C_2H_4$........... | .18 | .01 | .17 | | .40 | .06 |
| Carbon monoxide, $CO$.... | 18.28 | 21.05 | 18.72 | 16.01 | 21.00 | 16.91 |
| Hydrogen, $H_2$............. | 12.90 | 12.01 | 13.74 | 14.66 | 18.50 | 10.19 |
| Methane, $CH_4$............. | 3.12 | .49 | 3.44 | .98 | 2.20 | .66 |
| Nitrogen, $N_2$............. | 55.64 | 60.09 | 53.22 | 56.37 | 45.50 | 60.83 |

In both the foregoing types of producer, the extraction of the tar removes a large part of the heat value of the gas. If the tar can be sold at a good price this may not make much difference, but where the tar is thrown away the loss is sufficient to warrant the attempt to devise some means of converting this tar into gas of suitable quality for engine use. To this end *down-draft producers* are coming into general use and in them the gases are drawn down through the bed of coal and the tar is thereby decomposed into fixed, combustible gases.

Typical analyses of gases made from the same fuels in the up-draft (U. D.) and down-draft (D. D.) producer, the percentages being by volume, are given in the preceding table.

In the matter of steam raising it is questionable if better results are obtained by using the gas made from the coal than by firing the coal directly under the boilers, especially in the case of good grades of coal from subbituminous to anthracite, but many fuels, notably peat and some of the true lignites, that give indifferent results when fired directly under a steam boiler, give most excellent results when fired as gas. Further, almost any material containing carbon will yield fuel gas in the producer, even bituminous shale, saw-dust, wood pulp, cornstalks, and the like. It is practically impossible to predicate the yield of gas and the quality thereof of a coal from its analysis. Tests in the various types of producers are required for this purpose.

Strictly speaking, the gaseous fuels previously described are all producer gases, except natural gas, coal gas (illuminating gas), and the waste heat gases from beehive coke ovens. These various producer gases are not commonly used for steam raising under boilers, but are a direct source of power in internal combustion engines. It must be noted that as carbon monoxide is one of the most important heat-producing constituents of all these gases, extreme caution must be observed in inhaling them owing to their highly poisonous nature.

# BOILERS

## STEAM

### PROPERTIES OF STEAM

**Saturated Steam.**—If water is put in a closed vessel and heat is applied until boiling occurs and steam is given off, the pressure and the temperature of the steam will be the same as those of the water. The steam thus produced is known as *saturated steam;* that is, saturated steam is steam whose temperature is the same as that of boiling water subjected to the same pressure. Its nature is such that any loss of heat will cause some of the steam to condense, provided the pressure is not changed. Saturated steam that carries no water particles with it is called *dry saturated steam;* if it contains moisture it is called *wet steam.* At every different pressure, saturated steam has certain definite values for the temperature, the weight per cubic foot, the heat per pound, and so on. These various values, collected and arranged in order, form the table of the Properties of Saturated Steam, more commonly termed the Steam Table, which is given on the following pages.

The various properties of steam, with their symbols, as given in the Steam Table, are as follows:

The temperature, $t$, of the steam, which is the boiling point of the water from which the steam is formed.

The *heat of the liquid*, $q$, which is the number of British thermal units required to raise the temperature of 1 lb. of water from 32° F. to the boiling point corresponding to the given pressure.

The *latent heat of vaporization*, $r$, often termed the *latent heat*, which is the number of British thermal units required to change 1 lb. of water at the boiling point into steam at the same temperature.

The *total heat of vaporization*, $H$, often termed the *total heat*, which is the number of British thermal units required to raise 1 lb. of water from 32° F. to the boiling point for any given pressure and to change it into steam at that pressure. It is the sum of the heat of the liquid and the latent heat.

The *specific volume*, $V$, which is the volume, in cubic feet, of 1 lb. of steam at the given pressure.

The *density*, $w$, which is the weight, in pounds, of 1 cu. ft. of steam at the given pressure. It is the reciprocal of the specific volume.

## PROPERTIES OF SATURATED STEAM

| $p$ | $t$ | $q$ | $H$ | $r$ | $V$ | $w$ |
|---|---|---|---|---|---|---|
| 1 | 101.99 | 70.0 | 1,113.1 | 1,043.0 | 334.6 | .00299 |
| 2 | 126.27 | 94.4 | 1,120.5 | 1,026.1 | 173.6 | .00576 |
| 3 | 141.62 | 109.8 | 1,125.1 | 1,015.3 | 118.4 | .00844 |
| 4 | 153.09 | 121.4 | 1,128.6 | 1,007.2 | 90.31 | .01107 |
| 5 | 162.34 | 130.7 | 1,131.5 | 1,000.8 | 73.22 | .01366 |
| 6 | 170.14 | 138.6 | 1,133.8 | 995.2 | 61.67 | .01622 |
| 7 | 176.90 | 145.4 | 1,135.9 | 990.5 | 53.37 | .01874 |
| 8 | 182.92 | 151.5 | 1,137.7 | 986.2 | 47.07 | .02125 |
| 9 | 188.33 | 156.9 | 1,139.4 | 982.5 | 42.13 | .02374 |
| 10 | 193.25 | 161.9 | 1,140.9 | 979.0 | 38.16 | .02621 |
| 11 | 197.78 | 166.5 | 1,142.3 | 975.8 | 34.88 | .02866 |
| 12 | 201.98 | 170.7 | 1,143.6 | 972.9 | 32.14 | .03111 |
| 13 | 205.89 | 174.6 | 1,144.7 | 970.1 | 29.82 | .03355 |
| 14 | 209.57 | 178.3 | 1,145.8 | 967.5 | 27.79 | .03600 |
| 14.7 | 212.0 | 180.8 | 1,146.6 | 965.8 | 26.60 | .03760 |
| 16 | 216.32 | 185.1 | 1,147.9 | 962.8 | 24.59 | .04067 |
| 18 | 222.40 | 191.3 | 1,149.8 | 958.5 | 22.00 | .04547 |
| 20 | 227.95 | 196.9 | 1,151.5 | 954.6 | 19.91 | .05023 |
| 22 | 233.06 | 202.0 | 1,153.0 | 951.0 | 18.20 | .05495 |
| 24 | 237.79 | 206.8 | 1,154.4 | 947.6 | 16.76 | .05966 |
| 26 | 242.21 | 211.2 | 1,155.8 | 944.6 | 15.55 | .06432 |
| 28 | 246.36 | 215.4 | 1,157.1 | 941.7 | 14.49 | .06899 |
| 30 | 250.27 | 219.4 | 1,158.3 | 938.9 | 13.59 | .07360 |
| 32 | 253.98 | 223.1 | 1,159.4 | 936.3 | 12.78 | .07820 |
| 34 | 257.50 | 226.7 | 1,160.4 | 933.7 | 12.07 | .08280 |
| 36 | 260.85 | 230.0 | 1,161.5 | 931.5 | 11.45 | .08736 |
| 38 | 264.06 | 233.3 | 1,162.5 | 929.2 | 10.88 | .09191 |
| 40 | 267.13 | 236.4 | 1,163.4 | 927.0 | 10.37 | .09644 |
| 42 | 270.08 | 239.3 | 1,164.3 | 925.0 | 9.906 | .1009 |
| 44 | 272.91 | 242.2 | 1,165.2 | 923.0 | 9.484 | .1054 |
| 46 | 275.65 | 245.0 | 1,166.0 | 921.0 | 9.097 | .1099 |
| 48 | 278.30 | 247.6 | 1,166.8 | 919.2 | 8.740 | .1144 |
| 50 | 280.85 | 250.2 | 1,167.6 | 917.4 | 8.414 | .1188 |
| 52 | 283.32 | 252.7 | 1,168.4 | 915.7 | 8.110 | .1233 |
| 54 | 285.72 | 255.1 | 1,169.1 | 914.0 | 7.829 | .1277 |
| 56 | 288.05 | 257.5 | 1,169.8 | 912.3 | 7.568 | .1321 |
| 58 | 290.31 | 259.7 | 1,170.5 | 910.8 | 7.323 | .1366 |
| 60 | 292.51 | 261.9 | 1,171.2 | 909.3 | 7.096 | .1409 |
| 62 | 294.65 | 264.1 | 1,171.8 | 907.7 | 6.882 | .1453 |
| 64 | 296.74 | 266.2 | 1,172.4 | 906.2 | 6.680 | .1497 |
| 66 | 298.78 | 268.3 | 1,173.0 | 904.7 | 6.490 | .1541 |
| 68 | 300.76 | 270.3 | 1,173.6 | 903.3 | 6.314 | .1584 |
| 70 | 302.71 | 272.2 | 1,174.3 | 902.1 | 6.144 | .1628 |
| 72 | 304.61 | 274.1 | 1,174.9 | 900.8 | 5.984 | .1671 |
| 74 | 306.46 | 276.0 | 1,175.4 | 899.4 | 5.834 | .1714 |
| 76 | 308.28 | 277.8 | 1,176.0 | 898.2 | 5.691 | .1757 |
| 78 | 310.06 | 279.6 | 1,176.5 | 896.9 | 5.554 | .1801 |
| 80 | 311.80 | 281.4 | 1,177.0 | 895.6 | 5.425 | .1843 |
| 82 | 313.51 | 283.2 | 1,177.6 | 894.4 | 5.301 | .1886 |
| 85 | 316.02 | 285.8 | 1,178.3 | 892.5 | 5.125 | .1951 |
| 90 | 320.04 | 290.0 | 1,179.6 | 889.6 | 4.858 | .2058 |
| 95 | 323.89 | 294.0 | 1,180.7 | 886.7 | 4.619 | .2165 |
| 100 | 327.58 | 297.9 | 1,181.9 | 884.0 | 4.403 | .2271 |
| 105 | 331.13 | 301.6 | 1,182.9 | 881.3 | 4.206 | .2378 |
| 110 | 334.56 | 305.2 | 1,184.0 | 878.8 | 4.026 | .2484 |
| 115 | 337.86 | 308.7 | 1,185.0 | 876.3 | 3.862 | .2589 |
| 120 | 341.05 | 312.0 | 1,186.0 | 874.0 | 3.711 | .2695 |
| 125 | 344.13 | 315.2 | 1,186.9 | 871.7 | 3.572 | .2800 |
| 130 | 347.12 | 318.4 | 1,187.8 | 869.4 | 3.444 | .2904 |
| 135 | 350.03 | 321.4 | 1,188.7 | 867.3 | 3.323 | .3009 |
| 140 | 352.85 | 324.4 | 1,189.5 | 865.1 | 3.212 | .3113 |

TABLE—(*Continued*)

| p | t | q | H | r | V | w |
|---|---|---|---|---|---|---|
| 145 | 355.59 | 327.2 | 1,190.4 | 863.2 | 3.107 | .3218 |
| 150 | 358.26 | 330.0 | 1,191.2 | 861.2 | 3.011 | .3321 |
| 155 | 360.86 | 332.7 | 1,192.0 | 859.3 | 2.919 | .3426 |
| 160 | 363.40 | 335.4 | 1,192.8 | 857.4 | 2.833 | .3530 |
| 165 | 365.88 | 338.0 | 1,193.6 | 855.6 | 2.751 | .3635 |
| 170 | 368.29 | 340.5 | 1,194.3 | 853.8 | 2,676 | .3737 |
| 175 | 370.65 | 343.0 | 1,195.0 | 852.0 | 2.603 | .3841 |
| 180 | 372.97 | 345.4 | 1,195.7 | 850.3 | 2.535 | .3945 |
| 185 | 375.23 | 347.8 | 1,196.4 | 848.6 | 2.470 | .4049 |
| 190 | 377.44 | 350.1 | 1,197.1 | 847.0 | 2.408 | .4153 |
| 195 | 379.61 | 352.4 | 1,197.7 | 845.3 | 2.349 | .4257 |
| 200 | 381.73 | 354.6 | 1,198.4 | 843.8 | 2.294 | .4359 |
| 205 | 383.82 | 356.8 | 1,199.0 | 842.2 | 2.241 | .4461 |
| 210 | 385.87 | 358.9 | 1,199.6 | 840.7 | 2.190 | .4565 |
| 215 | 387.88 | 361.0 | 1,200.2 | 839.2 | 2.142 | .4669 |
| 220 | 389.84 | 363.0 | 1,200.8 | 837.8 | 2.096 | .4772 |
| 225 | 391.79 | 365.1 | 1,201.4 | 836.3 | 2.051 | .4876 |
| 230 | 393.69 | 367.1 | 1,202.0 | 834.9 | 2.009 | .4979 |
| 235 | 395.56 | 369.0 | 1,202.6 | 833.6 | 1.968 | .5082 |
| 240 | 397.41 | 371.0 | 1,203.2 | 832.2 | 1.928 | .5186 |
| 250 | 400.99 | 374.7 | 1,204.2 | 829.5 | 1.854 | .5393 |
| 260 | 404.47 | 378.4 | 1,205.3 | 826.9 | 1.785 | .5601 |
| 275 | 409.50 | 383.6 | 1,206.8 | 823.2 | 1.691 | .5913 |
| 300 | 417.42 | 391.9 | 1,209.3 | 817.4 | 1.554 | .644 |
| 325 | 424.82 | 399.6 | 1,211.5 | 811.9 | 1.437 | .696 |

The pressures, $p$, given in the first column are absolute pressures. The pressure registered by the gauge on the boiler is the *gauge pressure*, or the pressure of the steam above that of the atmosphere. The pressure of the atmosphere at sea level, with the barometer at about 30 in., is approximately 14.7 lb. per sq. in. Therefore, the *absolute* pressure at sea level is equal to the gauge pressure plus 14.7. In using the Steam Table, the atmospheric pressure, 14.7 lb. per sq. in., must always be added to the gauge pressure.

**Use of Steam Table.**—For any absolute pressure $p$ given in the first column of the Steam Table, the corresponding temperature $t$, total heat $H$, or other property is found in the same horizontal line, under the proper column heading; but if the pressure lies between two of the values given in the first column, the corresponding temperature, total heat, etc. must be found by interpolation, as illustrated in the following examples:

EXAMPLE 1.—Find the temperature corresponding to a pressure of 147 lb. per sq. in., absolute.

SOLUTION.—Referring to the Steam Table,

$$\begin{array}{ll} \text{for } p = 150 \text{ lb., } t = 358.26° \\ \text{and} \qquad\qquad \text{for } p = 145 \text{ lb., } t = 355.59° \\ \hline \text{Difference, 5 lb.,} \qquad 2.67° \end{array}$$

Difference for 1 lb. difference of pressure is $2.67° \div 5 = .534°$. 147 lb. − 145 lb. = 2 lb., the given difference of pressure; and for this, the difference in temperature is $2 \times .534° = 1.068°$ or $1.07°$, taking two decimal places. Hence, the increase of 2 lb. from 145 lb. to 147 lb. is accompanied by an increase in temperature of $1.07°$. Therefore, adding the increase $1.07°$ to the temperature $355.59°$ corresponding to 145 lb., the temperature for 147 lb. is $355.59° + 1.07° = 356.66°$.

EXAMPLE 2.—The pressure in a steam boiler as shown by the gauge is 87 lb. per sq. in. What is the temperature of the steam?

SOLUTION.—The absolute pressure is $87 + 14.7 = 101.7$ lb. per sq. in. This pressure, in the Steam Table, lies between the values 100 and 105.

$$\begin{array}{ll} \text{for } p = 105 \text{ lb., } t = 331.13° \\ \text{for } p = 100 \text{ lb., } t = 327.58° \\ \hline \text{Difference, 5 lb.,} \qquad 3.55° \end{array}$$

For 1 lb. change of pressure, the difference in temperature is $3.55° \div 5 = .71°$. From 100 lb. to 101.7 lb., the change of pressure is 1.7 lb., and the corresponding change of temperature is $.71° \times 1.7 = 1.207°$, or $1.21°$ as the values in the Steam Table contain but two decimal places. For 101.7 lb., therefore, the temperature is $327.58° + 1.21° = 328.79°$.

EXAMPLE 3.—What is the pressure of steam at a temperature of 285° F.?

SOLUTION.—From the Steam Table,

for $t = 285.72°$, $p = 54$ lb.
for $t = 283.32°$, $p = 52$ lb.

Difference, 2.40°,     2 lb.

From $t = 283.32°$ to $t = 285°$, the increase of temperature is $1.68°$. Now, as an increase of temperature of 2.40° gives an increase of pressure of 2 lb., the increase of 1.68° must give an increase of pressure of

$$\frac{1.68}{2.40} \times 2 \text{ lb.} = 1.4 \text{ lb.}$$

Hence, the required pressure is 52 lb. $+ 1.4$ lb. $= 53.4$ lb.

EXAMPLE 4.—Find, from the Steam Table, the total heat of 1 lb. of saturated steam at a pressure of 63 lb. per sq. in., gauge.

SOLUTION.—The absolute pressure is $63 + 14.7 = 77.7$ lb. per sq. in. From the Steam Table,

for $p = 78$ lb., $H = 1,176.5$ B. T. U.
for $p = 76$ lb., $H = 1,176.0$ B. T. U.

Difference, 2 lb.,     .5 B. T. U.
Difference, 1 lb.,     .25 B. T. U.

The difference between the given pressure and 76 lb. is $77.7 - 76 = 1.7$ lb. For a difference of 1.7 lb., the change of total heat is $1.7 \times .25 = .425$ B. T. U. Hence, for 77.7 lb., $H = 1,176.0 + .425 = 1,176.425$, say 1,176.4 B. T. U.

EXAMPLE 5.—Find the volume occupied by 14 lb. of steam at 30 lb. gauge pressure.

SOLUTION.—Absolute pressure $= 30 + 14.7 = 44.7$ lb. per sq in. From the Steam Table,

for $p = 44$ lb., $V = 9.484$ cu. ft.
for $p = 46$ lb., $V = 9.097$ cu. ft.

Difference, 2 lb.,     .387 cu. ft.

The difference for 1 lb. is $.387 \div 2 = .1935$. $44.7 - 44 = .7$ lb. actual difference in pressure. $.1935 \times .7 = .135$ difference in volume. As the pressure increases, the volume decreases; and to obtain the volume at 44.7 lb., it is necessary to subtract the difference .135 from the volume at 44 lb.; thus, for $p = 44.7$, $V = 9.484 - .135 = 9.349$ cu. ft. The volume of 14 lb. is $14 \times 9.349$ cu. ft. $= 130.89$ cu. ft.

EXAMPLE 6.—Find the weight of 40 cu. ft. of steam at a temperature of 254° F.

SOLUTION.—From the Steam Table, the weight $w$ of 1 cu. ft. of steam at 253.98 is .07820 lb. $254 - 253.98 = .02$. Neglecting the .02°, the weight of 40 cu. ft. is therefore $.07820 \times 40 = 3.128$ lb.

EXAMPLE 7.—How many pounds of steam at 64 lb. pressure, absolute, are required to raise the temperature of 300 lb. of water from 40° to 130° F., the water and steam being mixed together?

SOLUTION.—The number of heat units required to raise 1 lb. from 40° to 130° is $130° - 40° = 90$ B. T. U. Actually, a little more than 90 would be required but the above is near enough for all practical purposes. Then, to raise 300 lb. from 40° to 130° requires $90 \times 300 = 27,000$ B. T. U. This quantity of heat must necessarily come from the steam. Now, 1 lb. of steam at 64 lb. pressure gives up, in condensing, its latent heat of vaporization, or 906.2 B. T. U.; but, in addition to its latent heat, each pound of steam on condensing must give up an additional amount of heat in falling to 130°. As the original temperature of the steam was 296.74° F. (see Steam Table), each pound gives up by its fall of temperature $296.74 - 130 = 166.74$ B. T. U. Consequently, each pound of the steam gives up a total of $906.2 + 166.74 = 1,072.94$ B. T. U., and $27,000 \div 1,072.94 = 25.16$ lb. of steam will therefore be required to accomplish the desired result.

## SUPERHEATED STEAM

If saturated steam in contained in a vessel, out of contact with water, and heat is added to it, its temperature will begin to rise and its weight per cubic

foot will begin to decrease, provided the pressure remains constant. As more heat is added, the temperature rises farther above that of saturated steam at that pressure, and the steam is then called *superheated steam*. Superheated steam cannot exist in contact with water.

The following distinction is usually made between saturated and superheated steam: For a given pressure, saturated steam has one temperature and one weight per cubic foot, neither of which can change so long as the steam remains in immediate contact with water. Superheated steam at the same pressure has a greater temperature and less weight per cubic foot than saturated steam, and both the temperature and weight per cubic foot may vary while the pressure remains constant if the volume increases or decreases accordingly. In other words, both the pressure and the volume of superheated steam must be constant in order to maintain a constant temperature and a constant weight per cubic foot.

## QUALITY OF STEAM

**Moisture in Steam.**—The steam furnished by the average steam boiler is not dry saturated steam, but is usually wet steam. A good boiler should not show more than 2 or 3% of water in the steam. In a quantity of wet steam, or a mixture of steam and water, the percentage of dry steam, expressed as a decimal, is called the *quality of the steam*. For example, suppose that a certain boiler generates wet steam that contains 3%, or .03, of moisture; then the quality of the steam, or the percentage of dry steam, is .97. In other words, the quality of the steam is equal to 1 minus the percentage of moisture, expressed decimally, or
$$Q = 1 - m,$$
in which $Q$ = quality of steam;

$m$ = percentage of moisture, expressed decimally.

**EXAMPLE.**—What is the quality of steam that contains 2.7% of moisture?

**SOLUTION.**—Expressed as a decimal, 2.7% = .027. Then, substituting this value for $m$ in the formula, $Q = 1 - .027 = .973$.

**Heat in Wet Steam.**—The total heat contained in 1 lb. of dry steam is the sum of the heat required to raise 1 lb. of water from 32° F. to the boiling point and the heat required to change the boiling water into steam of the same temperature. That is, in the Steam Table, each value given in the fourth column is the sum of the values given in the third and fifth columns and lying in the same horizontal row. In a mixture of 1 lb. of steam and water at the same temperature there is less heat than in 1 lb. of dry steam at the same temperature; for all the water has not been changed to steam, and consequently the latent heat of 1 lb. of steam has not been utilized. Instead, there is present only that part of the latent heat which is used to evaporate the portion of the mixture that is dry steam, which is represented by the quality of the steam. Thus, using the symbols given in the Steam Table,
$$H = q + r \quad (1)$$
which is the formula for the total heat of 1 lb. of dry steam. But if the steam is wet, and $Q$ represents the quality of the steam, expressed decimally, the total heat of 1 lb., represented by $H_1$, is
$$H_1 = q + Qr \quad (2)$$

**EXAMPLE.**—What is the total heat of 10 lb. of steam at 150 lb. gauge pressure, if the steam contains 5% of moisture?

**SOLUTION.**—From the Steam Table, the heat of the liquid of 1 lb. of dry steam at 150 lb., gauge, or 150 + 14.7 = 164.7 lb., absolute, is $q$ = 337.84 B. T. U., and the latent heat of 1 lb. at the same pressure is $r$ = 855.71 B. T. U. As the moisture is 5%, the quality of the steam is 1.00 − .05 = .95. Then, applying formula 2, $H_1$ = 337.84 + .95 × 855.71 = 1,150.76 B. T. U.

## FLOW OF STEAM

**Weight of Steam Discharged.**—The number of pounds of steam that will flow continuously through a pipe of given diameter in 1 min. at specified pressure may be calculated by the formula
$$W = 87 \sqrt{\frac{w(P_1 - P_2)d^5}{L\left(1 + \dfrac{3.6}{d}\right)}},$$
in which $W$ = weight of steam discharged, in pounds per minute;

$w$ = weight of 1 cu. ft. of steam at pressure $P_1$;

$P_1$ = pressure of steam at entrance to pipe, in pounds per square inch;

$P_2$ = pressure of steam at discharge, in pounds per square inch;

$L$ = length of pipe, in feet;

$d$ = diameter of pipe, in inches.

In applying the preceding formula in determining the diameter of the steam pipe for an engine, it must be remembered that, in steam-engine work, the steam is drawn intermittently from the pipe. Thus, assume that an engine of 100 H. P., consuming 30 lb. of steam per H. P. per hr., cuts off at one-fourth stroke. In that case, the steam consumption per hour would be $100 \times 30 = 3,000$ lb. But as the steam used at each stroke is drawn into the cylinder during only one-fourth of the time required to complete the stroke, the 3,000 lb. of steam flows through the pipe in $\frac{1}{4}$ hr. Then, in order to determine the quantity of steam that would flow continuously at the same velocity at which it flows during admission to the cylinder, the actual steam consumption per hour should be divided by the fraction representing the cut-off and the quotient should be taken as the weight of steam discharged per hour. This value, divided by 60, should be substituted for $w$ in the formula. Thus, in the case mentioned, the amount of steam discharged per hour, flowing continuously at the same velocity as during the admission period, is $3,000 \div \frac{1}{4} = 12,000$, and the value of $W$ to be used in the formula is therefore $12,000 \div 60 = 200$ lb. per min. Knowing the pressures, the length of pipe, and the weight of the entering steam per cubic foot, different values of $d$ may be assumed, until a value is found that will give the necessary discharge $W$. This is the required pipe diameter.

## WEIGHT OF STEAM DELIVERED PER MINUTE THROUGH 100 FT. OF PIPE WITH 1 LB. DROP OF PRESSURE

| Initial Gauge Pressure Pounds per Square Inch | Nominal Diameter of Pipe, in Inches | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 3 | $3\frac{1}{2}$ | 4 | $4\frac{1}{2}$ | 5 | 6 | 7 | 8 | 9 | 10 |
| | Weight, in Pounds, of Steam Delivered per Minute Through 100 Ft. of Pipe with 1 Lb. Drop of Pressure | | | | | | | | | |
| 70 | 43.2 | 64.5 | 91.7 | 124.3 | 168.7 | 277.2 | 410.5 | 577.2 | 793.3 | 1,051.7 |
| 80 | 45.5 | 68.0 | 96.6 | 130.9 | 177.7 | 292.1 | 432.5 | 608.2 | 835.8 | 1,108.4 |
| 90 | 47.6 | 71.2 | 101.2 | 137.2 | 186.3 | 306.0 | 453.3 | 637.3 | 875.9 | 1,161.3 |
| 100 | 49.7 | 74.3 | 105.7 | 143.2 | 194.4 | 319.8 | 473.0 | 665.1 | 914.1 | 1,211.8 |
| 110 | 51.7 | 77.3 | 109.9 | 148.9 | 202.1 | 332.2 | 491.8 | 691.5 | 950.4 | 1,259.9 |
| 120 | 53.6 | 80.2 | 113.9 | 154.4 | 209.5 | 344.3 | 509.9 | 717.0 | 985.4 | 1,306.3 |
| 130 | 55.4 | 82.9 | 117.8 | 159.7 | 216.7 | 356.1 | 527.4 | 741.6 | 1,019.6 | 1,351.1 |
| 140 | 57.2 | 85.5 | 121.5 | 164.7 | 223.6 | 367.4 | 544.6 | 765.7 | 1,052.9 | 1,393.9 |
| 150 | 58.9 | 88.1 | 125.2 | 169.6 | 230.2 | 378.3 | 560.2 | 787.7 | 1,082.6 | 1,428.1 |

The approximate weights of steam delivered per minute through 100 ft. of pipe of various diameters, with a drop of pressure of 1 lb., are given in the foregoing table. On the whole, these values are slightly higher than those that would be obtained by the foregoing formula for the same conditions. If the drop of pressure is more or less than 1 lb., the value in the table must be multiplied by the square root of the drop, to obtain the discharge. Also, if the length of the pipe is more or less than 100 ft., divide 100 by the length, in feet, and multiply the square root of this quotient by the value given in the table. The following example illustrates this point.

EXAMPLE.—How many pounds of steam will be discharged per minute, with an initial gauge pressure of 120 lb. per sq. in., through a pipe 3 in. in diameter and 400 ft. long, with a drop of pressure of 2 lb.?

SOLUTION.—From the table, the amount discharged through 100 ft. of 3-in. pipe with a drop of 1 lb. and an initial pressure of 120 lb. per sq. in., is 53.6 lb. per min. But as the drop is 2 lb., the table value is multiplied by $\sqrt{2}$ and as the length is 400 ft., it must also be multiplied by $\sqrt{\frac{100}{400}}$. Hence, the discharge for the given conditions will be $53.6 \times \sqrt{2} \times \sqrt{\frac{100}{400}} = 37.9$ lb. per min.

Resistance of Elbows and Valves.—The presence of elbows, bends, and valves in a steam pipe increases the resistance to the flow of steam and thus increases the drop of pressure between the inlet and outlet ends. It has been found that the resistance caused by an elbow or a sharp bend is approximately

the same as the resistance of a length of pipe equal to 60 times the diameter, and that a stop-valve has a resistance equal to that of a length of pipe of 40 diameters. In using the foregoing formula for the weight of steam discharged, therefore, the value of $L$ should be the equivalent length of pipe, taking into account the bends and valves.

EXAMPLE.—What is the equivalent length of 300 ft. of 3-in. pipe containing four elbows and six stop-valves?

SOLUTION.—Each elbow has a resistance the same as that of 60 diameters of pipe, or $60 \times 3 = 180$ in. $= 15$ ft., and four elbows have the resistance of $4 \times 15 = 60$ ft. of pipe. Each stop-valve has a resistance that is equivalent to adding $40 \times 3 = 120$ in. $= 10$ ft. of pipe, and, as there are six valves, their combined resistance is that of $6 \times 10 = 60$ ft. of pipe. The equivalent length of pipe is, therefore, $300 + 60 + 60 = 420$ ft.

**Steam Pipes for Engines.**—In practice, the velocity of flow of steam in the supply pipes of engines and pumps is usually over 6,000 ft. per min., although it is increased to as much as 8,000 ft. per min. in some cases. For exhaust pipes, a common value is 4,000 ft. per min. The assumptions made are that the cylinder is filled with steam at boiler pressure at each stroke and that a volume of steam equal to the volume of the cylinder is released at each stroke, so that the flow is practically continuous. The areas of the steam and exhaust pipes may then be calculated by the formula

$$a = \frac{AS}{s},$$

in which    $a =$ area of steam or exhaust pipe, in square inches;
$A =$ area of cylinder, in square inches;
$S =$ piston speed, in feet per minute;
$s =$ velocity of steam in pipe, in feet per minute.

EXAMPLE.—Find the areas of the steam and exhaust pipes for an engine whose cylinder is 20 in. in diameter and whose piston speed is 450 ft. per min.

SOLUTION.—By the formula, the area of the steam pipe, assuming that $s = 6,000$ ft. per min., is

$$a = \frac{.7854 \times 20^2 \times 450}{6,000} = 23.6 \text{ sq. in}$$

Similarly, for the exhaust pipe, assuming that $s = 4,000$, the area is

$$a = \frac{.7854 \times 20^2 \times 450}{4,000} = 35.3 \text{ sq. in.}$$

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# BOILER PIPING

**Principal Considerations.**—The piping of an engine and boiler plant requires that careful attention be paid to all the details as well as to the general design, not only in order to make it suitable for the purpose, but also in order to reduce the likelihood of a breakdown. The main considerations regarding steam piping are the size of the pipes, the arrangement and construction of the piping system, the method of providing for expansion, and proper drainage.

**Materials for Pipes.**—Most of the piping for steam and water is built up of wrought-iron or steel pipe of standard size. The various grades of wrought-iron and steel pipe are known as *standard, extra strong,* and *double extra strong.* Both wrought-iron and steel pipe are used in the piping systems of power plants. Formerly, wrought iron was chiefly used, but of late steel has been employed, especially for the larger sizes of steam pipes. The two kinds are equally reliable when made into expansion bends, copper bends as a general rule being used only for very heavy work.



FIG. 1

**Expansion Joints.**—In installing steam piping, provision must be made for expansion and contraction, which ordinarily amounts to about $1\frac{1}{2}$ in. per 100 ft. of pipe. Generally, this may be provided for in the arrangement of the piping, but for great lengths that are straight, or nearly so, it is necessary to use expansion joints, which may be made in various ways. One form, shown in Fig. 1, is called the *slip joint.* The ends $a$ and $b$ of the sections of pipe come together in a stuffingbox $c$ in order to make a steam-tight joint. The stud bolts are extra long, so as to extend through holes in a flange $d$ riveted to the pipe $b$.
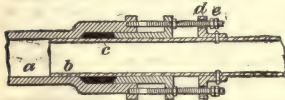
Check-nuts $e$ on the ends of the studs prevent the two ends of the pipe from being forced apart by the steam pressure. The nuts $e$ are not intended ordinarily to be in contact with the flange; their distance from the flange is adjusted so that the proper expansion may occur.

In Fig. 2 is shown a *corrugated expansion joint*, which is sometimes used on large exhaust pipes. It consists of a short section of flanged corrugated pipe, usually copper, which is put in the steam pipe wherever necessary. The elasticity of this section, due to the corrugations, permits expansion and contraction.



FIG. 2

**Expansion Bends.**—The best way of allowing for expansion is by using expansion bends, or bent pipes; but the space they occupy often limits their use. The forms of bends more commonly used are shown in Fig. 3, the trade name being given below each bend. Where a bent pipe is used, the radius $r$ of the bend should not be less than six times the diameter of the pipe, for wrought iron or steel; to secure the proper spring in bends used on long lines of piping, the radius should be greater than this. Bends of copper pipe may be of shorter radius, as copper yields more readily than iron or steel.



Elbow

Return bend

Double bend

Eye bend

Loop bend

FIG. 3

Bends made from iron or steel pipe must be bent while red hot. Iron and steel pipe bends generally have iron flanges fastened on; copper bends either have composition flanges riveted and brazed on, or have steel flanges, the edges of the pipe being turned over. The piping is usually installed so that it is under a slight tension when cold; when filled with steam, the expansion of the pipes removes the tension, and there is no stress on the pipe except that due to the steam pressure.

**Arrangement of Piping.**—The pipes and fittings must be so proportioned as to permit of free flow of steam or water. Water pockets should be avoided; and where such pockets are unavoidable, they must be drained to free them from water. By-pass pipes should be arranged around feedwater heaters, economizers, pumps, etc. The system must be so designed as to give perfect freedom for expansion and contraction.

Perfect drainage must be provided in order that all water of condensation shall be fully separated from the steam. Reliability is insured by careful design and superior workmanship, combined with the use of high-class materials and fittings and the judicious placing of cut-out and by-pass valves. Drainage is best effected by arranging the piping so that all the water of condensation will flow by gravity toward a point close to the delivery end of the pipe, and then providing a drip pipe at that point. A trap may be placed at the end of the drip pipe for automatic draining.
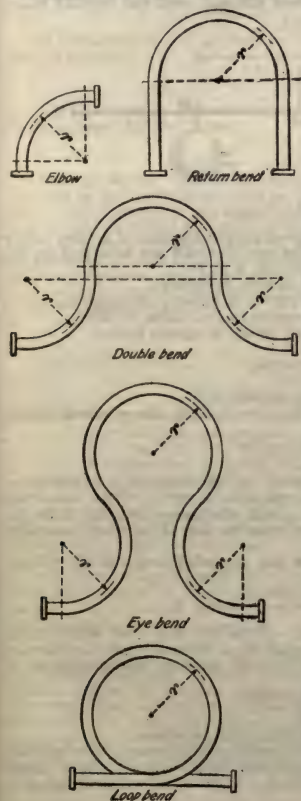
# BOILER FITTINGS

## SAFETY VALVES

The *safety valve* is a device attached to the boiler to prevent the steam pressure from rising above a certain point. When steam is made more rapidly than it is used, its pressure must necessarily rise; and if no means of escape is provided for it, the result must be an explosion. Briefly described, the safety valve consists of a plate, or disk, fitting over a hole in the boiler shell and held to its place by a dead weight, by a weight on a lever, or by a spring. The weight or the spring is so adjusted that when the steam reaches the desired pressure the disk is raised from its seat, and the surplus steam escapes through the opening in the shell.
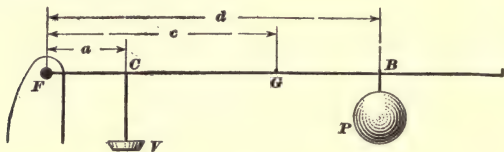


FIG. 1

**Weight of Ball for Lever Safety Valve.**—A simple diagram of a lever safety valve is shown in Fig. 1. The valve stem and the ball are attached to the lever at $C$ and $B$, respectively, and the fulcrum is at $F$.

Let $d = FB =$ distance from fulcrum to weight, in inches;
$c = FG =$ distance from fulcrum to center of gravity of lever, in inches;
$a = FC =$ distance from fulcrum to center line of valve, in inches;
$A =$ area of orifice beneath bottom of valve, in square inches;
$W =$ weight of ball $P$, in pounds;
$W_1 =$ weight of valve and stem, in pounds;
$W_2 =$ weight of lever, in pounds;
$p =$ blow-off pressure, in pounds per square inch.

Then, if the position of the ball $P$ on the lever is fixed, the required weight of the ball may be found by the formula

$$W = \frac{a(pA - W_1) - W_2c}{d}$$

**EXAMPLE.**—The area of the orifice is 10 sq. in., the distance from the valve to the fulcrum is 3 in., and the length of the lever is 32 in. The valve and stem weigh 5 lb., the lever weighs 12 lb., and the gauge pressure is 90 lb. What should be the weight $W$, if placed 2 in. from the end of the lever, assuming the lever to be straight?

**SOLUTION.**—In this case, $c = 32 \div 2 = 16$ in., and $d = 32 - 2 = 30$ in. Then, substituting in the formula,

$$W = \frac{3 \times (90 \times 10 - 5) - 12 \times 16}{30} = 83.1 \text{ lb.}$$

**Position of Ball for Lever Safety Valve.**—If the ball of a lever safety valve has a known weight and it is desired to find at what distance from the fulcrum it must be placed so as to give a required blow-off pressure, the formula to be used is

$$d = \frac{a(pA - W_1) - W_2c}{W}$$

in which the various letters have the same meanings as before.

**EXAMPLE.**—Suppose all the quantities to remain the same as in the solution of the preceding example, except that it is desired that the boiler should blow off at 75 lb. gauge pressure, instead of 90 lb. What will be the distance of the weight from the fulcrum?

**SOLUTION.**—Applying the formula

$$d = \frac{3 \times (75 \times 10 - 5) - 12 \times 16}{83.1} = 24.58 \text{ in.}$$

**Roper's Safety-Valve Rules.**—Some inspectors of the United States Steamboat Inspection Service prefer to have lever safety-valve problems worked

out by the rules that follow, known among American marine engineers as *Roper's rules*.

Let $A$ = area of valve, in square inches;
$D$ = distance from center line of valve to fulcrum, in inches;
$L$ = distance of weight from fulcrum, in inches;
$P$ = steam pressure, in pounds per square inch;
$W$ = weight of load or weight on lever, in pounds;
$V$ = weight of valve and stem, in pounds;
$w$ = weight of lever, in pounds;
$l$ = distance from fulcrum to center of gravity of lever, in inches.

Then, the pressure at which the safety-valve will blow off is found by the formula

$$P = \frac{WL + wl + VD}{AD} \qquad (1)$$

If the distance $L$ is known, the weight $W$ to be hung on the lever is found by the formula

$$W = \frac{APD - (wl + VD)}{L} \qquad (2)$$

The distance $L$ from the fulcrum to the point at which the weight $W$ is hung is found by the formula

$$L = \frac{APD - (wl + VD)}{W} \qquad (3)$$

**Area of Safety Valve.**—By area of safety valve is meant the area of the opening in the valve seat; that is, the area of the surface of the valve in contact with steam when the valve is closed. The size of the valve relative to the size of the boiler and the working pressure is prescribed by law in many localities, and must be made to conform to the law wherever such law is in existence. In localities having no law governing this matter, the size of the safety valve may be calculated by the accompanying formulas, which are based on practice and recommended by leading authorities.

For natural draft,

$$A = \frac{22.5G}{p + 8.62} \qquad (1)$$

For artificial draft,

$$A = \frac{1.406 \, w}{p + 8.62}, \qquad (2)$$

in which $G$ = grate surface, in square feet;
$p$ = steam, gauge pressure, in pounds per square inch;
$w$ = weight of coal burned per hour, in pounds;
$A$ = least area of safety valve, in square inches.

**Location of Safety Valve.**—The safety valve should be placed in direct connection with the boiler, so that there can be no possible chance of cutting off the communication between them. A stop-valve placed between the boiler and the safety valve is a very fruitful cause of boiler explosions. Again, the safety valve must be free to act, and to prevent it from corroding fast to its seat, it should be lifted from the seat occasionally. Care must be taken to prevent persons ignorant of the importance of safety valves from raising the blow-off pressure by adding to the weights or increasing the tension of the spring. To this end, the weights of lever safety valves are often locked in position by the boiler inspector.

## FUSIBLE PLUGS

*Fusible plugs* are devices placed in the crown sheets of furnaces, or in similar places, to obviate danger from overheating through lack of water. The plug often consists of an alloy of tin, lead, and bismuth, which melts at a comparatively low temperature. In many localities, the law requires that fusible plugs shall be attached to all high-pressure boilers.

The fusible plugs in common use are shown in section, on the next page. They consist of brass or iron shells threaded on the outside with a standard pipe thread. The plugs have some form of conical filling, the larger end of the filling receiving the steam pressure. The conical form of the filling prevents it from being blown out by the pressure of the steam. Fusible plugs applied from the outside differ from those applied from the inside, as shown.

**Location of Fusible Plugs.**—In the absence of local laws, the following rules issued by the Board of Boiler Rules of the State of Massachusetts may be adopted. Fusible plugs must be filled with pure tin, and the least diameter

shall not be less than $\frac{1}{2}$ in., except for working pressures over 175 lb., gauge, or when it is necessary to place a fusible plug in a tube, in which cases the least diameter of fusible metal shall not be less than $\frac{3}{8}$ in. The location of fusible plugs shall be as follows:



*Inside Type    Outside Type*
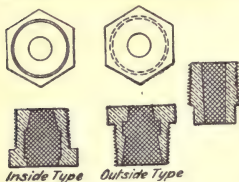
In horizontal return-tubular boilers, in the back head, not less than 2 in. above the upper row of tubes and projecting through the sheet not less than 1 in.

In horizontal flue boilers, in the back head, on a line with the highest part of the boiler exposed to the products of combustion, and projecting through the sheet not less than 1 in.

In locomotive-type or star water-tube boilers, in the highest part of the crown sheet and projecting through the sheet not less than 1 in.

In vertical fire-tube boilers, in an outside tube, placed not less than one-third the length of the tube above the lower tube-sheet.

In vertical submerged-tube boilers, in the upper tube-sheet.

In water-tube boilers of the Babcock & Wilcox type, in the upper drum, not less than 6 in. above the bottom of the drum and projecting through the sheet not less than 1 in.

In Stirling boilers of standard type, in the front side of the middle drum not less than 6 in. above the bottom of the drum and projecting through the sheet not less than 1 in.

In Stirling boilers of the superheated type, in the front drum, not less than 6 in. above the bottom of the drum, and exposed to the products of combustion, projecting through the sheet not less than 1 in.

In water-tube boilers of the Heine type, in the front course of the drum, not less than 6 in., from the bottom of the drum, and projecting through the sheet not less than 1 in.

In Robb-Mumford boilers of standard type, in the bottom of the steam and water drum, 24 in. from the center of the rear neck, and projecting through the sheet not less than 1 in.

In water-tube boilers of the Almy type, in a tube directly exposed to the products of combustion.

In vertical boilers of the Climax or Hazleton type, in a tube or center drum, not less than one-half the height of the shell, measuring from the lowest circumferential seam.

In Cahall vertical water-tube boilers, in the inner sheet of the top drum, not less than 6 in. above the upper tube sheet.

In Scotch marine-type boilers, in the combustion-chamber top, and projecting through the sheet not less than 1 in.

In dry-back Scotch-type boilers, in the rear head, not less than 2 in. above the top row of tubes, and projecting through the sheet not less than 1 in.

In Economic-type boilers, in the rear head, above the upper row of tubes.

In cast-iron sectional heating boilers, in a section over and in direct contact with the products of combustion in the primary combustion chamber.

In other types and new designs, fusible plugs shall be placed at the lowest permissible water level, in the direct path of the products of combustion, as near the primary combustion chamber as possible.

## CONNECTION OF STEAM GAUGE

A steam gauge should be connected to the boiler in such a manner that it will neither be injured by heat nor indicate incorrectly the pressure to which it is subjected. To prevent injury from heat, a so-called siphon is placed between the gauge and the boiler. This siphon in a short time becomes filled with water of condensation, which protects the spring of the gauge from the injury the hot steam would cause. Care should be taken not to locate the steam-gauge pipe near the main steam outlet of the boiler, as this may cause the gauge to indicate a lower pressure than really exists. In locating the steam gauge, care must also be taken not to run the connecting pipe in such a manner that the accumulation of water in it will cause an extra pressure to be shown.

## BLOW-OFFS

For the double purpose of emptying the boiler when necessary and of discharging the loose mud and sediment that collect from the feedwater, every boiler is provided with a pipe that enters the boiler at its lowest point. This

pipe, which is provided with a valve or a cock, is commonly known as the *bottom blow-off*. The position of the blow-off pipe varies with the design of the boiler; in ordinary return-tubular boilers, it is usually led from the bottom of the rear end of the shell through the rear wall. Where the boiler is fitted with a mud-drum, the blow-off is attached to the drum.

**Blow-Off Cocks and Valves.**—While in many boiler plants globe valves are used on the blow-off pipe, they have the disadvantage that the valve may be kept from closing properly by a chip of incrustation or similar matter getting between the valve and its seat, with the result that water may leak out of the boiler unnoticed. Plug cocks packed with asbestos are widely used, the asbestos packing obviating the objectionable features of the ordinary plug cock. Gate valves are also used to some extent, but have the same disadvantage as globe valves. In the best modern practice, the blow-off pipe is fitted with two shut-off devices. The one shut-off may be an asbestos-packed cock and the other some form of valve, or both may be cocks or valves, the idea underlying this practice being that leakage past the shut-off nearest the boiler will be arrested by the other.

**Protection of Blow-Off Pipe.**—When exposed to the gases of combustion, the bottom blow-off pipe should always be protected by a sleeve made of pipe, by being bricked in, or by a coil of plaited asbestos packing. If this precaution is neglected, the sediment and mud collecting in the pipe, in which there is no circulation, will rapidly become solid. The blow-off pipe should lead to some convenient place entirely removed from the boiler house and at a lower level than the boiler. Sometimes it may be connected to the nearest sewer. In many localities, however, ordinances prohibit this practice; the blow-off is then connected to a cooling tank, whence the water may be discharged into the sewer.

## FURNACE FITTINGS

**Bridge Wall.**—The bridge, also termed the *bridge wall*, is a low wall at the back end of the grate; it forms the rear end of the furnace and causes the flame to come into close contact with the heating surface of the boiler. It is usually built of common brick and faced with firebrick. The passage between the bridge and the boiler shell should not be too small; its area may be approximately one-sixth the area of the grate. The space between the grate and the shell should be ample for complete combustion, and the distance between the grate and the boiler shell may be made about one-half the diameter of the shell.

**Fixed Grates.**—The grate, which is nearly always made of cast iron, furnishes a support for the fuel to be burned and must be provided with spaces for the admission of air. The area of the solid portion of the grate is usually made nearly equal to the combined area of the air spaces.
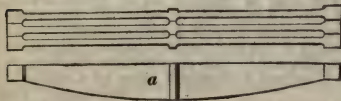


**FIG. 1**

The common type of fixed grate is made of single bars *a*, Fig. 1, placed side by side in the furnace. The thickness of the lugs cast on the sides of the bars determines the width of the open spaces of the grate. It is the general practice to make the thickness across the lugs twice the thickness of the top of the bar. For long furnaces, the bars are generally made in two lengths of about 3 ft. each, with a bearing bar in the middle of the grate. Long grates are generally set with a downward slope toward the bridge wall of about ⅜ in. per ft. of length.

For the larger sizes of anthracite and bituminous coal, the air space may be from ⅜ to ½ in. wide, and the grate bar may have the same width. For pea and nut coal, the air space may be from ⅜ to ¼ in., and for finely divided fuel, like buckwheat, rice, bird's-eye, culm, and slack, air spaces from ³⁄₁₆ to ⅛ in. may be used.



**FIG. 2**

The grate bar shown in Fig. 2, and known as the *herring-bone grate bar*, has in many places superseded the ordinary grate bar, because it will usually far outlast a set of ordinary grate bars. Herring-bone grate bars can be obtained in a great variety of styles and with different widths of air spaces. They are usually supported on cross-bars, and, like many other forms of grate
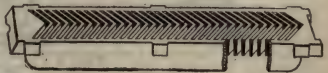
bars, may be arranged with trunnions, so as to rock the individual bars by means of hand levers.

A form of cast-iron grate bar for the burning of sawdust is shown in Fig. 3. The bar is semicircular in cross-section and is provided with circular openings for the introduction of air. Lugs are cast on each side of the bar to serve as distance pieces in providing air spaces between the bars.



**FIG. 3**
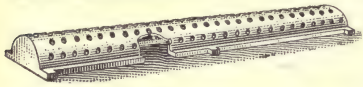
**Dead Plate.**—The front ends of the grate bars are usually supported on the dead plate, which is a flat cast-iron plate placed across the furnace just inside the boiler front and on a level with the bottom of the furnace door. The purpose of the dead plate is twofold: It forms a support for the firebrick lining of the boiler front, and a resting place on which bituminous coal may be coked before it is placed on the fire. To support the grate bars, the inner edge of the dead plate is either beveled or a lip is provided, as at a, Fig. 4.

**Objection to Stationary Grate Bars.**—The greatest objection to stationary grate bars is that with them the furnace door must be kept open for a considerable length of time when the fire is being cleaned. Cleaning fires when the boiler has a stationary grate not only severely taxes the fireman, but the inrush of cold air chills the boiler plates, thus producing stresses that in the course of time will crack them.

**Shaking Grates.**—There are on the market many designs of shaking grates for large steam boilers, differing chiefly in detail and arrangement. Usually the grate bars are hung on trunnions at each end and are connected together with bars to which are attached shaking rods that extend forwards through the furnace front. Levers or handles are attached to the shaking rods, and by working them back and forth the grate bars



**FIG. 4**

receive a rocking motion that breaks up the bed of coal on the grate and serves to shake the ashes through into the ash-pit. The fires may thus be kept clean without the necessity of opening the fire-doors.
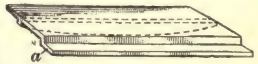
**Classes of Mechanical Stokers.**—A mechanical stoker is a power-driven rocking grate arranged so as to give a uniform feed of coal and to rid itself continuously of ashes and clinkers. The principal designs of mechanical stokers and automatic furnaces may be divided into two general classes, *overfeed* and *underfeed*.

**Overfeed Stoker.**—In overfeed stokers the fixed carbon of the coal is burned on inclined grates. The coal is pushed on to these grates, which are given a sufficiently rapid vibratory motion to feed it down at such a rate that practically all the carbon is burned before reaching the lower end, where the ashes and clinkers are discharged. In Fig. 5 is shown a sectional view of a stoker of this class. The coal is fed into the hopper a, from which it is pushed by the pusher plate b on to the dead plate c, where it is heated. From c it passes to the grate d. Each bar is supported at its ends by trunnions and is connected by an arm to a rocker bar i, which is slowly moved to and fro by an eccentric on the shaft s, so as to rock the grates back and forth; the grates thus gradually move the burning fuel downwards. The ashes and clinkers are discharged from the lower grate bar on to the dumping grate e. A guard f may be raised, as shown by the dotted lines, so as to prevent coke or coal from falling from the grate bars into the ash-pit when the dumping grate is lowered.
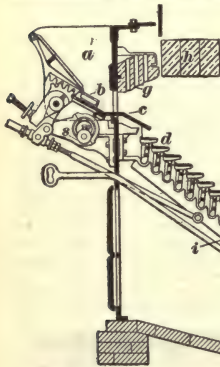


**FIG. 5**

Air for burning the gases is admitted in small jets through holes in the air tile *g*, and the mixture of gas and air is burned in the hot chamber between the firebrick arch *h* and the bed of burning coke below.

**Underfeed Stoker.**—The stoker shown in Fig. 6 illustrates the principle of operation and the construction of the underfeed stoker. Coal is fed into the hopper *a*, from which it is drawn by the spiral conveyer *b* and forced into the magazine *d*. The incoming supply of fresh coal forces the fuel upwards to the surface and over the sides of the magazine on the grates, where it is burned. A blower forces air through a pipe *f* into the chamber *g* surrounding the maga-
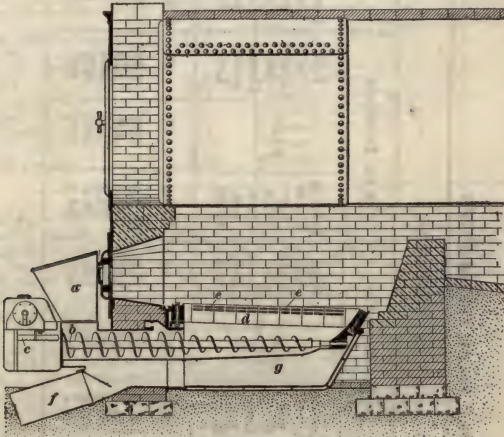


Fig. 6

zine. From *g* the air passes upwards through hollow-cast iron tuyère blocks and out through the openings, or tuyères *e*. The gas formed in the magazine, mixed with the jets of air from the tuyères, rises through the burning fuel above, where it is subjected to a sufficiently high temperature to secure its combustion. Nearly all the air for burning the coal is supplied through the tuyères, only a very small portion of the supply coming through the grate. The ashes and clinkers are gradually forced to the sides of the grate against the side walls of the furnace, from which they are removed from time to time through doors in the furnace front similar to the fire-doors of an ordinary furnace. In other words, owing to the construction of the underfeed stoker, the fire must periodically be cleaned from clinkers and the ashes removed by hand.

# COVERING FOR BOILERS, STEAM PIPES, ETC.

The losses by radiation from uncovered pipes and vessels containing steam are considerable, and in the case of pipes leading to steam engines, are magnified by the action of the condensed water in the cylinder. It therefore is important that such pipes should be well protected. The accompanying table gives the loss of heat from steam pipes naked, and covered with wool or hair felt of different thickness, the steam pressure being assumed at 75 lb., and the exterior air at 60°.

There is a wide difference in the value of different substances for protection from radiation, their values varying nearly in the reverse ratio to their conducting power for heat, up to their ability to transmit as much heat as the surface of the pipe will radiate, after which they become detrimental, rather than useful, as covering. This point is reached nearly at baked clay or brick.

## LOSS OF HEAT FROM STEAM PIPES

Outside Diameter of Pipe, Without Felt

| Thickness of Covering Inches | 2 In. Diameter | | | 4 In. Diameter | | | 6 In. Diameter | | | 8 In. Diameter | | | 12 In. Diameter | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Loss in Units per Foot Run per Hour | Ratio of Loss | Feet in Length per Horse-power Lost | Loss in Units per Foot Run per Hour | Ratio of Loss | Feet in Length per Horse-power Lost | Loss in Units per Foot Run per Hour | Ratio of Loss | Feet in Length per Horse-power Lost | Loss in Units per Foot Run per Hour | Ratio of Loss | Feet in Length per Horse-power Lost | Loss in Units per Foot Run per Hour | Ratio of Loss | Feet in Length per Horse-power Lost |
| 0 | 219.0 | 1.00 | 132 | 390.8 | 1.00 | 75 | 624.1 | 1.000 | 46 | 729.8 | 1.000 | 40 | 1,077.4 | 1.000 | 26 |
| ¼ | 100.7 | .46 | 288 | 180.9 | .46 | 160 | 187.2 | .300 | 154 | 219.6 | .301 | 132 | 301.7 | .280 | 92 |
| ½ | 65.7 | .30 | 441 | 117.2 | .30 | 247 | 111.0 | .178 | 261 | 128.3 | .176 | 225 | 185.3 | .172 | 157 |
| 1 | 43.8 | .20 | 662 | 73.9 | .18 | 392 | 66.2 | .106 | 438 | 75.2 | .103 | 385 | 98.0 | .091 | 294 |
| 2 | 28.4 | .13 | 1,020 | 44.7 | .11 | 648 | 41.2 | .066 | 703 | 46.0 | .063 | 630 | 60.3 | .056 | 486 |
| 4 | 19.8 | .09 | 1,464 | 28.1 | .07 | 1,031 | 33.7 | .054 | 860 | 34.3 | .047 | 845 | 45.2 | .042 | 642 |
| 6 | | | | 23.4 | .06 | 1,238 | | | | | | | | | |

A smooth or polished surface is of itself a good protection, polished tin or Russia iron having a ratio, for radiation, of 53 to 100 for cast iron. Mere color makes but little difference.

## CONDUCTING POWER OF VARIOUS SUBSTANCES
### (From Péclet)

| Substance | Conducting Power | Substance | Conducting Power |
|---|---|---|---|
| Blotting paper | .274 | Wood, across fiber | .83 |
| Eiderdown | .314 | Cork | 1.15 |
| Cotton or wool, any density | .323 | Coke, pulverized | 1.29 |
|  |  | India rubber | 1.37 |
| Hemp, canvas | .418 | Wood, with fiber | 1.40 |
| Mahogany dust | .523 | Plaster of Paris | 3.86 |
| Wood ashes | .531 | Baked clay | 4.83 |
| Straw | .563 | Glass | 6.60 |
| Charcoal powder | .636 | Stone | 13.68 |

Hair or wool felt has the disadvantage of becoming soon charred from the heat of steam at high pressure, and sometimes of taking fire therefrom. This has led to a variety of cements for covering pipes—composed generally of clay mixed with different substances, as asbestos, paper fiber, charcoal, etc. A series of careful experiments, made at the Massachusetts Institute of Technology showed the condensation of steam in a pipe covered by one of them, as compared with a naked pipe, and one covered with hair felt, was 100 for the naked pipe, 67 for the cement covering, and 37 for the hair felt.

The presence of sulphur in the best coverings and its recognized injurious effects make it imperative that moisture be kept from the coverings, for, if present, it will surely combine with the sulphur, thus making it active. Stated in other words, the pipes and coverings must be kept in good repair. Much of the inefficiency of coverings is due to the lack of attention given them; they are often seen hanging loosely from the pipe that they are supposed to protect.

## RELATIVE VALUE OF NON-CONDUCTORS
### (From Chas. E. Emery, Ph. D.)

| Material | Value | Material | Value |
|---|---|---|---|
| Loose wool | 3.35 | Wood, across the grain | .40 to .55 |
| Loose lampblack | 1.12 | Loam, dry and open | .55 |
| Goose feathers | 1.08 | Chalk, ground | .51 |
| Felt, hair or wool | 1.00 | Coal ashes | .35 to .49 |
| Carded cotton | 1.00 | Gas-house carbon | .47 |
| Charcoal from cork | .87 | Asbestos paper | .47 |
| Mineral wool | .68 to .83 | Paste of fossil meal and asbestos | .47 |
| Fossil meal | .66 to .79 |  |  |
| Straw rope, wound spirally | .77 | Asbestos, fibrous | .36 |
| Rice chaff, loose | .76 | Plaster of Paris, dry | .34 |
| Carbonate of magnesia | .67 to .76 | Clay, with vegetable fiber | .34 |
| Charcoal from wood | .63 to .75 | Anthracite powdered | .29 |
| Paper | .50 to .74 | Coke, in lumps | .27 |
| Cork | .71 | Air space, undivided | .14 to .22 |
| Sawdust | .61 to .68 | Sand | .17 |
| Paste of fossil meal and hair | .63 | Baked clay, brick | .07 |
| Wood ashes | .61 | Glass | .05 |
|  |  | Stone | .02 |

The preceding table is deduced from tests of raw materials contained in commercial pipe coverings and does not indicate the relative values of coverings into whose composition they enter. Mineral wool, a fibrous material made from blast-furnace slag, is a good protection, and is incombustible. Cork chips, cemented together with water glass make one of the best coverings known.

A cheap jacketing for steam pipes, but a very efficient one, may be applied as follows: First, wrap the pipe in asbestos paper, though this may be dispensed with; then lay from 6 to 12 strips of wood lengthwise, according to size of pipe, binding them in position with wire or cord, and around the framework thus constructed wrap roofing paper, fastening it by paste or twine. For flanged pipe, space may be left for access to the bolts, which space should be filled with felt. If exposed to weather, tarred paper should be used or the exterior should be painted. A French plan is to cover the surface with a rough flour paste, mixed with sawdust until it forms a moderately stiff dough. It should be applied with a trowel, in four or five layers each $\frac{1}{4}$ in. thick. If iron surfaces are well cleaned from grease, the adhesion is perfect; for copper, a hot solution of clay in water should be applied. A coating of tar renders the composition impervious to the weather.

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## BOILER FEEDING AND FEEDWATER

### INJECTORS

**Classification of Injectors.**—Injectors may be divided into two general classes, namely, non-lifting and lifting injectors. *Non-lifting injectors* are intended for use where there is a head of water available. When the water comes to a non-lifting injector under pressure, as from a city main, it can be placed in almost any convenient position close to the boiler. *Lifting injectors* are of two distinct types, called *automatic injectors* and *positive injectors*. As positive injectors generally have two sets of tubes, they are frequently called *double-tube injectors*. Automatic injectors are so called from the fact that they will automatically start again in case the jet of water is broken by jarring or other means. Positive, or double-tube injectors are provided with two sets of tubes, one set of which is used for lifting the water, and the other set for forcing the water thus delivered to it into the boiler. A positive injector has a wider range than an automatic injector and will handle a hotter feed-water supply; it will also lift water to a greater height than the automatic injector.

**Advantages and Disadvantages of Injectors.**—The advantages of the injector as a boiler feeding apparatus are its cheapness, compared with a pump of equal capacity; it occupies but little space; the repair bills are low, owing to the absence of moving parts; no exhaust piping is required, as with a steam pump; it delivers hot water to the boiler.

The disadvantages of the injector are that it will not start with a steam pressure less than that for which it is designed, and that it will stand but little abuse, being poorly adapted for handling water containing grit or other matter liable to cut the nozzles.

**Size of Injector.**—Most engineers prefer to select a size of injector having a capacity per hour about one-half greater than the maximum evaporation per hour in order to have some reserve capacity. The maximum evaporation,

### WATER DELIVERED BY INJECTORS

| Diameter of Throat Decimals of an Inch | Delivery, in Gallons per Hour, with a Pressure per Square Inch of | | | | |
|---|---|---|---|---|---|
| | 30 Lb. | 45 Lb. | 60 Lb. | 75 Lb. | 90 Lb. |
| .10 | 56 | 69 | 80 | 89 | 98 |
| .15 | 127 | 156 | 180 | 201 | 221 |
| .20 | 226 | 278 | 321 | 360 | 393 |
| .25 | 354 | 434 | 502 | 561 | 615 |
| .30 | 505 | 624 | 722 | 807 | 884 |

when not known, may be estimated in United States gallons by one of the following rules, which hold good for ordinary combustion rates under natural draft:

**Rule I.**—*For plain cylindrical boilers, multiply the product of the length and diameter in feet by 1.3.*

**Rule II.**—*For tubular boilers, either horizontal or vertical, multiply the product of the square of the diameter, in feet, and the length, in feet, by 1.9.*

**Rule III.**—*For water-tube boilers, multiply the heating surface, in square feet, by .4.*

**Rule IV.**—*For boilers not covered by the foregoing rules, multiply the grate surface, in square feet, by 12.*

**Rule V.**—*If the coal consumption, in pounds per hour, is known, it may be taken as representing the number of gallons evaporated per hour.*

No standard method of designating the size of an injector is followed by all makers; therefore, such an instrument must be selected from the lists of capacities published by the different makers.

## WATER REQUIRED PER MINUTE TO FEED BOILERS

| Horsepower of Boiler | Feedwater Gallons | Horsepower of Boiler | Feedwater Gallons | Horsepower of Boiler | Feedwater Gallons | Horsepower of Boiler | Feedwater Gallons | Horsepower of Boiler | Feedwater Gallons |
|---|---|---|---|---|---|---|---|---|---|
| 20 | 1.2 | 60 | 3.6 | 110 | 6.6 | 190 | 11.4 | 400 | 24.0 |
| 25 | 1.5 | 65 | 3.9 | 120 | 7.2 | 200 | 12.0 | 450 | 27.0 |
| 30 | 1.8 | 70 | 4.2 | 130 | 7.8 | 225 | 13.5 | 500 | 30.0 |
| 35 | 2.1 | 75 | 4.5 | 140 | 8.4 | 250 | 15.0 | 600 | 36.0 |
| 40 | 2.4 | 80 | 4.8 | 150 | 9.0 | 275 | 16.5 | 700 | 42.0 |
| 45 | 2.7 | 85 | 5.1 | 160 | 9.6 | 300 | 18.0 | 800 | 48.0 |
| 50 | 3.0 | 90 | 5.4 | 170 | 10.2 | 325 | 19.5 | 900 | 54.0 |
| 55 | 3.3 | 100 | 6.0 | 180 | 10.8 | 350 | 21.0 | 1,000 | 60.0 |

NOTE.—A. S. M. E. standard of 30 lb., or 3.6 gal., per H. P. per hr., evaporated from 100° F., to 70 lb. steam pressure per square inch.

**Location of Injector.**—An injector must always be placed in the position recommended by the maker. There must always be a stop-valve in the steam-supply pipe to the injector. While lifting injectors, when working as such, scarcely need a stop-valve in the suction pipe, it is advisable to supply it. When the water flows to the injector under pressure, a stop-valve in the water-supply pipe is a necessity. A stop-valve and a check-valve must be placed in the feed-delivery pipe, with the stop-valve next to the boiler. The check-valve should never be omitted, even if the injector itself is supplied with one. No valve should ever be placed in the overflow pipe, nor should the overflow be connected directly to the overflow pipe, but a funnel should be placed on the latter so that the water can be seen. This direction does not apply to the inspirator or to any other injector that has a hand-operated, separate over-flow valve. In the inspirator, the overflow pipe is connected directly to the overflow, but the end of the pipe must be open to the air. In general, where the injector lifts water it is not advisable to have a foot-valve in the suction pipe, as it is desirable that the injector and pipe may drain themselves when not in use. A strainer should be placed on the end of the suction pipe.

**Steam Supply to Injector.**—The steam for the injector must be taken from the highest part of the boiler, as it must be supplied with dry steam. Under no consideration should the steam be taken from another steam pipe. The suction pipe should be as straight as possible and must be air-tight. When connecting up an injector, the pipes should be cleaned by being blown out with steam before the connection is made, because if a small bit of dirt gets into the injector it will interfere seriously with its operation.

**Injector Troubles.**—In the following discussion of injector troubles, the suction pipe, strainer, feed-delivery pipe, and check-valve are considered as parts of the injector. When searching for the cause of a trouble, therefore,

the suction and delivery pipes should be carefully inspected as well as the injector.

1. *Failure to Raise Water*—The causes that prevent an injector from raising water are:

*Suction pipe stopped up*, due, generally, to a clogged strainer or to the pipe itself being stopped at some point. In case the suction pipe is clogged, steam should be blown back through the pipe to force out the obstruction.

*Leaks in suction pipe*, which prevent the injector forming the vacuum required to raise the water. To test the suction pipe for air leaks, plug up the end and turn the full steam pressure on the pipe; leaks will then be revealed by the steam issuing therefrom. Have the suction pipe full of water before steam is turned on, as the presence of small leaks will be revealed better by water than by steam.

*Water in the suction pipe too hot;* a leaky steam valve or leaky boiler check-valve and leaky injector check-valve may allow hot water or steam to enter the source of supply and heat the water until the injector refuses to handle it.

*Obstruction in the lifting or combining tubes;* or, the spills (or openings) in the tubes through which the steam and water escape to the overflow may be clogged up with dirt or lime.

2. *Injector Primes But Will Not Force*—In some cases an injector will lift water, but will not force it into the boiler; or, it may force part of it into the boiler and the rest out of the overflow. When it fails to force, the trouble may be due to one or the other of the following causes:

*Choked Suction Pipe or Strainer.*—If the suction pipe or strainer is partly choked, the injector will be prevented from lifting sufficient water to condense all the steam issuing from the steam valve. The uncondensed steam, therefore, will gradually decrease the vacuum in the combining tube until it is reduced so much that the injector will not work. The remedy, when the supply valve is partly closed, is to open it; when the suction pipe is choked, blow out the obstruction.

*Suction Pipe Leaking.*—The leak may not be sufficient to prevent the injector from lifting water, but the quantity lifted may be insufficient to condense all the steam, which, therefore, destroys the vacuum in the combining tube. A slight leak will simply cut down the capacity of the injector. In such a case an automatic injector will work noisily, on account of the overflow valve seating and unseating itself as the pressure in the combining tube varies, due to the leak.

*Boiler Check-Valve Stuck Shut.*—If the boiler check-valve is completely closed, the injector may or may not continue to raise water and force it out of the overflow; this depends on the design of the injector. If the boiler check is partly open, the injector will force some of the water into the boiler and the remainder out of the overflow. In case the check-valve cannot be opened wide, water may be saved by throttling both steam and water until the overflow diminishes, or, if possible, ceases. The steam should be throttled at the valve in the boiler steam connection. If a check-valve sticks, it can sometimes be made to work again by tapping lightly on the cap or on the bottom of the valve body.

*Obstruction in Delivery Tube.*—Any obstruction in the delivery tube will cause a heavy waste of water from the overflow. To remedy this, the tube will have to be removed and cleaned.

*Leaky Overflow Valve.*—A leaky overflow valve is indicated by the boiler check chattering on its seat. To remedy this defect, grind the valve on its seat until it forms a tight joint.

*Injector Choked With Lime.*—It is essential to the proper working of an injector that the interior of the tubes be perfectly smooth and of the proper bore. As in course of time the tubes become clogged with lime, the capacity of the injector decreases until, finally, it refuses to work at all. If the water used is very bad, it is frequently necessary to cleanse the tubes of the accumulated lime. This may be done by putting the parts in a bath consisting of 1 part of muriatic acid to 10 parts of water. The tubes should be removed from it as soon as the gas bubbles cease to be given off.

## INCRUSTATION AND CORROSION

**Incrustation.**—Broadly speaking, any deposit that is formed on the plates and tubes of a boiler is termed *scale*, or *incrustation;* it is caused by impurities that enter with the water and are left behind in the boiler when the water is evaporated. In passing through the soil, water dissolves certain mineral substances, the most important of which are carbonate of lime and sulphate of lime. Other substances frequently present in small quantities are chloride

of sodium, or common salt, and chloride of magnesium. The water also often contains other troublesome substances.

**Impurities in Feedwater.**—Some of the more common impurities found in feedwater, together with their properties, are as follows:

*Carbonate of lime* will not dissolve in pure water, but will dissolve in water that contains carbonic-acid gas. It becomes insoluble and is precipitated in the solid form when the water is heated to about 212° F.; the carbonic-acid gas is driven off by the heat.

*Sulphate of lime* dissolves readily in cold water, but not in hot water. It precipitates in the solid form when the water is heated to about 290° F., corresponding to a gauge pressure of 45 lb.

*Chloride of sodium* will not be precipitated by the action of heat unless the water has become saturated with it. As it generally is present in but very small quantities in fresh water, it will take a very long time for the water in a boiler to become troublesome, and with the ordinary blowing down of a boiler once a week or every 2 wk., there is little danger of the water becoming saturated with it. Consequently, it is one of the least troublesome scale-forming substances contained in fresh water.

*Chloride of magnesium* is one of the worst impurities in water intended for boilers, for while not dangerous as long as the water is cold, it makes the water very corrosive when heated, and when present in large quantities, it attacks the metal and rapidly destroys it.

*Organic matter* by itself may or may not cause the water to become corrosive, but will often cause foaming; when it is present in small quantities in water containing carbonate or sulphate of lime, or both, it usually serves to keep the deposits from becoming hard.

*Earthy matter*, like organic matter, is not dissolved in the water, but is in mechanical suspension. It is very objectionable, especially when it is clay, and when other scale-forming substances are present is liable to form a hard scale resembling Portland cement.

*Acids*, such as sulphuric acid, nitric acid, tannic acid, and acetic acid, are often present in the feedwater. The sulphuric acid is the most dangerous one of these acids, attacking the metal of which the boiler is composed and corroding it very rapidly. The other acids, while not so violent in their action as the sulphuric acid, are also dangerous, and water containing any one should be neutralized when it must be used.

**Formation of Scale.**—The small solid particles due to precipitation of substances in solution or matter in mechanical suspension, remain for a time suspended in the water, especially the carbonate of lime which will float on the surface of the water. These particles will gradually settle on the plates, tubes, and other internal surfaces. A large part of the impurities will be carried by the circulation of the water to the most quiet part of the boiler and there settle and form a scale. In a few weeks, if no means of prevention are used, the inner parts of the boiler may be covered with a crust from $\frac{1}{16}$ to $\frac{1}{2}$ in. in thickness.

**Danger of Scale.**—A scale $\frac{1}{32}$ in. or less in thickness is thought by many to be an advantage, as it protects the plates from the corrosive action of acids in the water. When, however, the scale becomes $\frac{1}{4}$ in. thick or more, heat is transmitted through the plates and tubes with difficulty, more fuel is required, and there is danger of overheating the plates. The chief danger from a heavy incrustation is the liability of overheating the plates and tubes. Scale also prevents a proper examination of the inside of the boiler, as it may hide a dangerously corroded piece of plate or a defective rivet head.

**Scale Containing Lime.**—The carbonate of lime forms a soft, muddy scale, which when dry, becomes fluffy and flourlike. This scale may be easily swept or washed out of the boiler by a hose, provided it is not baked hard and fast. A carbonate scale is much harder to deal with when grease is allowed to enter the boiler. The grease settles and mixes with the floury mass, making a spongy crust that remains in contact with the plates, being too heavy to be carried off by the natural circulation of the water. The sulphate of lime forms a scale that soon bakes to the plates.

**Kerosene as Scale Remover.**—Some substances seem to soften and aid in detaching scale. Of these, kerosene oil has met with much favor. Its action appears to be mechanical rather than chemical, the oil penetrating or soaking through the scale and softening and loosening it. It is somewhat useful, too, in preventing the formation of scale, enveloping the fine particles of the scale-forming substances that, after precipitation, float on the surface of the water for a little while. It seems that this prevents the particles from adhering firmly to one another and to the metal when they finally settle.

**Removal of Scale by Chipping.**—A hard scale, when once formed, is generally removed by chipping it off with scaling hammers and scaling bars; soft scale can be largely removed during running by a periodic use of the bottom and surface blow-offs, and the remainder can usually be washed out and raked out when the boiler is blown down and opened. In order to prevent the scale-forming substances deposited on the metal from baking hard, it is advisable to let the boiler cool down slowly until entirely cold preparatory to blowing off, whenever circumstances permit this to be done. This cooling process will generally take from 24 to 36 hr.

**Removal of Mud.**—Mud and earthy matter by itself will not form any hard scale, but will often do so when carbonate of lime and sulphate of lime are present. An accumulation of such matter can be prevented, and most of it can be removed, by a periodic use of the bottom blow-off, removing any remainder when the boiler is opened.

**Internal Corrosion.**—Corrosion of boiler plates may be defined as the eating away or wasting of the plates due to the chemical action of water. Corrosion may be internal and external. Internal corrosion may present itself as uniform corrosion, pitting or honeycombing, and grooving. In cases of uniform corrosion large areas of plate are attacked and eaten away. There is no sharp line of division between the corroded part and the sound plate. Corrosion often violently attacks the staybolts and rivet heads.
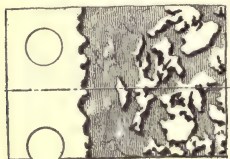


FIG. 1

**Pitting or Honeycombing.**—Pitting or honeycombing of the boiler plates is readily perceived. The plates are indented in spots with holes and cavities from $\frac{1}{32}$ to $\frac{1}{4}$ in. deep. The appearance of a pitted plate is shown in Fig. 1. On the first appearance of pitting, the surface so affected should be thoroughly cleaned and a good coating of thick paint made of red lead and boiled linseed oil should be applied. This treatment should be given from time to time to insure protection to the metal.

**Grooving.**—Grooving, which means the formation of a distinct groove, is generally caused by the buckling action of the plates when under pressure. Thus, the ordinary longitudinal lap joint of a boiler slightly distorts the shell from a truly cylindrical form, and the steam pressure tends to bend the plates at the joint. This bending action is liable to start a small crack along the lap, which, being acted on by corrosive agents in the water, soon deepens into a groove, as shown in Fig. 2.

**External Corrosion.**—External corrosion frequently attacks stationary boilers, particularly those set in brickwork. The causes of external corrosion are dampness, exposure to weather, leakage from joints, moisture arising from the waste pipes or blow-off, etc. External corrosion should be prevented by keeping the boiler shell free from moisture, and the stoppage of all leaks as soon as they appear.

Leakage at rivets and the calking edges of seams may be caused by the delivery of the cold feedwater on to the hot plates; another cause is the practice of emptying the boiler when hot and then filling it with cold water. The leakage in both cases is due to the sudden contraction of the plates.

In horizontal water-tube boilers of the inclined-tube type, external corrosion principally attacks the ends of the tubes, especially the back ends, close up to the headers into which they are expanded. In the course of time this will cause the tubes to leak around the expanded portion in the headers.



FIG. 2

If leaks are attended to as soon as they occur, no corrosion will take place, as the gases of combustion are harmless unless acting in conjunction with water or dampness, or unless the coal is rich in sulphur. Should, however, the ends of several tubes be found badly corroded but not yet leaking from that cause, the tubes should by all means be removed and replaced.

**Lamination.**—Sometimes what is called *lamination*, or the splitting of a plate into thin layers, is revealed by the action of the fire in causing a bag or blister to appear. Laminations due to slag and other impurities in the metal which become flattened out when the plates are rolled, are shown at *a*, Fig. 3.

Under the action of the heat the part exposed to the fire will form a blister which may finally open at the point *b* or *c*. If the laminated portion of the plate is small, it may be cut out and a patch put in its place. If there are a number of laminations in the same plate, it is advisable to put in a new plate.



FIG. 3

**Overheating.**—The heating of a plate beyond its normal temperature is called *overheating*, and may be caused by low water or by incrustation. When the plate is covered by a heavy scale, the plate becomes overheated, so that it yields to the steam pressure, forming a pocket, as shown in Fig. 4, which represents the shell sheet, or the sheet of a horizontal return-tubular boiler directly over the fire. If the pocket is not discovered in time for the repairs to be repaired, it stretches until finally the material becomes too thin to withstand the steam pressure; the pocket then bursts with more or less liability of an explosion. The vegetable or animal oils carried into the boiler from a surface condenser are particularly liable to cause the formation of pockets.

**Prevention of Incrustation and Corrosion.**—Incrustation can best be prevented by purifying the feedwater prior to its entering the boiler, but can be
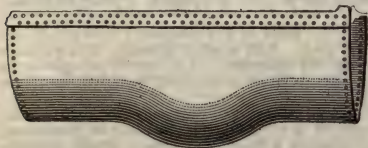


FIG. 4

fairly satisfactorily prevented by a chemical treatment of the water in the boiler. When the water contains large quantities of substances that float on the surface, mechanical means may be resorted to, using the surface blow-off at frequent intervals or some equivalent skimming device. Corrosion is prevented by neutralizing the acids in the water by an alkali. Corrosion due to a perfectly fresh water can be prevented by giving a protective coating to the metal, which may be a thick red-lead paint made up with boiled linseed oil. Sometimes organic substances containing tannic acid, such as oak bark, hemlock, or sumac, are used to loosen or prevent scale. They should not be used, as they are liable to injure the plates by corrosion. The accompanying table gives a list of scale-forming substances and the means of preventing or neutralizing them.

### SCALE-FORMING SUBSTANCES AND THEIR REMEDIES

| Troublesome Substance | Trouble | Remedy or Palliation |
|---|---|---|
| Sediment, mud, clay, etc. | Incrustation | Filtration<br>Blowing off |
| Readily soluble salts | Incrustation | Blowing off<br>Heating feed |
| Bicarbonates of lime, magnesia, iron | Incrustation | Addition of caustic soda, lime, or magnesia |
| Sulphate of lime | Incrustation | Addition of carbonate of soda or barium chloride |
| Chloride and sulphate of magnesium | Corrosion | Addition of carbonate of soda, etc. |
| Carbonate of soda in large amounts | Priming | Addition of barium chloride |
| Acid (in mine water) | Corrosion | Alkali<br>Heating feed |
| Dissolved carbonic acid and oxygen | Corrosion | Addition of caustic soda, slaked lime, etc. |
| Grease (from condensed water) | Corrosion | Slaked lime and filtering<br>Carbonate of soda<br>Substitute mineral oil |
| Organic matter (sewage) | Priming | Precipitate with alum or chloride of iron and filter |
| Organic matter | Corrosion | Same as last |

**Use of Zinc in Boilers.**—Zinc is much used in marine boilers for the prevention of both incrustation and corrosion. The zinc is distributed through the boiler in the form of slabs. About 1 sq. in. of zinc surface should be supplied for every 50 lb. of water in the boiler.

## TESTING OF FEEDWATER

**Testing for Corrosiveness.**—It is a good plan to occasionally test the feedwater and also the water in the boiler for corrosiveness. This may be done by placing a small quantity in a glass and adding a few drops of methyl orange. If the sample of water is acid, and hence corrosive, it will turn pink. If it is alkaline, and hence harmless, it will remain yellow. The acidity may also be tested by dipping a strip of blue litmus paper in the water. If it turns red, the water is acid. This method is not so sensitive as the previous one, which should be used in preference. If litmus paper is kept in stock, it should be kept in a bottle with a glass stopper, as exposure to the atmosphere will cause the paper to deteriorate. If the water in the boilers has become corrosive and corrosion has set in, the water in the gauge glass will show red or even black. As soon as the color is beyond a dirty gray or straw color, it is advisable to introduce lime or soda to neutralize the acid.

**Testing for Carbonate of Lime.**—Pour some of the water to be tested into an ordinary tumbler. Add a little ammonia and ammonium oxalate, and then heat to the boiling point. If carbonate of lime is present, a precipitate will be formed.

**Testing for Sulphate of Lime.**—Pour some of the feedwater into a tumbler and add a few drops of hydrochloric acid. Add a small quantity of a solution of barium chloride and slowly heat the mixture. If a white precipitate is formed, which will not redissolve when a little nitric acid is added, sulphate of lime is present.

**Testing for Organic Matter.**—Add a few drops of pure sulphuric acid to the sample of water. Then add enough of a pink-colored solution of potassium permanganate to make the whole mixture a faint rose color. If the solution retains its color after standing a few hours, no organic substances are present.

**Testing for Matter in Mechanical Suspension.**—Keep a tumblerful of the feedwater in a quiet place. If no sediment is formed in the bottom of the tumbler after standing for a day, there is no mechanically suspended matter in the water.

## PURIFICATION OF FEEDWATER

**Means of Purification.**—Water intended for boilers may be purified by settlement, by filtration, by chemical means, and by heat. Filtration will remove impurities in mechanical suspension, such as oil and grease, and earthy matter, but will not remove substances dissolved in the water. Chemical treatment of the water will render the scale-forming substances and corrosive acids harmless, and may be applied either before or after the water enters the boilers, but preferably the former. Purification by heat is based on the fact that most of the scale-forming substances become insoluble and are precipitated when the water containing them is heated to a high temperature.

**Purification by Settlement.**—For feedwater containing much matter in mechanical suspension, one of the simplest methods of purifying it is to provide a relatively large reservoir, or a large tank for small steam plants, where the impurities can settle to the bottom. While this method is fairly satisfactory in removing earthy matter, it will not clear the water of finely divided organic matter, which is usually lighter than the water and often so finely divided as to be almost dissolved in it.

**Purification by Filtration.**—Organic and earthy matter in mechanical suspension is most satisfactorily removed by a filter, passing the water through layers of sand, gravel, hay, or equivalent substances, or through layers of cloth. Hay and cloth are of service, especially where the feedwater contains oil or grease, as is the case where a surface condenser is used and the condensed steam is used over again.

**Purification by Chemicals.**—Chemical purification may take place before or after the water enters the boiler, the former method being somewhat more expensive. However, the purification is better carried out before the water enters the boiler, as the amount of impurities entering the boiler will be greatly reduced. The process adopted depends on the impurities.

**Use of Quicklime.**—When the water contains only carbonate of lime, it may be treated with slaked quicklime, using 28 gr. of lime for every 50 gr. of lime present in the water, the quicklime precipitating the carbonate of lime and being transformed into carbonate of lime itself during the process.

**Use of Caustic Soda.**—Water containing carbonate of lime may be treated with caustic soda, which precipitates the former and leaves carbonate of soda, which is harmless. For every 100 gr. of carbonate of lime 80 gr. of caustic soda should be added.

**Use of Sal Ammoniac.**—Sal ammoniac is sometimes added to water containing carbonate of lime and will cause the latter to precipitate. Its use is not advisable, however, because if used in excess there is danger of forming hydrochloric acid, which will attack the boiler.

**Treatment for Sulphate of Lime.**—While slaked lime will precipitate carbonate of lime, it will have no effect on sulphate of lime, and water containing the latter, either alone or in conjunction with carbonate of lime, must be treated with other chemicals. The most available chemicals for water containing both are carbonate of soda and caustic soda. These are often fed into the boiler and will precipitate the carbonate and sulphate of lime, requiring the sediment to be blown out or otherwise removed periodically.

**Quantity of Chemicals to Use.**—When treating water containing carbonate and sulphate of lime, caustic soda may be used either by itself or in combination with carbonate of soda, depending on the relative proportions of the lime compounds present in the water. The amount of caustic soda or carbonate of soda to be used per gallon of feedwater can be found as follows:

**Rule I.**—*Multiply the number of grains of carbonate of lime per gallon by 1.36. If this product is greater than the number of grains of sulphate of lime per gallon, only caustic soda is to be used. To find the quantity of caustic soda required per gallon, multiply the number of grains of carbonate of lime in 1 gal. by .8.*

**Rule II.**—*Multiply the number of grains of carbonate of lime per gallon by 1.36. If this product is less than the number of grains of sulphate of lime per gallon, multiply the difference by .78 to obtain the number of grains of carbonate of soda required per gallon. To find the amount of caustic soda required per gallon, multiply the number of grains of carbonate of lime in 1 gal. by .8.*

Example.—A quantitative analysis of a certain feedwater shows it to contain 23 gr. of sulphate of lime and 14 gr. of carbonate of lime per gallon. How much caustic soda and carbonate of soda should be used per gallon to precipitate the scale-forming substances?

Solution.—By rule I, $14 \times 1.36 = 19$ gr. As this product is less than the number of grains of sulphate of lime per gallon, rule II is to be used. Applying rule II, $(23 - 19) \times .78 = 3.12$ gr. of carbonate of soda, and $14 \times .8 = 11.2$ gr. of caustic soda.

**Use of Carbonate of Soda.**—Water containing sulphate of lime, but no carbonate of lime, may be treated with carbonate of soda. The amount of the latter that is required per gallon to precipitate the sulphate of lime is found by multiplying the number of grains per gallon by .78. When using soda, it is well to keep in mind that it will not remove deposited lime from the inside of a boiler. All that the soda can do is to facilitate the separating of the lime, that is, cause it to deposit in a soft state. This sediment must be removed periodically.

**Use of Trisodium Phosphate.**—For decomposing sulphate of lime, tribasic sodium phosphate, more commonly known as trisodium phosphate, is often used. This is claimed to act on the sulphate of lime, forming sulphate of sodium and phosphate of lime, the former of which remains soluble and is harmless, and the latter of which is a loose, easily removed deposit. Trisodium phosphate also acts on carbonate of lime and carbonate of magnesia, forming phosphate of lime and phosphate of magnesia, at the same time neutralizing the carbonic acid released from the carbonate of lime and magnesia, and the sulphuric acid released from the sulphates.

**Neutralization of Acids.**—Acid water can be neutralized by means of an alkali, soda probably being the best one. The amount of soda to be used can best be found by trial, adding soda until the water will turn red litmus paper blue.

**Purification by Heat.**—Carbonate of lime and sulphate of lime become insoluble if the water is heated, the former precipitating at about 212° F. and the latter at about 290° F. This fact is taken advantage of in devices that may be called combined feedwater heaters and purifiers; as they generally use live steam, they are also called *live-steam feedwater heaters.* As no feedwater heater can effect a direct saving of fuel except when the heat is taken from a source of waste, a live-steam feedwater heater can affect the fuel consumption but indirectly. This it does by largely preventing the accumulation of scale in the boiler and the attendant loss in economy due to the lowering of the rate of heat transmission through a plate heavily covered with incrustation.

### FEEDWATER HEATING

The feedwater furnished to steam boilers must be raised from its normal temperature to that of steam before evaporation can commence, and if not otherwise accomplished, it will be done at the expense of fuel that should be utilized in making steam. At 75 lb. gauge pressure, the temperature of boiling water is about 320° F., and if 60° is taken as the average temperature of feedwater, 320 − 60 = 260 B. T. U. is required to raise 1 lb. of water from 60° to 320°. It requires 1,151.5 B. T. U. to convert 1 lb. of water at 60° into steam at 75 lb. gauge pressure, so that the 260 B. T. U. required for heating the water represents 260 ÷ 1,151.5 = 22.6% of the total. All heat taken from a source of waste, therefore, that can be imparted to the feedwater before it enters the boilers is just so much saved, not only in cost of fuel but in boiler capacity.

**Types of Exhaust-Steam Feedwater Heaters.**—The impurities contained in the water will largely determine the type of exhaust-steam heater to be used in any given plant. These heaters are divided into two general classes, namely, open heaters and closed heaters.

An *open heater* is one in which the water space is open to the atmosphere. In a direct-contact open heater, the exhaust steam comes in contact with the water, which, by means of some one of a number of suitable devices, is broken into spray or thin sheets so that it will readily absorb the heat of the steam. In a coil heater, the exhaust steam passes through coils of pipe submerged in a vessel containing the water to be heated, and open at the top.

A *closed heater* is a heater in which the feedwater is not exposed to the atmosphere, but is subjected to the full boiler pressure. The steam does not come in contact with the water; the latter is heated through contact with metallic surfaces, generally those of tubes, that are heated by the exhaust steam.

**Selection of Heater.**—When the boiler feedwater is free from acids, salts, sulphates, and carbonates, so that no scale is formed at a high temperature the closed feedwater heater will be found satisfactory. Heaters of the coil type may be used with pure water, but should not be used with water that will precipitate sediment or scale-forming matter of any kind. The coil heater is very efficient as a heater, as the water circulating through the coils is a long time in contact with the surface surrounded and heated by the exhaust steam. Heaters of the closed type with straight tubes and a sediment chamber can be cleaned more readily than those having curved tubes, but the curved tubes allow more freedom for expansion and contraction. Heaters of the tubular type should have ample sediment chambers and may be used with water that contains organic or earthy matter, but not with water containing scale-forming ingredients. Carbonate of lime is likely to combine with earthy matter and form an exceedingly hard scale.

Heaters of the open exhaust-steam type have the advantage of bringing the exhaust steam in direct contact with the feedwater; some of the exhaust steam is condensed, thus effecting a saving in feedwater, and sediment and scale-forming ingredients, except sulphates of lime and magnesia, are precipitated or will settle to the bottom of the heater. The oil in the exhaust steam must be intercepted by special oil extractors, filters, or skimmers, generally combined with the heater and, by automatic regulation, sufficient fresh feedwater must be added to make up the total quantity required. When the system is properly arranged, all live-steam drips and discharges from traps are led to the heater.

# BOILER TRIALS

**Purposes of Boiler Trials.**—A boiler trial, or boiler test, as it is often called, may be made for one or more of several purposes, the method of conducting the trial depending largely on its purpose. The boiler trial may vary from the simplest one, in which the only observations are the fuel burned and the water fed to the boiler in a stated period of time, to the elaborate standard boiler trial, in which special apparatus and several skilled observers are essential. The object of a boiler trial may be to determine the efficiency of the boiler under given conditions; the comparative value of different boilers working under the same conditions; the comparative value of fuel; or the evaporative power, or horsepower, of the boiler.

**Observations During Trial.**—The essential operations of a boiler trial are the weighing of the feedwater and fuel, and the observation of the steam pressure, temperature of feedwater, and various other less important pressures

and temperatures. These observations should be made simultaneously at intervals of about 15 min.

**Weighing the Coal.**—The coal supplied to the furnace is weighed out in lots of 500 or 600 lb. It is a convenient plan to have a box with one side open placed on a platform scale. A weight is then placed on the scale beam sufficient to balance the box. The scale may then be set at 500 or 600 lb., the coal shoveled in until the beam rises, and then fed directly from the box to the furnace. After the test, the ashes and clinkers must be raked from the ash-pit and grate and weighed. This weight subtracted from the weight of the coal used gives the amount of combustible.

**Measurement of Feedwater.**—The amount of water evaporated in a test for comparative fuel values may be taken as equal to the amount of feedwater supplied without introducing any serious error. The most reliable method of measuring the feedwater delivered to the boilers is to weigh it.

**Standard of Boiler Horsepower.**—When making a horsepower or an efficiency test, a more elaborate method of procedure is required than for a comparative fuel-value test. The reason for this is that different boilers generate steam at different pressures, different feedwater temperatures, and different degrees of dryness; hence, to compare the performances of boilers so as to determine their comparative efficiencies, it is necessary to reduce the actual evaporation to an *equivalent evaporation* from and at 212° F. per lb. of combustible.

A committee of the American Society of Mechanical Engineers has recommended as a commercial horsepower *an evaporation of 30 lb. of water per hr. from a feedwater temperature of 100° F. into steam at 70 lb. gauge pressure*, which is equivalent to $34\frac{1}{2}$ units of evaporation; that is, to $34\frac{1}{2}$ lb. of water evaporated from a feedwater temperature of 212° F. into steam at the same temperature.

As 965.8 B. T. U. is required to evaporate 1 lb. of water from and at 212°, a boiler horsepower is equal to $965.8 \times 34\frac{1}{2} = 33,320$ B. T. U. per hr.

**Equivalent Evaporation.**—The equivalent evaporation is readily determined by means of the formula

$$W_1 = \frac{W(H - t + 32)}{965.8},$$

in which   $W$ = actual evaporation, in pounds of water per hour;
   $H$ = total heat of steam above 32° F. at observed pressure of evaporation;
   $t$ = observed feedwater temperature;
   $W_1$ = equivalent evaporation, in pounds of water per hour, from and at 212° F.

**EXAMPLE.**—A boiler generates 2,200 lb. of dry steam per hr. at a pressure of 120 lb. gauge; the temperature of the feedwater being 70° F.: (*a*) What is the equivalent evaporation? (*b*) What is the horsepower of the boiler?

**SOLUTION.**—(*a*) According to the Steam Table, the total heat $H$ corresponding to a gauge pressure of 120 lb. is 1,188.6 B. T. U. Applying the formula,

$$W_1 = \frac{2,200 \times (1,188.6 - 70 + 32)}{965.8} = 2,621 \text{ lb.}$$

(*b*) The horsepower is obtained by dividing the total equivalent evaporation by 34.5, the equivalent of 1 H. P., and is

$$2,621 \div 34.5 = 76 \text{ H. P., nearly}$$

**Factor of Evaporation.**—The quantity $\dfrac{H - t + 32}{965.8}$ that changes the actual evaporation of 1 lb. of water to the equivalent evaporation from and at 212° F. is called the *factor of evaporation.* To facilitate the calculating of equivalent evaporation, the accompanying table of factors of evaporation is inserted. The equivalent evaporation is found by multiplying the actual evaporation by the factor of evaporation taken from the table.

**EXAMPLE 1.**—A boiler is required to furnish 1,800 lb. of steam per hr. at a gauge pressure of 80 lb.; if the temperature of the feedwater is 48° F., what will be the rated horsepower of the boiler?

**SOLUTION.**—From the table, the factor of evaporation for 80-lb. pressure and a feedwater temperature of 40° is 1.214, and for the same pressure and a feedwater temperature of 50° it is 1.203; the difference is $1.214 - 1.203 = .011$. The difference of temperature is $50° - 40° = 10°$, and the difference between the lower temperature and the required temperature is $48° - 40° = 8°$. Then, $10° : 8° = .011 : x$, or $x = .009$; $1.214 - .009 = 1.205$. $1,800 \times 1.205 = 2,169$ lb., and $2,169 \div 34.5 = 63$ H. P., nearly.

# FACTORS OF EVAPORATION

## Gauge Pressures

Factors of Evaporation

| Temperature of Feedwater Degrees F. | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 120 | 140 | 160 | 180 | 200 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 32 | 1.206 | 1.211 | 1.214 | 1.217 | 1.219 | 1.222 | 1.224 | 1.227 | 1.231 | 1.234 | 1.237 | 1.239 | 1.241 |
| 40 | 1.198 | 1.203 | 1.206 | 1.209 | 1.211 | 1.214 | 1.216 | 1.219 | 1.223 | 1.226 | 1.229 | 1.231 | 1.233 |
| 50 | 1.187 | 1.192 | 1.195 | 1.198 | 1.200 | 1.203 | 1.205 | 1.208 | 1.212 | 1.215 | 1.218 | 1.220 | 1.222 |
| 60 | 1.177 | 1.182 | 1.185 | 1.188 | 1.190 | 1.193 | 1.195 | 1.198 | 1.202 | 1.205 | 1.208 | 1.210 | 1.212 |
| 70 | 1.167 | 1.172 | 1.175 | 1.178 | 1.180 | 1.183 | 1.185 | 1.188 | 1.192 | 1.195 | 1.198 | 1.200 | 1.202 |
| 80 | 1.156 | 1.161 | 1.164 | 1.167 | 1.169 | 1.172 | 1.174 | 1.177 | 1.181 | 1.184 | 1.187 | 1.189 | 1.191 |
| 90 | 1.146 | 1.151 | 1.154 | 1.157 | 1.159 | 1.162 | 1.164 | 1.167 | 1.171 | 1.174 | 1.177 | 1.179 | 1.181 |
| 100 | 1.136 | 1.141 | 1.144 | 1.147 | 1.149 | 1.152 | 1.154 | 1.157 | 1.161 | 1.164 | 1.167 | 1.169 | 1.171 |
| 110 | 1.125 | 1.130 | 1.133 | 1.136 | 1.138 | 1.141 | 1.143 | 1.146 | 1.150 | 1.153 | 1.156 | 1.158 | 1.160 |
| 120 | 1.115 | 1.120 | 1.123 | 1.126 | 1.128 | 1.131 | 1.133 | 1.136 | 1.140 | 1.143 | 1.146 | 1.148 | 1.150 |
| 130 | 1.104 | 1.109 | 1.112 | 1.115 | 1.117 | 1.120 | 1.122 | 1.125 | 1.129 | 1.132 | 1.135 | 1.137 | 1.139 |
| 140 | 1.094 | 1.099 | 1.102 | 1.105 | 1.107 | 1.110 | 1.112 | 1.115 | 1.119 | 1.122 | 1.125 | 1.127 | 1.129 |
| 150 | 1.084 | 1.089 | 1.092 | 1.095 | 1.097 | 1.100 | 1.102 | 1.105 | 1.109 | 1.112 | 1.115 | 1.117 | 1.119 |
| 160 | 1.073 | 1.078 | 1.081 | 1.084 | 1.086 | 1.089 | 1.091 | 1.094 | 1.098 | 1.101 | 1.104 | 1.106 | 1.108 |
| 170 | 1.063 | 1.068 | 1.071 | 1.074 | 1.076 | 1.079 | 1.081 | 1.084 | 1.088 | 1.091 | 1.094 | 1.096 | 1.098 |
| 180 | 1.052 | 1.057 | 1.060 | 1.063 | 1.065 | 1.068 | 1.070 | 1.073 | 1.077 | 1.080 | 1.083 | 1.085 | 1.087 |
| 190 | 1.042 | 1.047 | 1.050 | 1.053 | 1.055 | 1.058 | 1.060 | 1.063 | 1.067 | 1.070 | 1.073 | 1.075 | 1.077 |
| 200 | 1.032 | 1.037 | 1.040 | 1.043 | 1.045 | 1.048 | 1.050 | 1.053 | 1.057 | 1.060 | 1.063 | 1.065 | 1.067 |
| 210 | 1.022 | 1.027 | 1.030 | 1.033 | 1.035 | 1.038 | 1.040 | 1.043 | 1.047 | 1.050 | 1.053 | 1.055 | 1.057 |

**Example 2.**—What is the factor of evaporation when the feedwater temperature is 122° F. and the gauge pressure 72?

Solution.—In the table, under the column headed 70 and opposite 120 in the left-hand column is found 1.128; in column headed 80 and opposite 120 is found 1.131; difference is .003. In the same vertical columns and opposite 130 are found 1.117 and 1.120; difference is .003, same as before. Hence, for an increase of 10 lb. in gauge reading, there is an increase of .003 in the factor of evaporation, or an increase of .0003 for 1 lb. and of .0003 × 2 = .0006 for 2 lb. Therefore, for a feedwater temperature of 120° and 72 lb. pressure, the factor of evaporation is 1.128 + .0006 = 1.1286. The difference between the numbers opposite 120 and 130 in the two columns headed 70 and 80, respectively, is 1.128 − 1.117 = .011, and 1.31 − 1.120 = .011, showing that, for an increase of temperature in the feedwater of 10°, there is a decrease in the factor of .011 and for 1° a decrease of .0011, or for 2° of .0022. Hence, the value of the factor for a temperature of 122° and a gauge pressure of 72 lb. is 1.1286 − .0022 = 1.126.

**Boiler Efficiency.**—The efficiency of a boiler may be defined as the ratio of the heat utilized in evaporating water to the total heat supplied by the fuel. The efficiency thus calculated is really the combined efficiency of the furnace and boiler, as it is not easily possible to determine separately the efficiency of each.

The amount of heat supplied is determined by first accurately weighing the fuel used during the test and deducting all the ash and unconsumed portions. This weight, in pounds, is multiplied by the total heat of combustion of 1 lb. of the fuel, as determined by an analysis, the product being the total number of heat units supplied during the test under the assumption that combustion was perfect. The heat usefully expended in evaporating water is obtained by first weighing the feedwater and correcting this weight according to the quality of the steam; the corrected weight is then multiplied by the number of heat units required to change water at the temperature of the feed into steam at the observed pressure. The efficiency of the boiler, expressed as a per cent., may be found by the formula

$$E = \frac{A}{B},$$

in which

$E$ = efficiency of boiler;
$A$ = heat utilized in evaporating water;
$B$ = total heat supplied by fuel.

**Example.**—A boiler trial shows a useful expenditure of 186,429,030 B. T. U. and a total supply of 270,187,000 B. T. U. What is the efficiency of the boiler?

Solution.—Applying the formula,

$$E = \frac{186,429,030}{270,187,000} = .69 = 69\%$$

**Standard Code.**—For elaborate boiler trials, the standard code recommended by the American Society of Mechanical Engineers should be used.

## BOILER MANAGEMENT

### FILLING BOILERS

**Preparation for Filling Boiler.**—Before starting the flow of water into the boiler, the manhole plates or handhole plates that were removed preparatory to cleaning and overhauling must be replaced, and the blow-off valve must be closed. The gaskets, and also the surfaces with which they come in contact, should be examined to see that they are in good condition. It is customary to place a mixture of cylinder oil and graphite on the outer surface of each gasket, so that it may be removed without tearing. It is important that the manhole plates and handhole plates be properly replaced and secured in order to prevent leakage.

**Height of Water.**—In some cases the water can flow in and fill the boiler to the required height by means of the pressure that exists in the main supply pipe. In other cases, it may be necessary to use a hose or to fill the boiler with a steam pump or a hand pump. The boiler should be filled until the water shows half way up in the gauge glass.

**Escape of Air.**—While filling a boiler it is necessary to make provision for the escape of the contained air, as otherwise the pressure caused by the compression of the air may prevent the boiler from being filled to the proper height.

Most boilers have some valve that can be used for this purpose; a gauge-cock may be left open until water issues therefrom, when it may be closed. Sometimes the manhole plate, if the manhole is on top, is left off while filling a boiler.

## MANAGEMENT OF FIRES WHEN STARTING

**Precautions in Starting.**—After the boiler has been filled and before starting the fire, the attendant should see that the water column and connections are perfectly clear and free, that is, that the valves in the connections and the gauge-glass valves are open so that the water level may show in the glass; he should also see that the gauge-cocks are in good working order and should open the top cock or the safety valve; he should take care that the stress on the stop-valve spindle is relieved by just unscrewing the valve from the seat without actually opening it. He should make sure that the pump, or injector, or whatever device is used to feed the boiler, is in good working order, and ready to start when required.

**Starting the Fires.**—It is customary to cover the grates with a layer of coal first, and then to add the wood, among which may be thrown oily waste or other combustible material that may be at hand. To start the fire, light the waste or other easily ignited material and open the damper and ashpit doors to produce draft. Then close the furnace door. After the wood has started to burn well, spread it evenly over the grate and add a fine sprinkling of coal, until this in turn begins to glow, when more coal may be added and the fire occasionally leveled until the proper thickness of fuel has been obtained. Should the chimney refuse to draw, the draft can generally be started by building a small fire in the base of the chimney.

**Value of Slow Fires.**—When getting up steam, the fire should not be forced but, instead, should be allowed to burn up gradually. By forcing the fire, the plates or tubes that are nearest the fire suffer extreme expansion, while those parts that are remote from the fire are still cold; under such conditions the seams and rivets, and also the tube ends, which are expanded into the tube plates, are liable to be severely strained, and, possibly, permanently injured. It is not desirable to raise steam in any boiler, except in steam fire-engines, in less than from 2 to 4 hr., according to the size, from the time the fire is first started. When steam begins to issue from the opened top gauge-cock or the raised safety valve, as the case may be, the cock or the valve may be closed and the pressure still allowed to rise slowly until the desired pressure has been reached.

**Trying the Fittings.**—After the pressure at which the boiler is to run has been reached, and before cutting it into service, all the valves and cocks should be tried. The safety valve should be raised and its action noted; the water column should be blown out and the gauge-cocks tested; the feeding apparatus should be tried; and it should be noted particularly whether the check-valves seat properly and the valve in the feedpipe is open. All the accessible parts should be examined for leaks.

## CONNECTING BOILERS

**Cutting Boiler Into Service.**—Cutting a boiler into service is accomplished by opening the stop-valve, thus permitting the steam to flow to the engine or other destination. The stop-valve should be opened very slowly to prevent a too sudden change in the temperature and consequent expansion of the piping through which the steam flows, and also to prevent water hammer. The steampipe drain should be kept open until the pipe is thoroughly warmed up. In large plants with many boilers and long steam mains, it takes several hours to warm these pipes thoroughly by the slow circulation of the steam, but the main stop-valve should not be fully opened until these pipes are warm.

**Connecting Boilers to Main.**—Before connecting the different boilers of a battery to the same steam main, the precaution of equalizing the pressures in the different boilers must be observed in order to prevent a sudden rush of steam from one boiler to another. All the pressures should be within about 2 lb. before an attempt is made to connect the boilers.

**Changing Over.**—In plants where there are duplicate sets of boilers, one set being in operation while the other is undergoing repairs, overhauling, and cleaning, the method of *changing over*, or connecting, is as follows: Start the fires and raise steam in the boilers that are to be cut into service. Allow the pressure to rise in all to within 5 lb. of that which is in the boilers in operation. All arrangements before changing over should be made with a view of getting all the heat that can be obtained from the fires in the boilers that are to be cut

out. This can be accomplished by running until the fires have given up all their available heat for making steam, as indicated by the gradual fall in pressure when the dampers are wide open, and then making the change. While the fires in one set of boilers are burning low and the pressure is falling, the pressure in the boilers to be cut in is gradually rising and meeting, so to speak, the falling pressure of the set in operation. When the difference of 5 lb. is reached, change over. A man should be stationed at each stop-valve, and while one is being opened the other should be closed; the engine will continue running uninterruptedly while the change is being made.

## EQUALIZING THE FEED

When the boilers of a battery have been cut into service and hence are all connected together through the steam main, the regulation and equalization of the feedwater becomes an important factor. Each boiler has its own check-valve and feed stop-valve, and generally all the boilers are supplied from one pump, which is running constantly. The quantity of water admitted to each boiler is regulated by its feed stop-valve. When the water gets low in any boiler the feed stop-valve should be opened wider, while at the same time the feed stop-valves on one or more of the other boilers in operation may be closed partly and thus divert the feedwater to the one most requiring it. Some boiler plants have check-valves with an adjustable lift; in that case the feed is equalized by adjusting the lifts of the check-valves, the stop-valves being left wide open while running. It will be understood from the foregoing that the object in view is the maintaining of an equal water level in all the boilers through the manipulation of the feed stop-valves or check-valves. A boiler that is not doing its legitimate share in generating steam may be known by the fact that the feed stop-valve or check-valve on that boiler will be nearly, if not entirely, closed most of the time.

## FIRING WITH SOLID FUEL

The safe and economical operation of steam boilers calls for careful and intelligent management. The fires should be kept in such condition as to maintain the desired pressure and to burn the fuel with economy. Different fuels require different handling and hence only general rules can be given; much will depend on the skill and judgment of the attendant, who must himself discover in each case by actual trial the best method to pursue. The fires must be cleaned at intervals; the time and method of cleaning depend on the nature of the fuel and the rapidity with which it is being consumed, the style of grate in use, and the construction of the furnace.

**Cleaning of Fires.**—There are two methods employed in cleaning the fires: first, that of cleaning the front half and then the rear half; second, that of cleaning one side of the fire and then the other side. In the first method, previous to cleaning, green fuel is thrown on and allowed to burn partly until it glows over the entire surface. The new and glowing fuel is then pushed to the back of the furnace with a hoe, leaving nothing on the front half of the grate but the ashes and clinkers, which are then pulled out, leaving the front end of the grate entirely bare. The new fire, which was pushed back, is drawn forwards and spread over the bare half of the grate. The ashes and clinkers that are on the rear half of the grate are then pulled over the top of the front half of the fire and out through the furnace door; this leaves the rear half of the grate bare, which must be covered by pushing back some of the new front fire. The clean fire having been spread evenly, some new fuel must be spread over the entire surface.

The second method is substantially the same in principle, but the fire is pushed to one side instead of to one end of the furnace. The condition of the fires themselves and the nature of the service of the plant will determine just how often and at what time the cleaning of fires should take place. In general, the fires in stationary boilers require cleaning at intervals of from 8 to 12 hr. Fires require cleaning more often when forced draft is used than when working with natural draft.

Rapidity in cleaning fires is of great importance, as during the operation a large volume of cold air enters the furnace and chills the metallic surfaces with which it comes in contact; consequently, the boiler is damaged, however slightly. It is the greatest advantage of shaking grates that they allow the fire to be cleaned without opening the furnace door; the inrush of cold air and consequent chilling of the plates, etc. is thus avoided.

Before starting to clean fires, the steam pressure and the water level should be run up as high as is safe and the feed should be shut off in order to reduce

the loss in pressure while cleaning. The condition of the fire during cleaning and the opening of the furnace doors cause the pressure to drop quite rapidly, but the rapidity and the amount of drop will be reduced by taking the precautions mentioned and cleaning quickly.

The amount of drop in pressure while cleaning fires depends on several conditions. For example, with a boiler that has a small steam space and, in addition, is, too small for the work required of it without forcing, it is to be expected that the drop in pressure will be much more than if the reverse conditions exist. Furthermore, it may be necessary to clean fires while steam is being drawn from the boiler, instead of being able to clean at a time when the engine is stopped. In that case a greater drop must be expected than when cleaning while no steam is being drawn from the boiler. It is advisable when possible to do the cleaning at a time when no steam is being drawn from the boiler or when the demand for steam is light.

## UNIFORM STEAM PRESSURE

**Desirability of Uniform Pressure.**—The attendant should aim to carry the pressure in the boiler as uniform as possible. A steady steam pressure and a steady water level are conducive to economy in the use of a fuel because, with these conditions, in a properly designed plant there will be a fairly steady temperature in the furnace, which, under normal conditions, is sufficiently high to insure a thorough ignition of the volatile matter in the coal. With a constant demand for steam, a fluctuation in the steam pressure is caused by a change in the furnace temperature, assuming the feedwater supply to be constant, and whenever the steam pressure is down, the furnace temperature is low at the same time. In consequence of this, large quantities of the volatile matter in the coal often escape unconsumed and cause a serious loss of heat. Furthermore, with a steady steam pressure the stresses on the boiler are constant, and hence the life of the boiler will be increased and repair bills will be smaller than otherwise.

**Maintenance of Uniform Pressure.**—During the period of time between the cleaning of the fires, the pressure may be carried nearly uniform by manipulating the feed apparatus so that just the necessary amount of water constantly enters the boiler. Intermittent feeding is practiced under certain local conditions, as, for example, where there is an injector or a pump that is so large that it is impossible to run it continuously without increasing the height of the water level. In such a case, the feeding must be stopped just before firing, and is not resumed until the new fire begins to make steam, as indicated by the rise of pressure on the gauge. If the pressure tends to rise above the standard or normal the dampers must be partly closed and the quantity of feed increased, assuming in this case that no damper regulator is fitted and that, hence, the damper is regulated by hand. A damper regulator, systematic firing, and proper feeding are essential for carrying a practically uniform pressure. Should the pressure continue to rise, more green fuel must be thrown on, the damper closed, the feed increased, and only as a last resort should the furnace door be opened.

A uniform steam pressure cannot be kept without proper firing. To maintain such a pressure the following directions should be observed: Keep the fire uniformly thick; allow no air holes in the bed of fuel; fire evenly and regularly; be careful not to fire too much at a time; keep the fire free from ashes and clinkers; and do not neglect the sides and corners while keeping the center clean. Do not, however, clean the fires oftener than is necessary. Keep the ash-pit clear.

**Keeping Water Level Constant.**—In connection with the maintenance of a constant water level, the following instructions should be followed: On starting to work, remember that the first duty of the fireman is to examine the water level. Try the gauge-cocks, as the gauge glass is not always reliable. If there is a battery of boilers, try the gauge-cocks on each boiler.

## PRIMING AND FOAMING

**Priming.**—The phenomenon called *priming* is analogous to boiling over; the water is carried into the steam pipes and thence to the engine, where considerable damage is liable to take place if the trouble is not checked in time. There are several causes for priming, the most common ones of which are: insufficient boiler power, defective design of boiler, water level carried too high, irregular firing, and sudden opening of stop-valves.

When the boiler power is insufficient, the best remedy is to increase the boiler plant; the next best thing to do is to put in a separator, which, obviously,

will only prevent the entrained water from reaching the engine, and will not stop the priming.

Defective design of a boiler generally consists of a steam space that is too small or a bad arrangement of the tubes, which may be spaced so close in an effort to obtain a large heating surface as to interfere seriously with the circulation. In horizontal return-tubular boilers, a sufficiently large steam space can be obtained by the addition of a steam drum; sometimes the top row of tubes can be taken out to advantage, which permits a lower water level. Defective circulation in horizontal fire-tube boilers is difficult to detect and to remedy; if it is due to a too close spacing of the tubes, a marked betterment may be effected by the removal of one or two vertical rows of tubes. The remedy for a water level that is too high is to carry the water at a lower level.

**Evidences of Priming.**—Priming manifests itself first by a peculiar clicking sound in the cylinder of the engine, due to water thrown against the heads. In cases of very violent priming, the water will suddenly rise several inches in the gauge glass, thus showing more water in the boiler than there really is. When priming takes place, it can be checked temporarily as follows: Close the damper, and thereby check the fires until the water is quiet; the engine stop-valve should also be partly closed to check the inrush of water. Observe whether the water drops in the gauge glass, and then, if more feed is needed, increase the feed. To prevent damage to the engine, open the cylinder drains. Regular and even firing tends to prevent priming.

**Foaming.**—The phenomenon called *foaming* is not the same as priming, though frequently considered so. Foaming is the result of dirty or greasy water in the boiler; the water foams and froths at the surface, but does not lift. A boiler may prime and foam simultaneously, but a foaming boiler does not always prime. Foaming while taking place is visible in the gauge glass and is best remedied by using the surface blow-off. If no surface blow-off is fitted, the bottom blow-off may be used in order to get rid of the dirty water. Like priming, foaming will cause a wrong level to be shown, and hence the first thing to do in case of foaming is to quiet the water by checking the outrush of steam, either by slowing the engine down or by checking the fire, or by both.

## SHUTTING DOWN AND STARTING UP

**Preparations for Shutting Down.**—Before shutting down for the night it is advisable to fill the boiler to the top of the glass, so as to be sure to have sufficient water to start with in the morning. The presence of possible leaks through the valves, tube ends, or seams necessitates this course of action. Even if no leaks exist, it is good practice to do this, if for no other reason than to admit of blowing out a portion before raising steam in the morning. All the gauge-cocks should be tried and the water column should be blown out to insure their being free and clear.

**Banking of Fires.**—The fires may be banked at such a time that there will be about enough steam to finish the day's run, thus shutting down under a reduced pressure with only a remote possibility of its rising again through the night. If the fires are properly banked and the steam worked off while the feed is on, it will be remotely possible for the pressure to rise during the night to a dangerous extent. To bank the fires they should be shoved to the back of the grate and well covered with green fuel, leaving the front part of the grate bare, thus preventing any possibility of the banked fire burning up through the night.

**Closing Valves and Damper.**—The steam stop-valve, feed stop-valve, whistle valve, and other steam valves should be closed; the valves at the top and bottom of the gauge glass also should be shut off to prevent loss of water, etc. in case the glass break during the night. If there is a damper regulator, it should be so arranged that the damper may be left closed, but not quite tight, because a small opening must be left to permit the collecting gases from the banked fire to escape up the chimney; otherwise, there is danger that the gas will ignite and cause an explosion. It is very important to take this precaution and also to make a mark by means of which the distance the damper is open can be ascertained at a glance. In fact, a damper should be so made that when shut to the full extent of its travel there will be still sufficient space around it to allow the gas to escape. The damper regulator should be rendered positively inoperative in any manner permitted by its design so that when closed it will remain in that position until connected properly by the attendant in the morning.

**Starting the Fires.**—On entering the boiler room in the morning, the quantity of water in the boiler should first be noted. The gauge glass and the

gauge-cocks should be tried and the water level determined. After it has been found that the water is not too low, the banked fires may be pulled down and spread over the grates and allowed to burn up slowly, the damper regulator, if one is fitted, in the meantime having been connected.

**Blowing Down.**—While the fires are burning up and before the pressure begins to rise, the blow-off cock or valve should be opened and the boiler blown down; that is, a small quantity of the water should be blown out. This should be done every morning, so that any impurities in mechanical suspension in the water that settled during the night may be removed. Great care should be exercised while doing this so that too much water is not blown out; from 3 to 4 in. as shown by the gauge glass, is sufficient. Under no circumstances should the attendant leave the blow-off while it is open. Disaster to the boiler is liable to follow a disregard of this injunction. Next, all the valves, except the stop-valve, which were shut the night before should be opened and tried to see that they are free and in good working order.

## CARE OF BOILERS

**Safety Valves.**—Great care should be exercised to see that the safety valves are ample in size and in working order. Overloading or neglect frequently lead to the most disastrous results. Safety valves should be tried at least once every day, to see that they act freely.

**Pressure Gauge.**—The steam gauge should stand at zero when the pressure is off, and it should show the same pressure as that at which the safety valve is set when that is blowing off. If the pressures do not agree, the gauge should be compared with one known to be correct.

**Water Level.**—The first duty of an engineer before starting, or at the beginning of his watch, is to see that the water is at the proper height. He should not rely on glass gauges, floats, or water alarms, but try the gauge-cocks. If they do not agree with the water gauge, the cause should be learned and the fault corrected.

**Gauge-Cocks and Water Gauges.**—All gauge-cocks and water gauges must be kept clean. Water gauges should be blown out frequently, and the glasses and passages to them kept clean. The Manchester, England, Boiler Association attributes more accidents to inattention to water gauges than to all other causes put together.

**Feed-Pump or Injector.**—The feed-pump or injector should be kept in perfect order, and be of ample size. No make of pump can be expected to be continuously reliable without regular and careful attention. It is always safe to have two means of feeding a boiler. Check-valves and self-acting feed-valves should be frequently examined and cleaned. The attendant should satisfy himself frequently that the valve is acting when the feed-pump is at work.

**Low Water.**—In case of low water, immediately cover the fire with ashes (wet if possible) or any earth that may be at hand. If nothing else is handy, use fresh coal. Draw fire as soon as it can be done without increasing the heat. Neither turn on the feed, start nor stop engine, nor lift safety valve until fires are out and the boiler cooled down.

**Blisters and Cracks.**—Blisters and cracks are liable to occur in the best plate iron. When the first indication appears, there must be no delay in having the fault carefully examined and properly cared for.

**Fusible Plugs.**—When used, fusible plugs must be examined when the boiler is cleaned, and carefully scraped clean on both the water and fire sides, or they are liable not to act.

**Firing.**—Fire evenly and regularly, a little at a time. Moderately thick fires are most economical, but thin firing must be used where the draft is poor. Take care to keep grates evenly covered, and allow no air holes in the fire. Do not clean fires oftener than necessary. With bituminous coal, a coking fire, i. e., firing in front and shoving back when coked, gives best results, if properly managed.

**Cleaning.**—All heating surfaces must be kept clean outside and in, or there will be a serious waste of fuel. The frequency of cleaning will depend on the nature of fuel and water. When a new feedwater supply is introduced, its effect on the boiler should be closely observed, as this new supply may be either an advantage or a detriment as compared with the working of the boiler previous to its introduction. As a rule, never allow over $\frac{1}{16}$ in. of scale or soot to collect on surfaces between cleanings. Handholes should be frequently removed and surfaces examined, particularly in the case of a new boiler, until proper intervals have been established by experience.

The exterior of tubes can be kept clean by the use of blowing pipe and hose through openings provided for that purpose. When using smoky fuel, it is best to occasionally brush the surfaces when steam is off.

**Hot Feedwater.**—Cold water should never be fed into any boiler when it can be avoided, but when necessary it should be caused to mix with the heated water before coming in contact with any portion of the boiler.

**Foaming.**—When foaming occurs in a boiler, checking the outflow of steam will usually stop it. If caused by dirty water, blowing down and pumping up will generally cure it. In cases of violent foaming, the draft and fires should be checked.

**Air Leaks.**—Be sure that all openings for admission of air to boiler or flues, except through the fire, are carefully stopped; this is frequently an unsuspected cause of serious waste of fuel.

**Blowing Off.**—If feedwater is muddy or salt, blow off a portion frequently, according to condition of water. Empty the boiler every week or two, and fill up afresh. When surface blow cocks are used, they should be often opened for a few minutes at a time. Make sure no water is escaping from the blow-off cock when it is supposed to be closed. Blow-off cocks and check-valves should be examined every time the boiler is cleaned. Never empty the boiler while the brickwork is hot.

**Leaks.**—When leaks are discovered, they should be repaired as soon as possible.

**Filling Up.**—Never pump cold water into a hot boiler. Many times leaks, and, in shell boilers, serious weaknesses, and sometimes explosions are the result of such an action.

**Dampness.**—Take care that no water comes in contact with the exterior of the boiler, as it tends to corrode and weaken it. Beware of all dampness in seatings and coverings.

**Galvanic Action.**—Examine frequently parts in contact with copper or brass, where water is present, for signs of corrosion. If water is salt or acid, some metallic zinc placed in the boiler will usually prevent corrosion, but it will need attention and renewal from time to time.

**Rapid Firing.**—In boilers with thick plates or seams exposed to the fire, steam should be raised slowly, and rapid or intense firing avoided. With thin water tubes, however, and adequate water circulation, no damage can come from that cause.

**Standing Unused.**—If a boiler is not required for some time, empty and dry it thoroughly. If this is impracticable, fill it quite full of water, and put in a quantity of common washing soda. External parts exposed to dampness should receive a coating of linseed oil.

**Repair of Coverings.**—All coverings should be looked after at least once a year, given necessary repairs, refitted to the pipe, and the spaces due to shrinkage taken up. Little can be expected from the best non-conductors if they are allowed to become saturated with water, or if air-currents are permitted to circulate between them and the pipe.

**General Cleanliness.**—All things about the boiler room should be kept clean and in good order; negligence tends to waste and decay.

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# BOILER INSPECTION

## NATURE OF INSPECTION

The inspection of a boiler usually consists in an external examination of the complete structure, and of the setting if the boiler is externally fired, and an internal inspection. The examination of the boiler consists of an ocular inspection for visible defects, and a *hammer test* or sounding for hidden defects of plates, stays, braces, and other boiler parts. The hammer test is made by tapping the suspected parts with a light hammer and judging the existence and extent of defects from the sound produced by the blow. If the examination discloses marked wear and tear, a series of calculations is often required to find the safe pressure that may be allowed on the worn parts, using such formulas or rules as laws, ordinances, and regulations may prescribe. In the absence of officially prescribed formulas and rules, the inspector should use such rules as he deems in best accordance with good practice. The inspection is usually, but not always, completed by a so-called *hydrostatic test*, which is generally prescribed by official regulations.

## EXTERNAL INSPECTION

**Preparation.**—Before a boiler that has been in use can be inspected, it must be blown out and must be allowed to cool off. As soon as the water has been removed, the manhole covers, handhole covers, and washout plugs should be taken out and all loose mud and scale washed out with a hose. If the boiler is externally fired, the tubes must be swept and the furnace, the ash-pit, the smokebox, and the space back of the bridge wall must be cleaned out. Any removable insulating covering that prevents the inspector from having free access to the exterior of the boiler must be removed to the extent deemed necessary by him; it may even be necessary to take down some of the bricks of the setting.

**Inspection of Externally Fired Boilers.**—In the inspection of an externally fired fire-tube or flue boiler, the exterior is first examined. The seams are gone over inch by inch; the rivet heads and calking edges of the plates are carefully scrutinized for evidence of leaks; and possible cracks are looked for between the rivet heads, especially in the girth seams and on the under side of the boiler. The plates must also be examined for corrosion, bulges, blisters, and cracks. The heads are inspected for cracks between the tubes or flues, cracks in the flanges, leaky tubes, and leaks in the seams. The condition of the firebrick lining of the furnace and bridge and the top of the rear combustion chamber is noted while making the exterior examination of the underside of the boiler. Every defect that is found should be clearly marked. Attention must also be paid to the condition of the grate bars and their supports.

**Inspection of Internally Fired Boilers.**—The inspection of the shell and heads must be followed by examination of the fire-box or furnace tubes or flues, and of the combustion chambers if these are fitted inside the boiler. In fire-boxes, special attention must be paid to the crown sheet. The ends of the staybolts require close examination; if such ends are provided with nuts, these must be examined, as they are liable to loosen and are also liable to be burned off in time. Each staybolt should be tested for breakage, which is done by holding a sledge against the outside end of the staybolt and striking the inner, or fire-box, end with a light hammer; in making this test on the boilers of locomotives it is customary, when practical, to subject the boiler to an internal air pressure of from 40 to 50 lb. per sq. in. The internal pressure, by bulging the sheets, separates the ends of a broken staybolt, which renders it comparatively easy to find them by the hammer test.

**Inspection of New Boilers.**—As made in boiler shops, the external inspection of new boilers, whether they are internally or externally fired, and whether they are of the water-tube or the fire-tube type, usually consists in a thorough examination for visible defects and testing under water pressure to locate leaks. If a new boiler subject to official inspection during construction successfully passes such a hydrostatic test as the regulations prescribe, it will usually be permitted the working pressure it was designed for, the design having been approved officially before construction. The working pressure will be reduced, however, if the inspection discloses poor workmanship.

In the external inspection of water-tube boilers that have been in use, the tubes that are exposed directly to the heat of the fire must be particularly well examined for evidence of overheating. The plugs or handholes placed in headers to permit the insertion of the tubes and the cleaning of them are inspected for leakage, and the headers are inspected for cracks. Steam drums and mud-drums should be examined as carefully and for the same defects as the shells of externally fired fire-tube boilers. The fire-brick lining of the furnace, and the interior of the brick setting in general, as well as the baffle plates controlling the direction of flow of the gases of combustion, must be examined for cracks and any other defects. The external inspection of the setting can usually be made very rapidly, as everything is in plain sight.

## INTERNAL INSPECTION

**Preparation.**—Before the internal inspection is begun all loose mud should be washed out with a hose. In a horizontal return-tubular boiler and flue boiler, the shell plates and heads should be examined for corrosion and pitting; if the boiler has longitudinal lap seams, these should be inspected at the inside calking edge for incipient grooving and cracks. All seams should be examined for cracks between the rivet holes. Obviously, if the boiler is scaled to an appreciable degree, the scale must be removed before inspection. The tubes or flues should be examined for pitting, as well as for uniform corrosion. All braces should be inspected by sounding them with a hammer, and if they are attached by cotter pins, it should be seen to that these are firmly in place.

All defects found should be marked; it is good practice to make a memorandum of them as well. If any of the bracing seems to have worn considerably, it should be measured at the smallest part in order that the safe working pressure thereon may be calculated afterwards. To determine to what thickness a plate attacked by uniform corrosion has been reduced the inspector will have one or more holes drilled through the plate in the worn part to enable him to measure the thickness. These holes are afterwards plugged, generally by tapping out and then screwing in a plug.

**Inspection of Locomotive-Type Boilers.**—In internally fired boilers of the fire-box and locomotive type, particular attention must be paid to the crown bars, crown bolts, and sling stays; in boilers having the crown sheet stayed by radial staybolts, special attention is also paid to these. As a general rule, the inspector can make only an ocular inspection of most of them, as they are beyond his reach; where the outer sheets of the firebox contain inspection or washout holes above the level of the crown sheet, a lighted candle tied or otherwise fastened to a stick can usually be introduced through these holes from the outside by a helper. In inspecting above the crown sheet, the inspector should look for mud between the crown sheet and crown bars and sight over the top of the bars to see if any have been bent. As the inspector can reach from the inside of the boiler only a few of the staybolts staying the sides of the firebox, he must rely on the hammer test applied from the inside of the furnace for finding broken staybolts.

**Flues and Combustion Chambers.**—In boilers having circular furnace flues and internal combustion chambers, the top of the furnace flues must be carefully inspected for deposits of grease and scale, which are especially liable to be found if the feedwater is obtained from a surface condenser. Even a light deposit of grease on the furnace flue is liable to lead to overheating and subsequent collapse of the top. The tops of the combustion chambers, together with their supports, are usually easily inspected, there being ample space to reach every part.

**Inspection of Vertical Boilers.**—Vertical boilers as a general rule, except in the largest sizes, have no manhole to admit a person to the inside, and such internal inspection as is possible must be made through the handholes. Defects to be looked for are pitting and uniform corrosion of the shell and tubes near the usual water-line, and cracks in the heads between the tubes, the lower head being especially liable to show this injury.

## INSPECTION OF FITTINGS

**Inspection of Safety Valve.**—The safety valve requires very careful inspection. If this valve is known to leak, it should be reseated and reground before the hydrostatic test is made. After a boiler passes the hydrostatic test, the clamp locking the safety valve is removed, and by running the pressure up once more, the point at which the safety valve opens can be noted by watching the steam gauge, which is supposed to have been tested and corrected. If the safety valve does not open at the working pressure allowed or opens too soon, it is readjusted. If the safety valve is locked by a seal, as is often required by official regulations, the seal is applied after adjustment of the valve.

**Testing of Steam Gauge.**—The steam gauge should be tested before the hydrostatic test and at each inspection with a so-called boiler inspector's testing outfit. If the gauge under test is more than 5% incorrect most inspectors will condemn it although some will condemn gauges showing a much smaller error. In most cases the gauge can be repaired at small expense by the makers.

**Inspection of Water Gauge and Blow-off.**—The connections of water column and water-gauge glasses require examination in order to see that they are clear throughout their whole length. The blow-off pipe also requires examination in order to see that it is clear.

## SELECTION OF BOILERS

**General Requirements.**—When choosing a boiler, the facts to be kept in mind are:

1. The grate surface must be sufficient to burn the maximum quantity of coal expected to be used at any time, taking into consideration the available draft, the quality of coal, its percentage of ash, whether or not the ash tends to run into clinker, and the facilities, such as shaking grates, for getting rid of the ash or clinker.

2. The furnace must be adapted to burn the particular kind of coal used.

3. The heating surface must be sufficient to absorb so much of the heat generated that the gases escaping into the chimney will be not over 450° F. with anthracite, and 550° F. with bituminous coal.

4. The gas passages must be so designed and arranged as to compel the gas to traverse at a uniform rate the whole of the heating surface, being not so large at any point as to allow the gas to find a path of least resistance, or short-circuiting, or, on the other hand, so contracted at any point as to cause an obstruction to the draft.

When these elements are found in any boiler—and they may be found in boilers of many of the common types—the relative merits of the different types may be considered with reference to their danger of explosion; their probable durability; the character and extent of repairs that may be needed from time to time, and the difficulty, delay, and expense that these may entail; the accessibility of every part of the boiler to inspection, internal and external; the facility for removal of mud and scale from every portion of the inner surface, and of dust and soot from the exterior; the water and steam capacity; the steadiness of water level; and the arrangements for securing dry steam.

**Liability to Explosion.**—All boilers may be exploded by overpressure, such as might be caused by the combination of an inattentive fireman and an inoperative safety valve, or by corrosion weakening the boiler to such an extent as to make it unable to resist the regular working pressure; but some boilers are much more liable to explosion than others. When selecting a boiler, it is well to see whether or not it has any of the features that are known to be dangerous.

The plain cylinder boiler is liable to explosion from strains induced by its method of suspension, and by changes of temperature. Alternate expansion and contraction may produce a line of weakness in one of the rings, which may finally cause an explosion. A boiler should be so suspended that all its parts are free to change their position under changes of temperature without straining any part. The circulation of water in the boiler should be sufficient to keep all parts at nearly the same temperature. Cold feedwater should not come in contact with the shell, as this will cause contraction and strain.

The horizontal tubular boiler, and all externally fired shell boilers, are liable to explosion from overheating of the shell, due to accumulation of mud, scale, or grease, on the portion of the shell lying directly over the fire, to a double thickness of iron with rivets, together with some scale, over the fire, or to low water uncovering and exposing an unriveted part of the shell directly to the hot gases.

Vertical tubular boilers are liable to explosion from deposit of mud, scale, or grease, upon the lower tube-sheet, and from low water allowing the upper part of the tubes to get hot and cease to act as stays to the upper tube-sheet.

Locomotive boilers may explode from deposits on the crown sheet, from low water exposing the dry crown sheet to the hot gases, and from corrosion of the staybolts.

Double-cylinder boilers, such as the French elephant boiler, and the boilers used at some American blast furnaces, have exploded on account of the formation of a steam pocket on the upper portion of the lower drum, the steam being prevented from escaping from out of the rings of the drum by the lap joint of the adjoining ring, thus making a layer of steam about $\frac{1}{4}$ in. thick against the shell, which was directly exposed to the hot gases. In the case of vertical or inclined tubes acting as stays to an upper sheet, the upper part of the tubes may become overheated in case of low water; also, when there are stayed sheets, the stays are liable to become corroded.

In addition to these features of design, all boilers are liable to explosion due to corrosion. Internal corrosion is usually due to acid feedwater, and all boilers are equally liable to it. External corrosion, however, is more liable to take place in some designs of boilers than others, and in some locations rather than others. If any portion of a boiler is in a cold and damp place, it is liable to rust out. For this reason the mud-drums of many modern forms of boilers are made of cast iron, which resists rusting better than either wrought iron or steel. If any part of a boiler, other than a part made of cast iron, is liable to be exposed to a cold and damp atmosphere, or covered with damp soot or ashes, or exposed to drip from rain or from leaky pipes, and especially if such part is hidden by brickwork or otherwise so that it cannot be seen, that part is an element of danger.

The causes of boiler explosions may be summarized as follows: (1) Bad materials; (2) bad workmanship; (3) bad water, which eats away the plates by internal corrosion; (4) water lying upon plates, bringing about external

corrosion; (5) overpressure; (6) safety valves sticking; (7) water getting too low; (8) excessive firing; (9) hot gases, acting on plates above water level; (10) choking of feedpipes; (11) insufficient provision for expansion and contraction; (12) insufficient steam room and too sudden a withdrawal of a large quantity of steam; (13) getting up steam, or knocking off a boiler too suddenly; (14) allowing wet ashes to lie in contact with plates. The probable causes suggest their several remedies.

**Durability.**—The question of durability is partly covered by that of danger of explosion, but it also is related to the question of incrustation and scale. The plates and tubes of a boiler may be destroyed by internal or external corrosion and by being burned out. It may be regarded as impossible to burn a plate or tube of iron or steel, no matter how high the temperature of the flame, provided one side of the metal is covered with water. However, if a steam pocket is formed, or if there is a layer of grease or hard scale, so that the water does not touch the metal, the plate or tube may be burned. In a water tube that is horizontal, or nearly so, and in which the circulation of water is defective, it is possible to form a mass of steam that will drive the water away from the metal, and thus allow the tube to burn out. When considering the probable durability of a boiler, it is necessary to consider the same things as when investigating the danger of explosion. There are, however, many chances of burning out a minor part of a boiler without serious danger, to one chance of a disastrous explosion. Thus the tubes of a water-tube boiler, if allowed to become thickly covered with scale, might be burned out again and again without causing any further destruction at any one time than the rupture of a single tube. A new type of boiler should be considered with regard to the likelihood of frequent small repairs being necessary, as well as with regard to its liability to complete destruction. The most important of these considerations are: The circulation through all parts of the boiler must be such that the water cannot be driven out of any tube or from any portion of a plate, so as to form a steam pocket exposed to high temperature; there must be proper facilities for removing the scale from every portion of the plates and tubes.

**Repairs.**—The questions of durability and of repairs are, in some respects, related to each other; the more infrequent and the less extensive the repairs, the greater the durability. The tubes of a boiler, where corroded or burnt out, may be replaced and made as good as new. The shell, when it springs a leak, may be patched, but is then likely to be far from as good as new. When the shell corrodes badly it must be replaced, and to replace the shell is the same as getting a new boiler. Herein is the advantage of the sectional water-tube boilers. The sections, or parts of a section, may be renewed easily, and made as good as new, while the shell, being far removed from the fire and easily kept dry externally, is not liable either to burning out or to external corrosion. When considering the merits of a new style of boiler, with reference to repairs, it may be asked what parts of the boiler are most likely to give out and need to be repaired or replaced? Are these repairs easily effected, how long will they require, and, after they are made, is the boiler as good as new?

**Facility for Removal of Scale and for Inspection.**—The matter of facility for the removal of scale and for inspection has already been discussed to some extent under the head of durability. Some early water-tube boilers had no facilities for the removal of scale, it being claimed that they did not need any, because their circulation was so rapid. If there is scale-forming material in the water it will be deposited when the water is evaporated, and no amount or kind of circulation will keep it from accumulating on every part of the boiler, and in every kind of tubes, vertical, horizontal, or inclined. The nearly vertical circulating tubes of a water-tube boiler, in which the circulation is nine times as fast as the average circulation in the inclined tubes, have been found nearly full of scale; that is, a 4-in. tube had an opening in it of less than 1 in. in diameter. This was due to carelessness in blowing off the boiler, or to exceptionally bad feedwater, or both.

**Water and Steam Capacity.**—It is claimed for some forms of boilers that they are better than others because they have a larger water or steam capacity. Great water capacity is useful where the demands for steam are extremely fluctuating, as in a rolling mill or a sugar refinery, where it is desirable to store up heat in the water in the boilers during the periods of the least demand to be given out during periods of greatest demand. Large water capacity is usually objectionable in boilers for factories, especially if they do not run at night and the boilers are cooled down, because there is a large quantity of water to be heated before starting each morning. If *rapid steaming*, or the ability

to get up steam quickly from cold water, or to raise the pressure quickly, is desired, large water capacity is a detriment.

The advantage of large steam capacity is usually overrated. It is useful to enable the steam to be drained from water before it escapes into the steam pipe, but the same result can be effected by means of a dry pipe, as in locomotive and marine practice, in which the steam space in the boiler is very small in proportion to the horsepower. Large steam space in the boiler is of no importance for storing energy or equalizing the pressure during the stroke of an engine. The water in the boiler is the place to store heat, and if the steam pipe leading to an engine is of such small capacity that it reduces the pressure, the remedy is a steam reservoir close to the engine or a large steam pipe.

To secure steadiness of water level requires either a large area of water surface, so that the level may be changed slowly by fluctuations in the demand for steam or in the delivery of the feed-pump, or else constant, and preferably automatic, regulation of the feedwater supply to suit the steam demand. A rapidly lowering water level is apt to expose dry sheets or tubes to the action of the hot gases, and thus be a source of danger. A rapidly rising level may, before it is seen by the fireman, cause water to be carried over into the steam pipe, and endanger the engine.

**Water Circulation.—**Positive and complete circulation of the water in a boiler is necessary to keep all parts of the boiler at a uniform temperature, and to prevent the adhesion of steam bubbles to the surface, which may cause overheating of the metal. It is claimed by some manufacturers that the extremely rapid circulation of water in their boilers tends to make them more economical than others. However, proof is lacking that increased rapidity of circulation of water beyond that usually found in any boiler will give increased

## RATIO OF HEATING SURFACE TO HORSEPOWER AND TO GRATE AREA

| Type of Boiler | Ratio = $\dfrac{\text{Heating Surface}}{\text{Horsepower}}$ | Ratio = $\dfrac{\text{Heating Surface}}{\text{Grate Area}}$ |
|---|---|---|
| Plain cylindrical...... | 6 to 10 | 12 to 15 |
| Flue.............. | 8 to 12 | 20 to 25 |
| Return-tubular....... | 14 to 18 | 25 to 35 |
| Vertical............. | 15 to 20 | 25 to 30 |
| Water-tube.......... | 10 to 12 | 35 to 40 |
| Locomotive.......... | 1 to 2 | 50 to 100 |

economy. It is known that increased rate of flow of air over radiating surfaces increases the amount of heat transmitted through the surface, but this is because by the increased circulation, cold air is continually brought into contact with the surface, making an increased difference of temperature on the two sides, which causes increased transmission. But by increasing the rapidity of circulation in a steam boiler, it is not possible to vary the difference of temperature to any appreciable extent, for the water and the steam in the boiler are at about the same temperature throughout. The ordinary or "Scotch" form of marine boiler shows an exception to the general rule of uniformity of temperature of water throughout the boiler, but the temperature above the level of the lower fire-tubes is practically uniform.

**Heating Surface.—**In the various types of boilers, there is a nearly constant ratio between the water-heating surface and the horsepower, and also between the heating surface and the grate area. These ratios are given in an accompanying table. If the heating surface of a boiler is known, the horsepower can be found roughly; thus, if a return-tubular boiler has a heating surface of 900 sq. ft., its horsepower lies between $900 \div 18 = 50$ H. P. and $900 \div 14 = 64.3$ H. P., say about 57 H. P.

The heating surface of a boiler is the portion of the surface exposed to the action of flames and hot gases. This includes, in the case of a multitubular boiler, the portions of the shell below the line of brickwork, the exposed heads of the shell, and the interior surface of the tubes. In the case of a water-tube boiler, the heating surface comprises the portion of the shell below the brickwork, the outer surface of the headers, and the outer surface of the tubes. In any given case, the heating surface may be calculated by the rules of mensuration.

The following example will show the method of calculating the heating surface of a return-tubular boiler:

EXAMPLE.—What is the heating surface of a horizontal return-tubular boiler that has the following dimensions: Diameter, 60 in.; length of tubes, 12 ft.; internal diameter of tubes, 3 in.; number of tubes, 82.

SOLUTION.—Assume that two-thirds of the shell is in contact with hot gases or flame, and two-thirds of the two heads are heating surface.

Circumference of shell is $60 \times 3.1416 = 188.496 = 188.5$ in., say.

Length of shell is $12 \times 12 = 144$ in.

Heating surface of shell is $188.5 \times 144 \times \frac{2}{3} = 18,096$ sq. in.

Circumference of tube is $3 \times 3.1416 = 9.425$ in., nearly.

Heating surface of tubes is $82 \times 144 \times 9.425 = 111,290.4$ sq. in.

Area of one head is $60^2 \times .7854 = 2,827.44$ sq. in.

Two-thirds area of both heads is $\frac{2}{3} \times 2 \times 2,827.44 = 3,769.92$ sq. in.

From the heads must be subtracted twice the area cut out by the tubes; this is $82 \times 3^2 \times .7854 \times 2 = 1,159.26$ sq. in.

Total heating surface in square feet is

$$\frac{18,096 + 111,290.4 + 3,769.92 - 1,159.26}{144} = 916.64 \text{ sq. ft.}$$

## PROBABLE MAXIMUM WORK OF A PLAIN CYLINDRICAL BOILER OF 120 SQ. FT. HEATING SURFACE AND 12 SQ. FT. GRATE SURFACE

| | 2 | 3 | 3.5 | 4 | 4.5 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Rate of driving; pounds of water evaporated per sq. ft. of heating surface per hour ....... | 2 | 3 | 3.5 | 4 | 4.5 | 5 | 6 | 7 | 8 |
| Total water evaporated by 120 sq. ft. heating surface per hour, pounds...... | 240.00 | 360.00 | 420.00 | 480.00 | 540.00 | 600.00 | 720.00 | 840.00 | 960.00 |
| Horsepower 34.5 lb. per hour = 1 H.P. | 6.96 | 10.43 | 12.17 | 13.91 | 15.65 | 17.39 | 20.87 | 24.35 | 27.83 |
| Pounds water evaporated per pound combustible . | 10.88 | 11.30 | 11.36 | 11.29 | 11.20 | 11.05 | 10.48 | 9.48 | 8.22 |
| Pounds combustible burned per hour..... | 22.10 | 31.90 | 37.00 | 42.50 | 48.20 | 54.30 | 68.70 | 88.60 | 116.80 |
| Pounds combustible per hour per square foot of grate . | 1.85 | 2.65 | 3.08 | 3.55 | 4.02 | 4.52 | 5.72 | 7.38 | 9.73 |
| Pounds combustible per hour per horse-power....... | 3.17 | 3.05 | 3.04 | 3.06 | 3.08 | 3.12 | 3.30 | 3.64 | 4.16 |

The figures in the last line show that the amount of fuel required for a given horsepower is nearly 37% greater when the rate of evaporation is 8 lb. than when it is 3.5 lb.

The figures in the foregoing table that represent the economy of fuel, viz., Pounds water evaporated per pound combustible and Pounds combustible per hour per horsepower, are what may be called maximum results, and they are the highest that are likely to be obtained with anthracite, with the most skillful firing and with every other condition most favorable. Unfavorable conditions, such as poor firing, scale on the inside of the heating surface, dust

or soot on the outside, imperfect protection of the top of the boiler from radiation, leaks of air through the brickwork, or leaks of water through the blow-off pipe, may greatly reduce these figures.

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# CHIMNEYS

**Products of Combustion.**—The weight and volume of the various gases that enter into problems relating to combustion when measured at 32° F. and the average atmospheric pressure at sea level of approximately 14.7 lb. per sq. in., corresponding to a height of the mercurial barometer of 29.92 in. are given under the subject of Ventilation. To find the volume at any other temperature and pressure, the following formula is used,

$$v_1 = v_2 \frac{p_2 T_1}{p_1 T_2},$$

in which $v_2$ = volume corresponding to absolute pressure $p_2$ and absolute temperature $T_2$ (or $460 + t_2$);

$p_2$ = any given absolute pressure;

$v_1$ = volume at any other pressure $p_1$ and absolute temperature $T_1$ (or $460 + t_1$).

EXAMPLE.—What is the volume of 4 lb. of dry air at 75° F. and under an absolute pressure of 20 lb. per sq. in.?

SOLUTION.—From the table, it is found that 1 lb. of air, at 32° F. and 14.7 lb. per sq. in. absolute pressure, occupies 12.388 cu. ft., hence, under the same conditions, 4 lb. occupies $4 \times 12.388 = 49.552$ cu. ft. Substituting, in the formula, the values $v_2 = 49.552$, $p_2 = 14.7$ lb. per sq. in., $p_1 = 20$ lb. per sq. in., $t_1 = 75°$ F., we get     $v_1 = 49.552 \times \frac{14.7}{20} \times \frac{535}{492} = 39.6$ cu. ft.

Hitherto it has been considered that the fuel was burned in oxygen. When burning with air, the chemical reactions are the same, for the nitrogen in the air passes through the furnace unchanged. In calculations of temperature, however, account must be taken of the nitrogen, as it is heated by the combustion and therefore absorbs heat and causes the furnace to have a lower temperature than if oxygen alone were used.

The first table on page 447 gives the weights of air, water vapor, and saturated mixtures of air and water vapor at different temperatures, under the ordinary atmospheric pressure of 14.7 lb. per sq. in., or 29.92 in. of mercury.

EXAMPLE.—A coal whose heating value is 12,000 B. T. U. per lb., is burned with 20 lb. of air (not including water vapor) per lb. of coal. The relative humidity of the air is 90%, and its temperature is 92° F. How much heat is lost in the chimney gases on account of the moisture in the air, if the chimney gases escape at 512° F.?

SOLUTION.—From the table, it is found that 1 lb. of air will hold, when fully saturated, .03289 lb. of water vapor at 92° F.; hence, 20 lb. of air will hold 20 × .03289 = .6578 lb. of water vapor; at 90% relative humidity, it will hold .6578 × .90 = .59202 lb. water vapor.

The amount of heat absorbed in heating 1 lb. of water from 92° F. to 512° F. is: (a) From 92° to 212°, or through 120°, is 120 × 1 (specific heat of water) = 120 B. T. U.; (b) From 212° to 512°, or through 300°, is 300 × .48 (specific heat of superheated steam) = 144 B. T. U. The total absorption is 120 + 144 = 264 B. T. U. .59202 lb. water will absorb 264 × .59202 = 156.29 B. T. U. or $\frac{156.29 \times 100}{12,000} = 1.3\%$ of the heating value of the coal.

EXAMPLE.—How many cubic feet of dry air per pound of coal are used in the preceding example, if the air is at the mean atmospheric pressure of 14.7 lb. per sq. in.?

SOLUTION.—From the table, by calculation, it is found that 1 lb. of air at 32° and at the mean atmospheric pressure occupies $1 \div .0807 = 12.39$ cu. ft. At 92° it occupies $12.39 \times \frac{92 + 460}{492} = 13.9$ cu. ft.; 20 lb. will occupy $13.9 \times 20 = 278$ cu. ft.

EXAMPLE.—How many cubic feet of air must be delivered per minute by a fan, to drive 1,000 H. P. of boilers under the conditions of the preceding examples, if 4 lb. of coal is burned per H. P. per hr.?

SOLUTION.—     $\frac{4 \times 1,000 \times 278}{60} = 18,533$ cu. ft. per min.

## WEIGHT OF AIR, WATER VAPOR, AND SATURATED MIXTURES AT DIFFERENT TEMPERATURES

| Temperature Degrees F. | Weight of 1 Cu. Ft. of Dry Air at Different Temperatures Pound | Elastic Force of Water Vapor Inches of Mercury | Mixtures of Air Saturated With Vapor | | | | |
|---|---|---|---|---|---|---|---|
| | | | Elastic Force of Air in Mixture of Air and Vapor Inches of Mercury | Weight of 1 Cu. Ft. of Mixture of Air and Water Vapor | | | Weight of Vapor Mixed With 1 Lb. of Air Pounds |
| | | | | Weight of Air Pound | Weight of Vapor Pound | Total Weight of Mixture Pound | |
| 0 | .0864 | .044 | 29.877 | .0863 | .000079 | .086379 | .00092 |
| 12 | .0842 | .074 | 29.849 | .0840 | .000130 | .084130 | .00155 |
| 22 | .0824 | .118 | 29.803 | .0821 | .000202 | .082302 | .00245 |
| 32 | .0807 | .181 | 29.740 | .0802 | .000304 | .080504 | .00379 |
| 42 | .0791 | .267 | 29.654 | .0784 | .000440 | .078840 | .00561 |
| 52 | .0776 | .388 | 29.533 | .0766 | .000627 | .077227 | .00819 |
| 62 | .0761 | .556 | 29.365 | .0747 | .000881 | .075581 | .01179 |
| 72 | .0747 | .785 | 29.136 | .0727 | .001221 | .073921 | .01680 |
| 82 | .0733 | 1.092 | 28.829 | .0706 | .001667 | .072267 | .02361 |
| 92 | .0720 | 1.501 | 28.420 | .0684 | .002250 | .070717 | .03289 |
| 102 | .0707 | 2.036 | 27.885 | .0659 | .002997 | .068897 | .04547 |
| 112 | .0694 | 2.731 | 27.190 | .0631 | .003946 | .067046 | .06253 |
| 122 | .0682 | 3.621 | 26.300 | .0599 | .005142 | .065042 | .08584 |
| 132 | .0671 | 4.752 | 25.169 | .0564 | .006639 | .063039 | .11771 |
| 142 | .0660 | 6.165 | 23.756 | .0524 | .008473 | .060873 | .16170 |
| 152 | .0649 | 7.930 | 21.991 | .0477 | .010716 | .058416 | .22465 |
| 162 | .0638 | 10.099 | 19.822 | .0423 | .013415 | .055715 | .31713 |
| 172 | .0628 | 12.758 | 17.163 | .0360 | .016682 | .052682 | .46338 |
| 182 | .0618 | 15.960 | 13.961 | .0288 | .020536 | .049336 | .71300 |
| 192 | .0609 | 19.828 | 10.093 | .0205 | .025142 | .045642 | 1.22643 |
| 202 | .0600 | 24.450 | 5.471 | .0109 | .030545 | .041445 | 2.80230 |
| 212 | .0591 | 29.921 | .000 | .0000 | .036820 | .036820 | Infinite |

## OXYGEN AND AIR REQUIRED FOR THE COMBUSTION OF CARBON, HYDROGEN, ETC.

| Fuel | Chemical Reaction | Pounds $O$ per Pound Fuel | Pounds $N = 3.32 \times O$ | Air per Pound $= 4.32 \times O$ | Gaseous Product per Pound Fuel |
|---|---|---|---|---|---|
| Carbon to $CO_2$ | $C + 2O = CO_2$ | $2\frac{2}{3}$ | 8.85 | 11.52 | 12.52 |
| Carbon to $CO$ | $C + O = CO$ | $1\frac{1}{3}$ | 4.43 | 5.76 | 6.76 |
| Carbon monoxide to $CO_2$ | $CO + O = CO_2$ | $\frac{4}{7}$ | 1.90 | 2.47 | 6.76 |
| Hydrogen to $H_2O$ | $2H + O = H_2O$ | 8 | 26.56 | 34.56 | 35.56 |
| Marsh gas (methane), $CH_4$ to $CO_2$ and $H_2O$ | $CH_4 + 4O = CO_2 + 2H_2O$ | 4 | 13.28 | 17.28 | 18.28 |
| Sulphur to $SO_2$ | $S + 2O = SO_2$ | 1 | 3.33 | 4.32 | 5.32 |

The preceding table contains, in convenient form, the reactions involved in the combustion of various fuels, as explained more in detail before, as well as the weight of air, oxygen, and nitrogen, required to burn 1 lb. of the fuel,

and the weight of the products of combustion resulting therefrom. It is found, in practice, that if air is blown through a bed of hot anthracite or coke, and the resulting gases are analyzed, they always contain some carbon monoxide, showing imperfect combustion, unless they contain a considerable quantity of uncombined oxygen, or air. The excess of air required to effect complete combustion to carbon dioxide is usually not less than 50% of that theoretically necessary, so that about 17 lb. of air is required to insure the complete combustion of 1 lb. of carbon instead of 11.52 lb., the figure given in the table. It is probable, also, that more than 34.56 lb. of air is required to effect the combustion of each pound of hydrogen in a furnace, although, experimentally, one volume of oxygen and two volumes of hydrogen mixed together, or eight parts by weight of oxygen to one of hydrogen may be exploded by a spark, and converted into water vapor. The excess of air required in furnaces may be due to the presence of the great volumes of nitrogen and carbon dioxide, which dilutes the oxygen and makes it less active in causing combustion.

EXAMPLE.—How much air is required for the complete combustion of 1 lb. of coal containing 5% moisture, 20% volatile matter, 60% fixed carbon, 15% ash, assuming the volatile matter to be of the composition of marsh gas (methane), $CH_4$?

SOLUTION.—The molecular weight of marsh gas is $12+4=16$; hence, three-fourths of the weight of the volatile matter is carbon and one-fourth hydrogen. The carbon of the volatile matter is $\frac{3}{4}\times20=15\%$ of the fuel. The fixed carbon is given as 60%. The total carbon is $15+60=75\%$. The hydrogen of the volatile matter is $\frac{1}{4}\times20=5\%$ of the fuel.

As, from the table, 11.52 lb. of air is required to burn 1 lb. of carbon to $CO_2$, and 34.56 lb. of air is required to burn 1 lb. of hydrogen, the theoretical amount of air required to burn the fuel will be: For the carbon $.75\times11.52=8.640$ lb.; for the hydrogen $.05\times34.56=1.728$ lb.; making a total of 10.368 lb. If an excess of 50% of air is allowed, the amount will be $1.5\times10.368=15.552$ lb.

EXAMPLE.—How many cubic feet of dry air at 62° F. will be required in the preceding example?

SOLUTION.—The weight of 1 cu. ft. of air at a temperature of 62° F. and a barometric pressure of 29.92 is .0761 lb.; hence, 10.368 lb.$=10.368\div.0761$ $=136.24$ cu. ft. and 15.552 lb.$=15.552\div.0761=204.36$ cu. ft.

**Temperature of Ignition.**—Every combustible must be heated to a certain temperature, known as the temperature of ignition, or kindling point, before it will combine with oxygen, or burn. The accompanying table gives the temperatures of ignition of various fuels as determined by different authorities. It appears from this table that it requires a considerably higher temperature to ignite the gases distilled from coal than to ignite the coal itself, the temperature of ignition of the carbon being lower than that of the gases.

### TEMPERATURE OF IGNITION OF VARIOUS FUELS

| Fuel | Temperature of Ignition Degrees F |
|---|---|
| Marsh gas (methane), $CH_4$ | 1,202* |
| Carbon monoxide, $CO$ | 1,202 to 1,211 |
| Carbon monoxide, $CO$, in presence of a large quantity of carbon dioxide $CO_2$ | 1,292 |
| Ethylene (olefiant gas), $C_2H_4$ | 1,022 |
| Hydrogen | 1,031 to 1,130 |
| Anthracite | 925 |
| Semibituminous coal | 870 |
| Bituminous coal | 766 |
| Cannel coal | 668 |
| Soft charcoal, prepared at 500° F. | 650 |
| Sulphur | 470 |

*The temperature of ignition of marsh gas diluted with carbon dioxide and nitrogen in the proportions ordinarily found in a furnace is given by the French Coal Commission as 1,436° F.

**Temperature of Fire.**—Assuming that a pure fuel, such as carbon, is thoroughly burned in a furnace, all the heat generated will be transferred to the gaseous products of combustion, raising their temperature above that at which the fuel and the oxygen or air are supplied to the furnace. Suppose that 1 lb. of carbon is burned with 2⅔ lb. of oxygen, forming 3⅔ lb. of carbon dioxide, both the carbon and the oxygen being supplied at 0° F. The combustion of 1 lb. of carbon generates 14,600 B. T. U., which will all be contained in the 3⅔ lb. of carbon dioxide. The specific heat of carbon dioxide is .217 at constant pressure; that is, it requires .217 B. T. U. to raise the temperature of 1 lb. of carbon dioxide 1° F. To raise 3⅔ lb. of carbon dioxide 1° F. will require $3⅔ \times .217 = .7957$ B. T. U., and 14,600 B. T. U. will therefore raise its temperature $14,600 \div .7957 = 18,348.6°$ F. (approximately 18,350 F.) above the temperature at which the carbon and the oxygen were supplied. The temperatures thus calculated are known as theoretical temperatures, and are based on the assumptions of perfect combustion and no loss by radiation. The temperature of 18,350° is far beyond any temperature known, and it is probable that long before it could be reached, the phenomenon of dissociation would take place; that is, the carbon dioxide would be split into carbon and oxygen, and the elements would lose their affinity for each other.

The theoretical elevation of temperature of the fire may be calculated by the formula

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## HEAT AND PRODUCTS OF COMBUSTION OF BURNING CARBON

### Carbon Burned Partly to CO₂ and Partly to CO With Excess of Air

| Degrees F. Elevation of Temperature of Fire | Pounds Chimney Gases (Air and Carbon) | Per Cent. Excess of Air | Per Cent. Carbon Burned to CO | Per Cent. Carbon Burned to CO₂ |
|---|---|---|---|---|
| 3,328 | 18.28 | 50 | 20 | 100 |
| 3,375 | 15.52 | 40 | 40 | 80 |
| 3,350 | 12.99 | 30 | 60 | 60 |
| 3,323 | 10.67 | 20 | 80 | 40 |
| 3,139 | 8.60 | 10 | 100 | 20 |
| 2,743 | 6.76 | | | |

### Carbon Burned to CO₂ With Excess of Air

| Degrees F. Elevation of Temperature of Fire | Pounds Chimney Gases (Air and Carbon) | Pounds Air per Pound of Carbon | Per Cent. Air Supply Above 11.52 Pounds |
|---|---|---|---|
| 3,950 | 15.40 | 14.40 | 25 |
| 3,375 | 18.28 | 17.28 | 50 |
| 2,875 | 21.16 | 20.16 | 75 |
| 2,530 | 24.04 | 23.04 | 100 |
| 2,041 | 29.80 | 28.80 | 150 |
| 1,711 | 35.56 | 34.56 | 200 |

### Carbon Burned Partly to CO and Partly to CO₂ With Deficient Supply of Air

| Degrees F. Elevation of Temperature of Fire | B. T. U. Total Heat Generated | B. T. U. Heat Generated in Forming CO | B. T. U. Heat Generated in Forming CO₂ | Per Cent. Carbon Burned to CO | Per Cent. Carbon Burned to CO₂ | Pounds Chimney Gases (Air and Carbon) | Pounds Air per Pound of Carbon | Per Cent. Air Supply Below 11.52 Pounds |
|---|---|---|---|---|---|---|---|---|
| 4,858 | 14,600 | | 14,600 | 20 | 100 | 12.52 | 11.52 | 0 |
| 4,606 | 12,570 | 890 | 11,680 | 40 | 80 | 11.37 | 10.37 | 10 |
| 4,298 | 10,540 | 1,780 | 8,760 | 60 | 60 | 10.22 | 9.22 | 20 |
| 3,914 | 8,510 | 2,670 | 5,840 | 80 | 40 | 9.06 | 8.06 | 30 |
| 3,413 | 6,480 | 3,560 | 2,920 | 100 | 20 | 7.91 | 6.91 | 40 |
| 2,743 | 4,450 | 4,450 | | | | 6.76 | 5.76 | 50 |

$$\text{Elevation of temperature} = \frac{\text{B. T. U. generated by the combustion}}{\text{Weight of gaseous products} \times \text{their specific heats}}$$

It is evident from this formula that the rapidity of the combustion, or the time required to burn a given weight of fuel, has nothing to do with the temperature that may theoretically be attained. In practice, the temperature of a bed of coal in a furnace and that of the burning gases immediately above the coal are reduced, to some extent, by radiation; and as the quantity of heat radiated from a given mass of fuel is a function of the time during which it takes place, a considerable portion of the heat generated may be lost by radiation when the combustion is very slow. With ordinary rates of combustion, however, that is, of about 10 lb. of coal per square foot of grate surface per hour, and firebrick furnaces, the percentage of loss of heat by radiation is 1% or less, and the actual temperature that may be attained will be very nearly as high with that rate of combustion as with a rate of 20 or 40 lb.

The elevations of temperature given in the foregoing table were determined by means of the preceding formula, the specific heat of the chimney gases being taken as .24.

To burn 1 lb. of hydrogen, 8 lb. of oxygen is required, and there is also present $8 \times 3.32 = 26.56$ lb. of nitrogen, which is mixed with the oxygen in the air. The gaseous products are 9 lb. of water, in the shape of superheated steam (specific heat .48), and 26.56 lb. of nitrogen (specific heat .2438). The heat produced is 62,000 B. T. U. If the temperature of the atmosphere is 62° F., 150 B. T. U. is absorbed during the combustion in heating 1 lb. of water, $H_2O$, from 62° to 212° F. per lb., 965.8 B. T. U. in evaporating it at that temperature, and .48 $(T+t-212)$ in superheating it from 212° to the temperature $T+t$ of the fire, $T$ being the increase of temperature and $t$ the temperature of the atmosphere, which in this case is 62° F. All this heat may be recovered by condensing the steam and cooling the water of condensation to 62°. Therefore, the following equation is obtained:

$$62,000 = 9 \times [150 + 965.8 + .48 \times (T + 62 - 212)] + 26.56 \times .2438 T$$

which, being solved, gives $T = 4,873°$ F. $T + t = 4,935°$ F. Showing that hydrogen and carbon, when perfectly burned, give about the same maximum theoretical temperature.

By a process of reasoning similar to the preceding, the following formula is derived, to obtain the maximum theoretical temperature of the fire, when the fuel contains hydrogen and oxygen with a varying supply of air:

$$T = \frac{616C + 2,220H - 327O - 44W}{f + .02W + .18H},$$

in which  $T$ = elevation of temperature above that of atmosphere;
  $C$ = percentage of carbon in fuel;
  $H$ = percentage of hydrogen in fuel;
  $O$ = percentage of oxygen in fuel;
  $W$ = percentage of water in fuel;
  $f$ = pounds of dry gases of combustion ($H_2O$ excluded) per pound of fuel.

EXAMPLE.—What would be the temperature of the fire, the temperature of the atmosphere being 62° F., when burning a coal having the composition, excluding ash and sulphur, carbon 75%, hydrogen 5%, oxygen 10%, moisture 10%; the dry chimney gases amount to 20 lb. per lb. of this combustible, including the moisture?

SOLUTION.—Applying the formula,

$$T = \frac{616 \times 75 + 2,220 \times 5 - 327 \times 10 - 44 \times 10}{20 + .02 \times 10 + .18 \times 5} = 2,540° \text{ F.}$$
$$T + t = 2,540° + 62° = 2,602° \text{ F.}$$

EXAMPLE.—What is the maximum temperature attainable by burning moist wood of the composition carbon 38%, hydrogen 5%, oxygen 32%, nitrogen and ash 1%, moisture 24%; the dry gases are 15 lb. per lb. of wood, and the temperature of the atmosphere is 62° F.?

SOLUTION.—Applying the formula,

$$T = \frac{616 \times 38 + 2,220 \times 5 - 327 \times 32 - 44 \times 24}{15 + .02 \times 24 + .18 \times 5} = 1,403° \text{ F.}$$
$$T + t = 1,403° + 62° = 1,465° \text{ F.}$$

EXAMPLE.—What will be the temperature of a fire of Pocahontas coal analyzing carbon 84.22%, hydrogen 4.26%, oxygen 3.48%, nitrogen .84%, sulphur .59%, ash 5.85%, water .76%; the dry gases are 20 lb. per lb. of combustible, the heating value of the sulphur being neglected?

SOLUTION.—The combustible, carbon and hydrogen, is 88.48% of the coal, hence $f = 20 \times .8848 = 17.69$. Applying the formula,

$$T = \frac{616 \times 84.22 + 2,220 \times 4.26 - 327 \times 3.48 - 44 \times .76}{17.69 + .02 \times .76 + .18 \times 4.26} = 3,257° \text{ F.}$$

$$T + t = 3,257° + 62° = 3,319° \text{ F.}$$

When the combustion is perfect, and the furnace is entirely enclosed in walls of firebrick, highly heated, the temperatures calculated by the formula are nearly attained, the only loss being that due to external radiation. In ordinary practice, with the boiler immediately above the fire, the temperature is lowered by radiation, and also, when soft coal is used, by imperfect combustion.

**Estimation of Air Supply.**—The theoretical amounts of air required to burn the several combustible elements in a fuel were given in the table on page 447, but when burning coal in a furnace only a rough estimate of the quantity of air supplied may be obtained by direct measurement by an anemometer, or by counting the revolutions of a fan. The only available method of closely approximating the amount of air supplied, is by making a proximate analysis of the gases of combustion, taken from a point close to the furnace, but beyond the point of visible flame. If taken from the chimney, the gas may be of different composition on account of inward leaks of air through cracks in the brickwork.

The analysis of the gases gives the percentage of carbon dioxide, oxygen, and carbon monoxide, in this order, the quantity of these gases being determined by absorption; nitrogen is determined by difference; that is, the remainder after subtracting the sum of the other three gases from 100. Unburned hydrogen or hydrocarbon gases cannot be conveniently determined by ordinary analysis. If the combustion is complete, the percentage of nitrogen will always be found between 79 and 80. If it exceeds 80%, unburned hydrogen, or hydrocarbons are present, an error has been made in the analysis, or, possibly, the hydrogen of a gaseous fuel has been burned leaving the carbon unburned. Thus, in burning marsh gas (methane), $CH_4$, with an insufficient supply of air, the hydrogen only may be burned to water, $H_2O$, leaving the carbon unburned in the shape of soot, which is caught in a filter attached to the gas-collecting apparatus. The water, or water vapor, is condensed, and is not determined in the analysis, leaving the nitrogen that accompanied the oxygen alone to be determined. In this manner, the nitrogen in the gases may actually exceed 80%.

The following formula may be used for calculating the air supply:

$$\text{Pounds of dry air per pound of carbon} = \frac{3.032N}{CO_2 + CO}$$

in which $CO_2$, $CO$, and $N$ are percentages, by volume, of the dry gas.

EXAMPLE.—How many pounds of air are supplied per pound of carbon in burning a coal, if the gases analyze carbon dioxide 11.74%, carbon monoxide .10%, oxygen 7.71%, nitrogen 80.45%?

SOLUTION.—Substituting in the formula,

$$\frac{3.032 \times 80.45}{11.74 + .10} = 20.6 \text{ lb.}$$

**Production and Measurement of Draft.**—It is well known that any volume of gas is lighter when heated than the same volume of gas when cool. When hot gases pass into the chimney they have a temperature of from 400° to 600° F., while the air outside the chimney has a temperature of from 40° to 90° F. Roughly speaking, the air weighs twice as much, bulk for bulk, as the hot gases. Naturally, then, the pressure in the chimney is less than the pressure of the outside air. The production of draft and the satisfactory operation of a chimney depend on this pressure difference. The pressure of the draft depends on the temperature of the furnace gases and the height of the chimney. Chimney draft is affected by so many varying conditions that no absolutely reliable rules can be given for proportioning chimneys to give a certain desired draft pressure. The rules given for chimney proportions are based on successful practice rather than on pure theory.

The intensity of the draft may be measured by means of a water gauge such as is shown in the accompanying illustration. This is a glass tube open at both ends, bent to the shape of the letter **U**; the left leg communicates with the chimney. The difference in the two water levels $H$ and $Z$ in the legs represents the intensity of the draft, and is expressed in inches of water.

The draft produced by a chimney may vary from ¼ in. to 2 in. of water, depending on the temperature of the chimney gases and on the height of the chimney. Generally speaking, it is advantageous to use a high chimney and

as low a chimney temperature as possible. The draft pressure required depends on the kind of fuel used. Wood requires but little draft, say, $\frac{1}{4}$ in. of water or less; bituminous coal generally requires less draft than anthracite.

To burn anthracite slack, or culm, the draft pressure should be about $1\frac{1}{2}$ in. of water.

**Erection of Chimneys.**—The form of a chimney has a pronounced effect on its capacity. A round chimney has a greater capacity for a given area than a square one. If the flue is tapering, the area for calculation is measured at its smallest section. The flue through which the gases pass from the boilers to the chimney should have an area equal to, or a little larger than, the area of the chimney. Abrupt turns in the flue or contractions of its area should be carefully avoided. Where one chimney serves several boilers, the branch flue from each furnace to the main flue must be somewhat larger than its proportionate part of the area of the main flue.

Chimneys are usually built of brick, though concrete, iron, and steel chimneys are often used for those of moderate height. Brick chimneys are usually built with a flue having parallel sides and a taper on the outside of the chimney of from $\frac{1}{16}$ to $\frac{1}{4}$ in. per ft. of height. A round chimney gives greater draft area for the same amount of material in its structure and exposes less surface to the wind than a square chimney. Large brick stacks are usually made with an inner core and an outer shell, with a space between them. Such chimneys are usually constructed with a series of internal pilasters, or vertical ribs, to give rigidity. The top of the chimney should be protected by a coping of stone or a cast-iron plate to prevent the destruction of the brickwork by the weather.

Iron or steel stacks are made of plates varying from $\frac{1}{8}$ to $\frac{1}{2}$ in. thick. The larger stacks are made in sections, the plates being about $\frac{1}{4}$ in. thick at the top and increasing to $\frac{1}{2}$ in. at the bottom; they are lined with firebrick about 18 in. thick at the bottom and 4 in. at the top. Sometimes no lining is used on account of the likelihood of corrosion and the difficulty of inspection, and also because the inside of lined stacks cannot be painted.

On account of the great concentration of weight, the foundation for a chimney should be carefully designed. Good natural earth will support from 2,000 to 4,000 lb. per sq. ft. The footing beneath the chimney foundation should be made of large area, in order to reduce the pressure due to the weight of the chimney and its foundation to a safe limit.

**Height and Area of Chimneys.**—The relation between the height of the chimney and the pressure of the draft, in inches of water, is given by the formula

$$p = H\left(\frac{7.6}{T_a} - \frac{7.9}{T_c}\right),$$

in which

$$p = \text{draft pressure, in inches of water;}$$
$$H = \text{height of chimney, in feet;}$$
$$T_a \text{ and } T_c = \text{absolute temperatures of outside air and of chimney gases respectively.}$$

EXAMPLE.—What draft pressure will be produced by a chimney 120 ft. high, the temperature of the chimney gases being 600° F., and of the external air 60° F.?

SOLUTION.—Substituting in the formula,

$$p = 120 \times \left(\frac{7.6}{460+60} - \frac{7.9}{460+600}\right) = .859 \text{ in.}$$

To find the height of chimney to give a specified draft pressure, the preceding formula may be transformed. Thus,

$$H = \frac{p}{\left(\dfrac{7.6}{T_a} - \dfrac{7.9}{T_c}\right)}$$

EXAMPLE.—Required, the height of the chimney to produce a draft of $1\frac{1}{8}$ in. of water, the temperature of the gases and of the external air being, respectively, 550° and 62° F.

SOLUTION.—Substituting in the formula,

$$H = \frac{1.125}{\dfrac{7.6}{522} - \dfrac{7.9}{1,010}} = 167 \text{ ft.}$$
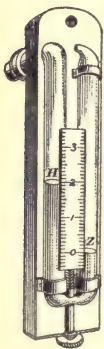
In determining the height of a chimney in cities, it should be borne in mind that the chimney must almost always be carried to a height above the roofs of surrounding buildings, partly in order to prevent a nullification of the draft by opposing air-currents and partly to prevent the commission of a nuisance.

The height of the chimney being decided on, its cross-sectional area must be sufficient to carry off readily the products of combustion. The following formulas for finding the dimensions of chimneys are in common use:

Let
$\quad$ $H$ = height of chimney, in feet;
$\quad$ $H.P.$ = horsepower of boiler or boilers;
$\quad$ $A$ = actual area of chimney, in square feet;
$\quad$ $E$ = effective area of chimney, in square feet;
$\quad$ $S$ = side of square chimney, in inches;
$\quad$ $d$ = diameter of round chimney, in inches.

Then,
$$E = \frac{.3\ H.P.}{\sqrt{H}} = A - .6\sqrt{A} \quad (1)$$

$$H.P. = 3.33 E \sqrt{H} \quad (2)$$

$$S = 12\sqrt{E} + 4 \quad (3)$$

$$d = 13.54\sqrt{E} + 4 \quad (4)$$

The table on page 454 has been computed from these formulas.

EXAMPLE.—What should be the diameter of a chimney 100 ft. high that furnishes draft for a 600-H. P. boiler?

SOLUTION.—Substituting in formula 1, $E = \dfrac{.3 \times 600}{\sqrt{100}} = 18$. Now, using formula 4, $d = 13.54\sqrt{18} + 4 = 61.44$ in.

EXAMPLE.—For what horsepower of boilers will a chimney 64 in. sq. and 125 ft. high furnish draft?

SOLUTION.—By simply referring to the table, the horsepower is found to be 934.

**Maximum Combustion Rate.**—The maximum rates of combustion attainable under natural draft are given by the following formulas, which have been deduced from the experiments of Isherwood:

Let
$\quad$ $F$ = weight of coal per hour per square foot of grate area, in pounds;
$\quad$ $H$ = height of chimney or stack, in feet,

Then, for anthracite burned under the most favorable conditions,
$$F = 2\sqrt{H} - 1 \quad (1)$$
and under ordinary conditions,
$$F = 1.5\sqrt{H} - 1 \quad (2)$$
For best semianthracite and bituminous coals,
$$F = 2.25\sqrt{H} \quad (3)$$
and for less valuable soft coals,
$$F = 3\sqrt{H} \quad (4)$$

The maximum rate of combustion is thus fixed by the height of the chimney; the minimum rate may be anything less.

EXAMPLE.—Under ordinary conditions, what is the maximum rate of combustion of anthracite if the chimney is 120 ft. high?

SOLUTION.—By formula 2,
$$F = 1.5 \times \sqrt{120} - 1 = 15\tfrac{1}{2} \text{ lb. per sq. ft. per hr.}$$

**Forced Draft.**—The use of forced draft as a substitute for, or as an aid to, natural chimney draft is becoming quite common in large boiler plants. Its advantages are that it enables a boiler to be driven to its maximum capacity to meet emergencies without reference to the state of the weather or to the character of the coal; that the draft is independent of the temperature of the chimney gases, and that, therefore, lower flue temperatures may be used than with natural draft; and in many cases that it enables a poorer quality of coal to be used than is required with natural draft. Forced draft may be obtained: (1) by a steam jet in the chimney, as in locomotives and steam fire-engines; (2) by a steam-jet blower under the grate bars; (3) by a fan blower delivering air under the grate bars, the ash-pit doors being closed; (4) by a fan blower delivering air into a closed fireroom, as in the *closed stoke-hold system* used in some ocean-going vessels; and (5) by a fan placed in the flue or chimney drawing the gases of combustion from the boilers, commonly called the *induced-draft system.* Which one of these several systems should be adopted in any special

## SIZE OF CHIMNEYS AND HORSEPOWER OF BOILERS

| 50 | 60 | 70 | 80 | 90 | 100 | 110 | 125 | 150 | 175 | 200 | Effective Area Square Feet | Actual Area Square Feet | Side of Square Chimney, Inches | Diameter of Round Chimney, Inches |
|----|----|----|----|----|-----|-----|-----|-----|-----|-----|------|------|----|----|
| | | | | | | | | | | | | | | |
| 23 | 25 | 27 | | | | | | | | | .97 | 1.77 | 16 | 18 |
| 35 | 38 | 41 | | | | | | | | | 1.47 | 2.41 | 19 | 21 |
| 49 | 54 | 58 | 62 | | | | | | | | 2.08 | 3.14 | 22 | 24 |
| 65 | 72 | 78 | 83 | | | | | | | | 2.78 | 3.98 | 24 | 27 |
| 84 | 92 | 100 | 107 | 113 | | | | | | | 3.58 | 4.91 | 27 | 30 |
| | 115 | 125 | 133 | 141 | | | | | | | 4.47 | 5.94 | 30 | 33 |
| | 141 | 152 | 163 | 173 | 182 | | | | | | 5.47 | 7.07 | 32 | 36 |
| | | 183 | 196 | 208 | 219 | | | | | | 6.57 | 8.30 | 35 | 39 |
| | | 216 | 231 | 245 | 258 | 271 | | | | | 7.76 | 9.62 | 38 | 42 |
| | | | 311 | 330 | 348 | 365 | 389 | | | | 10.44 | 12.57 | 43 | 48 |
| | | | 402 | 427 | 449 | 472 | 503 | 551 | | | 13.51 | 15.90 | 48 | 54 |
| | | | 505 | 539 | 565 | 593 | 632 | 692 | 748 | | 16.98 | 19.64 | 54 | 60 |
| | | | | 658 | 694 | 728 | 776 | 849 | 918 | 981 | 20.83 | 23.76 | 59 | 66 |
| | | | | 792 | 835 | 876 | 934 | 1,203 | 1,105 | 1,181 | 25.08 | 28.27 | 64 | 72 |
| | | | | | 995 | 1,038 | 1,107 | 1,212 | 1,310 | 1,400 | 29.73 | 33.18 | 70 | 78 |
| | | | | | 1,163 | 1,214 | 1,294 | 1,418 | 1,531 | 1,637 | 34.76 | 38.48 | 75 | 84 |
| | | | | | 1,344 | 1,415 | 1,496 | 1,639 | 1,770 | 1,893 | 40.19 | 44.18 | 80 | 90 |
| | | | | | 1,537 | 1,616 | 1,720 | 1,876 | 2,027 | 2,167 | 46.01 | 50.27 | 86 | 96 |

*Height of Chimney, in Feet* — *Commercial Horsepower*

case will usually depend on local conditions. The steam jet has the advantage of lightness and compactness of apparatus, and is therefore most suitable for locomotives and steam fire-engines, but it also is the most wasteful of steam, and therefore should not be used when one of the fan-blower systems is available, except for occasional or temporary use, or when very cheap fuel, such as anthracite culm at the mines, is used.

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# STEAM ENGINES

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## PRINCIPLES AND REQUIREMENTS

A good steam engine should be as direct acting as possible; that is, the connecting parts between the piston and the crank-shaft should be few in number, as each part wastes some power. The moving parts of an engine should be strong, to resist strains, and light, so as to offer no undue resistance to motion; parts moving upon each other should be well, truly, and smoothly finished, to reduce resistances to a minimum; the steam should get into the cylinder easily at the proper time, and the exhaust should leave the cylinder as exactly and as easily. The steam pipes supplying steam should have an area one-tenth the combined areas of the cylinders they supply, and exhaust pipes should be somewhat larger. The cylinder and the steam pipes and the boiler should be well protected. The engine should be capable of being started and stopped and reversed easily and quickly.

**Clearance.**—The term clearance is used in two senses in connection with the steam engine. It may be the distance between the piston and the cylinder head when the piston is at the end of its stroke, or it may represent the volume between the piston and the valve when the engine is on dead center. To avoid confusion, the former is called *piston clearance*, and the latter is termed simply *clearance.* Piston clearance is always a measurement, expressed in parts of an inch. Clearance, however, is a volume.

The clearance of an engine may be found by putting the engine on a dead center and pouring in water until the space between the piston and the cylinder head, and the steam port leading into it, is filled. The volume of the water poured in is the clearance. The clearance may be expressed in cubic feet or cubic inches, but it is more convenient to express it as a percentage of the volume swept through by the piston. For example, suppose that the clearance volume of a $12'' \times 18''$ engine is found to be 128 cu. in. The volume swept through by the piston per stroke is $12^2 \times .7854 \times 18 = 2,035.8$ cu. in. Then, the clearance is $128 \div 2,035.8 = .063 = 6.3\%$. The clearance may be as low as $\frac{1}{2}\%$ in Corliss engines, and as high as $14\%$ in high-speed engines.

Theoretically, there should be no clearance, because the steam that fills the clearance space does no work except during expansion; it is exhausted from the cylinder during the return stroke, and represents so much dead loss. This is remedied, to some extent, by compression. If the compression were carried up to the boiler pressure, there would be very little, if any, loss, as the steam would then fill the entire clearance space at boiler pressure, and the amount of fresh steam needed would be the volume displaced by the piston up to the point of cut-off, the same as if there were no clearance. In practice, however, the compression is made only sufficiently great to cushion the reciprocating parts and bring them to rest quietly.

It is not practicable to build an engine without any clearance, on account of the formation of water in the cylinder due to the condensation of steam, particularly when starting the engine. Automatic cut-off high-speed engines of the best design, with shaft governors, usually compress to about half the boiler pressure, and have a clearance of from 7 to 14%. Corliss engines require but very little compression, owing to their low rotative speeds; they also have very little clearance, as the ports are short and direct.

**Cut-Off.**—The *apparent cut-off* is the ratio between the portion of the stroke completed by the piston at the point of cut-off, and the total length of the stroke. For example, if the length of stroke is 48 in., and the steam is cut off from the cylinder just as the piston has completed 15 in. of the stroke, the apparent cut-off is $\frac{15}{48} = \frac{5}{16}$.

The *real cut-off* is the ratio between the volume of steam in the cylinder at the point of cut-off and the volume at the end of the stroke, both volumes including the clearance of the end of the cylinder in question. If the volume of steam in the cylinder, including the clearance, at the point of cut-off is 4 cu. ft., and the volume, including the clearance, at the end of the stroke is 6 cu. ft., the real cut-off is $\frac{4}{6} = \frac{2}{3}$.

**Ratio of Expansion.**— The ratio of expansion, also called the *real number of expansions*, is the ratio between the volume of steam, including the steam in the clearance space, at the end of the stroke, and the volume, including the clearance, at the point of cut-off. It is the reciprocal of the real cut-off. For example, if the volume at the end of the stroke is 8 cu. ft., and the cut-off is 5 cu. ft., the ratio of expansion is $8 \div 5 = 1.6$; in other words, the steam would be said to have one and six-tenths expansions. The corresponding real cut-off will be $\frac{5}{8}$.

Let
$e$ = real number of expansions;
$i$ = clearance, expressed as a per cent. of stroke;
$k$ = real cut-off;
$k_1$ = apparent cut-off;
$r$ = apparent number of expansions = $\dfrac{1}{k_1}$.

Then,
$$e = \frac{1}{k} \text{ and } k = \frac{1}{e} \quad (1)$$

$$k = \frac{k_1 + i}{1 + i} \quad (2)$$

**EXAMPLE.**—The length of stroke is 36 in.; the steam is cut off when the piston has completed 16 in. of the stroke; the clearance is 4%. Find the apparent cut-off, the real cut-off, and the real number of expansions.

SOLUTION.—Apparent cut-off = $\frac{16}{36} = \frac{4}{9} = .444$.

Real cut-off = $k = \dfrac{k_1 + i}{1 + i} = \dfrac{.444 + .04}{1 + .04} = \dfrac{.484}{1.04} = .465$.

Real number of expansions = $e = \dfrac{1}{k} = \dfrac{1}{.465} = 2.15$.

**Mean Effective Pressure.**—In order to find the horsepower of an engine, it is necessary to know the *mean effective pressure* (abbreviated M. E. P.),

which is defined as the average pressure urging the piston forwards during its entire stroke in one direction, less the pressure that resists its progress. If an indicator is not available, so that diagrams may be taken in order to determine the mean effective pressure of an engine, the value of this pressure may be estimated by the formula

$$P = .9[C(p+14.7)-17],$$

in which　　　$P$ = M. E. P., in pounds per square inch;
　　　　　　　$C$ = constant corresponding to cut-off, taken from following table;
　　　　　　　$p$ = boiler pressure, in pounds per square inch, gauge.

The foregoing formula applies only to a simple noncondensing engine. If the engine is a simple condensing engine, the formula should be altered by substituting for 17 the pressure existing in the condenser, in pounds per square inch.

## CONSTANTS USED IN CALCULATING MEAN EFFECTIVE PRESSURE

| Cut-off | Constant | Cut-off | Constant | Cut-off | Constant |
|---------|----------|---------|----------|---------|----------|
| $\frac{1}{6}$ | .545 | $\frac{3}{8}$ | .773 | $\frac{2}{3}$ | .943 |
| $\frac{1}{5}$ | .590 | .4 | .794 | .7 | .954 |
| $\frac{1}{4}$ | .650 | $\frac{1}{2}$ | .864 | $\frac{3}{4}$ | .970 |
| .3 | .705 | .6 | .916 | .8 | .981 |
| $\frac{1}{3}$ | .737 | $\frac{5}{8}$ | .927 | $\frac{7}{8}$ | .993 |

In this table, the fraction indicating the point of cut-off is obtained by dividing the distance that the piston has traveled when the steam is cut off by the whole length of the stroke; that is, it is the apparent cut-off. It is to be observed that this rule cannot be applied to a compound engine or to any other engine in which the steam is expanded in successive stages in several cylinders.

EXAMPLE.—Find the approximate mean effective pressure of a non-condensing engine cutting off at one-half stroke, if the boiler pressure is 80 lb., gauge.

SOLUTION.—According to the table, the constant corresponding to cut-off at one-half stroke is $C = .864$. Then, applying the formula, $P = .9 \times [.864 \times (80+14.7) - 17] = 58.34$ lb. per sq. in.

**Horsepower.**—The indicator furnishes the most ready method of measuring the pressures on the piston of a steam engine and, in consequence, of determining the amount of work done in the cylinder and the corresponding horsepower. The power measured by the use of the indicator is called the *indicated horsepower*. It is the total power developed by the action of the net pressures of the steam on the two sides of the moving piston. The indicated horsepower is generally represented by the initials I. H. P.

The part of the indicated horsepower that is absorbed in overcoming the frictional resistances of the moving parts of the engine is termed the *friction horsepower*. If the engine is running light, or with no load, all the power developed in the cylinder is absorbed in keeping the engine in motion, and the friction horsepower is equal to the indicated horsepower. This principle furnishes a simple approximate method of finding the friction horsepower of a given engine; as, however, the friction between the surfaces increases with the pressure, the power absorbed in overcoming the engine will be greater as the load on the engine is increased.

The difference between the indicated horsepower and the friction horsepower is the *net horsepower*. This is the power that the engine delivers through the flywheel or shaft to the belt or the machine driven by it, and is sometimes called the *delivered horsepower*. As the power that an engine is capable of delivering when working under certain conditions is often measured by a device known as a Prony brake, the net horsepower is frequently called the *brake horsepower*, abbreviated B. H. P.

**Finding the Indicated Horsepower.**—Knowing the dimensions and speed of the engine and the mean effective pressure on the piston, all the data for finding the rate of work done in the engine cylinder expressed in horsepower are at hand.

Let        $H$ = indicated horsepower of engine;

$P$ = mean effective pressure of the steam, in pounds per square inch;

$A$ = area of piston, in square inches;

$L$ = length of stroke, in feet;

$N$ = number of working strokes per minute.

Then,
$$H = \frac{P\ L\ A\ N}{33,000}$$

In a double-acting engine, or one in which the steam acts alternately on both sides of the piston, the number of working strokes per minute is twice the number of revolutions per minute. For example, if a double-acting engine runs at a speed of 210 R. P. M. there are 420 working strokes per minute. A few types of engines, however, are single-acting; that is, the steam acts on only one side of the piston. Such are the Westinghouse, the Willans, and others. In this case, only one stroke per revolution does work, and, consequently, the number of strokes per minute to be used in the foregoing formula is the same as the number of revolutions per minute. Unless it is specifically stated that an engine is single-acting, it is always understood, when the dimensions of a steam engine are given, that a double-acting engine is meant.

EXAMPLE.—The diameter of the piston of an engine is 10 in. and the length of stroke 15 in. It makes 250 R. P. M., with a mean effective pressure of 40 lb. per sq. in. What is the horsepower?

SOLUTION.—As it is not stated whether the engine is single or double acting, assume that it is double acting; then, the number of strokes is $250 \times 2 = 500$ per min. Hence,

I. H. P. = $\dfrac{P\ L\ A\ N}{33,000} = \dfrac{40 \times 1\frac{1}{4} \times (10^2 \times .7854) \times 500}{33,000} = 59.5$ H. P.

The indicated horsepower of a compound or triple-expansion engine may be calculated from the indicator diagrams in exactly the same manner as with any simple engine, considering each cylinder as a simple engine and adding the horsepowers of the several cylinders together. In taking the indicator cards from a compound engine, the precaution of taking the cards simultaneously from all cylinders must be observed, especially when the engine runs under a variable load, because, otherwise, an entirely wrong distribution of power may be shown, and there may also be a great variation between the indicated horsepower really existing and that calculated from diagrams taken at different times.

The indicated horsepower of compound engines is sometimes found by referring the mean effective pressure of the high-pressure cylinder to the low-pressure cylinder and calculating the horsepower of the engine on the assumption that all the work is done in the low-pressure cylinder. To do this, the mean effective pressures of the two cylinders are found from indicator diagrams; the mean effective pressure of the high-pressure cylinder is then divided by the ratio of the volume of the low-pressure cylinder to that of the high-pressure cylinder; and the quotient is added to the mean effective pressure of the low-pressure cylinder, the sum being the *referred mean effective pressure*. This sum is then taken as the mean effective pressure of the engine, and the area of the low-pressure piston as the piston area; with these data, the length of stroke and the number of strokes, the horsepower is computed as for any simple engine. In the case of a triple-expansion engine, the mean effective pressures of the high-pressure and intermediate cylinders are referred to the low-pressure cylinder and added to its mean effective pressure. Thus, suppose that in a 12″, 20″, and 34″×30″ engine the mean effective pressures are 82.3 lb., 27.8 lb., and 10.6 lb., respectively. Then, the referred mean effective pressure is $\dfrac{83.2}{8.03} + \dfrac{27.8}{2.78} + \dfrac{10.6}{1} = 10.4 + 10 + 10.6 = 31$ lb., and this value must be substituted for $P$ in finding the horsepower of the engine. While this method shortens the labor of computing the horsepower, it obviously does not show the distribution of work between the cylinders.

**Stating Sizes of Engines.**—The size of a simple engine, that is, an engine having but one cylinder, is commonly stated by giving the diameter of the cylinder, followed by the length of the stroke, both in inches. Thus, a simple engine having a cylinder 12 in. in diameter and a stroke of 24 in. would be referred to as a 12″×24″ engine, the multiplication sign in this case serving merely to separate the two numbers. The sizes of compound and multiple-expansion engines are designated in a similar fashion. Thus, a compound engine with a high-pressure cylinder 11 in. in diameter, a low-pressure cylinder

20 in. in diameter, and a stroke of 15 in. would be referred to as an $11''$ and $20'' \times 15''$ compound engine. In the same way, a $14''$, $22''$, and $34'' \times 18''$ triple-expansion engine would be one in which the diameters of the cylinders are 14 in., 22 in., and 34 in., and the stroke is 18 in.

**Mechanical Efficiency.**—The mechanical efficiency of an engine is the ratio of the net horsepower to the indicated horsepower; or it is the percentage of the mechanical energy developed in the cylinder that is utilized in doing useful work. To find the efficiency of an engine, when the indicated and net horsepowers are known, divide the net horsepower by the indicated horsepower.

**Piston Speed.**—The total distance traveled by the piston in 1 min. is called the *piston speed*. As it is customary to take the stroke in inches, to find the piston speed, multiply the stroke in inches by the number of strokes and divide by 12; or, letting $S$ represent the piston speed, $S = \dfrac{lN}{12}$, where $l$ is the stroke in inches. But $N = 2R$, where $R$ represents the number of revolutions per minute. Hence,

$$S = \frac{lN}{12} = \frac{l \times 2R}{12} = \frac{lR}{6}$$

**EXAMPLE.**—An engine with a 52-in. stroke runs at a speed of 66 R. P. M. What is the piston speed?

**SOLUTION.**—By the formula, $S = \dfrac{52 \times 66}{6} = 572$ ft. per min.

The piston speeds used in modern practice are about as follows:

| Type of Engine | Piston Speed Ft. per Min. |
|---|---|
| Small stationary engines | 300 to 600 |
| Large stationary engines | 600 to 1,000 |
| Corliss engines | 400 to 750 |
| Marine engines | 200 to 1,200 |

**Allowance for Area of Piston Rod.**—It is generally considered sufficiently accurate to take the total area of one side of the piston as the area to be used in calculating the horsepower of an engine. The effective area of one side of the piston is, however, reduced by the sectional area of the piston rod, and if it is important that the power be calculated with the greatest practical degree of accuracy, an allowance for the area of the piston rod must be made. This is done by taking as the piston area one-half the sum of the areas exposed to steam pressure on the two sides of the piston. Thus, if a piston is 30 in. in diameter with a 6-in. piston rod, the average area is

$$\frac{30^2 \times .7854 + (30^2 \times .7854 - 6^2 \times .7854)}{2} = 692.72 \text{ sq. in.}$$

If the piston rod is continued past the piston so as to pass through the head-end cylinder head; that is, if the piston has a *tailrod*, allowance must be made for the tailrod. Thus, with a piston 30 in. in diameter, a piston rod 6 in. in diameter, and a tailrod 5 in. in diameter, the average area is

$$\frac{(30^2 \times .7854 - 5^2 \times .7854) + (30^2 \times .7854 - 6^2 \times .7854)}{2} = 682.9 \text{ sq. in.}$$

**Cylinder Ratios.**—The cylinders of compound and multiple-expansion engines increase in diameter from the high-pressure to the low-pressure end, and it is customary to refer to their relative sizes by means of cylinder ratios. As all the cylinders have the same length of stroke, the volumes of the several cylinders are in proportion to the areas of the cylinders, and therefore in proportion to the squares of the diameters. The area of the high-pressure cylinder is taken as unity, and the other areas are referred to it, and the ratios of these areas, or the ratios of the squares of the diameters, are called the *cylinder ratios*. For example, a triple-expansion engine having cylinders 12 in., 20 in., and 34 in. in diameter will have the cylinder ratios of $12^2 : 20^2 : 34^2$, or $144 : 400 : 1,156$, which reduces to $1 : 2.78 : 8.03$; that is, the intermediate cylinder is 2.78 times as large as the high-pressure cylinder and the low-pressure cylinder is 8.03 times as large as the high-pressure cylinder. If there are two cylinders to one stage of expansion, as, for example, two low-pressure cylinders, the sum of their areas must be used in finding the cylinder ratios. Thus, if there had been two 24-in. low-pressure cylinders instead of one 34-in. cylinder, in the foregoing case, the cylinder ratios would have been $12^2 : 20^2 : 2 \times 24^2$, or $144 : 400 : 1,152$, which reduces to $1 : 2.78 : 8$.

# CONDENSERS

There are two types of condensers in general use, namely, the *surface condenser* and the *jet condenser*. In the former, the exhaust steam comes in contact with a large area of metallic surface that is kept cool by contact with cold water. In the latter, the exhaust steam, on entering the condenser, comes in contact with a jet of cold water. In either case, the entering steam is condensed to water, and in consequence a partial vacuum is formed. If enough cold water were used, the steam on entering would instantly condense and a practically perfect vacuum would be obtained were it not for the fact that the feedwater of the boiler always contains a small quantity of air, which passes with the exhaust steam into the condenser and therefore partly destroys the vacuum. To get rid of this air, the condenser is fitted with an air pump, which pumps out both the air and the water formed by condensation.

**Surface Condensers.**—In the surface condenser, the exhaust steam and the injection water are kept separate throughout their course through the condenser; and the condensed steam leaves the condenser as fresh water, free from the impurities contained in the injection water. The water of condensation from a surface condenser is therefore fit to be used as boiler feed, except that it contains oil used for cylinder lubrication, which can be eliminated by means of an oil separator. It is for this reason that the surface condenser, in spite of its greater complication, cost, size, and weight, as compared with the jet condenser, is used instead of the latter where the supply of injection water is unfit for use as boiler feed. Thus, the surface condenser is used altogether in marine work, except for vessels navigating clean, fresh water like that of the Great Lakes, in order to avoid the use of sea-water in the boilers.

In the surface condenser the steam may be outside and the water inside the tubes, or the reverse. If the water is inside the tubes, it should enter at the bottom of the condenser and be discharged at the top. This brings the coldest water into contact with the partly condensed steam, and the warmest water into contact with the hot entering steam. When the water is outside the tubes, it is necessary to fit baffle plates on the water side to force the water into a definite and regular circulation, and to prevent it from going directly from inlet to outlet and also to prevent the water from arranging itself in layers according to temperature, with the coldest water on the bottom and the hottest water on top. The outlet should be well above the top row of tubes. A solid body of water above the top row of tubes is thus assured, and the accumulation of a stagnant body of hot water in the top of the condenser is prevented by its being continually drawn off by the circulating pump and replaced by cooler water from beneath.

Air tends to accumulate in the top of the water side of a surface condenser. This is particularly inconvenient where the water is inside the tubes, as the air fills the top rows of tubes and excludes the water, destroying their value as cooling surfaces. To prevent this, an air valve must be provided, as high up on the water side as possible, by which the air can be drawn off when it becomes troublesome. Drain valves and pipes should be provided at the bottom.

As the condensed steam from the surface condenser is generally pumped back into the boiler as feedwater, it is desirable to have it as hot as possible; but it must be remembered that it is impossible to get the feedwater from the condenser at a higher temperature than that of saturated steam at the absolute pressure existing in the condenser.

It will be considerably cooler than this if, after being condensed, it is allowed to lie in the bottom of the condenser and give up its heat to the circulating water. The heat thus given up is a total loss, and should be avoided by connecting the air-pump suction to the lowest point of the condenser and by shaping the bottom of the condenser so that the water will drain rapidly into the air-pump suction.

**Cooling Water for Surface Condenser.**—The amount of cooling water required in the case of a surface condenser may be found by the formula

$$Q = \frac{H - (t - 32)}{t_2 - t_1},$$

in which $Q$ = number of pounds of cooling water required to condense 1 lb. of steam;

$H$ = total heat above 32° of 1 lb. of steam at pressure at release;

$t$ = temperature of condensed steam on leaving condenser;

$t_1$ = temperature of cooling water on entering condenser;

$t_2$ = temperature of cooling water on leaving condenser.

**EXAMPLE.**—Steam exhausts into a surface condenser from an engine cylinder at a pressure of 6 lb., absolute; the temperature of the condensing water on entering is 55° F., and on leaving it is 100° F.; the temperature of the condensed steam on leaving the condenser is 125° F. How many pounds of cooling water are required per pound of steam?

**SOLUTION.**—The total heat above 32° of 1 lb. of steam at 6 lb., absolute, from the Steam Table, is 1,133.8 B. T. U. Then, substituting the values of $H$, $t$, $t_1$, and $t_2$ in the formula,

$$Q = \frac{1,133.8 - (125 - 32)}{100 - 55} = \frac{1,040.8}{45} = 23.13 \text{ lb.}$$

**Injection Water for Jet Condenser.**—The quantity of injection water required for a jet condenser may be found by the formula

$$Q = \frac{H - (t - 32)}{t - t_1},$$

in which $Q$ = number of pounds of injection water required to condense 1 lb. of steam;

$H$ = total heat above 32° of 1 lb. of steam at pressure at release;

$t$ = temperature of mixture of injection water and condensed steam on leaving the condenser;

$t_1$ = temperature of injection water on entering the condenser.

**EXAMPLE.**—Steam is exhausted into a jet condenser from an engine cylinder at a pressure of 10 lb., absolute; the temperature of the injection water on entering is 60° F., and on leaving 140° F. How much injection water is required per pound of steam?

**SOLUTION.**—The total heat above 32° of 1 lb. of steam at 10 lb. absolute, from the Steam Table, is 1,140.9 B. T. U. Then, substituting the values of $H$, $t$, and $t_1$ in the formula.

$$Q = \frac{1,140.9 - (140 - 32)}{140 - 60} = \frac{1,032.9}{80} = 12.91 \text{ lb.}$$

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# ENGINE MANAGEMENT

## STARTING AND STOPPING

**Warming Up.**—About 15 or 20 min. before starting the engine, the stop-valves should be raised just off their seats and a little steam should be allowed to flow into the steam pipe. The drain cock on the steam pipe just above the throttle should be opened. When the steam pipe is thoroughly warmed and steam blows through the drain pipe, the drain cock should be closed and the throttle opened just enough to let a little steam flow into the valve chest and cylinder; or if a by-pass around the throttle is fitted, it may be used. The cylinder relief valves, or drain cocks, and also the drain cocks on the valve chest and the exhaust pipe should be opened, if the engine is non-condensing. If the cylinders are jacketed, steam should be turned into the jackets and the jacket drain cocks should be opened. While the engine is warming up, the oil cups and the sight-feed lubricator may be filled. A little oil may be put into all the small joints and journals that are not fitted with oil cups. The guides should be wiped off with oily waste and oiled. By this time the engine is getting warm. If the cylinder is fitted with by-pass valves, they should be used to admit steam to both ends of the cylinder. In general, all cylinders, especially if they are large and intricate castings, should be warmed up slowly, as sudden and violent heating of a cylinder of this character is very liable to crack the casting by unequal expansion.

An excellent and economical plan for warming up the steam pipe and the engine is to open the stop-valves and throttle valve at the time or soon after the fires are lighted in the boilers, permitting the heated air from the boilers to circulate through the engine, thus warming it up gradually and avoiding the accumulation of a large quantity of water of condensation in the steam pipe and cylinder. When pressure shows on the boiler gauge or steam at the drain pipes of the engine, the stop-valves and throttle may be closed temporarily, but not hard down on their seats. When this method of warming up the engine is adopted, the safety valves should not be opened while steam is being raised.

Stop-valves and throttle valves should never be opened quickly or suddenly and thus permit a large volume of steam to flow into a cold steam pipe or cylinder. If this is done, the first steam that enters will be condensed and a partial vacuum will be formed. This will be closely followed by another rush of steam

with similar results, and so on until a mass of water will collect, which will rush through the steam pipe and strike the first obstruction, generally the bend in the steam pipe near the cylinder, with great force and in all probability will carry it away and cause a disaster. This is called *water hammer* and has caused many serious accidents. Before turning steam into any pipe line or into a cylinder, all drain valves should be opened.

Another precaution that should be taken is the easing of the throttle valve on its seat before steam is let into the main steam pipe; otherwise, the unequal expansion of the valve casing may cause the valve to stick fast and thereby give much trouble. Even if a by-pass pipe is fitted around the throttle, it is better not to depend on it. As water is non-compressible, it is an easy matter to blow off a cylinder head or break a piston if the engine is started when there is a quantity of water in the cylinder.

**Oil and Grease Cups.**—The last thing for the engineer to do before taking his place at the throttle preparatory to starting the engine, provided he has no oiler, is to start the oil and grease cups feeding. It is well to feed the oil liberally at first, but not to the extent of wasting it; finer adjustment of the oiling gear can be made after the engine has been running a short time and the journals are well lubricated.

**Starting and Stopping Non-Condensing Slide-Valve Engine.**—A non-condensing slide-valve engine is started by simply opening the throttle; this should be done quickly in order to jump the crank over the first dead center, after which the momentum of the flywheel will carry it over the other centers. The engine should be run slowly at first, gradually increasing the revolutions to the normal speed. When the engine has reached full speed, the drain pipes should be examined; if dry steam is blowing through them, the drain cocks should be closed. If water is being delivered, the drain cocks should remain open until steam blows through and should then be closed.

To stop a non-condensing slide-valve engine, it is only necessary to shut off the supply of steam by closing the throttle, but care should be taken not to let the engine stop on the dead center. After the engine is stopped, the oil feed should be shut off and the main stop-valve closed. The valve should be seated, but without being jammed hard down on its seat. The drain cocks on the steam pipe and engine may or may not be opened, according to circumstances. It will do no harm to allow the steam to condense inside the engine, as the engine will then cool down more gradually, which lessens the danger of cracking the cylinder casting by unequal contraction. All the water of condensation should be drained from the engine before steam is again admitted to it.

**Starting and Stopping Condensing Slide-Valve Engine.**—In the case of condensing slide-valve engines, before the main engine is started, the air pump and circulating pump should be put into operation and a vacuum formed in the condenser; this will materially assist the main engine in starting promptly. Prior to starting the air and circulating pumps, the injection valve should be opened to admit the condensing water into the circulating pump; the delivery valve should also be opened at this time. If an ordinary jet condenser is used, no circulating pump is required, the water being forced into the condenser by the pressure of the atmosphere. If the air pump is operated by the main engine a vacuum will not be formed in the condenser until after the engine is started and at least one upward stroke of the air pump is made. In this case the injection valve must be opened at the same moment the engine is started; otherwise, the condenser will get hot and a mixture of air and steam accumulate in it and prevent the injection water from entering. When this occurs it is necessary to pump cold water into the condenser by one of the auxiliary pumps through a pipe usually fitted for that purpose; if such a pipe has not been provided, it may be found necessary to cool the condenser by playing cold water on it through a hose.

The operation of stopping a slide-valve surface-condensing engine is precisely similar to that of stopping a non-condensing engine of the same type, with the addition that after the main engine is stopped the air and circulating pumps are also stopped, and in the same way, that is, by closing the throttle, after which the injection valve and the discharge valve should be closed and the drain cocks opened. With a jet condenser, the operation of stopping the engine is the same as the above, with the exception that the injection valve should be closed at the same moment that the engine is stopped.

**Starting and Stopping Simple Corliss Engine.**—In the Corliss engine the eccentric rod is so constructed and arranged that it may be hooked on or unhooked from the eccentric pin on the wrist-plate at the will of the engineer.

After all the preliminary operations have been attended to, the starting bar is shipped into its socket in the wristplate and the throttle is opened. The starting bar is then vibrated back and forth by hand, by which the steam and exhaust valves are operated through the wristplate and valve rods; as soon as the cylinder takes steam, the engine will start. After working the starting bar until the engine has made several revolutions and the flywheel has acquired sufficient momentum to carry the crank over the dead centers, the hook of the eccentric rod should be allowed to drop upon the pin on the wristplate. As soon as the hook engages with the pin, the starting bar is unshipped and placed in its socket in the floor. The way to determine in which direction the starting bar should be first moved to start the engine ahead is to note the position of the crank, from which the direction in which the piston is to move may be learned. This will indicate which steam valve to open first; it will then be an easy matter to determine in which direction the starting bar should be moved. If the engine is of the condensing type, the same course of procedure in starting the air and circulating pumps should be followed as with the simple condensing slide-valve engine.

A Corliss engine is stopped by closing the throttle and unhooking the eccentric rod from the pin on the wristplate; this is done by means of the unhooking gear provided for the purpose. As soon as the eccentric rod is unhooked from the pin, the starting bar is shipped into its socket in the wristplate and the engine is worked by hand to any point in the revolution of the crank at which it is desired to stop the engine. The procedure is then the same as for the simple slide-valve engine. After stopping a Corliss condensing engine the same course should be followed as with a slide-valve condensing engine in regard to draining cylinders, closing stop-valves, etc.

**Starting and Stopping Compound Slide-Valve Engine.**—Before starting a compound engine, the high-pressure cylinder is warmed up in the same manner as a simple engine. To get the steam into the low-pressure cylinder is, however, an operation that will depend on circumstances. If the cylinders are provided with pass-over valves, it will be necessary only to open them to admit steam into the receiver and thence into the low-pressure cylinder. If the cylinders are not fitted with pass-over valves the steam can usually be worked into the receiver and low-pressure cylinder by operating the high-pressure valves by hand. Sometimes compound engines are fitted with starting valves, which greatly facilitate the operations of warming up and starting. Usually a compound engine will start upon opening the throttle.

If the high-pressure crank of a cross-compound engine is on its center and the low-pressure engine will not pull it off, it must be jacked off. If the pressure of steam in the receiver is too high, causing too much back pressure in the high pressure cylinder, the excess of pressure must be blown off through the receiver safety valve; if the pressure in the receiver is too low to start the low-pressure piston, more steam must be admitted into the receiver. If the engine is stuck fast from gummy oil or rusty cylinders, all wearing surfaces must be well oiled and the engine jacked over at least one entire revolution. If the cut-offs are run up, they should be run down, full open. If there is water in the cylinders, it should be blown out through the cylinder relief or drain valves, and if there is any obstruction to the engine turning, it should be removed.

If the crank of a tandem compound engine is on the center, it must be pulled or jacked off. If the high-pressure crank of a cross-compound engine is on the center, it may or may not be possible to start the engine by the aid of the low-pressure cylinder, depending on the valve gear and the crank arrangement. When the cranks are 180° apart, which is a very rare arrangement, the crank must be pulled or jacked off the center. When the cranks are 90° apart and a pass-over valve is fitted, live steam may be admitted into the receiver and thence into the low-pressure cylinder, in order to start the engine. When no pass-over is fitted, but the engine has a link motion, sufficient steam to pull the high-pressure crank off the center can generally be worked into the low-pressure cylinder by working the links back and forth. When no pass-over is fitted, but the high-pressure engine can have its valve or valves worked by hand, steam can be got into the low-pressure engine by working the high-pressure valve or valves back and forth by hand. If no way exists of getting steam into the low-pressure cylinder while the high-pressure crank is on a dead center, it must be pulled or jacked off.

If the air and circulating pumps are attached to and operated by the main engine, a vacuum cannot be generated in the condenser until after the main engine has been started. Consequently, in this case, there is no vacuum to help start the engine; therefore, if it is tardy or refuses to start, it will be

necessary to resort to the jacking gear and jack the engine into a position from which it will start. A vacuum having been generated in the condenser beforehand, the pressure in the receiver acting on the low-pressure piston causes the engine to start promptly, even though the high-pressure crank may be on its center.

Compound slide-valve engines, whether condensing or non-condensing, are stopped by closing the throttle, and, if a reversing engine, throwing the valve gear into mid-position. If the stop is a permanent one, the usual practice of draining the engine, steam chests, and receiver, closing stop-valves, stopping the oil feed, etc. should be followed. If the engine is intended to run in both directions in answer to signals, as in the cases of hoisting, rolling-mill, and marine engines, the operator, after stopping the engine on signal, should immediately open the throttle very slightly, in order to keep the engine warm, and stand by for the next signal. If the engine is fitted with an independent or adjustable cut-off gear, it should be thrown off; that is, set for the greatest cut-off, for the reason that the engine may have stopped in a position in which the cut-off valves in their early cut-off positions would permit little or no steam to enter the cylinders, in which case the engine will not start promptly, and perhaps not at all. While waiting for the signal, the cylinder drain valves should be opened and any water that may be in the cylinders should be blown out. When dry steam blows through the drains, the cylinders are clear of water.

When the signal to start the engine is received, it is only necessary to throw the valve gear into the go-ahead or backing position, as the signal requires, and to operate the throttle according to the necessities of the case, for which no rule can be laid down beforehand, as the position of the throttle will depend on the load on the engine at the time.

**Starting and Stopping Compound Corliss Engine.**—The operation of starting and stopping a compound Corliss engine is precisely similar to that of starting and stopping a simple Corliss engine. The high-pressure valve gear only is worked by hand in starting, the low-pressure eccentric hook having been hooked on previously. The low-pressure valve gear is worked by hand only while warming up the low-pressure cylinder. The directions given for operating the simple condensing engine apply to the condensing Corliss engine, so far as the treatment of the air pump, circulating pump, and condenser is concerned.

## POUNDING OF ENGINES

**Faulty Bearings.**—Loose journal brasses are the most frequent cause of pounding in engines. The remedy for pounding of this nature is obvious. The engine should be stopped and the brasses set up gradually until the pounding ceases. In the case of shaft journals, they may be set up without stopping the engine, provided the engineer can reach them without danger of being caught in the machinery.

It may so happen that the boxes or brasses are worn down until the edges of the upper half and those of the lower half are in contact and cannot be set up on the journal any farther; they are then said to be *brass and brass*, or *brassbound*. In a case of this kind, the journal must be *stripped*, as it is called, when the cap and brasses are removed from a journal. The edges of the brasses are then chipped or filed off, in order to allow them to be closed in.

It is a most excellent plan in practice to reduce the halves of the brasses so that they will stand off from each other when in place for a distance of $\frac{1}{8}$ to $\frac{1}{16}$ in. and to fill this space with hard sheet-brass liners from No. 20 to No. 22 Birmingham wire gauge in thickness, or even thinner. Should the journal become brass-bound, the cap may be slacked off and a pair of the liners slipped out without the necessity of stripping the journal.

In some instances journal-boxes are fitted with *keepers*, or *chipping pieces*, as they are sometimes called. These usually consist of cast-brass liners from $\frac{1}{4}$ to $\frac{1}{2}$ in. in thickness, having ribs or ridges cast on one side, for convenience of chipping and filing. These keepers are sometimes made of hardwood and are capable of being compressed slightly by the pressure exerted upon them during the setting-up process. When the boxes are babbitted, the body of the box is occasionally made of cast iron, in which case iron liners and keepers are used instead of brass ones.

In engines fitted with some types of friction couplings, there is a thrust exerted upon the shaft in the direction of its length. This will necessitate having a *thrust bearing*, or *thrust block*, as it is sometimes called. There are a number of types of thrust bearings, but the most common is the collar thrust, which

consists of a series of collars on the shaft that fit in corresponding depressions in the bearing. If these collars do not fit in the depressions rather snugly the shaft will have end play and there probably will be more or less pounding or backlash at every change of load on the engine. This can be remedied only by putting in a new thrust bearing and making a better fit with the shaft collars, unless the rings in the bearing are adjustable, in which case the end play may be taken up by adjusting the rings.

**Pounding in Cylinders.**—Pounding in the cylinders is frequently caused by water due to condensation or to that carried over from the boilers. This may be a warning that priming is likely to occur in the boilers or has already commenced. If the cylinders are not fitted with automatic relief valves, the drain cocks should be opened as quickly as possible and the throttle closed a little to check the priming.

Another source of pounding in the cylinder is a piston loose on the rod; this will result if the piston-rod nut or key backs off or the riveting becomes loose, permitting the piston to play back and forth on the piston rod. If due to backing off of the nut, the engine should be shut down instantly. There is generally very little room to spare between the piston-rod nut and the cylinder head; therefore, it cannot back off very far before it will strike and break the cylinder head. After the engine is stopped and the main stop-valve is closed, the cylinder head should be taken off and the piston nut set up as tightly as possible. As a measure of safety, a taper split pin should in all cases be fitted through the piston rod behind the nut or a setscrew should be fitted through the nut.

A slack follower plate or junk ring will cause pounding in the cylinder. It seldom happens that all the follower bolts back out at one time, but not infrequently one of them works itself out altogether. This is a very dangerous condition of affairs, especially in a horizontal engine. If the bolts should get end on between the piston and cylinder head, either the piston or the cylinder head is bound to be broken. Therefore, if there is any intimation that a follower bolt is adrift in the cylinder, the engine should be shut down instantly, the cylinder head taken off, the old bolt removed, and one having a tighter fit put in.

Broken packing rings and broken piston springs will cause noise in the cylinder, but it is more of a rattling than a pounding, and the sound will easily be recognized by the practiced ear. There is not so much danger of a breakdown from these causes as may be supposed, from the fact that the broken pieces are confined within the space between the follower plate and the piston flange.

Pounding in the cylinders of old engines is often produced by the striking of the piston against one or the other cylinder heads, due to the wearing away of the connecting-rod brasses. Keying up the brasses from time to time has the effect of lengthening or shortening the connecting-rod, depending on the design, and this change in length destroys the clearance at one end of the cylinder by an equal amount. The remedy is to restore the rod to its original length by placing sheet-metal liners behind the brasses; this obviously will move the piston back or ahead and restore the clearance. A rather rare cause of the piston striking the cylinder head is due to the unscrewing of the piston rod from the crosshead, in case it is fastened by a thread and check-nut. To obviate any danger, the check-nut should be tried frequently.

**Improper Valve Setting.**—The primary cause of another source of pounding is the improper setting of the steam valve, or possibly its improper design. In the case of improper setting of the valve, insufficient compression, insufficient lead, cut-off too early, and late release may all cause pounding on the centers.
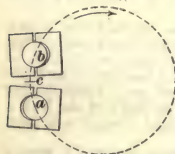
**Reversal of Pressure.**—The effect of a reversal of pressure is clearly shown in the accompanying illustration. With the crankpin at $a$ and the engine running in the direction indicated by the arrow, the connecting-rod is subjected to a pull, but after the crankpin has passed the dead center $c$, the connecting-rod is subjected to a push, in which case the rear brass, as shown at $b$, bears against the crankpin, while in the former case, as shown at $a$, the front brass bears against the crankpin. By giving a sufficient amount of compression, the lost motion in the pins and journals is transferred gently from one side to the other before the crankpin reaches the dead center. If the compression is insufficient, there will be pounding.

**Insufficient Lead.**—Insufficient lead causes an engine to pound because the piston has then little or no cushion to impinge on as it approaches the end of its stroke, and it is brought to rest with a jerk. A similar effect will be produced by a late release; the pressure is retained too long on the driving side of the piston. The ideal condition is that the pressures shall be equal on both sides of the piston at a point in its travel just in advance of the opening of the steam port. The position of this point varies with the speed of the piston and other conditions that only the indicator card can reveal.

**Pounding at Crosshead.**—The crosshead is a source of pounding from various causes, of which the loosening of the piston rod is one of the most common. There are several methods of attaching the piston rod to the crosshead. The rod may pass through the crosshead with a shoulder or a taper, or both, on one side of the crosshead and a nut on the other; or the rod may be secured to the crosshead by a cotter, instead of the nut; or the end of the rod may be threaded and screwed into the crosshead, having a check-nut to hold the rod in place. In the case first mentioned, the nut may work loose, which will cause the crosshead to receive a violent blow, first, by the nut on one side and then by the shoulder or taper on the other, at each change of motion of the piston; the remedy is to set up the nut. A similar effect will be produced if the cotter should work loose and back out. In case the piston rod is screwed into the crosshead and the rod slacks back, the danger is that the piston will strike the rear cylinder head. The check-nut should be closely watched. Pounding at the crosshead may be due to loose wristpin brasses, in which case they should be set up, but not too tightly. In case a crosshead works between parallel guides, pounding may be caused if the crosshead is too loose between the guides, and the crosshead shoes should therefore be set out.

If pounding results from the wearing down of the shoe of a slipper crosshead, a liner should be put between the shoe and the foot of the crosshead or the shoe should be set out by the adjustment provided.

**Pounding in Air Pump.**—Pounding in the air pump is generally produced by the slamming of the valves, caused by an undue amount of water in the pump, which will usually relieve itself after a few strokes. The pump piston, however, may be loose on the piston rod or the piston rod may be loose in the crosshead. A broken valve may also cause pounding in the air pump, all of which must be repaired as soon as detected.

**Pounding in Circulating Pump.**—In a circulating pump of the reciprocating type, pounding may be caused by admitting too little injection water, and the pounding may be stopped by adjusting the injection valve to admit just the right quantity. It may so happen, however, that the injection water is very cold, and to admit enough of it to stop the pounding in the circulating pump will make the feedwater too cold. To meet this contingency, an air check-valve is often fitted to the circulating pump to admit air into the barrel of the pump as a cushion for the piston; this check-valve may be kept closed when not needed to admit air. A broken valve, a piston loose on its piston rod, and a piston rod loose in the crosshead will all cause pounding in the circulating pump; they should be treated in the same manner as was specified for similar troubles in the air pump.

## HOT BEARINGS

Should any of the bearings show an inclination to heat to an uncomfortable point when felt by the hand, the oil feed should be increased. If the bearing continues to get hotter, some flake graphite should be mixed with the oil and the mixture should be fed into the bearing through the oil holes, between the brasses, or wherever else it can be forced in.

If, after trying the remedies just mentioned, the bearing continues to grow hotter, to the extent, for instance, of scorching the hand or burning the oil, it indicates that the brasses have been expanded by the heat and that they are gripping the journal harder and harder the hotter they get. At this stage, if the engine is not stopped or if the heating is not checked, the condition of the bearing will continue to grow worse, and may become so bad as to slow down and eventually stop the engine by excessive friction. By this time the brasses and journal will be badly cut and in bad condition generally, and the engine must be laid up for repairs.

After the simple remedies previously given have been tried and failed to produce the desired results, the engine should be stopped and the cap or key of the hot bearing should be slacked back and the engine allowed to stand until the bearing has cooled off. If necessary, the cooling may be hastened by pouring cold water on the bearing, though this is objectionable, as it may

cause the brasses to warp or crack. Putting water on a very hot bearing should be resorted to only in an emergency, that is, when an engine must be kept running. Water may be used on a moderately hot bearing without doing very much harm. It is quite common in practice, when sprinklers are fitted to an engine, to run a light spray of water on the crankpins when they show a tendency to heat, with very beneficial results.

**Dangerous Heating.**—Should a bearing become so hot as to scorch the hand or to burn the oil before it is discovered it is imperative that the engine should be stopped, at least long enough to loosen up the brasses, even though it is necessary to start up again immediately; otherwise the brasses will be damaged beyond repair and deep grooves will be cut into the bearing. If the brasses are babbitted, the white metal will melt out of the bearing at this stage. The engine will then be disabled, and if there is not a spare set of brasses on hand, it will be inoperative until the old brasses are rebabbitted or until a new set is made and fitted.

If it is absolutely necessary in an emergency to keep the engine running while a bearing is very hot, the engineer must exercise his best judgment as to how he shall proceed. After slacking off the brasses, about the best he can do is deluge the inside of the bearing with a mixture of oil and graphite, sulphur, soapstone, etc., and the outside with cold water from buckets, sprinklers, or hose, taking the chances of ruining the brasses and cutting the journal.

**Refitting Cut Bearing.**—The wearing surfaces of the brasses and journal must be smoothed off as well as circumstances will permit; but if the grooves are very deeply cut, it will be useless to attempt to work them out entirely, and if the brasses are very much warped or badly cracked, it will be best to put in spare ones, if any are on hand. If not, the old ones must be refitted and used until a new set can be procured. As for the journal, it is permanently damaged. Temporary repairs can be made by smoothing down the journal and brasses; but at the first opportunity the journal should be turned in a lathe and the brasses properly refitted or replaced with new ones.

**Newly Fitted Bearings.**—The bearings of new engines are particularly liable to heat, as the wearing surfaces of the brasses and journal have just been machined and hence are comparatively rough. The conditions just mentioned also exist with new brasses and the journals of an old engine. If a new engine, or one with new brasses is run moderately, in regard to both speed and load, and with rather loose brasses, there will be little danger of hot bearings, provided proper attention is given to adjustment and lubrication. This is what is familiarly termed *wearing down the bearings.*

**Faulty Brasses.**—When the brasses of an engine bearing are set up too tight, heating is inevitable. Often, an attempt is made to stop a pound in an engine by setting up the brasses when the thump should be stopped in some other way. The brasses should be slacked off as soon as possible. As a matter of fact, hot bearings should never occur from this cause.

Bearings may heat because the brasses are too loose. The heating is caused by the hammering of the journal against the brasses when the crankpin is passing the dead centers. The derangement is easily remedied, however, by setting up the cap nuts or the key. Most engineers have their own views regarding the setting up of bearings. One method is to set up the cap nuts or key nearly solid and then slack them back half way; if the brasses are still too loose, they are set up again and slacked back less than before, repeating this operation until there is neither thumping nor heating.

Another method of setting up journal brasses is to fill up the spaces between the brasses with thin metal liners, from No. 18 to No. 22 Birmingham wire gauge in thickness, and a few paper liners for fine adjustment. Enough of these should be put in to cause the brasses to set rather loosely on the journal when the cap nuts or keys are set up solid. The engine should be run for a while in that condition; then a pair of the liners should be removed and the brasses set up solid again. This operation should be repeated until there is neither thumping nor heating. It may require a week or more, and with a large engine longer, to reach the desired point. If this system is carefully carried out, there will be very little danger of heating. In removing the liners, great care should be exercised not to disturb the brasses any more than is absolutely necessary.

Warped and cracked brasses will cause heating, because they do not bear evenly on the journal, and hence the friction is not distributed evenly over the entire surface. If the distortion is not too great, the brasses may be refitted to the journal by chipping, filing, and scraping; but if they are twisted so much that they cannot be refitted, nothing will do but new brasses.

Brasses and journals that have been hot enough to be cut and grooved are liable to heat up again any time on account of the roughness of the wearing surfaces. As long as the grooves in the journal are parallel and match the grooves in the brasses, the friction is not greatly increased; but if a smooth journal is placed between brasses that are grooved and pressure is applied, the journal crushes the grooves in the brasses and becomes brazed or coated with brass, and then heating results. The way to prevent heating from this cause is to work the grooves out of the journal and brasses by filing and scraping as soon as possible after they occur.

Faulty workmanship is a common cause of the heating of crankpins, wrist-pins, and bearings. The brasses in that case do not bear fairly and squarely, even though they appear all right to the eye. A crankpin brass must fit squarely on the end of the connecting-rod and the rod itself must be square. If the key, when driven, forces the brasses to one side or the other and twists the strap on the rod, it will draw the brasses slantwise on the pin and make them bear harder on one side than on the other, thus reducing the area of the bearing surfaces. The same is true of the shaft bearings. If the brasses do not bed fairly on the bottom of the pillow-block casting or do not go down evenly, without springing in any way, heating will result

If the brasses are too long and bear against the collars of the journal when cold, they will most surely heat after the engine has been running a while. It is hardly possible to run bearings stone cold. They will warm up a little and the brasses will be expanded thereby, which will cause them to bear still harder against the collars. This, in turn, will induce greater friction and more expansion of the brasses. The evil may be obviated by chipping or filing a little off each end of the brasses until they cease to bear against the collars while running. A little side play is a good thing because it also promotes a better distribution of the oil and prevents the journal and brasses from wearing into concentric parallel grooves.

**Edges of Brasses Pinching Journal.**—Brasses, when first heated by abnormal friction, tend to expand along the surface in contact with the journal; this would open the brass and make the bore of larger diameter were it not prevented by the cooler part near the outside and by the bedplate itself. If the brass has become hot quickly and excessively, the resistance to expansion produces a permanent set on the layers of metal near the journal, so that on cooling, the brass closes and grips the journal. This is why some bearings always run a trifle warm and will not work cool. A continuance of heating and cooling will set up a bending action at the middle of the brass, which must eventually end in cracking it. Heating produced in this way may be prevented by chipping off the brasses at their edges parallel to the journal, as shown at *a* in the accompanying illustration, in which *b* is a section of the journal and *c* and *d* represent the top and bottom brasses, respectively.



**Hot Bearings Due to Faulty Oiling.**—It does not take long for a bearing to get very hot if it is deprived of oil. The two principal causes of dry bearings are an oil cup that has stopped feeding, either by reason of being empty or by being clogged up from dirt in the oil, and oil holes and oil grooves stopped up with dirt and gum.

The effect produced upon a bearing by an insufficient oil supply is similar to that of no oil, but in a less degree. Of course, it will take longer for a bearing to heat with insufficient oil than with none at all, and the engineer has more time in which to discover and remedy the difficulty.

Oils that contain dirt and grit are prolific sources of hot bearings. There is a great deal of dirt in lubricating oils of the average quality; therefore, all oil should be strained through a cloth or filtered, no matter how clear it looks. All oil cups, oil cans, and oil tubes and channels should be cleaned out frequently. Oil may be removed from the cups by means of an oil syringe, and all oil removed from the cups and cans should be strained or filtered before being used.

There are on the market many lubricating oils whose quality cannot be definitely decided on without an actual trial, and it is difficult to avoid getting a bad lot of oil sometimes. About the only safe way to meet this trouble is to pay a fair price to a reputable dealer for oil that is known to be of good quality, unless the purchaser is expert in judging oils or is able to pay a competent chemist to test them.
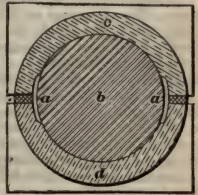
Bearings carrying very heavy shafts sometimes refuse to take the oil; or, if they do, it is squeezed out at the ends of the brasses or through the oil holes, and then the journal will run dry and heat. Large journals require oil of a high degree of viscosity, or heavy oil, as it is popularly called. Oil of this character has more difficulty in working its way under a heavy shaft than a thin oil has, but thin oil has not the body necessary to lubricate a large journal.

This difficulty may be met by chipping oil grooves or channels in the brasses. A round-nosed cape chisel, slightly curved, is generally used for this purpose; care should be taken to smooth off the burrs made by the chisel, which may be done with a steel scraper or the point of a flat file. The grooves are usually cut into the brass in the form of a **V** if the engine is required to run in only one direction; if it is to run in both directions the grooves should form an **X**. In the first instance, care must be taken that the **V** opens in the direction of rotation of the shaft; that is, the grooves should spread out from their junction in the same direction as that in which the journal turns. The oil grooves may be about $\frac{1}{4}$ in. wide and $\frac{1}{8}$ in. deep, and semicircular in cross-section.

**Grit in Bearings.**—Grit is an ever-present source of heating of bearings, and only by persistent effort can the engineer keep machinery running cool in a dirty atmosphere. The machinery of coal breakers, stone crushers, and kindred industries is especially liable to be affected in this way. Work done on a floor over an engine shakes dirt down upon it at some time or other; hence, all floors over engines should be made dust-proof by laying paper between the planks. If the engine room and firerooms communicate, and piles of red-hot clinkers and ashes are deluged with buckets of water, the water is instantly converted into a large volume of steam, carrying with it small particles of ashes and grit that penetrate into every nook and cranny, and these will find their way into the bearings sooner or later. Hot clinkers and ashes should be sprinkled, and the fireroom door should be closed while the ashes and clinkers are being hauled or wet down or while the fires are being cleaned or hauled. As an additional precaution, all open oil holes should be plugged with wooden plugs or bits of clean cotton waste as soon as possible after the engine is stopped, and should be kept closed until ready to oil the engine again preparatory to starting up. Plaited hemp or cotton gaskets should also be laid over the crevices between the ends of the brasses and the collars of the journals of every bearing on the engine and kept there while the engine is standing still.

**Overloading of Engine.**—The effects produced by overloading an engine are: The pressure on the brasses is increased to a point beyond that for which they were designed, the friction exceeds the practical limit, and the bearing heats. In case an engine is run at or near its limit of endurance, or if the journals are too small, it is wise and economical to have a complete set of spare brasses on hand ready to slip in when the necessity arises.

**Engine Out of Line.**—If the engine is not in line, the brasses do not bear fairly upon the journals. This will reduce the area of the bearing surfaces in contact to such an extent as to cause heating. If the engine is not very much out of line, matters may be considerably improved by refitting the brasses by filing and scraping down the parts of those that bear most heavily on the journal. If this does not answer, the heating will continue until the engine is lined up.

The crosshead guides of an engine out of line are apt to heat. The guides may also heat from other causes; for instance, the gibs may be set up too much. The danger of hot guides may be very much lessened by chipping zigzag oil grooves in their wearing surfaces and by attaching to the crosshead oil wipers made of cotton lamp wicking arranged so as to dip into oil reservoirs at each end of the guides if they are horizontal, and at the lower end if they are vertical. These wipers will spread a film of oil over the guides at every stroke of the crosshead.

**Effect of External Heat on Bearings.**—Bearings may get hot by the application of external heat. This may be the case if the engine is placed too near furnaces or an uncovered boiler, or in an atmosphere heated by uncovered steam pipes or other means. The excessive heat of the atmosphere will then expand the brasses until they nip the journals, which will generate additional heat and cause further expansion of the brasses, and so on until a hot bearing is the result. The remedy obviously depends upon the conditions of each case.

**Springing of Bedplate.**—If the bedplate of an engine is not rigid enough to resist the vibration of the moving parts, or if it is sprung by uneven settling or the instability of the foundation, the engine will be thrown out of line intermittently or permanently, and the bearings will heat. But it will do no good to refit the brasses unless the engine bed is stiffened in some way and leveled up.

**Springing or Shifting of Pillow-Block.**—The effect of the springing or shifting of the pedestal or pillow-block is similar to the springing of the engine bed; that is, the bearing will be thrown out of line, with the consequent danger of heating. As the pedestal is usually adjustable, it is an easy matter to readjust it, after which the holding-down bolts should be screwed down hard. If a pedestal is not stiff enough to resist the strains upon it and it springs, measures should be taken to stiffen it.

# STEAM TURBINES

The *turbine* is a machine by which the energy of a moving fluid, as steam or water, is transformed, producing a rotary motion. The rotating part of the turbine is cylindrical in form, comprising a shaft carrying a wheel, to which are fastened blades, also called *vanes* or *buckets*, against which the moving fluid impinges. This wheel, known as a *turbine wheel*, is enclosed in a casing. This form of motor is growing in popularity particularly in electrical work, the motor and generator being keyed on the same shaft, and for the following reasons:

1. The ability to use highly superheated steam, resulting in greater economy.

2. Reduced cost per unit capacity of the electrical generator, because of increased speed and less weight per horsepower.

3. Reduced floor space, resulting in less cost for land and power-station building.

4. Reduced cost of lubrication, as no cylinder oil is required and less oil is needed for bearings.

5. Saving in labor; engine oilers are not required, and one engineer can attend to more output than on reciprocating engines.

6. Reduced cost of foundations, as the turbine is balanced and has no reciprocating parts.

7. The turbine gives good steam economy over a wider range of load than the reciprocating engine; this is an important advantage in favor of the turbine, particularly for electric power stations where the load is variable. If it becomes necessary to operate a turbine unit at a comparatively light load, say, one-fourth or one-half load, the increase in steam consumption per horsepower per hour is not so great as it would be with a reciprocating engine under the same conditions. Also, a turbine unit will work more efficiently on overloads. The forces acting on the turbine wheels are continuous; hence, a uniform rotary motion is secured without the necessity of heavy flywheels.

**Types of Turbines.**—Steam turbines are of two general types, velocity turbines and pressure turbines. In the *velocity*, or *impulse, turbine* the rotation is produced by the direct impact or blow upon the blades of the turbine of steam issuing from a nozzle at high velocity, the action being the same as that of water in impulse water-wheels. Leading examples of this type of turbine are the De Laval, which is a *single-stage, expansion, velocity turbine*, in which all the expansion of the steam takes place in a single stage in one set of nozzles; the Curtis turbine, which is a *few-stage, expansion, velocity turbine*, in which the steam is expanded in two, three, four, or five stages; the Rateau turbine, which is a *multistage, expansion, velocity turbine*, in which the steam is expanded in many stages.

In the *pressure*, or *reaction, turbine*, the steam enters the central space and flows out through a series of guide vanes and then through the vanes of the moving wheel. In this type of wheel, the blades run full of steam, and there is a continual fall of pressure from the entrance of the steam until it leaves the turbine. The pressure turbine is always a multistage expansion turbine, the number of stages reaching 50 or 100. The leading American example is the Westinghouse-Parsons turbine.

When referring to the various stages of expansion in a turbine, it is customary to omit the term expansion and speak of the single-stage velocity turbine (instead of the single-stage expansion velocity turbine); the few-stage velocity turbine; the multistage velocity turbine.

In the turbines named, the Curtis turbine has a vertical shaft around which the blades rotate in a horizontal plane. In all the others the axis is horizontal and the blades rotate in vertical planes.

**Steam Consumption.**—The relation between the brake horsepower of the steam turbine at full load and the steam consumption is shown in the following table. The values in this table are taken from published tests of steam turbines that have attained the greatest commercial success. The turbines used saturated steam at from 115 to 140 lb. per sq. in., gauge pressure, and exhausted

into a vacuum of from 26 to 28.5 in. of mercury. Better results than those noted in the table can be obtained by the use of highly superheated steam.

The better the vacuum, the greater is the economy in the use of steam, both in the steam engine and in the steam turbine. A high vacuum is of greater value to the turbine, however, because the turbine can take advantage of a greater range of expansion. The degree of vacuum to be carried is a matter of dollars and cents; that is, it may cost more to create and maintain a high vacuum than may be saved in steam consumption. In a comparative test of a turbine and a triple-expansion engine under like conditions, it was found that, in the case of the reciprocating engine, little or nothing was to be gained by carrying a greater vacuum than about 26 in.; but the economy of the turbine in the use of steam increased rapidly as the vacuum was increased above 26 in. The conclusion is that high degrees of vacuum are more desirable for turbines than for engines.

**Comparison of Turbines and Engines.**—If the matter of steam consumption alone is considered, the average condensing turbine of less than about 700 H. P. is not so economical as the average compound or triple-expansion condensing engine, although the turbine may be preferred to the engine for other reasons. In larger sizes, however, and particularly in very large units, the economy of the turbine is very noticeable. The turbine possesses the ability to expand the steam to the lowest available condenser pressure without difficulty; but to do this in a reciprocating engine would require very

### STEAM CONSUMPTION PER HOUR OF TURBINES

| Brake Horsepower | Pounds of Steam Used | Brake Horsepower | Pounds of Steam Used |
|---|---|---|---|
| 100 | 18.2 | 600 | 15.3 |
| 200 | 17.5 | 700 | 14.8 |
| 300 | 16.9 | 800 | 14.3 |
| 400 | 16.3 | 900 | 13.7 |
| 500 | 15.8 | 1,000 | 13.2 |

large valves, and ports and heavy pistons, because of the great volume of steam to be handled at very low pressures.

**Finding Horsepower of Turbines.**—There is no way of finding the indicated horsepower of a steam turbine, because no form of indicator applicable to the turbine has been invented. Nor is any such instrument likely to be developed, owing to the very great difficulty of determining the energy given up to the blades of a turbine from a jet of steam. The usual way of finding the power of a steam turbine is to use a brake or a dynamometer and thus to determine the brake horsepower, or else to connect an electric generator to the turbine and measure the electrical output at the switchboard. In case the latter method is used, the efficiency of the generator and the turbine together is involved.

**Turbine Troubles.**—To obtain free running, it is necessary to allow clearance between the stationary and the moving rows of blades, as well as between the ends of the blades and the casing or the rotor. In impulse turbines, such as the Curtis and the Rateau, the clearance between the rows of blades is important; however, if it is made no greater than is necessary for mechanical reasons, the efficiency will not be affected seriously. In the reaction turbine, such as the Parsons, the clearance between the rows is of small consequence as compared with the clearance between the ends of the stationary blades and the rotor and between the ends of the moving blades and the casing. The former may vary from $\frac{1}{8}$ to 1 in. or more from the high-pressure to the low-pressure stage; but the tip clearance must be kept between a few hundredths and a few thousandths of an inch.

The stripping of the blades is one of the troubles to which turbines are subject. It may be due to the interference of the stationary and the movable blades, or to the rubbing of the blades against the shell or the rotor. In either of these cases the existing clearances are reduced by wear of the parts, shifting of the rotor, or unequal expansion of the rotor and the casing, until the blades touch and tear one another loose. The same result will occur if some foreign solid, as a stray nut or bolt, is carried along with the steam into the turbine. If a turbine is started too quickly, without being properly warmed up, the

sudden unequal expansion set up in the heavy casing and the lighter rotor may cause the blades to come in contact and be stripped. Stripping is claimed by some engineers to be more common in turbines in which the blades are not supported at their outer ends. To prevent it, some manufacturers apply shroud rings and metal lacings to the outer ends of the blades.

As there are no valves, pistons, or piston rings in the turbine to be maintained free from leakage, about the only thing that can affect the steam consumption is the condition of the blades. The blades of steam turbines are subjected to the cutting action of steam flowing at high velocities, and often carrying water particles with it. This cutting, or erosion, wears away the edges and surfaces of the blades. From the data available, it appears that the erosion is very slight if the steam is dry or superheated, no matter what velocities are used; but if the steam is wet, erosion will take place, and it will be greatly increased if the velocity of the steam is high. The horsepower is not affected to any great extent by blade erosion, according to the results of experience. In the case of a 100-H. P. De Laval turbine, the steam inlet edges of the blades were worn away about $\frac{1}{16}$ in., yet the steam consumption was only about 5% above that with new blades.

If the boiler supplying steam to a reciprocating engine primes badly, a slug of water may be carried over into the cylinder, resulting in a cracked piston or cylinder, a buckled piston rod or connecting-rod, or a wrecked frame. In case a steam turbine is used, however, the danger is greatly lessened. In turbines in which the blades are not supported at their outer ends, the water may cause stripping of the blades; but this is not very likely, as the blades at the high-pressure end of the turbine are short. A rush of water from the boiler has been known to bring a turbine almost to a stop without damaging the blades.

On account of the high speeds attained in turbine practice, the rotors are balanced accurately, so as to reduce vibration. But in spite of this careful balancing, vibration may manifest itself during ordinary running. It may be caused in any one of several ways, but the fundamental cause is lack of balance. If the rotor is warmed up too rapidly, the shaft or the wheels may be warped by unequal expansion, producing an unbalanced effect. The stripping of a blade or two will affect the balance of the wheel and tend to produce vibration. Even water carried into the turbine with the steam will bring about an unbalanced condition and will lead to vibration. When vibration is observed, it is well to reduce the speed a little, and to note whether this causes the vibration to cease. If it does, but comes back again as soon as the speed is increased, the source of the trouble should at once be determined.

**Operation of Turbines.**—If the steam turbine is a new one, or if it has been standing idle for a long period, it should not be started until it, together with its auxiliary apparatus, has been thoroughly inspected. The bearings should be properly adjusted and freed from dirt, and the entire lubricating system should be clean and filled with clean oil. The steam pipe from the boilers should be blown through, so as to clear it of any foreign matter that could be carried into the turbine by the steam. The governor mechanism should be examined, to see that it is in good order; the oil pump should be looked after, to ascertain whether it is in condition to maintain a continuous supply of oil; and, finally, before the turbine is started, the shaft should be turned over by hand, to insure that the rotor will turn freely in the casing.

A steam turbine should be started slowly, and before it is allowed to turn over under steam it should be warmed up. This is accomplished by opening the throttle valve just enough to let steam flow into the turbine. The drains should be kept open until the turbine is well started. The length of time required for warming up depends on the size of the turbine, a large unit requiring more time than a small one. As the warming up proceeds, the throttle may gradually be opened and the auxiliary machinery may be started. Once it has been started, the turbine should be brought up to speed slowly. If it is speeded up too rapidly, vibration will result. After the normal running speed has been reached, the load may be thrown on; but this, also, should be done gradually, to prevent a rush of water from the boiler with the steam.

If superheated steam is used, extra caution must be employed in starting, for during the warming up, with the throttle valve only slightly opened, the passing steam will be cooled considerably. But when the valve is opened wider, the greater volume passing will not lose so much of its superheat, and if care is not exercised the turbine will be subjected to sudden expansion because of the higher temperature of the steam. The main point in starting is to avoid any sudden changes of temperature in the turbine. If a turbine must be ready to be put in operation at short notice, steam may be allowed to flow

through it continually, by means of a by-pass around the throttle valve. It will always be warmed up, then, and can be brought up to speed with less danger and more rapidly.

The shaft or spindle of a turbine rotates at high speed, and therefore the bearings should be kept well lubricated; for if the oil supply fails, or if a bearing begins to heat because of grit carried into it, the resulting trouble will come very quickly. The presence of a hot bearing will usually be evidenced by the smell of burning oil or by the appearance of white smoke. When these signs are observed the oil supply should immediately be increased to the greatest possible amount. If this does not reduce the temperature of the bearing or prevent its further heating, the turbine should be shut down. To continue will result in burning out the bearing, and it is better to stop before this happens. The high speed of the shaft renders it impossible to nurse a hot turbine bearing as is done frequently in the running of reciprocating engines.

When shutting down a steam turbine, the throttle valve should be closed partly before the load is reduced, so as to prevent any possibility of racing when the load is finally taken off. The load may then be used as a brake to bring the rotor to a stop. When the throttle valve has been closed and the steam supply has been shut off completely, the auxiliary machinery may be stopped. If the load is taken off before the throttle is wholly closed the turbine may continue to rotate for $\frac{1}{2}$ hr., as the rotor is then running in a vacuum and under no load. The speed may be reduced by opening the drains and allowing air to enter the casing. The oil supply to the bearings must be continued until the turbine has come to rest, and the oil pump should be the last auxiliary to be stopped.

**Economy of Turbine.**—As there are no internal rubbing surfaces in the steam turbine, superheated steam may be employed without causing any of the lubrication troubles attending its use in reciprocating engines. Because of the greater amount of heat contained in 1 lb. of superheated steam, the economy of a turbine working with superheated steam is greater than that of one working with saturated steam; also, the efficiency is increased because the superheated steam causes less frictional resistance to the motion of the blades. To show the value of superheated steam in turbine work, it may be stated that 50° F. of superheat reduces the steam consumption about 6%; 100° F. of superheat reduces it about 10%; and 150° F. of superheat reduces it about 13½%. The use of high superheat, however, produces expansion of the rotor and the casing and may cause the blades to interfere; as a result, the usual degree of superheat in steam-turbine practice is 100° F., and seldom exceeds 150° F.

The steam turbine shows better economy than the steam engine when working with low-pressure steam in connection with a high vacuum; but when working with high-pressure steam and a vacuum of about 26 in., the engine is the more economical. As a consequence, a combination of the steam engine and the steam turbine has been adopted. The engine uses the high-pressure steam from the boilers and expands it to about atmospheric pressure. This exhaust steam then passes into the turbine, which exhausts into a condenser carrying a high degree of vacuum, and the expansion is carried to the extreme practicable limit. The turbine thus used in connection with an engine is termed an *exhaust-steam turbine.*

As the economy of the steam turbine is dependent so largely on the degree of vacuum carried, it is necessary for the engineer to watch the vacuum gauge closely. With reciprocating engines, the loss of 1 or 2 in. of vacuum may not be of much consequence; but in a turbine plant, where the vacuum is from 27 to 28 in., a loss of 1 or 2 in. will result in a considerable increase in the steam consumption. Because of the high vacuum employed, the difficulty of keeping pipes, valves, and glands from leaking is greater in turbine practice than in engine practice, but the greater economy obtained by keeping everything tight overbalances the increased care and labor.

**Care of Gears in De Laval Turbines.**—The De Laval Steam Turbine Company in their directions for operating their turbines state that in order to keep the gears in good condition the teeth should be cleaned occasionally when the turbine is not in service. They recommend that a wire brush and kerosene be employed for this purpose. At the same time the gear-case should also be thoroughly cleaned, and after the cleaning the gears should be well lubricated.

Should an engineer for any reason desire to take the gears out of the case, it is recommended that he secure special directions relating to their removal from the manufacturers. The same statement also applies to the adjustment of the gears, which need to be kept in perfect adjustment.

# RULES FOR STATIONARY ENGINEERS

If a gauge glass breaks turn off the water first and then the steam, to avoid scalding yourself.

Don't buy oil or waste simply because it is very cheap; it will cost more than a good article in the end.

When cutting rubber for gaskets, etc., have a dish of water handy, and keep wetting the knife blade; it makes the work much easier.

Don't forget that there is no economy in employing a poor fireman; he can, and probably will, waste more coal than would pay the wages of a first-class man.

An ordinary steam engine having two cylinders connected at right angles on the same shaft consumes one-third more steam than a single-cylinder engine, while developing only the same amount of power.

A fusible plug ought to be renewed every 3 mo., by removing the old metal and refilling the case; and it should be scraped clean and bright on both ends every time that the boiler is washed out, to keep it in good working order.

When trying a gauge-cock, don't jerk it open suddenly, for if the water happens to be a trifle below the cock, the sudden relief from pressure at that point may cause it to lift and flow out, thus showing a wrong height. Whereas, if it is opened quietly, no lift will occur, and it will show whether there is water or steam at that level.

Always open steam stop-valves between boilers very gently, that they may heat and expand gradually; by suddenly turning on steam a stop-valve chest was burst, due to the expansive power of heat unequally applied. The same care must be exercised when shutting off stop-valves; explosions have been caused by shutting a communicating stop-valve too suddenly—due to the recoil.

In order to obtain the driest possible steam from a boiler, there should be an internal perforated pipe (dry pipe, so called) fixed near the top of the boiler, and suitably connected to the steam pipe. The perforations in this pipe should be from one-quarter to one-half greater in area than that of the steam pipe.

If a glass gauge tube is too long, wet a triangular file with turpentine, then holding the tube in the left hand, with the thumb and forefinger at the place where it is to be cut, saw it quickly and lightly two or three times with the edge of the file. Take the tube in both hands, both thumbs being on the side opposite the mark, and 1 in. or so apart, and then try to bend the glass, using the thumbs as fulcrums, and it will break at the mark, which has weakened the tube.

A stiff charge of coal all over a furnace will lower the temperature 200° or 300° in a very short time. After the coal is well ignited the temperature will rise about 500°, and as it burns will gradually drop about 200°, until the fireman puts in another charge, when the sudden fall again takes place. This sudden contraction and expansion frequently causes the bursting of a boiler, and it is for this reason that light and frequent charges of coal, or else firing only one-half of the furnace at a time, should be always insisted on.

Be careful when using a wrench on hexagonal nuts that it fits snugly, or the edges of the nut will soon become rounded.

If a monkey-wrench is not placed on the nut properly, the strain will often bend or fracture the wrench.

The area of grate for a boiler should never be less than ⅓ sq. ft. per I. H. P. of the engine, and it is seldom advisable to increase this allowance beyond ½ sq. ft. per I. H. P.

The area of tube surface for a boiler should not be less than 2½ sq. ft. per I. H. P. of the engine.

The ratio of heating surface to grate area in a boiler should be 30 to 1 as a minimum, and may often be increased to 40 to 1, or even more, with advantage.

Lap-welded pipe of the same rated size has always the same outside diameter, whether common, extra, or double extra, but the internal diameter is of course decreased with the increased thickness.

A good cement for steam and water joints is made by taking 10 parts, by weight, of white lead, 3 parts of black oxide of manganese, 1 part of litharge, and mixing them to the proper consistency with boiled linseed oil.

To harden a cutting tool, heat it in a coke fire to a blood-red heat and plunge it into a solution of salt and water (1 lb. of salt to 1 gal. of water), then polish the tool, heat it over gas, or otherwise, until a dark straw and purple mixed color shows on the polish, and cool it in the salt water.

Small articles can be plated with brass by dipping them in a solution of 9¼ gr. each of sulphate of copper and chloride of tin, in 1¾ pt. of water.

Don't be eternally tinkering about an engine, but let well enough alone.

Don't forget that it is possible to drive a key with a copper hammer just as well as with a steel one, and that it doesn't leave any marks.

Keep on hand slips of thin sheet copper, brass, and tin, to use as liners, and if these are shaped properly, much time will be saved when they are needed.

A few wooden skewer pins, such as butchers use, are very useful for many purposes in an engine room.

In running a line of steam pipe where there are certain rigid points, make arrangements for expansion on the line between those points.

Arrange the usual work of the engine and firerooms systematically, and adhere to it.

Don't forget that cleanliness is next to godliness.

Rubber cloth kept on hand for joints should be rolled up and laid away by itself, as any oil or grease coming in contact with it will cause it to soften and give out when put to use.

When using a jet condenser, let the engine make three or four revolutions before opening the injection valve, and then open it gradually, letting the engine make several more revolutions before it is opened to the full amount.

Open the main stop-valve before the fires are started under the boilers.

When starting fires, don't forget to close the gauge-cocks and safety valve as soon as steam begins to form.

An old Turkish towel, cut in two lengthwise, is better than cotton waste for cleaning brass work.

Always connect the steam valves in such a manner that the valve closes against the constant steam pressure.

Turpentine well mixed with black varnish makes a good coating for iron smoke pipes.

Ordinary lubricating oils are not suitable for use in preventing rust.

It is possible to make a hole through glass by covering it with a thin coating of wax, warming the glass and spreading the wax on it; then scrape off the wax where the hole is wanted, drop a little fluoric acid on the spot with a wire. The acid will cut a hole through the glass, and it can be shaped with a copper wire covered with oil and rottenstone.

A mixture of 1 oz. of sulphate of copper, $\frac{1}{4}$ oz. of alum, $\frac{1}{2}$ teaspoonful of powdered salt, 1 gill of vinegar and 20 drops of nitric acid will make a hole in steel that is too hard to cut or file easily. Also, if applied to steel and washed off quickly, it will give the metal a beautiful frosted appearance.

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# COMPRESSED AIR

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## CLASSIFICATION AND CONSTRUCTION OF COMPRESSORS

An *air compressor* consists essentially of a cylinder in which atmospheric air is compressed by a piston, the driving power being steam, water, oil, gas, or electricity. Steam-driven compressors in ordinary use may be classed as follows:

1. *Straight-line* type, in which a single horizontal air cylinder is set tandem with its steam cylinder, and provided with two flywheels; this pattern is generally adapted for compressors of small size.

2. *Duplex* type, in which there are two steam cylinders, each driving an air cylinder, and coupled at 90° to a crank-shaft carrying a flywheel.

3. *Horizontal, cross-compound* engines, each steam cylinder set tandem with an air cylinder, as in 2.

4. *Vertical, simple,* or *compound* engines, with the air cylinders set above the steam cylinders.

5. *Compound* or *stage compressors*, in which the air cylinders themselves are compounded; the compression is carried to a certain point in one cylinder and successively raised and finally completed to the desired pressure in the others. They may be either of the straight-line or duplex form, with simple or compound steam cylinders. The principle of compound, or two-stage, air compression is recognized as applicable for even the moderate pressures required in mining. Compressors of class 5 are frequently employed, as well as classes 1, 2, and 3.

**Theory of Air Compression.**—The useful effect or efficiency of a compressor is the ratio of the force stored in the compressed air to the work that has been expended in compressing it; this probably never reaches 80% and often falls below 60%.

*Free Air* is air at ordinary atmospheric pressure as taken into the compressor cylinder; as commonly used, this means air at sea-level pressure (14.7 lb. per sq. in.) at 60° F. The *absolute pressure* of air is measured from zero, and is equal to the assumed atmospheric pressure plus gauge pressure. Air-compression calculations depend on the two well-known laws:

1. *Boyle's Law.*—The temperature being constant, the volume varies inversely as the pressure; or $PV = P'V' = $ a constant; in which $V$ equals the volume of a given weight of air at the freezing point, and the pressure $P$; $V'$ equals the volume of the same weight of air at the same temperature and under the pressure $P'$.

2 *Gay-Lussac's Law.*—The volume of a gas under constant pressure, when heated, expands, for each degree of rise in temperature, by a constant proportional part of the volume that it occupied at the freezing point; or, $V' = V(1 + at°)$, in which $a$ equals $\frac{1}{273}$ for centigrade degrees, or $\frac{1}{461}$ for Fahrenheit degrees.

Theoretically, air may be compressed in two ways, as follows:

1. *Isothermally*, when the temperature is kept constant during compression, and in this case, the formula $PV = P'V'$ is true.

2. *Adiabatically*, when the temperature is allowed to rise without check during the compression.

As the pressure rises faster than the volume diminishes, the equation $PV = P'V'$ no longer holds, and $\frac{P'}{P} = \left(\frac{V}{V'}\right)^n$, in which $n$ equals 1.406. The specific heat of air at constant pressure is .2375, and at constant volume .1689, and $n = .2375 \div .1689 = 1.406$.

In practice, compression is neither isothermal nor adiabatic, but intermediate between the two. The values of $n$ for different conditions in practice as determined from a 2,000-H. P. stage compressor at Quai de la Gare, Paris, are as follows: For purely adiabatic compression, with no cooling arrangements, $n = 1.406$; in ordinary single-cylinder dry compressors, provided with a water-jacket, $n$ is roughly 1.3; while in the best wet compressors (with spray injection), $n$ becomes 1.2 to 1.25. In the poorest forms of compressor, the value $n = 1.4$ is closely approached. For large, well-designed compressors with compound air cylinders, the exponent $n$ may be as small as 1.15.

**Construction of Compressors.**—Compressors are usually built with a short stroke, as this is conducive to economy in compression as well as the attainment of a proper rotative speed. In ordinary single-stage compressors, the usual ratio of length of stroke to diameter of steam cylinders is $1\frac{1}{2}$ to 1 or $1\frac{1}{4}$ to 1. In some makes, such as the Rand, the ratio is considerably greater, varying from $1\frac{1}{2}$ to $1\frac{3}{4}$ to 1, as in several large plants built for the Calumet & Hecla Mining Co. Many compressors have length and diameter of steam cylinders equal. The relative diameters of the air and steam cylinders depend on the steam pressure carried, and the air pressure to be produced. In mining operations, there is usually but little variation in these conditions. For rock-drill work, the air pressure is generally from 60 to 80 lb.

In using water-power, a compressor is driven most conveniently by a bucket impact wheel, such as the Pelton or Knight. The waterwheel is generally mounted directly on the crank-shaft, without the use of gearing. As the power developed is uniform throughout the revolution of the wheel, the compressor should be of duplex form, in order to equalize the resistance so far as possible. The rim of the wheel is made extra heavy, to supply the place of a flywheel. When direct-connected, the wheel is of relatively large diameter, as its speed of rotation must of necessity be slow. With small high-speed wheels, the compressor cylinders may be operated through belting or gearing. In most cases, however, the waterwheel may be large enough to render gearing unnecessary. Impact wheels may be employed with quite small heads of water, by introducing multiple nozzles. To prevent the water from splashing over the compressor, the wheel is enclosed in a tight iron or wooden casing. The force of the water is regulated usually by an ordinary gate valve. If the head is great, it may be necessary to introduce means for deflecting the nozzle, so that, when the compressor is to be stopped suddenly, danger of rupturing the water main will be avoided.

**Rating of Compressors.**—Compressors are rated as follows: (1) In terms of the horsepower developed by the steam end of the compressor, as shown by

indicator cards taken when running at full speed and when the usual volume of air is being consumed; (2) compressors for mines are often rated roughly as furnishing sufficient air to operate a certain number of rock drills; a 3-in. drill requires a volume of air at 60 lb. pressure, equal to 100 or 110 cu. ft. of free atmospheric air per minute; (3) in terms of cubic feet of free air compressed per minute to a given pressure.

As the actual capacity of a compressor depends on the density of the intake air, it will be reduced when working at an altitude above sea level, because of the diminished density of the atmosphere. The accompanying table gives the percentages of output at different elevations.

## EFFICIENCIES OF AIR COMPRESSORS AT DIFFERENT ALTITUDES

| Altitude Feet | Barometer Pressure | | Volumetric Efficiency of Compressor Per Cent. | Loss of Capacity Per Cent. | Decreased Power Required Per Cent. |
|---|---|---|---|---|---|
| | Inches Mercury | Pounds per Square Inch | | | |
| 0 | 30.00 | 14.75 | 100 | 0 | 0 |
| 1,000 | 28.88 | 14.20 | 97 | 3 | 1.8 |
| 2,000 | 27.80 | 13.67 | 93 | 7 | 3.5 |
| 3,000 | 26.76 | 13.16 | 90 | 10 | 5.2 |
| 4,000 | 25.76 | 12.67 | 87 | 13 | 6.9 |
| 5,000 | 24.79 | 12.20 | 84 | 16 | 8.5 |
| 6,000 | 23.86 | 11.73 | 81 | 19 | 10.1 |
| 7,000 | 22.97 | 11.30 | 78 | 22 | 11.6 |
| 8,000 | 22.11 | 10.87 | 76 | 24 | 13.1 |
| 9,000 | 21.29 | 10.46 | 73 | 27 | 14.6 |
| 10,000 | 20.49 | 10.07 | 70 | 30 | 16.1 |
| 11,000 | 19.72 | 9.70 | 68 | 32 | 17.6 |
| 12,000 | 18.98 | 9.34 | 65 | 35 | 19.1 |
| 13,000 | 18.27 | 8.98 | 63 | 37 | 20.6 |
| 14,000 | 17.59 | 8.65 | 60 | 40 | 22.1 |
| 15,000 | 16.93 | 8.32 | 58 | 42 | 23.5 |

EXAMPLE.—Calculate the volume of air furnished by an $18'' \times 24''$ compressor working at an elevation of 5,000 ft. above sea level, making 95 rev. per min., and having a piston speed of 380 ft. per min.

SOLUTION.— $9^2 \times 3.14 = 254.3$ sq. in. = piston area. $\frac{254.3}{144} \times 380 = 668.8$ cu. ft. = volume displaced per minute by the piston; deducting 10% for loss gives 602 cu. ft. At sea level at 80 lb. gauge pressure, this equals $\frac{15}{80+15} \times 602 = 95$ cu. ft. At an elevation of 5,000 ft., the output of a compressor would be $95 \times 84\% = 79.8$ cu. ft. per min.

**Cooling.**—Compressor cylinders may be cooled by injecting water into the cylinder, in which case they are known as *wet compressors*; or by jacketing the cylinder in water, when they are known as *dry compressors*.

## TRANSMISSION OF AIR IN PIPES

The actual discharge capacity of piping is not proportional to the cross-sectional area alone, that is, to the square of the diameter. Although the periphery is directly proportional to the diameter, the interior surface resistance is much greater in a small pipe than in a large one, because, as the pipe becomes smaller, the ratio of perimeter to area increases.

To pass a given volume of compressed air, a 1-in. pipe of given length requires over three times as much head as a 2-in. pipe of the same length. The character of the pipe, also, and the condition of its inner surface, have much to do with the friction developed by the flow of air. Besides imperfections in the surface of the metal, the irregularities incident on coupling together the lengths of pipe must increase friction. There are so few reliable

data that the influences by which the values of some of the factors may be modified are not fully understood; and, owing to these uncertain conditions, the results obtained from formulas are only approximately correct.

Among the formulas in common use, perhaps the most satisfactory is that of D'Arcy. As adopted for compressed-air transmission, it takes the form:

$$D = c \sqrt{\frac{d^5(p_1 - p_2)}{w_1 l}}$$

in which $D$ = volume of compressed air discharged at final pressure in cubic feet per minute;

$c$ = coefficient varying with diameter of pipe, as determined by experiment;

$d$ = diameter of pipe in inches (actual diameters of $1\frac{1}{4}$- and $1\frac{1}{2}$-in. pipe are 1.38 in. and 1.61 in., respectively; nominal diameters of all other sizes may be taken for calculations);

$l$ = length of pipe, in feet;

$p_1$ = initial gauge pressure, in pounds per square inch;

$p_2$ = final gauge pressure, in pounds per square inch;

$w_1$ = density of air, or its weight at initial pressure $p_1$, in pounds per cubic foot.

The values of the coefficients $c$ for sizes of piping up to 12 in. are:

| | | |
|---|---|---|
| 1 in............45.3 | 5 in............59.0 | 9 in............61.0 |
| 2 in............52.6 | 6 in............59.8 | 10 in............61.2 |
| 3 in............56.5 | 7 in............60.3 | 11 in............61.8 |
| 4 in............58.0 | 8 in............60.7 | 12 in............62.0 |

Some apparent discrepancies exist for sizes larger than 9 in., but they cause no very material differences in the results.

Another formula, published by Mr. Frank Richards, is as follows:

$$H = \frac{V^2 L}{10,000 \, D^5 a}$$

in which $H$ = head or difference of pressure required to overcome friction and maintain flow of air;

$V$ = volume of compressed air delivered, in cubic feet per minute;

$L$ = length of pipe, in feet;

$D$ = diameter of pipe, in inches;

$a$ = coefficient, depending on size of pipe.

Values of $a$ for nominal diameters of wrought-iron pipe:

| | | |
|---|---|---|
| 1 in......... .350 | 3 in......... .730 | 8 in.........1.125 |
| $1\frac{1}{4}$ in......... .500 | $3\frac{1}{2}$ in......... .787 | 10 in.........1.200 |
| $1\frac{1}{2}$ in......... .662 | 4 in......... .840 | 12 in.........1.260 |
| 2 in......... .565 | 5 in......... .934 | |
| $2\frac{1}{2}$ in......... .650 | 6 in.........1.000 | |

The values of $a$ for $1\frac{1}{4}$ and $1\frac{1}{2}$ in. pipe are not consistent with those for other sizes, for the reason already stated. When using this formula with its constants, the calculated losses of pressure are found to be smaller, and, conversely, the volumes of air discharged are larger, under the same conditions than those obtained from D'Arcy's formula.

It must be remembered that, within certain limits, the loss of head or pressure increases with the square of the velocity. To obtain the best results, it has been found that the velocity of flow in the main air pipes should not exceed 20 or 25 ft. per sec. When the initial velocity much exceeds 50 ft. per sec., the percentage loss becomes very large; and, conversely, by using piping large enough to keep down the velocity, the friction loss may be almost eliminated. For example, at the Hoosac tunnel, in transmitting 875 cu. ft. per min. of free air at an initial pressure of 60 lb., through an 8-in. pipe, 7,150 ft. long, the average loss including leakage was only 2 lb. A volume of 500 cu. ft. per min. of free air, at 75 lb., can be transmitted through 1,000 ft. of 3-in. pipe with a loss of 4.1 lb., while if a 5-in. pipe is used the loss will be reduced to .24 lb. The velocity of flow in the latter case is only 10 ft. per sec.

When driving the Jeddo mining tunnel, at Ebervale, Pa., two $3\frac{1}{2}$-in. drills were used in each heading, with a 6-in. main, the maximum transmission distance being 10,800 ft. This pipe was so large in proportion to the volume of air required for the drills (230 cu. ft. per min. of free air) that the loss was reduced to an extremely small quantity. A calculation shows a loss of .002 lb., and the gauges at each end of the main were found to record practically the same pressure.

A due regard for economy in installation, however, must limit the use of very large piping, the cost of which should be considered in relation to the

cost of air compression in any given case.  Diameters of from 4 to 6 in. for the mains are large enough for any ordinary mining practice.  Up to a length of 3,000 ft., a 4-in. pipe will carry 480 cu. ft. per min. of free air compressed to 82 lb., with a loss of 2 lb. pressure.  This volume of air will run four 3-in. drills.  Under the same conditions, a 6-in. pipe, 5,000 ft. long, will carry 1,100 cu. ft. per min. of free air, or enough for 10 drills.

A mistake is often made by putting in branch pipes of too small a diameter.  For a distance of, say, 100 ft., a 1¼-in. pipe is small enough for a single drill, though a 1-in. pipe is frequently used.  While it is, of course, admissible to increase the velocity of flow in short branches considerably beyond 20 ft. per sec., extremes should be avoided.  To run a 3-in. drill from a 1-in. pipe 100 ft. long, will require a velocity of flow of about 55 ft. per sec., causing a loss of 10 lb. pressure.

The piping for conveying compressed air may be of cast or wrought iron.  If of wrought iron, as is customary, the lengths are connected either by sleeve couplings or by cast-iron flanges into which the ends of the pipe are screwed or expanded.  Sleeve couplings are used for all except the large sizes.  The smaller sizes, up to 1¼ in., are butt-welded, while all from 1½ in. up are lap-welded, to insure the necessary strength.  Wrought-iron, spiral-seam, riveted, or spiral-weld steel tubing is sometimes used.  It is made in lengths of 20 ft. or less.  For convenience of transport in remote regions, rolled sheets in short lengths may be had.  They are punched around the edges, ready for riveting, and are packed closely—four, six, or more sheets in a bundle.

All joints in air mains and branches should be carefully made.  Air leaks are more expensive than steam leaks because of the losses suffered when compressing the air.  The pipe may be tested from time to time by allowing the air at full pressure to remain in the pipe long enough to observe the gauge.  A leak should be traced and stopped immediately.  When putting together screw joints, care should be taken that none of the white lead or other cementing material is forced into the pipe; this would cause obstruction and increase the friction loss.  Also, each length as put in place should be cleaned thoroughly of all foreign substances that may have lodged inside.  To render the piping readily accessible for inspection and stoppage of leaks, it should, if buried, be carried in boxes sunk just below the surface of the ground; or, if underground, it should be supported upon brackets along the sides of the mine workings.  Low points in pipe lines, which would form pockets for the accumulation of entrained water, should be avoided, as they obstruct the passage of the air.  In long pipe lines, where a uniform grade is impracticable, provision may be made near the end for blowing out the water at intervals, when the air is to be used for pumps, hoists, or other stationary engines.

For long lengths of piping, expansion joints are required, particularly when on the surface.  They are not often necessary underground, as the temperature is usually nearly constant, except in shafts, or where there may be considerable variations of temperature between summer and winter.

## LOSSES IN THE TRANSMISSION OF COMPRESSED AIR

To obtain compressed air, an engine drives a compressor, which forces air into a reservoir; the air under pressure is led through pipes to the air engine, and is there used after the manner of steam.  The resulting power is frequently a small percentage of the power expended.  In a large number of cases the losses are due to poor designing, and are not chargeable as faults of the system or even to poor workmanship.

The losses are chargeable, first, to friction of the compressor.  This will amount ordinarily to 15% or 20%, and can be helped by good workmanship, but cannot probably be reduced below 10%.  Second, a loss is occasioned by pumping the air of the engine room, rather than air drawn from a cooler place; this loss varies with the season, and amounts to from 3% to 10% and can all be saved.  The third loss, or series of losses, is caused by insufficient supply, difficult discharge, defective cooling arrangements, poor lubrication, and a host of other causes, in the compression cylinder.  The fourth loss is found in the pipe, it varies with every different situation, and is subject to somewhat complex influences.  The fifth loss is chargeable to a fall of temperature in the cylinder of the air engine.  Losses arising from leaks are often serious, but the remedy is too evident to require demonstration; no leak can be so small as not to require immediate attention.  An attendant who is careless about packings and hose couplings will permit losses for which no amount of engineering skill can compensate.

It is possible to realize 100% efficiency in the air engine, leaving friction out of our consideration, only when the expansion of the air and the changes of its temperature in the expanding or air-engine cylinder are precisely the reverse of the changes that have taken place during the compression of the air in the compressing cylinder; but these conditions can never be realized. The air during compression becomes heated, and during expansion it becomes cold. If the air immediately after compression, before the loss of any heat, was used in an air engine and there perfectly expanded back to atmospheric pressure, it would, on being exhausted, have the same temperature it had before compression, and its efficiency would be 100%.

But the loss of heat after compression and before use cannot be prevented, as the air is exposed to such very large radiating surfaces in the reservoir and pipes, on its passage to the air engine. The heat that escapes in this way, did, while in the compressing cylinder, add much to the resistance of the air to compression, and as it is sure to escape, at some time, either in reservoir or pipes, the best plan is to remove it as fast as possible from the cylinder and thus remove one element of resistance. Hence, compressors are almost universally provided with cooling attachments more or less perfect in their action, the aim being to secure isothermal compression, or compression having equal temperature throughout.

If air compressed isothermally is used with perfect expansion and the fall of temperature during expansion is prevented, 100% efficiency will be obtained. But air will grow cold when expanded in an engine, hence warming attachments have the same economic place on an air engine that cooling attachments have on an air compressor. In fact, attachments of this kind are found in large and permanently located engines, but their use on most of the engines for mine work is dispensed with, and the engines expand the air adiabatically, or without receiving heat.

The practical engineer, therefore, has to deal with nearly isothermal compression, and nearly adiabatic expansion, and must also consider that the air in reservoirs and pipes becomes of the same temperature as surrounding objects. Consideration must also be had for the friction of the compressor and the air engine. For the pressure of 60 lb., which is that most commonly used, the decrease in resistance to compression secured by the cooling attachments is almost exactly equaled by the friction of the compressor. Hence it is safe, when calculating the efficiency of the air engine, to consider the compressor as being without cooling attachments, and also as working without friction. The results of such calculations will be too high efficiencies for light pressures, which are little used; about correct for medium pressures, which are commonly employed; and too low for higher pressures, and will thus have the advantage of not being overestimated. This result is occasioned by the fact that, owing to the slight heat in compressing low pressures of air, the saving of power by the cooling attachments is not equal to the friction of the machine, but at high pressures, on account of the great heat, the cooling attachments are of great value and save very much more power than friction consumes.

In expanding engines, the expansion never falls as low as the adiabatic law would indicate, owing to a number of reasons, but if the expansion is considered as adiabatic, an error in calculations caused thereby will be on the safe side and the actual power will exceed the calculated power. Therefore, the compressor and engine may be considered as following the adiabatic law of compression and expansion, and as working without friction.

With this view of the case, the efficiency of an air engine, working with perfect expansion, stated in percentages of the power required to operate the compressor, can be placed as here shown for the various pressures above the atmosphere. As the efficiencies for the lower pressures are very much greater than for the high pressures, the conclusion is almost irresistible that to secure economical results air engines should be designed to run with light pressures.

| Pressure Above Atmosphere Pounds | Efficiency Per Cent. |
|---|---|
| 2.9 | 94.85 |
| 14.7 | 81.79 |
| 29.4 | 72.72 |
| 44.1 | 66.90 |
| 58.8 | 62.70 |
| 73.5 | 59.48 |
| 88.2 | 56.88 |

In the foregoing the pipe friction has been entirely neglected. A pressure of 2.9 lb. is credited with an efficiency of 94.85%; but, if the air were conveyed through a pipe, and the length of the pipe and the velocity of flow were such that 2.9 lb. pressure was lost in friction, the efficiency of the air, instead of being 94.85%, would be absolutely zero. It is the power that can be obtained from the air, after it has passed the pipe and lost a part of its pressure by friction, that must be considered when the efficiency of an apparatus is given.

The foregoing table of efficiencies with a loss of 2.9 lb. in the pipe, now gives different values for the efficiencies at the various pressures.

| Pressure Above Atmosphere Pounds | Efficiency Per Cent. |
|---|---|
| 2.9 | 00.00 |
| 14.7 | 70.44 |
| 29.4 | 68.81 |
| 44.1 | 64.87 |
| 58.8 | 61.48 |
| 73.5 | 58.62 |
| 88.2 | 56.23 |

It will be noticed that the light pressures have lost most by the pipe friction, 2.9 lb. having lost 100%; 14.7 lb. 11%, and 88.2 lb. only a trifle over one-half of 1%. It is also seen now 14.7 lb. is apparently the economical pressure to use. But a further careful analysis of the subject shows, that when the loss in the pipe is 2.9 lbs., then 20.5 lb. is the most economical pressure to use, and that the efficiency is 71%. But 2.9 lb. is a very small loss between compressor and air engine, and cases are extremely exceptional where the friction of valves, pipes, elbows, ports, etc. does not far exceed this. Yet, with these conditions, which are very difficult to fill, 20.5 lb. is the lightest pressure that should probably ever be used for conveying power, and 71% is an efficiency scarcely to be obtained.

Continuing the investigation and taking examples where the pipe friction amounts to 5.8 lbs., it is found that the following efficiencies correspond to the stated pressure:

| Pressure Above Atmosphere Pounds | Efficiency Per Cent. |
|---|---|
| 14.7 | 57.14 |
| 29.4 | 64.49 |
| 44.1 | 62.71 |
| 58.8 | 60.12 |
| 73.5 | 57.73 |
| 88.2 | 55.59 |

As friction increases, or, in other words, when more air is used and greater demands are made on the carrying capacity of the pipe, the pressure must be greatly increased to attain the most economical results. If the demands are such as to increase the friction and loss in pipe to 14.7 lbs., the air of 14.7 lb. pressure at the compressor is entirely useless at the air engine. The table will therefore stand thus:

| Pressure Above Atmosphere Pounds | Efficiency Per Cent. |
|---|---|
| 14.7 | 00.00 |
| 29.4 | 48.53 |
| 44.1 | 55.13 |
| 58.8 | 55.64 |
| 73.5 | 54.74 |
| 88.2 | 53.44 |

It is to be noticed that 88.2 lb. pressure has lost only about 3½% of its efficiency by reason of as high a friction as 14.7 lb., while the efficiency of the lower pressures has been greatly affected. As the friction increases the most efficient, and consequently, most economical, pressure increases. In fact, for any given friction in a pipe, the pressure at the compressor must not be carried below a certain limit. The following table gives the lowest pressures that should be used at the compressor, with varying amounts of friction in the pipe:

| Friction Pounds | Pressure at Compressor Pounds | Efficiency Per Cent. |
|---|---|---|
| 2.9 | 20.5 | 70.92 |
| 5.8 | 29.4 | 64.49 |
| 8.8 | 38.2 | 60.64 |
| 11.7 | 47.0 | 57.87 |
| 14.7 | 52.8 | 55.73 |
| 17.6 | 61.7 | 53.98 |
| 20.5 | 70.5 | 52.52 |
| 23.5 | 76.4 | 51.26 |
| 26.4 | 82.3 | 50.17 |
| 29.4 | 88.2 | 49.19 |

So long as the friction of the pipe equals the amounts there given, an efficiency greater than the corresponding sums stated in the table cannot be expected. In a case that corresponds to any of those cited in the table, the efficiency can be increased only by reducing the friction. An increase in the size of pipe will reduce friction by reason of the lower velocity of flow required for the same amount of air. But many situations will not admit of large pipes being employed, owing to considerations of economy outside of the question of fuel or prime motor capacity.

An increase of pressure will decrease the bulk of air passing in the pipe, and in that proportion will reduce its velocity. This will decrease the loss by friction, and, as far as that goes, a gain is obtained, but there is a new loss, and that is the diminishing efficiencies of increasing pressures. Yet as each cubic foot of air is at a higher pressure, and, therefore, carries more power, as many cubic feet will not be needed for the same work. It is obvious that with so many sources of gain or loss the question of selecting the proper pressure is not to be decided hastily.

As an illustration of the combined effect of these different elements, a very common case will be taken. The compressor makes 102 rev. per min., pressure is 52.8 lb., loss in pipe is 14.7 lb., machine in mine running at 38.2 lb., efficiency is 55.73%. As long as the friction of the pipe amounts to 14.7 lb., 52.8 lb. is the best pressure and 55.73% the greatest efficiency, but friction may be reduced by reducing the bulk of air passing through the pipe and if the cylinder of the air engine is reduced until it requires 47 lb. pressure to do the same work as before; the friction of pipe will then drop to 11.7 lb. The pressure on the compressor will rise to 58.8 lb., its number of revolutions will fall to 100, and the resulting efficiency will be 57.22%.

Another change of pressure on compressor to 64.7 lb. will decrease its revolutions to 93, friction to 8.8 lb., and its efficiency will rise to 57.94%. If the pressure is increased to 73.5 lb., there will be only 84 revolutions of compressor, 5.8 lb. loss in pipe, and an efficiency of 57.73%. In this last case the efficiency begins to fall off a little, and higher pressures will show less efficiency; but, in comparison with the first example, the same work will be done with a trifle less power and with a decrease of nearly 20% in the speed of the compressor.

Other common examples can be shown where an increase of pressure would result in wonderful increase in efficiency and economy. There are many cases where light pressures and high velocity in the pipe will convey a given power with greater economy than higher air pressures and lower speed of flow through the pipe. But these cases arise mostly when the higher air pressures become very much greater than are at present in common use. Therefore, when estimating the efficiency of the complete outfit, it is found that the pipe and the pressure are very important elements, and must be determined with care and skill to secure the most satisfactory results. As the volume and power of air vary with its pressure, the size and consequent cost of compressor for a certain work will also be affected by the pressure. To plan an outfit for a mine, due regard must be had to the cost of fuel or prime motor power, and also to the cost of compressor, pipes, and machinery, as the saving in one is often secured by a sacrifice in the other.

Next to determining the size of pipe, the skilful engineer has need of further care in the proper position of reservoirs, branches, drains, and other attachments, as only by the exercise of good judgment in this can satisfactory working be secured. The fact that, on account of the diminished density of the atmosphere at high altitudes, air compressors do not give the same results as at sea level, should also be taken into consideration when a compressor is to be installed in a mountainous region.

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## LOSS OF PRESSURE, IN POUNDS PER SQUARE INCH, BY FLOW OF AIR IN PIPES 1,000 FT. LONG

| Velocity of Air at Entrance to Pipe | | 1-In. Pipe | | | 2-In. Pipe | | | 2½-In. Pipe | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Meters per Second | Feet per Second | Loss of Pressure Pounds | Cubic Feet of Free Air Passed per Minute at 60 Lb. Gauge | Cubic Feet of Free Air Passed per Minute at 80 Lb. Gauge | Loss of Pressure Pounds | Cubic Feet of Free Air Passed per Minute at 60 Lb. Gauge | Cubic Feet of Free Air Passed per Minute at 80 Lb. Gauge | Loss of Pressure Pounds | Cubic Feet of Free Air Passed per Minute at 60 Lb. Gauge | Cubic Feet of Free Air Passed per Minute at 80 Lb. Gauge |
| 1 | 3.28 | .1435 | 6 | 7 | .0794 | 23 | 29 | .0574 | 32 | 41 |
| 2 | 6.56 | .6405 | 12 | 15 | .3050 | 46 | 59 | .2562 | 65 | 82 |
| 3 | 9.84 | 1.4545 | 18 | 22 | .7216 | 69 | 88 | .5818 | 97 | 124 |
| 4 | 13.12 | 2.5620 | 24 | 29 | 1.2566 | 93 | 117 | 1.0248 | 130 | 165 |
| 5 | 16.40 | 3.9345 | 29 | 37 | 1.9642 | 116 | 146 | 1.5738 | 163 | 207 |
| 6 | 19.68 | 5.4225 | 35 | 44 | 2.7120 | 139 | 175 | 2.1690 | 195 | 247 |
| 8 | 26.24 | 10.2480 | 47 | 59 | 5.0264 | 185 | 234 | 4.0992 | 260 | 330 |
| 10 | 32.80 | 15.7380 | 59 | 74 | 7.8568 | 232 | 294 | 6.2952 | 326 | 413 |

| | | 3-In. Pipe | | | 4-In. Pipe | | | 5-In. Pipe | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.28 | .0463 | 48 | 60 | .0347 | 86 | 109 | .0287 | 134 | 169 |
| 2 | 6.56 | .2092 | 96 | 121 | .1525 | 172 | 217 | .1281 | 268 | 239 |
| 3 | 9.84 | .4880 | 144 | 182 | .3608 | 258 | 326 | .2909 | 402 | 509 |
| 4 | 13.12 | .8381 | 193 | 243 | .6283 | 343 | 436 | .5124 | 537 | 678 |
| 5 | 16.40 | 1.3176 | 241 | 304 | .9821 | 429 | 544 | .7869 | 671 | 844 |
| 6 | 19.68 | 1.8080 | 289 | 364 | 1.3560 | 515 | 653 | 1.0845 | 805 | 1,017 |
| 8 | 26.24 | 3.3525 | 386 | 486 | 2.5132 | 687 | 871 | 2.0496 | 1,073 | 1,357 |
| 10 | 32.80 | 5.2704 | 480 | 607 | 3.9284 | 859 | 1,088 | 3.1476 | 1,342 | 1,696 |

| | | 6-In. Pipe | | | 8-In. Pipe | | | 10-In. Pipe | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.28 | .0232 | 193 | 244 | .0173 | 343 | 434 | .0143 | 537 | 680 |
| 2 | 6.56 | .1046 | 386 | 488 | .0762 | 687 | 864 | .0640 | 1,073 | 1,359 |
| 3 | 9.84 | .2440 | 579 | 633 | .1805 | 1,030 | 1,303 | .1455 | 1,610 | 2,039 |
| 4 | 13.12 | .4190 | 772 | 977 | .3141 | 1,373 | 1,736 | .2562 | 2,146 | 2,719 |
| 5 | 16.40 | .6588 | 965 | 1,221 | .4910 | 1,717 | 2,171 | .3934 | 2,683 | 3,399 |
| 6 | 19.68 | .9040 | 1,158 | 1,466 | .6780 | 2,060 | 2,605 | .5423 | 3,220 | 4,079 |
| 8 | 26.24 | 1.6762 | 1,544 | 1,954 | 1.2556 | 2,747 | 3,473 | 1.0248 | 4,293 | 5,438 |
| 10 | 32.80 | 2.6352 | 1,931 | 2,443 | 1.9642 | 3,434 | 4,342 | 1.5738 | 5,367 | 6,798 |

**Friction of Air in Pipes.**—Air in its passage through pipes is subject to friction in the same manner as water or any other fluid; therefore, the pressure at the compressor must be greater than at the point of consumption, in order to overcome this resistance. The power that is needed to produce the extra

pressure representing the friction of the pipe is lost, as there can be no useful return for it. The friction is affected by very many circumstances; it is increased in direct proportion to the length of the pipe and also in the square of the velocity of the flow of air. The pressure of the air does not affect it.

The losses by friction may be quite serious if the piping system is poorly designed, and, on the other hand, extravagant expenditure in pipe may result from a timid overrating of the evils of friction. A thorough knowledge of the laws governing the whole matter, as well as a ripe experience, is necessary to secure true economy and mechanical success. The loss of power in pipe friction is not always the most serious result. When a number of machines are in use in a mine, and the pipes are so small as to cause considerable loss of pressure by friction, there will be sudden and violent fluctuations in pressure whenever a machine is started or stopped. Breakages will be common occurrences, as the changes are too quick to be entirely guarded against by the attendant; perfectly even pressure at the compressor is no safeguard against this class of accidents. The trouble arises in the pipe, and the remedy must be applied there. A system of reservoirs and governing valves will regulate these matters and allow successful work to be done with pipes that would otherwise be entirely inadmissible.

The ordinary formulas for calculating the volume of air transmitted through a pipe do not take into account the increase of volume due to reduction of pressure, i. e., loss of head. To transmit a given volume of air at a uniform velocity and loss of pressure, it is necessary to construct the pipe with a gradually increasing area. This, of course, is impracticable, and in a pipe of uniform section both volume and velocity must increase as the pressure is reduced by friction. The loss of head in properly proportioned pipes is so small, however, that in practice the increase in volume is usually neglected. The table on page 482 gives the loss of pressure by flow of air in pipes calculated for pipes 1,000 ft. long; for other lengths, the loss varies directly as the length.

The resistance is not varied by the pressure, only so far as changes in pressure vary the velocity. It increases about as the square of the velocity, and directly as the length. Elbows, short turns, and leaks in pipes all tend to reduce the pressure in addition to the losses given in the table. An elbow with a radius of one-half the diameter of the pipe is as short as can be made.

## LOSS BY FRICTION IN ELBOWS

| Radius of Elbow | Equivalent Length of Straight Pipe |
|---|---|
| *Diameters* | *Diameters* |
| 5 | 7.85 |
| 3 | 8.24 |
| 2 | 9.03 |
| $1\frac{1}{2}$ | 10.36 |
| $1\frac{1}{4}$ | 12.72 |
| 1 | 17.51 |
| $\frac{3}{4}$ | 35.09 |
| $\frac{1}{2}$ | 121.20 |

# DESIGN, OPERATION, AND INSTALLATION OF AIR COMPRESSORS

With regard to the design, installation, and operation of air compressors, the following suggestions made by Mr. Alex. M. Gow, Mechanical Engineer, Oliver Iron Mining Co., and slightly enlarged, will be of interest.

**Design for Avoiding Explosions.**—Clearance space should be reduced to a minimum. Ingoing air should traverse as small a surface of hot metal as possible. Discharge valves and passageways should contain no pockets or recesses for the accumulation of oil. Cylinders and heads should be water-jacketed; in some cases piston water-cooling may be resorted to. Stage compression, with adequate intercooling should be employed wherever final pressure and first cost of installation will warrant. Discharge valves must be easy of access for cleaning and examination. There must be no excuse for dirty or leaky valves.

**Installation of Compressor.**—Air should be drawn from the coolest and cleanest place possible, and never from the engine room. Engine-room air is never cool nor clean and an open intake is a constant invitation to squirt oil in from a can. Around collieries, it is well to consider the washing of the air. Coal dust drawn in with the air, mixes with the oil and forms a substance that,

on heating, cakes and may take fire. The discharge pipe should be of ample size and have as few bends as possible. A thermometer, preferably recording, should be placed on the discharge pipe. Provision for aftercooling should be made, a water spray will answer, to be used when the thermometer indicates the necessity. The receiver should be provided with a manhole for cleaning, and a drain easy of access and ample in size. Automatic sight-feed lubricators should be depended on for regular lubrication, but in addition an oil pump may be installed for the introduction of soap and water in case of necessity.

**Operation of Compressors.**—High flash-test cylinder oil of the best obtainable grade should be used for regular lubrication of the air cylinders. Mr. L. A. Christian advises that the flash point be 625° F., and that the oil should be comparatively free from unnecessary volatile carbon compounds. Volatile hydrocarbons tend to reduce the flash point, and, mixing with the dust from the air, form combustible deposits in the receiver and outlet passages. Further, oil of low-flash test, on reaching the interior of the heated air cylinder will be vaporized and will pass out with the air into the receiver without affording any lubrication to the wearing surfaces. If the oil is too dense or is compounded with animal or vegetable oils, as is the case with many steam-cylinder oils, it will have a tendency to adhere to the discharge valves and passages, and, being subjected to the dry heat of the compressed air, will gradually change to a hard, brittle crust, which in time will completely choke up the air passages or will prevent the valves seating. The amount of oil to be fed into the air cylinder should be, if the machine makes less than 120 rev. per min., about 1 drop every 3 min. Kerosene should never be used to cut or eat away deposits of carbon, as is sometimes done, as its flash point is about 120° F. If the cylinders or air passages need cleaning, soapsuds made of 1 part of soft soap and 15 parts of pure water should be fed into the cylinder and the machine worked with a liberal solution instead of oil for a few hours or a day; then the blow-off valve of the receiver should be opened and the accumulation of oil and water drained off. After this treatment and before the machine is shut down, oil should be fed into the cylinder for an hour or so, in order that the valves and the parts connected with the cylinder may be coated with oil and thus prevent rust.

Discharge valves must be kept tight; to this end the use of the steam engine indicator is advised. The cards may not tell much about the conditions of the valves, but one of the greatest values of the indicator is the moral effect upon the engineer. The valves should be cleaned from dust and oil and frequently examined.

Accumulations of water and oil must be blown from the receiver and an internal examination made at stated intervals.

The thermometer on the discharge pipe should be watched like the steam gauge. Before it reaches 400° F., the after cooling spray should be put on, and all the water-supply pipes and the discharge valves examined.

The engineer in charge should be thoroughly instructed as to the possibility of an explosion, the dangers attendant upon the use of any but the prescribed oil, and the effect of leaky discharge valves. He should be instructed in the use of the steam-engine indicator and required to submit cards at stated intervals. He should record in the engine-room log the daily conditions of the machines under his charge. He should be given a wholesome respect for an air compressor, with imperative instructions to keep it clean, inside as well as out.

# ELECTRICITY

## PRACTICAL UNITS

In electric work, it is necessary to have units in terms of which to express the different quantities entering into calculations. The unit quantity of electricity flowing through a circuit is called a coulomb. A *coulomb* is the quantity of electricity that will deposit from a solution of silver nitrate through which it flows .001118 gram of silver.

**Strength of Current.**—The strength of current flowing in a wire may be measured in several ways. If a compass needle is held under or over a wire it will be deflected and will tend to stand at right angles to the wire. The stronger the current, the greater is the deflection of the needle. If the wire carrying the current is cut and the end dipped into a solution of silver nitrate,

silver will be deposited on the end of the wire toward which the current is flowing, and the amount of silver deposited in a given time will be directly proportional to the average strength of current flowing during that time. When the current flowing in a wire is spoken of, the strength of the current is meant.

The unit used to express the strength of a current is called the *ampere*. If a current of 1 amp. be sent through a bath of silver nitrate, .001118 g. per sec. of silver will be deposited. The expression of the flow of current through a wire as so many amperes is analogous to the expression of the flow of water through a pipe as so many gallons per second. If 1 amp. flows through a circuit for 1 sec., the quantity of electricity that has passed through the circuit during the 1 sec. is 1 coulomb; that is, 1 coulomb is equal to 1 amp. for 1 sec.

**Electromotive Force.**—In order that a current may flow through a wire, there must be an electric pressure of some kind to cause the flow. In hydraulics, there must always be a head or pressure before water can be made to flow through a pipe. It is also evident that there may be a pressure or head without there being any flow of water, because the opening in the pipe might be closed; the pressure will, however, exist, and, as soon as the valve closing the pipe is opened, the current will flow. In the same way, an electric pressure or electromotive force (often written E. M. F.) may exist in a circuit, but no current can flow until the circuit is closed or until the wire is connected so that there will be a path for the current.

The practical unit of electromotive force is the *volt*. It is the unit of electric pressure, and fulfils somewhat the same purpose as head of water and steam pressure in hydraulic and steam engineering. The electromotive force furnished by an ordinary cell of a battery usually varies from .7 to 2 volts. A Daniell cell gives an electromotive force of 1.072 volts. A pressure of 500 volts is generally used for street-railway work, and, for incandescent lighting, 110 volts is common.

**Resistance.**—All conductors offer more or less resistance to the flow of a current of electricity, just as water encounters friction in passing through a pipe. The amount of this resistance depends on the length of the wire, the diameter of the wire, and the material of which the wire is composed. The resistance of all metals also increases with the temperature.

The practical unit of resistance is the *ohm*. A conductor has a resistance of 1 ohm when the pressure required to set up 1 amp. through it is 1 volt. In other words, the *drop*, or fall, in pressure through a resistance of 1 ohm, when a current of 1 ampere is flowing, is 1 volt. 1,000 ft. of copper wire .1 in. in diameter has a resistance of nearly 1 ohm at ordinary temperatures.

**Ohm's Law.**—The law governing the flow of current in an electric circuit was first stated by Dr. G. S. Ohm, and is known as *Ohm's law*. It may be briefly stated as follows: *The strength of the current in any circuit is equal to the electromotive force divided by the resistance of the circuit.*

Let $E$ = electromotive force, in volts;
$R$ = resistance, in ohms;
$I$ = current, in amperes.

Then, $I = \dfrac{E}{R}$ $R = \dfrac{E}{I}$ $E = IR$

EXAMPLE 1.—A dynamo $D$ generating 110 volts, is connected to a coil of wire $C$ that has a resistance of 20 ohms; what current will flow, supposing the resistance of the rest of the circuit to be negligible?

SOLUTION.—As $E = 110$ volts and $R = 20$ ohms, by Ohm's law $I = 110 \div 20 = 5.5$ amp.

EXAMPLE 2.—If the resistance of the coil $C$ is 6 ohms, what electromotive force must the dynamo generate in order to set up a current of 15 amp. through it?

SOLUTION.—In this case the third formula will be used; that is, $E = 15 \times 6 = 90$ volts.

In case the current and electromotive force are known, the resistance of the circuit may be calculated by using the second formula.

EXAMPLE 3.—If the current in the previous examples were 8 amp. and the electromotive force of the dynamo 110 volts, what is the resistance of the circuit?

SOLUTION.— $R = 110 \div 8 = 13.75$ ohms.

**Electric Power.**—The electric power expended in any circuit is found by multiplying the current flowing in the circuit by the pressure required to force
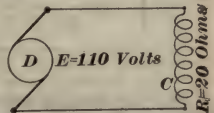
the current through the circuit. In other words, $W = EI$, where $W$ is the power expended, $E$ is the electromotive force and $I$ is the current. When $E$ is expressed in volts and $I$ in amperes, then $W$ is expressed in watts. The *watt* is the unit of electric power, and is equal to the power developed when 1 amp. flows under a pressure of 1 volt. The watt is equal to $\frac{1}{746}$ H. P.

Let
$$E = \text{electromotive force, in volts;}$$
$$I = \text{current, in amperes;}$$
$$R = \text{resistance, in ohms;}$$
$$W = \text{power, in watts;}$$
$$\text{H. P.} = \text{horsepower.}$$

Then,
$$W = EI = I^2 R = \frac{E^2}{R}$$

The energy used in forcing a current through the wire reappears in the form of heat; the heating effect of a current flowing in a conductor being proportional to the square of the current. Furthermore,

$$\text{H. P.} = \frac{EI}{746} = \frac{W}{746}$$

This relation is very useful for calculating power in terms of electric units. The watt is too small a unit for convenient use in many cases, so that the *kilowatt*, or 1,000 watts, is frequently used. This is sometimes abbreviated to K. W.

The unit of work is the *watt-hour*, which is the total work done when 1 watt is expended for 1 hr. For example, if a current of 1 amp. flows for 1 hr. through a resistance of 1 ohm, the total amount of work done is 1 watt-hour. A kilowatt-hour is the total work done when 1 K. W. is expended for 1 hr. It is about equivalent to $1\frac{1}{3}$ H. P. for 1 hr. The work done when 1 watt is expended for 1 sec. is called the *joule;* or 1 joule is expended in a circuit when 1 volt causes 1 amp. to pass through the circuit for 1 sec.

## ELECTRICAL EXPRESSIONS AND THEIR EQUIVALENTS

(*Arranged for Convenient Reference by C. W. Hunt*)

| 1 Watt | Rate of doing work | | 1 Watt-Hour | Quantity of work | |
|---|---|---|---|---|---|
| | 1. | amp. per sec. at 1 volt | | 2,654.28 | ft.-lb. |
| | .7373 | ft.-lb. per sec. | | .503 | mi.-lb. |
| | 44.238 | ft.-lb. per min. | | 1. | amp. hr.×1 volt |
| | 2,654.28 | ft.-lb. per hr. | | .00134 | H. P.-hr. |
| | .5027 | mi.-lb. per hr. | | $\frac{1}{746}$ | H. P.-hr. |
| | .00134 | H. P. | | | |
| | $\frac{1}{746}$ | H. P. | | | |
| 1 Kilowatt | Rate of doing work | | 1 Horsepower-Hour | Quantity of work | |
| | 737.3 | ft.-lb. per sec. | | 1,980,000. | ft.-lb. |
| | 44,238. | ft.-lb. per min. | | 375. | mi.-lb. |
| | 502.7 | mi.-lb. per hr. | | 746. | watt-hr. |
| | 1.34 | H. P. | | .746 | K. W.-hr. |
| 1 Horsepower | Rate of doing work | | 1 Ampere-Hour | Quantity of current | |
| | 550. | ft.-lb. per sec. | | 1 amp. flowing for 1 hr., irrespective of voltage | |
| | 33,000. | ft.-lb. per min. | | Watt-hour ÷ volts | |
| | 375. | mi.-lb. per hr. | | | |
| | 746. | watts | Torque | Force moving in a circle | |
| | .746 | K. W. | | Force of 1 lb. at a radius of 1 ft. | |

## CIRCUITS

The path through which a current flows is generally spoken of as an *electric circuit;* this path may be made up of a number of different parts. For example, the line wires may constitute part of the circuit, and the remainder may be composed of lamps, motors, resistances, etc. In practice, the two kinds of circuits most commonly met with are those in which the different parts of the circuit are connected in series and those in which they are connected in multiple or parallel.

**Series Circuits.**—In a series circuit, all the component parts are connected in tandem, so that the current flowing through one part also flows through the other parts; view (*a*) represents such a circuit made up of a different number

of parts. The current leaves the dynamo $D$ at the $+$ side and flows through the arc lamps $a$, thence through the incandescent lamps $l$, thence through the motor $m$ and resistance $r$, back to the dynamo, thus making a complete circuit. All these parts are here connected in series, so that the current flowing through each of the parts must be the same unless leakage takes place across from one side of the circuit to the other, and this is not appreciable if the lines are properly insulated. The pressure furnished by the dynamo must evidently be the sum of the pressures required to force the current through the different parts.

The most common use of this system is in connection with arc lamps, which are usually connected in series, as shown in (b). The objections to



(a)

(b)

(c)

(d)

*Trolley Wire*

250 Volts

*Rail*

(e)
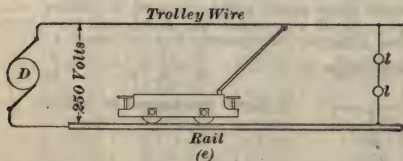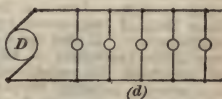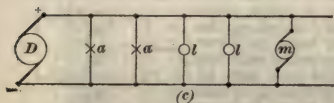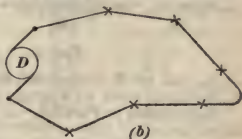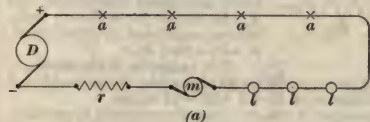
this system of distribution for general work are that the breaking of the circuit at any point cuts off the current from all parts of the circuit; also, the pressure generated by the dynamo must be very high if many pieces of apparatus are connected in series. In such a system, the dynamo is provided with an automatic regulator that increases or decreases the voltage of the machine, so that the current in the circuit is kept constant, no matter how many lamps or other devices are in operation. For this reason, such systems are often spoken of as constant-current circuits.

**Parallel Circuits.**—In a parallel circuit, the different pieces of apparatus are connected side by side, or in parallel, across the main wires from the dynamo as shown in (c). In this case, the dynamo $D$ supplies current through the mains to the arc lamps $a$, incandescent lamps $l$, and motor $m$. This system is more widely used, as the breaking of the circuit through any one piece of apparatus will not prevent the current from flowing through the other parts. Incandescent lamps are connected in this way almost exclusively. The lamps are connected directly across the mains, as shown in (d). Street cars and mining locomotives are operated in the same way, the trolley wire constituting one main and the track the other, as shown in (e). By adopting this system, any car can move independently of the others, and the current in each device may be turned off and on at will without affecting devices in other parallel circuits. In all these systems of parallel distribution, the pressure generated by the dynamo is maintained as constant as practicable, no matter what current the dynamo may be delivering. For example, in the lamp system, view (d), the dynamo will maintain a constant electromotive force of 110 volts. Each lamp has a fixed resistance, and will take a certain current ($110 \div R$ amperes) when connected across the mains. As the lamps are turned on, the current delivered by the dynamo increases, the pressure remaining constant. In street-railway work, the pressure between trolley and track is kept in the neighborhood of 500 volts, the current varying with the number of cars in operation. In mine-haulage plants, the pressure is usually 250 or 500 volts, the former being generally preferred as being less dangerous. Lamps may also be connected in series multiple, as shown in (e). Here the two 125-volt lamps $l$

are connected in series across the 250-volt circuit. Such an arrangement is frequently used in mines when lamps are operated from the haulage circuit.

Such circuits as those just described are called *constant-potential* or *constant-pressure* circuits, to distinguish them from the *constant-current* circuit mentioned previously.

## RESISTANCES IN SERIES AND MULTIPLE

**Resistance in Lines.**—If two or more resistances are connected in series, as in Fig. 1, their total combined resistance is equal to the sum of their separate resistances. If $R$ equals total combined resistance, and $R_1$, $R_2$, $R_3$ are the separate resistances connected in series, then,

$R_1$ $R_2$ $R_3$

Fig. 1

$$R = R_1 + R_2 + R_3.$$
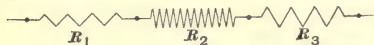
For example, if the separate resistances are $R_1 = 10$ ohms, $R_2 = 1$ ohm, and $R_3 = 30$ ohms, the three combined will be equivalent to a single resistance of $10 + 1 + 30 = 41$ ohms.

**Resistances in Parallel.**—If a number of resistances are connected in parallel, the reciprocal of their combined resistance is equal to the sum of the reciprocals of the separate resistances. In Fig. 2, three resistances are shown connected in parallel; therefore, the total resistance of such a combination must be lower than that of the lowest resistance entering into the combination. If the resistances in this case were all equal, the resistance of the three combined would be one-third the resistance of one of them, because a current passing through the three combined

$R_1$
$R_2$
$R_3$

Fig. 2

could split up equally between three equal paths, instead of having only one path to pass through. If $R$ represents the combined resistance, and $R_1$, $R_2$, $R_3$, and $R_4$, the separate resistances, the following relation is true for any number of resistances in parallel: $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4} +$ etc.
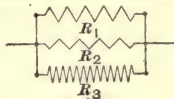
from which $R = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4} + \text{etc.}}$. If three resistances in parallel are

equal, $\dfrac{1}{R} = \dfrac{3}{R_1}$, or $R = \dfrac{R_1}{3}$.

EXAMPLE.—Three resistances of 3, 10, and 5 ohms are connected in parallel; what is their combined resistance?

SOLUTION.— $\dfrac{1}{R} = \dfrac{1}{3} + \dfrac{1}{10} + \dfrac{1}{5}$ or, $R = \dfrac{1}{\frac{10}{30} + \frac{3}{30} + \frac{6}{30}} = \dfrac{30}{19} = 1.58$ ohms.

If resistances in parallel are all equal, the current will divide equally through them; if not equal, the current in each path will be inversely proportional to the resistance of the path. In any case, the current through any path is equal to the difference of potential across the path divided by its resistance.

$A$

$B$
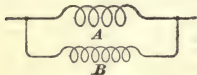
Fig. 3

**Shunt.**—When one circuit $B$, Fig. 3, is connected across another $A$, so as to form, as it were, a by-pass, or side track, for the current, such a circuit is called a *shunt*, or it is said to be in shunt with the other circuit

## ELECTRIC WIRING (CONDUCTORS)

**Materials.**—Practically all conductors used in electric lighting or power work are of copper, this metal being used on account of its low resistance. Iron wire is used to some extent for conductors in telegraph lines, and steel is largely used as the return conductor in electric-railway or haulage plants where the current is led back to the power station through the rails. The resistance of iron or steel varies from six to seven times that of copper, depending on the quality of the metal. Aluminum is coming into use in electric transmission. It is so much lighter than copper that it is able to compete with it as a conductor, even though its cost per pound is higher and its conductivity only about 60% that of copper.

## PROPERTIES OF ANNEALED COPPER WIRE; AMERICAN, OR BROWN & SHARPE, GAUGE

| Number B. & S. Gauge | Diameter, in Mils | Area in Circular Mils C. M. = $d^2$ | Weight Pounds | | Resistance per 1,000 Ft. International Ohms 68° F. | Current Capacity National Board Fire Underwriters Amperes | |
|---|---|---|---|---|---|---|---|
| | | | Per 1,000 Ft. | Per Mile | | Weather-Proof | Rubber-Covered |
| 0000 | 460.00 | 211,600.00 | 641.0000 | 3,380.000 | .0500 | 312 | 210 |
| 000 | 409.60 | 167,805.00 | 508.0000 | 2,680.000 | .0630 | 262 | 177 |
| 00 | 364.80 | 133,075.00 | 403.0000 | 2,130.000 | .0795 | 220 | 150 |
| 0 | 324.90 | 105,534.00 | 319.0000 | 1,690.000 | .1000 | 185 | 127 |
| 1 | 289.30 | 83,694.00 | 253.0000 | 1,340.000 | .1260 | 156 | 107 |
| 2 | 257.60 | 66,373.00 | 201.0000 | 1,060.000 | .1590 | 131 | 90 |
| 3 | 229.40 | 52,634.00 | 159.0000 | 841.000 | .2010 | 110 | 76 |
| 4 | 204.30 | 41,742.00 | 126.0000 | 667.000 | .2530 | 92 | 65 |
| 5 | 181.90 | 33,102.00 | 100.0000 | 529.000 | .3200 | 77 | 54 |
| 6 | 162.00 | 26,251.00 | 79.5000 | 420.000 | .4030 | 65 | 46 |
| 7 | 144.30 | 20,816.00 | 63.0000 | 333.000 | .5080 | | |
| 8 | 128.50 | 16,509.00 | 50.0000 | 264.000 | .6410 | 46 | 33 |
| 9 | 114.40 | 13,094.00 | 39.6000 | 209.000 | .8080 | | |
| 10 | 101.80 | 10,381.00 | 31.4000 | 166.000 | 1.0200 | 32 | 24 |
| 11 | 90.70 | 8,234.00 | 24.9000 | 132.000 | 1.2800 | | |
| 12 | 80.80 | 6,529.90 | 19.8000 | 104.000 | 1.6200 | 23 | 17 |
| 13 | 71.90 | 5,178.40 | 15.7000 | 82.800 | 2.0400 | | |
| 14 | 64.10 | 4,106.80 | 12.4000 | 76.200 | 2.5800 | 16 | 12 |
| 15 | 57.10 | 3,256.70 | 9.8600 | 52.000 | 3.2500 | | |
| 16 | 50.80 | 2,582.90 | 7.8200 | 41.300 | 4.0900 | 8 | 6 |
| 17 | 45.20 | 2,048.20 | 6.2000 | 32.700 | 5.1600 | | |
| 18 | 40.30 | 1,624.30 | 4.9200 | 25.900 | 6.5100 | 5 | 3 |
| 19 | 35.90 | 1,288.10 | 3.9000 | 20.600 | 8.2100 | | |
| 20 | 31.90 | 1,021.50 | 3.0900 | 16.300 | 10.4000 | | |
| 21 | 28.50 | 810.10 | 2.4500 | 12.900 | 13.1000 | | |
| 22 | 25.30 | 642.40 | 1.9400 | 10.300 | 16.5000 | | |
| 23 | 22.60 | 509.40 | 1.5400 | 8.100 | 20.8000 | | |
| 24 | 20.10 | 404.00 | 1.2200 | 6.400 | 26.2000 | | |
| 25 | 17.90 | 320.40 | .9700 | 5.100 | 33.0000 | | |
| 26 | 15.90 | 254.10 | .7690 | 4.000 | 41.6000 | | |
| 27 | 14.20 | 201.50 | .6100 | 3.200 | 52.5000 | | |
| 28 | 12.60 | 159.70 | .4840 | 2.500 | 66.2000 | | |
| 29 | 11.20 | 126.70 | .3840 | 2.000 | 83.5000 | | |
| 30 | 10.00 | 100.50 | .3040 | 1.600 | 105.0000 | | |
| 31 | 8.93 | 79.70 | .2410 | 1.270 | 133.0000 | | |
| 32 | 7.95 | 63.21 | .1910 | 1.010 | 167.0000 | | |
| 33 | 7.08 | 50.13 | .1520 | .801 | 211.0000 | | |
| 34 | 6.30 | 39.75 | .1200 | .635 | 266.0000 | | |
| 35 | 5.61 | 31.52 | .0954 | .504 | 336.0000 | | |
| 36 | 5.00 | 25.00 | .0757 | .400 | 423.0000 | | |
| 37 | 4.45 | 19.83 | .0600 | .317 | 533.0000 | | |
| 38 | 3.96 | 15.72 | .0476 | .251 | 673.0000 | | |
| 39 | 3.53 | 12.47 | .0377 | .199 | 848.0000 | | |
| 40 | 3.14 | 9.89 | .0299 | .158 | 1,070.0000 | | |

Most of the conductors used are in the form of copper wire of circular cross-section. Conductors of large cross-section are made up of a number of strands of smaller wire twisted together. For electrolytic plants, copper-refining plants, etc., copper bars of rectangular cross-section are frequently used. On account of the method used in supporting the larger sizes, large trolley wires are not always of circular cross-section.

**Wire Gauge.**—The gauge most generally used in America to designate the different sizes of copper wire is the American, or Brown & Sharpe (B. & S.). The sizes as given by this gauge range from No. 0000, the largest, .460 in. diameter, to No. 40, the finest, .003 in. diameter. Wire drawn to the sizes given by this gauge is always more readily obtained than sizes according to other gauges; hence, when selecting wire for any purpose it is always desirable, if possible, to give the size required as a wire of the B. & S. gauge. A wire can usually be selected from this gauge, which will be very nearly that required for any specified case.

The diameter of round wires is usually given in the tables in decimals of an inch and the so-called area of cross-section is given in terms of a unit called a circular mil. This is done simply for convenience, as it makes calculations involving the cross-section much simpler than if the square inch was used as the unit area. A *mil* is $\frac{1}{1000}$ in., or .001 in. A *circular mil* is the square of the diameter of a wire expressed in mils. A wire having a diameter of 1 in. has a sectional area of $.7854 \times 1^2 = .7854$ sq. in., and is said to have a sectional area of $1,000^2 = 1,000,000$ circular mils. Hence, *the area of cross-section of a wire in circular mils is equal to the square of its diameter expressed in mils.* CM is frequently used as an abbreviation for circular mils.

EXAMPLE.—A wire has a diameter of .101 in.; what is its area in circular mils?
SOLUTION.— .101 in. = 101 mils. Hence, $(101)^2 = 10,201$ CM.

### CARRYING CAPACITY OF COPPER CABLES

| Area Circular Mils | Current, in Amperes | | Area Circular Mils | Current, in Amperes | |
|---|---|---|---|---|---|
| | Exposed | Concealed | | Exposed | Concealed |
| 200,000 | 299 | 200 | 1,200,000 | 1,147 | 715 |
| 300,000 | 405 | 272 | 1,300,000 | 1,217 | 756 |
| 400,000 | 503 | 336 | 1,400,000 | 1,287 | 796 |
| 500,000 | 595 | 393 | 1,500,000 | 1,356 | 835 |
| 600,000 | 682 | 445 | 1,600,000 | 1,423 | 873 |
| 700,000 | 765 | 494 | 1,700,000 | 1,489 | 910 |
| 800,000 | 846 | 541 | 1,800,000 | 1,554 | 946 |
| 900,000 | 924 | 586 | 1,900,000 | 1,618 | 981 |
| 1,000,000 | 1,000 | 630 | 2,000,000 | 1,681 | 1,015 |
| 1,100,000 | 1,075 | 673 | | | |

### COMPARISON OF PROPERTIES OF ALUMINUM AND COPPER

| | Aluminum | Copper |
|---|---|---|
| Conductivity, for equal sizes........... | .54 to .63 | 1 |
| Weight, for equal sizes................ | .33 | 1 |
| Weight, for equal length and resistance.. | .48 | 1 |
| Price of bare wire, per pound, aluminum, 29 c.; copper, 16 c................ | 1.81 | 1 |
| Price, equal length and resistance, bare line wire........................ | .868 | 1 |
| Temperature coefficient, per degree F.... | .002138 | .002155 |
| Resistance of mil-foot, at 20° C......... | 18.73 | 10.5 |
| Specific gravity....................... | 2.5 to 2.68 | 8.89 to 8.93 |
| Breaking strength, equal sizes.......... | 1 | 1 |

The table on page 489 gives the dimensions, weight, and resistance of annealed copper wire. The weights given are, of course, for bare wire. The first column gives the B. & S. gauge number, the second the diameter in mils. The diameter in inches will be the number given in this column, divided by 1,000. The third column gives the area in circular mils, the numbers in this column being equal to the squares of those in the second column. The safe carrying capacity is also given. In case a conductor larger than that given in the table is required, stranded cables are used; these are made in various sizes. The first table on page 490 gives some of the more common sizes of stranded copper cables, with their allowable current capacity, while a comparison of the properties of aluminum and copper is given in the second table on page 490.

**Estimation of Resistance.**—The resistance of any conductor is directly proportional to its length, and inversely proportional to its area of cross-section, or $R = K \frac{L}{A}$, where $K$ is a constant. If $L$ is expressed in feet and $A$ is expressed in circular mils, the constant $K$ must be the resistance of 1 ft. of the wire in

### BREAKING STRENGTH OF COPPER AND ALUMINUM WIRES AND CABLES

| Wire Number B. & S. Gauge | Area Circular Mils | Breaking Strain, Pounds | | | |
|---|---|---|---|---|---|
| | | Copper, Solid | | Aluminum | |
| | | Annealed | Hard-Drawn | Solid | Stranded |
| 0000 | 211,600 | 5,650 | 8,310 | 4,320 | 6,830 |
| 000 | 167,805 | 4,475 | 6,580 | 3,430 | 5,420 |
| 00 | 133,079 | 3,550 | 5,226 | 2,720 | 4,290 |
| 0 | 105,592 | 2,800 | 4,558 | 2,150 | 3,410 |
| 1 | 83,695 | 2,225 | 3,746 | 1,710 | 2,700 |
| 2 | 66,373 | 1,775 | 3,127 | 1,355 | 2,143 |
| 3 | 52,634 | 1,400 | 2,480 | 1,075 | 1,700 |
| 4 | 41,743 | 1,115 | 1,967 | 852 | 1,350 |
| 5 | 33,102 | 885 | 1,519 | 657 | 1,070 |
| 6 | 26,251 | 700 | 1,237 | 536 | 850 |
| 7 | 20,817 | 550 | 980 | 426 | |
| 8 | 16,510 | 440 | 778 | 337 | |
| 9 | 13,094 | 350 | 617 | 267 | |
| 10 | 10,382 | 275 | 489 | 212 | |
| 11 | 8,234 | 220 | 388 | 167 | |
| 12 | 6,230 | 175 | 307 | 133 | |
| 13 | 5,178 | 135 | 244 | 105 | |
| 14 | 4,107 | 110 | 193 | 84 | |

question of 1 circular mil cross-section. The resistance of 1 mil-ft. of copper wire at 75° F. is about 10.8 ohms. Hence, for copper wire, $R = \frac{10.8L}{d^2}$, in which $d$ is the diameter in mils. This formula is easily remembered, and is very convenient for estimating the resistance of any length of wire of given diameter when a wire table is not at hand, or when the diameter of the given wire does not correspond to anything given in the table.

EXAMPLE.—Find the resistance of 1 mi. of copper wire .20 in. in diameter.
SOLUTION.—1 mi. = 5,280 ft.; .20 in. = 200 mils, or 40,000 C. M.; hence,
$$R = \frac{10.8 \times L}{d^2} = \frac{10.8 \times 5,280}{40,000} = 1.42 \text{ ohms}$$

The breaking strengths of copper and aluminum wires and cables are given in the preceding table. The ultimate strength of annealed copper is taken as 34,000 lb. per sq. in.; of hard-drawn copper, as 60,000 lb. per sq. in., except that for Nos. 000 and 00 it is taken at 50,000 lb., for No. 0 at 55,000 lb., and for No. 1 as 57,000 lb. The ultimate strength of aluminum is taken as 26,000 lb. per sq. in. The table gives the actual breaking strains to which a suitable factor of safety must be applied to give a safe working load.

# CALCULATION OF WIRES FOR ELECTRIC TRANSMISSION

**Direct-Current Circuits.**—No matter how large a wire may be, some energy must always be expended in forcing a current through it, because no conductor can be entirely devoid of resistance. It is true that the loss may be made as small as desired by using a very large conductor, but in practice this will not pay, because the interest on the cost of the copper will more than counterbalance the gain in the efficiency of transmission. In starting out, then, to estimate the size of wire to transmit a given amount of power over a given distance, one of the first things to be decided is the amount of power that may be allowed for loss in the line, because the greater the power lost the higher may be the line resistance, and hence the smaller the wire. The pressure required to force a current $I$ through a wire of a resistance $R$ is $I \times R$. This pressure is generally spoken of as the *drop*, for the reason that the pressure necessary to set up the current through the line is lost, and, consequently, the pressure falls off or drops from the dynamo to the receiving end of the line. In all cases, the pressure at the end of the line, or point where the power is delivered, is equal to the pressure at the dynamo less the drop in the line, and, conversely, the pressure that must be maintained by the dynamo in order to obtain a given pressure at the end of the line will be equal to the pressure at the receiving end plus the drop in the line.

This is shown in the accompanying illustration, where a dynamo $D$ supplies current to a motor $M$ situated 1 mi. distant. In order that the motor may operate properly, the pressure at its terminals must be kept constant at, say 500 volts; therefore, the pressure between $a$ and $b$ (the dynamo terminals) must be more than 500 volts by the drop or pressure necessary to force the current through the line. If the motor is taking very little current, that is, if it is running on a very light load, the current will be small, and hence the drop in the line will be small. In order, then, that the pressure at the motor may remain constant, or nearly so, the pressure at the dynamo must automatically increase as the load increases and the line must be designed with regard to the maximum current it has to carry. It will be supposed that the motor takes 50 amp. at full load and that the line wire has a resistance of .2 ohm per mi. The current has to pass through 2 mi. of wire (because it has to flow out through 1 mi. and back through 1 mi.), and hence encounters a resistance of .4 ohm. The drop in the line will then be $.4 \times 50 = 20$ volts, and in order to obtain a pressure of 500 volts at the motor, the pressure at the dynamo will have to be 520 volts. The loss of power in the line would be current $\times$ drop $= 50 \times 20 = 1,000$ watts, or about $1\frac{1}{3}$ H. P. The drop in an electric-transmission line is analogous to the loss in pressure due to the friction encountered by water flowing through a pipe line.
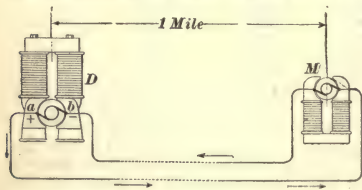


If, in the illustration just given, the wire had a resistance of .1 ohm per mi., the loss in the line would be halved, but the weight of copper required would have to be double the cross-section. The question as to whether it would pay better to invest more money in the line or to put up with the larger loss is something that must be determined in each case by the relative cost of power and copper.

In many cases, the loss allowed in the line is about 10% of the power to be delivered, though sometimes the loss may be allowed to run as high as 15% or 25%. This applies only to transmission lines. For local electric-light or power-distributing systems, the amount of drop allowed is usually about 2% for the former and 5% for the latter.

The problem of calculating line wires usually presents itself in the following form: Given, a certain amount of power to transmit over a known distance with a certain allowable loss, to determine the cross-section of the wire required.

Let   $P$ = power to be delivered, expressed in watts; $P$ will be equal to horsepower delivered at end of line multiplied by 746;

% = allowable percentage of loss in line, that is, percentage of power
delivered that may be lost in transmission;

$E$ = voltage at end of line where power is delivered;

$I$ = current at full load;

$L$ = length of wire through which current flows.

The cross-section of the copper conductor will then be given by the formula:

$$A = d^2 = \frac{10.8 \times L \times I \times 100}{E \times \%}$$

The circular mils will be $d^2$, and the corresponding size of wire may be found by consulting the wire table. It should be noticed, particularly, that in this formula, $L$ is the average length of conductor through which the current $I$ flows. The application of distance of transmission in the formula is shown in the following example:

EXAMPLE.—A mine pump, driven by an electric motor, is situated 2 mi. from the power station. The electric input of the motor at full load is 50 H. P., and the voltage at its terminals is to be 500. Estimate the size of line wire necessary to supply the motor, the allowable loss in the line being 15% of the power delivered.

SOLUTION.—The actual length of line through which the current will flow will be 4 mi., because the current has to flow out to the motor and back again. Then

$$I = \frac{watts}{E} = \frac{50 \times 746}{500} = 74.6 \text{ amp.}$$

Applying the formula,

$$A = d^2 = \frac{10.8 \times 2 \times 2 \times 5,280 \times 74.6 \times 100}{500 \times 15} = 226,880 \text{ C. M., nearly}$$

By consulting the wire table it is found that this calls for a wire a little larger than No. 0000, which has a cross-section of 211,600 C. M.; No. 0000 wire would probably be used in this case, as it is near enough to the calculated size for all practical purposes. In case the calculated size comes out larger than any size given in the table, a number of wires may be used in multiple to make up the required cross-section, or, what is better, a stranded cable may be used. These heavy stranded cables may now be obtained in different sizes, up to 2,000,000 C. M. cross-section.

If it were allowable to waste twice as much power in the line, or what is equivalent to having a line drop of 150 volts instead of 75 volts, the cross-section of wire required would be one-half of that just found. Such a large amount of loss would, however, be objectionable unless power was very cheap. A large drop in the line is in any case objectionable, because the voltage at the receiving end of the circuit will fall off greatly unless the voltage at the generating station is raised as the load comes on in order to compensate for the line drop. Most of the uses to which electricity is put, in mines or other places, require that the pressure at the point where the power is utilized shall be kept approximately constant. For example, in the case of incandescent lights, the lamps will fall off greatly in brightness if the pressure decreases even by a comparatively slight amount. Also, if motors are being operated, the speed will vary considerably if the pressure is not kept constant, and it may be stated, in general, that a large line loss tends to poor regulation at the end of the circuit where power is delivered.

From these considerations, it is evident that the size of wire to be used under given conditions is determined by the allowable amount of drop. In some cases, however, especially if the current is to be used near at hand, the size of wire so determined might not be large enough to carry the current without overheating. Of course, in such cases, the safe carrying capacity of the wire determines the size to be used, and the drop will be correspondingly less. The amount of current that a given wire can carry without overheating depends very largely on the location of the wire. For example, a wire strung in the open air will carry a greater current, with a given temperature rise, than the same wire when boxed up in a molding or conduit.

In order to keep down the size of wire required to transmit a given amount of power over a given distance, with a certain allowable loss, the current must be kept as small as possible. Now, for a given amount of power, the current can only be made small by increasing the pressure, because the number of watts, or power delivered, is equal to the product of the current and the pressure. As a matter of fact, if the pressure in any given case is doubled, the amount of copper required will be only one-fourth as great; in other words, for a given

amount of power transmitted, the weight of copper required decreases as the square of the voltage. It is at once seen, then, that if any considerable amount of power is to be transmitted over long distances, a high line pressure must be used or else the cost of copper becomes prohibitive. The use of high pressures in power transmission will be taken up in connection with alternating currents.

**Insulated Wires.**—For most overhead line work using modern voltages, weather-proof insulated wire is used. This wire is covered with two or three braids of cotton, and treated with insulating compound. For inside work, and in places where a better quality of insulation is required, rubber-covered wires are used. The accompanying table gives the approximate weight of weather-proof line wire. The cost of the wire per pound varies considerably, owing to variations in the price of copper.

### WEATHER-PROOF LINE WIRE (ROEBLING'S)

| Number B. & S. Gauge | Double Braid | | | Triple Braid | | |
| | Outside Diameter 32ds In. | Weight, in Pounds | | Outside Diameter 32ds In. | Weight, in Pounds | |
| | | Per 1,000 Ft. | Per Mile | | Per 1,000 Ft. | Per Mile |
| 0000 | 20 | 716 | 3,781 | 24 | 775 | 4,092 |
| 000 | 18 | 575 | 3,036 | 22 | 630 | 3,326 |
| 00 | 17 | 465 | 2,455 | 18 | 490 | 2,587 |
| 0 | 16 | 375 | 1,980 | 17 | 400 | 2,112 |
| 1 | 15 | 285 | 1,505 | 16 | 306 | 1,616 |
| 2 | 14 | 245 | 1,294 | 15 | 268 | 1,415 |
| 3 | 13 | 190 | 1,003 | 14 | 210 | 1,109 |
| 4 | 11 | 152 | 803 | 12 | 164 | 866 |
| 5 | 10 | 120 | 634 | 11 | 145 | 766 |
| 6 | 9 | 98 | 518 | 10 | 112 | 591 |
| 8 | 8 | 66 | 349 | 9 | 78 | 412 |
| 10 | 7 | 45 | 238 | 8 | 55 | 290 |
| 12 | 6 | 30 | 158 | 7 | 35 | 185 |
| 14 | 5 | 20 | 106 | 6 | 26 | 137 |
| 16 | 4 | 14 | 74 | 5 | 20 | 106 |
| 18 | 3 | 10 | 53 | 4 | 16 | 85 |

For high-tension lines, it is customary to use bare wires and insulate them thoroughly on special porcelain insulators. The ordinary weather-proof wire insulation is of little or no use as a protection when these high pressures are used, and it only makes the line more dangerous because of the false appearance of security that it gives. In many cases, it is also better to use bare feeders for mine-haulage plants, because the ordinary insulation soon becomes defective in a mine, and a wire in this condition is really more dangerous than a bare wire, because the latter is known to be dangerous and will be left alone.

## CURRENT ESTIMATES



FIG. 1
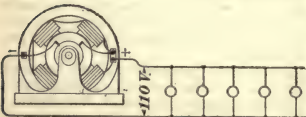
**Incandescent Lamps.**— Before calculating the size of wire required for any given case, it is necessary to know the current, and the method of getting at this will depend on what the current is to be used for. Incandescent lamps are usually operated on 110-volt circuits, as shown in Fig. 1, or on the three-wire system, as shown in Fig. 2. In the three-wire system, two 110-volt dynamos are connected in series so that the voltage across the outside wires is 220. The neutral wire *a* connects at

the point $b$ where the machines are connected together. The wire $a$ merely serves to carry the difference in the currents on the two sides of the system, in case more lamps should be burning on one side than on the other. The outside wires for such a system are calculated as if the lights were operated two in series across 220 volts. The middle wire is usually made equal in size to the outer wires. An ordinary 16-c. p. incandescent carbon-filament lamp requires about 55 watts for its operation; a 32-c. p. lamp requires about 110 watts. Tungsten lamps are designated by the watts required rather than by the candlepower light they give. Thus, a 25-watt tungsten lamp gives about 20 c. p. Hence, in the case of ordinary parallel distribution, as shown in Fig. 1, the dynamo will deliver about $\frac{1}{2}$ amp. for each 16-c. p. carbon lamp operated, and 1 amp. for each 32-c. p. carbon lamp. In the case of the three-wire system, each pair of 16-c. p. carbon lamps will take $\frac{1}{2}$ amp., and the total number of amperes in the outside wires will be one-fourth the number of lamps operated.
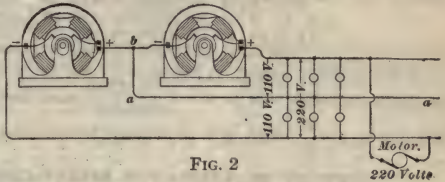
FIG. 2

EXAMPLE 1.—A certain part of a mine is to be illuminated by fifty 16-c. p. carbon lamps and ten 32-c. p. carbon lamps. This portion of the mine is 1,000 ft. from the dynamo room, and the allowable drop in pressure is 5%. The lamps are to be run on a 110-volt system. Find the size of wire required.

SOLUTION.—Fifty 16-c. p. carbon lamps require .... 25 amperes
Ten 32-c. p. carbon lamps require ..... 10 amperes

Total current..................... 35 amperes

$$\frac{10.8 \times 1,000 \times 2 \times 35 \times 100}{110 \times 5} = 137,454 \text{ C. M.,}$$

or about a No. 00 B. & S. wire.

EXAMPLE 2.—Take the same case, but suppose the lights to be operated on the three-wire system.

SOLUTION.—There will then be twenty-five 16-c. p. carbon lamps and five 32-c. p. carbon lamps on each side of the circuit, and the total current in the outside wires will be 17.5 amp. The voltage between the outside wires will be 220, and

$$\frac{10.8 \times 1,000 \times 2 \times 17.5 \times 100}{220 \times 5} = 34,363 \text{ C. M.,}$$

or about a No. 5 B. & S. wire.

If the central wire is made also of this size, this system would require three-eighths the amount of copper called for by the plain 110-volt system. There is the disadvantage that two dynamos are needed.

NOTE.—The length to be used in the wiring formula is the average distance traversed by the current in the conductor. For example, if, as in Fig. 3 (a), the lamps were all grouped or bunched at the end of the line, the length used in the formula would be twice that from $G$ to $A$, because the whole current has to flow out to $A$ through one main and back through the other. In other words, the whole current here passes through the whole length of the line. In case the load is uniformly distributed all along the line, as shown in (b), the current decreases step by step from the dynamo to the end. In such a case, the length or distance to be used in the formula is one-half that used in the former case, or simply the distance from the dynamo to the end, instead of twice this distance.

**Arc Lamps.**—Arc lamps are frequently run on constant-potential circuits, and usually consume from 400 to 500 watts. There are so many types of these lamps that it is difficult to give any current estimates that will be generally applicable. Enclosed arc lamps usually take from 3 to 5 amp. when run on 110-volt circuits.

**Motors.**—Practically all the motors used in mining work are run on the constant-potential system, either at 250 or 500 volts. The efficiency of ordinary motors will vary from 65% to 94%, depending on the size. The efficiency is greater with the larger machines, and, for the ordinary run of motors, it will

probably lie between 80% and 90%. By efficiency is here meant the ratio of the useful output at the pulley or pinion of the motor to the total input. The following table gives the efficiency of motors of ordinary size:

*Approximate Motor Efficiency*

$\frac{3}{4}$ to $1\frac{1}{2}$ H. P., inclusive = 70–75% efficiency
3 to 5 H. P., inclusive = 75–80% efficiency
$7\frac{1}{2}$ to 10 H. P., inclusive = 80–85% efficiency
15 H. P. and upwards = 85–90% efficiency

If the required output, in horsepower, of a direct-current motor is known, the input, in watts, will be $W = \dfrac{\text{H. P.} \times 746}{\text{efficiency}}$, and the current required at full load will be $I = \dfrac{W}{E}$, where $E$ is the voltage between the mains at the motor.

### CURRENT REQUIRED FOR DIRECT-CURRENT MOTORS

| Horse-power of Motor | Effi-ciency* Per Cent. | Watts Input | Amperes | | | | |
|---|---|---|---|---|---|---|---|
| | | | 110 Volts | 220 Volts | 250 Volts | 500 Volts | 550 Volts |
| 1 | 65 | 1,148 | 10.4 | 5.2 | 4.58 | 2.29 | 2.08 |
| 2 | 65 | 2,295 | 20.8 | 10.4 | 9.16 | 4.58 | 4.16 |
| 2½ | 65 | 2,870 | 26.0 | 13.0 | 11.45 | 5.72 | 5.21 |
| 3½ | 75 | 3,481 | 31.6 | 15.8 | 13.90 | 6.90 | 6.32 |
| 5 | 75 | 4,973 | 45.1 | 22.6 | 19.90 | 9.95 | 9.04 |
| 7½ | 80 | 6,994 | 63.5 | 31.7 | 27.90 | 13.95 | 12.70 |
| 10 | 80 | 9,325 | 84.6 | 42.3 | 37.20 | 18.60 | 16.90 |
| 15 | 85 | 13,165 | 119.8 | 59.9 | 52.70 | 26.40 | 23.90 |
| 20 | 85 | 17,553 | 159.6 | 79.8 | 70.20 | 35.10 | 31.90 |
| 25 | 90 | 20,770 | 189.0 | 94.5 | 83.10 | 41.60 | 37.80 |
| 30 | 90 | 24,864 | 225.8 | 112.9 | 99.40 | 49.70 | 45.20 |
| 40 | 90 | 33,232 | 302.6 | 151.3 | 133.00 | 66.50 | 60.50 |
| 50 | 90 | 41,540 | 378.0 | 189.0 | 166.20 | 83.10 | 75.60 |
| 75 | 90 | 62,310 | 567.0 | 283.5 | 249.30 | 124.80 | 113.40 |
| 100 | 93 | 80,215 | 729.0 | 364.5 | 320.50 | 160.30 | 145.70 |
| 125 | 93 | 100,269 | 912.0 | 456.0 | 401.00 | 200.50 | 182.30 |
| 150 | 93 | 120,322 | 1,094.0 | 547.0 | 481.00 | 240.70 | 219.00 |
| 200 | 94 | 158,510 | 1,442.0 | 721.0 | 634.00 | 317.00 | 288.00 |
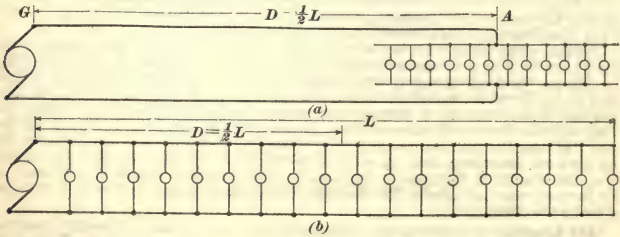


Fig. 3

**Conductors for Electric-Haulage Plants.**—In electric-haulage plants, the rails take the place of one of the conductors, so that, in calculating the size of feeders required, only the overhead conductors are taken into account. It

*Efficiencies are taken arbitrarily; a variation in these percentages will make proportionate changes in watts and amperes.

is a difficult matter to assign any definite value to the resistance of the track circuit, as it depends very largely on the quality of the rail bonding at the joints. If this bonding is well done, the resistance of the return circuit should be very low, because the cross-section of the rails is comparatively large. For calculating the supply feeders, the following approximate formula may be used:

$$\text{circular mils} = \frac{14 \times L \times I \times 100}{E \times \% \text{ drop}}$$

In this case, $L$ is the average length of feeder over which the power is to be transmitted. It will be noticed that the constant 10.8 appearing in the previous formulas has here been increased to 14. This has been done to allow, approximately, for the track resistance, but this constant might vary considerably depending on the quality of the rail bonding. If the load is all bunched at the end of the feeder, $L$ is the actual length of the feeder in feet. If the load is uniformly distributed, as it would be if a number of locomotives were continually moving along the line, the distance $L$ in the formula will be taken as one-half that used in the case where the load is bunched at the end. In other words, the whole current $I$ will only flow through an average of one-half the length of the line.
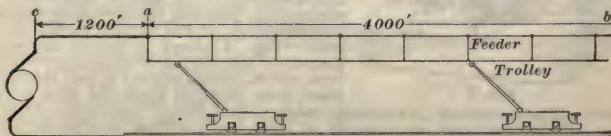


Fig. 4

EXAMPLE.—In Fig. 4, $ab$ represents a section of track 4,000 ft. long. From the dynamo $c$ to the beginning of the section, the distance is 1,200 ft. The trolley wire is No. 00 B. & S., and is fed from the feeder at regular intervals. Two mining locomotives are operated, each of which takes an average current of 75 amperes. The total allowable drop to the end of the line is 5% of the terminal voltage, which is 500 volts. Calculate the size of feeder required, assuming that the constant 14, in the formula, takes account of the resistance of the return circuit.

SOLUTION.—Since the locomotives are moving from place to place, the center of distribution for the load may be taken at the center of the 4,000 ft. The distance $L$ will then be $1,200 + 2,000 = 3,200$ ft. The total current will be 150 amperes; hence, we have

$$\text{circular mils} = \frac{14 \times 3,200 \times 150 \times 100}{500 \times 5} = 268,800.$$

This would require either a stranded cable or the use of two No. 00 wires in parallel from $c$ to $a$. From $a$ to $b$ we have the No. 00 trolley wire in parallel with the feeder; hence, the section of feeder $ab$ may be a single No. 00 wire. In many cases, the drop is allowed to run as high as 10%, because the loads are usually heavier, and the distances longer, than in the example given above.

## DYNAMOS AND MOTORS

### DIRECT-CURRENT DYNAMOS

A *dynamo*, or *generator*, is a machine for converting mechanical energy into electrical energy by moving conductors relatively to a magnetic field. An *electric motor* is a machine for converting electrical energy into mechanical energy by the relative motion between conductors carrying a current and a magnetic field. In the case of a dynamo a number of conductors are made to move across a magnetic field by means of a steam engine or other prime mover, and the result is that an electromotive force is set up in the conductors, and this electromotive force will set up a current if the circuit is closed.

In the case of a motor, a number of conductors are arranged so that they are free to move across a magnetic field, and a current is sent through these conductors from some source of electric current. The current flowing through these conductors reacts on the magnetic field and causes the conductors to move, thus converting the electrical energy delivered to the motor into mechanical

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energy. As far as mechanical construction goes, dynamos and motors are almost identical, and the operation of the motor is about the reverse of that of the dynamo.

Dynamos and motors may be divided into two general classes: Dynamos and motors for direct current; dynamos and motors for alternating current.



FIG. 1

*Direct-current dynamos* are those that furnish a current that always flows in the same direction. This kind of dynamo is largely used for incandescent lighting, and also for the operation of street railways. A dynamo generates an electromotive force by the motion of conductors across a magnetic field; hence, there must be a magnet of some kind to set up a magnetic field, known as the *field magnet*, or simply as the *field;* and a series of conductors arranged so that they may be moved or revolved in the magnetic field, known as the *armature.* The field is supplied by means of a powerful electromagnet which is magnetized by the current in the field coils. Fig. 1 shows a typical six-pole magnet of this kind; *B* are the magnetizing coils, which, when a current is sent through them, form powerful magnetic poles at *N* and *S.* The framework *A* is usually made of cast iron or cast steel. These field magnets may have any number of poles, but machines of ordinary size are usually provided with from two to eight poles.

The armature usually consists of a number of turns of insulated copper wire, arranged around the periphery of a ring or drum built up of soft iron sheets. Fig. 2 shows the construction of a typical armature of the ring type. The winding is divided into a number of sections, and the terminals connected to the *commutator.*

This commutator consists of a number of copper bars, insulated from each other by means of mica, the bundle of bars being clamped firmly into place



COMPLETE ARMATURE.
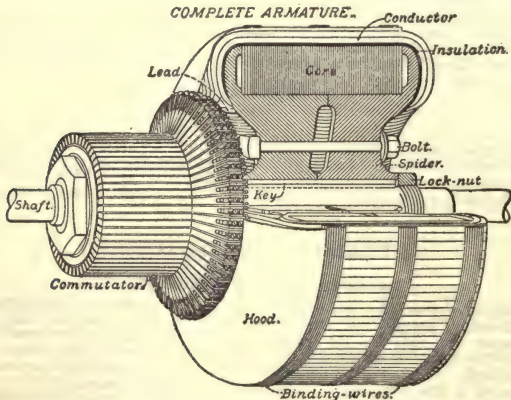
FIG. 2

and turned up to form a true cylindrical surface. The sections in the commutator correspond with those in the armature, and the use and operation of the commutator will be described later. The winding on the ring is endless, that is, it consists of a number of coils or sections *c*, Fig. 3, the end of one section

being joined to the beginning of the next, thus forming an endless coil. The construction of such a ring armature would be as shown in Fig. 2.

Suppose that the ring shown in Fig. 3 with its endless winding is rotated between the poles of a two-pole field magnet. Magnetic lines will flow from the N pole of the field magnet across through the iron core of the armature and enter the S pole on the other side. As all the conductors on the right-hand face of the ring are moving upwards, they will have an electromotive force generated in them in one direction, while the conductors on the left side will have an electromotive force in the opposite direction, because all the conductors on this side are moving downwards, or in the opposite direction, to those on the other side. These two opposing electromotive forces may be said to start at $a'$ and to meet at $a$, as shown by the arrowheads on the conductors, and will neu-



FIG. 3

tralize each other so that no current will flow through the windings of the armature. Suppose, however, that taps are connected at the points $a$ and $a'$, as shown by the dotted lines, and these taps connected to two conducting metal rings $r$ and $r'$, mounted so as to revolve with the armature. By allowing brushes $b$ and $b'$ to press on these rings, connection can be made with an outside circuit $d$, which may consist of a number of lamps or any other device through which it is desired to send a current. By putting in the taps at $a$ and $a'$, the two opposing electromotive forces may be set up a current through the common connections to the rings, and thence through the outside circuit. Current now flows in each half of the armature winding, unites at $a$, flows out by means of ring $r'$ and brush $b'$, thence through the outside circuit $d$ to brush $b$ and ring $r$, from whence it passes to $a'$, and thus completes the circuit. When the ring makes a half revolution from the position in the figure, the current in the outside circuit will flow in the opposite direction. In fact, an arrangement of this kind will deliver a current that will be periodically reversing in the outside circuit; it will be what is known as an *alternating current*.
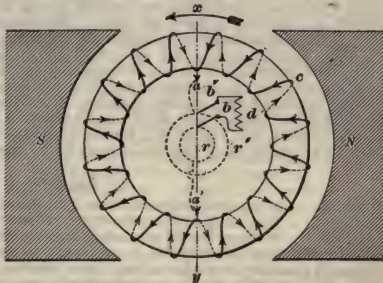
Instead of simply bringing out two terminals to rings, suppose the winding to be tapped at a fairly large number of points, and connections brought down



FIG. 4

to a number of metal bars insulated from one another, as shown in Fig. 4. When the armature is revolved the brushes will come in contact with successive bars and keep the outside circuit in such relation to the armature winding that the current will always flow through it in the same direction. Moreover, if the number of divisions in the armature is large, the current will fluctuate very little, being nearly as steady as that obtained from a battery. The arrangement made up of insulated bars is called the commutator, because it commutes or changes the relation of the outside circuit to the armature winding so that the current in the outside circuit

always flows in the same direction. All practical machines used for the generation of direct current must be provided with such a commutator. When alternating currents are used it is only necessary to use plain collector rings, as shown in Fig. 3.
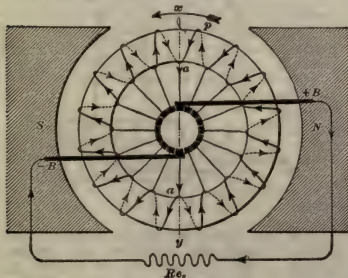
**Factors Determining Electromotive Force Generated.**—A dynamo should be looked upon as a machine for maintaining an electric pressure rather than as a machine for generating a current. A pump does not manufacture water—it merely maintains a head or pressure that causes water to flow wherever an outlet is provided for it to flow through. In the same way, a dynamo maintains a pressure, and this pressure will set up a current whenever the circuit is closed, so that the current can flow. The important thing to consider, therefore, is the electromotive force that the dynamo is capable of generating.

The electromotive force generated by an armature depends on the total number of magnetic lines cut through per second by the armature conductors. This means that (1) the faster the armature runs, the higher will be the electromotive force; (2) the greater number of conductors or turns there are on the armature, the higher will be the electromotive force; and (3) the stronger the magnetic field, the higher will be the electromotive force. The electromotive force in terms of these quantities may be written

$$E = \frac{nCN}{100,000,000}$$

in which    $n$ = speed, in revolutions per second;
            $C$ = number of conductors on face of armature;
            $N$ = number of magnetic lines flowing from one pole.

The constant 100,000,000 is necessary to reduce the result to volts. This makes it possible to make calculations relating to any two-pole dynamo, and with slight modification it is applicable to machines with field magnets having a number of poles.

**Field Excitation of Dynamos.**—In the earliest form of dynamo, the magnetic field in which the armature rotated was set up by means of permanent magnets. Permanent magnets are, however, very weak compared with electromagnets, which are excited by means of current flowing around coils of wire wound on a soft-iron core. As soon as the current ceases flowing around the coils of an electromagnet, the magnetism almost wholly disappears, but a small amount, known as the *residual magnetism*, remains. It is to this residual magnetism that the dynamo owes its ability to start up of its own accord and excite its own field magnets. When the armature is first started to revolve, a very feeble electromotive force is generated in it, but the armature is connected to the field coils in such a way that this small electromotive force is able to force a small current through the field coils, and thus set up a larger amount of magnetism in the field. This in turn increases the electromotive force in the armature, and the building-up process goes on rapidly until the dynamo generates its full pressure. There are three methods in use for supplying the field coils with current, and continuous-current dynamos are divided into three classes, according to the method used for exciting their fields. These three classes are: Series-wound dynamos, shunt-wound dynamos, and compound-wound dynamos.
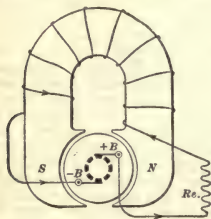


FIG. 5
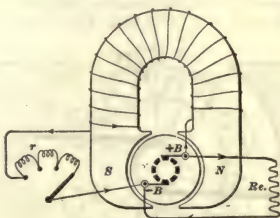


FIG. 6

In *series-wound dynamos* the field coils are connected in series with the armature, and all the current that passes through the armature also passes through the field and the outside circuit. This arrangement is shown in Fig. 5, where $N$ and $S$ represent the poles of the magnet, $+B$ and $-B$ the brushes, and $Re$ the outside circuit, which may consist of lamps, motors, or any other device in which it is desired to utilize the current. With an arrangement of this kind, and as long as the speed does not change, the electromotive force will increase as the current increases, because the field will become stronger. This will be true up to the point where the field carries all the

magnetism it is capable of, or, in other words, until it becomes saturated. After this point is reached, the electromotive force will increase very little with increase of current. In most of the work connected with lighting or power transmission, it is desirable to have the voltage remain nearly constant. For this reason, therefore, the series method of excitation has not been very largely used for dynamos. The only style of generator to which it has been applied at all generally is the arc-light dynamo, and these machines are provided with an automatic regulator of some kind to vary the voltage as desired. The series field winding has, however, been largely used in connection with the motors operated on constant-pressure circuits.

*Shunt-wound dynamos* have not been used largely of late years, although they were formerly very common. In this method of excitation, the field is connected as a shunt or by-pass to the armature; that is, the field winding is connected in parallel with the armature. This winding consists of a large number of turns of fine wire, so that its resistance is high and only a small part of the total current flows through it. Fig. 6 shows the connections for this kind of field excitation. An adjustable resistance $r$ is usually inserted in the field circuit, and by cutting this resistance in or out, the field may be weakened or strengthened and the voltage varied accordingly. With this type of machine, the current through the field does not vary greatly from no load to full load, and if the dynamo is well designed, the pressure at the brushes will keep approximately constant. The pressure will, however, always fall off more or less as the amount of current produced increases on account of the drop in the armature, due to its resistance, and also because of the tendency that the current in the armature has of weakening the field. The shunt winding is used quite largely for motors.

The *compound-wound dynamo* is the one most largely used for direct-current power and light distribution, and it is so called because the winding used for exciting the field is a combination of the series and shunt windings. The series winding serves to keep up the field strength while the load is increased, and thus keeps the pressure constant, or even makes it rise with increased load, if so desired. When the series winding is so adjusted that the pressure rises as the load is increased, the machine is said to be *overcompounded*. Fig. 7 shows the connections for such a machine. It will be seen that the shunt winding is connected as before, a field resistance or rheostat, not shown in the figure, being inserted for the purpose of adjusting the voltage. One brush connects directly to one terminal of the machine $+T$, while the other brush connects to one end of the series winding on the field. The other end of the series winding forms the other terminal $-T$, to



FIG. 7

which the outside circuit $Re$ is connected. By this arrangement, the shunt coil supplies a certain amount of initial magnetization that is augmented by the magnetism supplied by the series coils. Care must be taken to see that the current in the series coils circulates around the field in the same direction as that in the shunt coils, otherwise the electromotive force will fall off with an increasing load instead of keeping it up.
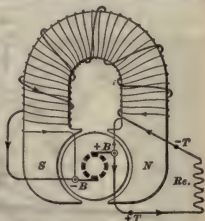
## DIRECT-CURRENT MOTORS

*Direct-current motors*, so far as their fundamental construction goes, are almost identical with direct-current dynamos. But as motors are often required to operate under very trying conditions, as in mine haulage or pumping plants or on the ordinary street car, the mechanical construction must be so designed as to enclose the working parts as completely as possible, and thus protect them from dirt and injury. The two kinds of motors most commonly used are the series and shunt varieties; compound-wound motors are only used for a few special kinds of work. Practically all the motors in use are operated from constant-pressure mains; that is, the pressure at the terminals of the motor is practically constant, no matter what load it may be carrying. Only constant-potential motors will be considered here.

**Principles of Operation.**—If the fields of an ordinary constant-potential dynamo are excited and a current supplied to the armature from some outside source, such as another dynamo $D$, Fig. 1, so that the current enters at $+B$, and passing through the winding in the direction indicated by the arrowheads, leaves at brush $-B$, all the conductors under the $S$ pole face, $b$, $c$, $d$, $e$, $f$, and $g$,

will tend to move *downwards*, and all those under the N pole face, j, k, l, m, n, and o, will tend to move *upwards*, as indicated by the small arrows.

These forces combine to produce a tendency of the armature to rotate about its axis as indicated by the large arrows, which tendency is called the *torque* of the motor.

The amount of this torque—which is usually expressed in pound-feet; that is, a certain number of pounds acting at a radius of a certain number (usually 1) of feet—depends on the strength of the field, the number of conductors, their mean distance from the axis of the armature, and the amperes in each conductor. In any given machine, the second and third conditions are constant, so that the torque depends on the strength of the field and the current.

If the armature is stationary, the electromotive force required to send the current through the winding is only that necessary to overcome the drop, which is due to the resistance of the winding. If the torque exerted by this current is greater than the opposition to motion, so that it causes the armature to revolve, the motion of the conductors through the field generates in them an electromotive force that is opposed to the electromotive force that is sending the current through the armature. This opposing electromotive force or *counter electromotive force* as it is called, then diminishes the effect of the applied electromotive force, so that the current is reduced, thereby reducing the torque. Should the torque still be greater than the opposition to motion, the speed of the armature will continue to increase, increasing the counter electromotive force, and thereby further reducing the current and the corresponding torque until the torque just balances the opposition to the motion, when the speed will remain constant.

At all times, the drop of potential through the armature is equal to the difference between the counter and the applied electromotive forces, and as the product of this drop and the current represents energy wasted, it is desirable
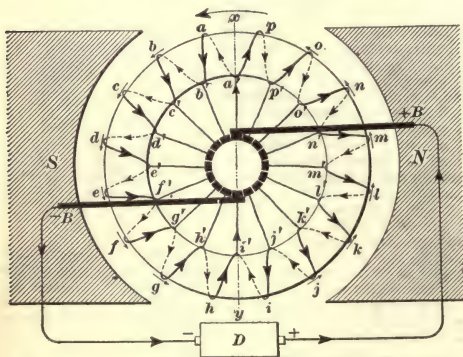


to make it as low as possible. In good motors of about 10 H. P. output, the drop in the armature is seldom more than about 5% of the applied electromotive force, and is less in larger machines. This being the case, it is evident that if the armature is at rest, so that it has no counter electromotive force, and is connected directly to the mains, a very large current will flow through it, which would be liable to damage the armature. On this account an external

Fig. 1

resistance, called a *starting resistance*, is connected in series with the armature when it is to be started. This resistance is made great enough to prevent more than about the normal current from flowing through the armature when it is at rest; as the armature speeds up and develops some counter electromotive force, this resistance is gradually cut out, until the armature is connected directly to the mains, and is running at normal speed.

The energy represented by the product of drop in the armature and the current is wasted; that represented by the product of the current and the rest of the electromotive force, that is, the counter electromotive force, is the energy required to keep the armature in motion.

Aside from the comparatively small amount of current required to furnish the torque necessary for overcoming the frictional losses in the motor itself, which are practically constant, the amount of current taken from the mains is directly proportional to, and varies automatically with, the amount of the external load; for, if this external load is increased, the current that has been

flowing in the armature cannot furnish sufficient torque for this increased load, so that the machine slows down. This decreases the counter electromotive force, which immediately allows more current to flow through the armature, increasing the torque to the proper amount. If the external load is decreased, the current flowing furnishes an excess of torque, which causes the speed to increase, increasing the counter electromotive force, and decreasing the current until it again furnishes only the required amount of torque.

As the counter electromotive force is very nearly equal to the applied, it is only necessary for it to vary a small amount to vary the current within wide limits. For example, if the resistance of a certain armature is 1 ohm, and it is supplied with current at a constant potential of 250 volts, then, when a current of 10 amp. is flowing through it, the drop is $10 \times 1 = 10$ volts, and the counter electromotive force is $250 - 10 = 240$ volts. Now, if the current is reduced to 1 amp., the drop is $1 \times 1 = 1$ volt, and the counter electromotive force is $250 - 1 = 249$ **volts; that is,** the counter electromotive force only varies $\frac{9}{240}$ or 3.75%, while the current varies $\frac{9}{10}$, or 90%.

As stated before, the field magnets of constant-potential motors are usually either shunt-wound or series-wound. If shunt-wound and supplied from a constant-potential circuit, the magnetizing force of the field coils is constant, giving a practically constant field. This being the case, the counter electromotive force is directly proportional to the speed, so that variations of the load make only slight variation in the speed. A shunt-wound motor is then (practically) a constant-speed motor.

With series-wound motors, the strength of the field varies with the current; if the load on such a motor is reduced, the excess of torque makes the armature speed up, but the increasing counter electromotive force produces a decreasing current, thereby decreasing the field strength, and, consequently, the armature must speed up to a much greater extent, in order to increase the counter electromotive force to the right degree, than would be necessary if the field were constant. If the load is increased, the increase in the current so increases the field strength that the speed must decrease considerably, in order to decrease the counter electromotive force by the right amount. The speed of a series-wound motor, then, varies largely with variations in the load.

An advantage of the series motor is that if a torque greater than the normal is required, it can be obtained with less current than with a shunt motor, since the increased current increases the field strength, which remains practically constant in a shunt motor, and the torque is proportional to both these factors.

It is not practicable to make the field strength of a shunt motor as great as is possible to get with a series motor, because it would require a very large magnetizing force, and with the shunt winding, this extra magnetizing force would have to be expended all the time, whether the strong field was required or not, which would be very wasteful; in the series-motor, however, this extra magnetizing force is only expended while it is needed.

A disadvantage of the series winding is that if all the load is taken off, the current required to drive the motor is very small, making a weak field, which requires such a high speed to generate the proper counter electromotive force that the armature is liable to be damaged. In other words, the motor will *race*, or *run away*, if the load is all removed. This cannot occur with the shunt motor as long as the field circuit remains unbroken.

On account of the foregoing features, shunt motors are used to drive machinery that requires a nearly constant speed with varying loads, or which would be damaged if the speed should become excessive, such as ordinary machinery in shops and factories, pumps, etc. Series motors are used on street cars, to operate hoists, etc., where, on account of the gearing used, the load cannot be entirely thrown off, except by opening the motor circuit, and the torque required at starting and getting quickly up to speed is much greater than the torque required when the car is running at ordinary speed.

**Speed Regulation of Motors.**—The speed of a motor may be varied by varying the applied electromotive force or the strength of the field. The simplest way to vary the applied electromotive force is to insert a resistance, in series with the armature, similar to the starting resistance. By varying this resistance, the applied electromotive force at the terminals of the motor is also varied, although the electromotive force of the supply circuit remains constant. It is evident that the energy represented by the product of the current and the drop through the resistance is converted into heat, and is thereby wasted; therefore, for great variations in speed, this method is not economical, though

often very convenient. The applied electromotive force may also be varied by varying the electromotive force of the generator supplying the current, but this can only be done where a single generator is supplying a single motor, or several motors, whose speed must all be varied at the same time; so that this method is only used in special cases.
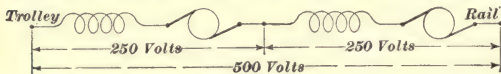


Trolley ⟨⟨⟨⟨⟩⟩ ⟨⟨⟨⟨⟩⟩ Rail

|← 250 Volts →|← 250 Volts →|
|←————— 500 Volts —————→|

FIG. 2

Varying the strength of the field gives a convenient method of varying the speed. If the strength of the field is lessened, the speed will increase, and if the field is strengthened, the speed will decrease. With shunt motors, the field may be weakened by inserting a suitable resistance in the field circuit, as in shunt dynamos; with series motors the same result may be obtained by cutting out some of the turns of the field coils or by placing a suitable resistance in parallel with the field coils. This method of regulation is also of limited range, as it is not economical to maintain the strength of the field much above or below a certain density. The resistance method described, being rather more simple, is generally used. The regulating field resistance must be so constructed as to remain in the circuit all the time the motor is running without getting hot enough to be thereby damaged.

For special cases, such as street-railroad work, various special combinations of the foregoing methods of regulation are used. One of the most common of these is known as the *series-parallel method*, and is the method of regulation generally used at present for operating street cars. This method is equivalent to the method of cutting down the speed by reducing the electromotive force applied to the motor, and is only applicable where at least two motors are used. It is also used, to some extent, in haulage plants. When a low speed is desired, or when the car is to be started up, the motors are thrown in series, as shown in Fig. 2, thus making the voltage across each motor equal to one-half the voltage



Trolley ⟨⟨⟨⟨⟩⟩ Ground
⟨⟨⟨⟨⟩⟩

|←————— 500 Volts —————→|

FIG. 3

between the lines, and cutting down the speed accordingly. When a high speed is desired, the motors are thrown in multiple, as shown in Fig. 3, and each motor runs at full speed because it gets the full line pressure. In practice, starting resistances are used in connection with the foregoing to make the starting smooth, but the two running positions are as shown, the motors being connected in series in the one case, and in parallel in the other.
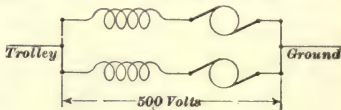
**Connections for Continuous-Current Motors.**—Fig. 4 shows the manner in which a shunt motor is connected to the terminals $+$ and $-$ of the circuit. The current through the shunt field does not pass through the resistance $R$
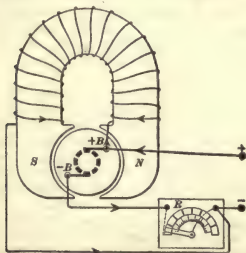


FIG. 4
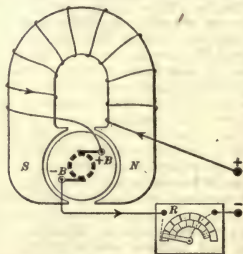


FIG. 5

which is connected in the armature circuit, as to keep the field strength constant, the full difference of potential of the supply circuit should be maintained between the terminals of the field coil. This would not be the case if the rheostat were included in the field circuit, for then the difference of potential

would be only that existing between the brushes $+B$ and $-B$. As on starting the motor this difference of potential is small, only a small current will flow through the field coils, which will generate such a weak field that an excessive current will be required to furnish the necessary torque for starting the motor.

When connected as shown, the field is brought up to its full strength before any appreciable current passes through the armature; so this difficulty does not arise. The current through the armature is gradually increased as the speed increases by gradually cutting out the resistance in the starting box.

As in a series motor the same current flows through both armature and field coils, the starting resistance may be placed in any part of the circuit. The diagram in Fig. 5 illustrates one method of connecting a series motor to the line terminals $+$ and $-$; here the starting or regulating resistance $R$ is placed between the $-$ line terminal and the brush $-B$ of the motor. To reverse the direction of rotation of a motor it is necessary to reverse either the direction of the field or the direction of the current through the armature. It is usual to reverse the direction of the current in the armature, a switch being used to make the necessary changes in the connections.
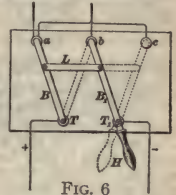


FIG. 6

Fig. 6 shows the connections of one form of reversing switch. Two metal bars $B$ and $B_1$ are pivoted at the points $T$ and $T_1$; one is extended and supplied with a handle $H$, and the two bars are joined together by a link $L$ of some insulating material, such as fiber. Three contact pieces $a$, $b$, and $c$ are arranged on the base of the switch so that the free ends of the bars $B$ and $B_1$ may rest either on $a$ and $b$, as shown by the full lines, or on $b$ and $c$, as shown by the dotted lines. The line is connected to the terminals $T$ and $T_1$, and the motor armature between $a$ and $b$, or vice versa, $a$ and $c$ being connected together.

When the switch is in the position shown by the full lines, $T$ is connected to $a$ by the bar $B$, and $T_1$ to $b$ by the bar $B_1$. If the switch is thrown by means of the handle $H$ into the position indicated by the dotted lines, $T$ is connected to $b$ by the bar $B$, and $T_1$ to $a$ by the bar $B_1$ and the connection between $c$ and $a$. The direction of the current through the motor armature, or whatever circuit is connected between $a$ and $b$, is thus reversed.

In order to reverse the current in the armature, the reversing switch must be placed in the armature circuit only. Fig. 7 represents the connection for a reversing-shunt motor $(a)$ and a reversing-series motor $(b)$; $+$ and $-$ are the



(a)     FIG. 7     (b)
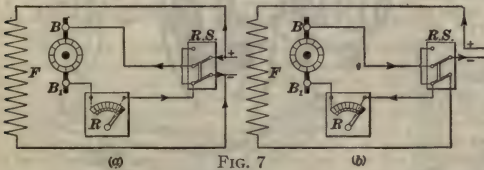
line terminals; $R$, the starting resistance; $B$ and $B_1$, the brushes of the motor, and $F$, the field coil of the motor. Some manufacturers combine the starting re-

sistance and reversing switch in one piece of apparatus. When connecting up motors, some form of main switch is used to entirely disconnect the motor from the line when it is not in use.

To prevent an excessive current from flowing through the motor circuit from any cause, short strips of an easily melted metal, known as *fuses*, mounted on suitable terminals, known as *fuse boxes*, are placed in the circuit. These fuses are made of such a sectional area that a current greater than the normal heats them to such an extent that they melt, thereby breaking the circuit and preventing damage to the motor from an excessive current. The length of fuse should be proportioned to the voltage of the circuit, a high voltage requiring longer fuses than a low voltage, in order to prevent an arc being maintained across the terminals when the fuse melts.

If desired, measuring instruments (ammeter and voltmeter) may be connected in the motor circuit, so that the condition of the load on the motor may be observed while it is in operation. All these appliances, regulating resistance, reversing switch, fuses, instruments, etc., are placed inside the main switch; that is, the current must pass through the main switch before coming to any of these appliances, so that opening the main switch entirely disconnects them from the circuit, when they may be handled without fear of shocks.

## ALTERNATING-CURRENT DYNAMOS

An *alternating-current dynamo* is one that generates a current that periodically reverses its direction of flow; as when an armature is provided simply with collector rings. This current may be represented by a curve such as that



FIG. 1

shown in Fig. 1. The complete set of values that the current, or electromotive force, passes through repeatedly is known as a *cycle*. For example, the values passed through during the interval of time represented by the distance *ac* will constitute a cycle. The set of values passed through during the interval *ab* is known as an alternation. An alternation is, therefore, half a cycle. The number of cycles passed through per second is known as the *frequency* of the current, or electromotive force.
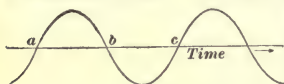
Alternating-current dynamos are now largely used both for lighting and power transmission, especially when the transmission is over long distances. The reason that the alternating current is specially suitable for long-distance work is that it may be readily transformed from one pressure to another, and in order to keep down the amount of copper in the line, a high line pressure must be used. Pressures much over 500 or 600 volts cannot be readily generated with direct-current machines, owing to the troubles that are likely to arise due to sparking at the commutator. On the other hand, an alternator requires no commutator or even collecting rings, if the armature is made stationary and the field revolving, as is frequently done. Alternators are now built that generate as high as 11,000 volts directly. If a still higher pressure is required on the line, it can be easily obtained by the use of transformers.

Alternating-current dynamos, like direct-current machines, consist of two main parts, the field and armature. Either of these parts may be the revolving member, and in many modern machines the armature, or the part in which the current is induced, is the revolving member. Fig. 2 shows a typical alternator of the belt-driven type, having a revolving armature. It is not unlike a direct-current machine as regards its general appearance. The number of poles is usually large, in order to secure the required frequency without running the machine at a high rate of speed. The frequencies met with in practice vary all the way from 25 to 150. The higher frequencies are, however, passing out of use, and at present a frequency of 60 is very common. This frequency is well adapted both for power and lighting purposes. When machines are used almost entirely for lighting work, frequencies of 125 or higher



FIG. 2

may be used. The frequency of any machine may be readily determined when the number of poles and the speed is known, as follows:

$$\text{Frequency} = \frac{\text{number of poles}}{2} \times \frac{\text{rev. per min.}}{60}$$

For example, if an eight-pole alternator runs at a speed of 900 rev. per min., the frequency will be

$$f = \tfrac{8}{2} \times \tfrac{900}{60} = 60 \text{ cycles per sec.}$$

Alternators may be divided into single-phase alternators and multiphase alternators.

*Single-phase alternators* are so called because they generate a single alternating current (as represented by the curve shown in Fig. 1). The armature is provided with a single winding and the two terminals are brought out to collector rings. Single-phase machines have been largely used for lighting work, but they are gradually being replaced by multiphase machines, because
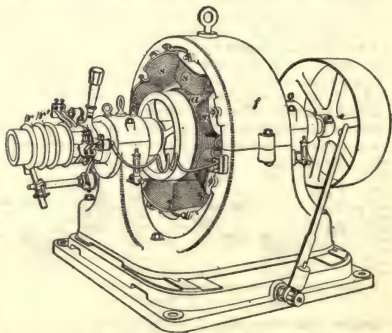
the single-phase machines are not well suited for the operation of alternating-current motors.

*Multiphase alternators* are so called because they deliver two or more alternating currents that differ in phase; that is, when one current is, say, at its maximum value, the other currents are at some other value. This is accomplished by providing the armature with two or more distinct windings which are displaced relatively to each other on the armature. One set of windings, therefore, comes under the poles at a later instant than the winding ahead of it, and the current in this winding comes to its maximum value at a later instant than the current in the first winding. In practice, the two types of multiphase alternators most commonly used are two-phase alternators, and three-phase alternators.

*Two-phase alternators* deliver two alternating currents that differ in phase by one-quarter of a complete cycle; that is, when the current in one circuit is at its maximum value, the current in the other circuit is passing through its zero value. By tapping four equidistant points of a regular ring armature, as shown in Fig. 3, and connecting these points to four collector rings, a simple two-pole two-phase alternator is obtained. One circuit connects to rings *1* and *1'*, the other circuit connects to rings *2* and *2'*. When the part of the winding connected to one pair of rings is in its position of maximum action, the electromotive force in the other coils is zero, thus giving two currents in the
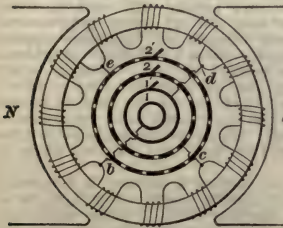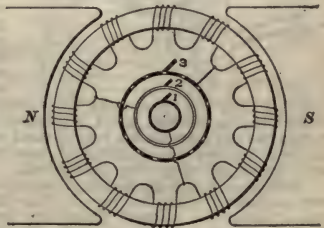


FIG. 3    FIG. 4

two different circuits that differ in phase by one-quarter of a cycle or one-half an alternation.

*Three-phase alternators* deliver three currents that differ in phase by one-third of a complete cycle; that is, when one current is flowing in one direction in one circuit, the currents in the other two circuits are one-half as great, and are flowing in the opposite direction. By tapping three equidistant points of a ring winding, as shown in Fig. 4, a simple three-phase two-pole alternator is obtained. Three mains lead from the collecting rings.

In order to have three distinct circuits, it would ordinarily be necessary to have six collecting rings and six circuits; but this is not necessary in a three-phase machine if the load is balanced in the three different circuits, because one wire can be made to act alternately for the return of the other two.

**Uses of Multiphase Alternators.**—Multiphase alternators are extensively used because alternating-current motors can be readily operated from two-phase and three-phase circuits. By using multiphase machines, motors can be operated that will start from rest under load, whereas with single-phase machines most motors have to be brought up to speed from some outside source of power before they can be made to run. For this reason, such machines are used for the operation of modern power-transmission plants. As far as the general appearance of three-phase machines goes, they are similar to ordinary single-phase alternators, the only difference being in the armature winding and the larger number of collector rings. The multiphase alternator is also adapted for the operation of lights, so that by using these machines both lights and motors may be operated from the same plant. They are well adapted for power-transmission purposes in mines, especially for the operation of pumping and hoisting machinery, because the motors operated by them are very simple in construction and therefore not liable to get out of order.

## ALTERNATING-CURRENT MOTORS

Alternating-current motors may be divided into synchronous motors and induction motors.

*Synchronous motors* are almost identical, so far as construction goes, with the corresponding alternator. For example, a two-phase synchronous motor



Fig. 1

will be constructed in the same way as a two-phase alternator. They are called synchronous motors because they always run in synchronism, or in step, with the alternator driving them. This means that the motor runs at the same frequency as the alternator, and if the motor had the same number of poles as the alternator, it would run at the same speed, no matter what load it might be carrying. This type of motor has many good points, and is especially well suited to cases where the amounts of power to be transmitted are comparatively large and where the motor does not have to be started and stopped frequently. Multiphase synchronous motors will start up from rest and will run up to synchronous speed without aid from any outside source. Most multiphase synchronous motors will not, however, start with a strong starting torque or effort, and will not, therefore, start up underload, and cannot be used in places where a strong starting effort is required. For this reason most synchronous motors are not suitable for intermittent work. Some special multiphase synchronous motors are so designed, however, that they may be used where a fairly strong starting effort is required.

*Induction motors* are so called because the current is induced in the armature instead of being led into it from some outside source. Fig. 1 shows a typical induction motor. There are two essential parts in these machines, viz., the *field*, into which multiphase currents are led from the line, and the armature, in which currents are induced by the magnetism set up by the field. Either of these parts may be the stationary or revolving member, but in most cases the field, or part that is connected to the line, is stationary. The stationary part of an alternating-current motor is called the *stator* and the rotating part, the *rotor*. Fig. 2 shows the construction of the stationary member or field. This consists of a number of iron laminations, built up to form a core and provided with slots around the inner periphery. Form-wound coils constituting the field winding are placed in these slots and connected to the mains. This winding is arranged in the same way as the armature winding of a multiphase alternator. When alternating currents differing in phase are sent through the winding, magnetic poles are formed at equidistant points around the periphery of the field, and the constant changing of the currents causes these poles to



Fig. 2

shift around the ring, thus setting up what is known as a revolving magnetic field. The armature, Fig. 3, consists of a laminated iron core provided with a number of slots, in each of which is placed a heavy copper
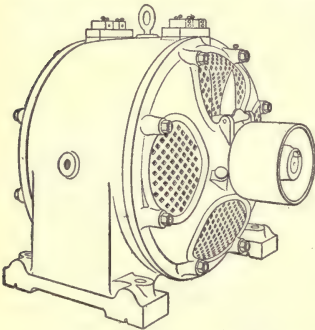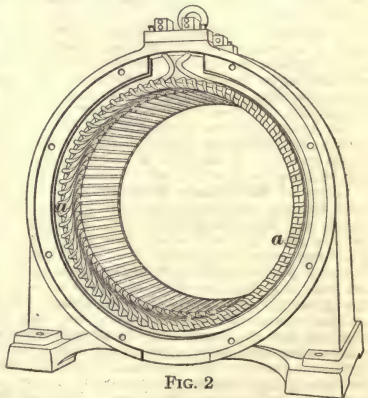
bar *b*. The ends of these bars are all connected together by two heavy short-circuiting rings *r* running around each end of the armature. The bars and end rings thus form a number of closed circuits. When such an armature is placed in the revolving field, the magnetism will cut across the armature conductors, inducing electromotive forces in them, and as the conductors are joined up into closed circuits, currents will flow in them. These currents will react on the field and the armature will be forced to revolve. Such an armature will not run exactly in synchronism, because if it did, it would revolve just as fast as the magnetic field, and there would be no cutting of lines of force. The speed drops slightly from no load to full load, but if the motor is well designed, this falling off in speed is slight.

The name *squirrel cage* is applied to a rotor of the type shown in Fig. 3. Another type of rotor used in induction motors is provided with a coil winding with ends brought out to three collector rings mounted on the shaft. By means of stationary brushes resting on these rings, the resistance in the circuit of such a rotor can be varied. Motors having such rotors are called *wound-rotor* motors, or *slip-ring* motors. Induction motors possess many advantages for mine work. Squirrel-cage motors having no sliding contacts operate with absolutely no sparking—a desirable feature for mine work. Such motors are also very simple in construction, and are therefore not liable to get out of order. They have the additional advantage over the synchronous motor in that they exert a strong starting effort, and, in fact, behave in most respects like any good shunt-wound direct-current motor. They are used quite successfully for all kinds of stationary work, such as pumping, hoisting, etc., but are not adapted for haulage purposes. Wound-rotor (slip-ring) motors are used where especially strong starting effort or variable speed is required.
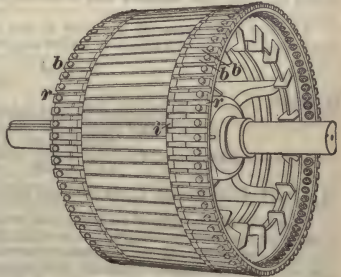


FIG. 3

At no load the speed of an induction motor is practically the quotient obtained by dividing the number of alternations of the current by the number of magnetic poles for which the motor field winding is connected. Thus with a 60-cycle current, or 7,200 alternations, a two-pole motor runs at 3,600 rev. per min., a four-pole motor at 1,800 rev., a six-pole motor at 1,200 rev., an eight-pole motor at 900 rev., a ten-pole motor at 720 rev., a twelve-pole motor at 600 rev., and a fourteen-pole motor at 514 rev. With a 25-cycle current, or 3,000 alternations, the corresponding speeds are 1,500, 750, 500, 375, 300, 250, and 214 rev. per min.

The full-load speed is less than the no-load, or synchronous, speed, and the decrease from no-load to full-load speed is called the *slip*. The slip depends on the resistance of the rotor circuit and on the load, and, the higher the slip with a given load, the lower the efficiency of the motor. In commercial motors, slip ranges in value from 2 to 10% of the no-load speed, depending on the size of the motor, large motors having low slip.

**Selection of Induction Motors for Mine Use.**—Induction motors are made for operating on single-phase, two-phase, and three-phase circuits. Single-phase motors can be used advantageously in small capacities up to, say, 10 H. P., for many purposes. They start with fairly strong effort and operate with efficiency and power factor approximately the same as two-phase and three-phase motors. They require but two line wires and one transformer to increase or decrease the voltage.

In cost and reliability of operation there is little choice between two-phase and three-phase motors, but because the latter require but three line wires as opposed to the four required by the former the resulting saving in copper has led to the more extensive use of the three-phase type. It should be noted, however, that two-phase systems require but two transformers where the voltage has to be changed as against three transformers for the three-phase system.

By the use of static transformers the voltage for induction motors can be made independent of the line voltage. For long-distance transmission lines

the generator voltage is usually *stepped-up* for transmission through the line, and the line voltage *stepped-down* for use in the motors at the end of the line. Large motors can be advantageously operated directly on the line if the voltage does not exceed 6,600, but great care must be taken to keep them dry. High-voltage motors cannot be used successfully in damp places; and low-voltage motors for use in such places should have waterproof insulation on the windings.

The choice of frequency generally lies between the two prevailing standards of 25 cycles and 60 cycles per sec. For railroad work the 25-cycle transmission system is preferred because a better regulation is obtainable and the rotary (synchronous) converters used in connection therewith cost less. For general lighting purposes, both arc and incandescent lamps operate better on 60-cycle circuits. When the voltage does not exceed 110, incandescent lamps are used without flickering on 25-cycle circuits; but when the voltage is high, frequency-changers should be employed to raise the frequency of that portion of the current used for lighting from 25 to 60 cycles. For the operation of stationary motors about mines, the 60-cycle system is generally preferred because the transformers are less costly, because the choice of motor speeds is much greater, and because this frequency is as well adapted to lighting as to power purposes.

The *full-load torque* of a motor is the turning effort it must develop when revolving at its rated speed in order to produce its rated horsepower; that is,

$$\text{H. P.} = \frac{2\pi T \times RPM}{33,000},$$

in which
$\pi = 3.1416$;
$T =$ torque, in foot-pounds;
$RPM =$ the number of revolutions per minute.

By transformation, $T = \dfrac{33,000\,HP}{2\pi \times RPM} = \dfrac{5,250\,HP}{RPM}$. Thus, the full-load torque

of a 10-H. P. motor at 1,700 rev. per min. is $\dfrac{5,250 \times 10}{1,700} = 30.9$ lb.-ft.

The starting torque of an induction motor is that developed at starting from a state of rest and is commonly considerably greater than the full-load torque. The pull-out torque is the maximum turning effort the motor can exert without stopping and is usually from 1.5 to 3 times the full-load torque. An induction motor cannot operate at more than full-load torque, except for short periods, without overheating.

Squirrel-cage induction motors have no wearing parts except the bearings, and these can be made dust-proof, so that these motors rarely need to be enclosed unless they are to be used in extremely dusty places. A free circulation of air through the interior of the motor is necessary to keep it cool. Ordinarily, the necessary circulation is secured by arranging the casing as shown in Fig. 1, where the openings are covered with gratings to prevent objects falling into and injuring the motor. Where the air is only moderately dusty, a supply of clean air may be drawn through a special pipe from outside the motor room and, after circulating through the motor, is discharged through another pipe. For use in very dusty or damp places, the openings in the casing (shown covered by gratings in Fig. 1) are closed with tight-fitting plates; these should be fitted with gaskets to keep out dampness. It must be remembered that this enclosing reduces the capacity of the motor for continuous operation owing to the heating through lack of interior ventilation; in fact, an enclosed motor can operate at full rating only intermittently.

Squirrel-cage motors are generally preferred to wound-rotor motors for driving machinery, such as mine fans, where the speed is constant. By providing a motor of this type with a high resistance rotor, its starting and pull-out torque with a given input of current can be made high, although its efficiency, when running under load, is correspondingly low. For intermittent service where a high torque is necessary for starting or while in operation, squirrel-cage motors with high-resistance rotors are sometimes desirable, as in driving elevators, conveyers, etc.

Wound-rotor motors are preferred for driving machinery such as hoisting engines, which must be frequently started and stopped with a high torque and where ability to change the speed is important. By means of a controller connected with the collector rings, the resistance in the rotor circuit can be varied at will, thus limiting the starting current and giving any desired speed. When driving machines other than hoisting engines where the engineer is constantly on duty, it should be noted that when a motor of this type is operated with external resistance in its rotor circuit, any change in the amount of the

load thrown upon it causes an inverse change in its speed so that, in these cases, also, the presence of an attendant may be necessary to keep the speed adjusted.

Induction motors are usually purchased under guaranteed heating limits and performance characteristics, such as efficiency, power factor, starting and pull-out torque, and, sometimes, slip. The heating limits, or rise in temperature of the windings, etc., of the motor when in operation are at present commonly guaranteed not to exceed 40° C. to 75° C. above the normal temperature; the amount of rise guaranteed depending on the style of motor, whether open, semi-enclosed, or enclosed, and on the nature of the service, whether under continuous or intermittent operation. High efficiency is desirable in a motor that is to be run continuously under a steady load, and the efficiency should be high at this load; the importance of high efficiency in such a motor increases with the cost of electricity. Efficiency is not of importance in a motor that operates intermittently, or where the cost of electricity is very low.

Low power factor indicates that some of the current in the generator and transmission line is useless or idle, in that the energy supplied the motors is less than is indicated by the line current. The actual energy that can be delivered by a generator within its guaranteed heating limits decreases with the power factor. Thus, a generator rated at 1,000 K. W. at 100% power factor would have an output of 900 K. W. at 90% power factor; and similarly, the carrying capacity of transmission lines is reduced in the same way. The power factor of a system depends on the power factors of the motors operating on the lines. A small number of motors operating at low power factors may not affect the general capacity appreciably, but as the number of motors is increased, the importance of operating them at high power factor increases, as the generator and lines may be overloaded with the current.

**Installation and Care of Induction Motors.**—When ordering a motor, the manufacturer should be advised whether it is to be placed on a side wall, suspended in a reversed position overhead, or used on the ordinary horizontal floor, or pedestal, mounting, as the arrangement of the housings, oil cups, etc., will be different in the several cases. The machine should be mounted in the driest, cleanest, and best-ventilated place possible, where it is in plain sight and within easy reach. The foundations should be solid enough to prevent vibration, masonry or concrete construction being preferred. The rails upon which the motor slides in being adjusted to position should be supported upon an absolutely even surface, so that the machine may rest properly upon them. When the motor is belt-connected to the driven machine, the axles of the two should be absolutely parallel, and the centers of the faces of the driving and driven pulleys should be exactly in line, so that the rotor will not be forced from its central position. If the motor is geared to the driven machine, the axles of the two must be parallel, the centers of the faces of the gear-wheels must be in the same line, and the distance between the centers of the gear-wheels must be exactly that demanded by the designs, that the pinion may mesh properly with the gear. If a thin piece of paper is passed through the gears while slowly turning the rotor, it should show by its crushing or marking, an even pressure across the full width of the tooth.

Before starting a motor for the first time, care must be taken that all circuits, connections, etc., are in accordance with the diagram furnished by the manufacturer. With small motors, enclosed type fuses are used in connection with the starter, but with larger motors circuit-breakers are used. Fuses that are not out during starting should have a current capacity at least 2½ times that of the motor. Very small motors require no starting apparatus and may be put in operation by closing the line circuit. In motors of 5 H. P. and upwards, the sudden throwing on of the entire current will cause fluctuations in the line current, which may interfere with the working of other motors on the same circuit and may even damage the motor windings.

To permit the application of low voltage at starting, which may be increased to full voltage after the motor has been brought up to speed, squirrel-cage induction motors are provided with auto or compensating starters. These starters differ in details of design, but in all cases they comprise the essential elements of two or three single-coil transformers for reducing the voltage, and a switch for changing the connections between the line, the transformers, and the motor. The transformers are provided with taps, and by adjusting the tapping-in point the voltage at starting may be varied. In the starting position, the switch connects the transformers to the line and the motor to the transformers. In the running position, the motor is connected directly to the line, no current passing through the transformers. Starters commonly have

but one starting position and open the circuit when passing from the starting to the running position. They are designed so that it is impossible to move from the off to the running position without passing through the starting position, or to introduce the starting connection in passing back from the running to the off position. The handles of most starters are moved clockwise to start and stop the motor, and can be turned in the reversed position only to return from the starting to the off position.

The power should not be thrown on in one rapid operation as such action will cause excessive starting current; rather, the handle should be left at the starting position for a short time, and then moved quickly either to the second starting position (if there is one) or to running.

When setting up a motor, the oil wells should be examined and if dirty should be blown out or cleaned with gasoline if they are badly clogged. After cleaning, they should be filled with a high grade of mineral oil, preferably dynamo oil, which flows easily and is readily carried up by the oil rings. Graphite and similar lubricants are not satisfactory for use in motors as they clog the oil ducts and interfere with the operation of the oil rings. When starting the motor (which may be turned by hand for testing purposes) care must be taken that the rings revolve and carry up the oil. At all times enough oil must be kept in the bearings that the rings may dip well below its surface. Bearings using oil and waste should be packed with a high-grade wool waste, which should fit closely against the shaft. Cotton or the poorer grades of wool waste become soggy and drop away from the shaft. The waste should be thoroughly saturated with oil, preferably by immersing it for 48 hr., and then letting it drip for 10 to 12 hr. Too much free oil will result in an overflow and the possible introduction of oil within the motor. Dirty oil should never be used; it should be replaced at once with clean oil, and to this end an oil filter is an excellent investment. Bearings using grease should be filled with a good quality of grease free from dirt or grit.

The distance between the centers of the driving and driven pulleys should be great enough to allow some sag in the belt. Kent gives the following general rules for belting. With narrow belts and small pulleys, the distance between centers should be 15 ft., with a sag in the loose side of the belt of $1\frac{1}{2}$ to 2 in. With belts of medium width and larger pulleys, the center distance of the pulleys should be 20 to 25 ft., and the sag, $2\frac{1}{2}$ to 4 in. With main belts on very large pulleys, the center distance should be 25 to 30 ft., and the sag from 4 to 5 in. If the distance between pulley centers is too great, the belt will flap unsteadily resulting in unnecessary wear of the belt and bearings; if the distance is too short, the severe tension required to prevent slipping will cause rapid wear and possible overheating of the bearings. The rules given represent good practice for long life of belt and bearings. Shorter distances must sometimes be used, in which cases the belt may be tighter, the belt and pulleys wider, or the pulleys larger and the belt speed greater. Very short belts will work satisfactorily if idler pulleys are used to increase the arc of contact between the belt and pulleys. Belts should not be run at a greater angle with the horizontal than 45° if this is possible, and never vertically. The belt should be just tight enough to avoid slipping or flapping; the slack side should have a gently undulating motion; the joints should be as smooth as possible, and a lapped joint should always trail, never lead over a pulley. A sidewise movement of the belt indicates poor alinement of the pulleys or unequal stretching of the edges of the belt.

When a bearing becomes hot, the cause of the trouble should be looked for at once. Some of the common causes of hot bearings are: Poor grades of oil, grit and dust in the oil well and bearings, stopping of the circulation through foreign particles in the oil grooves, an empty oil well, too tight or too heavy a belt, too much end thrust on the rotor due to poor alining or leveling, a sprung shaft, or worn or cut Babbitt metal in the bearings. The bearings of induction motors should be inspected daily, because the air gap between the rotor and stator being small, any excessive wear on the bearings may cause these parts to rub. Some of the larger sizes of induction motors are provided with adjustable bearings so that the rotor may be shifted to secure a uniform air gap, but in the smaller sizes the effects of wear can be overcome only by renewing the linings.

Dust and grit should not be allowed to accumulate around the windings and bearings. The motor should be regularly and thoroughly wiped and the dust blown from all its parts. The projecting portions of the stator coils of motors running in damp places should receive an occasional coating of waterproof insulating varnish.

The temperature of all induction motors will rise above that of the atmosphere while running under load. As long as the hand can be held continuously on the machine there is no danger, but as soon as the heat can be borne but for a few seconds, and particularly if the odor of burning oil is noticeable, the danger point has been reached. It will seldom be necessary to do more than supply the bearings with an abundance of fresh clean lubricant, care being taken that the oil or grease reaches the bearing surface; sometimes it may be necessary to remove excessive belt tension. If relief is not afforded in this way, a heavy oil should be poured directly on the journal if possible. If necessary, part or all of the load should be removed but the rotor should be kept in motion enough to prevent the bearing from becoming set or *frozen*.

When an induction motor is overloaded beyond its limit, it will stop or *pull-out*. Should a motor stop when it is not overloaded, and an examination of the bearings and air gap shows that the motor and stator are not rubbing, the stoppage may be due to abnormally low voltage in the supply circuit. The torque exerted by induction motors decreases as the square of the voltage; hence, a comparatively small drop in the voltage produces a large decrease in torque and the motor may come to a standstill if it happens to be carrying a heavy load at the time the voltage drops. To secure the best results from an induction motor, full voltage should be maintained and it is better to have the voltage too high than too low provided excessive heating does not result therefrom.

# TRANSFORMERS

*Transformers* used for raising the voltage are known as *step-up transformers;* those used for lowering the pressure are known as *step-down transformers.*

The transformer consists of a laminated iron core upon which two coils of wire are wound; these coils are entirely distinct, having no connection with each other. One of these coils, called the *primary*, is connected to the mains; the other coil, called the *secondary*, is connected to the circuit to which current is delivered. Fig. 1 shows the arrangement of



FIG. 1



FIG. 2

coils and core for a common type of transformer. The secondary coil is wound in two parts $S$ and $S'$, and the primary coil, also in two parts $P$ and $P'$, is placed over the secondary. $C$ is the core, built up of thin iron plates. Fig. 2 shows a weather-proof cast-iron case for this transformer. When a current is sent through the primary it sets up a magnetism in the core which rapidly alternates with the changes in the current. This changing magnetism
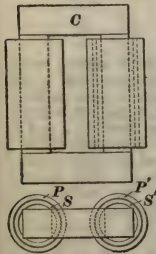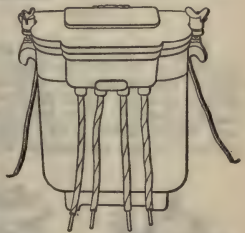
sets up in the secondary an alternating electromotive force, which depends on the number of turns in the secondary coil. If the secondary turns are greater than the primary, the secondary electromotive force will be higher than that of the primary. The relation between the primary electromotive force and secondary electromotive force is given by the following:

$$\text{Secondary E. M. F.} = \text{primary E. M. F.} \times \frac{\text{secondary turns}}{\text{primary turns}}$$

or,

$$\text{secondary E. M. F.} = \frac{\text{primary E. M. F.}}{\dfrac{\text{primary turns}}{\text{secondary turns}}}$$

The ratio $\dfrac{\text{primary turns}}{\text{secondary turns}}$ is known as the ratio of transformation of the transformer. For example, if a transformer had 1,200 primary turns and 60 secondary turns, its ratio of transformation would be 20 to 1, and the secondary voltage would be one-twentieth that of the primary. Transformers are made for a number of different ratios of transformation, the more common ones being 10 to 1 or 20 to 1. Of course, a transformer never gives out quite as much

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power from the secondary as it takes in from the primary mains, because there is always some loss in the iron core and in the wire making up the coils. The
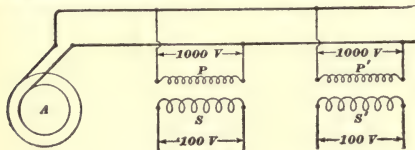

FIG. 3
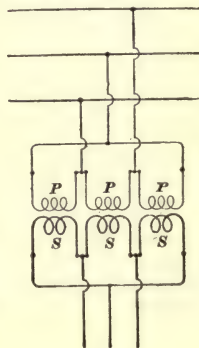

FIG. 4


FIG. 5
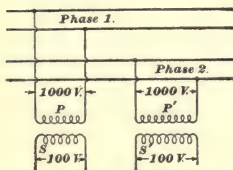
efficiency of transformers is, however, high, reaching as high as 97% or 98% in the larger sizes. Transformers are connected in parallel across the mains, and if they are well designed, will furnish a very nearly constant secondary pressure at all loads, when furnished with a constant primary pressure. Fig. 3 shows transformers connected on a single-phase circuit, Fig. 4 shows the connection for a two-phase circuit, and Fig. 5 shows one method of connection for a three-phase circuit.

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# ELECTRIC SIGNALING

## BATTERIES

*Batteries* are used for various purposes in connection with mining work, principally for the operation of bells and signals. The *Leclanché cell* is one that is widely used for bell and telephone work. It is made in two or three different forms, one of the most common being shown in (a) of the accompanying illustration. The zinc element of this battery is in the form of a rod $Z$, and weighs about 3 oz. The other electrode is a carbon plate placed in a porous cup and surrounded with black oxide of manganese, mixed with crushed coke or carbon. The electrolyte used in the battery is a saturated solution of sal ammoniac. The electromotive force of this cell is about 1.48 volts when the cell is in good condition. In another form of the cell, known as the *Gonda* type, black oxide of manganese is pressed into the form of bricks and clamped against each side of a carbon plate by means of rubber bands. The Leclanché type of cell will do good work if it is only used intermittently in circuits where the insulation is good and where there is no leakage causing the cell to give out current continuously. If current is taken from it for any length of time, it soon runs down, but will recuperate if allowed to stand.
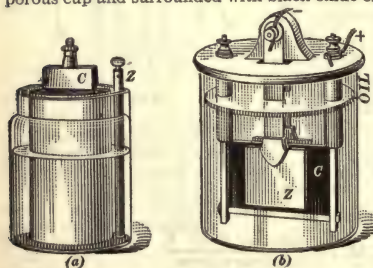

(a)    (b)

Dry cells are essentially the same as a Leclanché liquid cell, but the electrolyte is limited to the amount that can be retained in some absorbent material, such as paper, that is placed inside a zinc can which forms one electrode. The other electrode is a carbon rod in the center. The space between is filled with crushed coke and peroxide of manganese and the whole interior is saturated

## ELEMENTS OF PRIMARY BATTERIES

| Name | Anode | Electrolyte | Cathode | Depolarizer | E.M.F. | Remarks |
|---|---|---|---|---|---|---|
| Fuller | Amalgamated zinc | Sulphuric acid (very dilute) or water, in porous cup | Carbon | Sodium bichromate, 6 oz; sulphuric acid, 17 oz.; soft water, 56 oz. | 2.14 | Anode in porous cup with a little mercury. |
| Partz | Zinc | Sodium chloride or magnesium sulphate | Carbon | Same as in the Fuller battery | 1.9 to 2 | Gravity cell; resistance with sodium chloride, .5 ohm; with magnesium sulphate, 1 ohm. For closed-circuit work only; resistance 3 ohms. |
| Gravity Daniell | Zinc | Zinc sulphate, specific gravity 1.10 | Copper | Copper sulphate with copper sulphate crystals | 1.07 | For closed-circuit work only; resistance 3 ohms. |
| Leclanché | Zinc | Ammonium chloride saturated | Carbon | Peroxide of manganese | 1.48 | Carbon and depolarizer in porous cup; resistance, 4 ohms; for intermittent service. |
| Edison | Zinc | Caustic potash | Molded plates of cupric oxide held in copper frames | | .7 | Surface of electrolyte covered with layer of oil; resistance about .7 ohm, depending on size of cell. |
| Gordon | Zinc | Caustic soda | Flaky copper oxide in perforated copper cylinder | | .7 | For open- or closed-circuit working |
| Dry | Zinc | Ammonium chloride | Carbon | Peroxide of manganese | 1.4 | For intermittent service; internal resistance, .1 to .8 ohm. |

with the solution and the top sealed to prevent its evaporation. Dry cells are extensively used in place of wet Leclanché cells because they are as good and so much cheaper that it is economical to throw them away when exhausted and buy new ones instead of spending the time and money required to replenish the wet Leclanché cells. The internal resistance of cells not over 1 yr. old nor entirely exhausted varies from .1 to .8 ohm and the electromotive force from 1.3 to 1.5 volts.

In cases where the insulation is apt to be poor, as it often is in mines, it is best to use a battery that will stand a continuous delivery of current and that will at the same time operate all right on intermittent work or on work where the circuit is open most of the time. For work of this kind, cells of the *Edison* or *Gordon* type are excellent. View (*b*) shows the Edison cell. The elements consist of two zinc plates *Z* hung on each side of a plate of compressed cupric oxide *C*. The electrolyte is a saturated solution of caustic potash, which is kept covered with a layer of heavy paraffin oil, to prevent the action of the air on the solution. The voltage of the cell is only .7 volt, but its internal resistance is very low and its current capacity correspondingly large. The electrolyte used in the Gordon cell is also caustic-potash solution, and the two cells are much the same, so far as their general characteristics are concerned. The preceding table gives data relating to a number of different types of cell.

## BELL WIRING

The simple bell circuit is shown in Fig. 1, where *p* is the push button, *b* the bell, and *c* the cells of the battery connected up in series. When two or more



FIG. 1                                    FIG. 2
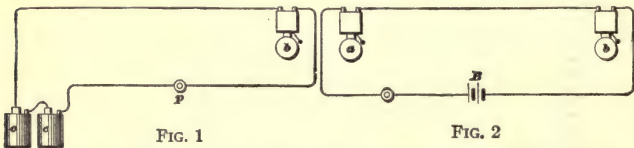
bells are to be rung from one push button, they may be connected in series, as shown in Fig. 2, or arranged in parallel across the battery wires, as at *a* and *b*, Fig. 3. The battery *B* is indicated in each diagram by short parallel lines, this being the conventional method. In the parallel arrangement, the bells are independent of each other, and the failure of one to ring will not affect the others; but in the series grouping, all but one bell must be changed to a single-stroke action, so that each impulse of current will produce only one movement of the hammer. The current is then interrupted by the vibrator in the remaining bell, the result being that each bell will ring with full power. The only change



FIG. 3                                    FIG. 4
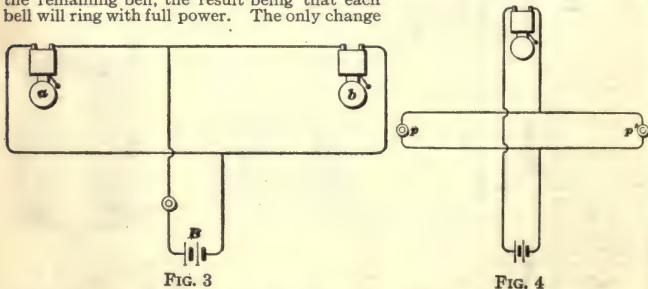
necessary to produce this effect is to cut out the circuit-breaker on all but one bell by connecting the ends of the magnet wires directly to the bell terminals.

When it is desired to ring a bell from one of two places some distance apart, the wires may be run as shown in Fig. 4. The pushes *p* and *p'* are located at the required points, and the battery and bell are put in series with each other.

A single wire may be used to ring signal bells at each end of a line, the connections being given in Fig. 5. Two batteries are required, $B$ and $B'$, and
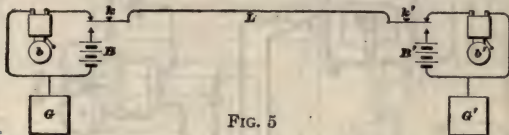


FIG. 5

a key and bell at each station. The keys $k$ and $k'$ are of the double-contact type, making connections normally between bell $b$ or $b'$ and line wire $L$. When one key $k$ is depressed, a current from one battery $B$ flows along the wire through the upper contact of the other key $k'$ to a bell $b'$ and back through the ground plates $G'$ and $G$.

When a bell is intended for use as an alarm apparatus, a constant-ringing attachment may be introduced, which closes the bell circuit through an extra
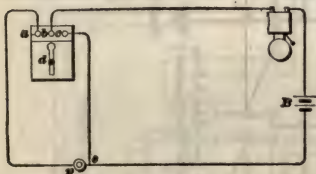


FIG. 6

wire as soon as the trip at a door or window is disturbed. In the diagram, Fig. 6, the main circuit, when the push $p$ is depressed, is through the automatic drop $d$ by way of the terminals $a$ and $b$ to the bell and battery. This current releases a pivoted arm which, on falling, completes the circuit between $b$ and $c$, establishing a new path for the current by way of $e$, independent of the push $p$. The bell will then ring until the drop $d$ is restored by some one or the battery becomes exhausted. For operating electric bells, any good type of open-circuit battery may be used; dry and Leclanché cells are largely used for this purpose.

**Annunciator System.**—The wiring diagram for a single annunciator system is shown in Fig. 7. The pushes *1, 2, 3,* etc., are located in various places, one side being connected to the battery wire $b$, and the other to the leading wire $l$ in communication with the annunciator drop corresponding to that place. A battery $B$ of two or three Leclanché cells is placed in any convenient location. The size of wire used throughout may be No. 18 annunciator wire.

Telephones are also used for signaling and communicating purposes. The mine telephone system performs two functions: It expedites the work, thereby lowering the cost of production, and it enhances the safety of the mine workers and mine property. Its value was realized first by some of the leading mining companies and several states have enacted mine laws requiring its use. The principles involved in a mine telephone system are identical with ordinary telephone practice, with such changes in details as are necessary to meet conditions existing in mines.

The telephone case in mine work must be damp-proof and dust-proof, and wiring must be of such a nature as to resist both dampness and the corroding influence of drops of acidulated mine water. It must also be



FIG. 7

suspended so as to be protected from injury due to falls of roof, cars, or the carelessness of employes.

It has been found that a first-class, long-distance, bridging telephone is the best type to use; *bridging telephones* are so called because they are bridged

Fig. 8

Switchboard    Arrester    Supt. Residence

Mine Boss Residence

Supt's. Office    Physician's Office

Engine House    Stables    Tipple

Main Entry

First Left Entry

First Right Entry

Second Left Entry

FIG. 9

FIG. 10

FOOT BELL

FROM ENGINE ROOM

TO ENGINE ROOM

HEAD

FOOT BELL

HEAD B.

HEAD BUTTON

FOOT BUTTON

TELEPHONE

1ST. LEVEL

2ND. LEVEL

3RD. LEVEL

4TH. LEVEL

5TH. LEVEL

6TH. LEVEL
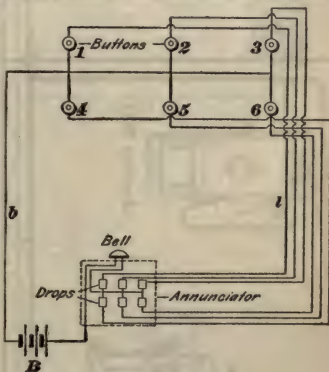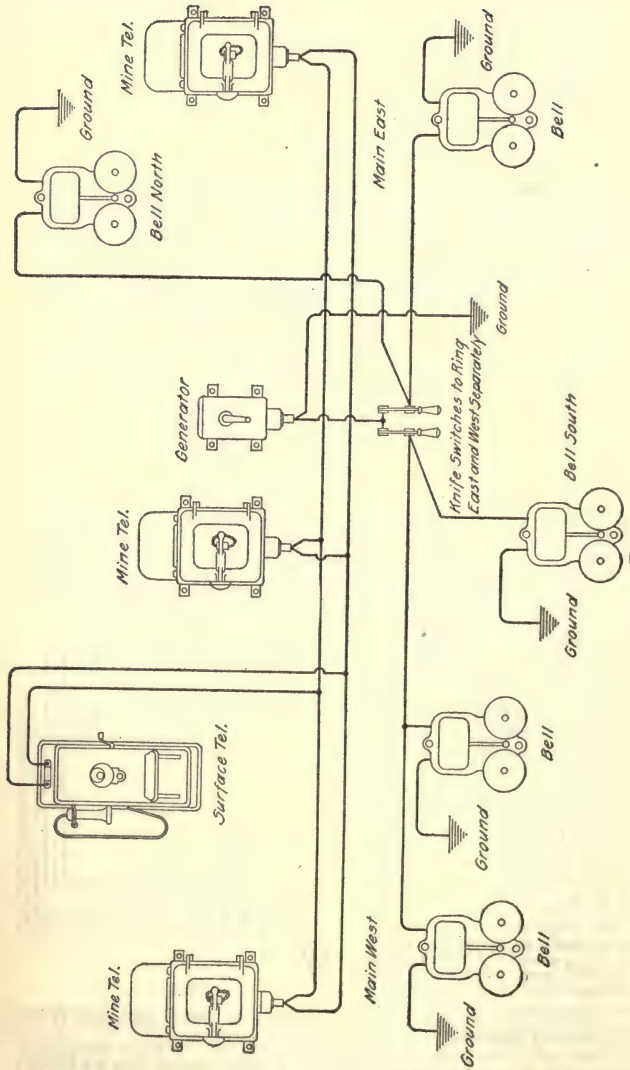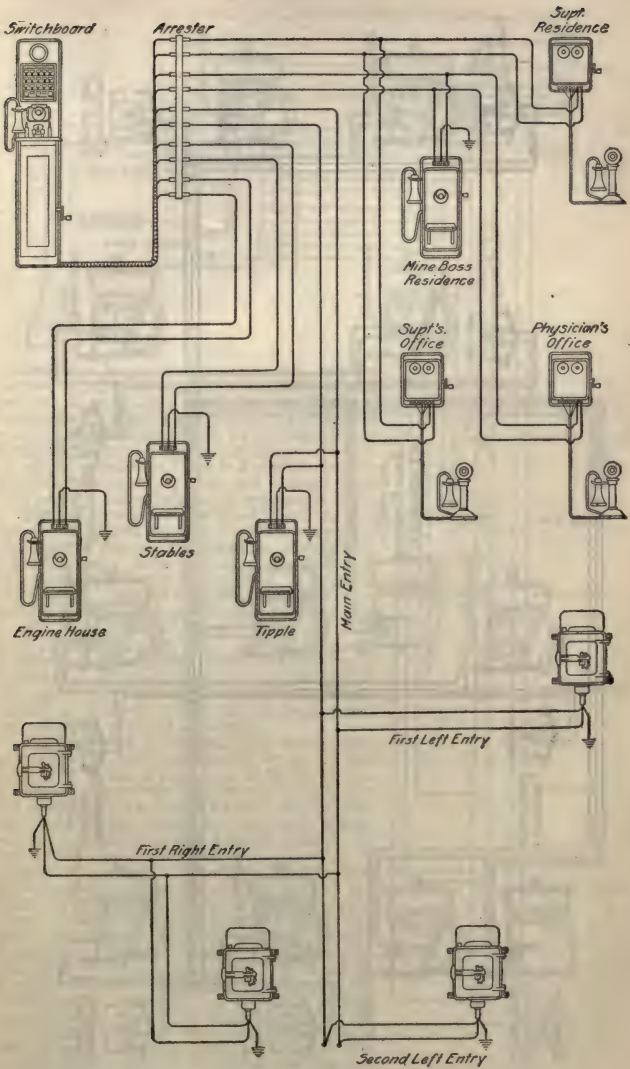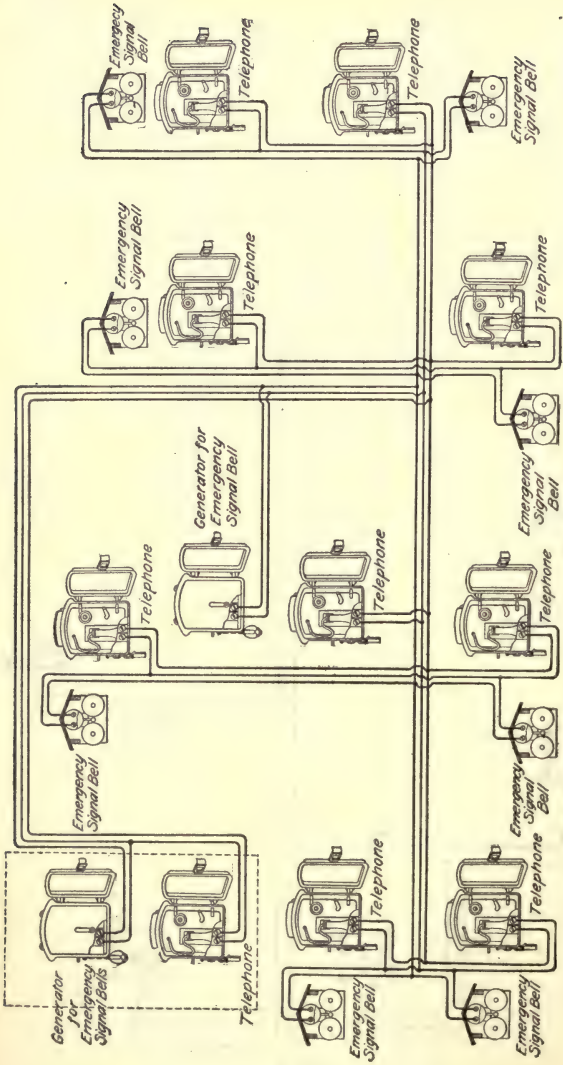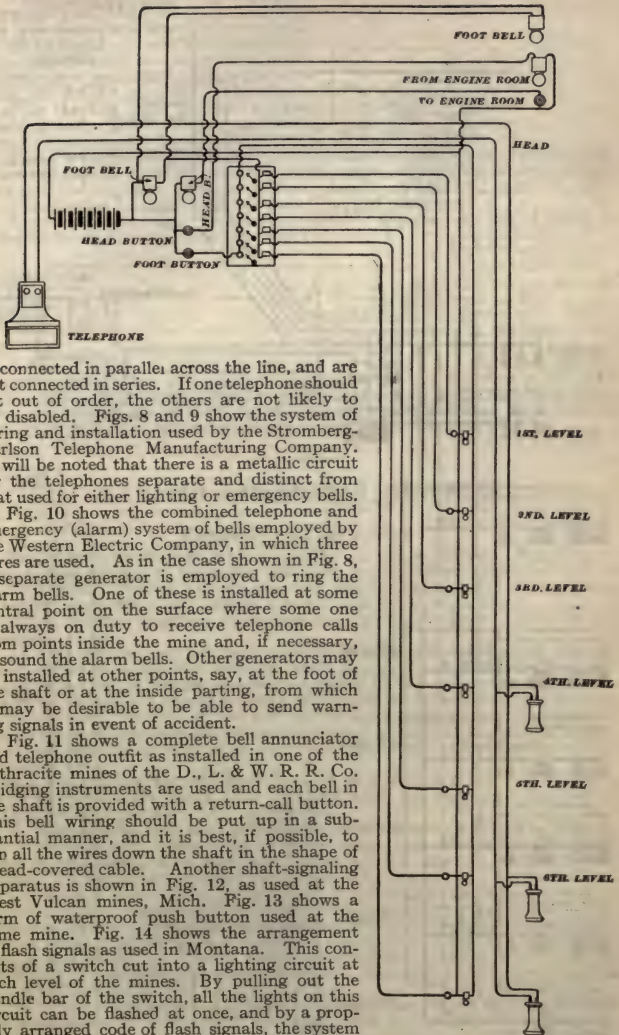
FIG. 11

or connected in parallel across the line, and are not connected in series. If one telephone should get out of order, the others are not likely to be disabled. Figs. 8 and 9 show the system of wiring and installation used by the Stromberg-Carlson Telephone Manufacturing Company. It will be noted that there is a metallic circuit for the telephones separate and distinct from that used for either lighting or emergency bells.

Fig. 10 shows the combined telephone and emergency (alarm) system of bells employed by the Western Electric Company, in which three wires are used. As in the case shown in Fig. 8, a separate generator is employed to ring the alarm bells. One of these is installed at some central point on the surface where some one is always on duty to receive telephone calls from points inside the mine and, if necessary, to sound the alarm bells. Other generators may be installed at other points, say, at the foot of the shaft or at the inside parting, from which it may be desirable to be able to send warning signals in event of accident.

Fig. 11 shows a complete bell annunciator and telephone outfit as installed in one of the anthracite mines of the D., L. & W. R. R. Co. Bridging instruments are used and each bell in the shaft is provided with a return-call button. This bell wiring should be put up in a substantial manner, and it is best, if possible, to run all the wires down the shaft in the shape of a lead-covered cable. Another shaft-signaling apparatus is shown in Fig. 12, as used at the West Vulcan mines, Mich. Fig. 13 shows a form of waterproof push button used at the same mine. Fig. 14 shows the arrangement of flash signals as used in Montana. This consists of a switch cut into a lighting circuit at each level of the mines. By pulling out the handle bar of the switch, all the lights on this circuit can be flashed at once, and by a properly arranged code of flash signals, the system can be used for communicating between the surface to any part of the mine, and between different portions of the mine.

| Return Wire |
| Grade Bell |
| Timber Cage |
| Cage for Men |
| East Skip |
| North Cable |
| West Skip |
| South Cable |
| Indicator Trip |

Hoisting Engine House.

Batteries.

North Bell West Skip.

South Bell East Skip.

Timber Cage.

Men's Cage.

North Cable Bell.

South Cable Bell.

Landing for Pockets.

Grade Bell

Battery.

Surface.

6th Level.

8th Level.

9th Level.

10th Level.

11th Level.

12th Level.

FIG. 12

FIG. 13

Mains

Switch

Push Button

Flash Station Lights

100"

200"

300'

400'

500'

FIG. 14

A system of signaling by which signals can be sent to the engine room from any point along the haulage road is shown in Fig. 15. The bare conductors $a$ and $b$ leading from the battery are supported on insulators parallel to each other along the roadside, and about 6 in. apart. A short iron rod, placed across the wires $a$, signals to the engineer, or by simply bringing the two wires together a signal may be sent. When the engineer hauls from different roads, the signaling system



FIG. 15

should be supplemented with indicators, so that when the bell rings the indicator will show from which point the signal came, and in case several signals were given at the same time, the engineer should not heed any until the indicator shows that a complete signal came from one place.
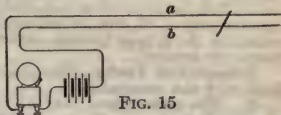
A system of signaling for showing whether or not a section of track is occupied by another motor is shown in Fig. 16. White lights indicate a clear
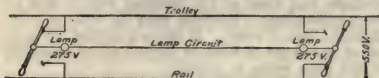


FIG. 16

track and darkness an occupied section. A single-center hinge, double-handle switch at each signal station is used and a touch of the handle throws the switch in the desired direction. The switches are placed in the roof, $4\frac{1}{2}$ ft.

above the rails within easy reach of the motorman. Each switch is provided with a spring (not shown in the figure) which, drawing across the center hinge when the handles are in their central position, insures a perfect contact when the switch is inclined toward either the trolley or rail-terminal plug.

# DYNAMO AND MOTOR TROUBLES

## SPARKING AT BRUSHES

Faults in dynamos and motors may be classed as follows: Sparking at the brushes; heating of armature, field coils, and bearings; noise; too high or too low speed. Besides, a motor may stop, fail to start, or may run backwards or against the brushes and a dynamo may fail to generate electricity.

**Brush Faults.**—Sparking at brushes may be due to some fault with the brushes.

1. The brushes may not have been set diametrically opposite one another because they were not set properly while at rest by counting the bars, by measurement, or by the use of reference marks on the commutator. Brushes can be set properly, if necessary, in an emergency while the machine is running by bringing the brushes on one side to the least sparking point by moving the rocker-arm, and then adjusting the brushes on the other side to the least sparking point by moving in or out the brush holder and clamping in new positions.

2. The brushes may not have been set at a neutral point. In this case, move the rocker-arm slowly back and forth until the sparking is stopped or reduced to a minimum.

3. The brushes may not have been properly trimmed. If sparking begins from this cause and the dynamo cannot be shut down, bend back and cut off the loose and ragged wires; retrim as soon as possible after the machine is shut down. If there are two or more brushes in each set they may be changed one at a time on a low-voltage machine while it is running. If there is a singing or hissing at the brushes apply a little mineral oil or better yet vaseline, or hold a piece of stearic-acid candle on the commutator a moment and then wipe off, leaving just a faint trace of oil or grease. To eliminate noise, it may be necessary to lengthen or shorten the brushes in the holders until a firm but gentle pressure is maintained free from vibration. Use only a cloth, never waste, to wipe off commutator.

4. The brushes may not be set in line. In this case, adjust each brush until the brushes rest on the same line and square with the commutator bar, bearing evenly throughout their width, unless purposely staggered. In case of a broken circuit in an armature winding, it is sometimes necessary to bridge the break temporarily by staggering the brushes until the machine can be shut down,

when it should be repaired; this is only a temporary makeshift to reduce sparking during a run when the machine cannot be stopped for repairs.

5. The brushes may not make good contact. In this case, clean the commutator of oil, dirt, or grit so that the brushes will bear properly on it and adjust by the proper tension screws and springs until a light but firm and even contact is secured.

**Commutator Faults.**—Sparking at the brushes may also be due to some fault with the commutator.

1. The commutator may be rough, worn in grooves and ridges or out of round. In any case, the commutator should be ground down with fine sandpaper (never emery in any form) laid in a piece of wood curved to fit the commutator, or a curved suitable stone, and finally polished with soft, clean cloth. If the commutator is too bad to grind down, it may be turned down with a special tool and rest while the commutator is turning slowly in its own bearings or the armature may be removed from the machine, placed in a lathe, and the commutator turned off with light cuts. An armature should have from $\frac{1}{16}$ to $\frac{1}{8}$ in. end motion so as to distribute the wear of the brushes evenly and to prevent their wearing ruts in the commutator. Brushes may be shifted sidewise occasionally to assist in the distribution of wear. If there is no end motion, the shoulders should be turned off of the shaft or filed or turned off of bearings until the armature has some free end play while in motion.

2. One or more commutator bars may be too high. A high bar should be forced down carefully with a wooden mallet or block of wood, care being taken not to bend, bruise, or injure the bar, and then tighten the clamping rings; if this does not remedy the fault, the high bar should be filed or trimmed down to the level of the other bars, or the commutator ground or turned down a little. High bars may cause the brush to jump or vibrate so as to sing.

3. One or more commutator bars may be too low. In this case, the commutator may be ground or turned down until no bearings are below a true cylindrical surface.

## HEATING OF ARMATURE, FIELD COIL, AND BEARINGS

**Heating of Armature.**—The heating of an armature may be due to the machine being overloaded, to a short circuit, a broken circuit, or a cross-connection; the causes and remedies for such conditions are given under the head of Miscellaneous Troubles. Moisture in the armature coils should be removed by drying out the coils with a slow heat secured by sending through the armature current that is regulated so as not to exceed the proper amount. If the moisture is so bad as to cause a short circuit or a cross-connection or to heat the armature too much, it may be dried out by the heat produced by its under current while running.

The heating of an armature may also be due to eddy currents in the armature coil. If the iron of the armature is hotter than the coils after a short run, the faulty armature core should have been more laminated and the laminations should have been better insulated from one another. There is no remedy but to rebuild the armature.

**Heating of Field Coils.**—The heating of field coils may be due to excessive current in the field circuit, eddy currents in the pole pieces, or moisture in the field coils. Excessive currents in the field circuit of a shunt machine may be reduced by decreasing the voltage at the terminals by reducing the speed or increasing the resistance of the field coils by winding on more wire, by rewinding with finer wire, or by putting a resistance in series with the field. Excessive current in the field circuit of a series machine may be reduced by shunting a portion or otherwise decreasing the current in the field coils or by taking off one or more layers of wire or rewinding the field coils with closer wire. Excessive current in a shunt or series machine may of course be due to a short circuit or from moisture in the coils acting as a short circuit.

Eddy currents in pole pieces may cause the pole pieces to become hotter than the coils after a short run. This is due to faulty construction or to fluctuating current; if due to the latter, the current should be regulated.

Moisture in the field coils may cause the coils to be lower in resistance than normal or it may cause a short circuit or a contact between the coils and the iron of the machine. The coil should be dried out, as already explained. Excessive current may be due to a short circuit or to moisture in the coils acting as a short circuit.

**Heating of Bearings.**—Heating of bearings may be due to not enough or a poor quality of oil. The remedy is to use plenty of oil and see that it is fed properly. Only the best quality of mineral oil, filtered clean and free from

grit should be used, and care must be taken not to flood the bearings so as to force the oil upon the commutator or into the insulation of the brush holders, as it will then gradually char and gather copper dust and form a short circuit. Vaseline, cylinder oil, or other heavy lubricant may be used if the ordinary oil fails to remedy the hot boxes; such lubricant should be used until the run is over, when the bearings should be cleaned and adjusted.

When ice is used to cool the bearings, care must be exercised not to let it get into the commutator or armature, which it may ruin unless they are water-proof as in the case of street-car and some other motors. A machine must never be shut down because of a hot bearing until all the remedies given therefor have been tried and proved of no avail. If it is absolutely necessary to shut down a machine, take the belt off as soon as possible, do not allow the shaft to stick in stopping, get the bearings out and cool off as soon as possible, but not in water, as this may ruin them. Then scrape, fit, clean, and polish the bearings and shaft and see if it can be turned freely by hand before putting on the belt and starting again. Use none but the best of mineral lubricating oil. New oil and oil from self-oiling bearings should be filtered before being used.

Heating of bearings may be due to dirt, grit, or other matter in bearings. In this case wash out the grit by flooding with clean oil until the run is over; then clean out the bearings, being careful, however, not to flood the commutator or brush holders. When the run is over, remove the cap of the bearing and clean the journals and bearings very carefully, then replace caps and lubricate well. Allow bearings to cool off naturally. It may be necessary to entirely remove the bearings and clean the grit away, polish all parts, and set up again.

Should rough journals or bearings cause hot bearings, polish the bearings in a lathe, remove cuts, scratches, and marks; then fit new bearings of Babbitt or other metal.

If journals are too tight in bearings, loosen the bolts in the cap of the bearing, put in very thin pieces of packing or sheet metal between the caps and the base, retighten the bolts until the run is over; then make the journal bearing smooth and so it can be rotated by hand. If necessary, turn down, smooth, and repolish the journal or scrape the bearings to a proper fit.

In the case of a bent or sprung shaft, bend it by carefully springing the shaft or turning it in a lathe.

If a bearing is out of line, loosen the foot of the bearing until the armature can be turned freely by hand with the belt off, being careful to keep the armature in the center of the polar space. Ream out the bolt in dowel-pin holes and fit new dowels to allow the new position to be retained when the bolts are drawn up tight. If the shaft must be raised or lowered, pack up or trim down the feet of the bearing to allow the proper setting.

Heating of bearings may be caused by end pressure of the pulley hub or shaft collars against bearings. The foundation should be level and the armature should have a free end motion. If there is no end motion, turn or trim off the ends of the bearings or hub on the shaft until the proper end motion is secured. Line up the shaft pulley and belt so that no end thrust is maintained on the shaft by a sidewise pull of the belt. The armature should have free end play while in motion.

If the heating of the bearings is caused by too great a load or strain on the belt, reduce the load so that the belt may be slackened and yet not slip; avoid vertical belts if possible. Use larger pulleys, wider and longer belts, run slack on top to increase adhesion and pull of belt without excessive tightening, so that a full load may be carried. Belts should be tightened just enough to drive a full load smoothly without that vibrating or flapping which may cause the lamps to flicker.

An armature out of center in polar space may cause hot bearings. The bearings may be worn out, thereby allowing the armature to move out of the center and to need replacing. Center the armature in the polar space and adjust the bearings to a new position, as already explained. File out the polar space to give equal clearance all around or spring the pole pieces away from the armature and secure it in place; this will be a difficult, if not an impossible, job in large machines.

## NOISE

If the armature strikes or rubs against the pole pieces, bend or press down the projecting wires and secure strongly in place with tie-bands or wire. File out the pole pieces where the armature strikes. The bearings may be worn

out, thus allowing the armature to move out of the center and may need readjustment. It is sometimes, though seldom, necessary to file out the polar space to give equal clearance all around, or to spring the pole pieces away from the armature and to secure it in place. This is a difficult, if not an impossible, job on large and rigid machines.

Collars or shoulders on the shaft or the hub or web of the pulley may strike or rattle against the bearings, because the bearings are worn out and too loose. New bearings may be required if the remedies given for bearings out of line and for loose screws, bolts, or connections do not remove the trouble.

If the noise is caused by loose screws, bolts, or connections, tighten them all to a firm bearing and keep them so by daily attention. The jar and movement of the machine tends to work screwed connections loose when they are not held by check-nuts, cotter pins, or some other device designed for that purpose.

Singing or hissing of the brushes may be stopped by the remedies described for brushes not properly trimmed. Sometimes, it may be necessary to apply a little mineral oil, preferably vaseline, or a piece of stearic-acid candle against the commutator and then wipe it off; just a faint trace of oil or grease is all that is necessary. The brushes may be adjusted in the holders until a firm, but gentle, pressure free from any vibration is secured. The trouble may be due to a faulty commutator, the remedies for which have already been given.

Flapping or pounding of belt joints will be remedied if the ends of the belt are properly laced or joined together, or an endless belt used.

If belts slip from overload, use larger pulleys, wider and longer belts, and run with the slack side of the belt on top.

To stop the humming of armature lugs, or teeth as they pass the pole pieces, slope the ends of the pole pieces, in order that the armature teeth shall not pass the edges all at once. Decrease the magnetism of the field or increase the magnetic capacity of the teeth.

## REGULATION

**Speed Too High.**—Too high a speed may cause the engine to fail to regulate with a varying load, in which case adjust the governor or other means of regulation. If this cannot be accomplished, get a better engine. The engine should regulate closely with proper steam supply from no load to full load.

A series motor may run too fast on account of receiving too much current for the load that it carries, and hence the motor runs away. In the case of a series motor on a constant-potential circuit, insert a resistance in series with the motor in order to cut down the current; or use a proper regulator or controlling switch, or change to automatic speed regulating motor.

The regulator may not be properly set, the proper amount of current may not be used, or the motor may not be properly proportioned and, therefore, may fail to regulate properly. The regulator should be adjusted to control the speed, the proper current, voltage, and rheostat should be used or get a motor properly designed for the work.

**Speed Too Low.**—It may be necessary to drive the dynamo with a better engine that will regulate better with proper steam supply from no load to full load. The motor may be overloaded, the causes and remedies for which have been previously given. There may be a short circuit in the armature, a striking or rubbing of the armature against the pole pieces, or an unusual amount of friction, all of which have been explained.

## MOTOR STOPS, FAILS TO START, OR RUNS BACKWARDS OR AGAINST THE BRUSHES

The stopping of a motor or its failure to start may be due to no load. The stopping may also be due to the motor's being greatly overloaded. In this case reduce the load to the proper amount that the motor is rated to carry.

Sometimes the stopping is caused by excessive friction, due to the heating of the bearings, the cause and remedies for which have been given. Open the switch and keep it open and the arm of the rheostat on the off-position while locating and eliminating the trouble; then close the switch and shift the arm gradually to the on-position to see if everything is correct. With a series motor no great harm will result from the motor stopping or failing to start; with a shunt motor on a constant-potential circuit the armature may and probably will burn out or the fuse will blow.

The stopping of the motor may be due to the circuit being open on account of the safety fuse being melted, a broken wire, a broken connection, the brushes not being in contact with the commutator or brush holder, or an open switch.

In any case, see that the switch is in good order and makes its connections properly. Then, if necessary, open the switch, locate and repair the trouble, and replace. A melted fuse should not be replaced until the fault is corrected, for otherwise the fuse will melt again when the motor is started up. If the open circuit is caused by a fault in the armature or with the brushes, the remedy has already been given. If the current should fail or be shunted off from the machine, open the switch, return the starting lever to its off-position, and wait until the current is again supplied to the line, testing from time to time by closing the switch and moving the starting lever to close the circuit.

When the trouble is due to a short circuit of the field, armature, or switch, test for and repair the trouble if possible, carefully looking over the insulation of binding posts and brush holders for poor insulation, oil, dirt, or copper dust. Such causes and remedies are given more fully under Armature Faults due to short-circuited coil.

When the trouble is due to wrong connections through the motor, connect up the motor properly, referring to a correct diagram of connections; if same is not to be had, try reversing the connections to brush holders or make other changes until the correct connections are secured for the direction of rotation desired; then connect up permanently.

## FAILURE OF DYNAMO TO GENERATE

**Reversed Residual Magnetism.**—The dynamo may fail to generate because the residual magnetism is reversed, owing to reversed current through field coils due to earth's magnetism, proximity of another dynamo, or too weak residual magnetism. In this case, a current should be sent from another dynamo or from a storage or primary battery through the field coils in the proper direction to correct the fault. The polarity may be tested by holding a compass needle as near as convenient to the center of each pole piece and the connections of any or all of the field coils may be changed until the proper polarity is obtained.

When the reversed residual magnetism is caused by reversed connections, connect properly for the direction of rotation desired, referring to proper diagram of connections if obtainable. See that connections for series coils (in compound dynamo) are properly made as well as those for the shunt coil. Make such changes in connection as may be necessary to give the desired and correct rotation.

If the brushes are not in their right position, shift them until evidence of improvement is secured. The position of the brushes for the best generation of energy should be clearly understood and is generally at or near the neutral point, as has already been stated.

**Short Circuit in Machine.**—When the failure of the dynamo is due to a short circuit in the machine, the procedure is the same as when a similar fault occurs in the motor.

**Short Circuit in External Circuits.**—If a lamp circuit or other device or part of a line is short-circuited or grounded, it may prevent the building up of the shunt, or compound, field of the dynamo. In this case, look for and remedy the short circuit before closing the switch. This fault, also, should be treated the same as similar faults in motors.

**Field Coils Opposed to One Another.**—If some of the field coils should be opposed to one another, reverse the connections of one or more of them and test the pole pieces with a compass. Alternate poles should show opposite polarity. If the pole pieces are found to be of proper polarity or are so connected as to give the proper polarity, and if the dynamo does not build up, try the remedies given under reversed residual magnetism due to a reversed current through the field coils, earth's magnetism, proximity of another dynamo or to too weakened residual magnetism. If current is not then produced in the proper direction, reverse field connections or recharge in proper direction.

**Open Circuit.**—When the failure of the dynamo to operate is due to an open circuit, it may be that the brushes are not in contact. The remedy for this fault has already been given.

In the case of a broken or melted safety fuse, open the switch, look for and repair the trouble and replace the fuse. A dead short circuit should blow the fuse and a new fuse should not be put in until this fault is removed, as it will simply blow the fuse again when the switch is closed. The remedy for this trouble is obvious.

If the external circuit is open or not properly connected, locate the trouble and repair it while the dynamo switch is open. The remedy for an open circuit in the armature has already been given.

**Overloaded Dynamo.**—If the dynamo fails to generate because of an overload, reduce the load. After the dynamo comes up to full voltage, as shown by lamp or voltmeter, close external circuits in succession, watching and regulating the voltage. If the load consists entirely, or partly, of incandescent lamps, shut off some of the lamps. If the insufficient voltage generated is due to too weak a field, gradually turn the regulating switch to cut resistance out of the field rheostat until the proper voltage is secured.

## MISCELLANEOUS TROUBLES

**Weak Magnetic Field.**—A weak magnetic field may be due to a broken circuit in the field, or to a short circuit of one or more coils, or to a dynamo not being properly wound, or without having the proper amount of iron. If a broken circuit is outside of the field coils where it is possible to get at it, it should be repaired. If the break is inside of a winding, that winding will have to be rewound. If the machine is not properly wound or does not contain the proper amount of iron there is no remedy except to rebuild the machine.

**Excessive Current in Armature Due to an Overload.**—An overload in a dynamo may be due to there being too many incandescent lamps on the circuit; or to a ground and leak from a short circuit on the line. In a motor, an overload may be due to excessive voltage on a constant-potential circuit, or excessive amperage on a constant-current circuit; it may also be due to friction or to too great a load on the pulley. The load should be reduced by degrees by cutting out a number of incandescent lamps on the circuit, or by removing a dead short circuit. A dead short circuit should blow the safety fuses or operate the circuit-breakers. The machine should be shut down, the fault located and repaired, and a new fuse put in or the circuit-breaker restored before starting again; fuse should not be inserted until the fault is corrected, as it is very likely to blow again on starting up the machine. A motor should be operated with the proper amount of current and no more, and a rheostat or controlling switch should be used for starting it. Trouble due to friction should be remedied by eliminating the excessive friction.

**Armature Faults.**—In the case of a short-circuited coil in the armature, look for copper dust, solder, or other metallic particles between the commutator bars. See that the clamping rings are perfectly insulated from the commutator bars and that carbonized oil, copper dust, or dirt is not causing the short circuit. In the case of a short circuit inside the armature, remove the armature from the machine and remove and rewind the defective coil; this may require rewinding the entire armature. Examine the insulation of the brush holders for the fault; dirt, oil, and copper dust may make a short circuit from the brush holders to the rocker-arm and thus short-circuit the machine.

In the case of a broken circuit in the armature, bridge the break temporarily by staggering the brushes until the machine can be shut down; then test out and repair. This is only a temporary makeshift in an endeavor to stop bad starting before the dynamo can be shut down. If the dynamo can be shut down, look for and repair the broken or loose connection on a commutator bar. If a coil is broken inside, rewinding of the armature is the only sure remedy, although the break may be temporarily bridged by hammering the disconnected bar until it makes contact across the insulating material with the next bar; this remedy is of doubtful value, and if done the bars must be repaired again when the fault is permanently remedied. Commutator lugs may be temporarily soldered together with or without a piece of heavy copper wire soldered to both bars, thus cutting out the broken coil. Care should be taken not to short-circuit a good coil, and thus cause sparking.

A cross-connection in the armature may have the same effect as a short circuit and is to be treated as such. Each coil should show a complete circuit without being crossed with any other coil.

## GENERAL PRECAUTIONS

Cleanliness about a dynamo-electric machine is imperative; dirt, oil, or copper dust may prove sources of great annoyance or damage. Small tubes, bolts, or pieces of iron must be kept away from the dynamo, as the magnetism may draw them into or cause them to fall upon the rotating armature and ruin it. Hence, loose articles of any kind must never be placed upon any portion of a dynamo-electric machine. It is preferable to use brass or copper oil cans, as they are non-magnetic. All connections should be clean and firm. All screws and bolts should be looked over daily and, if necessary, tightened. The brushes should not rest on the commutator when a dynamo is idle.

# GENERAL RULES FOR HANDLING ELECTRICITY

In coal-mining practice, the pressure, voltage, or difference of potential in or on any circuit, machine, or other piece of equipment, is frequently made a basis for classification. Low-pressure circuits are those in which the difference of potential at no time exceeds 250 to 300 volts; in medium-pressure circuits, the voltage does not rise above 550 to 650; and in high-pressure circuits, the voltage while above 650 is commonly less than 3,000. Long-distance transmission lines operate under voltages as high as 100,000 and installations at 150,000 volts are contemplated. However, pressures such as these are transformed in a special transformer house outside the mine into low or medium voltage before being conveyed underground, and, so, are not reckoned with in ordinary mine practice.

No voltage higher than medium (say, 550 volts) should be used anywhere in any mine for the actual operation of electrical machinery, and nothing above low voltage (say, 250 volts) should be employed at the face for operating coal- or rock-cutting machinery.

All high-voltage currents should be carried in properly insulated cables, either through bore holes drilled from the surface or along passageways upon which men do not travel, to a transforming station where they should be converted to low- or medium-pressure currents for farther conveyance to the point of application. In gaseous mines, high-voltage cables should be installed in the intake airway only, and high-voltage motors and transformers should be installed only in suitable chambers ventilated by a current of intake air that has not passed through or by a gaseous district. High-voltage transformers should have a normal capacity of not less than 5 K. W., and high-voltage motors should have a normal capacity of more than 15-brake H. P.

All metallic coverings and armoring of cables (except trailing cables) and the frames and bedplates of generators, transformers, and motors other than low-voltage portable motors, should be properly grounded, as should be the neutral wire of three-wire continuous-current systems.

When handling live wires or when making repairs to the live parts of machines, a person should wear rubber gloves or should stand upon a mat of rubber or other insulating material. Every mine should have a special map of the workings, made upon a sufficiently large scale (not less than 200 ft. to the inch) to show clearly the position of all wires, cables, conductors, transformers, trolley lines, switches, lights, and all fixed machines such as pumps, fans, etc. The plan should indicate the size, voltage, etc., of all motors and other apparatus, and the duty performed by each. In addition, the map should show the location of all signal and telephone wires, bells, telephones, and the like. In the event of a breakdown or in event of any portion of the equipment becoming alive, the current should be shut off, the trouble located, and repairs made at once.

All single switches, circuit-breakers, and other electric instruments should be mounted upon insulating bases of some suitable material. Switchboards should be set at least 3 ft. from the rib if the current is medium voltage, and 4 ft. if the current is high voltage; they should be accessible on all sides and combustible material should not be used near them. Insulating floors or mats should be provided. All high-potential feeder circuits with a capacity of 25 K. W. or over, should have, above ground, a switch on each pole, and an automatic circuit-breaker on one pole of direct-current systems, and on two poles of polyphase, alternate-current systems. Ground-return direct-current circuits should have a switch and circuit-breaker on the ungrounded side, and fuses may be substituted for circuit-breakers where the capacity of the line is 25 K. W. or less. High-potential, alternating, feeder circuits should have, at the surface, on each pole an oil-break switch provided with an automatic overload trip. All circuits should have an ammeter. Transformer rooms should be fireproof, and be provided with buckets of clean dry sand for use in case of fire, and no unauthorized person should be allowed to enter them. Where circuits enter or leave a transformer, they should be protected by circuit-breakers, etc., as on the surface.

While medium- or low-pressure wires leading into the mine may be bare, except in gaseous sections thereof, high-pressure wires should be enclosed in lead or other armored covers. Underground cables, except trailing cables, should be supported on insulators unless provided with a grounded metallic covering. The conductors connecting lamp and power supply should always be insulated. Lightning arresters should be provided at the generating

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station, at the mine mouth if this is 500 ft. from the station, and at intervals of not more than 1,000 ft. if this distance is exceeded. In gaseous mines or through gaseous portions of a mine, the potential should not be above medium and the currents should be brought in through the intake. Each pole at the junction of a branch and main circuit should have a switch of not less than 100 amp. capacity. One side of grounded circuits should be carefully insulated from the earth. Trolley lines should be placed as far to one side of the entry as possible and should be supported so that the sag between points of support does not exceed 3 in., except where the clear height of the wire above the rail is 5 ft. or more and where, the increased sag thus permissible, does not cause the trolley in passing to force the wire upon the roof. All wires, except telephone, shot-firing, and signal wires should be on the same side of the entry as the trolley wire. Where men are constantly compelled to work or pass under bare power wires less than 6½ ft. above the top of the rail, the wires should be set in channels cut in the roof or in inverted wooden troughs, the sides of which are not less than 5 in. deep. Branch trolley lines should be provided with some device by which the current may be shut off from them. Track rails should be of sufficient size to provide ample capacity for the return circuit, should be bonded rail to rail, and cross-bonded at intervals of not less than 200 ft., and should be frequently bonded to any air or water pipes where such exist, in order to eliminate difference of potential between the rails and pipes and to prevent electrolysis. Lighting wires should not be wrapped or tied about the stems or studs of trolley hangers, but should be inserted in holes drilled therein, held in place by a setscrew, and should be grounded into the track circuit. Lighting wires should be strung on porcelain or glass insulators and, unless protected with some insulating covering, should be strung at least 3 in. apart. All joints in conductors should be soldered, if at all possible, and joints in insulated wire should be carefully reinsulated. The exposed ends of cables, where they enter fittings, should be protected, so that moisture cannot enter the cable or the insulating material leak. All holes through which bare wires pass through metal frames or into boxes or motor casings, should be bushed with insulating material, which should be gas-proof where necessary. Extra precautions should be taken to see that power cables in shafts are highly insulated and very substantially secured in place. If a cable cannot sustain its own weight, it should be supported at intervals of not over 25 ft. by suitable grips. Hanging cables should be boxed in, but if that is not possible, they should be hung clear of the walls so that they may give and not break under a blow from falling materials. Cables and feed-wires should be strung so as to clear passing cars or motors by at least 12 in. and, if this not possible, should be protected by guards. Further, unless metal-covered, they should not be fixed to the ribs or timbers with uninsulated fastenings; and while repairs are being made in the entry or while blasting is going on, they should be protected from injury in some manner. Trailing cables should be protected with extra strong insulating material, should be frequently examined for defects, and if these are found, should be rejected until the proper repairs are made. Where such cables are divided at the motor, the split should be as short as possible, and they should be securely clamped to the frame of the motor, so they may not pull out from the connections. In gaseous portions of the mine, fixed, flame-proof, terminal boxes with a switch and fuse on each pole of the circuit should be provided where trailing cables are attached to the power lines. The switch should be operatable only from outside the box when it is closed, and should be so arranged that the trailing cables cannot be detached or removed when the switch is closed.

All switches, circuit-breakers, and fuses should have incombustible bases. Open-type fuses with terminals are permissible in non-gaseous parts of a mine, but where gas is generated they, and switches and circuit-breakers, must be inclosed in explosion-proof casings or must break under oil. Fuses should be marked with the maximum current they are allowed to carry and should be adjusted and replaced only by a competent person. Circuit-breakers should trip at from 50 to 150% of their rated capacity, and should be provided with an indicator showing at what current they are set to trip. On feeder lines, the circuit breaker should trip at the end of 10 sec. under an overload of 50%. Except on signal lines, all making or breaking of circuits should be done by means of switches, which should be so arranged that they cannot be closed by gravity; except that connections between gathering locomotives and mining machines and the trolley line may be made by means of hooks or similar devices.

Stationary motors and their starting resistance should have a fuse on one pole or circuit-breaking device where direct current is used, and on both poles where alternating current is employed, and should be provided with switches to cut off the power entirely. In gaseous parts of the mine, motors should be placed in a room ventilated by a separate split of intake air, or, if this is not possible, all current-carrying parts, starters, connections, terminals, and the like, should be enclosed in non-inflammable explosion-proof casings, which should be opened only by authorized persons when the power is off. Underground fan motors, not provided with a regular attendant, should be totally enclosed unless installed in a special room lined with incombustible material. In gaseous mines, a safety lamp should be provided with each machine and, on the first indication of gas, the machine should be stopped and the current cut off at the nearest switch and should not be turned on until the place has been made safe. Enclosed equipment should be regularly inspected and cleaned, motors once a week and switches once a month. In gaseous mines, a coal-cutting machine should not be operated unless the absence of gas has been proved by the use of a safety lamp, the operator should not leave the machine while it is in use, and tests for gas should be repeated at least every $\frac{1}{2}$ hr. If gas is found, the machine either should not be taken to the face, or if at the face the current should be at once cut off and the trailing cable disconnected from the power wires. A machine should never be left at the face unattended unless the power is cut off from the trailing cable. If arcing outside the machine is noticed or any defect is discovered in it, the trailing cables, etc., the power should be cut off at once and the machine put out of commission.

Electric haulage locomotives should not have a higher voltage than medium and should not be used in gaseous mines except upon an intake air-current fresh from the outside. Storage-battery locomotives are permissible in gaseous mines if the cells and other electric parts are enclosed in non-combustible explosion-proof casings.

Arc lamps should be of the enclosed type and are permissible in gaseous mines under the same conditions as electric haulage motors. The sockets of incandescent lamps should be of the weather-proof type, the exterior of which should be entirely non-metallic. Flexible lamp-cord connections should not be used except in the case of portable lamps and then only when the lamp and socket are enclosed in a heavy wire cage, which is attached to a handle through which the leading-in wires are carried. As a general rule, portable incandescent lamps of the ordinary type should not be used in gaseous portions of the mine, and in other parts thereof only for the inspection and repair of the machinery. Portable incandescent lamps, of the battery type, that have passed the requirements of the Bureau of Mines are permissible in any part of any mine. Standard incandescent lamps should be placed so that they cannot come in contact with combustible material, should be replaced by competent persons after having tested for the absence of gas, and in gaseous portions of the mine, unless ventilated by fresh intake air, should be protected by gas-tight fittings of strong glass. If the lamps are of 220 volts or higher, not over 8 c. p., and without tips, the gas-tight fittings are not necessary.

Electricity from grounded circuits should not be used for firing shots. Only trained men should be allowed to handle electric shot-firing apparatus. Shot-firing wires or cables should not be allowed to come in contact with light or power wires. Electric detonators and their leads should be of an approved type, should be kept in a dry place and should not be stored with other explosives. Shot-firing machines should be enclosed in a tight case inside of which all contacts, with the exception of the binding posts, should be made. The shot-firer alone should connect the leads to the battery and then only after the shot is actually and completely ready for firing, and all persons have sought a place of safety. After firing a shot, the leads should be at once disconnected from the current. If the shot has missed fire, the firing leads should likewise be disconnected, and no one should be allowed to approach the face until at least 5 min. thereafter.

Care should be taken to prevent signal and telephone wires from coming in contact with power or other wires, whether the same are insulated or not. The potential used for signal purposes in the gaseous parts of a mine should not exceed 24 volts, and bare wires should not be used except on haulage roads.

The electric relighting of safety lamps should be done in a special room not on the main return and where there is not likely to be an accumulation of gas. The relighting apparatus should be locked so that it cannot be handled by unauthorized persons, and all lamps should be carefully examined for defects before being reissued.

# INTERNAL-COMBUSTION ENGINES

## DEFINITIONS AND PRINCIPLES

**Internal-Combustion Engines.**—An internal-combustion engine is an engine in which power is generated by burning within the cylinder a mixture of air and gas or air and alcohol, kerosene, gasoline, or other liquid fuel. The burning of the fuel results in the production of gases of high temperature and pressure, which act directly on a piston that moves back and forth in a cylinder into which the air and fuel are admitted and from which the burned gases are discharged through suitable valves.

**Single- and Double-Acting Engines.**—Internal-combustion engines may be single or double acting. Engines in which gas is admitted to only one side of the piston are single acting, while those in which gas is admitted to each end of the cylinder alternately and is, consequently, burned first on one side of the piston and then on the other, are double acting. All haulage-motor engines and most stationary engines in which gasoline is the fuel are of the single-acting type; double-acting engines are sometimes used where gaseous fuel is available.

**Gasoline-Engine Cycles.**—As applied to a gasoline engine, the term *cycle* refers to the operations, or events, that take place within the cylinder from one explosion to the next, and by means of which the fresh charge is drawn into the combustion chamber and exploded and the exhaust gases expelled. These events always occur in the same order and are repeated after each explosion. The cycle on which an internal-combustion engine operates is one of the distinguishing features of different types.

In the first successful gas engine, the charge drawn into the cylinder under atmospheric pressure during part of the outward stroke was ignited when the piston had traversed about four-tenths of its stroke; the sudden rise in pressure due to the explosion of the gas was utilized to drive the piston to the end of its stroke and work was performed during the expansion of the hot gases. During the return stroke, the burned gases were driven from one end of the cylinder while a fresh charge was drawn in and ignited at the other end, the engine being of the double-acting type. Because of the extreme wastefulness of this engine, which was of the two-cycle type, a French scientist, Beau de Rochas, in 1870, proposed a new cycle of operations. This cycle was adopted and put into practical use by Otto, a German, who built his first compression engine in 1876; this engine is known by his name.

The Otto cycle, in its broad and strictly scientific meaning, is not concerned with the method of getting the combustible mixture into the cylinder nor that of expelling the hot burned gases. The steps of the cycle are as follows: Assume that, at the beginning of operations, the valves are closed, that the piston is at the position farthest out toward the crank-shaft, and that the cylinder is filled with a combustible mixture at atmospheric pressure. By forcing the piston inwards to the completion of the inward stroke, the charge will be compressed into the compression space, or combustion space. Now by igniting the compressed charge, the pressure will be increased still more by the heat of combustion. The pressure tends to drive the piston outwards, and as soon as the rotating crank-shaft has made the angle between the connecting-rod and crank sufficiently great, the pressure of the hot gases against the piston face will drive the crank-shaft. The burned gases expand to fill the increasing volume of the cylinder as the piston moves outwards and the pressure decreases. At the completion of the outward stroke, the exhaust valve is opened and the hot burned gases escape by expansion until the pressure falls to that of the atmosphere. This completes the Otto heat cycle.

The expulsion of the burned gases that remain in the cylinder at atmospheric pressure and the taking in of a fresh charge of combustible mixture is accomplished in two distinct ways, which are the foundation for the commercial names, *four cycle* and *two cycle*, as applied to gasoline engines.

**Four-Cycle Engines.**—A four-cycle engine is one in which four complete strokes of the piston are required to complete the cycle. In this engine the burned gases remaining in the cylinder after the exhaust valve has been opened and part of the hot gases removed by expansion are expelled in part by a

separate inward stroke of the piston, and a fresh charge is drawn into the cylinder through the inlet port by a separate outward stroke. Generally speaking, one event occurs during each of the four strokes of this cycle; that is, considering the stroke by which the charge is drawn into the cylinder as the first stroke, the mixture is compressed during the second stroke, burned during the third stroke, and the exhaust gases are expelled during the fourth stroke, after which the conditions are the same as at first and the cycle is complete. This type is sometimes known as the four-stroke Otto-cycle engine, and is in more general use than the two-cycle engine as it is much more economical in fuel.

**Two-Cycle Engines.**—A two-cycle engine is one in which only two strokes of the piston, corresponding to one revolution of the crank-shaft, are required to complete the cycle. In this cycle an explosion occurs on each downward stroke of the piston, the fresh charge being admitted and the exhaust gases expelled at or near the end of this stroke. Hence, for the same number of revolutions of the crank-shaft, there are twice as many explosions in the cylinder of a two-cycle engine as in that of a four-cycle engine. However, this does not mean that the power developed by a two-cycle engine is twice as great as that produced by a four-cycle engine of the same size and speed, for, on account of the inefficient *scavenging*, or cleaning, of the cylinder after the explosion and the lower compression pressure in the two-cycle engine, the
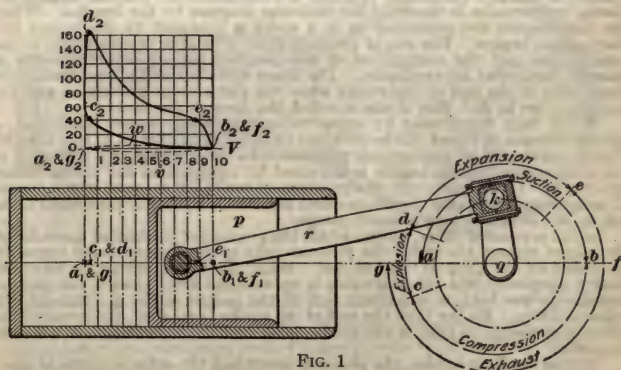


FIG. 1

explosions are not so powerful as in the four-cycle engine. It is generally estimated that a two-cycle engine of a certain size and speed will develop about 1.65 times as much power as a four-cycle engine of the same size and speed. This type is sometimes known as the two-stroke Otto-cycle engine.

**Application of Four-Cycle Principle.**—In the four-cycle engine, the first outward stroke is the *suction stroke*, the gas being driven into the cylinder by the pressure of the atmosphere or other pressure, because of the partial vacuum produced by the movement of the piston. This stroke fills the cylinder with a mixture of fuel and air at very nearly the pressure of the atmosphere. On the return stroke of the piston, all the openings leading from the cylinder are closed and the mixture is compressed. As the piston nears the end of this second stroke, which is known as the *compression stroke*, the igniter, or device by means of which the charge is fired, is operated in time to produce full ignition of the mixture at the end of the stroke. The pressure rises to three or four times that due to compression, and drives the piston forwards on its next outward stroke, which is known as the *power*, or *expansion, stroke*. Just before this stroke is completed, or as it is, the exhaust valve is opened, permitting the burned gas and uncombined air to escape to the atmosphere, and during the following inward stroke practically all of this waste material is expelled; this last is known as the *exhaust stroke*.

**Graphic Representation of Four-Stroke Cycle.**—The four strokes of the engine and the corresponding indicator diagram are shown in Fig. 1. Here,

$p$ denotes the piston; $r$ is the connecting-rod; $k$, the crankpin; $q$, the crankshaft. In the indicator diagram, the ordinates or vertical distances represent pressures, and the abscissas or horizontal distances denote the distance the piston has proceeded on its stroke. The pressures are measured from the line $OV$, which represents the pressure of the atmosphere. The line $Ovb_2$ is the suction line, and the line $b_2c_2$ is the compression line. At $c_2$, the charge is ignited, $c_2d_2$ is the explosion line; $d_2e_2$, the expansion line; $c_2f_2$, combined expansion and exhaust; and $f_2wO$ is the exhaust line. The pressures represented by the two lines $v$ and $w$ are slightly exaggerated, in order that the lines may be distinguished from the atmospheric line $OV$, which they follow very closely.

In the *suction stroke*, the crank-shaft turns in the direction of the arrow and the piston moves from the line $a_1$ to the position shown. The space between the end of the cylinder, when at the line $a_1$, and the cylinder head is called the *clearance space* or the *combustion chamber*. In this stroke, the inlet valve is open and the mixed air and gas is being drawn into the cylinder. The pressure within the cylinder drops slightly below the atmosphere, as shown by the line $v$. The valve remains open until the piston gets to the right-hand end of its stroke. The numbers at the left of the diagram represent the pressures, and those at the bottom the volumes, corresponding to the cross-lines opposite which they are written.

When the piston starts on its return stroke, the inlet valve is closed and the mixture is trapped within the cylinder and compressed. The rise of pressure during compression is shown in the indicator diagram by the line $b_2c_2$. When the compression has proceeded to $c_2$, a spark is produced by the igniter and combustion begins. The rise of pressure from $c_2$ to $d_2$ is therefore due to the compression and the combustion of the gas, but the maximum pressure is lessened somewhat by expansion. The flame spreads rapidly, and during the short time at the end of the stroke when the piston is practically at rest the pressure rises to $d_2$. This stroke is called the *compression stroke*.

It has been found that by compressing the charge before igniting it, a greater amount of power can be obtained from a given quantity of fuel than by simply burning it at atmospheric pressure. In other words, the efficiency of the internal-combustion engine is increased by compressing the charge before igniting it. Compressing the charge heats it; hence, on account of the danger of preigniting the charge the compression pressure is limited to from 60 to 75 lb. per sq. in., as shown by a pressure gauge.

In the *expansion stroke*, during which the pressure of the heated gases drives the piston toward the right, the pressure falls as the piston moves forwards, as shown by the drop in the line $d_2e_2$. When the expansion stroke has been nearly completed, the exhaust valve is opened and from $e_2$ to $V$ the drop of pressure is due both to expansion and to the escape of the gas through the exhaust valve. By the time the end of the stroke is reached, the pressure has fallen very nearly to that of the atmosphere, and the expanding gas has done its work.

During the next stroke, the piston is returning, the exhaust valve is open, and the gases are driven from the cylinder to prepare it for the reception of a new charge. There is a small rise of pressure during this stroke, due to the driving of the gas from the cylinder, indicated by the line $w$. At the end of the exhaust stroke, the exhaust valve closes, and the succeeding outward stroke begins a new cycle with the suction of a fresh charge of gas and air.

The series of operations that take place during the four-stroke cycle is as follows:

### FIRST REVOLUTION

*First Stroke.*—Outwards; suction; inlet valve open; pressure falls below atmosphere.

*Second Stroke.*—Inwards; compression; both valves closed; pressure rises; ignition before end of stroke, followed by explosion and rapid rise of pressure.

### SECOND REVOLUTION

*Third Stroke.*—Outwards; expansion; both valves closed; pressure falls; exhaust valve opens near end of stroke.

*Fourth Stroke.*—Inwards; exhaust; exhaust valve open; pressure rises very little above that of the atmosphere.

**Application of Two-Cycle Principle.**—Fig. 2 (*a*) illustrates the operation of a typical two-cycle engine, in which $p$ is the piston; $q$, the crank-shaft; $a$, the crank; $k$, the crankpin; $r$, the connecting-rod; $e$, the exhaust port; $o$, the inlet, or transfer, port; $b$, the passage leading from the crank-chamber to the cylinder; $s$, the inlet valve; $d$, a deflector on the end of the piston; and $i$, the part of the igniting device at which the spark is produced. The diagram of

pressures in the cylinder is shown in (b), while the diagram for the pressures in the crank-case is shown in (c).

The difference between the diagrams of this engine and that of the four-cycle engine should be carefully noted. When the piston is moving toward the cylinder head, it is compressing the mixture of gas and air, while at the same time it is drawing a new charge into the crank-case through the valve s. That portion of the diagrams given during this stroke is shown by full lines. In reality, the first part of the cycle must always be the suction into the crank-case before any mixture is taken into the cylinder. The line $Vfgh$ is identical with the compression and explosion line of the four-stroke cycle and covers the same series of operations; namely, compression to $f$, where ignition takes place, increase of the rate at which the pressure rises from $f$ to $g$, and the explosion line $gh$. While the piston is compressing the charge in the cylinder, the crank-case is drawing more fuel through the valve $s$, the pressure in the crank-case falling below the atmosphere, as shown by the line $v$ below $O'V'$.



(a)

(b)          FIG. 2          (c)
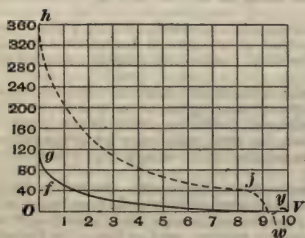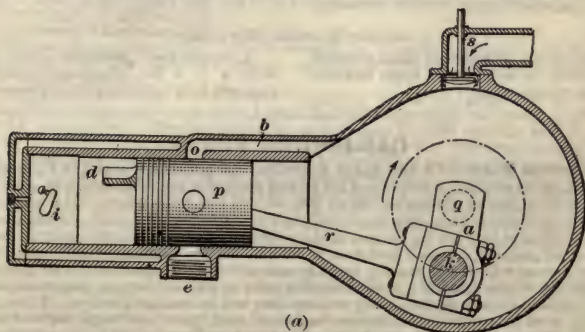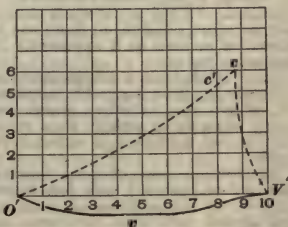
It should be noted that the diagram for the pressures in the crank-case have a different scale of pressures from the scale of the diagram for the pressures in the cylinder.

The next stroke moves the piston away from the head end, making the expansion stroke for the cylinder and the compression stroke for the crank-case, the inlet valve $s$ being closed. Before the exhaust port $e$ is uncovered, the portion of the indicator diagram from $h$ to $j$ for the cylinder and from $o'$ to $c'$ for the crank-case is drawn.

When the piston is very near the end of the outward stroke, both the inlet and the exhaust ports $o$ and $e$ are open; the exhaust gases escape from the exhaust port $e$ and the fresh charge enters through the by-pass $b$ and port $o$, and is thrown by means of the deflecting plate $d$ toward the cylinder head. The momentum of the column of exhaust gas as it leaves the cylinder is so

great that, unless there is considerable resistance in the exhaust passage, the pressure falls below that of the atmosphere, as shown by the small loop $w$, and is raised slightly, as shown by the loop $y$, when the fresh charge enters from the crank-case. If the engine is properly proportioned, none of the new mixture will escape at the exhaust port $e$, as it will be closed before the fresh charge has reached it. During this part of the stroke, the pressure in the crank-case rises from $c'$ to $c$ and then drops to $V'$, when the transfer port is opened. The following inward stroke compresses the new mixture in the cylinder and draws a new charge into the crank-case, thus beginning a new cycle.

The series of operations taking place during the two-stroke cycle are as follows:

| CYLINDER | CRANK-CASE |
|---|---|
| **FIRST STROKE, INWARDS** | |
| *Compression:* pressure rises; ignition near end of stroke, followed by explosion and rapid rise of pressure. | *Suction:* inlet valve open; pressure falls below atmosphere. |
| **SECOND STROKE, OUTWARDS** | |
| *Expansion:* pressure falls; exhaust followed by entrance of fresh mixture from crank-case. | *Compression:* pressure rises to from 4 to 8 lb.; charging cylinder; pressure falls to atmospheric pressure. |

# GAS-ENGINE FUELS

**Gaseous Fuels.**—Of the gases described on page 308 and the following pages, those generally employed for power purposes are: Natural gas, used at and within piping distance (150 to 200 mi.) of the wells where it is produced; water, or illuminating, gas, used in those cities having gas plants, although its application is limited by reason of its relatively high price; producer, or fuel, gas, used generally at iron and steel works; by-product gas, used at by-product coke ovens, which are usually built in connection with steel works or in some large city where there is a market for the coke as well as the gas. Gaseous fuels are suitable for use in stationary engines but not in haulage motors. They are rarely used in internal-combustion engines at coal mines, although natural gas, if the price is low by reason of the wells being in the coal fields, is sometimes used instead of coal under the boilers.

**Alcohol.**—Of the two kinds of alcohol, methyl, or wood, alcohol, $CH_4O$, and grain, or ethyl, alcohol, $C_2H_6O$, the former is not suited for use in internal-combustion engines as it apparently liberates acetic acid, which corrodes the cylinders or valves.

Grain alcohol has a specific gravity of .795, or 64° Baumé. It is obtained by distillation from vegetable substances containing sugar or starch, such as corn, wheat, rye, or other grains, potatoes, molasses, etc. When pure, it absorbs water more rapidly than it loses its own substance by evaporation. When diluted with 15% of water, it evaporates as if a single liquid and not a mixture.

The revenue laws of most countries require that grain alcohol must be denatured or rendered unfit for the manufacture of liquors before being sold as a fuel, by the addition of some poisonous or harmful ingredient such as wood alcohol, petroleum distillates, coloring matter, or the like. In France, the denaturing is accomplished by adding to 26 gal. of grain alcohol, 17 oz. of heavy benzine, and 10% of wood alcohol. The alcohols used are each of 90° strength or purity. To reduce the cost of the mixture below 38c., it is generally mixed with an equal volume of benzol containing 85% of benzine. In Germany, a fuel costing from 15 to 17½c. a gallon is made by adding to the grain alcohol 15% of benzol, no wood alcohol being used.

**Gasoline.**—Gasoline is produced by the distillation of petroleum, being among the first of the hydrocarbons to be given off in the manufacture of kerosene, or illuminating oil. Its boiling point varies from 158° to 176° F., its specific gravity from .66 to .67, and its density according to the Baumé scale from 80° to 78°.

Commercial gasoline is not a simple substance but a mixture of lighter and heavier products. It is rated according to its density by the Baumé scale. Owing to evaporation and other causes, the density of the gasoline as actually purchased is likely to be somewhat greater than its nominal rating and may test as low as 68°. The vapor of gasoline that forms over the liquid

consists chiefly of pentane, $C_5H_{12}$, having a specific gravity of .628; but the liquid gasoline consists of a mixture of hexane and heptane, the composition varying with the specific gravity of the gasoline.

A gasoline with a specific gravity of .683 and a boiling point of 154° F. has shown the following composition by analysis: hexane, 80%; heptane, 18%; pentane, 2%. The chemical composition is 83.8% carbon and 16.2% hydrogen; and the chemical formula is $41.86C_6H_{14}+6.48C_7H_{16}+C_5H_{12}$. This formula will aid in the calculation of the fuel value.

Commercial gasoline evaporates very readily at ordinary temperatures, but quite slowly in cold weather, and leaves small percentages of a heavier oil, which evaporates slowly or not at all. The vapor tension varies considerably with the temperature, but at 60° F. the vapor of commercial gasoline represents about 130 volumes of the liquid and sustains a water pressure of from 6 to 8 in. An explosive mixture of gasoline vapor and air is composed of the vapor of 1 part of liquid gasoline to from 8,000 to 10,000 parts of air by volume. The volume of the vapor will vary, but an average proportion will be 2.15 of gasoline vapor to 100 parts of air.

**Kerosene.**—Kerosene, or illuminating oil, the principal product of the distillation of petroleum and sometimes used in internal-combustion engines, boils at 302° to 572° F., has a specific gravity of .753 to .964, and a density of 56° to 32° Baumé.

Commercial kerosene varies in specific gravity (at 59° F.) from .760 to .820. Exceptionally light kerosene, such as the Pennsylvania light oil, has a specific gravity below .760. The boiling point of kerosene of .760 specific gravity is 302° F. and of kerosene of .820 specific gravity 536° F. Kerosene begins to give off vapor at from 100° to 120° F., and this vapor is mainly nonane, $C_9H_{20}$. Liquid kerosene is a mixture of decane, $C_{10}H_{22}$, with a little hexadecane, $C_{16}H_{34}$. The boiling points of these three liquids are as follows: nonane, $C_9H_{20}$, 277° F.; decane $C_{10}H_{22}$, 316° F; hexadecane, $C_{16}H_{34}$, 536° F. Average kerosene consists chiefly of decane. For the chemical action that takes place when kerosene is burned, that corresponding to the combustion of decane may be taken without appreciable error.

**Fuel, or Compound, Oils.**—Oils that are lighter than about 70° Baumé evaporate so rapidly that a large part is often lost before they reach the consumer. To reduce this loss on the part of the light oils and to make a market for the less salable heavy oils, the two are sometimes mixed and offered as fuel, or compound, oil or by some trade name. These mixtures are not to be confused with the fuel oil produced directly from wells and described on 395 and the following pages, which is crude petroleum. As the demand for the difficultly salable heavy oils varies, so will vary the composition of the artificial fuel oils into which they enter.

**Rating of Oil and Gasoline.**—In selecting gasoline, it is usually sufficient to know its density by Baumé's scale, this being the rating at which it is sold in the general market. For instance, "Gasoline 72 Baumé" means that the density of the gasoline is 72° of Baumé's hydrometer. Kerosene is generally rated by its flashing point. This point is the number of degrees of temperature to which it must be heated before the vapors given off from the surface of the oil will take fire from a flame held over the containing vessel. Thus, oil of 150° test is oil that will flash or take fire when heated to a temperature of 150° F. Kerosene, at ordinary temperatures, should extinguish a lighted taper when the taper is plunged into it.

**Baumé Hydrometer.**—The Baumé hydrometer shown in the figure consists of a glass tube, near the bottom of which are two bulbs. The lower and smaller bulb is loaded with mercury or shot, so as to cause the instrument to remain in a vertical position when placed in the liquid in the vessel $a$. The upper bulb $b$ is filled with air, and its volume is such that the whole instrument is lighter than an equal volume of water.

The point to which the hydrometer sinks when placed in water is usually marked, the tube being graduated above and below in such a manner that the specific gravity of the liquid can be read directly. It is customary to have two instruments: one with the zero point near the top of the stem, for use in liquids heavier than water; and the other with the zero point near the bulb, for use in liquids lighter than water.

**Comparative Value of Liquid Fuels.**—So far as their heating value per pound goes, there is not much to choose between kerosene and gasoline, each

## SPECIFIC GRAVITIES CORRESPONDING TO BAUMÉ READINGS FOR LIQUIDS LIGHTER THAN WATER

| Degrees Baumé | Specific Gravity | Degrees Baumé | Specific Gravity | Degrees Baumé | Specific Gravity | Degrees Baumé | Specific Gravity | Degrees Baumé | Specific Gravity |
|---|---|---|---|---|---|---|---|---|---|
| 20 | .9333 | 38 | .8333 | 56 | .7526 | 70 | .7000 | 79 | .6698 |
| 22 | .9210 | 40 | .8235 | 58 | .7446 | 71 | .6965 | 80 | .6666 |
| 24 | .9090 | 42 | .8139 | 60 | .7368 | 72 | .6931 | 81 | .6635 |
| 26 | .8974 | 44 | .8045 | 62 | .7290 | 73 | .6896 | 82 | .6604 |
| 28 | .8860 | 46 | .7954 | 64 | .7216 | 74 | .6863 | 83 | .6573 |
| 30 | .8750 | 48 | .7865 | 66 | .7142 | 75 | .6829 | 84 | .6542 |
| 32 | .8641 | 50 | .7777 | 67 | .7106 | 76 | .6796 | 86 | .6481 |
| 34 | .8536 | 52 | .7692 | 68 | .7070 | 77 | .6763 | 88 | .6422 |
| 36 | .8433 | 54 | .7608 | 69 | .7035 | 78 | .6730 | 90 | .6363 |

developing about 19,800 B. T. U. per lb. Kerosene, however, is about 10% heavier, so that 1 gal. of kerosene has more fuel value than 1 gal. of gasoline.

As compared with gasoline as a fuel for internal-combustion motors, alcohol exhibits several striking peculiarities. First, the combustion is much more likely to be complete. A mixture of 90°-alcohol vapor and air will burn completely when the proportion varies from 1 of the vapor with 10 of air to 1 of the vapor with 25 of air, thus exhibiting a much wider range of proportions for combustibility than is the case with gasoline. As the combustion is complete, the exhaust is practically odorless, consisting only of water vapor and carbon dioxide. Second, the inflammability of an alcohol mixture is much lower. This is due partly to the presence of water in the alcohol, which is vaporized with the alcohol in the engine and must be converted into steam at the expense of the combustion. For these reasons, the compression of an alcohol mixture is carried far above that permissible with a gasoline mixture, without danger of spontaneous ignition. The rapidity of combustion of alcohol in an engine is considerably less than that of a gasoline mixture, and for this reason the speed of alcohol engines must be somewhat slow.

With an engine of equal size, practically the same horsepower can be obtained when adapted to burning alcohol as when adapted to burning gasoline. This is true in spite of the fact that 1 lb. of alcohol contains considerably less heat energy than 1 lb. of gasoline, and it is explained by the fact that 1 lb. of alcohol requires much less air for its complete combustion than 1 lb. of gasoline. In other words, a larger quantity of alcohol than of gasoline is required to make 1 cu. ft. of explosive mixture. Approximately speaking, if there is no surplus air in either case, 1 lb. of gasoline will make 210 cu. ft. of explosive mixture, and 1 lb. of alcohol will make 120 cu. ft. As a matter of fact, a certain percentage of additional air is required, both for the most rapid combustion, and for the necessary economy of fuel. So far as can be judged, a somewhat greater proportion of air is advantageous with alcohol; but it seems to be clear that from 50 to 60% more alcohol than gasoline by weight is required to obtain the same power. On the other hand, alcohol is about 25% heavier than gasoline, so that 1 lb. of gasoline has 1¼ times the volume of 1 lb. of alcohol. Consequently, if the weight of alcohol needed for a given amount of work is 50% greater than the weight of gasoline, the volume of alcohol required will be only one-fifth greater, or in the proportion of 1.5 to 1.25.

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# TYPES OF INTERNAL-COMBUSTION ENGINES

**Internal-Combustion Engines at Mines.**—Internal-combustion engines, so far as their use at mines is concerned, may be placed in one of three general classes or groups; *stationary, portable,* or *haulage-motor engines.*

Those of the first class are permanently attached to their foundations and comprise hoisting and dynamo engines, engines used for operating station pumps, etc. They may be horizontal or vertical, may have one or more cylinders, and usually run at a speed of 300 to 400 rev. per min.; they are,
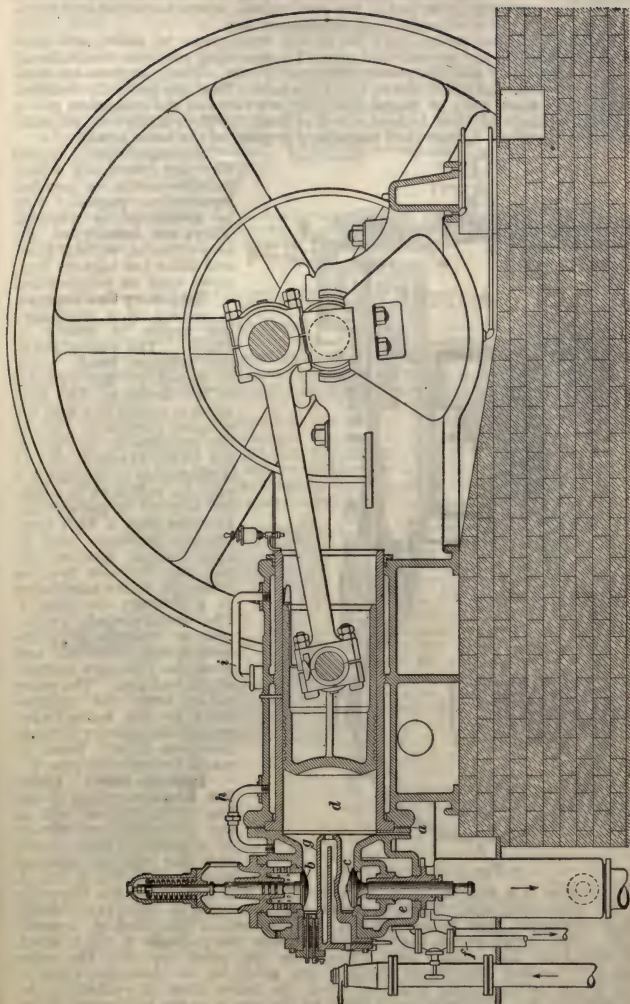
Fig. 1

therefore, relatively large and heavy for the amount of power developed. These engines are rarely found at coal mines where fuel is always cheap and water for boilers usually plentiful, but in the arid regions, where both fuel and water are scarce, they are in extensive and satisfactory use.

Portable engines, which may be moved from place to place, are quite commonly used in and around coal mines for operating concrete mixers, small pumps used in any one place only temporarily, etc. These engines are horizontal or vertical and usually have four cylinders. They are generally designed so that their speed may be varied, but they are rated at the maximum power they can produce at their highest speed of from 1,000 to 1,800 rev. per min. They are, therefore, lighter than stationary engines of the same power.

Haulage-motor, or, as they are commonly called, gasoline-motor, or gasoline locomotive, engines, may be horizontal or vertical. They are usually of the four-cylinder, vertical type, and differ but slightly from those used on automobiles.

**Stationary Gas Engines.** A section of a stationary gas engine is shown in Fig. 1. The valves are in the cylinder head, which is bolted to the cylinder at the flange $a$. The inlet valve is shown at $b$ and the exhaust valve at $c$. When the pressure is very great, the temperature due to compression in the space $d$ may be sufficient to ignite the mixture of gas and air. In order that a high compression pressure may be used without igniting the gas, the cylinder head is cooled by water introduced at $e$ through the pipe $f$. The water-cooled projection $g$ extends into the combustion chamber and cools the explosive mixture of air and gas. The water passes from the cylinder head to the water-jacket around the cylinder through the pipe $h$ and flows to waste through the pipe $i$.

**Haulage-Motor Gasoline Engines.**—One of the four cylinders, as well as the necessary mechanism of a gasoline engine suitable for use on haulage motors, is shown in Fig. 2.
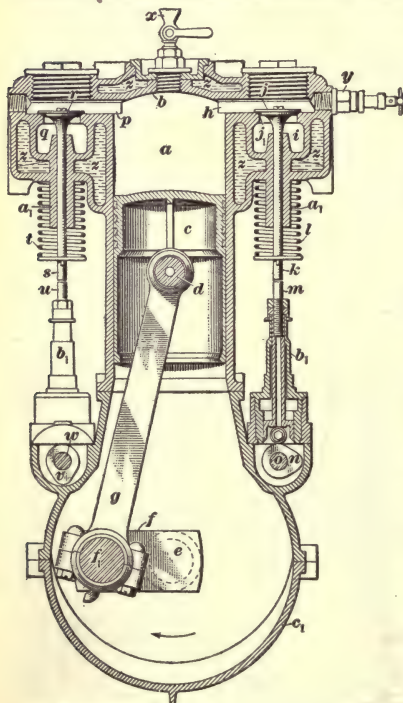


FIG. 2

The cylinder is $a$ and the cylinder head is $b$. The piston $c$ takes the place of the crosshead and therefore carries the wrist-pin $d$. The crank-shaft, crank, crankpin, and connecting-rod, are lettered $e$, $f$, $f_1$, and $g$, respectively. The charge enters the cylinder through the passages $h$ and $i$, which are closed by the conical inlet valve $j$ on the valve seat $j_1$. The valve stem $k$ is pressed downwards through the guide $a_1$, so that the valve is held closed by the spring $l$, except when the valve stem is pushed up by the push rod $m$. This push rod is lifted by the cam $n$ on the half-speed, or lay shaft $o$, and is held in position by the guide $b_1$. The letters from $p$ to $w$ mark the parts on the exhaust side of the engine corresponding to those marked by the letters $h$ to $o$ on the inlet side. The cup shown at $x$ serves the double

purpose of priming cup and compression relief valve. The spark plug is shown at $y$ and the water-jacket for cooling the various parts of the engine is at $z$. In some cases, both valves are placed on one side of the engine and are operated by cams on the same lay shaft.

The four-cylinder opposed engine differs from the vertical engine only in the arrangement of the cylinders. The shaft $a$, Fig. 3, has four cranks $b$. A



FIG. 3

cylinder $c$ with a piston $d$ and connecting-rod $e$ is placed opposite each crank. The valves and operating mechanism, which are placed above the cylinders, are not shown.

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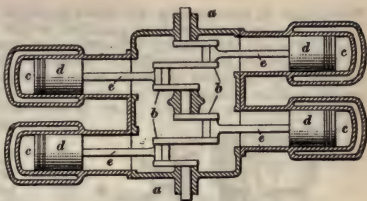
# CARBURETION AND IGNITION

**Carbureters for Constant-Speed Engines.**—When liquid fuel is used in internal-combustion engines, it must be reduced to a vapor or fine spray before it is introduced into the engine cylinder. The device by which this is done is called a *carbureter*, or *vaporizer*, one form of which, suitable for use in connection with stationary engines running at very nearly constant speed, is shown in Fig. 1.

The carbureter $a$ is attached to the side of the inlet pipe $b$. The fuel is pumped to the carbureter through the pipe $c$ into the reservoir $d$ from the side of which the nozzle $e$ is led into the inlet pipe in such a way that the surface of the fuel is just below the top of the nozzle. The surplus fuel overflows from $d$ and returns to the fuel-supply tank through the pipe $f$. The supply of fuel may be regulated by the needle valve $g$ and may be shut off by the valve $h$.
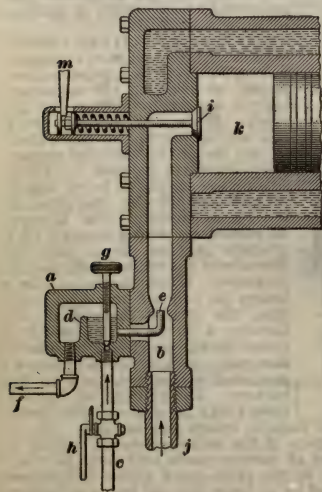


FIG. 1.

When the piston is moving out on the suction stroke, the inlet valve $i$ is opened and air is drawn in through the pipe $j$ into the combustion chamber $k$. The pipe $b$ is contracted at the level of the nozzle so that the velocity of the passing air is increased, with the result that some of the oil is sucked up from the nozzle and enters the cylinder as a fine spray or vapor mixed with the proper amount of air to secure its complete combustion.

**Carbureters for Variable-Speed Engines.**—A carbureter suitable for use with a variable-speed engine is shown in Fig. 2. In it, the spray nozzle $a$ and the tube $b$ are similar to the corresponding parts of the carbureter just described. The gasoline chamber $c$ contains a cork float that controls a small needle valve at the right through which the gasoline enters and which serves to maintain the fuel at a constant level. The flow of gasoline through the spray nozzle $a$ is regulated by the needle valve $d$ and the handle $e$. When set, this valve may be locked in position by the screw $m$. The main air inlet is at $f$ through the horn $g$ and the pipe leading to the engine is connected just above the throttle valve $j$.

As the speed of the engine is increased, the proportion of gasoline in the fuel mixture should be decreased. On the other hand, increased speed causes the air to flow more rapidly around

the nozzle $a$, thus taking up more gasoline and enriching the fuel mixture. To reduce the proportion of gasoline to the requirements of increased speed, the mixture is diluted by admitting air above the nozzle through auxiliary inlets closed by bronze balls $i$. When a certain degree of suction has been reached, one or more of these balls are lifted and air is admitted above the nozzle $a$, thus diluting the mixture. The balls are held in place by cages $k$ that are screwed into the body of the carbureter. The gasoline chamber may be drained by the cock $l$.



(a)



(b)
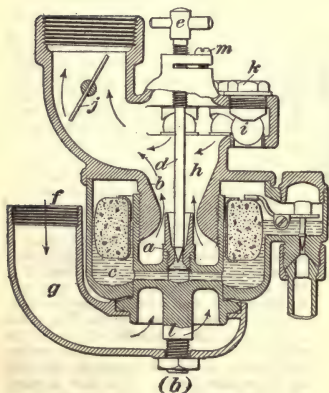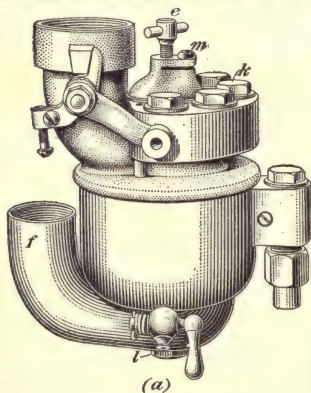
Fig. 2

**Make-and-Break Ignition.**—If the ends of two wires forming part of an electric circuit are brought in contact, thereby closing the circuit, and then quickly separated, a bright spark will be produced as the contact is broken. This phenomenon underlies the operative principle of what is known as the *make-and-break system of ignition*, with which it is necessary first to complete the electric circuit through the spark-producing mechanism, or *igniter*, and then break the circuit to obtain a spark for igniting the charge. In stationary gas-engine practice, the simplest kind of igniter uses city lighting current, with an incandescent lamp in series, in order to prevent the current from being too strong, and consists simply of a mechanical device for making and breaking the circuit in the combustion chamber at the proper moment. Batteries may be used with the make-and-break system of ignition by using a spark coil.

With a low-voltage current, such as that derived from a primary battery, a spark coil must be employed to produce the necessary electric tension or voltage for the spark. When a battery and spark coil are employed, the abruptness of the break between the contact point serves to increase the intensity of the spark, it being largely proportional to the sharpness of the circuit rupture. Fig. 3 shows an elementary wiring diagram for a primary ignition circuit, with the direction of the current indicated by an arrow. When the timing cam $a$ brings the points $b$ and $c$ into contact, the current flows from the battery $d$ through the switch $e$ (when closed)–spark coil $f$–insulated electrode $g$–rocking contact finger $h$–grounded contacts $i$, back to the battery. The grounded connections $i$ may be made to the frame of the machine, or any other convenient metallic return may be used.

**Jump-Spark Ignition.**—The mechanism of the make-and-break system of ignition requires a considerable number of moving parts that may be more or less objectionable. What is known as the *jump-spark system of ignition*, in which the primary current is converted by an induction coil into a secondary current of sufficiently high tension to cause a spark to jump an air gap may therefore be used. With this system, a revolving contact timer is employed in place of the snap cam. As there are no other moving parts, the whole apparatus is extremely simple.

In Fig. 4 are shown the essential elements of a jump-spark system of ignition. Here $a$ is the battery; $b$, a switch for opening the primary circuit when it is not in use; and $c$, a revolving timer turning at one-half the speed of the crank-shaft, if the engine is of the four-cycle type. The timer in the elementary apparatus shown consists of an insulating ring $d$, mounted on the shaft into which is dovetailed a copper or brass segment $e$ that is in electric connection, by a screw or otherwise, with the shaft $f$. A plate $g$ is mounted loosely on the shaft, so that it does not turn with it, but may be rocked about it through a suitable arc, say 45°. Mounted on this plate, and insulated from it, is a brush $h$ that bears against the insulating ring and makes contact with the metal segment at each revolution of the latter. The primary winding of the spark coil is represented by $i$, and $j$ is the ground on the engine. A trembler $k$ is



Fig. 3

provided so that the current may be rapidly interrupted. The trembler is for the purpose both of interrupting the current more rapidly than could be done with the timer and to produce a series of sparks in rapid succession instead of only a single spark.
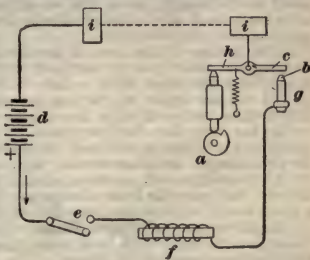
The course of the current is from the positive pole of the battery to the trembler–primary winding of spark coil–the engine frame $j$–contact $e$–brush of timer, when contact is made–switch $b$–negative terminal of battery. The negative terminal of the secondary winding of the coil is connected to the battery terminal of the primary winding, and the positive secondary terminal is connected to the insulated member of the spark device, or spark plug, from which, after jumping over the gap $l$, the current returns to the coil by way of the engine frame $j$ and primary winding. When the circuit is closed by the timer, a stream of sparks passes between the spark points $l$. For use with small, high-speed motors, the coil vibrator is frequently omitted, and a snap or vibrating form of timer is used that gives a quick break but only one spark.
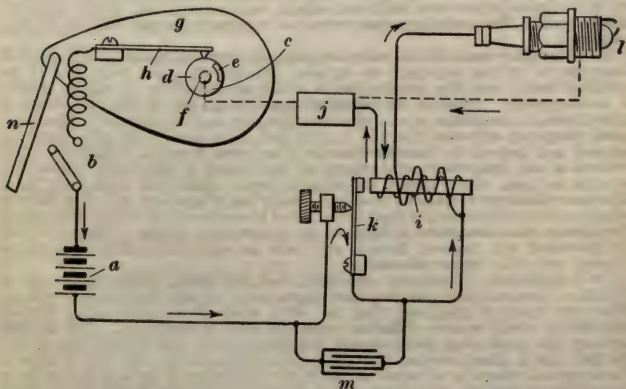


Fig. 4

The primary winding is provided with a condenser $m$, which serves the double purpose of increasing the abruptness of the circuit rupture, thereby increasing the intensity of the secondary spark, and of absorbing the current that otherwise would produce a hot spark at the trembler contacts, and soon

burn them out. The function of a condenser is to absorb the extra current induced in the primary coil at the moment of rupture. Under the primary system of ignition, it is precisely this extra current that produces the useful spark in the engine; but in the secondary system, this extra current is objectionable, because it dies down so slowly that it fails to induce a sufficiently intense spark in the secondary coil.

The change of the time of ignition is accomplished for different speeds by rocking the plate *g* to the right or left by means of the rod *n*, so that contact is made by the timer early or late in the revolution of the shaft.

To run an engine at varying speeds, it is necessary, in order to obtain the best results, to modify the time of ignition to suit the speed, making the time earlier for high than for low speed. It is also necessary to modify the time of ignition, according to the load the engine is carrying, if the engine is regulated by throttling. In other words, with a given speed, a charge will burn faster if highly compressed, as when a full charge is taken, than if only slightly compressed, as it may be if the charge has been much throttled. For these reasons, a great many engines are provided with means for varying the time of ignition.

**Requirements of Spark Plugs.**—The spark plugs on the market are of a variety of designs, each with some special features of advantage that may or may not be possessed by others. The chief requirements of a good spark plug are the following:

1. Where exposed to burning gas and oil vapors, the insulating material of porcelain or mica between the central electrode or stem, which is connected to the positive terminal of the coil, must not be too easily coated with carbon deposit. The electric resistance of any gas increases considerably as the gas is compressed, so that, although the current may jump between the proper spark points when the plug is in the open air, the resistance between these points may become so great when the plug is in the cylinder and the charge compressed, that the current will take an easier path through the carbon coating on the porcelain. Practically the same thing will happen if the porcelain is cracked, for the current will then take the direct route through the crack rather than the route from spark point to spark point through the compressed gas. The leakage through the carbon deposit must, therefore, be made as difficult as possible by giving the leaking current a considerable distance to travel; besides, special devices are sometimes employed to prevent the collection of carbon.

2. The plug must be easily cleaned of whatever carbon may be deposited on it. It must, therefore, be taken apart, reassembled, and made gas-tight easily; besides, the packing process must not endanger the porcelain more than necessary.

3. The plug must fit the standard sizes of threaded spark-plug holes and must not be unduly expensive to replace. Among the sizes most used is the so-called metric size, the proportions of which are based on the metric system of measurement. Most of the imported spark plugs are of this size, which is approximately the size of a $\frac{1}{2}$-in. pipe tap, but not tapered. American spark plugs are either of the $\frac{1}{2}$-in. or the $\frac{3}{4}$-in. pipe sizes. The pipe sizes are tapered and depend for tightness on the plug being screwed in tightly. This method is not altogether satisfactory, as the thread in the engine wears and permits leakage, which causes the plug to heat. Both the engine and plug tapers are liable to variations that may make one plug screw well into its hole while another catches only a few threads, and consequently is not so well placed for prompt communication of flame to the compressed charge. Plugs that are not provided with tapered threads are made gas-tight by gaskets of asbestos covered with thin copper sheathing.

It is desirable, though not essential, that the spark points should be of platinum, because they do not then burn away to any appreciable extent. When not made of platinum, they are often made of a special alloy of steel and nickel, which resists oxidation nearly as well as platinum. The air gap between the spark-plug points should not exceed $\frac{1}{32}$ in. nor be less than $\frac{1}{64}$ in.; the best size is about midway between these dimensions. In case a battery gives out and there is no other at hand, the engine may be kept going for a short time by pinching the spark-plug points a little closer together, to reduce the resistance offered by the gap.

## OPERATION OF INTERNAL-COMBUSTION ENGINES

**Engine Starters.**—Engines of less than 40 H. P. are usually started by turning over by hand, but larger engines are usually provided with some form of starting mechanism. The use of compressed air to start the engine is probably the most convenient. Its only disadvantage is the possibility of the air supply becoming exhausted by leaks in the tank or connections or through repeated failures to start the engine. This disadvantage is of course increased if the air compressor is operated from the engine itself or from a line shaft operated by the engine. It is of less consequence if the compressor is driven from a small auxiliary engine, the only difficulty in that case being the delay caused by the time required for charging the air tank.

The operation of compressed-air starters is much simplified if the engine is equipped with a mechanically driven and timed valve that admits the air to the cylinder during what is usually the expansion stroke of the engine, and allows it to escape during the regular exhaust stroke. In this case, all the operator has to do is to open the cock in the air pipe between the tank and the engine, and keep it open until the engine has attained a fair speed, when the air connection is shut off and the fuel supply is turned on. If no mechanically operated timing valve for compressed air is provided, the cock between the air tank and the engine must be opened and closed by hand, at proper intervals; this requires skill and watchfulness on the part of the attendant, since the cock must be closed except during the working stroke of the piston.

A very effective method, employed on large engines especially, is to admit air during every forward stroke of the piston and expel it during both the compression and the exhaust strokes. This is done by providing auxiliary cams that, when thrown in gear, will open the inlet valve during the suction and expansion strokes and the exhaust valve during the compression and exhaust strokes. After the engine is under way, the cams are disengaged and the engine is run in the regular manner.

In order that the engine may always be started promptly, the attendant must keep the air compressor and storage tank in good working order. The compressor requires lubrication at regular intervals, and the air valves must be kept tight to insure efficient service. The pressure in the tank should show no perceptible loss over night; and if it should fall to any great extent, the cause of the leak should be determined and the proper remedies applied. If the leak is located in the seams or rivets of the tank, they must be calked in the usual manner. In case the pipes or fittings between the tank and the engine are not tight, they must be screwed up or defective fittings replaced with perfect ones.

**Starting the Engine.**—The following rules for starting and stopping gasoline engines should be followed:

1. Attend to all lubricators and oil holes, always following the same order.
2. Apply a few drops of kerosene to the valve stems.
3. Open the gas-cock back of the rubber bag or regulator, or, when using gasoline, open the cock near the tank, and work the gasoline pump by hand until the liquid appears in the valve or overflow cup.
4. See that the electric igniter is properly connected, turn on the switch, and see that the spark is of proper intensity.
5. Turn the flywheel until the engine is at the beginning of the working stroke.
6. Open the fuel cock to the point that has been found most reliable for starting.
7. Throw the relief cam in gear or open the relief cock.
8. If a compressed-air or some other self-starter is employed, operate the device. If no starting devices are used, turn the flywheels rapidly until the engine starts.
9. Close the relief valve or disengage the relief cam and open the fuel cock to its full extent, gradually, as the speed of the engine increases.
10. Turn on the cooling water, if running water is used, or see that the tank is full and the cocks open if the tank system of cooling is employed.
11. Throw in the friction clutch or shift the belt to the tight pulley on the line shaft.

**Stopping the Engine.**—1. Disengage the friction clutch or shift the belt to the loose pulley on the line shaft.

2. Close the gas-cock near the rubber bag or regulator or the gasoline cock near the storage tank.

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3.  Close the gas or gasoline cock on the engine.

4.  Throw off the switch between the battery and the engine, or turn off the burner that heats the tube.

5.  Drain the water-jacket by closing the valve in the supply pipe and opening the cock that connects the bottom of the cylinder to the drain pipe. If water tanks are used, close the cocks in the water pipe and open the drain cock.

6.  Shut off all sight-feed lubricators.

7.  Clean the engine thoroughly, wiping off any oil or dust that may have accumulated on the engine.

8.  See that the engine stops in a position where the exhaust and inlet valves are closed. If necessary, turn the wheels by hand until this position is reached; it will protect the valve seats against corrosion.

**Lubrication.**—There are three essential properties that a good gas-engine cylinder oil must possess.

1.  It must have as high a fire-test as possible; that is, the temperature at which it gives off inflammable vapor should be as high as possible. In the best gas-engine cylinder oils, this temperature will be from 500° to 650° F., which is none too great considering the temperatures to which the oil is subjected when exposed to the burning charge in the cylinder.

2.  As the oil is vaporized by the heat, it should leave as little residue as possible. Any cylinder oil will leave some carbon deposit, which gradually accumulates on the inner walls of the combustion chamber and on the piston head and valves, but it is desirable that this accumulation should be prevented as far as practicable. If it becomes thick, especially if the compression is high or if the form of the combustion chamber is such that sharp corners are exposed to the heat of the flame, particles of the unburned carbon clinging to the walls or elsewhere may become heated to such a degree as to ignite the charge spontaneously before compression is complete.

3.  The oil should have a fairly high viscosity; that is, it should be quite thick, because the high temperature of the cylinder will cause any ordinary oil to run on the piston much like water and lose practically all its lubricating qualities.

It is often advisable to use a higher grade of oil in a high-speed engine than is necessary in a low-speed engine, owing to the greater rapidity of the explosions in the former and the consequently higher internal temperature. As the cylinder head of a high-speed engine is frequently cast in one piece with the cylinder, it is very hard to get at the combustion chamber to scrape the carbon deposit from it, and consequently it is well to use an oil that leaves as little deposit as possible.

For air-cooled cylinders, only the heaviest oil obtainable and with the highest possible fire-test should be used, and the oil tank should be placed near enough to the cylinder or exhaust pipe to insure that the oil will not refuse to feed in cold weather.

For ordinary water-cooled engines, except in the largest sizes, the grade of cylinder oil known as *heavy* is appropriate for summer use. In weather cold enough to cause this oil to stiffen, the next lighter grade, or *medium*, may be employed. In cold weather, if the medium oil does not feed freely, it is best to use a special oil suitable for use at low temperatures, though it is possible to thin the regular oil with kerosene or gasoline, to make it flow, and to increase correspondingly the feed of the oil cup or mechanical lubricator. It is best in every case to purchase oil that is known to be reliable. Besides, many manufacturers purchase and sell oils marked with their own labels, which they recommend for use in their engines. These oils may always be used with confidence in any engine of about the same character as that for which they are put up.

Should it be found impossible to obtain oil that is known to be suitable, the samples available may be tested for viscosity by putting a few drops of each on an inclined sheet of clean metal or glass, and noting the relative rapidity of their flow. The one that flows most rapidly has the least viscosity, and the one that flows most slowly has the greatest viscosity. The oils may be tested roughly for flashing point and for the carbon residue they leave by putting a little on a sheet of iron or tin plate, and heating gradually over a flame, taking care to move the plate over the flame so that all parts of it are evenly heated. The oils will become less viscous and will run on the plate, and for this reason two samples compared at the same time should not be placed too close together. They will gradually vaporize, leaving only a brownish and somewhat thick residue, which should be as small in amount as possible. A good, heavy oil

will vaporize almost completely, but will retain considerable body even at temperatures where an oil of low fire-test would be entirely burned away. Oils, either heavy or light, that leave any considerable amount of black, tarry residue should be avoided.

Although, strictly speaking, cylinder oil needs to be used only for cylinder lubrication, it is the almost universal custom to use the same oil for all bearings of the motor. This simplifies the lubrication and removes the danger of making a mistake in oiling the pistons. Cylinder oil is an excellent lubricant for bearings subjected to hard service, as those of the crankpins and crank-shaft. The bearings do not require as much oil as the pistons, four or five drops a minute usually being sufficient.

## ENGINE TROUBLES AND REMEDIES

**Hot Bearings.**—The causes of hot bearings and the remedies therefor are the same in internal-combustion as in steam and other engines.

**Misfiring.**—Either total or partial misfiring may be due to any of the following causes: Circuit not closed; weak battery; ignition fouled with soot; wet spark points; broken spark plug; grounded circuit, usually the secondary; broken wire or loose connection; trembler out of adjustment; igniter spring weakened or broken; rarely, a defective spark coil or condenser.

**Back Firing.**—The cause of back firing is in most cases due to the delayed combustion of a weak mixture containing an insufficient amount of fuel. The result of such a mixture is a weak explosion and slow burning, so that, during the entire exhaust stroke and even at the beginning of the suction stroke, there is a flame in the combustion chamber. The fresh charge will therefore be ignited by the flame of the delayed combustion of the previous charge; and, as the inlet valve is open at that time toward the air-supply pipe or passage, a loud report will be heard in the air vessel or in the space under the engine bed whence the air is taken. The remedy for this condition is to increase the fuel supply until the explosions become of normal strength.

Another cause of back firing may be the presence of an incandescent body in the combustion chamber, such as a sharp point or edge of metal, a projecting piece of asbestos packing, soot, or carbonized oil, and similar impurities accumulating in the cylinder. To stop back firing from these causes, any projections of metal or other material should be removed with a suitable tool and the walls of the combustion chamber made as smooth as possible, or the cylinder should be cleared of any soot or carbonized oil that may have gathered there.

Failure of the igniter to fire all charges admitted to the cylinder, or improper composition of the mixture resulting in the same way, will be indicated by heavy reports at the end of the exhaust pipe. One or more charges may in this manner be forced through the cylinder into the exhaust pipe, and the first hot exhaust resulting from the combustion of a charge will fire the mixture that has accumulated in the pipe.

On account of the shorter time between the opening of the exhaust port and the admission of the new charge in a two-cycle engine, there is much greater liability to back firing in an engine of that type than in a four-cycle engine. In a four-cycle engine, back firing will occur only when the inlet valve is off its seat; hence, back firing is more of an element of danger in four-cycle than in two-cycle engines. If there is no check-valve in the carbureter or vaporizer, and there is no direct opening to the atmosphere, the column of flame that would be blown through a carbureter or auxiliary air supply on account of back firing would be particularly dangerous because accumulations of gasoline vapor might thereby become ignited.

To be absolutely safe, a four-cycle engine having a float-feed carbureter not supplied with a check-valve should take its supply of air from some high point rather than from a point near the base. As the use of a check-valve in the carbureter would materially reduce the efficiency of the engine, it is rarely used. If a float-feed carbureter is used, and indications point to imperfect carburization, the carbureter should be examined carefully. If the float leaks, so that the height of gasoline is constantly above the desired level, or if the float does not cut off the supply where it should, it will be necessary to take the carbureter apart to ascertain the trouble.

Explosions in the muffler and exhaust piping are usually caused by the ignition of the gas accumulating from missed explosions due to weak mixtures or faulty ignition. They are not usually dangerous unless the muffler is large and is weakened by rusting inside or out.

Explosions in the carbureter are sometimes caused by the inlet valve sticking open and permitting the flame to communicate from the spark. More often it is due to improper mixture, which burns so slowly that flame lingers in the cylinder even after the exhaust stroke is completed and the inlet valve begins to open. Either a weak or a rich mixture will produce this result, though not always both in the same engine. Carbureter explosions are often attributed to the exhaust valve closing after the inlet valve opens, or to simple leakage of the inlet valve; but these are seldom the real causes.

**Preignition.**—Premature ignition, or preignition, while somewhat similar to back firing in its nature and origin, manifests itself in a different way and has a different effect on the action of the engine. Premature ignition, as usually understood, is the firing of the partly compressed mixture before the time fixed by the igniting mechanism. Its causes are similar to those that result in back firing, the effect being different in that the charge is ignited later than when back firing takes place, but before the end of the compression stroke. Preignition will cause the engine to lose power on account of the maximum pressure being exerted on the crank before it reaches the inner dead center and thus having a tendency to turn it in the wrong direction.

Besides the causes cited in connection with back firing, preignition may be due to any one of the following defects: Insufficient cooling of the cylinder, due either to shortage of cooling water or to the fact that portions of the water-jacket become filled with lime deposits or impurities contained in the water, thus interfering with proper circulation; compression too high for the grade of fuel used; imperfections in the surfaces of the piston end or valve heads exposed to the combustion, such as sandholes or similar cavities in which a small portion of the burning charge may be confined; electrodes or other parts of the engine exposed to the burning charge too light; or the piston head or exhaust-valve poppet insufficiently cooled and becoming red hot while the engine is running under a fairly heavy load.

Premature ignition manifests itself by a pounding in the cylinder, and, if permitted to continue, a drop in speed, finally resulting in the stopping of the engine. It will also put an excessive amount of pressure on the bearings, especially the connecting-rod brasses, and cause them to run hot even when properly lubricated. After a shut-down due to premature ignition and a short period during which the overheated parts are allowed to cool, it is possible to start again and run until the conditions of load will again cause the trouble.

The remedies to be applied, according to the source of the difficulty, are as follows: Increase the water supply until the cooling water leaves the cylinder at a reasonable temperature, which may vary with the fuel used, but which should never be over 180° F. Clean the water space and ports of any dirt or deposit so as to insure free circulation of the cooling water. Reduce the compression by partly throttling the air and fuel supply. Plug any sandholes or blowholes in the piston or valve heads, and make these surfaces perfectly smooth. Replace electrodes or other light parts with some capable of absorbing and carrying off the heat without becoming red hot. If necessary, arrange for cooling the piston by blowing air into the open end of the cylinder. If the head of the exhaust valve becomes too hot, it is a sign that it is not heavy enough, and it should be replaced by one with a head of sufficient thickness to carry off through the valve stem the heat imparted to it by the combustion. If a small particle of dirt lodges in a remote portion of the combustion chamber, the richer part of the charge may not reach it until the piston has traveled over a considerable portion of the compression stroke, and the resulting self-ignition may properly be called preignition. Every part of the combustion chamber should therefore be examined and all dirt removed.

**Carbureter Troubles.**—An engine may work improperly because the carbureter delivers no gasoline, an insufficient amount of gasoline, or too much gasoline to the cylinders. When little or no gasoline reaches the cylinders, the difficulty may be caused by: A failure to turn on the supply of gasoline, a clogged feedpipe, too light a float, failure of the needle valve to open wide enough, and obstructed nozzle. Too much gasoline in the carbureter is caused by too heavy a float or because the valve does not close tightly. In the latter case, the valve may need grinding or there may be grit between it and its seat.

**Compression Troubles.**—Either partial or total loss of compression may be due to any of the following causes: The valve stem may stick in the guide, in which case it should be washed with gasoline or kerosene and then well oiled; the valve stem may be weak or broken; the cylinder may be cracked; the piston ring may be broken or turned so that the slots are in line; the valve may need regrinding; water may leak into the cylinder through a bad joint.

# PROSPECTING

## OUTFIT AND METHODS

The prospector should have a general knowledge of the mineral-bearing strata, and should know from the nature of the rocks exposed whether to expect to find coal or not. He should also possess such a knowledge of the use of tools as will enable him to construct simple structures, and a sufficient experience in blacksmithing to enable him to sharpen picks and drills, or to set a horseshoe, if necessary.

**Outfit Necessary.**—The character of the prospecting being carried on will have considerable effect on the outfit necessary, which should always be as simple as possible. In general, when operating in a settled country, the outfit is as follows: A compass and clinometer for determining the dip and strike of the various measures encountered; a pick and shovel for excavating, and, where rock is liable to be encountered, a set of drills, hammer, spoon for cleaning the holes, tamping stick, powder and fuse, or dynamite fuse and cap; an aneroid barometer for determining elevations, and a small hand pick; the latter should weigh about 1¾ lb., and should have a pick on one end and a square-faced hammer on the other, the handle being from 12 to 14 in. long.

If the region under consideration has been settled for some time, there will probably be geological, county, railroad, or other maps available. These may not be accurate as to detail, but will be of great assistance in the work on account of the fact that they give the course of the railroads, streams, etc.

When operating in a mountainous region, away from a settled country, the following materials, in addition to those already mentioned, may be required: A donkey or a pony packed with a couple of heavy blankets, an A tent, cooking utensils, etc.; a supply of flour, sugar, bacon, salt, baking powder, and coffee, sufficient for at least a month. It is also well to take some fruit, but all fruit containing stones or pits should be avoided, as they are only dead weight, and every pound counts. For the same reason, canned goods should be avoided, on account of the large amount of water they contain. A healthy man will require about 3 lb. of solid food per day. Many prefer to vary the diet by taking rice, corn meal, beans, etc., in place of a portion of the flour.

Where game is abundant, a shot-gun or rifle will be found useful for supplying fresh meat. In regions abounding in swamps it becomes necessary to operate from canoes, or to take men for porters or packers, who carry the outfit on their backs or heads. These men will carry from 60 to 125 lb.

**Plan of Operations.**—When the presence of coal is suspected in a tract of land, a thorough examination of the surface and a study of the exposed rocks, in place, may result in its immediate discovery, or in positive proof of its absence; or it may result in still further increasing the doubt of, or the belief that, it does exist. The first procedure in prospecting a tract of land is to traverse it thoroughly, and note carefully any stains or traces of smut, and all outcrops of every description; and, whenever possible, take the dip and the course of the outcrop with a pocket compass. Any fossils should also be carefully noted, to assist in determining the geological age of the region. These outcrops are frequently more readily found along roads or streams than any other place on the tract. In traveling along the streams, the prospector should pay particular attention to its bed and banks, to see whether there are any small particles of coal or roof slate in the bed of the stream, or any stains or smut exposed along the washed banks. If small pieces of coal or roof slate are found in the stream, a search up it and its tributaries will show where the outcrop from which the find came is located. When the ravines and valleys are so filled with wash that no exposures are visible, and nothing is gained by a careful examination of them, the prospector must rely on topographical features to guide him.

In cases where there are no outcrops or any other surface indications, it would become necessary to sink shafts or test pits, or to proceed by drilling. The absence of any indication of coal in the soil may not prove that there is not an outcrop near at hand, for the soil is frequently brought from a distance, and bears no relation to the material underlying it. In like manner, glacial soil often contains debris transported from seams many miles away; but such

occurrences can usually be distinguished by the general character of the associated wash material.

Frequently, the weathered outcrop of a seam has been overturned or dragged back upon itself, so as to indicate the presence of a very thick deposit. For this reason, any openings made to determine the character of the material should be continued until the coal is of a firm character, and both floor and roof are well exposed. Sometimes, in the case of steeply pitching coal beds, the surface may be overturned for a considerable depth, so that it is difficult to tell which is the roof and which is the floor. Usually, if Stigmariæ are found in the rocks of one wall, it is supposed that this wall is the floor of the seam, while if Sigillariæ, fern leaves, etc. are found in the wall rock, it is probably the roof of the deposit. These indications are not positive proof, for both of these fossils may occur in either the top or bottom wall of a coal deposit, though they are usually found in the positions noted. Coal usually occurs in unaltered deposits, i. e., in rocks that have not undergone metamorphism, while metals and metallic ores usually occur in rocks that have undergone more or less metamorphism. This change may have been accompanied by heat and volcanic disturbances sufficient to render the rocks thoroughly crystalline, or it may simply have been the converting of limestone into dolomite. The prospector for coal usually avoids regions in which the rocks have been altered; but it should be remembered that a basin of unaltered stratified rocks containing coal seams may be found in the very midst of a high mountain range and surrounded by granitic and volcanic rocks, as in the Rocky Mountains.

When a prospector is operating in any particular region, it is best to study carefully the conditions of that region before proceeding, as such factors as lack of rain, frozen ground, etc. may have played an important part in determining the character of the outcrop and surface appearance of coal deposits. Experience obtained in one region is frequently very misleading when applied in another.

## COAL-BEARING FORMATIONS

**Outcrops.**—The presence of the outcrop of any bed may often be located by a terrace caused by the difference in the hardness of the strata; but as any soft material overlying a hard material will form a terrace, it is necessary to have some means of distinguishing a coal terrace from one caused by worthless material. Usually, the outcrop of a coal terrace will be accompanied by springs carrying a greater or less amount of iron in solution, which is deposited as ochery films upon the stones and vegetable matter over which the water flows. The outcrops of beds of iron or other ores are very frequently marked by mineral springs. Sometimes the outcrop of a bed will be characterized by a marked difference in the vegetation, as, for instance, the outcrop of a bed of phosphate rock by a luxuriant line of vegetation, the outcrop of a mineral bed by a lack of vegetation, the outcrop of a coal bed contained between very hard rocks by more luxuriant vegetation than the surrounding country, etc. Some indication as to the dip and strike of the material composing the bed may be obtained by examining the terrace and noting the deflections from a straight line caused by the changes in contour of the ground. If the variation occasioned by a depression is toward the foot of the hill, the bed dips in the same direction with the slope of the ground; but if the deflection is toward the top of the hill, the dip is the reverse from the slope of the ground, or into the hill. After any terrace or indication of the outcrop of a bed has been discovered, it will be necessary to examine the outcrop by means of shafts, tunnels, or trenches. The position of such openings will depend on the general character of the terrace. If the dip appears to be with the hill, a trench should be started below the terrace and continued to and across it; while if the dip appears to be into the hill, it may be best to sink a shallow shaft above the terrace.

**Formations Likely to Contain Coal.**—No coal beds of importance have as yet been found below the Carboniferous period, but coal may be looked for in any stratified or sedimentary rocks that were formed after this period, although the bulk of the best coal has been found in the Carboniferous period. As a rule, highly metamorphic regions and regions composed of volcanic or igneous rocks contain no coal. An examination of the fossils contained in the rocks of any locality will usually determine whether they belong to a period below or above the Carboniferous, and hence whether there is a probability of the formations containing coal. On account of this fact, the prospector should familiarize himself with the geological periods, and, by referring to any elementary

## GEOLOGICAL CHART FOR THE UNITED STATES

| Age | Era | Period | Epoch | Stage | Kinds of Rock |
|---|---|---|---|---|---|
| Age of Man | Cenozoic (Recent life) | Quarternary | Pleistocene | Present / Champlain / Glacial | Sand, gravel, clay, marl, peat |
| Age of mammals | | Tertiary | Neocene { Pliocene / Miocene } | | Sand, clay, marl, lignite |
| | | | Eocene | | Sand, clay, and lignite / Greensand |
| Age of reptiles | Mesozoic (Middle life) | Cretaceous Upper | Laramie | Livingstone / Denver / Laramie / Fox Hills | Sandstone, shale, coal |
| | | | Montana | Pierre | Coal, sandstone, shale |
| | | | Colorado | Niobrara / Benton | Sandstone, shale, coal / Sandstone and shale |
| | | Cretaceous Lower | Dakota | Dakota / Kootanie | Chalk and shale / Shale / Sandstone |
| | | Jurassic | Potomac—Como | *Wealden / *Oolite / *Lias | Fresh-water marls |
| | | Triassic | Newark | *Keuper / *Muschelkalk / *Bunter Sandstein | In the United States, beds of red and brown sandstones and shales with local beds of coal |
| Age of acrogens | Paleozoic (Ancient Life) | Carboniferous | Permian Coal measures | Greene County series / Monongahela series / Pittsburg series / Alleghany series | Shales, sandstones, coal beds, limestones, clays, and iron ores |
| | | | Mississippian or Lower Carboniferous | Mauch Chunk / Pocono | Red shale, sandstone / Sandstone, shale, limestone |
| | | Devonian | Chemung | Catskill / Chemung / Portage | Sandstone, shale, oil sands / Sandstone and shale / Sandstone and shale |

| Life | Era | | System | Group | Formation | Rock description |
|---|---|---|---|---|---|---|
| Age of fishes | Paleozoic (Ancient Life) | | Devonian | Hamilton | Genesee | Black shale |
| | | | | | Tully | Limestone |
| | | | | | Hamilton | Shales and sandstone (blue stone) |
| | | | | | Marcellus | Black shale |
| | | | | Corniferous | Gonidite | Limestone |
| | | | | | Corniferous | Cherty limestone |
| | | | | | Onondaga | Crystalline limestone |
| | | | | | Schoharie | Shaly sandstone |
| | | | | | Caudagalli | Shaly sandstone |
| | | | | Oriskany | | Sandstone |
| Age of invertebrates—mollusks, trilobites, echinoderms, corals, sponges | | | Silurian | Lower Helderberg | Upper Pentamerus | Limestone |
| | | | | | Deithyris shaly limestone | Shales and limestone |
| | | | | | Lower Pentamerus | Clay, limestone, shale |
| | | | | | Water lime | Red, gray, and green shales, vermicular limestone |
| | | | | Onondaga | Salina shales | Shale and limestone |
| | | | | Niagara | Niagara | Shale, limestone, and iron ore |
| | | | | | Clinton | |
| | | | | | Medina | Sandstone and shale |
| | | | | | Oneida | Conglomerate and sandstone |
| | | | Ordovician | Trenton | Hudson River (Cincinnati) | Shales and limestone |
| | | | | | Utica | Shales |
| | | | | | Trenton | Limestones, marbles, and shales |
| | | | | | | The Black River limestone |
| | | | | | | Bird's-eye limestone |
| | | | | Chazy | Chazy | Limestones and marbles |
| | | | | Calciferous | Calciferous | Calciferous sandstones, magnesian limestones |
| | | | Cambrian | Potsdam | Upper | Sandstone and quartzite |
| | | | | Acadian | Middle | Limestones, shales, sandstones |
| | | | | Georgian | Lower | Limestones, shales, sandstones |
| | Proterozoic | Precambrian — Algonkian | Keweenawan | | | Alternating series of sandstones, conglomerates, and volcanic rocks |
| | | | Upper Huronian | | | Metamorphic and igneous—granites, gneiss, schists, quartzites, marbles, limestones, slates, phyllites, iron ores, and jaspilites |
| | | | Lower Huronian | | | |
| | Azoic (Without life) | Precambrian — Archean | Laurentian | | | Igneous and metamorphic only. Granite, gneiss, and other crystalline rocks |

geology, with the most common fossils of each. The rocks most common in coal measures are sandstones, limestones, shale, conglomerates, fireclays, and, in some localities, the coal deposits are frequently associated with beds of iron ore.

The preceding table gives the names of the various geological eras, periods, epochs, and stages as they occur in the United States, together with the kinds of rocks characterizing each.

**Faults.**—Frequently a seam becomes faulted or pinched out underground, and it is necessary to continue the search by means of underground prospecting. If a fault or dislocation is encountered, the manner of carrying on the search will depend on the character of the fault. Where sand faults or washouts are encountered, the drift or entry should be driven forwards at the angle of the seam until the continuation of the formation is encountered, when a little examination of the rocks will indicate whether they are the underlying or overlying measures. In the case of dislocations or throws, the continuation of the seam may be looked for by *Schmidt's law of faults*, which is as follows: *Always follow the direction of the greatest angle.* It has been discovered that, in the majority of cases, the hanging-wall portion of the fault has moved down, and on this account such faults are commonly called *normal faults*. For instance, if the bed *ab*, of the accompanying figure, were being worked from *a* toward the fault, work would be continued down on the farther side of the fault toward *d*, until the continuation of the bed toward *b* was encountered. In like manner, had the work been proceeding from *b*, the exploration would have been carried up in the direction of the greatest angle, and the continuation toward *a* thus discovered. A *reverse fault* is one in which the movement has been in the opposite direction to a normal fault. Especially in the case of those mines where the material occurs as perpendicular or steeply pitching veins, faults are liable to displace the seam both horizontally and vertically, in which case it may be difficult to determine the direction of the continuation of the bed; but frequently pieces of coal or slate are dragged into the fault, and these serve as a guide to the miner, and indicate the proper direction for exploration. Where a bed or seam is faulted, its continuation can frequently be found by breaking through into the measures beyond, when an examination of the formation will indicate whether the rocks are those that usually occur above or below the desired seam.
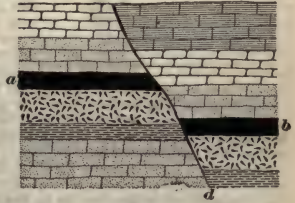
# EXPLORATION BY DRILLING OR BORE HOLES

**Earth Augers.**—When testing for coal seams that occur comparatively near the surface, hand augers may be employed to great advantage. A good form of hand auger consists of a piece of flat steel or iron, with a steel tip, twisted into a spiral about 1 ft. long, and having four turns. The point is split and the tips sharpened and turned in opposite directions and dressed to a standard width, usually 2 in. The auger is attached to a short piece of 1-in. pipe, and is operated by joints of 1-in. pipe, which are coupled together with common pipe couplings. The auger is turned by means of a double-ended handle having an eye in the center through which the rod passes.

The handle is secured by means of a setscrew. In addition to the auger it is well to have a straight-edged chopping bit for use in comparatively hard seams. This may be made from a piece of 1⅛-in. octagon steel, with a 2-in. cutting edge. The upper end of the steel is welded on to a piece of pipe similar to that carrying the auger. When the chopping bit is employed, it is necessary to have a heavy sinking bar, which may be made from a piece of solid 1¼-in. iron bar, fitted with ordinary 1-in. pipe threads on the ends. Prospecting can be carried on to a depth of from 50 to 60 ft. with this outfit. The number of men necessary to operate the rods varies from two to four, depending on the depth of the hole being drilled. When more than 30 ft. of rods are in use, it is usually necessary to have a scaffold on which some of the men can stand to assist in withdrawing the rods. When withdrawing the rods, to remove the dirt, they are not uncoupled unless over 40 ft. of rods are in use

at one time, and sometimes as many as 50 or 60 ft. are drawn without uncoupling.

**Percussion Drills.**—*Percussion*, or *churn*, drills are frequently employed in drilling for oil, water, or gas, and were formerly much used in searching for coal and ores, but, owing to the fact that they all reduce the material passed through to small pieces or mud, and so do not produce a fair sample, and to the fact that they can only drill perpendicular holes, they are at present little used in prospecting for either ore or coal.

### COST OF WELL DRILLING

| Size of Well Inches | Cost per Foot |
|---|---|
| 6 | $1.50 |
| 8 | 2.25 |
| 10 | 3.00 |
| 12 | 5.00 |
| 15 | 8.00 |

The cost and rate of drilling by means of a percussion or churn drill varies greatly, being affected much more by the character of the strata penetrated than is the case with the diamond drill. In the case of highly inclined beds of varying hardness, the holes frequently run out of line and become so crooked that the tools wedge, and drilling has to be suspended. For drilling through moderately hard formations, usually encountered in searching for gas or water, such as sandstones, limestones, slates, etc., the accompanying costs, from the American Well Works, Aurora, Ill., may be taken per foot for wells from 500 to 3,000 ft. deep for the central or eastern portion of the United States. This cost includes the placing of the casing, but not the casing itself.

When drilling wells for oil or gas to a depth of approximately 1,000 ft., using the ordinary American rig with a cable, the cost is sometimes reduced to as little as 65 c. per ft. for 6-in. or 8-in. wells; this is when operating in rather soft and known formations. From 15 to 40 ft. per da. of 24 hr. is usually considered a good rate of drilling, though in soft materials as much as 100 ft. may be drilled in a single day, and at other times, when very hard rock is encountered, it is impossible to make more than from 1 to 2 ft. per da.

**Percussion Core Drill.**—In order to overcome the chief objection to the use of percussion drills in coal prospecting work, an attachment is now provided which can be used in connection with the ordinary oil-well drilling outfit and by means of which a core of a coal seam may be recovered. A 6-in. hole is sunk with the ordinary tools until the vicinity of the bed to be cored is reached. The tools are then withdrawn, the bit and stem are removed, leaving the jars and rope socket attached to the cable, and the core drill is attached to the jars. This drill is a steel pipe about 14 ft. long provided with chisel-shaped cutting teeth, and within which is placed the core-barrel. The hole is carefully cleaned with the sand pump and its exact depth measured and recorded. This core-drilling attachment is lowered carefully into the hole until it rests on the bottom and drilling is resumed in the ordinary way at a moderate speed but with a stroke of from 15 to 18 in. After drilling 20 or 30 in. the tools are withdrawn; a slight jar is sometimes necessary to break the core loose. At the top of the hole, the tools are swung to one side, the core barrel with the contained core removed from the drill and another core barrel attached if a longer core is needed. Drilling operations are resumed while the core is being removed from the core barrel just brought up. The core barrels are of two lengths, the shorter ones being designed for use in hard formations like sand rock, limestone, etc.

**Core Drills.**—What are known as *core drills* are the only forms that have proved successful in drilling in any direction through hard, soft, or variable material. Even with core drills, many difficulties present themselves and demand careful study in adapting the form of apparatus to the work in hand, and in rightly interpreting the results obtained from any set of observations.

Core drills are of two main types, the diamond drill and the calyx drill. The two are essentially alike in consisting of a cutting bit attached to the end of a series of connected rods to which a rotary motion is applied by a steam, electric, compressed-air, or gasoline motor. In the diamond drill, the cutting bit consists of a hard steel cylinder in the bottom rim (both inner and outer edges) of which are set fragments of black diamond (bort, or carbonado) the edges of which slightly project beyond the metal surface of the bit. Being the hardest substance known, upon being rotated, the diamonds naturally cut out a cylindrical core of any rock penetrated.

The bit of the calyx drill is of two forms. For drilling in comparatively soft rock, it consists of a steel cylinder with chisel-shaped teeth that cut and

scrape away the rock. For drilling in harder rock, the bit is without teeth, being merely a ring of metal with a slot in the side through which chilled steel shot fed into the bore hole above find their way so that they may be rolled over the surface of the rock to be cut as the bit is rotated in the hole. While the following is intended primarily to guide in the selection, use, etc., of a diamond drilling machine, it is also applicable to the calyx drill.

**Selecting the Machine.**—It is not economy to employ a machine of large capacity in shallow explorations, as the large machines are provided with powerful motors, and hence do not work economically under light loads. When a large machine is operating small rods on light work, the driller cannot tell the condition of the bit, or properly regulate the feed. The machine should possess a motor of sufficient capacity to carry the work to the required depth, but where much drilling is to be done, it is usually best to have two or more machines, and to employ the small ones for shallow holes, and the large ones for deep holes.

All feed mechanisms employed in diamond drilling may be divided into two classes: Those that are an inverse function of the hardness of the material; this class includes friction, spring, and hydraulic feeds. Those in which the feed is independent of the material being cut, as in the case of the positive gear-feed. The first class is advantageous when drilling through variable measures in search of fairly firm material, which does not occur in very thin beds or seams. On account of the fact that this class of feed insures the maximum amount of advance of which the bit is capable in the material being cut, the danger is that the core from any thin soft seam may be ground up and washed away, without any indication of its presence having been given. The second class, or positive gear-feed, if properly operated, requires somewhat greater skill, but if used in connection with a thrust register, it gives reliable information as to the material being cut, and is especially useful when prospecting for soft deposits of very valuable material.

**Size of Tools.**—The size of tools and rods, and consequently the size of the core extracted, depends on the depth of the hole and the character of the material being prospected. When operating in firm measures, such as anthracite, hard rock, etc., it is best to employ a rather small bit, even when drilling up to 700 ft., or more, in depth. For such work, a core of from $1\frac{1}{8}$ to $1\frac{7}{8}$ in. is usually extracted. The rate of drilling with a small outfit is very much greater than with a large one, owing to the fact that there is a small cutting surface exposed, and the rate of rotation of the rods can be much greater. When prospecting for soft materials, such as bituminous coal, valuable soft ores, or for disseminated ores, such as lead, copper, gold, silver, etc., it is best to employ a larger outfit and extract a core 2 or 3 in. in diameter, and sometimes larger, even though a comparatively small machine is used to operate the rods.

**Diamond-Drilling.**—Drift of diamond-drill holes, or the divergence from the straight line, often becomes a serious matter. This trouble may be minimized by keeping the tools about the bit as nearly up to gauge as possible. Core barrels, with spiral water grooves about them, answer this purpose very well if they are renewed before excessive wear has taken place.

Surveying of diamond-drill holes may be carried on by either one of two methods, depending on the magnetic conditions of the district. Where there is no magnetic disturbance, the system developed by Mr. E. F. MacGeorge, of Australia, may be employed. This consists in introducing into the hole, at various points, small tubes containing melted gelatine, in which are suspended magnetic needles and small plummets. After the gelatine has hardened the tubes are removed, and the angles between the center line of the tube, the plummet, and the needle noted, thus furnishing the data from which the course of the hole can be plotted. This method gives both the vertical and the horizontal drift.

Where there is magnetic disturbance the needle cannot be used, but a system brought out by Mr. G. Nolten, of Germany, has been used extensively employed. In this case, tubes partly filled with hydrofluoric acid are introduced into the hole, at various points, and the acid allowed to etch a ring on the inside of the tube. After the acid has spent itself the tubes are withdrawn, and by bringing the liquid into such a position that it corresponds with the ring etched on the inside of the tube, the angle of the hole at the point examined can be determined. This method gives a record of the vertical drift of the hole only.

The value of the record furnished by the diamond drill depends largely on the character of the material sought. The core extracted is always of very small volume when compared with the large mass of the formation prospected, and hence will give a fair average sample only in the case of very uniform

deposits. The value of the diamond drill for prospecting may be stated as follows: *More dependence can be placed on the record furnished by the diamond drill when prospecting for materials that occur in large bodies of uniform composition than when prospecting for materials that occur in small bunches or irregular seams.* To the first class belong coal, iron ore, low-grade finely disseminated gold and silver ores, many deposits of copper, lead, zinc, etc., as well as salt, gypsum, building stone, etc. To the latter class belong small but rich bunches of gold, silver mineral, or rich streaks of gold telluride.

The arrangement of holes has considerable effect upon the results furnished. If the material sought lies in beds or seams (as coal), the dip of which is fairly well known, it is best to drill a series of holes at right angles to the formation. If the material sought occurs in irregular bunches, pockets, or lenses, it will be necessary to drill holes at two or more angles, so as to divide the ground into a series of rectangles, thus rendering it practically impossible for any vein or seam of commercial importance to exist without being discovered. Where the surface of the ground is covered with drift and wash material, it may be best to sink a shaft or drill pit to bed rock, and locate the machine on bed rock. After this, several series of fan holes may be drilled at various angles from the bottom of the pit. Owing to the upward drift of diamond-drill holes, the results furnished from a set of fan holes drilled from a single position would make a flat bed appear as an inverted bowl, or the top of a hill. On this account, it is best to drill sets of fan holes from two or more locations, so that they will correct one another. If fan holes from different positions intersect the same bed, a careful examination of them will usually furnish a check on the vertical drift of the holes.

The speed and cost of drilling depend on the hardness and character of the rock, the size of the hole, the depth of the hole, and the height of the derrick. Sedimentary rocks, such as sandstones, slates, and limestones are generally more rapidly drilled than the much harder and unstratified igneous rocks, and firm rocks than those that cave and require that the hole be cased. Cores of moderate size, say, up to $1\frac{1}{2}$ to 2 in. in diameter can be taken out more rapidly and hence at less labor cost (the chief item in drilling) than larger ones, and answer just as well for determining the nature and value of the rocks passed through. The deeper the hole, the more costly and the less the progress per shift, because of the time lost in pulling the drill rods and removing the cores. With deep holes the labor cost is materially reduced if the derrick is sufficiently high to permit unscrewing the drill rods at every fourth joint while raising them for the purpose of extracting the core.

The actual rate of drilling, including pulling the rods, removing the cores, etc., is dependent on the depth of the hole. In shallow holes, a rate of 2 ft. per hr. is fair; in holes of moderate depth, say, up to 700 ft., a rate of 1 ft. per hr. should be secured.

Prospecting with core drills is usually done under contract at an agreed sum per foot, which is determined by the number of holes, their average depth, size of core extracted, distance apart of the holes, etc. Contract prices for diamond drilling in the bituminous coal regions range from $2 to $2.75 per foot for extracting cores up to say, $1\frac{1}{2}$ in., where the holes range from 150 to 500 ft. in depth (averaging, say, 250 ft.), where from 1,200 to 2,000 ft. of drilling is required, and where the coal and water are furnished the contractor.

Where the drill is owned by an operating coal company, the cost of ordinary drilling should be less than $1 per ft., including labor, diamonds, and ordinary repairs.

**Calyx Drilling.**—The calyx drill will do essentially the same work as the diamond drill, except that it will not cut at angles of over 45° because the shot will roll to and remain at the lower side of the hole, and even this pitch is too flat for really satisfactory work. The gain in the use of the calyx over the diamond drill consists in the saving in cost of the abrasive material used. Black diamonds are now quoted as high as $90 a carat for the larger and better stones, so that a single bit will often cost from $750 to $2,000, according to size. There should be at least two bits in stock, so that one may always be in condition for use. If drilling is carried on a long distance from the base of supplies, three, four, or even more bits must be available, or else a diamond-setter must be employed. Setting diamonds is highly skilled labor and is paid accordingly.

When the material is so soft that the shot wedge or press into the bit with teeth is to be preferred to the shot bit, the latter coming in use when the rock is firm sandstone, limestone, and the like. The amount of shot used varies with the hardness of the material being drilled. Shale, slate, limestone, and ordinary sandstone may require from $\frac{1}{4}$ to $\frac{3}{4}$ lb. of shot per foot of hole. Very

hard sandstone, granite, quartz conglomerate, porphyry, taconite, and jasper require from $1\frac{1}{2}$ to 4 lb. per ft. Another material sometimes used is crushed steel, variously sold under such names as "diamondite," "abrasite," etc. While ordinarily inferior to chilled shot, and not giving such satisfactory results, for comparatively soft formations it is sometimes better than shot.

The calyx drill, like the diamond drill, is manufactured to be operated by hand or horsepower for use in boring shallow holes, and is also to be had mounted on a wagon-like frame with attached derrick for ease in transportation.

**Prospecting for Petroleum, Natural Gas, and Bitumen.**—Among the surface indications of petroleum and bitumen may be mentioned white leached shales or sandstones, shales burned to redness, fumaroles, mineral springs, and deposits from mineral springs. Also natural gas, springs of petroleum oil and naphtha, porous rocks saturated with bitumen, cracks in shale, and other rock partly filled with bitumen. Petroleum is never found in any quantity in metamorphic rocks, but always in sedimentary deposits. Bitumen can be told from coal, vegetable matter, iron, manganese, and other minerals, which it sometimes closely resembles, by its odor and taste, also by the fact that it melts in the flame of a match or candle, giving a bituminous odor. (Iron and manganese do not fuse, and coal and vegetable matter burn without fusion.) Bitumen is also soluble in bisulphide of carbon, chloroform, and turpentine, usually giving a dark, black, or brown solution. Frequently, springs or ponds have an iridescent coating of oil upon the surface. Sometimes iron compounds give practically the same appearance, but the iron coating can always be distinguished from the oil by agitating the surface of the water, when the iron coating will break up like a crust of solid material, while the oil will behave as a fluid, and tend to remain over the entire surface even when it is agitated.

Frequently, bubbles of gas are seen ascending from the bottoms of pools or creeks. These may be composed of carbureted hydrogen or natural gas, which is a good indication of the presence of petroleum or bitumen; they may be composed of sulphureted hydrogen or carbonic-acid gas. Carbureted hydrogen can be distinguished by the fact that it burns with a yellow luminous flame whereas sulphureted hydrogen burns with a bluish flame, and carbon dioxide will not support combustion, but, on the contrary, is a product of combustion. When carbureted hydrogen gas is discovered ascending from water, the bottom of which is not covered with decaying vegetation, it is almost a certain sign that there is petroleum or bitumen somewhere in the underlying or adjacent formations. If natural gas or bitumen is found upon the surface of shale, it is probable that the material ascended vertically through cracks in these rocks from porous strata below; while if it is found in connection with sandstones, it is probable that the material was derived from the porous sandstone itself. This is especially liable to be true if the sandstone has a steep pitch.

As a rule, deposits of bitumen or petroleum occur in porous formations overlaid by impervious strata, such as shales, slates, etc. Anticlines are more liable to contain such deposits, though they are not absolutely necessary to retain them, as at times portions of the underlying porous strata have been rendered impervious by deposits of calcium salts, silica, etc., and hence the petroleum or bitumen will be confined to the porous portions. Natural gas also occurs under similar conditions, but usually in anticlines only.

**Construction of Geological Maps and Cross-Sections.**—After the surface examination of a property is complete, the data should be entered on the best map procurable, or a map constructed. The scale depends on the size of the property, the complexity of the geological formation, the value of the property, and the material to be mined from it. The amount of work that it will pay to put on the survey will depend largely on the value of the property, more detail being justified in the case of high-grade properties. If a property 1,200 ft. ×3,000 ft. (the size of four U. S. metal mining claims) were to be surveyed and mapped with a scale of 1 in. equal to 100 ft., the map would be 12 in, ×30 in. A stratum 10 ft. wide on this map would appear as $\frac{1}{10}$ in. wide, which is about the smallest division that could be shown with its characteristic symbol; for greater detail, a larger scale, or larger scaled sheets of the most important portions of the deposit, will be necessary. If the geologist constructs the topographical contour map, he can take notes on the geology at the same time. When the boundaries of the property are being surveyed, certain points should be established, both vertically and horizontally, as stations in future topographical work. If the map is on government surveyed land, the government lines may be used for horizontal locations, but it will be necessary to determine the elevation of the different points. If the property

is much broken, it is well to run a few lines of levels across it, to establish points from which to continue the work. This work is usually done with a Y level and chain, the other details being subsequently filled in with a transit and stadia; the levels of the other points are taken by using the transit as a



FIG. 1

level, by vertical angles, by barometric observations, or by means of a hand level. Where lines of levels are run across the property in various directions it is best to run them so that they will cross the strike of the strata as nearly at right angles as possible, so that the profile thus determined may be used in constructing a cross-section. Sometimes, for preliminary work, simply a sketch map is all that may be necessary. All of the outcrop and exposures, together with their proper dip, should be entered on the map.

**To Obtain Dip and Strike From Bore-Hole Records.**—Before the results derived from bore holes are available for use in map construction, the dip and strike of the various strata must be ascertained. The process, in the case of stratified rock, is as follows: If three holes were drilled,

as at $A$, $B$, and $C$, Fig. 1, each intersecting a given bed, the strike and angle of dip of the bed may be obtained by reducing the results from the three holes to a plane passing through the highest point of intersection, which is at $A$. The hole $B$ intersected the bed at the distance $Be$, and $C$ at the distance $Cd$ below the point $A$. By continuing the line $CB$ indefinitely and erecting two lines $Be$ and $Cd$ perpendicular to it, each representing the distance from the horizontal plane through $A$ to the intersection of the strata, two points in the line $de$ are obtained, which line intersects $CB$ produced at $f$; $f$ is one point in the line of strike through $A$. In order to find the angle of dip, the perpendicular $Cg$ is dropped from the deepest hole $C$ upon the line of strike $Af$. The distance $Ch$, equal to $Cd$, is laid off at right angles to $Cg$, when the angle $Cgh$ gives the maximum dip. The results obtained from bore holes may thus be reduced to such form that the dips can be projected on the surface to obtain the line of outcrop for each stratum. Bore holes also furnish data for constructing underground curves in cross-sections of stratified rocks.

Having recorded on the map all exposures, whether surface or those obtained from underground work, draw the line of strike and the outcrops. Also construct a cross-section. If the seam is perpendicular, the outcrop will be a straight course across the map. If the bed is horizontal, the outcrop will correspond with a contour line. For beds dipping at any other angle, results between these limits will be obtained.

If the property being examined is cut by synclines or anticlines, the dips will not all be in the same direction, and if there is a dip along the axis of the synclines or anticlines, the construction of the map will be considerably complicated. Fig. 2 represents a plan or map on which there is an axis $xy$ toward which the strata dip from both sides. Outcrops are indicated at $A$, $B$, $C$, $A'$ and $B'$, each having a dip in the direction of the arrow. The lines $mn$, $op$, and $qr$ are contours. If the cross-



FIG. 2

section were constructed on the line $FG$, perpendicular to the axis $xy$, the various beds or deposits would be cut at such an angle as to show a thickness in the cross-section greater than that which actually exists. In order to show the actual thickness for each seam, the cross-section must be taken along the line perpendicular to the strike of the strata, which, in the present case, is

along the line *IHK*. In other words, the cross-section must be constructed in two parts. Where a general sketch is all that is necessary, a single cross-section with notes correcting the thickness of the seams may answer.

In order to construct the cross-section *IHK*, the outcrops *A*, *B*, *C*, *A'*, and *B'* must be projected to the points *a*, *b*, *c*, *a'*, and *b'*, this projection being along their contours. If the points on the line of the intended cross-section were not upon the contour, it would be necessary to project them on the plane of the cross-section, as shown in the figure, and then from the dip of the strata and the difference in elevation to obtain a corrected point along the line *IHK*.

The cross-section is constructed as shown in Fig. 3, each seam having its actual thickness as shown at the outcrop. If the upper surface of the cross-section is not a true profile of the surface, and the points are not projected in the plane on the cross-section, on this cross-section, according to their dips, there is considerable danger of exaggerating their thickness one way or the other.

On mine maps, the supposed course of the beds should be sketched in, subject to revision, as more data are brought out by later development work. Even in the case of stratified rocks, it is diffi-
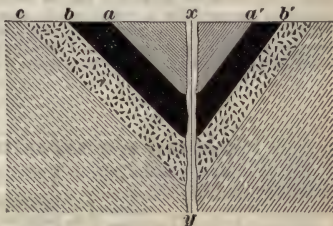


FIG. 3

cult to form a definite idea as to the underground conditions from surface indications, and, in the case of metamorphic or crystalline rocks, it is absolutely necessary to determine the underground conditions by drilling, or actual development work. If the property being examined is liable to become a large and valuable mining property, the original survey should be tied to monuments or natural landmarks, so that it can be checked by future observations, and these monuments or landmarks should become the basis of future and more careful mining surveys.

Some of the advantages of a careful geological examination of a property are that other materials of economic value would probably be discovered if any should exist on the porperty; also, such an examination of the property gives information as to the drainage system of the country that may be of great advantage in laying out the mine, and future exploration by drilling or sinking can be done to better advantage after a careful surface examination.

**Sampling and Estimating the Amount of Mineral Available.**—In many cases, it is necessary to do some development or exploration work before fair average samples can be obtained. The samples as taken should fairly represent the coal as it will be extracted. Such slate as would be sold with the coal should be included in the sample. When sampling any property it is well to divide the seam up into blocks, and sample each one separately. The samples may then be analyzed and an average obtained later, or the different samples may be mixed and an average analysis obtained. The amount of material broken for sample may vary from a few pounds to several tons. Large samples may be reduced by shoveling (that is, taking a proportionate number of shovelfuls for the sample, as every third or fourth shovelful). After the sample has been partly reduced, the operation may be carried on by quartering, which may be described as follows:

The coal is shoveled into a conical pile by throwing each shovelful on to the apex of the cone. After this, the cone may be reduced by scraping it down with a shovel, passing slowly around it. If the amount of material is small, a flat plate may be introduced into the cone, and the pile flattened by revolving the plate. The pile is then divided into quarters by drawing lines across it. After this, two alternate quarters are scraped out and shoveled away, and the other two quarters are left as the sample. The process may be repeated until the block has been sufficiently reduced. In shoveling away the discarded portions, care should be taken to see that the fine dust under them is brushed away also, as it often contains much of the impurity in the seams and its not being included might unduly increase the quality of the resulting sample. If the property being examined is a mine in active operation, samples may be taken from the working faces, and also from cars, loading chutes, etc. Usually, the samples from the face are kept separate from those

from the cars and loading chutes, the latter being intended as a check on the former.

The human factor always plays a large part in the value of a sample as finally selected, and hence it should be taken by a man who has had considerable experience in this class of work. For this reason, it is best to employ a mining engineer. One not accustomed to sampling very rarely undervalues a property, owing to the fact that it seems to be human nature to pick up a pure piece of coal, rather than the worthless bone or slate.

If a seam is penetrated by a number of bore holes, or by workings extended over a considerable area, it is fair to estimate that the material will run practically as exposed for a considerable area; but especially in the case of bituminous coal, it is a comparatively easy matter to form some estimate as to the amount of material available. The tonnage of coal seams per 1 in. and 1 ft. of thickness is given in the section upon weights of materials, etc.

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## DIAGRAM FOR REPORTING ON COAL LANDS

The following diagram will be useful as a guide in making out a report on a coal mining property:

**1. SITUATION AND SURROUNDINGS**
- 1. Name
  - 1. Location, if on surveyed land
  - 2. Nearest town or village
  - 3. Mineral district
  - 4. County, state, or territory
- 2. Distance and direction from one or more points

**2. TOPOGRAPHY**
- 1. Hills or mountains
- 2. Character of surface, vegetation, and timber
- 3. Streams and water supply
- 4. Elevations

**3. GEOLOGY**
- 1. Structure
  - 1. Rocks
    - 1. Stratified
    - 2. Crystalline
    - 3. Igneous
  - 2. Axes
    - Anticlines or synclines
  - 3. Faults
    - 1. Number
    - 2. Strike
    - 3. Dip
    - 4. Throw
  - 4. Dikes
    - 1. Number
    - 2. Strike
    - 3. Dip
    - 4. Filling
    - 5. Throw
  - 5. Horses
    - 1. Number and size
    - 2. Location
    - 3. Material
- 2. Geological period
- 3. Coal beds
  - 1. Number
  - 2. Thickness
    - 1. Reported
    - 2. Measured
    - 3. Average
    - 4. Uniformity
      - 1. Reported
      - 2. According to measurement
- 4. Quality of coal, specimens, appearance in mine, in cars
  - 1. Color, external, powder
  - 2. Luster
  - 3. Clearness from clay or sand, shale
  - 4. Sulphur
  - 5. Resin
  - 6. Firmness, size of lumps, air slaking
  - 7. Cleavage or fiber
  - 8. Coking
  - 9. Color of ashes
  - 10. Use: Gas, steam, domestic, forge, metallurgy, coking
  - 11. Analyses

## DIAGRAM FOR REPORTING ON COAL LANDS—*(Continued)*

4. **MINING**

  1. History
- 1. Dates of opening, abandoning, reopening, number of mines and names
- 2. Ownership
- 3. Superintendence

  2. Mine
- 1. Shaft, slope, or tunnel
- 2. Extent of workings
  - 1. Total depth
  - 2. Depth below water level
  - 3. Number of levels
  - 4. Extent of levels
- 3. Water pumps, size and kind, water cars, number and size, natural drainage
- 4. Ventilation, natural, furnace, fan (force or exhaust), sufficient or insufficient
- 5. Lighting, system used
- 6. Powder, kind and grade used
- 7. Explosive or noxious gases
- 8. Coal-cutting machines and power drills
- 9. Mode of working
  - 1. Room and pillar: (a) single entry, (b) double entry, (c) three or more entries
  - 2. Longwall: (a) advancing, (b) retreating
  - 3. Modifications of (1) and (2)
- 10. Rooms, pillars, dimensions, and general plan
- 11. Timbering
- 12. Roof, or hanging wall, strong or weak, air slakes or not
- 13. Floor, or foot-wall, hard or soft, creeps or not
- 14. Roads, rails, and cars
- 15. System of underground tramming
  - 1. Mules
  - 2. Electricity
  - 3. Compressed air
  - 4. Wire rope
  - 5. Chain
  - 6. Locomotive
- 16. System of hoisting; Cage, skip, cars.

5. **MAPS AND DRAWINGS**
- 1. Of the whole region
- 2. Of the underground workings
- 3. Sections
  - 1. Cross
  - 2. Longitudinal
  - 3. Columnar
    - 1. General
    - 2. Coal bed or other deposit
- 4. Buildings, works, or machinery
- 5. Explanation
  - 1. Scale
  - 2. North line, magnetic variation
  - 3. Date
  - 4. Maker
  - 5. Can buy, take, borrow, or have copied

6. **COKE OVENS**
- 1. History, ownership, etc.
- 2. Number
- 3. Character of ovens
- 4. Dimensions
- 5. Construction, materials, etc.
- 6. Operations
  - 1. Charge, quantity, etc.
  - 2. Working
  - 3. Discharging, quenching
- 7. Repairs
- 8. Quality of product (analyses, if any)
- 9. Disposition of by-products
- 10. Washery
  - 1. Construction
  - 2. Condition
  - 3. Capacity
  - 4. Water supply and consumption
  - 5. Quality of product

**DIAGRAM FOR REPORTING ON COAL LANDS**—*(Continued)*

7. DISPOSITION OF PRODUCT
- 1. Shipped
  - 1. As mined to
    - 1. Railroads
    - 2. Lake ports
    - 3. Tidewater
    - 4. Trade or jobber
  - 2. As coke
    - 1. Blast furnace
    - 2. Trade or jobber
- 2. Shipment
  - 1. Distance
  - 2. Roads
  - 3. Railroads
  - 4. Navigation

8. STATISTICS
- 1. Production
  - 1. Capacity, maximum and minimum
  - 2. Actual
    - 1. Daily, weekly, or monthly, in tons
    - 2. Yearly, in tons
    - 3. Average
- 2. Labor
  - 1. Whole number of workers
  - 2. Number of workers in each class
  - 3. Number of horses or mules
- 3. Prices
  - 1. Timber
  - 2. Tools
  - 3. Fuel
  - 4. Oil
  - 5. Powder
  - 6. Labor
    - 1. Day, different classes
    - 2. Contract or piece, yard or ton
  - 7. Carriage
  - 8. Local sales of product
  - 9. Value of plant
    - 1. Machinery
    - 2. Buildings
    - 3. Roads, tracks, etc.
    - 4. Rolling stock
    - 5. Supplies

9. SURFACE PLANT
- 1. Power plant
  - 1. Boilers
  - 2. Waterwheels
  - 3. Air compressors
  - 4. Steam and gas engines
  - 5. Electric plants
- 2. Shops
  - 1. Smith's shop
    - 1. Power for
    - 2. Number of forges
    - 3. Steam hammers
    - 4. Other tools
  - 2. Carpenter shop
    - 1. Power for
    - 2. Saws
    - 3. Lathes
    - 4. Other machines
    - 5. Benches and vises
  - 3. Machine shop
    - 1. Power
    - 2. Lathes
    - 3. Planers
    - 4. Shapers
    - 5. Drill presses
    - 6. Other tools
    - 7. Benches and vises
- 3. Powder houses
- 4. Offices
- 5. Dry or change houses
- 6. Storehouses
- 7. Boarding and dwelling houses
- 8. Stables
- 9. Shaft houses
- 10. Tipples
- 11. Pockets or slack bins
- 12. Company store
- 13. Timber yard and plant for preparing timber
- 14. Water
  - 1. City or commercial service
    - 1. Quality of water
    - 2. Sufficient or insufficient
    - 3. Pressure
  - 2. Company service
    - 1. Quality of water
    - 2. Sufficient or insufficient
    - 3. Gravity system
    - 4. Pumping system
      - 1. Direct
      - 2. Reservoir or stand pipe

**DIAGRAM FOR REPORTING ON COAL LANDS—(Continued)**

15. Lighting
- 1. Character { 1. Gas / 2. Electric }
- 2. Origin { 1. Commercial plant / 2. Company plant }
- 3. Sufficient or insufficient

16. Hoisting or winding plant
- 1. Steam engines
- 2. Compressed-air engines
- 3. Oil or gasoline engines
- 4. Electric motors
- 5. Water motors

17. Surface transportation
- 1. Railroad
  - 1. Gauge
  - 2. Total length
  - 3. Size of cars
  - 4. Number of cars
  - 5. Power used
  - 6. Number of motors
- 2. Wagon roads
  - 1. Character and surface of
  - 2. Length
  - 3. Number of wagons and teams

10. MISCELLANEOUS
- 1. Yearly income, last year, or for any year { 1. Gross / 2. Net }
- 2. Average cost per ton of material

11. CONCLUSIONS
- 3. Merits of property
  - 1. Deposits of value
    - 1. Quality of coal or coke
    - 2. Amount of coal in sight { 1. Gross / 2. Net }
    - 3. Value of coal in sight
  - 2. Value of plants and works
- 4. Advice
  - 1. Mining { 1. Continue present system / 2. Change system to——— }
  - 2. Disposition of product { 1. Ship as mined / 2. Coke and ship }
- 5. Local considerations
  - 1. Troubles
  - 2. Labor
  - 3. Supplies
  - 4. Climate
  - 5. Shipment facilities
  - 6. Markets

# OPENING A MINE

## GENERAL AND FINANCIAL CONSIDERATIONS

Usually the kind of opening through which the coal underlying a tract of land must be extracted is not a matter of choice but is fixed by the distance of the seam above or below the surface at the point where the plant must of necessity be built. If the seam outcrops near the selected site, the coal will be opened by a drift if the measures are horizontal or nearly so, or by a slope driven down the dip if the bed is inclined. If the coal does not outcrop, it must be opened by a vertical shaft, although, if the distance to the seam does not exceed, say, 50 to 100 ft., the mine may be opened by a rock slope; that is, by an inclined passageway driven downwards through the rock to intersect the seam.

In mountainous districts, the outcrop of a steeply pitching seam may be inaccessible by reason of its great elevation above the valley. In such a case, the mine may be opened by a rock tunnel driven from some point at the proper elevation in the valley across the inclined measures until the seam is met. In rare instances, the seam, while not outcropping within the limits of the property, lies but a short distance beneath the surface, and may be worked by *stripping* as explained under Methods of Working.

**Relation Between Investment and Cost of Production.**—Where a choice is possible, the relative cost of opening and equipping the mine by each type of opening (drift, slope, and shaft) must be considered in connection with the cost of operating through each before a selection can be made. If the capital is to be had, an increased investment is often warranted by a lower cost of production made possible through its use. The increased cost of driving and equipping one kind of opening as compared with another can usually be determined within fairly close limits, but it requires the skill and judgment acquired through long experience to be able to estimate, even approximately, in advance of actual developments how much more it will cost per ton to extract coal under one set of conditions than under another. The general method of making these calculations is shown in the following:

EXAMPLE.—There are 2,000,000 T. of coal in a property that it is proposed to extract in 10 yr. at the average rate of 200,000 T. per yr. A shaft costing $50,000 to sink and equip will effect an estimated saving of 5.25 c. per T. in the cost of production over a slope costing $20,000. If the money to open the property must be repaid in ten annual instalments with interest at 6%, which opening should be selected and what is the gain by so doing?

SOLUTION.—In the case of the shaft, there will be due at the end of the first year interest on the entire loan or $3,000 and one-tenth of the principal or $5,000, a total of $8,000; at the end of the second year there will be due interest on $45,000 or $2,700 and one-tenth of the remaining capital, a total of $7,700; and similarly each year thereafter until the end of the tenth year when the last payment of $5,300 for both principal and interest will be due. During the life of the mine there will have been paid out for principal and interest, the sum of $66,500, or an average of $66,500 ÷ 2,000,000 = 3.325$ c. per T. of coal produced.

In the same way it may be shown that the total cost for principal and interest for sinking and equipping a slope will be $26,600, or at the rate of 1.33 c. per T.

So far as the investment alone is concerned, the slope is the cheaper to the extent of $3.325 - 1.33 = 1.995$ c. per T., or very nearly $40,000 during the life of the property. On the other hand, mining through a shaft means a reduction in the cost of production of 5.25 c. per T. Hence, a net saving of $5.25 - 1.995 = 3.255$ c. per T., or a total saving of $.3255 \times 2,000,000 = \$65,100$ will be effected by sinking, equipping, and using a shaft.

**Relative Cost of Different Types of Opening.**—As drifts and slopes are driven through much softer material (coal), they are much less costly than shafts, which are sunk through rock; and, further, the returns from the coal extracted from them to some extent offset the cost of driving. Owing to the greater difficulty of handling materials on a pitch, to the added cost of pumping water, etc., slopes are more expensive to drive than drifts. Furthermore, the cost of the massive head-frames, powerful pumps and hoisting engines, etc., commonly required at shafts, and to but a slightly less extent at slopes, adds materially to the capital invested in equipment over that necessary for a drift. Therefore, from the standpoint of capital required, the choice of opening will be in the order, drift, slope, shaft.

**Cost of Production as Affected by Type of Opening.**—That type of opening is to be preferred which permits of the lowest cost of production for the same investment of capital. In the case of flat seams, the cost of production is materially less in a mine opened by a drift than in one opened by a slope or a shaft. This is largely due to the fact that in the latter cases, the delivery of the coal to the tipple is made in two stages; first, hauling it to the foot of the slope or shaft; second, hoisting it to the surface. In drift mines, the coal is hauled directly to the tipple in one operation and hoisting costs are thus saved. Furthermore, in drift mines the cost of pumping is usually negligible and that of handling men and supplies is a minimum. Flat seams are sometimes opened by a rock slope, but no reduction in operating costs is effected thereby unless the slope is short and the pitch such that long trips can be hauled and the men can walk to and from their work.

Whether a pitching seam is more cheaply operated through a slope driven from the outcrop on the coal or through a vertical shaft intersecting the seam at some depth below the crop cannot be determined without a careful study of all the factors concerned in the particular case. If the slope is comparatively flat it is to be preferred to a shaft, but if the pitch is so great that the mine cars must be hoisted on a slope carriage or is so very steep that the coal must be dumped at the foot of the slope into a gunboat or skip, a shaft is more cheaply operated.

In general, the cost of production through the three types of opening is in the same relative order as the cost of making openings; namely, drift, slope, and shaft, although, in the case of highly inclined slopes, the order will be, drift, shaft, and slope.

## LOCATION OF SURFACE PLANT

While every endeavor should be made to locate the mine opening and screening plant so that each may be operated with the greatest efficiency and economy, this is not always possible. Usually, the surface plant must be located to meet certain natural, business, or financial conditions, and the location of the mine opening is subordinated to the absolute necessity of preparing the coal in such a way that it is marketable. When locating the surface plant the following points have to be considered:

**Grades.**—The track grade should be such that the railroad cars will drop by gravity from one end of the siding to the other. Opinions differ as to the most desirable grades, but 1.5 to 2% from the end of the tail-track to the tipple is ample for the empties; and for loading under the tipple and thence to the end of the loaded track, 1.25% gives excellent results. Sometimes grades of 3% and more are met above a tipple, but such slopes are very dangerous, owing to the liability of a string of empties running down into the cars being loaded. When the grades are so flat that the cars will not run except when pinch bars are used, it will prove profitable to install some kind of car haul or car-spotting device operated by steam power.

**Length and Number of Sidings.**—The length of the sidings necessary for the storage of empty and loaded railroad cars will depend on the daily output of the mine and the number of sizes of coal shipped. The output of the mine depends on the ability of the company to sell the coal at a profit, and this, in a great measure, hinges on the quality of the product. A high-grade fuel, even at a high price, is more salable and is in steadier demand than a poor one.

There must be as much storage room for loaded cars below the tipple as there is for the storage of empty cars above it. If the coal is loaded in steel hoppers of 100,000 lb. (50 T.) capacity, averaging 40 ft. in length there will be required for each 1,000 T. of daily capacity $(1,000 \div 50) \times 40 = 800$ ft. of siding. To allow for switches and for cars of less capacity, it is better to assume 1,000 ft. of siding for each 1,000 T. of daily output. For a single-track tipple, that is, one under which there will be but one loading track and hence shipping mine-run coal only, there will thus be required 2,000 ft. of siding for each 1,000 T. of output. In the case of two- and three-track tipples (those loading two or three sizes of coal) if there is 2,000 ft. from the point of switch on the main line above the tipple to the corresponding point below it, there will be car-storage room for more than 1,000 T. capacity. This is because the second loading track will have, say, 1,500 ft., and the third, say, 1,000 ft. of available storage room, half above and half below the tipple. Owing to uncertainties and irregularities in train service, it is highly advisable to provide storage room for 2 da. car supply.

The width of bottom land required for the siding depends on the number of sizes of coal shipped. It is possible, but unusual, to ship but one size, that is, mine run or the unscreened output as mined. If the supply train pushes the empties above the tipple, they may be dropped down and loaded upon the same track. This will require about 16 ft. in width for the roadbed and ditches. Commonly, the empties are placed by the switch engine at the upper end of the mine branch railroad (or are dropped on the main line if the mine is situated thereon), and pass by gravity on to the mine siding or loading track, and down it to the tipple. The two tracks are commonly laid with their center lines 13 ft. apart, which requires a grade from 26 to 28 ft. wide, depending on the size of the side ditches. At the tipple, and for a few hundred feet above and below it where the cars are being handled during loading, the tracks should be laid with not less than 15 ft. between center lines; and if fast passenger traffic is passing on the main line, this distance should be 18 to 20 ft., unless a high fence is built between the mine and railroad tracks. This will require a bottom width of from 28 to 35 ft. For each additional size of coal shipped one track and 15 ft. of width should be allowed. Thus, if three sizes are made and the supply and first loading track require 30 ft. of width, there will be needed a total of $30 + 15 + 15 = 60$ ft. for the three loading and one supply track. While this width may be reduced in a tipple of steel construction, which spans the tracks and does not require bents between them, it is not advisable to reduce the distance between the supply and first loading tracks.

**Mining Plant.**—The space required for the mining plant, including in that term the tipple, boiler, and power house, car and repair shops, stable, supply house, fan, etc., will vary from a few hundred square feet at a small mine to several acres at a large one. The tipple must be located with respect to the tracks as explained. The fan is always near the mine and the car and repair shops and the stable are commonly placed near the mine mouth. The supply house is better placed near the tracks for ease in unloading supplies. If electric power is used and the boiler and power plant are placed near the tipple, cheap slack may be used for fuel, and the necessary power for running the fan, shop machinery, haulage motors, etc., may be cheaply conveyed to the mine at any reasonable distance. If compressed air is used for coal cutting and haulage, the air compressor may be placed at the mine mouth and operated by electricity generated at a distance. If room for the boiler and power plant is not to be had at the tipple, they must be located near the mine mouth, must use mine-run coal for fuel, and the power for operating the machinery must be transmitted to the tipple. For ease in supervision, the various units of the plant should be near to one another.

**Mining Village.**—As it is easier to hold and secure men if their homes are near their work, it is advisable to place the mining town (village, camp, or settlement) near the mine mouth, as this location will suit practically all the workers except, perhaps, the tipple hands. The space required for the town varies according to the number of men required to produce the desired tonnage, and this will depend on the thickness of the seam, its relative hardness, use or non-use of coal-cutting machinery, etc. Probably it is a fair estimate to assume that housing must be provided for from 150 to 200 families for each 1,000 T. of daily output. In addition, space must be provided for a store, a schoolhouse, several churches, a hall for assemblies, etc. It is now customary to provide playgrounds for the children and baseball fields and swimming pools, etc., for the men. Each house should be placed on a lot $50 \times 150$ ft., that there may be room for a kitchen garden in the rear. If $\frac{1}{4}$ A. can be given each house it is better, and the saving in insurance by having the houses farther apart is an item of importance. Two hundred houses, each on a lot $50 \times 150$ ft., can be built on 50 A., assuming the streets and alleys to take half as much space as the building lots. To the 50 A. must be added from 10 to 25 A. for stores, churches, schools, playgrounds, etc. Space for the village is easily obtainable in level country, but in mountainous districts the houses are too commonly perched along the hillsides in an unsightly fashion. In this case, it is advisable to secure the services of a competent landscape architect to secure the best and most artistic arrangement of the houses with respect to the irregularities of the surface. It is greater economy to pump water, haul supplies, etc., to a well-situated town, than to consider only the first cost and place the town in an undesirable location near a stream or the railroad tracks.

**Coke Ovens.**—If the coal, either as mine run or as slack, is coked on the property, more room is required than if it is all shipped to market, except in the unusual case where there are four or five loading tracks under the tipple. The space required for the ovens will depend on the tonnage of coke desired, whether the ovens are built in bank (single row) or in block (double row placed back to back), and on the width of yard required in front of the ovens to store the coke before it is loaded into cars. A beehive oven may be counted upon to yield an average of 2 to 2.5 T. of coke daily, depending on the size of the oven, kind of coal charged, etc. An output of 500 T. per day will, therefore, require 200 to 250 ovens. The distance apart, center to center, of the ovens will be equal to the diameter of the oven plus twice the thickness of the lining brick, or $2 \times 9 = 18$ in., plus from 4 to 6 in. between ovens in which clay is tamped. The spacing for ovens 12 ft. 6 in. inside diameter (a common size) will thence be 12 ft. 6 in. $+$ 1 ft. 6 in. $+$ (say) 6 in. $=$ 14 ft. 6 in. Allowing 5 ft. for the two end walls, the 200 to 250 ovens required for an output of 500 T. will be either $200 \times 14.5 + 5 = 2,905$ ft. or $250 \times 14.5 + 5 = 3,625$ ft. if built in bank, and one-half these lengths if built in block. The capacity of the railroad cars into which coke is loaded varies greatly. It is difficult to get the full 30 T. into box cars of 60,000 lb. capacity, but special *coke racks*, as they are called, will hold 50 T. Thus it will require ten to twenty cars, depending on their kind, to hold the assumed 500 T. of coke. If 750 ft. of clear trackage, or, say, 1,000 ft. to allow for switches, clearance, etc., is allowed both above and below the ovens, there will be ample room to hold the smallest cars used in the coke business. Thus, including the storage tracks, a bank of 200 to 250 ovens will be, say, 4,900 to 5,600 ft. long. If built in block about 1,000 ft. in total length will be saved.

The 12.5-ft. bank oven has a width of practically 16 ft. from the side hill against which it is built to the front, or mortar wall. The width of the coke yard in front of the ovens varies from as little as 10 ft., in cases where the coke is loaded out promptly, to as much as 30 ft. where, owing to uncertain market conditions, the coke must be piled on the yards pending sale. Further, there must be added, say, 14 ft. for the track upon which the cars are loaded. Hence, a row of bank ovens, exclusive of the space required for the supply track, will have a width of from $16 + 10 + 14 = 40$ ft. to $16 + 30 + 14 = 60$ ft.

The same sized oven arranged in block will require a width of about 33 ft. from the face of one front wall to the other. The two yards will vary between $2 \times 10 = 20$ ft. and $2 \times 30 = 60$ ft. in width, and the two loading tracks will require $2 \times 14 = 28$ ft. Thus, a block of beehive ovens of standard size will be 81 to 121 ft. wide, exclusive of the supply track.

It is well to have the tipple or coal bins at the upper end of the ovens, particularly if the grade is steep, so that the charging larry may run or may be assisted in running by gravity. This is not so important if mechanical haulage is used.

## LOCATION OF MINE OPENING

In level countries, it is almost always possible to so locate the mine with respect to the surface plant, that both mining and outside costs are a minimum. In hilly countries, however, the site of the surface plant is determined by the conditions just discussed, and the place for the mine opening may have to be decided by striking a balance between an increased or decreased cost of production on the one hand and a decreased or increased capital charge (interest and sinking fund) on the other.

When locating the mine opening, the following points should be considered: The opening should be at the lowest point of the seam; that gravity may assist both the underground haulage and drainage; haulage is almost always cheaper underground than on the surface and particularly so where the winters are severe and the fall and spring wet; the mine opening should be as near as possible to the tipple, as the concentration of the plant at one point tends to efficient management, and objectionable surface haulage is avoided.

**Flat Seams.**—A flat seam is commonly understood to be one in which the dip does not exceed 50 to 150 ft. to the mile, or, say, 1 to 3%; in all cases such a seam should be opened at the tipple by the necessary drift or shaft. It is desirable to have the dip of the seam in favor of the haulage, but if it is not, the more advantageous location of the opening and the saving in shaft haulage more than offsets the small increased cost for power in hauling up such slight grades. If the seam outcrops, an endeavor should be made to open it at *tipple height*. By tipple height is meant the distance between the rail on which the mine cars stand on the tipple and the rail on which the railroad cars are loaded. This distance varies according to the number of sizes of coal shipped. If mine-run coal only is loaded, this height need not exceed 16 to 18 ft. If three, four, or more sizes are loaded the height will be 30 or 32 ft., or even more, in order that the proper pitch may be given the screens over which the coal is sized. For modern tipples, a fair height is 32 ft. It will be understood that the outcrop of the seam is assumed to be at some elevation above the tipple platform in order that gravity may assist the movement of the loaded mine cars from the drift mouth to the dump. This elevation will be at the rate of 1 to 1.5 ft. per 100 ft. of distance. Thus, a drift mouth 35 ft. above the railroad and 300 ft. from the dump will afford a tipple height of $35 - (300 \times .01) = 32$ ft. if the down-grade from the mine is 1%. If the coal is not shipped but is coked at the plant, the mine cars may be run from an opening 60 to 75 ft. above the valley directly into bins into which they are dumped, the coal being drawn out below into larries and conveyed to the ovens. If the coal outcrops at more than tipple height, say, at an elevation of 100 or 200 ft., or more, it is commonly lowered on a self-acting or gravity incline. In other cases, the coal is dumped at the mine mouth into a retarding conveyer and carried in a continuous stream to the tipple, or the loaded cars are dropped to the tipple by a chain haul. In the Pocahontas region of West Virginia, in order to provide storage room for coal that the mine may be kept in operation while waiting for railroad cars to load the output, as well as to secure height for the slack bins, which are built in as an integral part of the tipple, it is customary to make the tipple height 60 ft. or more, even in those cases where the mine cars are dropped down on a self-acting incline. This permits of a very long chute in which may be stored enough coal to fill three, four, or even more railroad cars.

**Seams of Moderate Dip.**—Where the pitch of the seam does not exceed, say 30°, if the opening can be placed at the tipple, it is probably the better plan to open the property by a slope driven directly down the dip. The seam may then be developed by a series of levels driven both to the right and left on grades favorable to haulage, and the mine cars may be run directly from these levels to and up the slope. If a seam of this pitch is opened by a shaft, it will be necessary to lower all the coal produced on each level to the main level driven from the shaft bottom. While this lowering is commonly done on self-acting inclines at no expense for power, the machinery and track are costly to maintain and there must be several attendants for each incline. If the seam lays in the form of a syncline, whether it is better opened by one or more slopes driven down from the outcrop or by a shaft tapping the basin at its lowest point, should be made the subject of a careful calculation by these methods.

**Seams of High Dip.**—Where the dip of the seam is so great that the coal must be dumped into gunboats, much fine coal is made through this extra handling. While this is not objectionable where the entire output is coked or even in those cases where the slack alone is so treated, it is a source of loss at those mines where a large percentage of lump coal is desirable. In the latter case, the cost of production is less and the coal will reach the surface in better shape if the seam is opened from a vertical shaft by rock tunnels driven therefrom at regular intervals, as the underground haulage is less and the mine cars are loaded directly on to the shaft cages. As before, the best method for opening is to be decided after striking a balance between increased capital account and decreased cost of production and higher selling price because of the larger percentage of lump coal.

**Method of Working.**—As the system adopted for actually extracting the coal may have some effect upon the location and method of opening the mine, the section entitled Method of Working should be consulted in connection herewith.

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# DRIFTS

The size of a drift depends on the output desired, the size of the mine cars to be used, the character of the haulage, the thickness and character of the coal seam, and the character of the top and bottom rock. The height of the drift should not exceed the thickness of the seam, unless absolutely necessary, in order to avoid the expense of brushing (taking down) the roof or lifting (taking up) the floor. There should be at least 6 ft., and better, 6 ft. 6 in., from the top of the track tie to the roof or to the bottom of the timbers used for supporting the roof, so that the men who are employed at the mine mouth in handling the cars can walk without stooping.

The width of the drift depends on the purpose for which it is employed. If used solely for a manway, a width of 6 ft. is ample. If used solely as an intake airway, the drift should be as wide as economically possible, in order to reduce the friction and the velocity of the air and the power required to move it. Common widths for intake airways are 8 and 10 ft. When the drift is used for haulage, about the least width that can be used when any allowance is made for the safe passage of men is 8 ft., and 10 ft. is much better in view of the fact that the average haulageway is also an airway in which the mine cars form a serious obstruction to the ventilation, even if the opening is of good size.

When the drifts are used for haulage, it is a question whether the empty and loaded trips should run upon parallel tracks or whether each should have its separate opening. A two-track opening will have to be 16 to 20 ft. in width, and if the roof is at all poor the cost of timbering will be excessive. For this reason, and to prevent accidents to the employes from trips passing in opposite directions, many operators prefer distinct openings for the in-going and out-going traffic. In any case, posts (props) are to be avoided between tracks.

The grade of a drift is determined by the dip of the seam, but the perfect grade is one on which the pull required to return the empty car to the face is exactly equal to that needed to bring out the loaded car. A grade of from 1 to 1.5% in favor of the loads gives excellent results, but less will do if the track is well kept up and the car wheels are provided with roller or ball bearings to reduce friction.

A gutter or ditch is commonly dug along one rib, and is lined with tile or concrete or has a wooden trough laid in it if the bottom is soft and apt to erode. If the drift is being driven down hill, the water can be siphoned out for some time, after which a small steam, compressed-air, or electric pump must be used for the purpose.

In beginning a drift, an open cut is first started in the hillside. Its width at the face should be somewhat wider than the drift itself, and the two wings, or side walls, should diverge outwards so that the floor plan has somewhat the shape of a truncated **V**. This open cut should be continued until rock firm enough to be supported by timber is encountered, when a substantial set of timbers should be placed. These may be replaced after the drift is sufficiently
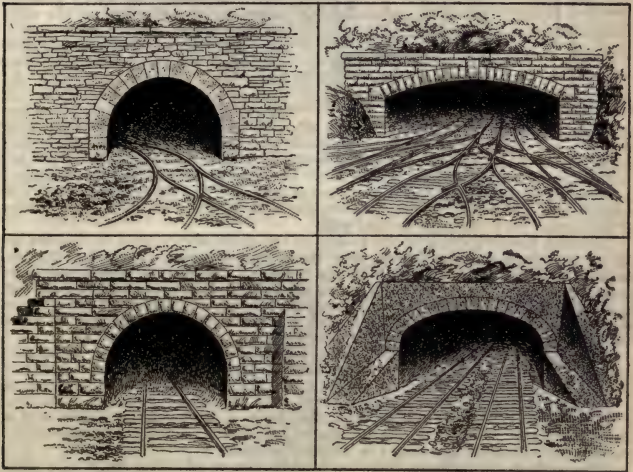


advanced by steel beams or by masonry or concrete. When the coal is reached, ordinary mining operations begin.

In order to prevent the hillside from washing into the cut and blocking the tracks at the drift mouth, the loose material above the opening as well as that forming the roof and sides is frequently held in place by masonry or concrete walls, arches, etc. Some of the forms employed are shown in the accompanying illustration.

## TUNNELS

### TUNNELS THROUGH LOOSE GROUND

Unless the outcrop of the seam is plainly exposed, it is well to put down a number of churn-drill holes along the supposed place of outcrop and at an elevation above that of the roof of the coal. These holes will determine the character of the covering immediately over the seam. If the roof rock is found in place but a short distance back of the toe of the crop, the drift may be begun by an open cut as just explained. If, however, the outcrop is covered with a layer of wash or drift, 25 or 50 ft., or more, in thickness, it is not generally possible (owing to the running of the sides, etc.) to proceed with an open cut. Instead, some one of the methods for tunneling through loose ground must be employed. These methods have been very completely developed in connection with railroad and underwater tunnels. Among the numerous processes are forepoling, wedging, the pneumatic process, the freezing process, and the use of metallic shields; the last three are described under the heading Shafts.

**Forepoling.**—Forepoling is used when driving through loose ground both at the outcrop of the seam, and underground where the coal seam is replaced by the clay, sand, and gravel of an old river bed. The process consists in driving sharpened pieces of narrow plank, or lagging, into the roof at a very slight pitch. The lagging rests on the collar of one timber set and is held firmly by having its end underneath the next timber toward the outside.

In the cut, *a* are the posts of sets of timbers, *b* the caps, and *e* the top bridging. The front ends of the spiles *g* from any given set rest on the bridging of the
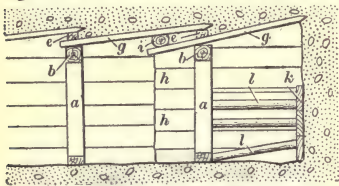


next advanced set, and the spiles for advancing the work are driven between the bridging and the set, as shown. To force the spiles into the ground, so as to provide room for the placing of the next set, tail-pieces *i* are placed behind the back end of the spiles as they are being driven. After the spiles have been driven forwards the desired amount, another set is placed, the tail-pieces knocked out, and the front end of the spiles allowed to settle against the bridging of a new set. Where the face is composed of extremely bad material, it may be necessary to hold it in place with breast boards *k* held in place by props *l* that rest against the forward timber set. In a similar manner the side lagging *h* is placed in position. When breast boards are used, it is generally necessary to employ foot and collar braces between the sets, so as to transfer the pressure of the breast back through several sets.

To start the forepoling at the mouth of the drift, several sets of timbers are set up and long lagging driven over them into the earth beyond. By balancing the pressure of the earth on the outside end, they are held up and enough earth removed to allow another set being placed to support the lagging nearer the tunnel face. While practicable in rather loose ground, this method is not available in material containing boulders, and is dangerous when used in quicksand.

**Wedging.**—It has sometimes been possible to drive through quicksand by using, in combination with forepoling, a number of wedges as shown in the accompanying illustration. Here, *a* are the posts of regular timber sets; *b*, the side planking; *c*, the spiling driven, as in forepoling, to support the top; *d*, the wedges; *e*, the tailing pieces; *f*, the floor; *g*, the bridging pieces; and *h*, the cap pieces. The set of timber shown below *e* is only placed temporarily, and is removed after the spile *c* is driven forwards.

The wedges *d* are driven into the face by means of a ram made of a piece of timber swung from the roof. They simply crowd the material away from
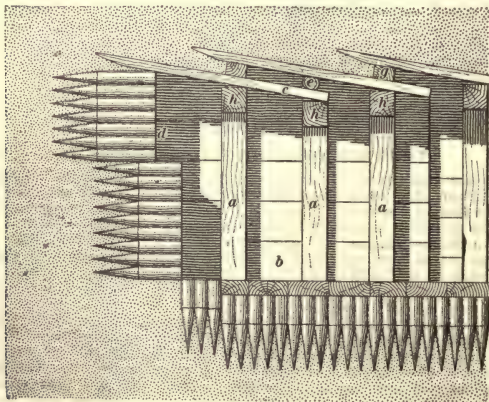


in front of the excavation; if the pressure becomes so great that they can be driven no farther, a few auger holes are bored into the face to relieve the pressure by allowing some of the material to flow into the drift. Wedges are

driven into the floor with a mallet as fast as those in the face advance, and are ultimately covered with a plank floor.

## TUNNELS THROUGH ROCK

Tunnels through rock, commonly called *rock tunnels*, are used in coal-mining operations to open a steeply pitching seam that extends upwards in the hills a considerable distance above drainage, by driving across the measures to the coal from some convenient elevation near the surface plant. They are also much used in mines where the strata are contorted, as in the anthracite regions of Pennsylvania, to connect two pitching and parallel seams, and to drive through anticlines or synclines in those cases where following the contour of the seam would result in the gangway being either too crooked or too steep for successful haulage.

The conditions governing the location, size, etc., of rock tunnels used to develop a property are the same as in the case of drifts. If the surface is firm, the tunnel may be started by an open cut; if it is not, forepoling or wedging must be resorted to. Underground tunnels, used to connect adjacent seams, etc., are usually made as small as is consistent with safety as they are used mainly for haulage.

**Arrangement of Drill Holes.**—In driving rock tunnels, the chief item of cost, under ordinary circumstances, is that of drilling. Where machine drills
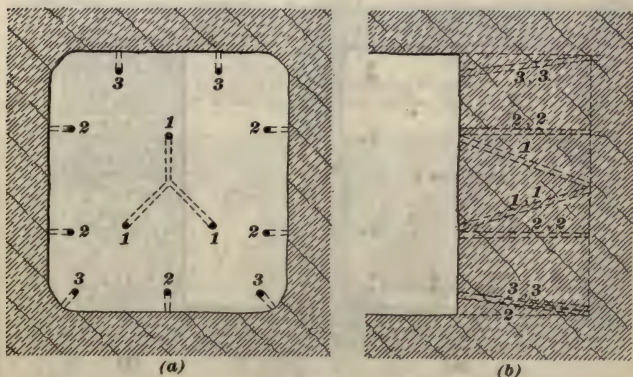


Fig. 1

are used, there will be more or less time consumed in shifting machines, for which reason it is deemed advisable, wherever headings are large enough, to use two machines and if possible on one column or bar. This is frequently also the case in shaft sinking. The machines should be so placed that the holes may be drilled methodically, so as to economize in powder and time.

In tunnel driving and in shaft sinking, there is always one free face, and in order to obtain two free faces it is necessary to first take out a cone or wedge of rock from the center or side of the heading. Holes put in for this purpose are termed *key holes*, or *cut holes*, and are fired simultaneously, in order to obtain the best effect of the powder and to save time.

In making key holes, the size of the heading and the hardness of the rock are to be considered. In soft rock, key holes in the bottom may be the best. In harder rock, the key holes may be arranged in circular form to take out a cone. The outer holes are then arranged more or less concentrically with the center cut holes, or more frequently the key holes are arranged in straight lines from top to bottom of the face so as to take out a wedge-shaped center cut. The enlarging holes are similarly arranged in straight lines parallel to the lines of key holes.

**American and European Practice.**—It is customary in some European countries to place the breaking-in holes so that they will not meet, in order

that there may be a wide end to the cavity broken out by them; American practice, however, is to make them meet so that the increased quantity of powder that may be inserted will have more effect. European practice is generally to use short holes; that is, one-half the width of the heading, while American practice is to make the holes about as deep as the heading is high. While the two systems call for about the same number of feet of drill hole, there must necessarily be considerable saving in time where drills are not changed as often as is required in European practice.

The American system is probably the better where labor is expensive, since it permits of quicker advance and keeps the shifts up to their work in better shape. It requires, however, more explosive, and is therefore best adapted for countries where explosives are cheaper than labor.

**Conical Center Cut.**—Fig. 1 (a) shows a rock heading 6 ft. by 7 ft. in medium-hard homogeneous rock. The key holes *1* are put in at an angle so that the bottom of the holes will meet at a depth of about 6 ft. from the face; view (b) shows a section of (a). Where drill holes meet in this manner, they form a chamber in which the powder can be placed in larger quantities and be more effective than in single drill holes. The direction of the second rows of holes is shown at *2*, and the third row at *3*. The holes are all drilled in one shift, and the blasting done and the broken rock removed by the following shift, It is customary to charge and blast holes by rounds, *1* being first fired.



(a)                                    (b)
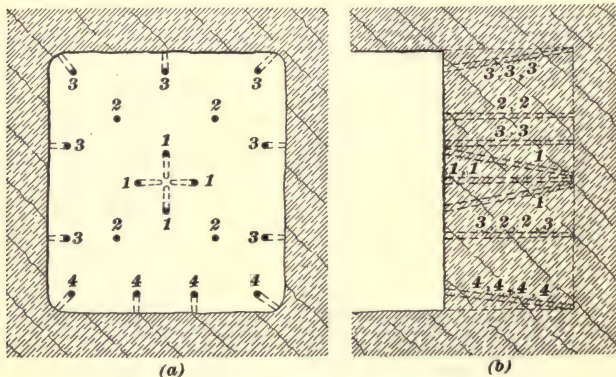
FIG. 2

and as soon as rock is cleared from this the enlarging holes *2* may be charged. It is sometimes customary to fire holes *2* and *3* together, but it is better to fire *2* and remove the dirt, and lastly *3*, as that gives a larger free surface to work on. There must be a wait, after each round is fired, for the air to change, but with an air hose the foul air can be driven out quickly, and while the rock is being removed the next round may be charged. The number *3* round is known as the squaring-up round, and while the holes are placed a short distance from the roof and floor they are kept, as nearly as is possible, parallel with the side walls. That they are not put in exactly horizontal is due to the inability of the drill runner to place the machine nearer the walls. The number and position of the drill holes for enlarging will depend on the size of the heading and the explosive used.

In some cases, the bottom holes *3* are fired after the upper holes *3*, as by that operation the debris is moved forwards making the work of the shovelers easier.

Fig. 2 (a) and (b) shows another arrangement where the rock is hard. The key holes *1* in this case are four in number and outside of them are four enlarging holes *2*. It might be possible to load and fire holes *3* and *4* together, but as the latter are in tension and the number *3* holes are assisted somewhat by gravity, better results will probably be obtained by firing number *4* holes last.

It is to be understood that what has been stated is not always the proper way to place holes; much depends on the force of the explosive, the hardness of the rock, and the evenness of its texture. The miner must be guided largely by experience and common sense, in finding exactly the best position. To drill extra hand holes for squaring up a heading is unsatisfactory and expensive, consequently, the key holes that furnish two free faces are the particular ones to be watched. Key holes should be placed as far apart as experiments show it to be necessary to give the best results, and when that distance has been found, it should not be varied.



FIG. 3

The Billy White Cut.—Three holes *1, 2,* and *3,* Fig. 3, are drilled straight into the breast, exactly in a line with one another, at a distance of 5 in., center to center, thus making the distance over all 12 in., assuming the holes are each 2 in. in diameter. Another hole *4* is drilled 7 in. to the right of the first three holes and in a horizontal line with hole *2.* Hole *5* is drilled 12 in. to the left of hole *2* and in the same line as hole *4.* These five holes
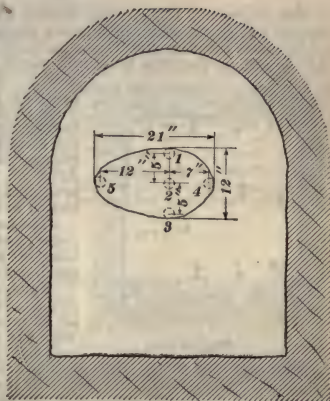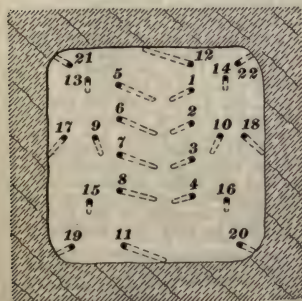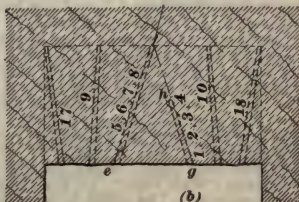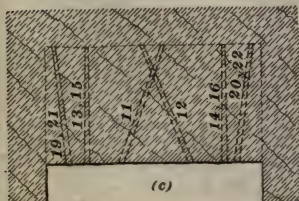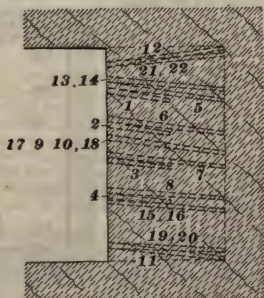


FIG. 4

complete the drilling of the cut. All are put in horizontally or looking down just enough to hold water.

The success of this cut lies mainly in drilling the holes in planes with one another and in shooting them in the proper rotation. Four of them are loaded in the usual manner, care being taken to cut the several fuses of slightly variable length so that the holes will go off in the following order; viz., *1, 3, 4,*



Fig. 5

and *5.* Hole *2* is drilled merely to provide space for hole *1* to break to, and is never loaded. Each shot creates more clear face for the following shot. After holes *1* and *3* have gone, the chamber is about 12 in. vertically by 5 in. horizontally and somewhat oval in section. After all the holes have been shot, there is produced a chamber about 12 in. by 21 in., as shown in the illustration.

The cut breaks as big at the bottom as at the collar, and to a depth of at least 6 ft. The full bore of the tunnel is obtained by the usual placing of holes that will break to this initial cut.

**Square-Cut Drilling and Blasting.**—A center cut is taken out the full height of the face and all subsequent holes are drilled in lines parallel with the side of the heading. Fig. 4 (*a*), (*b*), (*c*), and (*d*) illustrates the square-cut method

driven according to the European practice in blasting; that is, the holes are not more than one-half the width of the heading in depth. View (*a*) shows the face of the heading; (*b*) and (*c*) are horizontal sections, and (*d*) is a vertical section.

Two or four drills can be used to advantage in drilling the holes for the square-cut system. With strong hard rock, the diameter of the holes will be about 1¼ in. at the bottom, but the four holes *1, 2, 3, 4* will be somewhat shallower than the others. It will also be noticed that there are only three upwardly inclined or dry holes to be bored in this arrangement, as compared with four in the center-cut system, Figs. 1 and 2. As the drill holes are always slightly conical (because each succeeding drill is of smaller diameter than the preceding), the four shallower holes will be nearly, if not quite, 1¼ in. in diameter at the bottom. The entering wedge, Fig. 4 (*b*), is best removed in two stages: First, the part *e g h* by the breaking-in shots *1, 2, 3,* and *4,* and then the part *e f h* by the breaking-in shots *5, 6, 7,* and *8.* The order of firing the shots is as follows: First volley, *1, 2, 3,* and *4,* simultaneously; second volley, *5, 6, 7,* and *8,* simultaneously; third volley, *9, 10, 11,* and *12,* either simultaneously or consecutively; fourth volley, *13, 14, 15,* and *16,* either simultaneously or consecutively; fifth volley, *17, 18, 19, 20, 21,* and *22,* either simultaneously or consecutively. The effect is practically the same whether the enlarging shot holes are fired simultaneously or consecutively, on account of the fact that they are too far apart to assist each other, but to save time simultaneous firing is advisable.

**Side Cut in Heading.**—Sometimes there is a natural parting at one side of the heading, as when the heading is following a vein of ore, or a slip in the rock. In such a case, the side cut offers very important advantages and especially when only one rock drill is employed Fig. 5 illustrates a set of holes drilled to make an advance of 3 ft. 6 in. in a



Fig. 6

heading by means of a side cut. The order of firing would be as follows: First volley, *1* and *2,* simultaneously; second volley, *3, 4,* and *5,* consecutively; third volley, *6, 7,* and *8,* consecutively; fourth volley, *9, 10,* and *11,* consecutively.
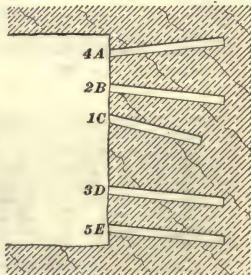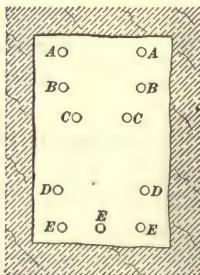
**Special Arrangement for Throwing Broken Rock from Face.**—Of course, no general rules can be laid down for drilling holes under all circumstances, as the rock may vary from point to point in the same drift or heading, and the seams or joints will always have an effect on the results. Fig. 6 illustrates a set of holes drilled in the face of a heading, which brings out another principle. In this case, the holes are fired in the order of their numbers, the holes *E* being fired last. It will be noticed that there has been an extra hole placed at the bottom, and these bottom holes are sometimes overcharged in order that the last shot may have a tendency to throw the broken rock away from the face. If the order had been reversed, and the upper shots fired last, the broken rock would be piled against the face of the heading in the very place where the drill would have to be set up for the next operation, and much valuable time would be lost in throwing back the broken material before the machine could be set up.

# SLOPES

A *slope* resembles a tunnel, a drift, or a shaft, depending on its inclination. A *flat slope* is treated, as to its size, method of driving, timbering, etc., essentially as a drift or tunnel, while a *steep slope* is treated like a shaft. Blasting in a slope is more difficult than in either a shaft or a tunnel, as the rock is said to *bind*, owing to the inclination of the strata; and, in general, it may be said that more holes and more powder are required for a slope than for either a tunnel or a shaft of equal size. The amount of increase in the width of the slope pillars as the slope descends depends on the degree of pitch of the slope, which determines the thickness of cover above the slope.

The removal of the material excavated from a slope is more difficult than in drifting or tunneling, and this difficulty increases with the inclination of the slope. When starting a steep slope, as when starting a shaft, the material is removed by the use of a windlass for drawing the car up the slope, or a small portable hoisting engine may be set up when the slope has been sunk but a few yards.

The drainage of a slope is accomplished by pumps located at or near the foot, small sinking pumps placed on trucks being generally used. In order not to be obliged to move a large pump too often while sinking, inspirators, which are easily moved as the work advances, are sometimes used at the slope bottom to throw the water up to the pump station.

The *timbering* of a slope differs from that of a drift or tunnel in the manner of setting the posts, which, in a slope, are *underset* or made to lean up the pitch from the normal position in the seam, or from a position perpendicular to the plane of stratification. The amount the post is underset and the manner of undersetting vary with the inclination of the seam.

**Safety Appliances.**—The necessity of safety appliances increases with the inclination of the slope. *Refuge*, or *shelter*, *holes* for the safety of the men engaged in the slope should be made in the sides; in some states, they are required by law, owing to the liability to accident to men by being caught and squeezed between the rib and a trip of cars, or by the breaking of the hoisting rope or car couplings, or the possibility of cars descending the incline before being attached to the rope.

*Safety blocks* are necessary at the knuckle or the head of all inclines, and in some states are required by law. They consist of pieces of heavy timber so arranged as to prevent cars descending the incline before all is ready and the signal given. They may be operated at the knuckle, by the topman, or headman, or from the engine room. The block is so arranged, by means of a spring pole, weight, or spring, that the ascending cars will pass it without difficulty, but it will automatically return to its place when the car has passed. At some slopes, safety blocks are arranged at regular intervals along the incline, for the purpose of preventing the descent of the car or cars if the hoisting rope should break.

A *derailing switch* is sometimes employed either instead of or in conjunction with a safety block. This is an automatic spring-pole switch similar to the switch used for turnouts in mine haulage, and permits the ascending cars to pass on the main track, but a descending car will be switched off on a side track. The derailing switch, like the safety block, may be operated from the knuckle or from the engine room, as desired.

The *safety dog* is a heavy trailing bar attached or coupled to the drawbar at the rear of the ascending car or trip of cars, and allowed to drag along the track as the car proceeds up the incline. The lower end, which drags on the

## DATA CONCERNING WELL-KNOWN SHAFTS

| Name | Location | Material Mined | Number of Compartments | Size of Hoisting Compartments | Size of Shaft Over All | Depth, Feet | Remarks |
|---|---|---|---|---|---|---|---|
| No. 5 shaft ..... | Wilkes-Barre, Pa. | Anthracite | 5 | 7' 6"×12' | 12' 0"×52' 0" | 1,039 | 2 hoistways, 1 pumpway, upcast 12'×14', 10', downcast 12'×12' |
| Hazleton shaft .. | Hazleton, Pa. | Anthracite | 5 | 7' 6"×12' 6" | 13' 10"×37' 0" | 326* | Pumpway 3'×12' 6" |
| Exeter red ash .. | { Exeter Boro, Luzerne Co, Pa. } | Anthracite | 6 | 7' 6"×12' 6" | 13' 10"×46' 2" | 578 | Pumpway 20'×12' 6"; airway 9' 6"×12' 6" |
| Leith mine..... | Uniontown, Pa. | Bituminous | 4 | 6' 10"×10' 4" | 12' 4"×23' 10" | 282 | 2 hoistways, 1 pumpway, 1 airway |
| Chicago, Wilmington & Vermilion Coal Co. | Thayer, Ill. | Bituminous | 2 | 8' 6"×4' 6" | 13' 8"×8' 6" | 292 | 2 hoisting compartments in main shaft; no airway in main shaft |
| Madison Coal Co., St. Louis, Mo......... | Divernon, Ill. | Bituminous | 3 | | (Inside) 17' 2"×8' 7" | 355 | Air chamber in main shaft, 1¼'×8' 7"; escape shaft, 17' 2"×8' 7", divided into two equal compartments, one for air and one for manway (stairway) |

| Name | Location | Material | No. comp. | Compartment dimensions † | Shaft dimensions | Depth, ft. | Remarks |
|---|---|---|---|---|---|---|---|
| Gilberton water shaft | Gilberton, Pa. | { Hoisting water and men } | 4 | { 7'×11' 8" and 7'×7' } | 14' 10"×19' 6" | 1,070 | 2 compartments used for hoisting water and two for men |
| General type | { Pa. Bituminous Region } | Bituminous | 3 | 7' 4"×10' 4" | 12' 4"×25' 6" | 50 to 550 | 2 hoistways, 1 airway |
| Centennial Eureka | Eureka, Utah | { Gold, silver, copper, lead } | 3 | 4' 2"×4' 2" | 5' 6"×12' 8" | 1,610 | 1 manway 2' 4"×4' 2" |
| Ontario | Park City, Utah | Silver | 3 | 4' 6"×5' 0" | 7' 0"×20' 0" | 1,500 | |
| Red Jacket | Calumet, Mich. | Copper | 6 | 6' 3"×7' 0" | 15' 6"×25' 0" | 4,900 | |
| Tamarack | Tamarack, Mich. | Copper | 5 | 5' 2"×7' 2" | 8' 10"×29' 2" | 4,615 | Still sinking |
| Anaconda | Butte, Mont. | Copper | 3 | 4' 6"×5' 0" | 6' 8"×20' 4" | | Still sinking |
| Butte and Boston | Butte, Mont. | Copper | 3 | 4' 0"×4' 6" | 6' 2"×8' 4" | | Still sinking |
| Hamilton | Iron Mountain, Mich. | Iron | 6 | 4' 8"×7' 0" | 9' 0"×23' 4" | 1,460 | 2 compartments 4' 8"×4' 4" for men, 2 for pipes 4' 8"×2' |
| Salisbury | Ishpeming, Mich. | Iron | 3 | 5' 6"×7' 0" | 9' 0"×22' 0" | 1,400 | Inclined, still sinking |
| Fayal Iron Co. | Eveleth, Minn. | Iron | 3 | 5' 0"×5' 8" | 6' 8"×19' 8" | | Inclined 66° from the horizontal |
| Consolidated California & Virginia Mining Co. | Virginia City, Nev. | Gold, silver | 3 | 4' 6"×5' 4" | 7' 8"×19' 0" | 2,500 | Pumpway 5' 4"×6' |
| Virginius | Revenue Mt., Colo. | Gold | 2 | | 4' 0"×8' 0" | 1,400 | |
| Isabella | Cripple Creek, Colo. | | 3 | | 4' 6"×13' 6" | 940 | |
| Average for large mines in | Colorado | | | | 5' 0"×15' 0" | | |
| Parker shaft | Franklin Furnace, N. J. | Zinc | 3 | 5' 0"×7' 0" | { 5' 0"×13' 6" and 7' 2"×19' 1" } | 980 | |
| General type | Joplin, Mo. | Zinc | 1 | | 5'×5' to 6'×9' | | |

* Depth completed, 1,150 ft.
† In the clear.

ground, may be either pointed or split. If the hoisting rope breaks, the weight of the car on the incline forces the dog into the floor, and the cars are either stopped or derailed.

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# SHAFTS

## INTRODUCTION

**Form of Shaft.**—A shaft may be *circular, elliptic, polygonal,* or *rectangular.* The first three forms are better adapted to withstand pressure than the rectangular, but they are more difficult to timber, and there is always a considerable area of the cross-section that is not available for hoisting. Such shafts are usually lined with brick, masonry, concrete, or metal instead of timber and are preferred in many European countries. The practice of lining shafts with concrete is growing rapidly and many of these have their sides and ends made as arcs of circles, so as to present an arch to the side and end pressures; the approximate section of the shaft is then elliptic. Rectangular shafts are either oblong or square, the former being the usual form for a hoisting shaft, while the latter is often used for a small prospect shaft, or for a second opening to be used as an escape shaft or an air-shaft. Rectangular shafts are usually not lined with masonry on account of the danger of the walls bulging from the pressure of the strata behind them, although a number of rectangular shafts have been lined with concrete; timber of sufficient size is generally used for the lining in these shafts, and when bulging takes place, any of these timbers can be taken out and replaced by others after the trouble has been removed.

**Compartments.**—A shaft is usually divided into two or more compartments, either by buntons or cross-timbers placed one above another and spaced from 6 to 8 ft. apart, or by solid partitions formed of 3-in. or 4-in. planking. If there are but two compartments, both of them may be hoistways or one may be a hoistway and the other a pumpway and ladderway. If there are three compartments, two of them are hoistways, and the third, and smaller, compartment, which is at the end of the shaft, is used for a manway and pumpway and for carrying steam or compressed-air pipes or electric wires into the mine.

**Size of Shafts.**—The size of a shaft depends on the use for which it is intended and is determined by the hoisting, drainage, and ventilating conditions at the given mine. Nothing is saved in sinking a shaft of too small dimensions, for the work of excavation is more easily accomplished in a large shaft, while the serious annoyance and limitations of a small shaft, and the great expense of enlarging a shaft already sunk, warrant a shaft of generous size. A *tight shaft* is one in which there is but little space between the curbing and the edge of the cage. In such a shaft, the cage acts like the piston of an air pump, moving the doors in the mine, and causing a general disarrangement of ventilation. In such a shaft, also, a very small amount of ice will interfere with hoisting. Shafts for coal mines vary in size from 5 ft.×10 ft. to 12 ft.×54 ft. inside the timbers. Shafts at metal mines vary in size from 5 ft.×5 ft. to 15 ft.×25 ft. The table on page 576 gives interesting data about some of the leading American shafts. The size of a hoisting shaft is determined by the output of material required, the depth of the shaft, the speed of hoisting, the size of the mine car, and the number of cars hoisted at one time.

**Width of Shaft.**—The width of the shaft depends on the size of the car to be hoisted. The length of the box of a mine car is determined by the formula

$$l = \frac{c}{bd},$$

in which

$l =$ inside length, in feet;
$c =$ capacity, in cubic feet;
$b =$ average breadth, in feet;
$d =$ depth, in feet, including the topping.

To the inside length of the car calculated by this formula, must be added the thickness of the end planks, each end being from 1 to 2 in. thick, and the length of the bumpers at each end of the car, from 4 to 10 in., according to the style of car used, in order to obtain the length of car, out to out of bumpers. To this must be added 6 to 8 in. for clearance between each end of the car and

the cage, and 6 to 9 in. more for clearance between each end of the cage and the shaft timbers, to obtain the width of the shaft in the clear.

Cars, for use in coal mines, vary from 4 to 6 ft. in width, from 5 to 10 ft. in length, and from 2 to 5 ft. in height. Their capacities vary from 1,000 to 8,000 lb. and their weight from 500 to 4,000 lb.

EXAMPLE.—Find the width of a shaft required for hoisting an output of 1,200 T. of bituminous coal per day of 8 hr. from a depth of 500 ft.; the seam is 5 ft. 6 in. thick, and has a good roof and floor; the specific gravity of the coal is 1.3.

SOLUTION.—Allowing 5% for delays, the net time of hoisting is $.95 \times (8 \times 60)$ $= 456$ min.; the output is

$$\frac{1,200 \times 2,000}{456} = \text{say, } 5,263 \text{ lb. per min.}$$

The speed of hoisting in a shaft 500 ft. deep varies from 25 to 40 ft. per sec. Assuming 25 ft. per sec., the time of hoisting one trip is $500 \div 25 = 20$ sec. Assuming 10 sec. for the time of caging and uncaging, the total time for each hoist is $20 + 10 = 30$ sec. Then 60 sec. $\div 30 = 2$ hoists per min., and if one car is hoisted at a time, the weight of material per hoist is $5,263 \div 2 = 2,632$ lb. of coal.

The weight of bituminous coal having a specific gravity of 1.3 varies from 45 to 50 lb. per cu. ft., when broken (loose). For the ordinary mine run, assume 48 lb. per cu. ft.; then the capacity of a car is $2,632 \div 48 =$ about 55 cu. ft.

Assuming that the depth of coal on the car, including topping, is 30 in. ($2\frac{1}{2}$ ft.) and the inside width 40 in. ($3\frac{1}{3}$ ft.), then the inside length is

$$l = \frac{c}{bd} = \frac{55}{3\frac{1}{3} \times 2\frac{1}{2}}; \quad \frac{55}{\frac{10}{3} \times \frac{5}{2}} = \frac{55 \times 6}{50} = 6.6 \text{ ft. (6 ft. 8 in.)}$$

Adding, to the inside length, 4 in. for the ends of the car and 12 in. for bumpers, the total length of car will be 8 ft. Then adding 3 in. clearance, between each end of the car and the cage and 9 in. at each end for shaft clearance, the required width of the shaft is 10 ft. in the clear.

**Length of Shaft.**—The length of the shaft must ordinarily be such as to provide for two hoistways, and a pumpway or m a n w a y. The width of each hoisting c o m p a r t m e n t should be such as to give at least 6 in. of clearance between the greatest width of the car, out to out, and the guides. Allowance must be made also for the width of buntons separating the two hoistways, the thickness of the guides, and the width of buntons separating the hoisting compartment from the pumpway. According to the size and depth of the shaft and the character of the strata, the thickness of the buntons will vary from 4 to 12 in. The size of the guides



FIG. 1

often employed in hoisting shafts is 4 in. $\times$ 4 in., and the guides are commonly spiked to the buntons.

In Fig. 1, the width of the car is shown as 40 in., out to out, while the clear width between the guides in each hoistway is 4 ft. 10 in., giving a clearance of 9 in. on each side of the car. The size of the guides is 4 in. $\times$ 4 in., making the total width of each hoistway 5 ft. 6 in. The buntons shown in the figure are 6 in. wide and the pumpway 5 ft. wide, making the total length of the shaft, in the clear, 17 ft. The width of the hoistway also depends on the number and size of cars hoisted at one time, and whether two cars are placed side by side on the cage, or one above the other on a double-deck cage.

Fig. 2 shows the cross-section of a shaft where two cars are hoisted side by side on the cage. The entire length of the shaft in the clear, including hoistways and pumpways, is 28 ft., giving two hoistways each 9 ft. 10 in. in the clear, between guides, and a pumpway 5 ft. in the clear. The guides are each 4 in.
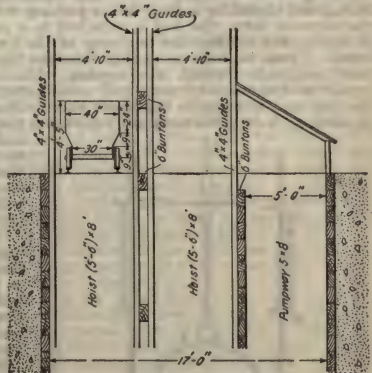
and the buntons 6 in. wide; the width of the cars is 46 in., giving a clearance of 8 in. on each side, and 10 in. between the cars. This is a very large shaft,
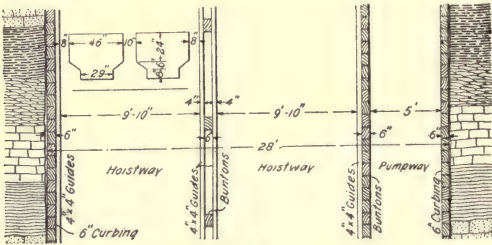


FIG. 2

being capable of accommodating an output of between 3,000 and 4,000 T. per da.

## SINKING TOOLS AND APPLIANCES

**Buckets.**—The buckets used for hoisting the material excavated in shaft sinking are usually made of boiler iron or steel, weigh from 150 to 500 lb. and hold from 2.5 to 14 cu. ft. or more. The bucket $C$, Fig. 1, is commonly swung between handles or bails $b$, which are attached to the hoisting rope by a special hook provided with a clip or extra link and pin for securing the hook fastening while the bucket is being dumped; or two hooks with the points facing and closed by a drop link $g$ passing over their necks may be used. In the common form, the bail is attached to a point below the center of gravity of the bucket so that the tendency of the bucket is to turn over and empty itself. These buckets are easily dumped but have been the cause of many fatal accidents through overturning while hoisting men and material. A bucket is often made by sawing off an oil barrel just above the second hoop from the top, and riveting to the lower part substantial eyes for securing the bail. Buckets with drop bottoms are sometimes used instead of the form described,



FIG. 1

and while a greater speed in removing material is claimed for them, they are dangerous on account of premature opening of the bottom and sinkers should not be allowed to ride on them.

**Bucket Guides.**—When the shaft is deep, the bucket has a tendency to twist or spin. This may be overcome by the use of guides and some form of sinking yoke. The guide ropes, which may be made from old but unkinked hoisting ropes $c$, Fig. 1, are either coiled on a drum and lowered as the sinking proceeds, or are hung from timbers across the top of the shaft. Large weights are attached to the lower end to keep them steady. The hoisting rope passes through a hole in the center of the *rider* (*monkey*, or *jockey*) $d$, which is an iron frame consisting of two legs joined together by a cross-bar, and encircling the two rope guides loosely at the four points $d$. At the bottom of the shaft timbers stop-blocks hold the rider while the bucket goes to the bottom of the shaft, thus keeping the rider and the guide ropes out of the way of the sinkers. As the bucket is hoisted, the rope socket picks up the rider when it is reached.
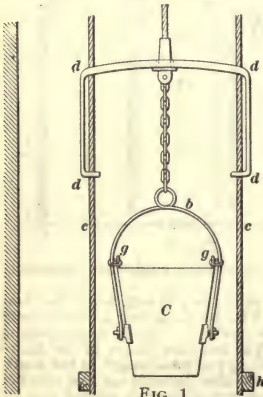
In some cases wooden, instead of rope, guides are used and the bucket is provided with a yoke. This consists of two wooden uprights carrying the guide shoes and two crosspieces holding the uprights together. In each crosspiece is a ferrule through which the rope passes, the bottom one being conical to receive the rope socket. At the bottom of the timbering, blocks are bolted to each guide to prevent the yoke passing below the timbers, while the bucket passes down to the bottom of the shaft. In using wooden guides and boxes (crossheads), the former must be parallel and have smooth joints to prevent the yokes from hanging while the bucket continues to descend. Should the yoke stick and subsequently be jarred loose and fall upon the bucket a serious accident may follow.

**Dumping Buckets.**—The bucket may be dumped automatically by placing a catch hook so that it engages one side of the bucket rim and tips it as the hoisting is continued, dumping the material either into a car or chute. Dumping the bucket while over the shaft is dangerous, as small stones may fall down the shaft through the hole provided for the rope in the shaft covering. It also throws a considerable strain on the head-frame, hoisting gear, and rope; and if an accident occurs to the hoisting rope while dumping, the bucket and its load may fall on the shaft cover with sufficient force to break through and fall to the bottom. A better arrangement is to swing the bucket clear of the shaft by means of a short snatch rope that hangs from a point in the top of the head-frame and at one side of the shaft opening. The hook is quickly put into the bail of the bucket as it comes up, and when slack is given by the engineer, the bucket is swung clear of the shaft and dumped or transferred to a car.

Several buckets are often used for hoisting material, and as soon as the bucket has passed through the shaft opening, a larry or truck running on a broad track that spans the shaft opening is pushed underneath it, the bucket is lowered on to the larry, the hooks are snapped, and an empty bucket attached in its stead. The larry is then moved to one side and the empty bucket lowered into the shaft.

**Engines and Boilers.**—When sinking small and shallow shafts, an ordinary contractor's hoist is commonly used in which an upright boiler is mounted on the same bedplate as a small hoisting engine. For larger and deeper shafts the engine and boiler are in separate units. The former is usually placed on a temporary foundation of heavy timbers and should be powerful enough to pick up the bucket at the bottom at any time without getting stuck on the center or having to run back for slack. The boiler is usually of the locomotive type. In some cases the permanent boiler and hoisting-engine plant are installed at the outset, and used to hoist the material while shaft sinking.

**Sinking Head Frame.**—The sinking head-frame is generally designed for temporary use only, though when an air-shaft or escape shaft is supplied with cages or a bucket for hoisting, the sinking tower, or head-frame, may be left in place after the shaft has been sunk. It is usually built of $8'' \times 8''$ or $10'' \times 12''$ pine timbers that are mortised and cross-braced, or tied, with heavy iron rods. Fig. 2 shows an unusual form of frame that was used in sinking one of the largest shafts ever sunk; it was 12 ft. $\times$ 54 ft. in cross-section.

Sinking frames are sometimes built of $2\frac{1}{2}'' \times 2\frac{1}{2}''$ angle iron; in some cases, these are cheaper than those made of timber, as they are put together with bolts and rivets, which can be easily removed with less damage to the parts than in the case of a timber frame put together with mortise and tenon. Railroad rails are laid across the shorter dimension of the shaft mouth midway of its length for a larry track.

A single sheave from 6 to 8 ft. in diameter rests on the tower, usually at a height of from 20 to 30 ft. above the ground, so that the bucket will hang in the center of the shorter dimension of the shaft. Instead of using a head-frame, a derrick is frequently used, at least until after the shaft has been sunk through the surface wash.

In order that the work of sinking may not interfere with the progress of the permanent work about and over the shaft, such as the erection of the main tower, or head-frame, and the building of the foundations for the permanent hoisting engine, buildings, etc., the temporary hoisting engine should be located at one end of the shaft (at the end opposite the manway if possible). The manway is divided from the hoisting compartments by a close partition of heavy timber. The buntons separating the two hoistways are put in later, or when the sinking is completed. The head-frame is set on the cross-sills, just inside the main sills, so as not to interfere with the erection of the outer posts of the permanent head-frame. By this arrangement, the hoisting of the excavated

material may continue uninterruptedly while the permanent head-frame and buildings are being erected.

The waste material hoisted out of the shaft is dumped about the shaft frame and about the foundations of the permanent machinery and a level surface is thus gradually built up. If the ground slopes away rapidly from the shaft, it may be necessary to build a trestle for the larry track. A smaller car is sometimes placed on a larger truck and run out on a trestle at right angles to the main dumping trestle.

**Shaft Coverings.**—In order to prevent material falling into the shaft, the top should be covered with 3-in. or 4-in. plank, excepting the portion that must
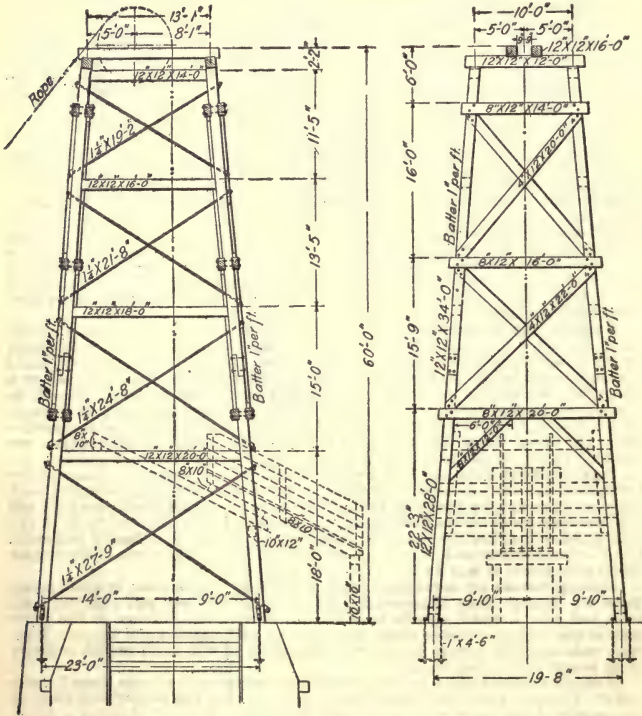


Fig. 2

be left open for the passage of the hoisting bucket. This opening may be simply covered by the larry, but it is better to have a pair of doors meeting in the middle and closing down flat, or as shown in Fig. 3. In the raised position, they rest on a triangular boxing at each end and may be so arranged that the ascending bucket will open the doors for its own passage, while they are closed by means of weights not shown; or the doors may be opened and closed by the levers shown in the figure. The balance weights should not hang inside the shaft as is sometimes done, for if the ropes break they will drop to the bottom. When the doors are closed, the hoisting rope passes through a small hole cut in the two edges of the doors.

**Ventilation and Lighting.**—An air-shaft of boards erected over the manway at the surface serves the double purpose of protecting the manway and ventilating the shaft by means of a natural air-current. The partition separating the manway from the hoistway should be kept close to the bottom of the shaft. If this does not produce a sufficient current of air, a steam jet or small blower, such as is used in a blacksmith's forge, may be employed. Where rock drills driven by compressed air are used in sinking, their exhaust will commonly provide an ample supply of pure air. In some cases a fire of wood or coal suspended in a bucket in the shaft, and known as a firebasket, is used to produce the circulation.

For general illumination at the shaft bottom, incandescent electric lamps protected by metal baskets, are to be preferred. They do not foul the air like oil lamps and may be drawn up out of the way while



Fig. 3

blasting. For individual use, portable electric or acetylene lamps are to be preferred to those using oil.
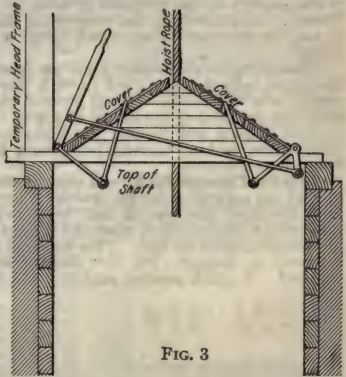
## SINKING THROUGH FIRM GROUND*

**Preliminary Operations.**—Where the seam is flat, the long side of the shaft should be made parallel to the loading tracks so that the chutes may be at right angles thereto. If the seam is inclined, the long side of the shaft should be, as nearly as possible, parallel to the line of dip of the coal; in which case curved loading chutes will be necessary unless the lay of the ground permits the tracks to be shifted into parallelism with the longer side of the shaft.

The shaft is staked out by driving eight stakes in line with the ends and sides of the shaft and outside the area likely to be disturbed by the sinking operations as shown in the cut. The lines formed by the side stakes are at a distance apart equal to the width of the shaft, and the distance between the lines joining the end stakes is equal to the length of the shaft. Cords stretched between the stakes will, at their intersections, determine the four corners *1, 2, 3, 4,* of the shaft.

Shallow trenches are dug on each end line and in them are laid the end-or cross-sills, which extend 6 to 8 ft. outside the shaft line on each side. Similar side or main sills are laid across the end sills and extend 4 or 5 ft. beyond each end line. The sills are of carefully selected 12″×12″ or 12″×16″ oak. They are fastened together where they cross one another by square boxings ½ in. to 2 in. deep, through which a heavy drift pin is passed. This framing is called a *shaft template* and its inner sides define the dimensions of the shaft in the clear. In order to prevent surface water running into the shaft these sills are frequently supported on blocking with clay packed beneath and around them.

**Sinking Through Earth and Loose Rock.**—When the material overlying the coal measures is composed of ordinary earth and loose rock free from water, the excavation to solid rock is carried on by means of ordinary long-handled shovels. After a depth of 8 or 10 ft. is reached, the earth must be thrown upon a staging whence a second gang of shovelers throw it to the



surface. The excavation is made larger than the inside dimension by an amount 2 or 3 in. more than the thickness of the lining. Thus, if the final size of the shaft is, say, 10 ft. × 26 ft., and the lining is 1 ft. thick, the extreme dimensions of the excavation will be, say, 12 ft. 4 in. × 28 ft. 4 in.
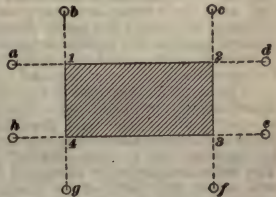
* See section on Timbering for methods of supporting excavations.

Plumb-lines are suspended in each corner of the shaft as a guide for the sinkers. These may be hung from a triangular-shaped piece of plank nailed in the corners of the template or from an iron plate screwed upon the surface thereof. In either case the supporting device is perforated with a small hole to receive the plumb-line, which hangs 4 to 6 in. from the face of the shaft lining.

The lining is commonly built up from the bottom as sinking progresses, the space behind being filled in with fine material to distribute the pressure evenly over the lining. In wet ground, the space behind the lining is rammed with clay to prevent the inflow of silt or fine sand between the timbers. The work of sinking and placing the lining is carried on alternately, the lining, except when sinking through rock, being kept close to the bottom, say, within 6 to 8 ft. at the most.

**Sinking Through Rock.**—As soon as the strata become hard or firm enough to hold the explosive charge, powder is employed and churn, hand, or machine drills are used, the type of drill depending on the character of the rock. In soft rock, a churn drill is used and a light lifting shot is employed to dislodge the material from its bed. This material is afterwards broken by wedges and hammers or sledges. For this class of work, a slow large-grained powder is required. A quick powder exploded in soft material will find vent by a single
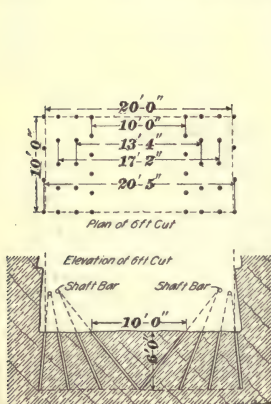


FIG. 1
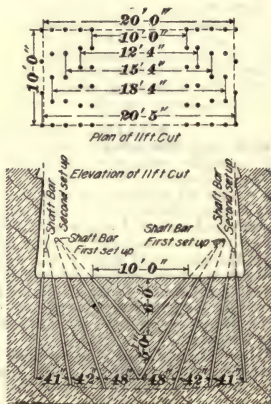


FIG. 2

rupture of the strata without exerting the lifting force on a great mass of material, as is done when a slower powder is used. If, however, the strata are full of seams and cracks, a small charge of a quick powder is used, since such rock will not confine the explosive force sufficiently to do effective work when a slow powder is used. In hard rock, dynamite is used, and power drills, operated by compressed air or steam and usually mounted on shaft bars, are employed.

The general position of the holes and their depth are about the same as described under Tunneling. The first shots in a level floor should be inclined at a fairly sharp angle with the floor, and are usually central in the shaft. These holes are often called *sumping holes;* their purpose is to start the excavation by blowing out a wedge-shaped piece of rock from the center of the floor. The holes are generally arranged in series, or rows, on each side of the center and across the width of the shaft, and are spaced an equal distance from one another. The general position of these holes is illustrated in Fig. 1, which also shows the position of the shaft bar on which the drills were mounted. The dimensions given are those that were employed in the sinking of a shaft in a white crystalline limestone. In this shaft, at first, only 6-ft. cuts were made, a single series of shots excavating the material to this depth. The depth of cut, however, was afterwards greatly increased by boring the side holes

deeper, as shown in Fig. 2, until the cuts averaged 11 ft., six successive cuts excavating the shaft a depth of 66 ft.

Fig. 3 illustrates the sinking of a $6' \times 18'$ shaft in limestone of varying hardness. The position of the shaft bar on which the drills were mounted is shown

at $a$, $a_1$, $a_2$, $b$, $b_1$, $b_2$. The two center rows of holes *1*, or the sumping holes, were drilled first, and each hole filled with from five to seven ½-lb. sticks of giant powder or dynamite, containing 50% of nitroglycerine. The depth of the holes varied from 3½ to 6 ft. Beginning at the center, the successive rows of holes, marked *1*, *2*, *3*, and *4*, respectively, on both sides of the shaft, were drilled, charged, and fired in pairs, the material being loaded and hoisted between each operation. The end holes required but four or five sticks of 40% dynamite apiece; the entire cut of twenty-six holes used from 50 to 60 lb. of dynamite, and excavated the material to a depth averaging from 3½ to 6 ft. The average quantity of 40% and 50% dynamite used in this material was 12 lb. per ft. of depth, or 3 lb. of dynamite per cubic yard of excavation. The sinking was carried on by three shifts of four men each, and the record of the sinking showed a depth of 100 ft. in 30 working da., or an average of 3⅓ ft. per da.
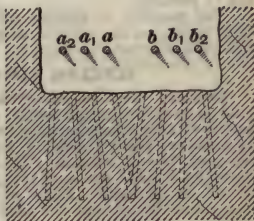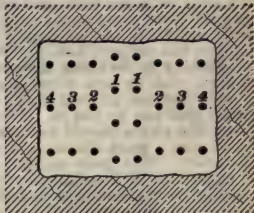


FIG. 3

**Long-Hole, or Continuous-Hole, Method.** As in sinking in rock, much time is ordinarily lost in drilling; and as machine drills cannot work close to the sides, ends, or corners of the shaft, the continuous-hole method is sometimes used. By this method, a number of diamond-drill holes are put down at definite distances apart, and from 100 to 300 ft. deep, over the area where the shaft is to be sunk. They are arranged in rows, from 3 to 4 ft. apart, with the outside rows close to the sides and ends of the shaft, so that they will nearly square it up and save much digging and trimming. They are then filled with sand or water, preferably the former. The sinkers prepare for the work of blasting by removing 3 to 4 ft. of sand from the holes and filling this space with explosives, which are tamped and fired.

Fig. 4 shows how the holes are arranged. The holes marked $a$ are first cleaned and fired to give a loose end to the holes $b$ on the outside, which are next cleaned out and fired. This work is continued until the bottom of the hole drilled by the diamond drill is reached, when another series of long holes is drilled.

This method probably originated from one that is sometimes used in the coal fields of the Central Basin. Shafts are sunk about the diamond-drill hole that has been used in prospecting, and from which the casing has been withdrawn, or is drawn as the sinking proceeds. The sinkers charge a section of the hole, using a false bottom, and blow out a center cut.
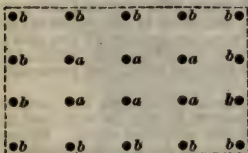


FIG. 4

When shafts are sunk to workings already opened, a diamond-drill or churn-drill hole is sometimes put down into the open works below, and this hole is kept open during sinking, thereby avoiding all hoisting of water. A long chain is used to clean out the hole when it becomes stopped up.

Both this plan and the long-hole plan are apt to cause crooked shafts, on account of the divergence of the drill hole from the vertical. The advantages of the long-hole system are that sinkers need not wait while holes are being drilled; and blasting can be done as soon as debris from shots is removed. The method is said to be very much quicker than the ordinary practice of using power drills driven by air or steam, but is more expensive.

Timbering is usually not required for securing the sides of the excavation when sinking in hard rock. Cross-buntons to support the cage guides, pipes,

wires, etc., are set in hitches in the face of the rock, and are spaced 6 or 8 ft. apart. They are carefully lined and placed vertically over each other and then tightly wedged. When sinking through soft shale or loose crumbling rock, a greater amount of timber is needed for securing the sides. The sides are trimmed with the pick; and when the material is dry, a close-fitting lining of 3-in. or 4-in. planking is sufficient, the thickness of the planking increasing with the depth of the excavation. In wet material, 4-in. timber should be used at the surface, 6-in. at 100 ft., and 8-in. at 200 ft.

**Sinking in Swelling Ground.**—Clay or marl that swells when brought in contact with air and water is difficult to excavate and support. There is no power that can prevent this swelling; it will burst any timber or break any frame that can be put in. When sinking a shaft or a slope under such conditions, the strata should be excavated for a certain depth back of the lining so as to give a good clearance between the formation and the lining all around the shaft. This space should be so arranged that a man can enter it and clear it from time to time as may be required. Drainage should be provided by cutting, in the hard pan or floor underlying such strata, a ditch connected by a pipe with the sump at the foot of the shaft. A good circulation of air should be made to travel around the space thus excavated so as to keep the clay as dry as possible.

The method of sinking through such ground does not differ materially from that used in other loose ground or rock, but the timbering of the excavation is of great importance.

## SINKING THROUGH RUNNING GROUND*

**Draining the Ground.**—Where beds of quicksand or loose water-bearing sand or gravel are supposed to occur, the ground should be thoroughly drilled before sinking operations are begun, and in localities where such deposits may be expected there should be kept on hand an ample supply of the materials required during sinking. Timber of different sizes should be framed and ready for instant use, and pumps and piping of the proper kind and capacity should be on hand. Eight or ten pointed pipes, with perforated ends, are sometimes driven into the sand 6 or 8 ft. apart and connected at their upper ends to a suitable pump. In some cases, a few hours' pumping draws off the water and the boiling sand settles and solidifies so that it may be removed with a shovel. Water can sometimes be drained from the soft ground within the area of the shaft into wells or small temporary shafts sunk adjacent to the larger shafts, thus leaving the sand within the shaft area compact and easily removable by shoveling.

When the watery sand is thus drained, there is a considerable decrease in volume of the material surrounding the sides of the shaft; the shaft lining is thus frequently robbed of all supporting material for a considerable distance up the shaft and begins to separate and sag, while the shaft may be swung out of line. This decrease in volume, or displacement of the strata, due to the draining off of the water, may be carried to such an extent that the surface of the ground will sink several feet over a large area surrounding the shaft. In removing the water, a large amount of sand is also removed; the effect of its removal is often not appreciated until too late. The sand contained in the water will often cut out the pump linings in a short time, and render the pump useless; but if a layer of straw or other light material is thrown into the shaft, it will form a mesh by which the sand will be largely filtered from the water.

The methods to be adopted when sinking through such material are particularly methods of timbering, or supporting, the sides of the excavation; and the excavation must be kept timbered close to the bottom of the shaft. There are, however, certain methods of sinking that are particularly applicable to such ground, as follows: *piling, forepoling,* the use of *shoes,* the *pneumatic process,* and the *freezing process.*

**Piling.**—A bed of quicksand or other soft material lying near the surface is often best treated by piling. If the bed is shallow, it may be sufficient to drive a single set of piles all around the site of the proposed shaft. Where thicker beds of quicksand occur, it may be necessary to drive several series of piles, each successive series being driven inside the former after the material has been excavated to a point near the bottom of the first piles driven. The second set of piles having been driven, the material within these is excavated to a point near the bottom of the piles, and, if necessary, a third set of piles is driven within the second. This method is illustrated in Fig. 1.

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*See section on Timbering for methods of supporting excavations.

After driving, the first set of piles *a* should be strengthened by timber frames or timber sets at their top and half-way of their length, as the material is excavated from the space they enclose. It is important that these frames should be promptly and braced by cross-buntons; they are supported by punch blocks *e*. As will appear from the figure, it will be necessary to set the first sets of piles back a sufficient distance from the shaft to allow for the decreased size of the excavation when each series of piles is driven. This distance is easily calculated when the depth of the sand beds is known (and this is given by the bore or drill hole *t* made beforehand).
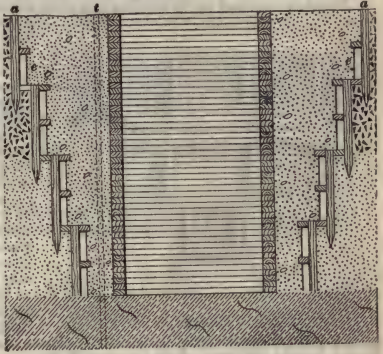
In some cases, the soil at the surface may be firm for a considerable depth, but underlaid by a flowing bed of quicksand. In this case, the excavation of the overlying soil may



FIG. 1

be done in the usual manner, and after this is lined or curbed the piles may be driven from the foot of the excavation in the same manner as from the surface. In this system of sinking through watery strata, the permanent shaft lining is built up as soon as the rock is reached. The space between the shaft lining and the piles is then filled with clay, where this can be obtained, or the timbers are backed with a sufficient thickness of cement, and this, in turn, with the material excavated.

**Forepoling.**—Fig. 2 shows a method of forepoling for sinking through quicksand, very similar to the method of forepoling described under Tunneling. Strong timber sets *j* are framed to the sides of the shaft. As each set is put in, it is suspended from the timbers *a* above by the light strips, or lath, *f*, while the punch blocks *b* are set between the frames to hold them apart. Two-inch planks with the ends sharpened are used for the spiles *k*, and are driven downwards in an inclined position behind the lower timber set. Before driving



FIG. 2

the spiles, the tail-pieces *c* are spiked to the lining just above the lower timber frame; the spiles are then driven as the excavation advances until their tops reach this tail-piece. Another set of timbers is then placed in position at the floor and tied to the timbers above, and the same operation repeated, driving the spiles and excavating the material as rapidly as possible. This process of forepoling may be carried on at any depth below the surface where the strength of the timbers will resist the pressure of the sand.
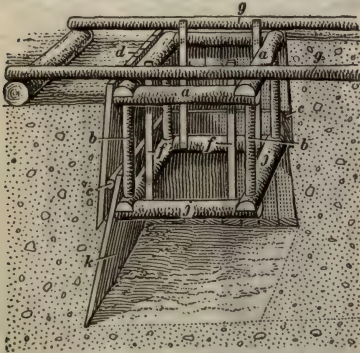
Where the sand is thicker and is found at a greater depth below the surface, the spiles are driven in at a flatter angle, the timber frames are placed somewhat closer and no tail-piece is employed, the tops of the spiles bearing against the timber above instead of against the tail-piece.

Fig. 3 illustrates the use of breast boards where the bottom has a tendency to rise and fill the shaft and must be planked to keep it down. The material

is removed a little at a time. A sump is carried ahead of the regular excavation by driving short piles and putting in a small frame.

Another method of forepoling requires the use of interlocking channel bars, as in Fig. 4. The shaft is started 2 ft. larger each way than the size desired,



and sunk in the ordinary manner to the sand; thus, an $8' \times 16'$ shaft must be started as 10 ft. $\times$ 18 ft. The lining forming the sides is composed of alternate channels $a$ and $b$. The channels $a$ have $Z$ bars $c$ riveted to them, which engage and interlock the edges of the channels. The channels $b$ have angle irons $d$ riveted to them, thus forming grooves in which the sides of the channels $a$ run. The corners of the shaft lining are made of three angles $e$ riveted together, as shown, which interlock with the side and end channels $a$ by means of the $Z$ bar attached to $a$. Heavier sections can be used, which would make the thickness of the metal

FIG. 3

about $\frac{1}{2}$ in. When sand is reached, these channels are set plumb in a solid frame inside of the shaft lining, and are driven vertically downwards through the sand to the solid material, if possible, before any sand is excavated. No one channel should be driven more than 2 ft. ahead of the rest. A perfect fitting anvil, or clinker, is used to protect the head of the channel bar while driving. Channels 12 ft. long are readily driven their entire length into the sand. The sheathing can be driven to varying depths by feeding in pieces from the top, thus driving the preceding one down, in the same manner that a follower is used when driving piling. The individual members, engaging and interlocking, slide on one another so that one can be driven at a time, and thus afford an opportunity to drive channels all around a boulder, should one be encountered. The channels interlock nearly watertight, and, by cementing above and below them, the water may practically be shut off. The channels take up about 5 in., while 6 in. should be allowed for timber. The price of this sheathing or lining is about $2.50 per sq. ft., or $120 per lin. ft. for an $8' \times 16'$ shaft. The channels are either left as a permanent lining or they may be drawn after a timber lining has been laid. They are cheaper than steel shoes or drums.
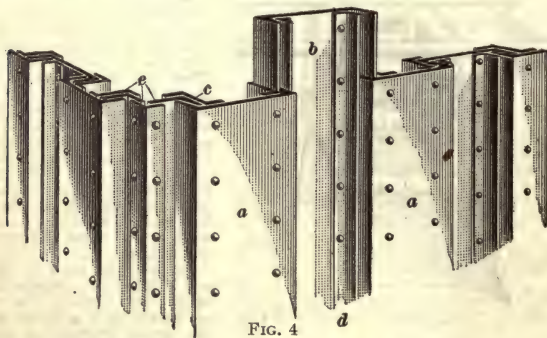


FIG. 4

**Shoes for Shaft Sinking.**—The shoe consists of a wooden or metal frame of the same size and shape as the shaft. Attached to its bottom is a beveled steel cutter that will sink easily through soft ground. While the shoe is usually open at both top and bottom, the top is sometimes closed with heavy

steel plates in order to resist the pressure of the sand from the shaft bottom. The upper part of the shoe is outside the shaft lining from 12 to 16 in., and the lower part is usually divided into compartments by braces.

In principle, the plan of sinking by a shoe is similar to the method of tunneling in soft ground with the use of an advance shield, except that shaft shoes in America are usually rectangular in shape, while the shield in tunnel driving is cylindrical. As the material is excavated from beneath the shoe, the shoe drops by its own weight or on account of pressure applied to its upper surface by weights laid on it or by means of jacks, generally the latter, thus walling back the sand, while the lining is being put in place. Only enough material is excavated from underneath the shoe and it is moved just far enough ahead to permit the placing of one set of timbers



FIG. 5

at a time; if planks are used for the shaft lining, they are put in flatwise. The shoe should descend uniformly at all points, and should be carefully leveled before the timber is placed.

The steel shoe shown in Fig. 5 is made of $\frac{1}{4}$-in. steel boiler plate braced as shown, has a height of 30 in. under the shaft timbers, and a sheet-iron lap 18 in. deep extending outside of the timbers. Fig. 6 shows it in position at the bottom of the shaft, as well as the manner of supporting it and controlling its descent. Four hooks, or claws, are provided, which may be screwed into the lower couplings, Fig. 7. To each of these hooks is fastened a strong chain attached to the frame of the shoe; by this means, the downward progress of the shoe is controlled, and there is less liability of its becoming wedged and thrown out of line.
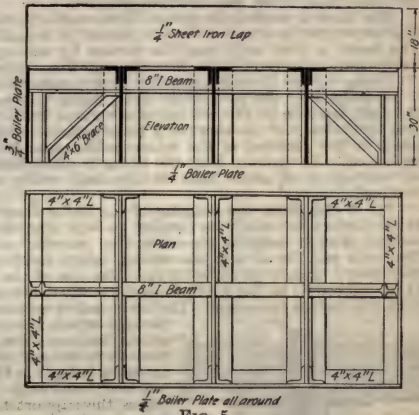


FIG. 6

One of the disadvantages of using the shoe is the fact that it is apt to be stopped by boulders, clay seams, or other obstructions, one part remaining stationary while the other goes down, thus throwing the shoe out of level and wedging it so tightly that it cannot be moved, and causing the shaft to be thrown out of line and perhaps abandoned. By means of the chains shown in Fig. 6, this difficulty is partly overcome, as by their use the shoe can be held stationary until the obstruction is removed. The chain may also be slacked at any time to allow the shoe to move.
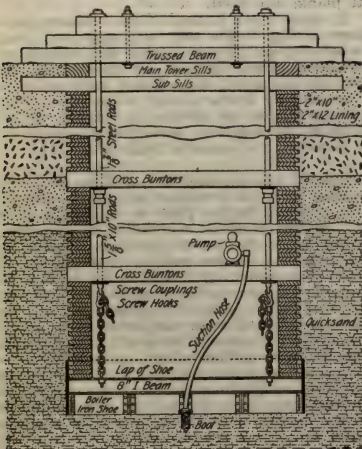
The cross-beams of the shoe frame furnish also a good support for the planks that are used in the shaft lining. As the shoe is lowered 2 in., or the thickness of a plank, the latter is slipped in place and spiked upwards from beneath, 40-penny nails being used for this purpose.

The shoe is sometimes forced downwards by the weight of the lining, if this rests directly on top of the shoe instead of hanging from the top of the shaft. The lining is then built from the surface by adding set on set, the increasing weight gradually forcing the shoe through the soft material.

Owing to the flowing character of the material being sunk through there is a tendency on the part of the shaft lining to settle and draw apart and for the shaft itself to be thrown out of a vertical line. This is due to the running into the bottom of the shaft of a large amount of the loose material from the sides, or the removal of the water in the sand by pumping or drainage, as explained. To remedy this, the lining is often hung from a strong frame at the surface or at some point in the shaft where a firm foundation can be obtained.

Fig. 6 illustrates the hanging of the lining from a frame or built-up beam at the surface, by steel rods coupled to each other in lengths of 10 ft., and supporting at each coupling a cross-bunton on which rests the intervening lining. The rods may be of any convenient length until the sand is reached, when their length should be about 10 ft. The size of the rods may vary from $1\frac{1}{2}$ to $2\frac{1}{2}$ in., according to the depth from the surface, the size decreasing as the depth from the surface increases. The lower end of each section of rod is passed through a hole in a cross-bunton $b$, Fig. 7, and an iron bearing plate, or washer, $a$ is placed over the end of the rod underneath the bunton. A screw coupling $c$ is then fitted to the end of the rod and screwed in place. This coupling furnishes the support for the next section of rod below, which is not, however, put in position until the excavation has reached the point where another cross-bunton is required. Until this time, the timbers of the shaft lining are supported by strips of lath nailed to their face, or by being spiked together from underneath when flat planks are used.
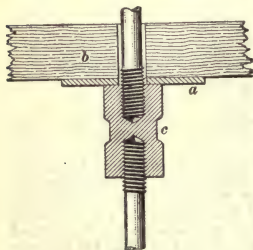
In some cases, instead of the built-up beam shown in Fig. 6, the rods are supported from a wooden truss fashioned after the style of an ordinary highway bridge. In other cases, the rods are supported from heavy railroad rails, steel I beams, or girders. The supporting frame should extend outwards beyond the shaft to solid ground so as not to be affected by shifting sands, and should be strong enough to support the weight of the sinking head-frame and sheaves, if necessary. These frames are commonly built of $12'' \times 12''$ to $16'' \times 16''$ white oak.

FIG. 7

**Pneumatic Process.**—The pneumatic process used for sinking shafts is commonly known as the *Triger method* after its inventor, and is an adaptation of the caisson method used in building bridge piers or driving tunnels through mud, as beneath a river. In this method, a cylinder of cast iron, made by successively adding one ring to another at the surface, is made to gradually sink into the loose ground, either by its own weight, by weights piled on top of the cylinder, or by means of pressure applied through jacks. In order to keep out the water from surrounding strata, compressed air is led into a closed chamber at the bottom of the iron cylinder, the pressure of the air being kept just sufficient to prevent an inflow of water and loose sand. This chamber forms the working space in which the material is excavated; above it, and connected to it by suitable trap doors, is another closed space, known as an *air lock*. This air lock, by means of trap doors above and below, gives a means of communication between the working chamber and the surface. A person enters it through the upper trap door; after closing this door he allows the compressed air from the working chamber to enter, by means of suitable valves, until the air has reached the same pressure as that in the working chamber or caisson; the lower trap door, which leads to the caisson, is then opened and he descends into the working chamber. In order to leave the caisson, the opposite procedure is adopted.

The excavated material can either be removed through the air lock, or it can be blown out through a pipe by means of air pressure after being mixed

with water. If only a few boulders are found during the sinking, they are carried down in the caisson and are hoisted out after solid material has been reached and the roof of the caisson cut away. If many boulders are encountered, they must be blasted and the pieces hoisted out through the air lock. In some cases, the metal casing on top of the caisson forms a sufficient lining for the shaft; in other cases, it is necessary to build a lining of timber or metal inside of this casing.

**Freezing Processes.**—In the freezing process, a sufficient thickness of the fluid material is frozen to form a substantial wall around the shaft so as to permit the excavation of the material enclosed within its area. Surrounding the shaft, a series of holes from 6 to 10 in. in diameter, is bored through the sand bed and cased with ordinary well casing; or if the sand is very fluid the casing may be driven through the sand. These holes if bored from the surface are usually vertical, but if bored from a point in the shaft a few feet above the bed of sand, they are inclined. They are not more than 3 or 4 ft. apart, in order to insure the thorough freezing of the sand between them. Inside these casing tubes, smaller ones, usually about 4 in. in diameter and closed at the bottom, are let down to the solid stratum, and the outer temporary casings withdrawn. The 4-in. tubes are closed at the top with metal cap pieces, and each contains a 1-in. tube that extends almost to the bottom. The 1-in. and the 4-in. tubes are connected at the surface to circular mains, each vertical tube being fitted with a screw-down stop-valve so that it can be cut off from the main.

There are two freezing processes distinguished by the character of the freezing medium. The *Pœtsch system* uses a brine composed of a solution of calcium chloride (or magnesium chloride) passed through a cooling machine on the surface, where its temperature is reduced to about 8° F. below zero. The solution of chloride of calcium is pumped through the smaller tube to the bottom of the hole, and then rises through the larger tube to the surface. In this process, the material is frozen first and hardest at the bottom where the greatest pressure is. Since this freezing mixture is much heavier than water, the pressure inside the pipes is greater than that outside, so that there is a tendency to burst the tube conveying the freezing solution, thus allowing it to escape into the sand outside and rendering it incapable of being frozen.

In the *Gobert system* anhydrous ammonia is sent down the inner tube (which is then usually made of copper) and allowed to vaporize in the tubes, thus freezing the ground directly instead of allowing it to cool a mixture that freezes the ground indirectly, as in the Pœtsch process. The ammonia gas is drawn off by a pump and reliquefied by compression and used over again. As the pressure is less inside than outside the tubes, if a leak occurs in the tube any water entering will be immediately frozen and the leak thus stopped.

The pipes may be driven well outside of the intended shaft area and a wall of earth frozen around the shaft, the central portion or shaft area being removed before it is frozen. In most cases, however, the ground has to be frozen solid and then blasted as though it were rock.

**Cementation Process.**—Cement injection is now replacing the various freezing processes for the sinking of shafts in water-bearing ground. The process appears to have been developed in the chalky formation of Northwestern France, but is suited for any fissured water-bearing rocks, although not for soft running ground such as quicksand. It consists of a number of bore holes sunk at equal intervals in the form of a ring surrounding the proposed site of the shaft. Cement and water, injected through these bore holes by means of a force pump, find their way into all the cavities and crevices of the ground surrounding each hole, in which the cement sets. As the cement from one hole penetrates the rocks surrounding it, that coming from the adjoining hole is encountered and a cemented, water-tight wall is formed around the proposed shaft. In France, the cost of sinking by this process is found to be about one-third that of the Pœtsch system.

The sizes of the holes through which the cement is forced vary from as much as 12 in. down to as low as 3 in. The larger holes are commonly sunk with an ordinary oil-well drilling outfit, but the smaller ones are put down with a diamond drill. The pressure under which the cement is forced into the ground varies from 800 to 1,200 lb. per sq. in. The holes are drilled to a depth of from 10 to 20 ft., and the cement injected. After it has set about 30 hr. the holes are drilled out, and sinking resumed. When solid non-water bearing rock is reached, cementation is discontinued. While sinking, the various drill holes are watched for signs of escaping water; if such is noted, cement injection is again injected into the holes. In some cases as an additional precaution, holes are drilled horizontally as the various water-bearing horizons are

encountered, and cement injected. The open spaces behind the permanent shaft lining are also filled with cement in the same way.

## OTHER METHODS OF SHAFT SINKING

The *Kind-Chaudron system* is applicable only to the sinking of circular shafts, and has been extensively used in Europe for sinking through strata with heavy feeders of water that prevent the use of the ordinary methods. The excavation is carried down to water level by the ordinary methods of sinking, and the shaft is lined to this point with timber or masonry. Boring is then commenced by means of a large trepan suspended in the shaft. The diameter of the excavation to water level must be sufficient to allow for the thickness of the walling or timbering, so that the latter will not interfere with the use of the trepan for sinking below this level. The excavation is effected in two or more successive operations. The first trepan used cuts a hole in the center of the shaft from 4 to 5 ft. in diameter; this is called the *guide pit* and is kept at least 35 ft. in advance of the second cut, which is made by enlarging the guide pit by means of a special trepan. During the entire boring, the water is allowed to accumulate in the hole, which often stands full, and the boring is done underneath the water.

The first *trepan*, or cutting tool, Fig. 1, consists of a horizontal wrought-iron bar $T$ having steel teeth $B$ attached below. The action of the cutting tool is the same as that of a churn drill. The trepan is suspended in the shaft by means of heavy iron rods attached to a large walking beam at the surface, and the weight is partly balanced by a counterpoise at the other end of the beam. An engine operates the beam, raising the rod a height varying from 10 to 20 in. and dropping it to the bottom. To avoid the shock caused by a cutting tool of such great weight, a slide bar similar to the jars in the American rope method of drilling is used. The trepan is turned by men who stand on a platform built above the level of the water in the shaft. When making this first cut, the hole is cleared by means of a sheet-iron sand pump about 6 ft. long, which is raised and lowered by the trepan rods.

The second cut is an enlargement of the first and is made with a trepan that usually weighs from 36,000 to 50,000 lb. It is quite similar to the first trepan, being formed of a wrought-iron bar having teeth attached to that portion of its length that exceeds the diameter of the guide pit. It is guided by means of a cradle, or iron bar, that fits closely within the excavation made by the smaller trepan. The teeth on the large trepan are so set that they cut the bottom of the annular portion surrounding the guide-bore pit into a sloping surface, so as to allow the fragments and cuttings to roll into the smaller shaft, where they are caught in a sheet-iron bucket previously lowered to the bottom of the guide-bore pit. Sometimes scrapers, which drag around after the trepan and sweep the material down the incline and into the bucket, are provided. The excavation having been made of the required size in two or more successive operations, the shaft is lined with iron tubbing, which is built in sections 4½ or 5 ft. high and added at the top as the whole is lowered from the surface.
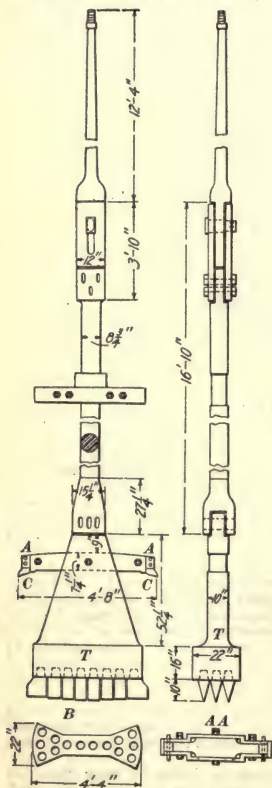


Fig. 1

To assist in lowering the great weight of the steel tubbing, it is provided with a water-tight bottom in which is a nozzle having a stop-cock by which a sufficient amount of water can be let into the tubbing to sink it gradually. The tubbing is thus floated in the shaft until it finally rests on the solid bed leveled to receive it. A special moss packing below the tubbing makes a watertight joint when the water is pumped out.



Fig. 2

The *Lippman system* differs from the Kind-Chaudron in that the shaft is bored the desired diameter at one operation by using the cutting tool shown in Fig. 2. The tools are made and the cutting teeth secured in a manner similar to that employed in the Kind-Chaudron system.
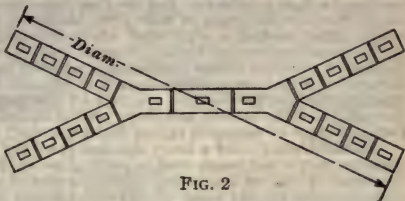
## ENLARGING AND DEEPENING SHAFTS

**Enlarging Shafts.**—Shafts may be enlarged by extending one end or one side of the shaft, for then timbering already in place is made use of, the alinement of the shaft is maintained, excavating is done easily, and less readjustment of hoisting sheaves, stops, etc.,



Fig. 1

is necessary. In order not to interfere with the hoisting of coal, the widening operations are commonly carried on at night.
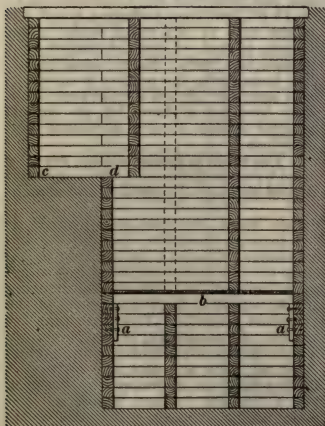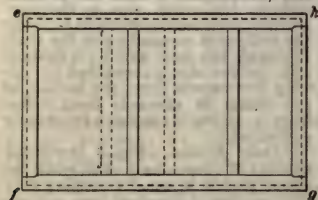
A method used for doing the work is shown in Fig. 1. Cleats *a* are nailed on the old lining and buntons *b* placed on them across the shaft; on these is laid a temporary platform on which the men work. The enlarging is begun on the surface and carried downwards, a section about 8 ft. high being taken out from each platform. The drillers work on the rock bench *cd* and load the waste directly into cars on the regular hoisting cage. The end *ef* is timbered and backed as in sinking a new shaft. The timber joints at the corners *g* and *h* are left undisturbed. Each alternate side timber is taken out for part of its length and a new timber dovetailed in between it and the timbers above and below, the parts being joined by a feather-edge joint. The dotted lines show the original position of the partitions and linings. These cannot be moved if mining operations are being carried on until the widening is completed for the depth of the shaft.

If both the length and breadth of the shaft are to be increased, mining operations must be suspended as the shaft will have to be entirely relined. Shafts have been enlarged and retimbered by filling them to the surface with cinders and ashes. The retimbering or enlarging begins at the surface, and the method, while costly, is often cheaper in the end than endeavoring to use one or more sides or ends of the old shaft.

**Deepening Shafts.**—*First Method.*—A false bottom of heavy timbers is provided in the sump as a resting place for the cage, and sinking is begun on

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the bottom of the sump. When the new seam is reached, a new sump is made, new guides are extended from the bottom upwards to meet the old guides, the false bottom is removed, and the cage ropes spliced, or new ones of sufficient length to allow the cages to hoist from the lower seam substituted for the old ropes. This method is often used where material is being hoisted during the day and sinking done at night. A small sinking cage is slung under the regular cage or a bucket is used instead, the material being hoisted to the old shaft-bottom level and there taken back into the old workings and gobbed. The disadvantages of this method are that all the water from the old sump drains through the false bottom and down on the sinkers at their work, and there is always danger of materials falling down the shaft on the sinkers.

*Second Method.*—At a short distance from the shaft bottom and on a passageway that is not much used, a steep slope *ab*, Fig. 2, or small shaft is sunk, the depth of sinking depending on the amount of rock necessary to be left as a support under the old sump while the deepening proceeds. At the foot of the slope a level heading *bc* is first driven to the opposite face of the shaft; the roof of this heading is strongly timbered by setting the collars in hitches cut in the sides, before the work of excavating the shaft below is commenced. When this is done, the excavation is begun and carried down in exact line with the shaft above, the material being removed by a hoisting bucket, operated by a windlass or temporary hoisting engine located at some point near the head of the slope. The further operation of sinking, timbering, etc., is the same as that previously described. When the sinking is complete and the shaft timbered, the main sump *s* is drained and the two shafts connected by driving downwards from the bottom of the sump, or upwards from below from a strong temporary staging erected at *c*.



FIG. 2

*Third Method.*—Fig. 3 shows the method of deepening a shaft while the upper part is in use, by opening only that portion of the shaft area not under the hoistway for a distance of 12 to 15 ft., and then widening it out the entire size of the main shaft. This leaves a roof of rock (*pentice*) that shields the men. When another lift has been sunk, the pentice is cut away and another started for the next drop. The hoisting is done by an underground engine or by a bucket and windlass.

The main hoisting engine may be used by setting out one of the cages and passing the hoisting rope through a hole drilled through the pentice and attaching it to the sinking cage.

**Upraising.**—Shafts are sometimes driven from the bottom upwards as when two parallel seams are to be worked through the same opening. From the labor standpoint the process is much cheaper, as there is no hoisting to do. The material extracted is generally stowed in the old workings below, but sometimes when room is not available it is sent to the surface. Before commencing to drive upwards, a careful survey is made to establish the four corners of the shaft in the mine immediately under the surface location. Four iron pins are driven in the bottom to mark these corners. If necessary, posts or timber cribs are set to secure the roof around the place before blasting is begun. When the excavation has proceeded upwards 8 or 10 ft. in the roof, the bottom is cleaned up, the pins located, and the shaft tested for alinement by hanging plumb-bobs in each of the four corners. Timbering is then begun by first setting a heavy square frame *f*, Fig. 4, in the roof, resting on substantial posts and sills, as shown in the figure. The inside measurements of the frame must correspond to the size of the shaft in the clear when timbered. This frame is exactly located by means of the plumb-bobs hanging over the four points previously established, and is then firmly wedged in place. The timbering of the shaft is built up on this frame after the ordinary manner of shaft timbering.

The timbering is carried as close to the roof as practicable, and a partition is carried up dividing the shaft into two compartments. This partition may later be used in the operation of the shaft as one of the permanent partitions, and should be located accordingly.

A heavy bulkhead is now constructed at the bottom of the shaft, and a chute arranged under the larger compartment $h$, by which the loose material excavated above and thrown into this compartment may be drawn and loaded as required. To control the descent of the loose material in this compartment, a door is arranged at the foot of the chute. The compartment $m$ serves the double purpose of a manway and airshaft, and for this purpose it is divided by a temporary partition. A ladder is constructed in the manway, by which the workmen travel up and down.

In the operation of upraising, the workmen ascend the manway by the ladder and stand on a temporary platform, or on the loose material that is allowed to fill the compartment $h$. The material is drawn from this compartment only as is required to furnish good
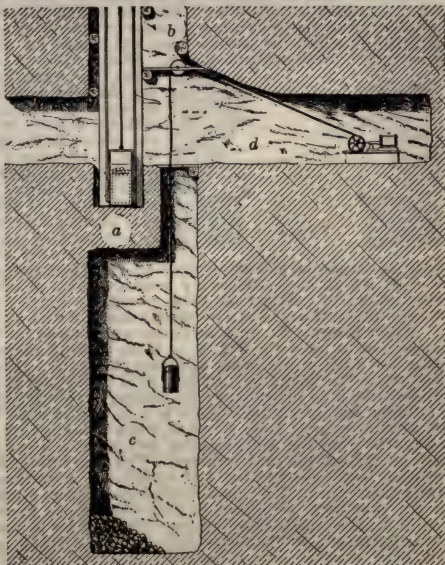


FIG. 3

standing room at the face. In upraising, the ventilation of the shaft is always more or less difficult, owing to the tendency of the smoke and hot bad air to remain at the top. The air compartment may be connected, by a box, to the main air-course while the manway is open to the return, or vice versa; by this means, a fair current of air may be maintained at the top of the shaft or upraise. At times, a small blower is used to blow the air into the face. When compressed air is used to operate the drills, there will be air sufficient for the ventilation of the upraise without making other provisions. The timbers required must be taken up the manway or the air compartment. When blasting, the manway and air compartments are covered with heavy planks, to avoid the material loosened by the blast falling down the shaft.

Of these methods of deepening shafts, those shown in Figs. 2 and 3 are generally employed, because it is unusual to have a lower seam open in advance of development; a condition that is assumed to exist when the method shown in Fig. 4 is used.
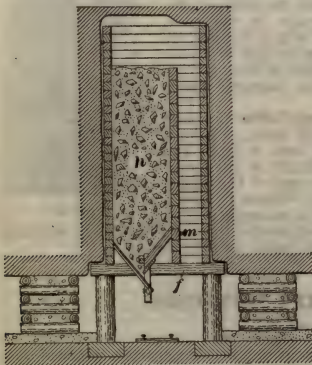


FIG. 4

## SHAFT DRAINAGE AND PUMPING

Surface water is kept out of the shaft by banking around the shaft sills the clay and other material taken out during sinking. The water pumped or hoisted from the shaft is carried away in tight wooden troughs that lead in the direction in which the surface dips, and extend far enough from the shaft to prevent the water from returning. Water within a comparatively short dis-



Fig. 1

tance from the surface can be drained from the shaft by sinking a well or small shaft adjacent to the main shaft. During the sinking, a hole, or sump, is excavated at one end or in the center of the shaft somewhat in advance of the general work. The water is either bailed out of this hole and hoisted in buckets, or a sinking pump of special form is employed. These pumps may be hung by hooks from the timbering, at any point or simply hung by ropes, and may be hoisted and lowered as desired. Instead of a special sinking pump, a small horizontal pump of ordinary pattern is often set up on a temporary staging, which is moved downwards as the work advances. Either of these pumps is connected with the steam and water pipes in the manway by short lengths of wire-wound rubber hose.

**Water Rings.**—To draw away the water made by the shaft, a notch is cut in the rock as shown in Fig. 1, or if the shaft is timbered *water rings*, or *curb rings*, are built in the lining as shown in Fig. 2. These catch the water running down the lining and conduct it to the corner of the shaft, from whence a pipe leads it to the sump at the bottom or to a lodgement or a coffer dam.

**Coffer Dams.**—A coffer dam is a section of solid lining designed to dam back water coming from a bed of water-bearing rock. Sufficient material is excavated from the water-bearing bed to allow a good cement backing to be inserted behind the shaft timbers. This excavation should be carried a short distance into the overlying and underlying impervious rock so as to form a water-tight joint. The space behind the timbers is filled with concrete either at the time they are placed or later through an opening left in them. The timber is often made much stronger and heavier at this point.

**Lodgements, or Basins.**—Lodgements, or basins, are openings from 6 to 8 ft. high, equal in width to the shaft, and driven, usually, from the end thereof. As they extend from 50 to 60 ft. back from the shaft they hold large quantities of water, which may be pumped thence to the surface instead of from the sump at the shaft bottom, thus effecting a large saving in power. They are commonly floored and lined with cement to prevent the water reaching lower levels through cracks in the rock. The mouth of the lodgement is closed with a timber, or concrete, dam, in which an opening large enough to admit a man is left. In the case of basins, no dam is necessary as it is made by excavating the floor of the lodgement to a sufficient depth to hold the water.



Fig. 2

**Sump.**—The shaft is carried far enough below the cage landing at the bottom to provide a catch basin, or sump, large enough to hold the water draining into it from the shaft and workings during 24 hr. The depth of the sump will be the height to which the suction end of the pump can draw water, say, 25 ft. at sea level. Where the mine makes much water, the area of the shaft is not sufficient to afford the required capacity, and the sump must either be extended at one end or a second sump, draining into the first, must be provided.

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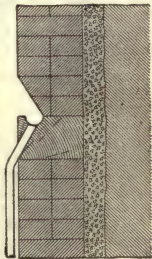
# SLOPE AND SHAFT BOTTOMS

## SLOPE BOTTOMS

At the foot of a slope, or at the landing at any lift, the entry is widened out to accommodate at least two tracks—one for the empty and the other for loaded cars. The empty track should be on the upper side of the entry, or that side nearer the floor of the seam, and the loaded track on that side of the entry nearer the roof of the seam.

Fig. 1 shows an arrangement of tracks often used. At a distance of 40 or 50 ft. above the entry, the slope is widened out to accommodate the branch leading into the entry loaded track. This branch descends with a gradually lessening inclination until nearly at the level of the entry it turns into the main loaded track. A short distance above the entry and below the switch $b$ a hinged bridge $d$ is placed, which, when lowered, forms a connecting platform or bridge by which the empty cars are taken off the slope. The empty track $e$ is about 6 ft. higher than the loaded track $l$, and is carried over it on a trestle.
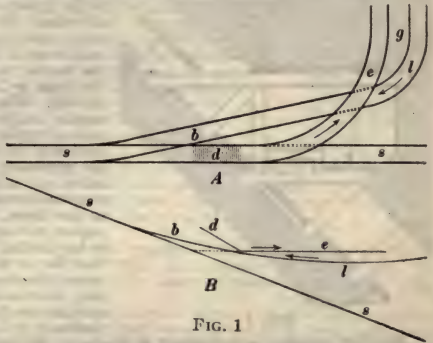
The figure shows the plan $A$ and profile $B$ as arranged for a single slope, or one side only of a slope taking coal from both sides. When coal is to be hoisted from this landing, the bridge is closed, the empty cars lowered in the slope run off over the bridge, the cars unhooked from the rope, and the hook and chain thrown down to the track below on which the loaded cars are standing; the loaded cars are then attached to the rope and hauled to the main track on the slope and hoisted. This plan can only be economically employed in a seam of moderate thickness that will not require the taking down of a large amount of the top. The cars can be handled on the landing by gravity.



FIG. 1
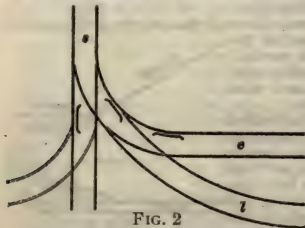
Fig. 2 shows an excellent method of laying switches in either thick or thin seams where the pitch does not exceed 20°. When there is only one track in the slope and coal is to be hoisted from both sides, the same arrangement is used on each side; but to avoid complications, such as crossings, etc., it is better to locate one of the switches on the main track farther down the slope, as indicated by the dotted lines. The empty track $e$ joins the loaded track $l$ before it reaches the slope track $s$.



FIG. 2

Fig. 3 shows a plan $A$ and profile $B$ of a switch used at the bottom of a slope. The figure shows one side only of the slope, the other side being similar. At the switch $a$ there is a pair of spring latches set for the empty track $e$ and which causes the empty cars coming down the slope to take this track. The empty cars pull the rope in to where it can be attached to the loaded cars, which are standing near the slope on the road $l$.
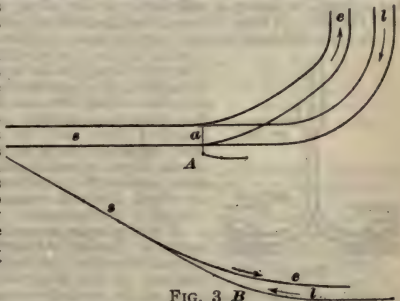
Fig. 4 shows a cross-section of the slope landing shown in Fig. 3 when the empty track $e$ is higher than the loaded track $l$, so that both the loaded and empty cars can be handled by gravity.



FIG. 3 $B$

When the pitch of the slope is so steep that the coal or ore falls out of the cars during hoisting, a gunboat is used or the cars are raised on a slope carriage—in either case, the arrangement of the tracks at lift landings is entirely different. With either a gunboat or a slope carriage, the arrangement of tracks on the slope is the same; but, in the former case, a connection between the slope and empty tracks is often advisable. When a gunboat is used, the empty tracks run direct to the slope, and a tipple, or dump, is placed on each side to dump the mine cars over the gunboat; but when the cars are raised on a slope carriage, the gangway tracks run direct (at right angles) to the slope, to carry the car to the cage or carriage. The floor of the cage is horizontal, and has a track on it that fits on the end of the entry track when the carriage is at the bottom, and this track is arranged with stops similar to those on cages used in shafts.
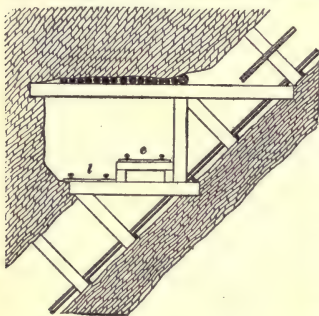


FIG. 4

Another common arrangement of tracks at the bottom of a slope is shown in Fig. 5. A branch is made by widening the slope out near the bottom, and this, being a few feet higher than the main track, is used to run off the empties by gravity. The loaded cars run in by gravity around the curve to the foot of the slope in position to be attached to the rope.

In ascending, the loaded car forces its way through the switch, or the switch may be set by a lever located at the foot of the slope. When the empty car descends, it runs in on the branch, where the chain is unhooked and thrown over in front of the loaded car, and runs around the curve into the entry by gravity.

It will be observed that in this plan the loaded car and, consequently, the bottom men, stand on the track in line with the slope, and are in danger from any objects falling down the slope, or from the breakage of the rope or couplings; but this can be obviated by making the bottom on the curve. The illustration shows only one side of the slope; the other side is, of course, similar.
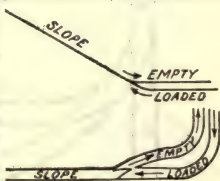


FIG. 5

All these plans necessitate the location of that part of the entry near the slope, in the upper benches of the coal or near the top rock. The gangway is then curved gently around toward the floor, so that, when it has been driven far enough to leave a sufficiently thick pillar, the bottom bench is reached and the entry is then driven along the bottom rock.

A very different bottom arrangement is shown by Fig. 6, which also represents a plan frequently adopted on surface planes. The two slope tracks are merged into one a short distance from the bottom of the slope, and on the opposite sides of the bottom two tracks curve around into the entry on opposite sides of the slope. As these branches curve into the main entry tracks, a switch sends off a side track for the empty cars. The switch on the slope is either set by the car—and this can be done because the next loaded car goes up on the same side on which the last empty descended—or by a lever located at the bottom.
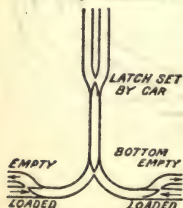


FIG. 6

It will at once be seen that in this plan no opportunity is afforded of handling the cars by gravity. The curved branches are made nearly level, and the momentum of the descending car, if quietly detached, is often sufficient to carry it partly or wholly around the curve, even against a slight adverse grade. The disadvantage of having the bottom in direct line with the slope (where there is danger from breakage and falling material) also obtains in this plan.

In the plan shown by Fig. 7, the grades may be so arranged that the cars can be entirely handled by gravity. The latches on the main-slope track may be closed automatically by a spring or weight, the loaded car running through them in its ascent on the slope, or both sets may be operated by a single lever at the bottom. The switch at the upper end of the central track (loaded) is set by a hand lever. All three sets may be linked together, so that they can all be properly set by a single lever. Reference to Fig. 5 will show that this is only a modification of that method. It requires space at the bottom for only three tracks, while Fig. 7 requires width to accommodate four tracks, and is objectionable because it is more complicated. The extra set of latches at the top of the central track, and the curvature of both main tracks into this central one, must inevitably cause much trouble and delay from cars jumping the track at this point.
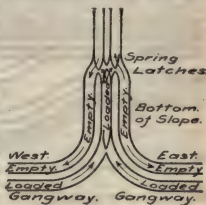


FIG. 7

The plan shown in Fig. 8 is open to many of the objections pertaining to some of those already described, and which need not be reiterated here. It can only be employed in thick seams, or in seams of moderate thickness lying at a slight angle or dip.

In planning the arrangement of tracks on a slope, it is advisable to place as few switches as possible on the slope itself, to keep the main track unbroken, to make the tracks as straight as possible, to have nothing standing at the bottom in direct line with the slope tracks, and to arrange the tracks so that cars are handled by gravity.

The arrangement of tracks near the top of the slope, and on the surface is often very similar to the bottom arrangements, as already described; but as all loaded cars (except rock and slate cars, which are run off on a separate switch) are to be sent off on one track, and all the empties come in on the same track to the head of the slope, and as there is usually abundance of room for tracks and sidings, these top arrangements are, in a measure, much more easily designed. In some instances, the two main-slope tracks run into a single track near the head of the slope—a plan somewhat similar to the bottom arrangement shown in Fig. 6—and the cars are then brought to the surface on one track, which, after passing the knuckle, bifurcates into a loaded and empty track. A similar arrangement is frequently adopted at slopes on which a carriage or gunboat is used. When the two main-slope tracks are continued up over the knuckle to the surface—the most common and best plan—the arrangement of tracks and switches may be planned entirely with a view to the quickest and most economical method of handling the cars.
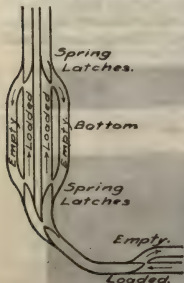


FIG. 8

**Vertical Curves.**—The vertical curves at the knuckle and bottom of a slope or plane should have a sufficiently large radius, so that when passing over them the car will rest on the rail with both front and back wheels. The wheel base of the car must be considered in adopting the radius for these curves, for if the curve is of too short a radius, there is danger of the car jumping the track every time it passes over the curve.

## SHAFT BOTTOMS

When coal is received on one side of the shaft only, an arrangement of tracks such as shown in Fig. 1 is often adopted, by which the empty car, when bumped from the cage $s$ by the loaded car, descends a short, sharp grade to $b$, and then by its own momentum ascends a short grade $c$ called a kick-back and returning by gravity passes through a spring latch at $b$, by which it is automatically switched and passes around the shaft by the track $g$ that connects with the empty track, which is from 2 to 3 ft. lower than the level of the loaded track. At times, the track leading around the shaft passes through a cross-over to an air-course or parallel entry occupied by the empty track, instead of returning to the main shaft bottom where the loaded track is located. Sometimes, the loaded track is in line with the center of the shaft, instead of as shown in the figure, the switch allowing the cars to pass to either cage, as desired.

Where there is not room back of the shaft for the length of track shown in the cut and where power is available, the empty tracks are brought together
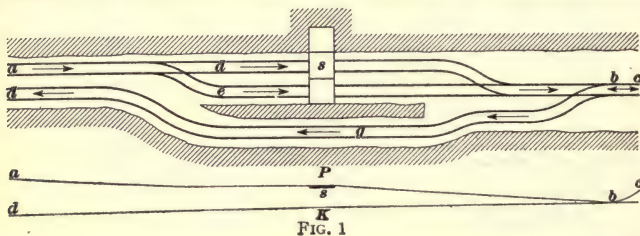


FIG. 1

as soon after leaving the cageway as possible. At the point of the spring latches is the foot of a power-driven chain haul, which hoists the empty cars to a sufficient elevation for them to run back by gravity along the line bd to a connection with the main entry track. The height through which the empties
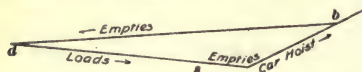


FIG. 2

are hoisted will vary from 4 to 8 ft. or more, and depends on the length and grade of the siding bd. A profile of the arrangement is shown in Fig. 2, in which s is the shaft, d the switch connecting the empty and loaded tracks on the main road, and b the switch back of the shaft between the car haul track and empty car siding bd.

When loads are caged from both sides of the shaft, the bottom arrangements are as shown in Fig. 3, or some modification thereof. The grades should
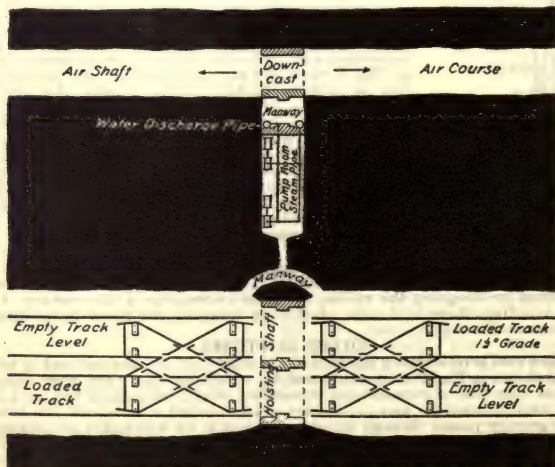


FIG. 3

be so arranged that from the inside latches of the crossings the empty track has a slight down-grade from the shaft, and the loaded track a slight

down-grade toward it. The crossings and the short straight piece of road close to the shaft should be level.

As it is often necessary to move cars from one side of the shaft to the other without stopping the hoisting, a narrow branch road connecting the tracks on opposite sides of the shaft should be cut through the shaft pillar, similar to that shown in *bgd*, Fig. 1.

Other shaft-bottom arrangements will be found in the section on Timbering.

## GENERAL BOTTOM DETAILS

In arranging tracks for shaft bottoms, at tops and bottoms of slopes, on coal bins, for mechanical-haulage landings, at foot of slopes or shafts, or in the body of the mine, it is customary to provide double tracks of sufficient length to hold the requisite number of cars for economically operating the plant and with sufficient distance from center to center of tracks, and from centers of tracks to sides of entries, to easily pass around the cars where it may be necessary, either in handling them, or in lubricating the wheels. For cars with a capacity of from $1\frac{1}{2}$ to 2 T., it generally requires an entry to be about 15 to 17 ft. wide in the clear for ordinary landings in the body of the mine, while at shaft bottoms the necessary width may attain 17 to 18 ft. in the clear, owing largely to location and local requirements. The curved crossovers connecting the tracks at shaft bottoms should be designed with radii of as great length as can be introduced, thereby giving an easy running track. They should not be less than from 20 to 50 ft. on center lines for ordinary gauge of tracks, i. e., 36 to 44 in.

On landings constructed in the body of the mine for the reception of empty and full cars handled by mechanical haulage from shaft or slope, and from this point transported by animal power to the various working places in the mine, a grade of about 1% in favor of the loaded cars to be handled by the stock will be found quite an assistance in delivering the cars to the haulage. The frogs and switches for these landings, as well as those required at the shaft or slope, should be formed of regular track rails, and can generally be arranged to be thrown by a spring or a conveniently located hand lever, as has been described, instead of being kicked to position, as was the custom at one time.

Besides these usual arrangements of shaft-bottom landings, at many plants the natural grades of the entries can be taken advantage of in designing convenient and economical methods for handling the mine cars. For instance, where the coal is to be hauled from the dip workings of a mine by some form of mechanical haulage, and a summit can conveniently be arranged for in the track on the same side of the hoisting shaft, at the proper distance therefrom, to accommodate the requisite number of loaded cars to be hauled, thus allowing them to run by gravity over, say, a 1% grade to the shaft, several empty-track arrangements can be made. The most simple form is to have the empty cars descend a short grade of from 4% to 5% when pushed from the cage by the succeeding full one. The momentum thus secured is quite sufficient to carry the car up an opposing grade of 1.5%. It again descends on the same track, and passing through an automatic switch, continues to the empty-car siding. From this latter point it is handled by the regular haulage machinery, and in its route passes around the shaft through an entry especially prepared for this arrangement. A shaft bottom so constructed is very economical to operate, requiring but few men to handle the cars.

Occasionally, it becomes more expedient to have a separate short haulage to draw the empty cars to the main haulage when it cannot be easily arranged to construct a complete gravity landing. Several other modifications of such a general design can be made. All the different devices, however, depend largely on the local requirements of the particular mine under consideration.

When endless-rope haulage is employed, it is generally found to be most convenient to have the landings for full and empty cars in the body of the mine reached by switches off of the main-haulage track, the cars coming on and leaving the main track at slight knuckles introduced in the track, in order to allow a place for the passing of the rope, which then moves along through a short cut or channel through the switch rails. The flanges of the cars pass over the rope in this manner without any injury to it.

**Mine Stables.**—In the location of the mine stable, the following points should be considered: The prompt rescue of the mules in case of accident; the ventilation of the stable by a separate split of fresh air without contaminating the air-current passing into the mine; the handling of the daily stable refuse and feed to and from the surface; water supply; distance from the stable to the working face. The stable is generally located near the bottom of the shaft

especially during the early development of the mine; though sometimes later, and after the workings have become extensive, the stable may be moved to some convenient second opening or air-shaft where the mules will be closer to the working face and can still be rescued promptly and fed and cared for economically. One arrangement is shown in the accompanying figure.

Usually no door is required at the entrance to the stable, but a regulator is placed at its rear end to control the supply of air entering from the main intake.

As a protection from fire, the posts for supporting the roof, as well as the partitions between the stalls, the doors, etc., are made of sheet iron. Very frequently, the walls and floors are coated with cement, which material, but with a wooden lining, is also often used in the construction of the feeding troughs. Stalls are commonly built so that there will be a passageway 2 to 3 ft. wide between the heads of the mules and the ribs. Commonly a track is laid back of the mules so that all manure, straw used for bedding, and other stable litter may be loaded into a mine car and sent to the surface each day. Wherever possible, underground stables should be provided with incandescent lights hung in metal wire baskets.

The question of water supply for the stables is sometimes a troublesome one. Where the mules cannot drink the mine water, as is usually the case, a supply of water must be piped to the stable from the surface. It is important to maintain a bar or chain at the entrance of the stable to prevent mules that get loose from wandering into other parts of the mine; the instinct of the mule will almost invariably lead him to the sump where he may be drowned.
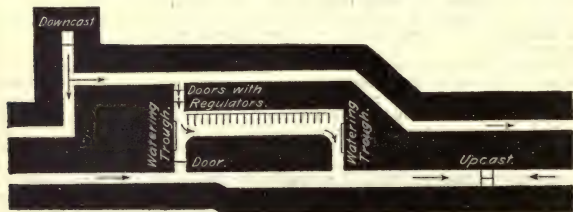


**Pump Room.**—The pump room frequently is located near the foot of the pump way of the main hoisting shaft. The use of a compartment of the hoisting shaft for pumping, however, often proves a serious inconvenience in the operation of the mine, owing to the exhaust steam filling the shaft and shaft bottom so as to interfere with the work of hoisting. With the pump room located in the shaft pillar between the downcast or air-shaft and the main hoisting shaft, this trouble is avoided.

It frequently happens that owing to the varying grades in the seam it is impracticable to drain all the mine workings to a sump at the shaft bottom. In such cases, a sump is often located at some convenient low point in the workings, and the pump room is then located at this point, and the water pumped to the surface through bore holes drilled for this purpose. The steam supplying the pump is likewise conducted in pipes from the boilers at the surface to the pump in the mine through a bore hole.

**Engine Room.**—The engine for rope haulage is often located at some point in the mine, and where steam is used for power it may be taken down the shaft and along the entry to the engine room, or down a bore hole that opens into the mine near the engine room. The engine may exhaust into a pipe leading up the shaft, or bore holes for this purpose may be sunk from the surface at the point where the engine is located. The engine room is an opening made in the shaft pillar or, if away from the shaft, in the entry pillar, which is then made larger to provide for the room. The roof over the engine room is well secured by solid timbers, or by steel I beams supported on brick or concrete walls at the sides of the room. The engine should be placed so that the pull of the rope will be as direct as possible.

**Lamp Stations.**—In a very gaseous mine where none but safety lamps are used in the workings, the lamp room or lamp station is generally located at the surface. In many gaseous mines, however, safety lamps are restricted to a portion of the workings only and naked lights are used in the other

portions of the mine. In such cases, lamp stations are frequently provided at some point on the main intake of the mine near the mouth of the entries or headings leading to these workings. Similar lamp stations, called *relighting stations*, are likewise often provided at different points on the main intake wherever safety lamps are used, where lights that have been extinguished may be relighted. A lamp station is a simple opening made in rib or pillar coal on the intake airway, where a strong current of pure air is passing, and where safety lamps may be kept or relighted when extinguished.

**Shanties.**—The various other shanties used in the operation of the mine, such as the mine-boss shanty, tool shanty, oil house, etc., as well as the wash rooms and hospital rooms, are simple openings made in the shaft or entry pillar, the size and arrangement depending on their use. Many mines now have wash rooms and hospital rooms at the shaft bottom, supplied with steam and water pipes, for the convenience of the men and for the care of the injured. The walls of these rooms, as also those of the mine-boss shanty, are often cemented and whitewashed, and the floors are also cemented so that they can be kept clean and comfortable. Tool shanties are often located at convenient points for the distribution of the tools to the company men, and sometimes there are blacksmith shops in the mine for the sharpening of the tools, though this is generally done at the surface.

**Manway About the Shaft.**—A small manway should encircle at least one end of every hoisting shaft. This manway is sometimes made by enlarging the shaft excavation by widening on the rib, but this is not a good plan. At other times, a narrow heading or passageway is driven in the solid coal from one side of the shaft to the other. A manway in the shaft pillar is objected to by some as endangering the shaft pillar, but allowance can be made for it in laying out the size of the shaft pillar, and it can be well timbered, if necessary, so as to run no risk of weakening the strata near the shaft. No hoisting shaft should be operated without such a manway, in order to avoid the risk to which the cager is exposed if obliged to pass under the moving cages.

## SURFACE TRACKS FOR SLOPES AND SHAFTS

The arrangement of the tracks on the surface naturally differs at every mine, owing to the different existing conditions. All surface roads should be so arranged that the loaded cars can be moved with the least possible power, always looking out for the return of the empties with as little expenditure of power as possible. To secure the running of the loaded cars from the mouth of the shaft or slope by gravity, a slight grade is necessary, the amount of which depends on the friction of the cars, which varies greatly. Care should be taken that an excessive grade is not constructed, or there will be trouble in returning the empties from the dump to the head of the shaft or slope.

The tracks connecting the top of the shaft and the tipple may be very short, or of considerable length, depending on the conditions at each mine. Usually from 20 to 60 ft. will be sufficient, although no definite rule can be given for this.

There are two general arrangements of tracks about the head of a shaft: First, where the loaded cars are removed from the cage and the empty cars placed upon it from the same side of the shaft; second, where the loaded cars are removed from one side of the shaft and the empty cars returned to the cages from the opposite side of the shaft.

In either case there are usually several empty cars on the platform ready to be put on the cages when the loaded cars have been removed.

Where the conditions are such that the loaded cars can be run by gravity to the dump, a good plan is to have a short incline, equipped with an endless chain, in the empty track. The empty cars can be run to the foot of this, hoisted by machinery to the top, and thus gain height enough to run them back to the shaft or slope by gravity.

At the Philadelphia and Reading Coal and Iron Co.'s Ellangowan colliery, where the tipple at the head of the breaker is above the level of the head of the shaft, the following plan is used: The loaded cars are taken off the east side of the cages, and run by gravity to the foot of the incline where the axles of the car are grasped by hooks on an endless chain and the car pulled up to the tipple. After being dumped, the car is run back from the tipple to the head of the incline, and is carried to the foot of the empty track of the incline by an endless chain. The foot of the empty track is several feet higher than that of the loaded track, and the cars are run by gravity around to the west side of the cages, and are put on from that side. The empty cars, as they run on the cage, have momentum enough to start the loaded car off the cage and

on toward the foot of the incline. There are a number of hooks attached to both the empty and loaded chain on the incline, and there are often several loaded and several empty cars on different parts of the plane at once. This arrangement permits of the hoisting of from 700 to 800 cars per day out of a shaft 110 yd. deep, with single-deck cages.

Another excellent arrangement for handling coal on the surface is the invention of Mr. Robert Ramsey, and has been adopted by the H. C. Frick Coke Co. and a number of other prominent operators. A description of this arrangement as applied at the H. C. Frick Coke Co.'s standard shaft is as follows: The landing of the shaft is made slightly higher than the level of the tipple, which is north of the shaft. South of the shaft is located a double steam ram, one ram being directly in line with the track on each cage. Directly in front of the rams is a transfer truck, worked east and west by wire rope. The loaded car on the cage is run by gravity to the tipple, where it is dumped by means of a nicely balanced dumping arrangement. As soon as it is empty it rights itself and runs by gravity alongside the shaft to the transfer truck, which carries it up a grade to a point directly in line with the cage that is at the landing, and one of the steam rams pushes it on the cage, and at the same time starts the loaded car off toward the tipple. This second loaded car is then returned by the same means to the opposite cage. The whole mechanism is operated by one man, by means of conveniently arranged levers, each of which is automatically locked, except when the proper time to use it arrives. It is therefore impossible for the topman to work the wrong lever and put an empty car into the wrong compartment of the shaft. Besides the one man at the levers, there is but one other man employed at the tipple, and his work is solely to look after the cars when dumping. All switches are worked automatically, and the average hoisting at this shaft is at the rate of 3 cars per minute. The shaft is about 250 ft. deep, and single-deck cages are used.

The Lehigh and Wilkes-Barre Coal Co. has a system in use at a number of collieries that has also proved very effective. In this system the loaded cars are run by gravity from the cage to the dump, and the empties are hauled from the dump back to a transfer truck by a system of endless-rope haulage. The transfer truck carries the car to a point opposite the back of the cage. The empty car runs by gravity to the cage, and its momentum starts the loaded car on the cage on its way to the dump. This system necessitates the employment of more topmen, but is a very good one. At the Nottingham shaft, which is 475 ft. from landing to landing, from 140 to 150 cars per hour are hoisted on single-deck cages.

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# METHODS OF OPEN WORK

No definite rules can be given for the selection of a method of mining that will cover all the conditions that may exist at any given mine. The system finally selected is that which will yield the maximum percentage of coal in the best marketable condition at the minimum of cost and danger.

All methods of working may be grouped under one of two heads or classes, viz., open work, or closed work.

*Open work* applies to the mining of those deposits that are either so thick or lie so near the surface that the material overlying them may be removed and the coal quarried out at a profit.

The advantages of this system are that all the coal may be extracted without any loss in pillars or through squeezes, and in the lumpiest condition; no timber is required; unprofitable underground workings do not have to be kept open and in repair; when required, a simple hoisting plant is used; there is less danger to the workmen from falls of roof and from blasting; there is practically no danger from fire; artificial lights are not required; mining can be done more economically, as larger faces are open, larger blasts can be used, and the amount of work accomplished per miner is greater, and better superintendence can be had; the health of the men is usually much better when working in the open; and, under proper conditions, the output can be increased almost indefinitely.

The chief disadvantage of open work is the possible reduction in output during the winter months owing to snow, the exposure of the men to the weather, etc. Further trouble may arise from flooding during the rainy season, and, unless the seam lies parallel to the surface, the cost of removing the over-burden soon becomes excessive.

The removal of the overburden is known as *stripping*, and may be carried on with or without the use of excavating machinery. When machinery is not used, the covering is removed with pick and shovel when it is earth, and by hand drilling and blasting when it is rock. This is the original method of stripping, probably first applied in the United States to the thick deposits of the anthracite region of Pennsylvania, and is limited in its application to those seams that are either very near the surface or are abnormally thick. Experience has shown that it will pay to remove, without the aid of machinery, 1 ft. in thickness of overburden for each foot in thickness of the underlying coal. Thus a seam 6 ft. thick will permit of the profitable removal of 6 ft. of cover, and one 25 ft. thick of 25 ft., possibly 35 ft., of cover. Mines of this class are known as *strippings* in Pennsylvania, and as *strip-pits* in the middle West.

**Steam-Shovel Mines.**—Steam-shovel mines are the result of the application of the familiar railroad contractor's steam shovel to stripping. Under favorable conditions, there is probably no cheaper method of mining. It is extensively used in the neighborhood of Park City, Utah, in metal mining and on the iron ranges of the Lake Superior region, where an output of 2,000 T. of ore per da. for a steam shovel and one locomotive has been reached.

The cost of removing 97,854 yd. of material from over a seam of anthracite (Pa.) was $1 a yd. of material stripped and $.516 per T. of coal obtained. The average depth of the stripping was 75 ft., and about two-thirds of the material removed was rock. Recent contracts in the same region have been let for as low as 25 c. a yd. for rock and 5 c. a yd. for earth. Where conditions are very favorable and a shovel of large size can be kept steadily employed, even lower average costs per yard (shale rock and dirt combined) may be had. The volume, in cubic yards, of overburden removed per long ton of coal extracted in recent Pennsylvania practice is 3.3, 3.8, 3.5, 3.6, and 3.0 to 1. In one extreme case, 5.4 cu. yd., and in another but 1.8 cu. yd. of overburden were removed per long ton of coal extracted.

In the Kansas field, where the surface is level and the seams horizontal, shovels of the largest size are employed to remove the covering to an average depth of 17 ft. (6 ft. to 24 ft.) from a seam that is but little more than 36 in. thick. The upper 6 ft. of cover is dirt, the second 6 ft. is soft shale or soapstone, underlying which is blue shale to the top of the coal bed. Where the seam has about 20 ft. of cover, the average steam shovel will, if employed pretty constantly, strip 12 to 15 A. a yr. at a cost of from 5 to 6 c. per cu. yd. The wage scale on an 8-hr. basis, is very close to $2.50, varying from $1.95 for waterboys to $3.05 for blacksmiths. Most of the workers receive $2.52 to $2.62 per da.

Near Danville, Illinois, where the conditions are very similar to those in Kansas, 38 to 40 ft. of overburden, of which 16 to 24 ft. is shale, is profitably removed from a coal seam 8 ft. thick. This is a fair general average for the district, although a ratio of 6 to 1 has been had.

The disposal of the overburden is frequently a matter of difficulty, particularly when it is thick. If it has to be transported to any great distance, the cost thereof may be prohibitory. If much water or sand occurs in the cover the cost of stripping is likewise increased. Strippings liable to overflow from flooded rivers are costly to operate and the workings should be protected by dams built of the overburden.

After the surface covering has been removed, a track is usually laid along the face of the stripping on the bottom of the workings, and the coal, after being blasted, is loaded into railroad cars by the steam shovel if it is shipped as mine run or into smaller cars for transportation to the tipple if it must be screened into sizes.

# METHODS OF CLOSED WORK

## INTRODUCTORY

By *closed work* is meant the mining and removal of the coal without the previous removal of the overburden. In general, the word *mine* is used to define a series of underground workings, and the words *stripping*, *strip-pit*, *open-cut*, *open-pit*, and the like, to what are more properly coal quarries of the nature just described.

No definite rules can be given for the selection of a method of mining that will cover all the conditions that may exist at any given mine. The system finally selected will be that which will result in the production of the maximum amount of coal per acre in the best marketable condition and at the minimum cost of extraction with the least danger to the workers.

**General Considerations.**—Some of the items to be considered in selecting a method of working are the thickness of the seam and the amount, location, and nature of its impurities; the use to which the coal is to be put; the character of the roof and floor; the amount of cover over the seam; the dip of the coal; the nature and direction of the cleat or cleavage of the seam; the character of the labor to be employed; the presence or absence of gas, etc.

1. *Roof Pressure.*—Of these items, the roof pressure is the most important, and a number of other causes are directly affected by it. The weight of the overlying cover will give a maximum roof pressure, but this may be so variously modified that the determination of the actual pressure is practically impossible, and estimates of this pressure must be based largely on practical experience; hence, rules for its calculation are of comparatively little value. One very essential point, however, must be borne in mind, i. e., that the direction of pressure is perpendicular to the bedding plane.

2. *Strength and Character of Roof and Floor.*—The strength of roof refers to the power of being self-supporting over smaller or larger areas. A strong roof permits larger openings, but increases the load on the pillars, thereby necessitating larger pillars. A weak roof requires smaller openings, and permits smaller pillars when the floor is good. A strong roof may yield and settle gradually, giving good conditions for longwall work, or it may be hard and brittle, and difficult to manage.

The character of floor influences largely the size of pillars. A soft bottom requires large pillars and narrow openings, especially when the roof is strong.

3. *Texture of Coal and Inclination and Thickness of Seam.*—Soft, friable coal requires large pillars, while a hard, compact coal requires only small pillars. The inclination and thickness of the deposit increase the size of pillars required, and also influence the haulage, drainage, timbering, method of working, arrangement of breasts, etc.

4. *Presence of Gas.*—The presence of gas in the seam or in the enclosing strata affects the system of working, as ample air passages must be provided, and provision must frequently be made for ventilating separately the different sections of the mine. Where the gas pressure is strong, and outbursts are of frequent occurrence, narrow openings are necessitated that render the working safe until the gas has escaped.

5. *Use to Which Coal is Put.*—If the coal is destined to be coked, a method of mining is to be preferred which results in the production of the largest possible amount of slack; whereas, if the coal is screened into sizes in the ordinary way, choice should be given to that system which results in the largest amount of lump coal.

When the longwall method is used, it is particularly important to have a constant market for the output, such as obtains if the mine is shipping fuel coal to a railroad at a fixed tonnage per day, as a few days' idleness may cause serious trouble at the face, even in temporarily closing it if the pressure is great.

6. *Character of Labor.*—While, in the room-and-pillar system of mining, the temporary or even long-continued stoppage of work in a portion or all of the working places, does not commonly have other effect than reducing the output and, consequently, the profits, under the longwall system the shutting down even for a few days of a comparatively few working places may cause

serious trouble in handling the pressure at the face. Hence the necessity, under the longwall system, of having not only a steady car supply as explained, but also steady men who will not lay off for one trivial excuse or another.

**General Systems of Mining.**—For purposes of classification the various systems of mining coal fall into one of two groups, as follows:

1. Systems in which the tract to be exploited is first penetrated by a series of two or more relatively narrow (8 to 10 ft.) entries (headings or gangways) from some of which entries are (usually) turned relatively wide (15 to 30 ft.) rooms (breasts or chambers) which are separated the one from the other by pillars of coal, and in which rooms the bulk of the output of the mine is obtained. This group may be further subdivided into the room-and-pillar, pillar-and-stall, and panel methods.

2. Systems in which entries are not driven, but in which all the coal is extracted in one operation from a continuous tract, the roof being allowed to fall or cave as fast as the coal is removed, haulage roads being maintained through the caved area by means of walls of stone built along their sides. The longwall system, largely used abroad, and but to a slight extent in the United States where the vast bulk of the coal is mined by the room-and-pillar method, is the single example of this second group.

The consideration of any system of mining requires a discussion of the following subjects: The system of mining as a whole, including the direction of driving the entries and rooms, the number and grade of the former, etc.; methods of supporting the roof and sides of excavations, see Timbering; methods and appliances for removing water from the workings, see Hydraulics; methods of bringing down the coal at the working face, see Explosives and Blasting; methods of transporting the coal from the face to the tipple, see Haulage and Hoisting; methods of supplying the working places with a current of fresh air from the surface, see Ventilation.

# ROOM-AND-PILLAR SYSTEMS OF MINING

## PRELIMINARY CONSIDERATIONS

In the room-and-pillar system of mining, the tract to be worked is divided into small districts, or blocks, by *main entries* and *cross-entries* intersecting one another at right angles or nearly so. The coal in each block is mined by turning off from the cross-entries a number of *rooms*.

That part of the coal which is left between the individual rooms and entries is called a *pillar*, and the pillars are pierced at more or less regular intervals by *cross-cuts* or *break-throughs* in part for the purposes of haulage, but most largely to provide a passage for the air-currents that a better ventilation of the working faces may be secured. The removal of the coal in driving the entries and rooms is called the *first working*, or more rarely the *advance working* or *working on the advance*.

Unless it is necessary to leave in the pillars to support the surface permanently they are commonly removed as soon as the rooms, either in part or all those turned from a single cross-entry, have been driven their full length; and unless the pillars can be removed the room-and-pillar system of mining is very wasteful of coal, as from 30 to 50% of the total amount must be left in the mine. The removal of the pillars is variously known as *second working*, *working on the retreat*, *robbing*, *drawing-back the pillars*, *pillar working*, *pillar drawing*, *pulling pillars*, etc. Fig. 1 shows a mine laid out on the double-entry room-and-pillar system of mining.

**Number of Entries.**—The haulageways in a bituminous mine are known as *entries* or *headings* and in an anthracite mine as *gangways*. There are several methods of arranging these passageways, known as *single-entry*, *double-entry*, *triple-entry*, etc.

The multiple-entry systems are expensive to drive, and the greater the number of entries the greater the expense, and are only used by companies with ample capital, large acreage of undeveloped coal, and a large and regular market.

In the *single-entry system*, now used but very rarely if at all, a single entry is driven ahead and rooms are turned from one or both sides of it. This entry, which is also the main haulage road, acts as an intake, the air being conducted along it to the last room, up which it passes to the break-through and *back* along the working faces to the first room and thence by a small air-course to the upcast. The circulation of air is liable, in this method, to be cut off at any

time by a fall of roof in the rooms, and the mine must be closed until the ventilation is restored.

1 The *double-entry system* of mining, illustrated in Fig. 1, may be considered the standard of American practice. The main entries are driven from the shaft bottom or from the drift mouth, and from these the cross- or butt-entries are driven, usually at right angles. The rooms may be turned directly off these
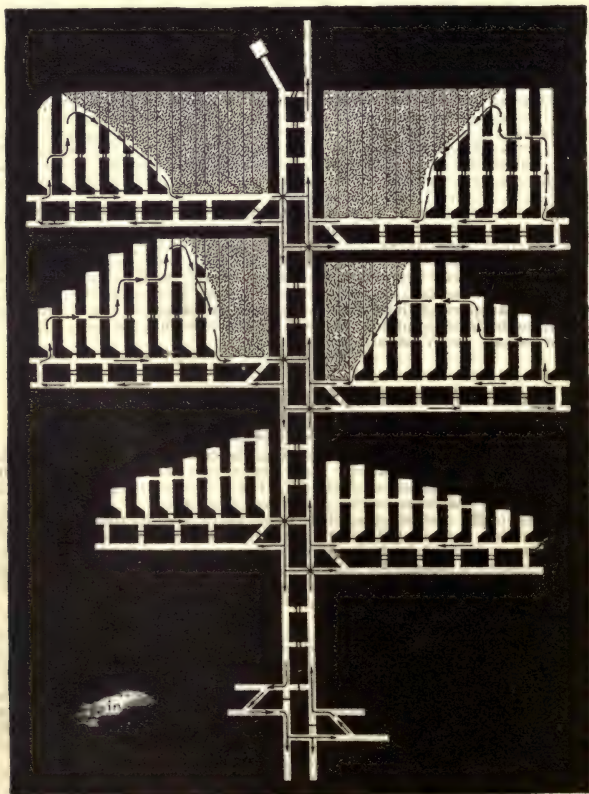


Fig. 1

cross-entries, or other entries may be driven at right angles to the cross-entries, and the rooms turned off them.

The advantages of the double-entry system are that in case of accident on one entry the other is available for escape, a fall of roof in one of the rooms does not obstruct the circulation in other portions of the mine, one or more of the pairs of cross-entries may be closed for any reason without in any way affecting other parts of the mine, and the entries may be driven ahead of room turning, as far as desired, for the purpose of prospecting the seam or to provide for a large number of extra working places that the output may be increased in event of a sudden demand.

In the *triple-entry system*, the center one of the three parallel entries is usually made the intake and the main haulage road for the mine, while the two side entries are made the return air-courses for their respective sides of the workings. Overcasts are usually built at the mouth of the center entry of each set of cross-entries to conduct the return current over the haulage road. Although this system requires a greater outlay because of the extra price paid for narrow work, it is often absolutely necessary in the working of a gaseous seam, to which it is particularly adapted. It is also used where it is not possible to drive a single entry of sufficient width for a double-track haulage road or where single or double entries of sufficient area to give the required quantity of air cannot be driven or economically maintained on account of poor roof or creep. Sometimes this system is applied to the main entries only, the cross-entries being driven double, as shown in Fig. 2.

In the *four-* or *quadruple-entry system* of mining, four parallel entries are driven. In this case, each side of the mine usually has its own intake (one of the center entries) and return (one of the outer entries). The center entries are the haulage roads, or one may be used for haulage and the other as a manway or traveling road for the men. Where high-speed endless-rope haulage is employed, this system is well adapted, the loads coming from the mine along one of the center entries and the empties entering it along the other. In some cases the center entries are not connected by breakthroughs, the right-hand and left-hand pairs of entries being used as the intake and return, respectively, of corresponding sides of the mine. This is equivalent to operating two distinct mines through the same main opening.

The *five-* or *quintuple-entry system* is the same as the preceding with the addition of one more entry, which is used as a manway.

**Size of Entries.** The size of an entry depends on its use, as well as on the thickness of the seam and the nature of the roof and floor. The cost of main-



Fig. 2

taining wide roadways under a bad roof or where the floor has a tendency to heave or where the coal is frail, often prevents their use and necessitates narrower openings. The thickness of the seam also affects the width of the roadways for a given output of coal, by reducing the height of ... in thin seams, and requiring a greater width of car, and consequently greater width of roadway for the same capacity or output. The amount of the daily output of coal is also a factor determining to a large extent the size of the haulage roads required. In a coal seam 6 ft. thick, with a good roof, and when the coal is clean and does not yield much gobbing material, the width of a single-track entry is generally from 8 to 10 ft. As the amount of material to be gobbed increases, the width is increased, if the roof will permit; but occasionally entries are driven only 6 ft. wide owing to poor roof. The general practice is, however, to drive entries as wide as the roof conditions will permit so as to avoid yardage as much as possible. In thin seams, where the roof must be taken down or the bottom lifted to provide headroom on the haulage roads, an entry is often driven 12 or 14 ft. wide where the conditions with respect to the roof and coal will permit. By this means, the cost of driving is paid by the coal taken out, there is no charge for yardage, and room is provided for stowing the waste material taken down from the roof. This waste is built along the side of the road as a pack wall, or *building*, as it is called. Fig. 3 shows a cross-section of an entry where the roof has been taken down to afford headroom and the waste built up at the side of the road.

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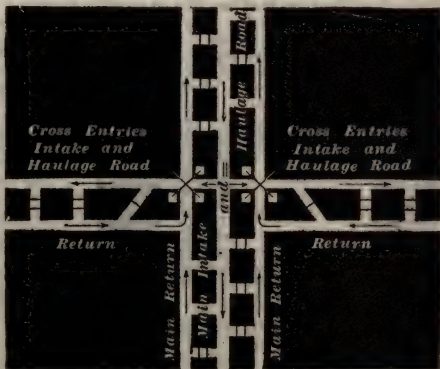
A common, but bad, practice is to make the haulage road, which is the intake, of a good height for passage of cars by tipping down the roof, but leaving the return airway of the height of the seam. Thus, if both entries are driven 10 ft. wide, and the roof in the intake is taken down to give a height of 6 ft. while the height of the return is that of the coal, or, say, 4 ft., the one entry has an area of 60 sq. ft. and the other one of 40 sq. ft., the disadvantages of which from the standpoint of proper ventilation are obvious. The return airway should have, at least, the same area as the intake.

Where practicable, the airways should be made in the form of a square instead of a parallelogram of the same area. Thus a square entry 10 ft. by 10 ft. in dimensions has an area of 100 sq. ft. and a perimeter of 40 ft.; whereas, an entry 5 ft. by 20 ft. in size while having the same area has a perimeter of 100 ft., and the friction of the air in passing through and the power required to produce ventilation is much greater than in the square entry.

Break-throughs between entries are usually made about the same width as the entry and at a distance apart determined by law or by the gaseous condition of the coal.

**Distance Between Entries.**—The distance between the center lines of the main entries is commonly made from 30 to 60 ft., which provides for a pillar from 20 to 50 ft. in thickness if the entries are 10 ft. wide as is usually the case. While the pillars are made as thin as possible to reduce the cost of narrow work in driving break-throughs, they must be sufficiently strong to withstand the effects of the pressure brought upon them by drawing the room and entry pillars on the cross-headings. Furthermore, as the main-entry pillars must last through the lifetime of the mine, they must be larger and stronger than those between the cross-entries, which are frequently pulled as soon as the rooms on them are worked out. The cross-entry pillars, which do not have to stand

so long, are frequently made 20 ft. thick, indicating a spacing of the center lines of 30 ft. when the entries are 10 ft. wide.

The distance between the center lines of the room entry of one pair of cross-entries and the return air-course of the next parallel pair of cross-entries is equal to the width of one entry plus the length of the room plus the thickness of the pillar left between the faces of the rooms driven from the next entry.
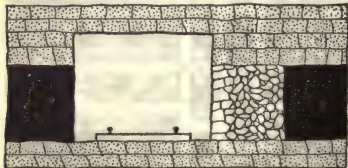
one entry and the return air-course of the next entry.

**Direction of Entries in Flat Seams.**—The direction of the main entries is determined by the shape of the property, the direction of the cleavage of the coal, etc. If the property is long and narrow, it effects an economy in haulage if the main entries are driven as near as possible and parallel to one of the longer sides so that all the cross- or room-entries may be on the same side of the haulage road. If the property is square, the main entries are commonly driven to divide the tract into two as nearly equal parts as possible, with room entries of the same length on each side of the main road. If the property is short and wide, in order to avoid excessive length of the cross-entries, two sets of main entries may be driven, which diverge toward opposite sides of the mine at a point a short distance inbye the drift mouth or the foot of the shaft or slope.

The influence of the direction of the cleavage of the coal upon that of the workings is more particularly noted under the head of Laying off Rooms, but it should be noted here that it is advisable to drive the headings parallel to one of the vertical cleavages and usually to the face. It is not unusual to demand an extra price per ton for mining coal in entries driven across the cleavage in those districts where this feature is pronounced. In such cases, it is a question for calculation whether the saving in haulage, etc., by having the headings divide the property on the lines just explained will offset the increased cost of driving them across the cleavage; assuming that the cleavage and property lines are not parallel.

So-called flat seams usually have a more or less decided dip, often rising to 3%, in one direction or another. Where possible, the main roads are driven either directly up or directly down the dip, so that the cross-entries on which the coal is produced may be level, and the haulage on a pitch is confined to that on the main entries.

If the property lies in a syncline, it is advisable to lay out the main roads in the basin, rather than across it, that the coal from the rooms may run down hill toward them.

**Direction of Entries in Inclined Seams.**—In pitching seams, the main roads are almost always driven either directly up or directly down hill, regardless of the direction of cleavage, etc. If the dip is not very marked, the main roads are frequently called *dip headings* or *rise headings* as may be, but when the dip is pronounced they are commonly referred to as the *slope, main slope, hoisting slope*, etc.

The cross-entries are driven to the right and left (either or both) of the slope, approximately on the strike of the seam, but with an up-grade of 1 to 2% to favor the haulage of the loaded cars, to insure drainage, etc.

If it is desired to drive the cross-entries on any given grade, the direction of the cross-entry may be found from the formula

$$\sin A = \frac{\tan x}{\tan y} = \frac{z}{\tan y},$$

in which     $A$ = angle between cross-entry and strike of seam;
             $x$ = pitch of cross-entry, in degrees;
             $y$ = pitch of seam, in degrees;
             $z$ = percentage of grade of cross-entry = $\tan x$.

**EXAMPLE.**—A slope pitches 15° in the direction N 25° 30′ E; what are the bearings of the right and left cross-entries if driven with a rising grade of 1%?

**SOLUTION.**—Substituting, $\sin A = \frac{z}{\tan y} = \frac{.01}{\tan 15°} = .03732$, whence $A = 2°\ 8′$. The strike has a bearing of S 64° 30′ E on the right of the slope and one of N 64° 30′ W on the left. The bearing of the right-hand cross-entry will be 64° 30′ − 2° 8′ = S 62° 22′ E, and of the left-hand entry, 64° 30′ + 2° 8′ = N 66° 38′ W.

**Alinement and Grade of Entries.**—As far as possible, the entries should be straight and of uniform grade in order to reduce the friction and wear and tear on the rolling stock, tracks, etc., to lessen the amount of coal shaken from the cars, which is subsequently ground into explosive dust, to diminish the number of accidents due to cars jumping the track, and to decrease the amount of power required for haulage.

Natural conditions are usually such in American mines that it is easily possible to secure straight tracks, although to produce a uniform grade it may be necessary to fill in swamps and cut down hills by shooting down the roof in the bottom of the dips and by taking up the floor at the top of a local rise. This grading is commonly confined to the main haulage road, which must usually last throughout the life of the mine, the cross-entries being allowed to follow the convolutions of the seam but with a slight upward grade.

Sharp angles are to be avoided on all entries, whether used for haulage or for air-courses, by substituting curves for angles or by means of diagonal roads or *cut-offs*.

In flat seams, the grades depend on the slight inclination of the seam that may exist. On the main haulage roads, the grade is the same as the pitch of the seam; and on cross-entries it is just sufficient for haulage and drainage purposes, say, with a rise toward the face of 1 to 1.5%. Uniform grades, on main roads, are commonly secured by brushing the roof and lifting the floor as explained; and on cross-entries, by following the strike of the seam, although the alinement thus secured may not be the best. In inclined seams, any desired grade can usually be obtained by altering the direction of the cross-entry, as described before.

**Rooms in General.**—Rooms are commonly turned off one side on an entry at a predetermined distance apart, the distance between the center lines of adjacent rooms being equal to the width of the room added to the thickness of the pillar between the rooms. They are usually opened by driving a narrow *neck* of about the width of the entry for a distance of 10 to 30 ft., depending on the roof pressure, after which the place is *widened out* more or less rapidly on one or both sides to *full room* width, which may be from 15 to 30 ft. or more, probably averaging 24 ft. in American mines. If the rooms are inclined to the entry, the necks must be longer than if they are at right angles to it, in order to furnish a sufficiently solid entry pillar. Room necks should not be driven any wider than need be and the seam should be undercut or sheared before blasting, that the minimum amount of powder may be required to bring down the coal, thus preventing shattering and consequent weakening of the entry pillar.

The tracks are generally laid along the straight rib (opposite to the *gob side*, or that side on which the room is widened) and at such a distance from it that there may be safe clearance for the miner between the side of a car and the rib. Sometimes the track is laid up the center of the room, in order to shorten the distance the coal has to be shoveled into the car at the face, in which case the room is commonly widened on both sides. In the case of very wide rooms, a track is sometimes laid up along each side. These tracks join at the neck of the room, that there may be but one room switch on the entry. Unless the mouth of the room is very wide the corner of the pillar on the entry and on the gob-side of the room is commonly rounded or beveled off to permit of a less abrupt curve on the track entering the place. Where necessary, the roof is supported over the track and at the face by props or props and caps (cross-bars). Any roof rock that falls and refuse from the seam are stowed on the wide side of the room; whence the term, gob side. The practice of throwing the fine coal resulting from mining (slack or bug-dust) into the gob cannot be too strongly condemned, as it will serve to furnish fuel for the propogation of a dust explosion.

The distance between centers of parallel and adjacent rooms as well as the width of the room and the thickness of the separating pillar, depend on the character of the roof, coal, and floor, and on the thickness of the coal and the amount of cover over it. No general rule for properly proportioning the width of room to the thickness of the pillar can be given, but points for consideration are given under the head of Timbering. The distances between the centers vary from 33 ft. to 80 ft. under different conditions. With 40-ft. centers, the pillars are usually 12 to 16 ft. wide, and the rooms 28 to 24 ft. wide. When the centers are farther apart than 40 ft., the pillars are often 20 to 30 ft. wide. Narrow rooms, about 12 ft. wide, are often driven and wide pillars of 60 to 70 ft. left between. The greater part of the coal is then got out by drawing
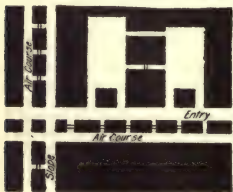


back these wide pillars, as explained later. When the room centers are 40 ft. or more apart, if the coal is soft the pillars are wide and the rooms narrow, but if the coal is hard the rooms are wide and the pillars narrow, provided that the roof and floor conditions will permit. The ratio between the width of the room and width of pillar in general decreases with an increase in depth below the surface. When an undue proportion of coal is mined in the first working, creeps are brought on, with all the accompanying evils of crushed coal, dilapidation of roadways and airways, extra cost for labor and material in repairing damages, and diminished production.

FIG. 4

The length of the rooms is governed by the distance decided on between entries. It is usual to make them from 150 to 300 ft. long, the former being preferable in thin beds and the latter in thick and steep pitching beds in order to avoid the expense of narrow work and cross-entry rails. The length of the rooms is also somewhat governed by the distance to which the coal can be economically hauled from the face to the entry, and by the gas present in the coal.

The distance apart of break-throughs depends on the amount of gas given off by the coal, on local practice, and on the mine laws of the state. Owing to the tendency of heated air, gas, etc., to accumulate at the face, rooms driven to the rise require the distance between break-throughs to be less than if the rooms are flat, while rooms driven to the dip require break-throughs at less frequent intervals than flat ones. Break-throughs should have the same cross-section as the other airways in the mine and should be turned and driven with as much care. Break-throughs between adjoining rooms should be driven in line so that the series of openings through the pillars formed by them can be used for haulage purposes when necessary.

**Double Rooms.**—When it is necessary to have a greater length of face than is afforded by a single room, double rooms, as illustrated in Fig. 4, may be driven. These rooms are connected to the entry by two necks and have two straight ribs with a track along each. Refuse is stored between the tracks, and where enough material is to be had, pack walls are built along the track so as to form two roadways leading to the face. The pillars are also wider than in the case of single rooms.

For the purposes of ventilation in gaseous seams, or as a protection against squeeze, to which a bed of coal may be especially liable, or for the purpose of

starting a long face for machine working, rooms are sometimes turned off of an entry as already described, but are opened into each other by the removal of the pillar in the first working, thus forming a continuous breast, as shown in Fig. 5.

**Rooms With Extra Entry Pillars.** Where there is excessive weight on the entry pillars, it is necessary, in order to keep open the entries, that these pillars be very large, or that a special pillar be left to protect the entry used as a haulage road, while the rooms are opened out from a parallel entry or cross-cut.



FIG. 5

Fig. 6 shows a method used at Danville, Illinois, where the coal is underlaid by a soft bottom, but has a strong cover. The weight of the cover would tend to force the pillars into the bottom and thus close up the entries. This is prevented by leaving the extra pillar e. The main entries a are driven and timbered for a double track; the cross-entries b are driven 10 ft. wide and the first room neck c is turned 10 ft. wide and driven up 15 ft. and then widened out on one side to a full width of 30 ft. A cross-cut d is driven 20 ft. wide and two more rooms are turned off this cross-cut, as shown; the fourth room is turned directly off the cross-entry, widened out on the right, and a cross-cut turned to the right as before and two more rooms are turned off this cross-cut, etc. The large entry coal pillars e, 40 ft. × 125 ft. in size, keep the weight off the cross-entries; and by making a rather thin pillar between the rooms,
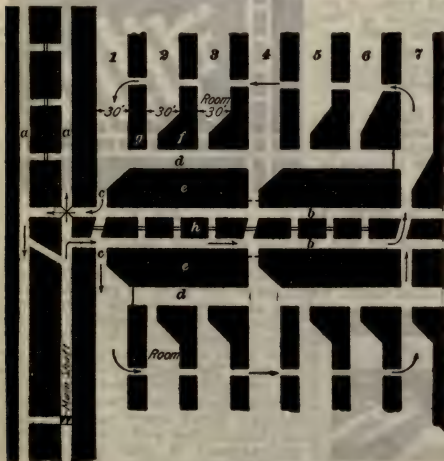


the weight is thrown on the face and made to assist in the mining. In the second working, these large pillars can be taken out, as well as the stumps f, the room pillars g, and the pillars h between the entries.

Fig. 7 shows another method of turning off the rooms in order to give additional support to the entries. Large pillars a are left above the entry b, and from each neck c turned off the entry b two roads are driven, one along each side of the room pillars e. This plan has been successfully used in a bed of hard coal with a strong bottom and a thick, strong roof.

**Inclination of Rooms to the Entry.** The direction in which rooms are driven with respect to the entries off which they are turned depends on the inclination of the bed, the cleat of the coal, and the nature of the overlying rock. When possible, the rooms are usually driven at a right angle to the entry. If the bed is flat, the rooms may be turned both to the right and to the left of

the cross-entries, and in such a seam, and where the entries are driven in pairs, a series of rooms is often driven off each entry of the pair. If the bed is inclined to any extent, the rooms are turned only to the rise of the higher entry, the other entry of the pair being used as an air-course. Rooms are usually not



FIG. 7

turned to the dip if much water will accumulate at the face. Where there is not much water to collect at the face, the rooms may be turned to the dip on a pitch as high as 6°, although, if the loaded car must be hauled out by a mule, the dip of the room toward the face should preferably not exceed 3° to 4°. If the loaded cars are pushed out of the rooms by hand, the road should dip from the face toward the entry, or should at least be level. A car can be controlled by spragging when pushed by hand until the inclination of the track is about 6°; that is, until the grade is about 10%.

As it is not usually practicable to haul empty cars up a grade of more than 6° to 8°, if the pitch of the bed is greater than this but less than about 12°, a suitable grade for haulage may be secured in the rooms by driving the rooms at an angle to the entry, as shown in Fig. 8. The method of working an inclined room does not differ from that used where the room is at right angles to the entry. The angle that the room makes with the entry should not be less than 30° or the entry pillar will not have the required strength, unless it is left very large. Where rooms are driven at an angle to the entry, the coal between the first room and the slope or main entry, as the case may be, is

worked out by means of cross-rooms $b$ driven off from the first room $a$ as shown. When the inclination of the bed is above 10° to 12°, the rooms are usually turned at right angles to the entry and the coal conveyed from the face to the entry and there emptied into the mine car.

Rooms driven to the dip are also sometimes driven at an angle to the entry where the inclination of the seam exceeds 3° and where mule haulage is used in the rooms.

The angle that a room should make with the entry in order to obtain a given grade of track in a seam having



FIG. 8

a given inclination is found by the method described under Direction of Entries in Inclined Seams, but placing $A$ = angle between room and cross-entry.

**Direction of Rooms as Determined by Cleat.**—In most coal seams there are vertical cleavages, called *cleats*, which cross the seam in two directions



FIG. 9

about at right angles to each other. The *face cleats* are the longer and usually the more pronounced, while the *end* or *butt cleats* are the shorter and more irregular. The cleat of the coal sometimes determines the direction in which the room should be driven, since the coal may break more easily on one cleat than another and thus produce a larger amount of coal for a given amount of undercutting. Fig. 9 shows rooms driven at various angles to the cleat and the name by which each is designated.

In driving face on, the room is driven so that the face is parallel to the face cleats, which are represented by the longer white lines, while the end cleats
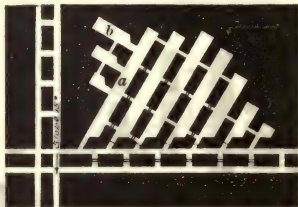
## DISTANCE FROM CENTER TO CENTER OF ROOMS OR BREASTS MEASURED ON ENTRY OR GANGWAY

| Angle Between Room and Entry Degrees | Width of Room + Thickness of Pillar, in Feet | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 |
| | Distance Measured on Entry, in Feet | | | | | | | | | | | |
| 90 | 20.0 | 25.0 | 30.0 | 35.0 | 40.0 | 45.0 | 50.0 | 55.0 | 60.0 | 65.0 | 70.0 | 75.0 |
| 85 | 20.0 | 25.1 | 30.1 | 35.1 | 40.2 | 45.2 | 50.1 | 55.2 | 60.2 | 65.3 | 70.3 | 75.3 |
| 80 | 20.3 | 25.4 | 30.5 | 35.5 | 40.6 | 45.7 | 50.8 | 55.8 | 60.9 | 66.0 | 71.1 | 76.2 |
| 75 | 20.7 | 25.9 | 31.1 | 36.2 | 41.4 | 46.6 | 51.8 | 56.9 | 62.1 | 67.3 | 72.5 | 77.7 |
| 70 | 21.3 | 26.6 | 31.9 | 37.2 | 42.6 | 47.9 | 53.2 | 58.5 | 63.9 | 69.2 | 74.5 | 79.8 |
| 65 | 22.1 | 27.6 | 33.1 | 38.6 | 44.1 | 49.6 | 55.2 | 60.7 | 66.2 | 71.7 | 77.2 | 82.8 |
| 60 | 23.1 | 28.9 | 34.6 | 40.4 | 46.2 | 52.0 | 57.7 | 63.5 | 69.3 | 75.1 | 80.8 | 86.6 |
| 55 | 24.4 | 30.5 | 36.6 | 42.7 | 48.8 | 54.9 | 61.0 | 67.1 | 73.3 | 79.4 | 85.5 | 91.6 |
| 50 | 26.1 | 32.6 | 39.2 | 45.7 | 52.2 | 58.7 | 65.3 | 71.8 | 78.3 | 84.9 | 91.4 | 97.9 |
| 45 | 28.3 | 35.4 | 42.4 | 49.5 | 56.6 | 63.6 | 70.7 | 77.8 | 84.9 | 91.9 | 99.0 | 106.1 |
| 40 | 31.1 | 38.9 | 46.7 | 54.5 | 62.2 | 70.0 | 77.8 | 85.6 | 93.3 | 101.1 | 109.0 | 116.7 |
| 35 | 34.9 | 43.6 | 52.3 | 61.0 | 69.7 | 78.5 | 87.2 | 95.9 | 104.6 | 113.4 | 122.1 | 130.8 |
| 30 | 40.0 | 50.0 | 60.0 | 70.0 | 80.0 | 90.0 | 100.0 | 110.0 | 120.0 | 130.0 | 140.0 | 150.0 |
| 25 | 47.3 | 59.2 | 71.0 | 82.8 | 94.6 | 106.5 | 118.3 | 130.1 | 142.0 | 153.8 | 165.6 | 177.5 |
| 20 | 58.5 | 73.1 | 87.7 | 102.3 | 117.0 | 131.6 | 146.2 | 160.8 | 175.5 | 190.1 | 204.7 | 219.3 |
| 15 | 77.3 | 96.6 | 115.9 | 135.2 | 154.5 | 173.9 | 193.2 | 212.5 | 231.9 | 251.2 | 270.5 | 289.8 |
| 10 | 115.2 | 144.0 | 172.8 | 201.6 | 230.4 | 259.2 | 287.9 | 316.7 | 345.5 | 374.3 | 403.1 | 432.0 |
| 5 | 229.5 | 286.9 | 344.2 | 401.6 | 459.0 | 516.3 | 573.7 | 631.1 | 688.4 | 745.8 | 803.2 | 860.5 |

are shown by the shorter ones. This is the general direction of driving rooms and is adopted where conditions permit. Fig. 1 shows that in the prevailing American practice the rooms are parallel to the main entries; hence main entries are face entries and cross-entries are driven parallel to the ends of the coal and are end-entries, although far more commonly called *butt-entries*. Face on is adopted where the face cleats are not as free or as numerous as the end cleats. Coal worked in this way breaks well, and the yield is perhaps larger for the same amount of undercutting than by any of the other methods, producing also a greater proportion of lump coal.

In working long horn, the opening is driven so that the face makes an angle less than 45° with the face cleats of the coal; the coal breaks in long slabs or wedge-shaped masses, giving rise to the name long horn. A face driven this way does not require the same amount of cutting; and if slightly inclined grip shots are used, good-sized lump coal is produced. If the coal works too freely face on, by this method support is given to the ends of the coal while being undercut.

In working half on, the rooms are driven at an angle of 45° with the cleats of the coal. The method is adapted to coals that break about equally well on the face and the end cleats.

In working short horn, the face of the room makes an angle between 45° and 0° with the face cleats. The method is adapted to the working of coal where the end cleats are so pronounced as to require the additional support given to the coal by this method when mining or undercutting. It bears the same relation to end-on work that long horn bears to face-on.

In working end on, the face of the room is at right angles to the face cleats, and, consequently, parallel to the end or butt cleats. This method, and short horn, are adapted to the working of coals under strong roof pressure. In general, the size of the coal and the yield are not as great as in face on or long horn.

As the face cleats are quite pronounced when rooms are driven end on, wide pillars are generally used.

Where there is much occluded gas at high pressure, the direction of the working face with respect to the face cleats of the coal is important, as a breast driven face on affords little or no opportunity for the escape of this gas, except

as it finds vent in violent outbursts. On the other hand, if this coal is worked end on, the face cleats are cut across and exposed and the gas escapes gradually and quietly. The method by short horn or half on, may be found to give good results in such a case, as the pressure of the gas is then made to do effective work in assisting to break down the coal.

**Direction of Rooms as Determined by Slips in the Roof.**—A roof slip is a line of weakness that was at some time a line of fracture in the rock and which may or may not have been filled subsequently, by infiltration with clay or other matter. Roof slips frequently occur in parallel lines in the rocks overlying coal seams; if this is the case, there is great danger from roof falls if the room face is parallel to the direction of the slips, for the miner cannot see the slip until too late to prevent accident by the falling of the slate or the sudden breaking down of the coal. By driving the room at an angle across the slips, not only is sufficient support given to the roof to prevent its breaking suddenly, but the presence of the slip is readily observed.

When the face is at right angles to the direction of the slips, there is not the same danger at the face as when the face is parallel to the slip, because the roof is better supported by the coal. The chief danger occurs when drawing back the pillars, for as the slips are parallel to the line of the pillars, a large fall may occur suddenly at any time by an unexpected cross-break. In any case, when driving under such roof, a larger amount of good timber is required.

## WORKING FLAT SEAMS

The general arrangement of a mine worked on the double-entry system according to the common American practice is shown in Fig. 1. The reasons for the direction, dimensions, etc., of the entries and rooms have been given in previous paragraphs. The general method is, of course, slightly changed locally to meet prevailing conditions, some of the modifications being given here under the names of the mining districts in which they are used.

**Pittsburg Region.**—The coal is worked with double entries, with cut-throughs between for air, and on face and butt entries are about 9 ft. wide, and the rooms 21 ft. wide and about 250 ft. long; narrow (or neck) part of room, 21 ft. long by 9 ft. wide; room pillars, 15 to 20 ft. wide, depending on depth of strata over the coal, which is from a few feet to several hundred feet. The mining is done largely by machines of various types. Coal is hard, of course, and, in many places, the roof immediately over the coal is also quite hard. There are about 4 ft. of alternate layers of hard slate and coal above the coal seam. Rooms are mined from lower end of butt as fast as butt is driven, the ribs being drawn as mining progresses. As the coal is harder than in the Connellsville region, thickness of coal pillar between parallel entries is somewhat less.

**Clearfield Region.**—The butt and face are not strongly marked in the B or Miller seam, the one chiefly worked in this region. Where possible, these cleavages are followed in laying out the workings, but the rule is to drive to the greatest rise or dip and run headings at right angles to the right and left, regardless of anything else. The main dip or rise heading is usually driven straight, and is raised out of swamps or cut down through rolls—very common here—unless they are too pronounced, when the heading is curved around them. The same is true of room headings, except that they are more usually crooked, not being graded except over very minor disturbances.

As the B seam rarely runs over 4 ft. in thickness, and is worked as low as 2 ft. 8 in., in the haulage headings the roof is taken down to give 5 ft. to 5 ft. 2 in. above the rail, or 5 ft. 8 in. to 5 ft. 10 in. in the clear. Where the resulting rock is taken outside, the headings are driven 10 ft. wide with 24 ft. of pillar, roof taken down in haulage heading but not in the air-course. Where the rock is gobbed underground, the haulage heading is 18 to 24 ft. wide, air-course 10 ft., pillar 24 ft., and roof taken down in haulage heading only. The thinner the coal, the wider the heading. It is more economical to haul the rock to daylight. The bottom generally consists of 3 ft. to 5 ft. of hard fireclay, frequently carrying sulphur balls.

In numerous places, the sand rock is immediately over the coal, but in most cases there is from 3 to 5 ft. of slate before the sand rock is reached. Room headings are driven 280 ft. apart, haul rock to daylight, heading 10 ft. wide with 24 ft. pillar to 10 ft. air-course, in which roof is left up. A 15 ft. to 25 ft. chain pillar is left between air-course and faces of rooms from the lower heading, every fourth to eighth of which is driven through to the air-course to shorten the travel of the air. The rooms are therefore 180 to 200 ft. long, and the men push the cars to the face, an important item in this thin coal.

**Rooms are 21 ft.** wide with a 15 ft. pillar, and a 15 ft. chain pillar is left between the first room on any room heading and the main heading, and roof is not taken down in rooms. Main-heading track is usually 30-lb. iron, room heading, 12 lb., and $2'' \times \frac{1}{2}''$ strap iron set on edge is used in the rooms in low coal. Mine cars hold from 600 to 800 lb. in low seams, and 1,500 to 2,000 lb. in the so-called thick seams; i. e., 3 ft. 8 in. to 4 ft. thick.

**Reynoldsville Region.**—The measures are very regular, and the method employed is the typical one shown in Fig. 1. The average thickness of the principal seam is $6\frac{1}{2}$ ft. and the pitch is 3° to 4°. The coal is hard and firm, and contains no gas; the cover is light, and on top of the coal there are 3 or 4 ft. of bony coal; the bottom is fireclay. Drift openings and the double-entry system are used. Both main and cross-entries are 10 ft. wide, with a 24-ft. pillar between. The cross-entries are 600 ft. apart, and a 24-ft. chain pillar is left along the main headings. The rooms are about 24 ft. wide and open inbye, the necks being 9 ft. wide and 18 ft. long. The pillars are from 18 to 30 ft. thick.

**West Virginia Region.**—The general plan of working the Pittsburg coal in the northern part of West Virginia is as follows: The coal measures vary from 7 to 8 ft. in thickness, and have a covering varying from 50 to 500 ft. The coal does not dip at any place over 5%. In most places the coal is practically level, or has just sufficient dip to afford drainage. The usual method of exploitation is to advance two parallel headings, 30 ft. apart, on the face of the coal. At intervals of 500 to 600 ft., cross-headings are turned to right and left, and from these headings rooms are turned off. These cross-headings are driven in pairs about 20 or 30 ft. apart. Between the main headings and the first room is left a block of coal about 100 ft. wide, and on the cross-headings there is often left a barrier pillar of 100 ft. after every tenth room.

The headings are driven from 8 to 12 ft. wide, and the rooms are made 24 ft. wide and 250 to 300 ft. long. A pillar is left between the rooms about 15 to 20 ft. wide. These pillars are withdrawn as soon as the panel of rooms has been finished. The rooms are driven in from the entry about 10 ft. wide for a distance of 20 ft., and then the room is increased in width on one side. The track usually follows near the rib of the room. Cross-cuts on the main and cross-headings are made every 75 to 100 ft., and in rooms about every 100 ft. for ventilation.

The double-heading system of mining and ventilation is in vogue. Overcasts are largely used, but a great many doors are used in some of the mines. Rooms are worked in both directions. This is the general practice when the grades are slight. When the coal dips over 1%, the rooms are driven in one direction only. In this case, the rooms are made longer, as much as 350 ft. It is the custom then to break about every third room into the cross-heading above (a practice ill advised). The floor of this bed of coal, being composed of shale and fireclay, often heaves, especially when it is made wet. Some trouble is at times experienced by having the floor heave by reason of the pillars being too small for the weight they support.

The dimensions of rooms and pillars given are for a mine (with covering 300 to 500 ft. thick) having a fairly good and strong roof. Where roof, bottom, and thickness of cover change, these dimensions are altered to suit the requirements. The main-heading pillars may be reduced to 30 or 40 ft.; the rooms may be made 15 ft. wide with 12 ft. pillars, and no barrier pillars may be left on the cross-headings.

The foregoing plan is very much followed in other parts of the state; at least an attempt is made to do so, but local disturbances often require changes in the plan. This plan is followed on some parts of New River, and also in the Flat Top field.

**George's Creek District, Md.**—Fig. 10 shows the method used in the George's Creek field, Maryland. The coal shows no indication of cleats, and the butts and headings can be driven in any direction. The main heading is driven to secure a light grade for hauling toward the mouth. Cross-headings making an angle of 35° to 40° are usually driven directly to the rise, and of the dimensions shown. Pillars are drawn as soon as the rooms are completed, being attacked on the ends and from the rooms on either side, the coal being shoveled to the mine car on a track in the room. Very wide pillars are split. No effort is made to hold up the overlying strata, and the entire bed is removed as rapidly as possible. An extraction of 85% of the bed is considered good work. A section of the seam is as follows: Roof coal, 10 in.; coal, 7 ft.; slate, $\frac{1}{2}$ in.; coal, 10 in.; slate, $\frac{1}{2}$ in.; coal, 10 in.; fireclay; slate. The top bench is bony and frequently left in place to prevent disintegration of the roof by the

air. Above this coal is from 8 to 10 ft. of *rashings*, consisting of alternating thin beds of coal and shale, that is very brittle, and requires considerable timber to keep it in place.

**Blossburg Coal Region, Pa.**—Coal is generally mined from drifts, but in a few cases by slopes. Fig. 11 shows the general method adopted; the breasts are run at right angles to the slips; the breast pillars are split by a center heading and taken out as soon as the breasts are finished. The gangway pillars
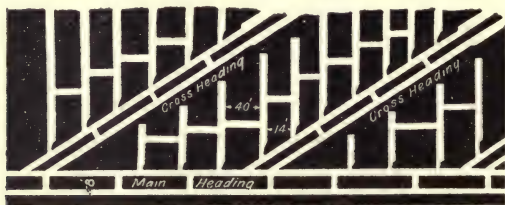


FIG. 10

are taken out retreating from the crop or boundaries of the property. The general average of the coal seams is not over 3½ ft., accompanied by fireclay and some iron ore. The dip of the veins is about 3%.

**Indiana Coal Mining.**—Fig. 12 shows the double-entry room-and-pillar methods as used in Indiana. The entries are generally 6 ft. high, 8 ft. broad, the minimum height required by law being 4 ft. 6 in. The rooms are from 21 to 40 ft. in width. The mines are generally shallow. The rooms are shown as widened on both ribs, but a more usual method in this locality is to widen the room on the inbye rib, leaving one straight rib for the protection of the road in the room.

**Iowa Coal Mining.**—The coal lies at a depth of 200 ft. below the surface, and is geologically similar to that of the Missouri and Illinois fields. It lies in lenticular basins extending northwest and southeast and outcropping in the larger river beds. The seams are practically level, non-gaseous, and generally underlaid by fireclay and overlaid by a succession of shales, sandstones,
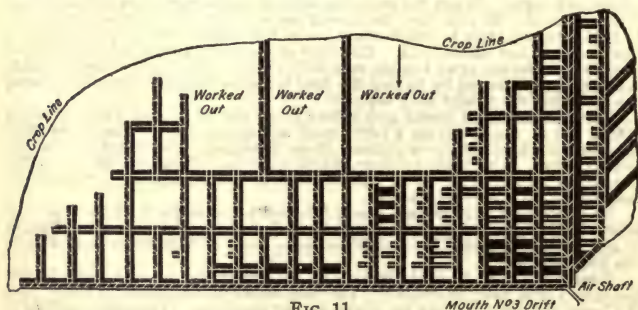


FIG. 11

and limestones, which are generally of a yielding nature, giving a strong, good roof for mining. There are three distinct seams, the lower one, which varies from 4 to 7 ft. in thickness, being the only one worked. The coal is a hard, brittle, bituminous that shoots with difficulty, but is excellent for steam and domestic uses. About Centerville, the coal has a distinct cleat, but elsewhere in the state it is lacking.

The entry pillars along the main roads are 6 to 8 yd. thick, for the cross-entries 5 to 6 yd., and for the rooms 3 to 5 yd. Room pillars are drawn in when approaching a cross-cut. Both room-and-pillar and longwall methods

are in use, with modifications of each. In the room-and-pillar method, the double-entry system is almost invariably used in the larger mines. Rooms are driven off each entry of each pair of cross-entries at distances of 30 to 40 ft., center to center; the rooms are 8 to 10 yd. in width, and pillars 3 to 4 yd. The rooms are narrow for a distance of 3 yd., and then widened inbye at an angle of 45° to their full width. They vary from 50 to 100 yd. in length, and the road is carried along the straight rib.

When double rooms are driven, the mouths of the rooms are 40 to 50 ft. apart, and they are driven narrow from the entry a distance of 4 or 5 yd. A cross-cut is then made connecting them, and a breast 16 yd. wide is driven up 50 to 60 yd. The pillar between each pair of rooms is 12 to 15 yd.

In pillar-and-stall work, the stalls are usually turned off narrow and widened inside, the pillar varying from 5 to 8 yd. The stalls are 30 to 40 yd. in length, and the pillars are drawn back. When the stalls are driven in pairs, the pillar 8 to 10 yd. in width is carried between them.

**Steep Rooms.**—Where the pitch is so great that mules cannot haul the car to the face for loading, a windlass may be used for the purpose, the handle of which is turned by the miner. If electric haulage is used, the motor may be blocked on the rails near the mouth of the room, and its cable reel or a special drum used to wind up a rope running over the necessary sheaves (pulleys) at the mouth and face of the room, which rope, attached to the end of the car,
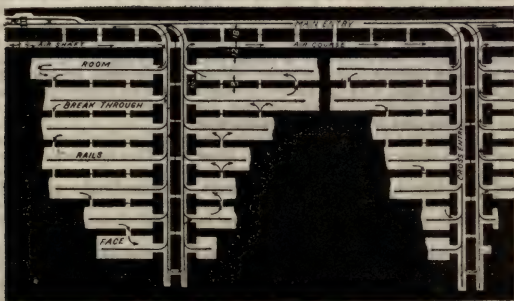


FIG. 12

hauls it up to the face. In other cases, self-acting inclines are used, in which the weight of the descending loaded car pulls the empty car to the face. This arrangement requires a double track in the room. Where roof conditions permit of only a single track, another pair of lighter rails may be laid between those of the regular room track and upon these a carriage with a counterweight will run. This counterweight is so adjusted that while the loaded car in descending will raise it to the face, its weight in descending will pull the empty car to the face.

Usually, a grade at which mules may pull the cars to the face is secured by inclining the rooms to the entry, the direction being determined by the formula given before. If the rooms are inclined and equipped with some one of the mechanical appliances just described, coal may be loaded at the face on much steeper pitches than would otherwise be the case.

## WORKING PITCHING SEAMS

**Difficulties in Mining on a Pitch.**—A soft friable coal when mined on a steep pitch has a tendency to *run;* that is, without any mining, it breaks freely from the face of the breast and slides down the pitch. Sometimes little or no work need be done in the breast after the chute has been widened out to form the breast, as sufficient coal thus breaks from the face from time to time to keep the breast full as the coal is drawn out through the loading chute; the coal continues to run until the breast breaks through into the upper gangway. The uncertainty that necessarily exists in regard to the flow of the coal

renders this method unreliable, although it is often adopted from necessity. One objection to this method is that the running of the coal cannot be controlled, and the widths of the breast and pillars cannot be maintained; the breast is often increased in width and much or all of the pillar coal runs out at the chutes, while at other times the width of the breast gradually decreases and ultimately the coal stops running. The miner must then go up into the breast and start the coal to running again by widening out the breast, or by placing one or more small shots in the coal; this is a dangerous operation, as the coal may come with a rush.

The coal on a steep pitch may not run sufficiently to do away with mining, but it may be so free as to require particular support at the face to prevent the coal from running sufficiently to injure the miners. The working of free coal on a heavy pitch requires skilled labor; and as gas usually issues from such coal in large quantities, safety lamps must often be used, thus increasing the danger from falls on account of insufficient light. The props used under these conditions should not be less than 6 in. in diameter and should be very firmly set. If the roof is strong and firm, these props may be taken out and moved forwards as required, thus saving labor and material.

In working coal by the battery method, the coal will sometimes become clogged and form an arch, which supports all the coal above the arch and allows the breast below to become empty as the coal is gradually drawn out through the loading chute. This condition is dangerous to the men working on top of the coal near the face, for if the arch suddenly gives away they may be carried down and buried in the coal. Such a slide is also apt to be very disastrous to the battery and the sides of the chute. To break down such an arch, an opening may be made in the side of the chute and the coal started to running by means of bars. Occasionally a small stick of dynamite is put under the coal and the arch loosened in this way. When this is done, an opening should be made from the side, and the miner should not, as he sometimes does, climb up the chute and after setting off his fuse trust to getting out before the coal begins to slide, as this is extremely dangerous.

After the face has been blasted down, lumps of coal will sometimes lodge in the manways alongside the chute and these must be similarly dislodged by means of bars or with dynamite. In returning to the face after a blast has been fired, the miner and his laborer should be exceedingly careful that the loose coal does not roll down the manway on them, and should also use great care to see that all loose coal in the face is barred down before they again begin work.

**Working Thick and Gaseous Seams That Run.**—In large seams, when the coal is soft and shelly or slippery, lies at an angle of more than 50°, and generates large quantities of firedamp, a source of danger is the sudden liberation of gas should a breast *run*. To meet these conditions, the air-course may be driven above the entry or gangway and used as a return, the fan being attached as an exhaust, and the working rooms or breasts ventilated in pairs. The inside manway of one of a pair of breasts is connected with the gangway for the intake, and the outside manway of the other breast with the return airway, giving each pair of breasts a separate split of the current. In collieries where this system of working is followed, the coal is soft. A new breast is worked up a few yards, but as soon as it is opened out, the coal runs freely and the manways are pushed up on each side as rapidly as possible, to keep up with the face. Two miners, one on either side, sometimes finish a breast without being able to cross to each other. The work is done exclusively with safety lamps, and when a breast runs the gas is liberated in such quantities that it frequently fills breasts from the top to the airway before the men can get down the manway on the return side. When the gas reaches the cross-hole, it passes into the return airway without reaching any part where men are working. Should a run of coal block a breast by closing the manway, it affects the current of one pair of breasts alone. As the gangway is the intake, leakage at the batteries passes into the breasts, as the cross-holes are above their level and the gas is thus kept above the starter when at the draw-hole. The gangway, chutes, and airway are supplied by wooden pipes, which connect with a door behind the inside chute. If a breast runs up to the surface, it does not affect the return airway, as it is in the solid.

Among the disadvantages urged against this system of working are the following:

It increases the friction, as the air must pass in the airway all the distance from the breast to the fan, the area of the airway being small in comparison to the gangway or intake.

As the faces of the breasts are so much higher than the return airway, the lighter gas must be forced down into the return against the buoyant power of its smaller specific gravity.

The reduction of friction obtained by splitting is neutralized by each split running up one small manway and down another; the advantage of running through several pillar headings and thus securing a shorter course being lost. This can be partly obviated by ventilating the breasts in groups, but the dangers avoided in splitting are increased.

Blackdamp, which accumulates in the empty or partly empty breasts, works its way down and mixes with the intake current, as there is no return current in the breast strong enough to carry it away, the return being closed in the airway.

All things considered, when the seam is soft and has a pitch of 40° and upwards, and emits large quantities of gas in sudden outbursts, as in running breasts, this system is the best that can be adopted.

**Working Thick Non-Gaseous Seams.**—The reverse of the system just described is followed at some collieries where the coal is hard and but little gas is encountered. The airway is driven over the gangway or against the top, the fan being used to force the air inward to the end of the airway. The air is distributed as it returns, being held up at intervals by distributing doors placed along the gangway.

Among the advantages claimed for this plan are the following:

As the pressure is outwards, it forces smoke and gas out at any openings that may exist from crop-hole falls or other causes.

The warm air from the interior of the mine returning up the hoisting slope or shaft prevents it from freezing.

As the current is carried from the fan to the end of each lift without passing through working places, the opening of doors as cars are passing, etc. does not interfere with the current.

If a locomotive is used, the smoke and gases generated by it are carried away from the men toward the bottom. Locomotives are generally used only from the main turnout to the bottom.

An objection to this system is that the gangway, as the return, is apt to be smoky. Starters and loaders are forced to work in more or less smoke, and even the mules work to disadvantage, while if gas is given off, it is passed out over the lights of those working in the gangway.

However, in places where there is but little gas, and airways of large area can be driven, this plan works very satisfactorily, and some of the best ventilated collieries are worked upon it.

An objection advanced by some against forcing fans is that they increase the pressure, thus damming the gas back in the strata. In case the speed of the fan is slacked off, the accumulated gas may respond to the lessened pressure and spring out in large volumes from its pent-up state. This argument, however, works both ways. An exhaust fan running at a given speed is taking off pressure, and if anything occurs to block the intake, the pressure is diminished, and the gas responds to the decrease on the same principle.

**Working Small Seams Laying From Horizontal to 10°.**—Two gangways may be driven, the lower or main gangway being the intake. Branch gangways should then be driven diagonally or at a slant, with a panel or group of working places on each slant gangway. Large headings should connect the panels. In this system, the air is carried directly to the face of the gangway and up into the breasts, returning back through the working places. The intake and return are separated by a solid pillar, the only openings being the slant gangways on which are the panels. The advantages of this plan are:

The main gangway is solid, with the exception of the small cross-holes connecting with the gangway above; these furnish air to the gangway and are small and easily kept tight. These stoppings should be built of brick, and made strong enough to withstand concussion.

A full trip of wagons can be loaded and coupled in each panel or section without interfering with, or detaining the traffic on, the main road; one trip can be loaded while another is run out to the main gangway for transportation to the bottom.

The only break in the intake current is when a trip is taken out from, or returns to, a panel; this can be partly provided against by double doors, set far enough apart to permit one to close after the trip before the other is opened. This distance can be secured by opening the first three breasts on a back switch above the road through the gangway pillar, or by running each branch over the other far enough to obtain the distance for the double doors.

If it is not desired to carry the whole volume of air to the end of the airway, a split can be made at each branch road. These will act as unequal splits in reducing friction, and, although not theoretically correct, are preferable to dragging the whole current the full length of the workings.

The objections urged to this plan are that it involves too much expense in the large amount of narrow work at high prices necessary to open out a colliery, that it necessitates a double track the whole length of the lift, and that the grade ascends into each panel or section. But the latter criticism falls, because the loss of power hauling the empty wagons up a slight grade is more than made up by the loaded wagons running down while the mules are away pulling a trip into another panel or section.

For a large colliery this is without doubt the best and cheapest system.

**Working Small Seams Laying at More Than 10°.**—In small seams lying at an angle of more than 10°, and too small to permit an airway over the chutes, it is more difficult to maintain ventilation. If air holes are put through every few breasts, and a fresh start obtained by closing the back holes, or if an opening can be gotten through to the last lift as often as the current becomes weak, an adequate amount of air can be maintained, because the lift worked can be used as the intake, and the abandoned lift above as the return. To ventilate fresh ground, the filling of the chutes with coal will have to be depended on, or a brattice must be carried along the gangway. This can be done for a limited distance only, as a brattice leaks too much air. As a rule, collieries worked on this plan are run along until the smoke accumulates and the ventilation becomes poor; then a new hole is run through and the brattice removed and used as before for the next section. This operation is repeated



(a)



(b)
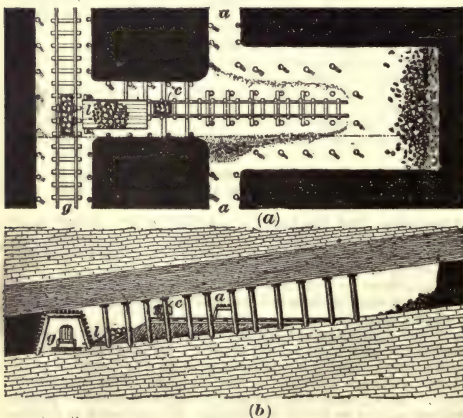
FIG. 13

until the lift is worked out. Sometimes, to make the chutes tight, canvas covers are put on the draw holes, but as they are usually left to the loaders to adjust, they are often very imperfectly applied. Then, as the coal is frequently very large, the air will leak through the batteries.

This plan works very satisfactorily if the openings are made at short intervals, say, as frequent as every fifth breast, but the distance is usually much greater to save expense. As the power of the current decreases as the distance between the air holes is increased, good ventilation is entirely a question of how often a cut-off is obtained.

An effective ventilation could be maintained in a small seam at a heavy angle by working with short lifts, say two lifts of 50 yd. instead of one of 100 yd., as at present. The gangways should be frequently connected, and one used as an intake and the other as a return. This would necessitate driving two gangways where one is now made to do, but the additional expense would be made up in the greater proportion of coal won.

**Buggy Breasts.**—For inclinations between 10° and 18°, that is, after mule haulage becomes impossible and until the coal will slide in chutes, buggies are often used. Fig. 13 shows a buggy breast in plan (a) and section (b). Coal is loaded into a small car or buggy c, which runs to the lower end of the breast and there delivers the coal upon a platform l, from which it is loaded into the mine car. The refuse from the seam is used in building up the track, and if there is not sufficient refuse for this, a timber trestle is used.

Another form of buggy breast is shown in Fig. 14. Here the coal is dumped directly into the mine car from the buggy. If the breast pitches less than 6°, the buggy can be pushed to the face by hand, but in rooms of a greater pitch, a windlass is permanently fastened to timbers at the bottom of the breast, while the pulleys at the face are temporarily attached to the props by chains, so that they can be advanced as the face advances. The rope used is from ½ in. to ⅝ in. in diameter, and any form of ordinary horizontal windlass can be used. With the windlass properly geared, one man can easily haul a buggy to the face of a breast in a few minutes' time. The buggy runs upon 20-lb. **T** rails spiked with 2½″×⅜″ spikes upon 2″×4″ hemlock studding sawed into lengths of 14 ft.

**Chutes.**—A chute is a narrow inclined passage down which the coal slides by gravity, or is pushed. When the pitch of the chute is between 15° and 30°, sheet iron is laid in it to furnish a good sliding surface for the coal. When the inclination is less than 20°, it is generally necessary to push the coal down the chute, as it does not then slide readily even on sheet iron. When the inclination of the chute exceeds 30°, coal will slide readily on a rock bottom without
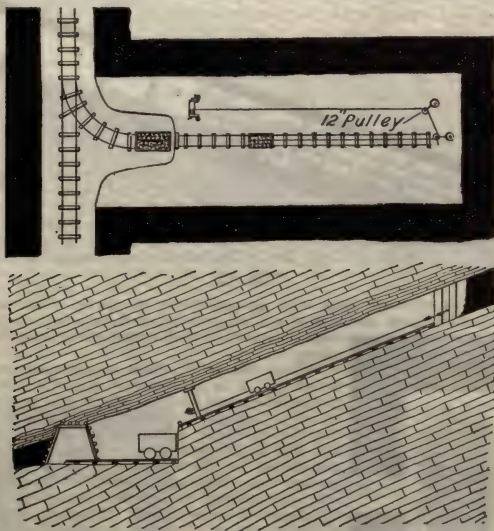


Fig. 14

the use of sheet iron. The use of chutes is therefore limited to seams whose inclination is greater than 15°, that is, to what are generally called *steep seams*.

When the inclination of the seam is less than 30° to 35°, a single chute is usually placed in the center of the room. The chute ends in a platform projecting into the entry, and from this platform the coal can be readily loaded into the mine car. The refuse made in mining is thrown on either side of the chute; and, if the pillars are to be robbed, this refuse should be kept as near to the center of the room as possible. Two chutes are sometimes employed, one along each rib, but as the cost of the second chute is considerable, it is not generally used unless it is required for purposes of ventilation.

Fig. 15 shows an inclined room with a sheet-iron chute a in the center. As the coal is mined at the face b, it is shoveled into the chute and slides by gravity to the platform c, from which it is shoveled into the mine cars on the track d. Rows of props e are frequently stood alongside the chute to keep up

the roof above it, and the gob is stored between the posts and the rib in the spaces $f$. The chute also acts as a manway, and by means of the props the
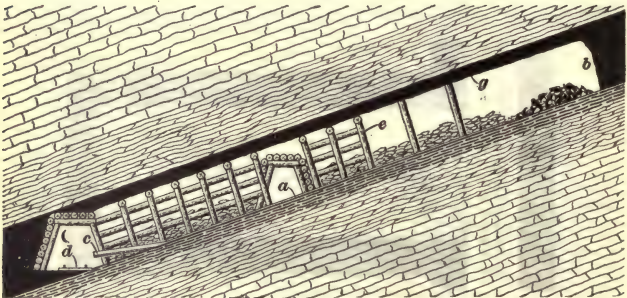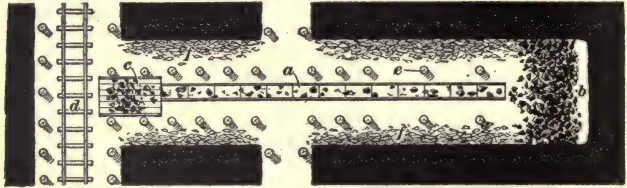


FIG. 15

miners are able to work their way up and down the room. The top coal $g$ is left up to help keep up the roof and may be taken down after the room has been driven to its full length or it may be left in place.
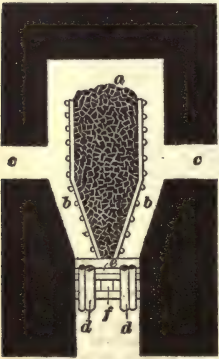
When the bed inclines at a greater angle than about 35°, it is necessary to provide a staging or platform of some kind on which the workmen can stand in mining the coal. A staging of timbers may be built and advanced as the face advances, but this is an expensive method, and it is generally better to allow the room to fill up with the broken coal, keeping the level of this broken coal just near enough to the face to provide a standing place for the workmen. The coal is supported at the bottom of the room by a bulkhead of heavy timber known as a *battery*, and the method of working is known as *working on battery*. Only enough coal is taken out through a chute at the bottom of the room to take off the excess of coal that cannot be thus stored in the room owing to the fact that the broken coal occupies about 75% more space than the same coal in the solid.

**Single-Chute Rooms.**—Fig. 16 illustrates the general form of a single-chute room. The coal $a$ is stored in the center and a manway $b$ is constructed up each side with props and planking for the purpose of ventilation and to afford access to the face for the workmen. Cross-cuts $c$ are made by driving through the pillar separating adjacent rooms. At the point where the room neck widens to full room width, a battery is constructed to hold back the coal in the chute. This is composed of



FIG. 16

strong posts $d$ set in the roof and floor of the seam as a support for the cross-timbers $e$; a small opening $f$, known as the *loading chute*, is left at the center of the battery and through this the coal is drawn as required. The manways $b$ are connected with the room neck by a small opening in the battery,

through which workmen pass in going to and from work. This opening is covered by a curtain so is to maintain the air-current along the face.

When the coal is drawn out through a central loading chute, the movement takes place principally in the coal lying near the center of the breast. If the roof is poor, the movement of the coal will not in this



FIG. 17    FIG. 18

way cause it to fall and mix with the coal; and if the floor is soft the props protecting the chute, and which are stepped into the floor, are not so liable to be unseated, closing the manway and blocking the ventilation. The surplus coal is sometimes thrown down the manways, instead of being drawn out at the bottom of the breast through a loading chute, leaving the loose coal in the center of the breast undisturbed until the limit is reached.

**Double Chute Rooms.**—Fig. 17 shows the arrangement of an inclined room in which the weight of the loose coal is supported mainly by a pillar of coal left along the entry. The coal is drawn out of the room by two loading chutes, one at each side of this pillar, and the workmen gain access to the manways along the room ribs through short manways driven through the entry pillars. Fig. 18 shows a similar arrangement to that shown in Fig. 17 except that the sides of the loading chutes are in line with the sides of the chute in the room, the manways are straight, and the loading chute and manway are in the same opening in the coal. This method has an advantage over that shown in Fig. 17, as it requires less cutting of the entry pillars.

An advantage of supporting the coal by a pillar at the bottom of the room, as shown in Fig. 24, is that there is less likelihood of a break occurring in the batteries, which would throw all of the coal on the gangway or airway, and thus close off these passages and interfere both with haulage and ventilation.

If the coal seam is mixed with rock that can be readily separated from the coal underground, this separation may be made on the platform $f$, Fig. 19 (a), the rock being left in the center of the room instead of the coal, as was illus-
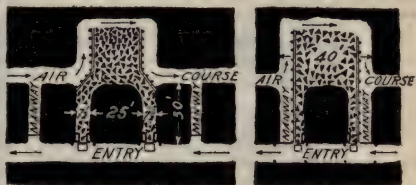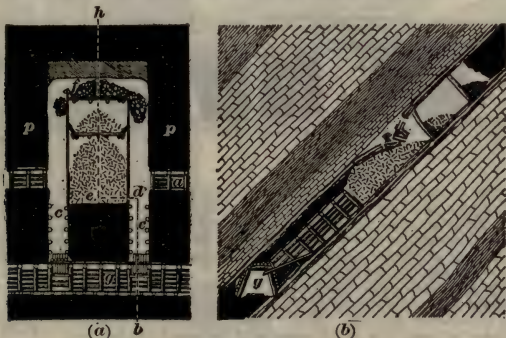


FIG. 19

trated in Figs. 16, 17, and 18. The coal is then thrown down the chutes $c$ and loaded into cars on the entry $g$. Fig. 19 (b) is a section through the room shown in (a) on the line $bdeh$. This method is also used where the coal is very gaseous and where it is not well, therefore, to keep broken coal stored in the

40

room. The air-current passes through the airway $a$ and up the chute $c$ to the face. This method is not adapted to very thick seams, as it is impracticable to build the necessary platforms in such seams.

The accompanying table gives approximate inclinations of the seam when the several methods just described are employed. These inclinations may be varied by local conditions.

## METHOD SUITABLE FOR USE IN INCLINED SEAMS

| Method | Inclination of Seam Degrees |
|---|---|
| Cars lowered by hand, or drawn by mule or motor, rooms on full pitch | 0–6 |
| Cars lowered by hand, or drawn by mule or motor, rooms at angle with pitch | 5–12 |
| Cars lowered by windlass, rooms on full pitch | 5–10 |
| Self-acting incline or jig road | 5–30 |
| Buggy system, thick seams | 10–18 |
| Sheet-iron chutes | 15–30 |
| Natural chutes with battery | 30 upwards |

The manways in steeply inclined rooms are constructed in two general ways. In a seam that is not over 6 to 8 ft. thick, the method shown in Fig. 20 may be used. The posts $a$ are set as shown and lined with plank; this partition forms the sides of the chute $b$ and leaves a manway $c$ between the chute and the rib.

In thick seams, inclined posts called jugulars $a$, Fig. 21, are used. These are set in hitches cut in the floor and the rib, so as to form a triangular passageway or manway $b$. The jugulars are lined with plank to form the sides of the center chute $c$, which is filled with loose coal or refuse.

As a general rule, in inclined seams as in flat seams, the rooms are driven up to within a short distance of the entry above, a chain pillar being left between the ends of the rooms and this entry; the width of this pillar varies with the character of the roof, floor, and coal, depth of cover, and inclination of the seam. To secure better ventilation, an occasional room is often holed through this pillar into the entry above; and where the coal has been worked
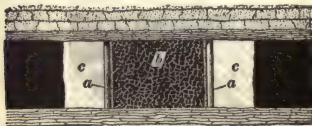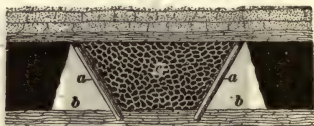


FIG. 20                    FIG. 21

out from the chambers above, and there is no water to interfere with the lower workings, many or all of the rooms are thus driven through to the upper entry. The chain pillar is sometimes drawn back with the other pillars. The distance between the successive lifts or entries varies with the conditions, but is usually about the same or slightly less than the distance between entries in flat workings under the same conditions.

**Battery Breasts.**—The methods of working by single and double chutes and batteries are adaptations of similar methods originally applied in connection with anthracite mining. Many modifications of these simple methods are used in order to meet the requirements of different pitches and different thicknesses of coal. The following are the most important of these modifications:

Fig. 22 shows an elevation ($a$) and cross-section ($b$) of a breast in a thick seam pitching about 60°. The seam is 16 ft. thick with several thin slate partings; the roof and floor are good, and the coal hard and firm. The gangway $g$ is driven on the strike of the seam, near the bottom of the coal and with sufficient

grade to insure drainage. A small airway $h$ is driven just below the top bench of coal, and is connected with the gangway by occasional openings not shown. This airway is often called a *monkey gangway*, or simply a *monkey*.
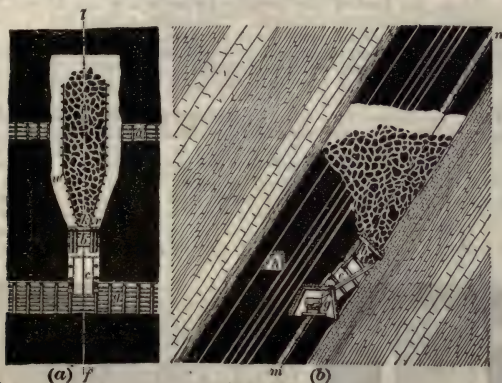


FIG. 22

A narrow opening, called a chute, is opened off the gangway and driven up on the floor of the seam a distance of about 5 yd., and at this point it is widened out gradually on both ribs, until the full width of the breast, 5 to 8 yd., is reached. A timbered chute $c$ conveys the coal into the car on the gangway $g$. A battery of timber $b$ is constructed at the point where the breast is widened out by setting double posts on each side of the center and close to the ribs;
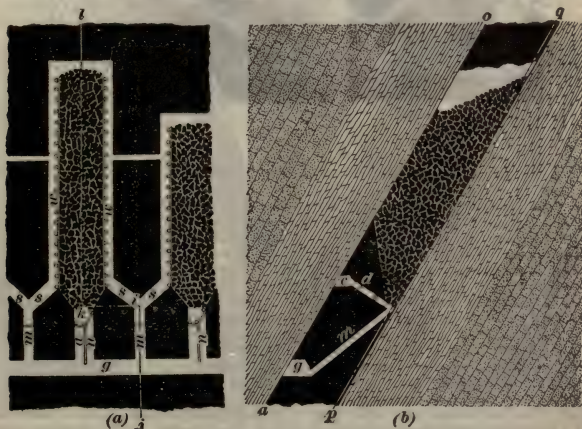


FIG. 23

these posts are firmly set in holes cut in the floor and the roof, and cross-timbers are then placed behind them, leaving only a small opening for the coal to pass through. A manway $w$ is constructed up each side of the chute, by

placing about 30 in. from each rib a line of posts which are firmly set in holes cut in the roof and floor and lined with plank to form the sides of the chute. An opening $r$ in the battery connects each manway with the chute $c$ and also affords access to the face of the breast. Cross-cuts $a$ are driven between adjoining breasts at points up the pitch. The breast is kept full of loose coal, on which the miner stands as he works at the face.

Fig. 23 represents in elevation ($a$) and cross-section ($b$) breasts driven to the full pitch of a thick seam whose inclination is about 60°. The gangway $g$ is driven on the strike and in the top of the seam while the airway $c$ is above the gangway and in the top coal.

The breasts are opened by a narrow opening 9 ft.×6 ft. driven up the pitch for a distance of 18 to 24 ft., this neck being gradualy widened out to the proper breast width, as shown. The section is taken on the lines $lk$ and $ij$, and the elevation is made on the line $pq$, and does not show, therefore, the headings $c$ and $d$ shown in ($b$). In the middle of the pillar between the loading chutes, a small manway $m$ is driven up a few yards, and then branches $s$ are turned off in both directions until intersection is made with the breasts on each side. From these points the manways $w$ are carried up on each side of the breast along the rib as shown. A narrow manway $n$ is usually made by planking off a por-
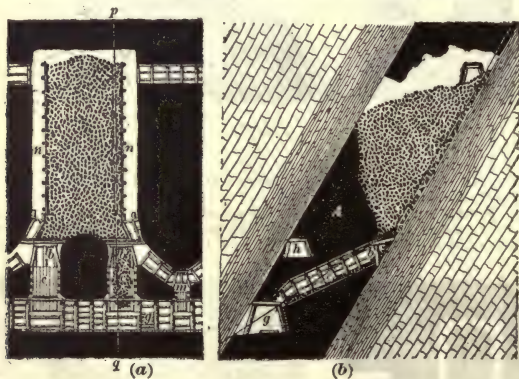


FIG. 24

tion of the opening so that the loaders may have free access to the battery at all times.

A small airway $d$ is driven from airway $c$ to the manway $m$, but cross-cuts between the airway and gangway are also necessary where much gas is given off during the working. The air-current passes along the gangway $g$ and returns along the faces of the breasts. The small airways $d$ and $c$ are not used when the breast is working, but if any accident in a breast manway $w$ blocks the ventilation, the air can be conveyed around the breast, through the airways $d$ and $c$ by simply removing stoppings. This plan is especially adapted to working thick steeply inclined seams of soft gaseous coal.

When the pitch of the bed is less than about 50°, the gangway $g$ is usually in the bottom coal, but for a greater inclination it is in the top coal, so that the flow of the coal may be more easily controlled.

Fig. 24 shows a method in which a loading chute $c$ is arranged on each side of the breast and a supporting pillar of solid coal is left in the mouth of each breast; the batteries $b$ are built near the upper side of this pillar. The gangway $g$ and the airway $h$ above it are driven in the top coal. The loading chutes $c$ are driven up from the gangway and across the full width of the seam at such an angle that the coal can be easily controlled in the chute. When the chutes $c$ reach the floor of the seam, they are widened out to breast width and at the same time the coal face is opened up to the top of the seam in a more or less vertical line, as shown in the cross-section ($b$). The manways $m$ are driven

through the gangway pillar between the breasts and are divided into two parts, as was described in connection with Fig. 23, a manway $n$ extending up each side of the breast. The advantage of the method illustrated in Figs. 23 and 24 over that shown in Fig. 22 is that the manways and the coal chutes are kept apart and there is therefore usually free access to the face at all times, even should there be a break in the coal chute.

By driving the gangway near the roof, as shown in Fig. 24, where the pitch is heavy the loading chute $c$ is more easily controlled, and the gangway is also less likely to be injured by a squeeze. The disadvantage of the method is the extra expense incurred in driving long chutes.

Fig. 25 is a sectional view of a thick seam of coal standing vertically and mined by room and pillar. The gangway or level $g$ is connected with the airway $h$ by the passages $c$, $d$, and $e$. The battery $b$ is at the inner end of the chute $c$ and near the foot of the vertical manway $m$, in which a ladder is placed. The passages $d$ and $e$ are for the purpose of ventilation; they also serve as manways to connect the gangway $g$ with the foot of the vertical manway $m$.

Fig. 26 is a section through what is called a *back breast* $p$ in thick anthracite seams. The regular breast $b$ having been mined out, the coal over the main gangway $g$ and monkey gangway $k$ is worked by opening a breast $p$ off the monkey gangway or



FIG. 25

off another gangway driven especially for the purpose of getting out this coal, and driven so that the coal may slide through chutes to the cars. Such a mode of working may enable a large proportion of the gangway stumps to be removed, which would be entirely lost otherwise.

**Working Contiguous Seams.**—Coal seams that are approximately parallel and are close together are said to be *contiguous*. The following points must be carefully considered in the working of contiguous seams: Thickness and character of the rock separating them; thickness of the seams; nature of the roof, floor, and coal of each seam; inclination of the strata; and general depth below the surface. The thickness of strata separating contiguous seams varies from a fraction of an inch to several feet. When this thickness does not exceed 2 or



FIG. 26

3 ft., the separating rock is called a *parting* and all the coal and rock are then usually taken out at the same time as one face of coal, or the face in the lower seam is kept a few yards in advance of that above. The waste material forming the parting is not removed in either case, but simply left where it falls, except on the roads, as the handling of so large an amount of waste would render the economical working of the coal impossible. At other times, the openings are first driven in the lower seam, and when these reach the limit of the workings the tracks in the rooms are taken up and the rock or slate parting is allowed to fall or is blasted down. The upper coal is then taken down.

When the thickness of the intervening rocks is greater, contiguous seams may be worked either by what are called *rock chutes* or by *cross-tunnels*. Fig. 27 shows a section of two seams, separated by a few yards of rock. Chutes from $4\frac{1}{2}$ to 7 ft. high and 7 to 12 ft. wide are driven in the rock from the gangway or level $g$ to the level $l$ in the seam

above, at such an angle that the coal will gravitate from the upper seam into the gangway g. The working, otherwise, is similar to that previously described.

Rock-chute mining contemplates the following sequence of operation:



FIG. 27

1. The opening of all gangways and airways in the lower seam, to develop coal as yet untouched in a thick seam lying a few feet above it.

2. Developing the thick bed by a regular series of rock chutes driven from the gangway below; workings being opened out from the chutes—as in ordinary pillar-and-breast working—the panel system or some other plan may be found better than pillar-and-breast workings.

3. Driving the breast to the limit of the lift and robbing out the pillars from a group of breasts as soon as possible, even if a localized crush is induced.

4. After one group of breasts is taken out and the roof has settled, opening a second series of chutes for the recovery of coal from any large pillars that were not taken out when the crush closed the workings.

5. While the work of recovering the pillar coal is in progress, a second group of breasts may be worked, and the process continued until all the area to be worked from that gangway has been exhausted. The same process is employed in opening lower lifts.

6. When all the upper bed of coal has been exhausted, the lower seam may be worked by the ordinary method. Workings in this seam may be carried on simultaneously with the upper bed, but to avoid the possibility of a squeeze destroying these workings, very large pillars must be left. After exhausting the upper seam, these pillars may be advantageously worked by opening one or two breasts in the center of each, and when these are worked to the upper limit, attacking the thin rib on each side, commencing at the top and drawing back.

When the roof of the lower bed is good, the cost of timbering and keeping open the gangways and airways will be considerably less than if these were driven in the upper seam, and this difference, in some cases, may be sufficient to pay for driving all the rock chutes.

There are three undetermined points in this connection, viz.: (1) The maximum distance between the two beds, or the length of rock chute that can be driven with satisfactory financial results. (2) The maximum dip on which such workings can be successfully opened. (3) The maximum thickness of the upper and also of the lower seam, which will yield results warranting the additional outlay when rock chutes are of considerable length.

Fig. 28 shows how one or more seams are worked by connecting them by a *stone drift*, or tunnel, driven horizontally across the measures, through which the coal from the adjacent seams is taken to the haulageway leading to the landing at the foot of the slope or shaft. Tunnels are sometimes driven horizontally through the measures from the surface, so as to cut one or more seams above water level.



FIG. 28

The lower seam of coal is worked from a gangway or level l, connected by a tunnel, or stone drift t, to the level or gangway g, in the thick seam. The stone drift may be extended right and left to open seams above and below the

thick seam. This tunnel, or stone drift, is never driven under a breast in the upper seam, but directly under the middle of the pillar.

In the upper and thicker seam, when the coal is very hard, a breast $b$ is worked to the limit and the loose coal nearly all run out through the chute $s$ into the gangway $g$. The monkey gangway $m$ is driven near the top as a return airway, and is connected to the upper end of the chute $s$ by a level heading $n$, and to the main gangway $g$ by a heading $v$. These headings are driven for the purpose of ventilation and to provide access to the battery in case the chute $s$ should be closed. In the lower seam, the breast is still being worked upwards in the ordinary way.

**New Castle, Colorado, Method.**—The following method is used at New Castle, Colorado, for highly inclined bituminous seams. The coals mined are only fairly hard, contain considerable gas, and make much waste in mining. Fig. 29 shows the method used for extracting the thicker vein to its full width of 45 ft., and the E seam 18 ft. thick, excepting that left for pillars. Rooms and pillars are laid out under each other in the two seams whenever practicable. Entries are along the foot-wall; 30 ft. up the pitch is an air-course. Rooms and breasts are laid out as shown in Figs. 17 and 18. In the thick vein, the manways go through the entry pillars to the air-course and thence along the
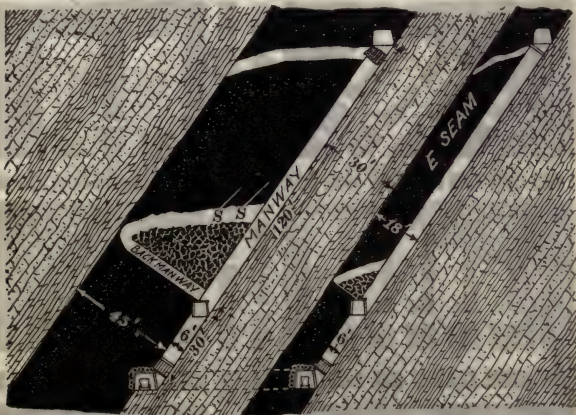


FIG. 29

ribs each side of the room, one manway to tne main entry serving for two double rooms. A lower bench of 6 ft. is first mined the full length of the rooms, 120 ft., side manways being protected by vertical or leaning props, bordered with 3-in. planks outside, and the chute or battery is then put in. At the top the rooms are connected by cross-cuts, and, occasionally, intermediate cross-cuts are required. The room is kept full of loose coal, only sufficient being drawn to keep the working floor at the proper height for the mining. When driven to the limit and with cross-cuts connected, the coal is all drawn out at the chutes, which have receptacles for rock and waste at their sides, to be picked out by the loaders. The next operation is to drive across the seam at the air-course until the hanging wall is reached, manways, called *back manways*, being maintained as before. A triangular section of coal is mined off, as shown in Fig. 17, and the room filled with loose coal. The full thickness of the seam is now taken off, shots being first placed at $SS$, coal being drawn out at the bottom as required. In robbing a pillar, the manways are moved back into the pillar each side 10 ft. or so, by mining on the lower bench as before, and holes are drilled into the roof with long drills, which bring down as much of the overhanging part as can be reached.

**Alabama Methods.**—Fig. 30 shows the common methods used in working the Alabama coals. The seams now working vary from 2 to 6 ft. thick, and

they pitch from 2° to 40°. Where the seams are thin, the coal is hard, and pillars of about 20 to 30 ft. are used to support the roof. The thick seams are soft and easily broken, and much larger pillars are left. The character of bottom and top varies; fireclay bottom and slate roof are usually found with the thick seams, and hard bottom and sandstone roof with the thin seams. The general plan of laying out the mine is to drive the slope straight with the pitch of the seam; this is usually on the butts of the coal. A single-track slope is 8 ft. wide, and a double-track slope 16 ft. Cross-headings are driven or turned from the slope water level every 300 ft.; air-courses are driven parallel on either side of the slope. Where an 8 ft. slope is driven, 30 ft. of pillar is left between the slope and airway, and for a 16 ft. slope, 40 ft. of pillar. The size of pillar, however, depends largely on the character of the roof and thickness and strength of coal. On the lower side of the heading, pillars from 20 to 60 ft. are left on the entry before turning the first room. The rooms are worked across the pitch on an angle of about 5° on the rail, as shown in A, when the coal does not pitch greater than 20°; where the pitch is greater, chutes are
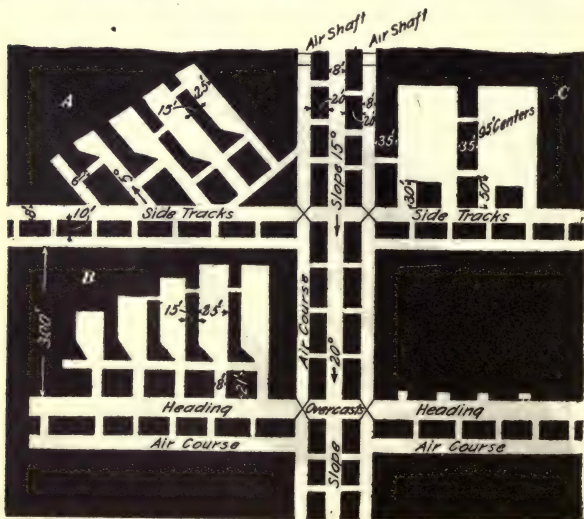


FIG. 30

worked and the rooms are driven straight up the pitch B. In a few cases where the pitch is not greater than 15°, double rooms are worked with two roadways in each room, as shown in C. A rope with two pulleys is used, and each track keeps the rib side of the room, the loaded car pulling up the empty on the opposite side of the room; distance between room centers, about 42 ft. Where single rooms are worked, the room is driven narrow (8 ft. wide) for 21 ft., when connections are made with the room outside of it; the room is then widened out to about 25 ft., sloping gradually until this width is attained; pillars of from 10 to 20 ft. thick are left between the rooms, and cross-cuts for ventilation are made about every 50 ft.; every third or fourth room is driven through to the entry above; pillars are then drawn back to the entry stumps or pillars. The average cover over the coal now working is from 100 to 600 ft. Air-courses usually have an area of 30 ft., and sufficient coal is taken out to give this area, the roof and bottom being left.

**Tesla, California, Method.**—The Tesla, California, method is shown in Fig. 31. The coal seam averages 7 ft. of clear coal, and pitches 60°. This

system was adopted in a portion of the mine to get coal rapidly; for, at this point, a short-grained, slate cap rock came in over the coal, making it difficult to keep props in place. The floor is a close blue slate and has a decided heaving tendency. The roof is an excellent sandstone. There is a small but trouble-some amount of gas. Two double chutes are driven up the pitch at a distance of 36 ft. apart, connected every 40 ft. by cross-cuts. One side of each chute is used for a coal chute and the other for a manway and air-course. At a distance of 12 yd. apart small gangways are driven parallel with the main mine gangways. These are continued from each chute a distance of 300 ft., if the conditions warrant it. The top line is then attacked from the back end and the coal is worked on the cleavage planes; the breast, or room, therefore consists of a 12-yd. face, including the drift or gangway through which the coal is carried to the chutes; a rib of coal (2 or 3 ft.) is left between the breasts to keep the rock from falling on the breast below. Thus in each breast the miners have a working face of about 15 or 16 yd., and as the coal is directed to the car by a light chute, moved along as the face advances, the coal is delivered into the cars at small cost, and but little loss results from the falling coal, as a minimum of handling is thus obtained. Immediately above each gangway, and starting from these main chutes, an angle chute is driven at about 45°, connecting with a part as the gangway chutes (30 ft), at an angle of 35°, and cross-cuts are driven the breast gangway above it, and into these chutes the coal from that breast
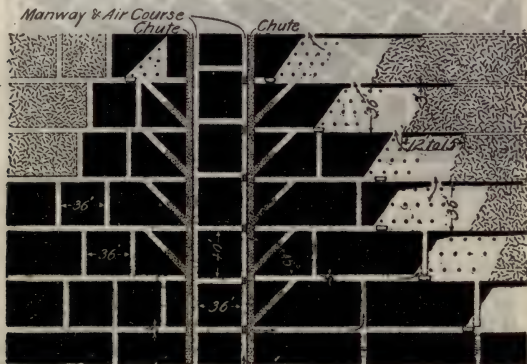


FIG. 31

is delivered, runs into the main chute, and from it is loaded into the mine cars in the main gangway. These angle chutes serve as a means of keeping the main chute full, and at the same time give each breast an opportunity to send out coal continuously. They also serve the purpose primarily intended, of saving the coal from breakage, by giving it a more gradual descent into the full chute. The breast gangways are driven 5 ft. wide. No timbers are needed in these gangways, as they are driven in the coal, except on the foot-wall or floor side, which, as before stated, is a firm sandstone. It is found safest to leave a rib of coal on the top of the breast 2 or 3 ft. thick, until the working face has passed on 12 or 15 ft., when this rib is cut out and thus all the coal extracted, the roof caving behind and filling in the opening. As cross-cuts are driven every 36 ft., ventilation is kept along the working faces, and a safe and effectual means of securing all the coal in the seam is thus attained.

Fig. 32 shows another system used in No. 7 vein at the same place. The seam averages 7 ft. of coal. The roof is shelly and breaks quickly, hence the coal must be mined rapidly.

In this system the gangway chutes are driven at right angles with the strike of the seam, 40 ft. up the pitch; a cross-cut 5 ft. × 6 ft. is then driven parallel with the gangway. From this cross-cut, chutes are driven at same distance every 40 ft. between chutes, for ventilation. After a panel of five or more chutes is driven up the required distance, work is commenced on the upper

outside pillar and the pillars on that line are drawn and the next line is attacked, and this is continued until the panel or block is worked down to the cross-cut over the gangway. About every 80 ft. in this level it is found advantageous
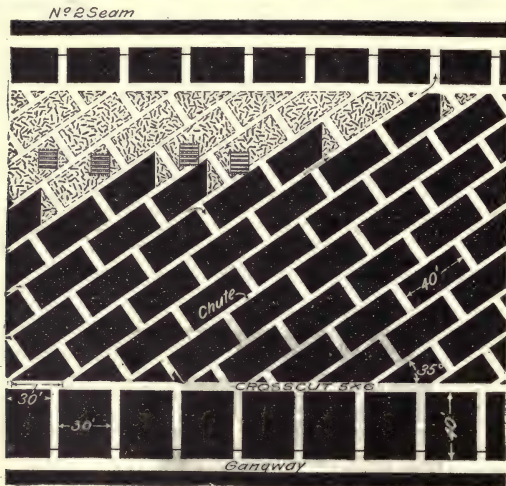


FIG. 32

to build a row of cogs parallel with the strike of the seam as the pillars are drawn. This serves to save the crushing of the pillars, and prevents any accidents from falls of rock. But few timbers are required by this system.

## PILLAR-AND-STALL SYSTEMS OF MINING

The *pillar-and-stall system*, also known as *post-and-stall*, *board-and-pillar*, or *stoop-and-room*, is a modification of the general room-and-pillar method in
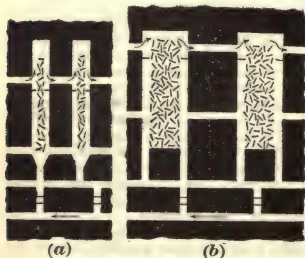


FIG. 1

which the openings, usually called *stalls* or *rooms* in America, are narrow, rarely exceeding 4 or 5 yd. in width. The pillars are at least as wide and usually wider than the stalls. In the *single-stall system*, the stalls are turned narrow off the entry as shown in Fig. 1 (a), and widened inside as described in room-and-pillar work. In the *double-stall system*, shown in (b), the openings are wider, and are similar in every respect to double rooms, except that the pillars separating the double stalls are generally wider in proportion to the width of the stalls than are the pillars separating rooms. In double-stall work, the openings are often 12 or 15 yd. in width, the roof being supported on good pack walls in the center of the stall; the pillars often reach a width of 30 yd. The pillar-and-stall system is adapted to weak roof and floor, to strong roof and soft bottom, to soft, brittle coal, and in general to conditions requiring ample support of the roof; the system is particularly useful in deep seams where the roof pressure is great.

**Connellsville Region.**—Fig. 2 shows the common method used in the Connellsville, Pennsylvania, region. The average dip is about 5%. The face and butt headings are driven, respectively, at right angles to each other on the face and the butt of the coal. The face headings leave the main butts about 1,000 ft. apart, while from these face headings, and 400 ft. apart, secondary butts are driven, and again from these butts on the face of the coal the rooms or wide workings are excavated to a length of 300 ft., this having proved the most convenient length for economical working. Room pillars have a thickness of 30 to 40 ft., while the rooms are 12 ft. in width and are spaced 42 to 52 ft. between centers, depending on depth of strata over the coal. The headings are 8 ft. wide, and in all main butts and faces the distance between centers of parallel headings is 60 ft., leaving a solid rib of 52 ft. A solid rib of 60 ft. is also left on the side of each main heading. The average thickness
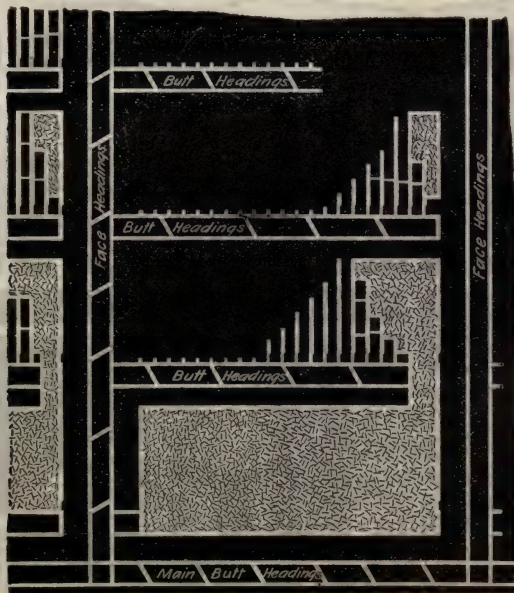


FIG. 2

of cover at the Leith mine, which is here described and which may be considered as a type of the region, is 250 ft., the overlying measures being alternate layers of soft shale and coal for 4 ft. The bottom is an 18-in. layer of hard fireclay and slate. These floor and roof materials are soft, and are easily disintegrated by air and water. At some mines, cover will reach as much as 700 ft., and the dip of 5% (as at Leith) is much heavier at some points on eastern outcrop, and will run as high as 12%, flattening off as the synclinal line of the basin is reached, until it is almost level. In some localities, the material below coal is hard limestone, requiring blasting to remove it, and at other places the roof slates are much more solid than at Leith, and not readily disintegrated. The method of drawing ribs is one of the advantages of the system, since it is harder to do successfully in a soft coal like the Connellsville than in hard coal. The coal itself is firm. When necessary to protect the top or bottom, 4 to 6 in. of coal is left covering the soft material.

The method just given is often applied to a whole series of butts (4 or 5) at once instead of to butt by butt, as shown in Fig. 2. In this case, work is started at the upper end of the uppermost butt and progresses, as shown, but, after cutting across the butt heading from which the rooms are driven, the butt heading itself and the upper rooms from the second butt, or that just before, are likewise drawn back by continuous slices being removed from the rooms of the upper butt, and on across the next lower butt, etc., all on an angle to the butts, and so continued as the operations progress, until another butt is reached, etc., thus gradually making a longer and longer line of fracture, which is only limited by the number of butts it is desired to include at one time in the section thus mined. This works very nicely and makes long
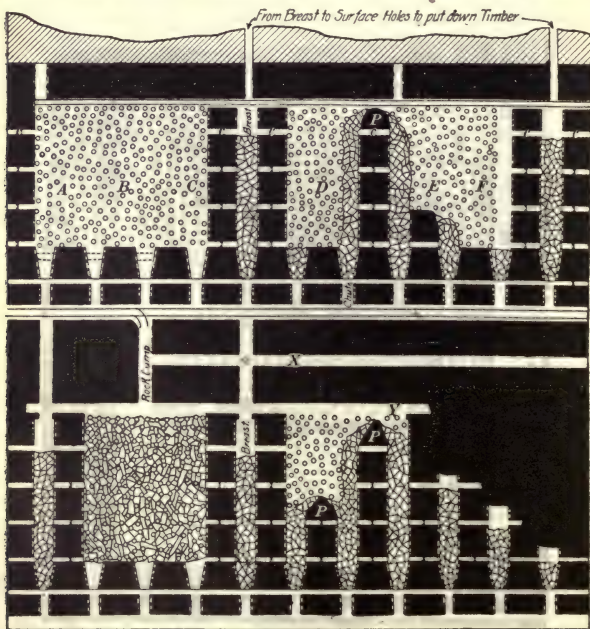


FIG. 3

even lines of fracture, the steps of the face of the workings (in the rib drawing) being about 30 ft. ahead of one another.

J. L. Williams Method.—The J. L. Williams method of working anthracite, Fig. 3, was applied successfully by the originator at the Richards mine, Mt. Carmel, Pennsylvania, and by it 90% of the available coal is said to have been obtained. The method is a pillar-and-stall method with the following distinguishing points: (1) Timbering the gob with props set not more than 6 ft. apart, to keep up the roof during the extraction of the pillars. (2) Making holes from the crop, for the delivery of timber into the workings. (3) Removing the pillars in shorter lifts than is possible when the roof is supported with culm pillars. (4) Keeping the gob open with timber for dumping the fallen rock, that would have to be sent to the surface if the breasts were flushed.

Both the floor and the roof of the mine were weak, so that it was not possible to make either the breasts or the pillars wide. In some cases, the floor consisted of 3 ft. of clod, and to prevent its lifting and sliding, every

alternate prop was put through the clod and its foot set in the slate beneath, while the other props were set on pieces of 2-in. plank 2 ft. in length to keep down the bottom. A small gangway $X$ is driven to take out the chain pillar, and $Y$ is a small gangway for taking in timber.

## PANEL SYSTEM OF MINING

In the *panel system*, the coal area is first blocked out by pairs of entries driven at right angles to one another if possible. As soon as the panel has been thus blocked out, the removal of the coal within the panel is begun by driving openings *a*, Fig. 1, from the cross-entries. These openings are connected by a cross-heading *b*, a suitable pillar being left between *b* and the cross-entry. Rooms, or stalls *c* are then opened off the heading *b* and driven almost the full length of the panel, only leaving suitable chain pillars *d* for the protection of the main and cross-headings enclosing the panel. After the rooms, or stalls, have been driven their full length, the pillars separating the stalls are drawn back, allowing the roof to fall as shown. The Connellsville method described

under the heading Pillar-and-Stall Systems of Mining, while closely resembling the Scotch and English pillar-and-stall method, may be considered a modification of the panel system.

When a panel is worked out, in order to close off the whole panel it is usually necessary only to put stoppings in the mouths of the openings *a*. A pipe is put through each stopping with a valve in the pipe on the outside of the stopping. As firedamp is often given off in the panel after it is worked out, these valves should be opened at regular intervals and the issuing air tested for firedamp with a safety lamp held a few inches from the mouth of the pipe, so that any escaping gas can mix with the fresh air. If gas is found, the valve is left open



FIG. 1

and the gas allowed to escape and should be led into the return; it is sometimes lighted at the pipe and allowed to burn off, but this is dangerous, for the flame may travel back through the pipe and explode the gas in the panel. A second pipe on which is a pressure gauge is sometimes placed in the stopping to test the pressure of gas behind the stopping, particularly when the gob is on fire and generating gases. If there is much pressure of gas behind the stoppings, the pipe through the stoppings should be left open when the men are not in the mine. In some cases, the pipe through the stopping is connected with a pipe laid along the entries and leading into the return air-current.

The term panel system is rather loosely applied in the United States to any system of mining in which the mine is divided into a series of blocks in which blocks the pillars are drawn and that section of the mine sealed off while operations are being carried on in adjacent blocks. Thus, a tract developed by a series of parallel cross- or butt-entries, say, 350 to 500 ft. apart, is often spoken of as being worked on the panel system when the respective butt entries are not connected and the room workings from one pair of entries are not driven through to the next parallel entry, the pillars being drawn as soon as the rooms and the entries reach their limit of length, or the coal between pairs of parallel entries may even be worked by the longwall method.

**Col. Brown's Method.**—Fig. 2 shows a panel system devised by Col. D. P. Brown, of Lost Creek, Pennsylvania, which gives good results in thick seams
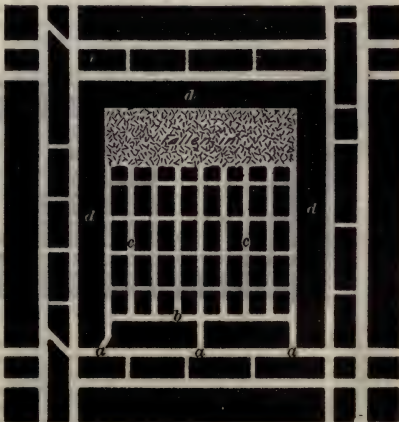
pitching from 15° to 45°, where the top is brittle, the coal free, and the mine gaseous. Rooms or breasts are turned off the gangway in pairs, at intervals of about 60 yd. The breasts are about 8 yd. wide, and the pillar between, which is about 5 yd. wide, is drawn back as soon as the breasts reach the airway near the level above. In the middle of each large pillar between the several pairs of breasts, chutes about 4 yd. wide are driven from the gangway up to the airway above. These are provided with a traveling way on one side, giving the miners free access to the workings. Small headings are driven in the bottom bench of coal, at right angles to these chutes, and about 10 or 20 yd. apart. These headings are continued on either side of the chutes until they intersect the breasts. When the chute and headings are finished, the work of getting the coal in the panel is begun by going to the end of the uppermost heading and widening it out on the rise side until the airway above is reached and a working face oblique to the heading is formed. This face is then drawn back to the chute in the middle of the panel. After the working face in the uppermost section has been drawn back some 10 or 12 yd., work in the next section below is begun, and so on down to the gangway, working the various sections in the descending order. Both sides of the pillar are worked similarly and at the same time toward the chute.

Small cars, or buggies, are used to convey the coal from the working faces along the headings to the chute, where it is run down to the gangway below
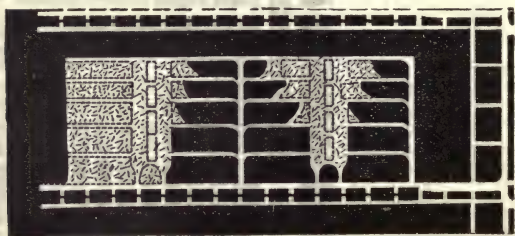


FIG. 2

and loaded into the regular mine cars. This system affords a great degree of safety to the workmen, because whenever any signs of a fall of roof or coal occur, the men can reach the heading in a very few seconds and be perfectly safe. A great deal of narrow work must be done before any great quantity of coal can be produced. The breasts are driven in pairs and at intervals, to get a fair quantity of coal while the narrow work is being done, and they are not an essential part of the system. It is claimed that the facility and cheapness with which the coal can be mined, handled, and cleaned in the mine more than counterbalance the extra expense for the narrow work.

The advantages of the panel system are: A more complete control of the ventilating current is possible, and the ventilation in any panel may be altered as circumstances may require; the powder smoke from each panel goes directly into the return air-current and does not go throughout the mine; an explosion or a fire occurring in one panel is usually confined to that panel; creeps or squeezes are of rare occurrence, and are confined to the panel in which they occur; the output of coal is better regulated and more reliable. The disadvantage of the system is the expense of entry driving, and the delayed extraction of the coal within the panel until the driving of the main and cross-headings has been completed.

# MINING AND BLASTING COAL

## SHOOTING OFF THE SOLID

Coal may be broken down at the working face by blasting from the solid; by blasting after having undercut or sheared the seam; and by a combination of the methods.

The term *solid shooting*, or *shooting off the solid*, is used to describe a method of working in which the coal is blasted from a solid face without previous

shearing or undercutting. It is practically the only method used in mining anthracite, and is also much used in bituminous mining. The chief labor in the production of coal by this method is the drilling of the holes for the powder and the loading of the coal into mine cars. The holes are drilled with a churn drill or with a rotary drill worked either by hand or by electric or compressed-air power.

The location of the holes, the depth to which they must be drilled, and their direction depend on the nature of the coal. If the coal is compact and without cleat, as is the case with anthracite, the holes are placed as they would be for a rock face worked under similar conditions. If the coal has a cleat, advantage must be taken of this to produce the maximum effect of the shot and to prevent the shot seaming out. The best method of blasting a particular coal can only be learned by experience.

Drill holes must be so placed that the explosion of the charges will increase the number of free faces (loose or open ends) exposed to the action of subsequent blasts and thus reduce the amount of powder otherwise necessary to bring down the coal.
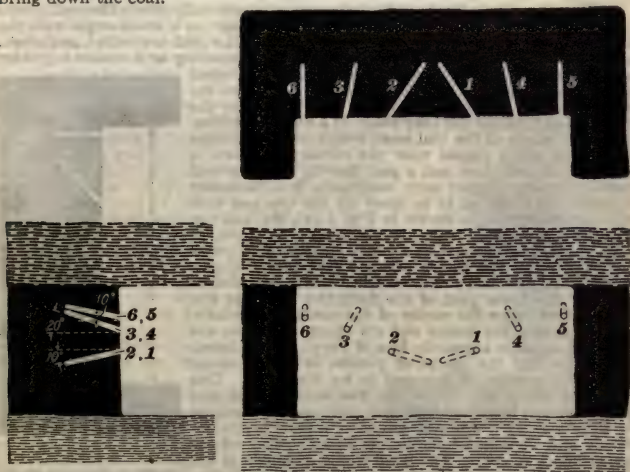


FIG. 1

Fig. 1 shows a common method of placing the shots in shooting off the solid used both for rooms and wide entries, where the coal is 5 to 7 ft. thick and is strong and close-grained and without cleats or partings which need be considered in the blasting. When firing with squib or fuse, the holes are exploded singly or in the numbered order, except *1* and *2*, which must be fired simultaneously. When firing with electric detonators, the shots are fired in pairs; first *1* and *2*; then *3* and *4*; and, lastly, *5* and *6*. The mutual reinforcing action of two charges when fired together is very noticeable in the case of holes pitching toward one another as *1* and *2*, which may be placed much farther apart and will break down much more coal when fired together than when fired singly. Shots *1* and *2* are sometimes called *breaking*, or *busting*, *shots*, as they break out the center and thus give loose ends for the shots *3* and *4*, which should take out the greater part of the coal. The shots *5* and *6* are placed about 10 in. from the ribs and are intended mainly to straighten the ribs; they are often inclined toward the rib. A cut from 4½ to 6 ft. deep across the face should be taken out by such a round of holes.

If any shot does not blow out the entire face from top to bottom, it is necessary to mine out the bottom or top coal that is left in order to square up the face in preparation for the next round of shots. Occasionally, a short hole,

or *plug shot*, is used to do this, but such a shot results in small coal, and a pick should preferably be used. The miner should aim to keep the center slightly



ahead of the sides in order to have free faces for the side shots. The only difference in the application of this method of placing the shots for a room or entry is that the shots are closer together in the case of an entry.
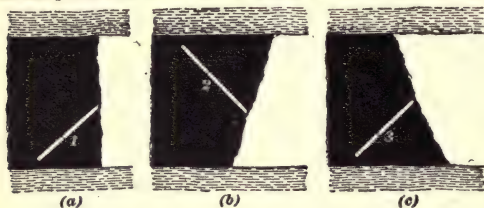
**Fig. 2**

When shooting fairly hard bituminous coals, especially where the coal breaks in wedge-shaped pieces, the holes should be inclined at a small angle with the face of the coal. Shots inclined to the face of the coal are called *grip shots*, and the shot is said to be *gripped* more or less according as it makes a greater or less angle with the face. When a shot is gripped too strongly, and the charge is located too deep on the solid, the force of the blast will be insufficient for the strength of the coal and may result in a blown-out shot.

If the center of the coal seam is soft or if it contains a parting, shots placed near the center, as shown in Fig. 1, may only tear out a gap, leaving the top and bottom intact. Under such conditions, it is necessary to place the shots so as to blast off the top and bottom alternately, as is shown in Fig. 2 (*a*), (*b*), and (*c*). The holes are placed across the face about as shown in Fig. 1, but are inclined at a much greater angle with the horizontal. In Fig. 2 (*a*), the coal face is shown vertical and the first round of drill holes is intended to take off the lower part of the face; the holes are run from a little below the center of the coal as shown at *1*, and, excepting the buster and rib shots, the holes are drilled diagonally across the face of the room and downwards, so that the charge of powder is placed across the strong portion of the coal to be displaced. This round of shots will throw out the bottom coal and leave the coal face standing with the top overhanging as shown in (*b*). The next round of holes *2* is run upwards and diagonally across the room to take off the upper bench of coal. The third round *3*, shown in (*c*), will be run downwards, and by thus alternately inclining the rounds upwards and downwards the face is advanced.



(*a*)

If the face is kept straight and center or buster shots must be used in connection with each round, an excessive use of powder is necessary and a larger amount of small coal is made than where an irregular face is carried, with the center in advance of the sides, or with either side in advance of the center. An irregular face provides a free side for the shots and allows the holes to be placed to greater advantage than where the face is kept straight. Fig. 3 shows a method of blasting off the solid that is applicable either to rooms or to wide entries under conditions similar to those given in connection with the method illustrated in Fig. 1. The coal is assumed to be from 5 to 7 ft. thick, strong, and close grained, and without partings or cleats that interfere with the blasting, and is under a strong roof. Fig. 3 (*a*) and (*b*) shows the location of the holes in the first round. These holes are placed about midway of the face vertically; they are inclined to the face about as shown in (*a*) and to the horizontal about as shown in view (*b*). Shot *1* is a buster shot, which takes
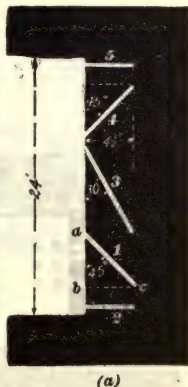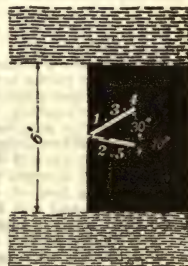


(*b*)

**Fig. 3**

out a piece *abc*; shot *2* is placed about 10 in. from the rib to straighten the rib; shot *3* takes out the greater part of the center coal, while *4* and *5* act similarly to *1* and *2*. After the straight face has thus been broken, the location of the subsequent shots is largely a matter of judgment, as so much depends on the conditions. No definite rules can be given, except that in solid coal the direction of the hole should be parallel to a free face if possible, though even this general rule will be greatly modified by cleats, partings, etc.

Fig. 4 shows approximately the appearance of the face after the shots shown in Fig. 3 have been fired. If there was a cleavage to the coal, a shot placed about as shown might blow out the piece of coal within the dotted line and thus provide two free faces with respect to which side shots could then be placed. If there was no cleavage and the coal was hard and solid, the shot would be placed nearer the previous shot *4*.

**Precautions in Solid Shooting.**—Where a single center shot, which is also known as the opening shot, such as is shown by the treble dotted line in Fig. 4, is employed to make an additional free face, its angle of grip and its length should bear such a relation to the strength of the coal that not to exceed 2 lb. of black powder (provided permissible powder is not used) will be required to bring the coal. This is commonly accomplished in coals where there are no marked cleats by limiting the angle between the straight face and the hole to 35°, in which case a 5-ft. hole, which is of the maximum allowable length, would have a line of least resistance 2 ft. 10 in.

long to work against. Where the cleats are favorable, the angle of grip may be as much as, but should never be greater than, 45°.

Gripping shots (see under E x p l o s i v e s) should not be permitted, as they are very apt to blow out, or to blow off the heel, leaving the toe in place, in which case a portion of the powder may burn in the air. On the other hand, holes should not be so pitched as to give them too thin a toe as the shot may blow out at the back leaving the heel standing.

Shots that are all dead (as would be *5* in Fig. 3 if fired alone) or are partly dead, should be prohibited. No shots should be drilled directly into the face and no shot should be drilled to such a depth that any portion of the charge is beyond the point where a perpendicular dropped from the drill hole will not cut a free face.

As a rule, shot holes should be parallel to a free face, as shown in Fig. 4, and should pitch a few degrees from the horizontal so as to cut across the bedding planes. It is advisable to incline the holes slightly in the direction of



FIG. 4

the length of the room, so that they may cut across the vertical cleavage planes. Shots should not be placed in soft streaks of coal, or shale, or mother coal, as they may blow out along these lines of weakness.

Shots should be well balanced with toe and heel of equal width, which should never be more than 5 ft. in any case and never greater than the thickness of the seam when less than 5 ft. A hole 5 ft. long, working against a toe and heel each 4 ft. wide, will give excellent results in seams 4 ft. thick and over.

Straight holes of uniform diameter give better results than crooked ones, and generally short holes are to be preferred to long ones. To ensure the holes being of a uniform size best adapted to the coal, the drills should be frequently tested.

Shots that cannot do their work when fired separately but depend for their successful action on the results accomplished by another shot or shots fired at the same time, and which are, consequently, known as *dependent* (or follow) shots, should not be permitted and are prohibited by law in some states. In Fig. 1, while shots *1* and *2* are strictly dependent shots, since either one fired alone would blow out, yet with proper judgment they may be safely used if fired at the same instant with an electric battery. The holes *3* and *4* are in every sense dependent as they are dead unless a free face has been made by firing *1* and *2*. Similarly, *5* and *6* are dependent on the successful firing of *3* and *4*. If fuse or squibs are used, *3* and *4* may explode a few seconds before the others, resulting in blown-out shots, to be followed by the detonation of *1*
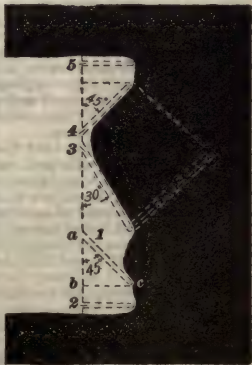
and *2* simultaneously which will do their work, the final explosion of *5* and *6* resulting in blown-out shots.    The reason for the failure to explode at the same instant is due to the impossibility of securing either squibs or fuse that will burn at exactly the same rate.    In electric firing with a blasting machine, holes *1* and *2* may be fired first, and then, after the smoke and dust have cleared away, *3* and *4* and *5* and *6* in separate pairs.    If delay-action detonators are used, all the holes may be fired in sequence in pairs by a single application of the current by using no-delay caps in *1* and *2*, first-delay caps in *3* and *4*, and second-delay caps in *5* and *6*.    However, one of the strongest arguments against solid shooting is that it is so entirely impossible to place the second of a series of drill holes until the results accomplished by the detonation of the first hole have been studied.    Good practice, then, demands that the center shot or shots (as *1* and *2*, Fig. 1) be fired first, and that the other shots be placed where needed after the face has been examined.

**Objections to Solid Shooting.**—The objections to solid shooting are two-fold:   It increases the percentage of slack coal produced and is dangerous to the men and mine, particularly where inexperienced or careless workers using black powder, are allowed to drill, charge, and fire their own shots, when, where, and how they please.

From an economic standpoint, the objection to solid shooting is that the usual employment of excessive charges of powder in poorly placed holes always leads to the production of an excessive amount of slack.    While this may be an advantage where the coal is coked, for ordinary commercial use the coal must be lumpy and as free from slack as possible.

The same causes that tend to produce an excessive amount of fine coal, also tend to the production of blown-out shots, and these, in turn, have been the cause of many mine explosions.    These dangers are largely reduced if permissible powders are used and particularly so if shot firers and electric blasting are employed.    They are reduced to the minimum if, in addition, the coal is undercut before blasting.

If the duties of the shot firer are limited to firing the holes that have been previously drilled, charged, and otherwise prepared by the miner, the possibility of damage to property is not reduced, and the danger to life is merely transferred from the miner to the shot firer.    Under these conditions the shot firer is killed in event of accident and not all the underground workers.

A distinct advance toward safety is made if the shot firers charge as well as blast the holes previously drilled by the miner, and are required to refuse to fire any and all holes that are improperly placed.    In theory, the method is perfect, but leads to many accidents in practice, as the shot firers, through mistaken friendship for the miner, often fire shots their better judgment must condemn.

The highest degree of safety is attained by employing an inspector who is independent of the miners to oversee the placing and drilling of all shot holes.    The inspector not only instructs the miner where to place the holes but determines their pitch, depth, and the amount of charge as well.    Before the miner leaves his place, the inspector examines it, and compels the drilling of other holes in place of those drilled contrary to his previously issued instructions. The inspector commonly places a marker or flag (a piece of paper) in the mouth of each hole to be fired, the charging and blasting being left to the shot firers.

Notwithstanding all precautions taken by inspecting the holes as described, the fatality rate among shot firers is needlessly high by reason of the failure of many of them to blast the holes properly.    Having a certain number of holes to charge and fire in order to complete their shift, the work is commonly done in from one-half to one-third the time that should be devoted to it; and this leads to carelessness in charging, in going back on delayed shots, and has led to serious mine explosions through blown-out shots igniting the dust thrown into suspension through rapid firing.    To protect the shot firers against themselves has arisen the custom of firing all the shots at one time by means of a current of electricity applied from some point outside the mine, and after all the men have left the workings.    This practice is not without danger to the mine, through its possible wrecking by a dust explosion caused by the detonation at a single instant of many hundred pounds of high explosives in an atmosphere charged with dust.

## BLASTING AFTER UNDERCUTTING

The object of mining or undermining the seam of coal previous to blasting is to secure the advantages of an additional free face.    The mining may be made in the bottom of the seam or in the fireclay underlying it (undercutting),

in a band or layer of shale or clay near the middle of the seam, or near the roof (topcutting) and may be done by a pick or by machinery. The depth of the undercut in seams up to 6 ft. or 8 ft. in thickness is commonly equal to the thickness of the seam where machinery is used, but is not usually much over 4½ ft. where the work is done with a pick. Where the cut is made with a chain machine, it has a uniform height from front to back of from 4 in. to 6 in. When made with a punching machine or with a pick, the height of the cut decreases from 14 in. to 18 in. at the front to 4 in. to 6 in. at the back.

The proper placing of the shot holes in a face that has been mined is a simple matter compared to the same work in solid shooting. The precautions previously given concerning the drilling of gripping, dead, and unbalanced shots, and charging with the proper amount of explosive must be heeded. As a rule, the depth of any hole should be about 6 in. less than the depth of the mining, and to secure this relationship between depth of mining and length o, shot hole, it is a common practice where the coal is undercut to a depth of, say, 7 ft., to make the drills furnished the miner of a maximum length of 6 ft. to 6 ft. 6 in.

In entries and in rooms not over 20 ft. wide where the coal is 6 ft. to 8 ft. thick, three shots placed as *1*, *2*, and *3*, in Fig. 1, should serve to bring down the coal. Where the place is more than, say, 2½ times as wide as the coal is thick, instead of a single center or bursting shot 1, two may be used, as *A* and *B*. The bursting shots, where the seam has been undercut to a depth of 6 or 7 ft are commonly but from 4 to 5 ft. in length and are usually given a pitch of from 5° to 10° downwards. In some cases, *1*



Fig. 1

may advantageously be placed nearer the roof in the same horizontal line with *2* and *3*, but is usually placed about as shown at two-thirds the height of the seam from the floor, and will break out a piece of coal with a cross-section approximately *G1H*. When two holes *A* and *B* are used and the place is not too wide, the coal broken will be about on the line *EABF*. The holes *2* and *3* are placed from 10 in. to 15 in. from the rib and so far below the roof that when drilled with an upward pitch of from 5° to 10°, their points will just clear the roof slate.
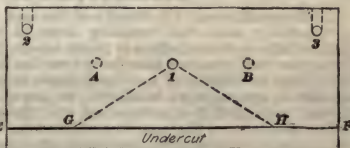
In some tough, tenacious coals, when undercut by chain machines and blasted with the comparatively slow-acting black powder, there is a tendency on the part of the coal to sit down on the undercut because its height is so small that, in falling, the coal does not gather sufficient impetus to roll forwards away from the face. In this case, the front part of the mining must be snubbed to a height of 18 in. to 24 in. either with a pick or by placing a series of snubbing, or pop shots, as *AB*, Fig. 2, in a row along the face. These shots require a very small amount of powder, are not usually more than 2 ft. deep and serve to break out a wedge-shaped piece of coal approximately along the lines *ABCD*. If the coal thus brought down is loaded out of the way, there will be ample room provided for the free fall of the rest of the seam.
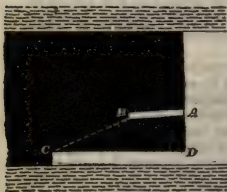


FIG. 2

When shots are fired by blasting machine or battery, shot *1* is fired first and the coal loaded out into cars. When two bursting shots *A* and *B* are necessary, they are fired simultaneously, and, as before, the place cleaned up. Shots *2* and *3* may then be fired together. Whet firing from a point on the surface is practised, the three shots are detonated at the one time, but good practice seems to indicate that no-delay caps should be used in *1* (or in *A* and *B*), and first-delay or even second-delay caps in *2* and *3*.

## COMBINED UNDERCUTTING AND SOLID SHOOTING

A method sometimes followed, which reduces the amount of undercutting in rooms, is shown in plan in the accompanying illustration. Here, the face *bc* with a width of from 8 ft. to 10 ft. is driven some 6 to 10 ft. in advance of the

rest of the room. Only the narrow face $bc$ is undercut and is shot down with the customary three holes. After the coal thus made is loaded out, two or at the most three light shots serve to bring the coal bounded by the faces $cd$ and $de$. The method is adaptable to pick mining, but where machines are used it would be much better to carry a straight face and undercut the room for the full width.



A method of mining formerly employed to some extent is known as *following the crack*, and consists in shearing to a depth of 2 ft., and firing a shot, called a *snubber*, placed close to the side of the cut, the hole being drilled 3 ft. deep, thereby locating the charge 1 ft. on the solid. The firing of the snubber cracks or crevices the seam along the rib. After the firing of the snubber, successive back shots are fired between these and the rib at the far side of the face, the coal being removed after the firing of each shot. The next shear or side cut is made at the place where the snubber, drilled 1 ft. on the solid, has cracked the coal. The dangers incident to the use of this method are apparent.

## UNDERCUTTING IN LONGWALL

In longwall work, the weight of the roof is made to act upon the face so as to materially assist the breaking down of the coal. Usually a succession of shallow cuts 6 or 8 in. deep is made across the entire face of the coal, thereby giving the roof pressure time to act on the face of the cut, the coal at the back being thus crushed and rendered more easy to mine. If, however, the roof pressure is excessive or is allowed to act on the face for too long a time, the crushed mining dirt is compacted and the undercutting made more difficult.

By machinery, the coal is undercut to the full depth in one cut. When the old-style, chain-breast machines are employed, the cockermegs, or cockers, are placed before the undercutting is started, and are removed and reset in turn as the machine advances. With the present, self-propelling, longwall machine, which can make a continuous cut of any length without resetting, the sprags are placed immediately behind the machine as fast as it pulls itself along the face. As these machines require 5 ft. or less of space between the face and the gob, they do not interfere with the timbering, gob, pack walls, etc.

## MACHINE MINING

The commercial use of coal-mining machines may be said to date from the year 1891 when the 545 machines in use undercut 6,211,732 T. of coal, or 6.66% of the entire output for the year, and at an annual rate of 11,398 T. per machine. In 1910, 13,254 machines undercut 174,012,293 short T. of coal, or 41.7% of the output, showing an annual capacity of 13,127 T. each. In 1910, the machines in use comprised 6,716 of the pick or puncher type, 5,973 of the chain-breast type, 518 longwall machines, and 47 of the shortwall type. It will be noted that the puncher machines formed more than one-half of the entire number used.

A universal mining machine has not yet been brought out, and one of the principal reasons for the failure of mining machines in a number of instances has been the attempt to use a machine under conditions to which it was not adapted. When a mining machine is designed and built to suit the conditions under which it is to be operated, it is safe to say that there are but few mines in which they cannot be successfully utilized. They are of particular advantage where there is a long working face and where the coal is over 3 ft. in thickness. Low seams require more undercutting for a given output than high seams. As a rule it has not been found economical to use machines in seams pitching over 12° to 15°, though pick machines have been used in mines having an inclination of 23°, the difficulty being not so much in the cutting as in moving the machine from place to place.

There are two general types of mining machines: *Pick*, or *puncher*, *machines* in which a steel pick attached to the end of a piston rod is given a reciprocating motion by means of compressed air; the action of the machine simulating that of a miner with a hand pick. *Cutter*, or *chain*, *machines* in which a series of teeth attached to a chain (usually) or to a disk or bar moved by an electric or compressed-air motor, scrape away the coal.

**Pick Machines.**—Pick machines may be divided into several classes according to their mounting. The ordinary type is mounted upon low wheels and is especially adapted for undercutting a straight face. For making a vertical sheer, the machine is mounted upon much larger wheels and is provided with

longer bits. Pick machines mounted upon wheels are not adapted to undercutting seams where the pitch is more than 10° because of the difficulty of keeping the machine up to the face. For pitches up to 40°, the machine is mounted on a post that is firmly wedged in the roof and floor, constituting what is commonly known as a *post puncher*, although the various manufacturers have their distinctive names. While the post puncher is able to work at practically any angle of pitch, in seams inclined at more than 40° shooting is generally done off the solid, the difficulty of moving weights of any kind on such steep slopes prohibiting the use of machinery. The post puncher is adapted for shearing or for undercutting in a horizontal line at any distance above the floor. One of the objections made to punchers, that they cut out too much coal or the cut is too high, making an excessive amount of slack, is overcome by the use of these machines, which will cut nearly as close to the floor as a chain cutter.

Punching machines directly operated by electricity have not met with success, but a punching machine in which the electric current operates a small air compressor, which in turn supplies the power for operating the pick are well and favorably known. In the Pneumelectric puncher, the motor and air-compressing mechanism are all in the one machine, which may be mounted upon wheels in the ordinary way or upon a post. In mines intending to use electric motors for haulage or in mines already wired, this machine will save the cost of special compressed-air piping.

The advantages claimed for pick machines are that they are able to cut around sulphur balls or other obstructions in the seam and are particularly suitable in places having rolls in the floor; that the exhaust air helps the ventilation, particularly in tight places; that there is no danger of igniting either gas or dust from short circuits of the electric arc; that they can be pointed at any horizontal angle and so can cut around and between posts which may be set close to the face; that they require less skilled labor than chain machines; and that they may be used for working pillars on which there is a squeeze, as they are light and can be easily handled and readily removed.

**Chain Machines.**—Chain machines consist of a low metal bed frame upon which is mounted a motor that rotates a chain to which suitable cutting teeth are attached. To operate chain machines to the best advantage, the coal should be comparatively free from pyrites. They also require more room than pick machines, and a space from 12 to 15 ft. in width is necessary along the face to work them to advantage. These machines have proved failures in some mines on account of the incessant jarring of the roof by the rear jack. Chain-cutter machines cannot be used to undercut coal when a squeeze is upon it. Coal seams that are comparatively level and free from pyrites and have a comparatively good roof can undoubtedly be more satisfactorily and economically cut with chain-cutter machines than with any other type.

The average height of cut is $4\frac{1}{2}$ to 5 in., and at this height, the chain-cutter machine makes only about 60% as much small coal as a pick machine. This is not always an advantage, as it does not always allow sufficient height for the coal to fall down after the cut is made. In a $3\frac{1}{2}$-ft. seam, three men are required to handle the machine to advantage.

Chain machines may be operated either by compressed air or electric motor and both are very extensively used. The original form of chain machine, after making its undercut, has to be moved across the face the width of a cut before another cutting can be made. This takes considerable time and in low seams, especially, involves much hard labor. To overcome these difficulties, what are known as *short-wall* machines are extensively used. These machines are set up in the same general way as the older type, but after making the first or sumping cut at one side of the face, instead of withdrawing the cutter and jacking the machine into a new position for another cut, the whole machine pulls itself across the face by means of a chain or wire rope attached to the opposite rib, cutting as it goes.

When a dirt or slate parting exists in the seam at such a distance above the floor that the bed is divided into two more or less nearly equal parts, it often is a material economy and leads to the production of clean coal if the cutting is done in the parting. When post punchers are used, this is easily done by raising or lowering the puncher on its supporting post, but cannot generally be done with chain machines unless they were supported upon a cribbing of logs, the building of which requires much time. The recently introduced *turret coal cutter* permits the undercutting of the seam by a chain machine at any height from 2 ft. to 5 ft. from the floor. The machine consists of a self-propelling truck with the necessary cable-reel to attach to the power line in

the entry. The cutting machine is mounted on a turntable carrying four heavy standards on which the machine is moved up and down to the desired height. After moving itself to the face of the working place, the machine is anchored, the cutter raised to the proper height, and the machine turned on the turret towards the right-hand rib and locked in position at an angle of about 15° with the track. After the cut is made on this setting, the cutter is swung across the face, cutting as it moves, until this angle with the room is about 20° to the left. The cutter is withdrawn, completing the left-rib cut, is swung parallel to the track, and is ready for moving to the next place. One advantage of this machine is that the entire cut may be made without removing it from the truck.

**Capacity of Coal Cutting Machines.**—So much depends on local conditions that it is almost impossible to give specific data or rates of working and costs of operating cutting machines, but the following figures are fair working approximations. In the case of chain machines, the amount of undercutting that can be done in a given time depends on the ability of the men running it, the character of the coal, and the presence of faults and impurities and other obstructions and hindrances to the work of cutting. Under favorable conditions, one machine operated by two men is recorded as having made 104 cuts, each 6 ft. deep, in 10 hr., these cuts being made in rooms and entries. This is exceptional cutting and much higher than the average. It is fair to say that a machine making from 36 to 45 cuts, 6 ft. deep, in 10 hr., is doing good work. An average of 40 cuts, 42 in. wide, will represent 140 lin. ft. of face, which for coal 6 ft. in height will make approximately 140 T. of coal per shift of 10 hr. Under unfavorable conditions, the result will not run nearly so high. There are cases where, with the chain breast machine, the undercutting of one room per shift of 10 hr. is considered good work. The capacity of the machine or the number of runs it will make differs in every locality and seam, and experience in the use of machines in a given locality is by far the best guide when the capacity of the machine is to be considered. The time it takes for a machine to make a cut is generally but a small part of the total time consumed in its operation, unless unfavorable conditions exist, as the time for moving and resetting the machines greatly exceeds the time of cutting. In cutting clean coal, a machine should make a cut its full depth in 4½ min., and 1 min. more will be required to withdraw the machine; the rest of the time is consumed in loading and unloading, moving and setting the machine in place ready for another cut, besides such delays as waiting to have rooms cleaned for operation, waiting for driver, and many small accidents that are liable to occur at any time. In the use of longwall machines, a record has been made of 500 lin. ft. of coal, cut 6 ft. deep, in 10 hr. This is an unusual record, as a longwall machine under ordinary conditions will not average more than 400 lin. ft. In a seam 4 ft. thick, a length of 400 ft., cut 6 ft. deep, makes approximately 280 T. of coal, which would be the average output of a machine per shift of 10 hr.; but under unfavorable conditions the output will, of course, be much less.

In the case of pick machines, a fair average of cutting for any one machine working in rooms would be 100 lin. ft. per da. of 10 hr., making a cutting 6 ft. deep. This in a seam 6 ft. thick makes about 100 T. of coal, which is the average output of a machine per day of 10 hr. The amount of coal that a loader can shoot down and load in a day varies, of course, with the thickness of the seam and the character of the coal, but an average for a day of 10 hr. is 12 to 15 T. In longwall work, as no changing of places is required, a fair average is 150 lin. ft., 6 ft. deep, per da., making about 150 T. per machine per da. The capacity of a pick machine for shearing depends largely on the height of the coal sheared. Working in coal of average height, eight cuts, sheared 6 ft. deep, in 10 hr. is considered fair work.

**Longwall Machines.**—Longwall machines are of two main types: (1) Disk and cutter-bar machines; (2) modified forms of chain machines. In the disk machine, a series of bits or cutters are inserted in the periphery of a cutter wheel or disk to which a horizontal rotary motion is imparted. The machine impels itself across a longwall face by means of a rope fixed at any convenient distance as in the shortwall machine. In the cutter-bar machine, the cutting is done by means of teeth set in a shaft about 2 in. in diameter set at right angles to the machine. The teeth are arranged so that one-fourth of them cut and three-fourths of them break the coal as the bar advances horizontally along the face. Lateral motion is secured by the use of a pair of rails laid along the face. One of these rails is notched and the other is plain. The machine is propelled by means of a toothed wheel meshing into the notches of the notched rail.

Longwall machines of the second class generally consist of a cutter arm of any desired length, depending on the depth of undercut needed. Around the necessary sprocket wheels on this arm is passed a chain bearing cutting teeth, the chain being given motion by a compressed air or electric motor of the type used for ordinary chain machines.

**Heading Machines.**—In the *Stanley header*, shown in the accompanying figure, the driving and the feeding mechanisms are placed on a massive frame $a$ that is mounted on wheels. When the machine is in place ready to commence work, it is held fast by the top jacks $b$ and the side jacks $c$. At the end of the main shaft $s$, an auger drill $e$ is placed for the purpose of steadying the cutter frame while the machine is working. The cutter frame consists of a large revolving cross-head $h$ carrying two arms $f$ in the ends of which the cutters or bits are set. The driving mechanism is so constructed that different rates of speed of the cutter frame can be produced as desired for a given rate of feed advance, on coals of varying hardness. As the main shaft rotates, it advances, turning the cutter frame. The bits cut out an annular groove from 3 to $3\frac{1}{2}$ in. wide, forming a complete cylinder of coal as deep as the arms $f$. When this is done, the machine is run back and the coal is taken down and loaded up. The machine is again set in place and another cylinder is cut out.

The cuttings are forced to the front of the annular groove by the scrapers on the arms $f$, and from here they are raked to one side of the machine by a helper whenever the revolving arms are not in the way. Lumps of coal that
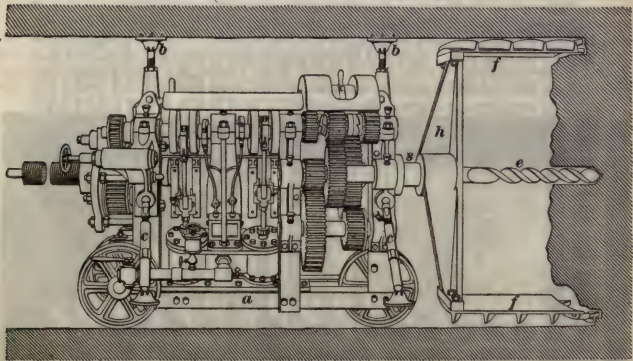


fall from the face are also drawn to the side by the helper, and finally loaded into a car. In many of these machines, however, the cuttings and the coal that falls while the machine is working are taken from the face by a friction worm and loaded into a mine car by means of a conveyer or elevator.

The principal use of this machine is for entry driving, where the work must be pushed rapidly. It is especially advantageous for prospecting a piece of coal, as an entry can be driven a long distance as a single entry, and with no other ventilation than that caused by the air used by the machine.

The Stanley header can cut out a cylinder of bituminous coal 4 ft. in diameter and 5 ft. in length in 15 min., and after making the necessary allowance for removals, a rate of advance equal to 75 ft. per shift of 10 hr. is accomplished. Where it is necessary to drive wide entries, two machines may be worked side by side, thus driving two parallel entries that nearly intersect each other. The thin pillar left between them can easily be cut out with a pick. If the coal could be removed as quickly as the cutting is done, the machine could advance an entry 12 ft. wide 25 ft. in 10 hr. The entries thus cut can be widened out to the desired height and width by the use of the pick. Impurities, such as sulphur balls, existing in coal hamper the use of the Stanley header; and when they are present its progress cannot be as rapid as in coal favorable to mining. An average cost per linear foot of entry for a cylindrical cut alone should not exceed 25 c,

**Machine Mining in Anthracite Mines.**—Owing to the exhaustion of the thicker seams and the rapidly increasing cost of mining the thinner seams, the anthracite operators in northeastern Pennsylvania have begun to use undercutting machines of the ordinary bituminous-coal mine type. The method of using the machines differs in no way from that employed in the soft-coal fields, although numerous ingenious expedients have been adopted to overcome local difficulties. The machines employed are mostly of the short-wall style that propel themselves along the face. In one place they are being used on a 15° pitch and can be used on pitches of as much as 20° to 25°. In these pitching places, after the sumping cut is made, an iron rail is placed back of the machine and parallel to the face of the room. This rail, which is held in place by jacks, holds the machine up to the face and along it the machine slides to the end of the cut. The rail is, of course, moved up after each cut is made and the coal shot down. The machine has the necessary power to pull itself up to the face and to make the sumping cut at the same time.

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# DRAWING PILLARS

The work of *drawing back*, or *robbing*, that is, removing the pillars left in the first working, should be commenced as soon as the rooms are worked up their full length, whenever this is possible. If this is delayed, and the openings left to stand for any length of time, the roof will settle heavily on the pillars and there will be danger of crushing them and thus losing the coal. Fig. 1 shows the way of drawing the pillars in rooms turned off one side of a single pair of entries where the drawing begins next to the main entries and progresses inbye.

The drawing of the pillars has been completed in rooms *1*, *2*, *3*, and *4*, down to the entry pillars *a*, which are left and these rooms are closed; the work on pillars *5*, *6*, and *7* is in progress; rooms *8* and *9* have reached the limit, and the
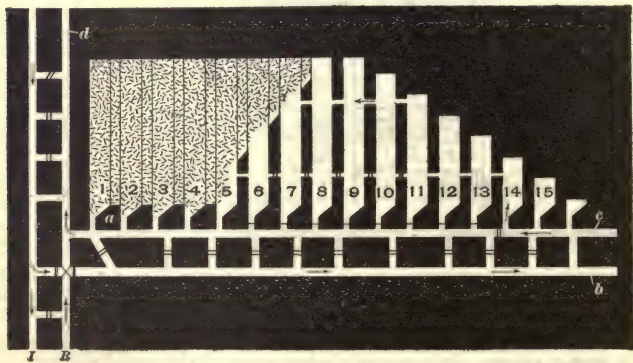


FIG. 1

work on the pillar separating rooms *8* and *9* will now begin. The rooms inside of number *9* have not been completed, and the last room on this pair of entries has just been turned. After the cross-entries *b* and *c* have been completed and all of the rooms off them completed and the pillars between the rooms drawn, the pillars between the entries *b* and *c* and the stumps *a* will be drawn back to the main entries *d*. It is advantageous to carry on the pillar drawing systematically and to keep the ends of the pillars in a line to avoid excessive pressure being brought on a single pillar by the drawing of the pillars on either side of it. Unless the ends of the pillars are kept in line, there is also increased danger from falls, and the work of drawing timber is made more difficult. If rooms are driven toward each other from adjoining cross-entries, a pillar should be left between the ends of the rooms and removed when the pillars between the rooms are taken out.

Fig. 2, page 635, shows the reverse method of drawing the ribs, that is, from the inby end of the entries or headings toward the haulage roads. The cross-headings are driven in pairs off the main headings, and off of one of these cross-headings other headings called *butt headings*, are driven. These butt headings divide the mine into panels and they are driven their full length, but so as to leave a chain pillar *c* between the end of the butt headings and the next pair of cross-headings. The rooms are then started from the inbye end of the butt headings, and as soon as the first or inbye room has been driven up to its full length, leaving a chain pillar between it and the next pair of butt headings, the pillars are drawn, as shown. As soon as the pillars have been drawn back to the butt headings, the pillars between the butt headings, and the
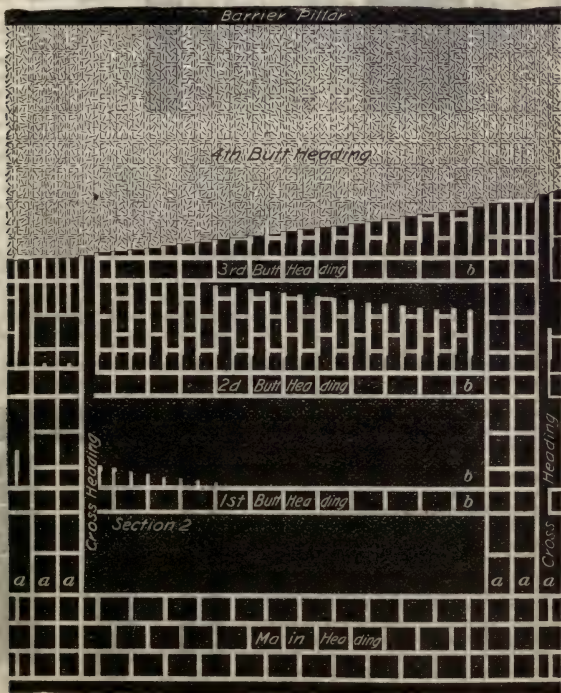


Fig. 2

chain pillar between the lower butt heading and the next lower tier of rooms are drawn back as shown. The advantage of this method over that illustrated in Fig. 1 is that both the room pillars and the entry pillars are drawn back in one operation, instead of the room pillars being first drawn and the entry pillars and stumps drawn subsequently. A greater portion of the total amount of coal is probably obtained in this manner when it is possible to carry it out.

The method of drawing back the pillars illustrated in Fig. 2 is similar to that just described, but it is extended over a much larger area. The cross-headings *a* are driven to the boundary of the property before the robbing is begun; from these headings the butt headings *b* are driven, and off these

narrow rooms, as shown. The pillars over the entire length of the property are then brought back at one time.

**Work of Drawing.**—When the work of drawing the pillar is to begin, a cut-through is driven from the face of each room to the face of the room adjoining so as to give a free face across the end of the pillar. There are a number of ways of attacking the pillar, the choice of a method being determined by local conditions and custom.



Fig. 3 illustrates one way of drawing back pillars separating wide rooms in which there is a track along each rib of the pillar. The work of drawing back the pillars is shown as having just begun, the pillars having been cut through at the face,
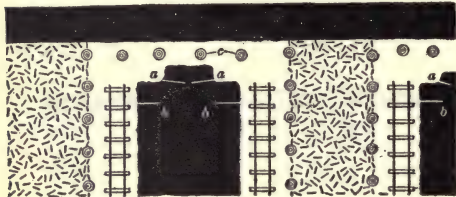
FIG. 3

and the first shots having removed the coal at each corner of the pillar. The second holes *a* will remove the remainder of the first slice across the pillar and the holes *b* the first cut of the second slice. The coal is thus removed in steps. A row of props *c* keeps up the roof along the face of the pillar. As the pillar is drawn back a sufficient distance, a second row of props similar to that shown at *c* is stood across the face of the pillar, parallel to the first row *c*, which is then withdrawn. In pillar drawing, the back timber should be drawn only so fast as to throw a proper weight on the pillar. If this weight is excessive, the end of the pillar is crushed. An excessive weight is also thrown on the pillars by leaving too much timber standing. Just how much timber to use can be determined only by experience. If the pillar is very wide, a slice or skip may be taken off it from the entry to the face before the pillar is cut across at the face.

When the pillars have been drawn back to the entry stump at the mouth of the room, where the room begins to narrow toward the neck, care should be taken to break the roof, if necessary, back to the entry pillar or stump. With a hard roof, it may be necessary to place one or two shots in the roof at this point. By this means the entry pillars are relieved of excessive pressure due to the settlement in the abandoned rooms, which have been closed by the drawing of the pillars.

Fig. 4 shows other methods very commonly used for drawing pillars in both flat and inclined seams, known as *splitting a pillar*. In the method shown in (*a*), the opening *b* is driven up the center of the pillar as wide as the strength of the roof will permit without crushing the pillars left between this opening and the rooms. Each of these small side pillars is then drawn back in slices by a method similar to that shown in Fig. 3. In Fig. 4 (*b*), the pillar is shown divided into a number of small pillars by cross-cuts *c*. Each of



FIG. 4

these small pillars is then divided lengthwise as shown at *d*. In (*c*), the pillar is divided up its full length by a narrow place *e*. This narrow place and the break-throughs *f* divide the original pillars into a number of small pillars *g*. These small pillars are next broken up by other cross-cuts *h*, leaving still smaller pillars, and these are then taken out by shots, as shown.

Fig. 5 shows some of the methods used in robbing the pillars in steep pitching, thick beds of anthracite. To get the coal out of the pillar at the left of *A*, a skip is taken off the side, as shown. Successive skips are thus taken off until the whole is removed, the miner keeping the manway open to the heading below as a means of retreat. The pillar between *A* and *B* is very similarly
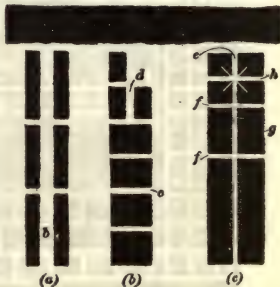
worked. To remove that between *B* and *C*, a narrow chute or heading is driven up the middle, and cross-cuts put to the right and left a few yards from the upper end. Shots are placed in the four blocks of coal thus formed, as shown, and they are fired simultaneously by battery. This operation is repeated in each descending portion unless the pillar begins to run. A pillar from which the coal has started to run is shown to the right of *C*.

**Delayed Pillar Drawing.**—The work of drawing pillars between the rooms is sometimes preferably delayed until the entries have been driven to the boundary and the rooms also worked up to that point, when the work of drawing pillars will be commenced at the boundary and proceed uniformly toward the mine opening. This may be necessary in the working



Fig. 5

of two beds separated by only a few feet of solid strata where a number of overlying beds are worked; or in certain cases where the bed is overlaid by water-bearing strata, and where the breaking of the roof rock would result in the flooding of the mine.

When this method is used, a constantly increasing extent of airways and roadways must be kept open and in repair, until the robbing begins, while the difficulties of ventilation are also increased. Again, the pillars first formed are last removed and there may be a loss from depreciation of the pillar coal due to weathering and also from the crushing of the pillars, unless much larger pillars are left than are required when the pillars are drawn as soon as the rooms are finished. With fairly thick and very soft coals, the rapid working up of the rooms and equally quick drawing of the ribs as soon as the rooms are driven their full distance, is essential to economical working; for delay in extracting ribs and pillars in such circumstances results in their getting crushed and the coal lost or largely ground to slack.

**Precautions in Pillar Drawing.**—When two or more overlying seams are worked simultaneously, the pillars in the lower seam should not be removed until the upper seams have been worked out and the pillars drawn.

It is not generally advisable to attempt to draw the pillars in a limited area surrounded by a district in which the pillars are not drawn, particularly under a hard roof, as an excessive weight will then be thrown on the pillars left standing and a disturbance set up that may extend a long distance from the immediate district from which the pillars are removed.

In very gaseous mines, the pillars are sometimes not taken out until the workings have reached a considerable distance from the shaft, in order that there may not be accumulations of gas in the gob and waste near the shaft, since it is often more difficult to prevent gas accumulations in robbed workings than when the pillars are left standing. If the coal is tender, the removal of the pillars should be delayed if the roof will not fall readily, because if they are taken out, excessive pressure may be brought on the entry pillars. In the case of bad roof, the pillars should be taken out as soon as possible, not only for economy, but also because, when the roof is bad and falls freely as the pillars are drawn, the débris soon sustains the superincumbent pressure and relieves the weight on the pillars next to the entries. Early drawing of pillars also concentrates the working district, and, excepting in a gaseous mine, reduces the area to be ventilated.

With a strong roof that does not break readily when the pillars are removed, great care must be taken that the removal of the room pillars does not bring sufficient weight on the entry pillars to crush them. A weak roof falls freely and soon fills up the gobs, thus partly sustaining the pressure from the roof and relieving somewhat the weight on the pillars along the side and main roads, due to the layers of rock immediately above the coal.

If the roof has fallen in the rooms before the work of drawing begins, the strata above the pillar can usually be kept up comparatively easily by props about the working face, without great danger to the miner, but where the roof remains over the rooms, excessive pressure is often thereby thrown on the pillars and the work of drawing is very dangerous and treacherous; under such circumstances, the whole pillar can rarely be removed, as it will usually crush before it can be taken out.

In drawing pillars, their ends should be kept in a straight line, for if they

are not, some pillars are subjected to greater pressure **than others,** valuable coal is lost, and the work is materially interfered with.

Especial attention should be paid to the effect of the removal of the pillars on the surface and the overlying strata, particularly if the latter are water-bearing or contain running materials, such as quicksand.

The work of drawing pillars is particularly dangerous where faults or slips are frequent in the roof, or top coal is to be taken down, or where pot bottoms, sink holes, boulders, etc. are of frequent occurrence in the roof, or where the workings underlie or approach buried valleys or extensive beds of quicksand.

Where the pillars are crushed and creviced, blown-out shots are liable to occur in their working. Undermining, in pillar work, should be done with caution. Pillar coal can sometimes be undermined with machines, but the practice is not general and hand work is usually depended on. Small stumps, or portions of pillars should not be left scattered through the gob, as they interfere with the uniform breaking of the top.

The conditions at different mines are so varied that no general rule can be laid down to suit all. There is probably no more dangerous work in mining than pillar drawing and the method adopted depends largely on local conditions and on the experience of the miner.

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# LONGWALL SYSTEM OF MINING

## SYSTEMS OF LONGWALL

In the longwall system of mining, the coal is taken out in a single oper-ation, the face of the workings advancing in an unbroken line or wall; no pil-lars of coal are left to support the roof which is allowed to fall and settle behind the miners as the workings advance. The accompanying illustration shows the general system of longwall followed in the United States. From the shaft, or point from which the workings are opened out, a number of roads are kept
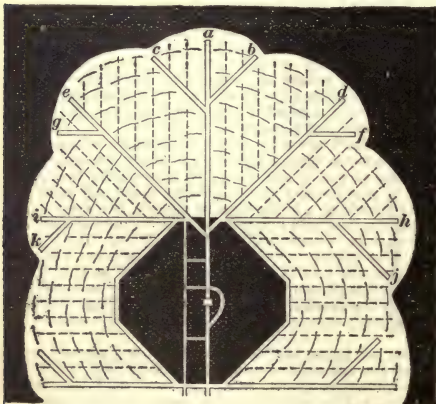


open for haulage pur-poses to the working face, as shown. Roads *a, d, e, h* and *i* are called *main roads,* or *main entries,* as they are kept open permanently. The roads *d* and *e* are sometimes called *diag-onal main roads,* or *main diagonal roads,* as they cut diagonally ac-ross the workings be-tween the straight roads *a* and *h* and *a* and *i.* Each of the b r o k e n straight lines represents a temporary road from the face to the nearest haulage road. The dis-tance between any two of these broken lines constitutes a *working place,* or a *room.* The *crossroads b, c, f, g, j, k* are driven at the main roads usually at an angle

of about 45°, and their purpose is to cut off the working rooms from time to time so that haulage distances may be reduced. The distance between cross-entries, or the limiting lengths, of the rooms is determined by the time it is possible to keep the *temporary roads* from the face to the main or cross-roads open.

The waste material from the seam, roof, or floor, is built in the form of *pack walls* along each side of the roadways and in the spaces between, from which the coal has been taken out. The pack walls, or packs, lining the road-ways, are called *road packs,* and those between the roads *gob packs.*

There are two general systems of longwall: *longwall advancing* and *longwall retreating*. In longwall advancing, the face is started from the foot of the shaft or from the inner side of the shaft pillar and advances toward the boundary of the property. As the haulage roads under this system are maintained by pack walls, it is often called the *gob-road system*. This is the method widely favored in those parts of the United States where longwall is used, as by means of it the mine is rapidly opened up and early returns are secured upon the capital invested. There is no expense for narrow work, and a minimum amount of timber is required.

In longwall retreating, narrow entries are driven through the solid coal to the boundary of the property and the longwall face is started at that point, the coal being taken out completely as the face is brought back toward the shaft. This is probably the better system of the two, provided the demand for coal and the capital available are such that the time may be taken to drive the entries. For fragile roof and soft coal, or a soft bottom, the method has many advantages, particularly for working seams lying at a great depth, as there is practically no expense for the maintenance of the haulage roads and the mined-out area is abandoned as soon as the coal is removed. The ventilation of the face is also more efficient and less expensive.

## CONSIDERATIONS AFFECTING THE ADOPTION OF LONGWALL

The points that must be considered before adopting the longwall method of mining coal, in preference to the room-and-pillar method, are as follows: The roof strata overlying the seam; depth of cover; nature, thickness, and inclination of the seam; nature of the floor or underlying strata; quantity of stowage or waste in the seam or contiguous strata; surface damage and the presence of water or gas in the seam or contiguous strata; supply of timber; labor conditions; and the transportation and marketing of the product.

**Roof Pressure.**—Although the longwall method is most successfully employed when the seam lies at a considerable depth below the surface, yet it has been successfully adopted under favorable roof strata where the depth of cover did not exceed 80 ft. The ideal roof in longwall is composed of tough, elastic, and pliable strata, that yield gradually by bending when the coal is removed, thereby causing a uniform settlement over the area mined and throwing a sufficient weight, or roof pressure, on the coal face to break the coal when the same has been mined or undercut.

The roof pressure depends on the depth and character of the cover or material overlying the seam. In the longwall method of mining, the weight of the strata overlying the seam is made to settle on the waste material or the pack walls that are built as the coal is removed, hence there is practically no limit of depth beyond which it cannot be worked.

In the room-and-pillar system, there is a practical limit to the width of opening that can be safely and economically kept open, and to the width of pillar that can be left to support the increasing roof pressure without crushing. These widths of opening and pillar determine the depth of the workings that can be mined by this method. As the width of pillar increases, the expense of driving the necessary cross-cuts increases, and the percentage of coal obtained in the first working is decreased. The limiting depth is not absolute, but will vary according to conditions, and as this depth is approached, pillar mining becomes more and more difficult and expensive. On this account, the longwall system is generally better adapted to the working of very deep seams than the room-and-pillar method.

**Nature of Coal Seam.**—Longwall is best adapted to a strong, tough coal that can be undercut to a moderate depth without breaking, and to seams of uniform and moderate thickness lying flat or nearly flat. Irregularities in the seam, such as sudden thickenings or thinnings, the presence of large masses of iron pyrites, black bat, etc., are unfavorable to the adoption of longwall, but in the room-and-pillar system can usually be cut through or around. In steep inclinations of the seam, longwall is not as successful as in flat seams, owing to the weight being drawn from the working face.

**Waste.**—In the longwall method, there is usually ample space for the storage underground of all waste rock, so that the expense and delay due to the haulage and hoisting of such rock is avoided. It is not always possible to avoid this expense and delay in room-and-pillar work. If there is not sufficient waste as the result of mining the coal by longwall, this method cannot be adopted to advantage unless it is practicable to bring material for the pack walls into the mine from the surface. This objection to the method is very apt

to apply in the case of thick seams where a large quantity of waste or gob is required for the packs.

For the working of a thin seam, on the other hand, longwall is particularly advantageous; for it is often absolutely essential for the successful working of a seam that all the coal be recovered, and that the expense for timber and maintenance of roadways be reduced to a minimum. This can often best be secured by the longwall method. Where plenty of waste material is present, there is little probability of a creep or squeeze in longwall work, except with a very soft bottom.

**Surface Damage, Water, Gas, Etc.**—In longwall working, damage to the surface is not as liable to occur, nor is it as great, as in the room-and-pillar system of mining, the subsidence of the surface, owing to the removal of the coal, being more gradual and uniform in longwall, and seldom producing the large breaks and caving in of the surface that are so common in room-and-pillar work. Consequently, the inflow of surface water is less in longwall work than in the room-and-pillar system.

The presence of gas in a seam is unfavorable to longwall work, for while the ventilation of the working face is as good, or better, in this system than in room-and-pillar work, it is less easily controlled, and a large quantity of gas liberated in one portion of the mine is not as easily confined to that portion of the mine in extended longwall work as in some forms of room-and-pillar work, unless some of the panel adaptations of longwall are used. In general, in longwall work, gas issuing at one point of the face is carried along the entire working face.

**Timber Supply.**—Where timber is scarce or expensive, longwall is particularly advantageous, owing to the small amount of timber required. In longwall, much more of the timber can generally be used again and again than in room-and-pillar work, as the props being drawn and set forwards as the face advances. On the other hand, in room-and-pillar work, a more abundant supply of timber is usually necessary, as the timbers cannot generally be used again.

**Labor and Trade Conditions.**—The longwall miner is a more highly skilled artisan than one who works in mines operated on the room-and-pillar system, and must be a steady, regular worker. If absent a day his place falls behind, making his work harder and his output less because of the excessive amount of slack that is likely to result from excessive roof pressure where the face falls behind. Longwall work is not adapted to conditions where regularity of output is not certain, owing to the frequent occurrence of strikes, to the scarcity of labor, or where market conditions are irregular or transportation facilities uncertain and liable at any time to cause an enforced idleness of the mine. Long seasons of idleness are not favorable to longwall work.

The chief advantages claimed for the longwall method of mining coal are as follows: Complete removal of the coal at a minimum expense, requiring a smaller capital; an earlier development on an extended scale is afforded than by any other method of mining, bringing earlier returns on the capital invested; the output of the coal is more uniform and of a better marketable size, yielding a better price; fewer roadways are required to be maintained for the same face of coal; there is no yardage for entry driving; less timber is required on the roadways and at the face; better ventilation of the working face is secured at less expense, a minimum quantity of air being required, and fewer doors, stoppings, and overcasts are necessary; there is less liability to accident from falls of roof, and there are no pillars to be drawn; less damage results to the surface in this method; the amount of surface water in the workings is generally less.

The disadvantages of the longwall method are as follows: To obtain the best results by this method, experienced longwall miners are required, or those familiar with the work, and ordinary labor cannot be used to the same advantage as is often done in room-and-pillar work; where a large amount of gas is present, the ventilation of a portion of the mine cannot be controlled or the section sealed off as in room-and-pillar work, where a panel system of working is adopted; a large amount of labor must be expended in the building of pack walls, this, however, is accomplished by cheap labor; the method is not practicable where periods of enforced idleness are liable to occur from any cause whatsoever; when the coal field is disturbed by faults, it is difficult or impossible to maintain a continuous longwall face. When the seam is thick and the roof hard, it is difficult to obtain sufficient packing material.

## LONGWALL WORKING IN FLAT SEAMS

**Scotch, or Illinois, Plan.**—The system of longwall mining extensively used in the interior coal basin of the United States is illustrated in Fig. 1, and is a modification of the Scotch, or 45° system of longwall, in that the diagonal roads make an angle of 60° instead of 45° with the main roads. A main entry $a$ for single track is driven from the hoisting shaft $H$ through the shaft pillar, which is left large enough to protect all buildings on the surface and to contain the air-shaft or escapement shaft $A$ and the stables, which are placed as shown. At the edge of the shaft pillar, entries $b$ are driven at right angles to the main entries. Diagonal roads $c$ are then driven at an angle of 60° with the main road $b$ and 300 ft. apart. After the work has progressed a certain distance, every other cross-road is discontinued and coal is hauled out through the remain-
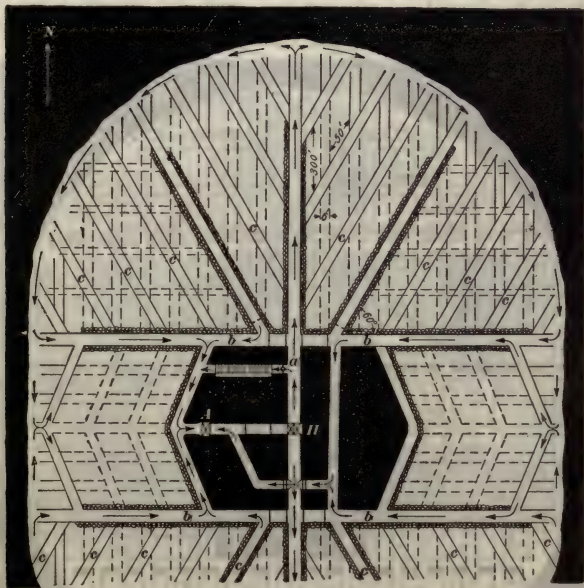


FIG. 1

ing roads; after the work has progressed still farther, the center cross-road only is used for hauling the coal out of each quarter of the mine, the coal being taken to this road through the rooms or along the face. The permanent haulage roads are shown in the plan with packs.

The main haulage roads are made 6 ft. high above the rails and 10 ft. wide; the cross-roads $c$ are 8 ft. wide. Pack walls not less than 4 ft. thick, and well built of strong slate, are erected along all main roads. In the first brushing along the roads, the miner takes down 18 in. to 2 ft. of slate for pack walls; and in the second brushing, the company men secure the roof and take down slate to make the roof 6 ft. above the rail. Heavy cross-bars and legs are used to support the roof after the final brushing on the main entries. The legs are given 1 ft. pitch in 6 ft. Permanent timbers or doors cannot be set until the roof has settled and no permanent timbers can be put in for a distance of 200 to 300 ft. back from the face. When turning off the roads $c$ from the main roads, an angle crib of some soft wood is put in so that it will give to the weight. Each room or working face is about 30 ft. wide, and as the circle of the working

face increases in size, the mine manager measures along the face and locates a new room, giving each two men about 30 ft. of face to work in. When the mine is first opened out, if water is present, a gutter is made in the entry around the shaft pillar below the coal and covered with railroad ties.

The air is first split at the bottom of the downcast shaft $H$ and again at the face of the main road $a$ on each side of the mine, thus giving each quarter of the mine a fresh current of air, a canvas door being used in each ross-road $c$ to direct the air along the working faces.

**Rectangular Longwall.**—Fig. 2 shows a rectangular, or square, longwall method, which, in some sections, is taking the place of the diagonal method just described. The entries and rooms are turned off at an angle of 90° instead of at 60°, as in Fig. 1. A pillar is left about the shaft and work begun in much the same way as in the Scotch plan. The room necks $a$ are made 10 ft. wide and are driven in from the haulageway 18 ft. The coal pillars $b$ are left in to protect the haulageways at the ends of the shaft pillar.
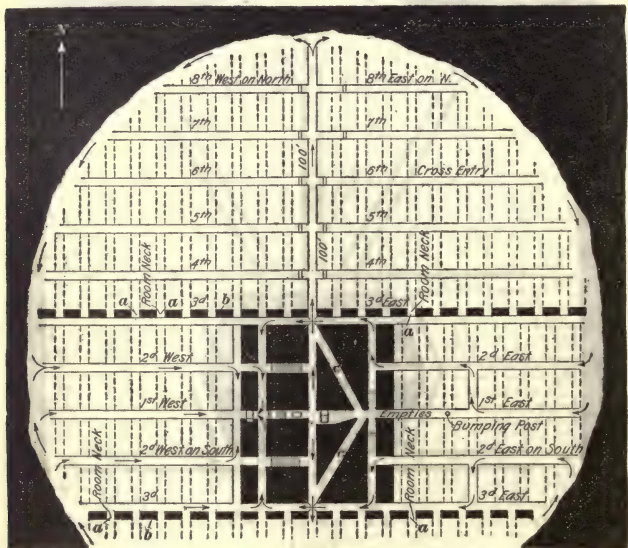


FIG. 2

The caging is done on one side of the shaft, empties are run to the bumping post shown and mules or motors pass through the small passages $c$ called *bolt holes* to the empties. The distance between the cross-entries is 100 ft. The main entry is single-tracked, and partings, or pass-bys, are made every 1,000 ft. by going into the mouth of one of the entries, passing up the first room to the next cross-entry, and then out to the main entry again. After the work has been opened to a certain extent, each alternate cross-entry is abandoned and later more of the cross-entries are abandoned, leaving only an occasional cross-entry for haulage purposes, thus decreasing the expense for maintaining haulageways. The last room on an entry is always kept open between cross-entries as a means of bringing coal from one entry to another.

An objection to the square system is the longer haulage roads required than in the diagonal system.

## LONGWALL WORKING IN PITCHING SEAMS

True longwall in which the face advances in a more or less circular form cannot be applied to pitching seams, as the face cannot be carried down hill because the packs would fall upon the miner. The general method of opening pitching seams is strikingly similar to that used in the room-and-pillar method. A pair of slopes is carried to the dip and to the right and left of them are turned pairs of levels. The coal between the pairs of levels is worked up hill on the longwall advancing system, the pillars between the pair of upper levels being taken out when the longwall face from the lower level reaches them. A barrier pillar is left throughout the life of the mine on each side of the slope, and these barrier pillars are extracted after the property is otherwise worked out.

As the movement of the loose coal down steep pitches would probably injure men working at a lower point on the same face, longwall faces in pitching seams are frequently laid out in steps. This step arrangement of the face is not used unless the pitch is so steep as to render it necessary as it increases the amount of small coal, does not permit of systematic regulation of the weight
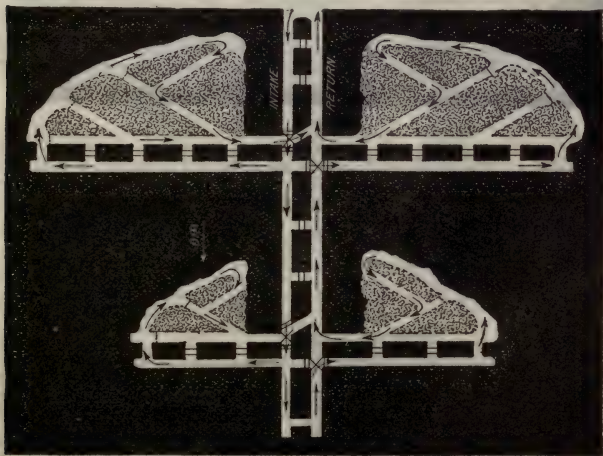


FIG. 1

over the entire face, and makes it more difficult to insure regularity of working and of advance in adjoining places.

A longwall face is usually advanced across the pitch in order to regulate the roof pressure on the coal; to decrease the grade on the roads; lessen the inclination of the face and thus reduce the danger in the work of mining while making the coal more easily handled at the face. Breaks and slips in the roof as well as the cleavage of the coal often require that the line of face should make an angle with the strike of the seam in order to give a better support to the roof at the face.

One of the chief difficulties in working longwall in pitching seams is the tendency of the roof to slip and sink back from the face, thus taking the pressure from the coal. For flat working it is comparatively easy to control the traveling weight on the coal; but as the inclination increases, there is considerable difficulty in preventing the broken roof masses from falling away from the coal in the direction of the dip, and then either sinking in front of the line of face, or, if they wedge and jam too tightly, they may bring an excessive pressure on the coal face and crush it. In inclined seams, it is very much more difficult to maintain the roads than in level seams, owing to the tendency of

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materials to slide down hill; for this reason, the principal level haulage roads are frequently protected on one or both sides by pillars of solid coal.

**Longwall on Low Inclination.**—In Fig. 1 is shown a longwall face on the advancing system, in which the gob roads are built across the pitch on a suitable grade for handling the cars by mule or by hand. This method can be employed in seams having an inclination varying from about 5° to 15°.
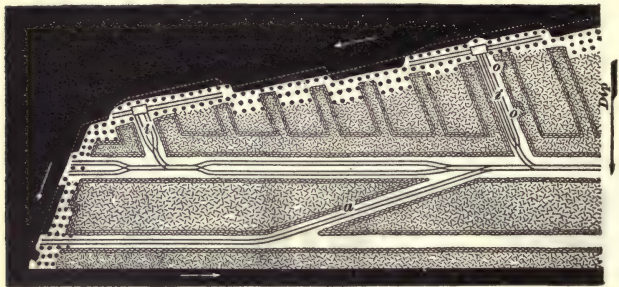


FIG. 2

Fig. 2 shows a pair of levels, the upper for haulage and the lower for drainage purposes. The levels are driven on the strike of a seam, and the long-wall face is arranged at a small angle with the line of strike. The coal is lowered from the face to the levels by the self-acting inclines $i$, which are provided with *safety holes*, or *manholes*, $o$ near the face. A slant road $a$ is driven from the haulage road to the lower, or drainage, level on such an angle that the coal may be drawn out of this level by a mule.

**Longwall When Inclination is Less Than 40°.**—Thin seams of coal having an inclination of about 40°, or less, are worked in Great Britain, France, and Belgium, by driving levels or lifts on the strike of the seam in each direction to the right and left of a main slope or incline. Both the advancing and the retreating methods are used. When the retreating method is used, the levels are driven to their limit or boundary, and the longwall face started at this point. In the advancing method, the longwall face is started at the slope, after leaving a sufficient pillar of solid coal for its protection. In either case, the upper rib of the level is marked off in sections, the length of which depends on the conditions for getting the coal to the roads. This length varies from a few yards, 15 or 20, up to 60 or 100. Fig. 3 represents the general plan where the advancing method is used.



FIG. 3

The coal is attacked by the usual longwall methods of undercutting and spragging the face in each section. In this system, the longwall face advances directly up the pitch or on the face cleats of the coal. The face in each section has one fast end that must be sheared. Each section is kept from 3 to 5 yd. in advance of the section next inby. The work in the first section started, $a$ being the most advanced, has two fast ends and is the most difficult. The waste of the seam is stowed in the space between the roads $b$ maintained by solid pack walls and placed in the center of each section, so as to provide a minimum distance the coal must be handled at the face. The pack walls are started at the level by building substantial cribs that are supported against slipping by timbers set in foot-holes cut in the roof and floor. The road from the face to the level is sometimes called a *jig road*.
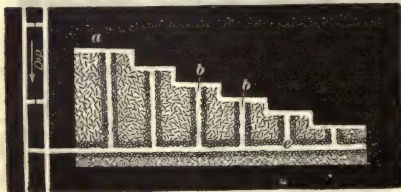
The coal is lowered from the face to the level by buggies operated by a wind-lass located either at the top or the bottom of the incline; when located at the bottom of the incline, the position of the windlass is permanent. The rope by which the buggy is raised and lowered passes over a block, or pulley, that



FIG. 4

s attached by a rope or chain to a timber at the head of the incline. Instead of the windlass, a form of wheel is often used for lowering and raising the buggy by hand. The rope is given a couple of turns around the wheel, the slack end being held in one hand, while the brake band is applied with the other hand by means of a lever to control the movement of the buggy or car down the incline. The buggies are emptied into cars along the level and the coal carried to the slope or shaft and hoisted to the surface.

It is possible to shorten the jig roads from the face to the main haulage road from time to time. This may be done as shown in Fig. 4, by building counterlevels *a* in the waste or gob parallel to the main level *b* either connecting with one of the jig roads *c*, which is kept open and operated as a main jig, or else these counterlevels may run to a landing on the main incline or slope.

**Longwall When Inclination is From 30° to 60°.**—The method shown in Fig. 5 may be used for a seam pitching from 10° to 60°, but it is particularly adapted to seams pitching from 30° to 60° and where the roof and floor are good and the roof pressure is moderate. In this method, owing to the steeper inclination or to the fact that the coal works better, the working face is advanced to the ends of the coal or parallel to the strike of the seam. The working face is broken into steps, or sections, usually about 20 yd. long, each section being kept from 5 to 8 yd. in advance of that above it to protect the miner from the falling coal in the upper sections. In each section, there are several miners, usually three or four for a face 20 yd. long, and each miner stands on a temporary platform, or on planks laid on props securely set in foot-holes cut in the roof and floor of the seam, or else stands on a gob built up from below.



FIG. 5

The platforms also serve to protect the miners working in the lower portions of each section, from the falling coal above them. Level roads are built in the waste or gob at the foot of each section. The coal is sheared at the upper end of each section and after being undercut it falls to the road at the bottom of the section where it accumulates and is loaded in cars or buggies and transported
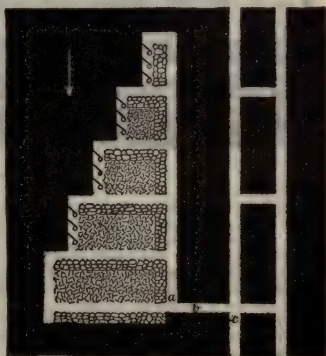
directly to the slope or to a jig road, or main incline $a$, down which it is taken to the level $b$, and then to the slope $c$. The lowest section of longwall face is started on the inby rib of the level $b$, the face extending a short distance below the level. A sufficient amount of coal is taken out on the lower side of the level to bring the breaks in the roof over the gob packs below the roadway. In deciding what breadth of coal should be worked on the lower side of the level, it is necessary to be guided largely by the direction of the first breaks in the roof, both on the rise side and the dip side of the opening-out places. It is generally found that the direction of these first breaks varies considerably from the direction of the main break.

When the first section has advanced over 5 or 8 yd., the next section above is started, and a second level built in the gob. In this manner, the upper sections are started consecutively one after another. As the face advances, the length of the level roads increases; so to avoid the unnecessary expense of maintaining these roads, they are cut off, from time to time, by an incline driven on the full pitch of the seam. It is often necessary to cut off these inclined planes by a counterlevel driven on the strike of the seam between the main
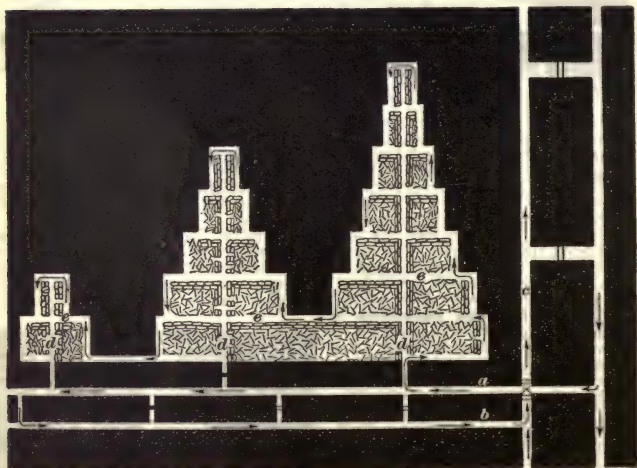


Fig. 6

levels, or built in the waste or gob as the face advances. When the method is used for comparatively flat seams, an inclined road is maintained at an angle with the main level instead of perpendicular to it, as shown.

The nature of the roof, floor, and coal at times necessitates greater protection against squeeze or creep than is afforded in the method just described. In this case, a pair of entries, $a$ and $b$, Fig. 6, are driven on the strike of the seam from the main slope or incline $c$ and roads, or chutes, $d$ turned off these levels at regular intervals and on the full pitch of the seam. A pillar of coal from 10 to 15 yd. thick is left for the protection of the level $a$ before opening out the longwall face on these rise roads. As shown in the figure, the face is opened to the right and left of each road $d$, cross-roads $e$ being maintained in the gob. The faces in each section, or lift, are advanced until they hole into each other, when the lift is abandoned, the cross-road being often filled with waste from other parts of the mine.

Where the roof is particularly frail or the bottom soft, especially with a heavy roof pressure, the method just described may be modified as follows: As before, the inclined roads, or chutes, $d$ are driven at regular intervals on

the full pitch of the seam, Fig. 7, and cross-levels *e* are then driven to the right and left of each chute until they hole into each other. At this point, the pillars separating the cross-levels are holed through on the pitch of the seam and the coal drawn back by longwall retreating. In both of the methods shown in Figs. 6 and 7, the coal is lowered down jig roads or inclines from the levels to the entry, whence it is taken to the main hoisting slope or shaft.
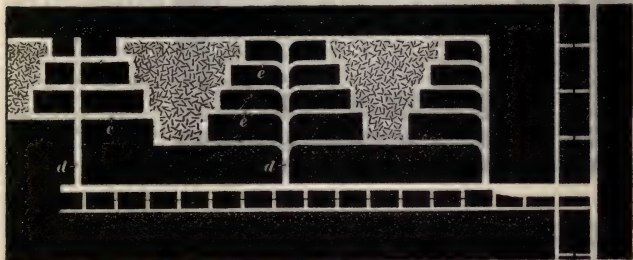


FIG. 7

**Longwall in Steeply Inclined Seams.**—The work in steeply inclined seams, as shown in Fig. 8, is similar to that described in connection with Figs. 5 and 6; but the length of each section of the face is only about 6 ft.; one miner works in each section. The coal is worked on end, the face being advanced in a direction parallel to the strike of the seam. Small shafts, or chutes, to carry the coal to the entry are built in the waste on the full pitch of the seam, one chute being built to three sections of the face. They are similar to the chute used in working steep seams by the room-and-pillar method, except that they are built in the waste instead of being driven in the coal. These chutes are kept full of coal, which is drawn as desired by opening a gate at the bottom. The miner
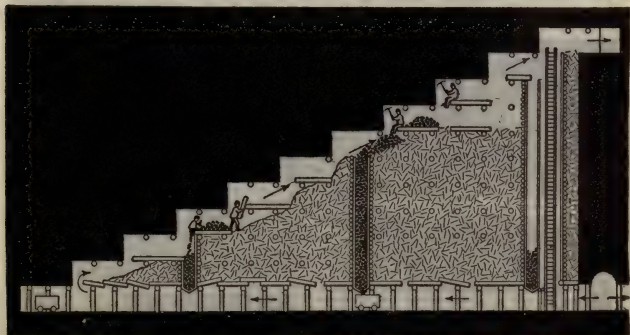


FIG. 8

stands on the gob, on planks, or on a temporary platform reaching from the gob to the face of the coal. In the three methods illustrated in Figs. 6, 7, and 8, the risk or danger from falling coal increases with the inclination of the seam.

## SPECIAL FORMS OF LONGWALL WORKING

**Longwall in Panels.**—With the ordinary longwall method, the mine cannot be divided easily into separate districts, as it is often desirable to do in working gaseous seams. In order to adapt longwall to the working of such

seams, it is necessary to restrict the length of continuous working face by dividing the mine into panels and carrying a longwall face in each panel, as shown in Fig. 1. Several rooms, or chambers, are turned off of a pair of entries and are connected at the inside of the entry pillar by being holed into each other. They are then driven up as a single breast by longwall advancing. The breast, however, has two fast ends, one on each side of the panel, which must be sheared as the face
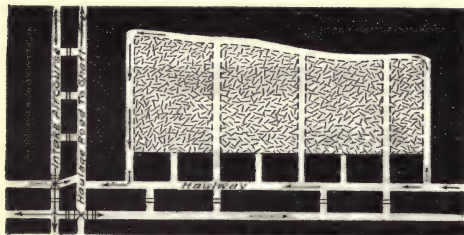


FIG. 1

is advanced. Each panel has a separate ventilating current and may be stopped off in case of a fire or a squeeze without interfering with the rest of the mine. The methods of working inclined seams previously described are practically panel methods.

The modified longwall system followed at Vintondale, Pa,. is shown in Fig. 2. The pitch of the seam is 8%, or about 4½°, and the coal is about 3 ft. 8 in. thick, the roof being blue slate, and the floor a hard fireclay. The cover is a fairly hard sandstone, 174 ft. thick. The main entry a, is driven directly up the pitch and centrally in the coal field, bisecting it between the outcrops. This main entry is paralleled by two airways b as shown, one on each side for the purpose of ventilating each side independently. On the right side of the figure, at c, is shown one method of laying out the faces parallel to the dip. At the end of the cross-entry d, an ordinary room is turned up the pitch and driven practically parallel to the main heading to a connection with the next pair of cross-entries inby, as shown at e. The outby rib of this wide room forms the longwall face, which is worked back toward the main entry. Empty mine cars are taken to the faces on the lower road of the
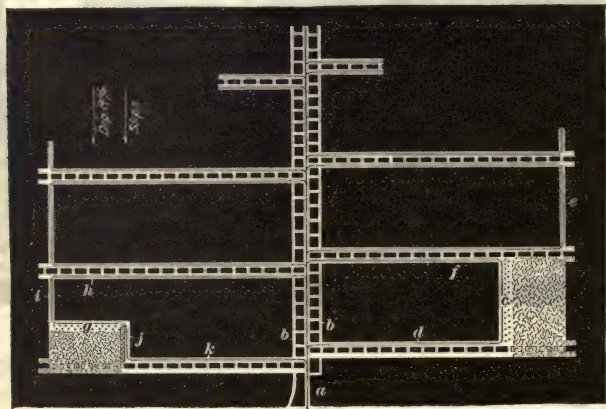


FIG. 2

inner cross-headings f and hauled along the face c, where they are loaded and hauled out on the upper road of the lower cross-headings d to the main heading.

On the left of the diagram at $g$, is shown the method of laying out faces so that they are 90° with the line of pitch, so that the full influence of any roof pressure due to the pitch is exerted fully on the gob. In this case, the empty mine cars are taken into the lower road of the upper pair of cross-entries $h$, dropped through the room $i$, which connects the pair of cross-entries, then to the face $g$ along which they are loaded, thence dropped through a gob roadway $j$ to the upper road of the lower pair of cross-entries $k$. The gob roadway $j$ has solid coal for one of its ribs, and is maintained by means of a substantial timber crib, built as the face progresses. On the left of the diagram, the faces advance directly up hill.

**Longwall in Thick Seams.**—A thick seam of coal is usually worked in several benches of moderate thickness. The longwall method is variously modified to suit the conditions, but the plan generally adopted, and which has given the best results in France, Bohemia, and other countries where such seams are worked, both with respect to the safety of the working and the percentage of coal obtained from the seam, is that of *close packing*, or completely filling with waste the space from which the coal is taken. Sufficient waste is not produced, ordinarily, in the working of the seam, and waste material is brought from the surface to fill in this space.



**(a)**

**(b)**
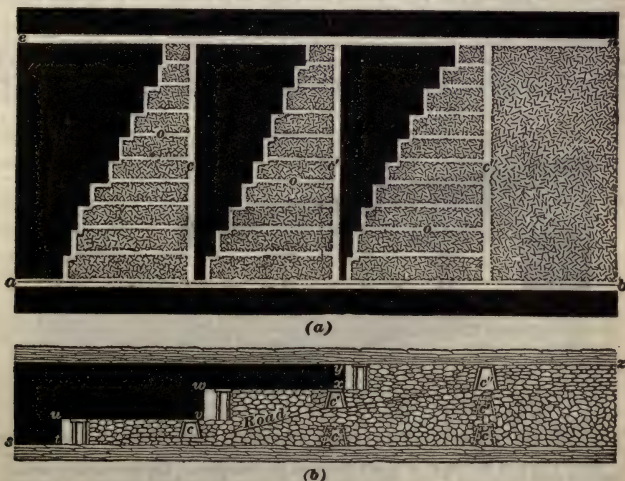
Fig. 3

In the working of flat seams by this method, a longwall face is carried forwards in each bench of the coal, the face in each bench being kept from 80 to 100 yd. in advance of that in the bench next above. Fig. 3 shows the general plan and cross-section of the workings at the face of a thick flat seam, the seam being divided, as shown, into three benches. The first mining is done in the lowest bench, in which the parallel main roads $ab$ and $en$ are driven and connected by the cross-roads $c$. The roads $o$ that lead to the face are protected by packs, and correspond to the temporary roads, or working places in the general longwall method. Above the cross-roads $c$ and from 80 to 100 yd. back from their faces are the cross-roads $c'$ in the second bench. Above the cross-roads $c'$ and from 80 to 100 yd. back from their faces are the cross-roads $c''$ in the third bench. As the temporary roads $o$ in the lower bench attain a length equal to the advance between the cross-roads $c$, they are cut off by a new cross-road in the lowest bench of the seam. At the same time, the corresponding temporary roads $o$ in each of the benches above and which are in the

same line with the roads o in the lowest bench reach their limits. The roof of the cross-roads c in the lowest bench is then ripped or taken down and the road is packed with waste on which a new cross-road c' is laid in the bench above. In like manner, the roof of each cross-road c' in the second bench is taken down, and the old road is packed with waste on which the new cross-road is laid in the bench above, as before. The main roads on each side are treated in the same manner, the road being graded from the cross-road in the lower bench to the cross-road in the upper bench, as indicated in the cross-section by the dotted line across the gob and marked *road*. This inclined road advances with the work in each bench.

**Longwall in Inclined Thick Seams.**—In the working of inclined thick seams, when the inclination of the seam is moderate, the method just described may be used for the removal of the coal. In steeper inclinations, a slope road is driven in the lower bench of the coal on the floor of the seam, and gangways, or levels, are driven to the right and left in the seam from this slope. Cross-drifts are then driven from these gangways across the seam to the roof rock, at intervals varying from 16 to 20 yd. At the roof, they are holed across from one to the other, and the coal drawn back, on the retreating method, in horizontal slices from 5 to 6 ft. in thickness. The face is also broken into steps between the cross-drifts, and as each slice of coal is taken out, it is often necessary to fill in the space with waste.

**Longwall in Contiguous Seams.**—In the application of longwall to contiguous seams, the methods of work at the face do not differ from those already described. Seams have been worked in England as one seam when separated by a slate parting 7 ft. in thickness. In this particular case, the lower seam was 7 ft., and the upper seam 2 ft., in thickness. A general method for work-
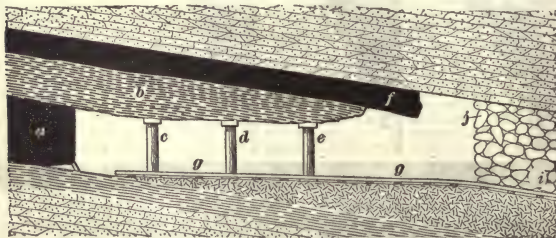


FIG. 4

ing contiguous seams is shown in Fig. 4, which is a cross-section of the working face. The lower seam a is worked first, the slate parting b being supported on three rows of props c, d, e set parallel to the face. By the withdrawal of the rear row of props e, the slate parting falls or is wedged down. The work of taking down the upper seam f then follows. The slate parting, as it falls, is broken up and leveled off, thus forming a convenient flat g at the top of the incline i. The roof above the incline is supported by pack walls j.

When contiguous seams are worked separately by the longwall method, either by driving cross-tunnels between the different seams, or by separate slopes or inclines, the longwall face in an overlying seam should generally be kept in advance of that in a lower one, as the working of an underlying seam by longwall will usually result in the crushing and crevicing of overlying seams to a certain extent.

## DETAILS OF LONGWALL WORKING

**Starting Longwall.**—There are two methods of starting longwall workings. In the first, the work of extraction begins at the shaft itself, the coal being taken out all around and its place filled with solid packs, leaving only space for the roadways. In the second method, a pillar of solid coal is left to support the shaft, cut only by the roadways. The longwall work is then started uniformly all around this pillar. Great care is needed in building the first pack

walls around the shaft pillar, to see that they are solidly built and well rammed, in order to break the roof over the coal. The system will not work rightly, however, until the breast has been advanced some distance from the pillar, so as to secure the benefit from the weighting action of the roof upon the coal face. The mining will be more difficult in the start, and in some exceptional cases it may even be necessary to place some light shots; this, however, should be avoided, if possible.

**Roadways.**—The general plan of laying out the roads is shown on pages 652, 655, and 656. The temporary roads connect the working places, or rooms, with the cross-roads, but as they are cut off from time to time by other cross-roads they are not protected by as substantial pack walls as the other roads. The length of the temporary roads may be different for each section of the mine, and must be determined in each case as the work progresses. A cross-road is started off the main road whenever the temporary road gives signs of closing. It will thus be seen that the distance between cross-roads, measured on the main entries, may not be a uniform distance for the different sections of the mine, and may even vary in the same section. Again, owing to a creep or crush closing some rooms, it may be necessary to turn a short stub road directly across the heads of such rooms or working places, the old roads in this case being gobbed tightly to counteract the effect of the squeeze. The distance between the temporary roads is decided mainly by the possibility of taking the cars along the face, which, in turn, depends on the clear width it is possible to keep open between the face of the coal and packs. When it can be done, roadways are laid along the face and protected by timbers. At the junction of the track along the face with the track running into the room, a turntable consisting of an iron plate that allows the car to be turned is sometimes used. This track may be made of oak mine rails spiked to cross-ties, or of light iron rails, held together by spreaders of $\frac{1}{4}'' \times 1\frac{1}{2}''$ strap iron.

If the mine car cannot be taken along the face and there is a hard, smooth bottom, the coal is often loaded on a sled, or buggy, which is dragged along the face to the head of the temporary road, where it is loaded into a car. The distance between the temporary roads will then depend on the distance to which the coal can be thus conveniently carried, but is usually from 40 to 60 yd., though it has reached 100 yd. in exceptional cases.

When it is necessary to shovel the coal to the road head where it is loaded or to wheel it in barrows along the face, the roads are made from 15 to 20 yd. apart, center to center.

In some mines a motor-driven conveyer laid along the face is used to carry the coal to one of the roads where it is loaded into mine cars. The conveyer is usually of the trough type, is moved forwards bodily as the face is advanced, and works well where the face is reasonably straight.

**Control of Roof Pressure.**—The removal of the coal and the slow advance of the working face is followed by a slow, but irresistible, downward movement of the cover or overburden, known as *settlement*. The immediate effect of this is to produce breaks in the roof strata over the area from which the coal has been taken, more or less parallel to the working face, and, consequently at right angles to the line of advance. The effect of removing the coal is to divide the overburden into two portions:

1. An underweight of broken material from 10 to 40 ft. thick which may be likened to heavy falls of the rocks immediately overlying the seam in room-and-pillar work. The weight of this broken material is small, compared with the overweight, and it may be temporarily supported by the timbers and by the face of the coal and the packs. By the withdrawal of the timber next the packs, the weight is thrown, or settles, forwards on the coal and breaks it. The amount of the underweight thrown on the coal face is controlled, as far as possible, by the posts set parallel to the face. The amount of timber, number of rows, and the distances apart of the rows and of the timbers in each row depend on the conditions at the face. For given conditions of roof and floor, more weight is thrown on the face of the coal by decreasing the amount of timber, while increasing the timber decreases the weight on the coal. With a hard roof and floor, the posts should be set on some soft material, or be provided with a thick soft cap that will yield and allow the post to take the weight gradually, or the post is sometimes tapered at the end for the same purpose.

2. An overweight due to the weight of the rocks from the top of that portion of them causing the underweight to the surface, and which is practically equal to the weight of the entire overburden. It is irresistible and its effect is to compress the pack walls and gob into spaces between the walls until the resistance offered to the weight is practically equal to that of the coal that has

been removed. The amount of settlement due to this overweight is regulated mainly by the proportion of pack walls to the entire area from which the coal has been removed and by the manner of building these pack walls.

The aim in longwall work is to so control the roof pressure that it may be just sufficient to break the coal from the face and yet not crush it. When this pressure can be controlled, it is done by means of the number and size of the pack walls, by rows of props placed parallel to the face, and by varying the open space left along the face between the coal and the pack walls. Experience under the given conditions alone determines in just what way and to what extent this control may be secured. The pack walls provide for a gradual and uniform settlement of the roof over the entire area from which the coal has been removed. They relieve the timbers at the face of a great part of the weight they would otherwise have to carry and permit the weight on the coal face to be regulated largely by means of timbers so that the coal may be properly broken and not crushed.

Excessive weight on the face of the coal is shown by increased hardness of the under clay and the increased difficulty of undercutting in it, and also by the crushing and nipping of the coal. This excessive weight may be due to too small packs and too little timbering, or to the attempt to carry too wide an area between the packs and the face. The remedy is to increase the amount of timber and the size or number of packs, or to decrease the distance between the face and the packs.

Too small a weight on the face of the coal is evidenced by the slow breaking of the coal. This may be due to too large a proportion of packs, to too many timbers, or to too narrow a space between the face of the coal and the packs. The remedy is to decrease the proportion of packs or the amount of timber, or to increase the open width at the face.

**Building Pack Walls and Stowing.**—Pack walls should be built large enough at first and kept well up to the face, to prevent the weight coming upon the timber and also to permit the roof to settle rapidly when the timber is taken out of the face. Often the roof will not stand this second movement without breaking, and possibly closing in the entire face. The face should therefore be kept in shape, and just as soon as there is room for a prop or chock, it should be put in immediately, and the pack walls likewise should be extended after each cut or web is loaded out.

As a general thing, the pack walls in the gob are not so wide as the road-side ones, particularly when the seam produces enough waste material to stow the *marches,* *cundies,* or *gobs,* between these pack walls. Usually about 50% of the cubic contents of the solid seam taken out will stow the spaces between the pack walls in thick pitching seams, where the entire gob must be completely filled or nearly so. No waste material, except such as will hasten spontaneous combustion, should be taken out of the mine to the surface.



**Timbering a Longwall Face.**—The method of timbering the working face depends on the nature of the roof, floor, coal, etc. The action of the roof on the coal face is regulated almost entirely by timber; consequently, when the coal is of such a nature as to require little weight to make it mine easily, the roof must be timbered with rows of chocks and, if necessary, a few props.

The ends of all stone packs nearest the face of the coal should be in line, and the ends of these pack walls should form a line parallel to the face of the coal. Timbers set at equal distances and in line along a longwall face are much more efficient in supporting the roof than irregularly set timbers. The accompanying figure shows the proper way of locating the pack walls and the face timber.

# EXPLOSIVES AND BLASTING

## CLASSIFICATION OF EXPLOSIVES

The characteristics of a good blasting explosive are: (1) Sufficient stability or difficulty of detonating by mechanical shock, and strength; (2) convenience in form and safety in handling; (3) absence of injurious effects upon the user.

Explosives are of two general classes: low explosives, or those discharged by fire, and high explosives, or those that require a detonator.

**Low Explosives.**—Ordinary gunpowder and black, or blasting, powder are low explosives. Blasting powder consists of 73 parts of Chili saltpeter (nitrate of soda, $NaNO_3$), 16 parts of charcoal, and 11 parts of sulphur. Blasting powder is graded according to the size of the grains passing through and over various sizes of round holes in sieves. In low explosives, the force of shattering effect is produced by the rapid combination of the oxygen of the saltpeter with the carbon of the charcoal; the sulphur is added merely to make the mixture more easily ignitable.

### SIZES OF GRAINS OF BLACK BLASTING POWDER

| Trade Name | Through | Over |
|---|---|---|
| CCC | $\frac{40}{64}$ | $\frac{32}{64}$ |
| CC | $\frac{34}{64}$ | $\frac{14}{64}$ |
| C | $\frac{24}{64}$ | $\frac{12}{64}$ |
| F | $\frac{20}{64}$ | $\frac{12}{64}$ |
| FF | $\frac{14}{64}$ | $\frac{7}{64}$ |
| FFF | $\frac{9}{64}$ | $\frac{6}{64}$ |
| FFFF | $\frac{8}{64}$ | $\frac{4}{64}$ |

**High Explosives.**—High explosives, which are very commonly called *dynamites* regardless of their composition, usually consist of a base or absorbent known as *dope*, which may or may not be an aid to the explosive, and an explosive proper. If the active agent is a liquid, as it is in the case of true dynamites, the dope is added to hold or absorb the explosive; if the active agent is a solid, the base is added to reduce the temperature of the gaseous products of combustion or to shorten the length of the flame produced by their burning, and, in general, to render their use safer. The high explosives generally derive their names from those of their active principle.

*Nitroglycerin* is prepared by slowly running glycerin into a mixture of concentrated nitric and sulphuric acids, the bath being stirred and kept cool during the process of mixing. It is a dense, oily liquid, white when pure but of a yellowish cast as found on the market. It is very poisonous, not only when taken through the mouth but also through the inhalation of its fumes, producing violent headaches which, however, tend to diminish in intensity after repeated exposures. Nitroglycerin may freeze after some exposure to a temperature of 52° F. It explodes when confined at 360° F. It takes fire at 306° F. and, if unconfined, usually burns harmlessly unless in large quantities, so that a part of it, before coming in contact with the air, becomes heated to the exploding point.

*Picric acid*, which forms the basis of many explosives, is made by treating carbolic acid (derived from coal tar) with concentrated nitric acid.

*Ammonium nitrate* is a salt similar to sodium and potassium nitrate and is largely used in the manufacture of a class of explosives known as *ammonium* (or *ammonia*) *nitrate* powders, *ammonia dynamite*, etc. It has been used in the United States for nearly 30 yr. in the manufacture of some dynamites, taking the place of sodium nitrate over which it has the advantage that upon exploding it goes completely into gases. It is also largely used here and abroad in the making of the so-called *permissible*, or *permitted*, *powder* for use in gaseous and dusty coal mines, for the reason that upon exploding it forms large amounts of water, and this lowers the temperature of all the products of combustion.

# EXPLOSIVES FOR ROCK WORK

Explosives for rock work in mining are all of the high-explosive type. The original form is dynamite, which consists of a base, usually wood pulp, with which the explosive or explosives are mixed.

**Straight Nitroglycerin Dynamite.**—By a *straight dynamite* is meant one that contains no other high explosive than nitroglycerin. Straight dynamites are graded according to the percentage of nitroglycerin they contain and their strength is made the basis of comparing the strengths of dynamites of other types. Thus, a 40%-ammonia dynamite means a dynamite that has the same strength as a 40% straight nitroglycerin dynamite. The Un ted States Bureau of Mines has given the subject of explosives considerable study, and in this connection the results of its investigations are freely used. The compositions of straight nitroglycerin dynamites are given in the accompanying table.

## COMPOSITIONS OF STRAIGHT NITROGLYCERIN DYNAMITES OF VARIOUS STRENGTHS

| Ingredients | 15% | 20% | 25% | 30% | 35% | 40% | 45% | 50% | 55% | 60% |
|---|---|---|---|---|---|---|---|---|---|---|
| Nitroglycerin............ | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| Combustible material* ... | 20 | 19 | 18 | 17 | 16 | 15 | 14 | 14 | 15 | 16 |
| Sodium nitrate.......... | 64 | 60 | 56 | 52 | 48 | 44 | 40 | 35 | 29 | 23 |
| Calcium or magnesium carbonate............ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

*Consisting of wood pulp, flour, and sulphur for grades below 40%; wood pulp only for other grades.

The straight nitroglycerin dynamites, as a class, develop greater disruptive or shattering force than any of the other commercial types of explosives, and for this reason they should be used, whenever the conditions permit, for producing shattering effects or for blasting very tough or hard materials.

**Slow, or Low-Freezing, Dynamites.**—The accompanying table shows typical compositions of low-freezing dynamites. The slow, or low-freezing, dynamites have the advantage of not freezing until exposed for a considerable time to a temperature of 35° F. or less, but, like all nitroglycerin explosives, after they become frozen they must be thawed before used in order to insure the most effective results.

## COMPOSITIONS OF LOW-FREEZING DYNAMITES OF VARIOUS STRENGTHS

| Ingredients | 30% strength | 35% strength | 40% strength | 45% strength | 50% strength | 55% strength | 60% strength |
|---|---|---|---|---|---|---|---|
| Nitroglycerin.................. | 23 | 26 | 30 | 34 | 38 | 41 | 45 |
| Nitrosubstitution compounds .... | 7 | 9 | 10 | 11 | 12 | 14 | 15 |
| Combustible material*.......... | 17 | 16 | 15 | 14 | 14 | 15 | 16 |
| Sodium nitrate................. | 52 | 48 | 44 | 40 | 35 | 29 | 23 |
| Calcium or magnesium carbonate. | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

*Composition similar to that in the straight nitroglycerin dynamites.

**Ammonia Dynamites.**—The ammonia dynamites, compared with the other dynamites, have the disadvantage of taking up moisture very readily, because ammonium nitrate is deliquescent, and care should be observed when they are stored or used in wet places. The following table shows typical compositions of ammonia dynamites:

## COMPOSITIONS OF AMMONIA DYNAMITES OF VARIOUS STRENGTHS

| Ingredients | 30% strength | 35% strength | 40% strength | 50% strength | 60% strength |
|---|---|---|---|---|---|
| Nitroglycerin.......... | 15 | 20 | 22 | 27 | 35 |
| Ammonium nitrate .... | 15 | 15 | 20 | 25 | 30 |
| Sodium nitrate........ | 51 | 48 | 42 | 36 | 24 |
| Combustible material*. | 18 | 16 | 15 | 11 | 10 |
| Calcium carbonate or zinc oxide.......... | 1 | 1 | 1 | 1 | 1 |

*Composition similar to that in the straight nitroglycerin dynamites of the grades below 40%.

**Gelatin Dynamites.**—The gelatin dynamites have been used to a large extent in wet blasting, as in the removal of obstacles to navigation and in deep workings, and as a general rule are best suited for these purposes. In the manufacture of these dynamites, the nitroglycerin is gelatinized by the addition of a small percentage of nitrocellulose. The jelly-like mass thus formed

## COMPOSITIONS OF GELATIN DYNAMITES OF VARIOUS STRENGTHS

| Ingredients | 30% strength | 35% strength | 40% strength | 50% strength | 55% strength | 60% strength | 70% strength |
|---|---|---|---|---|---|---|---|
| Nitroglycerin............. | 23.0 | 28.0 | 33.0 | 42.0 | 46.0 | 50.0 | 60.0 |
| Nitrocellulose............. | 0.7 | 0.9 | 1.0 | 1.5 | 1.7 | 1.9 | 2.4 |
| Sodium nitrate........... | 62.3 | 58.1 | 52.0 | 45.5 | 42.3 | 38.1 | 29.6 |
| Combustible material..... | 13.0 | 12.0 | 13.0 | 10.0 | 9.0 | 9.0 | 7.0 |
| Calcium carbonate........ | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |

is impervious to water and is of high density and plasticity. For these reasons, it is generally preferred for tunneling in hard rock. By the addition of different percentages of suitable absorbents the various grades of these dynamites are made.

The compositions of gelatin dynamites generally offered for sale in this country are given in the accompanying table. The combustible material

## ANALYSES OF HIGH EXPLOSIVES

| Constituent | 40% Strength Low-Freezing Dynamite | 40% Straight Nitro-glycerin Dynamite | 40% Strength Ammonia Dynamite | 40% Strength Gelatin Dynamite |
|---|---|---|---|---|
| Moisture.................. | 1.13 | .97 | .88 | 1.47 |
| Nitroglycerin.............. | 27.56 | 39.19 | 21.60 | 30.70 |
| Nitrocellulose............. | | | | .88 |
| Ammonium nitrate......... | | | 18.86 | |
| Nitrotoluene.............. | 10.13 | | | |
| Sodium nitrate............ | 51.54 | 49.53 | 46.04 | 54.27 |
| Wood pulp................ | 8.52 | 9.77 | 5.45 | 8.58 |
| Sulphur.................. | | | 4.85 | 3.08 |
| Zinc oxide................ | | | .88 | 1.02 |
| Calcium carbonate.......... | 1.12 | .54 | 1.44 | |

## ANALYSES OF MINE AIR AFTER BLASTING

| Class and Grade of Explosive Used | Quantity of Explosive Used Pounds | Time Sample Was Taken | Analyses of Mine-Atmosphere Samples | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | $CO_2$ | $O_2$ | $CO$ | $N_2$ | $H_2S$ | $H_2$ | $CH_4$ and $C_2H_4$ |
| 40% strength gelatin dynamite | 103.5 | Immediately after firing blast | 2.67 | 19.53 | .25 | 77.44 | .0 | .11 | .0 |
| | | 5 min. after blast | 1.75 | 19.73 | .12 | 78.34 | .0 | .06 | .0 |
| | | 10 min. after blast | .43 | 20.31 | .10 | 79.11 | .0 | .05 | .0 |
| 40% strength gelatin dynamite | 39.5 | Immediately after firing blast | .25 | 20.38 | .05 | 79.29 | .0 | .03 | .0 |
| | | 5 min. after blast | .13 | 20.70 | .00 | 79.15 | .0 | .00 | .0 |
| | | 10 min. after blast | .37 | 20.54 | .00 | 79.33 | .0 | .00 | .0 |
| New 40% strength gelatin dynamite | 28.5 | Immediately after firing blast | .10 | 20.80 | .00 | 79.20 | .0 | .00 | .0 |
| | | 10 min. after blast | | | | | | | |
| 40% straight nitro-glycerin dynamite | 22.0 | Immediately after firing blast | 1.65 | 20.40 | .40 | 79.10 | .0 | .20 | .0 |
| | | 2 min. after blast | .27 | 20.45 | .15 | 78.87 | .0 | .06 | .0 |
| | | 12 min. after blast | .08 | 20.80 | .00 | 79.12 | .0 | .00 | .0 |

in the 60 and 70% strength gelatin dynamite is wood pulp. Sulphur, flour, wood pulp, and sometimes resin are used in other grades. Some manufacturers replace a small percentage of the nitroglycerin in these explosives with an equal amount of ammonium nitrate. Ammonia-gelatin dynamites have somewhat the same advantages over straight gelatin dynamites that ammonia dynamites have over straight dynamites, except that they are no slower in velocity of detonation, but in certain classes of work the fumes are less objectionable.

**Comparative Analyses.** The analyses on page 669 show the composition of the four classes of high explosives, each being rated as a 40% dynamite. It will be noted that the only one of these analyses showing an approximate content of 40% of nitroglycerin is the straight dynamite in the second column, which is the basis of comparison.

**Products of Combustion.** The analyses of mine air taken at West Winfield, Pa., while blasting operations were being carried on in the usual way are given in the accompanying table. They are of interest in showing that where the ventilation is good, the contamination of mine air by the products of combustion of high explosives is very slight, even in the case of straight dynamites which yield a large amount of $CO$.

Dynamite is frequently condemned for producing injurious fumes, when, as a matter of fact, these fumes were made by the partial burning of the dynamite before its explosion, the dynamite having been lighted by the fuse before the fire reached the cap. An experienced person can readily distinguish between the fumes produced by burning dynamite, and those produced by detonation.

**Comparative Strength of Explosives.** — The following table gives the relative strengths of the different high explosives as determined from an extended series of tests

by the Bureau of Mines. In all cases 40% straight nitroglycerin dynamite has been taken as the standard with a value of 100%. The relative disruptive and propulsive effects here tabulated are shown graphically in the accompanying figure. Disruptive effect indicated represents averages of energies developed in the Trauzl lead-block, small lead block, and rate-of-detonation tests; propulsive effect indicated represents averages of ballistic-pendulum and pressure-gage tests. The figures given in this table are fairly consistent with general practice, and it is believed that the classification will serve

### RESULTS OF TESTS TO DETERMINE POTENTIAL ENERGY AND DISRUPTIVE AND PROPULSIVE EFFECTS OF EXPLOSIVES

| Class and Grade | Percentage Strength Representing Potential Energy | Average Percentage Strength Representing Disruptive Effect | Average Percentage Strength Representing Propulsive Effect |
|---|---|---|---|
| 30% straight nitroglycerin dynamite.. | 93.1 | 84.1 | 96.8 |
| 40% straight nitroglycerin dynamite.. | 100.0 | 100.0 | 100.0 |
| 50% straight nitroglycerin dynamite.. | 111.0 | 109.2 | 107.4 |
| 60% straight nitroglycerin dynamite.. | 104.0 | 119.8 | 114.9 |
| 60% strength low-freezing dynamite.. | 60.2 | 93.5 | 91.2 |
| 40% strength ammonia dynamite..... | 101.8 | 67.9 | 99.1 |
| 40% strength gelatin dynamite....... | 105.7 | 78.4 | 95.8 |
| 5% granulated nitroglycerin powder... | 67.6 | 21.6 | 53.3 |
| Black blasting powder.............. | 71.6 | 6.8 | 58.6 |

as a useful guide for comparing the practical value of explosives. It is worthy of note that the potential energy of 40% strength ammonia dynamite and of 40% strength gelatin dynamite (that is, the theoretical maximum work that these explosives can accomplish) is higher than that of 40% straight nitroglycerin dynamite, but that the disruptive and propulsive effects, which represent the useful work done as shown by actual tests, are less than those of 40% dynamite. While it is true that straight dynamites possess greater

EXPLOSIVE | ENERGY

Disruptive Effect, Per Cent. | Propulsive Effect, Per Cent.



Black powder (FFF).......

5% granulated powder...

40% ammonia dynamite..

40% gelatin dynamite.....

30% nitroglycerin dynamite

60% low-freezing dynamite

40% nitroglycerin dynamite

50% nitroglycerin dynamite

60% nitroglycerin dynamite
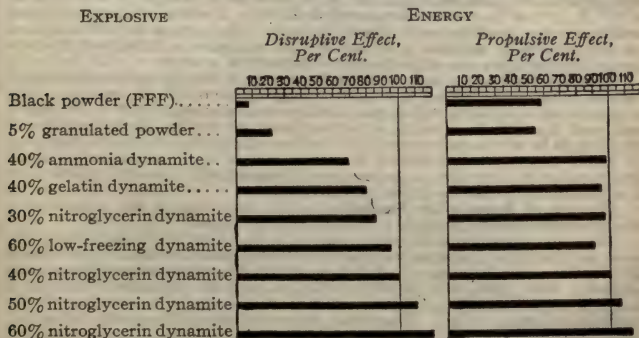
shattering effect than other standard types of explosives they are being rapidly displaced by the ammonia and gelatine explosives on account of the greater safety in handling characteristic of the latter. In hard, brittle rock, especially in chambered holes, the straight dynamites are more effective than the slower acting ones, but in softer rocks like sandstone, calcite, marl, etc., explosives of

the gelatin and ammonia types are very much more effective. Ordinary black blasting powder has only about one-third of the disruptive effect of granulated nitroglycerin powder.

## EXPLOSIVES FOR COAL MINES

As the object in practically all coal mines is to obtain as large a proportion of lump coal as possible, the quick-acting, powerful, high explosives are not well adapted to use. In their stead, the slower acting low explosives are greatly to be preferred. These coal mine explosives are of two general kinds: Black powder and the so-called permissible powders.

*Black powder* is an excellent explosive, but, unfortunately, is not always safe for coal mine use. Unless skilfully handled, its long flame is practically certain to ignite dust or methane when more than a small amount of these is present in the workings.

*Permissible,* or *permitted, explosives* have come into use in the United States within the past few years, although they have has been used in Europe for a much longer time. The original terms, *flameless explosive* and *safety explosive* have been dropped for the ones given, as none of them is absolutely flameless or absolutely safe.

In the United States, it is a function of the Bureau of Mines to determine by tests what explosives are relatively safe under the dangerous conditions of gas and dust prevailing in coal mines.

In determining this, the following rules are observed: The charge of explosive to be fired in tests 1 and 2 shall be equal in deflective power to $\frac{1}{2}$ lb. (227 grams) of 40% nitroglycerin dynamite of the composition given on page 669. Each charge shall be fired with an electric detonator, which will completely explode the charge, as recommended by the manufacturer. In order that the dust used in tests 2 and 3 may be of the same quality, it is always taken from the same mine, ground to the same fineness, and used while still fresh.

If a powder passes the three following tests made in the explosion gallery of the Bureau's testing station, at Pittsburg, Pa., it may be placed upon the list of permissible explosives.

**Test 1.**—Ten shots each with the charge described, in its original wrapper, shall be fired, each tamped with 1 lb.* of clay stemming, at a gallery temperature of 77° F., into a' mixture of gas and air containing 8% of gas (methane and ethane). An explosive is considered to have passed the test if no one of the ten shots ignites this mixture.

**Test 2.**—Ten shots each with the charge described, in its original wrapper, shall be fired, each tamped with 1 lb.* of clay stemming, at a gallery temperature of 77° F., into 40 lb. of bituminous-coal dust, 20 lb. of which is to be distributed uniformly on a wooden bench placed in front of the cannon and 20 lb. placed on side shelves in sections 4, 5, and 6. An explosive is considered to have passed the test if no one of the ten shots ignites this mixture.

**Test 3.**—Five shots each with 1$\frac{1}{2}$ lb. charge, in its original wrapper, shall be fired without stemming, at a gallery temperature of 77° F., into a mixture of gas and air containing 4% of gas (methane and ethane) and 20 lb. of bituminous-coal dust, 18 lb. of which is to be placed on shelves along the sides of the first 20 ft. of the gallery and 2 lb. to be so placed that it will be stirred up by an air-current in such manner that all or part of it will be suspended in the first division of the gallery. An explosive is considered to have passed the test if no one of the five shots ignites this mixture.

**Classes of Permissible Explosives.**—The following is the classification of permissible explosives made by the Bureau of Mines:

*Class 1. Ammonium-Nitrate Explosives.*—To Class 1 belong all the explosives in which the characteristic material is ammonium nitrate. The class is divided into two subclasses. Subclass *a* includes every ammonium-nitrate explosive that contains a sensitizer that is itself an explosive. Subclass *b* includes every ammonium-nitrate explosive that contains a sensitizer that is not in itself an explosive. When fresh, these explosives, if properly detonated, have the advantage of producing only small quantities of poisonous and inflammable gases, and are adapted for mines that are not unusually wet, and also for mines and working places that are not well ventilated.

*Class 2. Hydrated Explosives.*—To Class 2 belong all explosives in which salts containing water of crystallization are the characteristic materials. The

*Two lb. of clay stemming is used with slow-burning explosives.

explosives of this class are somewhat similar in composition to the ordinary low-grade dynamites, except that one or more salts containing water of crystallization are added to reduce the flame temperature. They are easily detonated, produce only small quantities of poisonous gases, and most of them can be used successfully in damp working places.

*Class 3. Organic Nitrate Explosives.*—To Class 3 belong all the explosives in which the characteristic material is an organic nitrate other than nitroglycerin. The permissible explosives listed under class 3 are nitrostarch explosives. They produce small quantities of poisonous gases on detonation.

*Class 4. Nitroglycerin Explosives.*—To Class 4 belong all the explosives in which the characteristic material is nitroglycerin. These explosives contain free water or an excess of carbon, which is added to reduce the flame temperature. A few explosives of this class contain salts that reduce the strength and shattering effect of the explosives on detonation. The nitroglycerin explosives have the advantages of detonating easily and of not being readily affected by moisture. On detonation some of them produce poisonous and inflammable gases equal in quantity to those produced by black blasting powder, and for this reason they should not be used in mines or working places that are not well ventilated.

## CARE OF EXPLOSIVES

**Storing Explosives.**—Dynamite cartridges should always be laid on the side and not stood on end, for in the latter position the nitroglycerin may ooze out from the dope and collect in the bottom of the cartridge. Dynamite should never be kept for any length of time (as in storage magazines) at a temperature greater than 75° F. It should be stored in a dry place having a reasonably uniform temperature. Magazines should be heated by means of hot-water or exhaust-steam pipes, never by a stove or live-steam pipes. There should, preferably, be two powder houses or magazines at a mine. The main magazine, holding the stock of explosive on hand, should be built sufficiently far from the plant or with some natural obstruction (such as a hill) between it and the plant that its accidental explosion may not injure the miners' village or the surface equipment. This magazine is commonly under the direct supervision of some one of the higher officials and is usually opened only to receive supplies in carload lots direct from the manufacturer and to withdraw the daily amount required by the men. Near the mine mouth is a smaller magazine, commonly called the powder house, to which the day's supply is taken from the main magazine, and where it is handed out to the men individually. All main magazines should be proof against high-power rifle bullets fired at short range. Storing explosives in large quantities in a mine is a bad and dangerous practice and in most states is prohibited by law.

The effect of storing fuse for several days at temperatures much above or below the normal is to greatly retard its rate of burning; commonly to the extent of 50% or more. It follows that fuse should not be stored in too warm a place as over boilers and steam pipes in winter, or in a tin box exposed to the direct rays of the sun in summer, nor should it be left in an unheated tool house when the temperature is much below the freezing point. Fuse is extremely difficult to dry out after wetting, and, as a general rule, fuse that has been stored in damp places or has in any way become wet should not be used in mining.

**Thawing Dynamite.**—Dynamite freezes at about 45° F., and when solidly frozen it is exploded with difficulty, and if it is exploded the detonation is only partial. It is dangerous to cut, break, or ram a frozen dynamite cartridge, as the frozen nitroglycerin crystals may explode. No attempt should be made to explode dynamite that has been frozen until it has been thoroughly thawed and is soft and plastic; many accidents occur through failure to observe this precaution. If incomplete detonation occurs, unexploded powder is often found in the holes or in the material blown down by the shot.

In cold weather, the cartridges should not be taken to the place where they are to be used until all the holes are ready to be loaded, and all cartridges should be soft and warm when charged into the holes. Dynamite that has been chilled, but not frozen, looses a large part of its efficiency. Many instances are on record in which some of the holes of a blast were loaded with warm, and others with frozen, or partially frozen, dynamite; the dynamite that had been warmed exploded and that which was frozen did not, and miners have subsequently been killed or injured by drilling into these misshots.

When thawing dynamite, it is necessary to use caution to keep the temperature from rising very high, as each degree rise is that much nearer the danger limit where extreme sensitiveness to shock prevails. The thawing of dynamite by placing it in a tight box surrounded by manure is a good method if the manure is fresh so that it is giving off heat. Dynamite should never be thawed before an open fire, on a shovel, in a tin can, or in an oven, for, while dynamite will very frequently burn in the open and when unconfined, it very often explodes. Also, it should never be thawed by immersion in hot water as that has a tendency to leach out the nitroglycerin and make it dangerous. The common practice of thawing dynamite cartridges by passing them through an oven or over a lighted candle is very dangerous.

When dynamite is being used on a large scale during the winter, it is well to provide a special thawing room, in which 1 or 2 days' supply can be kept ready for use. The room need not be of great size, say 12 ft. × 16 ft. The powder needed for the day's consumption can be carried in during the afternoon and left over night, the boxes simply being opened or the explosive taken out and put on shelves, the procedure depending on the time available. A thermometer should be consulted to insure that the temperature does not rise above 85° and is preferably kept between 75° and 80° F. If a brick or stone vault is made below the surface and tightly roofed over and banked with earth, dynamite may be kept in it all winter without freezing.

For handling smaller quantities of frozen dynamite, special thawing kettles are used to advantage. One device consists of a metal can having tubes that pass through it. The tubes are surrounded by water, and the whole so arranged that a miner's lamp or candle may be placed underneath the can to keep the water warm. When in use, the tubes are filled with sticks of dynamite, the space surrounding them is filled with water, and the cover slipped over so that the cartridges cannot fall out of the tubes. A lamp or candle placed under the can will soon heat the water sufficiently to thaw out the cartridges. These thawers, being portable, are very convenient and, filled with hot water, will keep dynamite in good condition for some time without being artificially heated. It is, however, of great importance that the water-receptacle should always contain water, otherwise an explosion may occur. A double thawing kettle commonly used consists of an outer kettle standing on legs and an inner kettle, which is held up by a bead around the edge, in which the cartridges are placed.

**Handling Explosives.**—While the dangerous practice still prevails to some extent of daily opening the main magazine and there handing out to the men all the explosives they may demand, this method has been very largely superseded by the much safer plan of taking the estimated total daily mine consumption to the small powder house near the opening where only enough for the day's work is given each miner. The amount of explosive is charged to the men either from entries made by the person in charge of the powder house or from written orders or receipts given by the men themselves. At some mines, the men purchase in advance several dollars' worth of so-called *powder checks*, which are pieces of metal stamped with a number of value (12½ c., 25 c., 50 c., etc.) and which may be exchanged at the powder house for explosives and other blasting supplies. After the men have taken their requirements, the explosive remaining is commonly returned to the main magazine, although this may not be done until late in the afternoon for fear it may be needed.

The amount of explosive that a miner may have in his possession or carry into the mine at one time is commonly regulated by law. In rare instances there is no limit placed upon this amount, although a miner will seldom carry in more than a single keg of 25 lb. Where there is a limit, a keg is purchased, but is kept in the magazine, and from it the miner draws his daily allowance of from 5 to 10 lb., which he carries to his working place in a metal canister, preferably of copper. Where permissible powders are used, a day's supply should not exceed 5 lb., as this amount of powder exploded in properly placed holes will, in a seam of average thickness and when undercut as it should be, bring down all the coal a man can load out on one shift.

It is a dangerous practice to carry large metal cans of black powder into the mine upon the shoulder or in mine cars where electric wires charged with current are strung along the roadway. Cars in which powder is being transported should be hauled by mules when the current is shut off the wires. At the working face, the powder can should be stored in a wooden box separate from the caps and fuse. The box should be kept locked when not in use and should be placed some 100 ft. from the face and 25 ft. from the track if this is possible.

Where shot-firers are employed who charge the holes, the mine is commonly divided into a series of districts of a size that two men can charge and fire all the holes therein in a reasonable time. The explosives estimated to be enough for the various districts are placed in separate boxes at the main magazine, conveyed to the nearest point to where they are to be used in a car hauled by a mule, and are there unloaded by the district shot-firer and his helper.

**Precautions When Handling Coal-Mining Explosives.**—The Bureau of Mines suggests that the following precautions should be observed in handling black powder:

Never open a metal keg of powder with a pick or metal object; use the opening provided by the manufacturer of the keg.

Never make up charges or handle cartridges or powder with an open light on the head; place the light at least 5 ft. away on the return-air side so that sparks from it will not fall into the powder.

Never allow powder or other explosive to remain exposed; keep it in a well-locked box at least 100 ft. from the working face and in an unfrequented place.

Never go nearer than 5 ft. to a powder box or powder when wearing an open light or when smoking.

Never use coal slack or coal spalls for stemming; it is dangerous. Use moistened clay, wet wood pulp, or other noninflammable material; even wet coal slack may, under some circumstances, cause an explosion.

Never withdraw a shot that has missed fire; drill a fresh hole at least 2 ft. from it but parallel to the old hole and fire this new hole. After the shot a careful search should be made for the unexploded charge to prevent its being struck by a pick and perhaps causing an explosion.

Never fire the hole the second time; if the first charge proves useless powder and labor are wasted in loading the hole a second time. Moreover, the first shot often cracks the coal so much that the second shot has a chance to blow out of the cracks, and thus a blown-out shot may result.

Never use iron or steel tampers or needles; have at least 6 in. of hard-drawn copper on the tamping end of the bar or, better still, use a hardwood tamping stick. The needle should be made entirely of hard-drawn copper.

Never tamp shots with an iron or steel scraper and do not push a cartridge into the drill hole with the scraper; the scraper rod should be tipped with at least 6 in. of brass or copper on the scraping end.

Never allow the point of the coal auger to become dull or to become of less than the standard gauge, so that a drill hole may be made with it into which the cartridge may always be pushed freely.

Never drill a hole past the loose end, chance, or cutting in solid shooting; if the coal has been undercut, do not drill beyond the undercutting. It is better to stop at least 6 in. short of the solid coal.

Never bore gripping holes; keep the holes parallel to the ribs or as nearly so as possible. Use the side gear on the machine if you can when boring a hole.

Never guess at the quantity of powder to be used; always measure it. This course is cheaper and better than guessing. Use cartridges rather than loose powder and make them of cartridge paper. Don't use newspaper for cartridge making.

Never place black blasting powder in the same drill hole with dynamite or a permissible explosive.

Never use short fuse; always have the fuse long enough to stick out at least 2 in. from the mouth of the drill hole. When the short fuse is lit any gas in the hole may be ignited, and this may result in a premature blast.

Never bite a piece of the match off the squib, nor oil it to make it burn faster.

Never use sulphur and gas squibs at the same working face.

Never light two or more shots at the same time.

Never fire shots in adjoining working faces at the same time.

Never return to a shot that has failed to explode until at least 10 min. after lighting it, if squibs were used, or 12 hr. after lighting if fuse was used. When shots are fired electrically be sure that all wires are disconnected from the battery, and wait at least 5 min. before returning to the face.

Never fire a rib or butt shot before a center or busting shot is fired; the opening shots should be fired first, in order to give the succeeding shots a chance to do their work.

Never drill a hole near the remaining portion of a former shot, nor near cracks and fissures made by previous shots, because there is great danger of the powder gases on explosion flying out of the loose coal or the cracks and igniting gas or dust in the mine air.

Never use squibs or any kind of fuse, except electric fuse, in mines that make inflammable gases.

Never fire a shot without making sure that the coal dust near by is well wet down.

Never light a dependent shot at the same time as another shot, and never fire a dependent shot until the first shot has broken properly.

Never fire a split shot; that is, never fire a hole that has been drilled into a mass of coal, cracked and shattered by a previous shot that failed to dislodge the coal.

The following additional precautions are to be observed in handling permissible explosives:

Never take more than 1 da. supply of permissible explosives into the mine at one time.

Never leave permissible explosives in the mine over night.

Never purchase permissible explosives not suited to the coal bed.

Never use weak detonators.

Never fire a charge until it has been completely and carefully tamped.

Never put black blasting powder and permissible explosives together in the same drill hole.

Never break the covering of a cartridge of a permissible explosive until ready to charge.

Never expect permissible explosives to yield entirely satisfactory results when coal is blasted off the solid.

Never expect the first blast with permissible explosives in a newly opened coal bed to be satisfactory. Several trials are often required before satisfactory results are obtained.

Never forget that permissible explosives are different from dynamite and entirely unlike black blasting powder.

Never use fuse to fire permissible explosives when it is possible to use electric firing.

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## FIRING EXPLOSIVES

### MEANS OF FIRING LOW EXPLOSIVES

Ordinary black blasting powder may be ignited by means of squibs, fuse, and electric squibs. High explosives, including permissible powders, are fired with fuse and caps, or, preferably, by means of electricity, and by means of an electric detonator; in the latter case the current is derived either from a battery or from a dynamo. Fuse and caps are commonly employed for firing single holes, and electric firing is used where several holes are fired at once, or in a volley, although portable electric blasting machines (formerly called batteries) are to be preferred even for firing single holes.

**Squibs.**—A squib (sometimes known as a *match, reed, rush, spire,* etc.) consists of a small paper tube that is filled with quick-burning powder and has a slow match attached at one end. The burning of the slow match gives the miner time to get to a place of safety between the time that he lights the match and the time that the flame reaches the quick powder. When the quick powder is ignited by the burning match, the squib shoots like a rocket through the hole that has been left in the tamping by the withdrawal of the needle into the blasting powder. Two kinds of squibs are in general use, *gas squibs* and *sulphur squibs*. In the gas squib, the match end is impregnated with a composition that does not flame when burning but glows throughout its length, and, for this reason, is supposed to be safe in an explosive mixture of methane and air. In the sulphur squib, the match end is dipped in sulphur and burns with a flame and somewhat faster than a gas squib. Since, in the rocket-like action that is necessary to propel the squib into the charge, large volumes of sparks are given off, neither type of squib can be safe in explosive atmospheres.

**Fuse.**—Fuse, sometimes called *safety fuse* or *Bickford's fuse*, from its inventor, consists essentially of a central core of fine-grained gunpowder wrapped about by threads of hemp, jute, or cotton. These threads are wound in two sets, the inner being known as the *spinning threads* and the outer as the *counter-threads* or, simply, *countering*. In *single-tape fuse*, the threads are wound with tape and then coated with tar and covered with fuller's earth or powdered talc to prevent sticking. *Double-tape fuse* is single-tape fuse wound with a second layer of tape, which is also tarred and powdered. Cotton or hemp fuse, not tape wound, is made, but is only suitable for use in absolutely

dry places and in hot climates. In cold countries, fuse covered with tar is apt to crack and thus become wet and misfire, while in hot countries it becomes sticky and unfit for use. For these reasons special fuses are manufactured for use in either arctic or tropical regions. For use under water, gutta-percha covered fuse has been made.

According to the work in which it is intended to be used, fuse may be divided into four classes. Fuse of the first class is suitable for dry work such as stump blasting and quarrying; it is usually untaped hemp and cotton fuse. Fuse of the second class is intended for damp work, as in coal mining, or in surface work where mud, rain, or dampness is encountered; it is commonly of the single tape variety. Fuse of the third class is suitable for very wet work, such as tunneling, shaft-sinking, etc. Fuse of the fourth class is designed for submarine work; double-tape, triple-tape, gutta-percha, and taped double-countered fuse belong to these classes. Owing to the large amount of carbonaceous material in the wrappings of fuse, the gases produced by its burning contain a large proportion of carbon monoxide, $CO$. Where numerous coils of fuse have caught fire and burned, as has happened in magazines and in the rooms of poorly ventilated mines, the gases evolved have been found to be particularly suffocating and poisonous.

The rate of burning of the better grades of American fuse has been determined by the Bureau of Mines to be very nearly 30 sec, per ft. of length, with a variation of some 10% either way.

**Electric Squibs.**—For use with black blasting powder only, electric squibs are made. They are similar to electric blasting caps in appearance, but the cap is made of paper instead of copper and the charge does not detonate but shoots out a small flame. They are made with iron wires 4, 5, 6, or 8 ft. long, and with copper wires of the same lengths as well as 10 and 12 ft. The wires are insulated and the space between them in the squib is bridged with fine platinum wire, which glows when the electric current is applied and furnishes enough heat to ignite the powder. They are designed to ignite the powder at the center of the charge, something that is obviously impossible with the ordinary paper squib.

## MEANS OF FIRING HIGH EXPLOSIVES

**Fuse and Caps.**—Dynamite and other detonating explosives, including permissible powders, may be fired by means of detonators or caps, but are best exploded by means of electric detonators of the strength prescribed for each one. *Caps, blasting caps, detonators,* or *exploders,* as they are variously called, consist of copper capsules, about as thick as an ordinary lead pencil, that are commonly charged with dry mercury fulminate or with a mixture of dry mercury fulminate and potassium chlorate that is compressed in the bottom of the capsule, filling it to about one-third its length. Several grades of these detonators are on the market, and they are differently designated by different manufacturers. A strong detonator is essential to securing the perfect explosion of permissible powders, etc., and for this purpose those of No. 6 strength, containing 15.4 gr. of charge, are recommended. Fulminate of mercury is extremely sensitive to heat, friction, and blows, and for these reasons blasting caps should be handled with as much care as dynamite, or a violent explosion may result.

The following precautions are recommended in handling them:

Never attempt to pick out any of the composition.

Do not drop caps or strike them with anything hard.

Do not step upon caps or place them where they may be stepped upon.

When crimping caps on to the fuse, take care not to squeeze the fulminate, and never crimp with the teeth.

Caps should be stored in a dry place and in a separate building from any other explosives.

Caps should not be carried into the mine with other explosives, or placed near other explosives except in a bore hole.

**Electric Detonators.** To overcome the dangers incident to the use of fuse and squibs, electric detonators (also called *electric blasting caps*) have been devised. These are simply ordinary detonators that have been fitted with the means of firing them with the electric current. This is done, as shown in the accompanying figure, by inserting within the caps two copper wires $d$, joined

at the inner ends by an extremely fine platinum or other high-resistance wire $e$, which becomes heated until it glows when an electric current is passed through it. This wire, known as the *bridge*, is set within the fulminate $b$, above which is placed a composition $f$ designed to hold the wires more firmly in place. The space above is filled and closed by means of a plug of sulphur or other waterproof composition which is poured in while soft and which is held in place by means of the corrugation $a$, in the shell of the cap. The copper wires $c$ beyond the cap are called *legs* or *wires* and are insulated, and come in lengths of 4 ft. increasing by 2 ft. to 30 ft. For firing charges of permissible powder or high explosives, the detonator used should be at least a No. 6 (double strength) with 15.4 gr. of fulminate. The precautions to be used in handling blasting caps apply with equal force to electric detonators. Heavy jars are particularly to be avoided as they may break the delicate bridge wire; and when this is broken the detonator is worthless. The wires must not be bent sharply or forcibly separated at the point where they enter the copper cap, as this may break or loosen the filling material and permit water to enter and damage the charge.

**Delay-Action Detonators.**—In some kinds of blasting, particularly in tunnel work and shaft sinking, it is necessary to blast the holes in sets or rounds, and it is a saving in time and adds to the safety of the operations if it will not be necessary for the men to return to the face, after the first round has been fired, in order to light the fuses in the second and subsequent rounds. This firing of the holes in sequence, or rounds, when fuse is used is accomplished by cutting the fuse to different lengths (assuming that it all has the same rate of burning per foot) and lighting all the holes as nearly as may be at the same time; the holes exploding in the order of the length of the fuse used. When electric firing is employed, the same result is accomplished with one application of the electric current by means of what are known as *no-delay*, *first-delay*, and *second-delay*, *electric blasting caps*. No-delay caps detonate at the instant the electric current passes through them. The first- and second-delay caps contain a slow-burning substance, which is ignited by the electric spark and which, after burning a short time, ignites the detonating composition below it. It is impossible from a commercial standpoint, to make the rate of burning of this slow-burning composition absolutely uniform and, consequently, *delay-action detonators* in the same circuit may not all explode at exactly the same instant. On the other hand, there is always a distinct time interval between the explosion of the no-delay caps and the quickest of the first-delay, and between the slowest of the first-delay and the quickest of the second-delay. These delay-action detonators are commonly made in the No. 6, or double strength, grade only.

# CHARGING AND FIRING EXPLOSIVES WITH SQUIBS OR WITH CAP AND FUSE

**Charging Black Powder and Firing With Squib.**—Black powder (blasting powder) is used in paper cases or cartridges, which are made and filled by the miner at the face and which are of slightly less diameter than the drill holes, in the bottom of which they are placed. If the hole is dry, these cartridges may be tamped hard enough to break the cartridge paper so that the powder will pack closely and fill all spaces, for the closer the powder is packed in the hole the greater will be the effect produced by the blast. In putting paper cartridges into holes, care should be taken not to break the paper until the bottom of the hole is reached. In case the hole is in seamy rock, a ball of clay is first put in the hole and then a clay bar driven into it, to spread it out and fill all crevices. If these crevices are not filled, the gases due to the explosion escape through them and much of the force of the explosive is lost. The *clay bar* is a good hickory or oak stick with a slightly pointed iron shoe at one end, or is an iron bar pointed at one end with an eye at the other for removal from the hole. To make the hole round, after using the clay bar, an auger may be turned in the hole and the surplus clay removed. Should the hole be wet, the same method of claying is followed, but the cartridges are well coated with miners' soap and not tamped so hard as to break them.

The *needle*, which is a copper rod about $\frac{1}{4}$ in. in diameter pointed at one end and provided with a handle at the other, is run into the cartridge, and about it the tamping is rammed in by means of a tamping bar, which has a groove through the head to accommodate the needle. The tamping bar is commonly of iron with a copper head of a diameter a little less than that of the drill hole. Where cartridges are not used and powder is charged in bulk, the tamping bar is made entirely of copper, since if iron comes in contact with rock or pyrites

it is apt to strike a spark and cause a premature explosion if loose powder is sticking to the sides of the hole. The danger from the use of metal tamping bars has been recognized in some states and their employment wisely prohibited by law. The tamping or stemming is put in and tamped little by little, being rammed hard to keep the needle hole open when the needle is removed.

In anthracite mines, the blasting barrel is often used instead of the needle, as tamping fine enough to pack is not obtainable unless the miner pounds up slate and coal. The *blasting barrel* is a steel or wrought-iron tube of about $\frac{1}{4}$ in. inside diameter; one end is inserted in the cartridge and the squib is fired through it to the powder. The rammer fits over the blasting barrel in the same way as over the needle. The blasting barrel is valuable where the tamping is damp or the hole slightly wet, but after each shot is fired it has to be recovered and if used again must be straightened.

The squib is inserted in the needle hole or blasting barrel with the match end outwards and after igniting the extreme end of the match with his lamp, the miner hastens to a place of safety. The practice of biting or breaking off the end of a squib, or unrolling it, or dipping it in oil in order to hasten its burning is extremely dangerous and is almost sure to result in a premature explosion.

**Firing Black Powder With Fuse and Cap.**—After cutting off, with a sharp knife, 1 or 2 in. of the fuse in order that the portion about to be used is dry and in good condition, the detonator (cap) is slipped over the end and is crimped

tightly in place by a form of pliers or pincers known as a *crimper*. The cap should never be crimped on the fuse with the teeth as an explosion may result, and the cap contains enough fulminate to blow off a man's head. When placing the cap in the cartridge, there is great diversity of opinion as to which of the many ways used is the best. Fig. 1 shows a very common method in which the cap is placed in the top of the cartridge and in the center with only about two-thirds of the cap embedded in the material of



FIG. 1

the cartridge. This is done to avoid the danger of its igniting the explosive and thus causing deflagration of the cartridge in place of detonation. An objection to a cap placed in the center of the cartridge is that the fuse is very apt to be bent and injured in the tamping, while it also interferes with the tamping.

Instead of placing the cap in the center of the end of the cartridge and tying the end of the paper wrapper about the fuse, an inclined hole is made in the end of the cartridge, as shown in Fig. 2, and the cap placed deep down in the charge. Instead of inserting the cap through the end of the cartridge, many manufacturers of explosives strongly recommend placing it in a hole in the side, as shown in Fig. 3. The fuse is tied in two places, a half hitch being taken around it.

A common, but bad, practice among miners is to make a hole in the side of a cartridge, place the cap and fuse in it, and bend back the fuse, as shown in Fig. 4 in order to prevent the cap being pulled out. The sharp bend in the fuse is sometimes sufficient to cause a break in the train of powder, resulting in a misfire.



FIG. 2   FIG. 3   FIG. 4

If the length of the cap is greater than the diameter of the cartridge and it is stuck through the side either diagonally or straight across, there is danger of its being jammed against the side of the hole when tamping, and thus cause a premature explosion. Care must be taken to insure that that portion of the cap which contains the fulminate is entirely within the charge; otherwise, the detonation of the explosive will be imperfect.

When tamping holes to be fired with fuse, hard bits of rock or slate mixed with the stemming may cut the fuse and cause a misfire, or the fuse may be kinked by a hump in the hole and thus cause either a misfire or a delayed shot. In order to use fuse with entire safety, it should be long enough and should be run through a blasting barrel. In gaseous mines, it is extremely important that the fuse extend beyond the end of the hole, for if it does not, when such a fuse is lighted any gas in the untamped part of the drill hole is certain to be set on fire at the same time and a premature explosion will follow.

If the work is wet, waterproof fuse should be used, the end of which should be protected by applying bar soap, pitch, or tallow around the edge of the cap where the fuse enters it.

**Charging and Firing Dynamite With Cap and Fuse.**—Blasting powder is commonly charged into the drill hole in one large cartridge, whereas dynamite and other high explosives, including permissible powders, are charged as a number of single and much smaller cartridges. When loading a hole with explosives of the latter type, the cartridges are placed in one after another and pressed, not rammed, in place with a wooden bar. If the cartridges at hand are less than the required size, the paper coverings may be split open with a knife and the explosive forced to fill the hole with the aid of the tamping bar. It is very important, if the full effect of the explosive is to be obtained, that the part of the hole in which the charge is located is completely filled, and that no air spaces are left between the charge and the walls of the hole.

The cartridge containing the cap is called the *primer*, and while it is usually the last or next to the last cartridge to be placed in the hole, it may be placed in the middle of, or even at the bottom of the charge, with the idea of insuring a more thorough explosion. This is theoretically correct, as the explosion acts equally in all directions, but while there may be some reason for firing a charge with black powder in this manner, there is no good reason for such practice when firing dynamite, except when firing holes consecutively, for the explosion of dynamite is so quick that there is no appreciable difference in the result, whether the cap is placed in the top or in the middle of the charge. There is, however, a decided objection to placing the cap in the middle or bottom of the charge when using common fuse, as there is a chance that the fuse will set fire to the dynamite and cause not only a loss of dynamite, but a premature explosion, which cannot be as thorough as if detonated by the cap. The primed



cartridge is pressed down until it rests on those already placed, and, after the cartridges are all pressed into place, the tamping is pressed lightly on the charge, care being taken not to explode the primer. Fig. 5 (a) shows a hole with the primer placed in the center of the charge and in which there is no bend in the fuse, a theoretical arrangement seldom found in practice. View (b) shows a common method of placing a charge with the primer on top and the cap placed in the side of the cartridge as illustrated in Fig. 3.

**Precautions When Tamping Explosives.** Before any shots are fired in a bituminous coal mine, the dust made by the augers should be scraped from the shot holes and, with the *bug dust* made by mining machines, should be loaded into tight cars and hauled outside. This is to prevent the possibility of a dust explosion either in the hole or in the bug dust immediately at the face. As the loading out of the dust throws considerable of the finest, and consequently

(a)      (b)

FIG. 5

most dangerous, particles into the air, some time, say $\frac{1}{2}$ hr., should elapse after this loading out is done before shots are fired. In very dusty mines, the face should be wet down with a hose or a liberal application of shale dust made before shots are fired. The material used for tamping should be of fine and uniform grain that it may be packed tightly and should not contain any substances, such as grains of quartz, pyrites, etc., that may strike fire while being rammed home. The depth of the stemming should be at least one-half that of the drill hole, and it should completely fill the hole from the explosive to the mouth. *Under no circumstances whatsoever should slack or bug dust from bituminous*
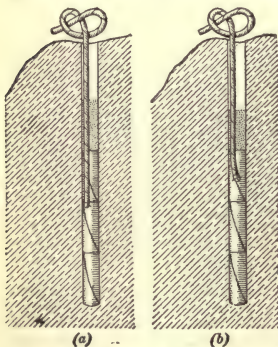
*coal be used for tamping.* Moist clay or brick dust sufficiently moistened to make it adhere form the best of tamping materials. Suitable stemming may be had by grinding and screening clay or shale rock, by digging it from a bank on the surface, or from the fireclay floor of the seam. The stemming is usually distributed in quantity by the company men after working hours, being left in a pile at the mouth of each entry or, more rarely, at the mouth of each room. For use, the miner makes the material into cartridges of the same diameter as those containing powder but only about 10 in. long. Where shot firers do the charging, the miner commonly makes and leaves at the face beneath each shot hole a sufficient number of these *dobe*, or *dummy*, *cartridges* to supply the tamping. Holes of large diameter require a proportionally greater length of tamping than smaller holes. The tamping should be rammed solid, in order to diminish the risk of a blown-out shot and to confine the powder tightly, that it may do its full work.

High explosives of the nitroglycerin type, which develop their full power instantaneously, usually require less tamping than powder, although the best results from their use are obtained by thorough tamping. The experiments of the Bureau of Mines prove that in the case of high explosives when the weight of the tamping is about that of the charge, the efficiency or work done by them is increased from 60 to nearly 80%, depending on the kind of stemming and the way it is used. While in deep downward-pitching holes, water makes a fair tamping, fine sand, clay, etc., are preferable and are generally used. The first 5 or 6 in. above the charge is filled in carefully so as not to displace the cap and primer; and then with a wooden rammer the balance of the stemming is packed in as solidly as possible, ramming with the hand alone, and not using any form of hammer.

## FIRING EXPLOSIVES BY ELECTRICITY

**Charging for Electric Firing.**—Figs. 1 and 2 illustrate the method of placing an electric exploder, or cap, in a cartridge of dynamite. The cap $a$ is placed either in the bottom or at the side of the cartridge, the hole to receive it having been made with a sharp stick or lead pencil. The best practice is to place the detonator so that it points to the bulk of the charge. After this is accomplished, the blasting wires $b$ are tied firmly to the cartridge, as illustrated at $c$. In firing dynamite by means of electricity, there is no danger of the wires setting fire to the powder, and hence the exploder can be placed well down in the cartridge. Sometimes, when a long charge in a very deep hole is to be fired, two or more electric exploders are used in the same charge, one cartridge containing an exploder being placed near the bottom of the hole and another at the top. The method of loading holes for firing by electricity is the same as that described for firing with fuses. As much care as possible should be taken to prevent the leading wires from coming in contact with the damp earth, also that in tamping the hole the wires do not become broken, or the covering materially injured, and that the wires are not brought into contact with each other or with the damp ground.



FIG. 1  FIG. 2  FIG. 3

Many miners have a bad practice of putting the cap of an electric exploder in obliquely and bending the wire over and securing the cap by a half hitch of the wire, as shown in Fig. 3, or, to make it worse, by two half hitches. So much force is used in making the half hitch that there is danger of a short-circuit being formed, or the sulphur filling in the electric cap may be broken, sometimes disarranging the wires in the cap and even breaking the fine platinum wire or bridge. In any event, the cement is so broken as to leave free passage into the cap of any water that may be contained in the hole. The platinum bridge of an electric cap is very small and delicate in order to be heated red hot by the very small current of electricity that is used to fire the cap, and any unusual strain on the wires may break the bridge, thus breaking the circuit and causing a failure of the shot. Furthermore, sharp bending of the copper wires may damage the insulation, very likely leaving bare wires touching, causing a short circuit and a failure of that particular cap. The bared wires, even if they do not touch, offer an opportunity for a short circuit through any moisture present, which will rob that particular cap of part of the current of electricity, while the next cap might get the full current. The result will be that the first cap will miss fire.
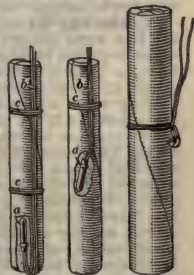
**Shot Firing With the Electric Blasting Machine.**—One method of electric blasting, as used in America, depends on the generation of a current of electricity by means of a small electric machine, which is really a small dynamo, the armature of which is made to revolve rapidly between the poles of the field magnets by means of a crank or by a ratchet. The machine, commonly, but incorrectly, called a *battery*, is shown in Fig. 4. It consists of a field magnet *a* and an armature *b* that revolves between the poles of the field magnet. The loose pinion *c* (the teeth of which engage the rack bar *d*) is arranged with a clutch, so that as the rack bar descends the pinion causes the armature *b* to rotate and generate a current. During the downward stroke of the rack bar, the connections are such that the current flows inside the machine without affecting the outside circuit. The current increases in strength until the rack bar strikes the spring *e*, which changes the connections so as to send the full strength of the current into the outside circuit and through the caps for firing the blasts. When more than one shot is fired at a time by means of a blasting machine the holes should be connected in series. Blasting machines are also called *push-down machines* and are rated by the number of electric detonators they can explode, as 4-hole machines, 50-hole machines, etc.
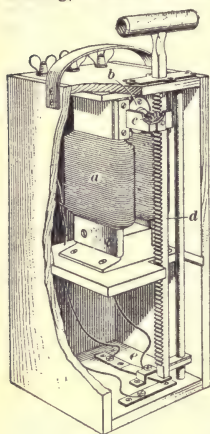


FIG. 4

**Connecting Wires.**—To connect the ends of two wires, scrape off the insulation for about 2 in. from each end and scrape the wires clean and bright. Then twist the ends together, as shown in Fig. 5. It is very important, to prevent misfires, that all connections are clean and well made, as one bad connection may cause all the holes to misfire.

**Connecting Up and Firing the Blasts.**—After the holes have been loaded, the fuse wires are left projecting from the holes, and are joined by connecting wires in such a manner as to leave one free wire at each end of the series to be fired, as at *b*, Fig. 6, the fuse wires *a* leading downwards to the charges to be fired.

After all is in readiness, the leading wires are connected to the loose ends *b*, and when every one has left the vicinity of the blast, the other ends of the lead wires or cables are attached to the blasting machine. Some blasting machines are provided with three



FIG. 5

screws on the outside, to which leading wires are attached. When only a small number of blasts are to be fired, one of the lead wires is attached to the middle screw and the other to the outside as illustrated in Fig. 6. When a large number of blasts are to be fired the lead wires are arranged as shown in Fig. 7, *a* being one series of charged holes, and *b* another. The wires on the outside are attached to the ends of the entire series, as in the previous case,
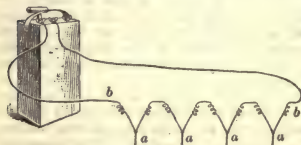


FIG. 6

while the wire from the central screw is attached to the center of the series of connecting wires as at *c*. By this arrangement, a large number of blasts can be fired with a single battery and the size of the lead wires very much reduced. The firing is accomplished by lifting the handle of the machine to its full height, and pushing it downwards with full force, until the rack attached to the handle reaches the bottom of the box and sends the current through the caps in the holes. When firing a hole by means of a blasting machine, the handle, or rack bar, should never be churned up and down, but should simply be given one vigorous stroke, as directed. Most blasting machines are made to fire with a downward stroke, but some fire with an

upward stroke of the handle. A machine should always be kept clean and never abused and played with. Its strength should be tested from time to time by means of a test lamp or rheostat. Test galvanometers are used to ascertain if any breaks exist in the circuit and the caps.

In Europe, another form of electric blasting machine is used. These magneto machines consist essentially of an armature revolving between the poles of a set of permanent magnets, are not unlike the American blasting machine in appearance, and are used in very much the same way.
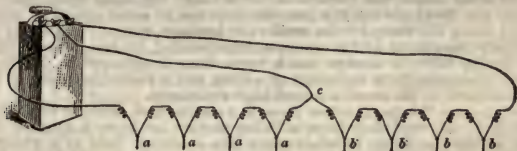


FIG. 7

**Firing With Dry Batteries.**—Firing charges of explosives by means of ordinary dry cells has been prohibited in foreign countries, because premature firing of detonators, and sometimes of the charge, has been caused by the wires coming in contact with the poles of the batteries. Safety-contact dry-cell batteries have lately been introduced in the United States and abroad. They are made with a spring-key contact, or with two safety-spring contact buttons, which are the poles of the battery. The two leading wires are laid on the buttons, which are at the same time pushed downwards. When the pressure of the thumbs is released the contact is broken. If the wires of a detonator accidentally come into contact with the poles of the battery, the current cannot be discharged unless both poles are pushed downwards. As these dry-cell batteries are cheap and easily portable in comparison with the blasting machine, they have become quite popular, but those without safety contacts have been the means of numerous fatal accidents through premature explosions due to the unintentional coming in contact of the wires with the binding posts before the men had left the face, or when carried in sacks with the explosives and detonators through a contact made between the battery and the detonator wires resulting in an explosion.

Small devices such as these can be used to fire only a few shots in one circuit, as those in a single room. The number of shots to be fired and the length of the leading wires and other conductors must be known, so that a battery of sufficient capacity may be selected. The strength of the battery may be tested by passing the current through a small electric lamp of known capacity and noting the brightness of the light given by the lamp. Or the battery current may be passed through a testing circuit whose resistance is equal to that of the circuit to be fired and which has in it one electric detonator, which must be put in a safe place. If the battery fires this detonator, it is strong enough and is in good condition.

**Precautions When Firing With the Electric Blasting Machine.**—To insure success when firing blasts by electricity, the following points should be observed:

1. The machine wires and primers should be suitable to each other; two kinds of primers must not be used in the same blast.

2. The blasting machine should be of sufficient power to fire all the caps or primers connected at one time; a blasting machine must not be loaded to its full limit.

3. The electric caps or primers should be kept in a dry place, and everything kept as clean as possible.

4. All the joints at connections and points of contact of the wires should be well made so that the wires cannot separate, and the surfaces should be clean; the joints in one wire must not touch those in another, and bare joints must not touch the ground.

5. The wires must not be kinked or twisted so as to cut the insulation during the process of tamping. If the insulation is cut, the fuse is useless for wet ground or a wet hole and should be laid to one side.

6. The operator's hands should not touch the terminals of the blasting machine when firing.

7. The blasting machine should not be connected to the leading wire or cable until every one is in safety.

8. The wire connections should be bound with insulating tape in damp places to avoid leakage and short-circuiting of the current.

9. Before firing the blast, the circuit should be tested by a galvanometer to insure that there is no break or short-circuit.

**Firing From Dynamo** —Instead of obtaining the necessary current to fire the blasts from a blasting machine or a dry-cell battery, it is a very common practice to derive it from the dynamo used in the power plant of the mine. While, in most cases, the current is carried into the mine upon special wires known as the *firing circuit*, the practice prevails to some extent of connecting up and firing the shots by the trolley, or haulage, wires. The latter practice is extremely dangerous and is commonly prohibited by law because the power on the trolley wires is always much more than is required to fire the shots. There is, consequently, a certainty of arcing and a probable fusing of the lead wire on account of its small size and high resistance.

In some cases the shots, either those in a single room or those in all the rooms in a district, are exploded from a firing station or from a number of firing stations within the mine; in other cases, the shots are fired as a whole from a firing cabin on the surface after all the men have been withdrawn from the
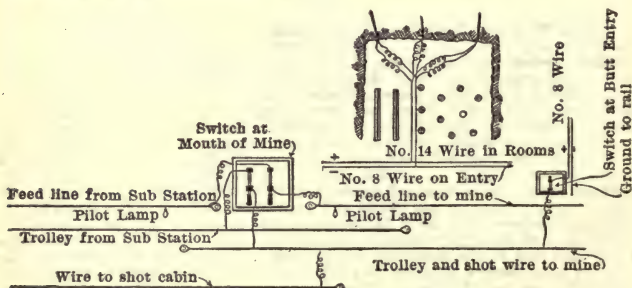


FIG. 8

workings. Fig. 8 shows the system in use at the mines of the Stag Cañon Fuel Company, Dawson, N. Mex. As the men enter the mine they are required to deposit a metal check at the shot-firing house outside, near the entrance. These checks are placed on a check-board and returned to the men as they come from the mine. A record of the working place of each check number is kept in the shot-firing house, and in case any check is uncalled for, the shot firer makes a search for the man until he is found. No shots are fired until it is known positively that no one is in the mine. The method of placing the shots is shown in Fig. 8. To insure safety against accidental discharge of the shots by electricity, there are two or more locked switch boxes in each mine, with throw-off switches, one at the mouth of the mine and at one or more stations inside the mine. After inspecting the inside connections with the shots to be fired, the shot firer en route from the mine makes connection at each of the switches mentioned. He then goes to the shot-firing cabin to turn on the electric current, but before doing so he turns on an electric signal light in a red globe, to warn all persons to remain away from the vicinity of the mouth of the mine; so that should an explosion occur within the mine, no one outside could be injured by flying debris. In connection with this system of shot firing, the company's rules provide that the undercutting must extend at least 6 in. beyond the back of the shot holes; that all shot holes must be at least 2½ ft. deep; that all dust must be removed from the shot holes before they are charged; that no more than five sticks of powder (which, at some mines, is in excess of the safe limit) shall be used in any one hole; that standing holes, or parts thereof, must not be recharged; that holes in tight corners must be at least 1 ft. from the rib at the back end of the hole; and that, in solid faces, shot holes must not be more than 6 ft. apart horizontally, and that not less than two such holes shall be fired.

All shot-firing wires should be well insulated. The size of wire used is commonly No. 6 on the principal line along the main entry; from a No. 8 to a No. 12 on the cross-, butt-, or side entries, depending on their length; and a No. 14 for the room connections. The connections between the cross-entry lines and the main line and between the rooms and cross-entry lines are made in parallel so that the failure of the shots to explode on a cross-entry or in any room or rooms on a cross-entry, will not prevent the detonation of any others. When two or more shots are to be fired in the same room, they are connected in series. The connecting up of the shots should proceed outwards from the working faces to the mine mouth; that is, the detonator wires should first be connected to the room wires; then the room wires to the cross-entry wires; and, finally, the cross-entry wires to those on the main entry. The room connections are usually permanent, unless it is desired to fire shots singly from the room mouth, but the connections on the main entry with the cross-entry circuits and at the foot of the shaft are made by switches. These switches are contained in locked boxes, the head shot firer alone being trusted with the keys.

When preparing for blasting, all the holes in all the rooms on No. 1 entry are first connected to their respective room firing circuits, then all the holes in all the rooms on No. 2 entry are connected, and so on until those on the innermost entry are connected. On the way out from the last entry, the shot firer opens the boxes and throws in the switches at the mouth of each room entry. At the foot of the shaft, or at the drift mouth, the final switch connecting the mine and firing circuits is thrown in. The current from the dynamo is then applied. After the shots have been fired, the shot firer returns to the mine, opening the various switches and relocking the boxes on the way. At the working places, an examination is made for fire, the room wires are placed out of the way of the loaders, and are disconnected from any detonators that may have failed to explode; the mine foreman being subsequently notified of the location of any misfires. During the day the entire shot-firing system is overhauled, defective insulations, etc., are repaired, and everything made ready for the work of the following shift.

**Firing Single Shots From the Surface** —From time to time various devices have been introduced to reduce or overcome the danger of a coal-dust explosion due to the detonation in a single blast of the very large amount of high explosives (often over 1,000 lb.) required in large mines. The logical procedure is to have a separate circuit from each working place to the firing cabin, but the cost of installing and maintaining the 400 or more wires necessary for this purpose in a mine of even moderate size, is prohibitory. The mine may be divided into two, three, or four, districts with the same number of firing circuits and four, six, or eight sets of wires, but beyond this subdivision it is not usually economically possible to go. In a recent device, what are known as *sparker boxes* are introduced in the firing circuit at the mouth of each room. The main-line wires *A* enter the box in the usual way but are separate from the wires *B* connecting one box with another. The room wires *C* are also distinct from either pair of wires *A* or *B*. By pushing a button, the room wires *C* are placed in circuit with the entry wires *A*, the wires *B* to the next box remaining dead. The application of the current at the surface detonates the holes in the room and at the same time releases the button so that the wires *A* and *B* become in circuit. A second application of the current fires the shots in the second room and arranges the mechanism in the sparker box at its mouth so that the current may pass to the third box, and similarly throughout the mine. The failure of the shots in any room to detonate does not interfere with the current passing from one box to another. According to the statements of a Western operator, this device has proved successful at his mine.

# SUBSTITUTES FOR BLASTING IN DRY AND DUSTY MINES

Before the introduction of permissible powders and the adoption of the various safety measures used in connection therewith, numerous substitutes for powder and divers means of bringing down coal without blasting were tried from time to time and with more or less success.

**Wedging Down Coal** —The plan of wedging down coal has been employed until quite recently and has its uses even at present. The seam is undercut in the usual way, and a series of drill holes but a short distance apart are placed

in a horizontal row near the roof in thin seams or between the roof and the floor in thick ones. If the seam has a pronounced bedding plane, an interstratified layer of slate, or another line of weakness along which it splits readily, the holes should be drilled in it. Wedges, placed in the holes and driven home by blows of a sledge will bring down the coal in large lumps. Other things being equal, the nearer the drill holes to one another, the easier is the coal brought down. If the coal is thin, hard, and blocky and adheres strongly to the roof, wedges will not work well, as the coal will break off short and but 1 in. or so in depth of the face will be removed at each application of the wedges. If the coal is soft, the wedge will be driven in flush with the face and will merely enlarge the size of the drill hole without bringing down any part of the seam. Seams having a tendency to split horizontally along a bedding, or other plane, are best adapted to wedging.

The wedges used are of various types. The common form is, of course, the ordinary single-piece wedge. The device known as *plug and feathers*, familiar to quarry men, has been largely used. This consists of two narrow triangular pieces of iron (the feathers) placed on opposite sides of the hole, between which a long wedge (the plug) is driven by blows of a sledge. As both the feathers and plug are tapered, the force of the blow is multiplied. To still further increase the force of the blow, *multiple wedges* have been used. They are essentially the same as the device just described, but there are four instead of two tapering feathers between which the plug is driven as before. In a special form of wedging machine, the plug and feathers are introduced in the back of the hole, the plug being drawn forwards by a combination of a screw and a lever. In a similar machine, in order to reduce the friction of two plane wedges sliding upon one another, the plug is made to run upon roller bearings between double feathers, and is drawn forwards from the back of the hole by the action of a screw and nut, driven by a ratchet and pawl.

In a French device used in rock work, a series of holes are drilled in a group in the face. The drill bit is replaced with a hammer head, which by compressed air drives in the plugs inserted between the feathers in the holes, breaking down and splitting up the rock.

**Hydraulic Cartridge.**—The hydraulic cartridge has been in use in many foreign mines for several years past, is absolutely safe in any atmosphere, and is remarkable for the large proportion of lump coal produced in comparison with blasting, but does not appear to be adaptable to breaking down all kinds of coal. The cartridge itself is a forged-steel cylinder of varying length and diameter, and having eight duplex rams or pistons arranged in a row, along one side. The size most generally in use is 21 in. long and 3 in. in diameter, and the expansion of the duplex piston is 2¼ in., giving a pressure of 60 T. The larger sizes are used only in very narrow-entry work or in extremely thick coal. The action of the pistons is simultaneous, the pressure being applied through the longitudinal passage in the steel cylinder at the back of the pistons. A liner of sheet iron, $\frac{1}{16}$ in. to $\frac{1}{8}$ in. thick, 1½ in. wide, and 22 in. long, is usually introduced in the lower side of the hole, to present a uniform bearing surface for the pistons and to prevent their sinking into the coal.

In the original machine the pressure was produced by the operation of a lever pump, but in the present type a powerful screw pump is used to compress the water and expand the pistons. The coal having been undercut and the holes drilled as near the roof as possible, and parallel to it, the cartridge, shown
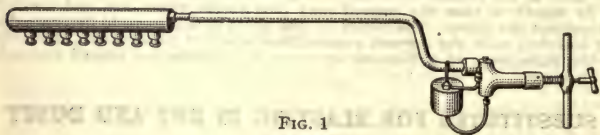


FIG. 1

in Fig. 1, is inserted in the hole, the pistons being telescoped and contained within the body of the cartridge. The small handle attached to the pump plunger is then slowly drawn in and out a few strokes until the cartridge is filled with water from the small reservoir, which is connected by a reinforced-rubber tube, and pressure is reached, the quantity of water thus used being about 1½ pt. The valves then being closed, the screw is operated by the handles and the compression rapidly increased. As the pistons are forced out the coal

begins to work and crack, and as the maximum expansion is approached, it falls in large lumps rarely exceeding one-man size. The release valve is then opened, the water returns to the reservoir, and the pistons are pushed back ready for the next hole. The pipe connecting the pump and cartridge is made in any required length suitable to the depth of the hole, which is usually about 6 in. less than that of the undercut. The number of holes necessary depends on the thickness of the seam, the nature of the coal, the partings, if any, and the adhesion of the roof rock; but in general it averages one hole for every 10 ft. of face in room-and-pillar work, and one hole for every 30 ft. of face in longwall work. The length of time necessary to bring down one hole varies with the location, breaking-in shots requiring from 10 to 15 min., and free-end shots from 3 to 5 min. In an English mine where 38,263 thrusts were made in 12 mo., the average cost of breaking down the coal was $2\frac{7}{10}$ mills per ton. A pressure of 3 T. per sq. in. is usually required in seams up to 4 ft. thick, and this gives a total pressure of 60, 90, and 150 T., respectively, on the three sizes of cartridges made.

**Lime Cartridges.**—The expansion of quicklime under the action of water has been used to some extent abroad as a substitute for powder in fiery mines. Ordinary limestone is calcined; that is, has the carbon dioxide driven off by burning in kilns, and is ground to a fine powder. The powder is compressed hydraulically into a cartridge having a groove running down its side. The cartridge is about 5 in. long and $2\frac{1}{2}$ in. in diameter, is wrapped in paper, and is kept in an airtight box. The seam is undercut and the holes drilled in the customary way. A perforated iron tube, $\frac{1}{2}$ in. in diameter and having a small external chan nelon the upper side, is inserted for the full length of the hole. Several cartridges are placed in each hole, the grooves in them fitting around the pipe, and the rest of the hole is tamped with clay. Water is forced into the tube by means of a small hand pump, and the water acting on the lime greatly expands its bulk, forcing down the coal. The process can be used only for certain classes of coal, and much care must be taken to keep the cartridges dry.

**Water Cartridge.**—The water cartridge is not a substitute for explosives, but is a means of using them in a less dangerous manner than as ordinarily employed. A cartridge of high explosive is placed in a skeleton case, *a*, Fig. 2, having a number of thin metal diaphragms, or wings *b*, which keep the cartridge in the center of the case *c* which contains water. A detonator is inserted in the cartridge and its wires connected up to the firing wires of a battery. After the detonator is placed, the outer case is filled with water and its mouth tightly tied around the wires *d*. There is, also, a guide wire *e* used to keep the cartridge lengthwise in the center of the case. The apparatus is delicate, requires a large size hole, and materially reduces the effect of the explosive.
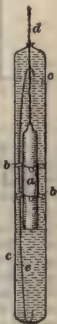
FIG. 2

# GENERAL CONSIDERATIONS AFFECTING BLASTING

**Definitions.**—A *free face*, or *free end*, is the exposed surface of a mass of rock, or of coal. Thus, *AB*, *BC*, etc., are free faces in the room of a coal mine shown in plan in Fig. 1, and represent parts of the coal detached from the main body of the seam by blasting or undercutting. That part of the seam in which the shot hole *6–7* is placed has but one free face *CD*, whereas those portions of the seam in which the holes *11–14* and *1–2* are placed have two free faces *AB* and *BC*, and *DE* and *EF*. In the latter cases, additional free faces could be secured by shearing into the rib in the direction of the length of the room at *A* and *F*, or at right angles to the room at *C* and *D*, or by undercutting the seam. When a block of coal has the maximum number of free faces, six, it is, of course, entirely detached from the rest of the seam.

The *heel* of a shot is the distance *1–E* from the mouth of the drill hole to the corner of the nearest free face *ED*; or is that portion *1–2* of the hole that is filled with the tamping; or is that portion *1–2–5–E* of the coal to be broken that is entirely outside the powder.

The *toe* of a shot is the distance *3–4* from the inner end of the hole to the adjacent free face measured at right angles to the direction of the hole; or is that portion of the hole *2–3* that is filled with powder; or is that part *2–3–4–5* of the seam to be broken lying between the powder and a free face.

The *line of least resistance* is the distance from the charge of powder in a hole to the nearest free face. In the case of the hole *11–14*, the line of least resistance

is *13–C.* In the hole *6–7*, the line of least resistance is not the line *7–8*, but a shorter line, a perpendicular from *7* to the face *CD*.
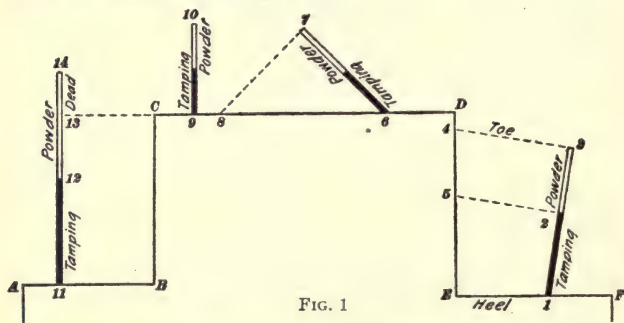


FIG. 1

A *dead hole* is one that extends into the solid coal beyond that part that can be broken by the maximum safe charge of explosive. Thus, the hole *11–14* is dead because the part *13–14* extends into the solid beyond the line of least resistance *13–C*. Sometimes such a hole is spoken of as being dead for 9 or 10 or any number of inches, depending on the length of the portion *13–14*. A definition taken from the mining laws of a Western state is: "A dead hole is a shot hole so placed that its width at the point (toe), measured at right angles to the drill hole, is so great that the heel is not strong enough to at least balance the resistance at the point (toe)." The hole *1–3* would be dead if the point *3* was so far to the right of its indicated position that the explosion of the charge would fail to break the portion of the coal marked *1–3–4–E*, and would result in a blown-out shot, or in merely breaking off a part of the heel. The hole *9–10* drilled directly into the solid is a dead hole.

A *gripping hole* is one whose direction is inclined away from the adjacent free face, or may be defined as one whose width at the toe is greater than at the heel. In the figure, both *6–7* and *1–3* are gripping shots. The degree of grip is indicated by the angle between the hole and the face it is intended to break. Thus, in the hole *6–7*, the greater the angle *7–6–8*, the stronger or greater is the grip. Similarly, in the hole *1–3*, the greater the angle between the lines *1–3* and *E–D*, the stronger is the grip. By increasing the angle mentioned, gripping holes become dead holes.

In a *balanced shot*, the width of the toe and heel are equal and are less than the length of the drill hole. The shot *1–3* would be balanced were its direction parallel to the free face *E–D*. Similarly, the shot *11–14* would be balanced did it not extend beyond *13*. A balanced hole will do the work expected of it with the minimum expenditure of powder.

Note that the definitions of the terms used in connection with drill holes are not uniform throughout the country; the foregoing are believed to be in accord with general practice.
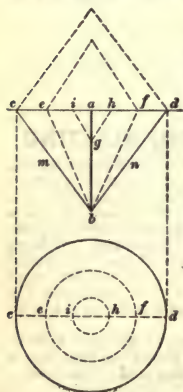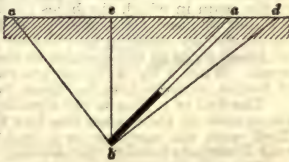


FIG. 2



FIG. 3

**Effect of Free Faces in Mining.**—The form of cavity produced when a single drill hole is fired in a mass of rock having one free face is usually that of a cone;

thus, in Fig. 2, if *ab* represents a vertical drill hole, the rock broken will theoretically have the form *cbd*, the line *cd* being the diameter of the base of a cone. If the strength of the explosive is not sufficient to overcome the tenacity of the rock to so large an extent as represented, the cone might have the form *ebf*, or *igh*. It is more than probable that a shot hole perpendicular to the face will break no rock at all but will result in a blown-out shot, as it is in the worst position to do effective work. This is because any pressure exerted in the direction of *m* or *n* is opposed by the resistance of an indefinite thickness of rock and the line of least resistance along which the force of the explosive naturally will act will be the resultant of the forces acting on *m* and *n*, or in the line of the drill hole.

When a hole is inclined in the face, as *ab* in Fig. 3, the line of least resistance *eb* is perpendicular to the face, and the cross-section of the piece of rock broken out will be approximately of the form *abc* and rarely that of *dbc*. Commonly, one edge will coincide with the drill hole and the other will be between the lines *eb* and *cb*. The angle *eab* is usually about 35° for the best results, and 45° is its



FIG. 4

limit. Less and less rock will be broken as the angle becomes less and when the direction of the drill hole *ab* is the same as that of the free face *ac*; that is, when the powder is placed on top of the ground, no rock will be broken out. Similarly, as the drill hole becomes more nearly perpendicular to the face the less will be the volume of the rock broken and when the hole is vertical as *eb*, it will, in the very great majority of cases, result in a blown-out shot.
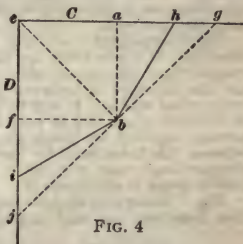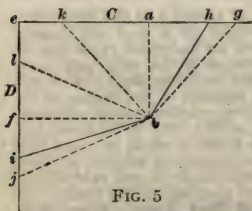


FIG. 5

The more free faces there are, the greater will be the ease with which an explosive will accomplish its work. Fig. 4 is a cross-section showing a hole *ab* placed in a rock having two free faces *C* and *D*. If there were but the one free face *C*, the force from the charge at *b* would break out the cone or crater *ebg*; if the face *D* were the only one exposed, the charge *b* would break out the crater *ebj*. With the charge *b* equally distant from the faces *C* and *D*, and of just the right size, the bounding surface between the two craters will coincide in the line *be*, but as the force of the explosion at *b* is divided between the two craters and a portion of it is reflected by the solid rock, the crater actually broken out will be approximately *hbi*; that is, a crater that is not equal to the sum of the two craters *ebg* and *ebj*.

If the charge *b*, Fig. 5, is located so that *bf* is greater than *ba*, the force acting on each face separately will break the craters *gbk* and *jbl*, the wedge-shaped piece *ekbl* not being included in either. With the charge *b* acting on both faces *C* and *D* together, part of the force is used in breaking down the mass *ekbl* and the crater broken out is bounded by the lines *hbi*, instead of by the lines *gbj*.

Similar reasoning may be applied to any increase in the number of free faces. The greater the number of free faces the larger the amount of material that can be broken



FIG. 6

down with a single shot; or what amounts to the same thing, a smaller charge will do the same amount of work, the greater the number of free faces, but the increased amount of material loosened will not be proportional to the increase in the number of free faces.

There is a general rule that the longest line of resistance should not exceed three-halves of the shortest line of least resistance if the maximum effect of

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the explosive is to be obtained. If possible, the shots should be placed so that the shortest line of resistance is horizontal and the longest vertical so that the weight of the rock may assist the breaking down.

It is evident, therefore, that in blasting it is advantageous to have as many free faces exposed as possible, not only on account of the decrease in the amount of powder required, but also because it is possible to obtain the material broken in larger lumps than when blasting is done with a single free face. This is advantageous particularly in coal mining where the lump coal is more valuable than the fine coal.

Fig. 6 represents two drill holes $h$, $h'$ drilled at a distance $W$ from the free face $AB$. If these holes are fired independently, each will break out approximately the same amount of material, as $mhn$ or $nh'o$. If they are fired together, and the distance $D$ between them is not too great, they will bring out the entire mass $mhh'o$. It will be seen from this that by firing two holes together an additional amount of rock $hnh'$ has been broken with the same amount of powder required to break the two masses $mhn$ and $nh'o$. The distance $D$ between the holes must be varied according to the character of the rock. In comparatively soft material it is less than in hard rock, though probably the limit is twice the distance $W$.

Fig. 7 illustrates another case. Here three holes $h$, $h'$, $h''$ have been drilled close together, and each one loaded with a charge the depth of which will be represented by $HE$. Any one of the holes, if fired separately, will not be able to break through the distance $W$, but, by firing the three together, the mass $ehh''gg'GHe'e$ may be removed at one shot. By this means, greater masses of rock can be removed with smaller drilled holes than is possible without the combined effect of the several charges.



FIG. 7

The form of cavity and the amount of material dislodged by a shot are largely theoretical and no universal rules can be given. Experience is the only safe guide in choosing the location and size of the holes and the amount of the charge of explosive, and an experienced miner will study the character of the rock to be blasted so as to place his holes at such an angle that he may get the maximum effect from them and avoid blown-out shots, and take advantage of slip and cleavage in the rock.

**Diameter of Shot Holes.**—When driving tunnels or sinking shafts, holes having a diameter between $\frac{3}{4}$ and $1\frac{1}{2}$ in. at the bottom give the most economical results in hard rock if they are charged with the strongest of the high explosives. When the rock is weaker, the explosive should be of less strength, but the diameter of the holes should be increased to $1\frac{1}{2}$ to $2\frac{1}{2}$ in. All holes should be of the same diameter, each should have an equal resistance of rock to work against, and each should be so placed that it will receive the greatest benefit from the free faces formed by blasting the holes previously fired.

**Amount and Kind of Explosive.**—The maximum pressure or effect that an explosive can develop is obtained when it entirely fills the space in which it is exploded; hence, the greatest efficiency is obtained when the charge fills the hole up to the tamping. There are no rules from which the amount of explosive necessary to be used in charging a given hole or series of holes can be determined. If the right amount of explosive has been used in a properly placed hole, upon detonation there will be a deep boom and the material will not be thrown with great force from the solid. If the charge is too heavy, there will be a sharp cracking report and the excess of explosive will throw the broken material away from the solid and will shatter it badly. If insufficient explosive is used or the hole has been badly placed, the rock will not be broken, but the tamping will be blown from the hole, resulting in a blown-out shot. The depth of the hole is to be considered when estimating the amount of powder to be used, as a shallow hole requires less powder than a deep one. In some parts of the bituminous coal field, where the seam is of moderate hardness and from 4 to 6 ft. thick, it is a common practice to make the holes from 2 to $2\frac{1}{2}$ in.

in diameter and to charge them with 2 to 6 lb. of black powder. Two lb. of black powder is not an excessive amount in a well-balanced hole, but any weight over this is dangerous, particularly if the mine is noticeably dusty or gaseous.

The length of the charge of explosive for single holes should not exceed from eight to twelve times the diameter of the hole; that is, a 1-in. hole should never have a charge of more than 12 in. of explosive placed in it. Where several holes are fired together this rule is sometimes slightly deviated from. It is usually best to employ a length of charge between these two limits, as, for instance, about ten times the diameter of the hole. After the proper relation between the diameter of hole and the length of charge has been determined by experiment for a certain diameter of hole under given conditions, it is safe to conclude that the same ratio of length of charge to diameter may be taken for other diameters. Thus, if it has been found that for a hole 2 in. in diameter the best results are obtained from a charge 24 in. long ($2 \times 12$), it may be assumed that in a hole $2\frac{1}{2}$ in. in diameter the charge should be $2\frac{1}{2} \times 12 = 30$ in. long. If the diameter of the hole $ab$, Fig. 4, remained the same and the length of the line of least resistance was increased, a place would soon be reached where the charge of explosive would fail to break out the rock. It is not good practice to have a powder charge occupy more than one-half the hole; hence, in order to increase the effect of a charge the diameter $ab$ of the hole should be increased; that is, as the distance $jb$ increases, the diameter of the hole $ab$ must be increased; or there must be a chamber formed at the lower end of the hole in which the powder charge is to be contained, so as to increase the size of the cone of throw toward any free face. This chamber is sometimes formed by using some form of expansion bit or reamer, but the usual custom is to introduce a small charge of high explosive that will enlarge the end of the hole. By continuing this process, an opening of sufficient size to contain the desired charge may be formed. This operation is called *chambering* or *squibbing.* Where large masses of soft material are to be loosened, it is common practice to use dynamite or nitroglycerine for chambering the hole and black powder for the blasting. Holes are sometimes drilled as much as 20 ft. deep and several kegs of powder introduced into the chamber formed by firing the high explosive. This method of blasting is used in open-cut ore mines, milling, and in steam-shovel mines; also in side cuts of earthworks.

When mining soft cleavable minerals, powerful explosives are not generally used, their effect being to shatter; on the other hand, in tenacious minerals, powerful explosives are used for their shattering properties. Less powerful explosives, such as gunpowder, are used where a rending action is desired, such as in mining coal, but they are not desirable in tough ore formations, because they break down the mineral in large pieces that, to be handled, must be block-holed and reblasted.

While the power of most explosives can be calculated, the theoretical power can never be obtained in practice. The factors that enter into the problem vary so widely and are so numerous that seldom can exactly similar results be obtained from blasts fired apparently under similar conditions. The weight of a mass of rock opposes the action of a blast, and this weight is assisted by the atmosphere that presses on it with a weight of 14.7 lb. per sq. in. at sea level. If the hole is damp or wet or the rock cold, the heat produced by the discharge and the power of the explosive will be decreased. Slips, joints, and cleavage planes affect the blast, as do also texture and structure of the rock The shape and location of the drill hole and the method with which it is charged and tamped are important factors.

Brittle rocks are easily fractured, while strong, compact rocks in which the cohesive powers are great are much harder to break. Plastic materials, like fireclay, which are neither brittle not tenacious, are very difficult to blast.

Fissures, or joints, and bedding planes, when open, have something the effect of free faces, and as a consequence they influence the best position for placing a blast. When possible, the charge of explosive should never be placed in contact with a fissure or bedding plane, but should be located in the firm rock, in order to avoid the escape of the gases through the fissure, that would thus reduce the effect of the blast. If it is possible to avoid it, a drill hole should not cross crevices and slips.

In compact and brittle rocks the limit of elasticity is soon reached and they rupture under the shock of exploding charges before there has been an appreciable enlargement of the drill hole. With porous rocks and sand, the blast tends to solidify the rock and to fill empty spaces, thereby increasing the size of the drill hole and decreasing the force of the explosion.

# SUPPORTING EXCAVATIONS

## INTRODUCTION

The various methods of supporting the surface overlying a seam from which the coal has been or is being removed may be roughly divided into those that use natural and those that use artificial materials for the purpose. The sole example of the use of natural means is afforded by the familiar American room-and-pillar system of mining in which natural pillars of coal are left, in the course of working, either to support the overlying rock indefinitely or throughout the life of the mine. There are numerous examples of the second method; the use of wood or steel posts or beams, called *timbering;* the use of built-up packs, cribs, etc., of timber, stone, or timber and stone; and the flushing of culm into the workings.

A distinction must be made between the methods and materials used for supporting the surface lying several hundred or several thousand feet above the coal bed, and those used to support the roof or that portion of overlying rocks usually but 1 to 2 ft. thick or in rare cases as much as 40 ft. The use of wooden or steel timbering is confined to supporting the comparatively thin roof rock, whereas pillars of coal, built-up cribs, and flushing are resorted to to sustain the weight of the many hundred feet of rock between the coal bed and the surface.

## COAL PILLARS

### GENERAL CONSIDERATIONS AFFECTING SIZE OF PILLARS

**Amount of Pillar Coal.**—The amount of coal left in the pillars for the support of the workings is generally expressed as a percentage, or a certain portion, of the total volume of the bed within the area included by the pillars. The term *pillar coal,* therefore, includes not only the coal left in the room pillars, but also that left in the pillars supporting the entries. The amount of coal left in the pillars in the first working varies widely under different conditions, but the best practice now counts on ample pillars in the first working so as to minimize the danger from squeezes. Many of the collieries in the anthracite region of Pennsylvania are now extracting but one-third of the coal in the first working, leaving two-thirds of all the coal as pillars to be taken out later as the different sections of the mine are worked up to the limit. In many of the mines in the Connellsville region of Western Pennsylvania, the rooms are only 12 ft. wide and the pillars from 60 to 72 ft. wide, so that only one-fifth to one-sixth of the coal is taken out in the first working, but the removal of the pillars begins as soon as the rooms have been driven their full length.

The proportion of coal left in the pillars along the entries to the amount of coal taken out in mining the entries is relatively larger than the percentage of pillar coal between the rooms, as the entry pillars usually have to stand a much longer time than the room pillars.

The amount of pillar coal left depends on the method of working the mine, on the nature of the coal, the top and the bottom, on the thickness of the coal, and the depth of cover, on the time of drawing the pillars.

**Practical Considerations Determining Size of Pillars.**—It is impossible to give exact rules or formulas for determining proper size of pillars and rooms that will be universal in their application. Each mine is a special problem, and in laying out the rooms and pillars it is well to find out what is the successful practice in the same field or in similar fields worked under the same conditions. Similar practice should not be followed blindly, as a great deal of the lack of progress in mining has been due to this copying of other methods. Still it is always well to find out how others have succeeded and why they have failed.

In general, the thicker the bed and the greater its depth below the surface, the wider must be the pillars and the narrower the opening. This rule is not invariable, however, for certain coals deteriorate very rapidly when exposed to the atmosphere and the pillars must be much larger than with hard, compact coals under similar conditions.

The length of time that a pillar must stand before it is to be drawn should be considered in determining the width of the pillar required. If a pillar is to be drawn in a short time, it need not be as large as one that is to be more permanent, as it is not subject to the disintegration due to pressure and to atmospheric effects.

Extremely large pillars are usually left to protect surface buildings, and also under swamps or large bodies of water, to avoid any possibility of a break in the roof through which the water can enter the mine.

Some coals are of such a nature that the sides and corners of pillars chip or split off when the coal is opened up, due to the disintegrating effect of the atmosphere, to the pressure of gas in the coal, or to the pressure of the roof. When this chipping or splitting off of pillar coal occurs, pillars of greater area are required.

Strong heavy strata, such as limestone or sandstone above the coal, that do not break and fall easily, act as a lever to crush the edge of the coal pillar and require larger pillars to prevent creep and squeeze than where the pressure is distributed over the pillar and not so much along the edge; under a friable roof, such as black shale or slate, that breaks and falls easily, and thus relieves the pressure on the edge of the pillar, a smaller pillar can often be used than under a strong compact roof, which brings the weight on the edge and constantly chips off the pillar. With a strong roof that does not break, there is danger from a movement of the strata over the pillar when robbing begins. A soft bottom requires a large pillar to prevent the heaving of the bottom. Faults, slips, and similar geological disturbances in the roof generally increase the size of the pillars, and also the difficulty and danger of drawing them.

If the floor is soft and the roof hard, a *creep* is likely to occur, and in such a case small pillars are so squeezed down into the floor as to be both troublesome and expensive to remove. If the floor is hard and the roof brittle, the latter will fall more or less in spite of all efforts, and the expense of cleaning up and timbering is great. If top and bottom are both strong, the weaker substance—the coal if left too long or in too small an amount—is crushed, and its value decreased or lost.

If the seam of coal is gaseous, the length of the pillar is decreased on account of the length of the cross-cuts or breakthroughs required to ventilate the rooms, thus necessitating a wider pillar than would otherwise be required.

While it is desired to have a certain excess of strength in the mine pillars, the expense of driving long cross-cuts through them for ventilating the openings, as well as the necessity for realizing as large a percentage as possible of coal in the first working, makes it desirable in many cases to use the minimum width of pillar required for the safe support of any given roof pressure.

When the room-and-pillar system is used, the best results as regards the percentage of coal won from a coal bed are undoubtedly obtained where narrow rooms are driven in the first working with ample pillars left between, and when the pillars are withdrawn as quickly as possible after the rooms are worked up their full length.

When two or more beds are worked at the same time, the width of pillar required in the lowest bed will, in most cases, determine the maximum width for all the overlying beds, for the pillars should be placed with their center lines vertically one above the other, and if any difference is made in the size of pillars in overlying beds the lower of the pillars should be the larger. This is the more important the closer the seams are together.

The *inclination* of the seam, although decreasing the normal pressure or the pressure perpendicular to the roof and floor, gives rise to a tendency of the roof to slide on the pillars when the coal is removed. This tendency greatly increases when the work of drawing the pillars is begun, especially if the roof is hard and fails to break. Although the decreased pressure in an inclined seam will call for a narrower pillar than for a flat bed, the tendency of the roof to slip necessitates an increased width over what will be required in a flat bed with the other conditions similar.

**Depth of Cover.**—The depth of cover, or the thickness of the rocks lying between the seam and the surface, is the prime factor, other things being the same, in determining the size of coal pillars. The table on page 694 gives the weight, in pounds per cubic foot, of the coal-measure rocks of the United States. For practical purposes, the weight of the overlying cover may be taken as 160 lb. per cu. ft. Thus, every $12\frac{1}{2}$ ft. in depth of cover causes a weight of 1 T. on each square foot of pillar. The pressure at a depth of 100 ft. will be 8 T. per squ. ft.; at 500 ft., 40 T.; at 1,000 ft., 80 T., and similarly for other thicknesses of cover.

## WEIGHT OF ROCKS

| Rock | Weight per Cubic Foot Pounds |
|---|---|
| Clay................ | 115 |
| Earth.............. | 100 |
| Gravel............. | 117 |
| Limestone......... | 165 |
| Sand.............. | 117 |
| Sand full of water.. | 120 |
| Sandstone......... | 150 |
| Shale............. | 162 |
| Slate............. | 175 |

**Crushing Strength of Anthracite.** The following table gives the crushing strength of anthracite as determined by experiments made under the auspices of the Engineers' Club, of Scranton, Pa. The results are averages of a large number of tests made on samples from a number of different beds and from different parts of the anthracite field. The test pieces were right prisms with a 2 in. square base, and of heights of 1, 2, and 4 in., respectively, corresponding to $a$, $b$, and $c$ in the headings in the table. These tests suggest the following conclusions in regard to the samples tested:

Although the area of the base, or the area pressed, is the same in each case, the total crushing load and the crushing load per square inch are not the same in the different samples, but vary, approximately, inversely as the square root of the height of the sample; that is to say, sample $c$ having four times the height of sample $a$ has approximately but one-half the crushing strength of the former. Other experients indicate that similar samples (samples whose heights and bases are proportional) have the same unit crushing load, or require the same crushing load per square inch of the area of the base, and the total crushing load in this case is proportional to the area of the base. For example, a cube measuring 2 in. on a side requires four times the total crushing load required by a cube measuring 1 in. on a side, but the unit crushing load or load per square inch of the area of the base is the same in each. If this can be conclusively proved for small test samples, it is fair to reason by analogy that the same rule holds for mine pillars, so that if *the strength of the pillar or the unit load supported is constant for similar pillars*, a pillar 40 ft. wide in a seam 20 ft. thick has the same strength as a 10-ft. pillar in a 5 ft. seam; or a 60-ft. pillar in a

## AVERAGE COMPRESSIVE STRENGTH OF ANTHRACITE

| | Sample $a$ | Sample $b$ | Sample $c$ |
|---|---|---|---|
| Total crushing load, in pounds..... | 23,000 | 16,348.000 | 11,416.0 |
| Crushing load per square inch, in pounds..................... | 5,750 | 4,087.000 | 2,854.0 |
| Load producing first crack in sample in pounds.................... | 3,022 | 2,025.000 | 1,875.0 |
| Height $h$...................... | 1 | 2.000 | 4.0 |
| $\sqrt{h}$........................ | 1 | 1.414 | 2.0 |
| Approximate relative crushing strength $\dfrac{1}{\sqrt{h}} =$ ............. | 1 | .707 | .5 |

15-ft. seam has the same strength as a 40-ft. pillar in a 10-ft. seam; the unit load supported, or the load per square foot of pillar, being the same in each case.

Also, if this be so, the crushing strength of any coal pillar, per square inch, can be found by multiplying the crushing strength of a unit cube 1 in. on a side by the square root of the ratio of the area of the base to the height of the pillar. The crushing strength of the unit cube as given by these experiments is 4,000 lb. The unit loads producing the first crack; that is, when the coal begins to scale off on the outside of the block, are approximately one-half of the unit crushing loads in each case, except in sample $c$, where the height of the sample is four times the width of the base. This form, however, need not be considered in the study of the crushing strength of mine pillars, which usually have a broad base as compared with the height of the pillar, and are represented more closely by samples $a$ and $b$. Hence, the unit load producing the first crack in mine pillars may be assumed as one-half of that obtained, or 2,000 lb.

**Crushing Strength of Bituminous Coal.**—There are no figures for bituminous coal similar to those given for anthracite, and owing to the great variation in the character of different bituminous coals, an average value cannot be given. In order to make similar calculations for bituminous coal, tests of the coal from the particular mine in question must be made.

## ROOM, ENTRY, AND SLOPE PILLARS

**Load on Pillars.**—As the removal of the coal throws the total load on the pillars, the roof pressure per square foot on the pillars is increased in the ratio of the total area of the opening and pillars to the area of the pillars. In the accompanying illustration, $o$ represents the width of an opening separated from other similar openings by pillars, the width of each pillar being $w$. A weight of cover equal to $w+o$ then rests on each pillar $w$, and if $L$ represents the roof pressure or load per square foot on each pillar,

$$L = 160d\frac{w+o}{w}$$

EXAMPLE.—Find the roof pressure at a depth of 900 ft. below the surface,

when the rooms are driven on 70-ft. centers with pillars 50 ft. wide between them and the width of the rooms is 20 ft.

SOLUTION.—In this case, $d=900$, $w+o=70$, and $w=50$. Substituting these values in the formula,



$L = 160 \times 900 \times \frac{70}{50} = 201,600$ lb. per sq. ft.; $201,600 \div 144 = 1,400$ lb. per sq. in.; $201,600 \div 2,000 = 100.8$ T. per sq. ft. This roof pressure must be below the safe crushing strength of the coal if the pillars are to stand and not be immediately ground to powder.

**Strength of Pillars.**—The safe strength of pillars may be estimated as one-half or one-third of the squeezing strength—that is, the point at which the first cracks appear—or one-fourth to one-sixth of the crushing strength, according to the conditions of mining; that is, using the values given in the table on page 694, the safe load for anthracite should not be estimated as greater than $2,000 \div 3$, say, 700 lb. per sq. in. under adverse conditions; or, under more favorable conditions, $2,000 \div 2 = 1,000$ lb. per sq. in. Expressed in tons per square foot, these values will vary from 50 to 70 T.

If the conclusions deduced from the experiments made on anthracite are verified by other experiments, the allowable unit load on a pillar of anthracite may be expressed by a formula as follows:

$$S = C\sqrt{\frac{w}{t}}$$

in which $S$ = unit load that can be supported, in tons per square foot;
$C$ = constant expressing safe crushing strength of a unit sample of anthracite, in tons per square foot;
$w$ = width of pillar, in feet;
$t$ = thickness of seam, in feet.

EXAMPLE.—Find the safe load that can be put on anthracite pillars having a width of 20 ft. in a seam 5 ft. thick.

Solution.—Substituting the given values in the formula, the safe load on these pillars is $S = 50 \times \sqrt{\frac{20}{5}} = 100$ T. per sq. ft. Ans.

**Width of Room Pillars.**—The strength of mine pillars, or the safe unit load they will support, must be at least equal to the roof pressure or load per square foot resting upon them; hence, to sustain the roof, $L = S$. Calling the percentage of room-pillar coal to be left in the mine $J$ (expressed decimally), $J = \dfrac{w}{w+o}$; multiplying the right-hand side of the equation for $S$ by 2,000 to reduce tons to pounds; and equating $L$ and $S$, and solving for $w$,

$$w = \left(\frac{160\ d}{2,000\ CJ}\right)^2 t;$$

in which $w$ = width of pillar, in feet;
$J$ = percentage of coal in room pillars;
$d$ = depth of cover, in feet;
$C$ = constant for safe unit crushing strength of coal;
$t$ = thickness of coal seam, in feet.

The percentage of pillar coal to be left between the rooms is often assumed, and it depends, of course, on the relative size of pillar and room openings. The safe width of opening is best determined by practical experience and judgment.

In the foregoing equations there are two variable quantities, $w$ and $o$, which may have any values. Hence, if the width of the room $o$ is assumed, the value for $w$ selected must satisfy the equations for both $J$ and $w$. This can only be accomplished by trial, as appears from the following example.

EXAMPLE.—Assume that a 16-ft. seam of anthracite lying 600 ft. below the surface is overlaid with alternate layers of shale and sand rock. Find the width of pillar that should be left between the rooms if the rooms are made 20 ft. wide.

SOLUTION.—Assume a width of pillar $w = 40$ ft. Substituting this and $o = 20$ ft. in the formula for $J$, $J = 66\frac{2}{3}\%$. Substituting this value of $J$ in the second formula, $w = 23 + $ ft.; very much less than the assumed value of 40 ft.

For a second trial, assume $w = 35$ ft., whence $J = 63 + \%$, and $w = 36.4$ ft. This second value is close enough for practical purposes, but another may be made by placing $w = 36$ ft., whence $J = 64 + \%$ and $w = 35.7$ ft.

From this, the pillars should be 36 ft. wide, and the rooms are driven on $20 + 36 = 56$ ft. centers.

For bituminous coal of medium hardness and good roof and floor, a rule often used is to make the thickness of room pillars, equal to 1% of the depth of cover for each foot of thickness of the seam, according to the expression

$$W_p = \frac{t}{100} D$$

in which

$W_p = $ pillar width;
$t = $ thickness of seam;
$D = $ depth of cover.

Then make the width of breast or opening equal to the depth of cover divided by the width of pillar thus found, according to the expression $W_o = \dfrac{D}{W_p}$, where $W_o = $ width of room.

Frail coal and coal that disintegrates readily when exposed to the air, and a soft bottom, may increase the width of pillar required as much as 50% of the amount just found; also, a hard roof may increase the same as much as 25%; while on the other hand, a frail roof or a hard coal or floor may reduce the width of pillar required 25%. The hardness of the roof affects both the width of pillar and width of opening alike, which is not the case with any of the other factors.

In the accompanying table, the weight thrown upon pillars at different depths by the removal of different proportions of coal is given:

## WEIGHT ON PILLARS AT VARIOUS DEPTHS

| Depth Seam Feet | Percentage of Coal Left in Pillars | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 90 | 80 | 70 | 60 | 50 | 40 | 30 | 20 | 10 |
| | Weight on Pillars in Pounds per Square Inch | | | | | | | | |
| 100 | 111 | 125 | 142 | 166 | 200 | 250 | 333 | 500 | 1,000 |
| 500 | 555 | 625 | 710 | 830 | 1,000 | 1,250 | 1,665 | 2,500 | 5,000 |
| 1,000 | 1,111 | 1,250 | 1,428 | 1,666 | 2,000 | 2,500 | 3,333 | 5,000 | 10,000 |
| 1,500 | 1,666 | 1,875 | 2,138 | 2,496 | 3,000 | 3,750 | 4,998 | 7,500 | 15,000 |
| 2,000 | 2,222 | 2,500 | 2,956 | 3,333 | 4,000 | 5,000 | 6,666 | | |
| 3,000 | 3,333 | 3,750 | 4,384 | 4,999 | 6,000 | 7,500 | | | |
| 4,000 | 4,444 | 5,000 | 5,912 | 6,666 | 8,000 | | | | |
| 5,000 | 5,555 | 6,250 | 7,340 | | | | | | |
| 10,000 | 11,110 | 12,500 | | | | | | | |

Slope Pillars.—A slope should have a pillar along its full length and theoretically the pillar should gradually increase in width from top to bottom as the

thickness of cover increases, but in practice this is seldom done and the slope pillar is the same width throughout. The frequency of squeezes on slopes indicates that this is faulty practice. The width of the slope pillar is sometimes prescribed by law.

There is not much danger of the draw destroying a slope sunk in the coal, except that due to mining in an underlying seam, because the line of the slope is in the same plane as the bed in which the mining is done and nearly at right angles to the plane of fracture, whereas in a shaft, the lines of fracture may reach or cross the line of the shaft, and in a pitching seam this danger is even greater than in a flat seam.

**Entry Pillars.**—Much that has been said with reference to room and slope pillars will apply with equal force to entry pillars. The chief factors determining the width of pillar required are the depth of cover, thickness, and character of coal seam and width of opening. The size of entry pillars, as of room pillars, is determined almost entirely by practical experience. The best practice advises leaving large pillars about the entries and all airways so as to avoid all possibility of a squeeze.

## SHAFT PILLARS

Great diversity of opinion exists among mining authorities as to the size of shaft pillars and the matter must be decided largely by local considerations and practical experience in the district in which the shaft is sunk. The shaft pillar should be large enough so that the effect of the draw cannot reach the shaft or the buildings on the surface about the shaft and thus interfere with its alinement. The same general rules apply to shaft pillars as to other pillars; namely, in general, the deeper the shaft and the thicker the seam the larger must the pillar be, while the harder the coal the smaller may the pillar.

**Pillars in Flat Seams.**—In flat seams, the size of the shaft pillar required depends chiefly on the depth of the seam below the surface, that is, the depth of cover, and to a less extent on the thickness of the seam. The rules given for determining the size of the shaft pillar afford widely varying results, owing to the varying conditions under which each rule was formulated, and for this reason that rule should be employed that seems best suited to the particular conditions of the case. These rules are given as formulas and the results obtained by applying them to determine the shaft pillars required at depths of 300 feet and 600 feet, respectively, are tabulated later.

Let  $D$ = diameter of round pillar, or side of square pillar, in yards;

$d$ = depth of cover, in yards;

$t$ = thickness of seam, in yards.

**Merivale's Rule.**—*Diameter of circular pillar, or side of square pillar, in yards, is equal to twenty-two times the square root of the depth of the shaft, in fathoms, divided by 50.*

$$D = 22 \times \sqrt{\frac{d}{2 \times 50}} \qquad (1)$$

**Andre's Rule.**—*Minimum diameter of circular pillar, or side of square pillar, 35 yd. to a depth of 150 yd.; add 5 yd. for each 25 yd. of additional depth.*

$$D = 35 + 5 \times \left(\frac{d - 150}{25}\right) \qquad (2)$$

**Wardle's Rule.**—*Minimum diameter of circular pillar, or side of square pillar, 40 yd. to a depth of 60 fath.; add 10 yd. for each 20 fath. of additional depth.*

$$D = 40 + 10 \times \left(\frac{d - 2 \times 60}{2 \times 20}\right) \qquad (3)$$

**Pamely's Rule.**—*Minimum diameter of circular pillar, or side of square pillar, 40 yd. to a depth of 100 yd., add 5 yd. for each 20 yd. of additional depth.*

$$D = 40 + 5 \times \left(\frac{d - 100}{20}\right) \qquad (4)$$

**Mining Engineering (London) Rule.**—*Radius of circular pillar, or half side of square pillar, in yards, is equal to 20 yd. plus one-tenth of the product obtained by multiplying the depth of shaft, in yards, by the square root of the thickness of the seam, in yards.*

$$D = 40 + \frac{d \times \sqrt{t}}{5} \qquad (5)$$

**Foster's Rule.**—*Radius of circular pillar, or half side of square pillar, in feet, is equal to three times the square root of the product of the depth of cover, in feet, and the thickness of the seam, in feet.*

$$D = 6 \times \sqrt{d \times t} \qquad (6)$$

**Dron's Rule.**—*Draw a line enclosing all surface buildings that should be protected by the shaft pillar. Make the pillar of such size that solid coal will be left in all around this line for a distance equal to one-third of the depth of the shaft.*

$$D = s + \frac{2d}{3} \qquad (7)$$

in which $s$ = diameter of circle, or side of square, in yards, at the surface.

**Hughes's Rule.**—*For the diameter of a circular pillar, or the side of a square pillar, allow 1 yd. for each yard in depth.*

$$D = d \qquad (8)$$

**Central Coal Basin Rule.**—In the Central Coal Basin of the United States, for shaft mines worked on the room-and-pillar method, the rule is: *Leave 100 sq. ft. of coal for each foot that the shaft is deep,* it being understood that a main entry of average width is driven through this pillar. *If the bottom is soft, the result given by this rule is increased by one-half.*

$$D = \sqrt{\frac{100 \times d}{3}} \qquad (9)$$

## SIZE OF SHAFT PILLAR OBTAINED BY USE OF SEVERAL FORMULAS

| Authority | Diameter of Side of Pillar Yards | |
|---|---|---|
| | Shaft 100 Yd. Deep | Shaft 200 Yd. Deep |
| Merivale.................................. | 22.0 | 31.00 |
| Andre..................................... | 35.0 | 45.00 |
| Wardle.................................... | 40.0 | 60.00 |
| Pamely.................................... | 40.0 | 65.00 |
| Mining Engineering (London)*.............. | 68.3 | 96.50 |
| Foster*................................... | 84.8 | 120.00 |
| Dron†..................................... | 100.0 | 166.66 |
| Hughes.................................... | 100.0 | 200.00 |
| Central basin............................. | 100.0 | 142.00 |

*The seam is assumed to be 2 yd. (6 ft.) in thickness.
†An allowance of 100 ft. has been made for the diameter of the circle, or side of the square, enclosing the buildings on the surface.

When using formulas 2, 3, and 4, negative results in the fractional part must be rejected, as the diameter of pillar cannot be less than the minimum diameter or side allowed by the rule. For example, it is useless to apply Andre's rule to depths less than 150 yd., Wardle's rule to depths less than 60 fath. (120 yd.), or Pamely's rule to depths less than 100 yd.

The foregoing table shows clearly that no hard-and-fast rule can be given for determining the size of shaft pillar required in any particular case. The rules stated, however, determine the size of pillar required within certain practical limits, and suited to different conditions of roof strata and are, therefore, useful and desirable. The presence of faults or slips in the roof makes larger pillars necessary.

**Pillars in Inclined Seams.**—The inclination of the seam increases the uncertainty in respect to the draw in the strata overlying the seam, making it more difficult to give any rule of universal application. The general practice in regard to the size of pillar required when the seam is inclined, is to increase the pillar on the rise side of the shaft, while that on the dip side of the shaft is often made the same as for a flat seam. To what extent it is necessary to increase the pillar on the rise side is largely a matter of experience and judgment in particular localities, and this is always the most reliable guide.

One method is to calculate the extent of the pillar on the dip side of the shaft by the rules given for flat seams, choosing for this purpose the rule that seems best suited to the conditions with respect to the character of the seam and overlying strata. The diameter of the circular pillar, or the side of a square pillar, thus obtained, will give the width of the pillar measured on the strike

of the seam, and half of this width will give the extent of the pillar measured below the shaft on the dip of the seam. Then, calling the width of the pillar $D$, the depth of the shaft $d$, and the inclination of the seam $a$, the extent of the pillar measured on the pitch of the seam may be taken as $\frac{D}{2} + \frac{3}{4} d \sin a$.

This rule is arbitrary, but approximates to a certain extent the condition relative to the inclination of the seam. All the rules and formulas given for determining the sizes of pillars, both in flat and inclined seams, are only suggestive of what is required, and must always be modified according to the experience and judgment of the person in charge of the work.

## PILLARS FOR MISCELLANEOUS PURPOSES

**Pillars for Supporting Buildings, Etc.**—Dron's rule for shaft pillars is probably the most practicable, as it provides for a given pillar of coal all around the buildings, etc., to be supported.

**Reserve Pillars.**—Extra large pillars of coal are often left at regular intervals in the workings; their purpose is to divide the mine into sections or districts so as to localize the effect of any squeeze that may start in one of these districts by breaking the roof at the reserve pillar. These pillars are usually equal to the width of one room and two pillars, and are formed by not driving one room as called for by the plan of the mine. They are taken out before the entry or gangway is abandoned.

**Chain Pillar.**—A chain pillar is usually left across the ends of a group of rooms to protect the gangway, or entry, toward which the rooms are being driven. The miners frequently drive their rooms too far and hole through into the next gangway in spite of the precautions that are taken to prevent this occurrence. To avoid the possibility of rooms being driven too far and holing through the chain pillar, a *cut-off room* is sometimes driven parallel to the entries or gangways. This place is driven wide enough to avoid the expense of yardage, and rooms driven from the next gangway are allowed to hole into it, thus avoiding the necessity of accurately measuring the length of the rooms and of carefully watching the miners to see that they do not exceed the limit allowed. The method also possesses the advantage of giving a regular width to the entry pillar and thereby avoiding the loss of a considerable amount of pillar coal when these entries are abandoned and their pillars drawn. When drawing back an ordinary chain pillar, any irregularity in the width of the pillar may cause a loss of some of the coal, which cannot occur when a cut-off room is driven as described.

**Barrier Pillars.**—The laws of some states require a pillar of coal to be left in each bed of coal worked along the line of adjoining properties, of such width, that, taken in connection with the pillar to be left by the adjacent property owner, it will be a sufficient barrier for the safety of the employes of mines on either property in case one should be abandoned and allowed to fill with water. These pillars are known as *barrier pillars*. The width of such pillars is determined by the engineers of the adjoining property owners and the mine inspector in whose district the properties are located.

An arbitrary rule for the width of barrier pillars, adopted by a number of coal companies and by the state mine inspectors of the anthracite districts of eastern Pennsylvania, is as follows:

**Rule.**—*Multiply the thickness of the deposit, in feet, by 1% of the depth below drainage level, and add to this five times the thickness of the bed.*

Thus, for a bed of coal 6 ft. thick and 400 ft. below drainage level, the barrier pillar will, according to this rule, be $(6 \times 400 \times .01) + (6 \times 5) = 54$ ft. wide.

The Bituminous Mine Law of Pennsylvania requires a thickness of 1 ft. of pillar for each $1\frac{1}{2}$ ft. of water head if, in the judgment of the engineer of the property and of the district mine inspector, this thickness is necessary for the safety of the persons working in the mine. The same law makes it lawful for any operator whose mine is endangered by an accumulation of water in abandoned workings located on an adjoining property, to drive a drift or entry protected by bore holes, across the barrier line, for the purpose of tapping and draining such water, and makes it unlawful for any person to attempt to, or in any way to obstruct the flow of such water to a point of drainage. The law also provides that no coal shall be mined within 50 ft. of any abandoned workings containing a dangerous accumulation of water, until such danger has been removed as described above.

## SIZE OF BARRIER PILLARS TO BE LEFT BETWEEN ADJOINING PROPERTIES*

| Thickness of Seam, in Feet, Mined From | Depth Below Water Level, in Feet | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 50 | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 | 550 | 600 | 650 | 700 | 750 | 800 | 850 | 900 | 950 | 1,000 | 1,050 | 1,100 | 1,150 | 1,200 | 1,250 | 1,300 | 1,350 | 1,400 | 1,450 | 1,500 |
| 3 | 15 | 17 | 18 | 20 | 21 | 23 | 24 | 26 | 27 | 29 | 30 | 32 | 33 | 35 | 36 | 38 | 39 | 41 | 42 | 44 | 45 | 47 | 48 | 50 | 51 | 53 | 54 | 56 | 57 | 59 | 60 |
| 4 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 | 46 | 48 | 50 | 52 | 54 | 56 | 58 | 60 | 62 | 64 | 66 | 68 | 70 | 72 | 74 | 76 | 78 | 80 |
| 5 | 25 | 28 | 30 | 33 | 35 | 38 | 40 | 43 | 45 | 48 | 50 | 53 | 55 | 58 | 60 | 63 | 65 | 68 | 70 | 73 | 75 | 78 | 80 | 83 | 85 | 88 | 90 | 93 | 95 | 98 | 100 |
| 6 | 30 | 33 | 36 | 39 | 42 | 45 | 48 | 51 | 54 | 57 | 60 | 63 | 66 | 69 | 72 | 75 | 78 | 81 | 84 | 87 | 90 | 93 | 96 | 99 | 102 | 105 | 108 | 111 | 114 | 117 | 120 |
| 7 | 35 | 39 | 42 | 46 | 49 | 53 | 56 | 60 | 63 | 67 | 70 | 74 | 77 | 81 | 84 | 88 | 91 | 95 | 98 | 102 | 105 | 109 | 112 | 116 | 119 | 123 | 126 | 130 | 133 | 137 | 140 |
| 8 | 40 | 44 | 48 | 52 | 56 | 60 | 64 | 68 | 72 | 76 | 80 | 84 | 88 | 92 | 96 | 100 | 104 | 108 | 112 | 116 | 120 | 124 | 128 | 132 | 136 | 140 | 144 | 148 | 152 | 156 | 160 |
| 9 | 45 | 50 | 54 | 59 | 63 | 68 | 72 | 77 | 81 | 86 | 90 | 95 | 99 | 104 | 108 | 113 | 117 | 122 | 126 | 131 | 135 | 140 | 144 | 149 | 153 | 158 | 162 | 167 | 171 | 176 | 180 |
| 10 | 50 | 55 | 60 | 65 | 70 | 75 | 80 | 85 | 90 | 95 | 100 | 105 | 110 | 115 | 120 | 125 | 130 | 135 | 140 | 145 | 150 | 155 | 160 | 165 | 170 | 175 | 180 | 185 | 190 | 195 | 200 |
| 11 | 55 | 61 | 66 | 72 | 77 | 83 | 88 | 94 | 99 | 105 | 110 | 116 | 121 | 127 | 132 | 138 | 143 | 149 | 154 | 160 | 165 | 171 | 176 | 182 | 187 | 193 | 198 | 204 | 209 | 215 | 220 |
| 12 | 60 | 66 | 72 | 78 | 84 | 90 | 96 | 102 | 108 | 114 | 120 | 126 | 132 | 138 | 144 | 150 | 156 | 162 | 168 | 174 | 180 | 186 | 192 | 198 | 204 | 210 | 216 | 222 | 228 | 234 | 240 |
| 13 | 65 | 72 | 78 | 85 | 91 | 98 | 104 | 111 | 117 | 124 | 130 | 137 | 143 | 150 | 156 | 163 | 169 | 176 | 182 | 189 | 195 | 202 | 208 | 215 | 221 | 228 | 234 | 241 | 247 | 254 | 260 |
| 14 | 70 | 77 | 84 | 91 | 98 | 105 | 112 | 119 | 126 | 133 | 140 | 147 | 154 | 161 | 168 | 175 | 182 | 189 | 196 | 203 | 210 | 217 | 224 | 231 | 238 | 245 | 252 | 259 | 266 | 273 | 280 |
| 15 | 75 | 83 | 90 | 98 | 105 | 113 | 120 | 128 | 135 | 143 | 150 | 158 | 165 | 173 | 180 | 188 | 195 | 203 | 210 | 218 | 225 | 233 | 240 | 248 | 255 | 263 | 270 | 278 | 285 | 293 | 300 |
| 16 | 80 | 88 | 96 | 104 | 112 | 120 | 128 | 136 | 144 | 152 | 160 | 168 | 176 | 184 | 192 | 200 | 208 | 216 | 224 | 232 | 240 | 248 | 256 | 264 | 272 | 280 | 288 | 296 | 304 | 312 | 320 |
| 17 | 85 | 94 | 102 | 111 | 119 | 128 | 136 | 145 | 153 | 162 | 170 | 179 | 187 | 196 | 204 | 213 | 221 | 230 | 238 | 247 | 255 | 264 | 272 | 281 | 289 | 298 | 306 | 315 | 323 | 332 | 340 |
| 18 | 90 | 99 | 108 | 117 | 126 | 135 | 144 | 153 | 162 | 171 | 180 | 189 | 198 | 207 | 216 | 225 | 234 | 243 | 252 | 261 | 270 | 279 | 288 | 297 | 306 | 315 | 324 | 333 | 342 | 351 | 360 |
| 19 | 95 | 105 | 114 | 124 | 133 | 143 | 152 | 162 | 171 | 181 | 190 | 200 | 209 | 219 | 228 | 238 | 247 | 257 | 266 | 276 | 285 | 295 | 304 | 314 | 323 | 333 | 342 | 352 | 361 | 371 | 380 |
| 20 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 | 290 | 300 | 310 | 320 | 330 | 340 | 350 | 360 | 370 | 380 | 390 | 400 |
| 21 | 105 | 116 | 126 | 137 | 147 | 158 | 168 | 179 | 189 | 200 | 210 | 221 | 231 | 242 | 252 | 263 | 273 | 284 | 294 | 305 | 315 | 326 | 336 | 347 | 357 | 368 | 378 | 389 | 399 | 410 | 420 |
| 22 | 110 | 121 | 132 | 143 | 154 | 165 | 176 | 187 | 198 | 209 | 220 | 231 | 242 | 253 | 264 | 275 | 286 | 297 | 308 | 319 | 330 | 341 | 352 | 363 | 374 | 385 | 396 | 407 | 418 | 429 | 440 |
| 23 | 115 | 126 | 138 | 150 | 161 | 173 | 184 | 196 | 207 | 219 | 230 | 242 | 253 | 265 | 276 | 288 | 299 | 311 | 322 | 334 | 345 | 357 | 368 | 380 | 391 | 403 | 414 | 426 | 437 | 449 | 460 |
| 24 | 120 | 132 | 144 | 156 | 168 | 180 | 192 | 204 | 216 | 228 | 240 | 252 | 264 | 276 | 288 | 300 | 312 | 324 | 336 | 348 | 360 | 372 | 384 | 396 | 408 | 420 | 432 | 444 | 456 | 468 | 480 |
| 25 | 125 | 138 | 150 | 163 | 175 | 188 | 200 | 213 | 225 | 238 | 250 | 263 | 275 | 288 | 300 | 313 | 325 | 338 | 350 | 363 | 375 | 388 | 400 | 413 | 425 | 438 | 450 | 463 | 475 | 488 | 500 |

*Each adjoining owner is to leave one-half of the pillar thickness required. The formula used in this case is: (Thickness of workings×1% of depth below drainage level)+(thickness of workings×5) = width of barrier pillar

## SQUEEZE AND CREEP

When the roof and floor are strong and unyielding and the pillars are insufficient to withstand the pressure thrown on them, they are filled with breaks and cracks, large pieces split off, and the pillars are finally crushed into small coal and the roof comes down. This is known as a *squeeze, thrust,* or *crush.*

When the material composing the floor or roof, or both, is soft and weak and the pillars left are too small, the weight on them causes the roof to sag or the floor to bulge upwards, or both. This result is known as a *creep.* The soft character of the floor or roof permits the pillars under the enormous roof pressure either to sink into the floor or to be forced into the roof, pressing out the softer material, which fills the openings. Fireclay is particularly susceptible to creep, and many of the fireclays that are hard when dry become extremely soft and plastic when moist; it is important to keep such a clay bottom dry.

The terms squeeze, thrust, crush, and creep are often incorrectly used synonymously. A squeeze and a creep may be going on at the same time. A squeeze or a creep does not generally come suddenly, but the pillars and timbers usually give evidence of the too great roof pressure by cracking and by pieces flaking off at the sides. The chipping or nicking of the pillar coal, indicating that the pillars are too small, should not be mistaken for the gradual spalling or chipping due to weathering alone. When pillars or timbers thus give evidence of increased pressure, they are sometimes said to be taking the weight. The coming of a squeeze is often first told by the departure of the rats from the affected area, as their sense of hearing is more acute than man's; next the coal begins to crack; and then the timbers split and crush.

**Stopping a Squeeze.**—When any sign of a squeeze appears, the pillars should be reenforced as much as possible by wooden chocks, or cribs, as here shown, and by supports of any kind that can be put up just outside of the part affected. If the action of the squeeze is slow, some of the pillars may be removed rapidly, which will allow the top to break and thus relieve the standing pillars of part of the weight.



The treatment of a squeeze should be determined by the inspection of an accurate and complete map of the workings. If the disturbed region cannot be isolated by timbering and building strong stoppings in all the roads round about it, the trouble may often be stopped with little expense by drawing out some of the timber already in place, and throwing the weight on some small outlying patches of coal that can, with advantage, be sacrificed to save the roads and pillars of the district affected. In many cases, such trouble can more quickly be arrested by helping it than by trying to prevent it. When once the roof becomes unsteady and the timbers are breaking and the floor is lifting, a force is operating that cannot be stopped by artificial means; it can, however, be directed by assisting it to find relief where the least damage will be done. If the roof does not break readily, dynamite should be used at different points to start the fall. By this means, the power of the squeeze may be broken and the danger of its spreading to adjacent workings lessened. The building of large cribs to avoid the disastrous results of squeeze often acts to increase the evil rather than to diminish it, especially if the cribs are placed at points where complete settlement is desired. The cribs are not easily removed, and serve as fulcrums by which the weight is carried forwards to other points. As permanent supports for the roof, cribs are of great advantage, but care should be taken to break the roof back of them when the weight comes on, in the same manner as over entry pillars, by the use of shots placed in the roof near them. Confining a squeeze to a certain limit is a difficult, expensive, and dangerous operation, requiring the utmost skill and care in every individual engaged in the work.

The creep will continue until the excavations are filled, and the whole becomes compact enough to resist the weight. This sometimes takes many months, but it is a sure result, whether the action is fast or slow. A creep cannot be resisted unless the space from which the coal has been removed is filled with other material like culm.

**Reopening a District Closed by Squeeze.**—Time should be given for the complete settlement of the roof before any attempt is made to reopen a district closed by a squeeze, for if work is begun before the action of the squeeze
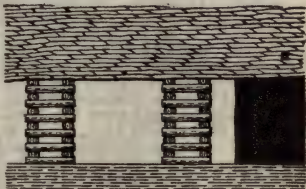
has wholly ceased, the movement will begin again and may extend to other parts of the mine. The work of reopening is expensive and seldom pays in thin seams unless the coal is very valuable. Where the entries are wholly closed, it is often possible to drive a new entry in the old pillars, or even across the pillars.

It is not usually economical to attempt to reopen old entries closed, or partially closed, by squeeze, as a larger amount of material must be handled, and more timber will be required than when a new opening is driven.

In the treatment of creep, it is usually better to excavate in the roof and leave the bottom undisturbed as the bottom often keeps working and fills up about as rapidly as it can be taken out.

Whenever practicable, the work of reopening can be done to better advantage by driving a pair of entries beyond the affected district, and coming back on the coal. By this means, the least affected portion of the district will be reached first and as much of the coal recovered as is found desirable; the demand for coal, however, will not always permit the adoption of this method.

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## FLUSHING OF CULM

In the anthracite regions of Pennsylvania, in Europe, and in South Africa, worked-out portions of the mine are now commonly filled with refuse material brought in on streams of water. This is done not only to support the roof over the workings but also to permit the recovery of the coal or the ore which would otherwise have to be left in the pillars.

Abroad, the material used for filling is very generally sand, but in Pennsylvania, the culm, or fine refuse from the breakers, washeries, etc., is commonly employed. In addition to culm, ashes from the boiler house, crushed slate, and the like are employed, either alone or, preferably, mixed with culm.

The plants for handling culm are more or less elaborate. The Dodson plant cost \$7,473.42 with a capacity of flushing 119 T. a da. and the Black Diamond plant, with a daily capacity of 287 T., cost \$6,280.12. They usually consist of crushers (where needed), troughs, conveyers, settling or mixing tanks, and in some instances storage tanks. Where culm alone is employed, it is usually brought from the breaker or culm bank by means of a scraper conveyer to a mixing tank, which may be anything from a simple oil barrel at a small operation to extensive wooden and concrete tanks at the larger ones. If a number of openings are being flushed at the same colliery, the mixing tank is generally set on a hill and is made of large size, pipes radiating from it to the various bore holes through which flushing is going on. Where coal is wet screened, the screenings from the breaker are generally caught in settling tanks so that the excess water flows away, the dewatered culm alone being elevated to the central mixing or distributing tank. At the Shenandoah City colliery, Shenandoah, Pa., all the waste material from the breaker is sent through the mine. The slate and screenings are brought out on separate conveyers. The screenings are dumped directly into the first of two flights of conveyers, which carry them to a distributing tower where enough water is added to flush them into the mine. The slate is carried first to a No. 3 Williams crusher where everything over $2\frac{1}{4}$ in. is broken and then conveyed back to be mixed with the breaker screenings. The ashes from the boiler house are run into the slate conveyer so that the lumps of clinker may be broken by the crusher and are thus mixed intimately with the culm and slate. When the breaker is not running the ashes pass to a concrete storage bin holding 1 wk.'s supply, from which they may subsequently be flushed when the breaker is running. The composition of the slush or sludge, the material used for flushing, is 50% screenings or culm, 44% slate, and 6% ashes.

The amount of water required for flushing depends on the material being flushed, the pitch of the seam and that of the pipe, and the distance to which the sludge has to be carried. At West Shenandoah, the proportion between water and screenings is made as nearly 2 to 1 as can be estimated. At this mine, the seam pitches 45° and less water is required than if the seam was flatter. At the Kohinoor colliery, 565 cu. yd. of culm were flushed daily with an expenditure of water of from 67 to 334 gal. per cu. yd. of culm; an average of 200 gal. of water per yd. of dry material. Experience has shown that from $1\frac{1}{2}$ to $1\frac{3}{4}$ lb. of water is required to flush 1 lb. of culm to level and down-hill places; 3 to 6 lb. of water to 1 lb. of culm to flush up-hill for heights varying from 10 to 100 ft. above the level of the shaft bottom. Any elevation of the pipe very materially increases the amount of water necessary. Mr. James B.

Davis, when superintendent of the Dodson and Black Diamond mines, ascertained by experiment that 1 cu. ft. of anthracite ground to culm can be flushed into a space of nearly $1\frac{1}{2}$ cu. ft., and it is therefore impossible to compress the culm more than one-third. In addition to acting as a filling and a support, to prevent squeezes and crushing, flushing has been advantageously used for fighting and sealing off mine fires. No instance has been recorded where spontaneous combustion has taken place in the flushed culm.

On the surface, the culm is generally conducted to the various openings through which it enters the mine in wooden troughs lined with sheet iron or terra-cotta, the latter being preferred. For handling culm, wooden pipe is preferred to iron, although the latter is, of course, largely used. The life of the pipe is dependent on the kind of material passing through it and the grade on which it is laid. Breaker screenings and culm will not wear a pipe as quickly as ashes and broken slate. A 6-in. iron pipe laid at West Shenandoah colliery, which carried crushed slate for a distance of 1,500 ft. on a considerable grade was worn out after passing 16,000 cu. yd., whereas wooden pipes carrying breaker screenings have lasted 1, 2, and 3 yr. Wrought-iron pipe is often used where ashes are being flushed separately. Wooden pipes should be turned after being in one position for a certain length of time. On very light grades, it may be necessary to turn them every 6 mo. On steep slopes, they had better be turned every few weeks to distribute the wear evenly over the interior and thus lengthen the life of the pipe. The life of wrought-iron pipe depends on the nature of the water used and the material flushed. With fresh water and small culm from the buckwheat screen, it lasts 18 mo.; when carrying culm from the bank, ranging from dust to pea coal and some chestnut, 9 mo.; and when mixed with ashes, 6 mo. The smaller the material the better.

The sludge is usually let into the mine through bore holes sunk with a rope drill from 6 in. to 8 in. in diameter, although the opening to a room driven to the crop, an old slope, or even an abandoned shaft have been used for the purpose. No rules can be given as conditions vary at each mine.

The handling of the culm underground will also vary from place to place. Basicly, the methods are the same in that brattices or dams are built at the mouths of the rooms or headings, the sludge let into the excavations through pipes and the water allowed to drain off through a pipe in the dam, leaving the solid material behind. If the sludge can be let in at the tops of pitching breasts it is naturally the best way, as they then fill by gravity. At several mines, gangways have been driven along the faces of a series of pitching breasts and along these gangways the sludge is conveyed in pipes, which are tapped at the face of each room. The mouths of the rooms on the main gangways are blocked with heavy bulkheads, so that the material is retained in the desired place. These bulkheads, or batteries, are built of material of a size and strength to suit the pitch of the seam. The steeper the pitch, the larger the props and the closer together are they placed. On pitches of 20° to 30° and upwards, 12-in. to 14-in. and larger props are used, set 2 ft. apart with braces between them. Care must be taken to place the props with their inside faces in line, so that the lining planks can be securely fastened. The lining or facing consists of a double layer of $1\frac{1}{2}$-in. plank securely hitched into the coal and all cracks stopped with hay. The tops and bottoms of the main posts are hitched into the roof and floor to a depth proportionate to the weight they will have to sustain.

The surplus water may be drained off in various ways. At some mines, small iron pipes are inserted in the bulkheads; in others, it merely drains off through the cracks. At Indian Ridge colliery and some others, the water drains off in wooden pipes placed in the brattices. These pipes are made of 1-in. boards and are 4 in. square inside. On their sides, saw cuts about 1 ft. long are made, four series of slots being made in each board. These slots allow the water to pass while holding the culm. Three, four, or six pipes may be placed in a battery as desired. Where the seam has not much pitch, a couple of square-board pipes are run up the breast. These are provided at the upper end with an upright branch pipe. When the dewatered material reaches the top of the upright, the pipe is closed with a board. Where the seam is flat, the flushing pipes are fastened along the roof for the full length of the chamber. Then as the chamber fills and the slush rises up to the pipe, a length of pipe is taken off and the flushing continued. In this case the seam cannot be filled completely, a little space always being left between the flush and the roof. After flowing for 1 or 2 hr. into one room, the slush is commonly turned into an adjacent room for the same length of time, during which interval the first place drains. The slush is then turned back to the first place while the second

one drains.  A room having a pitch of 5° or more can be filled with slush if it is poured in at the high end.   If ashes are used alone, a pitch of 10° or more is needed, as ashes, being porous, allow the water to drain away quickly and will not flow in a stream as readily as breaker screenings.  In caved areas, slush makes an excellent filling as it completely fills the spaces between the broken rocks.

The pipes used to convey the sludge underground are 6 in. in size and will carry any material coming to them, as the sludge has to pass through a 2¾-in. screen placed over the mouth of the bore hole.  A common trouble in flushing is to have the pipe become blocked.  This will happen if the pipe is not full of clear water without solids before flushing is begun, and if the pipe is not cleaned out with clear water after flushing is stopped.  Water must be turned into the pipe and be flowing before the slush enters and must be kept flowing after the slush is stopped.  The slush should be so mixed that there is a constant uniform flow.  Sudden rushes of slush are likely to block the pipe.  To quickly locate a block, holes are bored in wooden pipes and closed with plugs. The pipe is likely to clean itself out below any block, and by knocking the plugs out the blocks can be located and the pipes need be taken apart only at the block.

When roadways are cut through slush the sides will stand without timbering unless they become water soaked.  When drawing pillars after flushing, regular systems are followed as much as possible.  One method is to take out every third pillar, and it is mined from the gangway advancing, ventilation being maintained by means of a door on the gangway and a brattice up to the working face, as no breakthroughs can be maintained.  The coal pillars of the original working are now the rooms.  As soon as one is drawn, a battery is built at its mouth, and the empty space is flushed.  When one pillar is drawn the next one is mined, care being taken that the pillars on which work is being done are three apart and that all rooms in between are flushed.  When the pillars are irregular, no system can be followed.  Pillars in underlying seams can be taken out without disturbing those in seams above, but it is better to take them out simultaneously and have the rooms flushed in sections one over the other. When removing the pillars certain precautions have to be followed.  In doing this the face of the pillar along the gangway is attacked, and a road driven
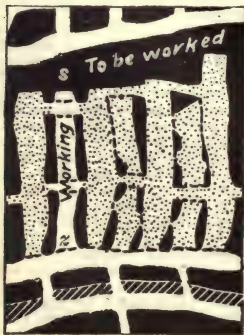


up through the pillar, splitting it, as shown at z in the accompanying illustration.  This road may be the full width of the pillar, but in general it is necessary to leave a narrow stump of coal on either side to keep up the fine flushed material in the adjoining breasts.  The thickness of this supporting coal depends entirely on the condition of the flushed material behind it. If that is fine, it will set firmly and form a compact mass that will not run.  In such a case, the pillar may be entirely taken out, leaving a vertical wall of solidly packed flushed culm.  When the flushed material is of a size larger than buckwheat, it will not set compactly, but will run when it is opened up, and when such material fills the adjoining breasts, the thin pillar of coal must be left to keep back the culm.  Timbers are placed flush up against the culm or the coal stumps, as the case may be, and if there is a tendency for the culm to run, lagging is placed behind the timbers.  In some cases, as much as 700 ft. of timber have been used per 100 ft. of pillar taken out.  As the pillar is removed, the top settles until it finally rests upon the flushed culm, and as the weight from the top and the pressure from the sides comes upon these props, they are broken, while the coal that has been left will also be crushed.  At the Black Diamond colliery, the props used are 16 ft. long, and at this colliery the top settles about 4 ft. if the flushed material is packed tightly before the roof pressure comes on it.  After this settling, new props 12 ft. in length are put in close up against the culm and the broken stump of the original pillar, and they serve to keep the road open up to the working face.

The Mammoth seam, ranging from 40 to 60 ft. thick, at the Kohinoor colliery is being mined by a flushing and slicing system.  A room is driven in the bottom bench and as soon as finished is flushed solidly full.  With the top

of the flushing as a floor, about 10 ft. more of coal is mined out, and this second space again filled with culm. In a similar manner the rest of the seam is sliced out and the openings flushed until the roof is reached. In working the first or bottom bench, the track is laid on the regular floor of the seam, but in working the remaining slices, a buggy or small car pushed by hand is used to carry the coal to a chute through which it is dumped into the regular mine car standing on the gangway.

The saving from flushing of culm over depositing it on the surface varies for the ordinary anthracite colliery from $5 to $15 per da. The average cost of putting in stoppings in a 9 ft. seam is given by Mr. James B. Davis as $9.50, including the material.

## BUILT-UP PACKS AND CRIBS

While the term *pack* is sometimes applied to any built-up structure of stone used to support the roof where the coal has been removed from a wide area, the word should be restricted in its meaning to the walls built up of roof slate, etc., used in longwall mining as a support for the roadways. In this system of mining, the packs must be carefully built, the larger and flatter stones used for facing, and all of them bedded in fine material so that the rock pressure may compact the pack into a solid mass. The face stones should be well bonded, and a number of them should extend back into the center of the pack so as to bind the face walls and prevent their bulging out when the weight comes on them.

*Cribs*, or *chocks*, consist of a square building of round logs built in log-cabin style and either notched into each other or pinned together at the ends. The interior is closely packed with loose slate, rock, spalls, etc., and when well built cribs offer substantial support to the roof. They are quite commonly employed to strengthen the corners of pillars that have been left too small; as permanent supports where the pillars have been entirely removed; and to cause a break to the surface and thus stop a squeeze. Cribs are of almost universal employment in longwall mines, and especially in those where there is a scarcity of material from which to build the necessary packs. Cribs are not adapted as permanent supports if the seam is more than a short distance beneath the surface, as they crush and may even be ground to powder under the enormous pressure at great depths.

**Strength of Packs and Cribs.**—In 1911, Prof. Frank P. McKibben made at the Fritz Engineering Laboratory of the Lehigh University, South Bethlehem, Pa., a series of important tests upon the crushing strength of various kinds and forms of artificial mine supports. These investigations were undertaken on behalf of Messrs. William Griffith and Eli T. Connor, appointed by the city of Scranton, Pa., to investigate mining conditions under that municipality and to suggest methods by which the surface might be supported over extended areas while, at the same time, mining and removal of coal was going on beneath.

The results of these investigations are summarized in the table on page 706, and while, naturally, made upon structures much smaller than those used in actual mining operations, give an excellent idea as to the relative value of the various types of support in common use. It will be noted that, in a general way, the tests were made upon two types of support. In the one, the supports consisted of cribs or piers of rock, or of rock and timber, or of piles of stone which were free to expand laterally. That is, they represented on a small scale the larger structures frequently built in open places in the mine upon the approach of a squeeze. The other type represented the various methods of filling in which the material used as a support is contained within the confines of a room in the mine and is not free to expand laterally, being held in place by the coal in the ribs and face.

Supports of the first class, whether artificial structures or piles of loose material, are not as strong as those of the second class except for small amounts of compression. Under heavy pressure and a consequent high percentage of compression, they are not to be recommended except for temporary use. It will be noted that circular piers, particularly if the interstices are filled with fine shovel stuff so that the voids are reduced to a minimum, are much stronger than the usual square or rectangular pier. Timber cogs, while stronger than square gob piers, are of short life owing to the decay of the wood, which is particularly rapid in warm, humid mine atmospheres. Concrete piers, as in test 14, give most excellent results up to a pressure sufficient to produce about 3% of compression and cracking, beyond which very little increased weight will grind them to powder. Test 6 was made upon slabs of sandstone, such

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## SUPPORTING STRENGTH OF VARIOUS FORMS OF DRY FILLING

| Number of Test | Kind of Material Composing the Artificial Supports | Per Cent. of Compression — Approximate Height, in Feet, of Column of Coal-Measure Rock 1 Ft. Square, Necessary to Compress Artificial Roof Supports | | | | | | Remarks |
|---|---|---|---|---|---|---|---|---|
| | | 1 | 3 | 5 | 10 | 20 | 30 | |
| 1 | Rectangular gob piers, ordinary construction | | 10 | 12 | 30 | 125 | 306 | Free to expand laterally |
| 2 | Circular piers of mine rock, well constructed | | 46 | 73 | 146 | 292 | 512 | |
| 3 | Timber cogs filled with gob, average construction | | 8 | 68 | 182 | 270 | 419 | |
| 4 | Loose pile of broken sandstone through 1¼-in. ring, 40% voids | | | 20 | 53 | 124 | 298 | |
| 5 | Pile broken sandstone, 40% voids, voids filled with sand | | | 21 | 53 | 186 | 465 | |
| 6 | Loose pile large broken sand rock, 45% voids | 12 | 48 | 66 | 121 | 351 | 492 | |
| 7 | Mine room filled with large broken sand rock, 50% voids | | 27 | 45 | 117 | 434 | 615[a] | |
| 8 | Mine room filled with broken sandstone, 40% voids | | 44 | 74 | 177 | 619 | 1,310 | |
| 9 | Mine room filled with broken sandstone, 40% voids filled with sand | | 46 | 77 | 325 | 6,000 | 8,860[b] | |
| 10 | Mine chamber filled with dry coal ashes, 64% voids | | 13 | 25 | 70 | 143 | 332 | |
| 11 | Mine room filled with dry river sand | 12 | 40 | 70 | 442 | 1,715 | 6,640 | |
| 12 | Mine room filled with river sand flushed in with water | 111 | 522 | 891 | 2,310 | 1,822 | 8,860[c] | |
| 13 | Mine chamber filled with coal culm flushed in with water | 32 | 118 | 190 | 472 | | 5,995 | |
| 14 | Concrete pier, 1 part cement, 7 parts sand and gravel; 5 mo. old | 117 | 1,092 | Gradually Cracked to Pieces Under Continuous Load Equal to 600 Ft. of Rock | | | | |
| | Resistance of flushed culm | 1.5 | 1.4 | 4.7 | 1 | 1 | | Comparative |
| | Resistance of flushed sand | 3.5 | 4.4 | | 5 | 4 | | |
| | Concrete piers | 3.6 | 9 | [d] | [d] | [d] | [d] | |

[a] 27 per cent. settlement.  [b] 23 per cent. settlement.  [c] 20¼ per cent. settlement.  [d] Worthless.

as would result from shooting down the roof over a wide area. The material was not packed, stowed, or built up artificially but was used as it fell and proved as good a support, practically, as the built-up circular gob pier used in test 2.

Tests 7, 8, and 9 represent unstowed material resulting from shooting down the roof in a room; this gives much greater support when the voids are filled with fine material; in fact, it is as strong as the best sand flushing under the maximum pressure. It will be noted that the sand flushing, so largely used abroad, affords a better support in the ratio of about 4 to 1 than the culm flushing commonly employed in the United States.

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# TIMBERING WITH WOOD

## GENERAL REMARKS

**Nature of Rock Pressure.**—The weight of the rocks overlying a coal seam exerts both a major and a minor pressure on the timbering. The major, or greater, pressure is practically irresistible and is due to the weight of all the overlying rocks from the coal seam to the surface, and at a depth of 1,000 ft. is equal to about 80 T. per sq. ft. It cannot be supported by timbering and cannot be entirely resisted by flushing culm into the workings. Solid masonry or concrete will alone withstand the major pressure. The minor pressure is caused primarily by the weight of the draw slate, which may vary from a few inches to several feet in thickness, producing a pressure of as much as, say, 500 lb. per sq. ft. In some places, to the weight of the draw slate must be added that of from 5 to 40 ft. of so-called soapstone, which crumbles on exposure to the weather and gradually falls until some solid stratum is reached. If as much as 40 ft. thick, this will cause an additional pressure of some 3 T. per sq. ft. It follows that mine timber is ordinarily designed to withstand a pressure of from, say, 50 lb. to 3 T. per sq. ft.

**Choice of Timber.**—Timber should be long grained and elastic, strong but not too heavy for easy handling in thick seams and in pitching places. Elasticity combined with the proper strength is of prime importance. The timber must be strong enough to resist the minor pressure for which it is designed and at the same time elastic so that, while resisting the minor pressure, it will bend without breaking and give warning of the approach of a major pressure caused, say, by a squeeze. Oak, beech, and similar hardwoods while very strong are heavy, and, being short grained, are not elastic, commonly breaking with but slight bending. The softer coniferous woods, such as pine, fir, and spruce, possess considerable strength and great elasticity and make the most desirable mine timbers. Very elastic timbers, such as cypress and willow, are not satisfactory for props because they bend like a bow without offering the resistance necessary to keep the draw slate in place even for a short time.

When selecting props, the principal points to be observed are: Straightness, slowness of growth as indicated by narrow annular rings, freedom from knots, indents, resin, gum, and sap.

## ROOM TIMBERING IN FLAT SEAMS

**Props.**—Single props are set to support the draw slate during the process of removing the coal. They are generally made of natural logs, cut and seasoned in the woods, with a diameter of from 6 to 18 in., depending on the weight of the roof and the thickness of the seam, and of a length some 2 in. less than the height of the coal. Where the seam varies in thickness in different portions of the mine, props of proportionate lengths are kept on hand.

If it is necessary to reduce the size of an individual stick, it is usually better to split than to saw it (thus forming a *split prop*), especially in the case of wood from coniferous trees, as splitting does not destroy the sap wood, or unduly injure the grain or fiber of the stick. If a prop must be shortened, the ends should be sawed off square and parallel, and not cut with an ax.

Props should be straight, have square ends, rest fully and firmly on the floor, have a cap piece between them and the roof, and be set perpendicularly.

The stronger the roof is, the stronger must be the props required, because the roof, if broken, is in much larger pieces; conversely, where the roof is broken and tender, the props set must be more numerous, and if these must be set so thick as to interfere with the carrying on of the work, or the ventilation, crossbar sets with lattice-work lagging must be substituted.

Where there is a strong roof and bottom, the props should be set so as to permit the roof to ease, or gradually settle down, or the bottom to heave, and thus prevent the breaking of the prop or prolong its usefulness as such. Under such conditions, they should not be driven very tight and caps of soft timber should be used, otherwise the prop will be bent and later on broken. To accomplish and extend the same purpose, tapered props have been introduced, which have given great satisfaction, both from a safe and economic standpoint. The face of the tapered end is usually about 3 in. in diameter and is about one-fourth of the area or section of the body of the prop.

In some localities, the butt end of the prop is placed toward the roof in order to afford more surface for the cap to rest on. This position is unstable and the stick is also harder to handle, but the butt end up gives greater bearing surface at the point where the prop is wedged and driven.

Whether posts should be set with their butt ends up or down is largely a matter of opinion, as practice differs in different localities. Some timbermen set the thick end down, while others set the larger end against the weaker stratum, whether it is top or bottom. The splitting or furring of the post is more apt to take place at the small end, and many prefer that this should occur at the bottom rather than at the top where the cap or other timbers are resting on the post.

Other things being equal, the strength of a prop varies directly as the square of the diameter and inversely as the length. The ratio of the diameter to the length of the prop, in order to have equal power of resisting compression and deflection, is 1 to 12. However, if by reason of physical defects, such as knots, splits, worm holes, or disease, the wood is weaker, the diameter of the post should be increased.

The crushing strengths of various American timbers are given in the section entitled Strength of Materials. For illustration, taking chestnut with an average strength of 5,300 lb. per sq. in., and allowing 75% of the material to be sound and straight in fiber, a prop with a cross-section of 16 sq. in. will support $.75 \times 16 \times 5,300 = 63,600$ lb. If the prop is 8 ft. long, the weight required to crush it will be $63,600 \div 8 = 8,000$ lb., about. This represents the weight of approximately 50 cu. ft. of draw slate, equivalent to that of a piece a little more than 7 ft. square and 1 ft. thick, or of a piece 3 ft. $\times$ 4 ft. and a trifle more than 2 ft. thick.

If the draw slate is not more than 3 in. thick, the best plan is to take it down and stow it in the gob, making no attempt to hold it by propping. When the slate is from 3 in. to 6 in. thick, a row of props is placed along the gob side at a sufficient distance from the track to allow cars to pass. Where the roof is poor, the arrangement shown in Fig. 1 is employed. This consists of an extra long cap set on the post at one end and set in a hitch cut in the coal at the other. Over the cap is driven lagging b, which extends from one set to another. The cross-bar is not usually jointed to the post, but is merely laid on top of it. The sticks used for this purpose are about 6 in. thick; sometimes split props are used, as the timbering is temporary and is needed only until the room is worked out.
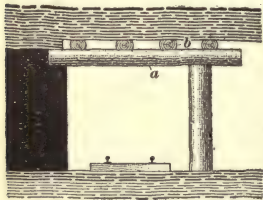
**Systematic Timbering.**—In systematic timbering, props are placed at regular distances apart, both in the direction of the length and in that of the width of the room, and are placed whether the appearance of the top does or does not indicate the necessity for support at the particular point where the prop is placed. The idea involved is that a much greater number of roof falls will be prevented if the props are placed systematically and symmetrically, than if their placing is left to the judgment of the miner. The H. C. Frick Coke Co., working the 8-ft. Pittsburg seam in the Connellsville region of Pennsylvania, in connection with rooms 10 ft. to 12 ft. wide and separated by pillars from 40 ft. to 80 ft. thick, have adopted a center-to-center spacing of 5 ft. posts of 4 ft. 6 in. Where rooms are of the usual width of 20 to 25 ft., there will be several series of posts set in parallel rows from 4 ft. 6 in. to 6 ft. apart as local experience dictates. Alternate parallel rows are set in such a way that a plan of the room timbering suggests the arrangement of the spots on the five of a suite of playing cards. Along the roadway, the props are commonly pro-

vided with a long cap, and sometimes with lagging as shown in Fig. 1. The printed rules of this company in regard to systematic timbering are:

" In rooms exceeding 10 ft. in width, posts must be set as near the center of the room as practicable, and the distance between centers must not exceed 4 ft. 6 in. In rooms where coal is mined by hand, the distance between the last post and the face must not exceed 6 ft. In rooms undercut by machine, the distance between the last post and the face shall be such as, in the opinion of the mine foreman and the mine inspector, affords the best protection for the workmen.

" In all rib or pillar drawing, where the coal can be reached without additional track, a line of posts not exceeding 4 ft. 6 in. between centers must be set in the working places, and when widening out, other posts not exceeding 4 ft. 6 in. between centers must be set parallel, and at right angles, to the first line. In rib or pillar drawing, where additional track must be laid when cutting over near the end of the rib or pillar, posts not exceeding 4 ft. 6 in. between centers must be set in line on both sides of the opening; and in the following named mines, cross-bars or collars must be set over them. The idea of setting these cross-bars across the track, where the roof is comparatively good, is that they may give warning, by their condition, of any unusual condition in the roof, as the presence of smooth slips or great weight; therefore, where the roof is usually of good character, they may be of lighter weight than where bad or dangerous conditions are known to exist.

" In all mines, when the gob is reached, a line of posts shall be set around its edge; the distance between such posts, or between the post and coal, must not exceed 4 ft. 6 in. between centers."

**Timbering Bad Roofs.**—Slips are vertical or inclined cracks reaching through the draw slate to a sound stratum above. Where the roof is known to contain slips, the props should be close together and kept as close to the face as possible. When the slip is vertical, a post is set immediately under it; if inclined, the post is set back from the crack a short distance so as to be more nearly under the place of greatest weight. The cap is usually larger and thicker than in ordinary prop timbering under a uniform roof, and is usually placed at right angles to the crack. Where the roof is very bad, the arrangement shown in Fig. 1 may be used, the
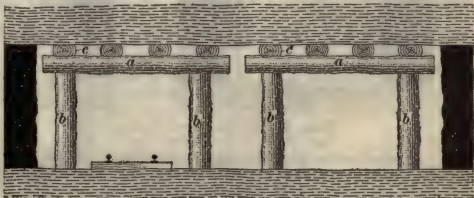


FIG. 2

slate being allowed to fall in the gob, or the form of timbering illustrated in Fig. 2 may be employed. The caps *a* are placed on the props *b*, but are not notched, being secured by the wedges *c*. In all cases, they should have a regular arrangement through the room. Props for timbering under a cracked roof should be about 8 in. in diameter and the caps about 6 in. thick. The props should be about 8 ft. apart, but the cross-bars can project 18 in. over their ends. After the room is completed and the track removed, such timbering may be drawn and saved for future use, the roof then being allowed to fall. It may be necessary, in some extreme cases where the cross-joints occur, to have two sets of cross-bars, one across the room and the other parallel to the ribs.

Where the slips are not visible, but are known to be present, some form of systematic timbering should be employed.

Timbering is varied to meet the conditions of the roof at each mine; therefore, at mines with good roofs, but little is needed, but it is always safe practice to timber under sags in the roof, as these suggest the bed of a stream of water in the past and an opportunity for its accumulation in the present; in fact, in flat beds, water is nearly always encountered in depressions of this kind. If the roof and bottom are both hard, the props are driven in as solidly as possible, the number of props used and their arrangement depending on the width of the opening and the nature of the roof, whether it is firm or shaly. In long wall work, where the roof is allowed to settle gradually, the props may be set on mounds of dirt. Where the roof of the bottom is soft, extra large cap

FIG. 3

pieces or foot pieces are used, so as to give as great a bearing surface as possible between the top or bottom and the props.

**Supporting the Face While Undercutting.**—As the coal is being undercut, it is usually necessary to support the web of coal over the miner's head to prevent its falling on him. The simplest method of doing this is by means of a *sprag a*, Fig. 3, which may be placed either at the opening of the undercut, or may be placed within the undercut.

The combination of timbers *bc* used for supporting the face is termed a *cockermeg*. It consists of two braces *b* between which a horizontal stick of timber *c* is placed along the face. If the angle that the braces *b* make with the vertical is less than 20°, they will not slip and they may be driven tightly against the roof and floor so as to bear against the stick *c*. If the braces *b* are placed at a greater angle than 20°, the ends should be put in hitches as shown.

## ENTRY TIMBERING IN FLAT SEAMS

**Two-Stick Sets.**—A two-stick set of timber, sometimes used for timbering entries, consists of two round or sawed sticks, wedged, but not framed, together
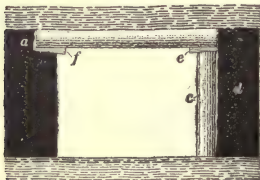


FIG. 1
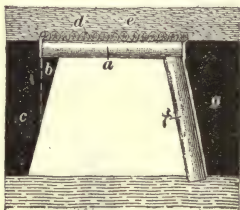


FIG. 2

The *post-and-bar* or *post-and-cap* arrangement is shown in Fig. 1. A hitch *a* 12 or 15 in. deep and only high enough to receive the cap is cut in the coal. The post *c* is then set close to the rib and the cap *b* placed in position. After tightening the post by driving the wedge *e*, the wedge *f* is fitted in place. Where the coal is soft, it is advisable to make the hitch *a* wide enough to receive a piece of heavy planking. By driving wedges between the cap and plank, a much better bearing surface is secured and the pressure thrown upon a greater area of coal.
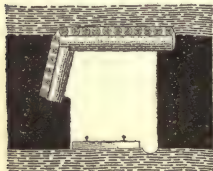


FIG. 3

Where the roof requires lagging, the arrangement shown in Fig. 2 is quite frequently employed, but is to be severely condemned. The ribs should be vertical as the overhanging coal will weather and fall into the entry, and the shoulder of coal *b* is so small that very little roof pressure will
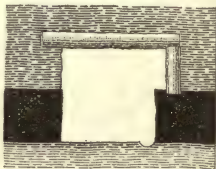


FIG. 4

break it off, causing it to fall. Further, it is not usually good practice to gouge out the roof slate to receive the lagging, as it weakens the roof and may open a seam of water. The leg *f* has too much batter to withstand heavy pressure and should be set vertically.

Instead of having the post the full height of the opening, a method sometimes used is to have short posts set on a ledge in the coal, Fig. 3, or on top of the coal when the top rock is taken down, as shown in Fig. 4.

**Three-Stick Sets.**—A set composed of two legs and a cap is the standard form of entry timbering in both wood and steel; such a set, used in a seam of gentle pitch, is shown in Fig. 4, page 714. The lagging is only necessary when the roof is weak and the coal at the sides of such a nature as to readily weather and fall into the roadway. Also, in flat seams where the pressure is almost entirely in a vertical direction, the legs are made plumb, or with a batter of about 1 in. to the foot. In Fig. 1, if the hitch *a* is omitted and a second leg placed under the cap, the ordinary form of entry timbering will be illustrated.



FIG. 5

A variant of the three-stick set, used extensively abroad for timbering wide entries, turnouts, etc., is shown in Fig. 5. The regular three-stick set is first put in place and is then reinforced with the timbers, *c*, *d*, and *e*. The system is costly and interferes to some extent with the ventilating currents, but is serviceable in timbering underground stables, engine rooms, and the like.
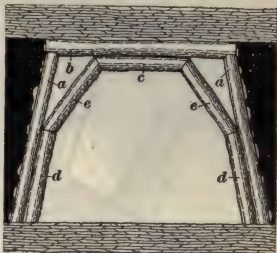
Certain materials composing the floor, such as fireclay, while giving little trouble as the entries are being driven, frequently soon afterwards soften under the action of the damp mine air and swell. The trouble is greater where it is necessary to lift the bottom or brush the top to get height for haulage. In ground of this character various modifications of the three-stick set are used, one of which is shown in Fig. 6. The timbers *a* and *b* running lengthwise of the heading are commonly 10 ft. long and extend over several sets, holding them together. These timbers are temporarily held in place while the struts *c* and *d* and the cap piece *e* are driven into position. The lower ends of the struts are inclined away from the legs to further strengthen the latter against pressure from the sides.



FIG. 6
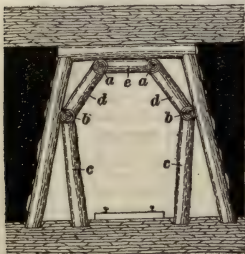
In timbering in such swelling ground, the best results appear to be obtained by using fairly strong timbers and excavating some of the material behind them whenever the swelling begins to exert too great a pressure. The lagging is usually light, sometimes 1-in. plank, is open in construction, and by its bending and breaking, gives warning that the sets are in danger of being crushed. In some cases skin-to-skin timbering is used; that is, heavy logs set in close contact, or any of the forms shown on pages 710 and 715 may be used when placed close together and closely lagged.

**Four-Stick Sets.**—A four-stick set of timbering is the same as a three-stick set, with the addition of a sill laid across the floor, which serves as a support for the ends of the legs and is designed to resist the upward pressure of the floor. A method of framing such sets is shown in Fig. 7. In some instances, the legs are tenoned to fit into a mortise in the sill, but this is unnecessary if the angle the leg makes with the vertical does not exceed 15°.



FIG. 7

When the ground is so heavy that this method of timbering is demanded, modern practice suggests the use of concrete or concrete-and-steel construction, particularly on main haulage roads designed to be open during the life of the mine, at shaft and slope bottoms, main partings, and other important places in the haulage system. Sets composed of structural steel shapes have been advantageously used in such places.
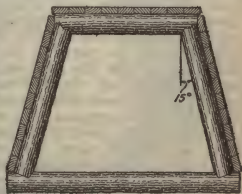
## ROOM TIMBERING IN PITCHING SEAMS

The general arrangement of the timbering at the mouths of rooms or breasts worked on a pitch, and which is designed primarily to keep the loose coal from rushing into the roadway rather than as a support for the roof, is shown under the title Working Pitching Seams.
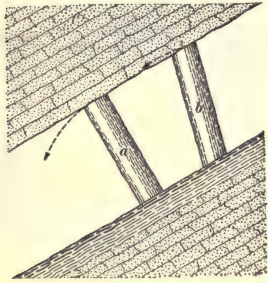


FIG. 1

In room timbering in pitching seams, the posts are not set quite at right angles to the roof, but are given a slight pitch, known as *underset*, up hill as shown in an exaggerated degree in *b*, Fig. 1. Any movement of the roof will cause the top of the prop *a* to move in the direction of the arc shown by the dotted lines and thus to fall down. The top of the prop *b*, moving in a similar arc, will, however, be bound more tightly against the roof.

The following table, from "Sawyer's Accidents in Mines," gives the maximum and minimum angles at which props should be set for varying inclinations. This table can be taken as a general guide, but it does not take account of the length of prop nor the varying amounts of movement of the top rock under different conditions.

### UNDERSETTING OF PROPS

| Rate of Inclination of Seam Degrees | Angle of Underset of Props | |
|---|---|---|
| | Minimum Degrees | Maximum Degrees |
| 6 | 0 | 1 |
| 12 | 0 | 2 |
| 18 | 1 | 3 |
| 24 | 1 | 4 |
| 30 | 2 | 5 |
| 36 | 2 | 6 |
| 42 | 2 | 7 |
| 48 | 3 | 8 |
| 54 and upwards | 3 | 9 |

To prevent the coal from falling on the roadway, it is also often necessary to place a series of props as shown in Fig. 2. The prop *a* is underset, its foot being placed in a hitch in the floor. Part of the draw slate *b* is taken down to prevent its falling and the remainder is held up by the prop *a* and cap *c*. To prevent the coal from falling into the gangway, the props are placed short distances apart and covered with stout lagging *d*. It is necessary to wedge the foot of the prop and drive the cap *c* in tightly; then, any movement of the roof will tighten the joint between the prop and the cap.
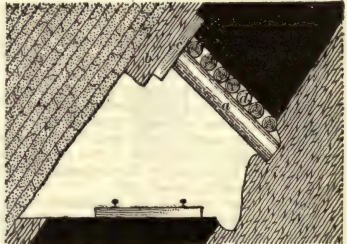
If the seam is very steeply inclined, so that there is danger of the cap between the roof and the prop slipping out, a hitch is cut in the roof rock so that the prop may



FIG. 2

have rock rests at each end. The pressure that the prop then has to sustain is from the coal, and the prop is in the position of a beam uniformly loaded

along its length. This system, shown in Fig. 3, is the better method in highly inclined beds, but the hitches cut in the roof must be at least 12 in. deep and the prop thoroughly wedged at both ends. The object of wedging timbers when placed in such positions is to give them stiffness, for if they bend they
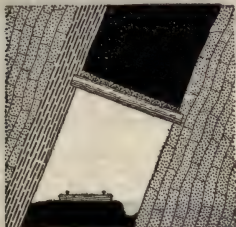


FIG. 3
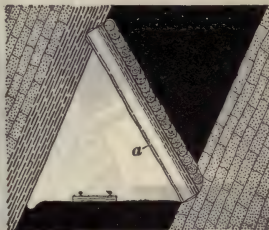


FIG. 4

will eventually break; by wedging the ends, the bending is, in a great measure, prevented.

Fig. 4 shows the method of timbering a wide coal bed at one mine in Pennsylvania. The logs *a* were about 20 ft. long and 18 in. in diameter at the top. They were placed 8 ft. apart and lagged with 8-in. round sticks. Handling these sticks and placing them were laborious operations and the method is not recommended.

The use of single props for timbering deposits exceeding 12 to 15 ft. in thickness is limited, as large, heavy timbers must be handled in such cases, making the system an expensive one; furthermore, the resistance of a prop to bending is not great when the length is more than twelve to fifteen times its diameter.

## ENTRY TIMBERING IN PITCHING SEAMS

**Two-Stick Sets.**—Fig. 1 shows a form of two-stick timbering that may be used where the seam has a slight pitch and is so thin that the floor must be blasted to secure entry height. To stiffen the collar, it must be thoroughly wedged at *d* and at its joint with the leg *e*, by wedges *f*. Lagging is necessary to prevent the coal falling into the entry. This form of timbering is good, provided the wedges *f* are tight so that the bulk of the pressure is transferred to the leg *e*. The beveled joint will become tighter as the side pressure from the coal increases.
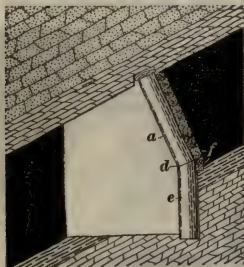


FIG. 1
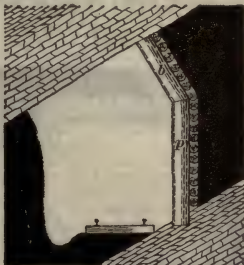


FIG. 2

A similar form of timbering is shown in Fig. 2, which is used in thicker seams where the roof is good. As the joint between the post *p* and collar *b* is by no means as strong as that shown in Fig. 1, this arrangement is more adapted to holding back the coal than to supporting the roof.

The arrangement in Fig. 3 is adapted to thin, highly inclined seams. Even if the top rock, or hanging wall $b$, is good, in order to prevent the leg $c$ being pushed into the roadbed, it is advisable to set its foot in a hitch in the coal and
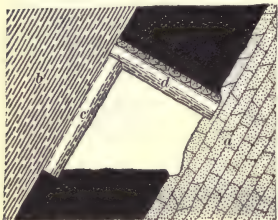


FIG. 3
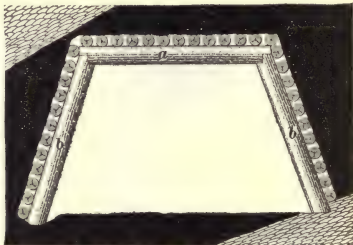


FIG. 4

to make a notched joint between it and the collar. The collars must be lagged to keep back the overhead coal, and, if the top rock $b$ is poor, the leg $c$ may be lagged as well. Seasoned lagging should be used in such places; split lagging with the flat side laid next the leg $c$ is preferable because the rounded side is stronger in compression than in tension.

**Three-Stick Sets.**—Fig. 4 shows the standard form of three-stick sets as used in seams of moderate pitch where the sides are weak and require the use of lagging. Where the sides are very weak the sets may be placed skin to skin, but they are commonly set 3 or 4 ft. apart.

Fig. 5 shows the method of timbering where the dip is great, the bottom hard, and the seam is not thick enough for full entry height. This method avoids the cost of taking out enough rock to get in a set of timber having legs of equal length. The shorter leg $l$ is given a firm hold on the rock bottom.

Fig. 6 shows a form of timbering used in pitching seams where the coal is soft and falls to a height greater than that required for the gangway. The leg $l$ on the high side is made long enough to reach up to the roof to support the lagging, which keeps the soft coal from continually sliding down into the gangway. The collar $c$ strengthens the leg $l$. The coal is allowed to fall off on the low side where no lagging is necessary.

Fig. 7 shows the method of timbering the levels in thin pitching seams, when the top is supposed to be weak. The legs $l$ and the collar $c$ are made of round timber about 12 in. in diameter, and are so jointed together that the
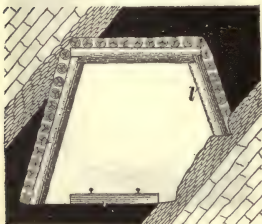


FIG. 5
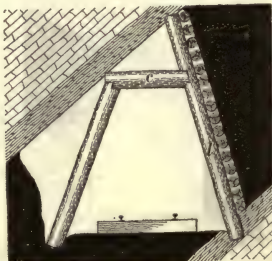


FIG. 6

collar $c$ will stand great pressure. The lagging $a$ consists of round poles taken direct from the woods, and usually from 3 to 6 in. in diameter. The poles are used to keep the loose coal and roof from falling between the sets of timbers,

which are from 3 to 5 ft. apart. Where the lateral pressure is slight, planks $p$ are used. The road is made level by filling in the low side with refuse $t$, as shown.

### SHAFT TIMBERING

**General Principles.** — The general arrangement of shaft timbers and some of the details thereof are illustrated under the title Opening a Mine. The nature and amount of the timbering will vary with the character of the ground penetrated.

In hard material, only such timbers are introduced as are necessary to furnish support to the guides, pipes, wires, etc. that are carried down the shaft. In loose material, the object of timbering is to give support also to the sides of the excavation. In watery strata, the pressure of the water behind the timber is another point that must be considered. Water encountered in the sink-

FIG. 7

ing of a shaft finds its way at once to the excavation or follows down behind the lining and collects in the bottom of the shaft, unless kept out by the shaft lining. If the lining is built tightly against the sides of the excavation, so as to impede or stop the flow altogether, the water rises behind the lining to the water level of the strata and the lining is subjected to a pressure dependent on the head of water. The strength of the lining must be sufficient to withstand this pressure. In such cases, the following formula may be employed to determine the thickness of white-pine lining that will resist a given head of water:

$$t = .016s \sqrt{d}$$

in which
$t$ = thickness of white-pine lining, in inches;
$s$ = clear unsupported span of timber, in inches;
$d$ = depth, or head, of water, in feet.

NOTE.—While in the statement of this formula white-pine timber is used, the same formula will give results that are practically correct for the other varieties of timber used in shaft linings.

It must be remembered that the head of water supported by the curbing does not mean the depth of the curbing below the surface, as the water rarely

FIG. 1

if, ever, heads to the surface.

EXAMPLE.—Find the thickness of white-pine curbing required for a coffer dam when the depth of the water head is 100 ft., the clear span of the end plates of the shaft being 7 ft.

SOLUTION.—Substituting the given values in the formula, $t = .016 \times (7 \times 12) \times \sqrt{100} = 13.44$; hence a 14-in. timber should be used.

**Timbering in Rock.**—Where a shaft, or a portion of a shaft, is excavated from hard-rock strata, the only timbering necessary is the cross-timbers, or buntons, to support the guides of the hoisting compartments of the shaft and the lines of pipes, or wires. The buntons $b$, Fig. 1, are set in hitches $h$ cut in the rock face and firmly wedged in line, one above the other, by wedges $w$. At times, the hitches are cut square and those on one side made deeper to permit the other end of the stick to be placed in the hole opposite.
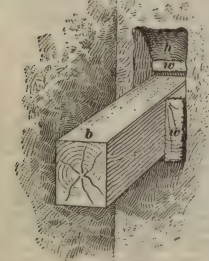
The buntons are spaced from 6 to 8 ft. apart, one above another, on each end of the shaft, and between the several compartments of the shaft. When it is desired to separate the compartments of the shaft, as in the caes of an air-

way or manway, planks are spiked to the buntons or built between them to form the partition.
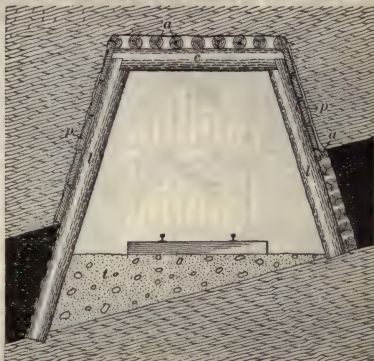
**Timbering in Loose Dry Material.**—In good ground, shafts have been sunk to a depth of 200 to 300 ft. by using $3'' \times 12''$ planking set on edge, but beyond this depth it is better to use 4-in. or 5-in. planks. When an especially soft, wet, or crumbling stratum is met, such as wet sand or fireclay, the planking is sometimes laid flatwise. If the sides of the shaft are not self-supporting and tend to crumble into fragments of varying size, if boulders that are likely to become detached are found, or if the strata are jointed and faulted, to preserve the shaft and to avoid accident from earth or rock falling to the bottom from the side walls, it is necessary not only to line the entire excavation with plank, but to support this planking by heavy timber sets placed inside the planking, as shown in Fig. 2.

These timber sets $a$ are placed at regular distances apart and are separated by the posts $b$. The lagging $c$, composed of closely fitting planks, may be driven in behind the timber sets or it may be first, placed in position and the timber sets or frames added afterwards. Cross-buntons $d$ are also inserted in each set to separate the compartments. Where a greater strength of timbering is required than is given by this form, the sets $a$ may be placed one on top of the other, i. e., skin to skin.

An open crib of timbers, similar to that shown in Fig. 3, may also be employed in loose ground, the openings between the timbers being gradually filled up compactly by the loose material. After the timbers have been placed in position, triangular strips, or corner pieces, $A$ are spiked to them in each corner of the shaft. This open crib may be built either from the top downwards or from the bottom upwards.

Instead of building the timbering from the top downwards, it is frequently built upwards from the bottom in sections of 10 to 15 ft., depending on the character of the ground. The bottom of the shaft is carefully leveled with a carpenter's level and straightedge; and, by measurements made from the plumb-lines hung from the shaft corners, a set of timbers is placed so that the inside is in line with the edge of the sills, or shaft templet. After the whole set is accurately leveled and joined, wooden wedges are driven between the timbers and earth at each corner. The wedges should be long and tapering, and should be driven into position while the set is held in place with a bar. Great care must be taken to get this first set level and in line with the shaft templet, as it is the foundation for the other sets. After this foundation set has been placed in position and wedged, another set is placed on it, leveled, and wedged in like manner. In this manner, the work is continued until the templet or next section of timbering is reached. If the sinker has measured correctly and has made due allowance for the number of sets required to close the distance between the shaft bottom and templet, his sets will close this space



FIG. 2
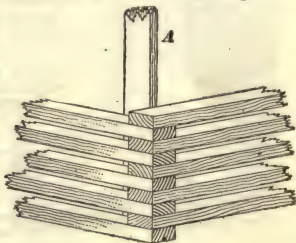


FIG. 3

exactly. The inside edges of the planking are brought flush with the inside edges of the templet, and iron straps, about 2½ in. × ¼ in. × 15 ft., provided with nail holes are hung from the surface downwards, connecting all the planking and hanging it from the templet. The straps or hangers are placed on the sides and ends of the shaft at distances of 2 to 3 ft. apart, and they break joints vertically as the timbering proceeds. If a small space is left between the last set and the templet and the planking does not close exactly, a *closing set* is necessary. For this purpose, a regular set is cut down to the required size by the rip saw or adz. However, the sinker should make his measurements and calculations so that no closing sets are required.

No cavities should be allowed to remain behind the timbering after it is completed, except in ground that swells. If cavities are found in the strata, or if more earth has been taken out than was necessary, the space must be filled with ashes, straw, etc.
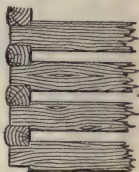


FIG. 4

**Timbering in Swelling Ground.**—A form of timbering often employed in swelling ground is a cribwork of heavy timbers, such as is shown in Fig. 4. These timbers are notched together after the fashion of a log cabin. One side of the timbers may be faced, so as to form the face of the shaft, but the back of the timbers is preferably left round. When the ground swells, the material more readily works out between the timbers, and can be removed from time to time, as it may be found necessary. An important feature of the work in dealing with swelling ground is to keep the material as dry as possible, because the moisture causes the swelling. In such ground, a space at least 6 in. wide is sometimes cut out all around the sides and ends of the shaft, and filled in loosely with moss, straw, sand, or ashes, allowance being made for the probable expansion of the ground. When the timbering, by bulging, shows signs of excessive pressure behind, the difficulty may be overcome by carefully removing two or more planks from the shaft at this point, and excavating such material as may be necessary, all around behind the timbers. The manway thus formed should be carefully drained by a pipe conducting the water to the sump or other lodgment. This manway should be timbered and cleaned out from time to time, as may be necessary; the bulged timbers of the shaft should also be replaced by good ones.



FIG. 5

**Timbering in Very Wet Ground or Quicksand.**—In wet ground, timbers should be closely joined. At times, it is desired to make a water-tight joint between each set of timbers to keep the water from entering the shaft; for this purpose, timbers have been laid in cement, but better results are obtained by backing the timbers with cement. A form of timbering that always gives good results, introduced for the first time in the sinking of the Ladd shaft at Ladd, Illinois, is that shown in Fig. 5, which illustrates a section of curbing passing through a stratum of quicksand, and through soft material overlying the same. At a point above the soft material, the 3″×8″ curbing plank *a* employed for the shaft lining is laid flatwise, increasing the thickness of the curbing from 3 to 8 in. When the quicksand is reached, the 8-in. plank is alternated by 6-in. plank, forming the corrugated backing shown at *b*; the effect of this rough backing is to clog the drainage that would otherwise find its way down the back of the curbing, and greatly reduces the amount of water entering the shaft.
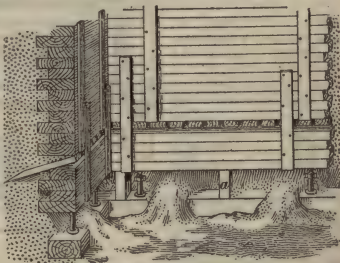


FIG. 6

The chief difficulty in sinking through quicksand is that arising from the flow of the soft material into the excavation before the timbers can be placed in position. To prevent this as far as possible, the excavation should be tim-

bered well down to the bottom of the shaft. Fig. 6 shows more or less accurately the inflow of sand and the method of setting the timbers. The lower timbers have been set, jacked up, and spiked. Blocks *a* are used to support the back of the lining. These blocks are knocked out by the next set of timbers when it is driven to its place. It is necessary to provide a temporary foundation for the jacks, which in this case is afforded by the sills shown. The form of lining employed is the alternate narrow and wide plank laid flatwise. To reduce the flow of sand temporarily, spiling has been driven between the timbers; but the spiles must be removed before they throw too much weight on the lining. To support the timber while the jacks under the set are being
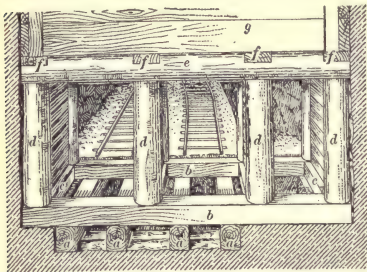


FIG. 7

lowered far enough for a new timber to be placed over them, cleats are spiked on the timbers as fast as each timber set is laid in place. If the timbers cannot be forced into place by hand or driven with a sledge, a jack, similar to those shown in Fig. 6, is used, being fastened to a piece of $6'' \times 6''$ or $8'' \times 8''$ timber, about 1 ft. shorter than the inside dimensions of the shaft.

**Square Frame at Foot of Shaft.**—When the bottom of the shaft is reached and the sump has been made by carrying the excavation several feet below the floor of the seam, a heavy substantial frame must be built for the support of the shaft timbers. The cage landing is first made by placing two heavy square timbers *a*, Fig. 7, under each hoistway. These timbers should be 10 in. $\times$ 12 in. or 12 in. $\times$ 16 in., according to the size and weight of the cage, and should occupy a position about under the rails on the cage. They are well bedded in the strata on each side of the shaft, and set low enough to make the floor of the cage, when the latter is resting on the timbers, level with the floor of the landing. When this has been done in each hoistway, heavy longitudinal sills *b* are laid over them, one at each side of the shaft; cross-timbers *c* are boxed into the sills to keep them the right distance apart and to form a solid frame for the cage landing. Substantial posts *d* are then set at the corner of each compartment. Heavy caps, or collars, *e* are framed to rest on these posts, and

cross-timbers *f* are boxed into these caps above. The whole frame is brought to such a height as will correspond to the height of the heading, and the shaft timbers, or lining *g* are made to rest on the top of this frame.

Underneath the cage timbers *a* heavy planks are inserted so as to cover the sump to prevent material from falling in and avoid the necessity of frequent cleaning. Without a cover, there is also the danger of animals falling



FIG. 8

into the sump and being drowned before they can be got out. This cover should be so arranged that it may be easily and quickly removed at any time.

**Square-Set Timbering.**—Square-set timbering is adapted to large shafts or heavy pressures. It is extravagant in the use of timber on account of both the size and the quantity of timber required. The form of joint is simple, as the
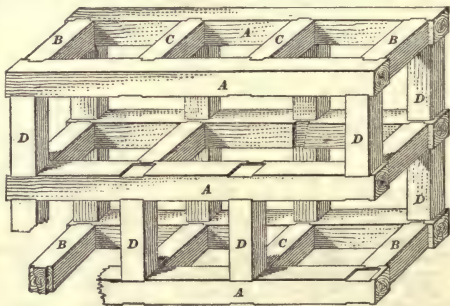
timbers are for the most part boxed slightly into one another. Fig. 8 shows the general construction of the timbering in a three-compartment shaft by square sets; in it, some of the timbers are omitted for the purpose of showing the form of joint employed. At A are shown the wall plates; at B, the end plates; at C, cross-bun-tons; and at D, posts or punch blocks or stud-dles. The joints may be varied as shown in Fig. 9.
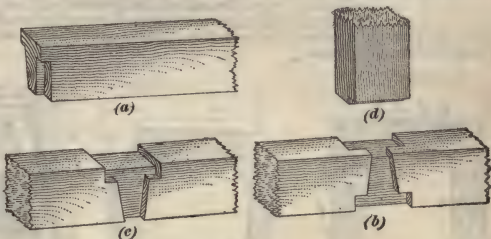


FIG. 9

With the joint shown in (b), the cross-bunton is put in place from below. The advantage of this is that, if the timbering must be kept close to the bottom while sinking, the bunton going in from below can be left out at first, so as to allow more room for the workmen. Fig. 10 shows another method of joining end and wall plates, the post F being boxed into the plates both above and below. In this figure, a 2-in. strip S is shown on which the lagging is to rest. Numerous other forms of joints are used in square-set timbering, but these will serve to illustrate the principle, namely, that as little of the timber should be cut out as possible, so as not to weaken the timber. When framing these timbers, regard must always be had to the manner in which they are put together in the shaft. When the timbering is done from the top downwards the sets are suspended by means of hanger bolts made of round-iron rods, bent to a hook shape on one end and having a thread and nut on the other end, Fig. 11.
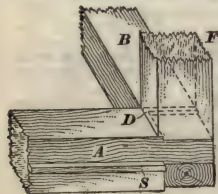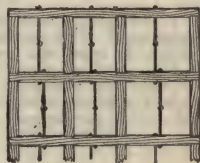


FIG. 10



FIG. 11

## MISCELLANEOUS FORMS OF TIMBERING

Fig. 1 shows a method of placing drift sets in the case of very heavy or swelling ground. Here a are the posts; c, the sills; b, the caps; d, the collar braces that bear against both the caps and the posts; e, foot or heel braces
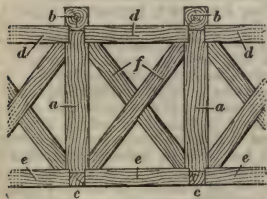


FIG. 1
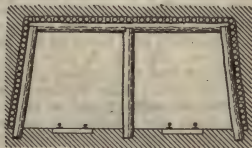


FIG. 2

that bear against both the sills and the posts; f, diagonal braces that are halved together and placed as shown.

Fig. 2 is a set employed in the case of an extra wide gangway, or parting, there being a post set under the middle of the cap. This form of set may be provided with a sill when the floor is soft.

Fig. 3 shows a form of drift set surrounded by bridging and used where such bad ground is encountered as to necessitate forepoling. At *a* are shown the posts; at *b*, the caps; at *c*, the sill of the regular set; at *d*, upright bridge pieces; at *e*, a horizontal bridge piece separated from the set proper by blocks *f* so as to provide spaces *h* around the regular set through which the spiles or forepoles can be driven.
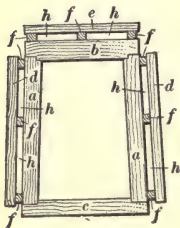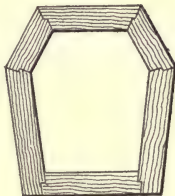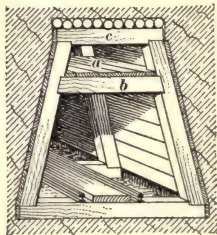


FIG. 3        FIG. 4        FIG. 5

Fig. 4 shows a form of drift set sometimes employed in very heavy or swelling ground. This method of framing the timbers shortens each piece and reduces the transverse strain on all the timbers.

Fig. 5 shows an ordinary drift set provided with a sollar for ventilation purposes. An additional brace *b* is placed parallel to the cap *c*, and this is covered with plank lagging *a*, so as to provide a passage above the regular drift, which may be used as a return air-course.

## FRAMING TIMBERS

**Limiting Angle of Resistance.**—In Fig. 4, page 714, the legs of the timber are inclined so that the pressure coming on the collar is transmitted equally to both legs. If the legs are placed at different angles, the pressure will bear unequally on them, the greater pressure coming on the leg making the smaller angle with the vertical. The tendency for the foot of a post to slip increases with the inclination; and if the angle between the post and the vertical is more than 20°, the post is apt to slip, but the legs are not apt to spread on a level rock surface when this angle is less than 20°. It is, of course, possible to block the foot of the leg against the side of the gangway, but even then the horizontal pressure against the foot of the post increases rapidly and it is advisable to keep the inclination of the legs less than 20° from the vertical.

When the pressure comes from four sides, four-stick timbering must be used. In some instances, the legs are tenoned for a mortise in the sill, but this is unnecessary if the angle the leg makes with the vertical does not exceed 15°, which, according to Morin's experiments, is the limiting angle of contact of oak on oak when the fibers of the moving surface are perpendicular to the surface of contact and those of the surface at rest are parallel to the direction of the motion. In Fig. 7, page 711, the moving surface is the lower end of the inclined leg and the surface at rest is the portion of the sill on which this end of the leg rests. The friction of two surfaces that have been, for a considerable time, in contact and at rest, is different not only in amount, but also in nature from the friction of surfaces in continuous motion. A jar or shock producing an almost imperceptible movement of the surfaces of contact causes the friction of contact at rest to pass to that which accompanies motion.

**Placing Timber Sets.**—It is important in the framing of a set of timbers to cut the joints between the two pieces of timber so accurately that the bearing surfaces are in close contact. The lower end of each leg must also be cut so as to be in contact, over its whole surface, with the floor in order to get the full benefit of the cross-section of the timber. When wedging the cap piece, care should be taken to drive the wedges as uniformly as possible over the full length of the cap piece; for if some wedges are driven more tightly than others, the

weight will be concentrated at these points, at which place the cap is apt to break. In placing wedges, care should be taken to secure the roof without throwing an unnecessary strain on any part of the timber set.

**Timber Joints.**—Fig. 1 shows a joint sometimes used to resist pressure from above rather than from the side. This joint is objectionable from the fact
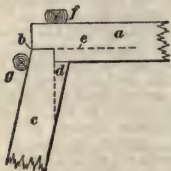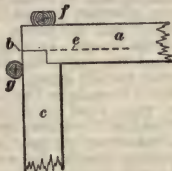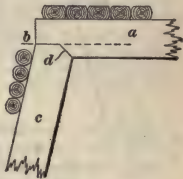


FIG. 1          FIG. 2          FIG. 3

that continued pressure on the collar $a$ will cause it to sag and thus raise the scarf $b$ of the joint from the post $c$, throwing all the weight on the part $d$. This has a tendency to split the post $c$ and the cap $a$, as shown by the dotted lines; if this occurs, the entire weight is thrown on the collar above the dotted line $e$, and on the part of the post to the left of $d$, the part $d$ and that below $e$ being useless in sustaining weight. The same bending trouble will take place, but to a less extent, if the timbers are joined as in Fig. 2, unless the wedge $f$ stiffens the collar sufficiently to prevent its bending; it is doubtful, however, if sufficient stiffening will occur when continued heavy pressure comes on the collar. In case the collar $a$ bends so as to open the joint $b$, the upper part above $e$ is useless for sustaining pressure. The side wedge $g$ is intended to keep the joint tight.

The joint in Fig. 2 is better able to withstand pressure from above than that in Fig. 1, for the pressure is along the fibers of the post $c$ and not across them.

The joints in Figs. 3 and 4 have proved very satisfactory in practice, as the timbers are not so apt to split as when the joints shown in Figs. 1 and 2 are used.

If the cap begins to sag, there is much less chance for the joint shown in Figs. 3 and 4 to open than there is with the joint shown in Figs. 1 and 2, as there are no sharp angles in the joint between the timbers $a$ and $c$. The pressure also comes on the faces $b$ and $d$ of the joint much more uniformly, and the absence of the heel or sharp corner in the joint also greatly reduces the tendency of the cap to split along the dotted line. In case the pressure is greater from the side than the top, the leg is given an inclination less than 20° from the vertical and a double-notched joint is made, as in Fig. 5. The foot of the leg is placed in a hitch in the floor to prevent its being pushed inwards.
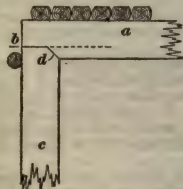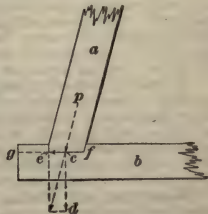


FIG. 4          FIG. 5          FIG. 6

If the leg $a$, Fig. 6, is let into the sill $b$ as shown, the pressure $p$ along the leg may be resolved into the two components $cd$ and $ce$. The more nearly vertical the leg $a$ is, the greater will be the component $cd$, which is resisted by the cross-grain of the wood, and the stronger will be the joint. The tendency of

46

the leg *a* to slip is also less the more nearly vertical *a* is. If the leg *a* is bent inwards, the heel *f* acts as a fulcrum of a lever and the corner *e* tends to split off the block above the line *eg*.

If the leg *a*, Fig. 7, is jointed to the sill *b* as shown, there is less danger of its slipping or of its splitting the timber than when the joint shown in Fig. 6 is used. The pressure *p* acting along the leg *a*, Fig. 7, can be resolved into the two components, one *cd* acting vertically and across the grain of the wood, and the other *ce* acting parallel to the grain of the wood. In this case, if the leg *a* bends toward the right, the tendency is for the heel *f* to split the sill along the line *fg*, but the length of wood fiber *fg* in this case is longer than the length of wood fiber *eg* in the joint shown in Fig. 6, and there is, therefore, not the same danger of the block above *fg* splitting off. Again, in case of a sudden shock, the wood tends to slide along the face *fh*, that is, perpendicularly to the direction in which the pressure is transmitted along the leg. The timber *a* could not, therefore, slide on the timber *b* as readily with the joint in Fig. 7 as it could with the joint in Fig. 6, where the angle of inclination between the faces of the timber is greater. In other words, with the joint in Fig. 7, there will be much more friction between the faces of the timber to oppose movement than with the joint shown in Fig. 6; and to start a movement of the leg, the jar must be much more severe.
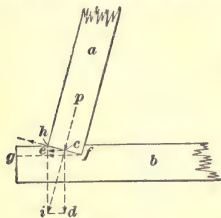


FIG. 7

The method of framing heavy shaft timbering is described under the head of Square-Set Timbering.

# CARE AND PRESERVATION OF TIMBER

## CUTTING AND STORING TIMBER

**Time to Cut Timber.**—The presence of much sap in the tree when it is cut causes the timber to decay more rapidly than it would otherwise, owing to the fermentation of the sap permitting the growth of fungi that feed on the life of the timber. In growing timber, the sap ceases to run about the middle of December and starts again about the middle of February. Timber cut, therefore, in the months of December, January, and February will contain the least sap and prove more lasting than the same timber cut at other times of the year. The work of cutting timber in winter gives employment also to farm hands during their idle season; moreover, the task of transporting timber on sleds to the mines or the railroads is a much easier one in winter than during the seasons when wagons must be used.

**Peeling.**—Peeling timber is a simple and inexpensive method of increasing its durability, and under some conditions is fairly effective. Bark retards the loss of moisture from timber, and unbarked wood therefore offers more favorable conditions for fungus growth than wood from which the bark has been removed. Moreover, the space between the bark and the wood is an excellent breeding place for many forms of wood-destroying insects. In dry workings, the life of timber may be increased from 10 to 15% by peeling, although in wet situations peeling seems to have little effect.

Besides increased durability, there are other advantages to be derived from the use of peeled timber. The bark of unpeeled timber often flakes off soon after placement, causing an accumulation of inflammable rubbish in the workings, which must be removed at some expense. To peel timber in the woods or at the shipping point effects a saving in freight and in cost of handling. With loblolly and shortleaf pine the weight of bark usually amounts to from 8 to 10% of the original green weight.

**Seasoning and Storing Timber.**—The durability of timber may in some cases be increased by seasoning. In dry, well-ventilated workings the life of seasoned timber is sometimes 25% greater than that of green timber. In wet locations, however, the effects of seasoning are counteracted by the reabsorption of moisture.

Whenever practicable, timber should be seasoned in the woods or at the shipping point, in order to realize, through loss of weight, a substantial saving in the cost of freight and handling.

The timbers should not be permitted to lie on the ground after seasoning operations are commenced, but should be placed on blocks so that they will be exposed to a circulation of air. The blocks should not be so far apart that the timbers will sag; and the timbers, if exposed to the sun, should be turned regularly, otherwise they may check or warp. Sawed timbers should be stacked up, with air spaces between the sticks; they should also be kept under sheds when seasoning and before they are taken below ground. If this is not possible, they should be stacked so that they will shed water.

The several lengths of timber should be stored together so that they can be readily obtained as required. To prevent warping and checking, the timber should not be seasoned too quickly, as is frequently the case when artificial heat is employed, or when the timber is exposed to a strong sun, especially when the circulation of air is not sufficient.

## PRESERVATION OF MINE TIMBER

**Destructive Agencies.**—The relative importance of the various destructive agencies affecting timber varies greatly with the conditions in the mines. Under average conditions, the different destructive agencies cause the following percentages of loss: Wear, 5%; breakage and fire, 20%; waste from all causes, 25%; decay and insect attack, 50%. Dry rot and fungus growth are diseases common to most timbers. Some timbers are more apt to be infested with insects and suffer from this cause more than others, owing, probably, to the nature of the wood or bark as furnishing food or nesting places for insects. Climatic conditions have much to do with this trouble; in some climates, the insects multiply rapidly and completely destroy the timber they infest. At times, the bark of the timber is completely filled with the eggs and the larvæ of insects, and must be removed in order to protect the timber from their inroads.

When wood is not properly seasoned, the sap is liable to ferment, especially in a dry, warm place, and *dry rot* occurs, beginning in the center of the stick and working outwards. In general appearance such a stick looks sound, but by thrusting a knife blade into it the damage is discovered. Fresh-air circulation, when the stick is away from decaying timber, is one preventive of dry rot, as in such situations the stick seasons.

When timber is placed in warm, moist air, *damp rot* takes place; this is the usual rot affecting mine timbers. It commences on the outside and gradually finds an entrance into the interior of the stick through some check. The destruction of a timber by damp rot is not so rapid as by dry rot and is noticeable from the fungus growth on the outside of the stick. In mines, dry rot occurs in the intake airways and in poorly ventilated workings, while damp rot occurs in the return airways and damp rooms. When fungus of the damp-rot species appears, it may be possible to save the timber and prevent the fungus reaching the heart wood by washing the stick down with lime or alum water from time to time.

**General Principles of Timber Preservation.**—The partial removal of sap will retard decay, for which reason timbers are sometimes submerged for several months, then removed and air-dried. A temperature of between 60° and 100° F. combined with moisture is favorable to decay; but mine timbers must often be placed where such conditions prevail. It may be possible, by special wood preservatives, to increase the life of timbers; but even then the sap must be either dried or removed, as wood covered with paint before being thoroughly seasoned will propagate dry rot in a warm, dry place, or damp rot in a moist, warm place. With good sound timber, creosoted joints will prolong its life, especially if the ends have been submerged in creosote a month or more. Different species of trees differ in their resistance to decay. Cedar, tamarack, and locust are more durable than pine, oak, or cypress, although in certain situations they may all have the same life. Contact with earth is particularly destructive to timber; and nearness to decaying timber is a source of disease. The principal means adopted to arrest the processes of decay and preserve timber are creosoting, salting, and charring the timber. The first two methods consist in impregnating the timber with cresote or a solution of salt so as to fill the pores. The acid acts to coagulate the albuminous matter of the sap. By this means, the pores of the wood are filled with a deposit of salt or with the coagulated albumen, which prevents the absorption of moisture and arrests the process of decay in the timber. The acid also destroys the organic life of the wood. In the third method, the charring of the ends and surface of the timber closes the pores of the wood and prevents the absorption of moisture; the charred surface of the wood will not then decay.

Attempts have been made to coat mine timber with some substance, as tar, to prevent or retard its decay; in other cases, the timber has been treated with chemicals with the same end in view. The objection to the use of creosote or tar for preserving mine timbering is that they make the timber more inflammable than it would otherwise be. Timbers are sometimes treated with solutions of the chlorides or the sulphates of the various metals. When a regular plant is installed for this work, timbers are first placed in specially prepared chambers from which the air is afterwards exhausted, and then the solution for preserving the timber is forced in under pressure, the exhausting of the air having reduced the pressure on the timber and opened the cells. After the preserving material enters the chamber, it is forced into the pores of the wood in such a manner as to thoroughly saturate it.

Before timber is treated with preservatives, it should be peeled, and, as a rule, thoroughly seasoned. All timbers should be cut and framed to their final dimensions and form before treatment, because the sawing and cutting of treated timber frequently exposes untreated surfaces to attack by wood-destroying organisms.

We are indebted to the reports of the U. S. Forest Service for the following information.

**Brush Treatments.**—A fairly effective and cheap treatment is to paint timber with two or three coats of hot creosote or some similar preservative. It is important to that the wood be seasoned before treatment, for otherwise checking may later expose untreated portions of the timber to fungus attack. Care should, moreover, be taken to get the preservative well into all checks, knot holes, and surface inequalities; otherwise decay is likely to develop at these points.

The amount of preservative required for treatments of this character is relatively small and no special equipment is needed. Brush treatments are therefore advisable when the amount of timber to be treated is too small to warrant the erection of even a small plant, or when it is necessary to restrict the initial cost of treatment to the lowest possible figure. The main disadvantage of brush treatments is that the slight penetration secured is not enough to insure the protection of the interior of the timber for any considerable period. The thin coating of treated wood may be broken or split, or fungus spores may enter through nail holes, checks, or splits, causing decay in the interior of the timber, while the outside appears sound.

**Open-Tank Treatments.**—A more effective method of treatment is the open tank. In this the timber is first immersed in a tank of suitable capacity containing the preservative, and the charge is then heated to a sufficiently high temperature to drive off a portion of the air and moisture contained in the wood. As excessive heating is likely to result in checks that will weaken the timber, and as large quantities of preservative may be lost by volatilization, the maximum temperature of the hot bath should not, in the case of creosote oils, exceed 220° F. and, if an aqueous salt solution is used, the temperature should be kept slightly below the boiling point of the solution. Following the hot bath, the timber is immersed in preservatives at a lower temperature, or it may be left in the hot liquid, which is allowed to cool.

The treatment of timber by the open-tank process insures a greater penetration of the wood by the preservative than does the brush method, and for this reason has proved more effective. In general, it is well adapted for the treatment of species that are easily impregnated.

**Pressure Treatments.**—With many species, a satisfactory treatment can be secured only by the use of pressure. The essential difference between the open-tank process and the pressure processes is that in the former atmospheric pressure is relied on to secure the penetration of the wood, while in the latter the preservative is forced into the timber by artificial means. Owing chiefly to the difficulty of impregnating many species of wood by the open-tank process, the pressure treatments are the most widely used.

Pressure processes may be employed for either full-cell or empty-cell treatment. The object of the former is to leave the treated portion of the wood completely filled with the preservative, while the latter aims to inject the preservative as deep into the timber but leave no free oil in the wood cells. The oldest process of full-cell pressure treatment with creosote is known as *Bethellizing*. A similar treatment with zinc-chloride solution is called *Burnettizing*.

**Comparison of Open-Tank and Pressure Treatments.**—Experiments made by the two principal processes upon different kinds of timber indicate:

1. Thoroughly seasoned loblolly and Pennsylvania pitch pine round mine timbers may be satisfactorily impregnated by the open-tank process.

2. Green timber of these species is much more difficult to treat than seasoned timber.

3. Satisfactory results may be secured in the treatment of seasoned western yellow pine by the open-tank process, the sapwood of this species being impregnated without difficulty.

4. In general, pressure treatments are more satisfactory than open-tank treatments. By the former, the time of treatment is reduced considerably and the preservative is more generally diffused through the timber.

5. Heart Douglas fir is impregnated with difficulty.

**Cost of Open-Tank Plant.**—The open tank is the simplest type of apparatus used for the impregnation of timber. The necessary equipment consists mainly of an uncovered tank provided with a device for submerging the timber. The tank may be so arranged that a fire can be built under it, but if a supply of steam is available it should be equipped with coils for heating purposes. If large timbers are to be treated, a derrick or gin pole is necessary for their convenient handling. A plant of this character, with an annual capacity of 100,000 cu. ft. may be erected at a cost of from $1,500 to $2,500. The low cost of an open-tank plant places it well within the reach of most mine operators, and this, perhaps, is its main advantage.

**Cost of Pressure Plant.**—The cost of pressure plants depends chiefly on their capacity. The total cost of a plant having a capacity of approximately 1,000 cu. ft. per run, or 750,000 cu. ft. per yr.* will amount to from $12,000 to $20,000. The following is a list of the main items of equipment: One horizontal treating cylinder, 65 ft. long, with inside diameter of 6 ft. 2 in, capable of withstanding an internal pressure of 175 lb. per sq. in.; two vertical measuring tanks, each of 15,000 gal. capacity; one storage tank of 50,000 gal. capacity; one hoist engine; one pressure pump, capacity 150 G. P. M. at 175 lb. pressure per sq. in.; one air compressor, capacity 460 cu. ft. free air per min. at 20 lb. pressure per sq. in.; sixteen cylinder cars; one zinc-chloride mixing tank of 2,000 gal. capacity. Special attention should be given to the design and construction of a storage yard of adequate capacity for both treated and untreated material, as handling the timber before and after treatment is an important factor in the cost of operation. It is also important to locate the plant at a convenient point in the mining district, so that treated timber may be readily distributed to points where it is to be used.

A pressure plant was designed by the Forest Service and erected by the Anaconda Copper Mining Co. at Rocker, Mont. Its capacity is about 570 cu. ft. of timber per run. The equipment is as follows: One treating cylinder 43 ft. long and 6 ft. in diameter, working pressure 100 lb. per sq. in.; one receiving tank, 47 ft. long and 5 ft. in diameter; two measuring tanks, 12 ft. in diameter and 17 ft. high; one general service pump, 360 G. P. M. at 125 lb. per sq. in.; one pressure pump, 60 G. P. M. at 125 lb. per sq. in.; one vacuum pump 7 in. × 10 in. × 6 in., and jet condenser; eighteen cylinder cars; one hoist engine. The total cost of the plant, erected, was approximately $15,000.

A plant designed by the Forest Service for the Tennessee Coal, Iron & Railroad Co., and erected at McAdory, Ala., has a capacity of about 830 cu. ft. of timber per run. The main equipment is as follows: One treating cylinder, 6 ft. in diameter and 65 ft. long, working pressure 100 lb. per sq. in.; two measuring tanks, 12 ft. in diameter and 18 ft. high; two storage tanks, 11 ft. in diameter and 36 ft. long; one settling tank, 16 ft. × 6 ft. × 4 ft.; one pressure pump, capacity 150 G. P. M. at 100 lb. per sq. in.; one air compressor, capacity 108 cu. ft. free air at 50 lb. per sq. in.; one air reservoir 3 ft. in diameter and 15 ft. long; one surface condenser, 230 sq. ft. of condensing surface; twenty cylinder cars; one derrick and hoist engine. The total cost of this plant, erected, including the necessary yard construction, amounted to approximately $12,000.

**Cost of Treatment.**—The unit of cost handling timber at open-tank plants is higher than at pressure plants, usually amounting to from 3 to 4 c. per cu. ft. at the former and from 2 to 3c. per cu. ft. at the latter. These figures include interest, depreciation, and operating charges, but not the cost of the preservative, which is by far the most important item. The accompanying table shows the approximate costs of the untreated and treated loblolly pine gangway and entry sets used in the Forest Service experiments in the mines of the Philadelphia & Reading Coal & Iron Co. One set consists of one 7-ft. collar, one 9-ft. leg, and one 10-ft. leg; average diameter of timber 13 in.; approximately 26 cu. ft. in one set.

*Annual capacity is based on two runs per day for 250 da.

## COST OF UNTREATED AND TREATED LOBLOLLY PINE GANGWAY AND ENTRY SETS PLACED BY THE PHILADELPHIA & READING COAL & IRON CO., IN COOPERATION WITH THE FOREST SERVICE

| Condition of Material Before Treatment | Method of Treatment | Preservatives Used | Unit Cost of Preservative | Amount of Preservative Used per Set | Cost per preservative | Cost per Set of Peeling, Seasoning, and Treating Timber* | Total Cost of Set in workings |
|---|---|---|---|---|---|---|---|
| Green unpeeled | Untreated | | | | | | $ 8.50 |
| Green peeled | Untreated | | | | | | 8.78 |
| Seasoned peeled | Brush-treated, two coats | Coal-tar creosote | 8c. per gal.† | 1½ gal. per set .5 lb. per cu. ft. | $1.12 | $.28 | 9.27 |
| Seasoned peeled | Brush-treated, two coats | Avenarius carbolineum | 70c. per gal.‡ | 1½ gal. per set .5 lb. per cu. ft. | 1.05 | .65 | 10.20 |
| Seasoned peeled | Impregnated | Water-gas-tar creosote | 7c. per gal.† | 10 lb. per cu. ft. or 30 gal. per set | 2.10 | .65 | 11.54 |
| Seasoned peeled | Impregnated | Coal-tar creosote, | 8c. per gal.† | 10 lb. per cu. ft. or 30 gal. per set | 2.40 | .94 | 11.84 |
| Seasoned peeled | Impregnated | Zinc chloride | 4c. per lb. dry salt | ½ lb. dry salt per cu. ft. or 13 lb. per set | .52 | .94 | 9.96 |

*Cost of treating includes cost of labor, fuel, and interest and depreciation on plant.
†Unit cost of creosote based on price of tank car lots of from 8,000 to 10,000 gallons.
‡Unit cost of carbolineum based on price of barrel lots

Below is given, in detail, the cost of untreated and creosoted 16 ft. by 8 in. Douglas fir shaft sets placed in the mines of the Anaconda Copper Mining Co. These sets contain 1,127 ft. b. m. of Douglas fir squared timbers from the Pacific coast, and 393 ft. b. m. of lagging. The average absorption secured in the treatment of these timbers amounted to 4.5 lb. of creosote per cu. ft.

### Cost of Untreated Sets

| | |
|---|---:|
| 1,127 ft. b. m. squared timbers, at $20.50 per 1,000 ft. b. m... | $25.36 |
| Framing timbers.............................. | 13.50 |
| Cost of lagging, at $15 per 1,000 ft. b. m................... | 5.90 |
| Switching and unloading charges....................... | .85 |
| Cost of placing set.................................. | 18.00 |
| Total cost of untreated set in place.................. | $63.61 |

### Cost of Treatment

| | |
|---|---:|
| Cost of treating, including interest, depreciation, fuel, and labor charges...................................... | $ 3.34 |
| Cost of creosote, at 15.6c. per gal.; absorption 4.5 lb. per cu. ft....................................... | 8.03 |
| Loading and unloading charges....................... | 1.23 |
| Total cost of treatment............................ | $12.60 |
| Total cost of treated set in place.................. | $76.21 |

These examples show that the cost of treating timbers, while a considerable item, does not, when taken in conjunction with the cost of the timber and its preparation and placement, form an unduly high proportion of the whole cost. The actual costs for other mines and other localities will, of course, differ more or less from the figures just given, but they serve to illustrate the relation between the cost of treated and untreated timbers under different conditions.

**Durability of Treated Timbers.**—Tests to secure data on the comparative durability of treated and untreated timber, begun by the Forest Service in 1906, in cooperation with the Philadelphia & Reading Coal & Iron Co., have been in progress for a sufficient period to produce results of practical importance. The experimental timbers were standard round gangway or entry sets, treated and untreated, each set consisting of a 9-ft. leg, a 10-ft. leg, and a 7-ft. cap or collar, the average diameter of the timber being 13 in. The species used were longleaf, loblolly, and shortleaf pine, Pennsylvania pitch pine, and red and black oaks.

The treated sets included loblolly and shortleaf pine treated by the brush method with creosote and carbolineum, by the open-tank method with various preservatives, and by the pressure method with creosote and zinc chloride. The accompanying table gives descriptions of the treated timbers.

Most of the timber was placed during 1906, 1907, and 1908; and inspections were made in December, 1907; March, 1909; and July, 1910.

Owing to the conditions in the various collieries, it was not always possible to make a complete inspection of all of the experimental timbers, nor was it possible in all cases to procure complete data on the cause of failure of individual pieces; but the condition of the timber as found in the various inspections offers sufficiently accurate data to warrant the following conclusions:

1. All of the untreated material failed within from 1 to 3 yr., while brush-treated timber remained serviceable for from 3 to 4 yr.

2. The life of untreated peeled loblolly and shortleaf pine was from 10 to 15% greater than that of similar unpeeled material.

3. In dry, well-ventilated workings the average life of untreated seasoned loblolly pine was approximately 25% greater than that of similar green material. In wet locations, seasoned timber did not appear to outlast unseasoned material.

4. Loblolly and shortleaf pine, brush-treated with coal-tar creosote and Avenarius carbolineum, proved to be from 50 to 100% more durable than similar untreated material. Moreover, brush-treated loblolly and shortleaf pine proved more serviceable than untreated longleaf pine, pitch pine, and red and black oak. Brush treatment with Avenarius carbolineum was somewhat more effective than similar treatment with coal-car creosote.

5. The condition of timber treated by the open-tank process with sodium and magnesium chloride, although not comparing favorably with that of timber similarly treated with other preservatives, was better than that of the brush-treated timbers.

**PEELED AND TREATED LOBLOLLY AND SHORTLEAF PINE GANGWAY SETS PLACED IN MINES OF PHILADELPHIA & READING COAL & IRON CO.**

| Condition Before Treatment | Method of Treatment | Preservatives Used | Amount Used | Number of Pieces |
|---|---|---|---|---|
| Seasoned | Brush treated, two coats | Coal-tar creosote | 1½ gal. per set (¾ lb. per cu. ft.) | 24 |
| Seasoned | Brush treated, two coats | Avenarius carbolineum | 1½ gal. per set (¾ lb. per cu. ft.) | 21 |
| Green | Open-tank process | Coal-tar creosote | 4 to 6 lb. per cu. ft. | 138 |
| Green | Open-tank process | Water-gas-tar creosote | 4 to 6 lb. per cu. ft. | 147 |
| Green | Open-tank process | Zinc chloride* | ¾ lb. dry salt per cu. ft. | 48 |
| Green | Open-tank process | Sodium and magnesium chlorides† | 1 lb. per cu. ft. mixture of dry salts | 39 |
| Seasoned | Open-tank process | Coal-tar creosote | 10 to 12 lb. per cu. ft. | 135 |
| Seasoned | Open-tank process | Water-gas-tar creosote | 10 to 12 lb. per cu. ft. | 36 |
| Seasoned | Open-tank process | Zinc chloride* | ¾ lb. dry salt per cu. ft. | 129 |
| Green | Bethell process (pressure) | Coal-tar creosote | 10 lb. per cu. ft. | 36 |
| Green | Burnett process (pressure) | Zinc chloride‡ | .3 lb. dry salt per cu. ft. | 87 |

\*Injected in a 6% aqueous solution.
†Injected in a 15% aqueous solution of a mixture consisting of 1 part magnesium chloride to 7 parts sodium chloride by weight.
‡Injected in a 2% aqueous solution.

6. Open-tank treatments of green timber with zinc chloride proved fairly effective, but the tests indicate that better results will be secured with seasoned material. About 13% of the green timber treated with zinc chloride by the open-tank process showed marked signs of decay after 4 yr., while no decay was found after the same period of service in seasoned material similarly treated.

7. With the exceptions noted, none of the impregnated timbers showed signs of decay after from 3 to 4 yr. service, although some of them had failed from crush and squeeze.

8. In some instances, impregnated timber, re-framed after treatment, showed signs of decay. This was probably due to the cutting away of treated material and the consequent exposure of untreated portions of the timber.

**Economy in Use of Treated Timbers.** — The original cost of a green unpeeled and untreated loblolly pine gangway set, including removal of old timbers and placement of new ones amounts to about $8.50. The average life of such a set is about 1 yr. and 4 mo. At the end of this period the simple interest charges on the expenditure amount to $.57, making the total cost up to that time $9.07, and to this must be added a replacement charge of $8.50. The total charges for the two installations and the maintenance and simple interest on the first installation up to this time amounted to $17.57. After a period of 2 yr. and 8 mo. the interest charges on the cost of the first installation amount to $1.14, and on the first replacement to $.57, making the total cost up to this time $18.71. A second replacement is then necessary, and if a number of sets are considered it is unlikely that all of them will fail at the same time. In 2 yr. the average total

charges against the untreated set amount to about $18.10. With a set brush-treated with creosote, on the other hand, the charges amount to $11.60, a saving of $6.50 due to the treatment. In 4 yr. this saving amounts to $13.80, which represents the difference between $33, the total cost of the untreated sets, and $19.20, the total cost of the brush-treated sets for that period. The tests further indicate that brush treatment with carbolineum proved more economical than brush treatment with creosote. The fact that the initial cost of the timber at different periods is considered to be the same makes the conclusions very conservative, as the price of mine timbers will unquestionably continue to rise. On the other hand, a certain salvage might have been allowed for removed props, which may be utilized for fuel or sawed into lagging. As, under the conditions of the experiment, failure from mechanical causes, such as crush and squeeze, was more common in treated than in untreated props, the former would have a greater salvage value, and the relative saving resulting from their use would be greater than that noted.

Because the impregnated timbers have not been in service long enough to enable their average life to be determined, most of them being still sound when last inspected, it is impossible to show the ultimate saving in money, resulting from their use. Even for the period since their installation, however, they have proved more economical than untreated or brush-treated material.

Not only will proper preservative treatment result in a direct saving in money, but it will make less timber necessary for any given working. Furthermore, the use of treated timber makes it possible to utilize many of the inferior and more rapid growing species, which, though possessing most of the requirements of high-grade structural timber, lack durability. Treated timber of these species has in many cases proved more serviceable than high-grade untreated material. Thus, in the Eastern and Southern States, treated loblolly and shortleaf pines may take the place of untreated longleaf pine, while treated red and black oaks may be substituted for untreated white oak. Douglas fir, which is now extensively used in the West, may in turn be replaced by treated hemlock, larch, or western yellow pine. Inferior grades of timber can usually be bought for less than higher grades, and an additional saving thus realized.

Timber that is to be treated should, whenever possible, be round instead of square, as the sapwood of most species is more easily impregnated than the heartwood. Moreover, the use of round timbers will do away with the cost of sawing and the consequent waste.

A further economy of waste may be effected by careful inspection, and a rigid condemnation of all unsound material. This is especially important where the timber is to be treated, for it is poor economy to apply an expensive preservative treatment to defective material.

The utilization of waste mine timbers has sometimes proved profitable. Sound sections of broken or partly decayed props have been sawed or split into laggings, planking, etc., and in some cases it has been found profitable to use this material for fuel or to sell it for pulp wood. In many cases such methods afford a considerable saving and also provide a means of disposal for waste.

**Summary.**—Results of recent investigations may be summarized as follows:

1. Decay is, in general, the agency most destructive to timber used in mines.

2. Although decay may often be retarded by peeling and seasoning, treatment with a suitable preservative is more effective.

3. The average life of green, unpeeled, and untreated loblolly-pine gangway sets, under the conditions of the experiments discussed, was less than 1½ yr. Brush treatments with creosote and carbolineum increased this to 3 and 4 yr. while impregnation treatments with zinc chloride and creosote left from 70 to 90% of the timbers untreated at the end of 4 yr.

4. The use of treated timber results in a saving in the cost of maintenance of workings and a reduction in the amount of timber required and makes possible the utilization of inferior species of wood.

5. Brush treatments are economical when the amount of timber to be treated will not warrant the erection of a small open-tank or pressure plant, or when only a short increase in service is required.

6. The open-tank process is adapted to the treatment of small quantities of easily impregnated timber. When a large amount of material is to be treated, a pressure process should be used.

7. Mine timbers impregnated with zinc chloride and creosote oils have shown the best results. Up to the present, no difference in their durability has been noted.

8.  An efficient system of inspection and careful supervision in the use of timber will reduce waste and result in considerable economy. Necessary waste can in many cases be utilized.

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## STEEL AND MASONRY SUPPORTS

### IRON AND STEEL PROPS

**Cylindrical Cast-Iron Props.**—Fig. 1 is a hollow iron prop made in two sections with a sleeve $a$. This prop has been used in longwall workings where it is necessary to draw the prop to let the roof sag. By knocking up the sleeve, the prop falls and can be pulled out of danger. In case one or the other section of the prop is buried by rock, it can usually be recovered by the chain attached, which is sufficiently strong for recovering the end from under the first fall. There is considerable danger of the cast-iron sleeve $a$ splitting when pressure comes on the prop.

**Steel H-Beam Props.**—Steel H-beam props, arranged as shown in Fig. 2, have been used to a limited extent in American mines. The section shown at $a$ is cut and forged to form level bearings; the cost of such props is relatively high. At $d$ is shown a design in which clips are formed by the use of $\frac{7}{16}$-in. screen plates such as are used at the breakers; in this case the **H** section is simply cut to length without other work; this is also the case with the sections shown at $b$ and $c$, where 2-in. plank or plain steel plates, $\frac{3}{8}$ or $\frac{1}{2}$ in. thick, are used for bearings. These props may be secured in place by wooden wedges.
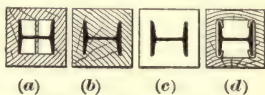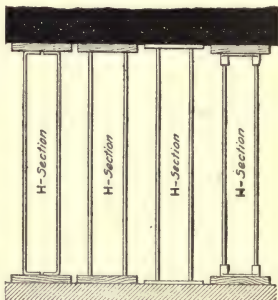


FIG. 1



FIG. 2

**Cast-Iron Posts With I-Beam Caps.**—Cast-iron posts with I-beam caps have been employed in a Staffordshire colliery in England. The posts were made hollow and flanged at the ends as in Fig. 3 (a). A cast-iron chair $a$ fits into the post and receives the cap $b$. The chair was made for a 50-lb. rail, which was reversed so that the head of the rail would slip into the horns of the chair as shown and the bottom of the rail be upwards. The lagging $c$ was of wood and above this were placed planks $d$ forming a double lagging filled in above with waste to make a tight joint with the roof rock. The planks $d$ placed on the lagging $c$ saved timber.

### STEEL ENTRY TIMBERS

**Standard Forms.**—A set of timbers, whether steel or wood, consists of two parts; the collar, or cross-beam, and the legs, or posts. Under certain conditions, the legs may be dispensed with and the collars, which ordinarily are rolled **I** beams, may be set in hitches cut in



FIG. 3

the coal. Or the collars may be set upon wooden or hollow cast-iron legs, or upon brick, masonry, or concrete walls. All of these methods have been successfully tried in many places at home and abroad. However, the I-beam collar or cap is generally, especially in entry timbering, set upon legs or posts of steel. The legs are of two general types—channel iron or **H** beam. The former is illustrated in Fig. 1, which is called by the makers, the Carnegie Steel Co., Style E. This form corresponds to installations in which steel I-beam collars have been used with wooden posts, the top of the post being cut to form a seat for the collar. The two channels forming the legs are
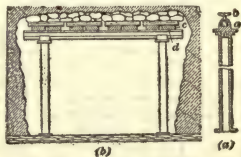
connected by bolts and separators, carry angle brackets at their tops on which the collar rests, and foot on a steel plate to which bars are riveted to hold

them firmly in place. Angles riveted to their webs transmit the load from the collar to the legs, and bent angle lugs prevent undue side motion. In this form of construction, there are but five pieces to be erected: the single collar beam, two legs, and two base plates. If the footing is good, even the base plates, shown in Fig. 2, can be eliminated or plain plates can be used instead of the riveted bases.

The second type of leg, illustrated in Fig. 3, consists of an **H** beam and in the combination shown is known by the makers as Style F. The lug angles at the top prevent side



FIG. 1

motion and the bearing plates are perfectly plain, as shown in Fig. 4, although base plates, Fig. 5, to which the legs may be bolted or riveted can be sup-

plied upon request. This is the simplest form of entry timber and perhaps the one in most general use. It should be noted, that where a broader bearing surface is necessary, **H** beams may be used for collars in place of the standard **I** beams.

Mr. R. B. Woodworth, in the Transactions of the Kentucky Mining Institute, Dec., 1911, states as follows: "With plain material at 1.25c. a lb. f. o. b. cars Pittsburg, and the usual cost for workmanship, the comparative costs of these styles are shown in the two tables that follow, from which



FIG. 2

can be seen at a glance how great a part attention to details may play in the economic use of materials by the avoidance of unnecessary work in

fabrication. The sets in each table are all of equivalent theoretical strength. In the first table, are given steel gangway supports for a very heavy double-track gangway, collar 17 ft. long between legs, legs 10 ft. 6 in. high in the clear, equivalent in strength to 24-in. round, longleaf, yellow-pine timbers.

In the second table are given the steel gangway supports for a single-track gangway, collar 10 ft. long between legs, legs 8 ft. high in the clear, equivalent in strength to 15-in. round, longleaf, yellow-pine timbers.

"Figures in these two tables do not include painting, cost of



FIG. 3



FIG. 4



FIG. 5

which varies with the kind of paint used, but may be estimated at $2 per T. additional. Styles A, B, D, and E, are various combinations of **I**-beam collars and channel iron legs, while styles F, G, C, and I have **H**-beam legs."

The details of another form of steel entry timbers are shown in Fig. 6, where the legs are pin-connected to the cap and not joined by angle bars. The cap piece *a* is an **I** beam, while each leg *b* consists of two channel beams that rest in a cast base *c* at the bottom. The top of each leg is fastened to the cap by means of the pin *d*, held in place by the split cotters *e*. Iron wedges *f* are also
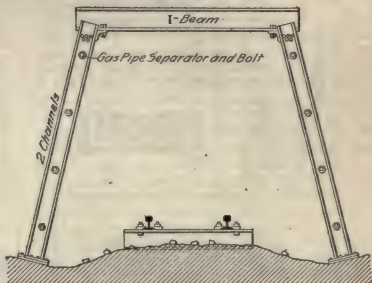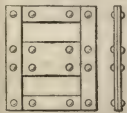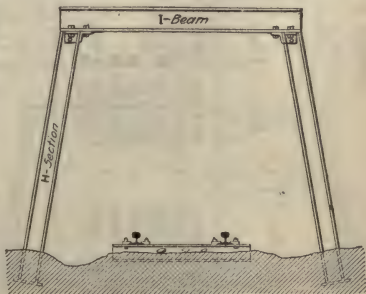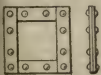
# STEEL GANGWAY TIMBERS

## Double Track

| Style | Size of Collar | Legs | | Weight per Set | | Cost per Set | |
|---|---|---|---|---|---|---|---|
| | | Kind of Leg | Weight per Foot-Pounds | Without Base Pounds | With Base Pounds | Without Base Plates | With Base Plates |
| B | 20-in. 65-lb. beam | Two 7-in. chan. | 14.75 | 1,930 | 2,030 | $36.82 | $39.48 |
| D | 20-in. 65-lb. beam | Two 7-in. chan. | 14.75 | 1,930 | 2,100 | 36.82 | 43.95 |
| C | 20-in. 65-lb. beam | One 8-in. H | 34.00 | 1,710 | 1,800 | 25.39 | 27.80 |
| A | 20-in. 65-lb. beam | Two 7-in. chan. | 14.75 | 1,930 | 2,500 | 36.82 | 58.50 |
| F | 20-in. 65-lb. beam | One 8-in. H | 34.00 | 1,690 | 1,730 | 25.81 | 26.88 |
| I | 20-in. 65-lb. beam | One 8-in. H | 34.00 | 1,730 | 1,780 | 25.22 | 27.12 |
| E | 20-in. 65-lb. beam | Two 7-in. chan. | 14.75 | 1,670 | 1,770 | 25.94 | 28.60 |
| G | 20-in. 65-lb. beam | One 8-in. H | 34.00 | 1,690 | 1,730 | 25.81 | 26.88 |

## Single Track

| Style | Size of Collar | Legs | | Weight per Set | | Cost per Set | |
|---|---|---|---|---|---|---|---|
| | | Kind of Leg | Weight per Foot-Pounds | Without Base Pounds | With Base Pounds | Without Base Plates | With Base Plates |
| A | 10-in. 25-lb. beam | Two 6-in. chan. | 10.5 | 765 | 945 | $14.81 | $23.17 |
| D | 10-in. 25-lb. beam | Two 6-in. chan. | 10.5 | 765 | 800 | 14.81 | 15.85 |
| B | 10-in. 25-lb. beam | Two 6-in. chan. | 10.5 | 765 | 810 | 14.81 | 16.12 |
| E | 10-in. 25-lb. beam | Two 6-in. chan. | 10.5 | 605 | 660 | 9.39 | 10.87 |
| F | 10-in. 25-lb. beam | One 5-in. H | 18.7 | 509 | 590 | 8.84 | 9.43 |
| G | 10-in. 25-lb. beam | One 5-in. H | 18.7 | 566 | 587 | 8.80 | 9.39 |
| C | 10-in. 25-lb. beam | One 5-in. H | 18.7 | 565 | 605 | 7.91 | 9.04 |
| I | 10-in. 25-lb. beam | One 5-in. H | 18.7 | 600 | 630 | 9.80 | 11.11 |

used to stiffen the connection between the cap and the leg. Several pinholes are made in both the legs and the collars, so that the same set may thus be used in several positions. In order that the legs may be given a desired batter



FIG. 6

the legs of the posts fit into a cast shoe $c$ that has a cylindrical bottom, and this bottom rests in a cast base $g$. This forms a very easily adjustable set, for by means of the base illustrated the legs can be given any desired batter, and the set is then stiffened by means of the wedges $f$.

Lagging used with steel timbering may be of wood or, better, of boiler plate, or corrugated sheets of buckle plates where extreme strength is needed. The thin concrete slab, however, may well be used where acid water is present, either plain or reinforced with expanded metal, wire-mesh reinforcement, or plain rods or wire.

For pump houses, as shown in Fig. 7, and for underground stables steel framing, particularly in connection with concrete lining, is daily growing in favor. In the pump house shown, the simple steel framing took the place of heavy framed timbers that it is necessary constantly to renew.

**Relative Cost of Steel and Wood Timbering.** — Owing to the movement of the strata overlying the pump room of the Dodson colliery, of the Plymouth, (Pa.) Coal Co., it was decided to retimber it with steel. The original wooden timbering consisted of 18-in. to 22-in. round sticks of white pine, yellow pine, and oak placed 2 ft. center to center. A great deal of trouble was experienced from these timbers becoming forced in close upon the pipe lines with the possibility of breaking them. As new timbers were placed, they were put in between the sets already in, so that eventually the pump room had timbers practically skin to skin. It is estimated that the entire pump room was retimbered in wood once a year.

The pump house is 100 ft. long, 8 ft. high in the clear, and 18 to 22 ft. wide. Beginning with April 16, 1910, the seventy wooden sets of mine timbers were replaced by forty-eight steel sets made up of 18-in., 55-lb., and 20-in. 65-lb. I-beam collars and 6-in. H-beam legs, weighing 23.6 lb. per ft. The last set was installed about December 15, 1910.

From the following statement it will be noted that the total cost for timbering once with wood was $2,415, and the total cost for timbering in steel $2,889.09, or a difference in first cost of not quite 20%. The steel cost at the mines slightly over two and a half times the cost of wooden sets, and it also cost 33¼% more for placing. Fewer sets were required, however, and the ultimate rate was thereby lessened.

The comparative cost of the two installations is shown in the statement below, prepared by Mr. Haddock:
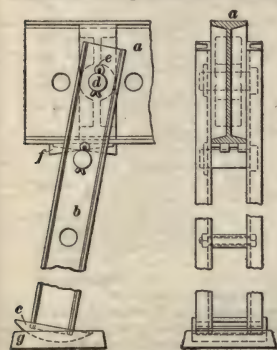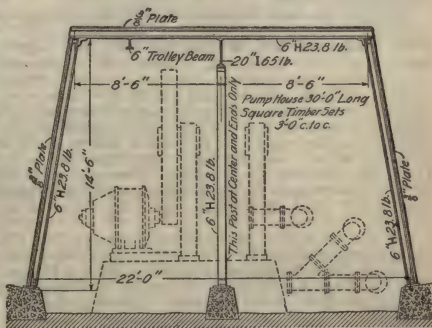


FIG. 7

### Wood

| | |
|---|---:|
| Number of sets............................................... | 70 |
| Average diameter of timber, inches......................... | 20 |
| Quality of timber, yellow pine and oak. | |
| Average weight per set, pounds............................ | 4,150 |
| Cost per set f. o. b. cars mines.......................... | $12.00 |
| Cost per set for placing.................................... | $22.50 |
| Cost per set in place....................................... | $34.50 |
| Total cost for timbering.................................... | $2,415.00 |
| Life of timber set, year .................................. | 1 |

### Steel

| | |
|---|---:|
| Number of sets............................................... | 48 |
| Size of collars, 18-in. beam, pounds...................... | 55 |
| Size of collars, 20-in. beam, pounds...................... | 65 |
| Size of legs, 6-in. **H** beam, pounds....................... | 23.6 |
| Quality of steel, structural grade. | |
| Average weight per set, pounds............................ | 1,483 |
| Cost per set f. o. b. mines............................... | $31.47 |
| Cost per set for placing................................... | $30.00 |
| Cost per set in place...................................... | $61.47 |
| Total cost for timbering.................................. | $2,889.09 |

The higher cost of placing the steel is due to three causes:

1. The charge of taking out the old timber, which, however, was insignificant, as the steel was placed a set at a time by forepoling ahead, the condition of the roof being very bad and there being loose material for an unknown distance above.

2. Great care was taken with the steel to line it up properly and provide a good base, which was made of a solid concrete wall built the full length of the pump room on each side. This solid concrete base is unnecessary with the wood and might have been omitted with steel, but its use means a real betterment in the construction.

3. The steel was placed without interfering with the operation of the pumps, which necessitated very careful handling and added something to what the expense would have been had the room been free from obstructions.

It is apparent that while the first cost of the steel construction is greater than that of wood, it will have much more than paid for itself if its life extends over 15 mo. only, and that every additional length of time it stands will mean that much less in cost of maintenance. The first steel after being in place 16 mo. showed no sign of deflection in the collars, and what is better, no evidence of fracture in the concrete where any overloading of the steel would immediately show.

In 1908, at their Maxwell colliery, the Lehigh and Wilkes-Barre Coal Co. timbered a double-track gangway with 20-in., 65-lb., **I**-beam collars 17 ft. long between legs, and 8-in. **H**-beam legs 10 ft. 6 in. high in the clear, weighing, with base plates, 1,720 lb. per set. These took the place of wood sets made of 24-in., round, yellow-pine timbers, the cost of which erected was $15 per set, weight 5,040 lb., and the life of which was 2½ yr. In view of their probable durability the steel sets were erected on concrete bases which added to the cost, which reached a total of $40 per set. Capitalized at 6% interest, the value of the steel sets at the end of 15 yr. will be $95.86 each, while the capitalized value of the six wooden sets needed in that time will be $153.56. At the end of the 15 yr. the steel will have a scrap value per set of $12.03, while the wood will be worth nothing, a saving by the use of steel of $69.73 per set or $4.65 per yr. The use of steel in English mines has effected a saving of 2c. per T. of coal mined.

At the No. 8 mine of the West Kentucky Coal Co., Sturgis, Ky., steel mine timbers are used in the new slope, both main heading and air-courses. Sets in use are Style F composed of 10-in., 25-lb., **I**-beam collar and 4-in., **H**-beam legs. Sets are spaced 3 ft. on centers and lagged with oak plank 3 in. thick on top and 2 in. thick on sides. Between the sets concrete is placed up to 4 ft. high. This makes a solid reinforced-concrete slope from the entrance to the point where ribs are hard and top good. According to figures furnished by Mr. W. H. Cunningham, general manager of the company, the comparative costs of wood and steel for this mine were:

Wood.—Yellow-pine creosoted; size 12 in.×12 in., 264 B. M. ft.; cost at Sturgis $10.56 per set; cost in place $15.70; weight 1,575 lb.

Wood.—Native white oak; size 12 in.×12 in., 264 B. M. ft.; cost at Sturgis $7.92; cost in place $13.06 per set; weight 1,340 lb.

Steel.—Cost of steel at Sturgis $9.75 per set; cost of placing $1; cost of concrete per panel $5.16; total cost in place per set, steel alone $10.75, steel concreted $15.91; weight of steel sets 425 lb.

The saving in the use of steel without concrete over native white oak was $2.31 per set; over yellow pine, $4.95. The excess cost of steel with concrete over white oak was $2.85 per set; over yellow pine, 21c. This favorable comparison is due to the high unit cost of the wood and to the elimination of waste. The safe uniformly distributed load on the wood collar is 19,200 lb., on the steel collar 26,000 lb. The safe compressive strength of the steel leg is 43,200 lb., while that of the wooden leg is 105,100 lb.; in the one case more than ample, in the other case out of all proportion.

**Advantages of Steel Timbering.**—Among the benefits coming through the use of steel timbering are:

1. *Reduced Excavation.*—In the Lehigh and Wilkes-Barre installation the same clear space inside was had with steel in an excavation 4 in. less in height and 32 in. less in width than that needed with wooden timbering; in the Dodson colliery, the space saved was 2 in. in height and 28 in. in width; and in the West Kentucky installation, 2 in. in height and 16 in. in width.

2. *Better Ventilation.*—The headings being larger, the ventilation is better, and further, the absence of all stages of decay common to wooden timbers, removes a serious source of vitiation of mine air.

3. *Less Dust-Catchment Area.*—Steel mine timbers afford a much less area for the lodgement of explosive coal dust than do wooden ones of the same strength and are much more easily cleaned.

4. *Fireproof Character.* One of the greatest advantages of steel timber consists in its absolute incombustibility, rendering it especially suitable for the construction of underground shanties, stables, pump and engine rooms, etc.

**Preservation of Steel Mine Timbers.** — The corrosion of structural-steel timbers within the mine is not so serious as above ground, as the conditions of temperature, humidity, etc. are practically uniform. Mine timbering steel should not be placed unpainted unless it is to be covered with concrete. If well painted before installation, steel timbering, so far as rust is concerned, should outlast the



FIG. 1

life of the mine. After 17 yr. of use, the steel timbers installed in the Sterns shaft, of the Susquehanna Coal Co. and in the pump room of the Hazleton shaft colliery, No. 40 slope, of the Lehigh Valley Coal Co., are still in use and in first-class condition. The steel timbering at the No. 1 shaft of the Spring Valley (Ill.) Coal Co., is in good condition after 18 yr. use. Structural steel does not have the opportunity to corrode as does the steel in track rails underground and consequently lasts indefinitely. Rails are laid where they come in direct contact with any acid mine water and, their tops being polished by passing car wheels, are in the most unfavorable position to resist corrosion.

The possible effect of acid mine water upon steel mine timbers has been exaggerated. Tests under working conditions show that the careful selection and application of good paint will prevent the destructive action of mine water. The paint should be applied in two coats, the first of which should be red lead or natural iron oxide and the second a good graphite. The coating should be applied to a clean surface and should be well rubbed in. The paints should be of the very best grade mixed in pure linseed oil, the weight of the paint, in pounds, per gallon of oil being about three times its specific gravity.
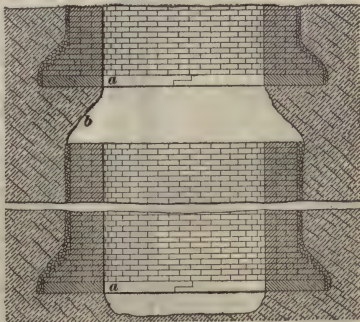
## MASONRY AND IRON SHAFT LININGS

**Masonry Shaft Lining.**—Masonry shaft lining, which may consist of brick, rock, or concrete, is used where timber is scarce or where the character of the

strata is such as to render timber lining impracticable. A section only of a shaft is sometimes thus lined, while the ordinary timber lining is used in the greater part of the shaft. These linings are usually laid on a wedging curb and are carried upwards in sections, as shown in Fig. 1. Each section is laid on a ring *a* of cast iron or timber resting on a temporary shelf or seat *b* cut in the rock. As the lower sections are built up, the shelf *b* supporting the masonry above is cut away in places and the masonry below carried up to furnish the necessary support for the upper section. In this manner, all the shelf is finally cut away and replaced by the masonry of the lower section.

**Tubbing.**—Tubbing is an English term applied to the metal, or sometimes to the timber, lining of a circular shaft, and is particularly used when such linings are employed to keep water from flowing into a shaft. The three kinds of metal tubbing are: (1) that which is made in sections with outside flanges and is simply wedged firmly into place by wedges placed between the tubbing



and the wall of the shaft; (2) that which is made in sections and bolted together on the inside both at the vertical and horizontal joints; (3) that which is made up of complete rings of cylinders bolted together by means of horizontal flanges.

The metal tubbing *a*, Fig. 2, consists of cast-iron segments varying in height from 18 to 36 in., according to the pressure to be resisted. The segments are flanged at top, bottom, and ends and $\frac{1}{2}$-in. pieces of pine are put between them as they are put in place, thus making tight joints both horizontally and vertically. To prevent breaking the metal lining by the pressure of air or gas behind it, the metal is perforated; these holes are loosely plugged, so that any particular pressure coming on them will force out the plugs. At *b* is shown a method of walling a circular shaft with brick, the brick being laid on a cast-iron wedge curb *s*.

Wood tubbing may be of two kinds: (1) planks 2 or 3 in. thick placed vertically and having edges like barrel staves; (2) thick blocks similarly beveled and placed vertically. At *c* is shown an example of plank tubbing. The planks have timber curves *m* placed inside them and spiked to them. The curves are kept apart by punch blocks *n* and are tied together and fastened to the shaft sills *l* by the stringers *r*. The sections of the shaft *b* and *c* are shown supported on a rock bench while the metal tubbing is being put in place below. When a shaft has been lined to the rock bench, this is cut away and the metal tubbing joined to the other portion of the shaft lining by small metal sections, called closers.

Fig. 2

The following formula is given by Mr. W. Galloway for calculating the proper thickness for cast-iron tubbing, or for cement or brick lining:

$$t = \frac{whd}{2(r + wh)}$$

in which $t$ = thickness of lining, in inches;
$d$ = internal diameter of shaft, in inches;
$h$ = head of water, in inches;

$w$ = weight of cubic inch of water = $\frac{62.5}{1,728}$ = .0361 lb.;

$r$ = $33\frac{1}{3}\%$ (one-third) of crushing load per square inch of material used.

The crushing strength of the material used should be determined in each case by experiment, but the following may be used as a fair average value:
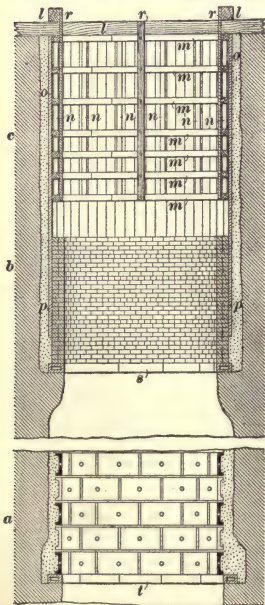
|  | Pounds Per Square Inch |
| --- | --- |
| Crushing strength of cast iron | 80,000 |
| Crushing strength of brick laid in lime mortar | 1,000 |
| Crushing strength of brick laid in cement and lime | 1,500 |
| Crushing strength of brick laid in best cement mortar | 2,000 |
| Crushing strength of concrete made from Portland cement and 1 mo. old | 1,000 |
| Crushing strength of concrete made from Rosendale cement and 1 mo. old | 500 |
| Crushing strength of concrete made from Portland cement and 1 yr. old | 2,000 |
| Crushing strength of concrete made from Rosendale cement and 1 yr. old | 1,000 |

EXAMPLE.—What should be the thickness of tubbing for a shaft 13 ft. in diameter at a depth of 800 ft.: (a) for cast iron? (b) for brick, assuming a mean crushing strength of 1,500 lb. per sq. in.? (c) for concrete made from Portland cement and 1 mo. old?

SOLUTION.—

$$(a) \quad t = \frac{.0361 \times 800 \times 12 \times 13 \times 12}{2 \times \left[\frac{80,000}{3} + (.0361 \times 800 \times 12)\right]} = \frac{27,031}{26,666 + 347} = 1 \text{ in}$$

$$(b) \quad t = \frac{.0361 \times 800 \times 12 \times 13 \times 12}{2 \times \left[\frac{1,500}{3} + (.0361 \times 800 \times 12)\right]} = \frac{27,031}{500 + 347} = 31.9, \text{ say } 32 \text{ in.}$$

$$(c) \quad t = \frac{.0361 \times 800 \times 12 \times 13 \times 12}{2 \times \left[\frac{1,000}{3} + (.0361 \times 800 \times 12)\right]} = \frac{27,031}{333\frac{1}{3} + 347} = 39.7, \text{ say } 40 \text{ in.}$$

## STEEL AND CONCRETE SHAFT LININGS

**Steel Sets.**—The use of steel alone and not in connection with concrete lining is unusual in shaft timbering. An illustration, however, is afforded by the shaft at the Mt. Lookout colliery, of the Temple Iron Co., and illustrated
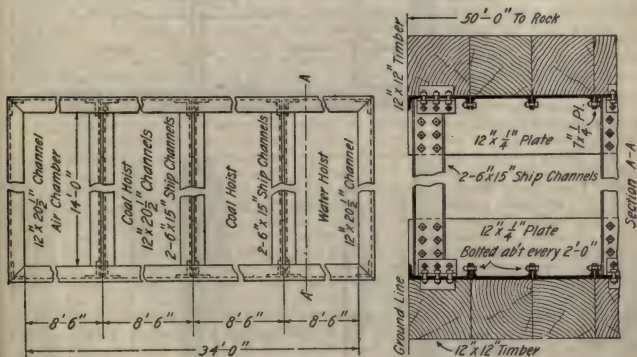


FIG. 1

in Fig. 1. The steel sets were used to reinforce the worn-out original wooden sets, which were left in place. The steel sets consisted of 12-in. channels set back to back, separated by anchor plates to catch in the wood of the original lining, and double 6-in. channels to take the place of the 10″ × 12″ buntons

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Reinforced Concrete Bunton

I-Beam Bunton

H-Beam Bunton

Channel Center Bunton

Yellow Pine Bunton

Channel End Bunton

FIG. 2

originally separating the compartments. It would seem that instead of channels, **H** sections should have been used as better adapted to resist compression than the channels, as well as being lighter and, hence, cheaper.

**Steel Buntons.**—Steel, in place of wood, is very commonly employed for buntons even in shafts that are not lined with concrete. Some of the various forms are shown in Fig. 2, and of these the **H** section appears the best and is the most generally used. Steel buntons are fireproof, but cost much more than wood; four times as much if a section as light as 35 lb. per ft. is used. On the other hand, with proper care, they will last indefinitely.

**Concrete and Steel Shaft Linings.** Practically all concrete-lined shafts are elliptic in section, the arch form being adopted as better able to withstand pressure than the rectangular. A full description of the concrete-lined shaft at the Filbert mine, of the H. C. Frick Coke Co., Fayette County, Pa., is given on page 230.

The rectangular, concrete-lined shaft of the Bunsen Coal Co., near Clinton, Ind., is shown in Fig. 3. The buntons are $6'' \times 8''$ concrete beams reinforced with a 6-in. 8-lb. channel. Where the shafts pass through soft ground, the ends are also reinforced with the same size channels. The lining is 6 in. thick through firm material and 12 in. through soft. The guides are $6'' \times 8''$ yellow pine bolted to the reinforced-concrete buntons. The partitions in the air-shaft at this mine are 8 in. thick and of reinforced concrete built with American Steel and Wire Co.'s No. 4 triangular-mesh reinforcement on $6'' \times 8''$ buntons reinforced with channels as noted.

In the elliptical shaft of the Tennessee Coal, Iron, and Railroad Co., at Pratt mine No. 13, near Ensley, Ala., a massive, unreinforced concrete lining is used with a minimum thickness of 15 in. through firm and 18 in. through soft material. The buntons are 6-in. steel **H** sections, weighing 23.8 lb. per ft., spaced 6 ft. apart. The guides are of the same material as the buntons and carry. bolted to them, cast-steel safety racks.



FIG. 3

# HOISTING

*Hoisting*, or *winding*, *engines* may be driven by hand, horse, or mechanical power. The mechanical power may be derived from engines, or motors, driven by steam, electricity, gasoline, compressed air, water, etc.

There are two general classes of hoists: single and double. In the former, there is but one cageway in the shaft and up this the cage and loaded car are hoisted by an engine. After the load is dumped at the surface, the cage and empty car descend through the same compartment, impelled by gravity, their speed being controlled by the brakes on the engine drum. In double hoists, there are two cages which travel in separate compartments, one ascending with the loaded car as the other descends with the empty car. Double hoists are the prevailing type, the use of single hoists being confined to prospecting shafts and to unimportant operations in the metal-mining districts.

There is no essential difference between stationary engines used for hoisting and for haulage. The chief distinction lies in the direction of application of the power generated by the engine. In hoisting engines, the power is applied vertically to raise a weight through a shaft; in haulage engines, the power is applied in a horizontal or approximately horizontal direction to move a weight along a track. Frequently, the same mechanism after having served its purpose as a hoisting engine is used for haulage, and vice versa.

The subject of hoisting ropes is discussed under the head of Wire Ropes.

## HAND- AND HORSE-POWER HOISTS

Hand- and horse-power hoists are of relatively small capacity and are almost entirely used for prospecting, shaft sinking, or the like.

The *windlass*, operated by one or two men, is frequently used for sinking small shafts to depths of about 75 ft., where the loaded bucket weighs but a few hundred pounds. In form, it is similar to the hoisting device used in connection with water wells and consists of a wooden barrel, about 8 in. in diameter and 4 or 5 ft. long, provided with a 1 to 1½-in. iron axle. This axle is supported at either end in **A**-shaped wooden standards nailed or mortised to a heavy timber base placed over the shaft. The necessary crank and handle is attached to each end of the barrel.

For hoisting heavier weights, *single-* or *double-geared iron crab winches* are used. In these, the power is transmitted to the drum or barrel by rack and pinion, so that one man can raise 1 T. or more, but at the expense of speed. These hoists are single and unbalanced, the bucket being hoisted by one or two men and descends by gravity; its speed is controlled by loosening or tightening the rope upon the drum.

For greater depths and heavier loads, *horse whims*, or *gins*, are used. These consist essentially of a drum mounted on a shaft to which are attached one or more cross-sweeps to each of which a horse or mule is hitched. Usually the whim is placed a little distance from the shaft and is so arranged that the movement of the car is regulated by two hand levers, which are connected to the driving gear in such a way that the movement of the drum may be stopped or reversed independently of the movement of the horse. One lever is moved to hoist and the other to lower the load, and through their use overwinding is prevented in case the animal does not stop on the instant.

In the better classes of whims, the drum is placed horizontally underground or below a platform to be out of the way of the horses, motion being imparted to it from a vertical shaft through beveled gearing. The vertical shaft is provided with, two, four, or six sweeps to each of which one or two horses may be hitched, so that as many as twelve animals may be used. With four horses, 90 T. have been hoisted 600 ft. in 10 hr.; and with eight mules 60 T. have been hoisted 900 ft. in the same time. Some whims are provided with gears for hoisting heavy loads at slow speed and lighter loads at high speed. Such machines, at slow speed will hoist 2,400 lb. 22 ft. per min.; and at fast speed, 950 lb. 55 ft. per min.

## STEAM-POWER HOISTING ENGINES

Hoisting engines are almost invariably of the duplex, or two-cylinder, type with cranks set at right angles to one another and therefore have no dead center; for which reason they can be quickly started from any position and run smoothly than single-cylinder engines.

Hoisting engines may be *simple* or *compound*, and *tandem* or *cross-compound*. The first is by far the most extensively used in the shallow shafts prevailing in the coal regions, where the shortness of the hoisting period and the frequent reversals of the engines are not conducive to the economical use of high-pressure steam expansively. In the metal-mining regions, where vertical lifts of 1,000 ft. are usual, of 2,500 ft. fairly common, and where several of from 4,000 to over 5,000 ft. exist, refinements in compounding, etc. are successfully used. Further, in metal-mining regions, the price of coal is such (from $4 to $10 and more per ton) that it is imperative to secure all the energy possible from each pound of fuel; while at coal mines and particularly at those where slack is used under the boilers the cost of power has not, until comparatively recent years, been thought worthy of consideration.

A hoisting engine may be of the slide-valve, piston-valve, or Corliss-valve type and may be condensing or non-condensing. For a large hoisting engine, a piston-valve gives a much better distribution of steam in the steam chest than a slide-valve, but not so good as a Corliss valve. A hoisting engine, to run condensing, should have an independent air pump and condenser; for if the air pump is operated by the engine it will stop when the engine stops and the vacuum will be lost, rendering the low-pressure piston, in some cases, inadequate to pick up the load at the beginning of the next hoist.

In a hoisting engine, the drum on which the hoisting rope coils takes the place of a flywheel, to a certain extent. The operation of hoisting is intermittent in character, and in some cases the engine is so connected that it will run only when operating the drum; in other cases it will run continuously, either empty or under some other load than the hoisting load, the work of hoisting being put on it, when needed, by means of a friction clutch connecting the engine with the drums. Where an engine runs continuously, its surplus power may be utilized for driving air compressors, fans, electric generators, and other machinery; and, by thus concentrating the power, a higher grade engine can be made available for hoisting purposes.

**Second-Motion, or Geared, Hoisting Engines.**—In a second-motion engine, power is transmitted from the engine shaft to the drum shaft through gearing. This engine is particularly adapted for portable hoists, such as are used in shaft sinking and similar temporary work, and for shallow mines, or mines where a small tonnage is raised. It is cheaper in first cost and in installation than a first-motion hoisting engine, as a smaller engine does the same work, but it cannot hoist as rapidly; there is also less liability of overwinding. Geared engines are used ordinarily where a hoisting speed of 750 ft. per min. or less is satisfactory, while first-motion engines are used for greater speeds. To hoist the same load, a first-motion engine must be three to four times as large as a second-motion engine, while the hoisting speed and cost will also be three to four times as much.

The relative number of teeth in the gears may be varied so that the piston speed may be made faster or slower or equal to that of the rope. The commonly used ratios vary from 1 to 3 to 1 to 5; that is, if the small gear-wheel on the engine shaft has, say, 20 teeth, the large gear-wheel on the drum shaft will have 60 to 100 teeth, depending on the ratio, and it will require from three to five revolutions of the engine to equal one of the drum.

If the ratio is exact, the teeth on the small gear come in contact with the same teeth on the large gear during every revolution and cause excessive wear. To equalize the wear, the number of teeth in the large wheel is commonly one less or more than that demanded by the exact ratio. Thus, if the engine is geared 1 to 5, while 20 teeth on the small wheel require 100 on the large wheel, either 99 or 101 would be used.

Hoists are occasionally built with metal teeth in the pinion and wooden teeth in the larger wheels. The larger wheels in such cases are cast with mortises, into which are driven maple cogs that are made secure by wedges. These wooden cogs, or teeth, wear well and are easily replaced when broken without seriously interrupting hoisting operations; they are almost noiseless. It a metal tooth breaks, the gear must be replaced, and hoisting must cease until this can be done. In *cut gears*, the teeth are finished by machine; this

adds slightly to the cost of these gears, but they are more serviceable than rough, cast gears and make less noise.

Geared engines may have single or double drums, the former being in general use at coal mines where the shafts are relatively shallow and the material is hoisted from one level. Single-drum engines are commonly used in balance, the drum being keyed directly to the shaft, one rope unwinding from the top of the drum as the other rope winds up beneath it. Double-drum engines may be used in balance, by leading the ropes as just indicated but on the separate drums; or they may be used independently, each drum hoisting as desired and both ropes leading on the drums alike, that is, both on top or both underneath.

The wearing surfaces in hoisting engines, especially the main bearings, should be made large and the engines proportioned to stand severe work. In the case of two wide-faced drums on the shaft, it is sometimes necessary to have a center bearing, which should be adjustable in every direction and kept as nearly in line with the other bearings as possible. Owing to the difficulty of keeping three bearings in line, and the danger of the shaft breaking in case the bearings are not in line, it is well, where practicable, to omit the center bearing and make the shaft as short as possible and ample in diameter.

**First-Motion, or Direct-Acting, Hoisting Engines.**—In first-motion hoists, a pair of engines (right- and left-handed) are used with their cranks on the ends of the same shaft as the drum, the cranks being set at angles of 90° with each other to prevent the engines stopping on a dead center. A direct-acting hoisting engine is used wherever the depth of the shaft or a large output demands a high speed of hoisting. In coal-mining practice, their use was formerly limited to the deepest shafts, but the large outputs required from modern mines have caused them to be introduced at comparatively shallow shafts.

# HOISTING ENGINES USING OTHER POWER THAN STEAM

**Compressed-Air Hoisting Engines.**—Where available, compressed air may be used in place of steam for power as there is no essential difference in the engines. Compressed air may be used exclusively or interchangeably with steam, and should be reheated before entering the engine cylinders. In the case of a compound engine, the reheater may be placed in the pipes leading to the high-pressure cylinder; or, still better, it may be placed before each cylinder; otherwise, the expansion will cause the moisture to freeze in the low-pressure cylinder and stop the engine.

Where a hoisting engine is located on the surface and a boiler plant is necessary, steam is generally preferable to compressed air, as the loss in efficiency due to compressing the air is avoided. If, however, water-power is available, it is frequently cheaper to use compressed air instead of steam, particularly if compressed air is also used at the mine for coal cutters or rock drills; or if for any reason it is necessary to place the hoisting engine at a distance from the boiler plant, as there is much less loss of efficiency in carrying compressed air than in carrying steam. For underground hoists, compressed air has many advantages, particularly in gaseous coal mines.

Further data on compressed air will be found under that heading and under the heading Haulage.

**Gasoline Hoisting Engines.**—Gasoline hoists are adapted for sinking prospecting shafts in mountainous districts where fuel is scarce and where an easily portable engine is desirable. They are not generally used for permanent hoists in mines of any great capacity. Their operation is essentially on the lines of gasoline haulage motors, which are described under the heading Haulage. The gasoline is injected into the engine cylinder in the form of a spray and is there mixed with air and ignited by means of an electric spark, producing an explosion that moves the piston. When starting the engine, the clutch is released and the engine is rotated by pulling over the flywheel until it has received the first impulse, which usually requires from one to two complete turns. After receiving the first explosion, the engine continues to operate, drawing in a supply of gasoline and air and igniting it with an electric spark. When operating the hoisting drum, the engineer first speeds up the engine and then throws the clutch that controls the hoisting drum. Drums must be well equipped with a powerful brake in the use of either gasoline or electric hoists, to avoid accident due to the possible failure of the power.

**Hydraulic Hoisting Engines.**—Hoisting engines using the direct energy of falling water as a source of power, while not infrequently employed in metal-mining districts, are not known in the coal fields.

**Electric Hoisting Engines.**—Electric hoists differ but slightly in mechanical construction from those operated by steam. Owing to the high speed of the ordinary motor, electric hoists are commonly double geared. The reduction between the armature shaft and the intermediate shaft is ordinarily about 1 to 4; between the intermediate shaft and the drum gear it varies according to the size of the drum and the hoisting speed desired.

Motors for heavy hoisting may be either of the alternating-current induction type or of the direct-current type. Alternating-current induction motors are discussed under the subject of Electricity with further notes under Haulage. When large direct-current motors are used at a distance from the power station, the power is transmitted by alternating current to the point of its application and is there transformed to direct current, usually by means of motor-generators.

The getting up of full speed (acceleration) and maximum load (coal and weight of entire rope) produce what is known as a peak, or high point, in the curve diagramming the power required from a hoisting engine; and the peak load is often double the average load upon the engines. In electric hoisting as the heating of the machines varies approximately as the square of the load
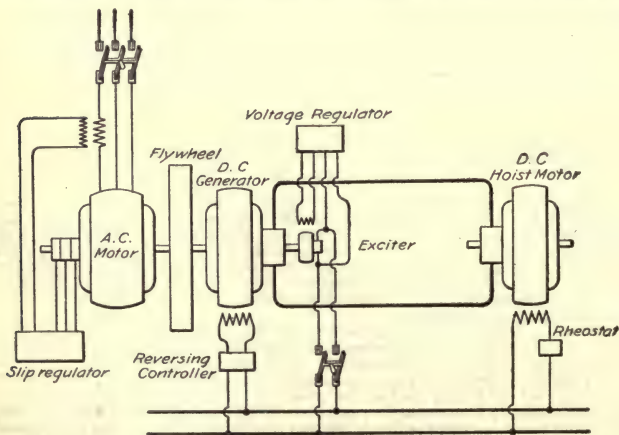


FIG. 1

it is important in order to reduce the size (and consequently the cost of the equipment) to make the load during the hoisting period as uniform as possible. The partial equalization of the load is accomplished through the use of some system of balanced hoisting, such as the Koepe described later. These systems, however, do not perfectly balance the load during all portions of the run, and various methods have been employed to produce what may be called an electric balance so that the input of electric energy may at all times be equal to the output of mechanical energy.

In the *Ilgner system*, shown in diagram in Fig. 1, a motor-generator set is used for supplying power to the hoist motor, which is of the shunt-wound direct-current type. The operation of the hoist is controlled by varying the voltage of the generator, to which it is directly connected electrically. By reversing the excitation of the generator, the direction of rotation of the motor is also reversed. A flywheel is connected with the motor-generator set and arrangements are made to automatically vary the speed of the set so that during peak-load periods the speed of the set is decreased, and part of the energy in the flywheel is used to assist the motor in driving the generator.

When the load drops below a certain value, the speed of the set is gradually increased and energy is again stored in the flywheel. By properly proportioning the flywheel, assuming that the cycle of operation remains constant, it is possible to keep the input to the hoisting plant within a few per cent. of the average load.

An objection to the Ilgner system of hoisting is the expensive nature of the hoisting plant, the addition of a motor-generator and flywheel increasing the cost considerably. To overcome this feature and still provide for the equalization of the input, a system has been introduced by the British Westinghouse Company which may be used under certain circumstances. This scheme is shown diagrammatically in Fig. 2. The hoist motor in this system may be either direct current or alternating current, depending on the source of supply. The diagram of connections shows the arrangement of the plant with an alternating-current source of supply. In parallel with the generators an equalizing outfit is arranged, which consists of the direct-current machine coupled to the flywheel, which is connected to the alternating-current system through a rotary converter.

This equalizing equipment can be located anywhere that may be convenient, it not being necessary to have it near the hoist. The operation is as follows: When the hoist load exceeds the value for which the regulator is set, the field
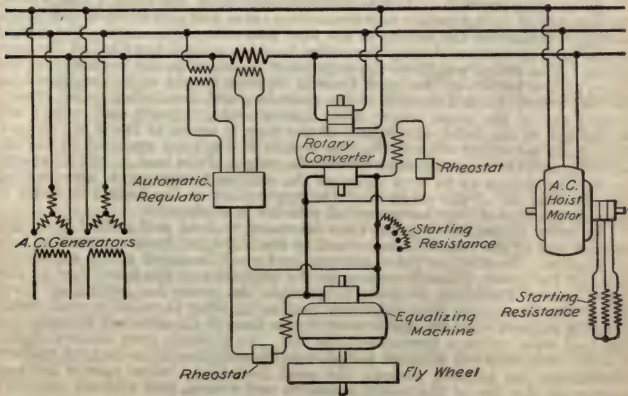


FIG. 2

of the equalizing machine is automatically strengthened, so that the speed tends to drop and the machine is driven as a generator by the flywheel and delivers energy through the rotary converters to the alternating-current system. The rate at which the energy is delivered is dependent on the operation of the regulator. When the demand drops below the value for which the regulator is set, the field of the equalizing machine is automatically weakened; this machine then runs as a motor and absorbs energy from the alternating-current system through the rotary converter and speeding up the flywheel. In this way the demand on the alternating-current system is kept practically constant. When this system is used with a direct-current source of supply, the rotary converter is omitted and the equalizing machine is connected directly to the line. This arrangement, however, does not provide for controlling the hoist motor, as does the Ilgner system, but it has the advantage that the equalizing machine has only to deal with the loads in excess of the mean value for which the regulator is set, and the cost is considerably reduced compared with the former system. It also has the advantage that a failure of any part of the equalizing equipment does not interfere with the operation of the hoist motor. The application of this system is practically confined to cases where the rheostatic control of the hoist motors offers no difficulties and where equalization of input is all that is required. In the case of very large plants, however, the

control question is of such importance that the Ilgner system is used almost exclusively.

One important feature in connection with electric hoisting is the ease with which safety devices can be arranged to prevent overwinding or overloading. In connection with systems using either a motor-generator flywheel or the Ilgner system of control, automatic devices have been arranged so that the rate of acceleration is limited and the hoist is automatically retarded independent of the operator, and as these devices are used every time the hoist is operated they are necessarily kept in order. With such arrangements, overwinding or starting up the hoist in the wrong direction is absolutely impossible, and in view of these features the German mining authorities have allowed the rate at which men may be hoisted to be increased from 1,200 ft. per min., which is the maximum with steam-operated hoists with the best safety gears, to 2,000 ft. per min. with electric hoists, and the question has been under consideration for some time of increasing this limit to 3,000 ft. per min.

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# BALANCED HOISTING

In hoisting through a double-compartment shaft and from a single level, the weights of the ascending and descending cages and cars balance each other, leaving unbalanced the weight of the load and that of the rope. The weight of the load is uniform during the entire hoisting period, but that of the rope is not. The entire weight of the rope must be raised at the beginning of the hoist; at the middle of the hoist there is no rope weight to be considered, as the two ropes (those attached to the ascending and descending cages, respectively) are of the same length and, consequently, balance each other; and at the end of the hoist, the weight of the rope is acting in favor of the load. In fact, in extremely deep shafts, the weight of the rope attached to the descending empty cage, may even exceed the weight of the load on the other cage to such an extent as to tend to cause the engines to run away, necessitating the shutting off of the steam and the application of the brakes in order to prevent the loaded cage being carried into the head-sheaves.

In order to provide for the time necessary to load and unload the cages at each end of the run, it is important to reach the full hoisting speed as soon as possible. Further, as the maximum load must be raised at the outset, the demand for power is much greater at the beginning of the hoist than at any other time. Therefore, in order to provide the power for getting up speed under the maximum load (the two producing what is called a peak load), hoisting engines must be made much larger than if the speed of hoisting and the load were uniform throughout the run. In hoisting from shallow shafts, it is not customary to attempt to balance the load (that is, to make it uniform throughout the hoisting period), the peak load being taken care of by proportioning the size of the steam cylinders and drums, and without any very serious increase in first cost or in operating expense.

Where the shaft is deep, the loads large, and the hoisting speed is necessarily great in order to produce a large tonnage, some system of counterbalancing is employed to render more uniform the demand for power upon the hoisting engine.

As an illustration of unbalanced hoisting, assume that a load of 4,000 lb. of coal is to be hoisted in a double-compartment shaft 1,000 ft. deep by means of a 1⅜-in. crucible-steel rope weighing 3 lb. per ft. (3,000 lb. in all) which winds upon a drum 7 ft. in diameter. It may be assumed, further, that the car and empty cage weigh 3,000 lb. each, and that friction is equal to 10% of the load exclusive of that of the rope. The friction adds to the load to be hoisted and decreases that to be lowered.

When the loaded cage is at the bottom of the shaft, the total weight to be hoisted is that of the coal, the empty car and the cage, a total of 10,000 lb. To this must be added 10% for friction, or 1,000 lb., and 3,000 lb. for the weight of the rope, or a grand total of 14,000 lb. As the drum has a radius of 3.5 ft., the total turning moment to be overcome by the engine is 14,000 × 3.5 = 49,000 ft.-lb. But the engine is assisted by the weight of the empty car and cage, or 6,000 lb. From this must be deducted 10%, or 600 lb. for friction, leaving a net load of 5,400 lb. assisting the engine, which is equivalent to a counterbalancing moment of 5,400 × 3.5 = 18,900 ft.-lb. Hence, at the beginning of the hoist the net load moment to be overcome by the engine is 49,000 − 18,900 = 30,100 ft.-lb.

At the end of the hoist the weight on the loaded-cage side is lessened by the weight of the rope, and is 11,000 lb. This is equal to a total load moment of $11,000 \times 3.5 = 38,500$ ft.-lb. On the empty side, the weight assisting the engine is increased by the weight of the rope, 3,000 lb., and is 8,400 lb. This is equal to a counterbalancing load moment of $8,400 \times 3.5 = 29,400$ ft.-lb. Hence the net load moment to be overcome by the engine is $38,500 - 29,400 = 9,100$ ft.-lb.

Since the load moment upon the engine varies from 30,100 ft.-lb. at the beginning of the run to 9,100 ft.-lb. at the end, in the assumed case the engines must exert more than three times as much power at one time as at another. Further, as the average load moment upon the engines is but 19,600 ft.-lb., and the maximum load moment is 30,100 ft.-lb., but about 65% of their average power is utilized.

**Tail-Rope Balancing.**—Counterbalancing the weight of the hoisting ropes may be accomplished by attaching to the bottom of one cage a rope of the same size and weight as the hoisting rope, passing it under a pulley or sheave wheel at the bottom of the shaft, and attaching the second end to the bottom of the other cage. As the length of the balance rope is twice that of either hoisting rope, the latter are exactly balanced. When this appliance is used, the weight to be hoisted is uniform throughout the run and is equal to that of the load and friction, because the cars and cages balance each other. Using the foregoing illustration, the net load moment at starting is 30,100 ft.-lb. From this must be deducted the load moment of the rope, which is $3,000 \times 3.5 = 10,500$ ft.-lb. Hence the net load moment, which is constant throughout the run, is $30,100 - 10,500 = 19,600$ ft.-lb. Hence, when the weight of the hoisting ropes is counterbalanced, but about 65% of the power is required to hoist as when the ropes are not balanced.

The use of the tail-rope gives its best results where hoisting is done from one level only, and in deep hoisting it is impracticable because of the extra weight which must be carried by the head-sheave axle and because of possible excessive swaying of the rope.

**Conical Drums.**—Conical drums are designed to make the work of the engine throughout the hoist as nearly uniform as possible. To accomplish this, when the cage is at the bottom of the shaft and the weight of the rope is added to that of the cage and its load, the rope winds on that part of the drum having the smallest diameter. As hoisting continues, the rope winds on a gradually increasing diameter of drum; and when the cage is at the top of the hoist and the load is only that due to the cage and the loaded car, the rope is winding on that part of the drum having the greatest diameter; in this way, the moment of the load at every point of the hoist remains approximately the same.

The ratio of the larger radius $R$ of a conical drum to the smaller radius $r$ is found as follows:

Let
$w_m$ = weight of material hoisted;
$w_c$ = weight of cage and car;
$w_r$ = weight of rope;
$R$ = large radius of drum;
$r$ = small radius of drum.

The moment of cage, car, load, and rope at the bottom of the shaft is $(w_c + w_m + w_r)r$. The moment of cage and car at the top of the shaft is $w_c R$. The net moment at beginning of hoist is $(w_c + w_m + w_r)r - w_c R$.

When the loaded cage has arrived at the top and the other cage has reached the bottom, the moment of cage, car, and load at the top is $(w_c + w_m)R$ and the moment of cage, car, and rope at the bottom is $(w_c + w_r)r$. The net moment at end of hoist is $(w_c + w_m)R - (w_c + w_r)r$.

Finally, placing the moment at beginning of hoist equal to that at end of hoist, and finding the value of the ratio of the two diameters which is equal to that of the two radii,

$$D = d \times \frac{2(w_c + w_r) + w_m}{2w_c + w_m}$$

**EXAMPLE.**—Using the foregoing illustration, $d = 7$ ft.; $w_m = 4,000$ lb.; $w_c = 3,000 + 3,000 = 6,000$ lb.; $w_r = 3,000$ lb.; it is required to find the larger diameter of a conical drum for balanced hoisting.

**SOLUTION.**—Substituting in the foregoing formula,

$$D = 7 \times \frac{2 \times (6,000 + 3,000) + 4,000}{2 \times 6,000 + 4,000} = 9.625 \text{ ft.} = 9 \text{ ft. } 7\tfrac{1}{2} \text{ in.}$$

From the equations representing the net load moment at the beginning of hoisting, the net load moment upon the engines is $(6{,}000+4{,}000+3{,}000)\times3.5$ $-6{,}000\times4.8125=16{,}625$ ft.-lb., which load moment is constant throughout the hoist as when counterbalancing with the tail-rope is used, but with no extra strain upon the head-sheave axle.

The use of the conical drum permits of the practical equalization of the load on the engine during the entire hoisting period, and requires less power in starting under load.

The disadvantages of the conical drum are as follows: To maintain a certain average speed of hoisting, the speed toward the end of the hoist is of necessity higher than the average and comes at a time when a slowing up should be taking place, so that more care must be exercised when making the landing. To prevent the rope from being drawn out of the grooves, the latter must be made deep and with a large pitch, thereby increasing the width of the face or length of the drum. In making a landing, when the rope is on the conical face, the rope must be kept taut, as any slackness will permit the rope to leave the groove, with the result that all the rope will pile up in the bottom grooves of the drum allowing the cage to drop into the mine, unless it is resting on the chairs. If there are several levels to be hoisted from, the equalizing of the load on the engines can only be realized for one level; for all other levels this advantage will be lost. For large depths, conical drums become very long and require correspondingly long leads from head-frame to drum. To hold the same amount of rope, conical drums are heavier than cylindrical ones, and as a result, the power required in starting the load is somewhat increased owing to the greater inertia of the rotating parts.
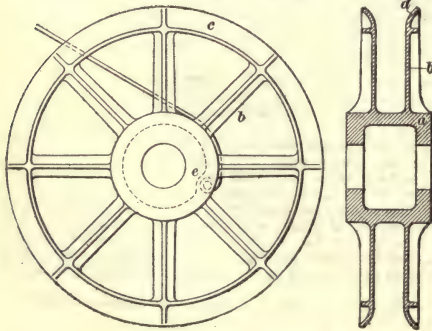


FIG. 1

Some of these disadvantages have been overcome by making a combination of cone and cylindrical drums. The drums are so designated that the landing takes place only when the rope is on the cylindrical portion of the drum. For deep hoisting, the greater diameter of the drum and its length must be inconveniently large if the load is equalized. The length and diameter can be reduced by making one-half of the drum cylindrical and by having the rope from each end wind on the same cylindrical portion of the drum. In all cases however, these modifications are made at the expense of the equalization of the load on the engines, and it is not possible to obtain the latter without including some serious disadvantage.

There are certain objections to both cylindrical and conical drums: their great size and weight, for large hoists, make them very expensive; their width necessitates placing the engines far apart, which adds to the cost of the engines, foundations, and buildings; the great weight of the drums is also objectionable, because it forms a large part of the mass to be put in motion and brought to rest at each hoist.

**Flat Ropes and Reels.**—To overcome the objections to conical and cylindrical drums, flat ropes wound on reels, Fig. 1, may be used. In this case the hub $a$ is increased in diameter, above what is necessary for strength, to such a size as is suitable to wind the rope on. It is then cored out from the inside, so as not to contain too great a mass of metal.

The arms $b$ of the reel extend radially from the hub to confine the rope laterally when it is all wound on the drum. These arms are connected at their outer ends by a continuous flange $c$, which is flared out, as shown at $d$, so as to take in the rope easily, if it is deflected at all sidewise.

In the larger-sized reels, the arms are bolted to the hub, and often the outer rim connecting the arms is omitted. Hardwood lining was formerly used on

the arms under the impression that the wear on the rope would be less than with bare iron arms, but sand and grit become embedded in the wood and grind the rope. Polished iron arms with rounded corners and lubricated with oil or tar are best. The end of the rope is fastened in a pocket *e* provided for it in the hub.

The rope winds on itself so that the diameter of the reel increases as the hoist is made and as the load due to the weight of the rope decreases. This serves to equalize the load due to the rope in the same manner as the conical drum. Two reels are generally put on the same shaft, and while one is hoisting from one compartment of the shaft the other is lowering into another compartment. The periphery of the hub where the rope winds should not be round but of gradually increasing radius, for if a flat rope is wrapped about a round hub the rope will have to abruptly mount itself at the end of the first revolution and so on for every revolution. The radius of the hub should increase at such a rate as to raise the rope an amount equal to its thickness in the first wrap, so that it will wind on itself without jar at the point of attachment, as well as on succeeding wraps.

In America, it is customary to wind on reels of small diameter, that is, starting at 3 or 5 ft. and increasing to 8 or 12 ft.; but several large plants have been built with reels starting at 8 ft. and increasing to 19 ft. In England, reels have been made starting at 16 ft. and increasing to 20 or 22 ft. Such large reels are easier on the rope but require large engines, as hoisting in balance is used to only a slight extent. The large reel is easy on the rope, both from the fact that it bends the rope but little and also gives less pressure on the bottom wraps, as each wrap adds to the pressure. These reels are driven by means of plain jaw or friction clutches.

The wear of a flat rope is excessive and the rope itself costs more than a round rope of the same strength, does not last as long, and requires more care and attention.

When calculating the dimensions of a reel for flat rope, the determination of the size of the rope and of the large and small diameters of the reel must proceed together. The smaller diameter *d* is that of the hub of the reel, and the larger diameter *D* is that of the last coil of rope after the reel is full. The large diameter *D* determines the length of the arms, *b*, Fig. 1.

EXAMPLE.—Using a factor of safety of 9, what size of flat rope will be required to hoist 5,000 lb. of material in a skip weighing 3,000 lb. through a two-compartment shaft 2,000 ft. deep; and what will be the dimensions of the reel?

SOLUTION.—The load to be hoisted, allowing 10% for friction, is 8,800 lb. Assuming a 6″ × ½″ rope with a breaking strength of 150,000 lb., this will weigh 5.1 lb. per ft., or 10,200 lb. for 2,000 ft. The total load will be 8,800 + 10,200 = 19,000 lb. The factor of safety will be 150,000 ÷ 19,000 = 7.8, which is too low under the assumed conditions.

The breaking strength of an 8″ × ½″ rope is 200,000 lb. This weighs 6.9 lb. per ft., or 13,800 lb. for the entire rope. With this larger rope, the total load will be 8,800 + 13,800 = 22,600 lb., and the factor of safety will be 200,000 ÷ 22,600 = 8.8, which is close enough for all practical purposes. Using the formula given under the heading Conical Drums,

$$D = d \times \frac{2 \times (3,000 + 13,800) + 5,000}{2 \times 3,000 + 5,000} = 3.5d$$

that is, the diameter when the last coil is wound on the reel must be 3.5 times that of the hub.

The area of the hub about which the rope is to coil is $\frac{1}{4}\pi d^2$, and the area included in the outer coil of rope is $\frac{1}{4}\pi D^2$; hence, the area of the annular space occupied by the rope is

$$\frac{1}{4}\pi D^2 - \frac{1}{4}\pi d^2 = \frac{1}{4}\pi (D^2 - d^2)$$

Such values for *D* and *d* must be chosen that the equation of moments, $D = 3.5d$, is satisfied, while the area of the annular space, $\frac{1}{4}\pi(D^2 - d^2)$, must correspond to the space occupied by the given rope when coiled. In the assumed case 2,000 ft. of rope ½ in. thick requires $\frac{2,000 \times 12}{2} = 12,000$ sq. in. in which to be coiled. To satisfy the equation of moments, *D* must equal 3.5d; hence, to satisfy both these conditions $\frac{1}{4}\pi[(3.5d)^2 - d^2] = 12,000$; whence $d = 37$ in., or 3 ft. 1 in.; and $D = 37 \times 3.5 = 129.5$ in. or 10 ft. 9½ in.

The dimensions of the reel will then be: diameter of hub 3 ft. 1 in.; width between flanges, 8½ in., allowing ¼ in. on each side of the rope for clearance; diameter of the flanges where they flare, 10 ft. 9½ in.

**Koepe System.**—In its lightest form, a drum requires a large amount of power to set it in motion, which power is absorbed by the brake and lost when it is brought to rest again. Furthermore, with deep shafts requiring long drums, the fleet, or angle that the rope makes with the head-sheave due to its traveling from one end of the drum to the other, is not only a disadvantage and possible cause of accident, but it is a source of wear. To overcome these objections and also the great cost of large cylindrical or conical drums, the Koepe system of hoisting, shown in Fig. 2, was devised by Mr. Frederick Koepe. A single-grooved driving sheave $a$ is used in place of a drum. The winding rope $b$ passes from one cage $A$ up over a head-sheave, thence around the sheave $a$ and back over another head-sheave, and down to a second cage $B$; it encircles a little over half the periphery of the driving sheave and is driven by the friction between the sheave and rope. A balance rope $c$ beneath the cages and passing around the sheave $d$ gives an endless-rope arrangement with the cages fixed at the proper points. The driving sheave is stronger than an ordinary carrying sheave, as it has to do the driving, and is usually lined
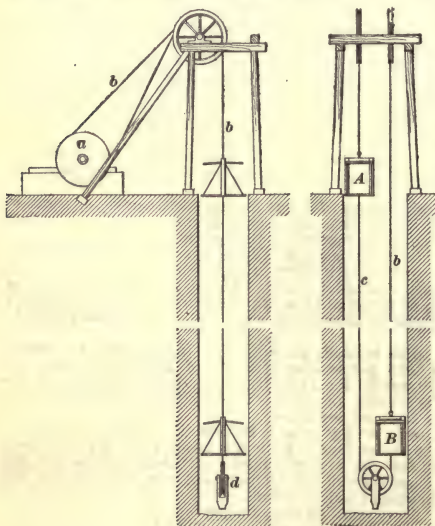


FIG. 2

with hardwood, which is grooved to receive the winding rope, the depth of the groove being generally equal to twice the diameter of the rope. Instead of being placed parallel, the head-sheaves are placed at an angle with each other, each pointing to the groove in the driving sheave, thus reducing the side friction of the rope on the sheaves.

The system has been in successful operation since 1877, and experiments made on it have determined that, with a rope passing only one-half turn around the drum sheave, the coefficient of adhesion with clean ropes is about .3. If the ropes are oiled, the adhesion becomes less, and sometimes slippage occurs, producing not only wear of the driving-sheave lining but giving an incorrect reading of the hoist indicator and thus possibly producing over winding, unless the

position of the cage is indicated by marks on the rope, or unless the engineer can see the cage.

At the end of the hoist, if the upper cage is allowed to rest on the keep, its weight and the weight of the tail-rope are taken from the hoisting rope, and there is then not enough pull on the hoisting rope to produce sufficient friction with the drum sheave to start the next hoist. To prevent this trouble, the keeps are dispensed with, or the rope is made continuous and independent of the cage. To do this, crossheads are placed above and below each cage and connected by ropes or chains outside of the cages. The bridle chains are then hung from the top crosshead, and when the cage rests on the keeps, the weight of the winding and tail-ropes remains on the driving sheaves.

With this system, only one driving sheave is necessary for the operation of two compartments, and it is light, inexpensive to build, and very narrow, admitting of a short sheave shaft and small foundations. This system permits a perfect balance of rope and cage, so that the work to be done by the engine is uniform, except for the acceleration, and consists only in lifting the material

and overcoming the friction. There is no fleeting of the rope between the driving sheaves and the head-sheaves.

The system has the following disadvantages, which prevent its being used to any considerable extent: Liability to slippage of the rope on the drum; if the rope breaks, both cages may fall to the bottom; hoisting from different levels cannot be well done, for, since the cages are at fixed distances from each other, the length of the rope is such that when one cage $A$ is at the top, the other cage $B$ is at the bottom. If hoisting is to be done from the bottom, this is satisfactory, but if hoisting is to be done from some upper level, cage $B$, which is at the bottom, must be hoisted to that level to be loaded before it can go to the top. Then, when cage $B$ goes to the top with its load, cage $A$ must go to the bottom, wait there while cage $B$ is being unloaded, and then be hoisted to the upper level to receive its load. For each trip, therefore, the time required for a cage to go from the bottom to the upper level and be loaded is lost; and two movements of the engines are necessary for a hoist instead of one.

**Whiting System.**—In the Whiting system, two rope wheels placed tandem are used in place of cylindrical or conical drums. As shown in Fig. 3, for a two-compartment shaft the rope passes from one cage $a$ up over a head-sheave $c$,
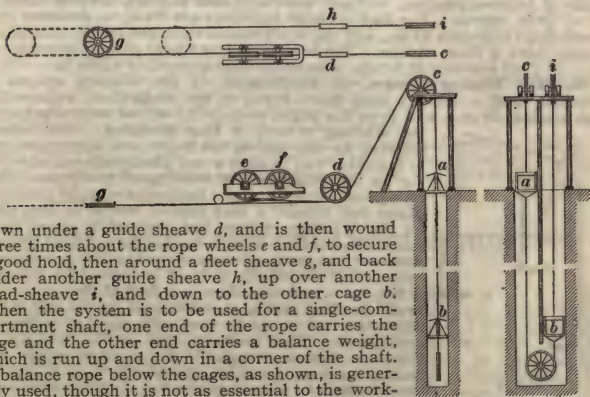


down under a guide sheave $d$, and is then wound three times about the rope wheels $e$ and $f$, to secure a good hold, then around a fleet sheave $g$, and back under another guide sheave $h$, up over another head-sheave $i$, and down to the other cage $b$. When the system is to be used for a single-compartment shaft, one end of the rope carries the cage and the other end carries a balance weight, which is run up and down in a corner of the shaft. A balance rope below the cages, as shown, is generally used, though it is not as essential to the working of the system, as it is in the Koepe system. When sinking a shaft, a balance rope cannot be

Fig. 3

used as it interferes with the work at the bottom of the shaft.

The drums or wheels $e$ and $f$ are light, inexpensive, and narrow, thus permitting short sheave shafts and small foundations. They are lined with hardwood blocks, each lining having three rope grooves turned in it. The main wheel $e$ is driven by a hoisting engine, which may be either first- or second-motion. The following wheel $f$ is coupled to the main wheel by a pair of parallel rods, one on each side, like the drivers of a locomotive. As the rope wraps about the wheels $e$ and $f$ three times, there are six semi-circumferences of driving contact with the rope, as compared with the one semi-circumference in the Koepe system, and there is no slipping of the rope on the wheels. The following wheel $f$ is best tilted or inclined from the vertical an amount equal, in the diameter of the wheels, to the pitch of the rope on the wheel, so that the rope may not run out of its groove and may run straight from one wheel to the other without any chafing between the ropes and the sides of the grooves.

The capacity of the wheels $e$ and $f$ is unlimited, while grooved cylindrical drums, conical drums, and reels will hold only the fixed length of rope for which they are designed.

As shown by the dotted lines, the fleet sheave $g$ is arranged to travel backwards and forwards, in order to change the working length of the rope from time to time to provide for an increased depth of shaft, and for changes in the length of rope due to stretching and when the ends are cut off to resocket the

rope.  The fleet sheave *g* is moved a distance equal to one-half the change in the length of rope.

Hoisting from intermediate levels can be readily done with the Whiting system; for instance, if the cage *a* is at the top and cage *b* at the bottom, and hoisting is to be done from some upper level, it is only necessary to run the fleet sheave *g* out, and thus shorten the working length of the rope until cage *b* comes up to the upper level.  It can then be loaded and go to the top.  While cage *b* goes to the top, cage *a* descends to the same level, where it can be loaded while cage *b* is being unloaded, and can then go directly to the top without any lost time, as is the case in the Koepe system.

The system permits a perfect balance of rope and cage, so that the work to be done by the engines is uniform, except for the acceleration, and consists only in lifting the material and overcoming the friction.

There is no fleeting of the rope, so the rope wheels can be placed as close to the shaft as may be desired.

This system was tried as early as 1862 in Eastern Pennsylvania, but it was not used extensively because hoisting from great depths was not necessary, since, for depths of less than 1,000 ft., cylindrical and conical drums are quite satisfactory.  Two of the Whiting hoists in the Lake Superior copper region are among the largest hoisting plants in the world.  Each of these consists of a pair of triple-expansion, vertical, inverted-beam engines, driving direct a pair of 19-ft. drums.  The high-pressure cylinders are 20 in. in diameter, the intermediate cylinders 32 in., and the low-pressure cylinders 50 in., and all six of them have a 72-in. stroke.  The rope used is a $2\frac{1}{4}$-in. plow-steel rope and hoists 10 T. of material at a trip, in one case from a depth of 4,980 ft.

**Modified Whiting System.**—A modification of the Whiting system is sometimes used in which a large drum keyed to the crank-shaft replaces the small tandem drums, and even the slight probability of the rope slipping in



FIG. 4

the Whiting system is thus obviated.  One rope is fastened to one end of the drum, and the other rope to the other end in such a way that while one is winding on the other will be winding off the drum.  One rope passes directly to the head-sheave while the other passes first around a fleet sheave, similar to that used for the Whiting system, but preferably placed horizontal, and thence to the head-sheave.  This system possesses the same advantages as the Whiting system except that the depth of hoist is limited by the size of the drum, and that there is a fleet of the rope.  Up to the limiting depth, as determined by the size of the drum, this system can be used with equal economy for any depth.  This hoist, as well as the Whiting, is therefore especially suitable for a place where one mining company operates several mines, for it enables the company to select one size for all their permanent work, with all the advantages that come from duplicate machinery.

**Despritz System.**—The general arrangement of the Despritz system is shown in Fig. 4.  A drum with a radius *T* is keyed to the same shaft as the rope drum with a radius *R*, and carries a small rope to which is attached a chain whose length *N* is one-half the distance between landings *H*.

The small rope is so wound on to its drum that at the commencement of the hoist the chain is suspended at full length in a small compartment specially provided.  Immediately the hoisting commences, the small rope starts to unreel, piling up the chain at the bottom of its compartment until the cages reach their point of passing midway in the shaft, at which instant the hoisting-rope loads balance, the piling up of the entire chain length is complete, and its load moment on the main shaft is zero.  At this instant, also, the small rope, carrying the chain, has all unreeled from its drum and its point of fastening is about to pass around underneath and take the rope into the position shown by the dotted line.

Hence, as soon as the cages have passed each other the chain rope begins to reel up again, extending the chain upwards until, at the termination of the hoist, it again hangs at full length, giving a load moment of opposite sign to that which it had at starting of the hoist.  That is, during the first half of the hoisting period the load moment of the chain on the drum shaft is plus, while during the second half of the period it is minus.

If $\qquad$ $W$ = weight of hoisting rope per foot;
$\qquad\qquad\qquad$ $w$ = weight of chain per foot;

$R$ = radius of rope drum, in feet;
$T$ = radius of chain drum, in feet;

$$w = \frac{2R^2W}{T^2}$$

**Monopol System.**—The system outlined in Fig. 5 is known as the Monopol. An auxiliary drum of diameter equal to the winding drums, is keyed to the main shaft and is of sufficient width to carry two relatively small ropes, one of which is underwound and the other overwound.

These ropes support a length of heavier balancing rope in the position shown; this balancing rope is usually a length of old hoisting rope of the size used in the hoisting operations and which has been discarded on account of wear. A glance at the diagram makes it evident that if this balancing rope is adjusted at the outset, so that each of its ends is in position opposite one of the cages, as shown, they will so remain whatever the position the cages take during a hoisting period, and the load moment on the drum shaft, so far as the hoisting ropes are concerned, will be equalized throughout. As in the former case, the weight of the small rope can be neglected.



FIG. 5

In practice either of the foregoing systems requires that a small compartment be provided for the accommodation of the balancing device. This is usually partitioned off from one end of the pump compartment of the main shaft at a nominal expense. As for the rest of the arrangements, almost any mechanic at the mines will find little trouble in providing whatever may be required for the installation.
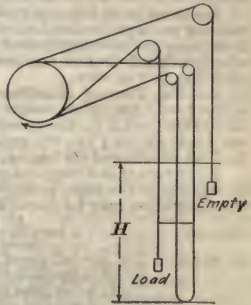
# CALCULATIONS FOR FIRST-MOTION HOISTING ENGINES

**General Considerations.**—Owing to the fact that many of the resistances that have to be overcome can only be estimated approximately, the determination of the horsepower required to hoist, as well as the dimensions of the engines required for that purpose, cannot be made exactly. The usual procedure is to calculate an average or minimum horsepower by means of some simple formula and then to add to this an amount that experience has indicated to be necessary to provide for uncertainties both in resistances to be overcome and in the future demands for power. The horsepower having been obtained, the actual design of the engines should be left to a skilled mechanical engineer; in fact, the entire matter, even including the calculation of the horsepower, is more properly the work of the engine builder than the mine superintendent.

The methods involved in the solution of hoisting-engine problems are best explained by the working out of the following example.

EXAMPLE.—What should be the horsepower and dimensions of a first-motion engine to hoist 1,500 T. of coal per da. of 8 hr. from a shaft 1,000 ft. between landings under the following conditions: One-half hour is allowed for delays of various kinds, 7 sec. for caging each trip, and 5 sec. each for acceleration and retardation; the car holds 5,000 lb. of coal and weighs 2,500 lb.; the cages weigh 4,000 lb. each; the drum is cylindrical, 8 ft. in diameter and 8 ft. wide; the head-sheaves are 8 ft. in diameter; the mean effective pressure of the steam in the cylinder is 100 lb. per sq. in., the plant has an efficiency of .85%; and the resistance of friction is taken to be the same as that required to move a weight equal to 5% of the total load?

SOLUTION.—1. *Hoisting Period.*—The operation of hoisting through a shaft may be divided into three periods. First, a period of acceleration, during which the load, rope drum, sheaves, moving parts of the engine, etc., are brought from rest to full speed. Second, a full- or constant-speed period during which the load is hoisted at the uniform velocity attained at the end of the period of acceleration. Third, a period of retardation, during which the load and moving parts are brought from full speed to rest.

In practice, the time required for acceleration is variously estimated as being equal to one-seventh of the net time of hoisting, as from 3 to 7 sec. depending on the depth of the shaft and speed of hoisting, as the time required for the drum to make, say, three revolutions, as the time required to hoist the cage 50 or 150 ft. starting from rest, etc. The period of retardation is usually taken to be equal in time to that of acceleration, although it may be a little shorter. The full-speed period is the longest of the three, and consumes about three-fourths of the net time of hoisting.

Because, in addition to raising the load, it as well as the drum, sheaves, and moving parts of the machinery must be accelerated or brought up to full speed from rest, the power required to hoist is very much greater during the period of acceleration than at any later time; and hoisting engines should be designed with this fact in view.

2. *Net Time of Hoisting.*—The gross time actually devoted to hoisting is $8 - \frac{1}{2} = 7.5$ hr. The weight of coal hoisted per hour $= 1,500 \div 7.5 = 200$ T. As 5,000 lb. $= 2.5$ T., the number of hoists per hour is $200 \div 2.5 = 80$. As there are 3,600 sec. in 1 hr., the gross time of hoisting per trip is $3,600 \div 80 = 45$ sec. As 7 sec. is allowed for caging, the net time of hoisting is $45 - 7 = 38$ sec.

3. *Speed of Hoisting.*—Assuming that the acceleration, that is, the increase in velocity of the cage, is uniform, the speed attained at the end of the period of acceleration may be found from the formula,

$$v = \frac{H}{t_n - t} = \frac{1,000}{38 - 5} = 30.3 \text{ ft. per sec}$$

in which

$v =$ full speed of hoisting, in feet per second;
$H =$ distance between landings, in feet $= 1,000$;
$t_n =$ net time of hoisting, in seconds $= 38$;
$t =$ time of acceleration, in seconds $= 5$.

As the cage starts from rest and attains a velocity of 30.3 ft. per sec. at the end of 5 sec., the space traversed during acceleration is $\frac{v}{2} \times t = 75.75$ ft., because the average velocity is one-half the final.

The acceleration, $a$, is found from the formula $a = v \div t = 30.3 \div 5 = 6.06$ ft. per sec. per sec. The space passed over during acceleration is not the same for each of the 5 sec. In the first second, the cage is raised a distance of but $\frac{1}{2}a$, or 3.03 ft.; during the second second, it is raised $\frac{1}{2}a + a = 9.09$ ft.; and during each succeeding second it is raised a distance $a = 6.06$ ft. more. The distances passed over in the single seconds making up the period of acceleration, are, respectively, 3.03, 9.09, 15.15, 21.21, and 27.27 ft., the sum of which is 75.75 ft. as determined before.

The velocity attained at the end of any particular second is found from the formula $v = at$. Thus, at the end of the fourth second, the velocity of the cage is $6.06 \times 4 = 24.24$ ft. per sec.; that is, if the accelerating force ceased to act at the end of the fourth second, the cage would enter the fifth second with sufficient velocity to carry it 24.24 ft. But during the fifth second, the accelerative force still acts, and adds $\frac{1}{2}a$ or 3.03 ft. to the distance traveled. Similarly, at the end of the fifth second, the cage, after traveling 27.27 ft. begins the sixth second with a velocity sufficient to carry it $27.27 + \frac{1}{2}a = 30.3$ ft. during that second without further acceleration.

4. *Revolutions of Drum per Minute.*—If $D$ is the diameter of the rope drum, in feet, the number of revolutions per minute made by it during the full-speed period may be found from the formula,

$$\text{rev. per min.} = \frac{v \times 60}{\pi \times D} = \frac{30.3 \times 60}{3.1416 \times 8} = \frac{1,818}{25.13} = 72.33, \text{ say, } 72.5$$

The total number of revolutions made by the drum during acceleration or retardation is $75.75 \div 25.13 = 3$, very nearly. The number of revolutions, or fractions of a revolution, made by the drum during the individual seconds of the acceleration period is found by dividing the distance passed over in any second by the circumference of the drum. In the problem, the approximate number of revolutions in each of the 5 sec. will be, respectively, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, and $1\frac{1}{10}$.

5. *Friction in Hoisting.*—In well-designed and well-built first-motion hoisting engines, the resistance due to friction should not exceed that due to raising a weight equal to 5% of the total load including the weight of the ropes, but in second-motion or geared hoists this resistance may amount to 7.5 to 10%.

The method of allowing for the effects of friction varies. By some, the friction is added to the load and treated as part of it; by others, the area of the steam cylinders is calculated on the basis of there being no friction and from

10 to 15% added to this area as an allowance for friction. Probably the first method is the more generally used.

In the problem, the total load to be moved is equal to the weight of the (coal+2 cages+2 cars+2 ropes) = (5,000+8,000+5,000+4,000) = 22,000 lb. At 5%, the resistance of friction is equivalent to raising a weight of 1,100 lb.

6. *Size of Hoisting Rope.*—The load on the rope is a maximum during the first period of hoisting because, in addition to raising the load and overcoming friction, there is a further resistance due to acceleration.

Assuming that the rope weighs 2 lb. per ft., when the cage is at rest just clear of the bottom, the load on the rope is equal to the weight of the (coal +cage+motion+rope) = (5,000+4,000+2,500+2,000 = 13,500 lb.) As soon as motion begins, the frictional resistance must be added, and the total load upon the rope is 13,500×.05+13,500 = 14,175 lb.

If the acceleration of gravity *g* is taken as 32.2 ft. per sec. per sec., the stress *P* on the hoisting rope during acceleration due to raising the weight *W* (load and friction) is

$$P = W + \frac{W}{g} \times a = 14,175 + \frac{14,175}{32.2} \times 6.06 = 14,175 + 2,667 = 16,842 \text{ lb.}$$

The factor of safety commonly used in American practice in calculating the strength of hoisting ropes is 5; hence, to sustain a load of 16,842 lb., the ultimate strength of the rope must be 84,210 lb. The manufacturers' tables show that the ultimate strength (breaking strength) of a 1¼-in. extra cast-steel rope weighing 2 lb. per ft. is 86,000 lb.; hence, it would be selected.

In choosing a rope of this size, no allowance is made for the strain due to bending around the drum and sheaves, which, if the same as the load on the rope, is 13,500 lb. The total strain on the rope during acceleration is, then, 16,842+13,500 = 30,342 lb., and the true factor of safety is 86,000÷30,342 = 2.83. Although this seems a small margin of strength for live loads, American practice seems to warrant it.

7. *Size of Drum.*—The manufacturers' tables indicate a minimum diameter of 5 ft. for the drum and sheaves to be used with a 1¼-in. rope. While a drum of this size might be used on contractors' hoists, a mine hoisting engine would have drums 8 ft. or more in diameter. The greater the diameter of the drum, the less will be its width for the same depth of hoist, the shorter will be the drum shaft, and the less the strain upon it. Also, the greater the diameter of the drum, the fewer will be the revolutions necessary to move the cage a given distance and the less will be the piston speed of the engine. Further, large drums reduce the bending strain on the rope and thus conduce to a longer life. On the other hand, during acceleration, the power required to overcome the inertia of the drum increases very rapidly with the increase in its diameter.

8. *Ordinary Method of Calculating Power Required to Hoist.*—The usual method of calculating the horsepower required to hoist is to multiply the unbalanced load, in pounds, by the speed of hoisting, in feet per second, and divide the product by 550. The unbalanced load is, in any case, taken as the weight of coal and one rope, or, in the problem, is 7,000 lb. To the unbalanced load is added the friction, which is variously estimated as 5 or 10% of the total weight being moved. Assuming the former figure, the resistance due to friction is 22,000×.05 = 1,100 lb., making the total load 8,100 lb. The speed of hoisting should be taken at 30.3 ft. per sec. (1,818 ft. per min.), which allows for the time lost in acceleration and retardation. With these data, the horsepower is (8,100×30.3)÷550 = 446.2.

Sometimes in calculating the speed of hoisting, the depth of the shaft is divided by the total time of hoisting, which gives a speed of 1,000÷38 = 26.3 ft. per sec. With this incorrect speed, the horsepower obtained is too low and is (8,100×26.3)÷550 = 387.3.

The horsepower, as thus obtained, is often divided by a factor representing the assumed efficiency of the plant, and sometimes is further increased by an arbitrary amount as an allowance for so-called contingencies. As will be shown in paragraph 12, the horsepower determined by this method, no allowance having been made for acceleration, is about one-half that really required to hoist.

9. *Piston Speed and Length of Stroke.*—The piston speed of hoisting engines varies from 300 to 500 ft. per min., a commonly accepted value being 500 ft. Because the engine makes two strokes for each revolution of the drum, if the piston speed is called *S*, the length of stroke *l* may be found from

$$l = \frac{S}{2 \times \text{R. P. M.}} = \frac{500}{2 \times 72.33} = 3.456 \text{ ft., say } 3.5 \text{ ft.} = 42 \text{ in.}$$

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In the foregoing, $S$ is taken as 500 ft. per min., and the revolutions per minute as determined in paragraph 4.

**10.** *Dimensions of Engine to Produce a Given Horsepower.*—After the power necessary to hoist and the piston speed have been determined, the total area of cylinder necessary to yield the calculated horsepower at the mean effective pressure may be found from the formula

$$A = \frac{33,000 \times \text{H. P.}}{PS} = \frac{33,000 \times 446.2}{100 \times 500} = 294.492 \text{ sq. in.}$$

**Because** there are two cylinders, each should have an area of $294.492 \div 2$ $= 147.246$ sq. in. The diameter $d$ of each cylinder will be $d = \sqrt{\dfrac{147.246}{.7854}}$ $= 13.7$, say, 14 in. As the length of stroke is 42 in., the use of a $14'' \times 42''$ engine is indicated.

By this method of calculation, no allowance is made for the power necessary to accelerate the load and machinery, which frequently amounts to as much as that required to raise the load. It is usually assumed that if one cylinder will develop sufficient power to start the load, the power from two cylinders will be great enough to accelerate it. This will commonly prove to be the case, even if no allowance is made for the efficiency of the plant; although, to provide for contingencies, it is advisable to make such an allowance. Thus, if a single cylinder must be of a size sufficient to raise the entire load, it must have an area of 294.492 or of $294.492 \div .85 = 346.46$ sq. in., depending on whether the plant is assumed to have an efficiency of 100% or of 85%. In the first case, $d = 19.4$, say, 20 in., and in the second case $d = 21$ in. From this, the dimensions of the engine will be 20 in. $\times$ 42 in. or 21 in. $\times$ 42 in., depending on the efficiency assumed for the plant. These dimensions are very nearly those obtained by the use of more accurate methods.

**11.** *Resistance and Force in Hoisting.*—The resistance that must be overcome in hoisting arises in part from raising the load and overcoming friction, and in part from accelerating the load and machinery, particularly the drum and sheaves.

The friction is constant throughout the run and may be considered as a portion of the load, or it may be included in an allowance made to cover the various uncertainties entering into all calculations of this nature. If considered as a portion of the load, it may be estimated as $22,000 \times .05 = 1,100$ lb.

The unbalanced load is the weight of the coal and the hoisting rope. The weight of the coal is constant throughout the run, but that of the rope decreases as the loaded cage ascends. When the cages pass in the shaft, the ropes attached to them are of equal length and their weights balance. Beyond the passing point, the length and weight of the rope attached to the empty cage becomes greater than that attached to the loaded cage and acts in favor of the engine. Hence, because the weights of the rope wound off and wound on the drum are equal, the unbalanced load at any instant is less than the original load by twice the weight of the rope wound on the drum. For convenience in calculation, the friction may be considered as part of the unbalanced load. At the beginning of hoisting, the unbalanced load including friction, is $5,000 + 2,000 + 1,100 = 8,100$ lb. At the end of the acceleration period when 75.75 ft. of rope, weighing 2 lb. per ft., has been wound upon and unwound from the drum, the original load is reduced by $2 \times 2 \times 75.75 = 303$ lb., and is 7,797 lb. At the end of the full-speed period, 924.25 ft. of rope will have been wound on the drum, and the load will be reduced by $2 \times 2 \times 924.25 = 3,697$ lb., and will be 4,403 lb. At the end of the period of retardation when the cage comes to rest at the landing, all the rope will have been wound on the drum, and the original load will be reduced by $2 \times 2 \times 1,000 = 4,000$ lb., and will be 4,100 lb. These loads, all having a positive sign, are entered in the first column of the following table.

The force, in pounds, required to accelerate the load is $\dfrac{W}{g} \times a$, in which $W$ is the total weight, or 22,000 lb., placed in motion; $a$ is the linear acceleration or 6.06 ft. per sec. per sec.; and $g$ is 32.2. The acceleration is then $\dfrac{22,000}{32.2}$ $\times 6.06 = 4,140$ lb. The retardation is equal to the acceleration, but has a negative sign; this force is entered in the second column of the table with its proper sign. The acceleration ceases as soon as the cage is moving at full speed.

The drum may be taken to weigh 25,760 lb., and the force required to accelerate it is $\dfrac{25,760}{32.2} \times 6.06 = 4,848$ lb.

The sheaves may be taken to weigh 805 lb. each, and the force required to accelerate them is $2 \times \dfrac{805}{32.2} \times 6.06 = 303$ lb.

The total force required to accelerate the drum and sheaves is, thence, $4,848 + 303 = 5,151$ lb., and is the same in amount but negative in sign during retardation. The algebraic sum of all the forces necessary to hoist and accelerate are given in the fourth column of the table. A negative force indicates that the resistance opposing it is acting to turn the drum and raise the load; hence, the brakes must be applied to prevent the engine running away.

### FORCES AND MOMENTS IN HOISTING

| Period | Raising Load and Friction | Accelerating | | Total Force or Resistance | Total Moment |
| --- | --- | --- | --- | --- | --- |
| | | Load | Drum and Sheaves | | |
| Beginning acceleration.. | $+8,100$ | $+4,140$ | $+5,151$ | $+17,391$ | $+69,564$ |
| End acceleration......... | 7,797 | 4,140 | 5,151 | 17,088 | 68,352 |
| Beginning full speed..... | 7,797 | | | 7,797 | 31,188 |
| End full speed.......... | 4,403 | | | 4,403 | 17,612 |
| Beginning retardation... | 4,403 | $-4,140$ | $-5,151$ | $-4,888$ | $-19,552$ |
| End retardation......... | 4,100 | 4,140 | 5,151 | 5,191 | 20,764 |

12. *Load Moments in Hoisting.*—The calculation of the area of the steam cylinder of a hoisting engine is based on the principle that the force resisting motion (or the resistance) multiplied by its lever arm, or the distance through which it acts, must equal the force producing motion multiplied by its lever arm; the forces being expressed in pounds and the lever arms (or distances) in feet.

The forces resisting motion at any instant are entered in the fourth column of the table. Each has the same lever arm, which is the radius $R$ of the drum, or 4 ft. When the forces are multiplied by this lever arm, they give the total moments, or load moments, in foot-pounds, which are given in the fifth column.

If the weights of the drum and sheaves are not known, the moments of the force required to accelerate them may be calculated by formulas suggested by Mr. Wilfred Sykes in the Transactions of the American Institute of Electrical Engineers. The formula for the drum is $I_d = 100R^2 \times$ width, and for sheaves $I_s = 25R^2$. The inertia $I_d$ or $I_s$, when multiplied by the angular acceleration $A_a$ of the drum and sheaves, gives the turning moment, which is the same as the load moment. Thus, in the present problem, as $R = 4$ ft. and the width of the drum is 8 ft., the inertia $I_d = 100 \times 4^2 \times 8 = 12,800$ lb. As there are two sheaves, their inertia will be $I_s = 2 \times 25 \times 4^2 = 800$ lb. The inertia of the drum and sheaves taken together is, therefore, 13,600 lb. The angular acceleration is equal to the linear acceleration $a \div$ radius of drum $= 6.06 \div 4 = 1.515$ radii per sec. per sec. From this, the turning moment or load moment of the drum is $13,600 \times 1.515 = 20,604$ ft.-lb.

Suppose that it is desired to calculate the total moment at the end of acceleration. The moment of the load is $7.797 \times 4 = 31,188$ ft.-lb., and that of the force required to accelerate the load is $4,140 \times 4 = 16,560$. The sum of these three moments is $20,604 + 31,188 + 16,560 = 68,532$ ft.-lb., which corresponds to the value in the last column of the table and which was determined from the weights of the drum and sheaves.

If, as shown in the accompanying figure, a base line $O O$ is divided to correspond with the length, in seconds of time, of the various hoisting periods there may be laid off above it the positive moments and below it the negative ones. If the points so determined are joined by lines, there will result the curve $a\,b\,c\,d\,e\,f\,g\,h$, which may be called the *curve of moments*. This curve shows graphically the variation in the load upon and the duty demanded of the engine from second to second during the hoisting period. At the start, the moments are a maximum, showing that the greatest power is required during the first few seconds. At the end of acceleration, there is a very great drop

in the load; in the present example it amounts to 55%. During the full-speed period, the load drops regularly until at its end, and coincident with the beginning of retardation, there is another great falling off in the load, which, with its moment, becomes negative and tends to turn the engine. A large amount of brake power is required during retardation; this power is supplied by reversing the engine and turning steam gently into the cylinders against the load and by the use of the brakes. As a result, for quick and heavy hoisting, an engine must be run at much above its economical load. In fact, during acceleration, the engine may be called upon to deliver more than twice the power required during full speed. In the present example, at the end of acceleration when the cage is moving with a speed of 30.3 ft. per sec., the horsepower exerted by the engine is $(17,088 \times 30.3) \div 550 = 941.4$. At the beginning of full speed, the horsepower is $(7,797 \times 30.3) \div 550 = 429.5$, or about 46% of what it was during acceleration. At the end of full speed, the horsepower drops to $(4,403 \times 30.3) \div 550 = 242.5$, and at the beginning of retardation there is a further drop to $(-4,888 \times 30.3) \div 550 = -269.3$ H. P.

It may be urged that the weight to be accelerated is not that of the entire load, or 22,000 lb., but rather that attached to the loaded rope, or 13,500 lb., because gravity accelerates the weight of the empty cage, car, and rope. If this line of reasoning is carried farther, it follows that the weight to be accelerated is that of the unbalanced load, or 7,000 lb. On the other hand, no account has been taken in the calculation for the power necessary to overcome the inertia of the crank, piston rod, etc., which in the aggregate is considerable, so that it is undoubtedly true that the dimensions of an engine calculated on the basis of accelerating a load of 22,000 lb. will be none too great for the work demanded of it. In fact, it would seem advisable in any case to add something to the area of the cylinders to provide for uncertainties in the resistances to be overcome and for future demands for power.
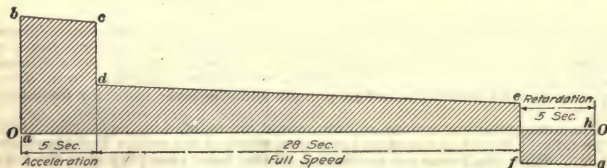


The variation in the load and moments is of the greatest importance in electric hoisting, because the motor is called on at the outset to furnish very much more power than at the end of the full-speed period. This reduces the load factor, which is the ratio between the total energy used in hoisting a trip under the given conditions and the energy required if the load and speed were uniform and the motor working at its full rated capacity; and a low load factor indicates low efficiency in the hoisting plant.

This great range in the demands for power, in the present example amounting to nearly 1,250 H. P. in 38 sec., also indicates the importance of counter-balancing the weight of the rope not only to reduce the unbalanced load to be raised, but also, and more particularly, to cut down the excessive power required to accelerate the load and machinery.

13. *Determination of Engine Dimensions From Load Moments.*—If the base line in the figure on this page is divided into revolutions instead of into seconds, and perpendiculars are drawn through the points of division, the average load moment for any revolution can be determined. This moment divided by the crank radius of the engine gives the average pressure on the crankpin for that revolution; from this, the required average piston pressure is obtained. If the initial steam pressure is known, the point at which the steam may be cut off at any revolution can be obtained, and the steam consumption per hoist can be accurately ascertained. Calculations of this nature, however, are properly the work of the mechanical engineer and not of the mine foreman or superintendent.

If the maximum load moments are known and the length of stroke has been determined by assuming a piston speed, as in paragraph 9, the proper area of cylinder may be found from the formula,

$$A = \frac{ML + MA}{PNCR_G}$$

in which   $A$ = area of a single cylinder, in square inches;

    $M_L$ = moment of unbalanced load, coal, one rope, friction;

    $M_A$ = moment of force required to accelerate total load, drum, two sheaves, machinery;

    $P$ = mean effective pressure of steam in cylinders, in pounds per square inch;

    $N$ = number of cylinders;

    $C$ = constant to reduce angular space passed through by crank to that passed through by piston in the same time = .64;

    $R_C$ = radius of crank circle, one-half length of stroke.

In the present problem, $M_L$ = 8,100 × 4 = 32,400 ft.-lb.; $M_A$ = (4,140 + 5,151) × 4 = 37,164; $P$ = 100 lb.; $N$ = 2; $C$ = .64; and $R_C$ = 42 in. ÷ 2 = 21 in. = 1.75 ft. Substituting in the formula,

$$A = \frac{32,400 + 37,164}{100 \times 2 \times .64 \times 1.75} = 310.55 \text{ sq. in.}$$

From this value of $A$, $d$ is found to be 19.99, say, 20 in. The length of stroke having been previously determined, the use of a 20″ × 42″ engine is indicated. In some cases an allowance has been made for the efficiency of the plant. In the present case, the efficiency has been assumed to be 85%; hence, the proper area for the cylinders will be 310.55 ÷ .85 = 365.35 sq. in., and $d$ = 21.6, say, 22 in. With this allowance, the dimensions of the engine will be 22 in. × 42 in.

It is not unusual in calculations of this nature to assume a ratio $r$ between the length of stroke (length of cylinder) $l$ and the diameter of cylinder $d$ such that $l = rd$. In this case, the diameter of cylinder may be found directly from a modification of the preceding formula. A common value for $r$ is 2, that is, the length of stroke is made twice the diameter of the cylinder. The formula for $d$ ($d^3$) follows, its application being shown by using the preceding data with a value of $r$ = 2:

$$d^3 = \frac{96(M_L + M_A)}{\pi r P N C} = \frac{96 \times (32,400 + 37,164)}{3.14 \times 2 \times 100 \times 2 \times .64} = 8,307.77 \text{ cu. in.}$$

From this, $d$ = 20.25 in., and $l$ = $rd$ = 2 × 20.25 = 40.5 in. A 21″ × 42″ engine would probably be selected, the extra power of the larger size being desirable.

# CALCULATIONS FOR SECOND-MOTION HOISTING ENGINES

When less than 200 to 250 H. P. is required to hoist, second-motion engines are commonly preferred to first-motion engines because of their smaller size, lower first cost, and generally greater ease in handling. Their small horsepower limits their use to shallow shafts, those, say, not over 250 to 350 ft. deep, where the net daily tonnage does not exceed 750 to 1,000, and where the speed of hoisting is not more than 500 to 750 ft. per min. The standard or stock sizes of second-motion engines are given in the following table, in which the hoisting speed is the rate of travel of the cage and the horsepower is based on a steam pressure of between 80 and 90 lb.

## STANDARD SIZES OF SECOND-MOTION HOISTING ENGINES

| Cylinders | | Horse-power | Hoisting Speed Feet per Minute | Cylinders | | Horse-power | Hoisting Speed Feet per Minute |
|---|---|---|---|---|---|---|---|
| Diameter Inches | Stroke Inches | | | Diameter Inches | Stroke Inches | | |
| 6¼ | 8 | 12 | 250 | 10 | 12 | 50 | 400 |
| 7 | 10 | 20 | 275 | 12¼ | 15 | 75 | 450 |
| 8¼ | 10 | 30 | 350 | 14 | 18 | 100 | 450 |
| 9 | 10 | 35 | 350 | 16 | 18 | 150 | 450 |
| 8½ | 12 | 40 | 375 | 18 | 24 | 200 | 550 |

**Dimensions of Second-Motion Engines.**—While the load moments may be determined in the same way as for first-motion hoisting engines, after which the size of the cylinders may be calculated, it is not customary to do so. The usual practice is to determine the horsepower by the method explained in paragraph 8, an allowance being made for the time spent in acceleration and retardation, and to select from the manufacturers' stock sizes an engine of slightly more than the calculated horsepower. Thus, if an engine of 185 H. P. is required, an engine with $18'' \times 24''$ cylinders and rated at 200 H. P. will be selected, the extra horsepower being desirable. As these engines cost from $40 to $60 per H. P. (list prices, subject to discount) depending on the accessories furnished, the cost of a few extra horsepower is not to be compared with the increased efficiency gained through the use of the larger size.

The necessary diameter of cylinder to yield a given horsepower may be calculated from the formula,

$$d^3 = \frac{792,000 \times \text{H. P.}}{\pi P r G \times \text{R. P. M.}}$$

in which $G$ = ratio of gearing, which is very commonly 3;
R. P. M. = revolutions of drum per minute;

and the other letters have the meanings previously given. The revolutions of the crank-arm per minute are equal to the revolutions of the drum multiplied by the gear ratio. That is, if the drum makes 50 rev. per min. and $G=3$, the revolutions per minute of the crank will be $G \times$ R. P. M. $= 3 \times 50 = 150$.

In the ratio $l = rd$, $r$ varies between 1.11 and 1.41, and should be taken to be essentially the same as the ratio in the table for engines of the required horsepower.

**EXAMPLE.**—What should be the dimensions of a second-motion engine to develop 200 H. P., when the mean effective pressure of the steam is 80 lb. per sq. in., the hoisting speed is 550 ft. per min., the drum is 6 ft. in diam., and the gear ratio is 3?

**SOLUTION.**—Here the revolutions per minute of the drum $= 550 \div (6 \times 3.1416) = 29.2$, about, and $r$ may be assumed as in the engine table to be as 18 : 24 or as 1 : 1.33. Substituting in the formula,

$$d^3 = \frac{792,000 \times 200}{3.14 \times 80 \times 1.33 \times 3 \times 29.2} = 5,412$$

From this, $d = 17.56$, say, 18 in., and $l = rd = 1.33 \times 18 = 24$ in. Hence the dimensions of the cylinders are 18 in. $\times$ 24 in.

In calculations relating to this type of engine, it is rarely necessary to determine the accelerating force. Sufficiently accurate results may be obtained by determining the horsepower required to hoist the unbalanced load (with friction at 10%) at the sustained speed, and then to make one cylinder large enough to do all the work. Where uncertainty exists as to the efficiency of the plant or as to future possible increased demands for power, the determined horsepower should be increased from 10 to 25% or more.

# HAULAGE

## RESISTANCES TO HAULAGE

**Total Resistance to Haulage.**—The total resistance $R$, in pounds, which must be overcome in bringing a car or a trip of cars from rest to full speed, may be represented by the expression,

$$R = F + C + G + I$$

in which $F$ = resistance due to friction, in pounds;
$C$ = resistance due to curvature, in pounds;
$G$ = resistance due to grade, in pounds;
$I$ = resistance due to inertia, in pounds.

When the trip is moving at a uniform speed, the resistance is

$$R = F + C + G$$

When the track is level, $G = 0$, and when it is straight $C = 0$.

**Resistance Due to Friction.**—The frictional resistance to haulage is due both to the friction of the wheels upon their axles, or *car resistance,* and to the friction of the wheels upon the rails, or *track resistance.* It is customary to consider these as one under the head of *train resistance.* On surface railroads, it is found that the train resistance increases with the speed and is materially

less for heavy than for light cars. The Baldwin Locomotive Works, quoting the experiments of Prof. E. C. Schmidt, gives the following formula for the resistance of freight cars, in pounds per ton,

$$F = 1.5 + \frac{106 + 2V}{W + 1} + .001 \ V^2$$

in which $\qquad W =$ weight of car, in tons;

$V =$ velocity of train, in miles per hour.

The same authority gives the resistance for light standard-gauge and narrow-gauge locomotives and tenders as

$$F = 5 + .004V^2$$

and for heavy standard-gauge locomotives as,

$$F = 4.3 + .003V^2$$

Both in mine- and surface-railroad practice, it is found that the train resistance depends on the diameter of the wheels, the kind of lubricant used and the amount of lubrication, the kind of axles and journal boxes, the condition of the track, the presence or absence of curves, etc. A large journal bearing reduces journal friction, and car wheels of large diameter roll more easily than small ones over inequalities in the rails. Flat and grooved wheels add materially to the resistance. With ordinary self-oiling wheels turning on fixed axles, as used in bituminous coal mines, many experiments have shown that the frictional resistance averages 30 lb. per T., or 1.5%, when the cars are properly oiled and kept in good repair. When the wheels are fixed and the axles revolve in self-oiling journal boxes, the resistance should be under 20 lb. per T., or 1%. In surface-railroad practice, at low speeds, the resistance will be from 4 lb. to 10 lb. per T., a working average being 6.5 lb. In mines where roller-bearing wheels are used, the resistance should not exceed 15 lb. per T., or .75%. Where the track is poor and wheels have much play on fixed axles, the resistance may easily amount to 50, 60, or, in extreme cases, to 100 lb. per T.

The track resistance is ordinarily but a small fraction of the total resistance and may be found from the formula,

$$F = \frac{CW}{r}$$

in which $\quad W =$ weight of car, in pounds;

$r =$ radius of car wheel, in inches;

$C =$ coefficient that, for iron wheels rolling on steel rails, is .02.

Thus the track resistance per ton of 2,000 lb., when 20-in. wheels are used, is $(2,000 \times .02) \div 10 = 4$ lb.

If a coefficient of friction $f$ is assumed, the total frictional resistance may be found from

$$F = fW \cos X$$

in which $\qquad W =$ weight of car;

$X =$ angle of inclination of track.

If the track is level, $X = 0$, $\cos X = 1$, and $F = fW$. Where the pitch is under 10%, say 5° 30′, no error of importance is made if this latter and more simple formula is used.

EXAMPLE.—What is the frictional resistance to moving a loaded mine car weighing 8,000 lb. on a level and on a slope of 20° when $f = 1.5\%$?

SOLUTION.—On a level track $X = 0$ and $F = fW = .015 \times 8,000 = 120$ lb. On the slope, $F = .015 \times 8,000 \times .937 = 112.5$ lb., about.

EXAMPLE.—Assuming the conditions of the preceding problem, what horsepower must be exerted to move the car at a rate of 2 mi. per hr. on a level track?

SOLUTION.—A speed of 2 mi. per hr. $= (2 \times 5,280) \div 3,600 = 2.9$ ft. per sec. The work per second required to overcome a resistance of 120 lb. is $120 \times 2.9 = 348$ ft.-lb. As 1 H. P. is equal to 550 ft.-lb. of work per sec., the required horsepower is $348 \div 550 = .63$.

**Resistance Due to Curvature.**—On surface railroads, it is customary to compensate for the resistance due to curvature by reducing the grade by .03 to .05 ft. per 100 ft. for each degree of curve. Thus, on an 8° curve, where the grade would otherwise be, say, 2%, it would be reduced to $(2.00 - .05 \times 8) = 1.60\%$.

This correction is not made on mine roads, where it is not generally possible or even necessary to do so. Centrifugal force pressing the wheels against the outer rail is the chief cause of curve resistance on surface railroads where the speed is high, the cars heavy, the curves of long radius, and where, in particular, the trucks being pivoted on king bolts, the axles are free to assume a position in the direction of the radius of the curve. On mine roads, however, where

the speed is low, the cars light, the curves of small radius, and the axles are fixed so that they cannot adjust their direction to that of the radius of the curve, the chief cause of curve resistance is the binding of the wheels against the rails.

The centrifugal force pressing the wheels against the outer rail of a curve may be found from the formula,

$$C = \frac{Wv^2}{gr}$$

in which
$W$ = weight of car or trip, in pounds;
$v$ = velocity of car or trip, in feet per second;
$r$ = radius of curve, in feet.

EXAMPLE.—A train weighing 400 T. is moving on an 8° curve at a speed of 50 mi. per hr. What is the pressure against the outer rail?

SOLUTION.— 400 T. = 800,000 lb. A speed of 50 mi. per hr. = $(50 \times 5,280) \div 3,600 = 73$ ft. per sec. The radius of an 8° curve is about 717 ft., and $g$ may be taken at 32.2 ft. per sec. per sec. Substituting,

$$C = \frac{800,000 \times 73^2}{32.2 \times 717} = 200,000 \text{ lb., approximately}$$

This is the total pressure of the train. If there are 8 cars each with two four-wheel trucks, the pressure per truck is $200,000 \div 16 = 12,500$ lb., and for each wheel bearing against the outer rail is 6,250 lb.

The following formula, based on experiments with mine cars moving at usual speeds, gives the approximate average resistance due to curvature:

$$C = \frac{bW}{5r}$$

in which $b$ = wheel base, in feet, and $C$, $W$, and $r$ have the meaning given in the preceding formula.

EXAMPLE.—What will be the resistance to the passage of a motor weighing 20 T. (40,000 lb.), and having a wheel base of 10 ft., around a curve with a radius of 200 ft.?

SOLUTION.—By substituting in the formula, $C = \frac{10 \times 40,000}{5 \times 200} = 400$ lb.

The preceding formula brings out the important point that, for equal resistance, the sharper the curve the less must be the wheel base of the car. Although not considered in the formula, it should be stated that for equal resistance, the sharper the curve the less should be the gauge of the track.

**Resistance Due to Grade.**—The grade or slope of a track may be expressed in various ways.

1. As an angle made by the track with the horizontal.
2. As the ratio between the rise, taken as 1, or unity, and the horizontal distance required to gain the rise; as 1 in 1, 1 in 5, etc.
3. As the rise in any horizontal distance, as a grade of 2 in. per ft., or of 6 in. per yd.
4. As a per cent.; this is a modification of 3, the horizontal distance being taken as 100 ft. Thus a 2% grade is one in which the rise is 2 ft. per 100 ft., or 105.6 ft. per mi.
5. As the rise in feet per mile of length of track; this is, also, a modification of 3, the horizontal distance being taken as 1 mi.

Of these ways of expressing grades, the second and third are awkward and practically obsolete. The first and fourth are in common use by mining engineers, and the fourth and fifth by civil engineers.

If $X$ = angle of slope, $V$ = vertical rise, $H$ = horizontal distance, and $S$ = slope distance, then,

$$\tan X = \frac{V}{H} \qquad \sin X = \frac{V}{S} \qquad \cos X = \frac{H}{S}$$

$V = H \tan X$ = $S \sin X$   $S = V \csc X = H \sec X$   $H = V \cot X = S \cos X$

The per cent. of grade is the tangent of the angle of inclination. A 10% grade = angle whose tangent is .10000 = 5° 43', about. A pitch of 7° 58', the tangent of which is .13995, equals a grade of .13995 × 100 = 13.995, say, 14%.

A 10% grade equals a rise of 5,280 × .10 = 528 ft. per mi. A grade of 460 ft. per mi. equals one of (460 ÷ 5,280) × 100 = 8.71%.

The resistance, due solely to the grade, which must be overcome in hauling up an incline at a uniform speed is

$$G = W \sin X$$

On a level track, where $\sin X = \sin 0° = 0$, $G = 0$ and there is no resistance due to the weight while the trip is moving at a uniform speed. In shaft hoist-

ing, where sin $X = $ sin $90° = 1$, $G = W$ and the resistance is equal to the total load.

If the grade is expressed in feet per mile, instead of degrees, the formula becomes,

$$G = W \times \text{grade in ft. per mi.} \times .3788$$

If the grade is expressed as a per cent. instead of in degrees, the formula becomes, $G = W \times$ per cent. of grade. This last relation is true only for comparatively flat grades, say, those not exceeding 10%, where the tangent and sine of the angle of inclination are practically equal.

## GRADE EQUIVALENTS

| Per Cent. | Feet per Mile | Degrees | Minutes | Per Cent. | Feet per Mile | Degrees | Minutes | Per Cent. | Feet per Mile | Degrees | Minutes |
|---|---|---|---|---|---|---|---|---|---|---|---|
| .01 | .53 | 0 | .34 | 2.80 | 147.84 | 1 | 36.23 | 6.40 | 337.92 | 3 | 39.72 |
| .02 | 1.07 | 0 | .69 | 2.90 | 153.12 | 1 | 39.67 | 6.50 | 343.20 | 3 | 43.14 |
| .03 | 1.58 | 0 | 1.03 | 3.00 | 158.40 | 1 | 43.10 | 6.60 | 348.48 | 3 | 46.56 |
| .04 | 2.11 | 0 | 1.37 | 3.10 | 163.68 | 1 | 46.54 | 6.70 | 353.76 | 3 | 49.99 |
| .05 | 2.64 | 0 | 1.72 | 3.20 | 168.96 | 1 | 49.97 | 6.80 | 359.04 | 3 | 53.41 |
| .06 | 3.17 | 0 | 2.06 | 3.30 | 174.24 | 1 | 53.41 | 6.90 | 364.32 | 3 | 56.83 |
| .07 | 3.70 | 0 | 2.41 | 3.40 | 179.52 | 1 | 56.84 | 7.00 | 369.60 | 4 | .25 |
| .08 | 4.22 | 0 | 2.75 | 3.50 | 184.80 | 2 | .27 | 7.10 | 374.88 | 4 | 3.67 |
| .09 | 4.75 | 0 | 3.09 | 3.60 | 190.08 | 2 | 3.71 | 7.20 | 380.16 | 4 | 7.09 |
| .10 | 5.28 | 0 | 3.44 | 3.70 | 195.36 | 2 | 7.14 | 7.30 | 385.44 | 4 | 10.51 |
| .20 | 10.56 | 0 | 6.88 | 3.80 | 200.64 | 2 | 10.57 | 7.40 | 390.72 | 4 | 13.93 |
| .30 | 15.84 | 0 | 10.32 | 3.90 | 205.92 | 2 | 14.01 | 7.50 | 396.00 | 4 | 17.35 |
| .40 | 21.12 | 0 | 13.75 | 4.00 | 211.20 | 2 | 17.44 | 7.60 | 401.28 | 4 | 20.77 |
| .50 | 26.40 | 0 | 17.19 | 4.10 | 216.48 | 2 | 20.87 | 7.70 | 406.56 | 4 | 24.19 |
| .60 | 31.68 | 0 | 20.63 | 4.20 | 221.76 | 2 | 24.30 | 7.80 | 411.84 | 4 | 27.60 |
| .70 | 36.96 | 0 | 24.07 | 4.30 | 227.04 | 2 | 27.73 | 7.90 | 417.12 | 4 | 31.02 |
| .80 | 42.24 | 0 | 27.50 | 4.40 | 232.32 | 2 | 31.16 | 8.00 | 422.40 | 4 | 34.44 |
| .90 | 47.52 | 0 | 30.94 | 4.50 | 237.60 | 2 | 34.60 | 8.10 | 427.68 | 4 | 37.85 |
| 1.00 | 52.80 | 0 | 34.38 | 4.60 | 242.88 | 2 | 38.03 | 8.20 | 432.96 | 4 | 41.27 |
| 1.10 | 58.08 | 0 | 37.81 | 4.70 | 248.16 | 2 | 41.45 | 8.30 | 438.24 | 4 | 44.68 |
| 1.20 | 63.36 | 0 | 41.25 | 4.80 | 253.44 | 2 | 44.89 | 8.40 | 443.52 | 4 | 48.09 |
| 1.30 | 68.64 | 0 | 44.69 | 4.90 | 258.72 | 2 | 48.32 | 8.50 | 448.80 | 4 | 51.51 |
| 1.40 | 73.92 | 0 | 48.13 | 5.00 | 264.00 | 2 | 51.75 | 8.60 | 454.08 | 4 | 54.92 |
| 1.50 | 79.20 | 0 | 51.56 | 5.10 | 269.28 | 2 | 55.18 | 8.70 | 459.36 | 4 | 58.33 |
| 1.60 | 84.48 | 0 | 55.00 | 5.20 | 274.56 | 2 | 58.60 | 8.80 | 464.64 | 5 | 1.75 |
| 1.70 | 89.76 | 0 | 58.43 | 5.30 | 279.84 | 3 | 2.03 | 8.90 | 469.92 | 5 | 5.16 |
| 1.80 | 95.04 | 1 | 1.88 | 5.40 | 285.12 | 3 | 5.46 | 9.00 | 475.20 | 5 | 8.57 |
| 1.90 | 100.32 | 1 | 5.31 | 5.50 | 290.40 | 3 | 8.89 | 9.10 | 480.48 | 5 | 11.98 |
| 2.00 | 105.60 | 1 | 8.75 | 5.60 | 295.68 | 3 | 12.32 | 9.20 | 485.76 | 5 | 15.39 |
| 2.10 | 110.88 | 1 | 12.18 | 5.70 | 300.96 | 3 | 15.74 | 9.30 | 491.04 | 5 | 18.80 |
| 2.20 | 116.16 | 1 | 15.61 | 5.80 | 306.24 | 3 | 19.17 | 9.40 | 496.32 | 5 | 22.20 |
| 2.30 | 121.44 | 1 | 19.06 | 5.90 | 311.52 | 3 | 23.59 | 9.50 | 501.60 | 5 | 25.61 |
| 2.40 | 126.72 | 1 | 22.49 | 6.00 | 316.80 | 3 | 26.02 | 9.60 | 506.88 | 5 | 29.02 |
| 2.50 | 132.00 | 1 | 25.93 | 6.10 | 322.08 | 3 | 29.45 | 9.70 | 512.16 | 5 | 32.42 |
| 2.60 | 137.28 | 1 | 29.36 | 6.20 | 327.36 | 3 | 32.87 | 9.80 | 517.44 | 5 | 35.83 |
| 2.70 | 142.56 | 1 | 32.80 | 6.30 | 332.64 | 3 | 36.29 | 9.90 | 522.72 | 5 | 39.23 |
|  |  |  |  |  |  |  |  | 10.00 | 528.00 | 5 | 42.64 |

**EXAMPLE.**—(a) What is the resistance opposing motion when a trip of cars weighing 10,000 lb. is moved upwards on a plane pitching 8°. (b) If the speed of the trip is 10 mi. per hr., what is the horsepower required to produce this motion when the coefficient of friction $f$ is .025?

SOLUTION.—(a) As the trip is moving on a straight track at a uniform speed, $A$ and $C$ are both equal to zero, and $R = F + G$. Substituting for $F$, and $G$ their values,

$$R = fW \cos X + W \sin X = (.025 \times 10,000 \times .99027) + (10,000 \times .13917)$$
$$= 247.57 + 1,391.70 = 1,639.27 \text{ lb.}$$

(b) A speed of 10 mi. per hr. = $(10 \times 5,280) \div 3,600 = 14.7$ ft. per sec. Whence the foot-pounds of work in 1 sec. = $1,639.27 \times 14.7 = 24,097.27$. But 1 H. P. = 550 ft.-lb. of work in 1 sec., hence to move the trip up the grade at a uniform speed of 10 mi. per hr. will require the expenditure of $24,097.27 \div 550 = 43.8$ H. P.

**Resistance Due to Inertia.**—The resistance due to the inertia of the trip is frequently called the starting resistance because it exists only during the period of acceleration and ceases as soon as the car is moving at a uniform speed. The force necessary to overcome this resistance is not uncommonly called the acceleration, although the word acceleration properly means the gain in the velocity of the body in feet per second per second under the influence of a constant force. During the time of its existence and application, the accelerating force, in haulage problems, may be considered as constant and is the same in value whether the load is moved horizontally, on an inclined track, or vertically.

The force $I$, in pounds, that constantly applied to a body will give it a specified velocity uniformly accelerated from rest at the end of a specified number of seconds, may be found from the formula,

$$I = Ma = \frac{W}{g}a = \frac{Wv}{gt} = \frac{Wv^2}{g2s}$$

in which $M = \dfrac{W}{g}$ = mass of body, and the other symbols have the meaning given them under Calculations for First-Motion Hoisting Engines.

EXAMPLE.—(a) Neglecting friction, what force is required to bring a loaded mine car weighing 5,000 lb. from rest to a uniform speed of 8 mi. per hr. at the end of 10 sec.? (b) What will be the maximum work per second and maximum horsepower during acceleration?

SOLUTION.—(a) Here $W = 5,000$ lb.; $t = 10$ sec.; $v = (5,280 \times 8) \div 3,600 = 12$ ft. per sec.; $a = 12 \div 10 = 1.2$ ft. per sec. per sec. Substituting,

$$I = \frac{W}{g}a = \frac{5,000}{32.2} \times 1.2 = 186 \text{ lb., about}$$

(b) Because the speed during acceleration increases from 0 to 12 ft. per sec., the maximum work is done during the last second, and is $186 \times 12 = 2,232$ ft.-lb. From this, the maximum horsepower is $2,232 \div 550 = 4.06$.

It is to be noted that the average work performed and the average horsepower required to perform this work during acceleration are one-half of the maximum called for during the last second.

EXAMPLE.—(a) What is the total resistance to be overcome in bringing a trip of 10 loaded mine cars weighing 5,000 lb. each, from rest to a speed of 8 mi. per hr. at the end of 10 sec.; the slope is 8°, and the track is laid in a 10° curve? (b) What is the maximum work and what is the maximum horsepower required to do the work during acceleration?

SOLUTION.—(a) The resistance to be overcome is equal to the sum of the individual resistances and is,

$$R = F + C + G + I = fW \cos X + \frac{Wv^2}{gr} + W \sin X + \frac{W}{g}a$$

In the problem, it may be assumed that $f = .02$; $W = 5,000 \times 10 = 50,000$ lb.; $X = 8°$; $g = 32.2$ ft. per sec. per sec.; $v = (5,280 \times 8) \div 3,600 = 12$ ft. per sec.; $r$ = radius of an 8° curve = 717 ft., about; $a = 12 \div 10 = 1.2$ ft. per sec. per sec. Substituting in the foregoing formula;

$$R = (.02 \times 50,000 \times .99027) + \left(\frac{50,000}{32.2} \times \frac{12^2}{717}\right) + (50,000 \times .13917) + \left(\frac{50,000}{32.2} \times 1.2\right)$$
$$= 990 + 312 + 6,958 + 1,863 = 10,123 \text{ lb.}$$

(b) The maximum work is at the end of the last second of acceleration when the trip has acquired its full speed and is $10,123 \times 12 = 121,476$ ft.-lb. As this work is performed in 1 sec., the horsepower is $121,476 \div 550 = 221$, about.

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# TRACKWORK*

**Choice of Grade.**—The grade of entries is rarely a matter of choice, but is determined by natural causes, particularly by the dip of the seam, the location of the main haulage road with respect to the property lines, by the direction and strength of the cleavage planes of the coal, etc.

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\* See further under Railroad Surveying.

Where possible the entries should be given a rising grade of from 6 in. to 1 ft. per 100 ft. to insure good drainage, as the flow in the ditches will be more or less impeded by material falling from overloaded cars, with road dirt, etc. At shaft and slope bottoms, unless the cars are fed along by a chain haul or some similar device, it is customary to give both the loaded and empty tracks a favoring grade of about 1.25%, even if the roof and floor have to be shot to do this.

Where capital is available, main haulage roads, which must last the life of the property and along all or most of the length of which all the coal must be moved, are graded with almost the same care as first-class surface railroads. More economical haulage is obtained where the grade is all in one direction, and not alternately up and down hill, because of the excessive power temporarily required to overcome the inertia and to bring the trip up to speed in pulling out of swamps. A motor will deliver to the drift mouth no more coal than it can pull over the sharpest grade, and for this reason it is not unusual to see the output of a motor reduced to 50% of what it would be were a little money spent in making the grades uniform.

**Curvature.**—Curves on mine tracks are generally designated by the length of their radius rather than by their degree of curvature, as is customary on surface-railroad work. In fact, a curve of 50 ft. radius is the sharpest that can be defined in terms of its deflection angle, because for it $\sin \frac{1}{2}D = \frac{50}{R} = 1$,

and $D = 180°$.

It is well to make all curves of as long radius as possible. Room turnouts are commonly given a radius of 25 to 35 ft.; turnouts from main-haulage roads to cross- or butt-entries, a radius of 60 to 100 ft.; and where a curve is necessary on a main-haulage road, its radius should be as great as possible, say, 150 to 200 ft., or more, in order to permit of high-speed traffic.

Rails on curves should not be sprung into place and then spiked, but should be bent to the required radius with a rail bender, or jim-crow, as explained under Railroad Surveying. The marked difference in length between the outer and inner rails on sharp mine curves may be found from the following rule:

**Rule.**—*Multiply the gauge of the track by the length of the curve and divide the product by the radius, all dimensions being given in feet.*

**EXAMPLE.**—What is the difference in length between the outer and inner rails on a curve of 50 ft. radius and 100 ft. long, when the gauge is 3 ft. 6 in.?

**SOLUTION.**—As 3 ft. 6 in. = 3.5 ft., the difference in length between the outer and inner rails will be $(3.5 \times 100) \div 50 = 7$ ft. 0 in.

With fixed axles and on a sharp curve, the running gear of a car or locomotive binds as the front wheel presses against the outside rail and the rear wheel against the inside rail. To overcome this, the difference in gauge between the car and the track is increased on curves. The amount of this increase depends on the gauge of the track, the wheel base of the car, and the radius of the curve, the maximum being limited, of course, by the tread of the wheels. Experiments have shown that, with a narrow-gauge track having sharp curves over which locomotives and cars with short wheel bases pass, a good rule is to widen the gauge of the track $\frac{1}{16}$ in. for each $2\frac{1}{2}°$ of curvature, that is, on a 40° curve the track gauge should be increased 1 in. On the very sharp curves frequently necessary in mines, the gauge should be widened as much as the wheel tread will allow, and in some cases it is well to lay guard-rails on the curves inside the rails, so that if one wheel mounts the track the other will not follow, but will pull it back on to the track.

In motor haulage, in passing around curves the centrifugal force crowds the outer wheel against the rails and tends to overturn the cars. To counteract this tendency, the outer rail is elevated by an amount proportionate to the speed of the trip and the sharpness of the curve.

In rope haulage, the pull of the rope on curves tends to overturn the cars inwards, the inner instead of the outer rail is elevated.

On a slope haulage, however, operated by a single rope, when the weight of the cars traveling on the slope is sufficient to draw the rope off the hoisting drum, the rails on curves should be elevated on the outside, as the centrifugal force only acts on the cars being lowered; the elevation in such a case should, however, be moderate, so as not to interfere with the trip when being drawn out by the rope, when, of course, the tendency is to tip the cars inwards. The table on page **764** gives the elevation of rail for different degrees of curvature and for a 42-in. track, assuming a speed of 10 to 15 mi. per hr.

## RAIL ELEVATION

| Degree of Curve | Radius of Curve Feet | Elevation of Outer Rail Inch | Degree of Curve | Radius of Curve Feet | Elevation of Outer Rail Inches |
|---|---|---|---|---|---|
| 1 | 5,729.6 | 1/8 | 10.0 | 573.7 | 1 1/8 |
| 2 | 2,864.9 | 1/4 | 12.0 | 478.3 | 1 7/16 |
| 3 | 1,910.1 | 5/16 | 15.0 | 383.1 | 1 3/4 |
| 4 | 1,432.7 | 7/16 | 18.0 | 319.6 | 1 13/16 |
| 5 | 1,146.3 | 9/16 | 20.0 | 287.9 | 2 1/16 |
| 6 | 955.4 | 11/16 | 60.0 | 100.0 | 4 1/4 |
| 7 | 819.0 | 13/16 | 100.0 | 60.0 | 4 1/2 |
| 8 | 716.8 | 7/8 | 180.0 | 50.0 | 4 1/2 |
| 9 | 637.3 | 1 | | | |

It is not generally advisable to elevate the rail more than $4\frac{1}{2}$ in., as it is not good practice to attempt to run trips around sharp curves at a high speed.

The rule for standard-gauge roads (4 ft. $8\frac{1}{2}$ in. or $56\frac{1}{2}$ in.) on surface and for speeds of 25 to 35 mi. per hr. is to elevate the outer rail $\frac{1}{2}$ in. for each degree of curvature. An approximate rule often given for narrower gauges

### TABLE OF RAILS
(*Carnegie Steel*

| Weight per Yard Pounds | Height of Rail and Width of Base. Inches | Length of Splice Bar Inches | Size of Bolt Inches | Size of Spike Inches | For One Joint | | Total Weight Complete Pounds | For 1,000 T. of Rail | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Weight of One Pair of Splice Bars Pounds | No. and Weight of Bolts and Nuts Pounds | | Number | | |
| | | | | | | | | Pairs of Splice Bars | Bolts and Nuts | Spikes |
| 110 | 6 1/2 | 34 | 3/4×4 1/2 | 5 1/2×9/16 | 99.50 | 5.60 | 105.10 | 1,892 | 11,350 | 61,760 |
| 100 | 5 3/4 | 34 | 3/4×4 1/2 | 5 1/2×9/16 | 87.00 | 5.36 | 92.36 | 2,075 | 12,450 | 67.710 |
| 95 | 5 9/16 | 34 | 3/4×4 1/2 | 5 1/2×9/16 | 80.80 | 5.36 | 86.16 | 2,184 | 13,100 | 71,270 |
| 90 | 5 3/8 | 34 | 3/4×4 1/2 | 5 1/2×9/16 | 74.00 | 5.20 | 79.20 | 2,305 | 13,830 | 75,230 |
| 85 | 5 3/16 | 34 | 3/4×4 1/2 | 5 1/2×9/16 | 68.13 | 5.20 | 73.33 | 2,441 | 14,640 | 79,660 |
| 80 | 5 | 34 | 3/4×4 1/2 | 5 1/2×9/16 | 63.13 | 5.05 | 68.18 | 2,593 | 15,560 | 84,640 |
| 75 | 4 13/16 | 34 | 3/4×4 | 5 1/2×9/16 | 58.50 | 4.96 | 63.46 | 2,766 | 16,590 | 90,270 |
| 70 | 4 5/8 | 34 | 3/4×3 3/4 | 5 1/2×9/16 | 54.64 | 4.76 | 59.40 | 2,963 | 17,780 | 96,720 |
| 65 | 4 7/16 | 24 | 3/4×3 3/4 | 5 1/2×9/16 | 35.55 | 3.18 | 38.73 | 3,192 | 12,770 | 104,200 |
| 60 | 4 1/4 | 24 | 3/4×3 3/4 | 5 1/2×9/16 | 32.40 | 3.12 | 35.52 | 3,458 | 13,830 | 112,900 |
| 55 | 4 1/16 | 24 | 3/4×3 3/4 | 5 1/2×9/16 | 28.90 | 3.12 | 32.02 | 3,772 | 15,090 | 123,100 |
| 50 | 3 7/8 | 24 | 3/4×3 1/2 | 5 1/2×9/16 | 25.50 | 3.00 | 28.50 | 4,149 | 16,600 | 135,400 |
| 45 | 3 11/16 | 20 | 3/4×3 | 5 1/2×9/16 | 18.75 | 2.90 | 21.65 | 5,148 | 20,590 | 150,500 |
| 40 | 3 1/2 | 20 | 3/4×3 | 5 ×1/2 | 16.10 | 2.90 | 19.00 | 5,790 | 23,160 | 169,300 |
| 35 | 3 5/16 | 16 1/2 | 5/8×2 1/2 | 4 1/2×1/2 | 12.10 | 1.74 | 13.84 | 6,618 | 26,470 | 193,500 |
| 30 | 3 1/8 | 16 1/2 | 5/8×2 1/2 | 4 ×1/2 | 10.45 | 1.74 | 12.19 | 7,722 | 30,890 | 225,700 |
| 25 | 2 7/8 | 16 1/2 | 5/8×2 1/2 | 4 ×1/2 | 5.70 | .97 | 6.67 | 9,264 | 37,060 | 270,800 |
| 20 | 2 5/8 | 16 1/2 | 1/2×2 | 3 1/2×7/16 | 4.86 | .91 | 5.77 | 11,580 | 46,330 | 338,500 |
| 16 | 2 3/8 | 16 1/2 | 1/2×1 3/4 | 3 1/2×7/16 | 4.36 | .865 | 5.225 | 14,480 | 57,920 | 423,200 |
| 14 | 2 1/16 | 16 1/2 | 1/2×1 3/4 | 3 ×7/16 | 3.44 | .865 | 4.305 | 16,540 | 66,180 | 483,600 |
| 12 | 2 | 16 1/2 | 1/2×1 3/4 | 3 ×7/16 | 3.44 | .865 | 4.305 | 19,300 | 77,200 | 564,100 |
| 10 | 1 7/8 | 16 1/2 | 1/2×1 1/2 | 2 1/2×5/16 | 2.60 | .45 | 3.05 | 23,170 | 92,680 | 677,300 |
| 8 | 1 11/16 | 16 1/2 | 1/2×1 1/2 | 2 1/2×5/16 | 2.00 | .45 | 2.45 | 28,960 | 115,800 | 846,400 |

(Note: "6 Bolts" is indicated in the Bolts and Nuts column for the upper rows; "4 Bolts" for the lower rows.)

is to make the elevation proportional to the gauge based on the amount given for standard gauge. Thus, for a 36-in. gauge, the elevation would be about two-thirds of the elevation for a 56½-in. gauge of the same speed and curve.

The elevations of the outer rail given in the table correspond to the middle ordinates of the respective curves for a chord of 20 ft. Hence, a common rule to determine the amount of the elevation of the outer rail, for a speed of 15 mi. per hr. for a 3-ft. gauge, is to measure the middle ordinate of a string 20 ft. long, stretched as a chord on the gauge line of the outer rail. For higher or lower speeds, make the length of the string proportional to the speed; thus, for a speed of 12 mi. per hr., use a 16-ft. string, for 9 mi. per hr., a 12-ft. string, etc. Also the elevation should be proportional to the gauge; thus, for a 30-in. gauge, use five-sixths of the above elevation, etc.

The general rule is to begin to elevate the rail a short distance before the curve begins, this distance depending on the amount of elevation required. It is, however, not always practicable to do this in mine work.

**Gauge of Track.**—The gauge of track selected must conform to local conditions. The thickness of the seam and the character of roof and floor determine in a general way the size of the haulage roads, and consequently the mine cars that pass through them. The question of economical haulage considers a minimum number of cars having a maximum capacity. As the car length is limited by the necessarily short wheel base to about 10 ft., and its height by the thickness of the seam and limit of easy hand loading, the remaining dimension, or the width of the car, is usually the variable factor. To obtain the maximum capacity required, the width of the car must be increased, thereby requiring a broader gauge for stability. A broader gauge

## AND ACCESSORIES
*Company)*

| For 1,000 T. of Rail | | | | For 1 Mi. Single Track | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Weight in Gross Tons | | | | Number | | | Weight, in Gross Tons | | | | | |
| Splice Bars | Bolts and Nuts | Spikes | Total Accessories | Pair of Splice Bars | Bolts and Nuts | Spikes | Splice Bars | Bolts and Nuts | Spikes | Total Accessories | Rail | Total Complete |
| 84.04 | 4.80 | 16.22 | 105.06 | 326 | 1,956 | 10,640 | 14.48 | .83 | 2.80 | 18.11 | 172.29 | 190.40 |
| 80.56 | 5.03 | 17.78 | 103.37 | 326 | 1,956 | 10,640 | 12.66 | .79 | 2.80 | 16.25 | 157.14 | 173.39 |
| 78.77 | 5.29 | 18.72 | 102.78 | 326 | 1,956 | 10,640 | 11.76 | .79 | 2.80 | 15.35 | 149.29 | 164.64 |
| 76.15 | 5.39 | 19.75 | 101.29 | 326 | 1,956 | 10,640 | 10.77 | .76 | 2.80 | 14.33 | 141.43 | 155.76 |
| 74.26 | 5.71 | 20.94 | 100.91 | 326 | 1,956 | 10,640 | 9.92 | .76 | 2.80 | 13.48 | 133.57 | 147.05 |
| 72.15 | 5.96 | 22.23 | 100.34 | 326 | 1,956 | 10,640 | 9.07 | .75 | 2.80 | 12.62 | 125.71 | 138.33 |
| 72.28 | 6.25 | 23.70 | 102.23 | 326 | 1,956 | 10,640 | 8.52 | .74 | 2.80 | 12.06 | 117.86 | 129.92 |
| 72.36 | 6.48 | 25.40 | 104.24 | 326 | 1,956 | 10,640 | 7.96 | .71 | 2.80 | 11.47 | 110.00 | 121.47 |
| 50.63 | 4.65 | 27.36 | 82.64 | 326 | 1,304 | 10,640 | 5.17 | .48 | 2.80 | 8.45 | 102.12 | 110.57 |
| 49.95 | 4.84 | 29.63 | 84.42 | 326 | 1,304 | 10,640 | 4.71 | .46 | 2.80 | 7.97 | 94.29 | 102.26 |
| 48.60 | 5.28 | 32.33 | 86.21 | 326 | 1,304 | 10,640 | 4.20 | .46 | 2.80 | 7.46 | 86.43 | 93.89 |
| 47.22 | 5.59 | 35.56 | 88.37 | 326 | 1,304 | 10,640 | 3.71 | .44 | 2.80 | 6.95 | 78.57 | 85.52 |
| 42.99 | 6.69 | 39.51 | 89.19 | 364 | 1,456 | 10,640 | 3.04 | .47 | 2.79 | 6.31 | 70.71 | 77.02 |
| 41.68 | 7.52 | 30.83 | 80.03 | 364 | 1,456 | 10,640 | 2.62 | .47 | 1.94 | 5.03 | 62.86 | 67.89 |
| 35.84 | 5.43 | 32.29 | 73.56 | 364 | 1,456 | 10,640 | 1.97 | .30 | 1.78 | 4.05 | 55.00 | 59.05 |
| 36.06 | 6.33 | 33.60 | 75.99 | 364 | 1,456 | 10,640 | 1.70 | .30 | 1.59 | 3.59 | 47.14 | 50.73 |
| 23.67 | 4.10 | 40.32 | 68.09 | 364 | 1,456 | 10,640 | .93 | .16 | 1.59 | 2.68 | 39.29 | 41.97 |
| 25.13 | 4.96 | 48.74 | 78.83 | 364 | 1,456 | 10,640 | .79 | .16 | 1.53 | 2.48 | 31.43 | 33.91 |
| 28.24 | 5.76 | 32.30 | 66.30 | 364 | 1,456 | 10,640 | .71 | .15 | .81 | 1.67 | 25.14 | 26.81 |
| 25.45 | 6.58 | 34.82 | 66.85 | 364 | 1,456 | 10,640 | .56 | .15 | .77 | 1.48 | 22.00 | 23.48 |
| 29.67 | 7.67 | 40.61 | 77.97 | 364 | 1,456 | 10,640 | .56 | .15 | .77 | 1.48 | 18.86 | 20.34 |
| 26.73 | 4.65 | 27.50 | 58.88 | 364 | 1,456 | 10,640 | .42 | .07 | .43 | .92 | 15.71 | 16.63 |
| 26.25 | 5.82 | 34.37 | 66.44 | 364 | 1,456 | 10,640 | .33 | .07 | .43 | .83 | 12.57 | 13.40 |

reduces wear and tear on tracks and rolling stock, and requires outside wheels, which are cheap, easy to lubricate, and easy to replace. Cars with a relatively narrow gauge run more easily around sharp curves, and they are generally made with inside wheels. With wheels inside the frame, the capacity of narrow-gauge cars may be made to almost equal that of cars of broader gauge, but they lack the stability of the latter. With narrow gauges, shorter ties can be used, reducing the amount of cutting in the bottom of a thin inclined seam, and leaving more room available for ditching and for gob room, but with a very narrow gauge too little room is given for the mules to tread, and they frequently slip on the rails or inclines and at curves.

The most common track gauges in coal mines are 30, 36, 42, and 48 in., but these are not absolute, as smaller and larger gauges are often employed. Gauges less than 26 in. make the cars top heavy and gauges more than 48 in. require large curves and extra wide haulage ways, room necks, etc.

In proportioning the gauge of a track to a given width of entry, provision should be made, if possible, to allow for a passageway between the car and the ribs, or at least between the car and one rib, so that a man and a mule can pass between the car and the rib.

**Rails.**—The size, that is, the weight per yard, of rails to be used depends on the nature of the traffic. Nothing is gained by having rails of too light a section. If the mine water is acid, light rails are soon eaten through, and in any case they are apt to spring and cause wrecks, the cost of cleaning up which will soon pay for the difference in cost of the heavier rails.

Main haulage roads, particularly where heavy motor or high-speed rope haulage is used, are very commonly laid with rails weighing 50 to 60 lb. per yd.

## TABLE OF RAILS
*(Carnegie Steel)*

| Weight per Meter Kilograms | Height of Rail and Width of Base. Millimeters | Length of Splice Bar Millimeters | Size of Bolt Millimeters | Size of Spike Millimeters | For One Joint | | | | For 1,000 Tonnes of Rail | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Weight of One Pair Splice Bars. Kilo. | | No. and Weight of Bolts and Nuts Kilograms | Total Weight Complete Kilograms | Number | |
| | | | | | | | | | Pairs of Splice Bars | Bolts and Nuts |
| 54.56 | 155.60 | 864 | 19.0×121 | 140×14.3 | 45.13 | | 2.54 | 47.67 | 1,854 | 11,124 |
| 49.60 | 146.00 | 864 | 19.0×114 | 140×14.3 | 39.46 | | 2.43 | 41.89 | 2,040 | 12,240 |
| 47.12 | 141.30 | 864 | 19.0×114 | 140×14.3 | 36.65 | | 2.43 | 39.08 | 2,146 | 12,876 |
| 44.64 | 136.50 | 864 | 19.0×108 | 140×14.3 | 33.57 | | 2.35 | 35.92 | 2,270 | 13,620 |
| 42.16 | 131.80 | 864 | 19.0×108 | 140×14.3 | 30.90 | 6 Bolts | 2.35 | 33.25 | 2,400 | 14,400 |
| 39.68 | 127.00 | 864 | 19.0×105 | 140×14.3 | 28.63 | | 2.29 | 30.92 | 2,550 | 15,300 |
| 37.20 | 122.20 | 864 | 19.0×102 | 140×14.3 | 26.54 | | 2.25 | 28.79 | 2,718 | 16,308 |
| 34.72 | 117.50 | 864 | 19.0×95.2 | 140×14.3 | 24.78 | | 2.16 | 26.94 | 2,916 | 17,496 |
| 32.24 | 112.70 | 610 | 19.0×95.2 | 140×14.3 | 16.10 | | 1.44 | 17.54 | 3,140 | 12,560 |
| 29.76 | 107.90 | 610 | 19.0×88.9 | 140×14.3 | 14.70 | | 1.42 | 16.12 | 3,410 | 13,640 |
| 27.28 | 103.20 | 610 | 19.0×88.9 | 140×14.3 | 13.10 | | 1.42 | 14.52 | 3,710 | 14,840 |
| 24.80 | 98.42 | 610 | 19.0×82.5 | 140×14.3 | 11.60 | | 1.36 | 12.96 | 4,080 | 16,320 |
| 22.32 | 93.66 | 508 | 19.0×76.2 | 140×14.3 | 8.50 | | 1.32 | 9.82 | 8,960 | 35,840 |
| 19.84 | 88.89 | 508 | 19.0×76.2 | 127×12.7 | 7.30 | | 1.32 | 8.62 | 10,080 | 40,320 |
| 17.36 | 84.13 | 410 | 15.9×63.5 | 114.3×12.7 | 5.50 | 4 Bolts | .79 | 6.29 | 11,520 | 46,080 |
| 14.88 | 79.37 | 410 | 15.9×63.5 | 101.6×12.7 | 4.74 | | .79 | 5.53 | 13,440 | 53,760 |
| 12.40 | 69.85 | 410 | 12.7×57.1 | 101.6×12.7 | 2.58 | | .45 | 3.03 | 16,140 | 64,560 |
| 9.92 | 66.67 | 410 | 12.7×50.8 | 88.9×12.7 | 2.20 | | .41 | 2.61 | 20,160 | 80,640 |
| 7.94 | 60.32 | 410 | 12.7×44.4 | 88.9× 9.5 | 1.98 | | .39 | 2.37 | 25,200 | 100,800 |
| 6.94 | 52.38 | 410 | 12.7×44.4 | 76.2× 9.5 | 1.56 | | .39 | 1.95 | 28,820 | 115,280 |
| 5.95 | 50.80 | 410 | 12.7×44.4 | 76.2× 9.5 | 1.56 | | .39 | 1.95 | 33,620 | 134,480 |
| 4.96 | 44.45 | 410 | 9.5×38.1 | 63.5× 7.9 | 1.18 | | .20 | 1.38 | 40,320 | 161,280 |
| 3.97 | 39.68 | 410 | 9.5×38.1 | 63.5× 7.9 | .91 | | .20 | 1.11 | 50,380 | 201,520 |

In motor haulage, it is recommended that the rails used weigh not less than 10 lb. per yd. for each ton in weight coming on a single driver. Thus, a 20-T., four-wheeled motor will have 5 T. on each wheel and will require a 50-lb. rail, which is light enough.

Side, cross, butt, or room entries, are commonly laid with 30- or 35-lb. rail, as the motors used for gathering are light and their speed low.

In rooms, 16-, 20-, and 25-lb. rails are used, depending on the weight of the loaded mine car and whether the gathering motor does or does not enter the rooms. Wooden rails, from 16 to 20 ft. in length and 3 in.×4 in. or 4 in.×5 in. in section, and nailed to the ties with wire nails were formerly much used but are now rarely seen. Room tracks, and in small mines the cross-entries, are not infrequently laid with light wooden rails upon which were nailed strips of strap iron about $\frac{1}{4}$ in.×$1\frac{1}{4}$ in. in section.

**Rule I.**—*To find the weight of rail, in long tons (2,240 lb.), required to lay 1 mi. of single track, multiply the weight of the rail, in pounds per yard, by* $\frac{11}{7}$, *or by 1.5714.*

**Rule II.**—*To find the weight of rail, in long tons (2,240 lb.), required to lay 1,000 ft. of single track, multiply the weight of the rail, in pounds per yard, by .29761.*

Thus, the weight of 70-lb. steel for 1 mi. and for 1,000 ft. of single track will be, respectively, $70 \times \frac{11}{7} = 110$ long tons and $70 \times .29761 = 20.833$ long tons. For lengths of track other than 1,000 ft., multiply the quantity required for 1,000 ft. by the ratio the given length bears to 1,000. Thus, for the materials required for 600, 1,580, and 4,000 ft. of track, multiply the quantities of fish-

## AND ACCESSORIES

*Company)*

| For 1.000 Tonnes of Rail | | | | | For 1 Kilometer, Single Track | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number | Weight in Tonnes | | | | Number | | | Weight in Tonnes | | | | | |
| Spikes | Splice Bars | Bolts and Nuts | Spikes | Total Accessories | Pairs of Splice Bars | Bolts and Nuts | Spikes | Splice Bars | Bolts and Nuts | Spikes | Total Accessories | Rail | Total Complete |
| 61,100 | 83.7 | 4.7 | 16.3 | 104.7 | 204 | 1,224 | 6,668 | 9.21 | .52 | 1.78 | 11.51 | 109.12 | 120.63 |
| 67,200 | 80.5 | 5.0 | 17.9 | 103.4 | 204 | 1,224 | 6,668 | 8.05 | .50 | 1.78 | 10.33 | 99.20 | 109.53 |
| 70,800 | 78.7 | 5.2 | 18.8 | 102.8 | 204 | 1,224 | 6,668 | 7.48 | .50 | 1.78 | 9.76 | 94.24 | 104.00 |
| 74,700 | 76.2 | 5.3 | 19.9 | 101.4 | 204 | 1,224 | 6,668 | 6.85 | .48 | 1.78 | 9.11 | 89.28 | 98.39 |
| 79,100 | 74.3 | 5.6 | 21.1 | 101.0 | 204 | 1,224 | 6,668 | 6.30 | .48 | 1.78 | 8.56 | 84.32 | 92.88 |
| 84,000 | 73.0 | 5.9 | 22.4 | 101.3 | 204 | 1,224 | 6,668 | 5.84 | .47 | 1.78 | 8.09 | 79.36 | 87.45 |
| 89,600 | 72.1 | 6.1 | 23.9 | 102.1 | 204 | 1,224 | 6,668 | 5.41 | .46 | 1.78 | 7.65 | 74.40 | 82.05 |
| 96,000 | 72.3 | 6.3 | 25.6 | 104.2 | 204 | 1,224 | 6,668 | 5.06 | .44 | 1.78 | 7.28 | 69.44 | 76.72 |
| 103,400 | 50.6 | 4.5 | 27.6 | 82.7 | 204 | 816 | 6,668 | 3.28 | .29 | 1.78 | 5.35 | 64.48 | 69.83 |
| 112,000 | 50.1 | 4.9 | 29.9 | 84.9 | 204 | 816 | 6,668 | 3.00 | .29 | 1.78 | 5.07 | 59.52 | 64.59 |
| 122,200 | 48.6 | 5.3 | 32.6 | 86.5 | 204 | 816 | 6,668 | 2.67 | .29 | 1.78 | 4.74 | 54.56 | 59.30 |
| 134,400 | 47.3 | 5.5 | 35.9 | 88.7 | 204 | 816 | 6,668 | 2.37 | .28 | 1.78 | 4.43 | 49.60 | 54.03 |
| 149,360 | 76.2 | 11.9 | 40.0 | 128.1 | 400 | 1,600 | 6,668 | 3.40 | .53 | 1.78 | 5.91 | 44.64 | 50.36 |
| 168,000 | 73.6 | 13.3 | 32.6 | 119.5 | 400 | 1,600 | 6,668 | 2.92 | .53 | 1.29 | 4.74 | 39.68 | 44.42 |
| 192,000 | 63.4 | 9.1 | 34.2 | 106.7 | 400 | 1,600 | 6,668 | 2.20 | .32 | 1.19 | 3.71 | 34.72 | 38.43 |
| 224,000 | 63.7 | 10.7 | 36.3 | 110.7 | 400 | 1,600 | 6,668 | 1.90 | .32 | 1.08 | 3.30 | 29.76 | 33.06 |
| 268,800 | 41.7 | 7.3 | 43.6 | 92.6 | 400 | 1,600 | 6,668 | 1.03 | .18 | 1.08 | 2.29 | 24.80 | 27.09 |
| 336,000 | 44.4 | 8.3 | 49.2 | 101.9 | 400 | 1,600 | 6,668 | .88 | .17 | .98 | 2.03 | 19.84 | 21.87 |
| 419,800 | 49.9 | 9.9 | 32.1 | 91.9 | 400 | 1,600 | 6,668 | .79 | .16 | .52 | 1.47 | 15.88 | 17.35 |
| 480,360 | 45.0 | 11.3 | 35.2 | 91.5 | 400 | 1,600 | 6,668 | .62 | .16 | .49 | 1.27 | 13.88 | 15.15 |
| 560,200 | 52.5 | 13.1 | 41.0 | 106.6 | 400 | 1,600 | 6,668 | .62 | .16 | .49 | 1.27 | 11.90 | 13.17 |
| 672,000 | 47.6 | 8.1 | 27.1 | 82.8 | 400 | 1,600 | 6,668 | .47 | .08 | .27 | .82 | 9.92 | 10.74 |
| 839,600 | 45.9 | 10.1 | 33.9 | 89.9 | 400 | 1,600 | 6,668 | .36 | .08 | .27 | .71 | 7.94 | 8.65 |

## WEIGHT OF RAILS, IN TONS OF 2,240 LB., REQUIRED TO LAY 1,000 FT. SINGLE TRACK

| Weight of Rail per Yard | Tons For 1,000 Ft. of Track | Weight of Rail per Yard | Tons For 1,000 Ft. of Track | Weight of Rail per Yard | Tons For 1,000 Ft. of Track |
|---|---|---|---|---|---|
| 8 | 2.381 | 40 | 11.905 | 75 | 23.321 |
| 12 | 3.571 | 45 | 13.393 | 80 | 23.809 |
| 16 | 4.762 | 50 | 14.881 | 85 | 25.298 |
| 20 | 5.952 | 55 | 16.369 | 90 | 26.786 |
| 25 | 7.441 | 60 | 17.858 | 95 | 28.274 |
| 30 | 8.929 | 65 | 19.346 | 100 | 29.761 |
| 35 | 10.417 | 70 | 20.833 | 110 | 32.737 |

plates, bolts, and spikes as well as rails, required for 1,000 ft. by .600, 1.58, and by 4, respectively.

It will be noted that each increase of 5 lb. per yd. in the weight of the rail, increases by 1.488 (1.5, nearly) T., the quantity required to lay 1,000 ft. of track.

The two following tables give the gross tons, of 2,240 lb., required for 1 mi., and the metric tonnes, or 2,204 lb. (about) required for 1 km. of single track, as well as the necessary fish-plates (splice-bars), bolts, nuts, and ties. It should be especially noted that above 50 lb. per yd. rails are 33 ft. long, the length adopted by the American Railway Association in 1908, the American Railway Engineering and Maintenance of Way Association in 1907, the American Society of Testing Materials in 1907, and the American Society of Civil Engineer's in 1908. The first table is based on 90% of rails to be 33 ft. long, and 10% not less than 24 ft. long, varying by even feet. Ties are to be placed to centers or 2,640 ties per mile. Rails below 50 lb. per yd. are furnished 30 ft. long, and 10% not less than 20 ft. long. No excess has been allowed. The second table is based on 90% of rails to be 10 m. long, with 10% varying down to 8 m. in length. Ties are to be spaced 600 mm. to centers or 1,641 ties per km. Rails below 24.80 km. per m. are furnished 5 m. long. No excess has been allowed. For 1,000 ft. of single track, there will be required sixty-eight 30-ft rails, 68 pairs of splice bars, 272 bolts when four are used per joint, or 408 bolts when six are used per joint.

**Ties.**—Main-entry ties should have at least a 5- to 8-in. face, and be 4 to 6 in. deep; their length will depend on the gauge of the track, but they should project from 8 to 12 in. on each side of the rail to give the roadbed stability and the ties a resting surface for the transmission of weight to the roadbed. The wood of main-track ties should be chestnut, oak, or hard pine. Locust ties are very serviceable, but it is not probable that they can be had in sufficient numbers to meet the demand. In case these woods are not to be had, other woods will naturally take their place, but if such is the case their faces should be enlarged. Sawed ties are not as durable as hewed ties with the bark removed.

On cross-entries where 20- to 30-lb. rails are used, the ties may have a 4- to 6-in, face and be 4 to 5 in. thick. In rooms, the ties need only be faced 3 or 4 in., or sufficient to form a flat surface for the rail to rest on.

Steel ties for mine use are a comparatively recent introduction. In some cases, channel or even I beams are used to which the rails are bolted, but the common form of steel tie consists of some special rolled shape, frequently corrugated, provided with a lug under which the outside edge of the base of the rail fits. The rail is held in position by a clip that presses against the inside base. In order to prevent slipping, the ties have small projections, on the bottom, that cut into the floor. Owing to their thinness, steel ties materially reduce the amount of brushing necessary in low seams.

## SIZES AND QUANTITIES OF SPIKES*

| Size Measured Under Head Inches | Average Number per Keg of 200 Lb. | Quantity | | | | Weight of Rail per Yard Pounds |
|---|---|---|---|---|---|---|
| | | Per 1,000 Ft. | | Per Mile | | |
| | | Pounds | Kegs | Pounds | Kegs | |
| 2½ × ⅜ | 1,342 | 300 | 1½ | 1,575 | 7⅞ | 8 to 16 |
| 3 × ⅜ | 1,240 | 324 | 1⅗ | 1,710 | 8½ | 16 to 20 |
| 3½ × ⅜ | 1,190 | 340 | 1¾ | 1,780 | 9 | 16 to 20 |
| 4 × ⅜ | 1,000 | 360 | 1⅘ | 2,090 | 10½ | 16 to 25 |
| 3½ × 7/16 | 900 | 445 | 2¼ | 2,350 | 11 | 16 to 25 |
| 4 × 7/16 | 720 | 550 | 2¾ | 2,910 | 14¾ | 20 to 30 |
| 4½ × 7/16 | 680 | 590 | 3 | 3,110 | 15½ | 20 to 30 |
| 4 × ½ | 600 | 670 | 3⅓ | 3,520 | 17⅞ | 25 to 35 |
| 4½ × ½ | 530 | 750 | 3¾ | 3,960 | 20 | 30 to 35 |
| 5 × ½ | 450 | 880 | 4⅝ | 4,660 | 23½ | 35 to 40 |
| 5 × 9/16 | 400 | 980 | 5 | 5,170 | 26 | 40 to 55 |
| 5½ × 9/16 | 375 | 1,112 | 5⅝ | 5,870 | 29½ | 45 to 100 |

NOTE.—In ordering spikes, a reasonable allowance should be made for waste For ordinary mine tracks with two spikes to the tie, divide by 2 the quantities given in the table. For other spacing than 2 ft., proceed as follows: For 30 in., multiply the quantity of spikes by .80; for 28 in., by .858; for 26 in. by .893; for 22 in., by 1.092; for 20 in., by 1.20; and for 18 in., by 1.334.

### NUMBER OF TRACK BOLTS IN A KEG OF 200 LB.

| Bolts Inches | Size of Nuts Inches | Bolts in Keg | Bolts Inches | Size of Nuts Inches | Bolts in Keg |
|---|---|---|---|---|---|
| ½ × 4½ | 1½ square | 195 | ½ × 2½ | 1 square | 654 |
| ½ × 4 | 1½ square | 200 | ⅝ × 2½ | 1¼ hexagonal | 170 |
| ⅝ × 3¾ | 1½ square | 208 | ⅝ × 3¾ | 1⅛ hexagonal | 237 |
| ⅝ × 3½ | 1½ square | 216 | ⅝ × 3½ | 1½ hexagonal | 228 |
| ⅝ × 4 | 1¼ square | 305 | ⅞ × 4 | 1⅛ hexagonal | 220 |
| ⅝ × 3½ | 1¼ square | 329 | ⅞ × 3½ | 1 hexagonal | 415 |
| ½ × 3½ | 1 square | 576 | | | |

### SPACES BETWEEN ENDS OF RAILS

| Temperature When Laying Track | Space to be Left Between Ends of Rails Inch | Temperature When Laying Track | Space to be Left Between Ends of Rails Inch |
|---|---|---|---|
| 90° above zero | 1/16 | 30° above zero | ¼ |
| 70° above zero | ⅛ | 10° above zero | 5/16 |
| 50° above zero | 3/16 | 10° below zero | ⅜ |

* In this table the ties are placed 2 ft. center to center and four spikes are placed in each tie.

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**FEET, BOARD MEASURE, IN MINE TIES OF VARIOUS LENGTHS**

| Size of Tie Inches | Length of Tie | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 4 Ft. 6 In. | 5 Ft. 0 In. | 5 Ft. 6 In. | 6 Ft. 0 In. | 6 Ft. 6 In. | 7 Ft. 0 In. | 7 Ft. 6 In. | 8 Ft. 0 In. |
| 3× 5 | 5.6250 | 6.2500 | 6.8750 | 7.5000 | 8.1250 | 8.7500 | 9.3750 | 10.0000 |
| 4× 5 | 7.5000 | 8.3333 | 9.1667 | 10.0000 | 10.8333 | 11.6667 | 12.5000 | 13.8333 |
| 5× 5 | 9.3748 | 10.4167 | 11.4583 | 12.5000 | 13.5416 | 14.5833 | 15.6250 | 16.6667 |
| 4× 6 | 9.0000 | 10.0000 | 11.0000 | 12.0000 | 13.0000 | 14.0000 | 15.0000 | 16.0000 |
| 5× 6 | 11.2500 | 12.5000 | 13.7500 | 15.0000 | 16.2500 | 17.5000 | 18.7500 | 20.0000 |
| 6× 6 | 13.5000 | 15.0000 | 16.5000 | 18.0000 | 19.5000 | 21.0000 | 22.5000 | 24.0000 |
| 5× 7 | 13.1250 | 14.5833 | 16.0418 | 17.5000 | 18.9583 | 20.4167 | 21.8750 | 23.3333 |
| 6× 7 | 15.7500 | 17.5000 | 19.2500 | 21.0000 | 22.7500 | 24.5000 | 26.2500 | 28.0000 |
| 7× 7 | 18.3748 | 20.4167 | 22.4582 | 24.5000 | 26.5416 | 28.5833 | 30.6250 | 32.6667 |
| 6× 8 | 18.0000 | 20.0000 | 22.0000 | 24.0000 | 26.0000 | 28.0000 | 30.0000 | 32.0000 |
| 7× 8 | 21.0000 | 23.3333 | 25.6667 | 28.0000 | 30.3333 | 32.6667 | 35.0000 | 37.3333 |
| 8× 8 | 24.0000 | 26.6667 | 29.3333 | 32.0000 | 34.6667 | 37.3333 | 40.0000 | 42.6667 |
| 7× 9 | 23.6250 | 26.2500 | 28.8750 | 31.5000 | 34.1250 | 36.7500 | 39.3750 | 42.0000 |
| 8× 9 | 27.0000 | 30.0000 | 33.0000 | 36.0000 | 39.0000 | 42.0000 | 45.0000 | 48.0000 |
| 9× 9 | 30.3750 | 33.7500 | 37.1250 | 40.5000 | 43.8750 | 47.2500 | 50.6250 | 54.0000 |
| 8×10 | 30.0000 | 33.3333 | 36.6667 | 40.0000 | 43.3333 | 46.6667 | 50.0000 | 53.3333 |
| 9×10 | 33.7500 | 37.5000 | 41.2500 | 45.0000 | 48.7500 | 52.5000 | 56.2500 | 60.0000 |
| 10×10 | 37.5000 | 41.6667 | 45.8333 | 50.0000 | 54.1667 | 58.3333 | 62.5000 | 66.6667 |

**NUMBER OF TIES PER 1,000 FT., AND PER MILE OF TRACK**

| Distance | Distance, Center to Center, in Inches | | | | | | |
|---|---|---|---|---|---|---|---|
| | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
| 1,000 ft. | 667 | 600 | 545 | 500 | 462 | 429 | 400 |
| 1 mi. | 3,520 | 3,168 | 2,880 | 2,640 | 2,437 | 2,267 | 2,112 |

EXAMPLE.—How many feet, board measure, in the ties required to lay 1,500 ft. of track, the ties being 6 ft. 6 in. long, 5 in.×6 in., and spaced 22 in. between centers?

SOLUTION.— 1,500 ft. = 1.5 thousands of feet. From the two tie tables, 1.5 × 546 × 16.25 = 13,308.75, say, 13,500 ft. B. M.

**Entry Switches.**— In large mines, the switches connecting the main-road tracks with those of the cross-entries are of the standard surface railroad type, as described under the heading Railroad Surveying. In smaller mines, the layout shown in Fig. 1 is commonly employed. It will be noted that what is known as a cast frog is used and that the latch rails are pivoted or bolted
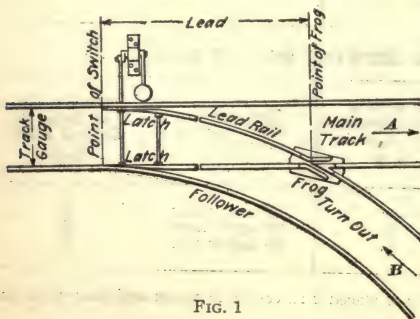


FIG. 1

and so are not sprung into place as on surface roads. Owing to the limited room for side tracks on mine roads, the lead of the switches is commonly much less than on surface roads and generally consists of but a single length of rail.

Where either motor or rope haulage is used, the lead is greater than where the hauling is done with mules. In this latter case, the switch rails may be any length up to about 15 ft., depending on the lead. On side entries, the length of the switch points is frequently but 2 or 3 ft., when they are known as latches. These latches may consist of a piece of bar iron tapered to a point so as to fit



FIG. 2

more snugly against the rail, and may be held in place by a bolt through a hole in the end.
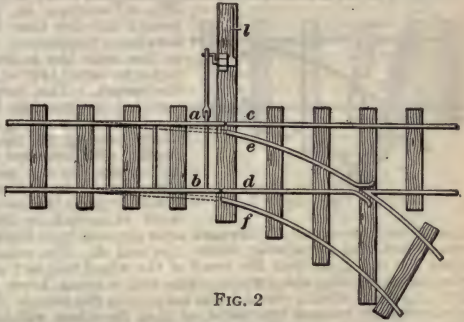
The stub switch shown in Fig. 2 is in common use where mule haulage is still employed, as it is much cheaper than the standard split switch, and answers every purpose where speeds are low. The rails *a* and *b* are free to swing from a point a little to the left of the last bridle, and their points slide on flat iron plates on the switch tie, which is broader and thicker than the other ties. The frog may be of the plate or cast type, or may be made by bending the rails composing the switch as shown in the figure.

**Frogs.**—On first-class main roads, either standard types of surface railroad frogs or plate frogs, as shown in Fig. 3, are employed. The latter consist of rails of the same section as those used on the mine track, shaped as shown in the



FIG. 3

cut, and riveted to a heavy iron plate, usually $\frac{1}{2}$ in. thick. The frog rails are fish-plated and bolted to the track rails, and the plate is spiked to the ties either at its sides or through holes in the plate. While these plate frogs may be made by any competent blacksmith, it is much cheaper to buy them.

One form of cast frog is shown in Fig. 4 and another placed in a turnout in Fig. 1. These frogs are cast in one piece and are inexpensive, but can only be used for temporary work, as they soon work loose from the tie and require constant attention. It is very difficult to get a straight workmanlike connection between them and the track rails, as they are not fish-plated thereto. When the spikes once work loose, the ties must be shifted to bring new wood under the lugs, as the spikes will not hold in the same holes. However, cast frogs are largely used on room entries where mule haulage is employed. The frog shown in Fig. 5 is made by welding or bolting together two rail ends



FIG. 4
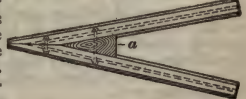


FIG. 5

beveled so that they fit properly. An oak block *a* placed in the frog angle between the rails helps to stiffen it.

**Room and Branch Switches.**—In mines of large capacity where motor haulage is employed on the cross-entries and gathering is done with motors, the room switches are laid as carefully as those on the main haulage road,

with plate frogs, points, etc. The following figures show various forms of simple switches often used where the cars are pushed by hand or hauled by mules, but which cannot be used where motor haulage is employed.

Fig. 6 shows a room switch with a cast-iron frog $f$ and fixed points $a$ and $b$.
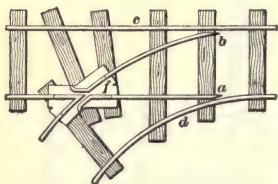


FIG. 6

The advantages of this switch are fixed points and the time saved when bringing cars from the rooms. Unless the point $a$ is in line with the main track, the point $b$ is liable to derail the car or cause it to run into the switch. This, however, can usually be avoided by making the rail $c$ somewhat lower than the rail $a$, thus causing the car while passing to cling to the rail $c$, and readily pass between the point $b$ and the rail $c$, and at the same time causing the wheel on the opposite side to take the rail $a$. Another great trouble experienced with this kind of switch is that where the wheels are allowed to remain on the cars after grooves have been worn in their treads, the wheel will invariably follow the rail $d$. The point $b$ should be higher than the rail $c$, so that the tread of the wheel will not strike the rail $c$ while the car is leaving the switch. The rail $c$ being lower than the rail $a$, it is obvious that when a car is to be taken into the switch, the driver will have to push the car toward the rail $d$, so that the wheel will take the rail $b$ and the flange of the wheel on the opposite side will pass between the point $a$ and the rail $d$. This form of switch is not applicable in the case of mechanical haulage, because it does not give an unbroken main line, which is essential to the steady movement of the trip.
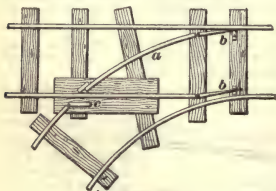


FIG. 7
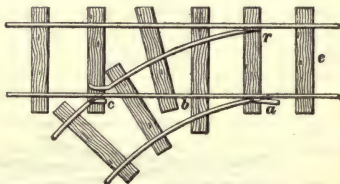


FIG. 8

The switch shown in Fig. 7 has loose latches $b$. Instead of a frog, a frog latch $c$ is used, which requires the lead rail $a$ to be raised a certain height above the rail of the main track, so that the latch $c$ can be thrown across this rail. The latch $c$ is held in position at one end by a strong bolt, and at the other end by a piece of iron spiked to a plank underlying the frog, as shown. By the use of this switch, the main track is broken only at the point of switch.

Fig. 8 shows a form of switch giving an unbroken main track. The lead rail of this switch has a fixed point; a frog latch $c$ is used similar to that shown in Fig. 7, and a switch latch $a$ is used on the follower rail. This latch has a slight projection on its under side to prevent its slipping off the rail of the main track when in use. This form of latch is undesirable, since, if, by the negligence of the driver the latch is not removed after being used, it will derail cars on the main road, since it is not easily pushed aside by a car passing out.
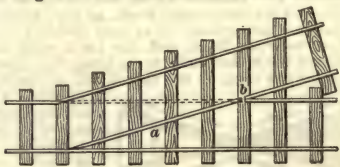


FIG. 9

Fig. 9 shows a rough arrangement where a turnout or any other condition requires the temporary use of a switch. The ordinary form for narrow gauges consists of a movable rail $a$, about 6 ft. long, pivoted on a center $b$. Where the curve is not great, this arrangement acts admirably where cars are pushed by hand, but for mule or locomotive haulage it is not recommended. The dotted line shows the position of the rail $a$ when the straight road is in use.

Fig. 10 shows a switch for permanent tracks in coal mines. No frog or latch is required. By turning the lever $h$, the throw rod $o$ moves the cranks $ikm$, so that the rails $r$ will face the rail $df$, the rail $n$ will face the rail $b$, and the rail $g$ will face the rail $f$. The lead and other rails can be reduced to any required length to suit circumstances. When the lead rail $fd$ is from 12 to 16 ft. long, and the other lengths are in proportion, the switch gives excellent results. It should not be made of less than 20-lb. rail, and heavier will be better. The objection to this switch is that the point of curve comes where the stub ends of the rails face each other and the angle formed causes the car to lurch.

**Diamond Switch.**—What is called a *diamond switch* in trackwork is a double crossover, such as is shown in Fig. 11 (*a*), (*b*), and (*c*). The laying out of a diamond switch is similar, as far as the calculation is concerned, to the laying out of a turnout switch. A simple method, and one that is often used in mine work, especially where track room is limited, is as follows:

Through a central point $a$, Fig. 11 (*a*), midway between the two tracks, draw two straight lines at right angles to each other, each making an angle of 45° with the track rails. Extend these lines until each intersects lines drawn through the points of switch $c_1$, $c_2$, $c_3$, and $c_4$ to which the latches come, if used, and at right angles to the track rails at these points. These intersecting lines are the centers of the turnout curves, and are marked $d_1$, $d_2$, $d_3$, and $d_4$, respectively. Where the diagonal lines cross the inner rails of the two tracks, $b_1$, $b_2$, $b_3$, and $b_4$ are the main-rail frog points.

It is evident that, in this construction, according to the distance between the track centers, the diamond-frog points, at each side of the diamond, will lie between the two tracks, as shown in (*c*), or they will be coincident with the two inner rails of these tracks, as shown in (*b*), or they will lie within the track rails, as shown in (*a*). The position of these frog points depends on the length of the crossing or the distance between the opposite switch points measured on the main rail, as compared with the distance between the track centers.

**Notes on Tracklaying.**—As explained under the heading Survey-ing, it is advisable to



FIG. 10

have the points (sights, sight plugs, strings, etc.) on which the entries are driven set such a distance from the rib that they come over one of the rails and may thus be used to aline the track. The distance the points are set from the rib depends on the width of the car but is commonly from 2 ft. to 3 ft. By keeping one rail in line with the plugs, the ample and uniform clearance demanded by the laws of most states may be maintained between the side of the car and the other rib.

It is not customary to lay the permanent track as fast as the entries are driven. Light rails spiked to widely spaced and unballasted ties laid on the floor are used at first, and after the entry has advanced three or four rail lengths (90 to 120 ft.) the permanent track is laid.

Before the rails are put down, the roadbed should be surfaced and brought to grade. While this is not always done, it is the better practice except where surfacing material must be brought from a distance, because by so doing the roadbed will be firm from the outset and will not require attention because of subsequent settling.

After the ties are placed at about the proper spacing, the rails are placed upon them and fish-plated together. The ends of the ties should be lined up so that on some one side their ends project a uniform distance, say 8 or 10 in. from the rail. A preliminary lining up is given by the sight strings before the rails are laid in position, but the final lining of the ends of the ties is done by measuring with a foot rule as the rail is about to be spiked to the tie. After the rail lengths on one side of the entry are spiked, the rails for the other side are laid in place and fish-plated together, and before being spiked are brought to gauge.

In first-class entry work the track is ballasted with crushed stone as in surface railroad practice and surfaced with finer material of the same kind.
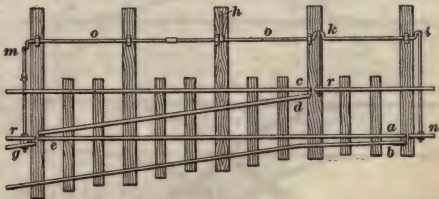
Neither draw slate nor bottom rock make satisfactory ballast, particularly if the mine is wet, as they soon disintegrate in to an impervious clay that holds water and becomes soft and mushy, a condition that results in the track soon being out of line and grade. Under no circumstances should coal or bone be used for ballast, as they soon are ground into powder and furnish an excellent material for the propagation of a dust explosion.

In side-entry work, where the track is not ballasted and the ties are laid upon a fireclay floor that is wet, they soon sink into it, and if animals travel over such a roadbed, it soon becomes muddy and affords an insecure footing.

Where depressions in the floor allow water to accumulate, this state of affairs is particularly apt to occur, so that especial attention should be paid to the ditching in order to drain off the water. If the water comes from the roof and drips on the track, the soft clay must be dug out and ashes substituted. The ashes may absorb the moisture and dry the fireclay to such an extent as to make the roadbed serviceable, but in case they do not, additional ties should be put in so that they are close together. If it is necessary to place the ties close together when the road is first laid, only a part of the ties are spiked to the rails until the track and bed have been put in shape; then the rails are spiked fast to the other ties. This forms a corduroy roadbed and will afford a fair roadbed and track, although it may need overhauling from time to time as the clay swells up and mixes with the cinders.

In some mines that have soft clay bottoms, it is the custom to lay mud sills of $3'' \times 12''$ plank parallel with the track and on these to place the cross-ties. The planks are sometimes placed skin to skin; the same care, however, is necessary in this as in the former case to provide for drainage and to prevent the clay oozing up between the planks.



FIG. 11.

The ordinary spacing of ties on main entries is from 18 in. to 2 ft., measured between the centers of the adjacent ties. In speaking of the spacing of ties, the distance between the edges of the ties is sometimes used instead of the distance between centers. Thus, if the ties have an 8-in. face and are spaced 2 ft. from edge to edge, there will be $2\frac{2}{3}$ ft. between the centers, which is too great a distance for a roadbed on which there is a heavy traffic.

On cross-entries where mule haulage is employed, a distance of 2½ to 3 ft. between centers is allowable where 30-lb. rails are used. Ties for room track have a 3- to 4-in. face and sufficient thickness to take the spike, and are placed from 3 to 4 ft. apart, where the cars are pushed by hand, but where motors enter the rooms the ties must be spaced as in entry work, but ballasting and grading is rarely necessary.

On room entries, it is advisable to lay the room switches at the same time the permanent track is put down. This prevents any subsequent interference with traffic and, further, the work can be done much better when the track-layers are not stopped by passing trips. If the rooms are driven on points set a uniform distance apart along the entry, the room sight plugs may be used by the tracklayer as a guide in placing the frogs. In many mines where the rooms are evenly spaced and a standard frog, etc. are used, the entire switch is purchased, all the lead, follower, switch, and point rails being cut to an exact length and properly drilled for plating and bonding. This is a great help toward securing a first-class track, as the frog at No. 1 room having been placed exactly, those on the rooms inside must of necessity come right.

Unless the pillars are to be drawn as soon as the rooms reach their limit, it is customary to take up the room tracks and in some cases the frogs on the entry, relaying them when pillar drawing is about to begin. Whether this is a wise policy depends on circumstances. In wet mines where the rails may be eaten by acid water, or where the roof is bad and the track is apt to be buried under falls, or when 5 or 10 yr., or more may elapse before pillar drawing begins, it is advisable to remove the rails from worked-up rooms and to use them elsewhere. But where the mine is comparatively dry, the roof good, and pillar drawing will begin in a year or two, the rails had better be left in place, as the cost of taking up and relaying them will more than offset the interest on the idle capital.

Rails should be stacked and not thrown in irregular piles. Three heavy timbers should be laid upon level ground in such a way that they support the ends and centers of the rails, which are placed side by side upon them. When one row of rails is filled, additional and lighter timbers are laid upon it, and another row filled, and so on, until the stock on hand is neatly piled, each length by itself. Ties should be stacked in a similar way to props, as explained under the heading Timbering. Both ties and rails are better kept outside the mine, a day's or a week's supply being brought in at intervals as needed.

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# ANIMAL HAULAGE

**Selection of Stock.**—While horses and ponies are generally used abroad for mine haulage, mules are preferred in the United States, as being hardier, less nervous, and more easily broken to their work. Large heavy mules with long backs and relatively short legs can exert their strength to greater advantage than short-bodied long-legged ones, although this is not always admitted. Mere weight is not an indication of strength, as it may be due to fat, but a good working weight, say up to 1,400 lb. without clumsiness or thick hocks, is to be desired in a mule. Perhaps the best mules come from Missouri and Kentucky and, for mine use, have an average weight of about 1,200 lb. and a height of about 16 hands.

Mules from 4 to 6 yr. of age that have been worked are easier to break to mine work than those without training of any kind. Mules are naturally influenced by changes of water, diet, altitude, etc., and before being tried in a mine should be given ample time to become accustomed to their new conditions. If this was always done, probably a much smaller number would be rejected as being unsuited to mine work, for a mule cannot be expected to work when it is not well.

While some mules give absolutely no trouble when first taken into the mine and will pull loads from the outset, the average mule has to be broken, and for this purpose should be handled by two men, the driver and an expert in managing stock. At first, the load should be light, the trips short, and the mule not worked for more than 2 or 3 hr. The load, length of trip, and number of hours worked may be increased daily. When properly harnessed and cared for and, above all, kindly treated, the average mule soon learns his duties, will back up to the trip without command, and will follow his driver's call or whistle.

**Feeding Mules.**—Mr. H. W. Hughes, gives the average daily ration at an English colliery for 80 horses averaging 15 hands high, the figures covering

a period of 8 yr., as follows: Grain, 7.25 lb.; bran, 9.25 lb.; hay, 18.75 lb.; total, 35.25 lb. The grain was composed of beans, 3 lb.; maize (corn), 2.75 lb.; and oats, 1.50 lb. The last item was hay, 14 lb.; clover, 1 lb.; and straw, 3.25 lb.

The American mule appears to be fed about two-thirds as much as the English horse. Mr. Chas. E. Bowron, gives the food allowance for mules at several mines in Alabama and Tennessee, the first figures being pounds of hay and the second, pounds of grain: 9.54 and 17.44, 7.92 and 19.20, 7.94 and 15.04, and 12.62 and 15.50. The total daily food allowance in these cases was 26.98, 27.12, 22.98, and 28.12 lb., respectively. It will be noticed that the weight of the hay was about one-half that of the grain, reversing the usual practice. Mr. Bowron further gives the allowance for army mules during the Spanish-American war as hay 14 lb. and grain 9 lb., and for the horses hay 14 lb. and grain 12 lb.

In the anthracite regions of Pennsylvania, the average ration for mules from 1,000 to 1,200 lb. in weight is 12 lb. of grain and 15 lb. of hay. The composition of the grain varies from two-thirds cracked corn and one-third oats, to equal proportions of each. Corn is richer in fat-producing elements than oats and is fed to give strength, but too much grain will cause acute indigestion, paralysis of the walls of the stomach, and usually results in death. A feed of bran once a week is recommended as a laxative; also a handful of pure coarse ground salt twice a week.

Mules should be fed three times a day, although some large companies feed but twice daily. On idle days, the food allowance may be reduced 25 to 30%. Hay is digested chiefly in the intestines and grain in the stomach, hence, if possible, a mule should be first watered, then given hay, and lastly grain. If the water is given last, it washes the food into the intestines before it is acted on by the gastric juices in the stomach. If the hay is given after the grain, it carries the grain with it into the intestines. This order of feeding is not always practicable in a mine and it is of advantage to place watering troughs about the mine so that the mules can be watered during the day while at work. As the feed is in the boxes when a mule is put in the stable at night, there should also be water in his water trough so that he can drink at intervals while feeding. Fresh food should never be placed on top of any left over from the previous feeding. A mule should have plenty of water the first thing in the morning, and care should be taken to have the water pure and the troughs clean.

**Care of Mules.**—Mine mules should have clean comfortable quarters, with pure water and food. Their feet and legs should be washed every night and their hocks dried; and they should be combed regularly. Extreme care should be taken that they are shod properly, and a competent shoer is imperative at mines where many mules are used. If the mine is too small to warrant the constant employment of a veterinarian, arrangements should be made for one to visit the stables monthly to look over the stock, paying particular attention to their feet.

The stable boss or mine foreman should inspect the harnessing of the mules before they begin work. All parts of the harness must fit properly, particularly the collar, which transmits the weight to the mule's shoulders; the hames, to which the traces are attached, should bear evenly upon the collar. The traces should be of equal length and free from knots; many insert a coiled spring between the trace and the car to take up the jar of starting.

Mules should not be worked more than one shift per day, and if overtime is necessary, should be given a chance to rest the next day. If stabled underground, they should, unless the expense is prohibitory, be brought to the surface from the close of work Saturday until Monday morning. This procedure is not only humane, but the fresh air with the chance to run and roll in the pasture and to nibble at the fresh grass, keeps the animals in health, adds to their efficiency, and prolongs their life.

Estimates of the length of the working life of mine horses and mules vary so widely that it seems impossible to give an average. Mr. Hughes, quoted before, gives the average useful life of horses working in English mines as more nearly 9 than 8 yr. This figure is based on 13 yr. experience at some large collieries and should be considered authoritative. Mules seem to have a much shorter working life than horses. Mr. Bowron assumes the average working life of a mule in the mines of the Birmingham, Alabama, district, to be 6 yr. The records of the Fairmont Coal Co. for 1905, show that in that year, 26% of their stock either died, was killed, or had to be disposed of on account of being crippled or worn out. This shows a working life of between 3½ and 4 yr. In

estimating the cost of mule haulage it is probably well to count on the life of a mule as 5 yr., and that 10% of the stock is in the stable either sick or temporarily disabled.

**Work of Mules.**—The amount of work that a mule can do is dependent on the strength and condition of the mule, the condition of the track and rolling stock, the relative sizes of paying and dead load hauled, length of trip, presence or absence of grades that may be for or against the loads, etc.    Mr. Bowron gives the following figures for mines in Alabama and Tennessee:

### WORK DONE BY MULES

| Group | Average Haul Mile | Average Output Tons | Average Ton-Miles per Mule | | Net Cost per Ton-Mile | Conditions |
|-------|-------------------|---------------------|---------|---------|----------------|------------|
| | | | Gross | Net | | |
| 1 | .32 | 513 | 12.4 | 6.9 | 35.7 | Unfavorable |
| 2 | .37 | 861 | 24.8 | 13.8 | 17.9 | Average |
| 3 | .78 | 502 | 41.4 | 23.0 | 10.7 | Best |
| 4 | .64 | 887 | 38.3 | 21.4 | 11.5 | Average |

The average haul is the distance traveled in bringing out the loaded car; the total haul is twice this.   In the columns headed Average Ton-Miles Per Mule, the figures under the heading Net are for the paying load of coal hauled. The figures in the column headed Gross are based on the assumption that the car weighs 40% as much as the coal carried, but is carried twice as far.   The mines in Group 1 are four in number with unfavorable conditions caused by short hauls, and steep adverse grades.   In all but one of these mines, the mules were employed solely for gathering.   In the seven mines in Group 2 and the two mines in Group 4, the mules were employed only for gathering, and average conditions prevailed, the mines being fully developed, the hauls of fair length, and the track in reasonable shape, etc.   In the three mines of Group 3, mules were used both for main-line haulage and gathering.   A comparison of Group 2 with Groups 3 and 4 shows that better results are obtained when the hauls are of fair length, as less time proportionately is taken up in changing trips.

It is estimated that a horse or mule will exert a tractive effort equal to one-fifth of its weight at a speed of 2 to 4 mi. an hr. for 1,000 to 1,200 hr. per yr., say for 4 to 5 hr. per da. in a mining year of 200 to 220 da.   In starting a load from rest, a much greater effort is exerted for a limited time.

When gathering single cars, where the most distant room is not much over ¼ to ¾ mi. from the parting, a mule should make two to three round trips per hour, and bring in fifteen to twenty-five loads per day.   In seams of moderate thickness where the cars hold, say, 1.5 T., this means the delivery of a paying load of from 22.5 to 37.5 T. per da.   In thick seams, where the load is 2.5 to 3 T., the production per mule will vary between 60 and 75 T.

In hauling from an inside parting to the drift mouth, where the distance is from ½ to ¾ mi., and the grades are such that a mule can haul two loaded cars, one animal will deliver from thirty to forty loaded cars per day, equivalent to a paying load of from, say, 45 to 100 T., depending on the size of the car.

These results cannot be obtained unless the management is competent, and sees to it that the rails, roadbed, and cars are in first-class condition, that the miners are properly distributed so that there are no unnecessary delays at the face in waiting for loads, and that the mules are well fed, well shod, and properly cared for.

When the seam is pitching and the entries are crooked with irregular grades and the track is soft, as in the anthracite fields of Pennsylvania, no average figures can be given because the conditions vary so from mine to mine, but, in general, the efficiency of a mule is about one-half that of the same animal in flatter and more regular seams.

**Cost of Mule Haulage.**—In order to compare the cost of haulage at one mine with that at another, haulage costs should be given in cents per ton-mile; that is, the cost of hauling 1 T. of coal 1 mi.   Further, the underground conditions should be known, for it is possible that there is greater efficiency where the cost is, say, 15 c. per T.-mi., than where it is but 10 c.

The cost of mule haulage is made up of three items; depreciation, feed and care, drivers' wages.

If ten working mules per day are required for a given tonnage, eleven must be provided, as one is practically certain to be laid up for the time being, either sick or crippled. If the life of an American mine mule is but 5 yr., and his cost is $250, $55 must be allowed annually per working mule for renewals, on the basis that 10% of the stock is idle.

If the mule is fed 12 lb. of corn and oats in equal proportions and 15 lb. of hay per day, his total food consumption will be 5,475 lb. of hay and 2,190 lb. each of oats and corn per year. At $25 per T. for hay, 45 c. per bu. of 32 lb. for oats, and 80 c. per bu. of 56 lb. for shelled corn, the individual cost of these items will be $68.44, $30.80, and $31.29, or a total of $130.53 per yr. Allowing for the feed of the idle mule, the annual cost per working mule will be $144.06. The wages of the stable boss, harness, shoeing, services of veterinarian, etc., will be fully $60 per yr. per mule at large mines and from $100 to $125 per yr. at small ones. Allowing for the idle stock, probably $90 per yr. is a reasonable charge. One working mule will therefore cost $55 for renewals, $144.06 for feed, and $90 for stable charges, etc., or a total of $289.06 per year of 365 da., or 79.2 c. per da. However, the mines do not run 365 da. per yr., the working days averaging about 220. On this basis the fixed charges per working mule per working day will be $289.06 ÷ 220 = $1.314. To this must be added the driver's wages, which at present vary from $1.75 to $2.50, averaging, say, $2.125, making the total cost per working mule per working day, $3.439. The cost per ton of coal shipped is found by dividing the cost of all the mules by the total tonnage. Thus, if ten mules at a total cost of $10 \times \$3.439 = \$34.39$ handle an output of 900 T., the cost per ton is $34.39 ÷ 900 = 3.821 c. If the average distance hauled is $\frac{3}{8}$ mi., the cost per ton-mile per ton of output is $3.821 \times \frac{8}{3} = 10.189$ c. Since each mule delivers an average of 90 T hauled $\frac{3}{8}$ mi., the ton-mileage per mule is $90 \times \frac{3}{8} = 33.75$. Assuming the cars to hold 2.5 T., the output requires the delivery of 900 ÷ 2.5 = 400 cars per da. If the cars weigh 1 T. each, the total weight handled by the ten mules per day will be, coal 900 T.; outbound loaded cars 400 T.; inbound empty cars 400 T.; or a total of 1,700 T. hauled $\frac{3}{8}$ mi. at a cost of $34.39. This is equivalent to 637.5 T. hauled 1 mi. for ten mules, or 63.75 T.-mi. per mule, at a cost of $34.39 ÷ 637.5 = 2.02 c. per T. gross (cars and coal) of material hauled, and $34.39 ÷ 637.5 = 5.4 c. per T.-mi.

The haulage costs given by Mr. Bowron in the preceding table are based on drivers' wages of $1.762; depreciation $25 per mule per year; feed and stable attendance 34.9 c. per da. for 365 da.; interest 3.3 c. per da. for 365 da.; making the total cost per working mule per working day, of which there were 276 in the year, $2.46.

In the reports of some coal mining companies, a charge of 50 c. per da. per mule is made for all items other than drivers' wages. This is, obviously, altogether too little.

**Safe Grade for Mule Haulage.**—While the grade against empties on the main haulways can be 1.5% the grade on cross-entries should not exceed .5 to 1%, where mules must gather cars in a hurry. If the mules are winded in taking in empties, the loaded cars must necessarily come out slower, so that the advantage gained by quick delivery of empty cars is offset by the loss of time in returning the loaded cars. Often mine mules are injured by winding them and then not giving them time to recover their breath for the return trip. The driver has not so much control over his car and animal that he can stop instantly, and if the mule lags or stumbles, the car will probably run against the mule and injure its legs. A safe down grade for mule haulage should not exceed 3% and great care will be needed in that case. On such steep grades, while the mule can pull up the ordinary mine car, the brakeman or driver should run the car down independent of the mule. A loaded mine car will slide on rails even with four wheels spragged when the grade is 6 to 8%, depending on the condition of the rails.

## SELF-ACTING INCLINES*

In a *self-acting incline*, otherwise known as a *gravity incline, gravity plane,* or simply, as an *incline*, the weight of a loaded car descending an inclined plane is utilized to raise an empty car. In hilly regions, where the coal seams outcrop at a considerable elevation above the valley, inclines are in common use to lower

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* See also Slope Bottoms.

the loaded cars from the mine to the tipple. They are also, but far less frequently, used underground to lower loaded cars from one level to another.

**Tracks, Switches, Etc.**—The tracks upon inclines may be arranged in one of three ways. The best arrangement consists in two separate tracks throughout the entire length of the incline. Where the amount of coal handled is not large or where the capital is not available, it is quite common to use three rails both above and below a parting or turnout midway of the incline where there are four rails. That is, both above and below the parting where there are two independent tracks, the middle rail of the three is common to both the ascending and descending tracks. In the third arrangement, while there are three rails above the parting, there are but two below it.

Self-acting switches set by the mine cars are provided at both the top and bottom of the incline where the rails join to form a single track, and similar switches are used at both ends of the turnout on three-rail inclines. Safety switches for derailing runaway cars are frequently used. These are usually of the *spring-latch type*, which are closed by the ascending empty car and open automatically after it has passed. They are thrown to permit the passage of the loaded car by means of a lever at the head of the incline, the connection between the switch and lever being made by wire or rods in the same manner as that employed on surface railroads to throw distant switches. Safety blocks are used at the top of the incline near the knuckle to prevent the cars running away down the incline before they are attached to the rope. The blocks may be heavy timbers placed by hand, or a more elaborate arrangement of iron set by levers or automatically by the cars.

The weight of the rail used should be proportioned to that of the loaded car. As the incline must last the life of the mine and all the coal shipped must pass over it, rails weighing less than 50 lb. per yd. are not to be recommended where the loaded car weighs as much as 5,000 to 6,000 lb. Lighter rails may be used for lighter loads, but are hardly to be advised as the smaller sections require a closer spacing of the ties, more attention on the part of the repairmen, and are more apt to be bent in case a runaway trip jumps the track. First-class rails should be used and the roadbed should have a uniform slope and should be well ballasted. In fact, incline tracks should be laid with as good material and with as much care as those on a main entry where motor haulage is employed. In order to prevent the track creeping or sliding down hill on steep pitches, every second or third tie is made long enough to extend across both tracks and its ends are anchored in hitches cut in the sides of the excavation, are braced by posts firmly planted in the ground, or are held by wire ropes or iron rods fastened to solid objects such as iron bolts sunk in a ledge of rock, large trees, etc. Square notches should be cut in the base of the rail at intervals of 10 or 15 ft. depending on the slope of the incline. These notches should come upon the long or anchor ties, to which the rails are in turn anchored by track spikes through the notches.

**Rollers.**—Rollers for supporting the rope should be set closer together than upon flat or upon steep inclines. Their distance apart varies from 12 to 18 ft., an average spacing being 15 ft. It is better to vary the spacing of the rollers by a foot or more each way from the average, as this will, in a very great measure, prevent the flapping of the rope which is so wearing upon it and the rollers and which is almost sure to occur when the rollers are exactly the same distance apart. The jolting of the cars shakes much coal from them, and this soon fills the spaces between the ties and clogs the rollers and prevents their turning. When the rollers are not free to revolve, the frictional resistance to the passage of the rope and the wear upon it and the rollers is very great; consequently, the track around the rollers should be cleaned frequently, sometimes as often as once or twice a day. With tracks in fair shape and rollers 12 to 15 ft. apart, actual tests have shown that the resistance due to the friction of the rope in running empty cars down grades of from 3.8 to 6.2%, varied from 6 to 15% of the weight of the rope.

**Ropes, Drums, Barneys, Etc.**—The strain on the hoisting rope is equal to the force required to accelerate and hoist the load and to overcome friction, and may be calculated by the formula $R = F + G + I$, as explained under the heading Resistance to Haulage, noting that the weight of the rope must be added to that of the car. As the wear on the rope from passing over idle rollers, trailing on the ground, and the jerks due to dropping the trip over the head of the incline is so great, a much higher factor of safety should be used than in the case of shaft hoisting ropes; 10 is none too large. Ropes for inclines are commonly made with six strands of seven wires each laid around a hemp center (see section on Wire Rope) and those of the lang-lay and locked-coil

types are often favored because of their larger wearing surface and consequent longer life. The rope is usually attached to the car by a chain 15 to 20 ft. in length, socketed or clamped to the rope and provided with a clevis or hook. Where the pitch is very steep and slope carriages or gunboats are used, bridle chains similar to those used on shaft cages are attached to the corners of the carriage or gunboat and to the rope at some point above the socket.

Where cars are attached to the rope by couplings they are subject to strains that rack them while being pushed over the knuckle; there is possibility of accident from their running over the head of the incline before the rope is attached; time is lost in hooking the cars to and unhooking them from the rope; and the rope is liable to be kinked and unduly strained in the hooking-on process. To avoid these causes of wear and danger, it is a common practice, where conditions are otherwise favorable, to attach to the end of the rope what is called a *barney*. The barney consists of a small car or truck running on light rails placed between those of the regular tracks of the incline. At the bottom of the plane, the barney passes into the *barney pit* so that the mine car may pass over it to the tipple. When cars are being hoisted, the barney comes out of the pit and pushes the cars ahead of it up the incline. Similarly, at the top of the incline, the cars are dropped against the barney and are held back by it as they are lowered. A barney may be built of heavy timber or iron and must have sufficient weight to prevent its being lifted from the track at the bottom of the incline by the weight of the cars pushing against it. The face of the barney should be covered for its full width and height by a sheet of heavy plate iron to afford a wide bearing for the bumpers of the car. Cars intended for use on inclines operated with barneys should have bumpers of larger face than usual so that they may not ride or interlock while being raised or lowered. To avoid the cost of a special track for the barney, one form of this device is provided with wheels which have double grooves on their face and a lateral play on their axles. For most of the trip the outer grooves of the wheels run on the same track as the cars, but at the bottom of the incline the inner grooves engage a supplementary barney track, the wheels are forced inwards on their axles to accommodate themselves to the narrower gauge, and the barney passes into the usual pit.

At the head of the incline, the ropes pass around a drum set, usually, at such an elevation that the cars may pass under it, one rope winding on the drum as the other winds off. The diameter of the drum is determined by the same rules that apply in the case of hoisting engines, and its width should be such that when the cars are in the middle of the incline (but not on a turnout in the case of three-rail inclines) the ropes will be in the centers of their respective tracks. If the head of the incline is built upon a trestle, in order to secure strength and steadiness and to avoid massive and expensive construction, it is usual to place the drum below instead of above the car tracks. Where the pitch is steep and a single drum is used upon which one rope winds on at the top and the other winds off at the bottom, the upper rope will be so high if the lead is short (that is, if the drum is near the knuckle) that the cars will be lifted from the track as they drop over the knuckle. This difficulty can be overcome by the use of two drums geared together so as to turn in opposite directions. This allows the ropes to run off the same side of their respective drums and at the same height above the track. The drums can then be placed such a distance back from the knuckle and such a distance above the track that the ropes are tangent to the rollers on the incline. This arrangement calls for the use of two brakes, one for each drum, which, however, may be placed close together on the same stand. Instead of drums, a pair of wheels each with a series of parallel grooves on its face, may be employed. The rope is in one piece and after making several turns around the pair of wheels has its end hitched to the loaded and empty trips respectively or to barneys. These wheels are set below the floor of the drum house at the head of the incline so that they revolve horizontally, and their axles are vertical and in the center line between the two incline tracks. They are made of such a diameter that the ropes are in the center of the tracks; that is, the diameter is equal to the distance between the center lines of the tracks. The greater the tension necessary to prevent the rope from slipping, the greater is the number of grooves; four or six are commonly used. Sometimes, to give a greater bearing surface between the rope and the wheels, the rope is lapped in the form of a figure 8; but this is to be avoided because of the undue bending strain thus thrown upon the rope.

The brakes controlling the speed of the drums must be very powerful, strongly constructed, and continually watched that they may be in good and effective condition. This is particularly necessary with steep pitches and

heavy loads, owing to the increase in the momentum as the pitch and the loads increase and to the fact that the load is constantly increasing as the rope on the load side winds from the drum while that on the empty side winds on it.

**Grades and Their Effects.**—On short inclines where the difference between the weight of the loaded and empty cars is considerable (say, those where the length does not greatly exceed 500 ft. and the load is equal to or greater than the weight of the car), a pitch of 3°, equal to a grade of 5.24%, may be sufficient to impart motion to the cars if they and the track are in first-class condition. Where the incline is long or where the weight of the empty car bears a greater ratio to that of the loaded car than that noted, pitches up to 10°, or 17.64%, may be necessary to start the trip. The greatest pitches on which cars can generally be run without spilling their contents over the end gate are about 15° (26.8%), for cars with topping to 20° (36.4%), for cars loaded level full. In some cases where the seam is thick enough to permit their use, cars with high backs are employed where the incline is steep. Where the pitch exceeds, say, 20° and where the breakage of the coal is not objectionable (as where the mine run coal is coked) the contents of several mine cars are dumped at the head of the incline into a large sheet-iron car, or tank, called a *gunboat* or, sometimes, a *skip*. These are not detached from the rope and are, of course, run in pairs in balance, and dump or discharge automatically at the foot of the incline into bins or upon screens. When breakage must be avoided and the pitch is too steep to permit the cars being lowered in the usual way, they may be placed upon a platform on wheels attached to the ropes. These platform cars are variously known as *slope carriages, dummys*, etc. When these are used, the tracks at both the head and foot of the incline are generally at right angles to those on the plane so that the car rests sidewise on the rails of the carriage. The loading and unloading of the carriage is performed in much the same way as on a shaft cage; that is, the loaded car pushes off the empty car at the top of the incline, and the empty car pushes off the loaded one at the bottom. If the car is placed on the carriage endwise instead of sidewise, particular attention must be taken to lock it securely in place.

If the cars are attached to the rope in trips, the drawbars must be continuous and they and the couplings must be of heavier metal than in hauling on a level. Further, the steeper the pitch and the longer the trip, the heavier must be the couplings. When barneys are used, special couplings are not necessary.

**Conditions Unfavorable to Use of Inclines.**—Inclines requiring for their operation a large number of cars, say, six or more, are expensive to operate and maintain. Barneys cannot be used as the bumpers are practically certain to interlock if the cars are pushed up a track that may not be in the best condition. Further, as the barney must generally be placed the length of a trip below the knuckle and the trip dropped down upon it, the strain upon the rope and drum from the impact of the cars is so great as to be extremely dangerous. Running a large trip attached to a rope also has objectionable features. The cars must be very heavy in all their parts to withstand the strain upon the couplings as the trip is pushed over the knuckle on the loaded side and as the slack between the empty cars is taken up at the foot of the incline. For this reason, the strain upon the rope and drum is great. The labor required at both the top and bottom to handle such a large number of cars is excessive, and its cost would pay a good interest on a rope haulage plant, retarding car haul, or electric motor for transporting the coal from the mine to the tipple.

Inclines give excellent results at mines of limited output where the plane is short and one-car trips can be run with barneys, but even under the most favorable conditions it is a question for consideration if they may not well be replaced by a retarding conveyer or car haul.

**Calculations for Self-Acting Inclines.**—Because the same number of cars is attached to each of the ropes, their weights balance and an incline will be self-acting if the weight of the coal to be lowered is greater than the weight of the rope to be raised together with the weight representing friction and other resistances. The following formulas may be used in solving problems connected with motion on self-acting inclined planes:

$$a = \frac{F}{W}g \ (\sin X - f \cos X) = \frac{v}{t}, \ v = at, \ t = \frac{v}{a} = \sqrt{\frac{2s}{a}}, \ \dot{s} = \frac{vt}{2}$$

These formulas, with the exception of the first, are the familiar ones applicable to all cases of uniformly accelerated motion; $s$, however, is the length of the incline. Because motion on an inclined plane is due solely to gravity that component of it $g \sin X$ that acts parallel to the plane is the measure of

the acceleration. But this constant acceleration is diminished by the constant resistance of friction $fg \cos X$; whence, the net acceleration is the difference of the two.

It is apparent that when $\sin X = f \cos X$, the plane is in equilibrium with the forces on the two sides balancing, because then $a = \dfrac{F}{W} \times g \times 0 = 0$, and when there is no acceleration there can be no motion. From the relation $\sin X = f \cos X$, there results $\dfrac{\sin X}{\cos X} = f$, whence $\tan X = f$. This is the general formula for equilibrium on an inclined plane, for when $\tan X$ is greater than $f$, the plane is self-acting; when $\tan X = f$, the plane is in equilibrium; and when $\tan X$ is less than $f$, the plane is not self-acting and it will require more force than the weight of the coal to overcome the friction. This formula is very valuable in making preliminary calculations.

EXAMPLE.—(a) If friction $f$ is 4% (.04), can a plane pitching 2° be made self-acting? (b) Friction remaining the same, can a plane pitching 8° be made self-acting? (c) With friction as before, what is the least pitch at which the plane will be self-acting?

SOLUTION.—(a) Here, $\tan X = \tan 2° = .03492$, is less than $f = .04000$ and the plane is not self-acting.

(b) Here, $\tan X = \tan 8° = .14054$, is greater than $f = .04000$, and the plane may be self-acting.

(c) Here it is required to find the angle of elevation of the plane when its tangent is known. Since $\tan X = f = .04000$, $X = 2° 17' 27''$. At any greater pitch, motion will result and the plane will be self-acting.

It does not follow, however, that because motion is possible that a plane can be operated in practice. Thus, in (c), while motion will result when the pitch is increased to, say, 2° 30', the number of cars that would have to be handled in a trip would be so very great as to be impracticable from an operating standpoint.

After it has been determined, by the use of the preceding formula, that the pitch is sufficient to permit of the plane being made self-acting, the number of cars required in a trip to start motion on the plane is commonly found from the formula,

$$N = \frac{R \, (\sin X + f \cos X)}{C \sin X - (C + 2E) f \cos X}$$

in which, $N$ = number of cars in trip;

$\quad\quad C$ = weight of coal in loaded cars, in pounds;

$\quad\quad E$ = weight of empty cars, in pounds;

$\quad\quad R$ = weight of rope attached to empty trip, in pounds.

In the formula, all the terms are known except $f$, which, while commonly taken as .025 may be very much less, especially on flat inclines where the rope sags on the ground between the rollers or where the track is in bad condition and the rollers so clogged with dirt that they do not turn.

EXAMPLE.—How many cars will it require to start the trip on an inclined plane 2,000 ft. long and pitching 8°, when the car that weighs 2,500 lb. carries a load of like amount, the rope weighs 2 lb. per ft., and the friction is 4 per cent. (.04)?

SOLUTION.—Substituting in the last formula, there results,

$$N = \frac{4,000 \times (.13917 + .04 \times .99027)}{2,500 \times .19317 - (2,500 + 2 \times 2,500) \times .04 \times .99027} = 14.1 \text{ cars}$$

From this, it will require fifteen cars to start the trip.

In the formula, $a = \dfrac{F}{W} g \, (\sin X - f \cos X)$, $a$ = acceleration in feet per second per second, produced by an unbalanced force $C$, which is weight of coal in descending cars, in setting in motion weight $W$, which is weight of coal + weight of cars + weight of rope $R$ + weight of drum $D$. The formula may, thence, be transformed to read

$$a = \frac{C}{(C + 2E + R + D)} g \, (\sin X - f \cos X)$$

In a general sense it may be said that the weight of the coal $C$ is the motive power that sets in motion all the other weights including its own.

EXAMPLE.—In the preceding example, assuming that the rope drum is 8 ft. in diameter and 10 ft. wide and weighs 32,000 lb., and that there are fifteen cars in a trip, (a) what will be the time of descent down the plane? (b) What is the final speed, in feet per second and in miles per hour? (c) Fur-

ther, assuming that the speed is not allowed to exceed 25 mi. per hr., what will be the capacity of the incline in an 8-hr. da., making the usual allowance for delays, etc.?

SOLUTION.—As there are fifteen loads of coal, $C = 15 \times 2,500 = 37,500$ lb., $2E = 2 \times 15 \times 2,500 = 75,000$ lb., $R = 4,000$ lb., $D = 32,000$ lb., and $(C + 2E + R + D) = 148,500$ lb. Substituting in the formula, $a = \dfrac{37,500}{148,500} \times 32.2 \times (.13917 - .04 \times .99027) = .81$ ft. per sec. per sec.

From this the time of descent is $t = \sqrt{\dfrac{2 \times 2,000}{.81}} = 70.3$ sec.

The final velocity reached at the foot of the incline will be $v = .81 \times 70.3 = 56.94$ ft. per sec., or $(3,600 \times 56.94) \div 5,280 = 38.8$ mi. per hr.

As the speed is limited to 25 mi. per hr., or 36 ft. per sec., the brakes must be applied at the end of $t = \dfrac{36}{.81} = 44.4$ sec., when the trip has traveled down the incline a distance of $s = \dfrac{36 \times 44.4}{2} = 799.2$ ft. As retardation and acceleration are equal, the time required to start and stop the trip will be 88.8 sec., and the distance traveled during these periods will be 1,588.4 ft. The remaining $2,000 - 1,588.4 = 412.6$ ft. of the trip will be made in $412.6 \div 36 = 11.4$ sec. The entire time of descent will be $88.8 + 11.4 = 100.2$ sec., or 1 min. 40 sec., practically.

If 50 sec. is allowed for handling the ropes between trips, a trip will require $2\frac{1}{2}$ min., and 24 trips will be made in 1 hr. As $(15 \times 2,500) \div 2,000 = 18.75$ T. of coal is lowered each trip, the capacity of the incline will be 450 T. per hr. If 1 hr. a day is lost in various delays, the incline will handle an average net daily output of $450 \times 7 = 3,150$ T.

In the example, the radius of gyration has been taken as equal to that of the drum. The true radius of gyration is probably about $.4R$, but the error arising through the use of the larger and more easily obtained value is on the safe side.

**Profile of Inclines.**—If an incline is a plane of uniform pitch, as commonly is the case, the weight of the descending trip constantly increases and that of the descending trip decreases, as one rope winds off and the other winds on the drum. Hence, the force $F$ and, consequently, the acceleration $a$ constantly increase and, therefore, the force that must be applied through the brake to keep the speed of the trip within the established limits greatly increases from the start to the end of the run.

The most easily operated incline, that is, one on which the force applied through the brake is constant, has a vertical tangent for such a distance below the knuckle that the trip in descending it acquires the proper velocity; it also has, from the end of this tangent to a point near the foot of the incline, the shape of a curve such that the velocity of the descending trip is constant and, hence, the brake resistance is uniform, and ends with a curve of such shape that the velocity of the trip at the end of the run is just sufficient to carry it to the tipple.

From the fact that after a trip has been started and has descended the incline some distance much less weight is needed to keep it in motion, if the upper end of the incline is given a greater pitch a much smaller trip may be set in motion. In fact, relatively flat inclines can often be made to work only by increasing the grade for from 25 to 75 ft. at the upper end to double or more than double what it is on the body of the plane. This increased grade is known as the *steep pitch*. All inclines, regardless of their pitch, terminate in a curve that is tangent to the plane at one end and to the tipple floor at the other. The object of this curve is to reduce the speed of the trip to an amount just sufficient to carry it to the dump.

# JIG PLANES

A *jig plane* is a modification of the self-acting incline in which the weight of a descending loaded car raises a counterweight running on wheels on a track, and the descending counterweight raises the empty car. The counterweight, or *balance truck*, runs on a track between and slightly below the rails of the car track, and its weight is made equal to one-half the sum of the weights of the loaded and empty cars or trips; that is, if the loaded car weighs 5,000 lb.

and the empty car 2,500 lb., the counterweight will weigh $(5,000+2,500) \div 2$ $=3,750$ lb.

In pitching seams, where the coal will not run by gravity in a chute or where it will be broken if allowed to run, the jig is often employed with advantage to lower loaded cars from the face to the gangway. For this purpose, a sheave is arranged at the head of the track, near the face, between two posts wedged firmly between the roof and floor. The rope is passed over this sheave, one end is fastened to the balance truck, and the other end is coupled to the car by any standard hitching. A brake wheel, securely fastened to one side of the sheave, and a strong lever furnish the means of controlling the motion of the car. Jig planes are generally used to drop single cars short distances. Where the grade is so flat that two or more cars must be employed to impart motion, a more cheaply installed and operated arrangement is generally possible. The rooms may be driven across the pitch on such a grade that a mule can pull the empty car to the face, or the car may be drawn up by a block and tackle operated by a mule or by a windlass, or by a rope passing over a sheave at the head of the room track, one end of the rope being fastened to the car and the other wound on a drum on the haulage motor, the drum being turned by current from the entry wires while the motor is blocked on the rails.

**Calculations for Jig Planes.**—While the formulas given under the head of Self-Acting Inclines apply equally to jig planes, it is better to determine the pitch of the track necessary to make the plane self-acting under the known conditions. If the necessary pitch is greater than that of the seam, a jig plane cannot be installed. The formula for the angle of pitch may be deduced from that given for $N$, as follows:

$$\tan X = \frac{F}{C-B-R}$$

in which, $F =$ friction in pounds $= (C+B+R) \times f$;

$C =$ weight of loaded car, in pounds;

$B =$ weight of balance car, in pounds;

$R =$ weight of rope, in pounds.

EXAMPLE.—It is desired to install a jig plane to lower a single loaded car weighing 5,000 lb. a distance of 200 ft. from the face to the entry. If the empty car weighs 2,000 lb., the rope 2 lb. per ft., and friction is taken as .025, what is the proper grade to make the plane self-acting?

SOLUTION.—Here, $F = (5,000+3,500+2 \times 200) \times .025 = 222.5$ lb.; $C = 5,000$ lb.; $B = (5,000+2,000) \div 2 = 3,500$ lb.; $R = 2 \times 200 = 400$ lb. Substituting in the formula,

$$\tan X = \frac{222.5}{5,000-3,750-400} = .26177$$

whence, $X = 14°\ 40'$. On any greater pitch than this, the plane will be self-acting, and on any lesser one it cannot be operated with one car, although it may be by two or more.

EXAMPLE.—With the conditions of the preceding problem, but with two-car trips, on what pitch will the plane be self-acting?

SOLUTION.—Here $C = 10,000$ lb.; $B = (10,000+4,000) \div 2 = 7,000$ lb.; $F = (10,000+7,000+400) \times .025 = 435$ lb.; and $R = 400$ lb., as before. Substituting,

$$\tan X = \frac{435}{10,000-7,000-400} = .16731$$

whence $X = 9°\ 38'$.

# SLOPES AND ENGINE PLANES

**Slopes.**—A *slope* is an inclined plane up which loaded cars are hauled on one track by an engine of any convenient type at the same time as a corresponding number of empty cars descend by gravity on a second and parallel track. A slope, in many of its features, is a combination of the self-acting incline and the standard double-compartment shaft. In all that relates to track, rollers, the use of barneys or gunboats, slope carriages, etc., a slope is identical with an incline; in all that relates to the motive power and the calculation of engine dimensions, a slope differs from a shaft only in that counterbalancing is not attempted and that the weights moved must be multiplied by $\sin X$ and the friction of the moving weights by $\cos X$ as in haulage on inclines and for the same reason. It should be remembered, however, that the inertia of the drum and sheaves is not influenced by the inclination of the slope.

Slopes are the usual means of opening and operating pitching seams, and are sometimes used underground in raising coal from a lower to a higher level.

While they are sometimes laid with three rails and a passing track, as some inclines, all first-class main slopes are double-tracked throughout. The profile of a slope is that of the dip of the seam slightly dished or concave at the bottom to reduce the speed of the descending trip. At the surface, the slope tracks are commonly extended upwards on a trestle having the same pitch as the seam so that the knuckle is at the tipple platform, in order that the cars, when released from the rope, may run by gravity to the dump.

The controlling factor in determining whether a certain grade is or is not adapted to slope haulage, is that it must be sufficiently sharp to allow the empty trip to run down by gravity dragging the rope after it. The proper angle of slope is found from the formula $\tan X = f$, for Self-Acting Inclines. The coefficient of friction $f$ should be made sufficiently great to allow for any increase in resistance due to the clogging of the rollers by material dropped from the cars. The pitch must also be steep enough to insure that the descending trip has at least the velocity imparted by the engine to the ascending trip; otherwise, the empty rope will buckle and double on itself.

Both first- and second-motion engines are used for hoisting on slopes; the choice between the two types is governed by the principles given under the head of Hoisting. Slopes up to 500 or 600 ft. in length and of moderate pitch are now generally operated with rope or chain hauls, the advantages obtained through the regular delivery of single cars to the dump through the use of these appliances more than offsetting their greater first cost.

**Engine Planes.**—An engine plane is a single-track slope, without central turnout, up which an engine pulls a loaded car as one operation and down which, as a second operation, the empty car descends by gravity, dragging the rope after it. There is frequent confusion in the use of the terms slope and engine plane and what is always and rightly known as engine plane when on the surface is miscalled a slope or a single-track slope when used underground.

The engines used on planes are commonly of the friction-clutch type. The drum is thrown in when it is desired to hoist and thrown out and allowed to turn on its shaft as the empty trip descends, its speed being controlled by powerful brakes. When hoisting on a plane, the load is that of the loaded trip and the weight of the rope, the latter diminishing uniformly as the rope winds on the drum. When lowering, the load at the outset is that of the empty trip but it is uniformly increased by the weight of the rope as the latter unwinds from the drum.

The question of hoisting the load is merely one of installing a sufficiently powerful engine, which must develop more power in engine plane than in slope haulage because in the former there is no balancing of the moving weights. In slope haulage, the inertia of the drum is overcome by the engine; but in engine-plane haulage, this is true only in the case of the ascending loaded trip. Hence, in engine-plane haulage the grade must be sharper than in slope haulage because the empty trip must overcome the inertia of the drum. In both forms of haulage, the length of the empty trip becomes constantly greater, and the unchanging weight of the empty trip may not always be sufficient to overcome the increasing rope resistance. For this reason and to attain an average speed of, say, 10 mi. per hr., the grade of an engine plane should not be less than 3%.

Flat slopes and planes are fully as unsatisfactory in operation as flat self-acting inclines. On the surface, their use may often be avoided by the employment of a motor running on a track laid on a flatter, but longer grade than the plane. If motors cannot be employed and a plane must be used, it is better to install a tail-rope or endless-rope system for its operation. Underground, where their use is local and the amount of material handled over them is not great, the use of flat slopes and planes is not so objectionable, although if short, and cheap power is to be had, a mechanical haul of some kind is to be preferred.

# ENDLESS-ROPE HAULAGE

In *endless-rope haulage* systems, a wire rope passes from the haulage-engine drum to and through the main entry to a sheave at the end of the workings, thence around the sheave and back along the main entry or a parallel entry to the engine drum. The ends of the rope are spliced together, forming an endless rope to which the cars are attached singly by grips or in trips pulled by a grip car.

As usually installed, the system requires two tracks, which may be laid in the same entry if the roof is sound enough to permit the necessary width.

If the roof is bad, it is usually cheaper to lay single tracks in parallel entries of standard width, using one road for inbound and the other for outbound traffic.

There are two general types of endless-rope haulage based on the speed of the rope. In the *low-speed system*, which is the original type, the rope moves continuously in the same direction at a rate of 2 to 4 mi. an hr.; there are two separate tracks; the cars are attached to the rope singly at intervals of 100 to 200 ft.; and, as far as possible, there are as many inbound empty cars as there are outbound loaded ones. In the *high-speed system*, the rope moves at a rate of 15 to 25 mi. per hr.; the cars are attached to the rope in trips of as many or more by means of a grip car; there is usually but one track, which is used for both inbound and outbound traffic; and the direction of motion of the rope is reversed to correspond to the direction in which the trip is being moved.

The low-speed system has been highly developed in England, where the cars, as well as the mine output, are relatively small, where the grades are undulating and the curves numerous, where many branches (cross-entries) must be worked from the main rope for long trips of cars, and where there is not storage room at the foot of the shaft for long trips of cars. While this requires much less power than the high-speed system and insures a uniform and regular delivery of loaded cars to the shaft bottom, it is costly in labor, as at least one attendant is required at each branch. Although at such low speeds there is but little danger of a car jumping the track, when such an accident does happen the damage may be serious to both cars and rope, as there is no attendant to signal promptly for the immediate stoppage of the rope.

The strain upon the rope in starting a car from rest varies as the square of the rope's velocity, hence the higher the speed the greater must be the diameter of the rope. Further, if the velocity is as great as 4 mi. per hr., the ropes become flattened, kinked, and unduly strained, much sooner than at velocities of 2 to 3 mi. per hr. For these reasons, the last-named speeds are preferred for slow-moving ropes. When attaching it to the rope, the attendant usually pushes the car by hand until it has acquired the velocity of the rope before gripping it to the rope. Grips for low-speed haulage are very largely of the automatic detaching type.

The high-speed system is in more general use in the United States where the cars and the required output are large; where the track is straight and the grades against the loads; where there are no branches; and where there is ample storage room for long trips of cars. The system is economical in labor, as no handling of the trip is necessary except at the points of origin and delivery, but it requires powerful engines to give the acceleration to the trip when started on the heavy grades that usually prevail where this system is used. The rope must be heavier than in the low-speed system, the rails should weigh 60 lb. or more per yard, the track must be kept clean and in perfect alinement, and the cars must be of the best type. The amount of coal that may be delivered by this system is practically only limited by the power of the engines, particularly if inbound and outbound trips are run at the same time. At the tipple, the long trips are fed into the dump by car hauls, imposing little if any extra labor over that required when the low-speed system is used.

Endless-rope haulage is particularly adapted to mines where the entries are level or have a slight uniform grade. In the low-speed system, the cars should be fastened to the rope at regular intervals to make the load on the engine constant and to insure a uniform delivery of cars to the tipple. However, owing to the delays incident to all mining operations, it is impossible to keep the cars regularly spaced and they become bunched. Thus, the cars may accumulate on a grade or on several grades in opposite directions and so throw a variable load upon the engine, which makes its regulation very difficult where the grades are not uniform in amount or direction. Where variable grades occur, the rope will lift from the track in low places and may lash to such an extent as to throw the cars from the track; while on an up-grade, the rope will bear heavily on the rollers, producing excessive wear and greatly increasing the friction.

**General Arrangement of Endless-Rope Haulage Systems.**—Fig. 1 shows the general arrangement of an endless-rope haulage system, which will answer for either low or high speed. The rope passes back of the engine drums *a* and *b* to the balance car *c*, where it is given a half turn around the sheave *d*. From *d* the rope passes back past the engine drums and through the mine, where it is supported on rollers, to the tail-sheave *e* which is carried on the balance car *f*. After making a half turn around this sheave *e*, the rope returns along the parallel track to the drum, thus completing the circuit. The balance cars are

intended to keep the rope tight. The passage of the rope under the pulleys $g$, over the pulleys $h$, and to one side of the pulleys $i$, is for the purpose of deflecting it to the center of the mine tracks. In the plan shown, the rope $k$ in going into the mine pulls the empty cars $l$ to the curve $m$ where they are unhooked and distributed to the working places. At the same point, but on the track $n$, the loaded cars are attached to the outgoing rope $p$ and hauled to the curve $q$ where they are unhooked and sent to the tipple.

**Endless-Rope Haulage Engines and Drums.**—The engines for an endless-rope haulage plant are almost invariably of the second-motion type and may be fitted with plain slide or Corliss valves; the latter is preferred if the grades are variable as they are more economical in the use of steam and are automatic. In the low-speed system, the engines, if run continuously in one direction, should use steam expansively, and, where the loads are variable, should be provided with a flywheel and governor.

There are generally two narrow drums, arranged tandem on separate shafts. One of the drums is driven by gearing and the other, frequently called the *follower*, is turned by the rope passing around it. The drums vary in diameter according to the size of rope, size of engine, length of haul, and should be of ample size to reduce the bending strain on the rope. The driven drum, or follower, is sometimes independent of the engine and made smaller than the driving drum; it is also sometimes permitted to run loose on its shaft so that the rope will lead properly from one groove to another. Drums with a concave rim are sometimes used, but with such there is considerable surging and jerking of the rope, and grooved drums are preferable. The best results appear to be obtained when the drums are placed 12 to 15 ft. apart, in order that there may be a slight sag to the rope and a better bite on the drum. The number of grooves on the driving pulley may be two, four, or more, depending on the strain coming upon the



Plan

Elevation
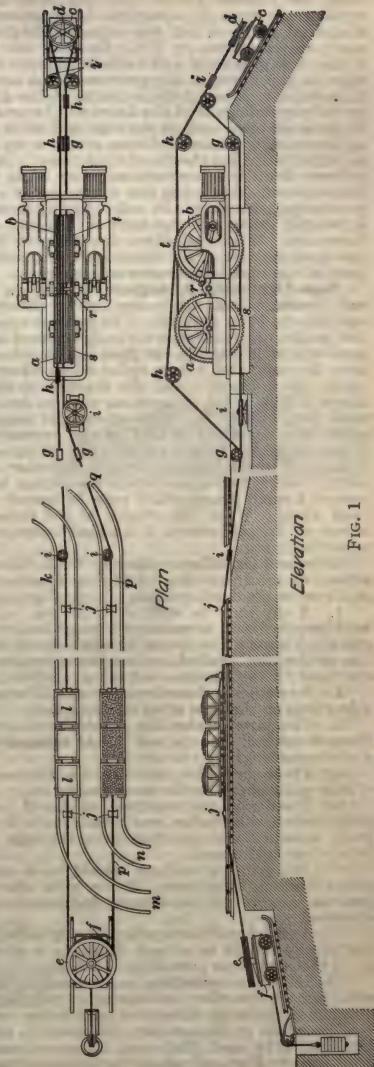
Fig. 1

rope; the follower pulley adds but little to the tension. To increase the tension, the rope is sometimes bent around the drums in the form of a horizontal figure 8, but the extra bending strain thus thrown on the rope materially shortens its life. Because the tension on the different turns of the rope gradually and uniformly decreases from the first to the last, the wear on the lining of the grooves is not equal. This results in time in the drum having as many diameters as it has grooves. The velocity of a point on the circumference of the drum will be greater on the unworn grooves than on the worn ones, and, as the rope and drum cannot travel at different speeds, an increasingly violent rubbing action is set up between the rope and drum, materially reducing the life of the former. To prevent this rubbing action, the follower drum is sometimes made up of as many single drums as there are grooves, each drum being free to revolve at a speed determined by that of the groove opposite it on the solid driving drum. There are various forms of *differential drums* on the market, designed to overcome the trouble under discussion. In some of them, the lining of the grooves is free to move upon the circumference of the drum and so to adjust itself to the variation in the speed of the rope from groove to groove.

**Rope-Tightening Arrangements.**—Owing to the stretching of the rope under loads, *balance-* or *tension- cars* or sheaves are placed at both ends of the rope line to keep it tight. The general arrangement is as shown in Fig. 1. The weight of the balance car is determined by experiment. Unless the track upon which the balance car runs has considerable length, a piece must be cut from the rope and a new splice made when the car reaches the end of the track. In order to avoid the expense of blasting out an inclined track as shown in Fig. 1, the balance car may be run on the grade of the entry and a more than usually heavy counterweight employed; in which case the only excavation necessary is that of the counterweight pit. The counterweight is then supported from a pulley wheel riding on a chain. One end of the chain is fastened to the balance car and the other end, after passing down into the pit to form a loop upon which the pulley wheel rides, is carried up to and around a small drum, which may be turned by worm-gearing operated by a hand wheel. When the stretch of the rope has allowed the counterweight to rest on the pit bottom, it is raised therefrom by winding some of the chain upon the drum.

**Grips and Grip Cars.**—Messrs. W. G. Salt and A. L. Lovatt, in the Transactions of the Institution of Mining Engineers (England), give the following as the seven essential qualities of a good grip, or *clip*, as it is called in England:

1.   A clip must be sufficiently strong, with a margin of safety, to do the work required and to withstand rough usage. If, however, the design of the clip is too strong, the desired results will not be obtained; for if the tub or tubs (car or cars) are derailed, serious damage might be caused to the rope or hauling machinery if the clip does not act as a safety valve.

2.   Its design and construction should be such that it can obtain and retain a firm grip on the rope when it is attached.

3.   The jaws of the clip should have a bearing of at least 70% on the circumference of the rope and should embrace all the strands of the rope within a minimum length of the clip jaw.

4.   There should be a good margin for wear and the clip should be capable of easy adjustment by the person using it.

5.   The design and construction should be as simple as possible; the fewer parts there are the better, and these should be such as to allow of the clip being easily attached and detached from the rope with certainty, the detachment being clean and certain.

6.   The gripping surfaces should be so arranged as not to kink the rope under working conditions. If the kink effect is reduced to a minimum the wear on the rope will be reduced, and consequently the life of the rope will be increased.

7.   A clip should be capable of being automatically detached from the rope, and ideally should be of such design as to work satisfactorily under any one or all of the conditions prevailing at a mine.

The following controlling factors must be taken into account in the adoption of a clip: Inclination, undulating or varying gradients, level roads, direction of roads (straight or otherwise); and under- or over-rope haulage.

The article by Messrs Salt and Lovatt is reprinted in *The Colliery Engineer* for Feb., 1914, and illustrates, compares, and criticizes the majority of the grips used in England, where the slow-moving endless rope system with single cars has been very successfully developed.

On undulating grades single grips have not proved universally successful, and it is usually necessary to have grips at each end of the car to prevent its running forwards or backwards and bending the rope at every change of grade.

Where the cars are run in trips instead of singly, grip cars are used. These are four-wheeled trucks carrying the grip below the platform. The grip is frequently of the jaw type with one fixed jaw; the movable jaw being brought down upon the rope by a lever or by turning a hand wheel. In handling heavy loads, the grip must be thrown in slowly so that the speed of the trip is gradually accelerated from rest to that of the moving rope. This is, perhaps, best accomplished by substituting for the jaw grip, the form used on street railways, some suspension bridges, etc. This grip consists essentially of three grooved wheels set in the same plane with their axles at the vertices of an equilateral triangle. The rope runs tangent to the upper edge of two wheels and tangent to the lower side of the other. As the axles of the wheels are brought together by suitable mechanism a greater and greater pressure is exerted upon the rope. At first the wheels revolve freely, but as the pressure is increased they grip the rope more and more firmly at the same time imparting an increasing velocity to the trip, until, when the grip wheels no longer revolve, the trip is traveling at the same speed as the rope. Where grip cars are employed, there is a device by which the frame carrying the grip wheels can be raised a sufficient height to clear the rollers. In fact, any grip to be serviceable must pass over the track rollers, deflection sheaves on curves, and the like. A *gripman* rides on the grip car not only to handle the grip, but also to release it and apply the brakes when needed.

**Rollers and Sheaves.**—When placing rollers on endless-rope haulage roads, the same precautions are to be followed as when setting them on inclined planes. The rollers are very commonly hollow, cast-iron cylinders 12 to 18 in. long, and 6 in. in diameter, with raised rims to prevent the rope running off them.



FIG. 2

Other rollers, 10 to 12 in. long, have a decidedly concave face to carry the rope to a central groove. At the bottom of a dip, it is usually necessary to have a roller in the roof to prevent the rope scraping against the roof; such a roller may be of the types just described. Too much care cannot be taken to see that the rollers are strongly and properly set and are in good condition and free to revolve at all times.

Sheaves are a type of roller with their axles vertical and are used to guide the rope around curves. When possible they are placed outside the rails on the inside of the curve; this arrangement permits the use of sheaves as large as 6 ft. in diameter, thus greatly prolonging the life of the rope above what it would be if the sheaves were in the center of the track where their diameter would be less than one-half the track gauge. The sheave shown in Fig. 2, which may be set with its axle inclined, is a very common form for use on the outside of curves. The planking *a* is intended to guide the rope to the sheave and may be covered with sheet iron held in place by bolts or nails with countersunk heads.

Where rollers or sheaves are placed outside the track on curves, some arrangement is necessary to prevent the rope catching under the head of the rail as it is deflected from the center line. A familiar arrangement consists of wedge-shaped pieces of wood spiked to the ties, the highest point of the wedge rising a little above the top of the rail. A wedge is placed on each tie and is set as close to the rail as possible, leaving clearance for the flange of the car wheel. The face of the wedge, along which the rope slips into position, may be covered with sheet iron held in place by spikes or bolts with countersunk heads. Numer-



(a)  FIG. 3  (b)

ous devices similar to the one illustrated in Fig. 3 are used for the same purpose. The finger *a*, the lower end of which is much the heavier, is pivoted at *b* so that it normally hangs vertical. When the rope presses against it, it assumes the position shown in (b), and after guiding the rope over the rails, falls back to its original position (a). Arrangements must be made at all switches to prevent the rope being caught in the frogs.
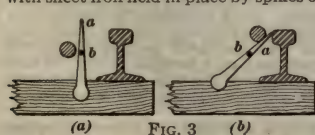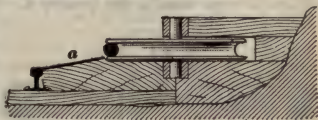
**Side-Entry Haulage.**—The endless-rope system is not readily adapted to haulage on side or cross-entries. As ordinarily arranged, each entry has its

own rope, which passes over a sheave at the face and makes a couple of turns over a grip or driving wheel at the mouth. On the main entry is placed a vertically turning wheel around which the main rope makes two turns to give it sufficient power to turn the driving pulley on the side-entry haulage; this it does by means of a friction clutch, which may be thrown in and out of gear. There is usually a reducing gear so that the speed of the side-entry rope is less than that of the main line. Loaded cars on the main line must be detached from the rope on arriving at a cross-entry, because of the wheel around which the main rope passes, and must be coupled on again after passing the entry. Similarly, cars to be delivered to a side entry or which are received therefrom must be uncoupled and coupled at the junction with the main line. On a level track or down grades, the cars on the main line will run past the branches under their acquired momentum, but on sharp up grades this may not be possible. In any case, much labor is required at junctions and the grip must be of a type that is quickly adjusted.

**Overhead Endless-Rope Haulage.**—The overhead endless-rope haulage system is a modification of the original low-speed type in which the rope is carried over the cars instead of under them. As the cars are spaced from 100 to 200 ft. apart, it is unusual for the rope to sag low enough to drag either on the track or on the rollers; hence, the wear on the rope is much less than when it runs under the cars.

Most of the grips or clips used with an under-running rope may be used with the overhead rope. A common form of *grab*, or *clutch*, is shown in Fig. 4, where the rope rests in the groove *a*. The friction due to the motion of the
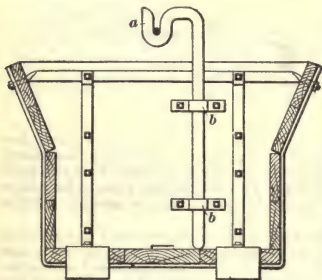


FIG. 4

rope in the grab causes it to turn slightly sidewise and grip the rope. The heavier the car or the steeper the grade, the more firmly will the grab take hold of the rope. As the grab is free to turn in the sockets *b*, it pulls the car on an up grade and holds it back on a down grade. When the cars reach their destination, they are automatically released by an increase in the down grade and by the gradual rise of the rope to an overhead sheave, which lifts the rope from the grab and allows the cars to run to the dump by gravity.

The overhead endless-rope system does not work very well on curves, which should be made as short as possible so that one large wheel is sufficient to carry the rope around

them. Branches may be worked by the overhead rope system by setting the driving pulley conveying power to the side-entry pulley so that it will revolve horizontally above the cars instead of vertically below them. As the rope rises to pass around the pulley, the cars are, of course, automatically detached and, after passing over the cross-entry switches by gravity, are automatically attached to the rope. The branch-entry driving pulley is operated by a friction clutch as in the case of an under-running rope.

**High Speed, or Reversing, Endless-Rope Haulage.**—In high-speed endless-rope haulage, the rope is only in motion when the cars are being moved. At the inside parting, the grip car pulling the trip is attached to the rope, which is put in motion upon receipt of the proper signal at the engine room outside the mine. At the tipple, a flying switch is usually made so that the cars can run in by gravity, and the rope is stopped. After attaching the empty trip to the grip car, the rope is started in the opposite direction and the empty trip hauled to the inside parting usually upon the same track used by the loaded trip, but sometimes upon a separate track laid in a parallel entry. Where but one track is used for traffic in both directions, the rope may be returned upon rollers laid on one side of the main entry, or the return air-course or manway may be used for the purpose.

There is no balancing of loads in this system and large and powerful engines are required because there may be fifty or more cars in a trip running at a speed of 15 to 25 mi. an hr. against grades of 3% and more. Balancing is possible when two tracks are used and an empty trip leaves the tipple for the mine at the same time a loaded trip leaves the inside parting for the tipple.

The high-speed and endless-rope and the tail-rope systems are used in the United States on straight hauls too steep for motors and where a large output is demanded.

**Endless-Rope Haulage on Inclines.**—Either the underneath or overhead endless-rope system is well adapted to lowering coal over an incline where the grades are not too steep. The rope is given a sufficient number of turns around the head-sheave to secure it against slipping. The head-sheave is a flat, grooved wheel, similar to the driving drum ordinarily used, and its speed is controlled by a brake. The diameter of the head-sheave and of the tail-sheave at the foot of the incline is equal to the distance between the center lines of the tracks, which is from 8 to 10 ft. Unlike ordinary incline haulage, the rope is in balance, and as there are the same number of cars on each side of the rope, they also will be in balance and the weight-producing motion, that of the coal, is constant throughout the run; hence, less powerful brakes are required than on ordinary inclines. Where the cars are not equally spaced, it may possibly be necessary to install a small engine at the head of the incline to keep the rope in motion at a speed of, say, 3 to 4 mi. per hr.

**Calculations for Low-Speed, Endless-Rope, Haulage Engines.**—In determining the horsepower required of low-speed, endless-rope, haulage engines it is usual to assume that the conditions demanded of the plant are perfectly fulfilled and then to add a liberal amount for the power required to overcome the irregularities and uncertainties always met in practice. Thus, it is assumed that there are as many inbound empty cars as there are outbound loaded ones and that they are equally spaced; that the track is level or has a uniform grade either for or against the loaded side; that the rope is continuously in motion and the cars will have the velocity of the rope when they are gripped thereto. In practice, it may happen that at some given instant there are no inbound empty cars, that the outbound loaded cars are badly bunched on a steep adverse grade, where, the rope having for some reason been stopped, they must be accelerated from rest to full speed. For these reasons, the engines for low-speed, endless-rope haulage should have from two to three times the horsepower calculated on the assumption that the conditions are perfectly fulfilled.

The load to be moved is that of the coal, for the cars and rope are in balance. To the weight of the coal must be added the friction based on the resistance to motion of the entire rope and all the cars. The force required to accelerate the system may be neglected.

EXAMPLE.—In a low-speed, endless-rope, haulage system 5,000 ft. long, the rope, which weighs 2 lb. per ft., has a speed of 3.5 mi. per hr. The grade is undulating, but averages 2% (1° 9′) against the loads. The mine car, which weighs 2,500 lb., holds 4,000 lb. of coal. If friction is estimated at 3%, what will be the theoretical horsepower of the engines to deliver 180 T. of coal per hr. to the tipple?

SOLUTION.—The speed of the rope is $(5,280 \times 3.5) \div 60 = 308$ ft. per min. There must be delivered to the tipple $180 \div 60 = 3$ T. or $3 \div 2 = 1.5$ cars of coal per minute. As the rope travels 308 ft. per min., the cars will be spaced 308 $\div 1.5 = 205.3$ ft. apart. There will be $5,000 \div 205.3 = 24.5$, say, 25 cars on each side of the rope.

The total weight in motion is, rope $2 \times 5,000 \times 2 = 20,000$ lb.; coal, 25 car-loads weighing $25 \times 4,000 = 100,000$ lb.; cars, $2 \times 25 = 50$, weighing $50 \times 2,500 = 125,000$ lb.; grand total, 245,000 lb. At 3%, the friction will be 7,350 lb.

The formula for the horsepower is,

$$\text{H. P.} = \frac{(W \sin X + F \cos X)S}{33,000} = \frac{(W \tan X + F)S}{33,000}$$

since, on such a flat grade and for practical purposes $\sin X = \tan X$ and $\cos X = 1$. $S$ is the distance traveled by the rope, in feet per minute.

As $W = 3$ T. = 6,000 lb. per min., $\tan X = $ grade in per cent. = .02, $F = 7,350$ lb. and $S = 308$ ft. per min., the theoretical horsepower required to keep the rope and cars in motion is

$$\text{H. P.} = \frac{(W \sin X + F)S}{33,000} = \frac{(6,000 \times .02 + 7,350) \times 308}{33,000} = 70, \text{ about}$$

If the rope should come to rest at a time when there were no inbound empty cars and the loads were bunched on an adverse grade, it is probable that 150 H. P., possibly more, might be required to get the system again in motion.

**Calculations for High-Speed, Endless-Rope, Haulage Engines.**—Because, in high-speed, endless-rope haulage, the entire system must be brought from

rest to full speed, the engines must be made sufficiently large to provide the power required for acceleration.

EXAMPLE.—A high-speed, endless-rope, haulage system is $1\frac{1}{2}$ mi. long. The rope, which weighs 2 lb. per ft., has a sustained speed of 15 mi. per hr. at the end of 60 sec., starting from rest. The grip car weighs 5,000 lb., the empty car weighs 2,500 lb. and carries 5,000 lb. of coal. The grade averages 2% against the loads and friction is 2.5%. If but one track is used for haulage, how many cars must be run in a trip to give an output of 200 T. per hr., and what must be the net horsepower of the engines?

SOLUTION.—The full speed of the trip is $(5,280 \times 1.5) \div 3,600 = 22$ ft. per sec. The acceleration is $a = v \div t = 22 \div 60 = .367$ ft. per sec. per sec., and the retardation will be the same. During the joint periods of acceleration and retardation, the trip will travel $2 \times (22 \times 60) \div 2 = 1,320$ ft. in 2 min. = 120 sec. The remaining $(5,280 \times 1.5) - 1,320 = 6,600$ ft. will be covered in $6,600 \div 22 = 300$ sec., and the total time required to haul out the loaded trip will be $120 + 300 = 420$ sec., or 7 min. If 3 min. is allowed for coupling and uncoupling and all delays, a trip will require 10 min. and six trips will be made per hour.

But for each loaded trip going out there must be an empty trip going in. Hence, on a single-track system but three loaded trips per hour will be delivered to the tipple. As each car carries $5,000 \div 2 = 2.5$ T., $200 \div 2.5 = 80$ loaded cars must be delivered each hour. The number of cars per trip will be $80 \div 3 = 27$.

The cars and the grip car will weigh $27 \times (5,000 + 2,500) + 5,000 = 207,500$ lb. The rope will weigh $2 \times (5,280 \times 1.5) \times 2 = 31,680$ lb. Hence, the total load will be 239,180 lb. As, on such a flat grade cos $X = 1$, the friction will be $239,180 \times .025 = 5,979.5$ lb., at a speed of 22 ft. per sec., the horsepower necessary to overcome friction will be $(5,979.5 \times 22) \div 550 = 239$ H. P.

To haul the trip up grade will require, since for practical purposes sin $X = $ tan $X = $ per cent. of grade, $(W$ tan $X \times 22) \div 550 = (207,500 \times .02 \times 22) \div 550 = 166$ H. P.

To accelerate the entire system from rest to a speed of 22 ft. per sec. in 60 sec., will require the outlay, in force, of

$$\frac{W}{g}a = \frac{239,180}{32.2} \times .367 = 2,726 \text{ lb.}$$

The horsepower required to accelerate will be $(2,726 \times 22) \div 550 = 104$ H. P.

Thence, the total net horsepower required to overcome friction, to raise the trip up the grade, and to accelerate the haulage system from rest to full speed, will be $239 + 166 + 104 = 509$ H. P. To allow for the friction of the engines, their efficiency, future demands for power, at least 600 H. P. should be provided, and 700 would be better.

NOTE.—If, in the example, the system was double tracked, six loaded trips of 14 cars each could be brought out per hour and as many equal size trips of empties hauled in; the capacity of the plant would then be increased from 81 to 84 cars. The horsepowers required to overcome the various resistances would be, for friction 171 H. P., for grade 56 H. P., and for acceleration 76 H. P., a total of 303 H. P. Thus, double tracking the system and reducing the loads, effects a saving in power of a trifle more than 40%. Similar economy may be had by reducing the speed from 15 to 7.5 mi. per hr., but running twenty-seven car trips as before. On the other hand, for the same expenditure of power, the capacity of a double-track system should be about 60% more than a single track.

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## TAIL-ROPE HAULAGE

**General Arrangement.**—The tail-rope system of haulage resembles the high-speed endless-rope system in that a single track is used upon which trips of cars are run at high speed (6 to 25 mi. per hr.) in opposite directions. It differs from the latter in that the rope is not continuous and that plain cylindrical drums and not driving and tightening pulleys are used on the haulage engine. There are two ropes used in this system; a main or haulage rope and a smaller tail-rope each winding and unwinding from its own drum, which revolves freely on its shaft and may be thrown in gear by friction clutches. When a loaded trip is to be hauled from the inside parting, the main rope is coupled to the first car and the tail-rope to the last. Upon signal, the main-rope drum is thrown in, the tail-rope drum is free on its shaft, and the engine started. The main rope, which is wound entirely on the drum at the completion of the run, pulls the trip from the mine and the trip drags the tail-rope after it unwinding it from its drum. The reverse process, the tail-rope

pulling the trip which, in turn, pulls the main rope, returns the empties to the mine.

The general plan of a tail-rope plant with the engines placed underground near the foot of the shaft $s$ is shown in the accompanying figure. Here $T$ is the drum for the tail-rope, $M$ that for the main rope, and $w$ are the sheaves, one of which is at the end of the main entry and one at the end of each side entry or district.



Each side entry has a rope reaching from its mouth to a tail-sheave $w$ at or near its inbye end and back again to the main entry; the rope is provided at each end with couplings similar to those on the main-entry ropes. When an empty trip is to be pulled into a side entry, the main tail-rope $n$ is uncoupled from the trip and the branch rope $f$ coupled in its place. The main tail-rope is uncoupled at $h$ and the branch rope $g$ coupled to it. The branch rope can now pull the empties and the main rope to the end of the side entry, and the reverse operation will haul the loads to the mouth of the side entry. If full trips are pulled from the side-entry parting to the shaft bottom, it is customary to disconnect the main-line ropes inbye $n$ and run the trips through to destination without stopping, effecting a considerable saving in power in having to accelerate the system but once.

**Engines, Drums, Etc.**—Tail-rope haulage engines are almost always second-motion or geared, as the loads and grades in mine haulage are commonly too variable to permit of the successful use of first-motion engines. Reversing gear is not needed, as the engines run continuously in the same direction, the direction of motion of the trip being controlled by throwing the proper drum in gear.

The drums are necessarily of much smaller diameter than those of hoisting engines, in order to keep the speed of the rope within the prescribed limits. The flanges of the drums are also deeper than usual, as several coils of rope must
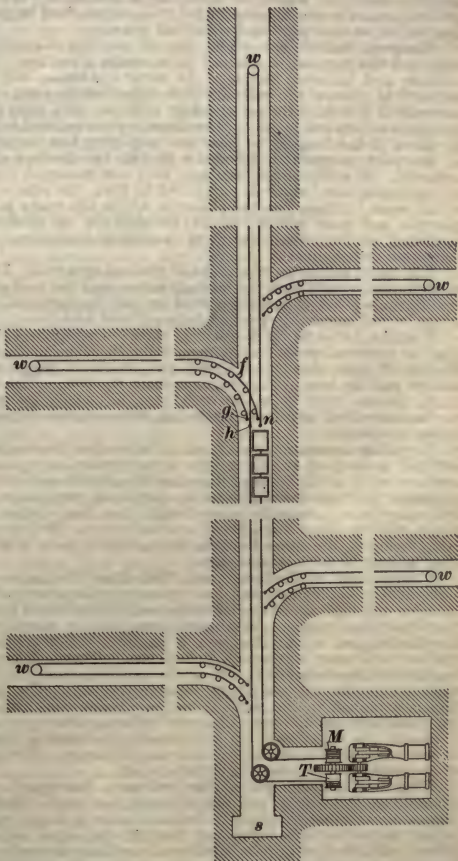
be wrapped upon one another, particularly if the haul is long. The brake power should be ample, and the engineman must use care not to permit the idle drum to turn too rapidly and thus pay off one rope more rapidly than the engine is winding the other rope on its drum. Indicators must be provided to show the location of the trip along the entry so that it may be stopped at the mouth of any of the side entries as well as at the end of the main road. They are also necessary where the direction of the grade varies, because, when the grade changes to one in favor of the trip, steam must be shut off and the brakes applied to prevent the trip over-riding the rope.

**Sheaves, Rollers, Etc.**—The sheaves used for changing the direction of the rope and the method of placing them under given conditions are the same in endless- and tail-rope haulage. In some instances, large wood-lagged drums are used in place of the ordinary iron deflection sheaves. These drums may be as much as 6 ft. in diameter and 2 ft. wide, set with their axles vertical. When the grooves become too deep, the drum is reversed end for end, thus subjecting all parts of the lagging to wear and requiring less frequent renewals. There is, however, a marked difference in the tail-sheaves employed in the two systems. In endless-rope haulage, they are mounted upon a counterbalance, or tightening car, which is employed to keep the rope at the proper tension; in tail-rope haulage, the tail-sheaves, which are made of as large diameter as possible and may revolve either horizontally or vertically, are very firmly anchored, as the proper degree of tension is maintained by manipulating the drums on the engine.

The track rollers are the same in the two systems. The tail-rope is usually carried along one side of the haulage entry near the roof and upon short rollers, the journals of which are supported by parallel uprights, 2 in. $\times$ 8 in. or a little larger, which are firmly wedged between roof and floor.

Usually, 15 to 20 ft. of chain is used between the end of the rope and the trip, to prevent injury to the rope through kinking when the trip is stopped. Knock-off links, detaching hooks, etc., many of which are automatic, are in use for rapidly uncoupling the rope from the trip. The selection of any automatic device must be made with care, as accidents may arise through the mechanism either acting at the wrong time or place or failing to act when and where it should.

Signaling apparatus must be installed so that the engineer may be promptly notified to stop or start the trip at any point in its course. At permanent stations, such as the main parting and the mouth and inbye end of side entries, signals of the push-button type or telephones are commonly used. To signal from any point along the main roads, two bare copper wires are suspended about 6 to 9 in. apart along the side of the entry. By bringing the wires together or by bridging the space between them with a piece of copper or iron, a signal may be sent to the engine room.

**Comparison of Endless- and Tail-Rope Haulage.**—Where the road is straight, the grades uniform, and no branches are worked, there is little choice between the high-speed endless-rope and the tail-rope haulage systems, except that the latter requires 50% more rope, but, on the other hand, it can use a simpler engine, as its direction of motion is not reversed. On curves, the endless rope must be carried around small sheaves placed in the center of the track; this arrangement is not nearly as satisfactory as in the tail-rope system where large sheaves are placed outside the rails. On down-grades, since the trip is attached to the rope at but one point (the grip car), there is always the possibility of its over-riding and injuring the rope. The tail-rope system is greatly the superior of the endless-rope in working branches, not only from the standpoint of labor, but of ease and efficiency. The horsepower demanded of the engines is the same in either system.

The chief advantages of the ordinary, low-speed, endless-rope, haulage system, are the regular delivery of single cars to the tipple permitting a better regulation of labor than where the cars come in large trips; economy in installation, as the low speed does not demand the use of such large engines or such heavy rails, ties, etc.; less liability of accident through the cars jumping the track; less cost for upkeep, as the wear on the roadbed and rolling stock is much less at low than at high speeds; less power, and consequently less fuel, is required; it is more easily extended; and requires one-third less rope.

The disadvantages of the low-speed endless-rope system are the cost of laying and keeping up two sets of tracks, which, where the roof is poor may be a serious matter even if parallel entries instead of one double-width entry are used; where branches must be worked, the cost for labor is high and the service is not satisfactory; and it is not so efficient on crooked roads with undulating

grades as the tail-rope system for the reasons just given. Also, when the workings become extensive and the haulageways long, the number of cars required to keep up the output is excessive, and the large number of cars considerably increases the strain on the rope and the power demanded of the engine.

The tail-rope system is at a disadvantage in those states where the speed of the trip is limited by law to 6 to 8 mi. per hr. Where the track and cars are in poor condition and the entries are crooked a speed of even 8 mi. per hr. may be excessive, but on modern roads, as maintained in first-class mines, and with proper rolling stock, speeds of 20 mi. per hr. and more are perfectly allowable; and high speed is an essential to the successful operation of tail-rope haulage.

**Calculations for Tail-Rope Haulage.**—The calculations for the horsepower required in tail-rope haulage are made in the same way as those in high-speed endless-rope haulage, provision being made for the power required to accelerate the drum. In the event of the grades being variable, the power should be sufficient to start and accelerate the trip on the most adverse grade.

# STEAM-LOCOMOTIVE HAULAGE

The large volumes of dense smoke given off by the fuel, combined at times with sulphur fumes and always with the exhaust steam, have prevented the extensive use of steam locomotives in the bituminous mines of the United States. For these reasons and because of the danger from fire, either through igniting the timbers or methane, the use of these locomotives has been prohibited by law in many states. In the anthracite regions of Pennsylvania, where, until recent years, the coal worked was much thicker than in the bituminous fields and where, also, the coal is smokeless, steam locomotives have not proved so objectionable, and they are still quite extensively used for underground haulage on return air-courses and on the surface for hauling between the mine and the breaker, etc.

The steam locomotive possesses the advantages of low first cost, of not requiring a power plant, as do compressed-air or electric locomotives, and of being readily understood and operated by the average mine mechanic. The last feature is of importance in securing an engine runner in outlying districts where skilled labor is scarce. The advantage of carrying their own power, that is, of being self-contained, is shared alike with compressed-air, gasoline, and storage-battery electric locomotives. For the reasons given in the first paragraph, they can only be used on return air-courses and not then if the amount of gas present is at all considerable or if the mine is liable to sudden outbursts of methane; hence, they are not adapted to gathering either from rooms or from side entries. This necessitates the use of some power other than steam for that purpose; and the use of two kinds of power is not generally economical and is to be avoided whenever possible.

**Steam Mine Locomotives.**—Steam mine locomotives are generally similar to the small locomotives used for switching on surface railroads. They have four or six drivers, no pony truck or tender, and carry their water in a saddle tank over the boiler and their coal in a box in the cab. When four drivers are used, all have flanges; when six drivers are used, the middle pair are not flanged in order that the locomotive may more readily pass around curves. The height of the locomotive should be from 14 in. to 16 in. less than the thickness of the seam, in order to give about 4 in. overhead clearance.

The wheel base of these locomotives, which must be as small as possible to permit of easy passage around the sharp curves common in mines, varies from 3 ft. 0 in. on the small four-wheel engine weighing 8,000 lb., to 8 ft. 6 in. on the largest six-wheel engine weighing 64,000 lb. These locomotives are provided with reversing gear, couplers at both ends, and the sand box is arranged to deliver sand to the rail when the engine is running in either direction.

The table on page 796 gives the manufacturers' dimensions, weights, power, etc., of standard four-wheel steam mine locomotives.

**Power of Steam Locomotives.**—The *tractive force*, or tractive effort, of a locomotive is the total force developed by it that is available for moving a load; this depends on the size of the cylinders, the diameter of the driving wheels, and the steam pressure. The tractive power of a locomotive is the measure of its ability to pull a load and depends on the adhesion between the driving wheels and the rails, which, in turn, depends on the weight of the locomotive and the coefficient of friction. Under ordinary conditions, and also on a wet,

**DIMENSIONS OF FOUR-WHEEL STEAM LOCOMOTIVES**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Cylinders { diameter, inches | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|            stroke, inches | 8 | 10 | 12 | 14 | 14 | 14 | 14 | 16 |
| Diameter of driving wheels, inches | 20 | 23 | 24 | 26 | 28 | 30 | 30 | 33 |
| Wheel base | 3'0" | 4'0" | 4'0" | 4'0" | 4'6" | 4'6" | 4'6" | 5'9" |
| Length over bumpers | 10'0" | 11'7" | 14'4" | 14'9" | 16'6" | 16'9" | 17'9" | 19'6" |
| Excess of width at cylinders over gauge of track | 1'11" | 2'1½" | 2'3" | 2'5½" | 2'7⅜" | 2'9¾" | 3'0" | 3'1½" |
| Height above rail, least desirable | 4'4" | 5'2" | 5'4" | 5'7" | 5'9" | 5'11" | 6'3" | 6'6" |
| Height above rail, least practicable | | 4'7" | 4'9" | 5'0" | 5'3" | 5'5" | 5'8⅜" | 6'0" |
| Weight in working order, pounds | 8,000 | 14,000 | 16,500 | 21,000 | 25,000 | 28,000 | 34,000 | 42,000 |
| Capacity of tank, gallons | 80 | 125 | 200 | 250 | 300 | 400 | 500 | 600 |
| Weight per yard lightest rail advised, pounds | 12 | 16 | 16 | 20 | 25 | 30 | 35 | 45 |
| Radius of sharpest curve advised, feet | 20 | 25 | 25 | 25 | 30 | 30 | 35 | 45 |
| Radius of sharpest curve practicable, feet | 12 | 15 | 15 | 15 | 16 | 16 | 16 | 25 |
| Boiler pressure per square inch, pounds | 140 | 140 | 140 | 140 | 140 | 140 | 140 | 140 |
| Tractive power, pounds | 1,190 | 1,860 | 2,915 | 4,100 | 4,820 | 5,560 | 6,720 | 8,320 |

sanded rail, the adhesion or *adhesive power* as it is generally called, is about one-fifth the weight of the locomotive; with favorable conditions and a dry rail without sand, it is about one-fourth; and on a well-sanded dry rail, about one-third the weight.

If the tractive force of a locomotive exceeds the adhesive power, the wheels will slip and part of the tractive power will be wasted. The power driving the locomotive and the weight on the driving wheels must, therefore, be properly proportioned to obtain satisfactory results, for if the cylinders of a steam or an air locomotive are too large the driving wheels will slip, showing that there is too much power for the weight. A locomotive in this condition is said to be *overcylindered*. On the other hand, if there is not enough power for the weight, so that the wheels will not move, the locomotive is said to be *undercylindered*. The sustained tractive power, that is, the power exerted by a locomotive when traveling at its normal speed and which is a measure of the load it will pull continuously is commonly taken as 25% of the weight of the locomotive; and this, whether the motive power is steam, compressed-air, gasoline, or electricity.

The *drawbar pull* of a locomotive is that portion of its tractive power that is available for pulling a load; that is, it is the total tractive power less the power required to move the machine itself. The drawbar pull may be measured by a dynamometer placed between the locomotive and the first car of the trip. The terms tractive power and drawbar pull are often used to express the same idea. While such use is never strictly correct, on a level track the weight of the locomotive is such a very small proportion of the weight it can pull, that the statement is practically true. Thus, the hauling capacity of a 42,000-lb. locomotive is given by the manufacturers as 1,255 T. on a level track. As the weight of the locomotive is but 21 T., or less than 2% of the weight it can pull, the reduction to be made from the tractive power to get the drawbar pull is of no practical importance. On an up grade, however, while the tractive power remains the same, the drawbar pull decreases rapidly on account of the increased amount of power required to run the locomotive itself.

On long hauls, the average drawbar pull required should be well within the rated pull, while, on short hauls and for intermittent service, a locomotive may be operated at its rated capacity. The sharpest grade against the loaded trip should be used in calculat-

ing the size of a locomotive, and on a short grade, a locomotive may be worked very close to the maximum adhesive power, or *slipping point*, as it is called.

The following empiric formula is commonly used by manufacturers to determine the tractive power of steam mine locomotives:

$$T = \frac{D^2 \times L \times .85p}{d}$$

in which

$T$ = tractive power, in pounds;
$D$ = diameter of cylinder, in inches;
$L$ = length of piston stroke, in inches;
$p$ = boiler pressure, in pounds per square inch;
$d$ = diameter of driving wheels, in inches.

From this formula, it is apparent if the steam pressure is increased or decreased, the tractive power will be similarly affected. Likewise, if the boiler pressure remains the same, an increase in the diameter of the driving wheels is accompanied by a decrease in the tractive power, and vice versa. Thus, on surface railroads, freight locomotives which must have great tractive power have relatively small drivers, whereas locomotives used in express service where speed and not ability to haul heavy loads is of the greatest importance, have large drivers. Taking the mean effective pressure of the steam in the cylinders as 85% of the boiler pressure (.85p) allows for the friction of the locomotive itself; hence, on level roads the drawbar pull may be taken to be the same as the tractive power calculated by the formula. The internal frictional resistance of locomotives is usually taken as 6.5 to 7.5 lb. per T. of the weight on the drivers, while the tractive power is taken as 300 to 450 lb. per T.

EXAMPLE.—What are the tractive power and drawbar pull of a steam mine locomotive, weighing 14,000 lb., with 6″×10″ cylinders and drivers 23 in. in diameter, when the boiler pressure is 140 lb. per sq. in., both on a level track and on a 2.5% grade?

SOLUTION.—By substituting in the foregoing formula, the tractive power on a level track is found to be

$$T = \frac{6^2 \times 10 \times .85 \times 140}{23} = 1,860 \text{ lb.}$$

The drawbar pull may be taken as the same.

On the grade, while the tractive power is the same, the drawbar pull is less than this by the amount of power required to move the locomotive itself up grade, or by $14,000 \times .025 = 350$ lb. The drawbar pull on a 2.5% grade is, thence, $1,860 - 350 = 1,510$ lb.

EXAMPLE.—How many loaded cars weighing 7,500 lb. each can be pulled up a 2% grade by a locomotive weighing 25,000 lb., if it has 9″×14″ cylinders, 28-in. drivers, and a steam pressure of 140 lb.; the friction of the mine cars being taken as 30 lb. per T.?

SOLUTION.—The tractive power, which may be calculated from the formula or taken from the table, is 4,800 lb. The resistance to motion, in pounds per ton, is for friction 30 lb. and for the grade $2,000 \times .02 = 40$ lb.; or a total of 70 lb. Whence the total tractive power of the locomotive on a 2% grade, when expressed in terms of car resistance, is $4,820 \div 70 = 69$ T., very nearly. Since the locomotive weighs 25,000 lb. = 12.5 T., the drawbar pull, in the same terms, is $69.0 - 12.5 = 56.5$ T. Because each car weighs 7,500 lb. = 3.75 T., the locomotive can pull up the grade $56.5 \div 3.75 = 15$ cars.

The method just given assumes that the friction of the locomotive is the same as that of the cars, which is not correct. The friction of the locomotive is only about one-quarter that of a car and, as stated, is allowed for in the formula by means of which the tractive power of the locomotive is calculated. The following method may, then, be preferred: The power required to move the locomotive up the grade is $25,000 \times .02 = 500$ lb. Whence, the drawbar pull is $4,820 - 500 = 4,320$ lb. The grade resistance per car is $7,500 \times .02 = 150$ lb. and the frictional resistance is $3.75 \times 30 = 112.5$ lb.; whence the total resistance per car is 262.5 lb., and the locomotive can haul $4,320 \div 262.5 = 16+$ cars.

In the foregoing calculations, no allowance is made for the power required to accelerate the locomotive and train, nor is such an allowance generally necessary because of the slow speed of mine traffic and because the excess of power that may be temporarily developed on a well-sanded rail is generally ample to produce the acceleration. However, it may be readily calculated when necessary.

EXAMPLE.—If, in the last example, it requires 1 min. to bring the trip from rest to the legal speed of 6 mi. per hr., how many cars can the locomotive pull up grade?

SOLUTION.—A speed of 6 mi. per hr. = $(5,280 \times 6) \div 3,600 = 8.8$ ft. per sec., and the acceleration is $a = v \div t = 8.8 \div 60 = .147$ ft. per sec. per sec. The power required to accelerate the locomotive is $\dfrac{W}{g}a = \dfrac{25,000}{32.2} \times .147 = 114$ lb. From this, the net drawbar pull is $4,320 - 114 = 4,206$ lb. The power required to accelerate a mine car is $\dfrac{7,500}{32.2} \times .147 = 35$ lb., and the total resistance of each car is $262.5 + 35 = 297.5$ lb. Hence, the locomotive can, on this grade, start a trip of $4,206 \div 297.5 = 14 +$ cars and bring them to a speed of 6 mi. per hr. at the end of 1 min. The gain in adhesion by sanding the rails will unquestionably permit of the locomotive accelerating the 15 cars which the preceding example shows it can pull up the grade.

**Speed of Steam-Locomotive Haulage.**—A locomotive cannot pull a maximum load at a maximum speed. As the load increases the speed decreases and vice versa, but no general rule or formula can be given which exactly fixes the relation between the load and the speed. Experience has shown that the relation between the load and speed of mine locomotives is approximately as follows:

Under usual track conditions, the speed in miles per hour attainable when hauling a train as heavy as the locomotive can start, will be equal to about one-fifth the diameter of the driving wheels, in inches.

If the load is reduced to two-thirds to three-fourths of the maximum, the speed will be about one-half the diameter of the driving wheels, in inches.

If the load is very light, say about one-eighth of the maximum, the speed will be equal to or greater than the diameter of the drivers, in inches.

Thus, a locomotive with 30-in. wheels which can start a load of, say, 800 T. on a level track, will pull this at a speed of $30 \div 5 = 6$ mi. per hr.; if the load is reduced to, say, 550 to 600 T., the speed will be increased to about $50 \div 2 = 25$ mi. per hr.; and if the load is still further reduced to about 100 T., the speed will rise to 30 mi. per hr., or more.

**Horsepower of Locomotives.**—The horsepower that may be developed by a steam locomotive is usually calculated from the formula,

$$\text{H. P.} = \frac{D^2 \times L \times .85p \times S}{d \times 375}$$

that is, the horsepower of a locomotive is equal to the tractive power multiplied by the speed $S$, in miles per hour, divided by 375.

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# COMPRESSED-AIR HAULAGE

As compressed-air locomotives are self-contained they, like steam and storage-battery locomotives, can travel wherever tracks are laid. They have the great advantage over steam locomotives that they cannot set fire to methane, timber, or coal dust, and therefore may safely be used in gaseous, heavily timbered, or dusty mines. There are no boilers or fireboxes, and hence fewer repairs are needed; there is no danger from boiler explosions and they are more easily and cheaply kept in repair than steam locomotives. As they do not give off any smoke or injurious gas, they may be used in any part of any mine, and for gathering, and their exhaust tends to improve the ventilation. On the other hand, their use requires the installation of a more or less expensive power plant, storage tanks or lines, pipe lines, etc., although only a part of the entire outlay is chargeable to haulage where compressed-air coal-cutting machines are used. They are not so high as steam locomotives of the same tractive power and may therefore be used in thinner seams, but their greater length prohibits their employment on sharp curves if the entries are narrow. They are, also, no exception to the rule that locomotives of all types work uneconomically on grades because of the large amount of power required to propel the motor itself. A compressed-air haulage plant will generally cost more than an electric plant of the same working capacity, but there is no danger of electric shock where it is used, nor of igniting methane or coal dust by the arc produced when electric power lines come in contact with one another or with metal.

A compressed-air haulage plant consists of an air compressor driven by a steam engine or by an electric or hydraulic motor, which compresses the air to a pressure of 1,000 lb. per sq. in. or more; a storage system for the air until

it is needed by the locomotive, which may consist of receivers or tanks, but is usually the pipe line connecting the compressor with the charging stations where the engine receives its supply of air; and one or more locomotives, which may be either simple or compound.

**Simple, or Single-Stage, Compressed-Air Locomotives.**—A compressed-air locomotive has the driving and running gear of a steam locomotive, but the boiler of the latter is replaced by a long, riveted, steel-plate, storage tank containing the supply of air under a pressure of from 800 to 1,000 lb. per sq. in. These tanks vary from 26 to 42 in. in diameter, from 7 to 22 ft. in length, and from 50 to 350 cu. ft. in capacity, their size depending on the air consumption of the locomotive, which is determined largely by the length of time it must run without recharging. When necessary to reduce the height of the locomotive, the single tank may be replaced by two or more smaller ones, the combined capacity of which is equal to the one large one. In cases where the length of the haul requires unusual storage capacity and the size of the locomotive is limited by the height or width of the entry or like practical considerations, it is customary to use pressures up to 2,000 to 2,500 lb. per sq. in. The single riveted-steel tank is then replaced by a series of very heavy seamless steel tubes about 9 in. in diameter; but these excessive pressures are very rarely used about mines.

The air passes from the high-pressure, main storage tank to the auxiliary tank or reservoir, which is about 9 in. in diameter and two-thirds as long as the main tank. In this it is held at the working pressure of, usually, 140 lb. per sq. in., and passes from it directly to the cylinders. Between the tanks is an automatic reducing valve and sometimes a stop-valve. The object of the former is to maintain automatically a uniform pressure in the auxiliary reservoir and consequently in the cylinders. The stop-valve, when used, is placed between the storage tank and the reducing valve and is controlled by the same lever as the throttle valve. When the latter is open the stop-valve is open, thus admitting air to the auxiliary reservoir only as air is drawn from that into the cylinders, and preventing leakage between the two.

The air passes from the auxiliary tank to the cylinders through a balanced throttle valve, which permits of the maintenance of a constant working pressure, prevents waste of air, and makes the locomotive more easily managed. A stop-valve is placed between the auxiliary tank and the throttle valve to prevent leakage of air past the latter into the cylinders.

These locomotives are provided with a safety valve, reversing gear, oil cups, whistle, head- and tail-lights, sand box, and the other accessories of steam locomotives. They may have either four or six drivers, the latter, because of their relatively long wheel base, being the better adapted to long straight hauls. The table on page 800 gives the dimensions of the standard, or stock, sizes of these locomotives as made by the H. K. Porter Company, Pittsburg, Pa., although they have been largely displaced by the two-stage motors described later.

**Reheating Compressed Air.**—The efficiency of compressed air can be greatly increased by reheating before admitting it into the cylinders, but in haulage machinery the added complication of the mechanism needed for the purpose is seldom justified by the saving in fuel or the increased efficiency, except where the haul is very long or where the price of fuel is high. The first condition rarely and the second never exists in coal-mining practice. The air may be heated before entering the cylinders by partially filling the auxiliary air-storage tank with water, which is kept hot by the injection of steam while the main tanks are being filled at the charging station. Indirectly, this method of heating increases the efficiency in another way; the moisture taken up by the air from the hot water improves the lubrication in the valves and cylinders. The cylinders of compressed-air locomotives are not lagged as are those of steam-locomotives and, further, they are sometimes cast with the outside corrugated so as to increase the surface exposed to the relatively warm mine air.

The air may be effectively heated in a device known as an interheater, which is explained under Compound Locomotives.

**Compound, or Two-Stage, Compressed-Air Locomotives.**—There is no essential difference in mechanical construction and accessories between the simple, or single-stage, and the compound, or two-stage, compressed-air locomotive. In the latter, the air in the auxiliary reservoir is under 250 lb. per sq. in. pressure (as opposed to 140 lb. in the simple locomotive) and passes to a single high-pressure cylinder through the usual stop and throttle valves. In the high-pressure cylinder, the air is expanded down from 250 to 50 lb.

## DIMENSIONS OF SINGLE-STAGE COMPRESSED-AIR LOCOMOTIVES

| | Four-Wheel Locomotives | | | | Six-Wheel Locomotives | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Cylinders { diameter, inches | 6 | 7 | 7 | 8 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| stroke, inches | 10 | 12 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 16 |
| Number of driving wheels | 4 | 4 | 4 | 4 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| Diameter of driving wheels, inches | 23 | 24 | 24 | 23 | 23 | 26 | 26 | 26 | 26 | 28 | 33 |
| Rigid wheel base | 3' | 4' | 4' | 4' | 4'8" | 5' | 5'6" | 5'6" | 5'6" | 6'3" | 7' |
| Length over bumpers | 10' to 13' | 12' to 15' | 12' to 15' | 13' to 17' | 17'5" | 17'5" | 19'5" | 19'5" | 19'5" | 21'6" | 24' |
| Width outside gauge at cylinders | 24" | 26" | 26" | 28" | 26" | 28" | 30" | 32" | 34" | 38" | 42" |
| Extreme height, least desirable | 4'8" | 5' | 5'3" | 5'6" | 4'8" | 4'10½" | 5'3½" | 5'5" | 5'7" | 6' | 6'6" |
| Capacity, main reservoir, cubic feet | 50 to 60 | 75 to 85 | 80 to 90 | 85 to 100 | 130 | 150 | 180 | 200 | 240 | 275 | 350 |
| Tank pressure, pounds per square inch | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 | 800 |
| Cylinder pressure, pounds per square inch | 140 | 140 | 140 | 140 | 150 | 150 | 150 | 150 | 150 | 150 | 150 |
| Weight in working order, pounds | 10,000 | 15,000 | 17,000 | 20,000 | 18,000 | 23,000 | 27,000 | 31,000 | 37,000 | 43,000 | 51,000 |
| Tractive power, pounds | 1,860 | 2,915 | 3,400 | 4,100 | 3,265 | 4,390 | 5,560 | 6,870 | 8,310 | 9,185 | 10,450 |
| Lightest rail advised, pounds | 20 | 25 | 25 | 30 | 20 | 25 | 25 | 30 | 35 | 40 | 45 |
| Radius of sharpest curve advised, feet | 25 | 30 | 30 | 35 | 25 | 30 | 35 | 35 | 35 | 40 | 50 |
| Radius of sharpest curve practicable, feet | 15 | 16 | 16 | 20 | 20 | 20 | 25 | 25 | 25 | 35 | 40 |

pressure and is reduced in temperature to about 140° F. below that of the atmosphere or, say, from 60° above to 80° below zero. In order to restore as much as possible of this lost heat to the air, the high-pressure exhaust passes into an interheater, which consists of a cylindrical tank surrounding a number of brass or aluminum tubes of small diameter. The exhaust from the low-pressure cylinder, by means of a device like the exhaust nozzle of a steam locomotive, draws the warm mine air through the thin tubes of the interheater, and the high-pressure exhaust which is circulating around and between these tubes is raised in temperature to within about 15° of that of the mine air or, say, to 45° F. After being heated, the air is led to the low-pressure cylinder or cylinders, where its pressure is reduced from 50 lb. to that of the atmosphere, and is then exhausted and used to draw air through the interheater, as explained.

In expanding from $250+14.7=264.7$ lb. pressure at $460+60=520°$ temperature to $50+14.7=64.7$ lb. pressure at $460-80=380°$ temperature, both pressure and temperature being absolute, one volume of exhaust air from the high-pressure cylinder will be increased to $\frac{264.7}{64.7} \times \frac{380}{520} = 2.99$ volumes. In the interheater, the temperature is increased from 380° to $460+45=505°$ absolute, and its volume from 2.99 to $2.99 \times \frac{505}{380} = 3.97+$, or to, say, 4 volumes. For this reason, the area of the low-pressure cylinder or cylinders is made four times as great as that of the high-pressure cylinder so that the work done in them may be the same.

The cylinders are arranged in one of two ways. In the smaller locomotives, there are single high-pressure and low-pressure cylinders, one on each side, the latter having twice the diameter or four times the area of the former. Consequently, the over-all width of the locomotive is greater on the low-pressure than on the high-pressure side. In the larger locomotives, there are two low-pressure cylinders arranged in tandem whose joint area is four times that of the single high-pressure cylinder but whose diameters are to that of the high-pressure cylinder as $1:\sqrt{2}$, that is, as $1:1.414$. The object of two, instead of one, low-pressure cylinders is to reduce the over-all width on the low-pressure side.

These locomotives are furnished with either four or six drivers, but the latter, owing to their long wheel base and consequent stiffness on sharp curves, are not recommended unless the haulage road is straight or the lightness of the rail requires that the weight of the engine should be distributed over a greater length of track. Where loaded cars must be pulled out of dip places to the entry or where empty cars must be pulled to the face of a pitching room, the locomotive is provided with a reel or drum holding from 300 to 1,000 ft. of wire rope, the drum being turned by a small engine operated by compressed air while the motor is blocked on the rails. For use in very narrow places, the cylinders are placed between the drivers, that is, the locomotive is inside connected. For exceptionally long hauls an extra supply of air may be carried in a tender but as the weight of the latter reduces the paying load the locomotive can haul, it is better practice to increase the pressure in the main storage tank 2,000 or 2,500 lb., as heretofore explained.

The gain in efficiency through using two-stage instead of single-stage expansion is very considerable, amounting to 40 and even 60% in cases.

**Dimensions of Two-Stage Compressed-Air Locomotives.**—The following table gives the dimensions of standard two-stage compressed-air locomotives manufactured by the H. K. Porter Company, Pittsburg, Pa. In all cases, the charging pressure is between 700 and 1,200 lb. and the working pressure is 250 lb. in the high-pressure and 50 lb. in the low-pressure cylinder. The tank pressure and capacity are adjusted by the manufacturer to suit local conditions and requirements. The weight of the locomotive for the same size of cylinders depends on the gauge of the track, capacity of the storage tanks, etc. In special cases, the width outside the gauge line may be made less than that given in the table. The height of the entry must be at least 2 in. more than that of the locomotive above the rail to provide the necessary headroom.

The first four locomotives, which have one high- and one low-pressure cylinder, make a class by themselves. By reason of their moderate storage capacity and lightness they are adapted to gathering and the short wheel base recommends them for use on crooked entries. They can be made for any width of track and their height, which is not given in the table, is less for wide then for narrow gauges.

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## DIMENSIONS OF TWO-STAGE COMPRESSED-AIR LOCOMOTIVES

| | Light Four-Wheel Locomotives | | | | Heavy Four-Wheel Locomotives | | | |
|---|---|---|---|---|---|---|---|---|
| Cylinders { Diameter, high-pressure, inches | 4½ | 5½ | 6 | 7 | 7 | 8½ | 9½ | 10 |
| Cylinders { Diameter, low-pressure, inches | 9 | 11 | 12 | 14 | 14 | 12 and 14 | 14 and 14 | 14 and 14 |
| Stroke, inches | 10 | 10 | 12 | 14 | 14 | 14 | 14 | 14 |
| Number of driving wheels | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| Diameter of driving wheels, inches | 22 | 23 | 24 | 26 | 26 | 26 | 26 | 26 |
| Rigid wheel base | 2' 9" | 2' 9" | 3' or 4' | 4' | 4' | 4' 6" | 4' 6" | 5' |
| Length over bumpers | 9' to 12' 6" | 10' to 12' 6" | 12' to 15' 6" | 12' to 18' 6" | 14' to 20' | 16' to 21' | 18' to 21' | 20' to 23' |
| Width outside gauge at cylinders, inches { High-pressure | 11 | 12¼ | 13 | 14 | 14 | 15½ | 16⅜ | 17¼ |
| Width outside gauge at cylinders, inches { Low-pressure | 12¼ | 14¼ | 15 | 16½ | 16¼ | 16¼ | 17½ | 17½ |
| Height above rail | | | | | 5' to 5' 10" | 5' to 5' 10" | 5' to 5' 10" | 5' 5" to 5' 10" |
| Main reservoir capacity, cubic feet | 20 to 60 | 30 to 75 | 60 to 104 | 60 to 120 | 97 to 160 | 114 to 210 | 160 to 275 | 160 to 290 |
| Weight in working order, pounds | 7,000 to 10,000 | 10,000 to 13,000 | 14,000 to 17,000 | 18,000 to 22,000 | 20,000 to 27,000 | 28,000 to 36,000 | 32,000 to 42,000 | 43,000 to 46,000 |
| Tractive power, pounds | 1,450 | 2,200 | 3,000 | 4,400 | 4,400 | 6,400 | 8,000 | 9,200 |
| Lightest rail advised, pounds per yard | 16 | 16 | 20 | 30 | 30 | 40 | 45 | 50 |
| Radius of sharpest curve advised, feet | 15 | 15 | 20 | 30 | 30 | 35 | 40 | 50 |
| Radius of sharpest curve practicable, feet | 12 | 12 | 15 | 20 | 16 | 18 | 20 | 25 |

Six-Wheel Locomotives

| | | | | |
|---|---|---|---|---|
| Cylinders { Diameter, high-pressure, inches | 7 | 8½ | 9¼ | 10 |
| Diameter, low-pressure, inches | 14 | 12 and 12 | 14 and 14 | 14 and 14 |
| Stroke, inches | 14 | 14 | 14 | 14 |
| Number of driving wheels | 6 | 6 | 6 | 6 |
| Diameter of driving wheels, inches | 26 | 26 | 26 | 26 |
| Rigid wheel base | 5' 6" | 5' 6" | 5' 6" | 6' 6" |
| Length over bumpers | 14' to 20' | 16' to 20' | 18' to 21' | 20' to 23' |
| Width outside gauge at cylinders, inches { High-pressure | 14 | 15½⅛ | 17¼ | 17¼ |
| Low-pressure | 16½ | 16⅛ | 17⅞ | 17⅞ |
| Height above rail | 5' to 5' 10" | 5' to 5' 10" | 5' to 5' 10" | 5' to 5' 10" |
| Main reservoir capacity, cubic feet | 97 to 160 | 114 to 210 | 160 to 275 | 160 to 290 |
| Weight in working order, pounds | 20,000 to 27,000 | 28,000 to 36,000 | 32,000 to 42,000 | 43,000 to 46,000 |
| Tractive power, pounds | 4,400 | 6,400 | 8,000 | 9,200 |
| Lightest rail advised, pounds per yard | 25 | 30 | 35 | 40 |
| Radius of sharpest curve advised, feet | 50 | 50 | 50 | 70 |
| Radius of sharpest curve practicable, feet | 30 | 30 | 30 | 50 |

The second four locomotives, all but the first of which have two low-pressure cylinders, have four drivers and are standard for main line service.

The last four locomotives are also standard for main line use, and are the same as the preceding four except that they have six drivers and consequently are stiffer on curves.

**Tractive Power of Compressed-Air Locomotives.**—The tractive power and drawbar pull of compressed-air locomotives are calculated by the same methods as those of steam locomotives. Since the efficiency of the air is increased by absorbing heat from the cylinders, pipes, etc., at seven-eighths cut-off, which is practically full stroke, the cylinder pressure may be taken as 98% of the auxiliary-tank pressure and not as 85% of the boiler pressure as in steam locomotives. For earlier cut-offs, the efficiency is less than this; but in any case, for equal cut-off, the efficiency of air is greater than steam. The formula for the tractive power of a compressed-air locomotive, as modified from that of a steam locomotive is, thence,

$$T = \frac{D^2 \times L \times .98 \times p}{d}$$

in which the symbols have the meanings given for steam locomotives, except $p$, which is the working pressure in the auxiliary tank.

The tractive power given in the preceding tables is calculated by this formula, but in practice, as in the case of all self-contained haulage locomotives, but 50 to 90% of the theoretical power can be continuously exerted owing to poor track, imperfect lubrication, etc.

**Locomotive Storage Tanks.**—The amount of air that a locomotive must carry at high pressure in its main storage tanks depends in part on the length of the run and in part on the tractive effort that it must exert. The length of the run is readily determined and is twice the distance from the charging station to the place where the loaded trip is made up. The tractive effort that the locomotive exerts depends on the weight of the empty and loaded cars that must be hauled on the inbound and outbound runs, respectively, and the grades that must be overcome. In order, then, to determine the net tractive effort at any part of the run, it is necessary to have a profile of the road showing the grades with their amount and length. The tractive effort required to overcome any grade determines the average cylinder pressure; this determines the point of cut-

off; and this, in turn, determines the volume of air consumed per stroke of the piston. The number of strokes of the piston made while moving any distance multiplied by the volume of air consumed per stroke, gives the total volume of air used during that part of the run. The sums of the volumes of air required to overcome the various grades gives the total amount of air required for the round trip; and to this 20% is added as an allowance for emergencies. Storage capacity is provided for this volume of air when compressed from four to six times the working or cylinder pressure. Favorable grades are of much more importance in compressed-air than in steam haulage, because in the former the locomotive must carry all its power (compressed-air) with it, while in the latter the power (steam) is generated in the locomotive itself while in motion.

Air is sometimes admitted to the cylinders throughout nearly full stroke and consequently, as the exhaust is at high pressure, the efficiency is lower than it should be. This practice is doubtless due to the tendency to use as small a motor as possible for the service required, on account of the limited headroom and narrow crooked gangways so common in mines. Better economic results are obtainable, however, by using the air expansively and increasing the size of the locomotive and the weight on the drivers; this is almost always done with large locomotives. Ample reserve power is available when necessary, because full tank pressure can be admitted to the cylinders in starting a heavy load, or in pulling on steep grades and sharp curves.

In using the air expansively, as can be done with properly proportioned cylinders, there should be no trouble from freezing of the moisture. Although the cold developed will produce a low cylinder temperature, as the initial working pressure is so much higher than that employed for pumps and other compressed-air machinery, the expanded air becomes relatively dry, and the force of the exhaust will be sufficient to keep the ports clear of accumulated ice. To this end, the exhaust ports should be large, straight, and short.

**Stationary Storage.**—The stationary storage system from which the main storage tanks of a compressed-air locomotive are charged may consist of a pipe line or of one or more storage tanks. For short hauls, the storage tank is probably more convenient, but under all ordinary conditions the pipe line from the air compressor to the locomotive charging station is made of a diameter and strength sufficient to serve as a storage reservoir. The volume of stationary storage is found from the formula,

$$V = v\frac{(p - p')}{(P - p)},$$

in which $V$ = volume of stationary storage, in cubic feet;
$v$ = volume of locomotive storage, in cubic feet;
$P$ = pressure in stationary storage, in pounds per square inch;
$p$ = pressure in locomotive storage, in pounds per square inch;
$p'$ = pressure in locomoitve storage at time of charging, in pounds per square inch.

**EXAMPLE.**—The storage tanks of a compressed-air locomotive have a capacity $v$ of 160 cu. ft. at a pressure $p$ of 800 lb. per sq. in. If, at the time of charging the pressure $p'$ in the locomotive storage tanks is 250 lb. and the pressure $P$ in the pipe line (stationary storage) is 1,000 lb., what must be the volume $V$ of the stationary storage?

**SOLUTION.**—By substituting the given values in the formula,

$$V = 160 \times \frac{(800 - 250)}{(1,000 - 800)} = 440 \text{ cu. ft.}$$

It should be noted that the pressure $P$ in the pipe line is, in this case, 200 lb. per sq. in. more than that in the locomotive storage tanks, in order that the latter may be charged as speedily as possible. The time of charging is about 1.5 min., during but 40 to 50 sec. of which the valve connecting the pipe line to the locomotive is open. The pressure $p'$ in the locomotive tanks has here been taken at 250 lb., the same as the working or cylinder pressure maintained in the auxiliary tank and to which it has been reduced from the charging pressure of 800 lb. in the main tank because of the air consumed by the locomotive in doing its work. On short hauls, by reason of the small consumption of air, the pressure of that remaining in the locomotive storage tanks at charging time may be considerably more than the working pressure as maintained in the auxiliary reservoir, but can hardly be less without reducing the tractive power of the locomotive.

In the preceding example, if the pipe line from the compressor to the charging station is, say, 4,000 ft. long, each linear foot must have a capacity of $\frac{440}{4,000} = .11$ cu. ft. in order to store the 440 cu. ft. of air. From the accompanying table of pipe suitable for compressed-air haulage plants, a $4\frac{1}{2}$-in. pipe, having a capacity of .1105 cu. ft. per ft. of length, will be required. In order that the time of charging may be as short as possible, the volume of the pipe line storage is usually made two to three times that of the locomotive tanks, although this ratio will depend on the line pressure and, in some measure, on the number of locomotives and charging stations, the frequency of charging, etc. In the example, the ratio of locomotive to pipe line storage capacity is 160 : 440 or 1 : 2.75. If there is but one charging station, the storage capacity is ample, and the compressor is of sufficient size and power, the drop in pressure due to charging one locomotive is practically certain to be recovered before another can be coupled to the charging station. If there are several charging stations, which may be in use at or about the same time, it is advisable to increase the storage capacity of the line unless a wait of a few minutes is not of importance.

## STANDARD STEAM AND EXTRA-STRONG PIPE USED FOR COMPRESSED-AIR HAULAGE PLANTS

| Trade Diameter Inches | Cubic Feet in 1 Lin. Ft. | Linear Feet Necessary to Make 1 Cu. Ft. | Steam | | Extra Strong | | Trade Diameter Inches |
|---|---|---|---|---|---|---|---|
| | | | Thickness Inch | Weight per Foot Pounds | Thickness Inch | Weight per Foot Pounds | |
| 2 | .0218 | 45.41 | .15 | 3.61 | .22 | 5.02 | 2 |
| $2\frac{1}{2}$ | .0341 | 29.32 | .20 | 5.74 | .28 | 7.67 | $2\frac{1}{2}$ |
| 3 | .0491 | 20.36 | .21 | 7.54 | .30 | 10.20 | 3 |
| $3\frac{1}{2}$ | .0668 | 15.00 | .22 | 9.00 | .32 | 12.50 | $3\frac{1}{2}$ |
| 4 | .0873 | 11.52 | .23 | 10.70 | .34 | 15.00 | 4 |
| $4\frac{1}{2}$ | .1105 | 9.05 | .24 | 12.30 | .35 | 17.60 | $4\frac{1}{2}$ |
| 5 | .1364 | 7.33 | .25 | 14.50 | .37 | 20.50 | 5 |
| $5\frac{1}{2}$ | .1650 | 6.06 | .26 | 16.40 | .40 | 24.50 | $5\frac{1}{2}$ |
| 6 | .1963 | 5.10 | .28 | 18.80 | .43 | 28.60 | 6 |

If the length and diameter and, hence, the volume $V$ of the storage line are determined by piping already in place, the pressure $P$ required in the pipe line to instantly charge the locomotive may be found by transposing the preceding formula, and is,

$$P = \frac{v(p - p') + Vp}{V}$$

EXAMPLE.—If the volume $v$ of the locomotive storage tanks is 160 cu. ft.; the full pressure $p$ in the locomotive storage tanks is 800 lb. per sq. in.; the pressure remaining in these tanks at the time of charging is 250 lb.; and the storage line is 4,000 ft. of $4\frac{1}{2}$-in. pipe, to what pressure must the air in the storage line be compressed to charge the locomotive?

SOLUTION.—A $4\frac{1}{2}$-in. pipe has a volume of .1105 cu. ft. per ft. of length. If this volume is taken as .11 cu. ft., that of the storage line 4,000 ft. long will be $4,000 \times .11 = 440$ cu. ft. $= V$. Substituting,

$$P = \frac{160 \times (800 - 250) + 440 \times 800}{440} = 1,000 \text{ lb. per sq. in.}$$

**Pipe Lines and Charging Stations.**—Pipe lines should be as straight as possible and should not be placed in a trench and covered, as leakage is then difficult to detect and general inspection is impossible. Expansion joints are not necessary underground where the temperature is practically uniform but one may be needed on the surface between the compressor and the mouth of the mine. The lengths of pipe should be connected by heavy, threaded,

screw couplings that are counterbored with an annular groove into which a strip of soft metal can be driven to stop any leakage. Flanged or union couplings should be placed at all charging stations and at intervals of 300 ft. or so along the line. The ends of the pipe are riveted into recesses in the flanges and are hammer-faced flush with the center bore. The flanges are also counterbored to hold a soft-metal or vulcanized-rubber gasket. The introduction of these flange couplings permits of the easy repair, extension, or alteration of the line. The pipe should be given one or more heavy coats of some non-corrosive paint.

A valve should be placed between the compressor and the pipe line, at each charging station and at convenient points along the line, so that the compressor or parts of the line may be inspected and repaired without loosing any of the air in the pipes. Where the line runs down a shaft, a heavy cast tee with several feet of pipe below it is placed at the bottom of the shaft to collect water. A waste valve is inserted at the bottom of the pipe to permit the water to be blown off.

The charging stations are simple in construction and repair. Attached to the pipe line by a flange is a special tee to which is fitted a $1\frac{1}{2}$-in. gate valve having a short nipple into which is screwed a Moran flexible joint to which is attached a short length of pipe. The Moran joint is of the ball-and-socket type so that the pipe may be turned in any direction necessary to couple up with the locomotive tanks. When not in use, the supply pipe is turned parallel to the track. The locomotive is also provided with a pipe having ball-and-socket joints and a gate valve opening into the locomotive tanks but which closes as soon as the gate valve on the pipe line is closed to cut off the pressure. A special coupling is used to connect the supply line and locomotive tanks. This coupling cannot be broken as long as any pressure remains in the pipes between the gate and check-valves, so a small globe valve is placed in the line immediately above the gate valve in order to bleed off the remaining air. All valves must be kept tight, packing must be replaced when worn out or lost, and all joints and connecting pipes must be supported to prevent undue stress coming upon them.

**Air Compressors for Haulage Plants.**—The capacity of an air compressor for a given plant depends on the number of locomotives, the capacity of their tanks, and the length of time between chargings. The number of cubic feet $F$ of free air (air at atmospheric pressure, 14.7 lb.) required to charge a locomotive may be found from the formula

$$F = \frac{(p - p')v}{14.7}$$

in which the letters have the meaning of those in the preceding formula. Having found $F$, the capacity of the compressor $C$ in cubic feet of free air per minute may be found from

$$C = \frac{nF}{t}$$

in which $n$ equals the number of charges in the time $t$.

EXAMPLE.—A haulage system requires that the single locomotive in use shall be charged three times an hour. The storage-tank pressure is 800 lb. and the pressure remaining in the tank at the time of charging is 250 lb. If the locomotive tanks have a capacity of 160 cu. ft., what must be the size of the compressor in cubic feet per minute?

SOLUTION.—Here $v = 160$, $p = 800$, $p' = 250$, $n = 3$, and $t = 60$ min. By substitution,

$$F = \frac{(800 - 250) \times 160}{14.7} = 6,000 \text{ cu. ft., about}$$

The required capacity of the compressor is,

$$C = \frac{3 \times 6,000}{60} = 300 \text{ cu ft. per min.}$$

Compressors for charging are of the three-stage type for pressures up to 1,000 lb. per in., and for higher pressures are usually four stage. They are provided with intercoolers as explained under Compressed Air.

The horsepower required to compress the air may be found from the following table. Thus, in the example, to compress 300 cu. ft. of air per min. to 800 lb. in a three-stage compressor will require $3 \times 32.5 = 97.5$ H. P.

## HORSEPOWER NECESSARY TO COMPRESS 100 CU. FT. OF FREE AIR TO VARIOUS PRESSURES AND WITH TWO-, THREE-, AND FOUR-STAGE COMPRESSORS

| Gauge Pressure Pounds | Horsepower Necessary | | | Gauge Pressure Pounds | Horsepower Necessary | | |
|---|---|---|---|---|---|---|---|
| | Two-Stage | Three-Stage | Four-Stage | | Two-Stage | Three-Stage | Four-Stage |
| 100 | 15.7 | 15.2 | 14.2 | 900 | 36.3 | 33.7 | 31.0 |
| 200 | 21.2 | 20.3 | 18.8 | 1,000 | 37.8 | 34.9 | 31.8 |
| 300 | 24.5 | 23.1 | 21.8 | 1,200 | 39.7 | 36.5 | 33.4 |
| 400 | 27.7 | 25.9 | 24.0 | 1,400 | 41.3 | 37.9 | 34.5 |
| 500 | 29.4 | 27.7 | 25.9 | 1,600 | 43.0 | 39.4 | 35.6 |
| 600 | 31.6 | 29.5 | 27.4 | 1,800 | 44.5 | 40.5 | 36.7 |
| 700 | 33.4 | 31.2 | 28.9 | 2,000 | 45.4 | 41.6 | 37.8 |
| 800 | 34.9 | 32.5 | 30.1 | 2,500 | | 43.0 | 39.0 |

## GASOLINE-MOTOR HAULAGE

**Construction of Gasoline Locomotives.**—Gasoline locomotives, except for the absence of a trolley pole, greatly resemble electric locomotives in appearance as all their moving parts, gasoline tanks, engines, etc., are enclosed in the same form of iron or steel casing in order to protect them from injury from falling roof, collision, etc. These locomotives are, at present, made in various sizes up to 20 T. in weight, those under 5 T. being generally used for gathering.

The engines are of the four-cycle type, and usually have four cylinders, although the larger ones may have six. The cylinders may be vertical or horizontal, the latter construction being necessary in mines where the head-room is limited. The engine shaft is placed lengthwise of the frame and is connected by gearing to a cross- or jack-shaft near the front end of the locomotive. From the cross-shaft, power is transmitted to the axle of the nearest pair of driving wheels either by spur gearing or by a chain passing around sprocket wheels. The two pairs of drivers are connected either by rods, as in steam or compressed-air locomotives, or by a chain drive passing over sprocket wheels on each axle. By either arrangement, each axle is a driving axle and the full power of the engines is utilized. There is no general or fixed ratio between the weight, in tons, of a gasoline locomotive and the horsepower developed by its engines. The various manufacturers, in their catalogs, indicate a ratio of from 6 to 8 to even 10 engine horsepower to each ton weight of the locomotive, but the engines are rated much below their capacity. Thus, an engine rated in the catalog as, say, 40 H. P. will commonly develop 50 to 55 or more H. P. upon brake tests, and will exceed this if its speed is allowed to exceed the 600 to 800 rev. per min., to which these engines are commonly limited.

The smaller locomotives are made with low and high gears and the larger ones, sometimes but not always, are made with low, intermediate, and high gears, which allows of two and three speeds, respectively, both forwards and reverse; and other speeds may be had by varying that of the engine. The low gear, giving a speed of from 3 to 5 mi. per hr., is used while bringing the trip from rest to full speed (accelerating) or while doing very heavy pulling, while the high (or intermediate) gear is used under all ordinary conditions. The speed changes are made by means of jaw clutches and the forward and reverse motions by means of friction clutches, both mechanisms being operated by levers from the engine cab. The transmission gearing is contained in an oil-tight casing or box through which a continuous flow of oil is circulated from the engine shaft, and which is in operation only while the locomotive is in motion.

After circulating through the water-jackets surrounding the cylinders, the cooling water passes to radiators or cooling tanks, being forced through the

system by a small pump operated by gearing from the main engine-shaft. The radiators are cooled by air from a small muitiblade fan placed on the forward end of the engine shaft. The gasoline fuel is carried in two seamless drawn-steel or copper tanks, which are provided with safety valves (thus giving each tank two valves) to prevent leakage. It is almost always arranged that the tanks cannot be removed or charged except at a regular charging station, which, in the case of drift mines, is always outside but near the opening. The charging is done in a few minutes by removing the empty tanks and replacing them with full ones, which hold about 5 gal. each.

The exhaust gases from the cylinders pass to some device intended to cool them and to prevent back firing and the escape of sparks and flame to the atmosphere. In most cases, this consists of a muffler provided with steel tubes, baffle plates, wire gauze, etc., the gases being sometimes led over water as an additional precaution against fire. The muffler also serves to make the motor as nearly noiseless as possible. In one type of locomotive, the exhaust is passed through a series of parallel perforated pipes contained in what is called a *deodorizing tank*, which is filled with a liquid preparation that extinguishes the flame and neutralizes the smell before the gases finally escape.

The more recent types of these locomotives are provided with a self-starter, which consists of an electric motor receiving current from a storage battery, and which, by means of reducing gear, can be made to drive the crank-shaft of the main engine. The storage battery is automatically charged by a generator driven by the main engine, and requires no attention. The chief advantage of the self-starter is that it allows the engines of the heavy locomotives, which are difficult to start by hand particularly when cold, to be shut down when the motor is not in motion, and to be readily started when the trip must be moved. Not only does this effect a material reduction in fuel consumed, but is of prime importance in that it prevents the pollution of the mine air by the exhaust gases if the engine is kept in motion while the locomotive is still, the common procedure when the self-starter is not used. The self-starting device is also made a source of current for operating electric headlights.

These engines are provided with headlights and tail-lights, a bell, one or more sand boxes, and a whistle operated by compressed air. All have efficient hand-brakes and some of the larger ones have a complete air-brake system, the air being supplied by a small compressor operated from the engine shaft, or otherwise. In this connection, the subject of Internal-Combustion Engines, on page 532, may be consulted.

**Hauling Capacity and Fuel Requirements.**—The maximum tractive power of a gasoline locomotive is exerted under low gear and may be taken as one-fifth of its weight under ordinary conditions and as one-quarter under favorable ones. The tractive power under high gear is, in the case of locomotives with two gears, about one-half that under low gear and should be made the basis of estimating the size or weight of a locomotive to meet the prevailing conditions. That is, power is sacrificed to gain speed. The drawbar pull, resistance of the cars, etc., are figured in exactly the same way as for steam locomotives.

Some choice in the matter of speed is offered. Thus the two-speed locomotives supplied by a leading manufacturer may be had with speeds of 3 and 6 or 4 and 8 mi. per hr., respectively. The 16-T. locomotive of the same maker is offered with four combinations of speeds on the low, intermediate, and high gears, respectively, of 3, 9, and 15, or 4, 12, and 20, or 5, 15, and 25, or 6, 18, and 30 mi. per hr. The relative speed ratios for low, intermediate, and high gear are 1 : 3 : 5, or between the low and intermediate 1 : 3 and between the intermediate and high 1 : 1⅔. While it is true that the increase in speed is accompanied by a loss in drawbar pull, yet these locomotives under ordinary conditions should pull at full speed under high gear the load they can start and accelerate under low gear. In mine practice, the speed is very commonly limited by law to 6 or 8 mi. per hr., but there seems no good reason why, in main-line haulage where the track and equipment are in first-class modern condition, that speeds of 20 and 30 mi. per hr. would not be perfectly safe.

The gasoline consumption of these locomotives depends on their size, the loads hauled, the grades, the length of shift, and whether they are operated continuously or intermittently. When operated continuously at full power, the engines will probably use a pint, or a little less, gasoline per horsepower per hour. Thus, a 10-T. locomotive with engines, say, of 62.5 H. P., will burn $62.5 \times 8 = 500$ pt. $= 62.5$ gal. of gasoline per 8-hr. shift. But these locomotives are usually over-engined, so that on the heaviest grades they rarely

exert more than three-fourths power. Further, no gasoline is used when descending a grade nor, if provided with a self-starting device, when standing waiting a trip, and but little is required for switching, etc. Experience has shown that it is very unusual for the engines to develop for an entire shift more than one-half their rated horsepower. In the example just cited, this would reduce the gasoline consumption from 62.5 to 31.25, say, 30 gal. per shift, which is at the rate of 3 gal. per T. of weight of the locomotive. This is a maximum. One manufacturer estimates the daily fuel requirements at 8 to 10 gal. for a locomotive with 25-H. P. engines (one weighing 3.5 T.), and as 10 to 20 gal. for one with engines of 50 H. P. (weighing 8 T.). Mr. Carl Scholz, speaking of the gasoline locomotives in his own mines, where average conditions prevail, says: "The average consumption of gasoline and oil for an 8-hr. shift on a 6-T. motor is about $2, gasoline costing 17 c. per gal." This would indicate a fuel consumption of 12 gal. per shift, perhaps a little more, as the amount paid for oil is not stated. At Gatliff, Tenn., a 5-T. motor uses 11 gal. of gasoline in 9 hr. Available figures indicate that under average working conditions the consumption of gasoline per shift is at the rate of 2 gal. per T. of weight of the locomotive, although in some cases it may be as high as 2.5 gal., and in rare instances 3 gal. It should be noted that the larger locomotives, particularly when fitted with self-starting devices, require relatively less fuel than the smaller ones, or those that must be started by hand. In comparison with steam locomotives, a manufacturer states that the cost of fuel is about the same for each type of motor for the same capacity.

**Cost of Gasoline-Locomotive Haulage.**—At the No. 2 entry of the Roane Iron Co., Rockwood, Tenn., the round-trip haul is 3 mi. with a uniform grade of 1.5% in favor of the loads. The average weight of the empty cars is 1,400 lb., of the loaded cars 3,640 lb., and the live load or weight of coal per car is 2,240 lb. Ten 20-car trips of empties are hauled in and the same number of loads hauled out from the mine in one shift. The inbound empties have a total daily weight of 140 T., the outbound loads one of 364 T.; the total daily weight moved is 504 T., and that of the coal delivered to the tipple is 224 T. The following are the details of the haulage costs by mule and by gasoline motor:

COST OF HAULAGE AT ROCKWOOD, TENN.

| By mules: | | |
|---|---:|---:|
| 4 drivers, at $1.65 | $6.60 | |
| 9 mules, at $.50 | 4.50 | $11.10 |
| | | |
| By motor: | | |
| 1 motorman, per day | $2.05 | |
| 1 coupler, per day | 1.65 | |
| 13 gal. gasoline, at 11½ c. | 1.50 | |
| 2 lb. carbide, at 4 c. | .08 | |
| ½ gal. gasoline engine oil, at 23 c. | .12 | |
| 1 gal. transmission case oil | .24 | $ 5.64 |
| | | |
| Saving by motor | | $ 5.46 |
| Or, 49.1%. | | |

The cost per ton of coal delivered to the tipple is 4.955 c. by mules and 2.518 c. by locomotive. 140 T. of empties are hauled into the mine a distance 1.5 mi. and 364 T. of loads are hauled out the same distance. This is equal to 504 T. hauled 1.5 mi. or 756 T. hauled 1 mi., at a cost of $11.10 or 1.468 c. per T.-mi. by mules and $5.64 or .746 c. per T.-mi. by motor. If the cost of hauling the live load of 224 T. of coal 1.5 mi. (equivalent to 336 T. moved 1 mi.) is considered, the cost per ton-mile by mules is 3.303 c. and by motor 1.678 c. In neither cost statement is any allowance made for depreciation, repairs, renewals, interest on the investment, etc. As the majority of these charges, particularly the first and third, are much greater in the case of mule than motor haulage, the difference in favor of the locomotive will be greater than that shown by the foregoing figures.

A 5-T. gasoline-haulage motor at the mines of the Southern Coal and Coke Co., Gatliff, Tenn., handles 500 net T. of coal daily in 9 hr., working 22 da. per mo. The average daily haul is 354 cars weighing 1,200 lb. when empty and a little more than 4,000 lb. when loaded. The haul is from an inside parting 2,500 ft., say ½ mi. from the tipple, on an undulating road, which varies in grade from 1.5% against the loads to 3.5% in their favor. The details of mule and motor haulage are as follows:

### Cost of Haulage at Gatliff, Tenn.

| | | |
|---|--:|--:|
| 1 gal. lubricating oil ......................... | $ .15½ | |
| ½ gal. gasoline engine oil. ..................... | .16½ | |
| 11 gal. gasoline, at 14 c. ...................... | 1.54 | |
| Carbide, use lights but little .................. | .15 | |
| Motorman ..................................... | 2.50 | |
| Trip rider .................................... | 1.83 | |
| Cleaning and repairs .......................... | .40 | |
| 6% int. on difference of cost of mules and motor. | .52 | |
| Extra upkeep on track over mules ............. | .50 | |
| Motor replaces 7 mules, at 42 c. per day for feed | | $ 2.94 |
| Three drivers ................................. | | 7.23 |
| Reduction to stable boss ...................... | | .35 |
| Daily saving by use of motor .................. | 2.76 | |
| | $10.52 | $10.52 |

The cost per ton of coal delivered to the tipple, on the basis of 500 T. per da., is 2.104 c. by mules and 1.552 c. by gasoline motor. The empty cars have a total weight of 212 T. and the loaded cars 708 T., making a total weight of 920 T. hauled ½ mi., which is equivalent to 460 T. moved 1 mi. The cost is, hence, 2.287 c. per T.-mi. for mule haulage and 1.687 c. for locomotive haulage. On the basis of the live, or paying, load only, the cost for transporting 500 T. ½ mi. (250 T. for 1 mi.) is 4.208 c. per T.-mi. by mules and 3.104 c. per T.-mi. by locomotive.

A gasoline locomotive at the mine of the Midvalley Coal Co., Wilburton, Pa., which displaces a steam locomotive and five mules, when making but 24 mi. per da., or about one-half its capacity, effects a saving over mule haulage of 32.2%. The locomotive is rated at 9 T. and uses 15 gal. of naptha per day at a cost of 10 c. per gal., or $1.50 for fuel. It is estimated that the consumption of gasoline would be 12 gal. per da., which, at 15 c. per gal., would cost $1.80. For a period of 6 mo., during which 2 hr. of each 9 hr.-da. were devoted to switching and were not properly chargeable to haulage, the average daily mileage for the loaded and empty cars was 12 for each. The empty cars weighed 2.5 T. and the total weight of them handled in 1 da. was 250 T. The loaded cars weighed 5.5 T., and the total weight of them handled in 1 da. was 550 T. The net weight of coal delivered to the mine mouth was, hence, 300 T. per da. The daily cost for motor haulage only was as follows:

### Cost of Haulage at Wilburton, Pa.

| | |
|---|--:|
| Wages of motorman and helper ......................... | $3.35 |
| 15 gal. of naptha at 10 c. ............................. | 1.50 |
| Lubricating oil ....................................... | .12 |
| Maintenance, $65.14 for 6 mo., 20 da. per mo. ........... | .54 |
| Total ......................................... | $5.51 |

The cost is, then, 1,837 c. per T. of coal delivered to the mine mouth, or but 1.429 c. per T. if the cost of the time spent in switching is deducted. From the figures furnished, there appear to have been an average of 12.5 trips per day, and the length of haul was not far from 1 mi. With this understanding, the cost per ton-mile was .687 c. for the combined weight of the loads and empties (800 T.) and 1.429 c. per T.-mi. for the 300 T. of coal delivered.

At the plant of the Shade Coal Mining Co., Windber, Pa., the haulage cost per ton of coal delivered to the tipple was 6.4 c. by mules and 3.15 c. by gasoline locomotive. On the ton-mile basis, the cost of mule haulage was 12.8 c. and of gasoline haulage 3.79 c. for the coal delivered to the tipple.

**Comparison of Gasoline and Other Types of Haulage Motors.**—Like other self-contained locomotives (steam, compressed-air, and storage-battery electric), the gasoline motor has the advantage over the ordinary electric mine locomotive operated through a trolley from overhead wires, that it can go anywhere in the mine that the tracks are laid. As compared with the compressed-air and overhead and storage-battery electric locomotives, it does not require a more or less expensive plant for the generation of power. As compared with the steam locomotive, it is as cheaply operated, does not so greatly befoul and befog the air with unpleasant or dangerous gases, and when the exhaust, carbureter, etc., are properly protected, is not so likely to ignite either coal dust or methane. It is questionable if the danger of igniting either of

these explosive agents is as great with a well-designed and well-managed gasoline motor as with the ordinary overhead-trolley electric locomotive.

The chief objections made to this type of locomotive relate to the cost of power and the difficulty of obtaining competent operators and to the danger to the health of the underground workers from the exhaust gases. When the power-plant charges are considered in the cost of electric or compressed-air locomotives, the cost of power for a gasoline locomotive will be found to be very much less than for the other two types. On the other hand, if the mine is already piped or wired for compressed-air or for electric coal-cutting machinery, some study will be required to determine if the haulage can be done more cheaply by gasoline motors than by those operated by the power already in use. Competent operators are readily obtained from outside workers familiar with motor trucks, the use of which as a substitute for horse-drawn wagons is rapidly increasing. Attracted by the better wages prevailing in the mine, a little training in the use of the gasoline locomotive and a familiarity with underground work, makes them first-class motormen. The complaint that gasoline locomotives will not take an overload as will electric locomotives does not seem well-founded. It is a question of proportioning the engine power to the weight of the locomotive, and a gasoline motor that has engine power enough to slip its drivers, will pull as great a tonnage as any other locomotive of the same weight.

It is unquestionably true, however, that a gasoline locomotive does discharge into the mine air a certain amount of obnoxious and harmful gases, carbon dioxide and carbon monoxide, respectively. At the same time, a definite amount of oxygen is withdrawn from the air and used in the combustion of the gasoline. The amount of these gases will depend on how well the engine is working, and this, in turn, will, in a very great measure, depend on the skill of the operator. The following analyses of mine air taken from workings where a gasoline locomotive was used are furnished by Mr. A. J. King, in an article read before the West Virginia Coal Mining Institute and reprinted in the Colliery Engineer for October, 1913; the samples having been taken by Mr. P. A. Grady, formerly mine inspector for the 12th District, West Virginia.

**ANALYSES OF MINE AIR AS AFFECTED BY EXHAUST OF GASOLINE LOCOMOTIVES**

| Number of Sample | $CO_2$ | $O_2$ | $CO$ | $CH_4$ | $N_2$ |
|---|---|---|---|---|---|
| 1 | .07 | 20.87 | .03 | .10 | 78.93 |
| 2 | .11 | 20.92 | .07 | .13 | 78.77 |
| 3 | .13 | 20.80 | .06 | .32 | 78.69 |
| 4 | .09 | 20.78 | .03 | .33 | 78.77 |
| 5 | .15 | 20.91 | .05 | .11 | 78.78 |
| 6 | .15 | 20.86 | .07 | .10 | 78.82 |

Sample No. 1 was taken 80 ft. ahead of the air at the face of a room in which the locomotive had been 5 min. with the engines running. The grade was 3% in favor of the loaded car and 6,000 cu. ft. of air per min. was passing on the entry.

Sample No. 2 was taken in the same place as sample 1, the locomotive having been run up to the face and had pulled out a loaded car.

Sample No. 3 was taken at the face 210 ft. ahead of the ventilating current, after the locomotive with its engines running had stood in the place for 5 min. and then come out; the fumes were noticeable.

Sample No. 4 was taken in the same place, the locomotive having run to the face and coupled to a loaded car which it pulled down a 3% grade in 1 min.

Samples Nos. 5 and 6 were taken on the entry, which had a cross-section of 15 ft. by 5 ft., an area of 75 sq. ft., and through which 6,000 cu. ft. of air per minute was passing. The samples were gathered after the locomotive had been made to perform hard work by running up the entry, which had a grade of 5%.

Mr. O. P. Hood, in the October, 1914, Bulletin of the American Institute of Mining Engineers, gives the following table, which shows the amount of $CO$ and $CO_2$, in cubic feet per minute, given off by gasoline locomotives with cylinders of various sizes, when running under both good and bad conditions.

## VOLUME OF CO AND CO₂ DISCHARGED BY GASOLINE LOCOMOTIVES, IN CUBIC FEET PER MINUTE

| Engine Cylinder Size Inches | Number of Cylinders | Speed Revolutions per Minute. | Piston Displacement Cubic Feet per Minute* | Maximum Probable Amount of Noxious Gases, in Cubic Feet per Minute, at 60° F. and 30 In. Barometer Produced With | | | | Cubic Feet of Air per Minute Required to Dilute Exhaust Gases to .10% of Air | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Good Carburation | | Bad Carburation | | Good Carburation | Bad Carburation |
| | | | | $CO$ | $CO_2$ | $CO$ | $CO_2$ | | |
| 4.75×5.25 | 4 | 800 | 172 | 2.61 | 6.80 | 9.91 | 3.65 | 2,610 | 9,910 |
| 5×5 | 4 | 600 | 136 | 2.06 | 5.37 | 7.84 | 2.88 | 2,060 | 7,840 |
| 5×5 | 4 | 800 | 182 | 2.76 | 7.18 | 10.48 | 3.86 | 2,760 | 10,480 |
| 5.5×5 | 4 | 800 | 218 | 3.30 | 8.60 | 12.56 | 4.62 | 3,300 | 12,560 |
| 5.5×5 | 4 | 600 | 165 | 2.50 | 6.51 | 9.50 | 3.50 | 2,500 | 9,500 |
| 6×6 | 4 | 700 | 275 | 4.17 | 10.86 | 15.85 | 5.85 | 4,170 | 12,850 |
| 6×7 | 4 | 500 | 229 | 3.47 | 9.04 | 13.19 | 4.85 | 3,470 | 13,190 |
| 6.5×7 | 4 | 500 | 269 | 4.07 | 10.63 | 15.50 | 5.70 | 4,070 | 15,500 |
| 6.5×8 | 4 | 650 | 399 | 6.04 | 15.76 | 23.00 | 8.46 | 6,040 | 23,000 |
| 7×7 | 4 | 500 | 312 | 4.73 | 12.33 | 17.97 | 6.62 | 4,730 | 17,970 |
| 7×7 | 6 | 500 | 468 | 7.08 | 18.49 | 26.97 | 9.92 | 7,080 | 26,970 |
| 8×7 | 4 | 500 | 407 | 6.16 | 16.08 | 23.45 | 8.62 | 6,160 | 23,450 |
| 8×6 | 6 | 500 | 610 | 9.24 | 24.10 | 35.14 | 12.93 | 9,240 | 35,140 |

* Piston displacement = area piston in square feet × stroke in feet × number of cylinders × revolutions per minute.

Referring to the West Virginia experiments in the first of the preceding tables and disregarding the $CO$, the greatest amount of gases foreign to normal air is met in the third sample, which shows the presence of .45% of $CO_2$ and $CH_4$ combined, neither of which gases is poisonous even in very much larger amounts than there shown. The dangerous gas is $CO$, the maximum amount of which that may be breathed for a short time and intermittently without injurious effects is commonly stated to be .10%. When breathed continuously, air should not contain more than .05% of this gas, and preferably but .025% unless those exposed to its effects are in good health and are not working violently. All the samples show less than the maximum, and the average of all is the safe minimum of .05%. All the samples appear to have been taken in the inner workings at places (the face of a room or entry) where the locomotive would not usually go more than, say, once an hour, and probably not more than five or six times a shift. In the intervals between trips, the $CO$ would soon diffuse even if there was no circulation of air, so that the mine cited was by no means in a bad condition although every effort should be made to keep the content of $CO$ to .05%. In the mine in question, this could readily have been done by increasing the volume of the air-current from 6,000 to 9,000 cu. ft. per min., which would have raised the velocity of the air from 80 to 120 ft. per min.; and neither the volume nor the velocity are high. Mr. King recommends that where gasoline locomotives are used, in addition to the air that would ordinarily be circulated through the entry, there should be a further amount of 1,000 cu. ft. per min. for each ton in weight of the locomotive.

Mr. King's requirements are considerably in excess of those of one of the leading manufacturers, who advocates that where the locomotive is in continuous operation there should be in circulation 800 to 1,000 cu. ft. of air per min. per ton of weight of the locomotive, depending on whether the hauling is done upon the intake or return. An occasional trip to a side entry or even to a working place beyond the air will not require a material increase in the quantity of air in circulation, as the normal volume of air in motion assisted by diffusion will carry off the $CO$ before a return trip is made to the same part of the workings. On the other hand, it is admitted that if the consumption of gasoline is the maximum, the air requirements will be doubled; but it is stated that it appears impossible to consume this maximum under ordinary normal conditions.

Mr. Hood's figures are of value in giving the quantities of harmful ($CO_2$) and poisonous ($CO$) gases exhausted per minute by gasoline-locomotive engines of various standard sizes when operating continuously with good and with bad carburation. But haulage motors are not working continuously under full load; in fact, they are idle so much of the time that the fuel consumption is commonly about one-third the maximum, in rare cases rising to one-half. Whether the air circulated shall be based on average or on extreme conditions is a question for the mine manager. If extreme conditions are to be provided for, the use of gasoline locomotives will be prohibited in many mines, where the assumption of average conditions would permit it. The advocates of gasoline haulage claim if extreme conditions (for no engines will be permitted to work badly for a longer time than that required to shut off the fuel supply) must be assumed in dealing with this type of locomotive, then electric haulage should be prohibited because of the possibility of a fall of roof bringing down the trolley wires with the consequent chance of a dust explosion through the electric arc almost certain to be produced; and compressed-air and wire-rope haulage should not be allowed as the compressor might explode or the trip jump the track.

The variation in the quantity of air required by a motor in constant use and, thus, consuming the maximum quantity of fuel, both when the carburation is good and when it is bad and when the proportion of $CO$ in the air is to be kept at .10% and at .05% may be illustrated in the case of two locomotives as follows: A 5-T. motor with 5″×6″ cylinders suitable for side-entry haulage will require 3,300 cu. ft. per min. of air when working properly if the $CO$ in the air is to be kept at .10% and 6,600 cu. ft. if the $CO$ is to be kept at .05%. The same motor, when working badly, will require nearly four times as much air or 12,560 and 25,120 cu. ft., respectively, depending on the allowable percentage of $CO$. Similarly, a 9-T. motor with 6.5″×8″ cylinders and adapted to main-line haulage will require 6,040 or 12,080, and 23,000 or 46,000 cu. ft. per min. of air, depending on whether the carburation is good or bad and whether the $CO$ is to be kept at .10 or .05%. The range in air requirements between the best and worst conditions of operation is from 3,300 to 25,120 cu. ft. per min, in the case of the smaller locomotive, and in the case of the larger one,

from 6,040 to 46,000 cu. ft. per min. Such an increase in the quantity of air, the ratio being 1 to 8, cannot be made from time to time as it may be temporarily needed, and must be permanently provided for in the ventilating scheme of the mine.

Gasoline locomotives have been in use too short a time to have permitted a solution of all the problems connected with their employment, and the following suggestions in their selection and management may be of value:

1. Buy a high-grade motor from a responsible manufacturer and be guided by his advice in its selection.

2. Do not use a larger motor than necessary to do the work; this will save in first cost, in fuel consumption, and particularly in the amount of $CO$ discharged.

3. Use only high-grade gasoline and employ only experienced motormen, who might be given a bonus for low fuel consumption; this will lessen both the fuel bill and the quantity of $CO$ admitted to the air.

4. If possible, arrange the main-line haulage so that the grades favor the outbound loads. If this is done, and the return air-current is made the haulage road, the engines may be shut down and the loads dropped out by gravity, saving in fuel and in $CO$ discharged. The empties will be taken in under power against the air, and the resultant velocity of the air-current will be that of the inbound locomotive added to that of the outbound air. If the velocities of the locomotive and the air-current are the same, this will double the quantity of air passing the locomotive and may provide enough excess air to take care of possible temporary bad carburation which is more apt to happen under full than under part load.

5. Do not have the speed of the locomotive and the air-current the same when they are moving in the same direction; to do so will cause a concentration of gas around the locomotive, which will prove harmful to the motorman.

6. Where possible, avoid pulling from dip workings unless the air supply is ample, because the maximum amount of gasoline is consumed and $CO$ produced when starting up grade under full load.

7. Do not use the locomotive ahead of the air either in rooms or entries more than necessary. It is in tight places that the effects of small amounts of $CO$ are the most marked. If compelled to enter such places, remain there as short a time as possible, and do not allow the miners to return to the face until some time has elapsed in order that the air-currents set up by the moving trip and diffusion may have an opportunity to dilute the $CO$ to the safe limit.

8. In event of carbureter troubles, shut off the gasoline instantly; an exception might be made when hauling against the full strength of the air-current.

9. Except in a strong air-current, do not allow the locomotive to stand with the engines running; this precaution is particularly to be observed in places ahead of the air. To this end, have the locomotive provided with a self-starting device.

10. If the men complain of sickness, remove the locomotive from that part of the mine until the reason for the trouble is found. Frequently a slight adjustment of the ventilating current made by opening or closing a regulator will remedy the trouble.

11. Overhaul and clean the locomotive thoroughly at the end of each shift. Under no circumstances is it safe to use a locomotive underground when the carbureter and ignition are out of order.

12. As the effects of $CO$ upon the system are dangerous, the percentage of it in the air must be kept as low as possible. Unfortunately, there are no simple tests for this gas that may be applied by the miner or the foreman, and the first indication of its presence is its ill effects. It might be well, then, at the time of installing gasoline-motor haulage to employ a competent chemist for a sufficient length of time to follow the locomotive to all parts of the mine to secure samples for analysis of the air in the places the motor has been. The expenditure of a few hundred dollars in taking and analyzing such samples will either satisfy the management that the use of gasoline motors is perfectly safe, or will suggest changes in the ventilating system or in the haulage schedules, or in the use of certain roads for haulage, etc., that, if carried out, will make the use of such motors unobjectionable.

**Purification of the Exhaust.**—No satisfactory way of getting rid of the $CO$ in the exhaust of gasoline locomotives has as yet been devised. It is probable that absorption by cuprous chloride ($Cu_2Cl_2$), the reagent used for the purpose in gas-analysis apparatus, is not practicable because of the expense.

Solutions of lime or caustic soda or even plain water will absorb $CO_2$ to a certain extent, and at the same time will remove the odor.

# ELECTRIC-LOCOMOTIVE HAULAGE
## GENERAL CONSIDERATIONS AFFECTING ELECTRIC HAULAGE

**Advantages and Disadvantages of Electric Locomotives.**—All well-designed and well-built haulage motors are practically equal in first cost, in the labor cost of running, and in the cost of repairs, therefore, as an effective machine for gathering and hauling coal, an electric locomotive possesses no marked advantages over one propelled by steam, gasoline, compressed air, or storage battery. The advantages and disadvantages commonly attributed to one type of locomotive as compared with another, are in reality due more to the power employed than to qualities inherent in the locomotive itself.

There is, however, one disadvantage possessed by the electric locomotive that does not exist in the case of self-contained locomotives propelled by steam, gasoline, or compressed air; it can be used only in those parts of the mine where trolley wires have been hung for conveying the power and where, at the same time, iron rails have been laid for the return circuit. This objection has been largely overcome through the use of a combination trolley and storage-battery locomotive, which can go anywhere in the mine; through the use of a cable-reel locomotive, which can go the length of the cable beyond the end of the trolley line; and through the use of the crab locomotive, which, while standing on the main road and drawing current from the trolley line can pull a car from a distance equal to the length of the rope carried on its rope drum.

So far as safety is concerned, the only locomotive that cannot possibly ignite either methane or coal dust is one operated by compressed-air; and it cannot cause death from shock. These are objections made rather unjustly to the electric locomotive and should be charged against the means of conveying power to it, the naked overhead trolley wire, and not to the locomotive itself which, in its modern form, is an eminently safe machine. It may be argued that the ventilating current should be sufficient to prevent the existence of dangerous quantities of methane either in entries or rooms, that accumulations of coal dust should be avoided, and that the mine should be watered or treated with shale dust, and that men should be careful and not come in contact with naked live wires, but while the first two conditions are successfully met in the majority of mines and there is usually little danger of an electric spark or arc igniting either methane or dust, there is always the possibility of this happening even in the best-managed mines. The dangers of shock or death through contact with live overhead wires can hardly be removed because of the seeming impossibility of teaching the average mine worker to be even reasonably careful. The possibility of setting fire to partitions, timbers, etc. is so slight if even moderate care in properly insulating the wires is taken as to be negligible. A danger chargeable to the locomotive itself may arise when an unusually heavy trip is started or when the customary load is pulled up a heavy grade. In both cases, it is usual to sand the rails and if too much sand is used, the contact between the locomotive wheels and the rails for the return circuit is broken, and the current will take a longer but easier path. This may result in serious shock to the motorman, or if the current passes from the locomotive through the drawbars of the cars and thence to the rail at some unsanded place, those riding in the cars may be injured, powder in metallic cans may be exploded, etc. The last is such a real danger, numerous accidents having happened therefrom, that in many states it is prohibited by law to transport powder in cars hauled by an electric locomotive. This last danger may be overcome in a very great measure by using a locomotive of a size proportioned to the work, and then not overloading it.

Where many power-consuming machines are used in and around a mine, rather than have each run by its own independent power generating engine it is cheaper and better to have a common power for all and to produce this at some central plant and transmit it to the various points of application, and it is the general adaptability of electricity to all power purposes that has led to its extensive use in the operation of haulage locomotives. Electric energy may be generated at any reasonable distance from the mine where power may be had and may be easily and cheaply transmitted to any place needed where it is available not only for the operation of haulage motors but also for that of coal-cutting machinery, pumps, ventilating fans, shot-firing systems, lighting, etc. Hence, in mines where many power-requiring machines are used, and particularly where these are scattered throughout the workings, electricity is a favorite source of energy and, being used for all other purposes, is used for haulage. On the other hand, in mines where, aside from haulage,

power is required only to run the ventilating fan, it will unquestionably prove cheaper in first cost and probably in operation to install a steam or gasoline locomotive. Compressed air may also be transmitted from a central plant and is available for the same purposes as electricity, except shot firing and lighting and is further absolutely safe under all underground conditions, but a compressed-air plant, including the piping, is more costly to install, extend, and operate than an electric plant.

**Current and Voltage.**—Direct current is generally used for electric haulage; the pressure most commonly used is about 250 volts, although 500 volts has been tried and is still used in some places. The objection to the higher pressure is the greater danger of injurious or fatal shocks, as well as the greater difficulty of insulating the wires from ground. The higher pressure can be profitably used only where all the passages through which the wires are strung are high or roomy enough to permit placing the wires where there will be little danger of contact with them, and dry enough to preserve the insulation.

**Electric Generators.**—If the power house is near the mouth of the mine, direct-current dynamos are generally used to generate the electric energy for haulage purposes, and at the pressure used in the mine, 250 or 500 volts as the case may be. It is frequently advantageous to locate the power house at some distance from the mine, so as to take advantage of a water fall to generate the power, or for other economic reasons; in such cases, in order to reduce the cost of line copper, it is customary to transmit a high-voltage alternating current to the mouth of the mine, or sometimes to the interior near where it is to be used, there transform it in step-down transformers, and convert it by means of rotary converters to direct current at ordinary mine voltage, after which it is treated in the mine installation precisely as would be the case with a direct current generated at the mouth of the mine.

**Classes of Electric Locomotives.**—According to the kind of current used, electric locomotives may be divided into direct-current, alternate-current, and storage-battery locomotives. The direct-current locomotives are those in general use in the United States and may be single or tandem. The standard form of direct-current locomotive is used for main-line haulage; the modifications of it used for gathering are known as combination, cable-reel, and rope-reel, or crab, locomotives. A special type, known as a rack or third-rail locomotive, is used on heavy grades.

## WIRING FOR ELECTRIC HAULAGE

**Arrangement of Power Lines.**—In a shaft mine or a steep slope, insulated feeder wires are run from the dynamo on the surface down the shaft or the slope, or occasionally down a bore hole, into the mine, where they are connected to the trolley wire and rails in the gangways; or, feeder wires may be continued along the haulage roads for a distance depending on the length of the haulage road and the amount of electric current that must be carried. Where



(a)   (b)   (c)

Fig. 1

the mine opening is a shallow slope or a drift, the power is sometimes carried into the mine by bare-wire conductors fastened at intervals to the caps, or legs, of the timbers. If the mine opening is wet, the power is transmitted through lead-covered cables. In shafts, the cables may be held in position by wooden brackets placed on the sides of the shaft, or suspended from the top by block and tackle, by means of which the cables may be moved up or down. In a wet shaft, the lead cable is carr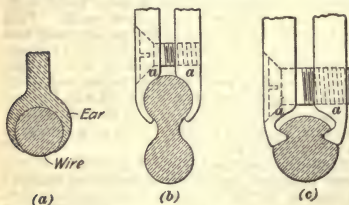ied far enough into the mine from the bottom of the shaft to be free from the shaft water, and is then connected with the bare wire used in haulage. A main switch should be provided at the foot of the shaft or the slope, so that the power can be turned off or on instantly.

**Shape of Trolley Wire.**—Trolley wire is made with round, figure-**8**, or grooved-cross-section, as shown in Fig. 1 (a), (b), and (c), respectively. The round wire, shown at (a), has been generally used for the purposes of mine haulage and for transmitting electric power into the mine. The preference for mine work now inclines to the grooved form shown at (c). This wire is sup-

ported by the clamp ears *a*, which fit into grooves in the sides of the wire just above the center. The figure-**8** wire, shown at (*b*), is liable to twist between supports and throw off the trolley; the round or the grooved wire, which is practically circular in section, may be twisted without interfering in any way with the trolley. When rounding curves, the figure-**8** wire is also more liable to pull or twist out of shape or out of the clamps entirely than either of the other shapes.

**Location of Wires.**—The trolley wire is located above the track, preferably along one side and from 6 to 15 in. outside the rail, so as to be out of the way of men and animals passing along the road. Where the roof is good, the trolley wire may be supported directly from it. The trolley construction should be of the most substantial nature, and the work of installation should be in charge of an experienced man, as the care and thoroughness with which this is done determine largely the successful operation of the plant. The mining laws of many states provide that when the haulage road is used as a traveling way, the trolley wire shall be set in an inverted wooden trough or boxing with sides from 3 to 5 in. deep. Various provisions are made for the safety of men compelled to pass and repass under the trolley wires at some particular point as at the foot of a shaft, at a parting, etc. At such places it is not unusual to compel the wires to be placed at their lowest point at least 6 ft. 6 in. above the top of the rail.

**Trolley Frogs.**—Fig. 2 (*a*) shows the under side of an overhead switch, or trolley frog, used to guide the trolley wheel from one wire to another. This is a simple **V** frog; it is shown in its natural position in (*b*). The trolley wires are held by clamps *b*, and the span, or supporting, wires are attached to the ears *a*. The frog must be placed with reference to the track so that the motion
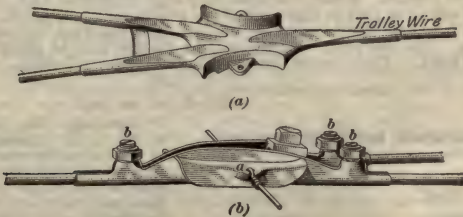


*(a)*

*(b)*

Fig. 2

of the locomotive, as it takes the switch, will have given the trolley an inclination in the right direction before the trolley wheel strikes the frog. The frog must also be considered. Or it will cause the wheel to leave the wire.

A simple method of finding the proper location for the trolley frog is shown in Fig. 3. Measure the distance from the point of switch *a* to the frog *c* and half way between these points make a chalk mark on the rail at *b*. Do the same on the straight rail, and mark the half-way point *d*. Stretch a line from *a* to *d* and one from *e* to *b*. Directly above their point of intersection *f* is the place for the frog.



Fig. 3

**Resistance of Steel Rails.**—As the rails form the return circuit for the electric current, they must be considered in connection with the voltage drop. The rail itself, on account of its large cross-section, has a large current-carrying capacity and the bonding should be done so that no appreciable drop will take place in the joints. The weight of rail, in pounds per yard, is fixed by traffic considerations and is usually determined by allowing 10 lb. per yd. for each ton of locomotive weight per driving wheel. Thus, a 10-T., four-wheel locomotive will have $10 \div 4 = 2.5$ T. on each driver and the required weight of rail will be $2.5 \times 10 = 25$ lb. per yd. This formula gives the minimum weight of rail, but much better results will be obtained by using the heavier rail recommended in the accompanying table.
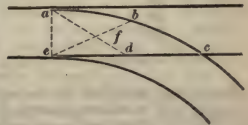
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## SIZES OF LOCOMOTIVES, RAILS, AND BONDS

| Weight of Locomotive Tons | Minimum Weight of Rail per Yard and Size of Bond | | Weight of Rail per Yard Recommended and Size of Bond | |
|---|---|---|---|---|
| | Rail Pounds | Bond Number | Rail Pounds | Bond Number |
| 3 | 16 | 4 | 20 | 0 |
| 4 | 16 | 4 | 25 | 0 |
| 5 | 16 | 4 | 25 | 0 |
| 6 | 16 | 4 | 30 | 0 |
| 7 | 20 | 0 | 40 | 00 |
| 8 | 20 | 0 | 40 | 00 |
| 10 | 25 | 0 | 45 | 00 |
| 15 | 40 | 00 | 50 | 0000 |
| 20 | 50 | 0000 | 60 | 0000 |
| 25 | 60 | 0000 | 80 | 0000 |

The resistance of steel rails to the passage of an electric current varies considerably with the composition of the metal. For the purpose of calculation it is, however, common to take the specific resistance of steel rails as twelve times that of copper. While this value may seem somewhat high, it is conservative and will allow for the slight additional resistance at the joints. By using the values from the following table, the rails can therefore be considered as continuous. The resistance values given are for two rails in parallel; that is, per mile of track. These values are based on the following formula:

$$\text{ohms per mile} = \frac{2.63}{\text{weight of rail per yard}}$$

### RESISTANCE OF STEEL RAILS

| Weight of Rail per Yard Pounds | Resistance per Mile of Track Ohms |
|---|---|
| 16 | .1642 |
| 20 | .1313 |
| 25 | .1051 |
| 30 | .0876 |
| 40 | .0657 |
| 45 | .0583 |
| 50 | .0525 |
| 60 | .0438 |
| 80 | .0328 |

**Bonding.**—The larger part of the track resistance occurs at the joints between the rails, and as the fish-plates do not form sufficient electric contact, the ends of the rails at the joints are always connected by a copper conductor known as a *bond*. The first of the two tables just given shows that the area of metal in the bond is essentially the same as that in the trolley wire.

There are many types of rail bonds, but these may be divided into two general classes; *protected bonds*, or those placed between the fish-plate and the rail, and *unprotected bonds*, which either span the fish-plate or are placed under the rail. All should be attached to the rail in such a way that the contacts between the copper and steel are clean and bright when made.
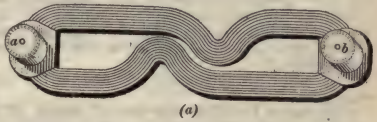
Fig. 4 (*a*) shows a protected bond of the *double-loop type*, shaped so as to give flexibility and at the same time allow openings for the track bolts. This bond is made of thin copper strips on which copper terminals *ab* are cast. After the terminals have been passed through the holes in the rail, they are compressed by a special screw compressor, which forces the metal out sidewise firmly against the sides of the holes. View (*b*) shows this bond in position, part of the fish-plate being cut away to expose the bond to view. The holes through the rail for track bolts show through the loops of the bond.

Fig. 5 shows one form of unprotected bond. The copper terminals are pressed into the holes in the rails by a powerful screw press, which expands
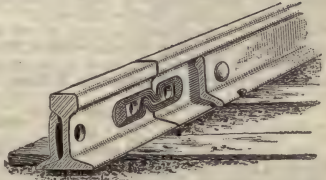
the metal in the hole so as to give complete contact with the rails. All bonds should be inspected frequently, as they may work loose.

A poor return circuit, which is due to poor bonding, is responsible for much of the motor trouble. If the voltage drops because of poor bonding or from any other cause the amperes will be increased with the result that the armatures will heat and possibly burn out. Poor bonding is commonly indicated by the marked drop in the illuminating power of the headlight (which then burns with a dull red glow) when a trip is started.

**Cross-Bonding.**—In addition to the regular bonding at the joints, the one line of rails should be electrically joined, or cross-bonded, to the other at intervals of not more than 500 ft. and by conductors of the size used for bonding. The object of cross-bonding is to still provide a complete return circuit in event of some of the rail bonds jarring loose. Instead of the standard form of cross-bonding both the D., L., & W. R. R. and the L. C. & N. Co. have successfully used old wire hoisting rope as a portion of the return circuit. The largest size of rope is used on the main haulage roads and 1½-in. rope on the branches. The rope is suspended near the bottom of the props which carry the feed wire and is bonded to the rails every 250 to 300 ft. Where the rope has to be spliced, the abutting ends, after being thoroughly cleaned and brightened, are inserted in the opposite ends of short pieces of lead pipe that are filled with solder. The attachments for bonding the rope to the rail are made by soldering a clamp to the rope, using the regular bonding hole on the rail. Tests have shown that the current-carrying capacity of a 1½-in. hoisting rope made of steel low in carbon and manganese is the same as that of a rail weighing 30 lb. per yd.



*(a)*

*(b)*

Fig. 4

**Feeders.**—Current is generally fed to the locomotive through an overhead trolley system with the track rails forming the return circuit as explained. In addition to the trolley wire, it is also almost always necessary to install feeders to reduce the drop in voltage. A *feeder* is a heavy insulated or bare copper cable suspended along one side of the heading and is tapped into the trolley wire at intervals along the route.

In the early days of electric mine haulage, the size of trolley wire was much smaller than now used, the size varying from No. 0 to No. 0000. The former is only used in small one- or two-locomotive installations, and experience has shown that a heavy trolley wire is of considerable advantage. For this reason the use of No. 0000 trolley wire is now very common.
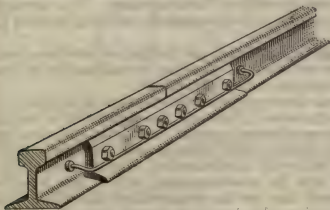


Fig. 5

The size of the feeders depends on the length of the haul, the distribution of the load, the current to be transmitted, and the permissible voltage drop. Excessive drop is a very common cause for complaint in a mine using electric haulage and it always pays to put sufficient copper in the feeders to prevent the voltage at the locomotives from falling to too low a value. Low line voltage makes it difficult to maintain the schedule and gives rise to trouble with the motors, to say nothing of the cost of the power loss. As an approximate rule, the voltage drop from the point of supply to any locomotive should be kept within 20%.

An approximate estimate of what the drop in the rails will be can easily

be formed at the outset by means of the table on page 818. The balance of the drop will then give that allowed for the trolley and feeders combined, and their cross-section can be determined from the following formula:

$$\text{Area, in circular mills} = \frac{10.8 \times L \times I}{D}$$

in which $L$ = distance between point of supply and load, in feet;

$I$ = maximum current, in amperes;

$D$ = drop in trolley and feeders, in volts.

From the value so found is subtracted the cross-section of the trolley, in circular mils, the result being the required size of the feeders. The calculation is easy, the only difficulty being the variation in the load both in magnitude and position.

In order to illustrate the method of calculation assume the following examples:

EXAMPLE 1.—Find the size of feeder, if the voltage is 500; rails, 40 lb.; trolley, No. 0000; length of road, 1 mi.; load, 400 K. W., bunched at end of line; permissible drop, 20% = 100 volts.

SOLUTION.—Resistance of 1 mi. of two 40-lb. rails = .0657; current = 400,000 ÷ 500 = 800 amp.; drop in rails = 800 × .0657 = 52.6 volts. This leaves a drop of 100 − 52.6 = 47.4 to take place in the trolley wire and feeder. Assuming the same conductivity of the material in these, their combined cross-section should be:

$$\frac{10.8 \times 5,280 \times 800}{47.4} = 965,000 \text{ cir. mils}$$

The trolley wire is No. 0000 and has a cross-section of 211,600 cir. mils. Deducting this from the total cross-section of 965,000, leaves 965,000 − 211,600 = 753,400 or about 750,000 cir. mils.

EXAMPLE 2.—Suppose that the total load of 400 K. W. is equally distributed, what size of feeder is required?

SOLUTION.—This is equivalent to an average load of 200 K. W. transmitted over the whole circuit. The drop in the rails is now only half the former value or 26.3 volts, leaving 100 − 26.3 or 73.7 volts to be consumed in the trolley and feeders. Their combined cross-section will therefore be:

$$\frac{10.8 \times 5,280 \times 400}{73.7} = 310,000 \text{ cir. mils}$$

Deducting from this 211,600 cir. mils for the trolley leaves only about 100,000 cir. mils, which corresponds to No. 0 feeder.

In making these calculations, attention must be paid to the carrying capacity of the wires and cables. This must be kept in mind, because if the lines are simply figured out on the basis of giving the allowable drop, the current may be sufficient to overheat the wires. In most cases, however, the size of wire necessary to keep the drop within the specified limits will be considerably larger than necessary to handle the current without overheating. It is always well, however, to compare the sizes obtained and the current carrying capacity, which will be found in the wire table.

By referring to Example 1, it is seen that there is no danger of overheating, but in Example 2 it will be necessary to increase the size considerably in the section nearest the station where the current value is too high.

The pressure at which the current is supplied to the motors is limited by considerations of safety. It would otherwise, of course, be desirable to use a higher pressure, because this would mean a lower current, less drop and smaller feeders for the same power. For this reason 500 volts is used in a few mine haulage systems, although 250 volts evidently is somewhat safer in operating.

In mines of ordinary capacity, it will be uneconomical to use the direct-current system only, when the current has to be transmitted for distances over 1 mi., and many mines have during the last few years been changing over their systems to a combination alternating current and direct current. That is, alternating current is generated and transmitted at a higher voltage to substations distributed along the tracks. In these substations, the alternating current is changed to direct current by means of synchronous converters. In this manner the 250-volt, direct-current supply can be brought near the centers of distribution and the losses in the lines, feeders, and rails are considerably reduced, also smaller size conductors can be used.

The following table shows the distance to which 100-K. W., three-phase current can be transmitted over different sizes of wires at different potentials, assuming an energy loss of 10%. A power factor of 85% is shown by the table.

## SIZES OF WIRES FOR THREE-PHASE TRANSMISSION SERVICE

Distance of Transmission for Various Potentials at Receiving End, in Miles

| No. B. & S. | Area in Circular Mils | Voltage of Line | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 2,000 | 3,000 | 4,000 | 5,000 | 6,000 | 8,000 | 10,000 | 12,000 | 15,000 |
| 6 | 26,250 | 1.32 | 2.98 | 5.28 | 8.27 | 11.92 | 21.12 | 33.1 | 47.68 | 74.50 |
| 5 | 33,100 | 1.66 | 3.75 | 6.64 | 10.40 | 15.00 | 26.56 | 41.6 | 60.00 | 93.75 |
| 4 | 41,740 | 2.10 | 4.74 | 8.40 | 13.15 | 18.96 | 33.60 | 52.6 | 75.84 | 118.96 |
| 3 | 52,630 | 2.54 | 5.96 | 10.16 | 16.55 | 23.84 | 40.64 | 66.2 | 95.36 | 149.00 |
| 2 | 66,370 | 3.33 | 7.51 | 13.32 | 20.85 | 30.04 | 53.28 | 83.4 | 120.16 | 187.75 |
| 1 | 83,600 | 4.21 | 9.48 | 16.84 | 26.32 | 37.92 | 67.36 | 105.3 | 151.68 | 212.00 |
| 0 | 105,500 | 5.29 | 11.92 | 21.16 | 33.10 | 47.68 | 84.64 | 132.4 | 191.72 | 298.00 |
| 00 | 133,100 | 6.71 | 15.11 | 26.84 | 41.97 | 60.44 | 107.36 | 167.9 | 241.76 | 377.75 |
| 000 | 167,800 | 8.45 | 19.04 | 33.80 | 52.85 | 76.16 | 135.20 | 211.4 | 304.64 | 476.00 |
| 0000 | 211,600 | 10.62 | 23.92 | 42.48 | 66.42 | 95.08 | 169.92 | 265.7 | 382.72 | 598.00 |

**EXAMPLE.**—What size of wires should be used to deliver 500 K. W. at 6,000 volts, at the end of a three-phase line 12 mi. long, allowing energy loss of 10% and a power factor of 85%?

**SOLUTION.**—If the example called for the transmission of 100 K. W. (on which the table is based), look in the 6,000-volt column for the nearest figure to the given distance, and take the size wire corresponding. But the example calls for the transmission of five times this amount of power, and the size of wire varies directly as the distance, which in this case is 12 mi. Therefore, look for the product 5×12=60 in the 6,000-volt column of the table. The nearest value is 60.44 and the size wire corresponding is No. 00, which is, therefore, the size capable of transmitting 100 K. W. over a line 60.44 mi. long, or 500 K.W. over a line 12 mi. long.

If it is desired to ascertain the size wires that will give an energy loss of 5%, or one-half the loss for which the table is computed, it is only necessary to multiply the value obtained by 2, for the diameter varies directly as the per cent. energy loss.

## VOLTAGES ADVISABLE FOR LINES OF VARIOUS LENGTHS IN THREE-PHASE TRANSMISSION

| Length of Line | Voltage |
|---|---|
| Up to 1 mile | 500 to 1,000 |
| 1 to 2 | 1,000 to 2,300 |
| 2 to 3 | 2,300 to 6,600 |
| 3 to 10 | 6,600 to 13,200 |
| 10 to 15 | 13,200 to 22,000 |
| 15 to 20 | 22,000 to 33,000 |

## DIRECT-CURRENT LOCOMOTIVES

**Number and Arrangement of Motors.**—Direct-current electric locomotives with two pairs of driving wheels may have one or two motors, while those with three pairs of drivers commonly have three motors.

In the single-motor locomotive, the armature, which is set lengthwise of the frame, is geared at each end to a driving axle with the result that both pairs of wheels revolve at the same time and with the same speed. This arrangement insures a high degree of adhesion with consequent strong tractive effort, together with perfect distribution of the weight on the drivers and good contact between the wheels and the rail.

There are two standard methods of mounting the motors in two-motor locomotives. In the tandem mounting, one motor is placed between the axles and the other between the forward axle and the front-end frame. In the central, or inside, mounting, both motors are placed between the axles. The tandem mounting permits of a short wheel base and is adapted for light- and medium-weight locomotives, which are commonly required to operate upon sections of the track having short-radius curves. The central, or inside, mounting requires a longer wheel base and is adapted for heavy locomotives used in main-line haulage, where the roads are commonly straight or with curves of long radius. With either arrangement, the locomotive frame is proportioned to give an equal distribution of the weight between both pairs of driving wheels. The motors may also be end mounted by placing each motor in the space between the axle and the forward and rear frames, respectively. This permits of a minimum wheel base, but is only used to meet very unusual conditions. An advantage claimed for the two-motor locomotive is that, in case of accident to one of the motors, the defective one may be disconnected and the machine run with one motor to the repair shop or it may be kept in operation although able to do less work. The advocates of the two-motor machine also claim for it higher efficiency and better speed control than is possible with the single-motor locomotive.

Six-wheel, three-motor locomotives of 15 to 25 T. weight may, to a certain extent, be used instead of the tandem locomotives described on page 826, for long and heavy runs over a main haulage road. Their application is, however, more or less restricted, as on account of their greater length they may not take the sharp curves usually found in mine work. Where this is not objectionable, locomotives of this kind have advantages, one being the possibility of using lighter rails than for a two-motor locomotive of the same weight, due to the equalization of the weight on all three pairs of driving wheels. To insure this, irrespective of any irregularities of the track, three-motor locomotives are supported from the journal-boxes, thus insuring at all times an even division of the load among the three motors. The equalizing system also furnishes a flexible suspension of the weight and produces an easy running locomotive, greatly minimizing the wear and tear on the track and roadbed. The center pair of wheels is generally furnished without flanges so as to prevent any binding on the curves.

**Construction of Motors.**—In the design and construction of mine locomotive motors, the following requirements are essential: Maximum capacity within the gauge limitations; large overload capacity; accessibility for inspection and repair; large bearing surface to minimize wear; protection against dust and moisture; accurate machining to insure interchangeability of parts; rugged construction to withstand rough usage.

The motors are always of the series type and should preferably be equipped with commutating poles. In this kind of motor, the same current passes through the main-series and commutating-pole field coils. The torque exerted and the speed at which it will run, depend on the flux entering the armature, the number of conductors on the armature, and the amount of current flowing in the winding. The flux, in turn, depends on the strength of the field magnets, which in their turn depend on the number of turns in the field coil and the amount of current flowing therein. The advantage of the commutating poles, which are connected in series with the armature, lies in the fact that the electrical and mechanical neutrals are made to coincide for all loads and for either direction of rotation, thus assuring good commutation under all conditions of operation.

The motor frames are split diagonally so that the upper part can be lifted off, exposing the interior for inspection. The bearing heads are securely clamped between the upper and lower frames, making it possible to readily take out the armature for repairs. The laminations, armature windings, and

commutator are all mounted on a common spider so that the shaft may be removed without disturbing them, and interchange can therefore readily be made.

The armature bearings are commonly of the Babbitt-lined, bronze-sleeve construction, designed for oil and waste lubrication. The Babbitt is of such a thickness that should it be melted from lack of lubrication, the shaft will be supported by the sleeves before the armature strikes the pole pieces.

The use of ball bearings in mine-locomotive motors is a new feature. Where they have been tried, they have given excellent results, and at present they bid fair to displace the plain bearings for this class of service. The principal advantage gained by the use of ball bearings is the small amount of lubricant required. This lubricant being vaseline or some similar grease in small amounts, there is very little possibility of its getting into the motor windings. There is a great advantage in this, as a large percentage of the motor troubles can be traced directly to oil having worked its way into the windings. When properly lubricated, ball bearings have another advantage, in that there is but a small amount of wear. This decreases the liability of the armature coming down on the pole faces with damaging results.

The field coils are held securely in place by spring-steel flanges, which are pressed against the coils by the pole pieces when the latter are bolted in place.

**Controllers.**—The controllers are of the rheostat magnetic blow-out kind. A commutating switch is incorporated in the reverse cylinder, the handle of which has four on-positions, two for each direction of motion, one with motors in series and the other with motors in multiple. The main and reverse cylinders are interlocked in the usual manner and the main cylinder provides for speed regulation with motors in series or multiple. This system of control by permitting motors to be started in multiple, allows them to exert their maximum tractive effort independently, so that the slippage of one motor does not affect the other—a valuable feature for starting heavy trains.

When the operating handle is in the off-position, all parts of the motor and rheostat equipment are dead and it is also impossible to retard the train by bucking the motors—a practice of motormen that is liable to cause trouble.

While single-end control may be considered standard, locomotives are often built with a controller at each end, a construction often of advantage.

**Frames.**—The general construction of electric mine locomotives involves two distinct forms, one in which the side frames are placed outside the wheels, and the other in which they are placed inside the wheels.

For a given track gauge, the outside frame allows the maximum space between the wheels for the motors and other parts of the equipment, renders the journal-boxes more accessible, and gives somewhat more space at the operating end for the motorman. The inside frame restricts, to a certain extent, the space between the wheels available for motors and other equipment, but allows for the minimum overall width, a construction that is necessary in those mines where the props are set close to the track or the space outside the rails is otherwise limited. The wheels being outside the frame, this locomotive in case of derailment is somewhat more readily replaced.

The locomotives may be supplied with side frames of cast iron or rolled steel plate, the latter construction being now the most generally used. The end frames as a rule consist of steel channels fitted with heavy wooden bumpers, except on large locomotives where cast-iron bumpers may be advantageous in order to get more dead weight. The bumpers and coupling devices must be designed to suit the mine cars.

**Wheels and Journals.**—The weight of the locomotive is ordinarily supported from the journal-boxes on heavy helical springs. The journals are somewhat similar to those on railway cars that have removable brasses and are lubricated from oil cellars filled with waste.

The construction of the journal-boxes is such that the brasses can be removed without disturbing the axles or frame stay-plates. On outside-frame locomotives, this is accomplished by jacking up the frame to relieve the pressure on the journal-box spring and removing two vertical retaining plates. The inside-frame journal-boxes are fitted with a removable oil cellar, which can be lowered for repacking.

Plate wheels are generally used for outside-frame locomotives, while for the inside-frame construction spoked wheels are used, in order to give access to the journal-boxes.

Chilled cast-iron and steel tires or rolled-steel wheels are in general use, the first named being, however, the more common. These are approximately 60% cheaper than steel-tired wheels and 45% cheaper than those of rolled

steel. The higher cost of the steel wheels is, however, largely offset by the fact that the treads can be refaced several times, if facilities are provided therefor in the repair shop.

A somewhat increased adhesion is generally realized by the adoption of steel wheels for mine service. While opinions differ greatly as to how much this really amounts to, it is generally conceded to be about 5%. A considerable portion of this increased effort is often found to be due to the greater weight of such wheels, and a comparison is difficult to make, as both the wheel tread and rail are always subject to wide variations caused by moisture, nature of the surface, amount and quality of sand used, etc.

**Brakes.**—Several kinds of brake mechanisms are in use. In an exceptionally strong and efficient one, the brake shoes are automatically locked in any position in which they are left by the operator without the use of pawls or ratchets. The brake shoes, made of cast steel, are removable in order to insure a long life and in addition exert a dressing action on the wheel tread.

In certain instances where there is a heavy down grade, in order to be sure of controlling the loaded trains, a special rail-grip brake is provided in addition to the usual wheel brakes. Jaws are arranged to press the shoes against a third rail, which is laid in the center of the track. This brake is powerful enough to stop a train within a distance of 100 ft. on an 8% grade, the train weighing 100 T., exclusive of locomotive, and running at a speed of 8 mi. per hr.

Sand riggings are always provided, and the sand boxes so arranged that the rails may be sanded ahead, when running in either direction.

**Trolleys.**—In the standard mine trolley, the wheel is mounted in a swiveled harp, which permits it to aline itself with the trolley wire, irrespective of the direction of the pole. The pole is of wood and the lower end is inserted in a swiveled base, which fits into sockets on either side of the locomotive. The force of the compressed spiral spring is so applied to the pole that the pressure of the trolley wheel against the wire is approximately uniform throughout the limits of vertical variation and the swivel harp permits a wide lateral variation of the wire. The pole being of wood is thoroughly nonconducting, and is so located that the motorman can easily handle and reverse it without leaving his position. The trolley cable terminates in a contact plug, which fits in a receptacle placed on each side of the locomotive so that the change from one side to the other is readily effected.

Some locomotives are made with two trolley poles, one at each side. This construction is convenient in those mines, where for any reason it is necessary to hang the trolley wire on the side opposite to the one where it is usually placed.

**Headlights.**—The headlights provided at each end of the locomotive, are usually each fitted with a 32-c.p. incandescent lamp, which gives sufficient illumination. A luminous-arc mine headlight is, however, manufactured, the mechanism of which is simple and requires little attention. The upper electrode is made of copper and lasts from 2,000 to 3,000 hr., while the lower one, which is made of a composition of magnetite, lasts from 50 to 75 hr.

**Capacity of Locomotives.**—Local conditions must be given a very careful study in laying out a system of electric mine haulage. Not only should the present output be considered but also the possibilities of increased output and longer hauls. The number of cars to be handled per trip and per hour, the time of lay-over, etc., must be correctly determined so as to result in the most efficient operation. It is also important that the main-haul locomotives have sufficient capacity to place on the parting enough empty cars per trip to serve the gathering locomotives simultaneously in order to prevent any reduction in the output from delays.

The amount of load that a locomotive is capable of hauling depends on the weight of the locomotive, the adhesion between the driving wheels and the track, the frictional resistance of the trailing load, and the curvature and gradients of the track.

The adhesion varies greatly, depending on the condition of the surfaces in contact, but experience has shown that with clean dry rails on a level track the coefficient of adhesion can safely be assumed to be 20% for cast-iron wheels and about 25% for steel-tired wheels. A 10-T. locomotive with steel-tired wheels, for example, will develop on a straight level track a maximum tractive effort of $10 \times 2,000 \times .25 = 5,000$ lb., before slipping the wheels.

With wet and slippery rails, when starting heavy trains or on steep grades, sand is used to increase the adhesion, which by this means may be increased to about 25 to 30% for cast-iron wheels and 30 to 33$\frac{1}{3}$% for steel-tired wheels.

Due to excessive wear of wheels and other undesirable effects when it is used too freely, sand should be limited in application to starting heavy trips and climbing the steepest grades. It is therefore not advisable to load a locomotive to its maximum tractive effort continuously, but about 10 or 15% reserve capacity should preferably be left.

Only moderate acceleration and retardation are as a rule required in mine-haulage service, .2 mi. per hr. per sec. being a sufficient value. This corresponds to a force of about 20 lb. per gross ton of the combined load and loco-motive. This factor, however, is usually neglected unless the train is to be started on a grade, as the slack can be taken up at the several couplings and thus only one car at a time is actually started. Quite steep grades exist also in the majority of cases, and the increased capacity of the locomotive to take care of these is usually greater than the percentage increase in weight of the locomotive demanded due to acceleration.

Where the service demands a high rate of acceleration, the weight of the locomotive must be increased accordingly. The unit of acceleration is generally taken as 1 mi. per hr. per sec., and the force required to accomplish this is about 95. lb. per T. above the frictional resistance.

*Frictional load resistance* is caused by the friction of the wheel treads and flanges against the rails and by the friction of the car journals. It may be as low as 10 lb. a T. or as high as 60 lb., depending on the nature and condition of the bearings, the size of rails, etc. For narrow-gauge roads with light rails and ordinary mine cars, from 20 to 30 lb. per T. is a fair figure.

For the locomotives, a resistance of from 12 to 15 lb. per T. is quite common, but this is generally such a small percentage of the total tractive effort that it can be neglected.

The resistance due to curves can generally be neglected unless the curves are very long or have a very short radius. Ordinarily, only a portion of the trip will be on a curve at one time, so that the drawbar pull to be added should be based only on the actual number of cars that are moving around the curve. Many grades in mining work are so short that only a part of the trip can occupy the up grade at one time, the balance of the trip being on a lesser grade, on a level, or on a down grade. By accelerating to a high speed as the hill is approached, quite steep grades of short length may be mounted without diffi-culty, and in such cases the locomotive can be worked close to the slipping point.

The resistance due to grades is always 20 lb. per T. for each per cent. grade and not only does a grade greatly increase the total train resistance, but it also reduces the available drawbar pull of the locomotive, for of the total tractive effort developed at the drivers, 20 lb. per T. for each 1% grade is consumed solely in driving the locomotive itself up the grade.

The size of a locomotive for a given load is therefore principally determined by the limiting grade. For example, assume a trailing load of 80 T., a frictional car and track resistance of 20 lb. per T., and a track that is practically level throughout with the exception of a stretch of 2% grade. The total train resistance on the level portion of the track is $80 \times 20 = 1,600$ lb., but on the grade it is $80 (20 + 2 \times 20) = 4,800$ lb., and in addition the force required for propelling the locomotive up the grade. A 4- or 5-T. locomotive can easily handle this on the level, while a 13- or 14-T. locomotive will be required to get it over the grade.

**Selection of Motors.**—Motors for mine locomotives are generally rated on the 1-hr. basis; that is, the load that they will carry continuously for 1 hr. without exceeding a certain specified temperature, usually 75° C. Standard equipments are furthermore so selected that the motors will develop the rated drawbar pull and speed of the locomotive on the above basis. Short overloads of 15 or 20% can generally be taken care of, while at overloads of about 25% the wheels will begin to slip.

The 1-hr. rating of a motor depends largely on the terminal capacity, while the real capacity is its ability to perform its cycle of operations during the entire day. The selection of the proper motor equipment on this basis, after its weight has been decided on, involves a complete knowledge of the profile of the road, the number of cars to be handled per trip and per hour, the weight of the empty and loaded cars and the frictional resistance. The motor capacity depends on the temperature that the windings will attain, and this in turn on the average heating value of the current. As this is proportional to the square of the current value, the average heating for an all-day service must be deter-mined from the square root of the mean square of the current.

A motor is selected from the various sizes that will fit the locomotive in question. and from the foregoing data and the characteristics of this motor

equipment, the current and speed are obtained for each part of the cycle. The current values are then squared and multiplied with the time during which they last. To allow for the extra heating produced by the acceleration and the switching and making up of trips at the ends of the run, about 10% should be added to the sum of the time-current-squared values for fairly long runs and about 15% for short runs. The sum of all these values is then divided by the total time, including lay-overs, and the result is the average squared current value. By taking the square root of this value, the root-mean-squared value of the current for the complete cycle is obtained. If the continuous capacity of the motor selected is below this value, a larger motor must be selected. As the motor curves usually give values for one motor, the locomotive and trailing weights, etc., should naturally be divided by two to give the weight each motor will be required to handle.

The tendency to use larger motors than formerly is quite common and is justified largely by the lower maintenance cost, but this can be carried too far, especially in small mines where the cycle of duty is such that the motors could not be overheated. In large mines, and especially for the long minehaulage duties, a careful comparison of the required duty and the motor characteristics should be made to insure a safe motor temperature. An approximate rule, easy to remember, is that a total motor capacity of about 10 H. P. is required for every ton the locomotive weighs.

**Tandem Locomotives.**—In the past, a mining locomotive was generally considered satisfactory so long as its motors could develop the torque required for the necessary traction. Owing to the relatively short and infrequent runs, heating was not the limiting feature, but as mine headings have increased in length to 6 and 7 mi. in some cases, the motors that were formerly good for runs of 1 and 2 mi. are no longer adequate for the longer service unless the loads are correspondingly reduced. A reduction in loads is impossible, because for the same output, the longer the runs the larger must be the trains, and larger trains means larger motor capacity. The space mine locomotives can occupy is limited by the gauge of the track, and the only way to increase the hauling capacity is either to run two locomotives in tandem or to use three-motor locomotives.

The weight of a large two-motor locomotive may furthermore be prohibitive due to the track construction. On well-laid tracks having 50- or 60-lb. rails, 25- or 30-T. four-wheel, two-motor locomotives will operate successfully, but where lighter rails exist, it is inadvisable to concentrate the weight on four drivers. Instead, therefore, of using a single 20-T. locomotive, two 10-T. locomotives coupled in tandem may be used, because while developing the same tractive effort, with this combination the weight will be distributed on eight driving wheels.

Cases are on record where large sums of money have been saved by the use of tandem locomotives, where the increased lengths of hauls or tonnage necessitated larger locomotive capacities. In one particular instance, it would have been necessary to widen the tunnel for many miles, while in another, several miles of track would have had to be relaid with heavier rails. It is extremely simple to couple the locomotives in tandem.

The first, or primary, locomotive is provided with a four-motor controller and the second, or secondary, locomotive, with a two-motor controller. The two are electrically interconnected so that there is a complete control of all the motors from the operating end of one. Similarly the brakes and sand valves of both locomotives can be operated from the same place.

The locomotives can also be operated singly as independent units by separation, which requires but a few minutes, and only involves the pulling out of the cable plugs, disconnecting the brake chain, and turning the primary brake stand parallel to the end frame.

**Cable-Reel Locomotives.**—Gathering locomotives of the cable-reel type are provided with a conductor in the form of a flexible insulated cable that can be connected to the trolley wire on the entry and through which current can be conveyed to the locomotive when it is necessary for it to go beyond the end of the trolley line. The arrangement is designed to do away with the cost of stringing wires in the rooms as well as to overcome the danger of shock to the miners.

The cable may be either single or double. The single cable is used where the rooms are laid with steel rails, which are bonded to form the return circuit. The double cable is used where the rails are of wood and the return circuit must be made through the cable itself.

The cable reel may be driven mechanically from the axle, or by an inde-

pendent motor. The mechanically driven cable reel is driven by a chain from the locomotive axle. As the locomotive moves ahead the cable is paid out automatically, being kept taut and the reel prevented from spinning by a friction device. As the locomotive returns from the face, the reel is wound through a clutch. In all cases, it is arranged that the tension on the cable cannot exceed a safe amount.

The motor-driven reel is generally preferred particularly for gathering on steep grades, because the motive power is independent of the axles, the cable is always taut and there is no danger of its being run over should the locomotive slide down grade with its wheels locked. The form and arrangement of the cable reel and its motor vary somewhat. In one standard type of gathering locomotive, the reel is drum shaped, is set above and on the end frames in such a way that it does not project above the main casing of the locomotive, and contains within it the necessary motor. In another standard type, the reel is flat and turns horizontally on ball bearings on top of the locomotive. The reel is driven through a double reduction gearing by a small, vertical, series-wound motor, the armature of which is, as a rule, provided with ball bearings. The motor is connected directly across the line, with a fuse and a switch inserted in the circuit, the former to protect against short-circuits and the latter in case for some reason it should be desired to open the circuit. A permanent resistance is also inserted in this circuit in order to limit the heavy rush of current that would take place when the locomotive is standing still. The motor, however, has sufficient capacity to permit its being stalled for any length of time without overheating.

The cable is generally about 500 ft. long, flexible, and heavily insulated to withstand the wear to which it necessarily is subjected. The inner end is connected to a collector ring on the underside of the reel and the outer end is fitted with a copper hook for attaching to the trolley wire. A carbon brush mounted on an insulated stud attached to the motor frame collects current from the ring from which it is conducted to the controller circuit.

The arrangement and design of the reel motor is such that at all times it will produce a tension on the cable. Thus, as the locomotive moves forwards, the counter torque will produce a tension in the cable and cause it to pay out evenly and drop along the roadbed without kinks. Owing to the braking effect of this counter torque, the reel will also come to a standstill when the locomotive stops; and as it starts on the return trip and the cable is slackened, the motor action will immediately come into play and the reel will commence to wind up the cable as the locomotive moves along, the peripheral rim speed of the reel being higher than the linear speed of the locomotive. The operation of the reel is thus entirely automatic and requires no controller, ratchet, or clutch to be handled by the motorman, but leaves the motorman free to give his entire attention to operating the controller and brakes and the proper running of the locomotive.

Gathering locomotives are equipped with a regular mine trolley so that they can be used in the same manner as regular hauling locomotives. When the cable reel is not being used and the locomotive is collecting current through the trolley pole in the regular way, the current flow through the reel motor is cut off by throwing the reel and trolley switch to the trolley side.

**Crab Locomotives.**—Crab, or traction-reel, locomotives carry a reel or drum mounted in a similar position to that of a cable-reel locomotive, but upon which is wound 350, 500, or more feet of wire rope. In operation, the locomotive remains on the entry with the brakes set, and the rope is dragged to the face and coupled to the loaded car by the motor helper; when the reel motor is started, the car is pulled to the entry. If the rope is long enough to reach from the entry to the face of the room, where it is passed around a sheave, and back again to the entry, this locomotive may be used to pull empty cars up a grade to the face. Crab locomotives are in general use in mines where the room track is too weak to sustain the weight of the motor, or where the working places are on such a pitch that the locomotive cannot propel itself in them.

**Combination Cable-Reel and Crab Locomotives.**—Gathering locomotives are sometimes built with both a cable and a rope reel. By the use of the cable, the locomotive itself can enter any place where the track is suitable and the grades are not too steep, and on heavy pitches the locomotive can stand on the entry and pull cars to it by means of the wire rope.

**Rack-Rail Locomotives.**—Traction locomotives may be used on short grades of 5%, but above that they are not to be considered. To handle trips on heavy grades without resorting to rope haulage, rack- or third-rail loco-

motives are often employed. In these, the teeth of steel gear wheels carried on the axle of the locomotive and turned by an electric motor, engage slots cut in an iron bar (the rack rail) laid between the track rails, thus mechanically pulling the locomotive forward up the grade. As the hauling capacity of the locomotive does not depend on its adhesion but on the horsepower developed by its motors, it may be made much lighter than the trolley locomotive with a corresponding gain in the weight it is able to haul.

The rack rail may be either live or dead. In the first case, the current for operating the locomotive is carried by the rack rail; in the second, the current is received from an overhead trolley wire and returns through the rails of the regular track.

A combination rack and traction locomotive is also made, which is arranged to run as a rack locomotive on grades and as a traction locomotive on a level, where no rack rail need be laid.

Rack-rail locomotives are planned on the unit system; that is to say, any number of units of 50, 100 H. P., etc., may be run as a single locomotive where the grades and the loads warrant it.

**Operation of Electric Locomotives.**—Before an electric mining locomotive is put into service, it should be inspected to see that all parts are in proper condition. It should be well oiled and the sand boxes should contain plenty of dry sand. The sand levers and brakes should be tried to see that they are operating satisfactorily, and the controller should be on the off-position before the trolley pole is put on.

When starting the locomotive, the current should be thrown on gradually and due consideration paid to the load that the locomotive is to haul. The slack in the couplings will often relieve the starting condition so that it will not be necessary to start all the cars in the train simultaneously. The controller should be advanced from one notch to another, quickly, being allowed to remain on one point until the locomotive has gathered speed to correspond, when it is moved quickly to the next notch, etc. If, however, the controller is advanced too rapidly and the wheels begin to slip the controller must not be thrown backwards one or two steps but must be thrown off quickly, completely, and advanced again in the usual manner. If the control is moved backwards slowly, arcing at the contact fingers may cause burning and blistering.

The controller is only intended for starting duty and the locomotive should not be run continuously with the controller on intermediate position, as this is liable to cause a burn-out of the resistance or other damage to the controller.

If the locomotive runs too fast with the controller in the on-position and the motors in parallel, the motors should be placed in series or the current thrown on for a short time and then off, letting the locomotive coast.

When it is necessary to brake, the controller should be thrown to the off-position before the brakes are applied. The controller should not be used for braking, by reversing the motors, except in case of emergency. This practice is sometimes resorted to, but is very severe on the motors, controllers, and in fact on the entire equipment. Reversing the motors when running at full speed is apt to break the gears and spring the armature shaft.

**Troubles of Electric Locomotives.**—1. *Failure to Start.*—The most common cause of a motor failing to start is broken connection in the electric circuit in the motors, the trolley, the track return, the circuit-breaker, controller, or resistance grids. If the open circuit is in the motors, the defective part can be located by raising the brushes of each motor commutator successively, with the controller in the multiple position and the current applied. If, however, neither of the motors will operate when so connected, the opening is in some other part of the electric circuit than the motors. An examination to determine this is best made by the use of a bank of lamps, one end of which is connected to the trolley wire and the other end applied to different parts of the circuit beginning with the trolley harps and taking the circuit step by step until the open circuit is passed.

When the open circuit is found to be in the field coils in one of the motors, it is necessary to cut this motor out of circuit and drive the locomotive with the other motor. Only half the customary load should then be hauled, although the locomotive will, to a great extent, protect itself, as the wheels connected to the driving motor will have a tendency to slip, which of course will determine the amount of load that the locomotive is capable of hauling. The defective motor is best cut out by removing its brushes.

Failure to start may also be due to faulty connections causing the motors to buck each other. This will cause a heavy current and the fuse or circuit-breaker will blow. It is readily corrected by reversing the brush leads on one

motor. Grounding the current may also prevent a locomotive from starting, while on the other hand mechanical troubles are often the cause; for example, the brakes may not be released, the gears may be broken, the bearings stuck or seized, etc.

If the locomotive jumps or does not start up smoothly, the trouble is generally short circuits in the starting resistance, wrong or open connections, controller troubles, etc.

2. *Excessive Heating.*—Heating may be due to the motors being overloaded when hauling heavy trips, and can then only be remedied by reducing the load or by providing larger locomotives.

Low voltage is a very common cause of a motor not developing its rated capacity causing overheating due to slower speed, breakdowns, etc. This may be the result of insufficient copper in the overhead wires, poor bonding of the rails, poor connections in the circuit or insufficient prime mover or generator capacity.

A short circuit in any armature turn will cause a circulation of heavy current therein, followed by excessive heating. This current is due to the transformer action of the field coils acting as primary and the short-circuited armature turns as secondary. The trouble can generally be detected by the smell of burning insulation or by the hand, as the short-circuited coils will be much warmer than the other part of the armature. As a temporary remedy, the short-circuited coils can be open circuited at the commutator and disconnected from it, the commutator being bridged at this point to close the gap.

Short-circuited field turns will cause the motor to speed up, particularly at light loads. This tendency to speed up will cause the motor to take an excessive current, causing overheating of the defective motor armature. The defective coil can be located by feeling with the hand, as it will be much cooler than the others. This is due to the reduced number of turns, which decreases the resistance of the coil and consequently the amount of loss therein. When a field coil is found to be short circuited so as to affect the operation of the motor, the coil should be removed and replaced by a new one.

Burn-out from excessive heating is also caused by the armature coming down on the pole faces. The remedy for this is, of course, only to give more attention to the motor bearings, keeping them properly lubricated and by frequently checking the air gap to see if the armature is getting dangerously close to the pole faces.

3. *Sparking.*—Excessive sparking at the brushes is frequently caused by an open circuit in the armature winding. Such sparking may often become so violent as to cause the motors to flash over at the commutator. An examination will show that the commutator segments, between which the open circuit occurs, are blackened and slightly burned. If the open circuit is not taken care of at once, it is liable to cause a flat spot on the commutator, requiring turning. Temporary relief can be had by bridging the open circuit at the commutator.

Short-circuited field turns, if affecting a large number of turns, are also liable to cause excessive sparking at the brushes.

Commutator troubles are a very common cause of sparking and commutators should be kept free from oil and dirt. If they become too rough from overheating and excessive sparking, it may be necessary to smooth them with sandpaper, and if this does not help, returning is the remedy.

Trouble with the commutators is often due to careless handling of the locomotive, such as operating it with a defective controller or a defective resistance. When a resistance is found to have a broken grid, a new one should be put in at once. The method sometimes resorted to of short circuiting a broken grid should not be allowed, except for temporary work, for when doing so, a large percentage of the resistance may be cut out of one or more of the steps, causing the motors to take excessive current when those points on the controller are reached. This will cause the locomotive to start with a jerk and very likely burn the commutator and brushes, besides being hard on the gears and other mechanical parts of the locomotive.

4. *Grounds.*—When a ground occurs in a motor, whether it is confined to the armature, field coils, or commutator, it will cause the circuit-breaker or fuse to blow, and it will not be possible to keep the circuit-breaker closed without holding it in, which should never be done.

Motors will also sometimes show a ground when tested with a voltmeter or a bank of test lamps, but otherwise will operate satisfactorily. It is then evident that there is a leakage path formed somewhere, and if the motors

are not inspected and thoroughly cleaned to remove this partial ground it is only a short time before a permanent ground can be expected.

When a ground occurs, the motor containing it should be cut out of service and the locomotive operated by the other motor until such time as the ground can be located and remedied.

## ALTERNATING-CURRENT LOCOMOTIVES

Alternating-current locomotives may be either single or three phase. Single-phase locomotives require but one overhead trolley wire, as in direct-current haulage, whereas three-phase locomotives require two trolley wires, the track rails forming the third leg of the circuit. Three-phase locomotives are not generally recommended for underground use because of the difficulty of maintaining and insulating two trolley wires, the increased complication of the switches where two wires are used, etc.

The single-phase locomotive, taking all its current from one phase of the supply system, produces an unbalanced load on the line, but this should not seriously affect a power station of good capacity. In extensive haulage installations, by taking the power for the various sections of the main line and for the various branches from different phases of the supply line, it is possible to practically balance the load.

The three-phase locomotive sometimes used at American mines for outside haulage, is similar in general construction and appearance to the direct-current machine. It has, however, either two trolley poles or a pantagraph trolley making sliding contact with the wires. These locomotives may be had up to 8 to 10 T. in weight and for the standard frequencies and voltages. They are provided with two-torque induction motors, with suitable starting resistances in the rotor circuit, so that reduced speeds may be had for starting (acceleration), switching, etc. As the induction motor is a constant-speed machine, the locomotive tends to maintain the speed for which it is geared regardless of the load or grades. The high-speed of the induction motors necessitates a double-gear reduction and consequently a different method of mounting than is used in the direct-current machine.

The advantage of the three-phase locomotive is in the saving in the cost of converters and the power lost in converting from alternating to direct current. The high voltages so often used with them are extremely dangerous.

## STORAGE-BATTERY LOCOMOTIVES

For gathering coal, storage-battery locomotives are recommended where the grades are not severe; where the speed does not exceed $3\frac{1}{2}$ to 4 mi. per hr.; where the hauls are short, say not over $\frac{1}{2}$ mi., and where the service is intermittent; that is, where the locomotive is idle a good portion of the time, as when waiting on empties or loads. These locomotives are not at present advised for main-line haulage where the travel is long because the practically continuous service requires a locomotive of a price and over-all dimensions that is commonly prohibitory. In size, these locomotives range from $2\frac{1}{2}$ to 10 T., a common size for gathering being 4 T.; however, a 20-T. locomotive of this type has been built. In the majority of cases, the battery is carried on the same truck as the motor and is an integral part of the locomotive, but in some of the larger machines designed for long hauls, heavy work, and the like conditions requiring more nearly continuous service, the batteries are carried on a trailing truck, tender, or battery car; the weight of which reduces the hauling capacity of the locomotive. Recharging is done at night with usually a little "livening up" during the noon hour or other idle times. Where separate battery cars are used, a fresh one may be coupled to the locomotive at the time the exhausted one is taken away to be recharged. One type of these locomotives is built to use current from the ordinary overhead trolley wire where such exists, thus saving the batteries for use in parts of the mine where current is not to be had. This locomotive is also arranged with suitable switches so that the batteries may be charged from the trolley circuit at the same time the motor is being run in the ordinary way.

The weight of a locomotive required to give the necessary adhesion to haul the load is calculated in the same way as for any other kind of locomotive. The calculation of the battery capacity or power is not easily made and requires a careful study of the grades, loads, and distances, from which may be calculated the foot-pounds of work the locomotive must perform. In this calculation, perhaps the most important point is estimating the ratio of the actual discharge rate of the battery cells to the normal rate of discharge. This

depends on the length of time the locomotive is developing the maximum drawbar pull or some other high value of the drawbar pull that is sustained for any considerable length of time. The value finally selected depends largely, if not entirely, on the judgment and experience of the individual. It is generally considered safe to make this ratio 1 : 3, although on flat grades where the maximum pull is exerted only at starting and for a second or two, the ratio may be as high as 1 : 5. The foot-pound of work may be reduced to kilowatt-hours on the basis of 1 ft.-lb. = .000000377 K. W.-hr. In ordinary estimates, it may be assumed that the kilowatt-hours per ton-mile of load are .125 for a level track, which includes the losses in the battery and locomotive.

# VENTILATION OF MINES

## CHEMICAL AND PHYSICAL PROPERTIES OF GASES

### CHEMISTRY OF GASES

**Matter and Its Divisions.**—Matter is the substance of which all things are composed and may be defined as anything that possesses weight or occupies space. There are three divisions of matter:

A *mass* is a body of matter of a size to be appreciable to the senses.

A *molecule* is the smallest particle of matter into which a mass may be divided by physical means; it is the smallest particle of matter that is capable of a separate existence. The exact size of a molecule cannot be determined but it is so small that the most powerful microscope would fail to recognize it. Lord Kelvin calculates that if a single drop of water was magnified until it appeared as large as the earth (approximately 8,000 mi. in diameter), the molecules in the drop would appear to have a size between that of a baseball and a small shot.

An *atom* is the smallest particle of an element that can enter into a chemical reaction and cannot further be divided. As a rule, atoms are incapable of existing in a free state, and are generally found in combination with other atoms, either of the same or of different kinds.

Atoms unite to form molecules, and molecules unite to form masses.

**Classes of Matter.**—An *element* is a mass of matter composed of the same kind of molecules which, in turn, are composed of the same kind of atoms. Thus, two atoms of hydrogen unite to form a molecule of hydrogen and an inconceivable number of molecules of hydrogen unite to form a mass, say, an ounce or a pound of hydrogen. In the case of an element, the mass, molecule, and atom are of the same kind.

A *compound*, or *chemical compound*, is a mass of matter composed of the same kind of molecules, but the molecules are composed of two or more atoms of different kinds. Thus a mass of methane is composed of molecules of methane, which are each composed of one atom of carbon and four atoms of hydrogen.

A *mixture* is a mass of matter composed of two or more different kinds of molecules, the one molecule being composed of different atoms than the other or others. Thus, afterdamp is a mixture of molecules of oxygen, nitrogen, carbon dioxide, carbon monoxide, and usually one or more other gases, the molecules of each of which are composed of characteristic atoms.

There are at present (1915) 83 definitely known elements having properties more or less clearly understood, together with a number more the identification or characteristics of which are in doubt.

**Forms of Matter.**—The atoms composing a molecule are held together by chemical affinity, and molecules composing a mass are held together by cohesion. In addition, molecules of all matter are acted upon by an opposing force, *repulsion*, which tends to drive them apart. Repulsion is not inherent in the mass, but is an induced or applied force that is largely the result of heat or the temperature of the body.

All matter exists in one of three forms, *solid*, *liquid*, or *gaseous*, according to the predominance of the attractive or the repulsive forces existing between the molecules. For example, water exists as ice, or in a solid form, when the attractive force exceeds the repulsive force between its molecules. As the temperature is raised or heat is applied, the ice assumes the liquid form due to the more rapid vibration of the molecules of which it is composed. In

other words, the repulsive force existing between the molecules is increased, and the result is a liquid. If the temperature is raised still further by applying more heat, the vibration of the molecules becomes yet more rapid, the repulsive force is increased between the molecules, and a gas or vapor called *steam* is formed.

**Changes in Matter.**—Matter cannot be destroyed but its form may be changed or, if a chemical compound, it may be broken up into its component elements. Changes affecting the form or state of matter brought about by physical causes, as heat, pressure, electricity, etc., and affecting only the molecules of a body are *physical changes;* changes affecting the atoms in a molecule, by which they are rearranged or combined in new ways are *chemical changes.* Physical changes always accompany chemical changes, a change in the arrangement or relations between the molecules of matter usually preceding a change in the arrangement of the atoms in the molecule. Thus, the change from ice to water to steam is a physical change due to heat; if the heat is still further increased, the molecules of water will be decomposed into hydrogen and oxygen gas, which is a chemical change.

**Symbols and Formulas.**—It is usual to express the names of the elements by letters called *symbols.* The letters selected are the first one of the name or the first and some letter following it. While in the majority of cases, the letters are taken from the common name of the element, in others the symbol is derived from its Latin or other name. Thus, the symbol for iron is $Fe$, derived from the Latin *ferrum,* and for tungsten is $W$, from *wolfram,* an earlier name. The symbols for antimony, gold, silver, tin, copper, sodium, potassium, and mercury, are similarly derived from the Latin. Two atoms of an element, as hydrogen, may be written either $2H$ or $H_2$.

A *formula* is the expression of the composition of a molecule by means of the symbols of the elements entering into it, the number of atoms of each kind in the molecule being denoted by subscripts. Thus, the formula for methane is $CH_4$, which indicates that a molecule of this gas is composed of one atom of carbon (symbol $C$) and four atoms of hydrogen (symbol $H$). When there are no subscripts, it is understood that but one atom is present in the molecule, as in $CO$, which is composed of one atom each of carbon and oxygen. Two molecules of methane would be written $2CH_4$, three molecules $3CH_4$, etc. The symbol for the element hydrogen is $H$, and the formula for the molecule of hydrogen is $H_2$, since each molecule of this gas contains two atoms as explained in the next paragraph.

**Atomicity of Elements.**—By atomicity is meant the number of atoms in a molecule. The rare atmospheric gases, argon, helium, krypton, neon, and xenon are *monatomic;* that is, their molecules contain but one atom. Hence, for these gases, the symbols $A$, $He$, $Kr$, $Ne$, and $Xe$, respectively, represent either one atom or one molecule.

The common atmospheric gases, hydrogen, nitrogen, and oxygen are *diatomic,* or their molecule is composed of two atoms. In these cases, the symbols for the atoms are, respectively, $H$, $N$, and $O$, and the formulas for the molecules are $H_2$, $N_2$, and $O_2$. While sulphur is *tetratomic* at temperatures of more than 800° C. and *hexatomic* at about 500° C. and its molecule is thence $S_4$ or $S_6$, it is commonly written $S$, as if monatomic, in questions of mine gases. The same is true of carbon, which is either diatomic or tetratomic.

**Chemical Reactions.**—A *chemical reaction* is any change in the arrangement of the atoms in a single molecule of a substance or in the atoms of several molecules of different substances brought about by external agencies. The agencies affecting the arrangement of the atoms in a molecule or molecules are heat, electricity, and chemical affinity. In any reaction, no matter is destroyed. There are always the same number and kind of atoms after the reaction as before it took place, but their combination, one with the other, to form molecules is different.

**Chemical Equations.**—A chemical equation is the expression of the equality between atoms before and after a chemical reaction takes place. The first, or left-hand, member of the equation gives the formula and the number of molecules or atoms of the substance acted upon or the two or more substances that react upon one another, while the second, or right-hand, member gives the formula and the number of molecules of the substance or substances formed by the reaction.

There are atomic and molecular equations. The former, which are the simpler, show the relation between the atoms concerned in a reaction. The atomic equation for the burning of carbon in air is written $C + 2O = CO_2$. The molecular equation for the same reaction is $C + O_2 = CO_2$. From the first

equation, when the weight of carbon burned is known, there may be calculated the weight of oxygen required for the combustion and that of the carbon dioxide formed. If the weights per cubic foot of $O$ and $CO_2$ are known, the volumes of these gases concerned in the reaction may then be calculated. The molecular equation, however, shows that the volume of oxygen consumed is the same as that of the carbon dioxide produced; hence, but one volume calculation need be made.

**Atomic Weight.**—The absolute weight of an atom is not known, but the relative weights of the atoms are known in most cases with a high degree of accuracy. As hydrogen gas is the lightest known substance, it is made the basis of comparison, and the relative weights of the atoms of the other elements are referred to it. Thence, the atomic weight of an element is the ratio between the weight of its atom and that of an atom of hydrogen. The atomic weight of oxygen = 15.88 when hydrogen = 1. Oxygen is a very common constituent of chemical compounds and hydrogen rather unusual; hence, for ease in calcu-

**THE ELEMENTS WITH THEIR SYMBOLS AND ATOMIC WEIGHTS**

$(O = 16)$

| Element | Symbol | Atomic Weight | Element | Symbol | Atomic Weight |
|---|---|---|---|---|---|
| Aluminum | $Al$ | 27.10 | Molybdenum | $Mo$ | 96.00 |
| Antimony | $Sb$ | 120.20 | Neodymium | $Nd$ | 144.30 |
| Argon | $A$ | 39.88 | Neon | $Ne$ | 20.20 |
| Arsenic | $As$ | 74.96 | Nickel | $Ni$ | 58.68 |
| Barium | $Ba$ | 137.37 | Niton | $Nt$ | 222.40 |
| Bismuth | $Bi$ | 208.00 | Nitrogen | $N$ | 14.01 |
| Boron | $B$ | 11.00 | Osmium | $Os$ | 190.90 |
| Bromine | $Br$ | 79.92 | Oxygen | $O$ | 16.00 |
| Cadmium | $Cd$ | 112.40 | Palladium | $Pd$ | 106.70 |
| Cæsium | $Cs$ | 132.81 | Phosphorus | $P$ | 31.04 |
| Calcium | $Ca$ | 40.07 | Platinum | $Pt$ | 195.20 |
| Carbon | $C$ | 12.00 | Potassium | $K$ | 39.10 |
| Cerium | $Ce$ | 140.25 | Praseodymium | $Pr$ | 140.60 |
| Chlorine | $Cl$ | 35.46 | Radium | $Ra$ | 226.40 |
| Chromium | $Cr$ | 52.00 | Rhodium | $Rh$ | 102.90 |
| Cobalt | $Co$ | 58.97 | Rubidium | $Rb$ | 85.45 |
| Columbium | $Cb$ | 93.50 | Ruthenium | $Ru$ | 101.70 |
| Copper | $Cu$ | 63.57 | Samarium | $Sa$ | 150.40 |
| Dysprosium | $Dy$ | 162.50 | Scandium | $Sc$ | 44.10 |
| Erbium | $Er$ | 167.70 | Selenium | $Se$ | 79.20 |
| Europium | $Eu$ | 152.00 | Silicon | $Si$ | 28.30 |
| Fluorine | $F$ | 19.00 | Silver | $Ag$ | 107.88 |
| Cadolinium | $Gd$ | 157.30 | Sodium | $Na$ | 23.00 |
| Gallium | $Ga$ | 69.90 | Strontium | $Sr$ | 87.63 |
| Germanium | $Ge$ | 72.50 | Sulphur | $S$ | 32.07 |
| Glucinum | $Gl$ | 9.10 | Tantalum | $Ta$ | 181.50 |
| Gold | $Au$ | 197.20 | Tellurium | $Te$ | 127.50 |
| Helium | $He$ | 3.99 | Terbium | $Tb$ | 159.20 |
| Holmium | $Ho$ | 163.50 | Thallium | $Tl$ | 204.00 |
| Hydrogen | $H$ | 1.008 | Thorium | $Th$ | 232.40 |
| Indium | $In$ | 114.80 | Thulium | $Tm$ | 168.50 |
| Iodine | $I$ | 126.92 | Tin | $Sn$ | 119.00 |
| Iridium | $Ir$ | 193.10 | Titanium | $Ti$ | 48.10 |
| Iron | $Fe$ | 55.84 | Tungsten | $W$ | 184.00 |
| Krypton | $Kr$ | 82.92 | Uranium | $U$ | 238.50 |
| Lanthanum | $La$ | 139.00 | Vanadium | $V$ | 51.00 |
| Lead | $Pb$ | 207.10 | Xenon | $Xe$ | 130.20 |
| Lithium | $Li$ | 6.94 | Ytterbium | $Yb$ | 172.00 |
| Lutecium | $Lu$ | 174.00 | Yttrium | $Yt$ | 89.00 |
| Magnesium | $Mg$ | 24.32 | Zinc | $Zn$ | 65.37 |
| Manganese | $Mn$ | 54.93 | Zirconium | $Zr$ | 90.60 |
| Mercury | $Hg$ | 200.60 | | | |

lating, chemists have found it advisable to consider the atomic weight of oxygen as 16, in which case that of hydrogen is 1.008, the ratio of 15.88 : 1 being the same as 16 : 1.008. When the atomic weights are based on oxygen = 16, those of all the elements must be multiplied by 1.008 if they have been determined on the basis hydrogen = 1. The foregoing list of atomic weights is based upon oxygen = 16, and is taken from the report of the International Committee of Atomic Weights for 1914. All the elements in the list are known, and there have been omitted therefrom sundry of the radioactive elements as actinium, polonium, radiothorium, etc., which are, as yet imperfectly identified.

**Molecular Weight.**—The molecular weight of any substance, elementary or compound, is equal to the sum of the atomic weights of the atoms in its molecule. It is customary, in all but precise calculations, to use the approximate rather than the exact atomic weights. The following are the approximate atomic weights generally used for the elements occurring in mine gases, the exact weight when oxygen = 16 being given in parenthesis: Carbon 12 (12); hydrogen 1 (1.008); nitrogen 14 (14.01); oxygen 16 (16); sulphur 32 (32.07). For illustration, the molecular weight of sulphuric acid, $H_2SO_4$, is found as follows:

| *Approximate* | *Exact* |
|---|---|
| $H_2 = 2 \times 1 = 2$ | $H_2 = 2 \times 1.008 = 2.016$ |
| $S = 1 \times 32 = 32$ | $S = 1 \times 32.07 = 32.070$ |
| $O_4 = 4 \times 16 = 64$ | $O_4 = 4 \times 16 = 64.000$ |
| Molecular weight = 98 | Molecular weight = 98.086 |

The following table gives the names, formulas, and molecular weights of the elementary (oxygen, nitrogen, and hydrogen) and the compound gases that may be met in mines. For all ordinary purposes, the approximate molecular weights may be used.

### FORMULAS AND MOLECULAR WEIGHTS OF COMMON GASES

| Name of Gas | Formula of Molecule | Molecular Weight When | | |
|---|---|---|---|---|
| | | $O = 16$ | $H = 1$ | Approximate |
| Acetylene............ | $C_2H_2$ | 26.016 | 25.82 | 26 |
| Carbon dioxide........ | $CO_2$ | 44.000 | 43.67 | 44 |
| Carbon monoxide..... | $CO$ | 28.000 | 27.79 | 28 |
| Ethane.............. | $C_2H_6$ | 30.048 | 29.82 | 30 |
| Ethylene............. | $C_2H_4$ | 28.032 | 27.82 | 28 |
| Hydrogen............ | $H_2$ | 2.016 | 2.00 | 2 |
| Hydrogen sulphide.... | $H_2S$ | 34.086 | 33.82 | 34 |
| Methane............. | $CH_4$ | 16.032 | 15.91 | 16 |
| Nitric oxide......... | $NO$ | 30.010 | 29.78 | 30 |
| Nitrogen............. | $N_2$ | 28.020 | 27.80 | 28 |
| Nitrogen dioxide...... | $NO_2$ | 46.010 | 45.66 | 46 |
| Oxygen............. | $O_2$ | 32.000 | 31.76 | 32 |
| Sulphur dioxide....... | $SO_2$ | 64.070 | 63.58 | 64 |
| Water, vapor......... | $H_2O$ | 18.016 | 17.88 | 18 |

**Percentage Composition.**—The actual weights of the various elements in a given weight of a chemical compound are proportional to the weights of the atoms of each element in a molecule of the compound.

**Example.**—What is the percentage composition of methane, $CH_4$, and how many pounds of carbon, $C$, and hydrogen, $H$, are there in 5 lb. of this gas?

**Solution.**—From the foregoing table, the weight of a molecule of $CH_4$ is 16, of which 12 parts by weight $(1 \times 12)$ is $C$, and 4 parts by weight $(4 \times 1)$ is $H$. From this, the percentage of $C$ in a molecule of $CH_4$ is $(12 \div 16) \times 100 = 75$; and of $H$, is $(4 \div 16) \times 100 = 25$.

In 5 lb. of $CH_4$ there are $5 \times .75 = 3.75$ lb. of $C$, and $5 \times .25 = 1.25$ lb. of $H$.

**Weights of Substances Concerned in Reactions.**—The actual weights of

the substances entering into any chemical reaction are proportional to the total molecular weights of the substances concerned in the reaction.

EXAMPLE 1.—(a) How many pounds of oxygen are required to burn 5 lb. of methane; (b) how many pounds of carbon dioxide and water vapor will be produced?

SOLUTION.—(a) The molecular equation for the reaction may be written
$$CH_4 + 2O_2 = CO_2 + 2H_2O$$
Molecular weights, $16 + 64 = 44 + 36$
Dividing by 16, $1 + 4 = 2.75 + 2.25$
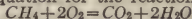
The molecular weights may be taken from the preceding table or may be calculated from the approximate atomic weights. Since the reaction is based upon a known weight of $CH_4$, the molecular weights are divided through by the molecular weight of $CH_4$ to reduce the relative weight of that gas to unity or 1. The reaction may be read: Four pounds of $O$ are required to burn 1 lb. of $CH_4$, the reaction producing 2.75 lb. of $CO_2$ and 2.25 lb. of $H_2O$. Since it requires 4 lb. of $O$ to burn 1 lb. of $CH_4$, to burn 5 lb. of $CH_4$ will require $5 \times 4 = 20$ lb. of $O$.

(b) Since 1 lb. of $CH_4$ in burning produces 2.75 lb. of $CO_2$ and 2.25 lb. of $H_2O$, 5 lb. of this gas will produce $5 \times 2.75 = 13.75$ lb. of $CO_2$ and $5 \times 2.25 = 11.25$ lb. of $H_2O$.

Note that the sums of the atomic weights on both sides of the equation are the same and equal to 80. Also that the actual weight of the substances burned is the same as that of the substances produced; thus 5 lb. of $CH_4 + 20$ lb. of $O = 25$ lb., and 13.75 lb. of $CO_2 + 11.25$ lb. of $H_2O = 25$ lb.

EXAMPLE 2.—How many pounds of carbon monoxide must be burned in oxygen to produce 10 lb. of carbon dioxide, and how many pounds of oxygen will be required?

SOLUTION.—The molecular equation for the reaction is
$$2CO + O_2 = 2CO_2$$
Molecular weights, $56 + 32 = 88$
Dividing by 88, $.636 + .364 = 1$
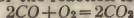
The molecular weights taken from the table or calculated are divided by 88, the weight of two molecules of $CO_2$, since it is the absolute weight of that gas that is required. It is apparent that to produce 10 lb. of $CO_2$, 10 $\times .636 = 6.36$ lb. of $CO$ must be burned in $10 \times .364 = 3.64$ lb. of $O$.

**Volumes of Gases Concerned in Reactions.**—As equal volumes of all gases contain the same number of molecules, all gaseous molecules are of the same size, whence the volumes of the gases concerned in any reaction are directly proportional to the number of molecules of the respective gases involved.

EXAMPLE 1.—How many cubic feet of oxygen will 100 cu. ft. of carbon monoxide consume in burning to carbon dioxide, and how many cubic feet of the latter gas will be produced?

SOLUTION.—The molecular equation is written
$$\overset{\text{II}}{2CO} + \overset{\text{I}}{O_2} = \overset{\text{II}}{2CO_2}$$
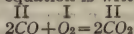
The Roman numerals written above the formulas for the gases, represent the number of molecules of each concerned in the reaction. Hence, two volumes of $CO$ combine with one volume of $O$ to produce two volumes of $CO_2$. In this reaction there has been a condensation since three volumes are reduced to two. On the other hand, there are six atoms on each side of the equation, and the molecular weights are 88 on each side. Since $CO$ combines with one-half its volume of $O$, it follows that 100 cu. ft. of $CO$ will combine with 50 cu. ft. of $O$ to form 100 cu. ft. of $CO_2$.

EXAMPLE 2.—How many cubic feet of oxygen are required for the complete combustion of 100 cu. ft. of methane, and how many cubic feet of carbon dioxide and vapor of water will be produced?

SOLUTION.—The molecular equation is written
$$\overset{\text{I}}{CH_4} + \overset{\text{II}}{2O_2} = \overset{\text{I}}{CO_2} + \overset{\text{II}}{2H_2O}$$
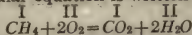
The volume of the $O$ will be twice that of the $CH_4$ and the volumes of the $CO_2$ and $H_2O$ will be equal, respectively, to those of the $CH_4$ and $O$. Hence, to burn 100 cu. ft. of $CH_4$ will require 200 cu. ft. of $O$, and there will be produced 100 cu. ft. of $CO_2$ and 200 cu. ft. of $H_2O$.

**Volumes of Gases When Burned in Air.**—When gases are burned in air, in order to compute the volume of the products of combustion exactly, account must be taken of the nitrogen in the atmosphere. The exact ratio by volume of the oxygen to the nitrogen in the air is $1 : 3.782$; that is, for every molecule of oxygen there are 3.782 molecules of nitrogen. From this, the formula for

air may be taken to be $(O_2+3.782N_2)$, and may be substituted for the molecule of oxygen $O_2$ in all reactions where it occurs. Where exactness is not required, it is usual to assume the $O : N$ ratio in the air as 1 : 4, and to write the formula $(O_2+4N_2)$. It should be noted that the foregoing are, strictly speaking, not formulas, but indicate, rather, the composition of a definite mixture of oxygen and nitrogen, which is known as *air*.

EXAMPLE 1.—What is the percentage, by volume, of methane in firedamp at its most explosive point?

SOLUTION.—By introducing the formulas for the ratio of oxygen and nitrogen in the air, the equation for the combustion of methane is,
$$CH_4+2(O_2+3.782N_2)=CO_2+2H_2O+7.564N_2$$

| Relative volumes, | 1 | $2\times(1+3.782)$ | 1 | 2 | 7.564 |
| Relative volumes, | 1 | 9.564 | 1 | 2 | 7.564 |
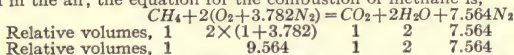
From this, one volume of $CH_4$ combines with 9.564 volumes of air and forms 10.564 volumes of firedamp. The proportion of methane in the mixture is $(1\div10.564)\times100=9.46\%$.

EXAMPLE 2.—Using the approximate $O : N$ ratio for air, what is the percentage composition of the afterdamp of an explosion of $CO$?

SOLUTION.—The molecular equation may be written
$$2CO+(O_2+4N_2)=2CO_2+4N_2$$
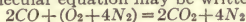
| Relative volumes, | 2 | 5 | 2 | 4 |

In the six parts of afterdamp there will be $\frac{2}{6}=\frac{1}{3}=33.33\%$ of $CO_2$ and $\frac{4}{6}=\frac{2}{3}=66.67\%$ of $N$.

**Weight and Volume of Gases in Reactions.**—When the volume, in cubic feet, of 1 lb. of gas is known, the volumes and weights of the gases concerned in a reaction may be obtained through the use of the ordinary formulas. The volume of 1 lb., in cubic feet, of the principal gases is given in a following table.

EXAMPLE.—Using the exact molecular weights when $O=16$, what are the weights and volumes, in cubic feet, of the gases involved in the burning of 1 lb. of carbon in oxygen?

SOLUTION.—The molecular equation is

Relative volumes, $\overset{1}{C}+\overset{1}{O_2}=CO_2$

Molecular weights, $12+32=44$

Dividing by 12, $1+2.67=3.67$
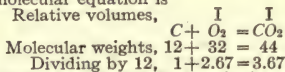
Inspection shows that 2.67 lb. of $O$ are required to burn 1 lb. of $C$, and that 3.67 lb. of $CO_2$ are produced; further, the volume of the $O$ required is the same as that of the $CO_2$ produced. From the table on page 837, the volume of 1 lb. of $O$ is found to be 11.208 cu. ft.; hence, 2.67 lb. will have a volume of $2.67\times11.208=29.93$ cu. ft. Further, as the volume of 1 lb. of $CO_2$ is 8.103 cu. ft., 3.267 lb. will occupy $3.67\times8.103=29.94$ cu. ft.

It will be noted that the volumes of oxygen and carbon dioxide as calculated are practically equal. This is as it should be as the relative volumes are the same, as is shown by the equation representing the reaction. In fact, in reactions between gases or into which gases enter, it is only necessary to calculate the volume of one of the gases; that of the others may be told from the relative volumes given by the equation. The volumes of the gases will always be equal or some simple multiple as 1, 2, 3, etc., of one another.

## PHYSICS OF GASES

**Avogadro's Law.**—*Equal volumes of all perfect gases, whether simple or compound, contain the same number of molecules when each are under the same conditions of temperature and pressure.* From this law it follows:

*The molecules of all perfect gases are of the same size.*

*A given volume of any perfect gas is as much heavier than the same volume of hydrogen as its molecular weight is greater than the molecular weight of hydrogen,* or, more simply, *the weight of 1 cu. ft. of any gas is proportional to its molecular weight.*

Avogadro's law and its two corollaries do no apply to either solids or liquids, and do not hold strictly true for all gases at all temperatures, but they are of much practical value in chemistry and physics. It has been found that the density and specific gravity of gases calculated on the assumption of the correctness of this law, do not in all cases, agree with the observed density and specific gravity.

**Density of Gases.**—The density of a gas is the ratio between the weight of a unit volume of the gas and that of the same volume of hydrogen, measured at a temperature of $32°$ F. and under a barometric pressure of 29.92 in. of mercury. Density is sometimes defined as the specific gravity of a gas referred

to hydrogen instead of to air as the standard. The following statements, based on the assumed correctness of Avogadro's law and the fact that the molecule of hydrogen is diatomic (composed of two atoms) are correct when the atomic weights are based on $H = 1$.

1. The density of any simple diatomic gas is equal to its atomic weight.
2. The density of any compound gas is equal to one-half its molecular weight.

The values in the table are calculated from the atomic weights and in numerous instances do not agree with the observed values, which they probably would do if Avogadro's law was strictly correct.

## DENSITY OF GASES AT 32° F. AND 29.92 IN. OF MERCURY

| Gas | Formula | Density | |
|---|---|---|---|
| | | Exact | Approximate |
| Acetylene............... | $C_2H_2$ | 12.910 | 12 |
| Air...................... | | 14.359 | 14 |
| Carbon dioxide........... | $CO_2$ | 21.835 | 22 |
| Carbon monoxide......... | $CO$ | 13.895 | 14 |
| Ethane.................. | $C_2H_6$ | 14.910 | 15 |
| Ethylene................ | $C_2H_4$ | 13.910 | 14 |
| Hydrogen............... | $H_2$ | 1.000 | 1 |
| Hydrogen sulphide........ | $H_2S$ | 16.915 | 17 |
| Methane................ | $CH_4$ | 7.955 | 8 |
| Nitric oxide............. | $NO$ | 14.890 | 15 |
| Nitrogen................ | $N_2$ | 13.910 | 14 |
| Nitrogen dioxide.......... | $NO_2$ | 22.830 | 23 |
| Oxygen.................. | $O_2$ | 15.880 | 16 |
| Sulphur dioxide........... | $SO_2$ | 31.795 | 32 |
| Water, vapor............. | $H_2O$ | 8.940 | 9 |

Air being a mixture and not a true gas has, strictly speaking, no density, but the values given are convenient in certain calculations.

Water vapor cannot exist at 32° or at any temperature below the boiling point unless the pressure is less than 29.92 in. The figures given are theoretical but, as in the case of air, are useful at times.

**Specific Gravity of Gases.**—The specific gravity of a gas is the ratio of its weight to that of an equal volume of air, measured at a temperature of 32° F. and a pressure of 29.92 in. of mercury.

## SPECIFIC GRAVITY, WEIGHT, AND VOLUME OF GASES AT 32° F. AND 29.92 IN. OF MERCURY

| Gas | Symbol | Observed Specific Gravity | Weight of 1 Cu. Ft. Pound | Volume of 1 Lb. Cubic Feet |
|---|---|---|---|---|
| Acetylene.................. | $C_2H_2$ | .9056 | .07309 | 13.682 |
| Air........................ | | 1.0000 | .08071 | 12.390 |
| Carbon dioxide............ | $CO_2$ | 1.5291 | .12341 | 8.103 |
| Carbon monoxide........... | $CO$ | .9670 | .07805 | 12.813 |
| Ethane.................... | $C_2H_6$ | 1.0494 | .08470 | 11.806 |
| Hydrogen.................. | $H$ | .0696 | .00621 | 177.904 |
| Hydrogen sulphide.......... | $H_2S$ | 1.1912 | .09614 | 10.401 |
| Methane................... | $CH_4$ | .5545 | .04475 | 22.346 |
| Nitrogen................... | $N$ | .9674 | .07808 | 12.807 |
| Olefiant gas (ethylene)...... | $C_2H_4$ | .9852 | .07952 | 12.575 |
| Oxygen.................... | $O$ | 1.1054 | .08922 | 11.208 |
| Sulphur dioxide...... ...... | $SO_2$ | 2.2131 | .17862 | 5.598 |

As in the case of the densities, the observed specific gravities determined by experiment, do not generally agree with the theoretical specific gravities determined from the weight of 1 cu. ft. of air and of hydrogen and the molecular weight of the gases. This want of agreement between the observed and calculated specific gravities will affect the weights per cubic foot and volumes per pound calculated from them. The preceding table is based on observed specific gravities and the weight of 1 cu. ft. of air of .08071 lb. at 32° F. and 29.921 in. of mercury pressure.

**Atmospheric Pressure.**—The pressure of the air upon an object on the surface of the earth is equal to the weight of the column of air extending from the object to the upper limits of the atmosphere, a distance variously estimated as from 45 to 200 mi. The pressure of the atmosphere decreases with the elevation of the place above sea level and increases with the distance below it. At sea level, when the temperature is 32° F., the atmospheric pressure is 14.697 lb. per sq. in. This pressure of 14.697 lb., which is commonly taken as 14.7 lb., is often called an *atmosphere.*

**Measurement of Atmospheric Pressure.**—The pressure of the atmosphere may be measured by the height of a column of air of uniform density, or that of a column of water (water gauge) or of mercury (barometer), necessary to produce such pressure. The following table gives the heights of the columns of these various substances necessary to produce a pressure of 14.697 lb. per sq. in. (one atmosphere) at a temperature of 32° F.

#### EQUIVALENT HEIGHTS OF COLUMNS OF AIR, WATER, AND MERCURY

| Pressure per Square Inch Pounds | Height of Column to Produce Pressure | | |
|---|---|---|---|
| | Air Feet | Water Feet | Mercury Inches |
| 14,697 | 26,220 | 33.942 | 29.921 |
| .491 | 876 | 1.134 | 1 |
| .433 | 772 | 1 | .882 |
| .036 | 64 | $\frac{1}{12}$ or 1 in. | .074 |

The pressure per square foot due to 1 in. of the water and mercury columns is 5.2 and 70.7 lb., respectively.

The height of the air column corresponding to 1 in. of the water gauge is, more exactly, 64.43 ft., at 32° F. and barometer 29.921 in. Note that in the following table the temperature is 60° and barometer 30 in.

#### CORRESPONDING MERCURY AND AIR COLUMNS, AND PRESSURE PER SQUARE FOOT FOR EACH INCH OF WATER COLUMN

| Water Gauge Inches | Mercury Column Inch | Air Column Feet (T. 60°, B. 30″) | Pressure Pounds per Square Foot | Water Gauge Inches | Mercury Column Inch | Air Column Feet (T. 60°, B. 30″) | Pressure Pounds per Square Foot |
|---|---|---|---|---|---|---|---|
| 1 | .0735 | 68 | 5.2 | 6 | .4412 | 407 | 31.2 |
| 2 | .1471 | 136 | 10.4 | 7 | .5147 | 475 | 36.4 |
| 3 | .2206 | 204 | 15.6 | 8 | .5882 | 543 | 41.6 |
| 4 | .2941 | 272 | 20.8 | 9 | .6618 | 611 | 46.8 |
| 5 | .3676 | 340 | 26.0 | 10 | .7353 | 679 | 52.0 |

## WATER COLUMN, AND PRESSURE PER SQUARE FOOT FOR EACH INCH OF MERCURY COLUMN

| Barometer Inches | Water Column Feet | Pressure Pounds per Square Inch | Barometer Inches | Water Column Feet | Pressure Pounds per Square Inch |
|---|---|---|---|---|---|
| 1 | 1.13 | .49 | 16 | 18.13 | 7.84 |
| 2 | 2.27 | .98 | 17 | 19.27 | 8.33 |
| 3 | 3.40 | 1.47 | 18 | 20.40 | 8.82 |
| 4 | 4.54 | 1.96 | 19 | 21.53 | 9.31 |
| 5 | 5.67 | 2.45 | 20 | 22.67 | 9.80 |
| 6 | 6.80 | 2.94 | 21 | 23.80 | 10.29 |
| 7 | 7.93 | 3.43 | 22 | 24.93 | 10.78 |
| 8 | 9.06 | 3.92 | 23 | 26.07 | 11.27 |
| 9 | 10.20 | 4.41 | 24 | 27.20 | 11.76 |
| 10 | 11.33 | 4.90 | 25 | 28.33 | 12.25 |
| 11 | 12.46 | 5.39 | 26 | 29.47 | 12.74 |
| 12 | 13.60 | 5.88 | 27 | 30.60 | 13.23 |
| 13 | 14.73 | 6.37 | 28 | 31.73 | 13.72 |
| 14 | 15.87 | 6.86 | 29 | 32.87 | 14.21 |
| 15 | 17.00 | 7.35 | 30 | 34.00 | 14.70 |

**Barometers.**—The aneroid barometer and its use in determining elevations is described on page 140. The *mercurial barometer* is often called the *cistern barometer;* or, when the lower end of the tube is bent upwards instead of the mouth of the tube being submerged in a basin, it is known as the *siphon barometer.* The instrument is constructed by filling a glass tube 3 ft. long, and having a bore of $\frac{1}{3}$ in. diameter, with mercury, which is boiled to drive off the air. The thumb is now placed tightly over the open end, the tube inverted, and its mouth submerged in a basin of mercury. When the thumb is withdrawn, the mercury sinks in the tube, flowing out into the basin, until the top of the mercury column is about 30 in. above the surface of the mercury in the basin, and after a few oscillations above and below this point, comes to rest. The vacuum thus left in the tube above the mercury column is as perfect a vacuum as it is possible to form, and is called a *Torricelli vacuum,* after its discoverer. There being evidently no pressure in the tube above the mercury column, and as the weight of this column standing above the surface of the mercury in the basin is supported by the pressure of the atmosphere, it is the exact measure of the pressure of the atmosphere on the surface of the mercury in the basin. If the experiment is performed at sea level, the height of the mercury will be found to average about 30 in.; at higher elevations it is less, while below this level, it is greater. Roughly speaking, an allowance of 1 in. of barometric height is made for each 900 ft. of ascent or descent from sea level (see Barometric Elevations). A thermometer is attached to each mercurial barometer to note the temperature of the reading, as it is customary in all accurate work with this instrument to reduce each reading to an equivalent reading at 32° F., which is the standard temperature for barometric readings.

Mercury expands about .0001 of its volume for each degree Fahrenheit. To reduce, therefore, a reading at any temperature to the corresponding reading at the standard temperature of 32° F., subtract $\frac{1}{10000}$ of the observed height for each degree above 32°; or, if the temperature is below 32°, add $\frac{1}{10000}$ for each degree.

Thus, 30.667 in. at 62° F. is equivalent to a reading of 30.555 in. at 32° F., since $30.667 - \frac{62-32}{10,000} \times (30.667) = 30.667 - .092 = 30.555$ in.

A scale is provided at the top of the mercury column with its inches so marked upon it as to make due allowance for what is called the *error of capacity.* In other words, the inches of the scale are longer than real inches, since the level of the mercury in the basin rises as it sinks in the tube, and vice versa. The top of the mercury column is always oval, convex upwards, owing to capillary attraction, and the scale is read where it is tangent to this convex surface.

**Relation Between Volume and Temperature of Gases.**—*The pressure remaining the same, the volume of a given weight of any gas is proportional to its absolute temperature.* (Gay-Lussac's, or Charles' law.)

The meaning of absolute temperature is explained on page 353. For general purposes, the absolute zero is taken as −460° and not at its exact value of −459.64° F.

If $V$ = volume of a gas at absolute temperature $T$;
$v$ = volume of same gas at absolute temperature $t$;

the proportion may be written

$$V : v = T : t \qquad (1)$$

EXAMPLE.—If 10,000 cu. ft. of air at 32° F. is heated to 60° F. in passing through a mine, what is the increased or expanded volume, the pressure remaining constant?

SOLUTION.—Here, $V = 10,000$, $T = 460 + 32 = 492$, $t = 460 + 60 = 520$, and it is required to find $v$; substituting in formula 1, $10,000 : v = 492 : 520$; whence,

$$v = 10,000 \times \frac{520}{492} = 10,569 \text{ cu. ft.}$$

**Relation Between Volume and Pressure of Gases.**—*The pressure remaining the same, the volume of a given weight of any gas is inversely proportional to its absolute pressure.* (Mariotte's, or Boyle's law.)

*Absolute pressure* is the pressure above that of a perfect vacuum to which a gas may be subjected and is equal to the pressure of the atmosphere at the particular time and place added to the pressure as recorded by a gauge or other instrument. Thus, at sea level and under ordinary atmospheric conditions, a gauge pressure of 100 lb. is equal to an absolute pressure of 114.697 lb. per sq. in. At a place 5,000 ft. above sea level, where the average reading of the barometer is, say, 24.9 in. corresponding to a pressure of 12.22 lb. per sq. in., 100 lb. gauge pressure is equal to 112.22 lb. absolute.

If $V$ = volume of a gas under an absolute pressure $P$;
$v$ = volume of same gas under an absolute pressure $p$;

then, $$V : v = p : P \qquad (2)$$

EXAMPLE 1.—It is estimated that the open and abandoned workings of a mine have a volume of 1,000,000 cu. ft. Should the barometer fall from 29.5 to 29.0 in., what volume of air and gas would be forced out of the gob and into the airways, the temperature remaining unchanged?

SOLUTION.—As the barometer measures absolute pressures, in this example, the volumes are inversely proportional to the readings of the barometer. Hence, $V = 1,000,000$, $P = 29.5$, $p = 29.0$, and it is required to find $v$; substituting in formula 2, $1,000,000 : v = 29.0 : 29.5$; whence,

$$v = 1,000,000 \times \frac{29.5}{29.0} = 1,017,250 \text{ cu. ft.}$$

The volume of gas and air forced into the airways will be 17,250 cu. ft.

EXAMPLE 2.—When the atmospheric pressure is 14.7 lb. per sq. in., how many cubic feet of free air must be compressed to a gauge pressure of 80 lb. to fill a cylinder having a capacity of 20 cu. ft., the temperature remaining unchanged?

SOLUTION.—A gauge pressure of 80 lb., under the given conditions, is equal to an absolute pressure of $80 + 14.7 = 94.7$ lb. Hence, $V = 20$, $P = 94.7$, $p = 14.7$, and it is required to find $v$; substituting in formula 2, $20 : v = 14.7 : 94.7$; whence,

$$v = 20 \times \frac{94.7}{14.7} = 128.84 \text{ cu. ft.}$$

**Relation Between Volume, Temperature, and Pressure of Gases.**—When both the temperature and pressure of a gas are changed, the change in volume is directly proportional to the change in absolute temperature (Gay-Lussac's law) and inversely proportional to the change in absolute pressure (Mariotte's law). By combining the formulas 1 and 2, there results,

$$V : v = Tp : tP \qquad (3)$$

EXAMPLE.—A certain volume of air measures 100 cu. ft. at 32° F. and a pressure of 14.7 lb. per sq. in.; what will be the volume of the air if the temperature is increased to 90° F., and the pressure reduced to 10 lb. per sq. in.?

SOLUTION.—Here, $V = 100$, $T = 460 + 32 = 492$, $P = 14.7$, $t = 460 + 90 = 550$,

$p = 10$, and it is required to find $v$; substituting in formula 3, $100 : v = 492 \times 10 : 550 \times 14.7$; whence,

$$v = 100 \times \frac{550 \times 14.7}{492 \times 10} = 164.33 \text{ cu. ft.}$$

**Relation Between Weight, Temperature, and Pressure of Gases.**—The weight of 1 cu. ft. of a gas is the reciprocal of its volume per pound, or $W = \frac{1}{V}$ and $w = \frac{1}{v}$, from which $V = \frac{1}{W}$ and $v = \frac{1}{w}$. Substituting the values of $V$ and $v$ in formula 3 and rearranging, there results,

$$W : w = tP : Tp \qquad (4)$$

EXAMPLE.—If 1 cu. ft. of carbon monoxide weighs .0781 lb. at 32°, barometer 29.92 in., what will be the weight of the same volume of gas at a temperature of 90°, barometer 28.00 in.?

SOLUTION.—Here $W = .0781$, $T = 460 + 32 = 492$, $P = 29.92$, $t = 460 + 90 = 550$, $p = 28.00$, and it is required to find $w$. Substituting in formula 4, $.0781 : w = 550 \times 29.92 : 492 \times 28.00$; whence,

$$w = .0781 \times \frac{492 \times 28}{550 \times 29.92} = .0654 \text{ lb.}$$

Another method of determining the weight of 1 cu. ft. of a gas at any temperature and pressure is given toward the end of the next section.

**Weight and Volume of Air and Gases.**—The weight of 1 cu. ft. of dry air at 32° F. and a pressure of 29.921 in. of mercury or 14.697 lb. per sq. in., is .08071 lb. avoir. Although not a true gas but a mixture of gases, the weight per cubic foot of air decreases as the temperature increases, and the pressure decreases, and vice versa.

The usual formula for finding, approximately, the weight $W$ of 1 cu. ft. of air when the temperature $t$, in degrees Fahrenheit, and the height $B$ of the barometer, in inches, are given, is,

$$W = \frac{1.3273 B}{460 + t} \qquad (5)$$

The denominator of the fraction is the absolute temperature, and 1.3273 is the weight of 1 cu. ft. of air under a pressure of 1 in. of mercury and at a temperature of 1° F., absolute ($-459°$ F.).

When the pressure $P$, in pounds per square inch, is given, $W$ may be found from

$$W = \frac{2.7 P}{460 + t} \qquad (6)$$

The factor 2.7 is obtained by dividing 1.3273 (formula 5) by the weight of 1 cu. in. of mercury, .4912 lb.

EXAMPLE 1.—What is the weight of 1 cu. ft. of dry air at 90° F., barometer 28 in.?

SOLUTION.—Substituting in formula 5,

$$W = \frac{1.3273 \times 28}{460 + 90} = \frac{37.1644}{550} = .06757 \text{ lb.}$$

EXAMPLE 2.—What is the weight of 1 cu. ft. of dry air at a temperature of 10° below zero, when the pressure is 10 lb. per sq. in.?

SOLUTION.—Substituting in formula 6,

$$W = \frac{2.7 \times 10}{460 + (-10)} = \frac{27}{450} = .06000 \text{ lb.}$$

When the specific gravity of a gas is known, its weight per cubic foot under any conditions of temperature and pressure may be found by first finding the weight of 1 cu. ft. of air under the same conditions, and multiplying this result by the specific gravity of the gas.

EXAMPLE.—The specific gravity of carbon monoxide is .967; what is the weight of 1 cu. ft. of this gas at 90° and 28 in.?

SOLUTION.—Using formula 5,

$$W = \frac{1.3273 \times 28}{460 + 90} \times .967 = .0676 \times .967 = .0654 \text{ lb. per cu. ft.}$$

This is the same result as was obtained in the example illustrating formula 4.

## VOLUME AND WEIGHT OF AIR AT SEA LEVEL AT DIFFERENT TEMPERATURES

| Temperature Degrees Fahrenheit | Volume of 1 Lb. Cubic Feet | Weight of 1 Cu. Ft. Pound | Temperature Degrees Fahrenheit | Volume of 1 Lb. Cubic Feet | Weight of 1 Cu. Ft. Pound | Temperature Degrees Fahrenheit | Volume of 1 Lb. Cubic Feet | Weight of 1 Cu. Ft. Pound |
|---|---|---|---|---|---|---|---|---|
| 0 | 11.583 | .08633 | 100 | 14.103 | .07091 | 260 | 18.135 | .05514 |
| 10 | 11.834 | .08450 | 110 | 14.355 | .06967 | 270 | 18.387 | .05439 |
| 20 | 12.086 | .08273 | 120 | 14.607 | .06846 | 280 | 18.639 | .05365 |
| 32 | 12.390 | .08071 | 130 | 14.859 | .06730 | 290 | 18.891 | .05294 |
| 40 | 12.590 | .07943 | 140 | 15.111 | .06618 | 300 | 19.143 | .05224 |
| 45 | 12.712 | .07864 | 150 | 15.363 | .06509 | 310 | 19.387 | .05158 |
| 50 | 12.843 | .07786 | 160 | 15.615 | .06401 | 320 | 19.647 | .05090 |
| 55 | 12.969 | .07711 | 170 | 15.867 | .06302 | 330 | 19.892 | .05027 |
| 60 | 13.095 | .07637 | 180 | 16.119 | .06204 | 340 | 20.151 | .04963 |
| 65 | 13.221 | .07564 | 190 | 16.371 | .06108 | 350 | 20.395 | .04903 |
| 70 | 13.347 | .07493 | 200 | 16.623 | .06016 | 360 | 20.655 | .04841 |
| 75 | 13.473 | .07422 | 210 | 16.875 | .05926 | 370 | 20.899 | .04785 |
| 80 | 13.599 | .07354 | 220 | 17.127 | .05839 | 380 | 21.159 | .04726 |
| 85 | 13.725 | .07286 | 230 | 17.339 | .05751 | 390 | 21.404 | .04672 |
| 90 | 13.851 | .07220 | 240 | 17.631 | .05672 | 400 | 21.663 | .04616 |
| 95 | 13.977 | .07155 | 250 | 17.883 | .05592 | 450 | 22.923 | .043624 |

**Diffusion of Gases.**—*The rate or velocity of diffusion between air and a gas, or between different gases, is inversely proportional to their specific gravities or densities.* (Graham's Law.)

*Diffusion* is the gradual mixing of one gas with another when bodies of them are brought into direct contact or when the wall of the vessel containing them is a porous membrane through which they can pass. Diffusion does not depend on stirring or mechanical mixing, although assisted thereby. Thus, when methane is given off at the floor of a seam, the tendency of the gas to rise owing to its extreme lightness greatly assists its rapid diffusion by bringing a greater number of molecules of air and gas in contact in a given time. A feeder in the roof or other high point may give off gas more quickly than diffusion can take place, particularly where the air-current is sluggish, in which case there will be formed a body of pure methane. Similarly, an accumulation of blackdamp may be formed near the floor or in some other low place where the current is feeble and the gas is given off more rapidly than it can diffuse.

Diffusion continues until the gases are uniformly mixed, and when so mixed the gases cannot be separated. As stated in Graham's law, the greater the difference in the specific gravities of two gases, the more rapidly will they diffuse or mix. Thus, carbon dioxide will mix with air more rapidly than will nitrogen.

The rate of diffusion of one gas with respect to another may be found by comparing their rates of diffusion with respect to air. Thus, the rate of diffusion of carbon dioxide with respect to methane is .812 ÷ 1.344 = .604, and of oxygen with respect to hydrogen is .949 ÷ 3.830 = .248.

The volumes of the various gases that will diffuse in the same time are proportional to their respective rates of diffusion. Thus, 1,344 volumes of methane will diffuse in the same time as 1,000 volumes of air or 812 volumes of carbon dioxide.

The rates of diffusion may also be calculated by comparing the densities of the gases with respect to hydrogen. The density of air and carbon dioxide are, respectively, 14.359 and 21.835, whence the rate of diffusion of carbon dioxide with respect to air is $\sqrt{14.359 \div 21.835} = \sqrt{.657614} = .811$, which agrees very closely with the observed rate of .812.

In the accompanying table, it will be noted that the observed and theoretical rates of diffusion agree very closely, except in the case of hydrogen sulphide.

## RATES OF DIFFUSION AND TRANSPIRATION OF GASES COMPARED TO AIR

| Gas | Specific Gravity | Rate of Diffusion | | Rate of Transpiration |
|---|---|---|---|---|
| | | Theoretical | Observed | |
| Hydrogen................. | .0694 | 3.7965 | 3.830 | 2.066 |
| Methane................. | .5545 | 1.3428 | 1.344 | 1.639 |
| Carbon monoxide........... | .9670 | 1.0169 | 1.015 | 1.034 |
| Nitrogen................ | .9674 | 1.0166 | 1.014 | 1.030 |
| Air.................... | 1.0000 | | | 1.000 |
| Oxygen................. | 1.1054 | .9511 | .949 | .903 |
| Hydrogen sulphide......... | 1.1817 | .9199 | .950 | 1.458 |
| Carbon dioxide........... | 1.5291 | .8087 | .812 | 1.237 |

**Occlusion and Transpiration of Gases.**—All coals in the seam contain a greater or less amount of various gases that are given off as the coal face is exposed in mining. It has commonly been supposed that these gases were occluded, or hidden, in the coal under great pressure, but there seems reason to doubt this as a universal rule (see under Formation of Methane). In any case, the escaping gases are not *occluded*, a term that refers to the probable condensation and perhaps existence of a gas in a quasi-metallic state in the pores of a metal, as hydrogen in the pores of the metals palladium or platinum. The conditions that have held the gas in the coal or adjoining rocks are largely closeness of grain in the coal and imperviousness of the clay in the roof shales. The pressure of the occluded gases is often as high as 10 to 40 or more atmospheres (see Properties and Sources of Methane).

*Transpiration* refers to the more or less steady outflow of gas from the pores of the coal at the working face. The rate of transpiration of the various mine gases, air being the unit, is given in the preceding table. Although the relative rates are not the same, the order of the gases in transpiration is the same as in diffusion, except in the case of the very heavy hydrogen sulphide and carbon dioxide. The rate of transpiration varies with the pressure under which the gas exists and decreases as the temperature decreases but not in the same ratio, and is independent of the specific gravity of the gas.

The rates of transpiration is of importance in determining the nature of the gas mixtures found in mines. Thus, 1,639 volumes of methane will transpire in the same time as 1,237 volumes of carbon dioxide and there is, thence, a tendency to increase the proportion of the former and decrease that of the latter in the airways. This difference in the rate of transpiration has made difficult the accurate determination of the different gases present in different coals. The principal occluded gases are methane, nitrogen, and carbon dioxide. In some coals, methane formed 93% of the occluded gas; in others, nitrogen formed 91%; while in others, carbon dioxide formed 54%. Oxygen rarely exceeds 4 or 5% and is usually much less. Analyses of occluded gases, both face and blowers, are given under Firedamp.

The transpiration of gas from coal seams varies widely in its nature, often being accompanied by a sharp crackling and a hissing sound; in extreme cases the pressure is so great as to dislodge the coal from the face. Usually, the gases issue without noise either from the pores in a newly exposed working face, or through blowers, which are the exposed ends of larger openings or crevices in the seam or its containing rocks (see Properties and Sources of Methane).

**Humidity.**—The amount of water, as vapor, that may be contained in a given volume of air depends on the temperature, and is greater at high than at low readings of the thermometer. When air contains all the moisture it can at any given temperature, it is said to be *saturated*. When the temperature of saturated air is lowered, some of the vapor is condensed and deposited upon surrounding objects in the form of drops of water. When the temperature is raised, the air is no longer saturated and is capable of taking up more moisture from the mine workings until it becomes saturated at the higher temperature. The gallons of water contained in 100,000 cu. ft. of saturated air is given in the following table.

## GALLONS OF WATER IN 100,000 CU. FT. OF SATURATED AIR AT TEMPERATURES FROM −20° F. TO +100° F.

| Temperature Degrees F. | Gallons per 100,000 Cubic Feet | Temperature Degrees F. | Gallons per 100,000 Cubic Feet | Temperature Degrees F. | Gallons per 100,000 Cubic Feet | Temperature Degrees F. | Gallons per 100,000 Cubic Feet | Temperature Degrees F. | Gallons per 100,000 Cubic Feet |
|---|---|---|---|---|---|---|---|---|---|
| −20 | .284 | 5 | 1.044 | 29 | 3.172 | 53 | 7.751 | 77 | 17.099 |
| 19 | .298 | 6 | 1.094 | 30 | 3.312 | 54 | 8.023 | 78 | 17.643 |
| 18 | .315 | 7 | 1.149 | 31 | 3.462 | 55 | 8.306 | 79 | 18.202 |
| 17 | .336 | 8 | 1.205 | 32 | 3.618 | 56 | 8.592 | 80 | 18.776 |
| 16 | .354 | 9 | 1.265 | 33 | 3.756 | 57 | 8.891 | 81 | 19.365 |
| 15 | .373 | 10 | 1.329 | 34 | 3.902 | 58 | 9.199 | 82 | 19.971 |
| 14 | .395 | 11 | 1.397 | 35 | 4.051 | 59 | 9.516 | 83 | 20.594 |
| 13 | .416 | 12 | 1.466 | 36 | 4.206 | 60 | 9.843 | 84 | 21.232 |
| 12 | .440 | 13 | 1.537 | 37 | 4.366 | 61 | 10.179 | 85 | 21.888 |
| 11 | .462 | 14 | 1.611 | 38 | 4.530 | 62 | 10.525 | 86 | 22.564 |
| 10 | .488 | 15 | 1.688 | 39 | 4.701 | 63 | 10.880 | 87 | 23.254 |
| 9 | .514 | 16 | 1.767 | 40 | 4.878 | 64 | 11.249 | 88 | 23.964 |
| 8 | .541 | 17 | 1.849 | 41 | 5.059 | 65 | 11.626 | 89 | 24.694 |
| 7 | .568 | 18 | 1.931 | 42 | 5.246 | 66 | 12.015 | 90 | 25.439 |
| 6 | .569 | 19 | 2.022 | 43 | 5.439 | 67 | 12.415 | 91 | 26.207 |
| 5 | .633 | 20 | 2.114 | 44 | 5.639 | 68 | 12.824 | 92 | 26.994 |
| 4 | .666 | 21 | 2.215 | 45 | 5.845 | 69 | 13.248 | 93 | 27.800 |
| 3 | .704 | 22 | 2.320 | 46 | 6.059 | 70 | 13.686 | 94 | 28.629 |
| 2 | .743 | 23 | 2.428 | 47 | 6.278 | 71 | 14.132 | 95 | 29.477 |
| 1 | .782 | 24 | 2.539 | 48 | 6.507 | 72 | 14.594 | 96 | 30.346 |
| 0 | .823 | 25 | 2.655 | 49 | 6.740 | 73 | 15.066 | 97 | 31.239 |
| +1 | .865 | 26 | 2.779 | 50 | 6.979 | 74 | 15.556 | 98 | 32.156 |
| 2 | .906 | 27 | 2.905 | 51 | 7.229 | 75 | 16.054 | 99 | 33.098 |
| 3 | .948 | 28 | 3.035 | 52 | 7.487 | 76 | 16.569 | 100 | 34.058 |
| 4 | .996 | | | | | | | | |

Relative humidity, which is often called humidity, is the ratio of the quantity of water vapor present to the quantity necessary to saturate the space occupied by the air, at the given temperature and pressure. When air is saturated, its relative humidity is 100%. At any relative humidity, the amount of water actually present in a given volume of air may be found by multiplying the amount required to saturate the air at the specified temperature by the relative humidity.

EXAMPLE 1.—A current of 60,000 cu. ft. of air a min. is entering a mine; its temperature is 20° F. and humidity 65%. What is the quantity of water brought into the mine in 1 min., in 1 hr., and in 1 da. of 24 hr.?

SOLUTION.—From the preceding table, 100,000 cu. ft. of saturated air at 20°, contains 2.114 gal. of water. At 65% relative humidity, this volume of air will contain 2.114×.65=1.3741 gal., and 60,000 cu. ft. will contain six-tenths of this quantity, or 1.3741×.6=.82446 gal., which is the quantity of water brought into the mine in 1 min. In 1 hr. there will be .82446×60 =49.4676 gal., and in a 24-hr. da. there will be 49.4676×24 =1,187.2224 gal.

EXAMPLE 2.—If, in example 1, the return air-current has a temperature of 65° and a relative humidity of 98%, what is the quantity of water absorbed from the mine workings in 1 min., 1 hr., and 1 da.?

SOLUTION.—At 98% humidity, 60,000 cu. ft. of air contains 11.626×.98 ×.6=6.83609 gal. At 20° F. and 65% humidity, the same quantity of air contains .82446 gal. (example 1); hence, there is absorbed by the air-current in its passage through the mine, 6.83609−.82446=6.01143 gal. a min.; 6.01143 ×60=360.6858 gal. an hr., and 360.6858×24=8,656.4692 gal. a da.

**Psychrometers or Hygrometers.**—A psychrometer, or hygrometer, is an instrument for measuring the quantity of aqueous vapor in the air. The standard type involves the determination of the temperature of evaporation and consists of two similar thermometers usually mounted on the same frame, one called the *dry bulb* and the other the *wet bulb*. Below the wet bulb is a small jar for water. In order to take an observation, a small muslin sack is

fixed around the wet bulb, its end extending down into the jar from which the water is drawn by capillary attraction. The thermometers are exposed to a current of air that has a velocity of 15 ft. or more per sec. If the relative humidity of the air is less than 100%, that is, if the air is not saturated, some of the water on the muslin sack is evaporated, and a certain amount of heat is absorbed by evaporation, and the temperature of the wet-bulb thermometer is reduced to a point at which the amount of heat absorbed by evaporation is just equal to that received from the surrounding air. From the readings of the two thermometers, by reference to suitable tables, the relative humidity and, consequently, the aqueous vapor per cubic foot of air may be calculated.

The *sling psychrometer* is the form in common use in mines. It consists of two thermometers mounted side by side in a case provided with a cover. The instrument has a handle at the top by which it may be given a whirling motion to secure a velocity of the wet bulb of 15 ft. a sec. or more. The muslin sack is moistened at the time of making the observation, which is repeated until the results agree.

The *hygrodeik* is a form of psychrometer in which the thermometers are attached to a fan-shaped wooden frame, on which are a series of curves and radial lines placed between the thermometers. By properly placing a pointer suspended from the upper part of the frame according to the readings of the two thermometers, the relative humidity may at once be taken from one of the scales, thus obviating any calculations.

In the *hair hygrometer*, the rapid change that takes place in the length of a strand of hair with changes in the amount of moisture in the air, is utilized to move a pointer by means of a delicately adjusted lever arm.

A *hygrograph* is an instrument by which the variations in relative humidity are automatically recorded. It consists of a hair psychrometer, the pointer of which carries on its end an inking arrangement so that it can trace a line on section paper wrapped on a revolving cylinder turned by clockwork. In this way, a permanent and continuous record is obtained. The readings of a hair psychrometer are not always reliable, and the instrument should be standardized by comparison with the regular wet-bulb instrument, at various temperatures and degrees of saturation.

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# MINE GASES

## ATMOSPHERIC AND MINE AIR

**Atmospheric Air.**—Sir William Ramsay gives the following as the composition of ordinary air:

### COMPOSITION OF PURE AIR

| Name of Gas | By Volume | | By Weight | |
|---|---|---|---|---|
| | Exact | Approximate | Exact | Approximate |
| Oxygen............ | 20.941 | 20.94 | 23.024 | 23.02 |
| Nitrogen.......... | 78.122 } | 79.06 | 75.539 } | 76.98 |
| Argon............. | .937 } | | 1.437 } | |
| Total............ | 100.000 | 100.00 | 100.000 | 100.00 |

In the column headed Approximate, the argon and nitrogen are considered as one gas, as is usually done. With the argon occur certain very rare gases similar to nitrogen whose proportions, by volume, Ramsay estimates to be: Helium, .0004%; krypton, .028%; neon, .0123%; xenon, .005%.

In addition to these normal gases, air always contains a certain amount of carbon dioxide and water vapor, the former averaging .03 to .04%, and the latter depending on the temperature and relative humidity. At certain times and in certain places, traces of ammonia, oxides of nitrogen, sulphur dioxide, and even hydrogen are found in air. The composition of the air is constant regardless of the altitude.

**Mine Air.**—Mine air does not differ from atmospheric air, except that in its passage through the mine, the ventilating current gives up a certain amount of oxygen to the coal and to the various processes of combustion (breathing of men and animals, burning of lamps, etc.), and receives a certain amount of other gases in place thereof; always carbon dioxide, usually methane, rarely carbon monoxide, and very rarely or in extremely minute amounts, ethane, olefiant gas, nitrous oxide, sulphur dioxide, hydrogen sulphide, hydrogen, and possibly hydrocarbons higher than ethane. The carbon dioxide, methane, and carbon monoxide are generally known as *mine gases*, as the others are found in very small amounts or under unusual conditions. The constituent of air that usually varies the most between the intake and the return is the absolute humidity, meaning by this term, the actual amount of water contained in a given volume of the air-current. Regardless of the relative humidity of the intake, that of the return is rarely less than 90% and is very commonly more than this; further, this is generally true in mines of any size whatever may be the outside temperature. That is, whether the air enters at 20° and 50% humidity or 80° and 95% humidity, it will usually leave the mine with a saturation of 90% or more at a temperature between 55° and 65°.

In passing through well-ventilated mines, the air-current will rarely lose more than .5 to .75% of oxygen and gain about the same amount of carbon dioxide and methane. At the larger American mines, a chemist is employed to regularly analyze the return air, and any deficiency in oxygen or dangerous increase in carbon dioxide and, more particularly, methane is promptly rectified by increasing the ventilating current.

In Great Britain, the mine code of 1914 provides: "A place shall not be deemed to be in a fit state for working or passing therein if the air contains either less than 19% of oxygen or more than 1¼% of carbon dioxide."

## OXYGEN

**Properties and Sources.**—Oxygen has an atomic symbol of $O$ and a molecular formula of $O_2$. It is but slightly soluble in water, 100 volumes of which take up 4.11 volumes of the gas at 32° and 2.83 volumes at 68°. It is tasteless, odorless, and colorless and is the supporter of life and combustion. Its chemical and physical constants have been given in preceding tables.

Oxygen is the most abundant element in nature, composing, by weight, 23.02% of the atmosphere, 85.79% of all water, and 47.17% of the rocks of the solid crust of the earth. It constitutes 49.85%, or practically one-half, of all matter existing between the upper limits of the earth's atmosphere and the lower depths of its crust. It is not produced by any of the ordinary chemical processes or changes going on in mines, but is absorbed or consumed in practically all of them. Small amounts are given off by the pores of the coal, by blowers, etc.

**Effect of Oxygen on Life.**—Oxygen is essential to life, forming in the lungs with the hemoglobin of the blood an unstable chemical compound known as *oxyhemoglobin*, which gives arterial blood its bright red color. In its passage through the body, the oxyhemoglobin parts with its oxygen and is converted into hemoglobin, which gives the familiar dark purple color to venous blood.

At rest, an average man breathes sixteen to eighteen times a minute and takes into the lungs at each respiration 30.5 cu. in. of air. When working moderately, there will be about twenty-five respirations a minute, which may be increased to as many as sixty when the work is very violent, as in running. Of the oxygen passing into the lungs, 10 to 35% is consumed in the processes of the body, the rest being exhaled with the nitrogen inhaled in the air together with the carbon dioxide formed in the tissues. Exhaled breath contains from 2.6 to 6.6% of carbon dioxide, with an average of about 4%. The proportion of carbon dioxide is less during sleep, although it changes but little with changes in the amount of oxygen in the air, as long as breathing is free.

The effect upon life of diminishing the proportion of oxygen in the air depends on whether its place is taken by nitrogen or carbon dioxide. When carbon dioxide is not present, the effects produced upon a miner by a simple deficiency in the percentage of oxygen are the same as those produced by the great diminution in atmospheric pressure at high altitudes (Haldane). Quoting further from Haldane: A diminution (of oxygen in the air) from 20.93% to 15% is by itself practically without effect on men, though a candle would be instantly extinguished in such air. As the diminution increases further, certain effects begin, however, to be produced. The first symptoms usually noticed are that any great muscular exertion is less easy, and that it is apt to cause slight dizziness and unusual shortness of breath. A person not exerting

himself will, as a rule, not notice anything unusual until the oxygen percentage has fallen to 10%. The breathing then usually begins to become deeper and more frequent, the pulse more frequent, and the face somewhat dusky. At 7%, there is usually distinct panting, accompanied by palpitations, and the face becomes a leaden blue color. At the same time the mind becomes confused and the senses dulled, although the person breathing the air may be quite unaware of the fact. Muscular power is also greatly impaired. At a slightly lower percentage, there is complete loss of consciousness . . . . In air containing no oxygen, loss of consciousness occurs within 40 sec. or less, without any previous warning symptom (see Effect of Carbon Dioxide on Life).

Pure oxygen may be inhaled for a long time without danger and is frequently administered in cases of suffocation, carbon monoxide poisoning, etc.

**Effect of Oxygen on Combustion.**—As the proportion of oxygen in the air is lessened, lamps burn more and more dimly until they finally go out. The amount of oxygen that will sustain combustion is influenced by the amount of moisture and carbon dioxide present. Thus, a lamp will be extinguished sooner when the air is moist and the oxygen has been replaced by carbon dioxide than when the air is dry and nitrogen is the only inert gas present.

A *residual atmosphere* is one that remains after a flame burning in it has so reduced the amount of oxygen that it is finally extinguished. The following table, from Clowes and the Bureau of Mines, gives the residual atmospheres in which flames of various burning substances were finally extinguished.

## COMPOSITION OF RESIDUAL ATMOSPHERES THAT EXTINGUISH FLAME

| Flame | Percentage Composition of Atmosphere | | |
|---|---|---|---|
| | $CO_2$ | $O_2$ | $N_2$ |
| Acetylene....................... | 6.30 | 11.70 | 80.50 |
| Alcohol, absolute................. | 4.35 | 14.90 | 80.75 |
| Candle........................... | 2.95 | 16.24 | 80.81 |
| Carbon monoxide................. | 12.25 | 13.35 | 74.40 |
| Coal gas......................... | 4.90 | 11.35 | 83.75 |
| Hydrogen........................ | | 5.50 | 94.50 |
| Methane......................... | 2.30 | 15.60 | 82.10 |
| Natural gas...................... | 3.25 | 13.90 | 82.85 |
| Paraffin, lamp oil................. | 3.00 | 16.60 | 80.40 |
| Wolf lamp, bonneted.............. | 3.00 | 16.50 | 80.50 |
| Wolf lamp, unbonneted........... | 3.00 | 15.82 | 81.18 |

Clowes sums up the results of his experiments as follows:

1. Wick-fed flames require atmospheres of very similar composition to extinguish them; while gas-fed flames require atmospheres of widely different composition.

2. Nitrogen must be added in larger proportion than carbon dioxide in order to extinguish the same flame (see Effect of Carbon Dioxide on Combustion).

3. The minimum proportion of extinctive gas that must be mixed with air in order to extinguish a flame is independent of the size of the flame.

4. The composition of an atmosphere that will at once extinguish a lamp placed in it is not the same as the residual atmosphere resulting from the lamp burning out in pure air. In the case of wick-fed flames, however, the difference is not great.

5. The flames of candles and lamps, when they are extinguished by burning in a confined space of air, produce an atmosphere of almost identical composition with that of air expired from the lungs.

6. The extinctive atmospheres produced by the combustion of the flames of candles and of lamps, and the air expired from the lungs after inspiring fresh air, are respirable with safety.

7. The extinction of an ordinary candle or lamp flame is not necessarily indicative of the unsuitability of an atmosphere to maintain life when it is breathed.

**Absorption of Oxygen by Coal.**—The experiments of the Bureau of Mines and of others show that coal bottled in air under ordinary atmospheric pressure rapidly absorbs oxygen until, after a few days, there remains often only 1 or 2% of oxygen in the free gas in the bottle. At the same time a certain proportion of the oxygen unites with the substances in the coal to form carbon dioxide, but this process, at least during the comparatively short time of the experiments accounts for only a small part of the oxygen entering the coal.

The effect of the gradual absorption of oxygen with the formation of greater or less amounts of carbon dioxide is to form blackdamp with the nitrogen remaining in the air, as explained under Blackdamp.

## NITROGEN

**Properties and Sources.**—Nitrogen has an atomic symbol of $N$ and a molecular formula of $N_2$. It is odorless, colorless, and tasteless, and supports neither life nor combustion. In the atmosphere, it serves to dilute the oxygen and, in combinations like ammonia, is a source of plant food. It is about one-half as soluble in water as oxygen, 100 volumes of water taking up 2.03 volumes of the gas at 32° and 1.40 volumes at 68°. The chemical and physical constants of nitrogen have been given in the preceding tables.

Small quantities of nitrogen are given off by the pores of the coal. It is the product of feeders and blowers, rarely as pure nitrogen, but mixed with considerable amounts of carbon dioxide, forming blackdamp. Feeders of methane, also, contain variable amounts of this gas. Small quantities may be present in the leakage from natural gas wells that have been drilled and improperly cased through the coal. It is found chiefly, and to the extent of 20 to 55%, in the products of combustion of explosives; the percentage depending on the kind of explosive and the conditions under which it is fired.

While composing 79.06% by weight of the atmosphere, nitrogen constitutes but .03% of the matter existing between the upper limit of the atmosphere and the lower limit of the earth's crust. Owing to the absorption of oxygen in the various chemical processes going on in mines, the proportion of nitrogen in mine air is usually greater than in that at the surface.

**Effect of Nitrogen on Life.**—Nitrogen is distinctly negative in its action and is not in any way poisonous. The effect of gradually increasing the percentage of nitrogen or, what is the same thing, decreasing the proportion of oxygen in mine air, is to cause death by suffocation (see Effect of Oxygen on Life).

**Effect of Nitrogen on Combustion.**—Pure nitrogen instantly extinguishes combustion of any kind. Increasing proportions of nitrogen in the air with the accompanying decreasing proportions of oxygen cause a flame to burn with diminishing brilliancy until it is finally extinguished (see Effect of Oxygen and Effect of Carbon Dioxide, Respectively, on Combustion).

Atmospheres deficient in oxygen and high in nitrogen and methane and, commonly, carbon dioxide, are very common in old workings. In these old workings, a lamp may not burn although breathing is without discomfort, but if sufficient fresh air is mixed with these gob atmospheres they may become highly explosive; or they may be explosive in one part of the workings and not in another.

## CARBON DIOXIDE

**Properties and Sources.**—Carbon dioxide, formerly known as *carbonic acid*, *carbonic-acid gas*, *blackdamp*, *chokedamp*, *stythe*, etc., has a formula of $CO_2$, is colorless and odorless, has a slight acid taste especially when dissolved in water, and is not combustible. It is very soluble in water, 100 volumes taking up 179.6 volumes of the gas at 32° and 90.1 volumes at 68°. The chemical and physical constants are given in preceding tables.

Under ordinary working conditions, carbon dioxide is produced in the mine by the breathing of men and animals, by the burning of oil, acetylene, gasoline, and explosives, by the decay of vegetable and animal matter, and by the slow oxidation of the coal. Small amounts of this gas are produced by feeders and blowers occurring in the roof, floor, or face, by transpiration from the coal being mined, by escape from the water of underground streams wherein it has been held under pressure, and by chemical reaction between water carrying carbonates in solution and acid minewater. However, the amount of carbon dioxide thus added to the air of a well-ventilated mine is insignificant and is, according to Haas, not over .02 to .03%, so that normal mine air should not contain over .06 to .07% of this gas.

Under unusual conditions, carbon dioxide is produced by spontaneous combustion of coal as in gob fires, by mine fires of any kind, or by explosions of methane and coal dust. While it may be formed by the burning or explosion of carbon monoxide, its origin in this way is highly improbable, as over 13% of monoxide is necessary for the purpose.

Owing to its high specific gravity, carbon dioxide naturally tends to collect along the floor and in dip workings, but this tendency is resisted by its diffusive properties, so that the gas is commonly distributed uniformly through the air of a working place. When concentrated at the floor it is because it is given off there faster than diffusion and the ventilating currents can remove it. As Haldane states, warm air laden with carbon dioxide from breathing and burning will keep at the top of the rise.

**Effect of Carbon Dioxide on Life.**—Carbon dioxide is not actively poisonous like carbon monoxide, hydrogen sulphide, etc., in that it does not combine with the hemoglobin of the blood or cause degeneration of the brain cells or the like, but in sufficient quantities it does have a toxic effect. That is to say, atmospheres in which a deficiency of oxygen is accompanied by a corresponding excess of carbon dioxide are more injurious than those where the oxygen deficiency merely results in increasing the proportion of nitrogen present. Messrs. Priestly and Haldane explain this as follows: "At ordinary atmospheric pressure, the breathing always regulates in such a way as to keep the percentage of carbonic acid ($CO_2$) in the air cells (alveoli) of the lungs constant. Each individual has his own exact percentage; but on an average there is about 5.6% of carbonic acid in the alveolar air of man. The regulation is almost astoundingly exact for each person.

"If air containing carbonic acid is breathed, the respirations become deeper, in such a way that the alveolar carbonic-acid percentage still remains practically the same, if possible. If, for instance, there is 2% of carbonic acid in the air, the breathing will need to be about 50% deeper than before. This difference would not be noticed by the person. If there is 5% of carbonic acid in the air, it requires much panting to keep the alveolar carbonic-acid percentage nearly constant; if there is 6 or 7%, it is, of course, quite impossible to maintain a normal alveolar carbonic-acid percentage, and great distress is produced, as the blood becomes abnormally charged with carbonic acid, to which the body is exquisitely sensitive.

"It is carbonic acid, and carbonic acid alone, which regulates our breathing under normal conditions. The supposed ill effects of a small percentage of carbonic acid in the inspired air are wholly imaginary, as a very slight increase in the depth of breathing at once compensates for the extra carbonic acid. So long as the efforts of the lungs to adjust themselves to the change in the amount of carbonic acid in the air are not attended with discomfort, the person is in no danger."

The same authority says that "carbon dioxide produces no very noticeable effect until it amounts to more than 3% (3% $CO_2$ and 97% air = 3% $CO_2$, 20.4% $O_2$, 76.6% $N_2$), which is more than is often met in mine air just extinctive of lights. With an increasing proportion, the breathing becomes noticeably deeper and more frequent; at about 5 or 6% (5% $CO_2$ and 95% air = 5% $CO_2$, 20% $O_2$, 75% $N_2$), there is marked panting accompanied by increased frequency of pulse. At about 10% (10% $CO_2$ and 90% air = 10% $CO_2$, 18.9% $O_2$, 71.1% $N_2$), there is violent panting, throbbing, and flushing of the face. Headache is also produced, especially noticeable on return to fresh air. Beyond 10%, carbon dioxide begins to have a narcotic effect, and at 25% (25% $CO_2$ and 75% air = 25% $CO_2$, 15.8% $O_2$, 59.2% $N_2$) death may occur after several hours; but as much as 50% may be breathed for some time without fatal effects, to judge from experiments on animals."

In commenting upon a certain blackdamp, Mr. E. M. Chance says: "With 10% of oxygen, the effects produced will be more dangerous from the deficiency of oxygen than from the 4% of carbon dioxide. In fact, the 4% of carbon dioxide will prolong life by creating deeper and more frequent respiration. In breathing atmospheres containing 2% or 3% carbon dioxide and, say, 10% or 12% oxygen, the respiration will be better and the man will be more resistant to the gas than if he breathed the air without the carbon dioxide."

Modern investigations show that a certain amount of carbon dioxide is distinctly stimulating to respiration and, in cases of suffocation, it is not unusual to administer the dioxide at the same time as oxygen, because the increased depth of breathing through the effort of the lungs to adjust themselves to the proper alveolar percentage of carbon dioxide, naturally draws into them more of the desired oxygen.

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**Effect of Carbon Dioxide on Combustion.**—Strictly, it is the lack of oxygen and not the presence of carbon dioxide or other inert gas that causes a lamp flame to dim and finally go out. However, the experiments of Haldane, Clowes, Clement, and others show that when a loss of oxygen is accompanied by an increase in carbon dioxide, a flame is extinguished more quickly than when a deficiency in oxygen merely causes an increase of nitrogen. The following table, abridged from Clement, shows the effects on the explosive range of methane by decreasing percentages of oxygen and increasing percentages of carbon dioxide. The explosions were made in a steel explosion chamber by an electric spark and are believed to approximate results that would be obtained in the mines. In any case, the amount of nitrogen in the air may be found by subtracting from 100%, the sum of the percentages of oxygen, carbon dioxide and methane.

## EXPLOSIVE RANGE OF MIXTURES OF METHANE AND CARBON DIOXIDE

| Atmosphere Contains | | Methane Explodes | | Atmosphere Contains | | Methane Explodes | |
|---|---|---|---|---|---|---|---|
| $O_2$ Per Cent. | $CO_2$ Per Cent. | Lower Limit Per Cent. | Upper Limit Per Cent. | $O_2$ Per Cent. | $CO_2$ Per Cent. | Lower Limit Per Cent. | Upper Limit Per Cent. |
| 19 | | 5.5 | 13.5 | 16 | 20.0 | 6.7 | 9.6 |
| 19 | 20 | 6.4 | | 16 | 30.0 | 7.4 | 8.1 |
| 19 | 50 | 8.0 | 10.8 | 15 | | 5.9 | 9.6 |
| 18 | | 5.7 | 12.8 | 15 | 16.0 | 6.9 | 8.7 |
| 18 | 20 | 6.4 | 11.8 | 15 | 21.0 | 7.5 | 7.5 |
| 18· | 48 | 8.5+ | 10.0 | 14 | | 6.2 | 8.2 |
| 17 | | 5.7 | 11.8 | 14 | 12.5 | 7.3 | 7.5 |
| 17 | 20 | 6.4 | 10.3 | 14 | 13.5 | 6.9* | 7.2* |
| 17 | 43 | 8.3 | 8.7 | 13 | | 6.3 | 7.1 |
| 16 | | 5.8 | 10.7 | 13 | 2.0 | 6.6 | 6.8 |

The effect of increasing percentages of carbon dioxide, that of the oxygen remaining unchanged, is to shorten the explosive range of methane by raising the lower and reducing the upper explosive limit. Immediately after the explosive limits coincide, there is so much carbon dioxide present that an explosion is not possible. The lower the oxygen percentage in the air, the less carbon dioxide is required to make the atmosphere inexplosive. Further, the dioxide appears to be more active in reducing the upper explosive limit than in raising the lower one.

According to J. F. Clement, the author of the foregoing table, the difference in the effects of carbon dioxide and nitrogen on the explosive limits of methane is due to the marked difference in the specific heats of the two gases. The mean molecular heats of carbon dioxide and nitrogen between 0° and 650° C. are, respectively, 10.6 and 7.2. A given volume of carbon dioxide will absorb, therefore, $10.6 \div 7.2 = 1.47$ times as much heat, practically 50% more, as the same volume of nitrogen while being heated to 650° C., the ignition temperature of methane. An amount of carbon dioxide may, thence, be eventually reached where its cooling effect is so great that the temperature of the surrounding air will be below the ignition point of methane.

Similar reasoning shows that the amount of heat required to raise the temperature of an excessive percentage of carbon dioxide may be so great that the products of combustion of an oil flame may be cooled below the ignition point of the oil; and when this is passed, the lamp goes out. As the cooling is gradual as the percentage of carbon dioxide increases, so is the diminution of the light given out by the lamp, which is proportional to the rate of combustion of the oil. A reduction of 1% in the oxygen normally in

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* Note that these are burnings and not explosions.

the air causes a loss of 30% in the light emitted by oil flame. Clowes states that when 2% of carbon dioxide is present, the hydrogen flame begins to change from reddish to pale blue, whereas 10% is necessary to reduce the size of an oil flame and 15% to extinguish it. A reduction in the oxygen to 16% causes the acetylene flame to become yellow.

**Blackdamp.**—The term blackdamp is commonly applied by the miner to carbon dioxide and by others to a mixture of that gas and air in which the proportion of carbon dioxide is so large that a lamp will not burn in it.

Strictly, there is no such gas as blackdamp, and the term is now used to describe a residual atmosphere, resulting from the slow absorption of oxygen by the coal and by the processes of oxidation always taking place underground. Was it not for the fact that in these processes of absorption and oxidation a certain amount of carbon dioxide is given off, the resulting mine atmosphere would be one of pure nitrogen. On this understanding, blackdamp is mine air deficient in oxygen but containing an excess of nitrogen, with (usually) or without carbon dioxide.

However, certain chemists like Haldane and those of the Bureau of Mines regard blackdamp as a more or less distinct gaseous mixture containing, on an average, from 90 to 85% of nitrogen and 10 to 15% of carbon dioxide, with seldom less than 5% or more than 20% of the latter. The chemical and physical properties of such a mixture, both by itself and when associated with air, will depend on the relative proportions of the two gases. Thus, a blackdamp mixture high in carbon dioxide will have a much more marked effect upon life and combustion than when the mixture is largely or entirely nitrogen; pure blackdamp is lighter than carbon dioxide and is lighter than air when the proportion of carbon dioxide is less than 5.25%. If a small amount of methane is associated with the blackdamp, the resulting mixture will almost always be lighter than air. As illustrating the variable composition of blackdamp, Haldane cites that at the Grotto del Cane, near Naples, as being pure carbon dioxide and that at a metal mine in Colorado as being pure nitrogen; between these extremes, the two gases may exist in any proportion.

The following analyses from Haldane illustrate the composition of air and blackdamp mixtures and the methods of computation.

In the first form, the amounts of the gases are reported in the ordinary way; in the second, the oxygen with the proper amount of nitrogen to form air is reported as such, and the remainder of the nitrogen together with all the carbon dioxide is reported as blackdamp.

### AIR FROM OLD WORKINGS, HAMSTEAD COLLIERY

| As Analyzed | | Air-Blackdamp Ratio | | |
|---|---|---|---|---|
| Oxygen | 3.35 | Air { Oxygen | 3.35 } | 16.00 |
| Nitrogen | 84.03 | { Nitrogen | 12.65 } | |
| Carbon dioxide | 5.25 | Blackdamp { Nitrogen | 71.38 } | 76.63 |
| Methane | 7.33 | { Carbon dioxide | 5.25 } | |
| Carbon monoxide | .04 | Methane | | 7.33 |
| | | Carbon monoxide | | .04 |
| | 100.00 | | | |
| | | | | 100.00 |

In the preceding analysis, the blackdamp is composed of 93.2% nitrogen and 6.8% carbon dioxide.

### AIR FROM A DISUSED END, DOLCOATH TIN MINE, CORNWALL

| As Analyzed | | Air-Blackdamp Ratio | | |
|---|---|---|---|---|
| Oxygen | 17.99 | Air { Oxygen | 17.99 } | 85.93 |
| Nitrogen | 78.83 | { Nitrogen | 67.94 } | |
| Carbon dioxide | 3.18 | Blackdamp { Nitrogen | 10.89 } | 14.07 |
| | | { Carbon dioxide | 3.18 } | |
| | 100.00 | | | |
| | | | | 100.00 |

In the preceding analysis, the blackdamp is composed of 77.6% nitrogen and 22.4% carbon dioxide.

In the section headed Firedamp are numerous analyses of gaseous mixtures given off by the face, blowers, feeders, etc., the majority of which contain blackdamp. The constituents of these, and similar mixtures, may be combined in the same way as the foregoing.

**Haldane's Blackdamp Indicator.**—Haldane's blackdamp indicator is based upon the fact that the distance above the bottom of a tube at which a lighted

taper is extinguished bears a constant relation to the amount of oxygen in the air or, conversely, to the amount of inert impurities present. The apparatus consists of a glass tube 7 in. long and $\frac{3}{4}$ in. in diameter with a vertical scale on its face which is graduated in percentages of blackdamp and the corresponding percentages of oxygen. The blackdamp graduations run from 0 to 10.5% and the corresponding oxygen graduations from 20.9 to 18.8%. A lighted taper, $\frac{1}{16}$ in. in diameter is introduced into the bottom of the tube and is moved upward until it is extinguished, when the percentage of blackdamp present is read directly from the scale. Since an excess of moisture or of nitrogen affects the flame in a similar way to carbon dioxide, the indicator records the sum of the inert impurities in the mine air. As methane elongates the flame, the device is better adapted to use in non-gaseous mines. The instrument is graduated to correspond with air saturated with moisture at 64° F. (about 2% of moisture) as this is a fair average of temperature and humidity conditions in mines. Provided the temperature and humidity do not vary greatly from those just named, it is found that the indications of the instrument may be relied upon to within about .2% of oxygen and 1% of blackdamp. The instrument may be graduated for any other conditions of temperature and humidity.

Below 18.8% of oxygen, the taper will no longer burn in the tube nor will it burn without it when the oxygen is below 18.2% (13% of blackdamp) unless held horizontally when it will burn until the oxygen has been reduced to about 17.2% (18% of blackdamp).

## CARBON MONOXIDE

**Properties and Sources.**—Carbon monoxide, formerly known as *carbonic oxide, carbonic-oxide gas,* and *whitedamp,* has a formula $CO$, is odorless, colorless, and tasteless, and supports neither life nor combustion. It burns with a bright bluish flame and is explosive when mixed with air in the proper proportions. The gas is so extremely poisonous that its presence in mine air indicates conditions that should be investigated and remedied at once.

The burning of coal dust suspended in the air or lodged on the roof, floor, and ribs, at the time of a mine explosion, is the greatest and most dangerous source of carbon monoxide, a gas that has probably caused more deaths, certainly more at one time, than have all the other mine gases combined.

Insignificant amounts of carbon monoxide are given off by certain coals either because they are occluded in the pores of the coal or because they result from the slow oxidation of the coal. Rollin T. Chamberlain investigated the gases given off by coals when bottled in a vacuum. The gases from small lumps from the Mansfield mine, Carnegie, Pa., contained .22% $CO$ at the end of 1 mo.; analyses made at the end of 10 and of 20 da. showing none of the gas. The gases from Monongah, W. Va., coal crushed to pass through a ten-mesh sieve, showed .20% of carbon oxide for the first 6 wk., .16% $CO$ for the next 10 wk., and 2.86% $CO$ for the final 10 wk. of the half year during which the tests were carried on. In these instances, the coal used was the Pittsburgh, or No. 8, containing over 30% volatile matter. It was found that the finer the coal, the more gas was given off. The amount of gas thrown into the mine air in this way is too small to be detected by analysis in the return.

Under all conditions of working, gasoline-haulage motors produce gases containing carbon monoxide. When the carburation is poor, the proportion of the gas is high and, if the ventilation is defective, may be a source of grave danger. Proper operation of the motor and sufficient ventilation will generally prevent trouble from this cause.

Carbon monoxide may be produced by the explosion of methane when the percentage of the latter gas in the air is between the limits of maximum explosibility and the upper explosive limit or between 9.46 and, say, 13%. The amount of the gas thus produced increases as the percentage of methane approaches the upper explosive limit (see Combustion Products of Methane).

Carbon monoxide is produced by the detonation of all kinds of explosives in amounts ranging from 2 to 35% of the volume of the gases resulting therefrom, depending on the composition of the explosive, the care used in handling it, and the conditions under which it was fired. Ordinarily, under proper conditions of management, the carbon monoxide produced in this way is not dangerous and cannot be detected in the return air.

The gases from black powder exploded in a vacuum contain from 2 to 12% of carbon monoxide, with an average of about 6%. In mining operations, however, the presence of coal dust will increase the proportion of the gas to 18 to 20% and to 35% in some cases. The proportion given off is found to

decrease as the pressure decreases and the amount of moisture present increases. Haas states that the gases from black powder will yield on an average 18% of the monoxide, and that 1 lb. of powder will produce 1 cu. ft. of the gas. Should 12 lb. of powder be exploded in a place 6 ft. × 15 ft., and such amounts are sometimes used in solid shooting, the gases, if confined within a distance of 10 ft. from the face, would cause the air to contain 1.33% of carbon monoxide; an amount that would be fatal in a few minutes.

In the case of a standing shot, the cracks and crevices of the coal are commonly filled with a combustible gas, which is frequently ignited by the lamp of a miner returning too soon to the face. It has commonly been supposed that this gas is carbon monoxide, but Mr. Haas suggests that it is probably some hydrocarbon distilled from the coal by the intense heat of the explosion, and that the monoxide produced was all burned immediately after the blast was discharged.

Some permissible explosives, notably those of the ammonium nitrate class, yield no carbon monoxide when detonated in a vacuum. However, when fired in the presence of coal dust, as in a mine, they will yield from 6 to 36% of the gas, the higher strength being extreme.

Abel and others have shown that the explosion of nitroglycerine does not yield any carbon monoxide, but the tests of the Bureau of Mines indicate the presence of as much as 28.4% $CO$ in the gases from exploding 40% strength nitroglycerine dynamite and 34.6% in the gases of the same explosive when of 60% strength. A 60% strength low freezing dynamite yielded as much as 47.4% $CO$, in the gases. The explosion products from ammonia and gelatine dynamites contain from 0 to 4% $CO$. However, when these explosives are improperly used, that is, when all or a part of the charge is burned and not exploded, the amount of carbon monoxide in the gases is greatly increased. In a test by the Bureau of Mines, the gases from a 40% strength gelatine dynamite contained 3% $CO$ when the explosive was detonated and 13.7% when it was burned.

The gases from selected permissible powders and dynamites should cause no trouble from carbon monoxide if the explosives are properly used with detonators of ample strength, if the air supply at the face is good, and if a reasonable time is allowed for the gases to mix with the air before returning to the face after a shot.

While the Bureau of Mines (Miner's Circular 14) states that natural gas does not contain carbon monoxide, from which it follows that the leakage from an improperly cased well drilled through the seam cannot be a source of this gas, other analysts have found from 0 to .4% $CO$ in natural gas from Pennsylvania and West Virginia, from 0 to .5% in that from Ohio and Indiana, and an average of 1% in that from Kansas.

Carbon monoxide is invariably present in the gases from gob fires. These fires are caused by the spontaneous combustion of the coal through the slow oxidation of piles of slack and bone coal left behind in the workings where the air supply is sufficient to furnish the necessary oxygen, but is not great enough to carry away all the heat formed by the chemical processes going on (see under Spontaneous Combustion and Mine Fires).

Should a gob fire burst into flame through an increase of the air supply, or should a fire in the ordinary sense start in the timber or in the coal, large volumes of carbon monoxide will be given off because, in the confined workings of a mine, it is not possible for there to be enough air present to furnish all the oxygen required to burn the carbon to carbon dioxide. Either a portion of the carbon is burned to the monoxide directly, or it is burned to the dioxide, a part or all of which is at once reduced to monoxide. Numerous deaths have been caused by the carbon monoxide from extensive mine fires being carried into the workings in mines, where under usual conditions, the air current was ample.

**Effect of Carbon Monoxide on Life.**—While perhaps not so poisonous as hydrogen sulphide and nitrogen monoxide, carbon monoxide is formed under so many more conditions and to a so much greater extent that it is commonly considered the most dangerous of mine gases.

The poisonous effect of the gas is due to its absorption by the hemoglobin of the blood (with which it forms an unstable chemical compound) which has 200 to 300 times the affinity for carbon monoxide that it has for oxygen, and to the extent that the blood is saturated with monoxide to that extent it cannot carry the oxygen required by the tissues. The Committee on Resuscitation from Mine Gases of the American Medical Association, in speaking of the effects of illuminating gas which contains much carbon monoxide, says: If the

amount of hemoglobin combined with carbon monoxide exceeds a figure between 60 and 70%, so that only 30 to 40% is available for the transportation of oxygen, and if the patient has been in this condition for $\frac{1}{2}$ hr. or more, degenerative processes will have started in the brain from lack of oxygen and death or serious nervous or mental impairment will certainly follow.  According to Wabner, when the saturation reaches 67% (two-thirds) death is practically certain and is inevitable when the saturation is 79%.  Haldane says that when death occurs gradually, the hemoglobin is usually about 80% saturated.

The absorption of carbon monoxide by the blood changes its color from deep purple red to a very characteristic pale or rose shade.  This distinctive coloring is noticeable in the lips and other parts of the body where the skin is thin, and for this reason the bodies of those who have died from monoxide poisoning commonly present a very lifelike appearance, entirely different from the pallid, leaden look accompanying death from other causes.

The effect of carbon monoxide is not cumulative, and for each per cent. of the gas in the air there is a fixed degree of saturation of the hemoglobin, as shown in the table on page 858.  Thus, breathing air containing .04% of carbon monoxide will result in the blood becoming 30% saturated; and indefinite exposure will not increase this degree of saturation unless the percentage of the gas in the air is increased.  Nasmith and Graham have shown that animals may be hardened to the effects of carbon monoxide so that after several weeks' exposure to gradually increasing proportions of the gas, they are able to live in atmospheres that would kill them at the outset.  In the case of man, however, the inhalation of small amounts of this gas while at work during the day does not render him immune, as the blood is purified at night by breathing fresh air.

Haldane says, "The symptoms of carbon-monoxide poisoning are essentially the same as those produced by air deficient in oxygen, and vary according to the degree of saturation reached by the blood.  The onset of the symptoms is very insidious, there being only slight shortness of breath and palpitations, but hardly any discomfort; and the senses, power of judgment and of movement, are commonly much impaired before the person is aware of anything being wrong.  In some cases there is much excitement, but often there is simple drowsiness and stupidity.  The symptoms are, in some respects, similar to those produced by alcohol.  One curious fact is that in carbon-monoxide, as in alcoholic, poisoning sudden exposure to cool fresh air may greatly increase the symptoms.  Death seems often to be immediately brought about by attempts to escape rapidly up inclines, ladders, etc.".  In several instances in American mines, the victims of a dust explosion have been able to make their way as much as $\frac{1}{2}$ mi. from their working places before being overcome by the carbon monoxide in the afterdamp.

The rapidity with which carbon monoxide will overcome a person and whether death or disablement will follow, depends on the percentage of gas in the air and the length of time it has been breathed, and on the rapidity of breathing which, in turn, depends on the violence of his exertions, and on the shortage of oxygen or presence of inert gases as carbon dioxide in the mine air.  When the blood is saturated with carbon monoxide to the extent of 50%, a man is too weak to walk.  As Haldane points out, the blood of a man will take up about 2 pt. of the monoxide, and of the gas inhaled about 60% is absorbed by the hemoglobin.  A man at rest inhales about 10 or 12 pt. of air a min., and should this air contain .10% $CO$, the absorption of the gas would be at the rate of .007 pt. a min., requiring 2.25 hr. to produce one-half saturation and total disability.  A man walking breathes about three times as much air as when at rest and would be disabled in 1 hr. or less, and when running in from 20 to 30 min.  As 50% saturation is the maximum with .10% $CO$, death does not follow.  Haldane's experiments showed that with .20% $CO$ in the air, it required 70 min. for the blood of a man at rest to become 50% saturated, and $\frac{1}{2}$ hr. when walking.  The same degree of saturation would probably be reached in 10 to 15 min. when running.  Since the maximum saturation for .02% $CO$ is a trifle over 70%, death is practically certain if the patient is unconscious when found and degeneration of the brain cells has set in.  If the air contains .30% $CO$, 50% saturation of the blood and unconsciousness will take place in $\frac{1}{2}$ hr. or less when at rest, in 10 to 15 min. when walking and proportionately less when running.  If at rest, with this percentage of carbon monoxide, the blood will become fully saturated (.30% $CO$ = 80% saturation) in a little more than $\frac{1}{2}$ hr., and death is certain.  According to Wabner, when there is 1% $CO$ in the air, saturation will require but 5 to 6 min., even supposing the ill-ventilated place does not already contain other

injurious gases or impoverished air, which will generally be the case in the pit. Death, of course, takes place.

Haldane gives the following as effects of percentages of carbon monoxide less than .10%: Even .01% may prove injurious where the oxygen is below 21% and where considerable quantities of carbon dioxide are present, as in afterdamp; .02% will produce a saturation of the blood to the extent of 20% resulting in a slight tendency to dizziness and shortness of breath on exertion; with .05% giddiness on exertion will result after $\frac{1}{2}$ hr. exposure; and with .07% there will be vertigo on the slightest exertion.

When the air is deficient in oxygen or contains notable amounts of carbon dioxide, breathing is deeper and more frequent and a greater amount of carbon monoxide is inhaled and absorbed in a given time than would otherwise be the case. Such atmospheres are common after dust explosions, and members of a rescue party entering them are either more quickly overcome by the same percentage of gas or are finally overcome by a less percentage than if the air, except for the presence of the monoxide, was otherwise normal. It must not be overlooked, however, that members of a rescue party have probably been exposed for some time to atmospheres containing appreciable amounts of this gas, so that when they enter a place where the content of carbon monoxide is fairly high, their blood is already partly saturated, and they naturally succumb more quickly than if entering from fresh air.

The percentage of this gas that may be safely allowed as a permanent constituent of mine air is very low indeed, certainly under .01%, as such air is commonly deficient in oxygen and contains carbon dioxide and methane as well. The maximum percentage that may be breathed for considerable time without producing permanent injurious after-effects is commonly placed at .05%. This percentage, although causing giddiness when working, results in a saturation of the blood not to exceed one-third, from which recovery, in fresh air, is rapid.

In severe cases of carbon-monoxide poisoning, death usually takes place in a few hours, although life may be prolonged for a few days or weeks; but recovery is not unknown. Such recovery is very slow and the patient may develop chronic pains in the head and legs, and impaired eyesight or disordered brain; and there appears to exist a tendency to pneumonia, etc. Even in mild cases, where complete recovery is made, the process is extremely slow, requiring from several hours to weeks, depending on the length of exposure; the milder attacks are accompanied by violent headaches, nausea, and vomiting, and the more severe ones by spasms, cramps, and the like, in addition.

The Committee of the American Medical Association, previously quoted, says further, "As hemoglobin gives up carbon monoxide very rapidly in atmospheres free from the gas, if the patient is still breathing when discovered, the mere inhalation of fresh air without the use of artificial respiration and the administration of oxygen will restore him unless the degeneration of the brain tissues has set in, in which case no amount of oxygen will prevent either death or serious mental derangement should the patient survive. Owing to the fact that it is impossible to tell at the time the patient is discovered or for an hour or two afterwards whether the critical point (degeneration of the brain cells) has been reached, it is of the greatest importance to begin resuscitation at once and preferably by administering oxygen, which removes the monoxide five times as fast as air alone." The Committee does not recommend the general use of appliances for mechanically stimulating breathing, the suction required to produce exhalation being very injurious to the delicate tissues of the lungs and say that the apparatus should be used not longer than 5 or 6 min. at a time, and should be alternated with the manual method alone, or in gas cases, combined with oxygen inhalation.

**Explosibility of Carbon Monoxide.**—Carbon monoxide will not support combustion but instantly extinguishes a flame placed in it. It is, however, explosive over a very wide range. Authorities differ as to the explosive limits, but they are generally taken to lie between 13 and 15.5% (lower limit) and 75% (upper limit) in air. For complete combustion by means of an electric spark, Blair fixes the limits at 16.5 and 75%, and Clowes gives 13 and 75% when the gas is ignited in the ordinary way and from below (see Relative Explosibility of Methane and Other Gases). The equation for the burning of carbon monoxide in air is $2CO + (O_2 + 3.782N_2) = 2CO_2 + 3.782N_2$; from this, the maximum explosive point is reached when there is 29.45% of the gas in air.

A certain amount of moisture is necessary for the explosion of carbon monoxide; it is claimed as much as 5% for the maximum explosive effects.

The reaction is a double one. First, $CO+H_2O=CO_2+H_2$; the hydrogen immediately combining with the oxygen in the air by the action, $2H_2+O_2=2H_2O$. If it is true that carbon monoxide enters into a dust explosion, which appears doubtful, in an absolutely dry mine an explosion of this gas would not be possible.

The explosive limits of whitedamp mixtures (air and carbon monoxide) may be lowered by the addition of a gas of a lower explosive limit, as methane or hydrogen, and this is true even when the percentage of the added gas is below its own explosive limit. Similarly, the addition of carbon monoxide to firedamp mixtures (air and methane) will raise the higher explosive limit of the last-named gas.

Explosions of carbon monoxide are very rare if they occur at all, not only because the lower explosive limit is so high but also because such large amounts of the gas are usually accompanied by still larger amounts of nitrogen or carbon dioxide, which so reduce the percentage of oxygen that there is not enough of it to combine with the monoxide in explosive proportions. Investigations appear to sustain the view of Mr. Haas that the burnings and explosions that sometimes follow the placing of a lamp in the crevices of a standing shot are due not so much to carbon monoxide given off by the powder as to hydrocarbons distilled from the coal by the heat of the explosion.

**Detection of Carbon Monoxide.—***Flame Test.*—In the laboratory, for equal percentages, carbon monoxide gives a flame cap identical with that of methane. The .30% which is exceedingly dangerous is without effect on the flame of the ordinary safety lamp and the 2%, which is the limit of detection by the skilled observer (very many firebosses cannot detect less than 3%), will produce unconsciousness in 30 sec. or less in one who is running, as a fireboss on his rounds. Further, in examining old workings where gob fires exist or in opening a mine that has been sealed to extinguish a fire, the observer will have been breathing for some time air containing gradually increasing amounts of the monoxide, so that his blood will be partly saturated and himself be more susceptible to its effects some time before he reaches a point where the gas will show in the safety lamp. It is unquestionably true that no observer, unless wearing some one of the types of rescue apparatus, ever detected carbon monoxide in the mine with a standard safety lamp and lived to describe his observations. In the Clowes hydrogen lamp, .25% of CO gives a plainly visible cap ½ in. in height, but as Mr. Clowes says it is impossible to distinguish between the cap made by the poisonous monoxide and the non-poisonous methane; so that the hydrogen flame test is not distinctive.

*Potain and Drouin Method.*—The apparatus employed by Potain and Drouin consisted of a long tapered tube connected with a supply of mine air to be tested and dipping to the bottom of an outer tube containing the reagent composed of 10 cu. cm. of a .01% solution of palladium chloride, and two drops of hydrochloric acid. When the mine air is drawn through the outer tube by an aspirator, the reagent (which is about 8 in. in depth) is discolored by the palladium precipitated by the carbon monoxide if present. The percentage of carbon monoxide is determined by comparing the volume of mine air taken with that of a standard mixture of air and carbon monoxide required to produce an equal discoloration in another similar bulk of the reagent. The method will detect the presence of .01% of CO.

### REACTION OF CO ON PtCl₄

| Percentage of CO in Air | Reaction Visible After | | Time in which Paper Turns Black Minutes |
|---|---|---|---|
| | Minutes | Seconds | |
| .010 | 11 | | 60 |
| .025 | 5 | | 32 |
| .050 | 3 | | 16 |
| .075 | 2 | | 12 |
| .100 | 1 | | 9 |
| .250 | | 44 | 6 |
| .500 | | 26 | 4 |
| .750 | | 20 | 3 |
| 1.000 | | 16 | 2 |
| 2.000 | | 15 | 2 |

*Simonis Method.*—In the Simonis method, mine air previously freed, if necessary, from the sulphides of ammonia or hydrogen by passing through a solution of copper sulphate or sulphuric acid, is drawn

through a tube in which is placed a strip of paper saturated with the chloride of either platinum or palladium. Carbon monoxide, if present, will decompose the chloride and turn the paper first brown and then black in a length of time proportional to the amount of gas present: .01% of $CO$ being detectible. The table gives the lengths of time required by various percentages of carbon monoxide in air to discolor and blacken the test paper saturated with platinum chloride which is rather more sensitive than palladium chloride. J. R. Campbell has modified the process by placing a wet sponge in the bottom of the receptacle, thus permitting the use of dry instead of moist test papers. Any number of papers may be prepared in advance for, in use, they will absorb enough moisture from the sponge to permit of the reaction. When used in the mine, air is forced through the apparatus by a pump or India-rubber inflator, but fairly good results may be obtained by merely exposing the instrument to the mine air for any desired length of time. As little as .01% of $CO$ may be thus detected.

*Use of Canaries or Mice.*—In recovery work after a mine explosion, the existence of dangerous amounts of carbon monoxide in the afterdamp is commonly determined by the effect of the gas upon mice or canaries carried in a cage by some one of the exploring party. When advancing with the air, the last man should carry the animals, and when moving against the air, the first man, so that they may be exposed to the air and its effects on them noted before the men enter it. Canaries are preferred to mice as they are more sensitive to the action of the gas, and their signs of distress while perched are more easily noted than those of mice who are apt to crouch in a corner of the cage. If a mouse is used, it must be made to move from time to time by tilting the cage, poking it with a stick, etc., so that, while moving, its symptoms may be noticed. The rate of breathing, number of heart beats, etc., in a mouse or canary, are so much more rapid (pulse about 700 to 1,000 beats a min.) than in a man, that the effects of the gas on them is much more rapid.

The accompanying table from the Bureau of Mines gives the effect on mice and canaries of varying percentages of carbon monoxide, and should be compared with the effect of similar amounts on men, as previously given. Thus, when exposed to an atmosphere containing .20% $CO$ it will require $\frac{1}{2}$ hr. for the blood of a man walking to become 50% saturated, at which stage the legs will give way; but a canary similarly exposed showed signs of distress in 1.5 min., and fell from his perch in 5 min. In using canaries as guides to the presence of the monoxide, certain facts must be remembered. In the excitement of rescue work, the unskilled observer may entirely fail to note in the canary the preliminary symptoms of poisoning that would be clear to the trained man;

### EFFECT OF $CO$ ON MICE AND CANARIES

| Mice | | Canaries | |
|---|---|---|---|
| Per Cent. $CO$ | Effect | Per Cent. $CO$ | Effect |
| .16 | Very slight distress at end of hour. | .09 | Very slight distress at end of 1 hr. |
| .20 | Distress in 8 min.; partial collapse in 15 min. | .12 | Weaker at end of 1 hr. than after exposure to .9% |
| .31 | Distress in 4 min.; collapse in 7½ min.; lost muscular power in 35 min. | .15 | Distress in 3 min.; fell from perch in 18 min. |
| .46 | Distress in 2 min.; collapse in 4 min. | .20 | Distress in 1½ min.; fell from perch in 5 min. |
| .57 | Distress in 1 min.; collapse in 2 min.; muscular power lost in 7 min.; death in 16 min. | .29 | Fell from perch in 2½ min. |
| .77 | Distress in 1 min.; muscular power lost in 6½ min.; death in 12½ min. | | |

and the ability of even trained men to detect these early symptoms varies considerably. There is also a marked difference in the resisting power to carbon-monoxide poisoning of canaries, some birds showing no more distress at the end of 6 min. than others did in 2 min. For this reason, either a bird should be selected which previous tests has shown to be sensitive to the gas, or several birds should be taken along. It would also appear that small percentages of carbon monoxide (say .10%) which in the course of 1 hr. or so seriously affect men, are often without any influence at all on the canary. For these reasons, while actual distress or disablement on the part of the bird are unfailing signs of the presence of the gas, these signs should not be waited for, and the display by any member of such symptoms of poisoning as dizziness, shortness of breath, weakness of the legs, impairment of sight, etc., should lead to the immediate withdrawal of the entire party.

*Blood Tests.*—The so-called blood test, while giving excellent results in determining whether carbon monoxide is or is not present in the air or blood, is not definite when the percentage of gas is required. For this determination, a chemical analysis should be made in the regular way. To make the blood test, one to three drops of blood secured from a healthy animal or by pricking the finger are shaken up with enough water to make about a 1% solution (1 part of blood in 100 of water). This results in a buff-colored solution, which is equally divided between two test tubes, which are corked. One test tube is set aside for comparison, and through the cork of the other are inserted two glass tubes, one of which is long enough to reach nearly to the bottom of the test tube, the end of the other being above the level of the solution. A glass bottle filled with the suspected air is connected with the long tube and suction applied at the end of the short one. This will exhaust the air from the bottle and cause it to bubble up through the test solution, which will be colored a distinctive pink shade if any carbon monoxide is present. The colors will be more distinctly visible if the two tubes are compared against a brightly lighted white background. The same result may be obtained by shaking one of the blood solutions for 10 min. or so in a bottle containing the suspected air. The presence of the gas in the blood may be told by preparing solutions of equal strength (same quantity of blood and water) of the blood of the supposedly poisoned person and that of a healthy person or animal; the latter for comparison, as before.

To obtain an approximate idea of the percentage of carbon monoxide in the air, Haldane recommends the following: Prepare a solution of normal blood and fill two test tubes with it as already described. Set one of the tubes aside, and thoroughly saturate the other with illuminating gas, which contains large quantities of the monoxide, by bubbling the gas through the solution and shaking them together for about 10 min. This will give a buff-colored solution containing no carbon monoxide and a pink solution saturated with it; and these are the two ends of the scale. A mouse is exposed in the suspected air in the mine for from 10 to 15 min. and then killed in the place, as its removal to fresh air, while alive, would greatly reduce the amount of the gas in its blood and thus invalidate the test. As much of the mouse's blood is taken as was required to make the test solutions, and it is diluted to the same degree. By comparing, against a white background, the color of the mouse-blood solution with that of the two test solutions, it can be approximately determined if its blood is $\frac{1}{10}$, $\frac{2}{10}$, etc., saturated. The amount of carbon monoxide in the air can then be determined from the following table.

### PER CENT. OF CARBON MONOXIDE IN AIR CORRESPONDING TO VARIOUS PERCENTAGES OF SATURATION OF BLOOD SOLUTION

| Blood Saturation Per Cent. | Carbon Monoxide in Air Per Cent. | Blood Saturation Per Cent. | Carbon Monoxide in Air Per Cent. |
|---|---|---|---|
| 10 | .012 | 60 | .12 |
| 20 | .025 | 70 | .19 |
| 30 | .040 | 80 | .30 |
| 40 | .060 | 90 | .70 |
| 50 | .080 | | |

As it is only necessary that the test solution should have a clearly perceptible color, one drop of blood may be sufficient. The test may be used to estimate the amount of the gas in the blood of a person suffering from gas poisoning, by taking as much blood from the patient as was used in making the test solution and diluting it to the same degree. As noted, if the patient has been unconscious for ½ hr. or more and his blood is found to be over 60% saturated, it is more than probable that degeneration of the brain cells has set in and that no amount of oxygen or other respiratory treatment would have effected his recovery.

By measuring the quantity of a normal solution of carmine that is required to color the mouse-blood solution to the same intensity as the solution saturated by means of illuminating gas, it is possible to make a very accurate quantitative determination of the amount of carbon monoxide present. The method, while delicate, is not used to any great extent, ordinary methods by absorption being preferred.

*Spectroscopic Methods.*—A portable pocket spectroscope for the detection of carbon monoxide has been devised by a German firm. A sample of the suspected air is obtained in the ordinary way and to it are added a few drops of a very weak solution of blood which is supplied with the instrument. After being well shaken the blood solution is examined with the spectroscope. Whether CO is present or not, two dark bands will be seen in the yellow and green parts of the spectrum between the Frauenhofer lines D and E, the position of which is specially marked on the scale of the spectroscope. If no carbon monoxide is present these bands will disappear on adding to the original solution a drop or two of colorless ammonium sulphide (diluted), and in their place and between them will appear one broad band which shades off at the edges. If carbon monoxide is present the two bands are but very slightly altered in appearance by the addition of ammonium sulphide and they are not replaced by a single band. In the one case the oxyhemoglobin is decomposed by ammonium sulphide and in the other the carboxyhemoglobin is not affected by that reagent.

## METHANE

**Properties and Sources.**—Methane, otherwise known as *light carburetted hydrogen, marsh gas, firedamp,* or simply as *gas,* has a formula $CH_4$, is odorless, colorless, tasteless, and non-poisonous, and will not support life or combustion. From the mining standpoint, it is the most important member of the paraffin series of gases whose general formula is $C_nH_{2n+2}$ in which $n$ is the number of atoms of carbon in a molecule of any one of these gases. Methane burns with a blue flame similar to that of carbon monoxide, and like that gas is explosive when mixed with air in the proper proportions.

Methane is given off by the pores of the coal in practically all mines, although often in amounts that can only be detected by careful analysis. Gas so contained in the coal is commonly, but incorrectly, spoken of as *occluded,* a term strictly referring to gas absorbed and possibly condensed in the pores of a metal, as hydrogen in palladium. The amount of gas given off by the seam is very variable, and depends, in a great measure, on the opportunity it has had to escape naturally from the coal. Thus, more gas may be expected in the inner workings than near the outcrop, in a shaft than in a drift mine, and at great than at shallow depths. In the deep European mines, the Prussian Firedamp Commission found that from 357 to 2,400 cu. ft. of methane are given off per ton of coal mined; and, in France, Chesneau reports 1,377 cu. ft. of gas per ton of coal mined at the Herin mine, Anzin, and 882 cu. ft. at Ronchamp. Similar figures for American mines are not available, but may be readily calculated when the output of the mine and the volume of the air current and its content of methane are known. Thus, in a mine producing 1,000 tons a day, when the return has a volume of 100,000 cu. ft. a min., which contains .30% of $CH_4$, the volume of gas per ton of coal mined is $(60 \times 24 \times 100,000 \times .003) \div 1,000 = 432$ cu. ft.

Many coals continue to give off gas for months after being mined and explosions of methane have occurred on shipboard by taking a lighted torch into the bunkers where the boiler coal is stored. Samples of coal bottled in a vacuum continue to give off gas for from ½ to 1½ yr., the gas escaping more rapidly from finely crushed than from lump coal. The volume of gas given off by the coal is much greater during the first few days after mining than later. From American coals, Chamberlain obtained a volume of gas equal to .9 to 2.3 times that of the sample before crushing. Lump coal yielded but .5 to .9 their volumes of gas under similar conditions. Analyses of the gases given off by coal will be found under Firedamp.

These so-called occluded gases exist under a pressure that is more or less proportional to the thickness of the cover upon the coal and, according to the European investigations recorded in the accompanying table, is frequently as great as 10 to 16 atmospheres (150 to 240 lb. per sq. in.). Pressures over 40 atmospheres, or about 600 lb. per sq. in., have been noted. The figures in the second column of the table are the depths of the boreholes, or the distance back from the face and in the solid coal where the pressure was measured.

### PRESSURE OF OCCLUDED GAS

| Name of Mine | Depth of Hole Feet | Pressure Pounds |
| --- | --- | --- |
| Elmore mine, main bed..................... | 8.53 | 4.36 |
| Hetton mine, Hutton bed................... | 8.98 | 6.96 |
| Eppleton mine, Hutton bed................. | 46.90 | 36.14 |
| Balden mine, Bensham bed................. | 31.85 | 71.41 |
| Harris Navigation mine.................... | 32.80 | 22.04 |
| Methyr Vale mine......................... | 49.20 | 39.67 |
| Celynen mine............................. | 54.48 | 68.32 |
| Harton mine (1,214 ft. deep).............. | 16.24 | 196.30 |
| Harton mine.............................. | 27.55 | 230.44 |
| Harton mine.............................. | 37.13 | 294.45 |

Methane is given off by blowers, otherwise known as *feeders* or *bleeders*, which are streams or jets of gas issuing from cracks and crevices in the coal face or in the floor or roof. These blowers seem to follow some well-defined line of fracture in the measures or some fissure or clay vein and, as they frequently give off gas under high pressure for many months, they are probably connected with some underlying reservoir or porous stratum filled with gas. In many cases, because of the high pressure, the gas in escaping makes a *singing* noise, which resembles the pattering of light rain upon leaves.

Methane is frequently found along the line of clay veins and similar irregularities, probably following the line of separation between the material filling the vein and the rocks through which it cuts. In many cases, the gas is found under high pressure just after the clay vein is pierced, whereas none, or but normal quantities are found between it and the crop line. Gas may be found within an area bounded by clay veins or by the dykes of igneous rock common in some of the western coal fields. In such cases, if the mine has not hitherto generated gas and, consequently, the necessary precautions to cope with it have not been made, the sudden piercing of a clay vein or dyke and the unexpected release of large volumes of gas has led to serious accidents.

Sudden outbursts of gas in large quantities and under great pressure sometimes take place. These may be caused by the gas finding its way through vertical crevices or cleats to cavities of considerable horizontal extent, so that the gas pressure is distributed over a wide area. As the mine openings approach such a locality, the pressure manifests itself by bursting the coal from its position in the face and throwing it into the entries, in some cases completely blocking them. These outbursts are frequently preceded and accompanied by thunderings and poundings, the former for several days before the gas actually escapes.

Whether the outbursts of gas so frequently accompanying bumps (bounces) are a cause or an effect of the throwing of large masses of coal into the workings as a result of a sudden rending of the pillars, is not satisfactorily decided. The weight of evidence, however, indicates that bumps are due to the sudden release of pressure in the overlying rocks, and that the shattering of the coal and roof, and sometimes the floor, sets free the contained gas and opens any pockets or crevices in which it may be stored. If pillars are left too small or their drawing has been carried too far, the tendency to slowly and regularly settle (and possibly to bring on a squeeze) may be resisted by a strong roof until such a time as the limit of its strength is reached when it will suddenly give and as suddenly crush the coal over an area of a few hundred square feet or several acres. When the resistance becomes adjusted to the pressure, the crushing will cease; therefore, unless the place is entirely closed, bumps may be repeated in the same or adjacent areas as the roof pressure accumulates. The outbursts mentioned in the first part of the paragraph produce results similar to bumps, but on a smaller scale.

In Belgium, large volumes of gas under very high pressure and accompanied by a peculiar porous coal dust are found in large chambers or pockets. When a reservoir of this kind is accidentally tapped by a heading, an outrush of large volumes of gas and coal dust occurs with great suddenness, filling the workings immediately, and causing enormous damage. Thus, according to Wabner, at the Agrappe colliery in Belgium, more than 500,000 cu. yd. of gas and a great mass of coal dust broke out into a heading at a depth of 2,000 ft., blocking up all the workings and the shafts, and taking fire at the pit mouth destroyed the head-frame and surface plant. After the first rush of gas had subsided, violent explosions occurred, completely closing the shafts and killing or injuring 112 men. Similar explosions have occurred in other mines in the same country. The Belgian measures have been subjected to very heavy horizontal pressure, which has resulted in the formations slipping over and past one another, the irregular line of fracture producing pockets in which the gas accumulated, and in which pockets were large quantities of dust resulting from the grinding of the coal by the rock movement.

Methane is often released in unusual quantities from the rocks during a squeeze or a creep, by a heavy fall of roof or a general breaking or settling of the overburden accompanying and following pillar drawing.

In some mining districts, the leakage under great pressure from improperly cased oil and gas wells drilled through the workings, is a source of methane as it constitutes 50% or more of natural gas.

Black powder does not usually produce any methane but many safety (permissible) powders and high explosives yield from a trace to as much as 5% of it when exploded in the laboratory. Analyses of the combustion products of explosives are given on page 670. When used in the mine in contact with coal, and particularly when slack is used for tamping, all powders yield important amounts of methane (5 to 10% or more of the gases of combustion), which may be in part distilled from the coal by the heat of the explosion, and in part result from chemical combination between the elements in the powder and those in the coal.

Methane is found in the gases from mine fires and in the afterdamp of explosions, where it is distilled from the coal by heat.

Analyses of combinations of methane and various gases will be found under Firedamp.

**Formation of Methane.**—Coal seams are formed by the slow alteration of vegetable matter in the presence of moisture and in the absence of air; heat generated by pressure, commonly assisting in the process. The alteration has resulted in the formation of various hydrocarbons, the chief of which is methane. While, in the course of ages, most of the gas has escaped through the overlying rocks, large quantities still remain. Methane may be formed wherever moist vegetable matter is protected from the air, as in swamps and peat bogs and the bottom of stagnant ponds, where it may be seen bubbling up through the water; whence the name *marsh gas*. It is suggested that methane is yet being formed in some seams where the processes of alteration have not been completed; but this has not been proved.

All methane does not originate in coal seams. It occurs in mines of salt and strontia, in beds of clay and sandstone, invariably accompanies the production of petroleum, and always forms the bulk of natural gas, even when these substances are found many hundreds of miles from any coal field. In mining regions, natural gas is often met in sands (more or less porous sandstones) several hundred to several thousand feet below the lowest coal bed. These facts have led Mr. Haas to hold, for northern West Virginia at least, that methane does not originate in the coal, but in the natural-gas sands many feet below. In these deep sands, the natural gas is under a pressure proportional to the depth, a pressure in many instances as great as 1,500 lb. and often reaching 2,000 to 3,000 lb. a sq. in. Because of this pressure, the gas is forced slowly upwards and meeting the seam, is held in the pores of the coal. This view of Mr. Haas' explains why the deeper seams are more gaseous than the higher ones, why one mine in a district may produce much gas and its neighbor but little, and why feeders from the floor and bottom of the seam are more lasting than those from the roof and the top of the bed.

**Occurrence of Methane in Mines.**—After prolonged exposure, the occluded methane is largely given off by the seam so that in active workings, at least, the gas is more commonly found at or near the face of the rooms and entries than nearer the drift mouth or the foot of the shaft. Unless diffused or removed by the air-current as fast as it escapes, the extreme lightness of methane tends to its concentration near the roof; and in workings being driven to the rise,

and it is commonly looked for there. When slowly given off near the floor, diffusion will distribute it uniformly through the air of the place; and when once mixed it will not again separate. The same is true of gas coming from the roof, but the diffusion is not so rapid. When issuing from the top, the gas under the influence of a gentle air-current will often flow in a thin sheet or layer next the roof, rising into and remaining in any cracks or crevices. In entries, gas will accumulate behind any obstruction in the air; thus, it will be found at the roof between the collars of parallel sets of timbers.

The volume and velocity of the air-current have much to do with the concentration of gas in one part or another of the workings. In rooms, where the air is apt to be stagnant, and particularly beyond the last break-through, accumulations of methane are usual. In such places, the gas often tails out or tails back from the face; that is, at the face it will extend down from the roof for, say, 3 ft. and thence gradually decrease in depth outwards towards the break-through where none may be found. Bodies of what is known as *standing gas* will accumulate in old workings where there is little or no circulation of the air. In some cases, the oxygen will be so reduced through absorption by the coal or the percentage of methane may be so high that an explosion is not possible in the old workings, but should the gas escape into the air-current an explosive mixture may result. Gas is very often a source of trouble in pillar drawing, accumulating above the falls of roof where it is entirely out of reach of the sluggish air-currents which, possibly, may be circulating through the gob. This pillar gas (as well as standing gas in old workings) may be forced into the airways with disastrous results by a heavy fall of roof or by the reduced atmospheric pressure, which is marked by a fall of the barometer.

**Effect of Methane on Life.**—As methane is not concerned with breathing, like carbon dioxide, and does not combine with the hemoglobin of the blood, like carbon monoxide, it is without active effect on life. When unusually large amounts are present, it acts in the same way as an excess of nitrogen, reducing the percentage of oxygen available for breathing which becomes more and more difficult and finally impossible. Wabner states that methane produces suffocation when constituting 50% of the inhaled air, while Guibal places the limit at 33%.

**Explosibility of Methane.**—Methane, like carbon monoxide, will either burn or explode when mixed in the proper proportions with air. The temperature of ignition of methane is variously stated to be 650° C. (1,202° F.) by Demanet, 740° C. (1,364° F.) by Brunck, 780° C. (1,436° F.) by Koehler. The Bureau of Mines gives the igniton temperature in air as 650° to 750° C. (1,202° to 1,382° F.); and in oxygen, 536° to 700° C. (1,033° to 1,292° F.).

The percentage of methane in the air necessary to produce either a burning or an explosion depends on the means taken to ignite the gas, on the presence of other combustible or inert gases, on the amount of oxygen and water vapor in the air, on the temperature and pressure, etc. With low percentages of methane, owing to the cooling effect of a large body of air, the burning will extend only a very short distance from the cause of ignition, as when the gas is detected in the flame of a safety lamp. As the percentage of gas increases, the inflammation extends farther and farther from its origin until it entirely permeates the gaseous mass. A point is then reached when the burning is instantaneous and an explosion occurs; this marks the lower explosive limit. Later, as the percentage of gas increases, the maximum explosive point is reached. When this is passed, the still increasing percentage of methane in the air is accompanied by explosions of less and less violence, until the higher, or upper, explosive limit is reached. From this point to the upper inflammable limit, the phenomena are the same, but in reversed order, to those leading from the lower inflammable to the lower explosive limit.

As it is a difficult matter to tell just when a very rapid burning becomes an explosion, authorities differ in the values assigned to the explosive limits of methane in air. Blair gives the limits as 6.1% and 12.8% for ignition by an electric spark when complete combustion results. Doctor Brunck states that below 6% and above 15% an explosion cannot take place. The Bureau of Mines notes that 5.5% of gas in air will explode, but not with violence, and that an incomplete burning sometimes results with as little as 4.5%. Similarly, the Bureau has secured slight burnings with as much as 20% of methane. Clowes places the explosive limits at 5 and 13% when the gas is ignited from below, and at 6 and 11% when ignition is from above, and remarks that the lower figures (5 and 6%) represent the least amount of gas that will always burn, and under certain conditions will explode. The maximum explosive point is reached when the air contains 9.46% of methane.

The determination of the exact explosive limits of the gas is not a matter of first importance, that is, it makes little difference in practical ventilation whether the lower limit is 5.5 or 7.1%, as either figure is far beyond the amount that can be tolerated in the presence of bituminous-coal dust, which is commonly fixed at not more than 1%, one of the largest producers requiring that the return air in its mines shall not contain more than .30% of methane.

Methane can only be ignited by a flame or an electric arc and not by dark heat alone, although explosions have been attributed, as at the Bellevue mine, Alberta, Can., to ignition of methane by sparks produced by rocks striking one another during a fall of roof. Sparks from the old-time flint mill, or wheel, used for lighting in gaseous mines are, also, believed to have originated gas explosions.

The explosion of methane is not instantaneous when it is raised to the ignition point, but requires an appreciable time interval. This fact is taken advantage of in the use of high and permissible explosives the flame of which is so short that notwithstanding its high temperature, it goes out before it has time to ignite the gas.

A decrease in the pressure makes the gas less explosive. According to Brunck, a mixture of air 7.5% of methane that explodes with violence at atmospheric pressure (30 in. of mercury) cannot be made to explode if the pressure is reduced to 8 in. of mercury. Similarly, an increase in pressure as well as one in temperature renders the gas more explosive; hence at great depths, where the barometer stands at, say, 33 to 34 in. and the thermometer registers possibly 100° F., a lower percentage of gas will be explosive than in surface workings.

As the other combustible gases have a much wider explosive range (see table below) and lower ignition temperature than methane, their presence increases the liability of a firedamp mixture to explode. On the other hand, inert gases like nitrogen and carbon dioxide by shortening the explosive range of methane and by their cooling effect on the products of combustion, materially reduce the liability of occurrence of such explosions. The presence of gases having a maximum explosive point greater than that of methane as well as that of inert gases, raises the maximum explosive point of firedamp. Since mine air is commonly deficient in oxygen, the explosive limits of methane in it are usually higher than in pure air, one authority placing them at 7.14 and 16.67%, respectively, and another stating that the maximum degree of explosibility is reached when the methane is one-eighth to one-ninth of the air, that is, from 12.5 to 11.1%.

The statement sometimes made that an addition of one-seventh (14.3%) of carbon dioxide to a volume of air and methane at its most explosive point ($CH_4$, 9.46%; air, 90.54%) entirely prevents its explosion seems contrary to the facts as the resulting mixture, which has the composition $CO_2$ 12.51%, $O_2$ 16.60%, $N_2$ 62.61%, $CH_4$ 8.28%, is well within the explosive limits as given in the following table.

It has been shown by Baker that an explosion of gas is not possible when it and the air are absolutely dry, moisture playing the part of a catalytic agent, as noted under Explosibility of Carbon Monoxide. However, the condition of absolute dryness does not exist in mines and cannot be brought about therein.

## LIMITING EXPLOSIVE MIXTURES OF VARIOUS EXPLOSIVE GASES WITH AIR

| Combustible Gas Used | Percentage of Gas Mixed With Air | | Method of Kindling |
| --- | --- | --- | --- |
| | Lower Explosive Limit | Upper Explosive Limit | |
| Methane...................... | 5 | 13 | Upwards |
| | 6 | 11 | Downwards |
| Coal gas, Nottingham............ | 6 | 29 | Upwards |
| | 9 | 22 | Downwards |
| Water gas, artificial............. | 9 | 55 | Upwards |
| Hydrogen...................... | 6 | 72 | Upwards |
| Carbon monoxide................ | 13 | 75 | Upwards |
| Ethylene, olefiant gas........... | 4 | 22 | Upwards |
| Acetylene..................... | 3 | 82 | Downwards |

The above table from Clowes gives the relative explosibility of methane and other gases. Clowes notes that the determination of the limits for acetylene is complicated by a somewhat explosive separation of the acetylene itself into its elements. The composition of the coal gas was: Olefins, 5.3%; hydrogen, 48.2%; carbon monoxide, 6.6%; methane, 34.2%; oxygen, .2%; nitrogen, 5.5%. The total combustible gases were 94.3%, inert gases 5.7%. The composition of the water gas was: Hydrogen, 49.6%; carbon monoxide, 40.8%; carbon dioxide, 2.6%; nitrogen, 7.0%. The total combustible gases were 90.4%, inert gases, 9.6%. Combustion was effected in tubes, by means of a Bunsen burner, the flame being held both below and above the body of the gas. The conditions, in the main, were not unlike those that might be met in the mine.

The probability of an explosion is greatest in the case of those gases having a wide explosive range, because it is possible to form with them a greater number of explosive mixtures. For this reason, methane with an explosive range of 5 to 8% is the least dangerous gas, and hydrogen, with a range of 67% is the most dangerous. Also, methane is less dangerous than the other gases because its temperature of ignition is higher, and it is more slowly kindled. However, methane is in the end the most dangerous gas from the viewpoint of possible explosions because of its general occurrence; the other gases are found but rarely or in very small amounts, and carbon monoxide, notwithstanding its wide range of explosibility, is dangerous solely because of its intensely poisonous properties.

**Firedamp.**—Firedamp originally meant the gas methane and later was modified to mean an explosive mixture of methane and air. Methane rarely occurs in a pure state being associated with small amounts of ethane, ethylene, nitrogen, etc., and the word firedamp is now very commonly defined as a mixture of explosive mine gases, very largely methane but often, if not always, mixed with other hydrocarbons and small amounts of inert gases. As, with the exception of the rare gas hydrogen, all mine gases are heavier than methane, the specific gravity of firedamp is higher than that of pure methane, and may reach above .7.

The following analyses, from Chamberlain, are of firedamp given off by coal when bottled in a vacuum.

### COAL FROM FACE, NAOMI MINE (GAS COAL, PITTSBURGH, PA., DISTRICT)

| Gas | First 3 Wk. | 4 Wk. Later | 10 Wk. Later | 9 Wk. Later |
|---|---|---|---|---|
| Carbon dioxide............ | 1.05 | .83 | .63 | .60 |
| Carbon monoxide.......... } | 1.03 | .29 | .84 | 1.13 |
| Olefins................... | | | | |
| Paraffins................. | 55.39 | 84.96 | 91.39 | 87.89 |
| Air { Oxygen............. | .51 | .35 | .50 | .40 |
| Nitrogen........... | 1.91 | 1.32 | 1.89 | 1.51 |
| Nitrogen, excess........... | 40.11 | 12.25 | 4.75 | 8.47 |
| | 100.00 | 100.00 | 100.00 | 100.00 |
| Relative volumes........... | .21 | .20 | .28 | .21 |

### COAL FROM FACE, NO. 1 NORTH SHAFT, NANTICOKE, PA. (ANTHRACITE)

| Gas | First Week | Second Week | Third Week | Fourth Week |
|---|---|---|---|---|
| Carbon dioxide............ | .02 | | .17 | |
| Olefins................... | .51 | .78 | .87 | .29 |
| Paraffins................. | 86.86 | 93.99 | 94.19 | 98.58 |
| Air { Oxygen............. | .04 | | | .07 |
| Nitrogen........... | .15 | | | .26 |
| Nitrogen, excess........... | 12.42 | 5.23 | 4.77 | .80 |
| | 100.00 | 100.00 | 100.00 | 100.00 |
| Relative volume............ | 1.22 | .99 | .89 | .81 |

In the preceding analyses, by olefins is meant gases of the type of ethylene, $C_2H_4$, and by paraffins, gases of the type of methane and ethane, $CH_4$ and $C_2H_6$, respectively. In the Naomi analysis, whatever carbon monoxide may have been present is grouped with the olefins, and its absence in the Nanticoke analysis is noticeable. Mr. Chamberlain states that certain differences observed in making the analyses can be explained by the presence of ethane (in the paraffins) in proportions of a trace to four parts of ethane to 100 parts of methane. So far as his tests went, the bituminous coals yielded more ethane than the single sample of anthracite (Nanticoke); in fact, the analyses of several anthracite gases indicated that instead of there being paraffins higher than methane, that small amounts of hydrogen were present.

The following table gives the composition and volume of gases (firedamp) issuing from some Welsh coals when heated in a vacuum to 212° F.

### GASES ENCLOSED IN THE PORES OF COAL AND EVOLVED IN A VACUUM AT 212° F.

(*Thomas*)

| Name of Colliery | Quality | $CO_2$ | $O$ | $CH_4$ | $N$ | Quantity Cubic Centimeters per 100 Gr. | Cubic Feet per Short Ton |
|---|---|---|---|---|---|---|---|
| Navigation........... | Steam | 13.21 | .49 | 81.64 | 4.66 | 250.0 | 80 |
| Dunraven............ | Steam | 5.46 | .44 | 84.22 | 9.88 | 218.0 | 70 |
| Cyfarthfa........... | Steam | 18.90 | 1.02 | 67.47 | 12.61 | 147.0 | 47 |
| Bute............... | Steam | 9.25 | .34 | 86.92 | 3.49 | 375.0 | 120 |
| Bonville's Court..... | Anthracite | 2.62 | | 93.13 | 4.25 | 555.0 | 178 |
| Watney's........... | Anthracite | 14.72 | .80 | 84.18 | 1.10 | 600.0 | 192 |
| Plymouth Iron Works | Bituminous | 36.42 | .80 | | 62.78 | 18 | 18 |
| Cwm Clydach....... | Bituminous | 5.44 | 1.05 | 63.76 | 29.75 | 55.1 | 18 |
| Bettwys............ | Bituminous | 22.16 | 6.09 | 2.68 | 69.07 | 24.0 | 8 |

The following table gives a series of firedamp analyses from Le Chatellier, some of them being of blowers, others from bore holes, etc.

### ANALYSES OF FIREDAMP

(*Le Chatellier*)

| Locality | $CH_4$ | $CO_2$ | $N$ | $O$ | Analyst |
|---|---|---|---|---|---|
| Dunraven mine, blowers.... | 96.70 | .47 | 2.79 | | J. W. Thomas |
| Dunraven mine, bore hole.. | 96.50 | .44 | 3.02 | | J. W. Thomas |
| Garswood mine............ | 84.16 | .86 | 12.30 | 2.65 | W. Kellner |
| Garswood mine, blowers.... | 88.86 | .41 | 8.90 | 1.83 | W. Kellner |
| Clamorgan mine, blowers... | 93.01 | .27 | 5.94 | .78 | W. Kellner |
| Dombran mine, blowers.... | 95.11 | .48 | 4.07 | .34 | Austrian Firedamp |
| Karwin mine.............. | 94.59 | .18 | 4.48 | .75 | Commission |
| Karwin mine, blowers...... | 99.10 | .20 | .70 | | |
| Hruschau mine............ | 79.16 | .19 | 17.04 | .61 | |
| Hruschau mine, blowers.... | 87.93 | .83 | 10.25 | .99 | |
| Peterswald mine, blowers... | 90.00 | .15 | 9.25 | .60 | |
| Segen Gottes mine........ | 83.51 | 1.17 | 15.02 | .30 | Sauer |
| Segen Gottes mine, bore hole | 87.16 | 1.11 | 11.73 | | Sauer |
| Liebe Gottes mine, bore hole | 77.69 | 3.77 | 18.48 | .06 | Sauer |

The following table from Schondorrf is entirely of firedamp from blowers.

## ANALYSES OF FIREDAMP FROM BLOWERS

| Locality | $CH_4$ | $C_2H_6$ | $H$ | $CO_2$ | $N+O$ |
|---|---|---|---|---|---|
| Bonifacious mine at Kray, Essen......... | 90.94 | | 1.40 | .30 | 7.36 |
| Consolidation mine at Schalk, Westphalia | 89.88 | | 5.84 | .67 | 3.61 |
| König mine at Neunkirchen, Saarbuck... | 84.89 | 1.62 | | .65 | 12.84 |
| Oberkirchen mine at Schaumburg........ | { 60.46 | 37.64 | 2.11 | 2.56 | |
| | { 93.66 | .88 | | .63 | 4.80 |
| Cavities in the roof, Lothringen mine at Castrop, Westphalia.................. | 27.95 | | 1.35 | .45 | 70.25 |
| New Iserlohn mine at Lawgendren, Westphalia............................. | { 4.75 | | .09 | 1.34 | 65.00 |
| | { 4.00 | .06 | | .40 | 95.00 |

The following analyses from Chamberlain are of feeder gas from American mines; the last being that of a gas issuing under high pressure from a standpipe at Luzerne, near Wilkesbarre, Pa. In this case, $C_nH_{2n}$ is the general formula for the olefin series of gases of which ethylene, or olefiant gas, is a type. When so used, the formula means that the gases, while present, were not separately determined. In the same way, the other paraffin gases, of which the general formula is $C_nH_{2n+2}$, were not determined but are included with the methane.

## ANALYSES OF FIREDAMP FROM FEEDERS

| Locality | $CH_4$ | $CO_2$ | $CO$ | $C_nH_{2n}$ | Air | $N_2$ Excess |
|---|---|---|---|---|---|---|
| Cardiff, Ill........ | 90.42 | 2.04 | | .20 | 6.02 | 1.32 |
| Cardiff, Ill........ | 92.17 | 2.39 | .26 | | 2.63 | 2.55 |
| Luzerne, Pa........ | 91.31 | 1.59 | .04 | .49 | 1.10 | 4.65 |

The following series of analyses of a firedamp mixture that could be explosive under proper conditions is given by Mr. G. A. Burrell. The analyses, made day by day, show the change in the atmosphere in an enclosed area in an anthracite mine, a part of which was sealed off to prevent entrance of a fire in progress in a neighboring district. The gases were not contaminated by the products of combustion of the fire as a heavy roof fall prevented the escape of the gases from the one to the other, and thus fairly represent the change in mine air away from the ventilating current. On the first two days, there was a leakage of air into the enclosed area, and after this was stopped the change in the imprisoned air was much more rapid.

## GASES FROM AN ENCLOSED AREA IN AN ANTHRACITE MINE

| Sample Number | Date | Percentage of | | | | |
|---|---|---|---|---|---|---|
| | | $CO_2$ | $CO$ | $O_2$ | $C_6H$ | $N_2$ |
| 1 | Oct. 31 | 2.2 | | 15.0 | 14.0 | 68.8 |
| 2 | Nov. 1 | 2.3 | | 14.6 | 18.1 | 65.0 |
| 3 | Nov. 2 | 2.6 | | 6.2 | 24.2 | 67.0 |
| 4 | Nov. 2 | 2.9 | | 5.7 | 29.3 | 62.1 |
| 5 | Nov. 3 | 2.8 | | 4.1 | 34.9 | 58.2 |
| 6 | Nov. 6 | 2.6 | | 5.0 | 53.0 | 41.4 |

The following analyses from Mr. Austin King give the composition of some firedamps from the Connellsville, Pa., bituminous-coal region. The very large

amounts of ethylene, carbon monoxide, and hydrogen in the first two samples is noticeable. Mr. King also furnishes the following analyses of gases escaping from bore holes drilled from the surface to drain the gob.

### ANALYSES OF FIREDAMP, CONNELLSVILLE REGION

| Samples Taken From | Percentage of | | | | | | |
|---|---|---|---|---|---|---|---|
| | $CO_2$ | $C_2H_4$ | $O_2$ | $CO$ | $H_2$ | $CH_4$ | $N_2$ |
| Drill hole in coal face......... | .30 | 1.50 | 1.60 | 1.30 | 7.35 | 79.60 | 8.25 |
| Crevice in roof, face main entry | 1.70 | 1.30 | 1.80 | .90 | 2.04 | 63.27 | 28.99 |
| Return airway of main entry.. | | | 18.40 | | | 6.50 | 75.10 |

### ANALYSES OF GAS FROM DRILL HOLES

| Remarks | $CO_2$ | $C_2H_4$ | $O_2$ | $CO$ | $H_2$ | $C_2H_6$ | $CH_4$ | $N_2$ |
|---|---|---|---|---|---|---|---|---|
| Hole, 525 ft. deep. | 2.50 | | 10.40 | | 1.35 | | 11.90 | 73.85 |
| Hole, 406 ft. deep. | 5.20 | | 3.20 | | | | 12.64 | 78.96 |
| Same, 1 yr. later.. | 1.80 | | 14.90 | | | | 2.20 | 71.68 |
| Hole, 474 ft. deep. | 4.60 | | 4.20 | .20 | 14.16 | | 8.80 | 67.52 |
| | 6.90 | .10 | 9.20 | .40 | | 1.60 | 38.30 | 48.50 |

Firedamp is usually found in the same relative positions in the workings as methane. However, the presence of large amounts of gas heavier than methane may make firedamp heavier than air and give it a tendency to collect on the floor. Thus, a gaseous mixture composed of 54.1% of methane and 45.9% of carbon dioxide has the same weight as air. Any increase in the percentage of carbon dioxide over 45.9 will make the mixture heavier than air and, if diffusion is not taking place, the firedamp will accumulate on the floor.

**Combustion Products of Methane.**—When pure methane is exploded in air the reaction is $CH_4+2(O_2+3.782N_2)=CO_2+2H_2O+7.564N_2$. The composition of the afterdamp is 15.1% $CO_2$, 12.3% vapor of water $H_2O$, and 72.6% $N_2$. As soon as the water vapor condenses, the remaining gas is a blackdamp with the composition 17.2% $CO_2$ and 82.8% $N_2$. This reaction holds good for all burnings and explosions of methane as long as there is an ample supply of oxygen; that is, from the lower inflammable limit (about 5%) to the maximum explosive point (9.46%).

As the percentage of methane in the air rises above the maximum explosive point and the supply of oxygen is less than that required for complete combustion, an increasing proportion of the methane is burned to carbon monoxide and hydrogen, the reaction probably being $2CH_4+(O_2+3.782N_2)=2CO+4H_2 +3.782N_2$. It is also probable that portions of the methane are burned to

### PRODUCTS OF EXPLOSION OF METHANE IN AIR

| Gas | Percentage of Methane in Air | | | |
|---|---|---|---|---|
| | 9.46 | 10.03 | 10.94 | 12.00 |
| Carbon dioxide............ | 15.10 | 10.16 | 8.35 | 4.80 |
| Carbon monoxide.......... | | 2.13 | 4.47 | 3.90 |
| Hydrogen................. | | 1.39 | 3.66 | 3.50 |
| Water.................... | 12.30 | | | |
| Nitrogen................. | 72.60 | 86.32 | 83.52 | 82.20 |
| Methane, etc............. | | | | 2.50 |

carbon monoxide without the formation of hydrogen by this reaction. $2CH_4 +3(O_2+3.782N_2) = 2CO+4H_2O+11.346N_2$. It is not possible to determine the percentage composition of the afterdamp in these cases as the relative proportions of the methane burned to carbon dioxide, and to carbon monoxide, and hydrogen, respectively, cannot be known.

The first column gives the composition of the afterdamp as calculated from the reaction for complete combustion. The second and third columns represent the results of investigations by Mr. G. A. Burrell, of the Bureau of Mines. The figures in the fourth column are from the French firedamp commission as quoted by Chamberlain. The sums total but 96.9% and indicate the presence of some unconsumed methane and other hydrocarbons. Mr. Burrell notes that no acetylene, olefin hydrocarbons, or unburned methane were found in the products of combustion.

The results of the French firedamp commission in some measure support the contentions of Doctor Broockmann, of Bochum, who, according to Wabner, has shown that the product of the imperfect combustion of methane is ethylene (olefiant gas, $C_2H_4$) and not carbon dioxide. The reaction for the explosion in oxygen would probably be written, $2CH_4+O_2=C_2H_4+2H_2O$. In discussing Doctor Broockmann's work, Chamberlain says: A mixture of air and firedamp containing from 10.8 to 13.5% of marsh gas he found to give two separate flames. Expecting to find some carbon monoxide formed as the result of the first flame, Broockmann removed the products of the first combustion, in an exploding mixture of methane and air, before the appearance of the second flame; but in no case found a trace of carbon monoxide. Instead, the heat of the first combustion converted the unconsumed methane into acetylene and hydrogen. These doubtless furnished the fuel for the second flame.

In the case of the formation of acetylene and hydrogen, the reaction for explosion in oxygen, would be, $2CH_4+O_2=C_2H_2+H_2+2H_2O$. From the evidence in hand, it would appear that the explosion of methane in a deficient air supply, that is when there is more than 9.46% methane in the air, results ordinarily in the formation of carbon monoxide and hydrogen, but that under certain conditions some acetylene and olefiant gas may be formed. One of these conditions for the formation of ethylene or acetylene would appear to be a percentage of methane approaching the upper explosive limit.

**Effect of Atmospheric Changes on Escape of Firedamp.**—The pressure of the atmosphere is not constant, but is subject to fluctuations depending on the condition of the atmosphere. Besides these, there are fluctuations that are more or less regular and are called *barometric variations*. There is both a yearly and a diurnal, or daily, variation. Of these two, the more important and the more regular is the daily variation, in which the barometer attains a maximum height from 9 to 10 o'clock A. M., and a minimum about 4 o'clock P. M. Other maximum and minimum readings are obtained at 10 P. M. and 3 A. M., respectively; but these are not as pronounced as those occurring in the daytime. The daily barometric variations range from .01 to .08 in.

A reduction in atmospheric pressure (fall of the barometer) permits, or tends to permit, an increase in the outflow of gas, and an increase in atmospheric pressure (rise of the barometer) reduces the outflow. Although at its point of escape face and blower gas are but little above atmospheric pressure, a short distance within the solid the pressure may be from five to forty or more atmospheres, whereas standing gas in old workings is under atmospheric pressure only. For this reason, a change in pressure has much more effect on the escape of gas from the gob than from the face, although the lengthening of the flame of a burning blower during periods of low barometer and its shortening when the barometer is high which are due to an increase and decrease, respectively, in the amount of escaping gas are well known.

In connection with barometric fluctuations the following points should be noted: Changes in the outflow of gas always take place before the corresponding change in the barometer, and from 1 to 6 hr. in advance thereof. This appears to be due to the fact that an air-and-gas column is much more sensitive to slight changes in pressure than one of mercury. Thus, the table on page 838 shows that 1 in. of mercury = .491 lb. a sq. in. = 876 ft. of air, and for a given change in the mercury column, the one in the air column is $876 \times 12 = 10,512$ times as great. One reason for the discontinuance of the barometer warnings in England was that they did not arrive until the danger was passed. In an extensive country like the United States, the well-known Daily Weather Maps of the Weather Bureau can be utilized in this connection. Thus, an area of low barometer, say, in Arizona, will probably not reach Illinois or

Pennsylvania for 24 to 36 hr., giving ample time to make any necessary changes in the ventilation. The reason why these maps have not received extended use is possibly due to the facts that the average American mine is shallow and non-gaseous, and provided with ample ventilation.

Increases in the outflow of gas due to working the mine should not be confused with those due to changes in atmospheric pressure. As pointed out by Messrs. Evans and Hutchinson, the amount of gas given off by the face is a minimum at the beginning of the shift, increases to a maximum at its end, and then decreases to a minimum at the beginning of the next morning shift. This is due to the fact that by day fresh faces of coal are constantly being exposed, and does not concern the escape of gas from the gob. For the same reason, a mine producing 2,000 or 3,000 tons a day gives off more gas than one mining but 1,000 tons. Rapidity of working, however, does not affect the outflow of gas from the gob.

The more rapid the change in the barometer, the greater will be the increase or decrease in the outflow of gas. When the pressure falls a certain amount and then remains constant for some time, the increased outflow of gas gradually diminishes as the pressure of the gas in the coal adjusts itself to that of the atmosphere. The outflow does not become as low as before the fall until the pressure returns to its original amount. The greater the pressure of the gas in the pores of the coal, the less will be the effect of atmospheric changes on its escape. Standing gas in old workings is more dangerous when changes of pressure take place than face or blower gas, as large volumes of it may be forced into the airways.

**Afterdamp.**—Strictly speaking, afterdamp and blackdamp are one and the same gaseous mixture ($CO_2$ and $N$), the one being formed with extreme rapidity by the explosion of methane or coal dust (or both), and the latter very slowly by the gradual absorption of oxygen and the giving off of carbon dioxide by the coal. Afterdamp, however, is a convenient term in indicating that the blackdamp mixture it defines is found after rather than before an explosion. Owing to the general insufficiency of oxygen for complete combustion and the universal presence of coal dust in mine workings, afterdamp probably always contains more or less carbon monoxide, and the presence of this gas may be used to distinguish it from blackdamp formed in the ordinary way. Unfortunately, it is not possible to give representative analyses of afterdamp, because in the time between an explosion and the arrival of a chemist, the mine atmosphere will have materially changed and correct samples cannot be obtained. Doctor Haldane says that afterdamp may be assumed to contain about 80 to 85% $N$, 12 to 14% $O$, 4 to 6% $CO_2$, .6 to 1.5% $CO$, together with small amounts of $SO_2$, $H_2S$, and unconsumed $CH_4$. Such an analysis indicates, roughly, a composition of 62% air, 38% blackdamp composed of 87% $N$ and 13% $CO_2$, and the amounts of the other gases previously named, and is a typical air-blackdamp mixture.

Afterdamp, at least by the time rescue parties arrive upon the scene, is very rarely extinctive of lights and so must contain at least, say, 17.5 to 18% $O$; considerably more than the figures given by Doctor Haldane. Neither is the atmosphere always extinctive immediately after an explosion as is proved by the numerous instances in which the lamps of the killed have been found either burning or exhausted of oil.

Whether the breathing of afterdamp results fatally or not depends almost entirely on the amount of carbon monoxide present, for, except for this gas, the afterdamp quoted from Dr. Haldane is not necessarily deadly, although extinctive of lights. In anthracite mines, where the dust is inert, carbon monoxide is derived from the imperfect combustion of methane, but in bituminous mines where, frequently, no gas is concerned in the explosion, the monoxide comes from the imperfect combustion of the carbon of the coal, from the reduction of carbon dioxide to carbon monoxide by incandescent carbon and, more likely, by the imperfect combustion of the various hydrocarbons distilled from the coal by the intense heat of the explosion. As no two explosions are alike in the relative proportions of methane (or coal dust) and air concerned, so does the amount of monoxide in the resultant afterdamp vary from amounts that are merely injurious to those that are almost immediately fatal, Doctor Haldane's figure of 1.5% probably well representing the average maximum.

In some cases, depending on the incompleteness of the explosion, the afterdamp will contain unburned methane, small amounts of hydrogen, and possibly traces of acetylene. The oxides of nitrogen may be found in minute quantities in afterdamp as a result of the burning of explosives and if black powder has

been one of these, traces of sulphur dioxide and hydrogen sulphide as well. Vapor of water is always a product of the explosion of hydrocarbon gases and, as stated, some moisture seems essential to the explosion of all gases.

**Detection of Methane.**—Some miners having long familiarity with methane claim to be able to feel the presence of large amounts of the gas, and others attribute to it a peculiar odor and a taste like apples. It is impossible to give any reason for these claims as the pure gas is odorless and tasteless and its effect on respiration is the same as that of nitrogen. The claim that the gas can sometimes be seen is not altogether unreasonable, the visibility being due to the irregular refraction of the light rays from a lamp while passing through layers of methane or of firedamp of varying densities.

The gas is universally detected in mine workings by the use of some form of safety lamp, in rare cases a so-called mechanical gas detector has been used. For accurate determinations of the methane content of the return air and the control of the ventilation of a mine as a whole, the larger corporations now cause chemical analyses of the mine air to be made daily or more often if necessary.

## THE RARER MINE GASES

**General Considerations.**—A number of gases, either by reason of their existence in mine air being disputed or doubtful, or by their occurrence under very unusual conditions or in such small amounts that they can be detected only locally and not when distributed through the ventilating current, are known as the *rarer mine gases*. These gases are ethane and ethylene and the higher members of the paraffin and olefin series of gases, and hydrogen, all of which may occur associated with methane in firedamp; acetylene, which is sometimes found as a combustion product of methane in a deficiency of oxygen; hydrogen sulphide and sulphur dioxide which may be formed by the burning of certain explosives, by mine fires, and by certain natural chemical processes; and several of the oxides of nitrogen, which are found in the combustion products of various explosives.

The part that any of these gases plays in the general scheme of mine ventilation is insignificant, and most of them are of little more than theoretical interest. Ethane, ethylene, and other members of their series are not easy to detect, and it is possible that their more general occurrence will be demonstrated when more accurate methods of analysis are employed. On the other hand, many competent chemists doubt their existence in mine air after special and exhaustive searches for them.

**Ethane and Other Paraffin Gases.**—Ethane, formula $C_2H_6$, the next higher than methane in the paraffin series of gases having the general formula $C_nH_{2n+2}$, is believed to be a fairly common constituent of firedamp, although its presence is denied or doubted by numerous competent observers. The Prussian Firedamp Commission reported a sample of firedamp containing 37.62% of ethane, but the maximum noted in American mines rarely exceeds .25 to .50%. Chamberlain found from a trace to 4 parts of ethane to 100 parts of methane in the gases given off by American coals when bottled in a vacuum, while Porter and Orvitz and Parr and Barker could find no trace of it in many samples of coal from Illinois and other states, and Brunck (Germany) and Harger (England) either could not find the gas or claim that its existence has not been definitely proved. As the gas is formed under the same natural conditions as methane, there seems no good reason why it should not be found in firedamp. The gas is given off by the face and by blowers, but it may enter the mine through leakage from wells of natural gas which usually contains from 3 to 6% of ethane and as high as 16% in the case of that piped to Pittsburgh, Pa., from the West Virginia fields.

Ethane is odorless, colorless, and tasteless, is slightly heavier than air, and is non-poisonous but, like methane and nitrogen, is suffocating in sufficient quantities. The gas is combustible, burning with a somewhat more brilliant flame than methane, and is explosive at a lower temperature (968° to 1,166° F.) according to the reaction $2C_2H_6 + 7O_2 = 4CO_2 + 6H_2O$. The explosive range of the gas does not appear to have been determined, but its point of maximum explosibility in air is fixed by Brunck at 5.6%. When present with methane, it makes the firedamp mixture more easily explosive, but not to a great extent owing to the relative difference in the amounts of the two gases.

Whether insignificant amounts of the next higher paraffin gases, propane $C_3H_8$ and butane $C_4H_{10}$, are or are not associated with ethane in the methane of firedamp is a very much unsettled question. The chemical and physical properties of these gases and their effect on firedamp mixtures are very similar

to those of ethane. In chemical analyses, they are commonly determined with the methane or ethane and are sometimes reported as paraffins.

**Ethylene and Other Olefin Gases.**—Ethylene, otherwise known as *ethene*, and *olefiant gas*, formula $C_2H_4$, is the principal member of the olefin series of gases having the general formula $C_nH_{2n}$. It is, perhaps, a rather more abundant constituent of some firedamps than ethane, although such a large maximum as 37.62% has not been reported for it. Chamberlain found .02 to .26% of ethylene and ethylene and carbon monoxide jointly, respectively, in feeder gas from Cardiff, Ill., and .49% of ethylene in gas escaping from a borehole near Wilkes-Barre, Pa. Small amounts of ethylene are not uncommon in natural gas; it is a usual constituent of producer and by-product coke-oven gas, and to its presence is due the illuminating power of coal gas distilled from coal.

Ethylene is colorless, has a slight ethereal odor, and is said to have a mild sweetish taste. It is a trifle lighter than air, is non-poisonous although suffocating like nitrogen and methane, is combustible, and is explosive between the limits of about 4 and 22% in air according to the reaction (in oxygen) $C_2H_4 + 3O_2 = 2CO_2 + 2H_2O$. The gas ignites and burns with a very brilliant flame at about 1,020° F., its point of maximum explosibility being reached when it constitutes 5.21% of the air.

As with ethane, the tendency of ethylene is to increase the explosibility of firedamp mixtures, but its influence is not as great as it rarely forms as much as 1% of a body of explosive gases.

It is possible that trifling amounts of propylene, $C_3H_6$, and butylene, $C_4H_8$, the next higher gases in the olefin series are present with ethylene. If so, they have not been separately determined but are included in the ethylene or are reported under the head of olefin or $C_nH_{2n}$ gases.

**Hydrogen.**—Hydrogen, with an atomic symbol of $H$ and a molecular formula $H_2$, is of quite general occurrence in small quantities in mines. Based on published analyses, the gas is not a common constituent of face or feeder gas, although Mr. Austin King reports 2.04% in the gases taken from a crevice in a mine roof and 7.35% in the gases from a hole drilled into the solid. From a trace to as much as 8 or 9% are found in the crevices of the coal face after blasting, where it is formed in part as an incomplete-combustion product of the powder and in part by distillation of the coal. It appears to be always present in amounts up to 2% or more in the gases from gob and ordinary mine fires, and is an almost universal constituent of the afterdamp of coal-dust explosions where it results from distillation from the coal or the incomplete combustion of methane and other hydrocarbons. It is found in natural gas sometimes to the extent of 25 to 30%, commonly forms nearly or fully one-half of coal gas, and is present in large amounts in all types of manufactured gas.

Hydrogen is odorless, colorless, and tasteless, is the lightest substance known, is non-poisonous but is suffocating like methane, etc. It is combustible and is explosive between the limits of 5 and 72% in air, having the greatest explosive range of any of the true mine gases. It ignites at a temperature between 1,030° and 1,130° F., and burns with an almost colorless bluish flame and with the production of intense heat, according to the reaction (in oxygen) $2H_2 + O_2 = 2H_2O$. As the water vapor condenses, the afterdamp from the explosion of pure hydrogen in air is nitrogen only.

As its explosive range is much wider and it occurs so much more frequently and abundantly, its effect in increasing the explosibility of firedamp mixtures is much more marked than that of any other gas.

**Acetylene.**—Acetylene, formula $C_2H_2$, is the only gas of the series having the general formula $C_nH_{2n-2}$, that is found in mines. It is reported only as a constituent of afterdamp resulting from the explosion of methane in a marked deficiency of oxygen. Acetylene, and the next higher gas in the same series, allylene $C_3H_4$, are minor constituents of coal gas.

Acetylene is colorless, tasteless, and has an ethereal odor that has by some been likened to that of geranium. It is slightly poisonous and is suffocating. It is combustible and is explosive between the limits of 3 and 82% in air, having, probably, the greatest explosive range of any gas. It ignites at about 900° F., and burns with an intensely brilliant flame and much smoke except in properly designed burners, according to the reaction (in oxygen) $2C_2H_2 + 5O_2 = 4CO_2 + 2H_2O$.

**Hydrogen Sulphide.**—Hydrogen sulphide, formula $H_2S$, and otherwise known as *sulphuretted hydrogen*, or *stinkdamp*, is colorless, has a sweetish taste and a very distinctive odor of decayed eggs, as little as .01% in air being capable of detection by this means. The gas is even more poisonous than

carbon monoxide, is combustible, and is explosive with violence when present to the extent of 12.5% in air, the point of maximum explosibility being reached at 14.2%. The reaction for its burning in oxygen is $2H_2S + 3O_2 = 2H_2O + 2SO_2$.

Hydrogen sulphide is produced in mines by the burning of black powder and those of the higher explosives of less than 40% strength that contain sulphur as a combustible. The amount of this gas in powder smoke varies from a trace to as much as 15 to 20% or more, the proportion, in a great measure, depending on the amount of moisture present. It is also formed by the decay of animal and vegetable substances containing sulphur, by the action of acid waters on metallic sulphides (particularly iron pyrites), by the heating of sulphides in the presence of moisture as in gob fires, by the distillation of coal containing sulphur when the hydrogen and sulphur unite directly, and by the reducing action upon sulphates of bacteria in foul and stagnant water. When absorbed by water, the gas is readily given off on stirring, and care should be taken not to disturb a pool known or supposed to contain it. Very small quantities of the gas are sometimes found in feeder and blower gas, and it is, at other times, carried into the mine in solution in water from the containing rocks.

The gas is an irritant poison affecting the nostrils and other approaches to the lungs, the eyes, brain, and all the tissues although, unless in large amount it does not appear to effect a change in the blood as does carbon monoxide. One of the first symptoms of very small amounts is irritation of the air-passages accompanied by an inflammation of the eyes, which feel as if full of dust. In larger amounts, according to Lehmann, the gas causes nausea, giddiness, cold skin, labored breathing, irregular action of the heart, and pains in the stomach. Death follows quickly after unconsciousness (which comes very rapidly in cases of hydrogen-sulphide poisoning), and is frequently accompanied by delirium, convulsions, and lockjaw. As little as .05% will prove fatal if breathed for some time, .07% in 1 hr., and .20% in a few minutes. The after-effects of this poisoning, like those of carbon monoxide, are frequently severe, long continued, and even permanent.

One of the immediate effects of hydrogen sulphide in dangerous quantities is a deadening of the sense of smell, which ceases to be a guide to the detection of the gas. A simple test for the presence of this gas may be made by dipping a piece of paper in acetate of lead (sugar of lead), and allowing it to dry. Upon exposure to air containing even a trace of the gas, the paper is immediately blackened by the formation of lead sulphide.

**Sulphur Dioxide.**—Sulphur dioxide, formula $SO_2$, appears to be formed during mine fires by the burning of coals containing pyrites by the reaction (in oxygen) $4FeS_2 + 11O_2 = 2Fe_2O_3 + 8SO_2$, and possibly by the decomposition of sulphates previously formed by the slow oxidation of pyrites according to the reaction, $2FeSO_4 = Fe_2O_3 + SO_2 + SO_3$.

Sulphur dioxide is colorless, has a suffocating irritating smell, and a pronounced taste. It is more than twice as heavy as air and is incombustible, sometimes being used instead of carbon dioxide to extinguish mine fires.

The gas is very soluble in water forming, it is believed, sulphurous acid by the reaction $H_2O + SO_2 = H_2SO_3$. As the gas is absorbed by the moisture in the throat, nostrils, etc., it is probably as sulphurous acid and not as sulphur dioxide that this gas acts on the system. The gas appears to decompose the red corpuscles of the blood, is fatal in extremely small amounts, and its symptoms in producing irritation of the air-passages, lungs, eyes, etc., with accompanying congestion, are similar to those of hydrogen sulphide. However, much smaller amounts of it than of the sulphide are dangerous. Thus, Lehmann says that as little as .001% produces some slight irritation of the mucuos membrane and respiratory organs, which is pronounced when the percentage rises to .003. In the case of rabbits, .04% causes congestion of the chest and inflammation of the air-passages and eyes, and .10% causes death in a few seconds.

**Nitric Oxide and Nitrogen Dioxide.**—When high explosives containing nitroglycerine and nitro-substitution compounds are burned rather than exploded, they do not yield carbon dioxide and nitrogen but their gases of combustion contain large quantities of carbon monoxide and nitric oxide, $NO$, which latter, on contact with air is at once converted into the red fumes of nitrogen dioxide $NO_2$, by the reaction $2NO + O_2 = 2NO_2$. Haldane shows that when nitroglycerine is exploded, the gases of combustion contain 63.2% $CO_2$ and 31.6% $N_2$, but when burned in the presence of its own gases, they contained 35.9% $CO$ and 48.2% $NO$. Clarence Hall found in the gases resulting from the burning of ordinary 40% strength gelatine dynamite, 13.7% $CO$, 11.3% $NO$, and .6% $NO_2$.

The conditions causing the burning, all or in part, of nitro-explosives instead of their detonation, are poor quality of the explosive, improper tamping, the use of detonators of insufficient strength of or defective fuses, and the actual burning of a quantity of the explosive by a fire carelessly started.

Haldane claims that nitrous fumes, $NO_2$, are even more poisonous than those of $H_2S$ and says that no gas met in mines is so treacherous in its effects. These fumes act as an irritant to the eyes, nose, and throat not dissimilar to that of hydrogen sulphide and sulphur dioxide, but whereas the after-effect of recovery from the last-named gas is a temporary catarrh, with nitrous fumes there is great danger of intensely acute bronchitis, which is often fatal. While temporary irritation and apparent recovery may be experienced at the time of exposure, these are frequently followed by the development of bronchitis in a few hours and this is frequently fatal in 2 da. Mice forced to breath air containing but .05% of these fumes for 1 hr., and which apparently recovered, died within 24 hr. of bronchitis.

Nitrous fumes are easily detected by their smell, even when greatly diluted, and care should be taken in returning to the face if this smell is at all noticeable. Minute traces of the gas may be detected by exposing in the suspected air a paper soaked in a solution of starch and potassium iodide; if any nitric oxide is present, the starch is at once turned blue by the iodine set free.

## EFFECT OF HEAT AND HUMIDITY ON MINE WORKERS

The mean annual underground temperature of the mines of the United States is probably about $10°$ more than that of the surface, and is between $60°$ and $65°$ F. The causes making for this increase in temperature are the heat given off by men and animals, by the burning of lamps, by the oxidation of animal and vegetable substances (as the decay of timber), by the slow oxidation of the coal, by gob fires whether active or smoldering, by the detonation of explosives, and by the interior heat of the earth. In deep mines, the effect of increasing heat with increasing depth is of prime importance in raising underground temperatures. Below the level of no annual variation which, in temperate climates is some 60 ft. below the surface, the temperature increases at a rate of from 1° F. in 100 ft. to 1° F. in 60 ft., about 1° F. for each 80 ft. of descent. A mine 1,000 ft. deep in a region as western Pennsylvania and Ohio, where the mean annual temperature is about 55°, may reasonably be expected to have an average temperature of $55° + \left(\frac{1,000 - 60}{80}\right) = 67°$, about, due to the interior heat of the earth alone. To this must be added an always uncertain amount, perhaps as much as 2° to 3°, for the heat arising from the other causes named.

Whether the temperature of the ventilating current will be greater or less than the temperature established by the preceding conditions depends on its initial temperature and its velocity. Radiation from the side walls is constant but slow and rapid currents of air will not absorb as much heat as those moving slowly; consequently, at the face or in any tight place the temperature is much, often 5° or 10°, greater than on the entries where the air is circulating freely. Haas found that in a certain mine entry 4,000 ft. long, over 30,000 B. T. U. a min. were radiated from the sides, which is equivalent to the heat derived from burning more than 1 T. of coal a day. In regard to the initial temperature of the air, Scholz found at certain mines in the Middle West that during summer, when the temperature outside averages 85° F. for 24 hr., with a maximum of 95° to 100°, the mine temperatures fluctuated between 70° and 78°; and that during winter, with a temperature of 40° to 50° outside, the mine temperatures ranged from 60° to 67°, the exact temperature necessarily depending somewhat on the extent of the mine. As the average annual temperature of the region in question cannot be far from 55°, it would appear that at all times (even in winter) the temperature of the mine is considerably above the annual for the place.

As long as the mine air is dry, or relatively so, men can work in temperatures as high as 90° and even 100° without much discomfort or impairment of health, as the perspiration is rapidly absorbed by the air-currents. But it has been shown that, regardless of the outside temperature and humidity, the return air is usually saturated to the extent of 90% or more, except in the arid regions of the West, where the humidity may fall to 85%. Haldane states that in still and moist air it is hardly possible for men to do continuous hard work at temperatures of 80° to 85° even when stripped to the waist. At temperatures

above 90°, by the wet bulb, it is only possible to work for short periods, and it becomes increasingly difficult to remain in the place even without working. Haldane found that at a temperature of 93° in still and saturated air and doing practically no work, his temperature rose 5° in 2 hr. and was still rising rapidly when he found it necessary to go out.

## SAFETY AND OTHER LAMPS

### PRINCIPLE AND ORIGIN OF SAFETY LAMPS

**Description.**—In a safety lamp, the flame of the burning illuminant is isolated from direct contact with the mine air by a wire gauze, or a glass and gauze cylinder, which is closed at the top where it is covered by a hood to which is attached a ring or hook for carrying. As a further means of isolating the flame, there may be two or more gauzes with an air space between each, and in practically all lamps the outer gauze is surrounded by a shield, called a *bonnet*, which is provided with perforations or slots. The various parts of the lamp are securely held together by the necessary standards and screw, soldered, or riveted joints.

**Dates of Discovery.**—The principle of isolating the flame of the lamp was evolved by Dr. William R. Clanny in the spring of 1813, although his, the first safety lamp, did not receive its final and successful trial until Oct. 16, 1815. The principle of the bonnet was demonstrated on Nov. 28, 1815, by George Stephenson; and on Dec. 15 of the same year, Sir Humphrey Davy announced the use of the wire gauze.

**Principles of the Safety Lamp.**—Although the last to be made public, the principle discovered by Davy is the first in importance. This principle is that, while a wire gauze of fine mesh entirely surrounding the flame will permit the free entrance of air within the lamp, yet in its outward passage through the gauze the burning gas is broken up into a series of fine jets and is so reduced in temperature by the cool metal that its flame is extinguished and, hence, cannot ignite firedamp mixtures outside the lamp.

In Stephenson's lamp, the burning gas was extinguished not by a cool metal gauze, but by bringing it in contact with the inert products of combustion that were held in the upper part of the lamp between the bonnet and the gauze. While, in modern lamps the bonnet plays in a greater or less degree the part for which it was originally intended, its chief use is to prevent the direct impact against the gauze of air-currents of high velocity which might extinguish the lamp or, what is more dangerous, might force the flame against or through the gauze and thus cause an explosion. When the blanketing effect of the original bonnet is desired, it is now generally accomplished through the use of double or triple gauzes as in the Marsaut lamp.

Safety lamps are not absolutely safe in the sense that they may be burned indefinitely in explosive mixtures of gas and air. In comparison with the unprotected candles that they replaced or with modern open lights, they are relatively safe in that the warning of the presence of a dangerous amount of gas afforded by its burning within the lamp, gives the miner time to withdraw before an explosion takes place.

**Early Classification of Safety Lamps.**—Safety lamps were formerly divided into two general classes, those designed for testing for gas and those intended for working lamps, the construction of the former being such that they were the more sensitive to gas. This distinction in usage and construction is now rarely made, and at any particular mine the same kind of lamp is commonly used by fireboss and miner alike. The reason for this is that none but special lamps in the hands of skilled observers can detect the less than 1% of gas that is dangerous in the presence of explosive coal dust. Such being the case, there is nothing to be gained in providing a fireboss with a dangerous lamp (all sensitive lamps are unsafe) to detect 2.5% of gas when a safe lamp will detect, say, 3%; as these proportions when coal dust is present and equally harmless when it is not.

**Approved Safety Lamps.**—An approved safety lamp possesses those features that a mining department or legislature declares essential in lamps to be used within its jurisdiction. The features considered essential vary, but both here and in Europe, to be approved, a lamp must have a bonnet. The Davy lamp, not being safe with or without a bonnet, is not permitted in Europe, but is allowed in a few American states for gas testing, although the number of states permitting it is decreasing.

## SAFETY-LAMP CONSTRUCTION

**Specifications.**—Mr. J. W. Paul sums up the structural requirements of safety lamps as follows:

1. The framework should be rigid and well made so that it will not get out of shape when roughly handled;

2. If the lamp has a glass chimney, the upright rods (standards) should be of such number and so spaced that a straightedge, or ruler, placed against any two adjacent rods will not touch the glass;

3. If a lamp has no bonnet, the gauze should be protected with rods in the same manner as the chimney, as indicated in 2;

4. The lock should be such as will require, when locked, a special device for unlocking;

5. The glass chimney should have a smooth and even wall throughout, should be of the best quality, and should have its ends ground truly parallel and at right angles to the axis of the chimney. The chimney should bear the trade-mark of the manufacturer;

6. When the lamp is assembled there should be no openings between the outside and the interior of the lamp except those in the gauze or other heat-absorbing device, such as a perforated plate or cylinder in which the size of the perforations corresponds with that of the gauze openings;

7. The handle of the lamp should be either an open ring or a hook, strongly made and not easily bent in the hand;

8. The construction of the lamp should be such that its parts are made in standard uniform sizes and fit so intimately that should any part be omitted in assembling its absence would be easily detected by the most casual inspection;

9. There should be an expansion ring or equivalent device used with the glass chimney so that the chimney when heated can expand without breaking any part of the lamp;

10. In the selection of a safety lamp, carefully examine each of the disassembled parts to ascertain defects or improper construction; if any such is discovered, the entire lamp should be rejected.

**Design of Safety Lamps.**—As pointed out by Hughes, Marsaut and others have shown that a certain relation should exist between the volume contained within a lamp and the gauze surface open for the escape of the products of combustion resulting from an internal explosion, as experiments have proved that the ignitions of explosive mixtures outside the lamp by explosions within it become less frequent as the open surface of the gauze is enlarged.

Marsaut also proved that: (1) A lamp of small diameter (such as a Davy) does not readily pass an explosion, as the volume of gas that can be exploded is insignificant. (2) A lamp without a glass is more secure against the effects of internal explosions than a lamp with a glass cylinder, as the latter confines the gases at the time of an explosion and acts like a cannon; it is, therefore, advisable to reduce both the height and the diameter of the glass. (3) A wire gauze of conical shape is more secure against the transmission of internal explosions than is one of cylindrical shape and of the same capacity. (4) Gases resulting from combustion play a certain part in preventing external explosions and it might, therefore, not be advisable to guide them by a chimney. (5) A descending current of feed-air prevents the filling up of glass lamps with an explosive mixture, and occasions the formation of an inexplosive and elastic cushion at the bottom of the lamp.

**Materials of Construction.**—With the exception of the gauze and glass, the various parts of safety lamps are made of brass or, where lightness is required, of aluminum or magnalium. Where iron enters into the construction, it is usually in the standards and hood.

**Safety-Lamp Gauzes.**—The main gauze of safety lamps is of 28 mesh; that is, there are 28 openings in 1 lin. in. or 784 openings in 1 sq. in. If made of No. 28 (B. W. G.) wire .014 in. in diameter as is usual, 1 sq. in. of the gauze will be about two-thirds (.6151 sq. in.) metal and one-third (.3849 sq. in.) openings. As exceptions to the use of this standard gauze, those in the Marsaut and Chesneau lamps have 934 and 1,264 openings per sq. in., respectively.

Gauzes are commonly made of iron wire, although copper is sometimes used. The latter is rather more durable than iron as it does not rust or burn out so quickly, but it becomes hot and passes the flame sooner than iron as it has a higher specific heat.

The gauze cylinder must not exceed a certain size, which Davy fixed in his original lamp as 2 in. in diameter and 7 in. high (contents 22 cu. in.), otherwise the burning of the large volume of gas within it will heat the gauze, and par-

ticularly the top, to a point where it will no longer cool the flame sufficiently to prevent its igniting gas outside the lamp. In modern lamps, the diameter of the gauze is about the same as that of the original Davy, but its height is less and varies from 4 to 5 in., as the lower part is replaced with glass. In lamps built on the Eloin principle, that is in those lamps that take air in through ports below the flame and are thence known as underfeed or under-draft lamps, as all the gauze is available for the discharge of the combustion products, it may be made much smaller than in a lamp of ordinary construction. The Ashworth-Hepplewhite-Gray lamp, Fig. 1, e, page 885, is an example of a lamp with a relatively small gauze.

As the top of the gauze receives the full effects of the flame, it is often reinforced by what is known as a gauze cap or smoke gauze. This consists of a cylinder of standard gauze closed at the top, which fits snugly over the main gauze for about one-third its length. The upper part of this cap is sometimes crimped or indented so that it may not be pushed too far down upon the main gauze as it is desirable to leave a small space between the tops of the two.

The gauze of the early lamps was always cylindrical, but in many modern lamps it is in the form of a truncated cone; a shape commonly followed where there is more than one gauze.

**Safety Lamp Glasses.**—Although Clanny and Stephenson used glass in front of their original (1815) lamps to increase the light-giving power, the present form of safety lamp in which a glass cylinder entirely surrounds the flame and is surmounted by a cylinder of gauze is due to Dr. Clanny who appears to have combined ideas original with both Davy and Stephenson. In a general way, modern lamps may be said to be Davy lamps in which the lower portion of the gauze cylinder is replaced with one of glass; or to be Stephenson lamps, the perforated metal cylinder being replaced by one of gauze.

Glasses are commonly cylindrical, but in the Ashworth-Hepplewhite-Gray lamp they have the shape of an upward-tapering truncated cone. This form allows the upward diffusion of the light, at least in pary, and thus permits of a closer inspection of the roof without having recourse to the very dangerous practice of turning the lamp on one side.

**Multiple Gauzes.**—Some safety lamps are made with two or even three gauzes, one within the other, with a small air space between; and lamps so made are known as multiple-gauze lamps. The inner gauze is always conical but the outer gauze may be cylindrical or conical with a little more slope than the inner one so that there may be more air space between the gauzes at the top than at the bottom. The intention of multiple gauzes is to interpose one or more curtains or screens of inert gases between the flame of any gas burning in the lamp and the outside air. These screens are formed by the retention of the products of combustion in the spaces between the several gauzes. The multiple gauze is the modern development of the original Stephenson principle of preventing the outward passage of the flame by smothering it in inert gases, and adds greatly to the safety of the lamp.

The effect of these gas screens is to impede the free upward and outward passage of the products of combustion simultaneously reducing the amount of oxygen admitted to the flame and the amount of oil burned in a given time. In other words, they reduce the draft and thus increase the tendency of the lamp to smoke and diminish the illuminating power. The reduction in illuminating power increases with the number of gauzes. Thus, the Marsaut lamp, Fig. 1, d, page 885, with three gauzes has an illuminating power 20 to 25% less than the same lamp with two gauzes. In lamps admitting air on the Eloin principle and which, in consequence, have a strong natural draft, the reduction in illuminating power through the use of multiple gauzes is considerabiy less than in lamps drafted in the ordinary way. With multiple gauzes, the gauze cap (smoke gauze) is rarely used.

**Safety-Lamp Bonnets.**—A bonnet consists of a metal cylinder entirely surrounding the gauze of the safety lamp, and is intended to prevent the direct impact of air-currents of high velocity against the gauze, as explained under Principles of the Safety Lamp. The surface of the bonnet is usually smooth, but in the Wolfe lamp, Fig. 1, f, page 885, it is corrugated. The bonnet is perforated or slotted with a varying number of holes arranged in various ways and which are designed to permit access of air to and egress of combustion products from the lamp. In some bonnets, the slots are indented on one side or arranged on the corrugations (Wolfe lamp) so that the air enters tangentially and not directly against the gauze. In the early Davy lamp, the screening effect of the bonnet was secured, but only in part, by the use of a semicircular shield that could be slipped in front of the flame when moving against the air.

The fewer the perforations or slots, the greater the blanketing effect of the bonnet and the more nearly it approaches a series of gauzes in its influence on the circulation of air within the lamp and on its light-giving power. Lamps with tight-fitting bonnets are easily extinguished when exposed in high percentages of gas, the flame being smothered by the large volume of combustion products held back by the imperfect circulation.

In the Ashworth-Hepplewhite-Gray lamp, a double bonnet is used, and double- and triple-gauze lamps are always bonneted for general use and commonly so when employed for gas testing.

**Circulation of Air in Safety Lamps.**—Air is admitted to safety lamps in three ways.

In the Davy lamp, Fig. 1, *a*, page 885, the air enters at the bottom of the gauze, which extends below the top of the wick tube, and the products of combustion pass upwards and out through the top of the gauze.

In underdraft lamps (Eloin principle) as in Fig. 1, *e*, page 885, the air enters below the flame through gauze-protected ports, and the products of combustion follow the same course as in the Davy lamp.

In the majority of lamps, Fig. 1, *b*, *c*, and *d*, page 885, the air enters at the base of the gauze above the glass and must pass downwards to the flame.

Lamps of the first two classes, in which the air follows what may be called the natural course (as in an ordinary chimney) are sensitive to small amounts of gas, and are apt to flame readily, are adapted to gas testing but not to general use, and are unsafe in air-currents of any but very low velocity unless provided with bonnets or multiple gauzes. When so equipped, underfed lamps are excellent for general use, but are not so sensitive to gas as otherwise.

In the third class, the air in passing down to the lamp flame conflicts with the ascending products of combustion with the formation of eddy currents, which may cause the lamp to flicker and smoke, thus making it less sensitive to gas and decreasing its illuminating power. In the Mueseler lamp, Fig. 1, *c*, page 885, the flame is surmounted with a conical sheet-iron chimney, which increases the draft and causes the air to circulate in a natural course. Following the arrows, the air enters through the base of the gauze, passes down beside the chimney to the wick, and the products of combustion pass up the chimney and thence out through the upper part of the gauze (see Mueseler lamp). It should be noted that a bonnet or a series of multiple gauzes interferes with the rapidity but not with the direction of air circulation.

In order to detect thin layers of gas near the roof without the dangerous necessity of turning the lamp on one side, of waving it to and fro, or of brushing the top air down upon it with a cap, several special constructions are employed. In the Ashworth-Hepplewhite-Gray lamp, the standards are hollow and air can, when needed, be drawn down through them by closing the regular entrance ports at the base of the lamp. A device for the same purpose that may be attached to ordinary lamps consists of an L-shaped pipe about ½ in. in diameter, the short arm of which is attached to a special port in the lamp below the flame while the long arm projects upwards into the gas. An improvement on the preceding is used by Mr. Joseph Smith, general superintendent of the Stag Cañon Fuel Co., at Dawson, N. Mex. The device consists of a small, double-acting pump the discharge end of which may be directly connected to the base of any of the standard forms of safety lamps. By means of a number of 5-ft. lengths of ¼-in. gas pipe which may be screwed together until their combined length is sufficient to reach the top of the highest falls and then attached to the suction end of the pump, samples of air from otherwise inaccessible places may be drawn into and through the lamp. For the same purpose Sir William Garforth uses a rubber bulb with a strong metal nozzle. In the base of the safety lamp is a tube with a self-closing valve. When testing for gas, the bulb is placed in a cavity in the roof or other place where gas is suspected, and is filled with the firedamp by compression in the usual way. The gas is prevented from escaping by holding a finger over the nozzle and the lamp is taken to some safe place where the end of the bulb is inserted in the tube in the lamp, opening the valve in so doing, when the gas may be squeezed upon the flame.

**Wick Tubes, Wicks, Etc.**—The wick tube of a lamp may be round or flat. When flat, one side is commonly made with one or more grooves to reduce the friction when the wick is adjusted in height and to provide a space for the circulation of the air that the oil may ascend. The top of the tube should be set about ½ in. above the base of the glass for, if set too low, the shadow cast on the ground by the body of the lamp is increased; and if set too high, the amount of light diffused upwards is decreased. In many lamps, in one side of the wick tube is a narrow slot in which the point of the picker is inserted to

adjust the wick. This slot should be as narrow and as short as possible, otherwise the oil will be vaporized and possibly ignited at the side rather than at the top of the tube. In some lamps, the wick remains stationary and the height of the flame is adjusted by raising or lowering a sheath that fits over the wick tube; and, in other lamps, the wick is contained in an adjustable sheath sliding within a fixed wick tube. In either case, the sheath is adjusted by turning a screw attached to the bottom of a shaft passing through the oil chamber.

Wicks are round or flat to correspond with the wick tube. They are made of strands of cotton yarn very lightly twisted or plaited (flat wicks) to form a bulk but slightly greater than the inside dimensions of the wick tube. Wicking should be thoroughly dried before use, as moisture impedes the flow of oil and reduces the illuminating power of the flame.

The picker used for cleaning the wick should sweep the entire top of the wick tube with a motion somewhat inclined to the horizontal.

**Igniters, or Relighters, for Safety Lamps.**—An igniter is a device for relighting a safety lamp without opening it. In the igniter commonly used with the Wolf lamp, the match is a narrow strip of paraffined paper in which are inserted small lumps of fulminate at intervals of about $\frac{1}{2}$ in. The coiled match is contained in a flat metal box inserted in a special receptacle in the bowl. The igniter proper consists of a piece of spring steel doubled on itself, one end of which is provided with fine teeth to engage the match. By raising the rod attached to the igniter, the teeth catch in the match and push its end slightly above the level of the wick. When the rod is suddenly pulled downwards by means of the head at the bottom of the oil chamber, the teeth of the igniter explode one of the fulminate caps thus igniting the match, which burns until level with the top of the wick tube. In a similar igniter, a friction match is held against a feed-screw by a steel spring. The upper part of the feed-screw carries a wheel with sharp teeth that strike against and ignite the match when a button on the lower end of the feed-screw is turned. In other igniters, phosphorus is used in place of fulminate to light the match.

Electric relighters are used in some standard lamps. Where light oils, as naptha, giving off vapors are burned, a low-tension current is used to heat to incandescence a platinum wire placed immediately over the wick tube. For the heavier colza and seal oils, a series of sparks produced by a high-tension current are passed over the wick tube. The current is taken into the lamp through a carefully insulated conductor passing through the bowl, the body of the lamp furnishing the return circuit. (See Protector Lamp and Hailwood Lamp.)

The property of alloys of cerium of sparking when brushed by a milled wheel has been taken advantage of in the design of safety-lamp igniters. The alloy commonly employed is one of iron and cerium which, when struck by the milled wheel after the manner of a flint against steel, often throws off unburned particles of metal, which may lodge against the gauze. If these are subsequently ignited, sufficient heat may be developed to fire the gas outside the lamp. To overcome this difficulty, the American Safety Lamp Co. uses an alloy of cerium and magnesium and a positive igniting device. In the older igniters of this type, a series of small sparks are struck by turning the milled wheel until the lamp is lit. In the improvement, a spring attached to the igniter is compressed by turning the head of a stem projecting below the bowl. When the tension of the spring reaches a certain amount, the wheel is released and revolves with such rapidity that the spark is practically a continuous flame, which ignites the wick at once.

Many have advised against the relighting of lamps in the presence of explosive amounts of gas, and particularly so if the lamp burns naphtha because, in a few seconds after being extinguished, the gauze may become filled with highly combustible vapors that may explode and pass the flame when the igniter is applied. The long series of tests made with the Wolf lamp go to show that this claim is not well founded. Others object to placing an igniting device in the hands of miners and irresponsible boys and hold that an electric relighter, which can only be applied at a lamp station, is to be preferred to other types.

**Locks for Safety Lamps.**—Locks for safety lamps are made on one of three general plans.

1. The lamp may be locked by a screw pin, catch, or similar device that may be opened by a key. While locks of this type cannot be opened accidently, they may be readily unlocked by any one even without a key. This form of lock is now rarely used.

2. The lock may be constructed so that it may be opened by any one, but any attempt to do so extinguishes the light or is revealed in some way.

3. The lamp may be locked by a device operated by electricity or compressed air, and can only be opened by means of special appliances kept in the lamp room at the surface or at a relighting station in the mine.

To the second class belongs the lead-plug lock. The lower part of the lamp is encircled with a movable ring to which is attached a hinged lock that drops over a projecting lug on the bowl. A lead plug is inserted in the lug and is punched flat; the punch used for the purpose stamping the latter or date for the day on the lead.

In the "Protector" lock, the wick tube is surrounded by a close-fitting collar of the same height held in place by a steel pin (lock bar) fastened by a piece of spring steel. In order to remove the lock bar, it is necessary to unscrew the bowl to reach the spring, but in so doing the wick tube (and wick) is drawn down through the collar and the lamp extinguished. In other locks of this class, the unscrewing of the bowl brings into action a cap, or extinguisher, that smothers the flame.

In the third class are several magnetic and compressed-air locks. In the Wolf lock, the tooth on the end of a pawl pivoted at its center is forced by a spring into a socket in the bowl when the latter is screwed into place. To unlock the lamp, the poles of a powerful horseshoe magnet are applied to poles in the base ring of the lamp. The current passes through the spring into the pawl, which causes the end opposite to the tooth to move inwards thus releasing the tooth from its socket and permitting the bowl to be unscrewed. When the lamp is released from the magnet, the spring forces the tooth into its original position so that when the bowl is screwed into place, it is locked automatically.

In the Hailwood lock, the ring holding the glass is provided on the under side with ratchet teeth into which engages an iron lock-bolt, which is held in an upright position by a strong spring resting on a movable iron guard plug, the latter resting on a solid shoulder formed in a recess in the bowl. To unlock the lamp, the nose of an electromagnet in which a current is generated by operating a treadle is placed against the guard plug and is pressed upwards. As soon as the plug comes in contact with the lock-bolt, it secures a strong magnetic hold upon it. Depressing the pedal draws down the guard plug and with it the lock-bolt, releasing it from the teeth in the glass ring and permitting the unscrewing of the bowl. The lamp may be locked automatically by simply screwing up the bowl.

To open the ordinary air lock, the suction end of a small air pump is applied to the mouth of the recess in which the lock-bolt fits. On operating the pump by a treadle, the vacuum created draws the bolt outwards against the pressure of the spring holding it in place, and permits unscrewing the bowl. The Hailwood air-lock differs from the preceding in that the positive pressure (not suction, or a vacuum) of compressed air is used to force back the bolt. In this lock, the spring holding the bolt in place may be made to withstand a pressure as high as 250 lb. so that the pressure of the air in the power mains is not sufficient to open the lamp.

**Oils for Safety Lamps.**—Because of their colorless sensitive flame, alcohol and even hydrogen are burned in some special forms of safety lamps designed for the determination of small percentages of methane. Ordinarily, some kind of illuminating oil is used both in working and in testing lamps.

The principal illuminating oils of vegetable origin are pressed from the seeds of the cotton and rape plants, the crude oil being treated with acid, washed, etc., to remove various mucilaginous substances that would otherwise cake on the wick. The purified or refined oils are commonly known as winter oils as their temperature of solidification is much below that of the crude product.

Refined cottonseed oil has a specific gravity between .922 and .926 and solidifies at from about 33° to 50° F.

*Rape*, or as otherwise called, *colza oil* is extracted from the seeds of several species of the genus Brassica of the Cruciferæ, or mustard, family. These plants are extensively cultivated in all parts of the world, except in the United States, for the illuminating oils contained in them. The species commonly cultivated are *Brassica napus*, or rape, the *B. campestris*, or rutabaga, and, to a less extent, the *B. oleracea*, or cabbage. The oils extracted from these plants differ slightly in their properties but are all sold as rape or colza oil, the latter name being derived from the word cole, or kohl, meaning cabbage. Colza oil has a specific gravity of .913 to .915 and solidifies at about 15° F., con-

siderably below cottonseed oil. The illuminating power of vegetable oils is low and may be increased and the incrustation of the wick decreased by the addition of one-half their volume of kerosene (ordinary coal oil).

Whale and seal oil extracted from the blubber of the respective animals are largely used in safety lamps, but not to the same extent as lard oils. Animal oils, like vegetable oils, do not possess great illuminating power, although this depends on their purity. The British Accidents in Mines Commission recommends the use of a mixture of one-third refined petroleum (kerosene) and two-thirds rape or seal oil as being cheaper and having the same illuminating power as the best colza oil while not forming such a hard cake or crust on the wick. The Commission considered seal as superior to colza oil in maintaining a more uniform height of flame for a longer time without retrimming.

Of the distillation products of petroleum, the so-called light oils are largely used in safety lamps because of their high illuminating power. The oils used and the temperatures at which they are distilled are, gasoline below 140° F., naphtha between 140° and 230° F., and benzine between 230° and 302° F.; kerosene, which is distilled between 302° and 572° F., is used only when mixed with non-volatile animal and vegetable oils.

As these oils are highly volatile, their volatility decreasing in the order of the temperatures at which they are distilled, their fumes have been considered as a source of danger, but this has been disproved by long and safe use of lamps burning naphtha and by the researches of Watteyne and Stassart in Belgium. These gentlemen found that when benzene was used, there was a slightly greater tendency of the lamps to heat and, in some cases, of the glasses to crack, but these in no way involved a passing of the flame or a deterioration of the lamp. Their photometric observations showed that the average illumination of the best oil (animal or vegetable) fed lamp was but .4 candlepower as opposed to .87 candlepower of the benzene-burning lamp with underfeed draft. Special tests have shown that the Wolfe lamp burning naphtha or benzene is safe under any conditions of use; thus, when the oil vessel of a burning lamp was heated to 180° F., the lamp was extinguished and without danger.

The illuminating power of safety-lamp oils varies so widely according to the purity of the oil, the kind of lamp used, the conditions of burning, etc., that it is not possible to give exact figures as to their relative light-giving value. The following figures are average of many determinations of the light power of various oils burned in a Clanny lamp when referred to a standard candle burning 120 grains of spermaceti an hour: Standard candle, 1.00; English rape oil, .32; best quality colza oil, .47; seal oil, .35; two parts of rape oil and one of kerosene, .30; various grades and makes of so-called safety lamps oils, .51, .43, and .48, respectively.

**Illuminating Power of Safety Lamps.—**The illuminating power of a safety lamp depends on the illuminant used and on the construction of the lamp. As its flame is not surrounded with glass, the Davy gives less light than any other lamp except the Stephenson and so (aside from being unsafe) is unsuited for working purposes.

### RELATIVE ILLUMINATING POWER OF SAFETY LAMPS

| Lamp | Candle-power | Lamp | Candle-power |
|---|---|---|---|
| Ashworth-Hepplewhite-Gray | .75 | Hailwood, burning naphtha | 1.00 |
| Clanny | .34 | Marsaut, three gauzes | .45 |
| Davy, common | .15 | Marsaut, two gauzes | .55 |
| Davy, Jack | .08 | Mueseler, Belgian | .36 |
| Davy, in case | .16 | Mueseler, English | .32 |
| Evan Thomas | .43 | Stephenson | .10 |
| Hailwood, burning oil | .67 | Wolf, burning naphtha | 1.00 |

A free circulation of air, which is best secured by an underfeed draft, insures a better supply of oxygen, removes the combustion products more quickly, and thus increases the lighting power of the lamp. Bonnets and multiple gauzes (see the two types of Marsaut lamp in the table) while increasing the safety of the lamp, reduce its illuminating power through impeding the circulation.

The table gives the relative illuminating power of various safety lamps referred to a standard candle (burning 120 gr. of spermaceti an hour) as unity. The oils used were mostly colza or seal oil, and the results are averages only and are not to be taken as exact and absolute.

According to Hughes, Marsaut found that the illuminating power of a lamp when the oil chamber is made of brass is but 70% of that of the same lamp when the chamber is made of iron, which appears to be due to the greater heat conductivity of the brass by reason of which the lamp bottom gets much hotter than if it was made of iron and the oil becomes viscous and will not flow.

## TESTING FOR METHANE

**Desirable Features in Lamps for Testing and for General Use.**—The following are considered desirable features in a safety lamp for gas testing:

1. The flame should be clear, steady, and free from smoke, that the gas cap may be more plainly observed and, to make the indications afforded by the cap of value, atmospheric conditions should be the same within and without the lamp. Alcohol and naphtha, particularly when burned in lamps with underfeed draft, afford a better and less smoky flame than animal or vegetable oils burned in a lamp drafted above the glass in the ordinary way, see under Circulation of Air and Oils for Safety Lamps.

2. A construction such that when the lamp is exposed to air-currents of high velocity, the flame will not be blown against or through the gauze to its injury or cause the ignition of gas outside the lamp. This is secured through the use of bonnets, multiple gauzes, etc.

3. There should be no bright surface behind the flame, reflections from which may interfere with the visibility of the cap. Secured by giving the metal parts a dull finish, careful selection of the glass, etc.

4. Ability to detect thin layers of gas near the roof, see Circulation of Air in Safety Lamps.

5. A scale for measuring the height of a flame cap so as to more accurately determine the percentage of gas in the air. The use of the scale is based on the assumption that a cap of given height always corresponds to a definite percentage of gas in the air. While this may be true in the laboratory where the scale is adjusted to flames obtained by burning known proportions of pure methane in pure air, it is rarely, if ever, true in the mine where deficiency of oxygen, the presence of blackdamp and coal dust, and varying conditions of pressure, temperature, and humidity tend to alter the cap for the same percentage of gas in the air. That is, a cap, say, 1 in. in height obtained in the mine indicates a different per cent. of gas, and usually a greater one, than the same cap in the laboratory. Further, the conditions within the lamp where the air is more or less mixed with combustion products cannot be the same as in the mine.

The following are essential features in lamps for general use, that is, in working lamps:

1. The lamp should give the maximum light consistent with safety; this is secured by the use of the proper illuminant and construction.

2. The lamp should be safe in air-currents of high velocity; this is secured through the use of bonnets, multiple gauzes, etc.

3. The lamp should be strong in all its parts so that it may not be easily broken through careless handling or in minor accidents, and should be simple in construction so that it can easily be taken apart for cleaning and as easily assembled when it is done.

4. The lamp should be capable of being securely locked so that it cannot be opened by unauthorized persons or at any but some appointed place.

5. The lamp should not be as sensitive to gas as one used for testing. A lamp that rapidly fills with flame in the presence of explosive mixtures and must be as rapidly removed therefrom to prevent the passage of the flame through the gauze or the extinction of the light, requires constant watching, and is unfitted for a working lamp. The same construction that makes a lamp safe in strong air-currents also makes it less sensitive to gas.

6. The lamp should diffuse light upwards so that the roof may be inspected without turning the lamp on one side.

7. The lamp should be provided with an appliance for relighting without opening it. The tests of Watteyne and Stassart showed that the explosion relighter causes external explosions in rare instances, but that the phosphorus igniter does not. It has been demonstrated that, as long as the glass is not broken, the gauze punctured, etc., internal relighting is safe provided a proper igniter is used.

**Testing for Gas.**—When a safety lamp is brought into an atmosphere containing methane, the presence of the gas is indicated by a bluish halo or cap surrounding and surmounting the lamp flame. The ordinary lighting flame of the lamp is rarely used in gas testing as it does not show a cap but merely an increase in length in the presence of methane, and this increase cannot well be measured unless the length of the flame in fresh air is first observed. The same is true of what is sometimes called an intermediate flame about one-half the height of the ordinary flame.

The flame commonly used for testing is made by screwing down the wick until the yellow color of the wick has disappeared and nothing but a faint blue cap remains on the burner. This is sometimes spoken of as a cap, cap-flame, blue cap, testing flame, non-luminous flame, etc., and is from $\frac{1}{8}$ to $\frac{1}{4}$ in. high, depending on the illuminant used, type of lamp, etc.

A blue cap, wholly or partially visible, and which should not be confused with that due to gas, is frequently seen above the testing flame. This is commonly called the *fuel cap*, and has been supposed to be due to the burning of the volatile products of the lamp fuel driven off by the heat. Briggs, however, has shown that the fuel cap is the outer of the three layers or parts into which any flame may be divided, occurs with solid as well as with liquid fuels, and that it is intensified only by the vapors given off when the lamp is hot. The fuel cap appears as a halo and, in order not to be deceived by it, the observer should become accustomed to its appearance in fresh air.

To test for methane, hold the lamp in an upright position with one hand and with the other screening the eyes from the body of the flame, slowly raise the lamp toward the roof and watch closely for the first appearance of the cap. When this is observed, the lamp should be promptly but cautiously drawn down, while the distance of the lamp from the roof is noted; this gives the depth of the gas in the place. When sufficient gas is present, or the lamp is raised too quickly, the entire gauze sometimes fills with flame; a condition known as *flaming*. When this occurs, the lamp must be handled with great care. An explosion of gas within the lamp is very likely to take place when it is withdrawn from a body of gas into fresh air, and this may be communicated to the outside gas unless the lamp is properly made. Bonneted lamps are more liable to internal explosions than those not bonneted, but, owing to the restricted circulation in the lamp, are far less likely to pass the flame to the outside.

**Height of Gas Cap.**—With the same lamp, burning the same illuminant, with the same wick, and using the same height of flame, as long as the air is pure and is not contaminated with carbon dioxide or excess nitrogen, there is a fixed height of cap for each per cent. of methane present. However, if the lamp, illuminant, wick, or height of flame is changed, or if the proportion of inert gases in the mine air is varied, there will be a change in the height of the cap made by the same percentage of gas so that, unless all the conditions are constant or are known, it is not possible to tell whether a cap of a certain height is due to the presence of, say, 1 or 2.5 % of methane. Prof. G. R. Thompson, Leeds University, gives the following table for the heights of gas caps in different lamps, using different oils, etc.

### HEIGHT OF GAS CAPS IN DIFFERENT LAMPS

| Lamp No. | Wick and Oil Used | Percentage of Gas in Mixture | | |
|---|---|---|---|---|
| | | 1 | 2 | 3 |
| | | Height of Cap, Inches | | |
| 1 | Circular wick, .23 in. in diameter, burning paraffin oil | .200 | .30 | 0.4 |
| 2 | Flat wick, .65 in. wide, burning naphtha (boiling point, 55°C.) | .520 | .67 | 1.1 |
| 3 | Flat wick, .55 in. wide, burning colza oil | .175 | .50 | 1.0 |

The following figure shows the height, in centimeters and in inches, of the gas cap corresponding to various percentages of gas and pure air as observed

in the naphtha-burning lamps now so generally used for testing and general purposes.

The gas cap in lamps like the Davy of Clanny burning sperm or colza oil are so very small and difficult to distinguish when the percentage of methane is reduced to 2.5 %, that it is considered impossible for the most skilled observer to detect less than 2 % of gas with these lamps, and 3 % is about the limit for the average man. For small percentages of methane the naphtha flame is much more sensitive than the oil, and with it as little as 1 % of gas may be detected.

To avoid adjusting the flame in each working place, fire bosses should carry a second lamp with normal flame or, better, may use a storage-battery portable electric lamp.
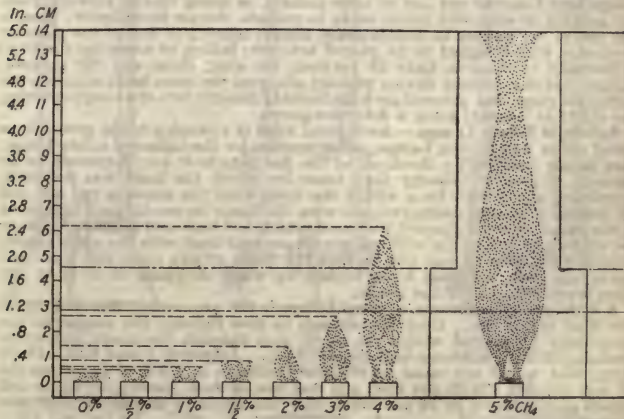


## CARE OF SAFETY LAMPS

**Cleaning Lamps.**—The following suggestions in regard to cleaning standard naphtha-burning safety lamps are selected from J. W. Paul.

1. Each workman should have his own lamp, marked with a distinctive number corresponding to that on a hook or the lamp rack. When a lamp is turned in at the end of a shift, if it is in bad condition or has been tampered with, it should be set aside for later detailed examination. If the lamp is returned in normal condition, it should be unlocked and the bowl, gauzes, and globe loosened and hung on a rack.

2. All removable parts should be detached and the fount sent to the filling station, which should be separated from the lamp room by fireproof partitions with iron or steel drop doors.

3. The gauze should be brushed inside and out and blown, preferably with compressed air, until all wires are clean, holes freed from dirt, etc. Gauzes with broken wires, enlarged holes, etc., should be crushed and thrown aside. New gauzes should be thoroughly burned to remove the grease in order to prevent flaming on the outside in the presence of an explosive mixture of gas and air.

4. The glass should be wiped with a damp and dried with a clean, dry cloth until free from all oil or moisture. Gaskets should be whole, should fit, and should be free from grit or dirt.

5. The bonnet should be brushed until free from soot or dust.

6. The lower-ring gauze of underdraft lamps should be brushed and if holes or broken wires are found, should be discarded or sent to the repair shop.

7. The igniter should be tested to see if it is in working order, whether it is supplied with tape (match), and whether it fits in its receptacle so that there is no unnecessary opening from the outside to the inside of the lamp.

8. Only enough gasoline should be used to saturate the cotton in the bowl,

and the outside thereof should be wiped clean. The use of special filling tanks is is to be recommended. Naphtha or gasoline of the best quality should be used; its specific gravity should be 0.70 to 0.72.

9. Before the lamp is assembled, the picker should be in condition for use and should not hang below the bottom of the bowl or it may be bent.

10. After the lamp has been assembled the wick should be lit, adjusted to a low flame, and the tightness of the joints tested by blowing against them; leakage will be shown by the wavering of the flame. Compressed-air coils in which the lamp may be placed are recommended.

11. It is advisable to place the lighted lamp in a testing box containing an explosive atmosphere.

**Assembling Lamps.**—Some of the common errors made in assembling lamps are: Leaving out one or both gaskets, or using broken gaskets; placing gaskets in underfed lamps so as to exclude the air from below; leaving out one of the gauzes in double-gauze lamps; placing on top of the glass an expansion ring designed to be placed below it; failing to screw up the bowl sufficiently to make a tight fit between the glass globe and the gaskets; leaving out the igniting device without plugging the stem hole; omitting the deflection rings that prevent air from blowing directly into the lamp; omitting the shield or bonnet; using a defective gauze.

**Failure of Safety Lamps.**—Aside from want of attention to the precautions noted under Assembling Lamps, other reasons for the failure of safety lamps, that is, the ignition by the lamp of the gas outside it are: Exposure to air currents of greater velocity than those for which the lamp was designed; permitting the gas to burn within the lamp until the gauze is red hot and its cooling property destroyed; allowing the lamp to smoke until the pores of the gauze become clogged with soot which will heat and possibly burn, or the coating of the gauze with oil, grease, or coal dust with the same result; holding the lamp on one side so that the flame strikes upon and heats the gauze; allowing the glass to be broken by water dropping on it or in any other way; puncturing the gauze.

A little-recognized cause of lamp failure is the presence of fine and explosive dust in the air. Dust fine enough to pass through the meshes of the gauze may be ignited within the lamp, pass through the gauze and ignite firedamp or dust mixtures outside the lamp. Ashworth found that a Davy lamp, that would not cause an external explosion in 4.5% of gas when the air was moving 370 ft. per min. and was free of dust, passed the flame and caused an explosion in 10 sec. when only the ordinary amount of dust was floating in the air.

**Relighting Stations, Lamp Houses, Etc.**—In large mines it is customary to have lamp stations with a man in attendance to see to the relighting, renewal, etc., of the safety lamps. These stations are usually located at the mouth of each principal entry and are the headquarters of the district fire boss when not on his rounds.

Lamp houses, where safety lamps are received from and delivered to the men and where they are cleaned, repaired, etc., vary in size and completeness of equipment depending upon the number of lamps handled daily At almost all mines it is customary to store the oil in a separate building or in a fireproof room, pumping out the daily requirements into one or more filling tanks in the lamp room proper. At large mines, the lamp room is fitted with revolving brushes and a small air compressor for cleaning and testing lamps, a gas testing box, etc. The system employed in handling lamps varies widely. Generally the lamps are numbered and, where possible, the lamp number and that of the miner's check correspond. Lamps are often turned in merely by being hung on a hook with a number corresponding to that by the lamp, to which they are returned by the lampman in time for the morning shift. In other cases, the lamps are placed in separate compartments in a large cabinet. The compartments are sometimes locked, each miner having his own key, and commonly are open on the back or lamp room side so that the lampmen may easily remove the lamps for cleaning and as easily return them to the right place. Not infrequently, the lamps are handed out personally through a window, in the fashion of ordinary supplies.

## STANDARD TYPES OF SAFETY LAMPS

**Davy Lamp.**—The Davy lamp (Fig. 1, *a*, page 885) consists of the usual oil vessel, or bowl, to which is secured by three standards a gauze cylinder surmounted by a gauze cap (smoke gauze). The cylinder is made of standard wire gauze, is generally $1\frac{1}{2}$ in. in diameter and, with its cap is about 6 in.

high. Air enters all around the lamp, below the flame, and passes out at the top of the gauze as shown by the arrows. In the *fire-boss Davy*, the oil chamber is quite small and the gauze does not exceed 5 in. in height; in the *pocket Davy*, the gauze is 3½ to 4 in. high.
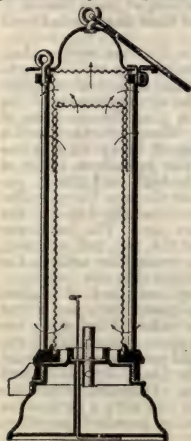


(a) Davy
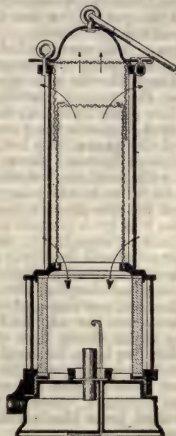
(b) Clanny
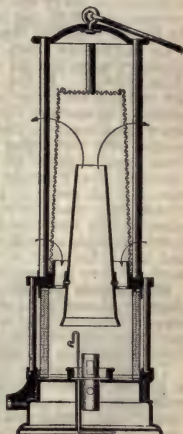
(c) Mueseler
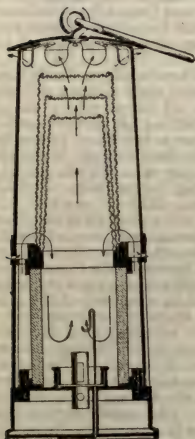
(d) Marsaut
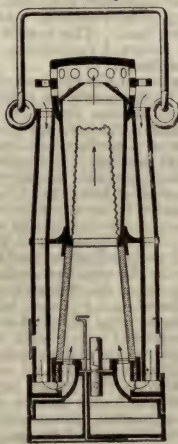
(e) A-H-G

(f) Wolf

Fig. 1

Davy lamps are often provided with an upward-sliding metal shield encircling the gauze for two-thirds of its circumference as a protection in

strong air currents. Owing to the free admission of air these lamps give a good flame for testing and are sensitive to gas. At one time they were in universal use for gas testing, but they are so very unsafe that their use for any purpose is prohibited in Europe and in most of the United States. The unbonneted Davy will pass the flame in air currents moving at more than 6 ft. per sec. or 360 ft. per min. (4 mi. per hr.), a less speed than that maintained by the average fire boss in making his rounds.

In the *tin-can Davy*, the gauze was surrounded by a tin case with a glass window; later, the tin can was replaced by a brass case having an all-around glass window. In the *Jack Davy*, the tin case was replaced by a glass cylinder (either within or without the gauze) reaching the entire length of the gauze. In another form, a low glass cylinder, held in place by a spring or screw, was made to slide up or down over the gauze. When provided with a bonnet over the upper part of the gauze, this last form of Davy was, at one time, very popular in the United States. Some of the numerous types of bonneted Davy lamps have withstood air velocities as high as 1,200 ft. per min. (14 mi. per hr.) and even more in the case of the tin-can Davy. The *Scotch Davy* was distinguished by the greater diameter (nearly 3 in.) of the gauze which was closed at the top by a conical copper cap. The lamp was provided with a hook at the side, instead of a ring at the top, for carrying, and had a flat wick with a small shield beside it as a protection against the wind. Davy lamps are designed to burn animal or vegetable oils only.

**Stephenson Lamp.**—The original Stephenson lamp consisted of a glass chimney closed by a perforated copper cap and surmounted by a perforated copper shield. The space between the cap and the shield filled with the inert products of combustion which, from lack of oxygen, extinguished the flame of any gas burning within the lamp before it could reach the outside air.

The more modern lamp of this name resembles the Davy in appearance as it uses the same gauze, but without the gauze cap. Instead, within the main gauze is a conical glass chimney, closed at the top with a perforated copper cap, which may be raised from the bottom so as to admit air more freely at the base of the flame.

Stephenson lamps are often called *Geordie lamps.*

**Clanny Lamp.**—The original Clanny lamp consisted of a cylindrical metal case (height about 3 times the diameter) the front of which was replaced with glass and in which an ordinary candle burned. Air was forced into the lamps by a bellows, through a water seal. The top of the lamp was closed by a tapering copper cap like an inverted funnel, the opening in which was too small to permit passage of the flame.

The simplest form of the modern Clanny lamp is essentially a Davy lamp in which the lower portion of the gauze is replaced with a glass cylinder about 2 in. high, as in Fig. 1, *b*, page 885. As in all lamps where the air enters above the gauze (see arrows), there are conflicting air currents which interfere with the formation of a perfect cap, cause a tendency to smoke, and unfit the lamp for delicate testing. The unbonneted Clanny is not safe in air currents moving over 8 ft. per sec. or 480 ft. per min. (5½ mi. per hr.). There are, however, many forms of bonneted Clanny lamps, some of which are safe in currents moving 2,000 ft. or more per min. (22¾ mi. per hr.).

**Evan Thomas Lamp.**—There are several lamps of this name, all of which are modifications of the Clanny. The original Evan Thomas lamp was of the underdraft type, being provided with a double steel bonnet above a double glass chimney. Air was drawn in at the top, descended between the bonnets and glasses, and entered the lamp below the flame through gauze-protected ports. The lamp was of excellent illuminating power, but the tendency of the glass to crack by the heat of the gas burning within it led to its abandonment.

The present lamp is a bonneted Clanny with the addition of a device to control the air, which commonly takes the form shown in the Deflector Lamp Fig. 2. In another form, a deflector ring placed around the base of the lamp, throws the entering air upward. The gauze is protected by a very deep gauze cap at the top between which and the top of the bonnet are deflectors throwing the products of combustion downward. The result of this construction is that only a very small part of the gauze is exposed to the action of gas burning within the lamp and the retention of the products of combustion, on the Stephenson principle, in the upper part of the lamp, materially adds to its safety in explosive mixtures. This last form of lamp is said to have safely withstood an explosive current moving at the rate of 3,200 ft. per min. (36.3 mi. per hr.). The lamp burns oil and gives a good light, but has a tendency to smoke.

**Deflector Lamp.**—A deflector lamp is, strictly, not a distinct type but is any one of the standard lamps to which is added a device known as a deflector that is designed to control the direction of the air currents. As shown in Fig. 2, where it is applied to a bonneted two-gauze Marsaut lamp, the deflector consists of a brass shield *a* midway between the outer gauze and the bonnet and about 1½ in. high. About ⅓ in. above the top of this shield is the bottom of an angle ring *b*. This ring fits closely to the gauze, its top flange entirely closing the space between the gauze and the bonnet. The air follows the course shown by the arrows and is thrown directly upon the flame. As the air is heated by passing over the warm deflector and gauze, the draft, rate of combustion of oil and, consequently the illumination are improved. The deflector is said to fit the lamp for burning in air containing a much higher percentage of carbon dioxide than is otherwise possible.



FIG. 2

**Bull's Eye, or Mauchline, Lamp.**—The bull's eye, or Mauchline, lamp is a Clanny in which the glass is replaced by a metal cylinder fitted up like a bull's eye lantern, with a lense at one end and a reflector behind the flame at the other. In each side, at the height of the flame, is a gauze-protected circular port through which gas caps may be observed. The lamp is not generally bonneted, throws a good light directly ahead and was designed, primarily, for the use of mine surveyors.

**Marsaut Lamp.**—The Marsaut lamp, Fig. 1, *d*, is a Clanny lamp with two or, usually, three conical gauzes to afford protection against strong air currents and, particularly, internal explosions. The lamp has a tendency to smoke and heats quickly, but the confinement of the products of combustion between the gauzes, on the Stephenson principle, adds much to the safety of the lamp. The unbonneted lamp is considered safe in air currents moving not faster than 600 ft. per min. (7 mi. per hr.). The bonneted Marsaut is one of the safest of lamps and easily withstands velocities of 3,000 ft. per min. (34 mi. per hr.), and more. The table on page 880 shows that the two-gauze lamp has about 22 % more illuminating power than that with three gauzes.

**Mueseler Lamp.**—The Mueseler Lamp, Fig. 1, *c*, page 885, is a Clanny lamp with an interior conical sheet-iron chimney which increases the draft, separates the products of combustion from the entering air, increases the security of the lamp against internal explosions and decreases its tendency to flame. The unbonneted type, shown in the figure, may be considered safe in an air current having a velocity not greater than 600 ft. per min. (7 mi. per hr.).

There are two types of the bonneted Mueseler, the Belgian and the English, which differ only in the dimensions of the chimney. In the former, which is the official or approved lamp in Belgium, the chimney must have a total height of 4.6 in. and must be so placed that its bottom is .85 in. above the top of the wick tube and its top 3.55 in. above the horizontal gauze which, surrounding the chimney at the level of the top of the glass, divides the upper from the lower part of the lamp. The object of the horizontal gauze is to prevent any burning gas passing upwards between the chimney and the main gauze. The chimney of the English Mueseler is set higher above the flame than that of the Belgian and the area of its upper end is much larger. The Belgian lamp, in the tests of the Royal Accidents Commission (English), was extinguished without harm after a few seconds exposure to explosive air currents moving at the rate of 2,880 ft. per min. (32.7 mi. per hr.), while, in every instance, the English lamp caused an explosion; in fact, the English lamp failed when the velocity exceeded 1,000 ft. per min. (11.3 mi. per hr.). The Mueseler lamp is sensitive to air currents striking it obliquely, and these sometimes blow the air circulating in the lamp from its regular course with danger of an explosion. The lamp is easily extinguished if held at a slight angle from the vertical as the products of combustion then pass upwards
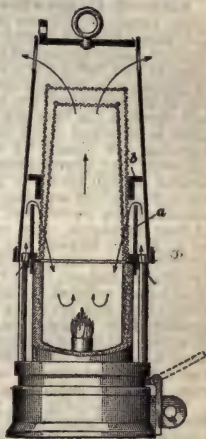
between the chimney and the gauze and, mixing with the entering air, smother the flame. The statement that the Mueseler lamp is safe in air currents moving 100 ft. per sec., or 68.1 mi. per hr., seems hardly credible.

**Ashworth-Hepplewhite-Gray Lamp.**—The Ashworth-Hepplewhite-Gray lamp, familiarly known as the A-H-G or as the Gray lamp, is shown in Fig. 1, *e*, page 885. The lamp is a bonneted Clanny with underdraft, air entering through gauze-protected ports or through a gauze ring entirely surrounding the lamp at and below the level of the flame. Admission of air to the gauze ring or gas ports is through the standards which are hollow. The openings at the top of the standards are closed by a plate which may be revolved over them, and those at the base are closed by slides. When used for testing, the cover plate is revolved until the tops of the standards are open and the bottom openings are closed by slipping the slides down over them, the air then following the course shown by the arrows. This construction permits the testing of thin layers of gas near the roof. When used as a working lamp, the top openings are closed by the cover plate and the slides at the base of the standards are pushed up. In some types of this lamp there are three instead of four standards, only one of which is hollow, the others being of thin wire so as not to impede the light. The conical glass and short conical gauze permit the upward diffusion of light. In some cases the steel bonnet is cylindrical instead of as in the figure, but it is always closed by a truncated cone which reduces the area of the top of the opening and better controls the circulation and prevents downward currents. The opening in this cone is protected by a perforated dome or cap which, in some lamps, is extended downwards like an ordinary bonnet to the level of the top of the glass. The lamp is intended for burning colza or similar oils, has high illuminating power (see following table), is said to be safe in air currents traveling 6,000 ft. per min. (68.1 mi. per hr.), and is generally considered a most excellent lamp.

**Wolf Lamp.**—Fig. 1, *f*, page 885, shows the Wolf lamp as used in the United States. The lamp burns naphtha or gasoline and, hence, assisted by its construction, gives a maximum of light and permits the detection of small percentages of methane. It uses a magnetic lock (see Locks for Safety Lamps), has an internal ignition device (see first paragraph on Igniters, or Relighters), and has an underdraft.

The lamp is of the Clanny type with the double conical gauzes of the Marsaut, and usually has a corrugated bonnet, the openings in which are so arranged that air currents strike tangentially and not directly upon the gauze. Air for combustion enters through gauzes or a gauze ring at the base of the glass chimney, the openings being protected from direct contact of air currents by a baffle ring. The wick is held in a sheath moving within the wick tube and may be adjusted by turning the screw at the base of the lamp.

The baffle ring and bonnet are made in various forms so that the external appearance of Wolf lamps varies considerably. Some of these lamps have the overdraft of the Clanny; in some the igniter box is circular; in others, a lead in place of a magnetic lock is used; some are arranged to burn colza; others burn alcohol, have the Chesneau scale attachment, and are adapted for testing for small percentages of gas. While such variations and adaptations are common in the lamp as used in Europe, they are practically unknown in the United States.

The Wolf is a most excellent lamp, and is safe in air currents containing 9 % of methane when moving 3,600 ft. per min. (41 mi. per hr.).

**Protector Lamp.**—The Protector lamp is not a separate type but is a modified Clanny, Marsaut, or Mueseler lamp designed to burn colzaline, a light oil obtained by the purification of colza; and is provided with an electric igniter. The wick tube, or burner, is double with a narrow annular space between the two tubes. In the inner tube is a stationary cotton wick extending down into the bowl which contains a piece of sponge for absorbing and retaining the oil fuel. The lamp is lit electrically by means of a platinum wire connected to two terminals, one of which is connected with a contact on the bottom of the bowl, the framework of the lamp forming the return circuit. A low-tension current from a battery is sufficiently powerful to heat the wire to the ignition point of the colzaline vapor, which, as it forms, passes up the annular space between the inner and outer wick tubes and is burned around the head of the stationary wick. The flame is regulated by screwing the bottom of the lamp up or down, and is extinguished if an attempt is made to unscrew the bottom completely, as explained under Locks for Safety Lamps (Protector Lock).

**Hailwood Lamp.**—The Hailwood lamp is designed to burn naphtha or

gasoline. It is essentially a bonneted Clanny lamp with the double gauze of the Marsaut and an underdraft protected by a baffle ring. The lamp is provided, as desired, with either the magnetic or the compressed-air lock (described under Locks for Safety Lamps). The lamp is relit in a special gas-proof, gauze-protected chamber by means of an electric spark which jumps the gap between the top of the wick tube and the end of an upright insulated copper wire through which the current enters the lamp. The wick tube is flat and is surrounded by a sliding sheath by means of which the lampman may adjust the flame as desired but which is so arranged that it is impossible for the miner to raise the flame to such a height that it may be drawn through the gauze. The ordinary, or burning, wick is fed with naphtha by a permanent feeding wick which extends down into the bowl and which is pressed against the ordinary wick by a spring. Owing to its construction and to the illuminant used, the lamp gives a most excellent light and also permits the detection of low percentages of methane. In the Belgian government tests the lamp successfully withstood air currents containing 8 % of methane when moving 900 m., or 2,952 ft., per min. (33.5 mi. per hr.), regardless of the angle at which the air struck the lamp.

The Hailwood oil-burning lamp differs in a few details from the naphtha-burning lamp just described. The picker, which is of copper and through which an electric current may be passed, is provided with two prongs or arms, one of which is used to snuff or trim the lamp wick and the other to convey the current to the wick tube when it is desired to relight the lamp. Other lamps of this name, designed more particularly for gas testing, are of the Mueseler type but with a glass instead of a metal chimney on the back and inner side of which is a piece of metal as a background against which to better view the gas caps.

## SPECIAL TYPES OF SAFETY LAMPS

**Clowes Hydrogen Lamp.**—The Clowes hydrogen lamp is essentially an A-H-G lamp with a somewhat taller chimney and an attached device for burning hydrogen. A seamless copper tube is inserted in the bowl beside the wick tube and is connected either below or at the side of the lamp with a small portable cylinder *a* (Fig. 3) containing hydrogen. The cylinder, which is about 5 in. long and 1 in. in diameter, is attached to the lamp by the clip *b* and the screw *e*. In testing for gas, the valve *d* is opened, the hydrogen enters the lamp and is ignited at the mouth of its burner by the oil flame, which is then pulled down by the picker until it is extinguished. By means of the valve *d*, which regulates the supply of hydrogen, the height of the testing flame is adjusted until its top coincides with a scale not shown in the figure, the adjustment being made in air free from methane. The scale consists of a number of cross-bars in a ladder-like frame placed in front of the flame. The heights of the cross-bars (which appear as dark lines against the flame) mark the heights of the gas caps corresponding to various percentages of pure methane burning in pure air. The hydrogena ttachment is designed to render possible the detection of $\frac{1}{4}$ to 3 % of gas; for higher percentages, the oil flame is used. With $\frac{1}{4}$ % of gas the cap is $\frac{1}{2}$ in. high, and with 2 % of gas about $1\frac{1}{2}$ in.
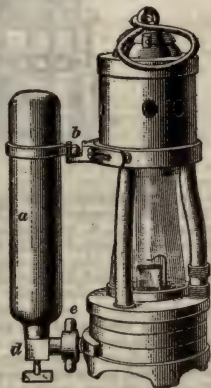


FIG. 3
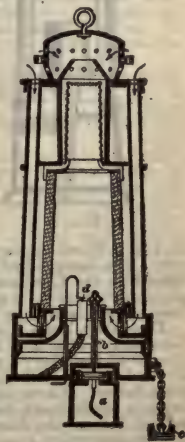


FIG. 4

**Stokes Alcohol Lamp.**—The Stokes alcohol lamp, Fig. 4, is a modification of the A-H-G lamp in which a small alcohol flame is introduced beside the

regular oil flame. The small alcohol bowl *a* is screwed beneath the regular oil bowl and is provided with a long wick tube *b*. When the screw plug *c* is removed and the alcohol lamp screwed in place its wick is lit by the oil flame *d* which is then extinguished by drawing down the wick with the picker. In other respects the lamp is the same as the Clowes. The alcohol is not as persistent as the hydrogen flame and is more easily extinguished in gas; on the other hand, it is more stable in gas than the oil flame but is more easily blown out by the wind. The lamp is designed to detect from ⅓ to 3 % of gas.

**Pieler Lamp.**—The Pieler lamp, Fig. 5, is essentially a Davy lamp with an exceptionally tall gauze, and is arranged to burn alcohol. The flame is surrounded by a short conical metal bonnet or shade reaching up about 2 in. from the bottom of the gauze, and into coincidence with the top of which the tip of the lamp flame is brought by adjusting in fresh air. Affixed to the lamp standards is a slotted metal plate, each slot marking the height of gas cap corresponding to a certain percentage of pure gas in air. The lamp in the cut is designed to indicate percentages of gas from ¼ to 1½ %, increasing



FIG. 5                          FIG. 6
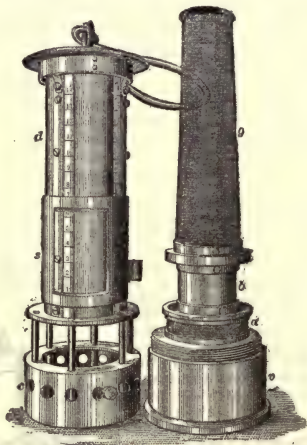
by ¼ %. When the cap reaches the top of the scale plate, 2 % of gas is present. The lamp illustrated is unbonneted. The bonneted Pieler greatly resembles the Chesneau lamp, Fig. 6, in external appearance, as the scale plate is replaced by a glass plate inserted in the bonnet, on which are etched lines corresponding to the height of the flame caps.

The Pieler lamp is extensively used in Austria (where it originated) and Germany, but is of limited application in the United States. The lamp, like all of the Davy type, is unsafe in air currents of any but low velocity. Beyond 2 % of gas, the lamp is useless as the gauze, which is 8 in. high fills with flame. Where the gaseous conditions are unknown, it is advisable to make a preliminary test with an ordinary lamp, as the Pieler is practically certain to pass the flame if placed in an explosive mixture of gas and air.

The absorptive power of the cotton, with which the bowl of the lamp is filled, is commonly great enough to modify the height of the flame cap and consequently to affect the accuracy of the determinations. Furthermore, after a determination has been made, the heat remaining in the gauze assists in the volatilization of the naturally volatile alcohol, so that for 20 to 30 min., or until it thoroughly cools down, the lamp cannot be used for a second test as there will be an artificial atmosphere of alcohol vapor within the gauze.

**Chesneau Lamp.**—The Chesneau lamp, Fig. 6, of French origin, is a bonneted Clanny with underdraft and is designed to burn alcohol. In the

figure, *c* is one of the openings through which air has access to the gauze ring surrounding the lamp below the wick tube. The cylinder *a* plays the same part as the conical shield in the Pieler lamp, and the lamp flame, when a test is to be made, is adjusted to the level of its top, that is, to the base of the main gauze. The gauzes in this lamp have 1,264 openings per sq. in. In the front of the bonnet is inserted a mica window *d* with a scale on either side. One scale is graduated in millimeters for measuring the height of the gas cap, and the other is graduated in the corresponding percentages of gas. A sliding shield *d* can be adjusted to the exact height of the cap, thus permitting of much more precise readings.

The Chesneau lamp is much superior to the Pieler in that it is safe in air currents moving over 2,000 ft. per min., and requires but 30 to 90 sec. to cool down between tests; but, as in the Pieler lamp, the accuracy of the tests is somewhat interfered with by the absorptive power of the cotton in the bowl. It should be noted, however, that in the Chesneau as in the Pieler lamp so long as the physical condition of the cotton is the same as when the lamp was standardized, its indications are accurate.

**Stuchlick Acetylene Safety Lamp.**—The Stuchlick acetylene safety lamp, an Austrian invention, is essentially a Clanny lamp designed to generate and burn acetylene gas. The bowl of the lamp is double and consists of an outer carbide box in a groove in which slides an interior water vessel, the two being connected by a flexible siphon tube. The water vessel can be raised and lowered within the carbide box, and is held in position by a spring and screw. The lamp, which weighs about 3 lb., is held together by screwing on the bowl.

After the carbide box is two-thirds filled with calcium carbide, the water vessel is pushed down to its lowest position and filled with water. The lamp is then assembled. In the lowest position of the water box the level of the water in it is below that of the opening into the carbide box, and the generation of acetylene is not possible. As the water box is raised, water flows through the siphon tube into the carbide box and the acetylene then given off enters the burner through a small pipe within the flexible tube. Any excess of gas is carried back into the water vessel and thence to the open air, the flexible pipe, which has a hydraulic joint, acting as a safety valve. In testing for gas, the flame is adjusted for height by a screw which moves the gas nipple in the burner.

One per cent. of methane in air is easily detected by the green halo which surrounds the acetylene flame, which, in dangerous percentages of gas, is extinguished by the products of combustion. One-third pound of calcium carbide and one filling of the water box will furnish a light for 8 hr. at a less cost than benzene.

**Tombelaine Acetylene Safety Lamp.**—The Tombelaine lamp is an underdraft and bonneted Clanny with double gauzes, the inner of which extends downward over the flame similarly to the Mueseler chimney so that any sudden enlargement of the acetylene flame will not break the glass. The bowl of the lamp is double, the inner cylinder holding the carbide and the outer the water. In use, the inner cylinder after being filled with carbide is screwed into place and the bottom of the lamp placed in water which flows into the water holder through openings designed for the purpose. The amount of water entering the carbide chamber, the flow of acetylene to the burner, and the height of flame are regulated by a thumbscrew in the base of the lamp. The lamp weighs 1.5 kg. (3.3 lb.), has an illumination of about 6 c.p., and will burn 11 hr., with a slightly greater consumption of acetylene than the Stuchlick lamp.

## GAS INDICATORS AND GAS-SIGNALING DEVICES

**Use and Principles.**—Gas indicators are designed to more exactly determine the amount of methane in mine air than does the ordinary safety lamp. These devices are based on the use of some one of the well-known physical or chemical properties of gases, such as: The difference in the density or in the rate of diffusion of methane and air; the heat generated by the burning of methane or the contraction in volume of its products of combustion; the increased brilliancy of platinum or palladium wire, or their increased electrical resistance when heated in the presence of methane; the absorption of methane by platinum or palladium sponge, etc.

In addition to the foregoing are colored glass, various chemicals, loops of wire, etc., the use of which is intended to make more distinctly visible the gas cap formed in the ordinary safety lamp.

While, in the main, based upon correct principles, these devices, with few exceptions, have been discarded as being too cumbersome or costly; as requiring too much time or skill in their manipulation; or as being too inaccurate under practical mining conditions while meeting the perfect ones in the laboratory. This last objection is the most serious, and it is clear that any apparatus which is standardized under certain atmospheric conditions and for pure methane and pure air, as it would be in the laboratory, cannot give correct readings in the mine where the atmospheric conditions may be and usually are widely different and where, above all, the air is certain to be more or less deficient in oxygen and contaminated with nitrogen, carbon dioxide, etc.

In a signaling device or system numerous gas indicators placed at points in the workings where methane is apt to accumulate are electrically connected with some central station, as the superintendent's office. When methane is present, the indicators become operative, closing their respective circuits and thus ringing bells or moving pointers at the central station.

Gas-signaling systems have never proven satisfactory. The indicators fail for the reasons previously stated and, as they are not instantaneous in their action, never give warning of gas until some time after it has accumulated. They are particularly at fault in that they announce the presence of gas only at points where an indicator is placed and, as it is impossible to entirely cover the workings with indicators, a dangerous accumulation of gas may exist within a few feet of one of these appliances. Further, indicators of the type that glow in the presence of methane are dangerous, even when enclosed in safety-lamp gauze.

Experience thus far has shown that for the detection of gas at the face nothing is better adapted than a standard safety lamp, and for accurate percentage determinations, chemical analysis should be relied upon.

**Liveing Indicator.**—The Liveing indicator consists of two coils or spirals of platinum wire of equal electrical resistance enclosed in separate glass-ended cylinders set facing each other and about 4 in. apart. One of the cylinders, which is tightly sealed, is filled with pure air, and the other is made of standard safety-lamp gauze. Between the spirals is a wedge-shaped mirror for reflecting their image upward through the small glass window of the box containing the apparatus. By applying suction (by the mouth or a small air pump) to the end of a rubber tube attached to the box, mine air is drawn into the apparatus through another tube which is made long enough to reach places not readily accessible, as cavities in the roof, etc.

When an electric current, generated by turning the handle of a small magneto placed in the bottom of the box, is passed through the spirals they glow with equal intensity, if no methane is present. If, however, gas is present, the spiral within the gauze cylinder will glow more brightly. The mirror is then moved until the images of the two spirals as viewed through the window appear of equal intensity, when the percentage of gas may be read from a graduated scale over which the mirror passes.

Repeated heating of the gauze-encased spiral alters its electrical conductivity to such an extent that it soon becomes necessary to adjust the zero point of the scale in fresh air before a test is made. This is done by heating up the coils and shifting the mirror until the images appear of equal brightness; the zero of the scale is then made to coincide with the position of the mirror.

**Coquillon's Indicator.**—Coquillon's indicator consists of a glass tube in which is a loop of palladium wire that can be heated to incandescence by a small battery contained within the same box as the tube. If a measured quantity of mine air containing methane is introduced into the tube and the electric current is applied, all the gas will be burned and the contraction in volume of the products of combustion is a measure of the percentage of gas present. This indicator is really an apparatus for making a rapid analysis of mine air.

**Le Chatelier's Indicator.**—Le Chatelier's indicator, while differing from Coquillon's in a few minor details, is chiefly distinct in the use of platinum for palladium wire.

The indicators devised by Maurice, Monier, and some others are based upon the same principle as Le Chatelier's.

**Turquand's Indicator.**—Turquand's indicator consists of a glass U-tube of fine bore about one-half filled with mercury. The ends of the tube are inserted in a metal block in such a way that there is a small space between the top of each arm of the tube and the bottom of a porous stopper inserted

in the corresponding holes in the block. In one of these spaces is an absorbent to distinguish between methane and carbon dioxide, and in the other is a coil of palladium wire that can be made to glow by passing an electric current through it. Normally, the mercury stands at the same level in the two arms of the tube, but when methane or other hydrocarbon gases that are absorbed by hot palladium wire enter through the porous stopper, heat is generated and the thread of mercury is pushed down one leg and up the other, the difference in level of the columns of mercury being a measure of the amount of gas present.

The Swan indicator is of this type, but employs the expansion of a column of mercury by the heat liberated by the absorption of methane by a glowing platinum wire, to show the percentage of gas present upon a graduated scale.

**Ralph's Indicator.**—Ralph's indicator employs a differential galvanometer, in one coil of which is a piece of platinum wire enclosed in standard safety-lamp gauze so that it may safely be exposed in air containing methane. When no gas is present and an electric current is passed through the apparatus, the needle or indicator of the galvanometer is not deflected as the resistance of the two coils is the same. When methane is present, its absorption by the platinum wire increases the resistance in that coil, and the needle is deflected by an amount proportional to the percentage of gas in the air.

This indicator is the basis of some signaling systems in which the variation in resistance is made to extinguish a distant light, to place a distant buzzer in action, etc.

**Garforth-Walker Indicator.**—That the amount of methane required to make a platinum wire glow when an electric current is passed through it is proportional to the thickness of the wire is made the basis of an indicator devised by Mr. S. F. Walker for Sir William Garforth. Several small glass tubes are fixed inside the protecting glass of any portable electric lamp. The tubes are arranged to be easily replaced, are enclosed in safety-lamp gauze, and contain platinum wire of a gauge or thickness proportional to the percentage of gas in air that each particular tube is intended to indicate. At the entrance to each tube is a self-closing valve that can be pushed open by the metal nozzle of a rubber bulb which is filled with the mine air to be tested, the insertion of the nozzle automatically switching on the current to the wire. After sufficient time is allowed for the wire to reach the proper temperature, the contents of the bulb are squeezed out, the wire glowing if the percentage of methane corresponding to the particular tube is present. In testing, the tube with the coarsest wire, which indicates the greatest percentage of gas, is used first; if the percentage of gas is not great enough to make this wire glow, tubes with successively finer wires are used until one is found that is sensitive to the percentage of gas present. Palladium may be used to advantage in place of platinum wire. This apparatus is still in the experimental stage.

**Ansell's Indicator.**—Ansell's indicator consists of a cylindrical chamber, one side of which is formed by a movable diaphragm of porous unglazed earthenware and the other by a stationary graduated dial or plate to the center of which is pivoted a hand or needle, the apparatus somewhat resembling an aneroid barometer in appearance. When exposed to a mixture of methane and air, the gas diffuses through the porous diaphragm more rapidly than the air passes out, causing a difference in pressure on the two sides with a resultant outward movement of the diaphragm. By means of mechanism, the motion of the diaphragm is made to move the needle over the circular scale on the edge of the dial on which the percentage of methane may be read off. After making a test, the needle must be set back to zero by exposing the apparatus in pure air, a process requiring considerable time.

In some indicators of this type the diaphragm is placed midway between the ends, one of which is porous while the other carries the scale.

The action of all indicators of this type becomes absolutely unreliable if a deficiency of oxygen or an excess of carbon dioxide or moisture or a rise or fall of temperature affects the density of the air, as the rate of diffusion is then changed from that prevailing when the instrument was standardized. Clowes has shown that when 4 % of coal gas and 3 % of methane was present, the indicator showed but 1.71 % of the latter gas; also, that when absolutely pure air was heated, the indicator showed it to contain 4 % of gas, and that when the same air was cooled the indicator gave a minus reading of the same amount.

The indicators devised by Libin, McCutcheon, Webster, and some others are of this type.

**William's Methanometer.**—William's methanometer, which resembles the previously mentioned Ralph indicator, consists of a pair of thermo-electric couples connected with a galvanometer. Each couple is enclosed in porous material, that surrounding one of them being impregnated with platinum-black to absorb methane. The couples may be brought to the temperature at which platinum-black is most sensitive to methane by means of a battery current and, if no gas is present, the needle of the galvanometer remains stationary. If, however, gas exists, the temperature of the couple surrounded by platinum-black will be raised through its absorption of methane, its resistance will be changed and the needle of the galvanometer will be deflected in proportion to the amount of gas present. The indicator is made in portable form for fire bosses and may be used as the basis of a signaling system when connected to the necessary wires for transmitting the indications of the needle to a distance.

**Aitkin's Indicator.**—In the Aitkin indicator two thermometers are suspended side by side in the same frame, the bulb of one being covered with platinum-black (spongy platinum). In pure air the readings of the thermometers will be the same, but if methane is present its absorption by the platinum-black causes a rise in temperature which is indicated by the proper thermometer and which is a measure of the percentage of gas. The apparatus is not accurate in that the platinum-black rapidly deteriorates through the absorption of moisture and through the deposition of dust on its surface and, further, because platinum is insensitive to low percentages of gas when cold.

An indicator of this type was at one time attached to the Sussmann portable electric lamp.

**Beard-Mackie Sight Indicator.**—The Beard-Mackie sight indicator is a device for making visible and more accurately measuring the height of the caps made by small percentages of gas. While it may be applied to any safety lamp, it is commonly used in connection with the Davy or with those having an underdraft. The indicator, which resembles a ladder in appearance, consists of two upright standards between which are strung a series of fine wires, the lowest being of brass and the others of platinum. The standards are soldered to a brass washer which fits over the neck of the wick tube or, better, when it is pivoted so that the indicator may be swung into the flame only when a test is to be made. The latter construction prevents the sooting of the wires through continued contact with the flame, which is the greatest drawback to the use of this indicator. In testing, the lamp flame is adjusted in fresh air until the lowest, or standard, wire is just aglow. The platinum wires are so spaced that when $\frac{1}{2}$% of methane is present the lowest of them will glow, when 1% is present both the first and second, and similarly up to the last wire which indicates 3% of gas.

**Brigg's Wire Loop.**—Brigg's wire loop which is intended to delumine, or remove the color from the lamp flame in order that the gas cap may be more distinctly visible, may be applied to any safety lamp and is not patented. The device consists of a piece of 22-gauge copper wire bent into a loop, the longer axis of which is equal in width to the wick of the safety lamp. The loop is supported upon an upright brass standard extending through the lamp bowl so that it may be swung in or out of the flame. In testing, the flame is left at its normal working height. As soon as the loop is swung into it the flame, this, without being reduced in size, loses its yellow color and the cap, if methane is present, in as little as $\frac{1}{4}$% of gas becomes visible and that of $\frac{1}{2}$% of gas may be measured.

By dipping the loop in a solution of chloride of zinc a green coloration is imparted to the lamp flame, to the so-called fuel cap (if present) and, to a less extent, to the gas cap. This action improves the indications considerably, but the chloride of zinc soon burns off. The loop, however, sometimes gives a fairly strong green flame without this treatment, especially if it has not been used for some days, and it generally gives a very faint one.

The flame may be intensified by introducing 1 or $1\frac{1}{2}$% of carbon tetrachloride into the lamp oil or naphtha, at a cost of roughly $\frac{1}{4}$ c. a shift. The tetrachloride does not affect the working flame, but when the loop is moved into it the flame becomes green from the copper chloride and the gas cap, if present, a bright blue from the copper oxide formed.

**Cuninghame-Cadbury Indicator.**—In the Cuninghame-Cadbury indicator a small sheet of asbestos is secured on a holder in much the same way as the Brigg's wire loop so that it may be moved in or out of the normal, or but slightly reduced, flame of the lamp. The asbestos, which is twice steeped

in a strong solution of carbonate of soda, is placed about $\frac{1}{8}$ in. above the wick, and enters the flame for about two-thirds the thickness of the same. In pure air a slight fuzzy yellow, or orange, halo will appear around the flame toward the upper part of the asbestos. If methane is present, the halo will be surmounted by a yellowish conical cap the length and distinctness of which depends on the per cent. of gas. In some cases a perpendicular scale for measuring the he ght of gas caps is attached to the snuffer pin so that it can be temporarily moved into position when testing. It is stated that the use of this device, which is not patented and may be attached to any safety lamp, permits the detection of as little as $\frac{1}{2}$ % of gas.

**Colored Glass Indicators.**—In order to cut off the yellow light and thus render the cap more distinctly visible, Mr. A. L. Steavenson suggests that a sheet of blue glass be interposed between the flame and the eye, or that a pair of blue glass spectacles be worn while testing.

To do away with the reflection of the flame and cap in the polished glass of the ordinary safety lamp which often interferes with the accuracy of the test, a strip of dull-surfaced metal may be placed back of the flame. The same result may be obtained by making a strip of soot or smoke down one side of the glass after it has been cleaned. As an additional precaution, all metallic surfaces, such as the bowl, standards, etc., that can in any way reflect the flame may be given a dull finish.

**Forbes Indicator.**—The Forbes indicator consists of a brass tube about 6 in. long in which moves a piston the position of which, in terms of the per cent. of methane in the air, is indicated by a pointer and scale. In the mouth of the tube is fixed a tuning fork that makes 512 vibrations a second and emits a corresponding sound when fresh air is forced through the tube by moving the piston. If the density of the air is lowered by the presence of methane, the length of stroke of the piston and the corresponding volume of air required to produce a note of equal depth is not the same as in pure air, the difference being measurable on the scale. In making tests an allowance is necessary for changes in the density of the air due to variations in temperature.

**Firedamp Whistle.**—In the firedamp whistle the attempt is made to estimate the percentage of methane in the air by the difference in the sound emitted by a metal pipe when air currents of different densities are blown through it. When pure air is used, the pipe, which is about 10 in. long and $2\frac{1}{2}$ in. in diameter, emits a certain tone, but as the density of the mine air decreases as the proportion of methane in it increases, the tones become higher and tremulous. The device, which is an adaptation of the Forbes indicator, can hardly be considered reliable, as changes in temperature and pressure, or a deficiency in oxygen, or an increase in carbon dioxide or nitrogen, will affect the density of the air as well as changes in the methane content.

**Hardy Indicator.**—In the Hardy indicator there are two separate pipes, alike in every respect, one of which is blown with pure air and the other with the mine air to be tested. The number of vibrations per second made by the pipes is made the measure of the percentage of methane present. The same objections apply to this indicator as to the Forbes and to the firedamp whistle.

**Shaw Gas-testing Machine.**—In the Shaw machine, a graduated beam operated by a crank and connecting arm raises and lowers the pistons in two vertical cylinders known as the air and gas cylinders, respectively. The larger, or air, cylinder is fixed in position, while the smaller, or gas, cylinder may be shifted along a graduated slide in such a way that the length of travel of its piston and consequently the volume of its discharge may be varied. Both cylinders discharge into a small combustion cylinder in front of which a gas jet is burning and one end of which is movable outward against a gong. As a preliminary operation it is necessary to determine the percentage of some readily available gas (as illuminating gas) that must be mixed with pure air in order to produce an explosion. To do this, pure air and pure illuminating gas are pumped from their respective cylinders until the mixture delivered to the combustion chamber is feebly explosive as evidenced by a slight ringing of the gong when the mixture is lit and exploded by the gas jet. This requires several determinations, much shifting of the gas cylinder to secure the proper ratio of gas to air, and consumes a good deal of time.

If mine air in place of pure air is pumped from the air cylinder into the combustion chamber, less and less illuminating gas will be required to make

the mixture explosive as the percentage of methane in the air increases. A rubber bag containing mine air is connected with the air cylinder and the position of the gas cylinder shifted until the mixture from the two cylinders as delivered to the combustion chamber is of the same explosive intensity as the mixture of pure air and gas. From the position of the gas cylinder in either case may be calculated the percentage of illuminating gas required to produce an explosion both with pure and with mine air. A simple proportion then gives the percentage of methane in the mine air.

This apparatus is bulky, expensive, and slow, but at one time had a considerable following through intensive advertising. It is no longer used.

**Hauger and Pescheux Gas-signaling Apparatus.**—In the signaling apparatus devised by Hauger and Pescheux, an extremely sensitive balance carries on one end a tightly sealed vessel of pure air and on the other a tray of the same area and weight as the air vessel. If the composition of the atmosphere is changed in any way, its density will vary according to the percentage of gas invading it and, as the composition of the air in the closed vessel is unaltered, the equilibrium of the balance will be destroyed. If the gases invading the atmosphere are lighter than air the air chamber will descend, but if they are heavier than air it will ascend. Attached to the beam is a needle dipping in a cup of mercury which, immediately on disturbance of the balance, closes an electric circuit that may be made to ring a bell or set a danger signal at any distance from the apparatus. To allow for the disturbing influences of changes in atmospheric temperature and pressure, two compensators are attached to the beam. One of these is an aneroid barometer which acts on a multiplying lever which in turn changes the position of a rider which slides along a thread. To compensate for changes in temperature, a bimetallic spiral is made to act on a lever which, in its turn, shifts the position of a rider on the beam.

**Low Gas-signaling Apparatus.**—The low signaling apparatus consists of two wires arranged in V-shape and held in tension by a bar and spring. One wire is of platinum and carries at short intervals lumps of spongy platinum; the other is of iron and brass in such proportions that its coefficient of expansion is the same as that of the platinum for equal changes of temperature. As long as the wires contract and expand equally they are kept in tension by the spring referred to, but should the platinum wire sag by reason of its more rapid expansion, an arrangement of springs and multiplying mechanism closes an electric circuit which may be made to give a determined signal at any distance. When the apparatus is exposed to hydrocarbon gases they are absorbed by the spongy platinum, the platinum wire is heated, expands and sags, and rings the alarm as stated.

## ELECTRIC SAFETY LAMPS

The ordinary safety lamp is subject to many disadvantages, and several explosions have been traced to imperfections in these lamps or to their unintentional breakage. To obviate these disadvantages many varieties and models of so-called electric safety, lamps all of which employ a small storage battery, have been devised.

Such lamps must provide safety against ignition of mine gases, a steady and uninterrupted production of light for at least one shift and should be of practically foolproof construction. Since such lamps are exposed to extremely rough usage in the hands of inexperienced men, even slight mechanical or electrical weaknesses may result in a total failure of the light supply. Furthermore, to guard against the opening of the lamp while in the mine most such lamps are provided with some means, such as a lock, which prevents anyone from tampering with or dismounting the apparatus. These locks may be either in the nature of an ordinary padlock or a type of magnetic lock such as is often used on naphtha-burning safety lamps.

**Points of Danger in an Electric Safety Lamp.**—Experiments both in this country and in Germany have demonstrated that the only point of danger in a portable electric lamp is the glowing filament. Sparks obtained by the making or breaking of the electric circuit are not of sufficient strength to ignite an explosive mixture. It is unnecessary, therefore, to provide against sparking at the switch or other connections between the battery and the lamp. The lamp filament under ordinary conditions is enclosed in a vacuum bulb, and the danger of igniting mine gases is present only when this bulb is broken without rupturing the filament. Several methods may be employed for preventing such a contingency. The two most commonly used, however, are a spring which instantly forces the lamp out of its socket, thus instantly

breaking electrical connections, and a fuse which blows the moment the bulb is fractured.

**Types of Electric Safety Lamps.**—There are two general types of electric safety lamp. These may be designated as hand lamps and cap lamps; the former strongly resembles in appearance an oil-burning safety lamp, while the latter is modelled after the open-flame cap lamp extensively employed in this country. Although many varieties of each have been placed on the market, all models of the same type strongly resemble each other and a description of one will apply with only minor variations as to constructional features to all lamps of its particular type.

**The Ceag Lamp.**—The Ceag lamp won first prize in the competition conducted by the British government in 1912, since which time it has been accepted by practically all European governments and has been approved by the U. S. Bureau of Mines. This lamp is illustrated in Fig. 7. As described by Mr. H. O. Swoboda its construction is as follows:

The bulb is covered with a heavy glass dome $D$, which is protected by four heavy steel rods $H$, held together by a sheet-steel roof $I$. A substantial hook is attached to this roof. Thus the miner can either stand the lamp on the ground or hang it to a post in the immediate neighborhood of his working place. The bottom part, made of heavy corrugated galvanized sheet steel, contains the storage battery. By turning the upper part on the lower the miner can turn the light on and off. The incandescent lamp rests in a socket which is pressed upward by a spiral spring $O$ against another spring $P$ between the bulb and the glass dome $D$, providing a complete spring support and preventing breakage even with the most severe shock. Electric connection is established for one pole through the socket spring $O$ and for the other pole by another smaller spring $E$ inside the socket spring and insulated from it. In case the bulb breaks the socket spring pushes the socket upward, and as the inner spring does not expand as much as the socket spring the circuit is interrupted. Another safety device has been added, but it is not shown in this illustration. This consists of a fuse which blows the moment the bulb of the incandescent lamp is broken. This eliminates the possibility of obtaining sparks or getting the filament to glow in case the miner should attempt to push the bulb back into its normal position. It also protects the battery from being short-circuited for any length of time in case the leads to the bulb have become short-circuited during the accident.



FIG. 7

The rotating movement of the upper part of the lamp upon the lower is limited by a soft-iron pin $M$, which acts as a magnetic lock. This pin can be withdrawn in the charging room by a strong electromagnet, and when this is done the upper and lower parts of the housing separate and the battery can be removed for charging. The storage battery consists of a single round lead cell with concentric electrodes inside a cylindrical vessel $A$ covered with a waterproof lid of the same material. The holes in the terminal sockets contain bushings made of acidproof metal into which removable terminals $P_1$ and $P_2$ are fitted. These terminals are pressed upward by the terminal springs $W_1$ and $W_2$ against the contact segments $K_1$ and $K_2$ of the switch, carrying in this manner the current to the incandescent lamp. Terminals and springs can be easily taken out and cleaned by washing in warm water. In charging storage batteries gases develop which must have an opportunity to escape. It is therefore impossible to make the cells air-tight. An ordinary opening would allow the acid to run out in case the cell were upset. The center of the cell is therefore equipped with a celluloid tube $B$ which communicates by means of a small side tube $F$ with the upper part of the cell where the gases collect. The gases, therefore, may pass from the cell
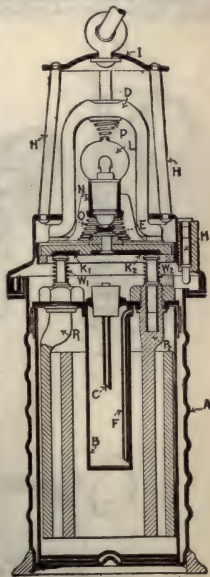
through the side tube $F$ and finally through the center tube $C$ to the open, while any particles of acid will be deposited in the cylinder $B$. Even if the cell is turned upside down no acid can escape and the lamps will burn upside down without leaking.

The weight of this lamp in standard size is 5 lb. Its height, not including hanger, is $10\frac{1}{4}$ in., while its largest diameter is $3\frac{3}{4}$ in. The lamp consumes 0.85 amp. at 2 volts. The battery has a capacity of 16 amp.-hr. and the maximum charging current should not exceed 2 amp. This general design, however, is built in four sizes, ranging from $1\frac{1}{2}$ to 5 lb. in weight and in capacities ranging from 4 to 16 hr. for one discharge and producing a light ranging from 0.75 to 3 c.p.

**Special Forms of the Ceag Lamp.**—A number of modifications of the Ceag lamp have been developed. Lamps are made for rescue parties, cages,
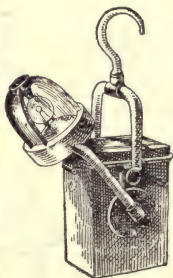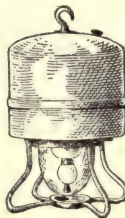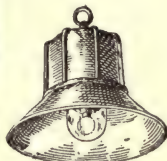


FIG. 8          FIG. 9          FIG. 10          FIG. 11
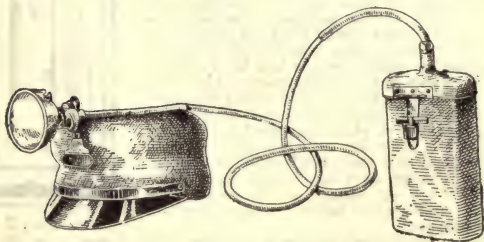


FIG. 12

powder magazines, shaft lighting, shaft inspection, loading places, blasting; also for head and tail lamps of trips. The lamp shown in Fig. 8 is similar to the standard design, but has the incandescent lamp mounted on one side and combined with a reflector projecting the light in one direction. This lamp is used for inspection and for a head and tail lamp. It is made in the same capacities as the standard lamp.

The shaft lamp shown in Fig. 9 is arranged with an adjustable arm carrying the incandescent lamp and is made to furnish from 8 to 24 c.p., burning from 7 to 12 hr. on one charge, according to size.

Fig. 10 is another type of shaft lamp without the adjustable arm. It is made for 8 to 12 c.p. and to furnish light from 10 to 15 hr. on one charge.

Fig. 11 is constructed to project light downward. It is built for from 8 to 32 c.p. and for a length of discharge of from 7 to 15 hr.

**Cap Lamps.**—Naked-flame cap lamps have long been used in this country, and it is but natural that the miner should desire an electric lamp of similar utility. The lamps developed to meet this demand are essentially of two

parts, the storage battery being carried on the belt while the lamp proper is attached to the cap, the two being connected by a suitable flexible cable. Such a lamp is shown in Fig. 12, while Fig. 13 shows the general principles of construction. The incandescent lamp bulb is mounted inside a parabolic reflector provided with a lens. A ball joint may also be incorporated in the design, permitting the wearer to direct the light beam where it is most needed. The flexible cable connecting the battery to the lamp is heavily insulated and in addition is armored at both ends where the liability of bending short is the greatest. Furthermore, an alloy with a low melting point is employed in this cable so that in case of accident to the lamp and the possible short-circuiting of the battery, this alloy will melt and destroy the short-circuit before a sufficient temperature has been attained to render the ignition of mine gases possible.
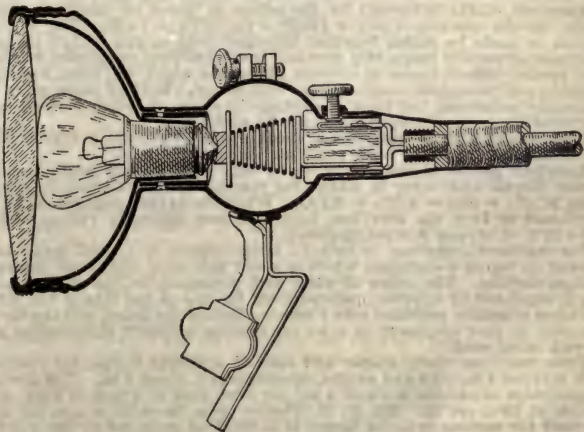


Fig. 13
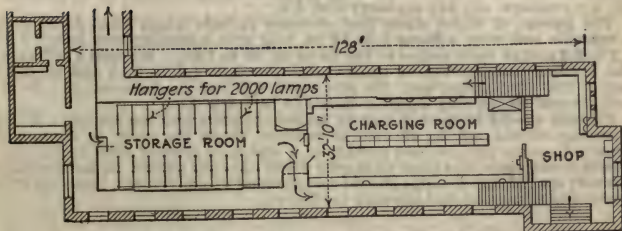


Fig. 14

**Charging Stations.**—After a shift in the mine an electric safety lamp must be left at the lamp house to be cleaned and recharged. Special charging racks have been designed for this purpose, one or more charging circuits being employed. Each circuit should be equipped with a switch, a rheostat and an ammeter. The rheostat should be provided with surplus resistance so that less than a full complement of cells may be charged. A portable voltmeter of suitable capacity, say 3 volts, should also be provided so that readings may be taken on each individual cell. After charging, cleaning and reassembling, the lamps are placed in special racks from which the miners remove them when starting a new shift.

Electric charging stations or lamp houses, particularly if many lamps are to be handled, follow the same general principles of design as do those where oil-burning safety lamps are used. No special arrangements need, however, be made for the storage and handling of dangerous inflammable oils. Fig. 14 shows a lamp house designed to accommodate 4,000 lamps. It contains a charging room with 20 racks, three small motors for buffing and cleaning and a distribution board with a watt-hour meter. There is also a store room for receiving the lamps when they are ready for service, and a repair shop. A small room is also employed for a number of ordinary safety lamps to be used by the fire bosses. To secure reliability of service it is essential that care and intelligence be employed in the maintenance and repair of any electric safety lamp.

### ACETYLENE LAMPS

Acetylene, $C_2H_2$, is formed by the action of water on calcium carbide, $CaC_2$, by the reaction $CaC_2 + 2H_2O = Ca(OH)_2 + C_2H_2$. The calcium carbide is made by fusing together lime and coke in the electric furnace. Commercial carbide frequently contains small amounts of calcium sulphide and, rarely, minute traces of calcium phosphide which will form hydrogen sulphide and phosphide, respectively, with water. Both of these gases are extremely poisonous, but their percentage in mine air, when derived from acetylene lamps, is so insignificant as to be negligible.

While the carbide of magnesium yields about 50% more acetylene than the carbide of calcium, it is too costly for commercial use.

Acetylene ignites at 896°F. and burns with an extremely white flame to carbon dioxide and vapor of water, the reaction for combustion in oxygen being $2C_2H_2 + 5O_2 = 4CO_2 + 2H_2O$. When the oxygen content of the air is reduced to 16% the acetylene flame becomes distinctly yellow, and at 12 to 13% of oxygen it is extinguished.

The standard acetylene, or carbide, lamp consists of a small water tank screwed on top of a container which is about half filled with small lumps of calcium carbide. The inflow of water and, consequently, the production of gas, is regulated by a valve operated from the top or the side of the lamp. The lamp is usually provided with a reflector behind the flame, and the burner is similar to that employed in a jet for burning ordinary illuminating gas, no wick being used.

While there are many shapes and sizes of acetylene lamps, the common form is about 4 in. high and weighs, when charged, about 6 oz. The average consumption of carbide is about 4 oz. per 8-hr. shift at a cost of 2.5 c. with carbide at 10 c. per lb. Mr. G. W. Pfeiffer gives the relative cost per man per shift for various types of lamps at a Mexican mine, where the price of materials is greater than in the United States, as: Mixture of coal- and lard-oil, 6 c.; miners' oil, 15¾ c.; acetylene, 5 c.

The Bureau of Mines states that ordinary carbide lamps when fitted with a reflector and with a flame 1 to 1½ in. long give a candlepower head-on of 4.2 to 6.2 and at right angles to the flame of .87 to 1.45. Without a reflector, the head-on candlepower of these lamps averages 1.9 to 2.15 and at right angles 1.9. In comparison, the maximum average candlepower of miners' and drivers' oil-burning lamps is stated to be 1.4 to 1.9.

Carbide should be kept in tightly sealed canisters and the contents of the container should not be thrown at random about the mine as there will usually be some unconsumed carbide in it which, in contact with water, may generate sufficient acetylene to start a fire if this gas should be accidentally ignited; special metal tanks should be provided at convenient intervals into which the exhausted carbide may be thrown.

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## EXPLOSIVE CONDITIONS IN MINES

In the ventilation of gaseous seams, the air current may be rendered explosive by the sudden occurrence of any one of a number of circumstances that cannot be anticipated. Among these are the following: (1) Derangement of the ventilating current. (2) Sudden increase of gas due to outbursts, falls of roof, feeders, fall of barometric pressure, etc. (3) Presence of coal dust thrown into suspension in the air, in the ordinary working of the mine, or by the force of blasting at the working face, or by blown-out, or windy shots. (4) Pressure due to a heavy blast, or any concussion of the air caused by closing of doors, etc. (5) Rapid succession of shots in close workings. (6) Accidental discharges of an explosive in a dirty atmosphere.

Any or all of these causes may precipitate an explosion at any moment. Hence, the condition of the air current should be maintained far within the explosive limit. The explosive conditions vary considerably in different coal seams. The nature of the coal and its enclosing strata, its friability and inflammability, together with the character of its occluded gases, determine, to a large extent, the explosive conditions in the seam. Experience in any particular seam or district must always be the best guide and furnish the best standard for determining the exploding power of any given lamp flame. For example, a 2-in. flame may be comparatively safe in a small mine where the coal is hard and not particularly inflammable, while a 1½-in. flame cap would be considered unsafe in mines where the conditions are more favorable to the generation of gas and formation of coal dust. The daily output of the mine and the general care that is enforced upon the miners at the working face are factors that should always be considered and taken into serious account in determining explosive conditions.

**Derangement of Ventilating Current.**—The flow of the air current must be uniform and continuous. Doors must be kept closed, since the mere setting open of a door, for a short period of time, may be enough to make an explosive condition possible. Any contemplated change in the current, by the erection of brattices, air bridges, stoppings, etc., should be carefully considered before the work is begun, and every precaution adopted to secure the safety of the men. Derangement of the current may occur through a fall of roof upon the main airway, by which the area of the airway is reduced, which results in the reduction of the quantity of air traversing the mine. If this fall is not noticed at once, serious results may happen. The utmost vigilance is therefore required on the part of fire bosses and all connected with mine workings. The failure of the ventilating apparatus is another source that gives rise to the derangement of the current. As a rule, furnaces are not now employed for the ventilation of gaseous seams. There are, however, some furnaces in use in such seams, and these require constant attention lest the fire should burn low. Upon any accident occurring to the ventilating machinery, notice should at once be given to the inside foreman, and the men withdrawn as rapidly as possible.

**A sudden increase of gas** may occur at any time in a gaseous seam, owing to an outburst, which suddenly yields a large volume of gas and may render the mine air in that section extremely explosive. The men working on the return of such a current must be hastily withdrawn, and all open lights extinguished. A heavy fall of coal in the mine workings or in the airways, or the tapping of a large gas feeder, produces the same effect in a less degree. The nearer to the face of the workings the fall of roof takes place, the more liable it is to be followed with a large flow of gas, inasmuch as the gas near the face has not had time to drain off, as in the case of old workings. This fact is always true in reference to new workings in a gaseous seam. The gas continues to flow freely for a considerable period, when its flow gradually decreases until it about ceases. When a large feeder has been tapped, it may be plugged for a time, if necessary, but the better practice is to allow it to flow freely and diffuse into the air current, which should be sufficiently increased to dilute the quantity of gas given off and to render it inexplosive. The men upon the return air should be notified. It is dangerous practice to light these feeders.

When there is a large area of abandoned workings in the mine, any considerable fall of barometric pressure is usually followed by an outflow of gas from the gobs or waste places of the mine. A fall of 1 in. in 5 hr. represents a very rapid decrease of barometric pressure. At all large collieries there is, or should be, a good standard barometer located upon the surface near the shaft. In many cases, these barometers are self-recording, and are often provided with an automatic alarm that gives warning whenever a fall of barometric pressure occurs. This warning should at once be conveyed to the men in the workings, and every precaution adopted to avoid evil results. The fact is fairly well established that a fall of atmospheric pressure is not followed by an outflow of gas from the mine workings for the space of, say, 3 hr. after such fall occurs. This statement must be regarded with caution, however, as it largely depends on the condition and extent of the abandoned workings. Where these are full of gas, its expansion affects the condition of the airways much more quickly than in cases where these workings places are partly ventilated.

**Effect of Coal Dust in Mine Workings.**—According to the inflammability of the coal the presence of coal dust in a finely divided state becomes a

dangerous factor. Certain coals are friable and easily reduced to fine dust which in the course of ordinary operations becomes stirred up and is suspended in the air. For a long time it was a much disputed question whether the presence of this dust was a dangerous factor unless some gas was also present in the atmosphere. Evidence secured in numerous investigations following explosions that have occurred during recent years have established the explosibility of coal dust when acted upon by a flame of sufficient intensity, beyond the slightest doubt. The exact action of the flame on such dust is but imperfectly understood, but the action once started is continuous as long as the explosive medium is available. Naturally, quantities of methane will greatly increase the violence, but its presence is unnecessary to produce an explosion.

Regarding the prevention or the checking of gas and dust explosions the Bureau of Mines in numerous experiments has proved that an explosion cannot originate from thoroughly wet coal dust, but that it is not easy to wet piles of coal dust even with well-humidified air currents. This is an important feature. When a saturated air current passes through a mine it dampens the roof, floor, and sides, but the coal dust itself when in accumulations appears to repel moisture; even with long exposure dust like that from the Pittsburgh seam takes up only 1 or 2 % of moisture, though the walls and floor may become damp. The surprising result of experiments makes it evident that it is necessary to remove coal-dust accumulations, so that, after a passageway has been well dampened, any particles of dust falling on wet surfaces will themselves become wet. It has been observed after some dust-explosion disasters that the explosion has traversed entries in which there was standing water along the bottom, but on the other hand examination of the benches and projections along the sides of such entries has disclosed quantities of dry dust. Also it has been observed that timbers frequently carry on their upper surfaces quantities of dust sufficient to propagate an explosion. Consequently, the Bureau emphasizes two precautions, namely, *first remove all accumulations of dry dust and then keep the entries wet or use a coating of rock dust. There will then be little danger of explosion.*

One of the principal mediums for distributing coal dust about the mines is the mine car. It is often loaded so high that the coal strikes the timbers or roof and is so jarred that it falls to the roadway where it is ground to a powder by men and mules. Tight cars should be used wherever possible. Gateless cars are used in Europe except in Wales and Scotland and revolving tipples are employed for dumping them. By proper arrangement the coal is thus discharged with little breakage. In the case of a downcast shaft the shaking screens in the tipple should not be placed immediately adjacent to the shaft and if they are already near it, vacuum dust collectors should be installed over the screens and chutes.

Otherwise, a large quantity of dust may be drawn down the shaft. In a certain mine in England in which rock dust was used to counteract the danger of coal dust a thick film of coal dust was observed on top of the rock dust, the deposit extending for a distance of 500 or 600 ft. from the shaft. Had it not been for the light-colored rock dust the deposit could not have been seen. The coal dust had been collecting for only 2 mo. subsequent to the time when the rock dust had last been laid. This mine has since put in vacuum dust collectors over its screens. In many of the recently built European plants it is the practice to place the screening plant 100 to 200 ft. distant from the downcast shaft.

The Bureau of Mines in *Technical Paper* No. 56 has outlined a number of preventive methods to be used in soft-coal mines for fighting the coal-dust danger. Naturally, there is but one way to prevent any coal-dust explosions, and that is to wet or wash down all rooms or haulageways where coal dust is likely to accumulate and to keep such places in a moist condition. This is often impossible and impractical, but the following methods suggested by the Bureau of Mines are all commendable.

**Humidifying the Air Current.**—With the humidifying system the intake air current is so saturated or supersaturated as to carry the moisture into the mine in minute but constant quantities every minute of the day. The amount of water vapor that air will carry or support varies with its temperature. For example, if 3,300 cu. ft. of air will support 1 lb. of water at freezing or 32°F. then at 62° it will support 3 lb. and a current of air of 3,300 cu. ft. per min. entering a mine at 32°F. would absorb moisture up to its capacity at 62° or whatever the temperature may be. Thus ordinarily the current of air takes up and carries away 20 lb. per min. or about 2½ gal. per min., which

would be over 3,500 gal. per day. This going on for months makes a mine more and more dry. No ordinary sprinkler system will entirely overcome this for the air will only absorb moisture. As it becomes heated it expands and dries. On the other hand, moisture in saturated air entering a mine at a higher temperature than that within will condense over the sides and roof of the haulageways and working places, thereby depositing water instead of withdrawing it. This principle seems to be the solution for the coal-dust problem from a humidifying standpoint. In this connection, the Colorado Fuel and Iron Co. installed radiators and steam pipes in the intake of its coal mines in southern Colorado, the radiators to raise the temperature of ingoing air, and the steam pipes to inject the necessary moisture in the form of steam.

The percentage of saturation obtained will depend upon the volume of air entering the mine and its temperature, the heating surface of the radiator and the amount of moisture supplied. That is, the larger the volume and the lower the outside temperature, the greater the heating surface and amount of moisture will have to be to give the same results.

Another method of supplying a mine with a preheated and humidified ventilating air is suggested by the operation and tests of an evaporative condenser installed at the central power plant of a



FIG. 1

group of mines near Pittsburg, to handle the exhaust steam from turbo generators. This suggestion is made in contradistinction to the steam jet and steam coil heating method.

The condenser referred to for the purpose of humidifying air, consists of a nest of 900 vertical, $1\frac{1}{2}$-in. diameter copper tubes, 19 ft. long, fixed top and bottom in suitable headers. The tubes are housed in on two sides, as
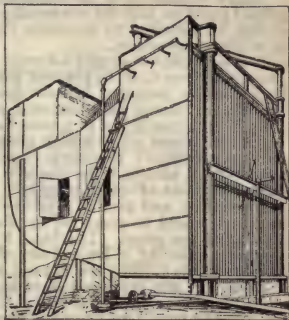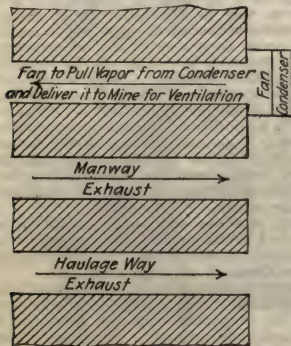


FIG. 2.—Arrangement of Entries, Fan Blowing
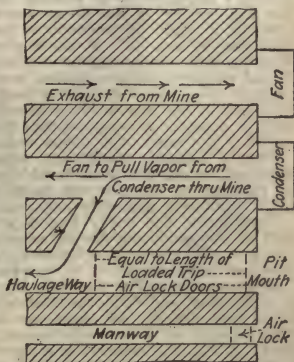


FIG. 3.—Arrangement of Entries, Fan Exhausting

shown in Fig. 1, one side being left open for the admission of air, which is drawn in and around the tubes, by a fan placed opposite the open side. The vapor generated by the evaporation of water, with which the tubes are mechanically wetted, is picked up by the air as it passes around the tubes. Where the arrangement of a mine's power equipment will permit, it is suggested that the usual mine fan be made to perform the double service of

drawing the air around condenser tubes, where it takes on heat and moisture in proportion to the work of condensation, as well as through the mine By test performance, the amount of water evaporated per pound of steam. condensed is approximately 1 lb. Figures 2 and 3 will indicate the arrangement of the connections to a condenser, fan, and mine, when blowing or exhausting.

Many variations of the condenser as described could be used for the suggested purpose, but the evaporative kind seems especially adapted, inasmuch as the air required to maintain its efficiency obtains its heat and humidity in a single operation.

It is conceded that cold air must be heated and have a sufficient amount of moisture given to it, to prevent it from absorbing moisture from the mine, as it gradually becomes heated during its passage through the air-courses, thereby increasing its moisture-carrying capacity. The assumption is made that, if the air supplied to a mine be heated to the mine's normal temperature, and that be also given a high relative humidity, it will issue from that mine having practically the same temperature and humidity. It is further assumed that, should it be possible to sufficiently heat and humidify the required amount of ventilating air, somewhat in excess of the mine's normal temperature, and give it a proportional burden of humidity, an amount of moisture would be given off by the air, as its temperature is adjusting itself to that of the mine. The basis for the last assumption lies in the nil effect that any quantity of heat given off by the ventilating air, so treated, would have towards raising the normal temperature of a mine.

However, any interchange of heat that might take place, from air to the walls of a mine, would tend to diminish the moisture-carrying capacity of the air, and would result in the deposit of a certain amount of moisture. The possibility of conditioning sufficient ventilating air, and the amount of exhaust steam required to perform the work, can be judged from the result of a series of problems, the results of which can be displayed graphically by means of curves.

By virtue of a suitable condenser, the air used as a vehicle to carry off the vapor, equal to the amount of water placed on the tubes necessary to effect condensation within them, produces in one operation a preheated and humidified atmosphere. It seems possible to adjust the degree of heat and humidity imparted to air passing through such a condenser to such a degree that the comfort of the miner would in no way be affected. By so doing, however, it might be necessary to forego some inches of vacuum which might otherwise be available at the engine in order to maintain the adjustment. Where a sufficient horsepower of exhaust steam is not at hand, the amount of coal required under a boiler, working at 60% efficiency, to produce low-pressure steam used in coils to heat air, is approximately 6½ T. per 24 hr., for a unit of 100,000 cu. ft. of ventilating air per min. In such an arrangement, it must be understood that the air is humidified by a second operation and does not come into direct contact with the steam formed in the boiler.

From a hygienic standpoint, mine ventilating air treated in the manner described would, to a considerable extent, bring about the same results that are claimed for devices now being used to condition air used to ventilate public buildings, assembly halls, and many up-to-date residences.

Considering the many factors entering into the successful operation of such an air-tempering device, when adapted to the general mine proposition, it is quite difficult to draw definite conclusions; however, the foregoing matter possesses sufficient merit to warrant consideration of mine operators.

**Hygrometers.**—The use of the hygrometer is in its infancy for observations in coal mines and while the complete rotary sling hygrometer or psychrometer is undoubtedly the most accurate for obtaining humidity readings, it is too delicate an instrument to carry around underground. The hygrometer shown in Fig. 4 is inclosed in a carrying case which converts it into a pocket instrument that is not liable to become broken when carried about in the mine. The wet and dry thermometers are inserted in each side of a split cylindrical case which is readily closed or opened by a handle. It is easy to swing but it is not so quick as the sling hygrometer. The thermometers are mounted on springs to lessen the danger of breakage, and this, with the case, makes a handy arrangement for underground observations.

Recording hygrometers, giving a single record of the relative humidity have been in use for a long time. Engineers have appreciated the distinct advantage of having a record of both the dry-bulb temperature and the wet-bulb temperature independently but simultaneously on the same chart.

Such an instrument has the added advantage in the ease with which its accuracy can be checked with a standard thermometer at all times. The importance of proper conditions of temperature and humidity is being more and more appreciated in its effect on coal dust.

The recording hygrometer illustrated in Fig. 6 consists of two sensitive bulbs mounted in tandem back of the case, the wet bulb being jacketed and kept moist by maintaining water at a constant level in a trough beneath the bulb. The pen arms are attached directly to shafts concentric with the
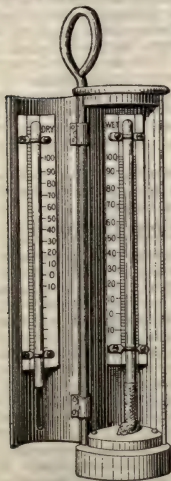


FIG. 5.



FIG. 4.—Hygrometer Open



FIG. 6.—Recording Hygrometer
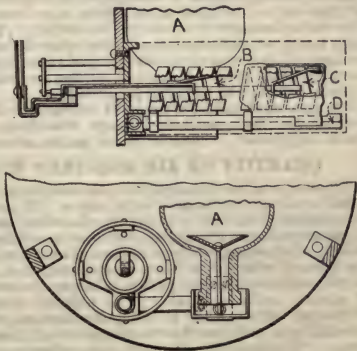
helical tube bulbs. The case is mounted on a swivel bracket enabling the swinging of the instrument at right angles to the wall or support, and giving easy access to the inverted glass bottle serving as a water reservoir. It is made to cover ranges between the freezing and boiling points of water (32° to 212°F. or 0–100°C.).

The principle of the construction of the recording hygrometers is shown in Fig. 5. In the cut *A* represents the reservoir, the tube *B*, the dry bulb which records the atmospheric temperature, and *C* the wet bulb which is covered with a special jacket leading down into trough *D*, containing water. The bulb *C* is always cooler than *B* due to the evaporation of the water. The evaporation increases or diminishes according to the amount of moisture in the air. Taking the difference between the two thermometer readings and consulting the table that is sent with the instrument the relative humidity is quickly ascertained.
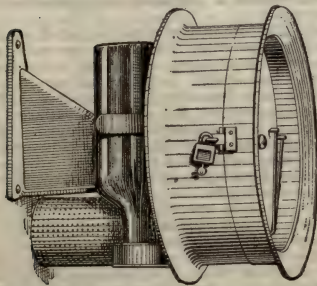
**Pressure as Affecting Explosive Conditions.**—Gaseous mixtures that are not explosive in the ordinary condition of a mine, often become explosive under the momentary pressure to which they are subjected by heavy blasting, and, in some instances, this may occur from the concussion of the air caused by the quick shutting of a door. In the latter case, however, the

explosive condition of the air would necessarily have to be close to the limit, in order for such a slight occurrence to precipitate an explosion. The factor of pressure as increasing the explosiveness of gaseous mixtures should be considered and constantly borne in mind.

**Rapid Succession of Shots in Close Workings.**—It constantly happens that two, three, or more shots are fired by means of fuse or touch squibs in a single chamber or heading, where the circulation of air is not always the best. The practical effect is that a considerable quantity of carbonic-oxide gas, $CO$, is produced by the firing of the first shot, and this gas does not have time to diffuse or become diluted by the air current before it is fired by the flame of the following shots. An explosion may often be precipitated by such an occurrence, if the workings are at all dusty. Two shots at the most are all that should be fired at one time in a close chamber or heading.

Mine explosions are commonly the result of the ignition of firedamp with an open lamp, or coal dust exploding after being set in motion by an explosion of gas, blown out shot, fall of roof or a rush of air and an electric arc. Numerous other cases of explosions are recorded but are not in the class commonly referred to as mine explosions.

Before the coal-dust theory was advanced and proven, it was believed that wherever the greatest damage was done, was the point where the explosion originated. This is by no means always the case.

## QUANTITY OF AIR REQUIRED FOR VENTILATION

The quantity of air required for the adequate ventilation of a mine cannot be stated as a rule applicable in all cases. Regulations that would supply a proper amount of air for ventilation of a thick seam would be found to cause great inconvenience if applied without modification to the workings in a thin seam. Likewise, the ventilation of an old mine with extended workings, a large area of which has been abandoned, and in many cases not properly sealed off, will require, naturally, a larger quantity of air per capita than a newly opened mine or shaft. The natural conditions existing in rise and dip workings, with respect to the gases that may be liberated or generated in those workings, call for the modification of the quantity of air required in each case. For example, dip workings, where much blackdamp is generated, will require a larger quantity of air, or higher velocity at the working face, to carry off such damps; and rise workings, liberating a large amount of marsh gas, will likewise require a higher velocity at the working face. On the other hand, a reversal of these conditions, such as a large quantity of marsh gas being liberated in dip workings, or a similar amount of blackdamp being generated in rise workings, will require a comparatively low velocity of the air at each respective working place.

**Quantity Required by State Laws.**—The quantity of air required by the laws of the several states is generally specified as 100 cu. ft. per man per min., and in many cases an additional amount of 500 cu. ft. per animal per min. is stated. This quantity is in no case stated as the actual amount of air required for the use of each man or animal, but is only the result of experience, as showing the quantity of air required for the proper ventilation of the average mine, based on the number of men and animals employed. The number of men employed in a mine is an indication of the extent of the working face, while the number of animals employed is an indication likewise of the extent of the haulage roads, or the development of the mine. These amounts refer particularly to non-gaseous seams.

The Bituminous Mine Law of Pennsylvania specifies that there shall be not less than 150 cu. ft. per min. per person in any mine, while 200 cu. ft. are required in a mine where firedamp has been detected.

The Anthracite Mine Law of Pennsylvania specifies a minimum quantity of 200 cu. ft. per min. per person. Each of these laws contains modifying clauses, which specify that the amount of air in circulation shall be sufficient to "dilute, render harmless, and sweep away" smoke and noxious or dangerous gases. Some mining companies specify the amount of air that must pass the last breakthrough and that the breakthrough shall not be more than a certain distance from the face of room or heading. One such company operating several mines in a region where mine explosions are fairly frequent has never had an explosion. Its rule is to have 12,000 cu. ft. of air per min. passing the last breakthrough which must not be over 100 ft. from the working face.

**Quantity of Air Required for Dilution of Mine Gases.**—To determine this requires a knowledge of the quantity of gas generated or liberated in the

workings. The quantity of air for dilution should be ample, and should be such as not to permit the condition of the current to approach the explosive point. The ventilation should be ample at the face.

**Quantity of Air Required to Produce the Necessary Velocity of Current at the Face.**—This consideration modifies considerably the quantity of air required for the ventilation of thick and thin seams. The velocity of the current is dependent not only on the quantity of air in circulation, but on the area of the air passage. This area is quite small in thin seams, and often very large in thick seams. As a result, the velocity is often low at the face of thick seams, and insufficient for the proper ventilation of the face, although the quantity of air passing into such a mine may be very large. A certain velocity of the current is always required in order to sweep away the gases. This velocity depends on the character of the gases and the position of the workings. Heavy damps are hard to move from dip workings where they have accumulated; and, likewise, lighter damps accumulating at the face of steep pitches are hard to brush away, and the velocity of the current in these cases must be equal to the task of driving out these gases.

## ELEMENTS IN VENTILATION

The elements in any circulation of air are (a) horsepower, or power applied; (b) resistance of the airways, or mine resistance, which gives rise to the total pressure in the airway; (c) velocity generated by the power applied against the mine resistance.

**Horsepower or Power of the Current.**—The power applied is often spoken of as the *power upon the air*. It is the effective power of the ventilating motor, whatever this may be, including all the ventilating agencies, whether natural or otherwise. The power upon the air may be the power exerted by a motive column due to natural causes, or to a furnace, or may be the power of a mechanical motor. The power upon the air is always measured in foot-pounds per minute, which expresses the units of work accomplished in the circulation.

**Mine Resistance.**—The resistance offered by a mine to the passage of an air current, or the mine resistance, is due to the friction of the air rubbing along the sides, top, and bottom of the air passages. This friction causes the total ventilating pressure in the airway, and is equal to it. Calling the resistance $R$, the unit of ventilating pressure (pressure per square foot) $p$, and the sectional area of the airway $a$, we have, $R = pa$; that is to say, the total pressure is equal to the mine resistance.

**Velocity of the Air Current.**—Whenever a given power is applied against a given resistance, a certain velocity results. For example, if the power $u$ (foot-pounds per minute) is applied against the resistance $pa$, a velocity $v$ (feet per minute) is the result; and since the total pressure $pa$ moves at the velocity $v$, the work performed each minute by the power applied is the product of the total pressure by the space through which it moves per minute, or the velocity. Thus, $u = (pa)v$.

**Relation of Power, Pressure, and Velocity.**—The relation of these elements of ventilation is not a simple relation. For example, a given power applied to move air through an airway establishes a certain resistance and velocity in the airway. The resistance of the airway is not an independent factor; that is to say, it does not exist as a factor of the airway independent of the velocity, but bears a certain relation to the velocity. *Power always produces resistance and velocity*, and these two factors always sustain a fixed relation.

This relation is expressed as follows: The *total pressure or resistance varies as the square of the velocity*; i.e., if the power is sufficient to double the velocity, the pressure will be increased 4 times; if the power is sufficient to multiply the velocity 3 times, the pressure will be increased 9 times. Thus, we observe that a change of power applied to any airway means both a change of *pressure* and a change of *velocity*.

Again, since the power is expressed by the equation $u = (pa)v$, and since $pa$, or the total pressure, varies as $v^2$, the work varies as $v^3$. From this it follows that, if the velocity is multiplied by 2, and, consequently, the total pressure by 4, the work performed $(pa)v$ will be multiplied by $2^3 = 8$. *We thus learn that the power applied varies as the cube of the velocity.*

## MEASUREMENT OF VENTILATING CURRENTS

The measurement and calculation of any circulaton in a mine airway includes the measurement of (a) the velocity of the air current, (b) of pres-

sure, (c) of temperature, (d) calculation of pressure, quantity, and horsepower of the circulation.

These measurements should be made at a point in the airway where the airway has a uniform section for some distance, and not far from the foot of the downcast shaft or the fan drift.

**Measurement of Velocity.**—For the purpose of mine inspection, the velocity of the air current should be measured at the foot of the downcast, at the mouth of each split of the air current, and at each inside breakthrough, in each split. These measurements are necessary in order to show that all the air designed for each split passes around the face of the workings.

The measurement of the velocity of a current is most conveniently made by means of the anemometer. This instrument consists of a vane placed in a circular frame and having its blades so inclined to the direction of its motion that 1 ft. of lineal velocity in the passing air current will produce 1 revolution of the vane. These revolutions are recorded by means of several pointers, each having a separate dial upon the face of the instrument, the motion being communicated by a series of gearwheels arranged decimally to each other. Most anemometers are provided with a large central pointer that makes 1 revolution for each 100 revolutions of the vane. The dial for this pointer is marked by 100 divisions, which record the number of lineal feet of velocity. In very accurate work with the anemometer, certain constants are used as suggested by the instrument maker, but these constants are of little value in ordinary practice and are of doubtful value even in more accurate observations.

The measurement of the velocity of an air current must necessarily represent only approximately the true velocity in the airway. The air travels with a greater velocity in the center of the airway, and is retarded at the sides, top, and bottom by the friction of these surfaces. Hence, the air to a large extent rolls upon these surfaces, which naturally generates an eddy at the sides of airways. When measuring the air, the anemometer should be held in a position exactly perpendicular to the direction of the current, and moved to occupy different positions in the airway, being held an equal time in each position, or it may be moved continuously around the margin of the airway, and through the central portion.

The person taking the observation should observe the caution of not obstructing the area of the airway by his body, as the area is thereby reduced, and the velocity of the current increased. The area of the airway is accurately measured at the point where the observations are taken.

To obtain the quantity of air passing (cubic feet per minute), multiply the area of the airway, at the point where the velocity is measured, by the velocity.

EXAMPLE.—The anemometer gives a reading of 1,320 ft. in 2 min., the height of the airway is 6 ft. 6 in., and its average width 8 ft. 8 in. What volume of air is passing in the airway per minute?

$$6\tfrac{1}{2} \times 8\tfrac{1}{4} \times \frac{1,320}{2} = 37,180 \text{ cu. ft. per min.}$$

The **measurement of the ventilating pressure** is made by means of a water column in the form of a water gauge.

**Water Gauge.**—The water gauge is simply a glass U-tube open at both ends. Water is placed in the bent portion of the tube, and stands at the same height in both arms of the tube when each end of the tube is subjected to the same pressure. If, however, one end of the tube is subjected to a greater pressure than the other end, the water will be forced down in that arm of the tube, and will rise a corresponding height in the other arm, the difference of level in the two arms of the

FIG. 7

tube representing the water column balanced by the excess of pressure to which the water in the first arm is subjected. An adjustable scale graduated in inches measures the height of the water column. The zero of the scale is adjusted to the lower water level, and the upper water level will then give the reading of the water gauge. One end of the glass tube is drawn to a narrow opening to exclude dust, while the other end is bent to a right angle, and passing back through the standard to which the tube is attached, is cemented into the brass tube that passes through a hole in the partition or brattice, when the water gauge is in use. The bend of the tube
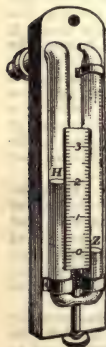
is contracted to reduce the tendency to oscillation in the height of water column (see Fig. 7).

When in use, the water gauge must be in a perpendicular position. It may be placed upon a brattice occupying a position between two airways, as shown at $A$, Fig. 8. The brass tube forming one end of the water gauge is inserted in a cork, and passes through a hole bored in the brattice. The water gauge must not be subjected to the direct force of the air current, as in this case the true pressure will not be given. Fig. 8 shows the instrument as occupying a position in the breakthrough, between two entries. It will be observed that the water gauge records a difference of pressure, each end of the water gauge being subject to atmospheric pressure, but one end in addition being subject to the ventilating pressure, which is the difference of pressure between the two entries. The water gauge thus enables us to measure the resistance of the mine *inbye* from its position between two airways. If placed in the first breakthrough, it measures the entire resistance of the mine, but if placed at the mouth of a split, it measures only the resistance of that split. It never measures the resistance *outbye* from its position in the mine, but always *inbye* (see Calculation of Pressure).
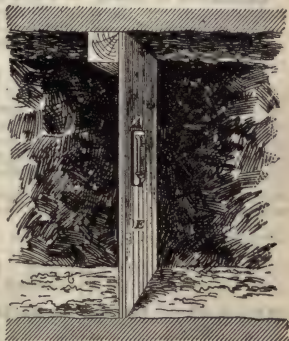


FIG. 8

**Calculation of Mine Resistance.**—The mine resistance is equal to the total pressure $pa$ that it causes. This mine resistance is dependent upon three factors: ($a$) The resistance $k$ offered by 1 sq. ft. of rubbing surface to a current having a velocity of 1 ft. per min. The *coefficient of friction k*, or the *unit of resistance*, is the resistance offered by the *unit* of rubbing surface to a current of a *unit* velocity. This unit resistance has been variously estimated by different authorities (see following table). The value most universally accepted, however, is that known as the Atkinson coefficient (.0000000217). ($b$) The mine resistance, which varies as the square of the velocity. ($c$) The rubbing surface. Hence, if we multiply the unit resistance by the square of the velocity, and by the rubbing surface, we will obtain the total mine resistance as expressed by the formula $pa = ksv^2$.

TABLE OF VARIOUS COEFFICIENTS OF FRICTION OF AIR IN MINES.

| | Pressure per Sq. Ft. Decimals of a Pound. |
|---|---|
| J. J. Atkinson's treatise | .0000000217 |
| A. Devillez in Ventilation des Mines: | |
| Forchies | .000000008211 |
| Crachet-Picquery | .000000008928 |
| Grand Baisson | .000000008611 |
| Average of 2, 3, and 4 | .000000008585 |
| Used in Ventilation des Mines | .000000009511 |
| Arched Tunnels | .000000002113 |
| Along a working face of coal | .000000014266 |
| G. G. Andre, Atmosphere of Coal Mines | .000000022424 |
| Peclet, Cheminee (Devillez, p. 112) | .000000003697 |
| D. K. Clark | .000000002272 |
| According to Goupilliere's Cours d' Exploitation des Mines, Vol. II, p. 389: | |
| D'Aubuisson | .000000001955 |
| Navier | .000000001872 |
| W. Fairley | .00000001 |
| J. Stanley James | .0000000929 |
| D. Murgue | .000000008242 |

It will be observed that J. J. Atkinson's coefficient is greatly in excess of any other, with the exception of Andre's. Fairley's is derived from an

average taken between Atkinson, Devillez, and Clark, and, undoubtedly, it is an exceedingly simple coefficient to work out calculations with, as it will save a great mass of figures. James, in his work on colliery ventilation, reduces the coefficient still further on the authority of the Belgian Mine Commission, but he gives a most unwieldy figure to use.

Atkinson's figure is the one most in use, and if it is too high, it errs on the side of safety, and it is always advisable to have plenty of spare ventilating capacity at a mine. For this reason, and until a regular and thorough investigation, made by a commission of competent men, provides a standard coefficient, we prefer to abide by Atkinson's coefficient, and it is used in all our calculations.

**Calculation of Power, or Units of Work per Minute.**—If we multiply the total pressure by the velocity (feet per minute) with which it moves, we obtain the units of work per minute, or the power upon the air. Hence, $u = pav = ksv^3$, which is the fundamental expression for *work per minute*, or *power*.

**The Equivalent Orifice.**—This term, often used in regard to ventilation, evaluates the mine resistance, or, as will be seen from the equation given below for its value, expresses the ratio that exists between the quantity of air passing in an airway and the pressure or water gauge that is produced by the circulation. This term was suggested by M. Daniel Murgue, and refers to the flow of a fluid through an orifice in a thin plate, under a given head. The formula expressing the velocity of flow through such an orifice is $v = \sqrt{2gh}$; multiplying both members of this equation by $A$, and substituting for the first member $Av$, its value $q$, we have, after transposing and correcting for *vena contracta*, $A = \dfrac{q}{.62\sqrt{2gh}}$, in which .62 is the coefficient for the *vena contracta* of the flow. Reducing this to cubic feet per minute and inches of water gauge represented by $i$, we have, finally, the equation $A = .0004 \times \dfrac{q}{\sqrt{i}}$. By this formula, Murgue has suggested comparing the flow of air through a mine to the flow of a fluid through a thin plate; since, in each case, the quantity and the head or pressure vary in the same ratio. Thus, applying this formula to a mine, Murgue multiplies the ratio of the quantity of air passing (cubic feet per minute) and the square root of the water gauge (inches) by .0004, and obtains an area $A$, which he calls the equivalent orifice of the mine.

**Potential Factor of a Mine.** (*Proposed by T. J. Beard.*)—Equations 8 and 27 pages 370–371, give, respectively, the pressure and the power that will circulate a given quantity of air per minute in a given airway. These equations may be written as equal ratios, expressed in factors of the current and the airway, respectively; thus, $\dfrac{p}{q^2} = \dfrac{ks}{a^3}$, and $\dfrac{u}{q^3} = \dfrac{ks}{a^3}$, which show that the ratio between the *pressure* and the square of the quantity $it$ circulates in any given airway is equal to the ratio between the *power* and the cube of the quantity $it$ circulates. Solving each of these equations with respect to $q$, we have the following:

With respect to pressure,

$$q = \left( a \sqrt{\frac{a}{ks}} \right) \sqrt{p}.$$

With respect to power,

$$q = \left( \frac{a}{\sqrt[3]{ks}} \right) \sqrt[3]{u}.$$

Hence we observe that, in any airway, *for a constant pressure*, the quantity of air in circulation is proportional to the expression $a\sqrt{\dfrac{a}{ks}}$; and, *for a constant power*, the quantity is proportional to the expression $\dfrac{a}{\sqrt[3]{ks}}$, which terms are called the *potentials* of the mine with respect to pressure and power, respectively; and their values $\dfrac{q}{\sqrt{p}}$ and $\dfrac{q}{\sqrt[3]{u}}$ are the potentials of the

THE FOLLOWING TABLE OF WATER GAUGES WILL BE FOUND OF ASSISTANCE IN CALCULATING THE AMOUNT OF AIR REQUIRED FOR MINE WORKINGS

| Water Gauge | Ventilating Pressure, Pounds per Sq. Ft. | Pressure, Ounces per Sq. Ft. | Pressure, Ounces per Sq. In. | Foot-pounds in 1,000 Cu. Ft. of Air | Horsepower in 1,000 Cu. Ft. of Air | Water Gauge | Ventilating Pressure, Pounds per Sq. Ft. | Pressure, Ounces per Sq. Ft. | Pressure, Ounces per Sq. In. | Foot-pounds in 1,000 Cu. Ft. of Air | Horsepower in 1,000 Cu. Ft. of Air |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.52 | 8.32 | 0.05 | 520 | 0.016 | 5.1 | 26.52 | 424.32 | 2.94 | 26,520 | 0.800 |
| 0.2 | 1.04 | 16.64 | 0.12 | 1,040 | 0.031 | 5.2 | 27.24 | 435.84 | 3.02 | 27,240 | 0.825 |
| 0.3 | 1.56 | 24.96 | 0.17 | 1,560 | 0.048 | 5.3 | 27.56 | 440.96 | 3.04 | 27,560 | 0.835 |
| 0.4 | 2.08 | 33.28 | 0.23 | 2,080 | 0.063 | 5.4 | 28.08 | 449.28 | 3.10 | 28,080 | 0.850 |
| 0.5 | 2.60 | 41.60 | 0.28 | 2,600 | 0.079 | 5.5 | 28.60 | 457.60 | 3.17 | 28,600 | 0.866 |
| 0.6 | 3.12 | 49.92 | 0.34 | 3,120 | 0.081 | 5.6 | 29.12 | 465.92 | 3.23 | 29,120 | 0.884 |
| 0.7 | 3.64 | 58.24 | 0.40 | 3,640 | 0.110 | 5.7 | 29.64 | 474.24 | 3.30 | 29,640 | 0.900 |
| 0.8 | 4.16 | 66.56 | 0.46 | 4,160 | 0.128 | 5.8 | 30.16 | 482.56 | 3.38 | 30,160 | 0.913 |
| 0.9 | 4.68 | 74.88 | 0.51 | 4,680 | 0.141 | 5.9 | 30.68 | 490.88 | 3.40 | 30,680 | 0.930 |
| 1.0 | 5.20 | 83.20 | 0.58 | 5,200 | 0.159 | 6.0 | 31.20 | 499.20 | 3.44 | 31,200 | 0.940 |
| 1.1 | 5.72 | 91.52 | 0.63 | 5,720 | 0.173 | 6.1 | 31.72 | 507.52 | 3.50 | 31,720 | 0.962 |
| 1.2 | 6.24 | 99.84 | 0.70 | 6,240 | 0.188 | 6.2 | 32.44 | 519.04 | 3.55 | 32,240 | 0.983 |
| 1.3 | 6.76 | 108.16 | 0.75 | 6,760 | 0.205 | 6.3 | 32.76 | 524.16 | 3.61 | 32,760 | 0.993 |
| 1.4 | 7.28 | 116.48 | 0.80 | 7,280 | 0.225 | 6.4 | 33.28 | 532.48 | 3.67 | 33,288 | 1.000 |
| 1.5 | 7.80 | 124.80 | 0.86 | 7,800 | 0.236 | 6.5 | 33.80 | 540.80 | 3.73 | 33,800 | 1.024 |
| 1.6 | 8.32 | 133.12 | 0.92 | 8,320 | 0.252 | 6.6 | 34.32 | 549.12 | 3.81 | 34,320 | 1.042 |
| 1.7 | 8.84 | 141.44 | 0.99 | 8,840 | 0.268 | 6.7 | 34.84 | 557.44 | 3.87 | 34,840 | 1.056 |
| 1.8 | 9.36 | 149.76 | 1.03 | 9,360 | 0.283 | 6.8 | 35.36 | 565.76 | 3.92 | 35,360 | 1.071 |
| 1.9 | 9.88 | 158.08 | 1.10 | 9,880 | 0.299 | 6.9 | 35.88 | 574.08 | 3.98 | 35,880 | 1.083 |
| 2.0 | 10.40 | 166.40 | 1.15 | 10,400 | 0.315 | 7.0 | 36.40 | 582.40 | 4.01 | 36,400 | 1.103 |
| 2.1 | 10.92 | 174.72 | 1.21 | 10,920 | 0.331 | 7.1 | 36.92 | 590.72 | 4.10 | 36,920 | 1.120 |
| 2.2 | 11.44 | 183.04 | 1.27 | 11,440 | 0.348 | 7.2 | 37.64 | 602.24 | 4.17 | 37,640 | 1.140 |
| 2.3 | 11.96 | 191.36 | 1.32 | 11,960 | 0.362 | 7.3 | 37.96 | 607.36 | 4.19 | 37,960 | 1.150 |
| 2.4 | 12.48 | 199.68 | 1.38 | 12,480 | 0.378 | 7.4 | 38.48 | 615.68 | 4.24 | 38,480 | 1.166 |
| 2.5 | 13.00 | 208.00 | 1.44 | 13,000 | 0.390 | 7.5 | 39.00 | 624.00 | 4.30 | 39,000 | 1.181 |
| 2.6 | 13.52 | 216.32 | 1.50 | 13,320 | 0.409 | 7.6 | 39.52 | 632.32 | 4.40 | 39,520 | 1.193 |
| 2.7 | 14.04 | 224.64 | 1.56 | 14,040 | 0.425 | 7.7 | 40.04 | 641.12 | 4.45 | 40,040 | 1.213 |
| 2.8 | 14.56 | 232.96 | 1.61 | 14,560 | 0.441 | 7.8 | 40.56 | 648.96 | 4.50 | 40,560 | 1.230 |
| 2.9 | 15.08 | 241.28 | 1.68 | 15,080 | 0.456 | 7.9 | 41.08 | 657.28 | 4.56 | 41,080 | 1.242 |
| 3.0 | 15.60 | 250.60 | 1.72 | 15,600 | 0.471 | 8.0 | 41.60 | 665.60 | 4.63 | 41,600 | 1.262 |
| 3.1 | 16.12 | 257.92 | 1.80 | 16,120 | 0.490 | 8.1 | 42.12 | 673.92 | 4.68 | 42,120 | 1.274 |
| 3.2 | 16.64 | 266.24 | 1.84 | 16,640 | 0.504 | 8.2 | 42.84 | 685.44 | 4.70 | 42,840 | 1.300 |
| 3.3 | 17.16 | 274.56 | 1.89 | 17,160 | 0.520 | 8.3 | 43.16 | 690.56 | 4.76 | 43,160 | 1.308 |
| 3.4 | 17.68 | 282.88 | 1.96 | 17,680 | 0.536 | 8.4 | 43.68 | 698.88 | 4.82 | 43,680 | 1.320 |
| 3.5 | 18.20 | 291.20 | 2.01 | 18,200 | 0.551 | 8.5 | 44.20 | 707.20 | 4.87 | 44,200 | 1.340 |
| 3.6 | 18.72 | 299.52 | 2.08 | 18,720 | 0.567 | 8.6 | 44.72 | 715.52 | 4.96 | 44,720 | 1.360 |
| 3.7 | 19.24 | 307.84 | 2.13 | 19,240 | 0.583 | 8.7 | 45.24 | 723.84 | 5.02 | 45,240 | 1.370 |
| 3.8 | 19.76 | 316.16 | 2.20 | 19,760 | 0.600 | 8.8 | 45.76 | 732.16 | 5.08 | 45,760 | 1.386 |
| 3.9 | 20.28 | 324.48 | 2.25 | 20,280 | 0.614 | 8.9 | 46.28 | 740.48 | 5.16 | 46,280 | 1.400 |
| 4.0 | 20.80 | 332.80 | 2.30 | 20,800 | 0.631 | 9.0 | 46.80 | 747.80 | 5.20 | 46,800 | 1.415 |
| 4.1 | 21.32 | 341.12 | 2.36 | 21,320 | 0.646 | 9.1 | 47.32 | 758.12 | 5.25 | 47,300 | 1.432 |
| 4.2 | 21.84 | 349.44 | 2.42 | 21,840 | 0.661 | 9.2 | 48.04 | 768.64 | 5.28 | 48,040 | 1.452 |
| 4.3 | 22.36 | 357.76 | 2.46 | 22,360 | 0.677 | 9.3 | 48.36 | 773.76 | 5.33 | 48,360 | 1.465 |
| 4.4 | 22.88 | 366.08 | 2.54 | 22,880 | 0.693 | 9.4 | 48.88 | 782.08 | 5.43 | 48,880 | 1.481 |
| 4.5 | 23.40 | 374.40 | 2.58 | 23,400 | 0.709 | 9.5 | 49.40 | 790.40 | 5.45 | 49,400 | 1.500 |
| 4.6 | 23.92 | 382.72 | 2.65 | 23,920 | 0.724 | 9.6 | 49.92 | 798.72 | 5.50 | 49,920 | 1.513 |
| 4.7 | 24.44 | 391.04 | 2.71 | 24,440 | 0.740 | 9.7 | 50.44 | 807.04 | 5.56 | 50,440 | 1.525 |
| 4.8 | 24.96 | 399.36 | 2.80 | 24,960 | 0.753 | 9.8 | 50.96 | 815.36 | 5.66 | 50,960 | 1.542 |
| 4.9 | 25.48 | 407.68 | 2.83 | 25,480 | 0.773 | 9.9 | 51.48 | 823.68 | 5.72 | 51,480 | 1.560 |
| 5.0 | 26.00 | 416.00 | 2.87 | 26,000 | 0.789 | 10.0 | 52.00 | 832.00 | 5.74 | 52,000 | 1.600 |

NOTE.—The table is based on 1 cu. ft. of water weighing 62.4 lb. A column of water 1 in. in depth and 1 sq. ft. in area equals 5.2 lb.

current with respect to pressure and power, respectively. These factors, it will be observed, evaluate the airway, as they determine the quantity of air a given pressure or power will circulate in that airway (cubic feet per minute). By their use the relative quantitites of air any given pressure or power will circulate in different airways are readily determined. The rule may be stated as follows:

**Rule.**—*For any given pressure or power, the quantity of air in circulation is always proportional to the potential for pressure, or the potential for power, as the case may be.*

This rule finds important application in splitting (see Calculation of Natural Splitting). In all cases where the potential is used as a ratio, the *relative potential* may be employed by omitting the factor $k$; or it may be employed to obtain the pressure and power in several splits by multiplying the final result by $k$ (see Formulas 26, 27, etc., page 914).

EXAMPLE.—20,000 cu. ft. of air is passing in a mine in which the airway is 6 ft. × 8 ft., and 10,000 ft. long, under a certain pressure; it is required to find what quantity of air this same pressure will circulate in a mine in which the airway is 6 ft. × 12 ft., and 8,000 ft. long.

Calculating the potential $X_p$ with respect to the pressure for each of these mines, or airways, we have, using the relative potential,

$$X_1 = 6 \times 8 \sqrt{\frac{6 \times 8}{2(6+8) \times 10,000}} = .62845, \text{ and } X_2 = 6 \times 12 \sqrt{\frac{6 \times 12}{2(6+12) \times 8,000}}$$

$=1.1384$. Since the ratio of the quantities is equal to the ratio of the potentials with respect to pressure, in these two mines, we write the proportion $20,000 : q_2 : : .62845 : 1.1384$, and $q_2 = \dfrac{20,000 \times 1.1384}{.62845} = 36,229$ cu. ft. per min.

EXAMPLE.—20,000 cu. ft. of air is passing in a mine in which the airway is 6 ft. × 8 ft., and 10,000 ft. long, under a certain power; it is required to find what quantity of air will be circulated by this same power in a mine in which the airway is 6 ft. × 12 ft., and 8,000 ft. long.

We calculate the potential $X_u$ with respect to power for each of these mines, using, as before, the relative potential. Thus, $X_1 = \dfrac{6 \times 8}{\sqrt[3]{2(6+8) \times 10,000}}$

$= .7337$, and $X_2 = \dfrac{6 \times 12}{\sqrt[3]{2(6+12) \times 8,000}} = 1.0905$. Then, in this case, since the ratio of the quantities is equal to the ratio of the potentials with respect to power, we write the proportion, $20,000 : q_2 : : .7337 : 1.0905$, and $q_2 = \dfrac{20,000 \times 1.0905}{.7337} = 29,726$ cu. ft. per min.

The following table will serve to illustrate the use of the formulas employed in these calculations. It will be observed that there are several formulas for quantity, and for velocity, and for work or horsepower, but in each respective case the several formulas are derived by simple transposition of the terms of the original formula, and are tabulated here for convenience. Choice must be made in the use of any of these formulas, according to the known terms in each example. Thus, an example may ask: What pressure will be produced in passing a given quantity of air through a certain mine, the size and length of the airways being given? We then use the formula $p = \dfrac{ksq^2}{a^3}$. But if the question asks what quantity of air a given pressure will produce in this same mine, we use the formula $q = \sqrt{\dfrac{pa}{ks}} \times a$. It will be observed that this second formula is a simple transposition of the first.

In like manner the question may be asked, what power will produce a certain quantity of air in a certain airway; and the expression used, in this case, is $u = \dfrac{ksq^3}{a^3}$. Or the question may be asked, what quantity of air will be produced in a given airway by means of a certain power or work applied to the airway. In this case the formula used is $q = a \sqrt[3]{\dfrac{u}{ks}}$. If the question asks for the power required to produce a given velocity in a given airway,

the formula employed is $u = ksv^3$. All of these formulas are derived by combining the simple formulas $p = \dfrac{ksv^2}{a}$, $q = av$, and $u = qp$.

To illustrate the use of the formulas, we take as an example an underground road, 5 ft. wide by 4 ft. high, and 2,000 ft. in length, and calculate the value of each symbol or letter, assuming a velocity of 500 ft. per min.

| | Symbol | Value of Symbol for this Particular Example |
|---|---|---|
| Area of airway (5 ft. × 4 ft.)............... | $a$ | 20 sq. ft. |
| Horsepower*............................. | $h$ | 2.959 H. P. |
| Coefficient of friction†................... | $k$ | .0000000217 lb. |
| Length of airway......................... | $l$ | 2,000 ft. |
| Perimeter of airway, 2(5 ft. + 4 ft.)........ | $o$ | 18 ft. |
| Pressure (lb. per sq. ft.)................... | $p$ | 9.765 lb. |
| Quantity of air (cu. ft. per min.)........... | $q$ | 10,000 cu. ft. |
| Area of rubbing surface................... | $s$ | 36,000 sq. ft. |
| Units of work per minute (power).......... | $u$ | 97,650 ft.-lb. |
| Velocity (ft. per min.).................... | $v$ | 500 ft. |
| Water gauge.............................. | $i$ | 1.87788 in. |
| Equivalent orifice of the mine............. | $A$ | 2.919 sq. ft. |
| Potential for power....................... | $X_u$ | 217.16 units |
| Potential for pressure..................... | $X_p$ | 3,200 units |
| Weight of 1 cu. ft. of downcast air........ | $w$ | .08098 lb. |
| Motive column (downcast air)............. | $M$ | 120.5 ft. |
| Depth of furnace shaft.................... | $D$ | 306.77 ft. |
| Average temperature of the upcast column.. | $T$ | 350°F. |
| Average temperature of the downcast column. | $t$ | 32°F. |

* A horsepower is equal to 33,000 units of work.
† This coefficient of friction is an invariable quantity, and is the same in every calculation relating to the friction of air in mines.
NOTE.—The water gauge is calculated to five decimal places to enable all the other values to be accurately arrived at. In practice it is only read to one decimal place.

## FORMULAS

On the right side of each formula the various calculations, based on the example given, are worked out in figures.

| To Find: | No. | Formula | Specimen Calculation |
|---|---|---|---|
| Rubbing surface of an airway. (Sq. ft.) | 1 | $s = lo$ | $2{,}000 \times 18 = 36{,}000$ sq. ft. |
| Area of an airway. (Sq. ft.) | 2 | $a = \dfrac{q}{v}$ | $\dfrac{10{,}000}{500} = 20$ sq. ft. of area |
| Velocity. (Ft. per min.) | 3 | $v = \dfrac{q}{a}$ | $\dfrac{10{,}000}{20} = 500$ ft. |
| | 4 | $v = \sqrt[3]{\dfrac{u}{ks}}$ | $\sqrt[3]{\dfrac{97{,}650}{.0000000217 \times 36{,}000}} = 500$ ft. |
| | 5 | $v = \sqrt{\dfrac{pa}{ks}}$ | $\sqrt{\dfrac{9.765 \times 20}{.0000000217 \times 36{,}000}} = 500$ ft. |
| | 6 | $v = \dfrac{u}{pa}$ | $\dfrac{97{,}650}{9.765 \times 20} = 500$ ft. |

| To Find: | No. | Formula | Specimen Calculation |
|---|---|---|---|
| Pressure. (Lb. per sq. ft.) | 7 | $p = \dfrac{ksv^2}{a}$ | $\dfrac{.0000000217 \times 36{,}000 \times 500^2}{20} = 9.765$ lb. |
| | 8 | $p = \dfrac{ksq^2}{a^3}$ | $\dfrac{.0000000217 \times 36{,}000 \times 10{,}000^2}{20^3}$ $= 9.765$ lb. |
| | 9 | $p = \dfrac{u}{q}$ | $\dfrac{97{,}650}{10{,}000} = 9.765$ lb. |
| | 10 | $p = Mw$ | $120.58 \times .08098 = 9.765$ lb. |
| | 11 | $p = 5.2i$ | $5.2 \times 1.87788 = 9.765$ lb. |
| | 12 | $p = \dfrac{q^2}{X_u{}^3}$ | $\dfrac{10{,}000^2}{217.16^3} = 9.765$ lb. |
| | 13 | $p = \dfrac{q^2}{X_p{}^2}$ | $\left(\dfrac{10{,}000}{3{,}200}\right)^2 = 9.765$ lb. |
| Water gauge. (In.) | 14 | $i = \dfrac{p}{5.2}$ | $\dfrac{9.765}{5.2} = 1.87788$ in. |
| Resistance of an airway. (Total pressure, lb.) | 15 | $pa = ksv^2$ | $.0000000217 \times 36{,}000 \times 500^2 = 195.3$ lb. |
| | 16 | $pa = \dfrac{u}{v}$ | $\dfrac{97{,}650}{500} = 195.3$ lb. |
| Quantity. (Cu. ft. per min.) | 17 | $q = av$ | $20 \times 500 = 10{,}000$ cu. ft. |
| | 18 | $q = \dfrac{u}{p}$ | $\dfrac{97{,}650}{9.765} = 10{,}000$ cu. ft. |
| | 19 | $q = \sqrt{\dfrac{pa}{ks}} \times a$ | $\sqrt{\dfrac{9.765 \times 20}{.0000000217 \times 36{,}000}} \times 20$ $= 10{,}000$ cu. ft. |
| | 20 | $q = \sqrt[3]{\dfrac{u}{ks}} \times a$ | $\sqrt[3]{\dfrac{97{,}650}{.0000000217 \times 36{,}000}} \times 20$ $= 10{,}000$ cu. ft. |
| | 21 | $q = X_u \sqrt[3]{u}$ | $217.16 \times \sqrt[3]{97{,}650} = 10{,}000$ cu. ft. |
| | 22 | $q = \sqrt[3]{X_p{}^2 u}$ | $\sqrt[3]{3{,}200^2 \times 97{,}650} = 10{,}000$ cu. ft. |
| | 23 | $q = X_p \sqrt{p}$ | $3{,}200 \times \sqrt{9.765} = 10{,}000$ cu. ft. |
| Units of work per minute, or power on the air. (Ft.-lb. per min.) | 24 | $u = avp$ | $20 \times 500 \times 9.765 = 97{,}650$ ft.-lb. |
| | 25 | $u = qp$ | $10{,}000 \times 9.765 = 97{,}650$ ft.-lb. |
| | 26 | $u = ksv^3$ | $.0000000217 \times 36{,}000 \times 500^3$ $= 97{,}650$ ft.-lb. |
| | 27 | $u = \dfrac{ksq^3}{a^3}$ | $\dfrac{.0000000217 \times 36{,}000 \times 10{,}000^3}{20^3}$ $= 97{,}650$ ft.-lb. |

| To Find: | No. | Formula | Specimen Calculation |
|---|---|---|---|
| Units of work per minute, or power on the air. (Ft.-lb. per min.) | 28 | $u = h\,33{,}000$ | $2.959 \times 33{,}000 = 97{,}650$ ft.-lb. |
| | 29 | $u = \dfrac{q^3}{X_u{}^3}$ | $\dfrac{10{,}000^3}{217.16^3} = 97{,}650$ ft.-lb. |
| | 30 | $u = \dfrac{q^3}{X_p{}^2}$ | $\dfrac{10{,}000^3}{3{,}200^2} = 97{,}650$ ft.-lb. |
| Horsepower. | 31 | $h = \dfrac{u}{33{,}000}$ | $\dfrac{97{,}650}{33{,}000} = 2.959$ H. P. |
| Power potential. (Units.) | 32 | $X_u = \dfrac{a}{\sqrt[3]{ks}}$ | $\dfrac{20}{\sqrt[3]{.0000000217 \times 36{,}000}} = 217.16$ units. |
| | 33 | $X_u = \sqrt[3]{\dfrac{q^2}{p}}$ | $\sqrt[3]{\dfrac{10{,}000^2}{9.765}} = 217.16$ units. |
| | 34 | $X_u = \dfrac{q}{\sqrt[3]{u}}$ | $\dfrac{10{,}000}{\sqrt[3]{97{,}650}} = 217.16$ units. |
| Pressure potential. (Units.) | 35 | $X_p = a\sqrt{\dfrac{a}{ks}}$ | $20\sqrt{\dfrac{20}{.0000000217 \times 36{,}000}} = 3{,}200$ units. |
| | 36 | $X_p = \dfrac{q}{\sqrt{p}}$ | $\dfrac{10{,}000}{\sqrt{9.765}} = 3{,}200$ units. |
| Equivalent orifice. (Sq. ft.) | 37 | $A = \dfrac{.0004\,q}{\sqrt{i}}$ | $\dfrac{.0004 \times 10{,}000}{\sqrt{1.87788}} = 2.919$ sq. ft. |
| Motive column, downcast air. (Ft.) | 38 | $M = D \times \dfrac{T-t}{459+T}$ | $306.77 \times \dfrac{350-32}{459+350} = 120.5$ ft. |
| | 39 | $M = \dfrac{p}{w}$ | $\dfrac{9.765}{.08098} = 120.5$ ft. |
| Motive column, upcast air. (Ft.) | 40 | $M = D \times \dfrac{T-t}{459+t}$ | $306.77 \times \dfrac{350-32}{459+32} = 198.7$ ft. |
| | 39 | $M = \dfrac{p}{w}$ | $\dfrac{9.765}{.04915} = 198.7$ ft. |

**Variation of the Elements.**—In the illustration of the foregoing table, we have assumed fixed conditions of motive column, as well as fixed conditions in the mine airways. It is often convenient, however, to know how the different elements, as velocity $v$, quantity $q$, pressure $p$, power $u$, etc., will vary in different circulations; since we may, by this means, compare the circulations in *different* airways, or the results obtained by applying different pressures and powers to the *same* airway. These laws of variation must always be applied with great care. For example, before we can ascertain how the quantity in circulation will vary in different airways, we must know whether the *pressure* or the *power* is constant or the same for each airway. The following rules may always be applied:

**For a constant pressure:** $v$ varies as $\sqrt{\dfrac{a}{lo}}$; $q$ varies as $a\sqrt{\dfrac{a}{lo}}$ (relative potential for pressure).

**For a constant power:** $v$ varies as $\dfrac{1}{\sqrt[3]{lo}}$; $q$ varies as $\dfrac{a}{\sqrt[3]{lo}}$ (relative potential for power).

**For a constant velocity:** $q$ varies as $a$; $p$ varies as $\dfrac{lo}{a}$; $u$ varies as $lo$.

**For a constant quantity:** $v$ varies *inversely* as $a$; $p$ varies *inversely* as $X_u{}^3$ (potential for power); $u$ varies *inversely* as $X_u{}^3$ (potential for power) or directly as $p$.

**For the same airway:** The following terms vary as each other: $v$, $q$, $\sqrt{p}$, $\sqrt[3]{u}$.

### SIMILAR AIRWAYS

$r$ = length of similar side, or similar dimension

**For a constant pressure:** $v$ varies as $\sqrt{\dfrac{r}{l}}$; $q$ varies as $r^2 \times \sqrt{\dfrac{r}{l}}$; $r$ varies as $lv^2$, or $\sqrt[5]{lq^2}$.

**For a constant power:** $v$ varies as $\dfrac{1}{\sqrt[3]{lr}}$; $q$ varies as $r \times \sqrt[3]{\dfrac{r^2}{l}}$; $r$ varies as $\dfrac{1}{lv^3}$, or $\sqrt[7]{lq^3}$.

**For a constant velocity:** $q$ varies as $r^2$; $p$ varies as $\dfrac{l}{r}$; $u$ varies as $lr$; $r$ varies as $\sqrt{q}$, $\dfrac{l}{p}$, or $\dfrac{u}{l}$.

**For a constant quantity:** $v$ varies *inversely* as $r^2$; $p$ and $u$ vary *inversely* as $\dfrac{r^5}{l}$; $r$ varies as $\dfrac{1}{\sqrt{v}}$, $\sqrt[5]{\dfrac{l}{p}}$, or $\sqrt[5]{\dfrac{l}{u}}$.

### FURNACE VENTILATION

$p$ (motive column) varies as $D$; $q$ varies as $\sqrt{D}$

### FAN VENTILATION

It has been customary in calculations pertaining to the yield of centrifugal ventilators to assume as follows: $q$ varies as $n$; $p$ varies as $n^2$; $u$ varies as $n^3$.

More recent investigation, however, shows that when we double the speed we do not obtain double the quantity of air in circulation; or, in other words, the quantity does not vary exactly as the number of revolutions of the fan. Investigation also points to the fact that the efficiency of centrifugal ventilators *decreases* as the speed *increases*. To what extent this is the case has not been thoroughly established. The variation between the speed of a fan and the quantity, pressure, power, and efficiency, as calculated from a large number of reliable fan tests, may be stated as follows:

**For the same fan,** discharging against a *constant* potential: $q$ varies as $n^{.97}$. $p$ varies as $n^{1.94}$. *Complement of efficiency* $(1-K)$ varies as $n^{.425}$.

The efficiency here referred to is the *mechanical efficiency*, or the ratio between the effective work $qp$ and the theoretical work of the fan.

**Quantity Produced by Two or More Ventilators.**—In the development of a mine, it often happens that the means used for producing a ventilating current becomes inadequate for the production of the quantity of air required as the extent of the workings increases. To increase the circulation, it is often proposed to duplicate the ventilating apparatus in use by adding another fan or furnace similar to the one already in operation. This means an increase of ventilating power, which, of course, produces an increase in the quantity of air in circulation. Assuming that no change is made in the course of the circulation of the air through the mine, any increase of quantity will require an increase of power in proportion to the cube of the ratio in which the quantity is increased, as is shown by the following comparison of power and quantity for a given airway:

If $u_1$ represents the power on the air for a given airway when a quantity $q_1$ is circulating, $u_1 = \dfrac{ksq_1{}^3}{a^3} = \left(\dfrac{ks}{a^3}\right) q_1{}^3$; if $u_2$ represents the power on the air

when a quantity $q_2$ is circulating through the same airway, $u_2 = \dfrac{ksq_2^3}{a^3} = \left(\dfrac{ks}{a^3}\right) q_2^3$; then,

$$\frac{u_1}{u_2} = \frac{\left(\dfrac{ks}{a}\right) q_1^3}{\left(\dfrac{ks}{a^3}\right) q_2^3}$$

As the same airway is considered in each case, $k$, $s$, and $a$ are the same and by canceling, $\dfrac{u_1}{u_2} = \dfrac{q_1^3}{q_2^3}$ or $u_1 : u_2 = q_1^3 : q_2^3$; that is, for the same airway, the power is proportional to the cube of the quantity, or the ratio between the powers for two quantities of air equals the cube of the ratio between the quantities. For example, if the quantity is to be doubled, the quantity ratio is then 2 and the power ratio is $2^3 = 8$. That is to say, it will require eight times the power to double the quantity of air in the same mine or airway. This shows that two fans of the same size and running at the same speed will not produce double the quantity of air circulated by one of these fans alone.

When two or more ventilating motors are employed, it is evident that the total power producing the circulation is equal to the sum of the powers of the several motors.

Now, calling the quantities produced by several motors working separately on the same airway $q_1$, $q_2$, etc., the powers of these several motors $u_1$, $u_2$, etc., and the total quantity produced when all the motors are working together,

$Q$, $Q^3 \left(\dfrac{ks}{a^3}\right) = q_1^3 \left(\dfrac{ks}{a^3}\right) + q_2^3 \left(\dfrac{ks}{a^3}\right) +$ etc. Or, dividing both members of the

equation by $\dfrac{ks}{a^3}$, $Q^3 = q_1^3 + q_2^3 +$ etc., and, finally,

$$Q = \sqrt[3]{q_1^3 + q_2^3 + \text{etc.}}$$

This formula shows the quantity of air produced by the combined action of two or more ventilating motors working on the same mine or airway, and which, when working alone, produce the quantities $q_1$, $q_2$, etc., in the same mine or airway.

EXAMPLE.—A fan ventilating a certain mine is capable of producing 42,600 cu. ft. of air when operated alone, and another fan ventilating the same mine will produce 57,400 cu. ft. when working alone; what quantity of air will be produced in this mine when both fans are in operation, assuming that the general conditions in the mine remain the same?

SOLUTION.—Substituting the given quantities in the formula, and calling the unknown quantity $x$, the total quantity of air produced by the combined action of the two fans

$$x = \sqrt[3]{42,600^3 + 57,400^3} = 64,300 \text{ cu. ft. per min.}$$

The installation of two fans side by side at the mouth of the same return is very unusual, but the installation of a second fan called a "booster" at some point in the interior of the workings is a fairly common practice. Boosters unquestionably increase the amount of air in circulation by aiding the main fan to overcome the frictional resistances of exceptionally long air-courses, but their operation is expensive for the reasons explained. They are permissible in ventilating headings that will have but a short life, or even main workings which are shortly to be abandoned, but should not be tolerated as part of the permanent equipment of a mine. It is frequently the case that a booster is installed when the cleaning up of the return air-courses to their proper normal width would have permitted the main fan to have supplied the necessary quantity of air, and more cheaply.

## DISTRIBUTION OF AIR IN MINE VENTILATION

When a mine is first opened, the air is conducted in a single current around the face of all the headings and workings, and returns again to the upcast shaft, where it is discharged into the atmosphere. As the development of the mine advances, however, it becomes necessary to divide the air into two or more splits or currents. This division or splitting of the air-current is usually accomplished at the foot of the downcast, or as soon as possible after the current enters the mine. There are several reasons why

the air current should be thus divided. The most important reason is that the mine is thereby divided into separate districts, each of which has its own ventilating current, which may be increased or decreased at will. Fresh air is thus obtained at the face of the workings, and the ventilation is under more perfect control. It often happens that certain portions of a mine are more gaseous than others, and it is necessary to increase the volume of air in these portions, which can be readily accomplished when each district has its own separate circulation. Again, the gases and foul air are not conducted from one district to another, but each district is supplied with fresh air direct from the main intake. Should an explosion occur in any part of the mine, it is more apt to be confined to one locality when a mine is thus divided into separate districts. Another consideration is the reduced power necessary to accomplish the same circulation in the mine; or the increased circulation obtained by the use of the same power.

**Requirements of Law in Regard to Splitting.**—The Anthracite Mine Law of Pennsylvania specifies that every mine employing more than 75 persons must be divided into two or more ventilating districts, thus limiting the number that are allowed to work on one air current to 75 persons. The Bituminous Mine Law of Pennsylvania limits the number allowed to work upon one current to 65 persons, except in special cases, where this number may be increased to 100 persons at the discretion of the mine inspector.

**Practical Splitting of the Air Current.**—When the air current is divided into two or more branches, it is said to be *split*. The current may be divided one or more times; when split or divided once, the current is said to be traveling in *two splits*, each branch being termed a split. The number of splits in which a current is made to travel is understood as the number of separate currents in the mine, and not as the number of *divisions* of the current.

**Primary Splits.**—When the main air current is divided into two or more splits, each of these is called a *primary split*.

**Secondary Splits.**—Secondary splits are the divisions of a primary split.

**Tertiary Splits.**—Tertiary splits result from the division of a secondary split.

**Equal Splits of Air.**—When a mine is spoken of as having two or more *equal splits*, it is understood to mean that the length and the size of the separate airways forming those splits are equal in each case. It follows, of course, from this that the ventilating current traveling in each split will be the same, inasmuch as they are all subject to the same ventilating pressure. When an equal circulation is obtained in two or more splits by the use of regulators, these splits cannot be spoken of as equal splits.

**Unequal Splits of Air.**—By this is meant that the airways forming the splits are of unequal size or length. Under this head we will consider (a) *Natural Division of the Air Current*; (b) *Proportionate Division of the Air Current*.

**Natural Division of the Air Current.**—By *natural division of air* is meant any division of the air that is accomplished without the use of regulators; or, in other words, such division of the air current as results from natural means. If the main air current at any given point in a mine is free to traverse two separate airways in passing to the foot of the upcast shaft, and each of these airways is free or an open split, i.e., contains no regulator, the division of the air will be a *natural division*. In such a case, the larger quantity of air will always traverse the shorter split of airway. In other words, an air current always seeks the shortest way out of a mine. A comparatively small current, however, will always traverse the long split or airway.

**Calculation of Natural Splitting.**—It is always assumed, in the calculation of the splitting of air currents, that the pressure at the mouth of each split, starting from any given point, is the same. Since this is the case, in order to find the quantity of air passing in each of several splits starting from a common point, the rule given under Potential Factor of a Mine is applied. This rule may be stated as follows:

*The ratio between the quantity of air passing in any split and the pressure potential of that split is the same for all splits starting from a common point. Also, the ratio between the entire quantity of air in circulation in the several splits and the sum of the pressure potentials of those splits is the same as the above ratio, and is equal to the square root of the pressure.*

Expressed as a formula, indicating the sum of the pressure potentials $(X_1 + X_2 + \text{etc.})$ by the expression $\Sigma X_p$, this rule is $\dfrac{Q}{\Sigma X_p} = \dfrac{q_1}{X_1} = \sqrt{p}$. Hence,

$p = \dfrac{Q^2}{(\Sigma X_p)^2}$ and $u = \dfrac{Q^3}{(\Sigma X_p)^2}$ express the pressure and power, respectively, absorbed by the circulation of the splits. These are the basal formulas for splitting, from which any of the factors may be calculated by transposition. They will be found illustrated in the table at the end of this section. We will give here two examples only, showing the calculation of the natural division of an air current between several splits. We have, from the above formulas,

$$q_1 = \frac{X_1}{\Sigma X_p} Q.$$

**EXAMPLE.**—In a certain mine, an air current of 60,000 cu. ft. per min. is traveling in two splits as follows: Split $A$, 6 ft.$\times$8 ft., 5,000 ft. long; split $B$, 5 ft.$\times$8 ft., 10,000 ft. long. It is required to find the natural division of this air current.

Calculating the relative potentials for pressure in each split, we have

$$\left. \begin{array}{l} \text{for split } A, \ X_1 = 48\sqrt{\dfrac{48}{2(6+8)\,5,000}} = .8888 \\[2mm] \text{for split } B, \ X_2 = 40\sqrt{\dfrac{40}{2(5+8)\,10,000}} = .4961 \end{array} \right\} \text{ and } \Sigma X_p = 1.3849;$$

and substituting these values, we have,

$$q_1 = \frac{.8888}{1.3849} \times 60,000 = 38,506 \text{ cu. ft. per min.};$$

and

$$q_2 = \frac{.4961}{1.3849} \times 60,000 = 21,494 \text{ cu. ft. per min.}$$

**EXAMPLE.**—In a certain mine, there is an air current of 100,000 cu. ft. per min. traveling in three splits as follows: Split $A$, 6 ft.$\times$10 ft., 8,000 ft. long; split $B$, 6 ft.$\times$12 ft., 15,000 ft. long; split $C$, 5 ft.$\times$10 ft., 6,000 ft. long. Find the natural division of this current of air.

Calculating the respective relative potentials with respect to pressure, we have

$$\text{for split } A, \ X_1 = 60\sqrt{\frac{60}{2(6+10)\times 8,000}} = .9185;$$

$$\text{for split } B, \ X_2 = 72\sqrt{\frac{72}{2(6+12)\times 15,000}} = .8314;$$

$$\text{for split } C, \ X_3 = 50\sqrt{\frac{50}{2(5+10)\times 6,000}} = .8333.$$

Adding these potentials, we have $\Sigma X_p = .9185 + .8314 + .8333 = 2.5832$. Then, applying the foregoing rule, we have

$$q_1 = \frac{.9185}{2.5832} \times 100,000 = 35,556 \text{ cu. ft. per min.};$$

$$q_2 = \frac{.8314}{2.5832} \times 100,000 = 32,184 \text{ cu. ft. per min.};$$

and

$$q_3 = \frac{.8333}{2.5832} \times 100,000 = 32,260 \text{ cu. ft. per min.}$$

$$\text{Total,} \quad \overline{100,000}$$

**Proportional Division of the Air Current.**—It continually happens that different proportions of air are required in the several splits of a mine than would be obtained by the natural division of the air current. It is usually the case that the longer splits employ a larger number of men, and require a larger quantity of air passing through them. They, moreover, liberate a larger quantity of mine gases, for which they require a larger quantity of air than is passing in the smaller splits. The natural division of the air current would give to these longer splits *less* air, and to the shorter ones a *larger* amount of air, which is directly the reverse of what is needed. On this account, recourse must be had to some means of dividing this air proportionately, as required. This is accomplished by the use of regulators, of which there are two general types, the *box regulator* and the *door regulator*.

**Box Regulator.**—This is simply an obstruction placed in those airways that would naturally take more air than the amount required. It consists of a brattice or door placed in the entry, and having a small shutter that can be opened to a greater or less amount. The shutter is so arranged as to allow the passage of more or less air, according to the requirements. The box regulator is, as a rule, placed at the end or near the end of the return air-

way of a split. It is usually placed at this point as a matter of convenience, because, in this position, it obstructs the roads to a less extent, the haulage from the back entry in this split being carried over to the main haulway, through a crosscut, before this point is reached. The difficulty, however, can be avoided, in most cases, by proper consideration in the planning of the mine with respect to haulage and ventilation. The objection to this form of regulator is that, in effect, it lengthens the airway, or increases its resistance, making the resistance of all the airways, per foot of area, the same. It is readily observed that, by thus increasing the resistance of the mine, the horsepower of the ventilation is largely increased, for the same circulation. This is an important point, as it will be found that the power required for ventilation is thus increased anywhere from 50 % to 100 % over the power required when the other form of regulator can be adopted.

**Door Regulator.**—In this form of regulator, which was first introduced by Beard, the division of the air is made at the mouth of the split. The regulator consists of a door hung from a point of the rib between two entries, and swung into the current so as to cut the air like a knife. The door is provided with a set lock, so that it may be secured in any position, to give more or less air to the one or the other of the splits, as required. The position of this regulator door, as well as the position of the shutter in the box regulator, is always ascertained practically by trial. The door is set so as to divide the area of the airway proportionate to the work absorbed in the respective splits. The pressure in any split is not increased, each split retaining its natural pressure.

**Calculation of Pressure for Box Regulators.**—When any required division of the air current is to be obtained by the use of box regulators, these are placed in all the splits, save one. This split is called the *open*, or *free, split*, and its pressure is calculated in the usual way by the formula $p = \dfrac{ksq^2}{a^3}$. The natural pressure in this open split determines the pressure of the entire mine, since all the splits are subject to the same pressure in this form of splitting.

First, determine in which splits regulators will have to be placed, in order to accomplish the required division of the air. Calculate the natural pressure, or pressure due to the circulation of the air current, for each split, when passing its required amount of air, using the formula $p = \dfrac{ksq^2}{a^3}$. The split showing the greatest natural pressure is taken as the free split. In each of the other splits, box regulators must be placed, to increase the pressure in those splits; or, in other words, to increase the resistance of those splits per unit of area.

EXAMPLE.—The ventilation required in a certain mine is:

split $A$, 6 ft. × 9 ft., 8,000 ft. long; 40,000 cu. ft. per min.
split $B$, 5 ft. × 8 ft., 6,000 ft. long; 40,000 cu. ft. per min.
split $C$, 9 ft. × 9 ft., 8,000 ft. long; 10,000 cu. ft. per min.
split $D$, 6 ft. × 8 ft., 10,000 ft. long; 30,000 cu. ft. per min.

In which of these splits should regulators be placed, to accomplish the required division of air, and what will be the mine pressure?

Calculating the pressure due to friction in each split when passing its required amount of air, we find,

for split $A$, $p = \dfrac{.0000000217 \times 2(6+9)8,000 \times 40,000^2}{54^3} = 52.92$ lb. per sq. ft.;

for split $B$, $p = \dfrac{.0000000217 \times 2(5+8)6,000 \times 40,000^2}{40^3} = 84.63$ lb. per sq. ft.;

for split $C$, $p = \dfrac{.0000000217 \times 2(9+9)8,000 \times 10,000^2}{81^3} = 1.176$ lb. per sq. ft.;

for split $D$, $p = \dfrac{.0000000217 \times 2(6+8)10,000 \times 30,000^2}{48^3} = 49.45$ lb. per sq. ft.

Split $B$ has the greatest pressure, and is therefore the free split. Box regulators are placed in each of the other splits to increase their respective pressures to the pressure of the free split or the mine pressure. Therefore, the mine pressure in this circulation is 84.63 lb. per sq. ft.

**The size of opening in a box regulator** is calculated by the formula for determining the flow of air through an orifice in a thin plate under a certain head or pressure. The difference in pressure between the two sides of a box regulator is the pressure establishing the flow through the opening, which

corresponds to the head $h$ in the formula $v = \sqrt{2gh}$. This regulator is usually placed at the end of a split or airway, and since the regulator increases the pressure in the lesser splits so as to make it equal to the pressure in the other split, the pressure due to the regulator will be equal to the ventilating pressure at the mouth of the split, less the *natural* pressure or the pressure due to friction in this split. Hence, when the position of the regulator is at the end of the split, the pressure due to friction in the split is first calculated by the formula $p = \dfrac{ksq^2}{a^3}$, and this pressure is deducted from the ventilating pressure of the free or open split, which gives the pressure due to the regulator. This is then reduced to inches of water gauge, and substituted for $i$ in the formula $A = \dfrac{.0004q}{\sqrt{i}}$. The value of $A$ thus obtained is the area (square feet) of the opening in the regulator.

EXAMPLE.—50,000 cu. ft. of air is passing per min. in a certain mine, in two equal splits, under a pressure equal to 2 in. of water gauge, and it is required to reduce the quantity of air passing in one of these splits, by a box regulator placed at the end of the split, so as to pass but 15,000 cu. ft. per min. in this split. Find the area of the opening in the regulator, assuming that the ventilating power is decreased to maintain the pressure constant at the mouth of the splits after placing the regulator. The size and length of each split is 6 ft. × 10 ft. and 10,000 ft. long.

The natural pressure for the split in which the regulator is placed will be

$$p = \frac{ksq^2}{a^3} = \frac{.0000000217 \times 2(6+10)10,000 \times 15,000^2}{(6 \times 10)^3} = 7.233 \text{ lb. per sq. ft.}$$

Then, $\dfrac{7.233}{5.2} = 1.4$ in. of water gauge (nearly), due to friction of the air current in this split. And, $2 - 1.4 = .6$ in. water gauge due to regulator. Finally, $A = \dfrac{.0004q}{\sqrt{.6}} = \dfrac{.0004 \times 15,000}{\sqrt{.6}} = 7.746$ sq. ft., area of opening.

**Size of Opening for a Door Regulator.**—The sectional area at the regulator is divided proportionally to the work to be performed in the respective splits according to the proportion $A_1 : A_2 : : u_1 : u_2$. Or since $A_1 + A_2 = a$, we have $A_1 : a : : u_1 : u_1 + u_2$, and $A_1 = \dfrac{u_1}{u_1 + u_2} \times a$. This furnishes a method of proportionate splitting in which each split is ventilated under its own natural pressure. The same result would be obtained by the placing of the box regulator at the intake of any split, thereby regulating the amount of air passing into that split, but the door regulator presents less resistance to the flow of the air current. The practical difference between these two forms of regulators is that in the use of the box regulator each split is ventilated under a pressure equal to the natural pressure of the open or free split, which very largely increases the horsepower required for ventilation of the mine; while in the use of the door regulator each split is ventilated under its own natural pressure, and the proportionate division of the air is accomplished without any increase of horsepower. This is more clearly explained in the following two paragraphs, and the table showing the comparative horsepowers of the two methods.

**Calculation of Horsepower for Box Regulators.**—By the use of the box regulator, the pressure in all the splits is made equal to the greatest *natural* pressure in any one. This split is made the open or free split, and its natural pressure becomes the pressure for all the splits, or the *mine* pressure. This mine pressure, multiplied by the total quantity of air in circulation (the sum of the quantities passing in the several splits), and divided by 33,000, gives the horsepower upon the air, or the horsepower of the circulation. Thus, in the first example given on page 920, in which for split $B$ the pressure $p = 84.63$ lb. per sq. ft. and the total quantity of air passing per minute is 12,000 cu. ft., we have

$$h = \frac{84.63 \times 120,000}{33,000} = 307.745 \text{ H. P.}$$

**Calculation of Horsepower for Door Regulators.**—In the use of the door regulator, each split is ventilated under its own natural pressure, and, hence, in the calculation of the horsepower of such a circulation, the power of each split must be calculated separately, and the sum of these several

powers will be the entire power of the circulation. For the purpose of comparison, we tabulate below the results obtained in the application of these two methods of dividing the air in the above example

| Splits | Natural Division | Required Division | Horsepower | |
|---|---|---|---|---|
| | | | Door Regulator | Box Regulator |
| Split $A$, 6 ft. × 9 ft., 8,000 ft. long. | 28,277 | 40,000 | 64.145 | 102.582 |
| Split $B$, 5 ft. × 8 ft., 6,000 ft. long. | 22,360 | 40,000 | 102.582 | 102.582 |
| Split $C$, 9 ft. × 9 ft., 8,000 ft. long. | 47,423 | 10,000 | .356 | 25.645 |
| Split $D$, 6 ft. × 8 ft., 10,000 ft. long. | 21,940 | 30,000 | 44.955 | 76.936 |
| Totals.................... | 120,000 | 120,000 | 212.038 | 307.745 |

## SPLITTING FORMULAS

The following table of formulas will serve to illustrate the methods of calculation in splitting. The example assumes the same airway as that given on page 912 and used to illustrate the table of formulas, pages 913, 914 and 915 but the air current is divided, as specified in the table:

NATURAL DIVISION
**Primary Splits.**—Split (1) = 4 ft. × 5 ft., 800 ft. long. Split (2) = 4 ft. × 5 ft., 1,200 ft. long.

| To Find: | No. | Formula | Specimen Calculation |
|---|---|---|---|
| Potential for pressure. | 35 | $X_p = a \sqrt{\dfrac{a}{ks}}$ $\Sigma X_p = (X_1 + X_2 + \text{etc.})$. | (1) $20\sqrt{\dfrac{20}{.0000000217 \times 14,400}} = 5,060.$  (2) $20\sqrt{\dfrac{20}{.0000000217 \times 21,600}} = 4,131.$  $5,060 + 4,131 = 9,191.$ |
| Natural division. | 41 | $q = \dfrac{X_p}{\Sigma X_p} \times Q.$ | (1) $\dfrac{5,060}{9,191} \times 10,000 = 5,505$ cu. ft.  (2) $\dfrac{4,131}{9,191} \times 10,000 = 4,495$ cu. ft. |

Or the natural division may be calculated from the pressure at the mouth of the several splits by using Formula (23); thus,

| | 23 | $q = X_p \sqrt{p}.$ | (1) $5,060\sqrt{1.1838} = 5,505$ cu. ft.  (2) $4,131\sqrt{1.1838} = 4,495$ cu. ft.  See formula (42). |
| Pressure | 42 | $p = \dfrac{Q^2}{(\Sigma X_p)^2}.$ | $\dfrac{10,000^2}{9,191^2} = 1.1838$ lb. |
| Power | 43 | $u = \dfrac{Q^3}{(\Sigma X_p)^2}.$ | $\dfrac{10,000^3}{9,191^3} = 11,838$ units. |
| Quantity | 44 | $Q = \Sigma X_p \sqrt{p}.$ | $9,191\sqrt{1.1838} = 10,000$ cu. ft. |
| | 45 | $Q = \sqrt[3]{(\Sigma X_p)^2 u}.$ | $\sqrt[3]{9,191^2 \times 11,838} = 10,000$ cu. ft. |
| Increase of quantity due to splitting. (Pressure constant.) | 46 | $Q = \dfrac{\Sigma X_p}{X_{p-o}} \times q_o.$ | $\dfrac{9,191}{3,200} \times 10,000 = 28,722$ cu. ft. |
| Increase in quantity due to splitting. (Power constant.) | 47 | $Q = q \sqrt[3]{\left(\dfrac{\Sigma X_p}{X_{p-o}}\right)^2}.$ | $10,000\sqrt[3]{\left(\dfrac{9,191}{3,200}\right)^2} = 20,205$ cu. ft. |

| To Find: | No. | Formula | Specimen Calculation |
|---|---|---|---|
| Relative potential for pressure | 35 | $X_p = a\sqrt{\dfrac{a}{s}}$. | (1) $20\sqrt{\dfrac{20}{14,400}} = .7471.$ (2) $20\sqrt{\dfrac{20}{9,000}} = .9428.$ (3) $20\sqrt{\dfrac{20}{7,200}} = 1.0541.$ (4) $20\sqrt{\dfrac{20}{5,400}} = 1.2172.$ |
| Natural division | 48 · 41 | $q_1 = \dfrac{Q - q_2}{Q}$. <br><br> $q_2 = \dfrac{}{1+X_1\sqrt{\dfrac{1}{X_2{}^2}+\dfrac{1}{(X_3+X_4)^2}}}$ <br><br> $q = \dfrac{X_p}{\Sigma X_p}\times Q$ | (1) $10,000 - 5,388 = 4,612$ cu. ft. See Formula (48). <br> (2) $\dfrac{10,000}{1+.7471\sqrt{\dfrac{1}{.9428^2}+\dfrac{1}{(1.0541+1.2172)^2}}} = 5,388$ cu. ft. <br> (3) $\dfrac{1.0541}{(1.0541+1.2172)}\times 5,388 = 2,500$ cu. ft. <br> (4) $\dfrac{1.2172}{(1.0541+1.2172)}\times 5,388 = 2,888$ cu. ft. |
| Pressure | 49 | $p = k\left(\dfrac{Q}{X_1+\dfrac{1}{\sqrt{\dfrac{1}{X_2{}^2}+\dfrac{1}{(X_3+X_4)^2}}}}\right)^2$ | $.0000000217\left(\dfrac{10,000}{.7471+\dfrac{1}{\sqrt{\dfrac{1}{.9428^2}+\dfrac{1}{(1.0541+1.2172)^2}}}}\right)^2 = .8290$ lb. |
| Power | 50 | $u = k\left(\dfrac{Q^3}{X_1+\dfrac{1}{\sqrt{\dfrac{1}{X_2{}^2}+\dfrac{1}{(X_3+X_4)^2}}}}\right)^2$ | $.0000000217\left(\dfrac{10,000^3}{.7471+\dfrac{1}{\sqrt{\dfrac{1}{.9428^2}+\dfrac{1}{(1.0541+1.2172)^2}}}}\right)^2 = 8,290$ units. |
| Quantity | 51 | $Q = \left(X_1+\dfrac{1}{\sqrt{\dfrac{1}{X_2{}^2}+\dfrac{1}{(X_3+X_4)^2}}}\right)\sqrt{\dfrac{p}{k}}$ | $\left(.7471+\dfrac{1}{\sqrt{\dfrac{1}{.9428^2}+\dfrac{1}{(1.0541+1.2172)^2}}}\right)\times\sqrt{\dfrac{.829}{.0000000217}} = 10,000$ cu. ft. |

**Secondary Splits.**—(1) 4 ft.×5 ft., 800 ft. long. (2) 4 ft.×5 ft., 500 ft. long. (3) 4 ft.×5 ft., 400 ft. long. (4) 4 ft.×5 ft., 300 ft. long.

The calculation is often shortened, when many splits are concerned, by using the *relative potential*, omitting the factor $k$; but the final result must then be multiplied by $k$ to obtain the pressure or power; or, these factors must be divided by $k$, when finding the quantity, as in formulas (49) to (51).

### PROPORTIONATE DIVISION

**Primary Splits** (only).—(1) 4 ft.×5 ft., 800 ft. long = 3,500 cu. ft. (2) 4 ft.×5 ft., 1,200 ft. long = 6,500 cu. ft.

| To Find: | No. | Formula | Specimen Calculation | |
|---|---|---|---|---|
| Pressure due to friction. | 13 | $p = \dfrac{q^2}{X_p^2}$ | (1) | $\dfrac{3,500^2}{5,060^2} = .47845$ lb. |
|  |  |  | (2) | $\dfrac{6,500^2}{4,131^2} = 2.4757$ lb. |

To accomplish this division of air, the pressure in split (1) must be increased by means of a regulator to make it equal to the pressure in the free or open split (2), and, hence, the pressure due to the regulator is equal to the difference between the natural pressures in these splits.

| Pressure due to the regulator in split (1). | 53 | $p = p_2 - p_1$ | $2.4757 - .47845 = 1.99725$ lb. | |
| Area of the opening in regulator. | 37 | $A = \dfrac{.0004q}{\sqrt{i}}$ | $\dfrac{.0004 \times 3,500}{\sqrt{\dfrac{1.99725}{5.2}}} = 2.259$ sq. ft. | |

**Secondary Splits.**—(1) 4 ft.×5 ft., 800 ft. − 3,500 cu. ft. (2) 4 ft.×5 ft., 500 ft. − 6,500 cu. ft. (3) 4 ft.×5 ft., 400 ft. − 4,000 cu. ft. (4) 4 ft.×5 ft., 300 ft. − 2,500 cu. ft.

NOTE.—When using the relative potential, multiply the result by $k$, to obtain the pressure, or the power.

| Pressure due to friction. Free split—second-pressure. | 13 | $p = \left(\dfrac{q}{X_p}\right)^2$ | (1) $.0000000217\left(\dfrac{3,500}{.7471}\right)^2 = .47848$ lb. |
|---|---|---|---|
|  |  |  | (2) $.0000000217\left(\dfrac{6,500}{.9428}\right)^2 = 1.0314$ lb. |
|  |  |  | (3) $.0000000217\left(\dfrac{4,000}{1.0541}\right)^2 = .31248$ lb. |
|  |  |  | (4) $.0000000217\left(\dfrac{2,500}{1.2172}\right)^2 = .091546$ lb. |

Since the natural pressure in (3) is greater than that in (4), (3) is the free split, and its natural pressure is the pressure for the secondary splits. The pressure for the primary splits is then found by first adding the pressures in (2) and (3), and if their sum is greater than the natural pressure in (1), it becomes the pressure for the primary splits, or the mine pressure. If the natural pressure for (1) is the greater, this is made the free split, and *its* natural pressure becomes the primary or mine pressure. In this case, the secondary pressure must be increased by placing a regulator in split (3).

| Primary or mine pressure. |  | $p_2 + p_3$ | $1.0314 + .31248 = 1.34388.$ | |
|---|---|---|---|---|
| Pressure due to the regulators. |  | $p_3 - p_4$ $(p_2+p_3) - p_1.$ | (4) $.31248 - .091546 = .220934$ lb. (1) $(1.0314 + .31248) - .47848$ $= .86540$ lb. | |
| Areas of openings in the regulators. | 37 | $A = \dfrac{.0004q}{\sqrt{i}}$ | (4) $\dfrac{.0004 \times 2,500}{\sqrt{\dfrac{.220934}{5.2}}} = 4.8514$ sq. ft. | |
|  |  |  | (1) $\dfrac{.0004 \times 3,500}{\sqrt{\dfrac{.8654}{5.2}}} = 3.4328$ sq. ft. | |

# METHODS AND APPLIANCES IN THE VENTILATION OF MINES

**Ascensional Ventilation.**—Every mine, as far as practicable, should be ventilated upon the plan known as ascensional ventilation. This term refers particularly to the ventilation of inclined seams. The air should enter the mine at its lowest point, as nearly as possible, and from thence be conducted through the mine to the higher points, and there escape by a separate shaft, if such an arrangement is practicable. Where the seam is dipping considerably and is mined through a vertical shaft, the upcast shaft should be located as far as possible to the rise of the downcast shaft as possible. The intake air is then first conducted to the lowest point of the dip workings, which it traverses upon its way to the higher workings. In the case of a slope working where a pair of entries is driven to the dip, one being used as the intake and the other the return, there being cross-entries or levels driven at regular intervals along the slope, the air should be conducted at once to the inside workings, from which point it returns, ventilating each pair of cross-entries from the inside, outwards. Where the development of the cross-entries or levels is considerable, their circulation is considered separately, and a fresh air split is made in the intake at each pair of levels. In all ventilation, the main point to be observed is to conduct the air current first to the inside workings, from whence it is distributed along the working face as it returns toward the upcast.

**General Arrangement of Mine Plan.**—Every mine should be planned with respect to three main requirements, viz.: (a) *haulage;* (b) *drainage;* (c) *ventilation.* These requirements are so closely connected with one another that the consideration of one of them necessitates a reference to all. The mine should be planned so that the coal and the water will gravitate towards the opening, as far as possible. There are many reasons, in the consideration of non-gaseous mines, why the haulage should be effected upon the return airways. The haulage road is always a dusty road, caused by the traveling of men and mules, as well as by the loss of coal in transit, which becomes reduced to fine slack and powder. If the haulage is accomplished upon the intake entry or air-course, this dust is carried continually into the mine and working places, which should be avoided whenever possible. When the loaded cars move in the same direction as the return air, the ventilation of the mine is not as seriously impeded. It is often the case that fewer doors are required upon the return airway than upon the intake, which is a feature favorable to haulage roads. Again, in this arrangement, the hoisting shaft is made the upcast shaft, which prevents the formation of ice, and consequent delay in hoisting in the winter season. The arrangement, however, presupposes the use of the force fan or blower, since if a furnace or exhaust fan is employed, a door, or probably double doors, would have to be placed upon the main haulage road at the shaft bottom, which would be a great hindrance.

In the ventilation of gaseous mines, however, other and more important considerations demand attention. The gaseous character of the return current prevents making the return airway a haulageway. In such mines, the haulage should always be accomplished upon the intake air, as any other system would often result in serious consequences. In such gaseous mines, men and animals must be kept off the return airways as far as this is possible.

As far as practicable, ventilation should be accomplished in sections or districts, each district having its own split of air from the main intake, and its own return connecting with the main return of the mine. Reference has been made to this under Distribution of the Air in Mine Ventilation. This splitting of the air current is accomplished preferably by means of an air bridge, either an under crossing or an over crossing. There are, in general, three systems of ventilation, with respect to the ventilating motor employed: (a) *natural ventilation:* (b) *furnace ventilation;* (c) *mechanical ventilation.*

**Natural ventilation** means such ventilation as is secured by natural means, or without the intervention of artificial appliances, such as the furnace, or any mechanical appliances by which the circulation of air is maintained. In natural ventilation, the ventilating motor or air motor is an air column that exists in the downcast shaft by virtue of the greater weight of the downcast air. This air column acts to force the air through the airways

of the mine. An air column always exists where the intake and return currents of air pass through a certain vertical height, and have different temperatures. This is the case whether the opening is a shaft or a slope; since, in either case, there is a vertical height, which in part determines the height of air column. The other factor determining the height of air column is the difference of temperature between the intake and return. The calculation of the ventilating pressure in natural ventilation is identical with that of furnace ventilation, which is described later.

**Ventilation of Rise and Dip Workings.**—We have referred to the air column existing either in vertical shafts or slopes as the motive column or ventilating motor. Such an air column will be readily seen to exist in any rise or dip workings within the mine, and may assist or retard the circulation of the air current through the mine. It is this air column that renders the ventilation of dip workings easy, and that of rise workings correspondingly difficult, depending, however, on the relative temperature of the intake and return currents; the latter usually is the warmer of the two, which gives rise to the air column. The influence of such air columns must always be taken into account in the calculation of any ventilation. This is often neglected.

The influence of air columns in rise or dip workings, within the mine, becomes very manifest where, from any reason, the main intake current is increased or decreased. For example, a mine is ventilated in two splits, a *rise* and a *dip* split; a current of 50,000 cu. ft. of air is passing in the main airway, 30,000 cu. ft. passing into the dip workings, and 20,000 into the rise workings. A fall of roof in the main intake airway, or other cause, reduces the main current from 50,000 to 35,000 cu. ft. Instead, now, of 21,000 cu. ft. going to the dip workings and 14,000 to the rise workings, we find that this proportion no longer exists, but that the dip workings are taking *more* than their proportion of air, and the rise workings *less*. Thus, the circulation being decreased to 35,000 cu. ft., the dip workings will probably take 25,000 cu. ft., and the rise workings 10,000 cu. ft. On the other hand, had the intake current been *increased* instead of *decreased*, the *rise* workings would then take more than their proportion, while the *dip* workings would take less. The reason for this distribution is evident; suppose, for example, the intake or mine pressure is 3 in. of water gauge, and in the dip workings there is $\frac{1}{2}$ in. of water gauge acting to assist ventilation, while a like water gauge of $\frac{1}{2}$ in. in the rise workings acts to retard ventilation. The effective water gauge in the dip workings is therefore $3\frac{1}{2}$ in., while the effective water gauge in the rise workings is $2\frac{1}{2}$ in., or they are to each other as 7 : 5. If, now, the mine pressure is decreased to, say, 2 in., the effective rise and dip pressure will be, respectively, $2\frac{1}{2}$ in. and $1\frac{1}{2}$ in., or as 5 : 3. We observe, before the decrease, the dip pressure was $\frac{7}{5}$, or 1.4, times the rise pressure, while after the decrease took place in the mine pressure, the dip pressure became $\frac{5}{3}$, or 1.66, times the rise pressure. The relative quantities passing in the dip split before and after the decrease took place, as compared with the quantities passing in the rise split, will be as the $\sqrt{1.4}$ : $\sqrt{1.66}$, showing an increase of proportion. Now, instead of a *decrease* taking place in the mine pressure, let us suppose it is *increased*, say, from 3 in. to 4 in. The effective pressures in the dip and rise workings will then be, respectively, $4\frac{1}{2}$ in. and $3\frac{1}{2}$ in., or they will be to each other as 9 : 7, instead of 7 : 5. Here we observe that the dip pressure is $1\frac{2}{7}$, or 1.15, times the rise pressure, instead of 1.4. The relative quantities, therefore, passing in the *dip* split, before and after the increase of the mine pressure, as compared with the quantities passing in the rise split, will be in the ratio of $\sqrt{1.4}$ : $\sqrt{1.15}$, showing a decrease of proportion. We observe that any alteration of the mine pressure by which it is increased or decreased does not affect the inside dip or rise columns, and hence the disproportion obtains. In case of a decrease of the mine pressure, the dip workings receive more than their proportion of air, and in case of an increase of the mine pressure, they receive *less* than their proportion of air.

**Influence of Seasons.**—In any ventilation, air columns are always established in slopes and shafts, owing to the relative temperatures of the outside and inside air. The temperature of the upcast, or return column, may always be assumed to be the same as that of the inside air. The temperature of the downcast, or intake column, generally approximates the temperature of the outside air, although, in deep shafts or long slopes, this temperature may be changed considerably before the bottom of the shaft or slope is reached, and

consequently the average temperature of the downcast, or intake, is often different from that of the outside air. The difference of temperatures will also vary with the seasons of the year. In winter the outside temperature is below that of the mine, and the circulation in shafts and slopes is assisted, since the return columns are warmer and lighter than the intake columns for the same circulation. In the summer season, however, the reverse of this is the case. The course of the air current will thus often be changed. When the outside temperature approaches the average temperature of the mine, there will be no ventilation at all in such mines, except such as is caused by accidental wind pressure.

In **furnace ventilation** the temperature of the upcast column is increased above that of the downcast column by means of a furnace. The chief points to be considered in furnace ventilation are in regard to the arrangement and size of the furnace. Furnace ventilation should not be applied to gaseous seams, and in some cases is prohibited by law. It is, however, in use in many mines liberating gas. In such cases the furnace fire is fed by a current of air taken directly from the air-course, sufficient to maintain the fire, and the return current from the mine is conducted by means of a dumb drift, or an inclined passageway, into the shaft, at a point from 50 to 100 ft. above the seam. At this point, the heat of the furnace gases is not sufficient for the ignition of the mine gases. The presence of carbonic-acid gas in the furnace gases also renders the mine gases inexplosive. In other cases where the dumb drift is not used, a sufficient amount of fresh air is allowed to pass into the return current to insure its dilution below the explosive point before it reaches the furnace.

**Construction of a Mine Furnace.**—In the construction of a mine furnace, a sufficient area of passage must be maintained over the fire and around the furnace to allow the passage of the air current circulating in the mine. The velocity of the current at the furnace should be estimated not to exceed 20 ft. per sec. and the entire area of passage calculated from this velocity. Thus, for a current of 50,000 cu. ft. of air per min., the area of passage through and around the furnace should be not less than

$$\frac{50,000}{60 \times 20} = 41\tfrac{2}{3} \text{ sq. ft.}$$

This is a safe method of calculation, notwithstanding the fact that the velocity of the air is often much more than 20 ft. per sec., yet the volume of the air is largely increased owing to the increase of temperature.

The length of the furnace bars is limited to the distance in which good firing can be accomplished, and should not exceed 5 ft. The width of the grate will therefore determine the grate area. The grate area must, in every case, be sufficient for the heating of the air of the current to a temperature such as to maintain the average temperature of the furnace shaft high enough to produce the required air column, or ventilating pressure, in the mine. The area $A$ of the grate of the furnace is best determined by the formula

$A = \dfrac{34}{\sqrt{D}} \times \text{H. P.}$, in which $A$ = grate area in square feet; H. P. = horse-

power of the circulation; and $D$ = depth of shaft in feet. The horsepower for any proposed ventilation may always be determined by dividing the quantity of air (cubic feet per minute) by the mine potential $X_u$, and cubing and dividing the result by 33,000; thus

$$\text{H. P.} = \left(\frac{q}{X_u}\right)^3 \times \frac{1}{33,000}.$$

The furnace should have proper cooling spaces above and at each side; upon one side, at least, should be a passageway or manway. The furnace should be located at a point from 10 to 15 yd. back from the foot of the shaft, at a place in the airway where the roof is strong. This is well secured by railroad iron immediately over the furnace. A good foundation is obtained in the floor, and the walls of the furnace carried up above the level of the grate bars, when the furnace arch is sprung. If possible, a full semicircle should be used in preference to a flat arch. The sides and arch of the furnace should be carried backwards to the shaft; this is necessary in order to prevent ignition of the coal. The walls and arch are constructed of firebrick a sufficient distance from the furnace, and afterwards of a good quality of hard brick; the shaft is also lined with brick or protected by sheet iron a sufficient height to prevent the ignition of the curbing.

**Air Columns in Furnace Ventilation.**—As previously stated, natural ventilation and furnace ventilation are identical, in so far as in each the venti-

lating motor is an air column. This air column is an imaginary column of air whose weight is equal to the difference between the weights of the upcast and downcast columns. The upcast and downcast columns in furnace ventilation are sometimes referred to as the primary and secondary columns, respectively. The *primary* or *furnace column* is, in nearly every case, a vertical column, and consists of a single air column whose average temperature is easily approximated. According to the manner of opening the mine, whether by shaft, slope, or drift, the *secondary* column may be a *vertical column* in the shaft, an *inclined column* in the slope, or an *outside air column* in case of a *drift* opening. Again, it is to be observed that in case of a slope opening where the top of the furnace shaft is much higher than the mouth of the slope, and the dip of the slope is considerable, the *secondary column* consists of *two* columns of different temperatures, an *outside air column* and the *slope column*. These two parts of the secondary column must be calculated separately, and their sum taken for the weight of the secondary column. The level of the top of the furnace shaft determines the top of both the primary and secondary columns, whether these columns are in the outer air or in the mine. The weight of the upcast or primary column is largely affected by its gaseous condition. For example, if the return current from the mine is laden with blackdamp $CO_2$, its weight will be much increased, since this gas is practically $1\frac{1}{2}$ times as heavy as air, while, if laden with marsh gas, or firedamp mixture, its weight will be considerably reduced. These causes decrease and increase, respectively, the ventilating pressure in the mine.

**Inclined Air Columns.**—In a slope opening, the air column is inclined; it is none the less, however, an air column, and must be calculated in the same manner as a vertical column whose vertical height corresponds to the amount of dip of the slope. Fig. 9 shows a vertical shaft and a slope, the air column in each of these being the same for the same temperature. The air column in all dips and rises must be estimated in like manner, by ascertaining the vertical height of the dip.
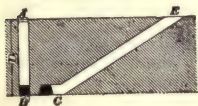


FIG. 9

**Calculation of Ventilating Pressure in Furnace Ventilation.**—The ventilating pressure in the mine airways, in natural or in furnace ventilation, is caused by the difference of the weights of the primary and secondary columns. Air always moves from a point of higher pressure towards a point of lower pressure, and this movement of the air is caused by the difference between these two pressures. In this calculation each column is supposed to have an area of base of 1 sq. ft. Hence, if we multiply the weight of 1 cu. ft. of air at a given barometric pressure, and having a temperature equal to the average temperature of the column, by the vertical height $D$ of the column, we obtain not only the weight of the column but the pressure at its base due to its weight. Now, since the ventilating pressure per square foot in the airway is equal to the difference of the weights of the primary and secondary columns, we write

$$p = \left(\frac{1.3253 \times B}{459+t} - \frac{1.3253 \times B}{459+T}\right) \times D$$

**EXAMPLE.**—Find the ventilating pressure in a mine ventilated by a furnace, the temperatures of the upcast and downcast columns being, respectively, 350°F. and 40°F., the depth of the upcast and downcast shafts being each 600 ft., and the barometer 30 in.

Substituting the given values in the above equation, we have

$$p = 1.3253 \times 30 \times 600 \left(\frac{1}{459+40} - \frac{1}{459+350}\right) = 18.32 \text{ lb. per sq. ft.}$$

**Calculation of Motive Column or Air Column.**—It is often convenient to express the ventilating pressure $p$ (pounds per square foot) in terms of air column or motive column $M$, in feet. The height of the air column $M$ is equal to the pressure $p$ divided by the weight $w$ of 1 cu. ft. of air, or $M = \dfrac{p}{w}$.

The expression for motive column may be written either in terms of the upcast air or of the downcast air, the former giving a higher motive column than the latter for the same pressure, since the upcast air is lighter than that of the downcast. As the surplus weight of the downcast column of air produces the ventilating pressure, it is preferable to write the air column in terms of the downcast air, or, in other words, to consider the air column as being located in the downcast shaft, and pressing the air downwards and

through the airways of the mine. If we divide the expression previously given for the ventilating pressure by the weight of 1 cu. ft. of downcast air $\left(\dfrac{1.3253 \times B}{459 + t}\right)$, we obtain for the motive column, after simplifying, $M = \left(\dfrac{T - t}{459 + T}\right) \times D$, which is the expression for motive column in terms of the downcast air.

If, on the other hand, we divide the expression for the ventilating pressure by the weight of 1 cu. ft. of upcast air $\left(\dfrac{1.3253 \times B}{459 + T}\right)$, we obtain $M = \left(\dfrac{T - t}{459 + t}\right) \times D$, which is the expression for motive column in terms of the upcast air.

**Influence of Furnace Stack.**—To increase the height of the primary or furnace column, a stack is often erected over the mouth of the furnace shaft. The effect of this is to increase the ventilating pressure in the mine in proportion to the increased height of the primary column, and to increase the quantity of air passing in the mine in proportion to the square root of this height. Thus, the square root of the ratio of the heights of the primary column, before and after the stack is erected, is equal to the ratio of the quantities of air passing before and after the erection of the stack. Or, calling these quantities $q_1$ and $q_2$, and the height of stack $d$, we have

$$\sqrt{\frac{D + d}{D}} = \frac{q_2}{q_1} \quad \text{or} \quad q_2 = \sqrt{\frac{D + d}{D}} \times q_1.$$

## MECHANICAL VENTILATORS

A large number of **mechanical ventilators** have been invented and applied, with more or less success, to the ventilation of mines. The earliest type of ventilator was the *wind cowl*, by which the pressure of the wind at the surface was brought to bear effectively upon the mine airways by the action of a cowl whose mouth could be turned toward the wind; this was naturally very unreliable. The *waterfall* was also extensively applied at one time, but its application could only be made where there was a reliable source of water supply, and where the drainage of the mine could be effected through a tunnel, or where the mine opening could be placed in connection with such a waterfall outside of the mine. Where these conditions are obtained, as is the case in some mountainous districts, the waterfall is still in use, as it is an effective means of ventilation, and is economical. Its application, however, must be limited to the ventilation of small mines. The *steam jet* is another mechanical device for producing an air current in the mine. The steam is allowed to issue from a jet at the bottom of an upcast shaft, and, by the force of its discharge, causes an upward current in the shaft. Its use, however, is very limited, and is practically restricted to the ventilation of shafts while sinking. In this connection it may be mentioned, however, that the discharged steam from the mine pumps, where practicable, may be conducted into the upcast shaft; or the discharge pipe from the pumps may be carried up the upcast shaft, its heat increasing the temperature of the shaft, and thereby increasing the motive column and the ventilation.

**Fan Ventilation.**—Mechanical motors of this type present two distinct modes of action in producing an air current: (*a*) by propulsion of the air; and (*b*) by establishing a pressure due to the centrifugal force incident to the revolution of the fan. Fans have been constructed to act wholly on one or the other of these principles, while others have been constructed to act on both of these principles combined.

**Disk Fans.**—The action of this type of fan resembles that of a windmill, except that in the latter the wind drives the mill, while in the former the fan propels the air or produces the wind. This type of fan consists of a number of vanes radiating from a central shaft, and inclined to the plane of revolution. The fan is set up in the passageway between the outer air and the mine airways. Power being applied to the shaft, the revolution of the vanes propels the air, and produces a current in the airways. The fan may force the air through, or exhaust the air from, the airways, according to the direction of its revolution. This type of fan is most efficient under light pressures. It has found an extensive application in mining practice, and has taken a large number of devotees, but has been replaced to a large degree in the ventilation of extensive mines. This type of fan acts wholly by propulsion.

**Centrifugal fans** include all fans that act solely on the **centrifugal principle,** and those that combine the centrifugal and propulsion principles. The action of the fan, whether by centrifugal force alone, or combined with propulsion, depends on the form of the fan blades. In this type of fan, the blades are all set at right angles to the plane of revolution, and not inclined, as in the disk fan just described. The blades may, however, be either radial blades, sometimes spoken of as paddle blades, or they may be inclined to the radius either forward in the direction of revolution, or backward. When the blades are radial, the action of the fan is centrifugal only. The inclination of the blades backward from the direction of motion gives rise to an action of propulsion, in addition to the centrifugal action of the fan. The blades in this position may be either straight blades in an inclined position, as in the original Guibal fan, or they may be *curved* backward in the form of a spiral, as in the Schiele and Waddle fans.

Centrifugal fans may be (*a*) exhaust fans or (*b*) force fans or blowers. In each, the action of the fan is essentially the same; i.e., to create a difference of pressure between its intake or central opening, and its discharge at the circumference. The centrifugal force developed by the revolution of the air between the blades of the fan causes the air within the fan to crowd towards the circumference; as a result, a rarification is caused at the center and a compression at the circumference, giving rise to a difference of pressure between the intake and the discharge of the fan.

**Exhaust Fans.**—If the intake opening of the fan be placed in connection with the mine airways, and the discharge be open to the atmosphere, the fan will act to create rarefaction in the fan drift leading to the mine, which will cause a flow of air through the mine airways and into and through the fan. In this case, the fan is exhausting, its position being ahead of the current that it produces in the airway. The atmospheric pressure at the intake of the mine airways forces the air or propels the current toward the depression in pressure existing in the fan drift caused by the fan's action.

**Force Fans and Blowers.**—If the discharge opening of the fan be placed in connection with the mine airways, a compression will result in the fan drift owing to the fan's action, and the air will flow from this point of compression through the airways of the mine, and be discharged into the upcast, and thence into the atmosphere. The ventilating pressure in the case of either the exhaust fan or the force fan is equal to the difference of pressure created by the fan's action. In the former case, when the fan is exhausting, the absolute pressure in the fan drift is equal to the atmospheric pressure less the ventilating pressure, while in the latter case, when a fan is forcing, the absolute pressure in the fan drift is equal to the atmospheric pressure increased by the ventilating pressure. This gives rise to two distinct systems of ventilation, known as (*a*) *vacuum system* and (*b*) *plenum system.*

**Vacuum System of Ventilation.**—In this system, the ventilation of the mine is accomplished by creating a decrease of pressure in the return airway of the mine. This decrease may be created by the action of an exhaust fan, as just described, or by the action of a furnace. In either case, the absolute pressure in the mine is below that of the atmosphere, or, we may say, the mine is ventilated under a pressure *below* the atmospheric pressure. This system has many points of advantage over the plenum system, and for years was considered by many the only practicable system of ventilation. Its application, however, is controlled by conditions in the mine with respect to the gases liberated, the arrangement of the haulage system, etc.

**Plenum System of Ventilation.**—In this sytem, the air current is propelled through the mine airways by means of the compression or ventilating pressure created at the intake opening of the mine. This ventilating pressure may be established by a fan, waterfall, wind cowl, or any other mechanical means at hand. In this system, the absolute pressure in the mine is above that of the atmosphere; or, as we say, the mine is ventilated under a pressure *above* the atmospheric pressure.

**Comparison of Vacuum and Plenum Systems.**—No hard-and-fast rule can be made to apply in every case, as each system has its particular advantages. In case of a sudden stoppage of the ventilating motor at a mine, there is, in the vacuum system, a rise of mine pressure, instead of a fall, and the gases are driven back into the workings for a while, while, in the plenum system, any stoppage of the ventilating motor is followed at once by a fall of pressure in the mine, and mine gases expand more freely into the passageways at the very moment when their presence is most dangerous. This point must be carefully considered in the ventilation of deep workings. In

shallow workings, the plenum system is often advantageous, especially if there is a large area of abandoned workings that have a vent or opening to the atmosphere, either through an old shaft or through crevices extending to the surface. Every crevice or other vent becomes a discharge opening by which the mine gases find their way to the surface, and the gases accumulating in the old workings are driven back into the workings, and find their way to the surface instead of being drawn into the mine airways, as would be the case in an exhaust system. Any given fall of the barometer affects the expansion of mine gases to a less extent in the plenum system than in the vacuum system, but this small advantage would not give it consideration in determining between the adoption of the one or the other of these two systems; regard must be had, however, to other conditions more vital than this. In the ventilation of gaseous seams, owing to the necessity of making the intake airway the haulage road, the exhaust system has usually been adopted, as the main road is thereby left unobstructed by doors.

## TYPES OF CENTRIFUGAL FANS

We shall only mention the more prominent types of fans that have been or are still in use, giving the characteristic features, as nearly as possible, of each fan. Many fans have been built, however, combining many of the features that originally characterized a single type of fan.
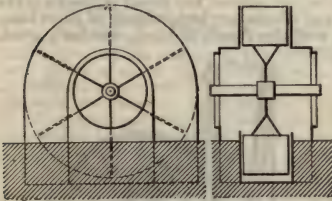


FIG. 10

FIG. 11

**Nasmyth Fan.**—Fig. 10 is the original type of fan representing straight paddle blades radiating from the center, which is its characteristic feature. This was probably the earliest attempt to apply the centrifugal principle to a mine ventilator, and although not recognized at the time, the fan embodied some of the most essential principles in centrifugal ventilation. It possessed certain disadvantages, however, chief of which was a contracted central or intake opening. The blades, also, were straight throughout their entire length, being normal both to the inner and outer circles of the fan, and thus did not provide for receiving the air without shock at the throat of the fan. The depth of Nasmyth's blades equaled one-half the radius of the fan, which was, under ordinary conditions of mine practice, far too great, and gave the fan a low efficiency.

**Biram's Ventilator.**— About 1850, Biram attempted to improve upon the Nasmyth ventilator by reducing the depth of blade so that it was but one-tenth of the radius. The blades were



FIG. 12

straight, as in Nasmyth's ventilator, but inclined backwards from the direction of motion at a considerable angle. A large number of these blades were employed. This fan was run at a considerable speed, but proved very inefficient. It depended more on the effort of propulsion given to the air than on the centrifugal principle, as the depth of the blade was as much too small
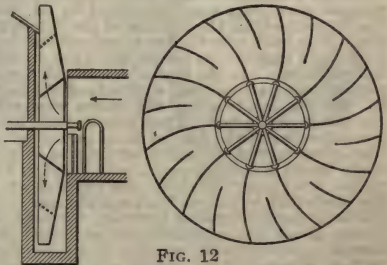
as that of Nasmyth's was too great.   The intake or central opening in this fan was as contracted as in the former type.   See Fig. 11.

**Waddle Ventilator.**—In this fan, Fig. 12, the inventor attempted to reenforce the discharge pressure at the circumference against the pressure of the atmosphere.   The discharge took place all around the entire circumference of the fan, which was entirely opened to the atmosphere.   The blades were curved backward from the direction of motion in spiral form.   The width of the blade decreased from the throat toward the circumference, so as to present an inverse ratio to the length of radius.   Thus, the area of passage between the fan blades was maintained constant from the throat to the circumference of the fan.   The purpose of this was to maintain the velocity of the air through the fan constant, and to fortify the pressure due to the fan against the atmospheric pressure at the point of discharge.   The essential features of the Waddle ventilator were, therefore, curved blades tapered towards the circumference, and a free discharge into the atmosphere all around the circumference.   This type is the best type of the open-running fans having no peripheral casing, and discharging air into the atmosphere all around the circumference.

**Schiele Ventilator.**—This ventilator, Fig. 13, was constructed on the same principles as the Waddle ventilator just described, but differed from the latter, as the discharge was made into a spiral chamber surrounding the fan and leading to an expanding or évasé chimney.   There was some advantage in this feature, as it protected the fan against the direct influence of the atmosphere, and reduced the velocity of discharge; but, in each of these fans, the intake opening was contracted, and the depth of blade was very great, yielding a comparatively low efficiency.
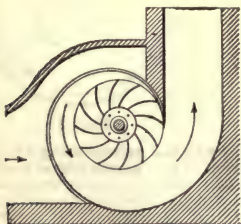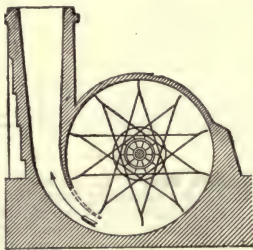


FIG. 13                                   FIG. 14

**Guibal Ventilator.**—The next important step in the improvement of centrifugal ventilators was introduced by M. Guibal, who constructed a fan, Fig. 14, embodying the features of the Nasmyth ventilator, with the addition of a casing built over the fan to protect its circumference.   This casing was, however, a tight-fitting casing, and as such, differed very materially from the Schiele casing.   In the Guibal fan the blades were arranged upon a series of parallel bars passing upon each side of the center and at some distance from it.   By this construction, the blades were not radial at their inner edge or the throat of the fan.   They were curved, however, as they approached the circumference of the fan, so as to be normal or radial at the circumference. The advantage of this construction was to give a strong skeleton or framework to the revolving parts, and, further, each blade was inclined to the radius at its inner extremity, the effect of which was to receive the air upon the blade with less shock than was the case in the Nasmyth ventilator.   The intake or central opening, however, was very contracted, and the tight-fitting casing about the circumference prevented the effective action of the fan during a considerable portion of its revolution.   The fan was supplied with an évasé chimney, which was a feature of the Schiele fan, but vibration was so strong that a shutter was required at the cutoff below the chimney, to prevent it.   This shutter was made adjustable, and is known as the *Walker shutter*, having been applied to the fan later.

The Guibal ventilator presents some important and valuable features in the protecting cover, and in the blades meeting the outer circumference radially, and in the air being received with less shock than before.   On the

whole, it has proved a very efficient ventilator, although much work is lost by reason of its contracted central orifice and tight casing, where the same is used.

**Murphy Ventilator.**—Fig. 15 consists of twin fans supported on the same shaft and set a few feet apart. Each fan receives its air on one side only, the openings being turned towards each other. This ventilator is built with a small diameter, and is run at a high speed. The blades are curved backwards from the direction of motion. The intake opening is considerably enlarged; a spiral casing generally surrounds the fan, and in every respect this fan makes an efficient high-speed ventilator. It has received considerable favor in the United States, where it has been introduced into a large number of mines.
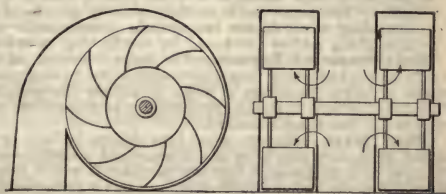
**Capell Ventilator.** —Perhaps none of the centrifugal ventilators have been as little understood in regard to their princi-



FIG. 15

ple of action as the Capell fan. The fan is constructed along the lines of the Schiele ventilator, but differs from it in the manner of receiving its intake air and delivering the same into the main body of the fan. Here, and revolving with it, is a set of smaller supernumerary blades. These blades occupy a cylindrical space within the main body of the fan, and are inclined to the plane of revolution so as to assist in deflecting the entering air through small ports or openings into the main body of the fan, where it is revolved and is discharged at the circumference into a spiral space resembling that surrounding the Schiele fan. The larger blades of this fan are curved backwards as the Schiele blades, but are not tapered toward the circumference. The fan is capable of giving a high water gauge, and is efficient as a mine ventilator. The space surrounding the fan is extended to form an expanding chimney. The fan may be used either as an exhaust fan or a blower. The best results in the United States have been obtained by blowers. In Germany, where this fan is in general use, there are no blowers.
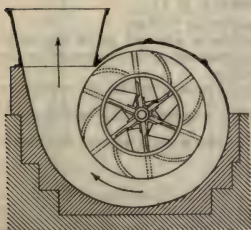


FIG. 16
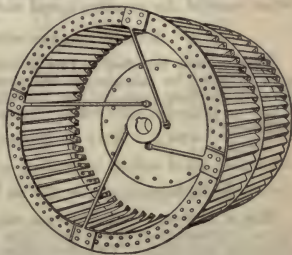


FIG. 17

**Sirocco Fan.**—This Fig. 17 is the original multi-blade fan, having forwardly inclined blades. The blades, therefore, act upon air at rest, relatively to their own path which is at right angles to that of the incoming stream.

Thomas Chester, chief engineer of the American Blower Co. gives numerous formulas for use in connection with these fans. These are given here in detail.

Sirocco fan blades have a forward inclination; that is to say the outer tips are in advance of the inner edges in the direction of rotation and in consequence air is thrown off at a higher velocity than the peripheral speed. This enhanced velocity is responsible for the remarkable volumetric and

manometric efficiencies of the Sirocco fan, these values usually being around 250 % and 100 % respectively.

The high mechanical efficiency is due to the following features:

1. Large inlet area.
2. Uniform action over the whole periphery, due to the large number of blades.
3. Absence of whirlpool or vortex motion of the entering air before reaching the fan blades, thereby avoiding the expenditure of power on unnecessary work.
4. Better stream lines, as the air leaves blades tilted forwards in very nearly the same path as that already given off by the impeller and traveling towards the fan outlet, consequently minimizing the power-absorbing eddies produced by conflicting streams.

**Direct-connected Engines.**—For large volumes of air against ordinary resistances when using fans direct connected to engines it is necessary to use wheels of large diameters in order to obtain the peripheral speeds required. This calls for fans of special proportions as the widths under these conditions are usually less than standard.

**Other Drives.**—Sirocco mine fans driven by belt or ropes or directly connected to motors can be made of standard proportions which means that in the case of a single-inlet fan the peripheral width is equal to one-third the wheel diameter and a double-inlet fan has a peripheral width two-thirds the wheel diameter.

**Method of Determining Fan Diameter.**—The following formulæ are used in designing these fans, the volume being in cubic feet per minute and W.G. being the mine resistance in inches of water.

Single-inlet, diameter in inches equals $\dfrac{.5\sqrt{\text{volume}}}{\sqrt[4]{\text{W.G.}}}$

Double-inlet, diameter in inches equals $\dfrac{.354\sqrt{\text{volume}}}{\sqrt[4]{\text{W.G.}}}$

Taking as an example 200,000 cu. ft. per min. against 3-in. mine resistance, a double inlet exhauster being required, the formula for a fan of this type gives a diameter of 120 in. and the impeller would therefore be $120 \times 80$ in. Impeller diameters in inches are in multiples of 6 so that in event of either formula giving an intermediate diameter such as 118 in., the nearest standard size should be taken which in this case would be 120 in.

**To Ascertain Fan Speed Required.**—Having determined the diameter of wheel needed the rotative speed can be found as follows:

$$\text{Revolutions per minute} = \frac{10,800\sqrt{\text{W.G.}}}{\text{diameter in inches}}$$

**Horsepower Needed.**—The power consumption is arrived at in the ordinary manner by calculating the theoretical horsepower or horsepower in the air and dividing same by the mechanical efficiency.

In the example referred to previously the horsepower in the air would be $\dfrac{200,000 \times 3}{6,346} = 94.5$. With a Sirocco fan of this capacity the mechanical efficiency could safely be figured at 75 % so that the actual power consumption or brake horsepower would be $\dfrac{94.5}{.75} = 126$.

**Size of Motor.**—In selecting a motor for work of this character it should be borne in mind that the actual mine resistance cannot be predetermined with absolute accuracy. The resistance offered by a mine to the flow of the required quantity of air may be less than anticipated, with a consequent increase in the volume handled by the fan and a correspondingly increased power consumption, so that to provide a margin of safety the factor .6 should be used. This would indicate that a motor capable of developing $\dfrac{94.5}{.6} =$ say 158 B. H. P. should be installed.

**Evase Stack.**—The foregoing is based on the supposition that the fan would be equipped with an evase stack and the effective length in feet measured along the stack axis from the cutoff should be 5 times the square root of the water gauge in inches. In the example under consideration the effective length would be $5\sqrt{3}$ or say 8 ft.

**Maximum Inlet Velocity.**—The minimum inlet area of a Sirocco fan is at the inner ends of the inlet cones, the diameter of each being .875 the wheel diameter. In the case cited the minimum area would be 60.1 sq. ft. for each inlet or a total of 120.2 sq. ft. The maximum inlet velocity would therefore be 1,660 ft. per min.

**Loss at Inlets.**—Due to the right-angle turn made by the air entering a fan inlet the velocity energy at this point is almost entirely lost but as the velocity head would only be .171-in. W.G. this could be considered satisfactory and it would not pay to increase the size of fan to make a reduction of this loss.

The velocity head is the height of a column of water that can be supported by a stream of air moving at any given velocity. This can readily be calculated by using as a unit the air velocity required to sustain 1 in. of water. This velocity can be figured from the formula $K = 60 \sqrt{\dfrac{2gh}{12}}$ where $g$ is 32.16 the acceleration due to gravity and $h$ is the ratio between the weight of a cubic foot of water at 62°F. and the weight of a cubic foot of air under the conditions respecting temperature, humidity and barometric pressure prevailing.

**Standard Air.**—United States Navy Department engineers figure on standard air as 70°F. and 70 % relative humidity and in this condition at sea level it weighs .07465 lb. per cu. ft. Water at 62°F. weighs 62.355 lb. per cu. ft. so that the constant $K$ for standard air is 4,015 and an air velocity of 4,015 ft. per min. would be equivalent to 1-in. W.G.

**Inlet Velocities.**—Taking the case under consideration the velocity head or equivalent water gauge of the air entering the fan inlets is $\left(\dfrac{1,660}{4,015}\right)^2$ or .171-in W.G. as previously stated. The equivalent velocity head for any requirement can be worked out by using the constant $K$ as shown and the following table gives the maximum inlet velocities of standard Sirocco fans for various resistances with the impeller diameters figured as recommended.

| Mine Resistance, In. | Max. Inlet Velocity, Ft. per Min. | Mine Resistance, In. | Max. Inlet Velocity, Ft. per Min. |
|---|---|---|---|
| 1 | 960 | 4¾ | 2,090 |
| 1¼ | 1,070 | 5 | 2,140 |
| 1½ | 1,175 | 5¼ | 2,195 |
| 1¾ | 1,270 | 5½ | 2,250 |
| 2 | 1,355 | 5¾ | 2,300 |
| 2¼ | 1,440 | 6 | 2,350 |
| 2½ | 1,515 | 6¼ | 2,400 |
| 2¾ | 1,590 | 6½ | 2,445 |
| 3 | 1,660 | 6¾ | 2,490 |
| 3¼ | 1,730 | 7 | 2,535 |
| 3½ | 1,790 | 7¼ | 2,580 |
| 3¾ | 1,855 | 7½ | 2,625 |
| 4 | 1,920 | 7¾ | 2,670 |
| 4¼ | 1,975 | 8 | 2,715 |
| 4½ | 2,030 | 8¼ | 2,760 |

**Special Fans.**—Fans of standard proportions can be used for all the mine resistances given in the above table as the most severe duty stated requires an inlet velocity of 2,760 ft. per min. which is equivalent to .47-in. velocity head. When large volumes of air are to be handled against high resistances, however, the power reduction which can be obtained by installing a larger and more expensive fan than indicated by the formulæ frequently justifies the greater cost as each horsepower saved represents approximately $100 per yr. Cases of this kind and those involving fans for high altitudes should be submitted to mine fan specialists.

**Equivalent Orifice.**—No mine should be equipped with any style of fan having a minimum inlet area less than twice the equivalent orifice, if anything like good efficiency is desired. The equivalent orifice of a mine varies

directly as the volume of air passed per minute and inversely as the square root of the resistance, so that with the same mine conditions prevailing the equivalent orifice remains the same even with the fan speed altered and the volume increased or decreased.

Reverting to the requirement of 200,000 cu. ft. per min. against 3-in. mine resistance previously considered, the equivalent orifice is found to be 46.2 sq. ft. by using the well-known formula

$$\text{Equivalent orifice in square feet} = .0004 \times \frac{\text{vol. cu. ft. per min.}}{\sqrt{\text{W.G.}}}$$

In other words a ventilator drawing air through an opening of 46.2 sq. ft. in a thin plate would encounter just the same resistance as when exhausting the same volume, 200,000 cu. ft. per min., from a mine offering a resistance of 3-in. water gauge.

As already noted the fan selected for this requirement has a minimum inlet area of 120.2 sq. ft. or 2.6 times the equivalent orifice of the mine.

**Murgue's Formula.**—It will doubtless be instructive at this point to examine the equivalent orifice equation developed by M. Daniel Murgue as given above. Using the established value 62 % as representing the effective area of an opening in a thin plate allowing for vena contracta, the equivalent orifice of 46.2 sq. ft. is found to have an effective area of 28.6 sq. ft.

Dividing the volume, 200,000 cu. ft., by the effective area 28.6 the velocity is found to be 7,000 ft. per min. Using $K$ the constant for standard air as per U. S. Navy practice namely 4,015 ft. per min., the equivalent water gauge is found by dividing the square of the air velocity through the effective area of the equivalent orifice by the square of the constant $K$

$$\left(\frac{7,000}{4,015}\right)^2 = 3.04\text{-in. water gauge.}$$

The small discrepancy indicates that Murgue used a constant slightly higher than 4,015 ft. per min. so that he evidently figured on air having a little less weight than is considered standard by the Navy Department engineers. This would readily be accounted for if he assumed a higher relative humidity than 70 %.

**Sullivan Reversible Fans.**—The Sullivan fan is reversible. The operation of reversing is secured in a manner that is considered extraordinarily simple and safe. It is by the use of a steel hood swing by a gear and pinion controlled by a hand wheel. This has an advantage in that a smaller and simpler housing is needed than is commonly the case. The fan itself is of the multi-blade pattern with the double wheel, a double inlet and cone-shaped deflectors for changing the direction of the air with minimum friction and loss in power. They are of relatively small diameter and of high speed.

It must be noted that the action of the hood is entirely independent of that of the fan. It hangs in bearings concentric with those of the fan wheel
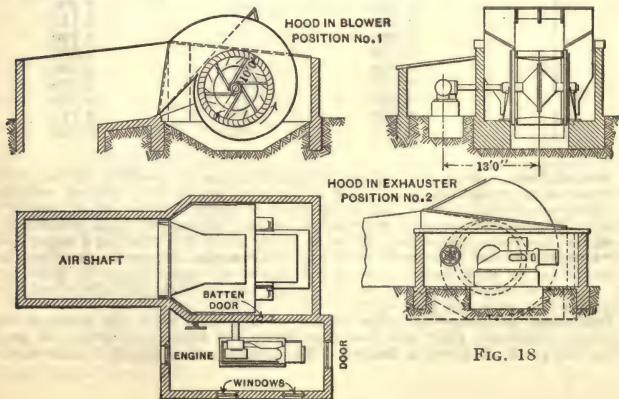


FIG. 18

SULLIVAN FANS; SIZES, WEIGHTS, DIMENSIONS

| Diam. of Fan Wheel, Ft. | Total Weight Fan only, Lb. | Wheel Dimensions | | Width of Blade, In. | Shaft Diameter, In. | | Length of Each Bearing, In. | Thickness of Sheets, In. | | | | Spokes (each side) | | No. of Blades, Both Sides | Foundation and Walls for D. C. Fan and Engine | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Width, Ft. | Inlet, Ft.-in. | | Hubs | Bearings | | Blades | Cent. Wall Rings | Side Rings | Hood | No. | Diam., in. | | No. Brick in Walls | Cu.Yd. Concrete in Fd'n |
| 6 | 10,000 | 3½ | 4-8 | 5½ | 5 | 4½ | 11 | 3/16 | 1¼ | 3/16 | 3/16 | 6 | ¾ | 72 | 3,000* | 7* |
| 8½ | 26,000 | 5 | 7-0 | 7½ | 8 | 7½ | 18 | ¼ | ¼ | ¼ | 3/16 | 6 | 1 | 88 | 15,500 | 50 |
| 10 | 42,000 | 6 | 8-6 | 9 | 10 | 9 | 21 | 3/16 | ¼ | ¼ | 3/16 | 6 | 1½ | 96 | 20,000 | 90 |

* For Fan only. Engine not included.

DETAILS, BELTED FANS

| Size of Fan, Ft. | Outboard Bearings | | Foundation and Walls, Fan and O. B. Bearing | |
|---|---|---|---|---|
| | Size, In. | Weight, lb. | Brick in Walls | Concrete Foundation, Yd. |
| 6 | 3½ × 9 | 600 | 12,000 | 20 |
| 8½ | 6 × 14 | 1,000 | 13,000 | 34 |
| 10 | 7½ × 18 | 1,500 | 18,000 | 56 |

Motor or engine foundation is not included in these figures for the belt-driven fans.

### RATINGS.　6-FT. AND 8½-FT. FANS

#### 6-ft. Fan

| Capacity, Cu. Ft. Air P.M. | | Water Gauge, In. | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ¼ | ½ | ¾ | 1 | 1¼ | 1½ | 1¾ | 2 | 2½ | 3 | 3½ | 4 | 4½ | 5 | 5½ | 6 |
| 10,000 | R.P.M. | 90 | 105 | 119 | 134 | 148 | 163 | 178 | 192 | 221 | 251 | 280 | 309 | 338 | 367 | 396 | 425 |
| | H.P. | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 19 | 21 | 23 | 25 | 27 | 29 | 31 | 33 |
| 20,000 | R.P.M. | 133 | 146 | 159 | 172 | 185 | 199 | 212 | 225 | 251 | 277 | 303 | 330 | 356 | 382 | 408 | 435 |
| | H.P. | 13 | 15 | 16 | 17 | 19 | 20 | 22 | 23 | 26 | 29 | 32 | 34 | 37 | 40 | 42 | 45 |
| 30,000 | R.P.M. | 177 | 189 | 201 | 212 | 224 | 236 | 248 | 260 | 283 | 307 | 330 | 357 | 384 | 410 | 437 | 448 |
| | H.P. | 17 | 18 | 20 | 22 | 24 | 25 | 27 | 29 | 33 | 36 | 40 | 43 | 47 | 50 | 53 | 57 |
| 40,000 | R.P.M. | 220 | 231 | 241 | 252 | 262 | 273 | 284 | 294 | 315 | 337 | 358 | 379 | 400 | 421 | 442 | 463 |
| | H.P. | 20 | 22 | 24 | 26 | 29 | 31 | 33 | 35 | 39 | 44 | 48 | 52 | 56 | 61 | 66 | 69 |
| 50,000 | R.P.M. | 263 | 272 | 282 | 291 | 301 | 310 | 319 | 329 | 348 | 366 | 385 | 404 | 423 | 442 | 461 | 480 |
| | H.P. | 23 | 26 | 28 | 31 | 34 | 36 | 38 | 41 | 46 | 31 | 56 | 61 | 66 | 71 | 76 | 81 |
| 60,000 | R.P.M. | 307 | 315 | 324 | 332 | 341 | 349 | 357 | 366 | 383 | 399 | 416 | 433 | 450 | 467 | 483 | 500 |
| | H.P. | 27 | 29 | 32 | 35 | 38 | 41 | 43 | 47 | 53 | 59 | 64 | 70 | 76 | 82 | 87 | 93 |
| 70,000 | R.P.M. | 350 | 357 | 365 | 372 | 380 | 387 | 395 | 402 | 417 | 432 | 447 | 462 | 477 | 492 | 507 | 523 |
| | H.P. | 30 | 33 | 36 | 40 | 43 | 46 | 49 | 53 | 60 | 66 | 72 | 79 | 86 | 92 | 99 | 105 |
| 80,000 | R.P.M. | 393 | 400 | 406 | 413 | 420 | 427 | 433 | 440 | 453 | 467 | 480 | 494 | 507 | 520 | 534 | 548 |
| | H.P. | 33 | 37 | 40 | 44 | 48 | 51 | 54 | 59 | 66 | 74 | 80 | 88 | 96 | 103 | 110 | 117 |
| 90,000 | R.P.M. | 437 | 443 | 449 | 455 | 461 | 467 | 473 | 479 | 491 | 503 | 515 | 527 | 539 | 551 | 563 | 575 |
| | H.P. | 37 | 40 | 44 | 49 | 53 | 57 | 60 | 65 | 73 | 81 | 89 | 97 | 105 | 113 | 121 | 129 |
| 100,000 | R.P.M. | 480 | 485 | 491 | 496 | 502 | 507 | 512 | 518 | 528 | 539 | 550 | 561 | 572 | 583 | 594 | 605 |
| | H.P. | 40 | 44 | 49 | 53 | 58 | 62 | 66 | 71 | 80 | 88 | 97 | 106 | 115 | 124 | 132 | 141 |

H. P. tabulated are I. H. P. steam eng. dir. conn. or motor output belted.

NOTE.—The power ratings are shown in their present form, rather than as net H. P. delivered to the fan shaft, on account of the variation in efficiency with each change in engine speed.

### 8-ft. 6-in. Fan

| Capacity, Cu. Ft. Air P.M. | | Water Gauge, In. | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | 1 | $1\frac{1}{4}$ | $1\frac{1}{2}$ | $1\frac{3}{4}$ | 2 | $2\frac{1}{2}$ | 3 | $3\frac{1}{2}$ | 4 | $4\frac{1}{2}$ | 5 | $5\frac{1}{2}$ | 6 |
| 25,000 | R.P.M. | 56 | 65 | 74 | 83 | 91 | 100 | 109 | 118 | 136 | 153 | 171 | 189 | 207 | 224 | 242 | 260 |
| | H.P. | 17 | 20 | 22 | 24 | 27 | 29 | 32 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 |
| 50,000 | R.P.M. | 80 | 88 | 97 | 105 | 114 | 122 | 131 | 139 | 156 | 173 | 190 | 206 | 223 | 240 | 257 | 274 |
| | H.P. | 24 | 27 | 31 | 34 | 38 | 41 | 44 | 48 | 55 | 62 | 68 | 75 | 82 | 89 | 95 | 102 |
| 75,000 | R.P.M. | 104 | 112 | 119 | 127 | 134 | 142 | 150 | 157 | 172 | 188 | 203 | 218 | 233 | 248 | 264 | 279 |
| | H.P. | 31 | 35 | 40 | 44 | 48 | 52 | 57 | 62 | 71 | 79 | 87 | 96 | 104 | 113 | 121 | 129 |
| 100,000 | R.P.M. | 128 | 135 | 142 | 148 | 155 | 162 | 169 | 176 | 189 | 203 | 216 | 230 | 244 | 257 | 271 | 285 |
| | H.P. | 38 | 43 | 48 | 53 | 58 | 64 | 69 | 74 | 84 | 94 | 104 | 115 | 125 | 135 | 145 | 156 |
| 125,000 | R.P.M. | 152 | 158 | 164 | 170 | 176 | 182 | 189 | 195 | 207 | 219 | 231 | 244 | 256 | 268 | 280 | 292 |
| | H.P. | 45 | 51 | 57 | 63 | 69 | 75 | 81 | 87 | 99 | 111 | 123 | 135 | 147 | 159 | 171 | 183 |
| 150,000 | R.P.M. | 176 | 181 | 187 | 192 | 198 | 203 | 208 | 214 | 225 | 235 | 246 | 257 | 268 | 279 | 289 | 300 |
| | H.P. | 52 | 59 | 66 | 73 | 80 | 87 | 93 | 100 | 114 | 128 | 142 | 156 | 169 | 183 | 197 | 210 |
| 175,000 | R.P.M. | 200 | 205 | 209 | 214 | 219 | 224 | 228 | 233 | 242 | 252 | 261 | 271 | 280 | 289 | 299 | 309 |
| | H.P. | 59 | 67 | 75 | 82 | 90 | 98 | 106 | 113 | 129 | 144 | 160 | 175 | 191 | 206 | 222 | 237 |
| 200,000 | R.P.M. | 224 | 228 | 232 | 236 | 240 | 245 | 249 | 253 | 261 | 269 | 277 | 286 | 294 | 302 | 310 | 319 |
| | H.P. | 66 | 75 | 83 | 92 | 100 | 109 | 118 | 126 | 143 | 161 | 178 | 195 | 212 | 229 | 247 | 264 |
| 225,000 | R.P.M. | 248 | 251 | 254 | 257 | 260 | 264 | 267 | 270 | 276 | 282 | 288 | 295 | 301 | 307 | 313 | 320 |
| | H.P. | 73 | 83 | 92 | 102 | 111 | 121 | 130 | 140 | 159 | 178 | 197 | 216 | 235 | 254 | 273 | 291 |
| 250,000 | R.P.M. | 272 | 275 | 278 | 281 | 284 | 287 | 290 | 293 | 300 | 306 | 312 | 318 | 324 | 330 | 336 | 342 |
| | H.P. | 80 | 90 | 101 | 111 | 121 | 132 | 142 | 152 | 173 | 193 | 214 | 235 | 255 | 276 | 297 | 318 |

H. P. tabulated are I H.P. steam eng. dir. conn. or motor output belted.

### 10-ft. Fan Ratings

#### 10-ft. Fan

| Capacity, Cu. ft. Air P.M. | | Water Gauge, In. | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ¼ | ½ | ¾ | 1 | 1¼ | 1½ | 1¾ | 2 | 2½ | 3 | 3½ | 4 | 4½ | 5 | 5½ | 6 |
| 25,000 | R.P.M. | 46 | 63 | 78 | 90 | 100 | 110 | 119 | 126 | 141 | 154 | 167 | 178 | 189 | 199 | 209 | 218 |
| | H.P. | 20 | 24 | 28 | 31 | 34 | 37 | 40 | 42 | 47 | 52 | 57 | 61 | 65 | 69 | 73 | 77 |
| 50,000 | R.P.M. | 52 | 69 | 82 | 93 | 103 | 112 | 121 | 128 | 144 | 157 | 169 | 180 | 192 | 202 | 212 | 221 |
| | H.P. | 24 | 30 | 34 | 38 | 42 | 46 | 50 | 54 | 61 | 68 | 74 | 80 | 86 | 92 | 98 | 104 |
| 75,000 | R.P.M. | 62 | 75 | 87 | 98 | 107 | 116 | 124 | 132 | 147 | 159 | 172 | 183 | 194 | 204 | 214 | 223 |
| | H.P. | 30 | 36 | 42 | 47 | 51 | 56 | 61 | 65 | 74 | 83 | 92 | 100 | 108 | 116 | 124 | 132 |
| 100,000 | R.P.M. | 74 | 84 | 94 | 104 | 113 | 121 | 129 | 137 | 150 | 163 | 174 | 186 | 197 | 207 | 216 | 225 |
| | H.P. | 37 | 43 | 49 | 55 | 61 | 66 | 72 | 77 | 88 | 99 | 109 | 119 | 129 | 139 | 149 | 159 |
| 125,000 | R.P.M. | 90 | 97 | 103 | 113 | 121 | 129 | 136 | 143 | 156 | 167 | 178 | 190 | 200 | 210 | 219 | 228 |
| | H.P. | 44 | 50 | 57 | 64 | 70 | 77 | 83 | 90 | 102 | 114 | 126 | 138 | 151 | 163 | 175 | 188 |
| 150,000 | R.P.M. | 107 | 112 | 117 | 124 | 131 | 138 | 144 | 150 | 163 | 174 | 184 | 194 | 204 | 214 | 224 | 232 |
| | H.P. | 51 | 58 | 65 | 73 | 80 | 88 | 95 | 102 | 117 | 131 | 145 | 159 | 173 | 187 | 201 | 214 |
| 175,000 | R.P.M. | 123 | 127 | 132 | 137 | 143 | 148 | 154 | 160 | 170 | 180 | 190 | 200 | 210 | 219 | 229 | 237 |
| | H.P. | 59 | 67 | 74 | 82 | 91 | 99 | 107 | 115 | 131 | 147 | 163 | 179 | 195 | 211 | 227 | 242 |
| 200,000 | R.P.M. | 140 | 144 | 147 | 150 | 155 | 160 | 164 | 169 | 179 | 189 | 199 | 208 | 217 | 226 | 235 | 243 |
| | H.P. | 66 | 75 | 83 | 92 | 101 | 110 | 119 | 128 | 146 | 174 | 182 | 200 | 217 | 235 | 253 | 271 |
| 225,000 | R.P.M. | 158 | 160 | 163 | 166 | 169 | 173 | 177 | 181 | 189 | 199 | 208 | 217 | 225 | 234 | 242 | 249 |
| | H.P. | 73 | 83 | 93 | 102 | 112 | 121 | 131 | 141 | 161 | 181 | 201 | 220 | 240 | 260 | 279 | 299 |
| 250,000 | R.P.M. | 174 | 177 | 179 | 181 | 183 | 187 | 190 | 193 | 201 | 210 | 218 | 227 | 235 | 243 | 249 | 257 |
| | H.P. | 81 | 91 | 102 | 112 | 123 | 133 | 144 | 155 | 176 | 198 | 220 | 241 | 263 | 285 | 306 | 327 |
| 275,000 | R.P.M. | 192 | 194 | 196 | 198 | 200 | 202 | 204 | 207 | 214 | 222 | 230 | 237 | 244 | 253 | 259 | 265 |
| | H.P. | 88 | 100 | 111 | 123 | 134 | 145 | 157 | 168 | 182 | 215 | 239 | 262 | 286 | 310 | 333 | 356 |
| 300,000 | R.P.M. | 209 | 211 | 212 | 213 | 215 | 217 | 219 | 223 | 228 | 235 | 240 | 247 | 255 | 262 | 268 | 275 |
| | H.P. | 96 | 108 | 120 | 132 | 145 | 157 | 170 | 183 | 208 | 233 | 258 | 283 | 309 | 334 | 360 | 385 |
| 325,000 | R.P.M. | 227 | 228 | 229 | 230 | 231 | 233 | 235 | 237 | 242 | 248 | 254 | 261 | 267 | 273 | 279 | 285 |
| | H.P. | 103 | 117 | 130 | 143 | 156 | 170 | 183 | 196 | 223 | 251 | 279 | 305 | 332 | 360 | 387 | 414 |
| 350,000 | R.P.M. | 243 | 244 | 245 | 246 | 247 | 248 | 250 | 253 | 257 | 263 | 268 | 273 | 278 | 284 | 290 | 296 |
| | H.P. | 111 | 125 | 139 | 153 | 167 | 182 | 196 | 211 | 239 | 269 | 298 | 327 | 356 | 385 | 414 | 443 |

H. P. tabulated are I. H. P. steam eng. dir. conn. or motor output belted.

so that it may be easily revolved without stopping the fan. The operation is so simple that anyone about a mine is enabled to reverse the current instantly and to know positively that the operation is completed.

The foregoing types of fans are given to show the general designs that are now in use. There are numerous makes now on the market, each with one or more modifications of these general types. Most of these produce results worthy of investigation. To describe them all in detail is impractical in a pocketbook of this size.

The following table of capacities may be valuable to those having installations of the standard moderate-speed fan as manufactured by Crawford and McCrimmon.

| Diameter of Fan, Ft. | Width of Blades, In. | Size of Driving Engine | Revolutions per Minute | Maximum Capacity, Cubic Feet per Minute |
|---|---|---|---|---|
| 8 | 32 | 5 × 10 | 180 | 25,000 approx. |
| 10 | 40 | 6½ × 12 | 150 | 40,000 approx. |
| 12 | 48 | 8 × 13 | 150 | 65,000 approx. |
| 14 | 48 | 8 × 16 | 150 | 75,000 approx. |
| 15 | 60 | 10 × 20 | 120 | 90,000 approx. |
| 16 | 60 | 10 × 20 | 120 | 100,000 approx. |
| 18 | 72 | 10 × 24 | 100 | 150,000 approx. |
| 20 | 84 | 12 × 24 | 100 | 200,000 approx. |

**The Position of Any Fan, Etc.,** whether used as an exhaust or blower, should be sufficiently removed from the fan shaft or drift to avoid damage to the fan in case of explosion in the mine. Even in non-gaseous mines, the fan should be located a short distance back from the shaft mouth, to avoid damage due to settlement. Con-nection should be made with the fan shaft by means of an ample drift, which should be deflected into the shaft so as to produce as little shock to the current as possible. In case of gaseous seams, explosion doors should be provided at the shaft mouth. The ventilator at every large mine should be arranged so that it may be converted from an exhausting to a blowing fan at short notice. This is managed by housing the central orifices or intake of the fan in such a man-ner as to connect them directly with the fan drift. A large door *ab*, Fig. 19, is arranged at the foot of the expanding chimney, the latter being placed between the fan and the shaft. This door, when the fan is exhausting, is in the lower position *ab*, and then forms a portion of the spiral casing leading to the chimney. When the fan is blowing, how-ever, the door is swung upwards



FIG. 19

so as to occupy the position *ac*, being tangent to the cutoff at *c*, thereby closing the discharge into the chimney and causing it to enter the fan drift behind the door. At the same time, the positions of the two doors, *ed* and *fd*, in the fan drift, are changed to *et* and *fs*, respectively, to open the fan drift to the discharge from the fan, and to close the openings leading from the fan drift to the housing upon each side of the fan, while another set of doors *A A* upon each side of the fan, in the housing, which were previously closed tightly, are now set wide open to admit the outside air to the intake
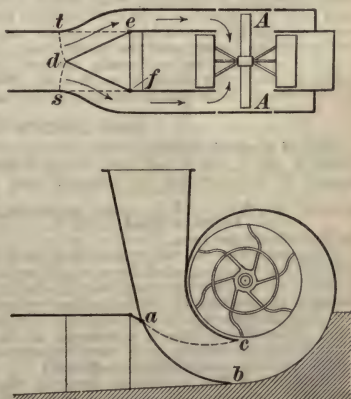
openings of the fan. The fan is thus made to draw its air from the atmosphere, and discharge it into the fan drift, instead of drawing its air from the fan drift and discharging into the chimney, as before.

**The manometrical efficiency** of a fan is the ratio between its effective and theoretical *pressures*. It has been assumed that the theoretical pressure due to the fan's action is given by the equation $h = \dfrac{u^2}{g}$, or $i = \dfrac{u^2 \times 1.2 \times 12}{g \times 1,000}$, $u$ being, as before, the tangential speed (feet per second), and $g$ the force of gravity (32.16); $h$ = head of air column in feet; $i$ = water gauge in inches.

**Mechanical efficiency** is a term applied to the ratio between its effective and theoretical *powers*. In estimating the efficiency of a ventilator, it is customary, though incorrect, to estimate the theoretical power of the fan from an engine card taken from the steam cylinder of the fan engine. The efficiency of the steam engine is this confused with the efficiency of the ventilator. Mr Beard gives the following formula for the theoretical work of the fan per minute: $U = .001699 \dfrac{m^3 - 1}{m^3} \sqrt{V} \, R^3 b n^2$, in which $m$ = ratio between outer and inner diameters of fan ($D = md$), and $V$ = velocity (feet per minute) of air in fan drift; $R$ = outer radius of fan blades (feet); $b$ = breadth of fan blades (feet); $n$ = number of revolutions of fan per minute. If we divide the power upon the air, as determined by the expression $qp$, by the theoretical work given in the last equation, we obtain the value of the coefficient of efficiency. According to this formula the efficiency of the ventilator changes with the speed, decreasing as the speed increases, but not in the same ratio. An expression for the coefficient of efficiency of a ventilator is given by Beard as follows: $K = \dfrac{163,600^2}{cX^3 + 163,600^2}$.

The factor $c$ is a constant of design whose value may vary from 2 to 7, but for an ordinary design, the value $c = 4$ may be taken. This factor has reference to the equipment of the machine with respect to its efficiency for passing an air current through itself with least resistance. Thus, where the ventilator is to be equipped with intake blades for the deflection of the air current into the machine, and with straight radial blades having only a forward curve at the lip of the blade to avoid the shock of the entering air against the revolving blades, and the spiral casing starting a short distance upon the cutoff and extending uniformly around the circumference of the fan, the value of this constant may be 2 or 3. Where none of these accessories to the efficiency of the fan is employed, the value of $c$ may be as high as 7.

## FAN CONSTRUCTION

**Size of Central Orifice.**—The velocity of the intake should vary between 1,000 ft. and 1,500 ft. per min., while 1,200 ft. may be used for fan calculations. If $d$ = diameter of opening, and $q$ = quantity of air passing per minute, $d = \sqrt{\dfrac{q}{1,200 \times .7854}}$ for single-intake fans, and $d = \sqrt{\dfrac{q}{2,400 \times .7854}}$ for double-intake fans.

Upon entering the fan the air travels in a radial direction; this change of direction is accompanied by a slight reduction of the velocity, hence the throat area of the fan must be slightly in excess of the intake area. The *throat* is the surface of the imaginary cylinder that has for its two bases the two intake openings of the fan, and for its length the width of the fan, = $\pi db$. [The throat area is commonly made 1.25 times the total area of the intake orifices, which gives for breadth of blade $b = \frac{5}{8}d$ for double-intake, and $b = \frac{5}{16}d$ for single-intake.—*Beard*.]

**Diameter of Fan.**—Murgue assumes the tangential velocity of the blade tips ($u$) to create a depression double that due to the velocity as expressed by the equation $H = \dfrac{u^2}{g}$, or if the manometrical efficiency $= K$, and the effective head produced $= h$, $h = KH = K\dfrac{u^2}{g}$, or $u = \sqrt{\dfrac{gh}{K}}$. From this equation, the tangential velocity (feet per second) may be calculated for any given effective head $h$. This effective head $h$ is the head of air column effective in producing the circulation in the airway. To convert the effective head of air column into inches of water gauge ($i$), we have $h = \dfrac{1,000}{1.2 \times 12} i$. Having found the tangential speed required in feet per second

this is multiplied by 60, to obtain the speed in feet per minute, and dividing this result by the desired number of revolutions per minute, or the desired speed of the ventilator, the outer circumference of the fan blades is obtained. No reference is made in the equation to the quantity of air in circulation, which is determined from the equivalent orifice of the mine and of the fan by the equation $V = \dfrac{.65\sqrt{2K}au}{\sqrt{1 + \dfrac{a^2}{o^2}}}$, in which $V$ = volume of air (cubic feet per second); $a$ = equivalent orifice of the mine; $o$ = the equivalent orifice of the fan. M. Murgue also uses the equation $h = \dfrac{Ku^2}{g\left(1 + \dfrac{a^2}{o^2}\right)}$, and suggests that the value of $K$ for any particular type of machine should be first decided, after which the tangential speed required to produce any given effective head of air column ($h$) is easily calculated for the formula $u = \sqrt{\dfrac{gh}{K}}$. The breadth of the blade is left largely to judgment, while this method of calculation gives the same size of fan for any given effective head desired, regardless of the quantity of air to be circulated, which is the same as saying that the ventilator will present the same efficiency when a large amount of air is crowded through its orifice of passage as when a smaller amount of air is necessary.

Mr. Beard uses the following formulas for determining the several dimensions of a ventilating fan:

$$m = \sqrt[3]{\dfrac{Q}{n^2\sqrt{V}}\left(c + \dfrac{163,600^2}{X^3}\right) + 1};  \qquad D = \dfrac{m}{\sqrt{m^3 - 1}}\dfrac{3,770\sqrt{p}}{n^4\sqrt{K^2V}};$$

$$b = \dfrac{385,000,000p}{(m^3 - 1)n^2K\sqrt{QV}};  \qquad e = \dfrac{\sqrt{m^3 - 1}}{170m}\sqrt[4]{\dfrac{X^3K^2V}{p}};$$

in which $m = \dfrac{D}{d}$, which is the ratio between the outer diameter of the fan blades $D$ and the inner diameter of the blade $d$, which equals the diameter of the intake orifice; $b$ = width of fan blade; $e$ = expansion of spiral casing at point of cutoff.

The other symbols stand for the same quantities as previously indicated.

**Curvature of Blades.**—It was at one time supposed that the curvature of the blades should be such that the radial passage of the air current would be undisturbed by the revolution of the fan; but fans constructed on this principle gave no adequate results, and the theoretical spiral thus developed was entirely abandoned. A certain curvature of the blade backward, however, is assumed by many to increase the efficiency of the fan. This has not been proved in practice, but the effect of the backward curvature appears simply to necessitate a higher speed of revolution in the fan, in order to obtain the same results as are obtained with radial blades.

The Guibal blade, radial at its outer extremity, or normal to the outer circumference, and curved forward in the direction of motion, at its inner extremity, so that the lip of the blade approaches tangency to the throat circle, seems to be an effective blade in centrifugal ventilation.

**Tapered Blades.**—The object of the taper is to produce a constant area of passage from the throat to the circumference of the fan, and thus prevent the reduction of the velocity of the current in its passage through the fan. This feature presents an attempt similar to that attempted by the curvature of the blades, to hasten the passage of the air through the fan. It has not been proved, however, to have produced any beneficial result, except in the strengthening of the discharge pressure against the atmospheric pressure, in *open-running fans*. On the other hand, the slowing up of the air in its passage through a covered fan has by no means been proved a detriment, but is assumed by many to be an advantage, inasmuch as the air thus remains longer within the influence of the fan blades.

The **number of blades** depends on the size of the fan. An increased number strengthens the fan's action at the circumference, or supports the air at that point, and thus prevents the backlash or the reentry of air into the fan, due to the eddies occurring at the circumference when the blades are too far apart. To a certain extent, the number of blades is modified by the speed

of revolution, high-speed motors requiring a somewhat lesser number, while low-speed motors require more. In any case, the number of blades should not be so great as to abnormally increase the resistance to the air current. In general, the distance upon the outer circumference from tip to tip of the fan blades should be from 2 to 3 times the depth of the blade.

The **spiral casing** gradually reduces the velocity of the air and reduces the shock incident to the discharge of the air into the atmosphere. The spiral casing should be so proportioned that the velocity of the flow from the fan blades will be maintained constant around the entire circumference, and this should not be less than the velocity of the blade tips. The expansion $e$ of the casing at the cutoff should be such as to provide a velocity of the air at this point equal to the velocity of the blade tips, according to the

equation $e = \dfrac{Q}{\pi D n b}$, in which $D$ = diameter of fan; $n$ = number revolutions per minute; $b$ = breadth of fan blade.

The **évasé chimney** reduces the velocity of the air, as it is discharged into the atmosphere, to a minimum. The chimney should be sufficiently high to protect the fan from the effect of high winds, but should not extend too far above the fan casing, the point of cutoff being situated below this, at about the level of a tangent to the throat circle at its lower side.

**High-speed and Low-speed Motors.**—The question of speed of the ventilating motor is largely an open one, inasmuch as the same work may be performed by a small ventilator running at a high speed as is performed by a large ventilator running at a low speed.

It is important to design a mine ventilator at a speed such as to admit of its being increased in case of emergency. If the ventilator has been designed at a high speed, a demand for an increase of speed cannot be met as readily as when the ventilator is designed at a medium or low speed; in other words, the exigencies of mine ventilation demand that a ventilator shall be capable of greatly increased speed.

**Fan Tests.**—A large number of fan tests have been made, from time to time, on different types of fans and under different conditions, with respect to the resistance against which the fan is operated, and the quantity of air required, and the speed of the ventilator. The experiments have resulted, to a large extent, in tabulating a mass of contradictory data. The conditions that affect the yield of the centrifugal ventilator are so numerous, and the tabulation of the necessary data has been so often neglected in these experiments, as to render them practically useless for the purpose of scientific investigation. In conducting a reliable fan test, the following points should be observed: (1) Take the velocity, pressure, and temperature of the air at the same point in the airway, as nearly as practicable. This point should be selected near the foot of the downcast shaft, or in the fan drift at a suitable distance from the fan, to avoid oscillations of pressure and velocity. (2) The area of the fan drift should be uniform for a suitable distance in each direction from the point of observation, and this area should be carefully measured. (3) Take the velocity readings at different positions in the airway, so as to obtain an average reading over the entire sectional area. Do not interpose the body in this area so as to decrease the sectional area of the airway. (4) Take outside temperature of the air and the barometric pressure at the time of making the test. (5) The intake and discharge openings of the fan should be protected against wind pressure. (6) At least three observations should be made, at as many different speeds of the ventilator, and the number of revolutions of the fan carefully observed and recorded for each observation.

Mr. R. Van A. Norris (*Trans.* A. I. M. E., vol. XX, page 637) gives the results of a large number of experiments performed upon different mine ventilating fans. This table, like all other tabulated fan tests, shows a large amount of contradictory data. The conclusions drawn by Mr. Norris from these tests are interesting and would be given here excepting that they might be misleading if considered apart from the description of the experiments and the discussion leading up to the conclusions.

## CONDUCTING AIR CURRENTS

**Doors.**—A mine door is used for the purpose of deflecting the air current from its course in one entry so as to cause it to traverse another entry, at the same time permitting the passage of mine cars through the first entry. The essential points in the construction of a mine door are that it shall be hung from a strong door frame in such a manner as to close *with* the current. The

door should be hung so as to have a slight fall. If necessary, canvas flaps may be supplied to prevent leakage around the door, and particularly at the bottom. *Double* doors are used on main entries at the shaft bottom, or at any point where the opening of the door causes a stoppage of the entire circulation of the mine. Such doors should be placed a sufficient distance apart to allow an entire trip of mine cars to stand between them, so that one of the doors will always be closed while the other is open.

**Stoppings.**—Stoppings are used to close breakthroughs that have been made through two entries, or rooms, for the purpose of maintaining the circulation as the workings advance; also to close or seal off abandoned rooms or working places. Stoppings must be air-tight and substantially built. A good form of stopping is constructed by laying up a double wall of slate, having about 8 or 10 in. of space between the two walls. This space is filled, as the building progresses, with dirt taken from the roadways and other fine material. A still better plan is to build the stopping of brick or concrete. Hollow tile has been used as a material for stoppings and appears to be admirably adapted to this purpose.

**Air Bridges.**—An air bridge is a bridge constructed for the passage of air across and over another airway, this being called an *overcast*; or, the crossing may be made to pass under the airway, this being called an *undercast*. In almost every instance, overcasts are preferable to undercasts for several reasons. An undercast is liable to be filled with water accumulating from mine drainage; it is also liable to fill with heavy damps from the mine, when the ventilation is sluggish, and to offer considerable resistance to the free passage of the air current. An undercast can never be maintained as air-tight as an overcast, on account of the continual travel through the haulageway or passageway leading over it. This continual passing over the bridge causes a fine dust to sift into the airway and mingle with the air current. All these objections are overcome in the construction of the overcast.

An **air brattice** is any partition erected in an airway for the purpose of deflecting the current. A thin board stopping is sometimes spoken of as a brattice; but the term applies more particularly to a thin board or canvas partition running the length of an entry or room and dividing it into two airways, so that the air will be obliged to pass up one side of the partition and return on the other side of the partition, thus sweeping the face of the heading or chamber. Such a temporary brattice is often constructed by nailing brattice cloth or heavy duck canvas to upright posts set from 4 to 6 ft. apart along one side of the entry a short distance from the rib.

**Curtains.**—These are sometimes called canvas doors. Heavy duck, or canvas, is hung from the roof of the entry to divide the air or deflect a portion of it into another chamber or entry. Curtains are thus used very often previous to setting a permanent door frame. They are of much use in long-wall work, or where there is a continued settlement of the roof, which would prevent the construction of a permanent door; also, in temporary openings where a door is not required.

# MINE FIRES

**Means Employed.**—There are two general methods of extinguishing mine fires; namely, flooding and smothering. It is generally considered better practice and cheaper to smother a fire than to drown it, since in the latter case the water used for quenching must be pumped out of the mine after the fire is extinguished. In cases, however, where sealing off cannot be accomplished or where for any reason it is not effective, flooding may be the only means of quenching the fire.

**Isolating the Section.**—Where fire exists in a portion of a mine only and it is expedient to keep the remainder of the mine in operation the section involved may be isolated by means of a suitable dam or dams sufficiently strong to withstand the greatest head of water that may possibly come upon them. Such dams may be built of wood with clay or other filling of brick, masonry, concrete or reinforced concrete. In most mines it will be found, however, that the reinforced concrete will be the cheaper structure to build if the head of water to be resisted is at all high.

When all necessary dams have been constructed, water may be turned into the portion of the mine affected and the fire thus effectually quenched. The water may be poured down a regular mine opening such as a slope or shaft or one or more bore holes may be sunk from the surface for this purpose.

In many instances a mixture of water and clay, mud, silt, culm or other mine refuse may be used for quenching fires. Such a flush (often called slush) has the advantage of baking into a more or less solid mass under the action of the heat which it encounters and is consequently preferred by many engineers.

**Methods Employed in Sealing off Fires.**—In the majority of instances it is not necessary to resort to flooding a mine, particularly if the fire is promptly discovered and quick action is taken. Particularly in the more easily inflamed bituminous coals, fires often originate in the process of shooting down. These, if discovered promptly, may be sealed off without seriously endangering the rest of the mine or any considerable loss in output. Fig. 1 shows a fire of this kind and the position of the necessary stoppings for sealing it off.

Mining men differ somewhat as to whether a fire of this kind should be first sealed on the intake or the return. It is probable, however, that in the majority of instances it is preferable to seal up first the intake. If, however, there are several openings to the fire all of which must be closed and through oversight or delay the fire has gained considerable headway or if a strong feeder of gas is the primary cause of such a fire, or if the sealing is to be undertaken a considerable distance from the fire, it might under certain circumstances be wise to seal the return first. This procedure should at least be given careful consideration.

In fighting mine fires in general the following points should be kept in mind: Attack the fire as soon after its origin as possible, extinguish it directly, that is, with fire-hose and water if possible, and if it is safe to do so. Never allow a gas feeder of any magnitude discharging in rear of the fire, that is, on the intake side to continue its discharge into the air current reaching the fire, even though this gas discharge may be diluted far below the explosive mixture. If possible conduct the gas from such a feeder direct to the return. Erect all seals as close to the fire as possible. No more air should be allowed to reach the sealing point than is necessary while the work is being done. What air is carried to this point should be directed by brattice and curtains close around the men. Men erecting a seal should never raise their heads close to the roof and thus inhale an overdose of smoke.
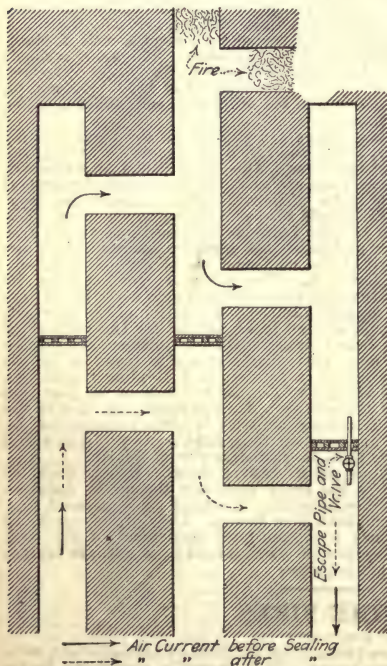
**Stopping Materials.**—The materials employed in building a seal may be the same as those used for ordinary stoppings, that is, rock packs covered with cement, mortar or even clay, brick walls, concrete or reinforced concrete, hollow tile, etc. As a general rule, however, it will be found that a double wooden partition with clay tamping between can be built up more rapidly than any other type of stopping—and speed is important. A good



Fire....

Escape Pipe and Valve

→ Air Current before Sealing
⇢ " " after "

FIG. 1

type of stopping of this nature is shown in Fig. 2. This should be set into floor, ribs and roof, and the joint around the complete periphery made as nearly air-tight as possible with clay tamping. It is frequently advisable on discovering a fire to start a few men to preparing the places selected for the various stoppings while others collect necessary material and as soon as possible begin the erection of the first seal. The intake to the fire should be short-circuited and a careful watch kept upon the smoke as a heavy fall of rock may drive this smoke and the products of combustion back onto the workers unless they receive warning in time to get away. The materials will consist of the timber posts, which should be at least 6 in. square (10 in. square is preferable), the necessary planking, brattice cloth to cover the entire stopping and the clay or earth for the filling and for a daubing coat on the outside if necessary. Both sides of the stopping should be carried up simultaneously together with the clay filling which should be carefully tamped in place.

The last stopping is always the most difficult to build, and it is frequently advantageous to here erect a temporary brattice inside the stopping proper to keep back as much as possible the smoke and gases from combustion.



2" Plank→ Stiff Clay 10"x10" or 6"x6" Posts····

←·············· 12' ·············→
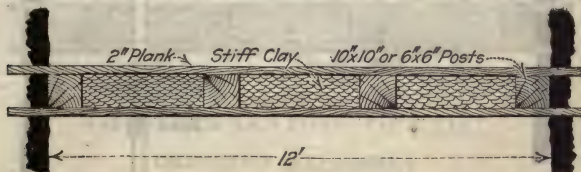
Fig. 2

This brattice as well as the stopping itself should contain a section approximately 24 × 34 in. which may be afterwards easily removed, allowing the entrance of a fire boss or other person to ascertain the condition of the affected area. A pipe, say 4 in. in diameter, or more, and provided with a valve, should be built into this last stopping to allow the escape of the heated air in the fire area (see Fig. 3). Otherwise the expansion of this air may push out one of the stoppings. Throughout the entire process of building the last stopping extreme care should be exercised that no workman raise his head into the dense smoke, thus inhaling gas.

When all passages leading to the fire have been sealed, the ventilation up to the stopping should be restored, thus allowing the largest available amount of air to circulate to all the working places in that portion of the mine, thus sweeping out the smoke and gases which may have accumulated there while the ventilation was restricted. A careful inspection should then be made to see that such places are fit for operation. Near the seal containing the escape pipe and valve a canvas brattice or tight curtain with suitable means of entrance should be constructed, so that the gas coming through the pipe may be periodically tested. This test may be made each evening after the workmen have left the mine. For the first few days the gas issuing from the pipe will be afterdamp, after which firedamp will begin to appear. The showing of this gas will get stronger from day to day and during the night or at such times as no men are at work in the mines the valve may be left open. After several days the gas issuing from the pipe will be sufficiently strong to give a cap on the lamp when the latter is held some distance from the pipe. From the indication of this gas it may be judged when it is advisable to open the sealed area. This will probably not exceed 18 days to three weeks from the time of sealing up, provided, of course, that the fire was promptly discovered and that only a comparatively small volume of air exists in the sealed area.

**Unsealing after the Fire is Out.**—In unsealing a supposedly extinguished fire, it is well to curtain off other districts of the mine so as to prevent the escaping gases from reaching them; also to confine the return from the fire to one air-course. The previously mentioned manhole opening in the last seal is first broken open. During this process there should be no flame, not even that of a safety lamp in line with the opening as the gas emitted will be strongly inflammable. After this opening has been made in the return a similar one may be opened in the intake and a current of fresh air thus

passed through the heretofore sealed area. With the air current through this fire area established, observers may be stationed at intervals along but outside of the return at such points that a safety lamp may be pushed into the return airway periodically to test the air. These tests should be made at intervals. When no showing of gas appears in the return from the fire area it is time enough to make a thorough inspection of the section which has been sealed up. When this is found to be safe for working, all the stoppings may be removed, the place cleaned up if necessary, fallen or burned timbering renewed and work resumed.
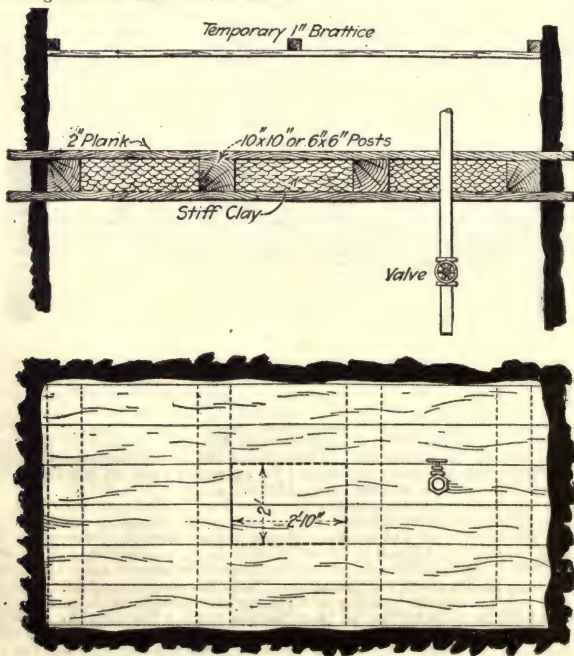


FIG. 3

**Gob fires** are due to the spontaneous ignition of coal, and are most likely to occur in pack walls and gobs where there is an insufficiency of air. Ample ventilation is the best preventive.

**Spontaneous Combustion.**—According to Prof. Able, Dr. Percy, and Prof. Lewes, the causes of the *spontaneous ignition* of coal are: *First*, and chiefly, the condensation and absorption of oxygen from the air by the coal, which of itself causes heating, and this promotes the chemical combination of the volatile hydrocarbons in the coal and some of the carbon itself with the condensed oxygen. This process may be described as self-stimulating, so that, with conditions favorable, sufficient heat may be generated to cause the ignition of portions of the coal. The favorable conditions are: A moderately high external temperature; a broken condition of the coal, affording the fresh surfaces for absorbing oxygen; a supply of air sufficient for the purpose, but not in the nature of a strong current adequate to remove the heat; a considerable percentage of volatile combustible matter or an ex-

tremely divided condition. *Second*, moisture acting on sulphur in the form of iron pyrites. The heating effect of this second cause is very small, and it acts rather by breaking the coal and presenting fresh surfaces for the absorption of oxygen.

**Coal Storage.**—Prof. Lewes gives the following recommendations for the storage of coal: "The coal store should be well roofed in, and have an iron floor bedded in cement; all supports passing through and in contact with the coal should be of iron or brick; if hollow iron supports are used, they should be cast solid with cement. The coal must never be loaded or stored during wet weather, and the depth of coal in the store should not exceed 8 ft., and should only be 6 ft. where possible. Under no condition must a steam or exhaust pipe or flue be within or near any wall of the store, nor must the store be within 20 ft. of any boiler, furnace, or bench of retorts. No coal should be stored or shipped to distant points until at least a month has elapsed since it was brought to the surface. Every care should be taken during loading or storing to prevent breaking or crushing of the coal, and on no account must a large accumulation of small coal be allowed. These precautions, if properly carried out, would amply suffice to entirely do away with spontaneous ignition in stored coal on land."

When the coal pile has ignited, the best way to extinguish the fire is to remove the coal, spread it out, and then use water on the burned part. The incandescent portion is invariably in the interior, and when the fire has gained any headway usually forms a crust that effectually prevents the water from acting efficiently.

# THE PREPARATION OF COAL

## CRUSHING MACHINERY

The object of crushing coal is usually to reduce it in size preliminary to washing, a better separation of impurities being secured in the jig if the particles are of nearly the same size. At the same time the coal is crushed the larger pieces of slate, sulphur, etc., are separated by the action of the machine from the lumps of coal and are, also, more effectively removed in the subsequent washing. In the case of anthracite coal it is reduced in size to meet the requirements of trade which changes in its demands from time to time, requiring a maximum of stove sizes today and a maximum of nut sizes tomorrow, and at no time being able to consume the lump as it comes from the mines.

Jaw crushers, ball and tube mills, stamps, etc., commonly employed in metallurgical operations do not find a place in coal preparation. Coal is almost invariably broken down in size by some form



Fig. 1

of *rolls*, a few forms of which are:
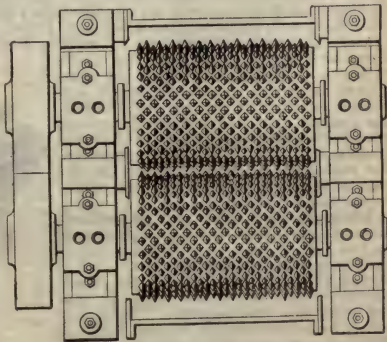
**Cracking Rolls.**—This is a general name applied to rolls having teeth, which are usually made separate and inserted. These rolls, Fig. 1, are employed for breaking the coal, the object being to break the material into angular pieces with the smallest possible production of very fine material. The principal field for cracking rolls is in the preparation of anthracite coal, and the exact style or design of the roll depends largely on the physical condition of the coal under treatment. In most cases the rolls are constructed with an iron cylinder having steel teeth inserted, the size, spacing, and form of the teeth depending on the size and physical condition of the material to be broken. Cracking rolls vary from 12 to 48 in. in diameter and from 24

to 36 in. in face width. The teeth of the larger sizes are from 3 to $3\frac{1}{2}$ in. high, and of the smaller 1 in. or less.

The average practice in the anthracite regions of Pennsylvania is to give the points of the teeth a speed of about 1,000 ft. per min., though the speed in different cases varies from 750 to 1,200 ft. per min. One of the largest anthracite companies has a standard roll speed of 97.5 R. P. M. for the main rolls and 124.5 R. P. M. for the pony rolls. The harder the coal the faster the rolls can be run. If run slow and overcrowded, the rolls will make more



culm than when driven at a proper speed. One advantage of comparatively fast driven rolls is that the higher speed has a tendency to free the rolls by throwing out, by centrifugal force, any material lodged between the teeth. In one test it was found that less fine coal was produced at 800 ft. per min., but that the rolls blocked at this speed and hence had to be driven 1,000 ft. per min.

In one case a pair of main rolls 24 in. in diameter, 36 in. face, running at 1,000 ft. per min., handled 2,500 T. of coal in 24 hr. A pair of 19 in.$\times$24 in. main rolls run at 1,000 ft. per min. handled 300 T. mine run in 10 hr.

A well-known maker of rolls for crushing bituminous coal gives a speed of 100 to 150 R. P. M., according to the output required, for rolls 24 in. in diameter and 33 in. long. As a rule, cracking rolls are never run up to their full capacity, as is the case with crushing rolls.

(a)    (b)

FIG. 2

The *form of the teeth* varies greatly, but, as a rule, the larger rolls have straight pointed teeth of the sparrow-bill or some similar form, Fig. 2a. The old curved, or hawk-billed, teeth, Fig. 2b, have now gone almost wholly out of use.

On small sized rolls, rectangular teeth with a height equal to one side of the square base are frequently employed, and these may be cast in segments of manganese or chrome steel.

**Corrugated rolls** have teeth or corrugations extending their entire length. They were first introduced by Mr. E. B. Coxe, at Drifton, Pa., but they have not come into general use owing to the fact that, while they break one coal fairly well, in most cases it has been found that a continuous edge causes too much disintegration along its length, while a point splits the coal into three or four pieces only, all the cracks radiating from the place where the point strikes, thus producing very much less culm. Another advantage possessed by the toothed rolls is that if anything hard passes through the corrugated roll and breaks out a piece of the corrugation, the entire roll is ruined, while, in the case of the toothed rolls, any one of the teeth may be replaced.



FIG. 3

**Disintegrating rolls and pulverizers** are sometimes used to reduce coking coal to the size of corn or rice before introducing it into the ovens. One roll is driven at double the speed of the other, the slower roll acting as a feed-roll, and the other as a disintegrator. The slower roll is commonly driven at from 1,800 to 2,000 ft. per min. peripheral speed, and the faster roll at from 3,600 to 4,000 ft. per min. The teeth are always fine, rarely being over $\frac{3}{8}$ in. high. In some cases, the inner roll is provided with a series of saw teeth from $\frac{1}{4}$ in. to $\frac{3}{8}$ in. high and having about $\frac{3}{8}$-in. pitch, the individual teeth being set so as to form a slight spiral about the body of the roll. The other roll is provided with teeth having their greatest dimension in the direction of rotation, so that they tend to cross the teeth on the opposite roll. These teeth are also set so as to form a slight spiral, and thus prevent blocking. In other cases, the teeth on both rolls are set in the form of quite a steep spiral.

**Hammers.**—For the reduction of coal, crushers employing hammers have been used, Fig. 3. The crushing chamber is usually of a circular or barrel
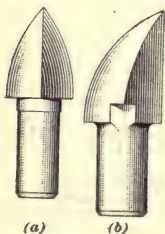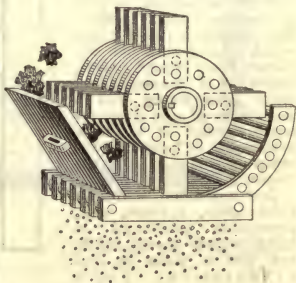
form, and the crushing is done by means of hammers pivoted about a central shaft. These swing out by centrifugal force and strike blows upon the coal to be broken. When it is reduced sufficiently fine, it is discharged through bars or gratings at the lower portion of the machine. This style of machinery is usually employed in preparing coal for coke ovens, thus occupying the same field as the disintegrating rolls. A No. 3 pulverizer of this type will crush 50 to 75 T. per hr. run of mine, down to ½ in., or it will crush 100 T. per hr. of slack. Such a machine occupies about 8 sq. ft. of floor space and requires 25 to 30 H. P. to run it.

**Miscellaneous Forms of Crushers.**—Most crushers can be classed under one of the previous heads, but there are some forms that depend on the material itself to do the crushing. For instance, in the preparation of coal for coke ovens, there has been a combined crusher and separator invented that may be described as follows: A large horizontal drum or cylinder, provided with screen openings around its periphery, is mounted in a horizontal position. The coal to be separated is fed into one end and is caught by shelves or plates projecting radially into the cylinder. These lift the material to the upper side, from which it falls by gravity and strikes the bottom, thus crushing the softer parts. The sulphur and slate, being harder than the coal, are not crushed by the same height of fall, and hence, by a proper adjustment of the diameter of the cylinder, the coal may be crushed and discharged through the screen while the slate and sulphur will pass out at the opposite end of the cylinder.

## SIZING AND CLASSIFYING APPARATUS

**Stationary Screens, Grizzlies, Head-Bars, or Platform Bars.**—These are the various names given to an inclined screen employed for removing the fine material from the run of mine so that only the coarse portion will be passed to the crushers. At concentrating works always and sometimes at anthracite coal breakers, the term *grizzly* is employed, and a common form is shown in Fig. 4. This is composed of flat bars held apart by cast-iron washers through which the bar bolts are passed to hold the entire frame together. Grizzlies are usually placed at an angle of from 45° to 55°, and ordinarily they are from 3 to 6 ft. wide and from 8 to 12 ft. long, the amount of space between the bars depending on the size of the run-of-mine material and on its subsequent treatment.



FIG. 4

In the anthracite coal breakers, the terms *platform bars* or *head-bars* are usually employed, and these bars are made of 1½-in. to 2-in. round iron placed at an inclination of 5 in. to 1 ft., the spacing depending on the size of coal it is desired to make in the breaker.

A standard size for a bituminous lump screen (the bars are called a screen) for Ohio, Pennsylvania, Indiana, and Illinois is 12 ft. long and 6 ft. wide over the screen surface. The screen consists of 6 bearing bars 4 in. by ¾ in. of soft steel and 39 steel screen bars, Fig. 5, with 1⅛ in. clear space between bars. In Iowa, the same sized bar is used, but the space between the bars is 1⅜ in. In the other Western and Southern States there is apparently no standard.

*Adjustable Bars.*—The top of the bar is cylindrical and projects beyond the web which supports it, so that any lump which passes through the upper part will fall freely without jamming. The two ends of the bar are V-shaped and fit into similarly shaped grooves, so that the bars can be set at distances from each other varying with the sum of the width of the bases of the triangles, the usual opening being about 4 in. These bars are generally 4 ft. long, but they can be of any size.



FIG. 5

*Finger bars* are screen bars that are fixed at one end only, and the bars are narrower at the lower end than at the top, so that the spaces between them are wider at the bottom than at the top, thus giving less tendency for pieces of material to become wedged between the bars.

*Movable* or *oscillating bars* are screen bars that are attached to eccentrics at their lower ends, the eccentrics of adjoining bars being placed 180° apart.
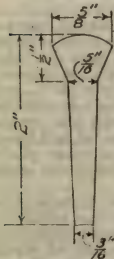
This movement throws the material forward and the bars do not, therefore require nearly the same inclination as fixed bars.

**Shaking screens** have an advantage in that the entire area of the screen is available for sizing, and hence a greater capacity can be obtained from a given area of screening surface. They also occupy less vertical height than a revolving screen. In coal breakers they are particularly applicable where the coal is wet and has a tendency to stick together. The principal disadvantage of the shaking screen is that the reciprocating motion imparts a vibration to the framing of the building. For anthracite coal, the screens usually have an angle or pitch of from $\frac{1}{2}$ in. to 2 in. per ft., the average being about $\frac{3}{4}$ in. per ft. These screens are run at from 90 to 280 shakes per min., the average being about 200 shakes per min. or 100 rev. per min. for the camshaft. The throw of the eccentric or cam varies from 2 in. to 5 in.

The *capacities* of shaking screens operating on anthracite coal have been given as follows. The parties giving these figures advise the use of 140 R. P. M. for the camshaft.

For broken and egg coal, $\frac{1}{4}$ sq. ft. per T. for 10 hr.

For stove and chestnut coal, $\frac{1}{3}$ sq. ft. per T. for 10 hr.

For pea and buckwheat coal, $\frac{3}{4}$ sq. ft. per T. for 10 hr.

For birdseye and rice, $1\frac{1}{4}$ sq. ft. per T. for 10 hr.

**Size of Mesh.**—The following perforations have been adopted by two of the largest anthracite coal companies as the dimensions for the holes in shaking screens to produce sizes equivalent to those produced by revolving screens:

MESH FOR SHAKING SCREENS

| Kind of Coal | Lehigh Valley Coal Co. | Phila. & Reading Coal & Iron Co. | | Kind of Coal |
|---|---|---|---|---|
| | Round, In. | Round, In. | Square, In. | |
| Steamboat....... | ............ | $5\frac{3}{8}$ | 5 | Steamboat. |
| Lump........... | $4\frac{1}{2}$ | $4\frac{1}{4}$ | 4 | Large broken. |
| Broken......... | $3\frac{1}{4}$ | $3\frac{1}{4}$ | $2\frac{3}{4}$ | Small broken. |
| Egg............ | $2\frac{1}{16}$ | $2\frac{1}{4}$ | 2 | Egg. |
| Stove.......... | $1\frac{5}{8}$ | $1\frac{1}{2}$ | $1\frac{3}{8}$ | Stove. |
| Chestnut ....... | $1\frac{1}{16}$ | $\frac{7}{8}$ | $\frac{3}{4}$ | Chestnut. |
| Pea........... | $\frac{5}{8}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | Pea. |
| Buckwheat...... | $\frac{11}{32}$ | $\frac{5}{16}$ | $\frac{1}{4} - \frac{3}{8}$ | Buckwheat. |
| Rice........... | $\frac{3}{16}$ | $\frac{3}{16}$ | $\frac{3}{8}$ | Rice. |

**Revolving Screens, or Trommels.**—The screen is placed about the periphery of a cylinder or frustum of a cone. The material to be sized is introduced at one end; the small size passes through the screen, and the other size is discharged from the other end. If the form is cylindrical, it is necessary to place the supporting shaft on an incline so that the material will advance towards the discharge end. The inclination of the shaft determines the rapidity with which the material will be carried through the screen. The advantage of the conical screen is that the shaft is horizontal and hence the bearings are simpler. This is a very decided advantage in many plants where the machinery must of necessity be crowded into a minimum space and be hard to get at.

Revolving screens are frequently jacketed, that is, two or more screens are placed concentrically about the same shaft, the inmost one being the coarsest, and each succeeding screen serving to make additional separations. This method reduces the space necessary for a given amount of sizing machinery. In other cases, a long cylindrical screen has a coarse mesh near its discharge end and finer mesh near the entrance end, thus making two or more through products as well as the overproduct. The disadvantage of jacketed screens is that the necessarily slow speed of the inmost screen reduces the capacity of the entire combination, so that if rapid work is essential, it is better to use fairly large-diameter screens placed one after the other in place of jacketed screens. Another disadvantage is that, to renew the inner jackets, it is often necessary to remove the outer ones.

The disadvantages of having two or more sizes of wire cloth on one screen are that the fine-meshed screen near the head is worn out rapidly, as all the material both coarse and fine passes over it, while, when separate screens are employed, each screen has to deal with its through or oversized product, all coarser material having been removed.

**Speed.**—The periphery of a revolving screen should travel about 200 ft. per min. In the case of very fine material, screens are sometimes run faster than this.

The following have been adopted as standard speeds for screens by one of the largest anthracite coal companies:

### SPEED OF SCREENS

| | Rev. per Min. | | Rev. per Min. |
|---|---|---|---|
| Mud screens | 8.87 | Big screens | 8.52 |
| Counter mud screens | 15.49 | Pony screens | 10.87 |
| Cast-iron screens | 11.25 | Buckwheat screens | 15.30 |

**Duty of Anthracite Screens.**—The following table gives the number of square feet of screen surface required for a given duty in the case of revolving screens working upon anthracite coal:

| | | |
|---|---|---|
| Egg coal, | 1 T. per 1 | sq. ft. per 10 hr. |
| Stove coal, | 1 T. per 1¼ | sq. ft. per 10 hr. |
| Chestnut coal, | 1 T. per 1½ | sq. ft. per 10 hr. |
| Pea coal, | 1 T. per 2 | sq. ft. per 10 hr. |
| Buckwheat coal, | 1 T. per 2¾ | sq. ft. per 10 hr. |
| Rice coal, | 1 T. per 3¾ | sq. ft. per 10 hr. |
| Culm, | 1 T. per 5 | sq. ft. per 10 hr. |

These figures may be reduced from 20 % to 30 % for very dry or washed coal.

**Revolving Screen Mesh for Anthracite.**—A standard mesh for revolving screens for sizing anthracite coal was adopted some years ago, but it is only approximately adhered to and a considerable variation from the standard is found throughout the anthracite region.

The following are probably as nearly standard meshes for revolving screens for sizing anthracite coal as can be given:

### MESH FOR SIZING COAL

| | | |
|---|---|---|
| Culm | | passes through $\frac{1}{12}$-in. mesh. |
| Birdseye | passes over ¹⁄₁₂-in. mesh, | and through ¹⁄₁₆-in. mesh. |
| Buckwheat | passes over ¼-in. mesh, | and through ⅛-in. mesh. |
| Pea | passes over ½-in. mesh, | and through ¼-in. mesh. |
| Chestnut | passes over ⅞-in. mesh, | and through 1⅝-in. mesh. |
| Stove | passes over 1⅝-in. mesh, | and through 2 -in. mesh. |
| Egg | passes over 2 -in. mesh, | and through 2¾-in. mesh. |
| * Grate | passes over 2¾-in. mesh, | and out end of screen. |
| * Special grate | passes over 3 -in. mesh, | and out end of screen. |
| * Special steamboat | passes over 3 -in. bars, | and through 6-in. bars. |

**Hydraulic Classifiers.**—The separation of materials by this class of machinery depends upon the law of equally falling bodies, which may be stated as follows: *Bodies falling free in a fluid, fall at a speed proportional to their weight divided by the resistance.* From this it will be seen that small masses of a heavy mineral will fall as rapidly as large masses of a light mineral, owing to the fact that the weight increases as the volume and the resistance only as the area, so that if a quantity of iron pyrites and coal of various sizes were introduced into water, it would settle into approximate layers, each composed of relatively large pieces of coal and relatively small pieces of iron pyrites. This same action would be true in the case of any minerals differing in specific gravity.

The **Jeffrey-Robinson coal washer, Fig. 6,** which operates on the principle of the Spitzkasten, consists of a steel chamber $B$ in the form of an inverted cone, inside of which are projecting arms and stirring plates $C, C$ revolved by a driving gear $A$. The water supply enters at the bottom from the water pipe $P$ through perforations $M$. The coal is introduced through a chute $S$ and is kept in a continual state of agitation by the current of water, and being lighter than the impurities, it passes out through the overflow $K$ onto the conveyors $E, F$ and through the chutes $X, X$, while the water and sludge

---

* These sizes and "lump" size are seldom made, and there is no uniformity whatever in the sizes called by these names.

drain through the hopper into the sludge tank G, whence, if necessary, the same water can be again pumped by the pulsometer H back into the washer. (As mentioned elsewhere, it is poor practice to use this water over again when it is desired to decrease the percentage of sulphur in the washed product as greatly as possible.)

The heavy impurities sink to the bottom into the chamber J and when this is full the upper of the two valves shown is closed and the lower valve is opened to discharge the refuse.

The following data in regard to one of these washers is given by Mr. J. J. Ormsbee in the Transactions of the A. I. M E. These results were obtained at the Pratt Mines, Alabama, with a plant having a nominal capacity of 400 T. per day. By washing slack that passed between screen bars placed ¾ in. in the clear, the washed coal contained 42% less ash than the unwashed coal, the reduction in sulphur was 15%, while the volatile matter was increased 4%, and the fixed carbon 5%. With coal passing over ¾-in. perforations, the results were a reduction of 48% in ash, 15% in sulphur, and a gain of 5% in volatile matter and 6% fixed carbon. These results indicate that the washer is better adapted to large sizes than to fines. The amount of water used per ton of washed coal was 35.1 gal. and the cost was 2.25 c.



FIG. 6.

per T. for washing 400 T., itemized as follows: Labor at washer, $2.00; labor at boiler, fuel, etc., $4.00; repairs and supplies, $3.00; total, $9.00.
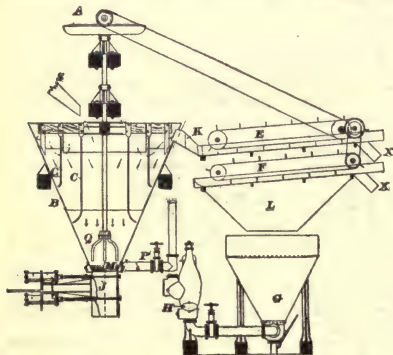
The **Scaife trough washer** consists of a semicircular iron trough 2 ft. in diameter and 24 ft. long. Inside is a series of fixed dams or partitions that can be made higher or lower, as required, by means of plates. A shaft running the entire length of the trough and turning in babbitted journals carries a number of stirring arms or forks and is given a reciprocating motion by a connecting rod attached to a driving pulley at its center. The coal is fed with water at the upper end of the trough, and by the action of the flowing water and the agitation of the arms, the slate, pyrites, and other impurities settle at the bottom and are caught behind the dams, while the clean coal passes over the dams and out at the lower end of the trough. When the spaces behind the dams are filled, feeding is stopped and the refuse in the dams quickly dumped. This form of washer is particularly successful with coal mixed with fireclay. One washer handles from 75 to 100 T. of coal per day, and one man can attend to six washers. Each washer requires less than 1 H. P. to operate it. The larger the coal, the greater must be the slope and the quantity of water used.

**Jigs.**—This is a general term applied to that class of machines in which the separation of the mineral from the impurities takes place on a screen or bed of material and is effected by pulsating up-and-down currents of a fluid medium.

There are a number of different methods in use for driving the pistons that cause the pulsations of the water in jigs. Some of these use plain eccentrics, giving the same time to both the up and the down strokes of the pistons, while others employ special arrangements of parts, which give a quick down stroke and a slow up stroke, thus allowing the water ample time to work its way back through the bed without any sucking action from the piston. This tends to make a better separation in some cases than the use of the plain eccentrics.

**Stationary Screen Jigs.**—This class is illustrated by Fig. 7, which shows a 3-compartment jig. The separation takes place on screens supported on

wooden frames $g$, and is effected by moving the water in each compartment so that it ascends through the screen, lifting the mineral and allowing it to settle again, thus giving the material an opportunity to arrange itself according to the law of equally falling particles. Each compartment is composed of two separate parts, one containing the screen on the support $g$ and the other adjoining it and arranged so that the piston in it may impart the necessary pulsations to the water. These pistons are equally loose fitting
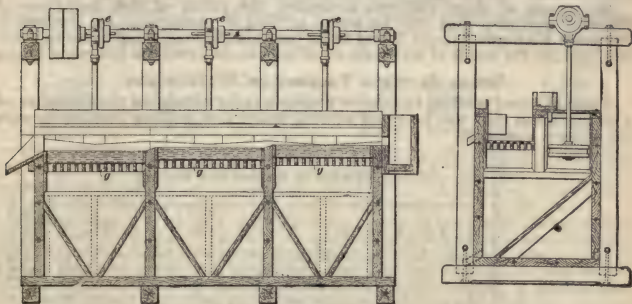


FIG. 7

and are operated by the eccentrics $e$ on the shaft $s$. Jigs should be fed with approximately sized material, when the impurities will accumulate near the bottom on the screen and the coal will be carried over the discharge.

The **Heberle gate,** Fig. 8, acts as follows: $a$ is a U-shaped shield fastened against the inside of the jig and held in place by a band $b$, the ends of which are drawn down into the form of bolts and pass through the sides of the jig, where they are secured with suitable nuts. The shield $a$ may be raised or lowered by loosening the band $b$. The discharge takes place through the opening $f$ in the side of the jig, the size and position of the opening being regulated by slides $c$. The impurities $k$ rest on the screen $e$ supported by a grating $d$, while the coal $i$ occupies a higher position. The shield $a$ prevents the coal from flowing out through the opening $f$, while the impurities flow along the screen and rise to a height somewhat lower than the top of the coal in the jig, when they are discharged through the opening $f$ over the spout $h$, as shown at $p$. The coal is usually discharged over the dam at the end of the jig.
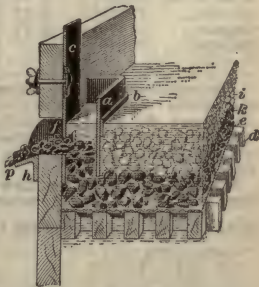


FIG. 8

**Theory of Jigging.**—By far the most exhaustive investigations on the theory of jigging carried on in America are those of Prof. Robert H. Richards, of the Massachusetts Institute of Technology, and the greater part of the following theoretical discussion is based on his several theoretical papers published in the Transactions of the American Institute of Mining Engineers.

Four laws of jigging are given by the several authorities: (1) The law of equal settling particles, under free settling conditions; (2) the law of interstitial currents, or settling under hindered settling conditions; (3) the law of acceleration; (4) the law of suction.

The first of these is the most important, but the others are elements that cannot be disregarded in connection with jigging.

**Equal Settling Particles.**—Rittinger gives the following formulas to represent the relation between diameter of grains and rate of falling in water for irregularly shaped grains:

$$V = 2.73\sqrt{D(\delta-1)}, \text{ for roundish grains;}$$
$$V = 2.44\sqrt{D(\delta-1)}, \text{ for average grains;}$$
$$V = 2.37\sqrt{D(\delta-1)}, \text{ for long grains;}$$
$$V = 1.92\sqrt{D(\delta-1)}, \text{ for flat grains,}$$

in which $V$ = velocity in meters per second; $D$ = diameter of particles in meters, and $\delta$ = specific gravity of the minerals.

By means of these different formulas, the ratios of the diameters of different particles that will be equal settling in water can be computed. Professor Richards has not found these formulas to hold in all cases in practice, and, as the result of elaborate experiments, he gives the following table:

#### EQUAL SETTLING FACTORS OR MULTIPLIERS

Table of equal settling factors or multipliers for obtaining the diameter of a quartz grain that will be equal settling under free settling conditions with the mineral specified.

| | Specific Gravity | Velocity in Inches per Second | | | | | | | | | Rittinger's Multipliers |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
| | | Author's Multipliers | | | | | | | | | |
| Anthracite.. | 1.473 | .500 | .352 | .225 | .213 | | | | | | .288 |
| Epidote..... | 3.380 | 1.57 | 1.35 | 1.05 | 1.13 | 1.50 | 1.61 | 1.56 | 1.56 | 1.47 | 1.45 |
| Sphalerite.. | 4.046 | | 1.46 | 1.05 | 1.17 | 1.62 | 1.64 | 1.68 | 1.66 | 1.56 | 1.85 |
| Pyrrhotite.... | 4.508 | | 1.73 | 1.29 | 1.48 | 2.00 | 2.22 | 2.26 | 2.13 | 2.08 | 2.14 |
| Chalcocite.... | 5.334 | | 1.90 | 1.47 | 1.62 | 2.07 | 2.28 | 2.41 | 2.44 | 2.17 | 2.64 |
| Arsenopyrite | 5.627 | | 1.90 | 1.57 | 1.89 | 2.42 | 2.56 | 2.72 | 2.84 | 2.94 | 2.82 |
| Cassiterite... | 6.261 | | 2.11 | 1.79 | 2.00 | 2.73 | 2.93 | 3.03 | 3.05 | 3.12 | 3.32 |
| Antimony... | 6.706 | | 2.71 | 2.00 | 2.00 | 2.73 | 2.93 | 3.03 | 2.98 | 3.00 | 3.48 |
| Wolframite.. | 6.937 | | 2.71 | 1.83 | 2.07 | 2.86 | 3.04 | 3.21 | 3.28 | 3.26 | 3.64 |
| Galena...... | 7.856 | | 2.71 | 1.83 | 2.26 | 3.00 | 3.42 | 3.65 | 3.76 | 3.75 | 4.01 |
| Copper..... | 8.479 | | 2.71 | 2.00 | 2.36 | 3.00 | 3.20 | 3.58 | 3.76 | 3.75 | 4.56 |
| Quartz........ | 2.640 | | | | | | | | | | |

The significance of the above table is as follows: If a piece of anthracite of a certain size falls in water with a velocity of 4 in. per sec., a piece of quartz .213 times the diameter of the anthracite will fall with the same velocity. If a piece of copper of a certain size falls with a velocity of 7 in. per sec., a piece of quartz 3.58 times as large as the copper will fall with the same velocity.

**Interstitial Currents, or Law of Settling under Hindered Settling Conditions.**—If $d$ equals the diameter of a falling particle, and $D$ that of the tube in which it falls, the larger the fraction $\dfrac{d}{D}$, the greater will be the retardation
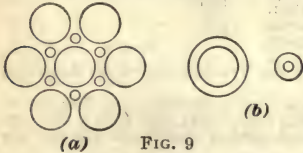


(a)        (b)        FIG. 9

or loss of a velocity by the particle. When this fraction equals 1, the particle stops. If, in Fig. 9 (a), the larger circles represent say particles of quartz and the smaller circles equal settling particles of galena, then if these mixed particles are settling together or are held in suspension by a rising current of water, each particle may be considered to be falling in a tube, the walls of which consist of the surrounding particles. Substituting a circle in each case for the imaginary tube, we have Fig. 9 (b) representing the conditions for galena and quartz, the outer circle in each case representing the imaginary tube. Evidently, $\dfrac{d}{D}$ is much smaller for the galena than for the quartz, and

it will therefore be much less impeded in its fall than the quartz; hence, the particles of galena found adjacent to the particles of quartz will be smaller than the ratio that the law of equal settling particles under free settling conditions would indicate. Application of this principle is found when a mass of grains is subjected to a rising current of sufficient force to rearrange the grains according to their settling power and the grains are said to be treated under hindered settling conditions, as on the bed of a jig.

**Interstitial factors,** or multipliers for obtaining the diameter of the particle of quartz that under hindered settling conditions will be found adjacent to and in equilibrium with the particle of the mineral specified, are the following:

| | | | | | |
|---|---|---|---|---|---|
| Copper...... | 8.598 | Cassiterite... | 4.698 | Pyrrhotite.. | 2.808 |
| Galena...... | 5.842 | Arsenopyrite. | 3.737 | Sphalerite.... | 2.127 |
| Wolframite... | 5.155 | Chalcocite... | 3.115 | Epidote..... | 1.628 |
| Antimony..... | 4.987 | Magnetite... | 2.808 | Anthracite.. | .1782 |

These signify that, after pulsion has done its work on a jig bed, for example, where quartz and anthracite are being jigged, the grains will be so arranged that the grains of quartz are .1782 times the diameter of the grains of anthracite that are adjacent to and in equilibrium with them.

**Acceleration.**—A particle of galena that is equal in settling to the particle of quartz reaches its maximum velocity in perhaps one-tenth the time required by the quartz. The oft-repeated pulsations of a jig, therefore, give the galena particles a decided advantage over the quartz, placing beside the quartz, when equilibrium is reached, a much smaller particle of galena than we should expect according to the law of equal settling particles.

**Suction** acts to draw down through the screen small grains, mainly of the heavier mineral, which are distributed among large grains. It increases as the length of plunger stroke, with the difference in specific gravity of the two minerals, and with the diminishing of the thickness of the bed on the sieve, whether of the heavier mineral only or of both minerals. The law of suction seems to be that jigging is greatly hindered by strong suction where the two minerals are nearly of the same size, the quickest and best work then being done with no suction; but when the two minerals differ much in size of particles, the quartz being the larger, strong suction is not only a great advantage, but may be necessary to get any separation at all. Experiments have indicated an approximate boundary between grains that are helped and those that are hindered by suction; namely, if the diameter of the quartz particles is equal to or greater than 3.52 times the diameter of the other mineral particles, then separation is helped by suction; if less, separation is hindered. This value 3.52 is approximate only, and it will differ with the fracture of the quartz under consideration; if the quartz grains are much flattened, it will have a large value.

**Removal of Sulphur from Coal.**—The object of washing coal is to remove the slate and pyrites, thus reducing the amount of ash and sulphur. Many forms of washers easily and cheaply reduce the slate from 20 % in the coal to 8 % of ash in the coke, but it is much more difficult to reduce 4 % of sulphur in the coal to 1 % or less of sulphur in the coke. Sulphur occurs in the coal in three forms, as hydrogen sulphide, calcium sulphate, and pyrite. The first is volatile and is removed in coking, the second cannot usually be removed by preliminary treatment, and it is the removal of the third form with which washing has to do. The presence of water in the coke ovens apparently assists the removal of the sulphur; but wet coals require a longer time for coking than dry, and, therefore, pyrite should be removed as far as practicable before charging the coal into the coke ovens. The pyrite in coal as it comes from the mine seems to be in particles even finer than those of the coal dust. This impalpable powder or flour pyrites floats in air or water. This being the case, the common practice of using the water over and over again in a washery cannot give the best results in the removal of sulphur, as some flour pyrites will be carried back each time and remain with the washed coal. Experiments made by Mr. C. C. Upham, of New York City, show that the critical size at which an almost complete division of the coal and pyrites takes place varies with coals from different districts and beds, and in laying out coal-washing plants, the proper fineness of crushing should be determined beforehand by careful experiment.

## PREPARATION OF ANTHRACITE

Under[*] the well-known conditions of the anthracite field, the general methods of preparation may be summarized under three classes: namely, (I) dry preparation; (II) dry and wet preparation; and (III) wet preparation; of which the one to be adopted depends on the quality of coal to be mined.

Class I (Fig. 10) is employed when the seams of coal mined are dry, or are practically free from impurities, or where the benches of slate occurring in the seams cleave free from the coal, and may be removed during hand loading, and the run-of-mine contains generally not over 7 or 8 % of rock or slate, which may be removed by hand-picking or by dry mechanical separators.
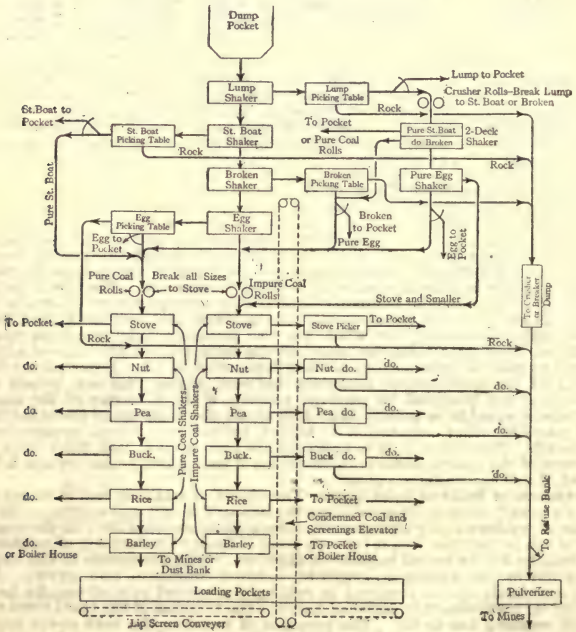


Fig. 10.

Class II (Fig. 11) is employed when the run-of-mine contains a high percentage of impurities, including rock, slate, and bone. This percentage may be as high as 55 %, but the run-of-mine must contain large lumps of pure coal, which can be handled as a separate product, as in the first class. The sizes smaller than lump are sized and cleaned, using water to wash the product, to improve its appearance, and to remove the impurities by jigging.

Class III (Fig. 12) is adopted when the run-of-mine is high in impurities and shows a discoloration, as is the case near the outcrop of the vein, or when the entire product comes from wet, dirty seams, requiring a thorough washing to remove the dirt and discoloration.

Class I presents the ideal breaker, with the advantages of low costs of installation, operation, and maintenance. Moreover, shipments of dry coal

[*] The Preparation of Anthracite, Paul Stirling, E. M. in *Trans.*, Amer. Inst. Ming. Engrs., vol. XLII (1911).

are very desirable to the trade, as they are free from the risk of the freezing of coal in cars, and the subsequent trouble of unloading it.

Class II retains to some extent the advantage of dry coal-shipments, but is higher in first-cost, operation, and maintenance than Class I or III.

Class III permits no dry shipments, and is higher in first-cost, operation, and maintenance than Class I.
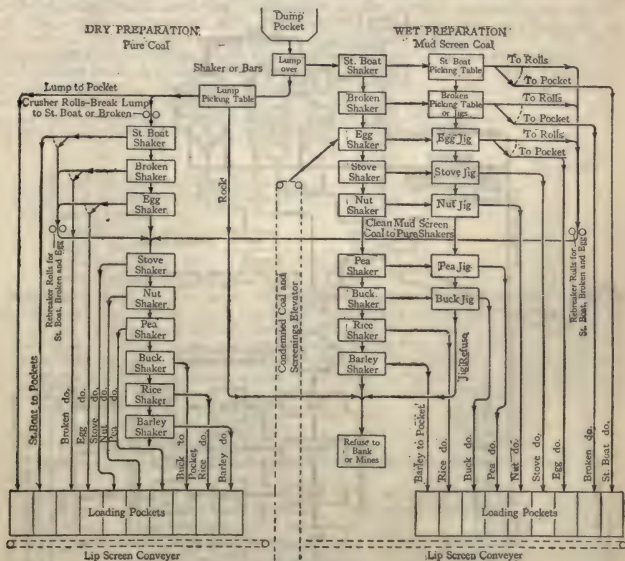


FIG. 11.

## PREPARATION OF BITUMINOUS COAL

**Sizes.**—Bituminous coal is not, as a rule, prepared with as great care as is anthracite. In some regions where the coal is used mainly as a domestic fuel and where competition is strong the coal is carefully sized and in many instances the smaller grades are washed before being sent to market. Unlike anthracite soft coal ignites with ease but has a tendency to spaul off small pieces during the process of burning. It is therefore in most cases unnecessary to carry sizing to as fine a point as with hard coal. The grade of bituminous which brings the highest price is the lump. There appears to be no fixed rule or even well-established custom concerning the sizes into which the run-of-mine is prepared. In general outside of southern Illinois not more than four or five grades are made. These and their approximate sizes are shown on the following table:

| Grade | Through Circular Perforation | Over Circular Perforation |
|---|---|---|
| Slack | ¾ in. | |
| Nut | 1¼ in. | ¾ in. |
| Egg | 4 in. | 1¼ in. |
| Lump | | 4 in. |

Another grade which has made its appearance recently is called cobble and is intermediate between the egg and the lump, passing through a 6-in. to 7-in. circular opening and over (say) a 3½-in. to 4-in. circular opening.

Many other grades such as 1½-in. lump, 2½-in. lump, etc., are prepared in certain fields. These are produced either by passing the coal over bar screens or perforated shaking screens. The coal which passes over being known as the lump and that which passes through as slack.

**Method of Preparation.**—Bituminous coal is usually sized on some type of shaking screen, two general varieties being in use: the inclined shaking



Fig. 12.

screen and the horizontal screen. The horizontal screen has only one representative. This is patented and is known as the Marcus. The motion imparted to the screen by means of a special driving head is a reciprocation back and forth with a non-uniform rapidity, that is, one stroke being made more quickly than the other. The coal is thus caused to travel along the screen and over the perforations unaided by gravity. This type of screen may be readily used as a picking table and possesses some other advantages. The inclined shaking screen which has a uniform rapidity of oscillation in both directions is either hung from above or supported from below. If hung

from above, it may be suspended either by links or connecting rods which pivot upon pins at either end or may be hung by flexible wooden supports rigidly attached at both top and bottom. If supported from below it is usually either placed on rollers or carried by flexible wooden supports similar to those used for suspension. Large screens sizing run-of-mine coal into three or four grades are usually given a stroke of 5 to 6 in., the latter being the more common. The speed usually varies from 90 to 120 R. P. M. of the driving shaft and the inclination of the screen is usually about 15° but this varies somewhat with the district where the coal is produced on account of the means employed in mining and the fracture of the coal. Frequently the screening plate is not made flat but in a series of steps, each one tending to turn over the lumps of coal and dislodge any fines which may be riding thereon. Such screens are known as lip screens and the perforations are frequently slots instead of circular openings. The following table shows the inclination of various fixed screens and other apparatus over which the coal may slide in a tipple:

|  | Inclination, Degrees |
|---|---|
| Fixed lip chute screens for 6-in. egg (slot $\frac{3}{4} \times 1\frac{1}{2}$ in. tapered) | 30 |
| Fixed lip chute screen for 3-in. nut (slot $\frac{1}{2} \times \frac{3}{4}$ in. tapered) | 32 |
| Dump chute | 30 |
| Standard bar screens | 28 |
| Weigh pan | 38 |
| Mine-run chute 40 in. diam. (circular) | 28 |
| Slack hopper and chute | 45 |

**Screening Area.**—The area of a shaking screen should be sufficient to size the coal but not so large that the material is needlessly handled. The area of the various perforated plates does not vary directly with the output of the mine. There is no means of calculating the area necessary but experience has proved that the following sizes of plates are ordinarily well adapted to a tipple handling 1,000 T. of coal per 8-hr. day where this coal is passed over a bar screen or grizzly before reaching the shaking screens:

| | |
|---|---|
| 1$\frac{1}{4}$-in. round perforations | 96 sq. ft. |
| 2$\frac{1}{2}$- to 3-in. perforations | 96 sq. ft. |
| 5- to 6-in. circular perforations | 24 sq. ft. |

If no bar screen is employed the areas above given should be increased about 30%.

Any screen designed to prepare 2,500 T. or more of run-in-mine run coal in 8 hr. should be equipped with a feeder. The areas of the screens of a 2,500-T. mine would be as follows:

| | |
|---|---|
| 1$\frac{1}{4}$-in. circular perforations | 160 sq. ft. |
| 2$\frac{1}{2}$- to 3-in. circular perforations | 160 sq. ft. |
| 5- to 6-in. circular perforations | 40 sq. ft. |

The areas of a screen plate for a capacity of 4,000 T. in 8 hr. would be as follows:

| | |
|---|---|
| 1$\frac{1}{4}$-in. circular perforations | 190 sq. ft. |
| 2$\frac{1}{2}$- to 3-in. circular perforations | 190 sq. ft. |
| 5- to 6-in. circular perforations | 40 sq. ft. |

**Shaker Screens for Small Sizes.**—It is sometimes advantageous when handling large capacities to make a double separation, that is, screen out of the run-of-mine the egg coal and smaller in one operation and then size this material into the egg, nut, slack and other grades in a secondary process. The chief difference in the design of shaker screens for handling run-of-mine and those making smaller sizes is in the pitch and speed of the screen. Small coal screens need not be given a pitch in excess of 1$\frac{1}{2}$ in. in a foot. The speed of such a screen should be about 120 R. P. M. of the driving shaft and the throw of the eccentric or crank should be about 3 in. Such screens are frequently used for preparing material which will pass through a 2-in. circular opening; that which passes over this opening being known as 2-in. lump. In such a case the following grades are made:

| Name of Grade | Through Circular Perforation | Over Circular Perforation |
|---|---|---|
| No. 2 nut | 2 in. | 1$\frac{1}{4}$ in. |
| No. 3 nut | 1$\frac{1}{4}$ in. | $\frac{3}{4}$ in. |
| No. 4 pea | $\frac{3}{4}$ in. | $\frac{1}{16}$ in. |
| Duff | $\frac{1}{16}$ in. | |

**Screen Feeders.**—Two types of screen feeders are in general use, the reciprocating and the continuous feed. The reciprocating feeder always leaves a cushion of coal in the hopper for the succeeding dump to strike. With this type of feeder, however, coal from two or three dumps is mixed in the hopper and it is often impossible to dock for impurities. The reciprocating pan of such a feeder oscillates longitudinally at a speed about 60 R. P. M. of the driving shaft while the stroke is usually about 8 in. This type of feeder subjects the coal to a more or less pronounced grinding action, frequently causing unnecessary degradation.

The continuous feeder is a heavy steel-plate apron, the plates being usually beaded. It is provided with a dump chute and hopper at one end receiving the coal from the weigh pan while the sides are from 2 to 4 ft. high to prevent spillage. With this type of feeder it is possible to thoroughly inspect each carload of coal as it leaves the weigh pan and docking is made easy without delaying hoisting.

**Tipple Design.**—The efficiency, capacity and cost of sizing coal depends in large measure upon the design of the tipple and sizing apparatus. As reliability is of greater importance than extreme efficiency, simplicity should always be sought in the design of a bituminous preparation plant. The arrangement of such plants differs somewhat in different localities and with the wage agreement with the miners' organization if any exists. Taking the most complicated case of a shaft mine, the process and equipment is somewhat as follows:

The coal is delivered by self-dumping cages to bar screens if the mine is operated on a lump basis or to the weigh pan if operated on a run-of-mine basis. In the former instance the bar screens discharge to the weigh pan and in the latter instance the weigh pan discharges to the bar screens. If these screens discharge to the weigh pan the screenings pass through and are delivered by chute to the shaking screens thus preceding the lump coal. This is the better process so far as the shakers are concerned as a less screening area will be required. After passing the shaking screens the lump, cobble if any, egg and frequently nut coal are picked on separate picking tables. The egg and lump are, however, frequently picked together before final separation is made. The larger sizes are delivered to railroad cars on separate tracks either by telescoping chutes, side shaking chutes or hinged loading booms of either the apron or belt conveyor type, frequently the loading boom, or rather the horizontal portion of it, is utilized as a picking table. Side shaking chutes for the delivery of the larger sizes often contain degradation screens which deliver to a conveyor, which in turn discharges the coal to the head of the sizing screen or to the slack car.

**Washing Bituminous Coal.**—In many localities bituminous coal, particularly the smaller sizes, is washed in order to remove the sulphur, fire clay, slate and other detrimental material. While differing somewhat in detail, the process as well as the machinery employed is similar to that employed with anthracite.

# HANDLING OF MATERIAL

**Anthracite Coal.**—The following may be taken as average figures for the angle or grade of chutes for anthracite coal, to be used where the chutes are lined with sheet steel: For broken or egg coal, $2\frac{1}{2}$ in. per ft.; for stove or chestnut coal, $3\frac{1}{2}$ in. per ft.; for pea coal, $4\frac{1}{2}$ in. per ft.; for buckwheat coal, 6 in. per ft.; for rice coal, 7 in. per ft.; for culm, 8 in. per ft.

If the coal is to start on the chute, 1 in. per ft. should be added to each of the above figures; while if the chutes are lined with manganese bronze in place of steel, the above figures can be reduced 1 in. per ft. for coal in motion, or would remain as stated to start the coal. When the run-of-mine is to be handled, as in the main chute, at the head of the breaker, the angle should be not less than 5 in. per ft., or practically $22\frac{1}{2}°$ from the horizontal. If chutes for hard coal are lined with glass, the angle can be reduced from 30% to 50%, depending somewhat on the nature of the coal. In all cases, the flatter the coal, the steeper the angle must be, on account of the large friction surfaces exposed, compared with the weight of the piece. If chutes are lined with cast iron, the angle should be about the same as that employed for steel, though sometimes a slightly greater angle is allowed.

The following tables are printed through the courtesy of the Link-Belt Engineering Co., Philadelphia, Pa.;

### WEIGHTS AND CAPACITIES OF STANDARD STEEL BUCKETS

| Chain | Size of Bucket, In. | Weight of Bucket, Lb. | Capacity of Bucket, Lb. | Capacity of Elevator. 100 Ft. per Min. | | Number of Drawing |
|---|---|---|---|---|---|---|
| | | | | Lb. per Min. | Net Tons per Hr. | |
| ⅝ In. Dodge | 12× 9×11¾ | 18½ | 11 | 1,100 | 33.0 | 5,357 |
| | 14× 9×11¾ | 22½ | 12½ | 1,250 | 37.5 | 5,357 |
| | 18× 9×11¾ | 27 | 16½ | 1,650 | 49.5 | 5,357 |
| | 24× 9×11¾ | 36 | 22 | 2,200 | 66.0 | 5,357 |
| ¾ In. Dodge | 12×10×16¼ | 20 | 19 | 1,380 | 41.4 | 5,357 |
| | 18×10×16¼ | 29 | 28½ | 2,072 | 62.2 | 5,357 |
| | 24×10×16¼ | 38 | 38 | 2,760 | 82.8 | 5,357 |
| | 30×10×16¼ | 46½ | 47½ | 3,450 | 103.5 | 5,357 |
| | 18×12×16¼ | 31 | 33 | 2,400 | 72.0 | 5,357 |
| | 24×12×16¼ | 40 | 44 | 3,200 | 96.0 | 5,357 |
| | 30×12×16¼ | 48 | 55 | 4,000 | 120.0 | 5,357 |

Buckets taken ¾ full. Buckets continuous. 1 lb. of coal = 34 cu. in.

### ELEVATING CAPACITIES OF MALLEABLE IRON BUCKETS
Table gives tons (2,000 lb.) of pea coal per hour at 100 ft. per min.

| Buckets | | Capacities | | Distance between Buckets in In. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Size, In. | Wt., Lb. | Cu. In. | Lb. | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 2¾× 4 | 0.75 | 15 | 0.48 | 2.16 | 1.73 | 1.44 | 1.23 | 1.08 | | | | |
| 3½× 5 | 1.50 | 31 | 0.97 | 4.36 | 3.49 | 2.91 | 2.49 | 2.18 | 1.94 | | | |
| 4 × 6 | 2.00 | 51 | 1.57 | 7.06 | 5.65 | 4.71 | 4.04 | 3.53 | 3.14 | 2.83 | | |
| 4½× 7 | 2.56 | 75 | 2.33 | 10.38 | 8.39 | 6.99 | 5.99 | 5.19 | 4.66 | 4.19 | 3.81 | |
| 5 × 8 | 3.56 | 102 | 3.15 | ..... | 11.34 | 9.45 | 8.10 | 7.09 | 6.30 | 5.67 | 5.15 | 4.72 |
| 6 ×10 | 5.47 | 185 | 5.73 | ..... | ..... | 17.19 | 14.73 | 12.88 | 11.46 | 10.31 | 9.38 | 8.59 |
| 7 ×12 | 8.97 | 287 | 8.90 | ..... | ..... | ..... | 22.88 | 20.02 | 17.80 | 16.02 | 14.56 | 13.35 |
| 7 ×14 | 11.41 | 295 | 9.14 | ..... | ..... | ..... | ..... | 20.56 | 18.28 | 16.45 | 14.95 | 13.71 |
| 10 ×18 | | | | | | | | | | | | |

Weight of 1 cu. ft. of pea coal = 53.5 lb.  32.3 cu. in., or .0187 cu. ft. = 1 lb.

### CONVEYING CAPACITIES OF FLIGHTS AT 100 FT. PER MIN.
(Tons of Pea Coal per Hour)

| Size of Flight, In. | Horizontal | | | | Inclined | | |
|---|---|---|---|---|---|---|---|
| | | | | | 10° | 20° | 30° |
| | Every 16 In. | Every 18 In. | Every 24 In. | Lb. Coal per Flight | Every 24 In. | Every 24 In. | Every 24 In. |
| 4×10 | 33.75 | 30 | 22.5 | 15 | 18.0 | 14.25 | 10.5 |
| 4×12 | 42.75 | 38 | 28.5 | 19 | 24.0 | 18.00 | 13.5 |
| 5×12 | 51.75 | 46 | 34.5 | 23 | 28.5 | 22.50 | 16.5 |
| 5×15 | 69.75 | 62 | 46.5 | 31 | 40.5 | 31.50 | 22.5 |
| 6×18 | ..... | 80 | 60.0 | 40 | 49.5 | 40.50 | 31.5 |
| 8×18 | ..... | 120 | 90.0 | 60 | 72.0 | 57.00 | 48.0 |
| 8×20 | ..... | ..... | 105.0 | 70 | 84.0 | 66.50 | 56.0 |
| 8×24 | ..... | ..... | 135.0 | 90 | 120.0 | 96.00 | 72.0 |
| 10×24 | ..... | ..... | 172.5 | 115 | 150.0 | 120.00 | 90.0 |

NOTE.—These ratings are for continuous feed.  2,000 lb. = 1 T.

## HORSEPOWER FOR BUCKET ELEVATORS

$$H.P. = N \times \frac{H\omega}{d}.$$

$N$ = number taken from table;
$H$ = height of elevator in feet;
$\omega$ = weight of material in one bucket;
$d$ = distance apart of buckets, in inches.

| Revolutions per Minute | Diameter of Head-wheels | | | | | | Revolutions per Minute |
|---|---|---|---|---|---|---|---|
| | 22 In. | 24 In. | 26 In. | 28 In. | 30 In. | 32 In. | |
| 10 | .064 | .070 | .075 | .080 | .087 | .093 | 10 |
| 12 | .077 | .083 | .090 | .097 | .104 | .111 | 12 |
| 14 | .089 | .096 | .106 | .114 | .121 | .130 | 14 |
| 16 | .102 | .111 | .121 | .130 | .140 | .148 | 16 |
| 18 | .115 | .125 | .136 | .146 | .157 | .167 | 18 |
| 20 | .128 | .139 | .151 | .162 | .174 | .186 | 20 |
| 22 | .140 | .153 | .166 | .179 | .191 | .204 | 22 |
| 24 | .153 | .167 | .181 | .195 | .209 | .223 | 24 |
| 26 | .166 | .181 | .196 | .211 | .226 | .242 | 26 |
| 28 | .179 | .195 | .211 | .227 | .244 | .260 | 28 |
| 30 | .191 | .209 | .226 | .244 | .261 | .279 | 30 |
| 32 | .204 | .223 | .241 | .260 | .278 | .297 | 32 |
| 34 | .217 | .237 | .256 | .276 | .296 | .316 | 34 |
| 36 | .230 | .251 | .271 | .292 | .313 | .334 | 36 |
| 38 | .242 | .265 | .287 | .309 | .331 | .353 | 38 |
| 40 | .255 | .279 | .302 | .325 | .348 | .372 | 40 |

## PITCH AT WHICH ANTHRACITE COAL WILL RUN, IN INCHES PER FOOT

| Kind of Coal | Sheet Iron | | Cast Iron | Glass | | Glass | |
|---|---|---|---|---|---|---|---|
| | Start on | Continue on | Start on | Start on | Continue on | Start on | Continue on |
| | Dry | | | | | Wet | |
| Broken slate.......... | 5¼ | 4⅜ | 5⅝ | 3¼ | 3 | | |
| Dry egg slate.......... | 5¼ | 4⅜ | 5¼ | 3¼ | 3 | | |
| Dry stove slate........ | 5¼ | 4⅜ | 5¼ | 3¼ | 3 | | |
| Dry chestnut slate..... | 5 | 4⅜ | 5¼ | 3¼ | 3 | | |
| Broken coal........... | | | | 2⅝ | 2¼ | | |
| Egg coal.............. | 3⅜ | 3 | 3⅜ | 2⅜ | 2¼ | 2¼ | 1¾ |
| Stove coal............ | 4¼ | 4¼ | 4¼ | 3 | 2⅜ | 2¼ | 2¼ |
| Chestnut coal......... | 4¼ | 4¼ | 4¼ | 3 | 2½ | 3 | 2½ |
| Pea coal.............. | 5½ | 5 | 5¼ | 3¼ | 2½ | 2¼ | 2 1/16 |
| Buckwheat No. 1...... | | | | 3⅜ | 3¼ | 3¼ | 3¼ |
| Buckwheat No. 2...... | | | | 3½ | 3¼ | 3¼ | 3¼ |
| Buckwheat No. 3...... | | | | 4½ | 3¾ | 7⅞ | 4¼ |
| Buckwheat No. 4...... | | | | 4⅝ | 4¼ | 4⅝ | 4¼ |

HORSEPOWERS FOR COAL CONVEYORS (COAL INCLUDED)
Speed, 100 ft. per min. Conveyors, 100 ft. long. Standard steel troughs.

| Size of Chain | Size of Flights, In. | Horizontal | | Inclined | Size of Chain | Size of Flights, In. | Horizontal | | Inclined |
|---|---|---|---|---|---|---|---|---|---|
| | | 12 In. between Flights | 18 In. between Flights | 12 In. between Flights | | | 16 In. between Flights | 24 In. between Flights | 16 In. between Flights |
| $\frac{1}{4}$ in. or $\frac{3}{8}$ in. Dodge | $4 \times 10$ | $2\frac{1}{2}$ | 2 | 3 | $\frac{3}{4}$ in. Dodge | $5 \times 15$ | 4 | $3\frac{1}{2}$ | $4\frac{1}{2}$ |
| | $4 \times 12$ | 3 | $2\frac{1}{2}$ | $3\frac{1}{2}$ | | $6 \times 18$ | 5 | 4 | $5\frac{1}{2}$ |
| | $5 \times 12$ | $3\frac{1}{2}$ | 3 | 4 | | $8 \times 18$ | 7 | 5 | 8 |
| | $5 \times 15$ | $4\frac{1}{2}$ | $3\frac{1}{2}$ | $5\frac{1}{2}$ | | $8 \times 20$ | 8 | 6 | 10 |
| | $6 \times 18$ | $5\frac{1}{2}$ | $4\frac{1}{2}$ | $6\frac{1}{2}$ | | $8 \times 24$ | $9\frac{1}{2}$ | 7 | $11\frac{1}{2}$ |
| | | | | | | $10 \times 24$ | $12\frac{1}{2}$ | 8 | 14 |

HORIZONTAL PRESSURE EXERTED BY BITUMINOUS COAL AGAINST VERTICAL
RETAINING WALLS PER FOOT OF LENGTH



Surface horizontal $\begin{cases} \text{total pressure} & = 6.37d^2 \\ \text{pressure lowest, ft.} & = 6.37(2d-1) \end{cases}$

Surface sloping $\begin{cases} \text{total pressure} & = 10d^2 \\ \text{pressure lowest, ft.} & = 10(2d-1) \end{cases}$

Angle of repose = $35°$

$d$ = height of wall in feet or $ab$

BITUMINOUS

| Depth $ba$ in Ft. | Horizontal Surface, $bm$ | | Sloping Surface, $be$ | | Depth $ba$ in Ft. | Horizontal Surface, $bm$ | | Sloping Surface, $be$ | |
|---|---|---|---|---|---|---|---|---|---|
| | Total Pressure | Pressure Lowest, Ft. | Total Pressure | Pressure Lowest, Ft. | | Total Pressure | Pressure Lowest, Ft. | Total Pressure | Pressure Lowest, Ft. |
| 1 | 6.4 | 6.4 | 10 | 10 | 26 | 4,305 | 325 | 6,760 | 510 |
| 2 | 25.0 | 19.0 | 40 | 30 | 27 | 4,641 | 338 | 7,290 | 530 |
| 3 | 57.0 | 32.0 | 90 | 50 | 28 | 4,993 | 350 | 7,840 | 550 |
| 4 | 102.0 | 45.0 | 160 | 70 | 29 | 5,358 | 363 | 8,410 | 570 |
| 5 | 159.0 | 57.0 | 250 | 90 | 30 | 5,733 | 376 | 9,000 | 590 |
| 6 | 229.0 | 70.0 | 360 | 110 | 31 | 6,122 | 389 | 9,610 | 610 |
| 7 | 312.0 | 83.0 | 490 | 130 | 32 | 6,523 | 401 | 10,240 | 630 |
| 8 | 407.0 | 96.0 | 640 | 150 | 33 | 6,935 | 414 | 10,890 | 650 |
| 9 | 516.0 | 108.0 | 810 | 170 | 34 | 7,362 | 427 | 11,560 | 670 |
| 10 | 637.0 | 121.0 | 1,000 | 190 | 35 | 7,778 | 440 | 12,250 | 690 |
| 11 | 770.0 | 134.0 | 1,210 | 210 | 36 | 8,253 | 452 | 12,960 | 710 |
| 12 | 917.0 | 146.0 | 1,440 | 230 | 37 | 8,754 | 465 | 13,690 | 730 |
| 13 | 1,076.0 | 159.0 | 1,690 | 250 | 38 | 9,193 | 478 | 14,440 | 750 |
| 14 | 1,248.0 | 172.0 | 1,960 | 270 | 39 | 9,682 | 490 | 15,210 | 770 |
| 15 | 1,433.0 | 185.0 | 2,250 | 290 | 40 | 10,192 | 503 | 16,000 | 790 |
| 16 | 1,630.0 | 197.0 | 2,560 | 310 | 41 | 10,669 | 516 | 16,810 | 810 |
| 17 | 1,840.0 | 210.0 | 2,890 | 330 | 42 | 11,236 | 529 | 17,640 | 830 |
| 18 | 2,063.0 | 223.0 | 3,240 | 350 | 43 | 11,797 | 541 | 18,490 | 850 |
| 19 | 2,298.0 | 236.0 | 3,610 | 370 | 44 | 12,331 | 554 | 19,360 | 870 |
| 20 | 2,548.0 | 248.0 | 4,000 | 390 | 45 | 12,968 | 567 | 20,250 | 890 |
| 21 | 2,809.0 | 261.0 | 4,410 | 410 | 46 | 13,478 | 580 | 21,160 | 910 |
| 22 | 3,083.0 | 274.0 | 4,840 | 430 | 47 | 14,100 | 592 | 22,090 | 930 |
| 23 | 3,369.0 | 287.0 | 5,290 | 450 | 48 | 14,679 | 605 | 23,040 | 950 |
| 24 | 3,669.0 | 299.0 | 5,760 | 470 | 49 | 15,275 | 618 | 24,010 | 970 |
| 25 | 3,981.0 | 312.0 | 6,250 | 490 | 50 | 15,925 | 631 | 25,000 | 990 |

Weight of coal = 47 lb. per cu. ft.

## HORIZONTAL PRESSURE EXERTED BY ANTHRACITE COAL AGAINST VERTICAL RETAINING WALLS PER FOOT OF LENGTH



Surface horizontal $\begin{cases} \text{total pressure} &= 9.78d^2 \\ \text{pressure lowest, ft.} &= 9.78(2d-1) \end{cases}$

Surface sloping $\begin{cases} \text{total pressure} &= 14.22d^2 \\ \text{pressure lowest, ft.} &= 14.22(2d-1) \end{cases}$

Angle of repose $= 27°$

$d$ = height of wall in feet.

### ANTHRACITE

| Depth $ba$ in Ft. | Horizontal Surface, $bm$ Total Pressure | Pressure Lowest, Ft. | Sloping Surface, $be$ Total Pressure | Pressure Lowest, Ft. | Depth $ba$ in Ft. | Horizontal Surface, $bm$ Total Pressure | Pressure Lowest, Ft. | Sloping Surface, $be$ Total Pressure | Pressure Lowest, Ft. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 9.78 | 9.78 | 14.22 | 14.22 | 26 | 6,611.1 | 498.78 | 9,612.8 | 725.21 |
| 2 | 39.12 | 29.34 | 56.88 | 42.66 | 27 | 7,129.5 | 518.35 | 10,366.0 | 753.67 |
| 3 | 88.02 | 48.90 | 127.98 | 71.10 | 28 | 7,667.6 | 537.90 | 11,149.0 | 782.10 |
| 4 | 156.48 | 68.46 | 227.52 | 99.54 | 29 | 8,225.0 | 557.46 | 11,988.0 | 810.54 |
| 5 | 244.50 | 88.02 | 355.50 | 127.98 | 30 | 8,802.0 | 577.01 | 12,797.0 | 839.00 |
| 6 | 352.08 | 107.58 | 511.92 | 156.42 | 31 | 9,398.5 | 596.59 | 13,655.0 | 867.41 |
| 7 | 479.22 | 127.14 | 696.78 | 184.86 | 32 | 10,015.0 | 616.14 | 14,561.0 | 895.86 |
| 8 | 625.92 | 146.70 | 910.08 | 213.30 | 33 | 10,650.0 | 635.70 | 15,486.0 | 924.30 |
| 9 | 792.18 | 166.26 | 1,151.82 | 241.74 | 34 | 11,306.0 | 655.26 | 16,439.0 | 952.70 |
| 10 | 978.00 | 185.82 | 1,422.00 | 270.18 | 35 | 11,980.0 | 674.81 | 17,420.0 | 981.19 |
| 11 | 1,183.38 | 205.38 | 1,720.62 | 298.62 | 36 | 12,675.0 | 694.39 | 18,429.0 | 1,009.60 |
| 12 | 1,408.32 | 224.94 | 2,047.68 | 327.06 | 37 | 13,389.0 | 713.94 | 19,467.0 | 1,038.10 |
| 13 | 1,652.82 | 244.50 | 2,403.18 | 355.50 | 38 | 14,123.0 | 733.50 | 20,533.0 | 1,066.50 |
| 14 | 1,916.88 | 264.06 | 2,787.12 | 383.94 | 39 | 14,875.0 | 753.07 | 21,629.0 | 1,095.00 |
| 15 | 2,200.50 | 283.62 | 3,199.50 | 412.38 | 40 | 15,648.0 | 772.63 | 22,752.0 | 1,123.40 |
| 16 | 2,503.68 | 303.18 | 3,640.32 | 440.82 | 41 | 16,440.0 | 792.20 | 23,904.0 | 1,151.80 |
| 17 | 2,826.42 | 322.74 | 4,109.56 | 469.26 | 42 | 17,252.0 | 811.74 | 25,084.0 | 1,180.30 |
| 18 | 3,168.72 | 342.30 | 4,607.28 | 497.70 | 43 | 18,083.0 | 830.73 | 26,293.0 | 1,208.70 |
| 19 | 3,530.58 | 361.86 | 5,133.42 | 526.14 | 44 | 18,934.0 | 850.86 | 27,530.0 | 1,237.20 |
| 20 | 3,912.00 | 381.42 | 5,688.00 | 554.58 | 45 | 19,804.0 | 870.41 | 28,793.0 | 1,265.60 |
| 21 | 4,313.00 | 400.98 | 6,271.00 | 583.26 | 46 | 20,695.0 | 889.99 | 30,090.0 | 1,294.00 |
| 22 | 4,733.50 | 420.54 | 6,882.50 | 611.46 | 47 | 21,605.0 | 909.54 | 31,412.0 | 1,322.30 |
| 23 | 5,173.70 | 440.10 | 7,522.50 | 639.90 | 48 | 22,533.0 | 929.10 | 32,763.0 | 1,350.90 |
| 24 | 5,633.30 | 459.67 | 8,190.70 | 668.35 | 49 | 23,482.0 | 948.66 | 34,143.0 | 1,379.40 |
| 25 | 6,112.60 | 479.22 | 8,887.50 | 696.79 | 50 | 24,450.0 | 968.21 | 35,550.0 | 1,407.90 |

## COST OF UNLOADING COAL

Coal is generally unloaded from railroad cars into the hold of a vessel by some form of unloader, which usually raises the car bodily and dumps it directly into the hold of the vessel. In this way the cost of unloading has been reduced to a very small figure, and the speed of unloading greatly increased. The cost of unloading is given by the makers of the Brownhoist as varying from 2½ c. per T. up to 4¼ c. per T.; deducting in each case 2 c. for

trimming the coal in the vessel, the actual cost of loading varies from $\frac{1}{2}$ c. to $2\frac{1}{2}$ c. per T., depending on the conditions. Along the Lakes it is customary to pay a premium of $\frac{1}{8}$ c. per T. to all connected with the loading, for all coal loaded in excess of 2,500 T. per day and 1,800 T. per night. The Brownhoist has a guaranteed capacity of at least 300 T. per hr., but this has been greatly exceeded in practice. The McMyler end dump has a record of 4.65 T. per min., and the McMyler side dump of 8.41 T. per min. These figures apply to the lake cities of the U. S.

The C. W. Hunt Co., West New Brighton, N. Y., gives the following figures for handling coal along the Atlantic seaboard: The cost of shoveling coal by hand in the hold of the vessel into ordinary iron buckets is about 6 to 7 c. per T. of 2,000 lb.; the cost for iron ore, phosphate rock, or sand, about 10 % less. The cost of shoveling coal and hoisting it out of vessel to the wharf with an ordinary hoist with manila rope is 12 to 13 c. per T., so that the hoisting costs about the same as the shoveling. The cost for both shoveling and hoisting with a steam engine is 10 to 11 c. per T. The cost when using a steam shovel or grab bucket for taking up coal out of the vessel varies greatly in different classes of vessels, but usually runs from about $1\frac{1}{2}$ to 5 c. per T., averaging about 3 c. After the coal is hoisted, it can be carried into storage with an automatic railway or other efficient plant, at a cost of about 1 to $1\frac{1}{2}$ c. per T. For great distances, a cable railway or a conveyor can be used, which handles the material about as cheaply as for short distances, but the cost of plant is greatly increased.

In unloading anthracite from cars on a trestle into pockets or on the ground, the loss on all sizes is 2 to 3 % when the coal is not resized; when it is resized the loss is $8\frac{1}{2}$ to 9 %.

The cost of stocking and unloading anthracite by the Dodge system is given by Mr. Piez, as follows:

| Year | Engine Service, Stocking and Lifting, per Ton. Cents. | Office Expense, per Ton. Cents. | Steam, Wages and Fuel, per Ton. Cents. | Labor, Dumping and Lifting, per Ton. Cents. | Repairs, per Ton. Cents. | Supplies, per Ton. Cents. | Total, per Ton. Cents. |
|---|---|---|---|---|---|---|---|
| 1895 | .87 | .29 | .97 | 2.67 | .78 | .25 | 5.83 |
| 1896 | .78 | .30 | .82 | 2.19 | .90 | .27 | 5.26 |
| 1897 | .69 | .32 | .62 | 1.88 | .97 | .16 | 4.64 |

## BRIQUETING

**Machines Employed.**—Fuel, fuel dust, and other products may be briqueted by a number of different styles of machines, but all these may be divided into two classes, briquet and eggette machines. The eggette machines have a pair of rollers, the faces of which are provided with semispherical or semiovoid depressions. The material that is fed between these rolls crowds into the openings of the two rolls, thus forming small nodules. The material is mixed with a suitable binder before being fed to the rolls, and the eggettes are received on any suitable form of traveling belt or chute and removed for drying or storage. This style of machine has not been used to any great extent in this country. The briqueting machines all act more or less on the principle of the brick machine, having some kind of a die or mold into which the material is crowded. The material is either pressed as it is being fed into the mold or subsequently by some form of plunger. For some materials, common brick machines, such as are used in the manufacture of building brick, are employed, while in others special forms are necessary.

**Briqueting of Fuel.**—Fuel briquets have not come into general use in the United States for two reasons: (1) on account of the great amount of cheap fuel available, which has prevented the utilization of culm, coal dust, etc.; and (2) on account of the lack of or high price of suitable bonding material. This latter condition is now being removed by the introduction of byproduct coke ovens, from which supplies of coal tar can be obtained. Aside from peat and certain kinds of brown coal, and possibly some caking coals, it is neces-

sary to employ a binder in the making of any fuel briquets. This is especially true in the case of anthracite coal. The present tendency is to employ no inorganic bonding materials, as they increase the ash. The material to be briqueted should be as clean and free from dirt or slate as possible, and the particles should be of practically uniform size, the most satisfactory product being from coal crushed to about ¼ in. cube size. The coal must be thoroughly mixed with bonding material and then subjected to a heavy pressure. One advantage claimed for briquets is that they can be made of such a form as to occupy less space than the original fuel. The French navy has found it possible to store 10 % more briquets than coal in a given space, and also that the loss by breakage and pulverization is very much less. Under favorable conditions, fuel can be briqueted for 20 c. per T., and the following are some of the advantages claimed for these briquets: They are sound throughout and will not decrepitate while burning, thus reducing the loss by fine material working through the grates. The binder if properly selected, renders the briquets practically waterproof, so that they are not injured if kept in storage, do not evolve combustible gases, nor ignite from spontaneous combustion. There is no fine material mixed with the briquets, and hence a more uniform fire can be maintained with them.

**Briqueting of Flue Dust.**—Flue dust from iron blast furnaces has been successfully briqueted in a number of instances. One firm employs a common brick machine, making bricks 2½ in.×4½ in.×9 in. With this machine, they mix the flue dust with 3 % of lime and 3 % of cement, the lime acting as a flux in the furnace. These machines work with comparatively light pressure. When regular briqueting machines, producing round bricks and employing high pressures are employed, no cement need be used, the flue dust being mixed with 4 % to 6 % of lime. The flue dust is first carefully screened from hard lumps and then mixed warm with milk of lime in a mixer, after which it is put through the press, and the briquets are then placed in drying ovens and subjected to heat from the gases of a boiler or furnace plant, the temperature not to exceed 300°F. For moderate sized briquets, about 6 hours' drying is sufficient. Just before the briquets are quite dry, they are loaded into barrels and taken direct to the blast furnace, with as little handling as possible. The results have been very satisfactorily compared with the ore replaced. The flue dust itself frequently contains 30 % to 40 % metallic iron and more or less carbonaceous matter. It is also stated that at a large furnace plant the cost of making and handling should not exceed $1 per T.

Another firm, figuring on a basis of 130 T. per 24 hr., and using 3 % lime in the solution, gave the following figures:

| | |
|---|---|
| 4 T. lime, $3.00 per T. | $12.00 |
| 2 machine tenders (day and night), 12 hr. at $2.50 | 5.00 |
| 2 laborers (day and night), 12 hr. at $1.75 | 3.50 |
| Oil and waste | 2.00 |
| Wear on machinery | 1.50 |
| Interest on cost of plant | 1.00 $25.00 |

This is less than 20 c. per T. This estimate does not take into consideration the cost of power, which would be about 35 H.P., nor does it take into consideration hauling of material to plant and removing of briquets.

CUBIC FEET OCCUPIED BY 2,000 POUNDS OF VARIOUS COALS
(*Link-Belt Engineering Co., Philadelphia, Pa.*)

| Varieties | Broken | Egg | Stove | Chestnut | Pea |
|---|---|---|---|---|---|
| Lackawanna, anthracite | 37.10 | 36.65 | 34.90 | 34.35 | 37.25 |
| Garfield red ash, anthracite | 37.30 | 36.95 | 36.35 | 36.35 | 37.50 |
| Lykens Valley, anthracite | 37.55 | 37.25 | 37.55 | 37.25 | 38.50 |
| Shamokin, anthracite | 38.05 | 37.70 | 37.25 | 37.25 | 38.50 |
| Plymouth red ash, anthracite | 34.90 | 34.85 | 34.75 | 34.70 | 36.90 |
| Wilkes-Barre, anthracite | 34.95 | 34.35 | 33.75 | 34.00 | 36.90 |
| Lehigh, anthracite | 33.30 | 33.80 | 33.55 | 32.55 | 33.05 |
| Lorberry, anthracite | 34.65 | 34.20 | 33.80 | 33.55 | 35.20 |
| Scranton, anthracite | 35.35 | 35.20 | 34.60 | 33.30 | 34.95 |
| Pittston, anthracite | 35.45 | 34.95 | 34.35 | 33.70 | 35.50 |

| | | | |
|---|---|---|---|
| Cumberland, bituminous | 36.65 | Pocahontas, bituminous | 34.00 |
| Clearfield, bituminous | 33.55 | American cannel, bituminous | 41.50 |
| New River, bituminous | 40.15 | English cannel, bituminous | 42.30 |

# SAFETY AND FIRST AID

The following directions for first aid to the injured were prepared by Dr. George H. Halberstadt of Pottsville, Pa., Surgeon for the Philadelphia & Reading Coal and Iron Co. They cover in a concise manner the principles of the excellent detailed instructions that have resulted in the great efficiency of the P. & R. C. and I. Co.'s first-aid corps.

## RULES FOR FIRST-AID CORPS

1. The work is limited to first aid.
2. In all serious cases summon a doctor. On his arrival, if requested, give him all possible assistance.
3. Keep your presence of mind and do not, by your manner, indicate to the patient the seriousness of the injury.
4. Examine the mouth and remove any foreign substance.
5. In all serious injuries cut off the clothing to properly inspect and dress wounds. For slight injuries begin removal of the clothing from the sound side, and to replace it begin on the injured side.
6. Place the patient in a comfortable position, preferably on his back.
7. When vomiting occurs turn the patient on his side, neck extended and head low, so that the vomited material will not enter the wind pipe.
8. Never attempt to wash or cleanse a wound; do not touch it with the fingers; cover up quickly every wound with sterile dressing and secure this with a clean bandage. This will prevent infection and blood poisoning. The old common practice of applying a chew of tobacco to stop bleeding, and the use of handkerchiefs or pieces torn from clothing in binding up open wounds is a source of serious danger.
9. Do not handle the part of the dressing that will come into contact with the wound.

10. Stop severe hemorrhage from arms and legs with a tourniquet applied above the wound, then apply sterile dressings securely held with bandage. If the dressings control the hemorrhage, loosen tourniquet but leave it in position to be retightened if necessary; always watch for secondary hemorrhage.

A tourniquet may be made of the material contained in a first-aid cabinet or a handkerchief can be used. Lay a firm and even compress or pad over the artery if you can readily locate it. The artery in the thigh runs along the inner side of the muscle in front near the bone, as shown by a dotted line on Fig. 1. A



FIG. 1        FIG. 2

little above the knee it passes to the back of the bone. In case of injuries at or above the knee, apply the compress higher up, on the inner side of the thigh, along the line of the artery, as for instance, at the point *P*. When the leg is injured below apply the compress at the back of the thigh just above the knee at *P*, Fig. 2. The artery in the arm runs down the inner side of the large muscle in front, quite close to the bone as shown in Fig. 3. Lower down it is further forward, towards the bend of the elbow. It is most easily



FIG. 3

compressed a little above the middle at *P*. In all cases apply the tourniquet as close above the wound as possible.

Tie the cloth or the handkerchief around the limb, covering the compress, put a bit of stick between the bandage and the limb and twist it as in Fig. 4. until it is just tight enough to stop the bleeding, then put one end of the stick under the handkerchief as shown in Fig. 5 to prevent untwisting.

11. Check bleeding from all parts of the body by dressings and bandages. Both roller and triangular bandages are used in first-aid work. The roller bandages are made of various widths, and 5 yd. long. Triangular bandages are furnished in first-aid equipments are triangular pieces of cotton cloth, measuring 4 ft. 2 in. on the base, and 2 ft. 9 in. on the sides. Similar bandages can be made by cutting a yard of muslin diagonally. The advantages of triangular

bandages are: they can be folded to make bandages of any desired width, and can also be used as slings. Triangular bandages furnished in first-aid outfits have illustrations printed on them showing their application to different parts of the body.

12. Cover up every injured person with blankets, regardless of the weather. Use hot-water bottles and an ambulance stove, when necessity requires them. When hot-water bottles are used always have clothing or woolen blankets between them and the patient.

13. Do not, under any circumstances, use the so-called antiseptic lotions, salves, balsams, or any material that is likely to infect the wound.



FIG. 4     FIG. 5

14. Splint every injury from toe to hip and from fingers to shoulders. Always support forearm with sling.

15. Notify at once the patient's family or the hospital of the character of the injury and probable time of arrival at destination.

**Shock.**—Shock accompanies all serious accidents. The skin is cold and clammy; the pulse is weak, rapid and often intermittent; the breathing irregular; vomiting and retching are common; eyelids half open; pupils dilated; and there may be any degree of insensibility. Check hemorrhage at once. Cover patient with blankets and surround him with hot-water bottles. The hot-water bottles must not come into contact with the skin, have a layer of blanket or clothing intervene.

When able to swallow give hot black coffee or a half teaspoonful of aromatic spirits of ammonia in a half tumbler of water. Alcoholic stimulants should not be given unless ordered by the doctor. If vomiting occurs, turn patient on his side and see that no vomited material is left in the mouth. When the injury is to the skull give no stimulants.

**Burns and Scalds.**—Remove clothing with knife or scissors. Do not break the blisters. Cover with picric acid gauze; this with a layer of cotton and secure with bandage. A solution of baking soda, one tablespoonful to a quart of boiled water, may be applied on gauze, if picric acid gauze is not at hand.

**Heat Prostration and Heat Exhaustion.**—In heat prostration, the face is flushed, the skin dry and hot, pulse may be rapid or slow, and the heat of the body excessive; and patient may be unconscious. Remove patient to a cool place, loosen the clothing, lessen the heat of the body and head by cold applications.

In heat exhaustion, the face is pale, the skin cold and clammy, breathing shallow, pulse weak and rapid. Loosen the clothing, apply heat to the surface of the body, and when able to swallow, give hot black coffee or tea.

**Convulsions.**—Lay patient on back, loosen clothing, place blanket or coat under his head. Do not attempt to resist the convulsive efforts; stand by and protect him from doing injury to himself. If he is biting his tongue, force a stick between his back teeth and release it.

**Artificial Respiration.**—This is administered for drowning, for electric shocks, suffocation by smoke, illuminating gas or any mine gas.

**The Schaefer Method.**—Lay the subject on his belly, with arms extended as straight forward as possible, as shown in Fig. 6, and with face to one side so that the nose and mouth are free for breathing. Let an assistant draw forward the subject's tongue. If possible, avoid so laying the subject that any burned or injured places are pressed upon. Do not permit bystanders to crowd around and shut off the fresh air.

Kneel straddling the subject's thighs and facing his head; rest the palms of your hands on his loins (on the muscles of the small of the back), with thumbs nearly touching each other (Fig. 6) and with fingers spread over the lowest ribs.

With arms held straight (Fig. 7), swing forward slowly so that the weight of your body is gradually brought to bear upon the subject. This operation, which should take from 2 to 3 sec., must not be violent and internal organs may be injured. The lower part of the chest and also the abdomen are thus compressed, and the air is forced out of the lungs.

Now immediately swing backward so as to remove the pressure, but leave your hands in place, thus returning to the position shown in Fig. 6.
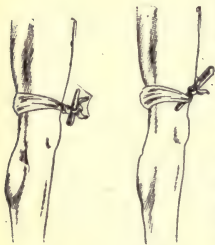
Through their elasticity, the chest walls expand and the lungs are thus supplied with fresh air.

After 2 sec. swing forward again. Thus repeat deliberately 12 to 15 times a minute the double movement of compression and release; a complete respiration in 4 or 5 sec. If a watch or clock is not visible, follow the natural rate of your own deep breathing; swing forward with each expiration, and backward with each inspiration. While this is being done, an assistant should loosen any tight clothing about the subject's neck, chest, or waist.



FIG. 6

Continue artificial respiration, if necessary, 2 hr. or longer without interruption until natural breathing is restored, or until a physician arrives. Even after natural breathing begins carefully watch that it continues. If it stops, start artificial respiration again. During the period of operation, keep the subject warm by applying a proper covering and by laying beside his body bottles or rubber bags filled with warm (not hot) water. The attention to keeping the subject warm should be given by an assistant or assistants.
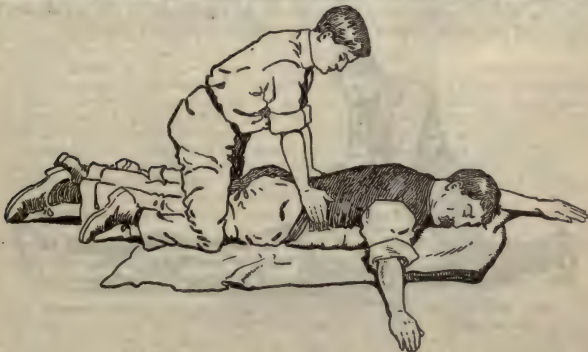


FIG. 7

**The Sylvester Method.**—*Loosen clothing.* Place the patient on his back with folded coat under the shoulders to extend the neck. Kneel at his head. Grasp the forearms just below the elbows with each hand. To expand the chest, draw the arms outwards and upwards to the sides of the head, as in Fig. 8. To expel the air from the chest, bring patient's arms to sides and front of chest as in Fig. 9, and exert firm compression on lower border of ribs. This should be done 15 times a minute. The tongue, when the patient is in this condition, may drop back. The mouth can be pried open with a stick, and the tongue drawn forwards with the fingers covered with a handkerchief.

With either method of artificial respiration the administration of oxygen can be given at the same time. The tube, preferably, is placed in the nose. If the Draeger pulmotor is at hand, the mask can be adjusted to the face and used synchronously with artificial respiration, or the pulmotor may be used alone.

**Treatment for Electrical Shock.**—An accidental electric shock usually does not kill at once, but may only stun the victim and for a while stop his breathing. The shock is not likely to be immediately fatal, because:



FIG. 8

(A) The conductors may make only a brief and imperfect contact with the body.

(B) The skin, unless it is wet, offers high resistance to the current.

Hope of restoring the victim lies in prompt and continued use of artificial respiration. The reasons for this statement are:

(A) The body continuously depends on an exchange of air, as shown by the fact that we must breath in and out about 15 times a minute.



FIG. 9

(B) If the body is not thus repeatedly supplied with air, suffocation occurs.

(C) Persons whose breathing has been stopped by electric shock have been restored after artificial respiration has been continued for approximately 2 hr.

**Rescue from Electrical Contact.**—Break the circuit immediately.

1. With a single quick motion separate the victim from the live conductor. In so doing avoid receiving a shock yourself. Many have, by their carelessness, received injury in trying to disconnect victims of shock from live conductors.

Observe the following precautions:

(*A*) Use a dry coat, a dry rope, a dry stick or board, or any other dry non-conductor to move either the victim or the wire, so as to break the electrical contact. Beware of using metal or any moist material. The victim's loose clothing, if dry, may be used to pull him away; do not touch the sole or heels of his shoes while he remains in contact; the nails are dangerous.

(*B*) If the body must be touched by your hands, be sure to cover them with rubber gloves, mackintosh, rubber sheeting or dry cloth; or stand on a dry board or on some other insulating surface. If possible, use only one hand. If the victim is conducting the current to the ground, and is convulsively clutching the live conductor, it may be easier to shut off the current by lifting him than by leaving him on the ground and trying to break his grasp.

2. Open the nearest switch, if that is the quickest way to break the circuit.

3. If necessary to cut a live wire, use an ax or a hatchet with a dry wooden handle, or properly insulated pliers.

After release from contact get patient's face or hand in contact with ground, so as to discharge any retained current. Summon a doctor at once. Immediately begin artificial respiration by the Schaefer or Sylvester methods.

**Fractures.**—Fractures of bones can be divided into simple and compound. The simple fracture is a break of the bone without an opening to the surface of the skin. A compound fracture is a break of the bone with an opening to the surface of the skin. Fractures of the bones of the arms and legs can usually be detected by the position the limb assumes, and the loss of power and control over the limb. The limb should be straightened gently by pulling on the foot or hand firmly, and fixed in position by padded splints, securely fastened with bandages.

In compound fractures do not attempt to replace a protruding bone. Cover the wound with sterile dressings and then apply splints. Fractures of the bones of the head and body should be fastened with dressings and bandages.

In the absence of regular splints, straight pieces of wood of proper length, well padded can be used in emergencies.

**Dislocations.**—Dress all dislocations of joints in the position they assume with bandages drawn only sufficiently tight to render the transportation of the patient less painful.

**Drowning.**—After removal from the water, elevate patient's hips, shake him gently and press on chest as in Shafer method for about 2 min. to allow the water to run out of chest. The mouth should be cleansed and artificial respiration continued for an hour or more. The patient's body should be warmed by the application of hot-water bottles.

**Foreign Body in Eye.**—Have patient face the light. Examine the eye, when the foreign body is not imbedded, remove it with a clean handkerchief or gauze. Evert the upper lid by taking hold of the lashes, have patient look down and apply pressure with stick to skin of upper lid. The under surface of the upper lid is where most loose foreign bodies are found. Do not try to scrape off imbedded foreign bodies. If a foreign body cannot be removed without scraping wait for a physician. Do not use cocaine.

**Foreign Bodies in Throat.**—Stand patient on his feet and head and a few smart blows on back will usually dislodge foreign body.

**Foreign Body in Nose.**—If it cannot be blown out by patient, summon a doctor.

**Foreign Body in Ear, Insects, Etc.**—A teaspoonful of water or olive oil will drown an insect. Hardened wax better be removed by a doctor.

## METHOD OF MOVING INJURED PERSONS

**One Bearer.**—To lift patient erect: Turn patient on his face. The bearer steps astride body, facing towards head, with hands in arm pits, he lifts the patient to his knees, then clasping hands over abdomen he lifts him to his feet. He then with left hand seizes patient's left wrist (Fig. 10), drawing left arm about his (the bearer's) neck holds it against his left chest, the patient's left side resting against his body and supports him with his right arm about the waist.

**To Place Patient Across the Back.**—The bearer with his left hand seizes the right wrist of the patient (Fig. 11) and draws the arm over his head and down upon his left shoulder, then shifting himself in front, stoops and clasps the right thigh with his right arm placed between the legs, his right hand seizing the patient's right wrist; lastly, he, with the left hand grasps the patient's left hand and steadies it against his side, when he rises.

To place patient across shoulder: The bearer clasps his hands about the

patient's waist, shifts himself to the front, facing him, and stooping places his right shoulder against the abdomen; he passes his right hand and arm between the thighs (Fig 12), securing the right thigh, and with his left grasps



FIG. 10



FIG. 11



FIG. 12



FIG. 13

the patient's right hand, bringing it from behind under his (bearer's) left armpit, when, the wrist being firmly grasped by his right hand he rises. This position leaves the left hand free.

In lowering patient from these positions the motions are reversed. Should a patient be injured in such a manner as to require these motions to be

conducted from his right side, instead of left, as laid down, the change is simply one of hands, the motions proceed as directed, substituting right for left, and vice versa.

**By Two Bearers.**—Bearers face each other on right and left of patient. Raise patient to sitting posture, clasp hands behind patient's back, while the other hands are passed under patient's thighs and fingers interlocked as in Fig. 13. A third bearer can support the legs.

**By Three Bearers.**—Lay patient on back; the bearers kneel on knees nearest the patient's feet. Pass arms under shoulders, hips and knees. Raise patient to bearers' knees; bearers rise and carry patient facing them in elbows. To lower patient to litter, reverse movements.

**Emergency Supplies.**—Accidents very commonly happen where supplies are not available for proper treatment of injuries. In such cases makeshifts may be devised from material at hand. Splints may be made from



FIG. 14

strips of bark removed from a post; by splitting a post or cap; from the staves of nail or spike kegs; from powder kegs cut in strips with an ax, and even thin and long pieces of slate, sprags, etc. Bandages for holding splints in place may be improvised from clothing torn into strips.

A very common form of emergency stretcher is made from two coats as shown in Fig. 14. The sleeves of two coats are turned inside out and the coats placed on the ground with their lower edges touching. A pole or long drill is placed through the sleeves on each side, the coats are buttoned up and the buttoned side turned down. This stretcher is commonly made by one man grasping a drill in either hand, and his companion pulling his coat off over his head. Stretchers may be made from doors, plank nailed together, etc., but the coat-stretcher described above is probably the easiest and quickest to make and the most comfortable for the injured person.

# MINE SAFETY

**Safety First.**—Although the motto of "safety first" has been generally adopted by many American coal mining firms it may or may not have a real significance. Unless this or a similar motto is to be strictly lived up to it is better not adopted. The chief cause of mining accidents is of course falls of roof and coal. These can be guarded against only by vigilance, care and an adequate amount of carefully placed timber.

**Systematic Timbering.**—The roofs of mines differ widely from one field to another and even frequently in different mines of the same field or even in different portions of the same mine. Experience has demonstrated that it is often safer to set an excess of timbers, that is, more than is deemed necessary under the existing conditions than to trust to the judgment of even experienced men. The spacing of such timbering will vary considerably throughout different fields but will in the main bear a considerable resemblance to Fig. 1 which has proved eminently successful in the prevention of accidents in the Pocahontas field of West Virginia. No props or timbers as a general rule should ever be set without cap pieces.

**Adequate Supervision.**—Experience has proved that to attain the greatest measure of safety to the mine workers adequate supervision must be given to each working face. For this purpose one assistant mine foreman (sometimes called face foreman or roof inspector) should be employed for each 20 to 40 men, this number varying somewhat with natural conditions and with the area over which the miners are scattered. The fundamental idea of employing the assistant mine foreman is that he shall be able to visit each working face within the territory intrusted to his care at least twice during each shift. It is his duty whenever he sees a dangerous condition of any kind,

whether from loose coal, loose roof, inadequate timbering or what not, not only to instruct the workmen in that place to rectify the condition at once but to himself remain until the dangerous condition has been rendered absolutely safe.

**The Premium System and Company Rules.**—In order to encourage the foremen and assistant foremen to be constantly on the alert for danger and dangerous practices and to see to it that the laws of the state and the rules of the company are implicitly obeyed, one plan which has been adopted is a premium system arranged on a merit and demerit basis so that the official who has a clean record, that is, one without a serious or fatal accident, is paid a bonus at the end of the month. If this record is clean for six months a special bonus is paid which continues monthly as long as the record is clean. The United States Coal & Coke Co., with works in Mc-Dowell County W. Va., which has adopted this system with marked success, furnishes each foreman and assistant foreman with a book of instructions which he is supposed to carry at all times and which contains the rules of the company which he is expected to see obeyed. Although many of these rules may be considered as being applicable only to the field in question several of the more important ones are here given. Doubtless many others will suggest themselves to wide awake officials operating in other localities.



Fig. 1

The last breakthrough of every pair of headings, whether working or not, must have at least 12,000 cu. ft. of air per min. passing through it.

Assistant foremen must examine each working place in their district, and mark each place visited with date of month before allowing men to enter them. Each idle place must also be examined each day and a record of the examination must be made daily in the book provided for this purpose.

When the fan has been stopped for 1 hr. or more for any reason, all places must be thoroughly examined by assistant foremen before allowing men to enter them and a record of the examination must be made in the record book.

All permanent brattices must be built of incombustible material, concrete preferred.

Air measurements must be made weekly and be recorded in the book provided for that purpose.

Assistant foremen must carry Pieler testing lamps when making examinations, and all final tests for gas must be made with this lamp. When an ordinary safety lamp will not detect gas, try the Pieler lamp.

The presence of dust, as well as gas, must be noted in the record book.

Dust must not be allowed to accumulate in working places nor anywhere in the mine.

All dusty places on haulage roads must be sprinkled daily.

No charge of any explosive in any hole shall exceed 2 lb.

Assistant foremen must not fire any shot that is improperly drilled, drilled on the solid, or improperly tamped.

An assistant foreman must not fire any shot within 3 hr. after detecting gas in any place in his district.

Where more than one shot is to be fired in any place, one shot only shall be charged, tamped and fired, and an examination of the place must be made before firing the second or following shots. Only one shot shall be fired at a time. Assistant foremen must carefully examine working places after shooting before allowing work to begin.



All trolley frogs are to be placed directly above point where inside of curved rail, is 6" from inside of main line rail.
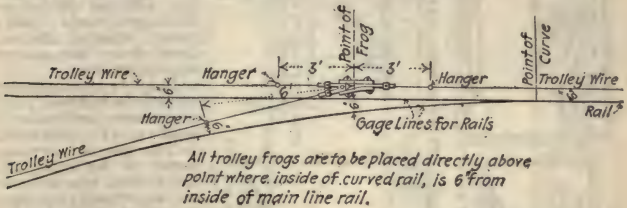
FIG. 2

No explosives except flameless explosives approved by the U. S. Bureau of Mines shall be used.

If necessary, water dust at working face before shooting.

Shots must be fired by battery and not from machine or trolley wire.

All haulage roads on which more than one car is hauled per trip shall be at least 5 ft. high above the rail, and there shall be at least $2\frac{1}{2}$ ft. clearance between any part of a car, and the side of the heading at all places.



300 Ft. Radius, Hangers 6" From Track

250 Ft. Radius, Hangers 6" From Track

150 Ft. Radius, Hangers 6" From Track

100 Ft. Radius, Hangers 6" From Track
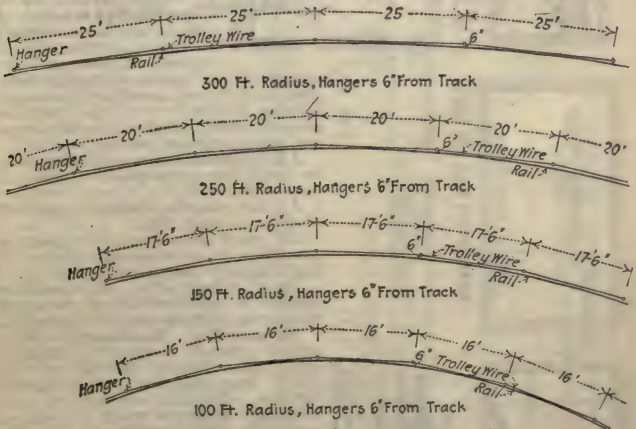
FIG. 3

All rash, slate, etc., is to be kept cleaned off haulage roads.

Permanent track must be bonded as it is laid.

Where any dangerous slate is found, an assistant foreman must take it down at once before work is done near it or anyone or any trip allowed to pass under it.

Loaders must post their working places. Proper arrangement and placing of caps, posts, etc., is shown on standard plan which must be strictly followed. Posts must be set in straight lines and be vertical. Cap pieces must be wedged tightly against roof with wedge between post and cap.

FIG. 4.

TO BE USED WHERE HEADROOM IS BETWEEN 6 AND 7 FT.

4x8 Hardwood Block, Length in direction of heading. Fasten to plugs in roof with drift bolt

Sure Grip Clamp

St 4 fastened to plugs by drift bolts

Trolley Wire

4x8x3 Hardwood Block

NOTE Wood plugs to be driven in roof and hangers or hanger-supports fastened to these by drift bolt

TO BE USED WHERE HEADROOM IS LESS THAN 6 FT. ABOVE RAIL

2" Wooden Plugs

Hanger fastened to wooden plugs by drift bolt

Type 6 Barn hanger

TO BE USED WHERE HEADROOM IS NOT OVER 6 FT.

WHERE ROOF IS OVER 8 FT. HIGH

Type 6 pipe bracket hanger

1½" Pipe

ARRANGEMENT WHERE HEADING USED AS HAULWAY OR HAULAGE JOINS HEADING

Trolley Wire

Boards swung on hinges

Boards swung on hinges

Assistant foremen must not O. K. nor allow any place to be cut where posts are more than 6 ft. from the face at the bottom.

No permanent timbering will be allowed on haulage roads unless under special instructions from the general superintendent.

Loaders must clean up slate falls in their working places, and must be paid extra for this work.

The general arrangement of hangers, etc., for trolley wire is shown on accompanying plans (see Figs. 2, 3, 4 and 5) which must be strictly followed.

Trolley wire must be hung 6 in. outside of rail and must be as nearly parallel to it, both horizontally and vertically as it can be.

Feeder cables must be supported at intervals of 20 ft. on barn hangers and special "Gem" insulators. Cables are to be placed 12 in. outside of trolley lines, and must be connected to them at intervals of 200 ft. Cables must be properly dead ended with cable clamps and insulated turn buckles.

All insulated cables and wires must be kept free from grounds, same as bare wires.

All men must be checked in and out of the mine each day.

All work excepting such repairs as cannot be done while operating or at night must be suspended on Sundays.

No machinery of any kind must be allowed to operate unless all gears and dangerous portions are fully guarded.

**Safeguarding Machinery.** — The second greatest cause of accidents in coal mines results directly or indirectly from machinery, mechanical devices, electrical conductors, etc. Although there are many exceptions, accidents with machinery generally arise from one or more of five causes:

(a) Falls from ladders, platforms, runways, etc., around machines.

(b) Coming into contact with moving machine parts.

(c) Electric shocks.

(d) The failure of a machine part.

(e) Mismanipulation of hand-operated controlling devices (valves, levers, switches, etc.).

The remedy for the class of accidents

under (*a*) is simple and if carefully executed, quite effective. Ladders, stagings, platforms, runways, etc., should be made abundantly strong to carry any weight which may be placed upon them; ample railings should be provided on all platforms, stairways, etc., and non-slipping feet on movable ladders.

Coming into contact with moving elements of machines is a prolific source of mining accident. When buying new machinery it is wise to specify that such machines should be properly safeguarded before delivery. Particularly dangerous are exposed gear trains, revolving bolt heads, set screws, splines, open keyways and the like. Such machine parts should be eliminated so far as possible, e.g., ordinary square-head set screws should be replaced with hollow flush-head set screws. In addition, revolving or moving dangerous machine parts should be thoroughly guarded. Guards for machine parts or machines in general may be of many materials and many types of construction. Wooden guards are vastly better than none. Pipe or structural shape railings have a legitimate application in safeguarding belts, silent chains and the like. Probably the most satisfactory guards are, however, constructed of a structural shape framework over which heavy woven wire or expanded metal is placed. Such guards are shown in Figs. 6, 7, 8, 9 and 10. Some firms have constructed quite efficient guards as well as some presenting a respectable appearance from worn-out perforated plates used on shaker screens. For this purpose, however, the perforations should not exceed about 2½ in. in diam-



FIG. 5
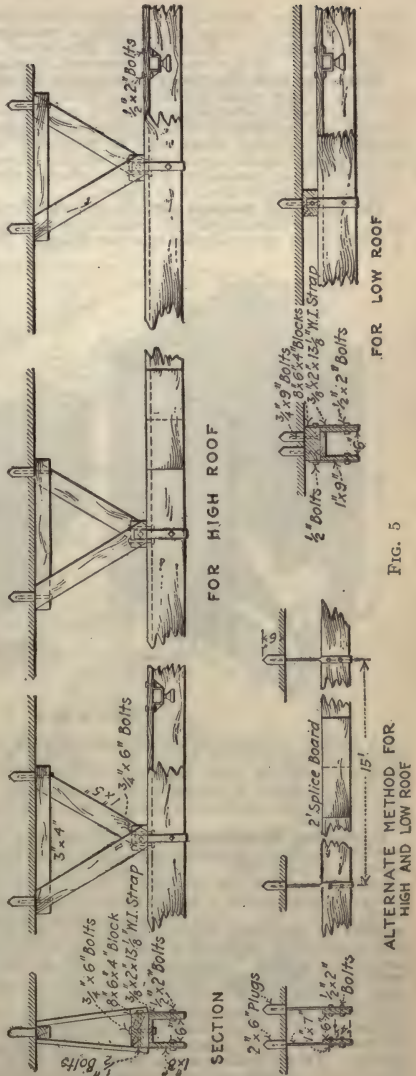
eter as a larger opening may admit a man's hand. Experience has proved that it is better to make the guarding of machinery to which access is necessary for oiling, etc., a little difficult of removal rather than extremely easy to take apart. If it is too easily removed, it may not be replaced.

**Protecting from Electricity.**—Generally speaking an air gap of sufficient width is a sure protection against electric shocks. The only safe way in which to treat an electrical conductor, regardless of insulation or the voltage carried, is to consider it as if it carried a high potential and was devoid of all insulation. Trolley wires may be portected from accidental contact by trolley guard boards, a good type of which is shown in Fig. 5. In all locations where men may pass under such wires during the day's work, as at turnouts, such boards should always be placed.

**The failure of machine parts** is an accident which it is difficult to anticipate. There is practically no means of determining the existence of a flaw in a welded pipe or a cold shut in a cast fitting. The danger from the failure of



GUARD FOR CENTRIFUGAL PUMP COUPLING

DEEP WELL PUMP CAGE

GUARD FOR PICKING TABLE GEARS
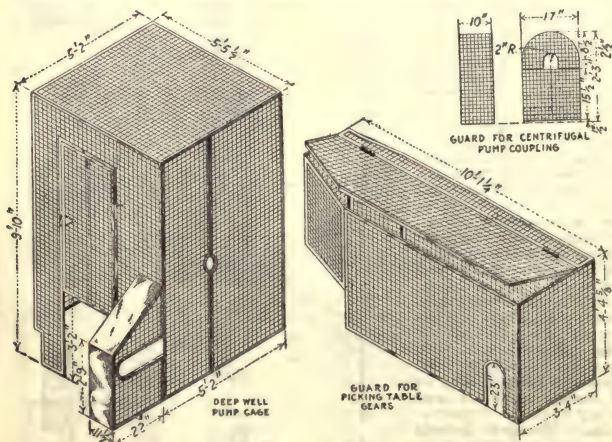
FIGS. 6 TO 8

such parts is in many instances quite as much from the release of the fluid carried (steam, air, hot water, etc.), as from flying fragments. In certain instances breaking parts, dangerous in themselves, may be rendered harmless by the installation of proper shields or retainers. Probably the best known of such devices are water glass protectors on boilers and safety collars on emery wheels.

**Preventing Mismanipulation of Controlling Devices.**—Many machines, machine parts and electrical equipment cause accidents through mismanipulation. Among these might be mentioned overwinds, the closing of electric circuits while men are repairing parts thereof; the unintentional opening of valves allowing steam or hot water to enter boilers in which repair men are at work, and the like. Overwinds may be prevented by reliable overwind preventors or regulating devices attached to the engine which will not allow a safe speed to be exceeded or the landing to be passed. Many such devices are on the market. Switches controlling electric circuits should be provided with means for locking them open, the key being carried by the chief repair man. In the case of such circuits as trolleys or the cables feeding them an additional precaution consists in placing an intentional temporary short circuit, e.g., a crowbar, mine drill, piece of wire, etc., between the trolley and the rail a few yards away from the repair men in the direction of the power supply. Valves leading to the interior of boilers, feed-water heaters and the like wherein repairs are being made should either be locked in the closed position by means of a chain and padlock or some other means should be taken

PLAN

Tightener Pulley

Motor Pulley

Belt

Compressor Wheel

Floor

SIDE VIEW

Fig. 9

15½" on Gage Line

Fly Wheel

Chain Wheel

C.I. Base Plate

DIAGRAM OF ASSEMBLED GUARD

DETAILS OF GUARD
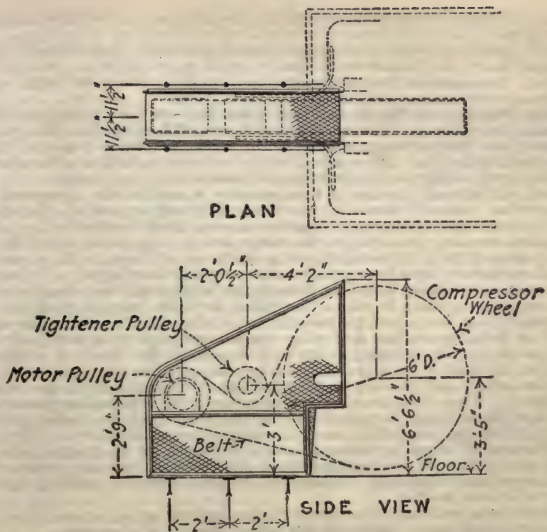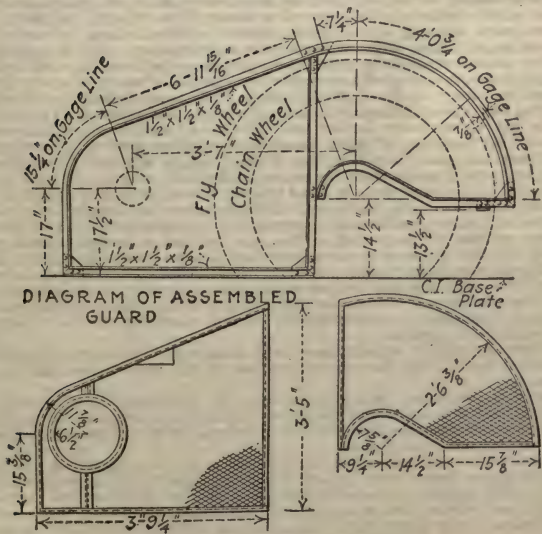
Fig. 10

which will render it impossible for either steam or hot water to be turned in upon them.

The foregoing are only a few suggestions of the many means which may be employed to insure the safety of workers around coal mines. Many others will doubtless suggest themselves and be found advisable to install under specific circumstances.

**Safety Practices of the H. C. Frick Coke Co.**—In the proceedings of the American Institute of Mining Engineers, vol. 51, page 345, Thomas W. Dawson gives the following résumé of the safety practices employed by the H. C. Frick Coke Co. These are all well worthy of careful consideration by the official who really wishes to make his mines as safe as possible.

Every official and foreman of the company is continually impressed with the fact that "safety" should be the first consideration, and all officials and their subordinates are brought together as one great committee on safety. Pamphlets showing the duties of the miner and the manner in which he may protect himself from danger and giving safety regulations for those working around machinery have been printed and generally distributed. Permanent danger signs are placed wherever there is the least possibility of an accident. When men are working in shafts, the "Men in Shaft" sign is placed so that no accident can be caused by mistake in moving cages. A similar sign is placed on hoisting engines and other machinery when it is being repaired. When workmen are cleaning or making repairs to the inside of a boiler, a "Man in the Boiler" sign is displayed outside and the steam valve for this boiler is locked and the key carried by one of the men until the work is completed. When coke-drawing machines are being repaired or cleaned, the "Do Not Move" sign is placed on the controller and the trolley wheel is locked and the key carried by one of the repair men until the work is finished. "No Clearance" signs are conspicuously displayed at all points about the plants where there is no clearance for a man between moving cars and obstructions of any character. Bridge guards and overhead warning signs are placed wherever needed.

In the mines, guide signs in various languages are posted at road junctions and on traveling ways, indicating the safest way out of the mine.

All machinery is safely guarded. These guards include locking devices for handwheels of valves, safety locks for electric switches, guards for water gauges; safety gaskets to be inserted in steam blow-off and feed-water connections when cleaning and repairing boilers; safety locking device for self-dumping cages; soap lubrication for air compressors; wagon guard and dumping platform for swing-gate mine cars; spooling device for tail ropes on haulages; stiles or protected crossings over rope and sheaves where necessary for men to pass; improved safety catch for cages; device for positively rectifying wagon catches on car hauls; self-closing hinges for shaft gates; steel galleries for runways over boilers, and safety platforms for operating electric larries. "Do Not Touch" signs are used about electric wires, indicating voltage of current; and "Do Not Pass Under" signs are used where there is danger in passing underneath structures. Steel doors are provided to drop over shafts which have wooden head frames or coal bins above them, should these wooden structures catch fire. The company has originated a device for automatically controlling high-pressure air compressors. When the temperature of the discharge air in the pipe reaches a predetermined point, showing that the pressure is excessively high, it acts on the thermometer and recording device, thus closing an electric circuit and energizing a solenoid. This moves over a tripping device, which opens the pilot valve, releasing the steam pressure on one side of the regulating piston. Thereby the valve on the steam feed pipe is automatically closed, shutting off the steam and stopping the compressor.

All hoisting engines are equipped with an automatic overwinding device, which acts directly on the engine, cutting off the steam and applying the brakes.

When it is necessary to clean the sump at the bottom of the shaft, the cages are hoisted to a clearance height and secured by iron pins, through holes in the guides; these pins are attached to the guides by chains, which prevent their removal when not in use.

At the surface landings of all shaft mines, a device is installed which prevents the gates from being opened when the cage is not in position at the landing. All hoisting compartments of shafts are lined at the cage ends. All cages and safety catches are periodically inspected, tested, and a written report made of the inspection. In no case is a hoisting rope kept in service

longer than 2½ yr., even though apparently safe and in good condition. Frequent inspection of air shafts must be made to keep them open and free at all times from ice and other obstructions. A fire boss must make this examination and travel either up or down such shaft once each day, the mine foremen once each two weeks, and the superintendent once a month.

The company's rules require that in mines generating explosive gas not less than 500 cu. ft. of air per min. per person employed in the mine shall be provided at the intake and this must be so distributed that there will not be less than 300 cu. ft. per min. per person employed in each split at the working faces. No mine shall have at the intake less than 300 cu. ft. of air per min. per person employed, and at the working places at least 150 cu. ft. per min. per person employed. Measurements of air supplied are carefully made and reported to the general office once each week. Local officials at mines generating gas are required to keep air up to the working faces and to such other places where explosive gas might be encountered. At a number of the larger and more recent plants, the ventilating fan is operated by two engines, one on each end, and either of them powerful enough to operate the fan in case of failure of the other. All ventilating systems in the mines are ascensional.

The Clowes hydrogen test lamp is used in all mines generating gas, for testing purposes. Samples of air are taken in gaseous mines and sent in copper cans to the company's laboratory, where they are analyzed. The results of the analyses are reported to the general office and to the mine. If these show a percentage of explosive gas which might have been detected by the Clowes lamp, the party making the test and reporting no gas is required to make an explanation.

Boreholes are frequently drilled from the surface to release any dangerous accumulations of explosive gas in the gob, where these cannot be removed by the mine ventilation. Shot firers have been employed to do all blasting by battery, and inspect all places where shots have been fired to see that there is no fire or other danger thereafter. Only the safest permissible explosives are used, and all tamping is done with clay.

All safety-lamp mines are examined on Sundays, holidays and lay-off days, and all safety mines which have been idle for more than two consecutive days are examined before operations are renewed. In the larger mines, wherever safety lamps are used, auxiliary escapeways are provided. In some instances these are stair shafts from the surface to the mine, placed in the active working sections, and used also for additional ventilation. In other cases, means of escape are provided by having connections between mines, which are closed by double iron doors. Frequent examinations are made to see that these doors are always in condition for use. Where coal dust occurs, a system of pipes and a supply of water under sufficient head and all necessary appliances are provided to dampen thoroughly the floor, sides, and roof of all parts of dry mines.

On rope haulage, a device is provided for disengaging the rope from the trip as soon as it is given slack. Brakes are provided for all mine cars and 2½ ft. clearance is provided on all haulageways on one side; this side being indicated by a wide whitewashed strip on the rib.

Systematic timbering systems are devised and strictly followed. Printed regulations cover the system of timbering in rooms, headings, and in rib and pillar drawing; these are worked out to suit conditions at the various mines. Timbering is not set without caps or cross-bars.

All mines have complete mine-telephone systems. Stables, pump rooms, haulage-engine rooms, shaft bottoms, underground offices and all such places where men might congregate are of fireproof construction and are kept clean and neat. No open lights are allowed in any building. Cans are provided for the reception of oily waste, grease, small quantities of oil, etc. All electric wiring is carefully inspected twice each year. All bare power lines underground and on the surface are properly guarded for their entire length by a neat wooden guard, so as to prevent the workman or his tools from coming in contact with the same. For the same reason, trolley wires for coke-drawing machines are placed at a sufficient height to make contact with tools unlikely. A system of checking men in and out of the workings is maintained at all of the mines. All abandoned places in the mines are fenced off.

The company employs four mine inspectors, one of them acting as chief. It is the duty of these men to visit each mine and thoroughly inspect it at least once every 60 days. When an accident occurs in or about any mine, the chief mine inspector promptly visits the scene of the accident, gathers all of the data he can relative thereto and makes a sketch of the surroundings.

This sketch is put into permanent form, blueprinted and sent to each mine with a circular letter, giving a full account of the accident and making suggestions for the prevention of similar ones. This is discussed at the meeting of the local officials at each plant. Once each week, the superintendent of each plant and his subordinates meet and discuss mine conditions and operations in general and especially matters pertaining to the safety of their employees. The discussions of these meetings are reported to the General Superintendent each week. General meetings are held at stated intervals at the general office, which are attended by the superintendent of each plant and heads of departments.

Projections for mine workings are made far in advance of the actual work, and the haulage and ventilating problems are planned so that when the mine is developed the best system is in use. Specifications are written for each mine, stating where and how the mining is to be done. The officials of the company make detailed inspections at intervals, insuring that their instructions and the best methods are actually followed.

A safety committee of three or four men is appointed at each mine, which inspects periodically the working places, roadways, ventilation and any other things which in its opinion might be the cause of an accident. The committee reports in writing to the superintendent of the mine, who forwards the same to the general office. These suggestions are immediately acted upon and all dangers reported, should there be any, are removed as quickly as possible. Three rescue and first-aid stations are maintained at the different plants of the company, which are fully equipped with the best apparatus and accessories obtainable. About 400 men have been thoroughly trained and qualified in both rescue and first-aid work, local contests being held by the different teams at various times.

Emergency hospitals, fully equipped, have been provided at a number of the largest mines.

Tests are made frequently for gas above roof falls in gobs. Work is prohibited in any place in which gas is found, until after it has been removed. Mine inspectors instruct all new employees about the dangers of their work.

## MINE-RESCUE WORK

Mine-rescue work is usually understood to mean the rescue of men or the recovery of bodies after a mine explosion, mine fire or other disaster. In the case of explosions or fires it implies the use of so-called rescue apparatus consisting of oxygen helmets, mouth-breathing apparatus, etc.

**Organization.**—The organization for mine-rescue work will differ so widely with local conditions that but little may be said on the subject in a book of this kind. Careful preparation and training of a rescue crew here means everything. While rescue work may be undertaken by untrained men it is necessarily much slower and more dangerous than where a well-trained, well-disciplined and thoroughly reliable team of helmet men are at hand. As to the selection, organization and training of a helmet crew the U. S. Bureau of Mines has done much valuable work along this line, and every mine or group of mines under one management should avail itself of the instruction, training and advice of the mine-rescue experts on the various mine-rescue cars maintained by the government. At least four helmets and preferably six should be available at all times for immediate use, and the organization of the team should be such that not more than half its members should be underground at any one time, that is, with a rescue team containing 12 men, 6 should at all times be available for instant call to service, and at least 4 of these should be thoroughly familiar with the underground workings. All should be men of good physique, sound heart and of known reliability, nerve and coolness.

**First Steps after a Disaster.**—It is extremely important that immediately upon the occurrence of a mine disaster, such as an explosion, that the proper steps toward rescue be taken promptly. Here again much will depend upon circumstances and local conditions, but in a general sense the following is necessary. Call the helmet men together, summon aid from nearby mines, summon the nearest government mine-rescue car. It is important that exploration work start as promptly as possible. Consequently the helmet men should precede all others into the mine. There are two other important considerations which require careful attention; the ventilation must be restored as quickly as possible and the means of communicating with the underground workings must be kept intact or repaired immediately. If the mine is a shaft operation the hoisting cages if damaged should be repaired as

quickly as possible or if this is out of the question a temporary means of raising and lowering men and material must be had. For this purpose some mines keep on hand an emergency cage so that if the regular cages are put out of commission, men at least may still be raised and lowered by the emergency apparatus.

**Reversing the Air Current.**—An explosion usually causes more or less havoc with the underground ventilating system, that is, brattices are frequently destroyed, stoppings broken down, etc. There is a wide diversity of opinion among engineers and mine men in general concerning the advisability of reversing the air current, that is, changing a blowing fan to an exhauster and *vice versa* after a mine disaster. This is a question, the expediency of which had better be thought out before the explosion occurs, or in any instance the current should not be reversed without due and careful consideration. Men attempting to find their way out of the mine after a disaster are apt to be guided largely by the air current flowing. They are in utter darkness unless of course they are provided with safety lamps or electric lamps and will naturally move against the air current. If this current is reversed it will in many instances drive the foul and poisonous gases resulting from the explosion directly upon them. There may however be instances wherein it would be advisable to reverse the current.

**The Work of Recovery.**—Mine-rescue and recovery work requires above all else a strong and careful leader. This leader should if possible be known at least by reputation to all the men engaged in the work. He should be a man whom all can respect and trust.

After it has been ascertained that the fan is in working order and at work (an auxiliary fan may be used if necessary), and a means of access to the mine is established, the helmet men with their mice, canaries or other means of testing the gas may enter the mine. These men are to the main body of the rescuers what the scouts are to an army—in large measure at least they constitute the department of security and information. Their work will be to explore the mine, ascertain the presence of dangerous gas and bring to a point of safety any living men that may be found. Unhelmeted men should follow them, restoring the ventilation as they go. This usually requires the building of a considerable amount of temporary bratticework and stoppings and material therefor (boards, plank, posts, canvass, nails, spikes, etc.) must be provided. No one should be allowed to enter the mine merely through curiosity. Whoever enters should be immediately put to work. The person directing recovery work may do so either from the surface, the foot of the shaft or some other convenient point, communicating with his various lieutenants either by word of mouth, by telephone if possible or in some instances by messenger at frequent intervals.

The person directing rescue and recovery work should be careful in his selection of lieutenants. The ventilation apparatus is perhaps the most important of all machinery. It must be kept going, or if a shutdown is absolutely necessary, this must be anticipated, a sufficient amount of time to allow all men, helmeted as well as unhelmeted to be withdrawn, from the mine before the air current is actually stopped. It is well therefore to place an experienced man at the fan, whose sole duty it shall be to keep it in operation. If necessary, this man should have all the helpers he may require. The hoisting apparatus is also important, but if possible the regular hoist man should stick to his post. There should be appointed a gang whose duty should be to secure and bring to the mouth of the mine the materials necessary for bratticing. If the mine is electrically lighted a competent electrician with a requisite number of helpers should be put to work repairing or establishing lighting conditions. The brattice men following the helmet crew should be under an experienced brattice builder who is competent to see that the work is done properly and rapidly; in many instances also a man with such assistance as he may need may be employed to transport the various materials from the mine entrance to the point where they are needed. Furthermore, since mine rescue work usually lasts for several hours or even days a commissary department should be established so that food such as sandwiches, hot soup and particularly hot coffee may be served to the men at work at regular intervals and the coffee whenever they desire it. Men skilled in first aid as well as physicians should also be on hand to give prompt and efficient treatment to any men that may be found alive. The helmet men will of course remove few if any dead bodies so long as there is even hope of finding living people in the mine. Once a man is found alive he should be promptly taken to a point where at least reasonably pure air is available.

In selecting his lieutenants heading the various gangs or groups of men above mentioned, the man in charge should use careful discretion and delegate to each subordinate the work with which that particular man is most familiar. Thus a mine official of even high standing such as a superintendent might be given a job of caging at the ground landing if he were known to have had successful previous experience at that kind of work. The man in charge of the transporting of material from the shaft bottom to the point of use might be a mine superintendent or he might be a motor boss, depending on his previous experience in the transportation department. The main idea is that every man in charge of a gang should know the work which he is called upon to do, know it well and be a person that can be depended upon. The success of mine-rescue work, that is, the recovery of living men, will often depend much upon the coolness, good judgment and persistence of the man or men in charge of this work. The careful mine official will therefore think out and decide many possible mine-rescue problems before the actual time of disaster arrives.

## MINE-RESCUE APPARATUS

Mine-rescue apparatus, so-called, is of two general types: (a) breathing apparatus, used by the rescue crew, and (b) resuscitation apparatus used by or rather on the people recovered.

**Breathing Apparatus.**—There are two types of breathing apparatus in general use in the United States. These are known respectively as helmet apparatus and mouth-breathing apparatus. The helmet apparatus consists of a metal helmet which may be strapped over the face and be rendered airtight by an inflatable gasket which fits under the chin and extends upwards completely encircling the front portion of the head or by other means.
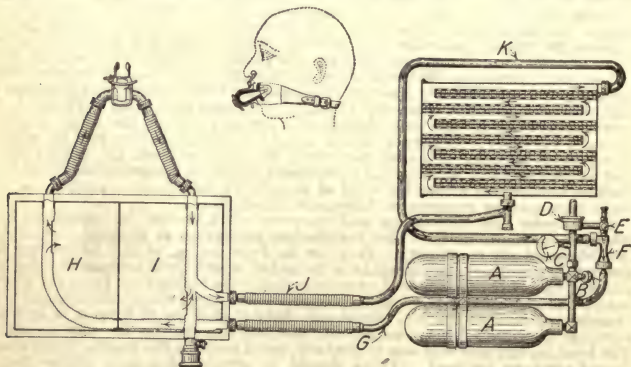


FIG. 1

The rear portion of the head is protected by a leather apron. The mouth-breathing apparatus is exactly similar except that in place of the helmet a mouthpiece which is provided with a device to close the nostrils of the nose is strapped onto the head. The oxygen containers, pipes, breathing bag and regenerator are at least similar if not identical in the two types and sometimes are made interchangeable.

The operation of the instrument is simple and may be readily understood from the diagrammatic drawing Fig. 1. The oxygen tanks $AA$ are connected together and the flow of oxygen is regulated by the valve $B$. Opening this valve allows the compressed oxygen in the tanks to flow to the pressure gauge $C$ and to the reducing valve $D$, which is fitted with a safety valve $E$. The oxygen is reduced to a predetermined pressure in the regulating valve and next passes to the injector $F$. It then flows through the pipe $G$ to the inhalation compartment of breathing bag $H$ and from thence to the mouthpiece or helmet. After being exhaled from the lungs the gas passes to a

second compartment or exhalation bag *I* which is part and parcel of the inhalation bag but separated therefrom by a partition. From here it passes through the pipe *J* to the regenerator. This is provided with potash in a granular form which is arranged in wire gauze trays around which the exhaled oxygen passes and from which the carbon dioxide is absorbed by the chemical. It then passes through the pipe *K* to the injector where it is reoxygenated and again passed to the breathing bag *H* to be inhaled. Under ordinary conditions the oxygen tanks contain sufficient compressed oxygen for 2 hr. of hard work. A smaller regenerator is sometimes employed for practice work in the smoke room, thus reducing the cost of each practice. When the mouth-breathing apparatus is used, it is advisable to supply the wearer with smoke goggles to protect the eyes in case work is being done in any gas which would tend to irritate them.

The operation of the helmet is practically the same as that of the mouth-breathing apparatus above described except that the helmet is substituted in place of the mouthpiece. The mouth-breathing apparatus is somewhat lighter and simpler both in construction and operation than is the helmet. The helmet, however, possesses the advantage of allowing the wearer to talk with his companions which is difficult, if not impossible, with the mouth-breathing apparatus.

Oxygen for use with mine-rescue apparatus may be purchased in large cylinders from whence it may be transferred to the small cartridges of the breathing apparatus by means of a suitable hand pump. Such pumps are usually made double-acting and will compress the oxygen in the small tanks to approximately 120 atmospheres.

**Self Rescuer.**—A small type of breathing apparatus for use in noxious or poisonous gases and holding a charge of oxygen sufficient for 30 min. work is known as the self rescuer. This apparatus is compact, weighs about 6½ lb., can be quickly adjusted to the wearer and does not require previous training. This apparatus is shown diagrammatically in Fig. 2. Here *S* represents the oxygen cylinder, *U* the closing valve, *P* the potash or regenerating cartridge, *A* the breathing bag, *L* the respiration pipe which is provided with the rubber mouthpiece *M*. The apparatus is suspended by a strap around the neck while a canvas apron fastened around the waist holds the apparatus in place. The exhaled air flows through the respiration pipe *L* into the regenerator *P* where it is subjected to the action of the carbon-dioxide-absorbing chemicals. Freed from the products of respiration the air enters the breathing bag *A* where a fresh oxygen supply is provided from the cylinder *S*. The regenerated air is again drawn through the potash cartridge where it is once more subjected to purification and is inhaled through the pipe *L*. Care must be taken with any of these instruments to provide fresh potash cartridges for regenerating the air. Fresh cartridges when shaken will rattle, spent ones will not.
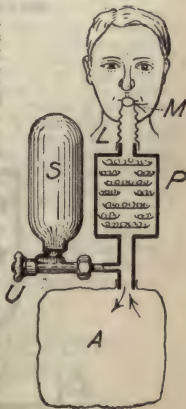


Fig. 2

**Resuscitation Apparatus.**—In cases of partial asphyxiation from poisonous gas or drowning, severe electric shock, etc., a means of compelling the patient to breathe is necessary. This may be supplied either by artificial respiration according to the Schaefer or Sylvester method or by some variety of resuscitation apparatus. One of the simplest of resuscitation apparatuses is shown in Fig. 3 and is known as the lungmotor. This may be arranged to administer either atmospheric air only or atmospheric air enriched with oxygen either from a charged tank or an oxygen generator, or pure oxygen from the same source. The lungmotor consists of two air pumps which are operated simultaneously but which are connected together only by the two flexible tubes leading to the face mask. After the face mask has been adjusted and strapped in place and the adjustment for the size of the patient made by turning the pin *A* so as to give the proper length of stroke to the two pumps the handle of the machine is simply worked up and down at the normal rapidity of breathing; the operator may judge this from his own respiration. Air is thus gently but positively forced into and withdrawn from the patient's lungs, the lungs meanwhile being maintained at their normal inflation. In

NEWBORN
5 YEARS
10 YEARS
15 YEARS
OR SMALL ADULT
ADULT AVERAGE
ADULT LARGE
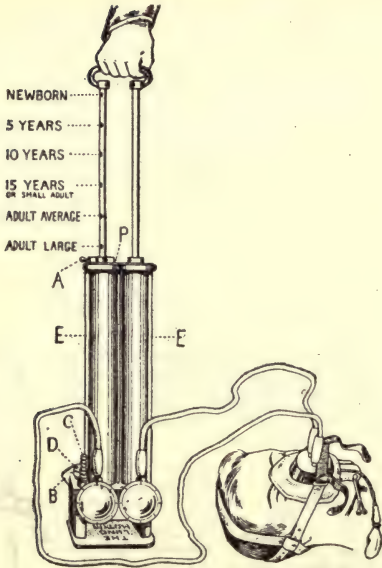
FIG. 3
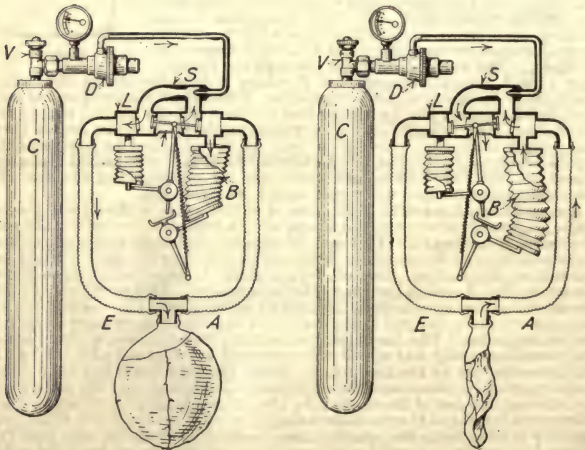


FIG. 4

case it is desired to administer to the patient an atmosphere richer in oxygen than atmospheric air a charged tank of oxygen or an oxygen generator may be connected to the nipple *C* which forms the oxygen inlet. An adjustment of the mixing valve *B* renders it possible to administer all air, all oxygen, or any desired mixture of the two. This apparatus is light, positive in action, easily portable and is not dependent for operation upon a supply of oxygen either compressed in tanks or generated as required.

Another type of apparatus which has been used to a considerable extent is known as the pulmotor. The motive power for this machine is the oxygen compressed in the cylinder *C* (see Fig. 4). When the valve *V* is opened the full pressure of the oxygen in the tank is exerted upon valve *D*. It is here reduced to 75 lb. per sq. in. and at this pressure passes to the injector *S*. Here the oxygen is fed at the rate of ⅓ cu. ft. per min. through the tube *L*. The injector is so arranged, however, that while accomplishing this function it will create a suction through a line connected to the outside air, thus drawing in a large volume of atmospheric air, mixing it with the oxygen and forcing it through tube *E* to the lungs, which are represented by the bag at the bottom of the figure. This action continues until the lungs are inflated to a measured amount. There being no valve or other obstruction to prevent, this increase of pressure acts through the tube *A* which leads to the bellows *B*. As soon as the pressure attains .29 lb. per sq. in. this pressure forces the head of the bellows outward reversing the pulmotor valve *L* through the medium of a tension spring. The suction action of the nozzle *S* is now cut off from the outside circuit and is carried through the return air tube *A* which is connected through the face mask to the lungs. Air from the lungs is thus exhausted to the outside air until a proper vacuum amounting to .37 lb. per sq. in. is developed in the lungs, when the bellows contracts under the action of the vacuum throwing the valve back to its original position and starting the cycle of operations over again. This action proceeds at the rate of from 14 to 18 strokes per min.

The entire equipment, including several sizes of face masks, is packed in a wooden carrying case for convenience in transportation. This apparatus has the advantage of being automatic in action but is somewhat complicated. It has, however, been used to a wide extent and with considerable success.

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# TABLE OF NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS

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## EXPLANATION

**Given an angle, to find its sine, cosine, tangent, and cotangent**
To find the sine, cosine, tangent, and cotangent of 37° 24′, look in the table of natural sines along the tops of the pages, and find 37°. Glancing down the left-hand column marked (′), until 24 is found, find opposite this 24 in the column marked sine and headed 37°, the number .60738; then .60738 = sin 37° 24′. Similarly, find in the column marked cosine and headed 37°, the number .79441, which corresponds to cos 37° 24′. So, also, find in the column marked tangent and headed 37°, and opposite 24′, the number .76456; and in the column marked cotangent and headed 37°, and opposite 24′, the number 1.30795.

In most of the tables published, the angles run only from 0° to 45° at the heads of the columns; to find an angle greater than 45°, *look at the bottom of the page and glance upwards, using the extreme right-hand column to find minutes,* which begin with 0 at the bottom and run upwards, 1, 2, 3, etc., up to 60.

To find the sine of 77° 43′, look along the bottom of the tables until the column marked sine and marked 77° is found. Glancing up the column of minutes on the right until 43′ is found, find opposite 43′ in the column marked sine at the bottom and marked 77°, the number .97711; this is the sine of 77° 43′. Similarly, the cosine, tangent, and cotangent may be found.

To find the sine, cosine, tangent, or cotangent of an angle whose exact value is not given in the table:

**Rule.**—*Find in the table the sine, cosine, tangent, or cotangent corresponding to the degrees and minutes of the angle. For the seconds, find the difference of the values of the sine, cosine, tangent, or cotangent taken from the table between which the seconds of the angle fall; multiply this difference by a fraction whose numerator is the number of seconds in the given angle and whose denominator is 60.*

*If sine or tangent, add this correction to the value first found; if cosine or cotangent, subtract the correction.*

EXAMPLE.—Find the sine, cosine, tangent, and cotangent of 56° 43′ 17″.

SOLUTION.—Sin 56° 43′ = .83597. Sin 56° 44′ = .83613. As 56° 43′ 17″ is greater than 56° 43′ and less than 56° 44′, the value of the sine of the angle lies between .83597 and .83613; the difference equals .83613 − .83597 = .00016; multiplying this by the fraction $\frac{17}{60}$, .00016 × $\frac{17}{60}$ = .00005, nearly, which is to be added to .83597, the value first found, or .83597 + .00005 = .83602. Hence, sin 56° 43′ 17″ = .83602.

Cos 56° 43′ = .54878; cos 56° 44′ = .54854; the difference equals .54878 − .54854 = .00024, and .00024 × $\frac{17}{60}$ = .00007, nearly. Now, as the cosine is desired, this correction must be subtracted from cos 56° 43′, or .54878; subtraction, .54878 − .00007 = .54871. Hence, cos 56° 43′ 17″ = .54871.

**Given the sine, cosine, tangent, or cotangent, to find the angle corresponding**

If the sine of an angle is .47486; what is the angle? Consulting the table of natural sines, glance down the columns marked sine until .47486 is found, opposite 21′ in the left-hand column and under the column headed 28°. Therefore, the angle whose sine = .47486 is 28° 21′, or sin 28° 21′ = .47486.

To find the angle corresponding to a given sine, cosine, tangent, or cotangent whose exact value is not contained in the table:

**Rule.**—*Find the difference of the two numbers in the table between which the given sine, cosine, tangent, or cotangent falls, and use the number of parts in this difference as the denominator of a fraction.*

*Find the difference between the number belonging to the smaller angle and the given sine, cosine, tangent, or cotangent, and use the number of parts in the difference just found as the numerator of the fraction just mentioned. Multiply this fraction by 60, and the result will be the number of seconds to be added to the smaller angle.*

EXAMPLE.—Find the angle whose sine equals .57698.

SOLUTION.—Looking in the table of natural sines, in the column marked sine, it is found between .57691 = sin 35° 14′ and .57715 = 35° 15′. The difference between them is .57715 − .57691 = .00024, or 24 parts. The difference between the sine of the smaller angle, or sine 35° 14′ = .57691, and the given sine, or .57698, is .57698 − .57691 = .00007, or 7 parts. Then, $\frac{7}{24}$ × 60 = 17.5″, and the angle = 35° 14′ 17.5″, or sin 35° 14′ 17.5″ = .57698.

The cosecant of an angle is equal to the reciprocal of its sine, and the secant is equal to the reciprocal of its cosine. Hence, to multiply a quantity by the cosecant, divide it by the sine; or, to divide it by the cosecant, multiply it by the sine. Similarly, to multiply a quantity by the secant of an angle, divide it by the cosine; or, to divide it by the secant, multiply it by the cosine.

| ′ | 0° | | 1° | | 2° | | 3° | | 4° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | |
| 0 | .00000 | 1. | .01745 | .99985 | .03490 | .99939 | .05234 | .99863 | .06976 | .99756 | 60 |
| 1 | .00029 | 1. | .01774 | .99984 | .03519 | .99938 | .05263 | .99861 | .07005 | .99754 | 59 |
| 2 | .00058 | 1. | .01803 | .99984 | .03548 | .99937 | .05292 | .99860 | .07034 | .99752 | 58 |
| 3 | .00087 | 1. | .01832 | .99983 | .03577 | .99936 | .05321 | .99858 | .07063 | .99750 | 57 |
| 4 | .00116 | 1. | .01862 | .99983 | .03606 | .99935 | .05350 | .99857 | .07092 | .99748 | 56 |
| 5 | .00145 | 1. | .01891 | .99982 | .03635 | .99934 | .05379 | .99855 | .07121 | .99746 | 55 |
| 6 | .00175 | 1. | .01920 | .99982 | .03664 | .99933 | .05408 | .99854 | .07150 | .99744 | 54 |
| 7 | .00204 | 1. | .01949 | .99981 | .03693 | .99932 | .05437 | .99852 | .07179 | .99742 | 53 |
| 8 | .00233 | 1. | .01978 | .99980 | .03723 | .99931 | .05466 | .99851 | .07208 | .99740 | 52 |
| 9 | .00262 | 1. | .02007 | .99980 | .03752 | .99930 | .05495 | .99849 | .07237 | .99738 | 51 |
| 10 | .00291 | 1. | .02036 | .99979 | .03781 | .99929 | .05524 | .99847 | .07266 | .99736 | 50 |
| 11 | .00320 | .99999 | .02065 | .99979 | .03810 | .99927 | .05553 | .99846 | .07295 | .99734 | 49 |
| 12 | .00349 | .99999 | .02094 | .99978 | .03839 | .99926 | .05582 | .99844 | .07324 | .99731 | 48 |
| 13 | .00378 | .99999 | .02123 | .99977 | .03868 | .99925 | .05611 | .99842 | .07353 | .99729 | 47 |
| 14 | .00407 | .99999 | .02152 | .99977 | .03897 | .99924 | .05640 | .99841 | .07382 | .99727 | 46 |
| 15 | .00436 | .99999 | .02181 | .99976 | .03926 | .99923 | .05669 | .99839 | .07411 | .99725 | 45 |
| 16 | .00465 | .99999 | .02211 | .99976 | .03955 | .99922 | .05698 | .99838 | .07440 | .99723 | 44 |
| 17 | .00495 | .99999 | .02240 | .99975 | .03984 | .99921 | .05727 | .99836 | .07469 | .99721 | 43 |
| 18 | .00524 | .99999 | .02269 | .99974 | .04013 | .99919 | .05756 | .99834 | .07498 | .99719 | 42 |
| 19 | .00553 | .99998 | .02298 | .99974 | .04042 | .99918 | .05785 | .99833 | .07527 | .99716 | 41 |
| 20 | .00582 | .99998 | .02327 | .99973 | .04071 | .99917 | .05814 | .99831 | .07556 | .99714 | 40 |
| 21 | .00611 | .99998 | .02356 | .99972 | .04100 | .99916 | .05844 | .99829 | .07585 | .99712 | 39 |
| 22 | .00640 | .99998 | .02385 | .99972 | .04129 | .99915 | .05873 | .99827 | .07614 | .99710 | 38 |
| 23 | .00669 | .99998 | .02414 | .99971 | .04159 | .99913 | .05902 | .99826 | .07643 | .99708 | 37 |
| 24 | .00698 | .99998 | .02443 | .99970 | .04188 | .99912 | .05931 | .99824 | .07672 | .99705 | 36 |
| 25 | .00727 | .99997 | .02472 | .99969 | .04217 | .99911 | .05960 | .99822 | .07701 | .99703 | 35 |
| 26 | .00756 | .99997 | .02501 | .99969 | .04246 | .99910 | .05989 | .99821 | .07730 | .99701 | 34 |
| 27 | .00785 | .99997 | .02530 | .99968 | .04275 | .99909 | .06018 | .99819 | .07759 | .99699 | 33 |
| 28 | .00814 | .99997 | .02560 | .99967 | .04304 | .99907 | .06047 | .99817 | .07788 | .99696 | 32 |
| 29 | .00844 | .99996 | .02589 | .99966 | .04333 | .99906 | .06076 | .99815 | .07817 | .99694 | 31 |
| 30 | .00873 | .99996 | .02618 | .99966 | .04362 | .99905 | .06105 | .99813 | .07846 | .99692 | 30 |
| 31 | .00902 | .99996 | .02647 | .99965 | .04391 | .99904 | .06134 | .99812 | .07875 | .99689 | 29 |
| 32 | .00931 | .99996 | .02676 | .99964 | .04420 | .99902 | .06163 | .99810 | .07904 | .99687 | 28 |
| 33 | .00960 | .99995 | .02705 | .99963 | .04449 | .99901 | .06192 | .99808 | .07933 | .99685 | 27 |
| 34 | .00989 | .99995 | .02734 | .99963 | .04478 | .99900 | .06221 | .99806 | .07962 | .99683 | 26 |
| 35 | .01018 | .99995 | .02763 | .99962 | .04507 | .99898 | .06250 | .99804 | .07991 | .99680 | 25 |
| 36 | .01047 | .99995 | .02792 | .99961 | .04536 | .99897 | .06279 | .99803 | .08020 | .99678 | 24 |
| 37 | .01076 | .99994 | .02821 | .99960 | .04565 | .99896 | .06308 | .99801 | .08049 | .99676 | 23 |
| 38 | .01105 | .99994 | .02850 | .99959 | .04594 | .99894 | .06337 | .99799 | .08078 | .99673 | 22 |
| 39 | .01134 | .99994 | .02879 | .99959 | .04623 | .99893 | .06366 | .99797 | .08107 | .99671 | 21 |
| 40 | .01164 | .99993 | .02908 | .99958 | .04653 | .99892 | .06395 | .99795 | .08136 | .99668 | 20 |
| 41 | .01193 | .99993 | .02938 | .99957 | .04682 | .99890 | .06424 | .99793 | .08165 | .99666 | 19 |
| 42 | .01222 | .99993 | .02967 | .99956 | .04711 | .99889 | .06453 | .99792 | .08194 | .99664 | 18 |
| 43 | .01251 | .99992 | .02996 | .99955 | .04740 | .99888 | .06482 | .99790 | .08223 | .99661 | 17 |
| 44 | .01280 | .99992 | .03025 | .99954 | .04769 | .99886 | .06511 | .99788 | .08252 | .99659 | 16 |
| 45 | .01309 | .99991 | .03054 | .99953 | .04798 | .99885 | .06540 | .99786 | .08281 | .99657 | 15 |
| 46 | .01338 | .99991 | .03083 | .99952 | .04827 | .99883 | .06569 | .99784 | .08310 | .99654 | 14 |
| 47 | .01367 | .99991 | .03112 | .99952 | .04856 | .99882 | .06598 | .99782 | .08339 | .99652 | 13 |
| 48 | .01396 | .99990 | .03141 | .99951 | .04885 | .99881 | .06627 | .99780 | .08368 | .99649 | 12 |
| 49 | .01425 | .99990 | .03170 | .99950 | .04914 | .99879 | .06656 | .99778 | .08397 | .99647 | 11 |
| 50 | .01454 | .99989 | .03199 | .99949 | .04943 | .99878 | .06685 | .99776 | .08426 | .99644 | 10 |
| 51 | .01483 | .99989 | .03228 | .99948 | .04972 | .99876 | .06714 | .99774 | .08455 | .99642 | 9 |
| 52 | .01513 | .99989 | .03257 | .99947 | .05001 | .99875 | .06743 | .99772 | .08484 | .99639 | 8 |
| 53 | .01542 | .99988 | .03286 | .99946 | .05030 | .99873 | .06773 | .99770 | .08513 | .99637 | 7 |
| 54 | .01571 | .99988 | .03316 | .99945 | .05059 | .99872 | .06802 | .99768 | .08542 | .99635 | 6 |
| 55 | .01600 | .99987 | .03345 | .99944 | .05088 | .99870 | .06831 | .99766 | .08571 | .99632 | 5 |
| 56 | .01629 | .99987 | .03374 | .99943 | .05117 | .99869 | .06860 | .99764 | .08600 | .99630 | 4 |
| 57 | .01658 | .99986 | .03403 | .99942 | .05146 | .99867 | .06889 | .99762 | .08629 | .99627 | 3 |
| 58 | .01687 | .99986 | .03432 | .99941 | .05175 | .99866 | .06918 | .99760 | .08658 | .99625 | 2 |
| 59 | .01716 | .99985 | .03461 | .99940 | .05205 | .99864 | .06947 | .99758 | .08687 | .99622 | 1 |
| 60 | .01745 | .99985 | .03490 | .99939 | .05234 | .99863 | .06976 | .99756 | .08716 | .99619 | 0 |
| ′ | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | ′ |
| | 89° | | 88° | | 87° | | 86° | | 85° | | |

| ′ | 5° Sine | 5° Cosine | 6° Sine | 6° Cosine | 7° Sine | 7° Cosine | 8° Sine | 8° Cosine | 9° Sine | 9° Cosine | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .08716 | .99619 | .10453 | .99452 | .12187 | .99255 | .13917 | .99027 | .15643 | .98769 | 60 |
| 1 | .08745 | .99617 | .10482 | .99449 | .12216 | .99251 | .13946 | .99023 | .15672 | .98764 | 59 |
| 2 | .08774 | .99614 | .10511 | .99446 | .12245 | .99248 | .13975 | .99019 | .15701 | .98760 | 58 |
| 3 | .08803 | .99612 | .10540 | .99443 | .12274 | .99244 | .14004 | .99015 | .15730 | .98755 | 57 |
| 4 | .08831 | .99609 | .10569 | .99440 | .12302 | .99240 | .14033 | .99011 | .15758 | .98751 | 56 |
| 5 | .08860 | .99607 | .10597 | .99437 | .12331 | .99237 | .14061 | .99006 | .15787 | .98746 | 55 |
| 6 | .08889 | .99604 | .10626 | .99434 | .12360 | .99233 | .14090 | .99002 | .15816 | .98741 | 54 |
| 7 | .08918 | .99602 | .10655 | .99431 | .12389 | .99230 | .14119 | .98998 | .15845 | .98737 | 53 |
| 8 | .08947 | .99599 | .10684 | .99428 | .12418 | .99226 | .14148 | .98994 | .15873 | .98732 | 52 |
| 9 | .08976 | .99596 | .10713 | .99424 | .12447 | .99222 | .14177 | .98990 | .15902 | .98728 | 51 |
| 10 | .09005 | .99594 | .10742 | .99421 | .12476 | .99219 | .14205 | .98986 | .15931 | .98723 | 50 |
| 11 | .09034 | .99591 | .10771 | .99418 | .12504 | .99215 | .14234 | .98982 | .15959 | .98718 | 49 |
| 12 | .09063 | .99588 | .10800 | .99415 | .12533 | .99211 | .14263 | .98978 | .15988 | .98714 | 48 |
| 13 | .09092 | .99586 | .10829 | .99412 | .12562 | .99208 | .14292 | .98973 | .16017 | .98709 | 47 |
| 14 | .09121 | .99583 | .10858 | .99409 | .12591 | .99204 | .14320 | .98969 | .16046 | .98704 | 46 |
| 15 | .09150 | .99580 | .10887 | .99406 | .12620 | .99200 | .14349 | .98965 | .16074 | .98700 | 45 |
| 16 | .09179 | .99578 | .10916 | .99402 | .12649 | .99197 | .14378 | .98961 | .16103 | .98695 | 44 |
| 17 | .09208 | .99575 | .10945 | .99399 | .12678 | .99193 | .14407 | .98957 | .16132 | .98690 | 43 |
| 18 | .09237 | .99572 | .10973 | .99396 | .12706 | .99189 | .14436 | .98953 | .16160 | .98686 | 42 |
| 19 | .09266 | .99570 | .11002 | .99393 | .12735 | .99186 | .14464 | .98948 | .16189 | .98681 | 41 |
| 20 | .09295 | .99567 | .11031 | .99390 | .12764 | .99182 | .14493 | .98944 | .16218 | .98676 | 40 |
| 21 | .09324 | .99564 | .11060 | .99386 | .12793 | .99178 | .14522 | .98940 | .16246 | .98671 | 39 |
| 22 | .09353 | .99562 | .11089 | .99383 | .12822 | .99175 | .14551 | .98936 | .16275 | .98667 | 38 |
| 23 | .09382 | .99559 | .11118 | .99380 | .12851 | .99171 | .14580 | .98931 | .16304 | .98662 | 37 |
| 24 | .09411 | .99556 | .11147 | .99377 | .12880 | .99167 | .14608 | .98927 | .16333 | .98657 | 36 |
| 25 | .09440 | .99553 | .11176 | .99374 | .12908 | .99163 | .14637 | .98923 | .16361 | .98652 | 35 |
| 26 | .09469 | .99551 | .11205 | .99370 | .12937 | .99160 | .14666 | .98919 | .16390 | .98648 | 34 |
| 27 | .09498 | .99548 | .11234 | .99367 | .12966 | .99156 | .14695 | .98914 | .16419 | .98643 | 33 |
| 28 | .09527 | .99545 | .11263 | .99364 | .12995 | .99152 | .14723 | .98910 | .16447 | .98638 | 32 |
| 29 | .09556 | .99542 | .11291 | .99360 | .13024 | .99148 | .14752 | .98906 | .16476 | .98633 | 31 |
| 30 | .09585 | .99540 | .11320 | .99357 | .13053 | .99144 | .14781 | .98902 | .16505 | .98629 | 30 |
| 31 | .09614 | .99537 | .11349 | .99354 | .13081 | .99141 | .14810 | .98897 | .16533 | .98624 | 29 |
| 32 | .09642 | .99534 | .11378 | .99351 | .13110 | .99137 | .14838 | .98893 | .16562 | .98619 | 28 |
| 33 | .09671 | .99531 | .11407 | .99347 | .13139 | .99133 | .14867 | .98889 | .16591 | .98614 | 27 |
| 34 | .09700 | .99528 | .11436 | .99344 | .13168 | .99129 | .14896 | .98884 | .16620 | .98609 | 26 |
| 35 | .09729 | .99526 | .11465 | .99341 | .13197 | .99125 | .14925 | .98880 | .16648 | .98604 | 25 |
| 36 | .09758 | .99523 | .11494 | .99337 | .13226 | .99122 | .14954 | .98876 | .16677 | .98600 | 24 |
| 37 | .09787 | .99520 | .11523 | .99334 | .13254 | .99118 | .14982 | .98871 | .16706 | .98595 | 23 |
| 38 | .09816 | .99517 | .11552 | .99331 | .13283 | .99114 | .15011 | .98867 | .16734 | .98590 | 22 |
| 39 | .09845 | .99514 | .11580 | .99327 | .13312 | .99110 | .15040 | .98863 | .16763 | .98585 | 21 |
| 40 | .09874 | .99511 | .11609 | .99324 | .13341 | .99106 | .15069 | .98858 | .16792 | .98580 | 20 |
| 41 | .09903 | .99508 | .11638 | .99320 | .13370 | .99102 | .15097 | .98854 | .16820 | .98575 | 19 |
| 42 | .09932 | .99506 | .11667 | .99317 | .13399 | .99098 | .15126 | .98849 | .16849 | .98570 | 18 |
| 43 | .09961 | .99503 | .11696 | .99314 | .13427 | .99094 | .15155 | .98845 | .16878 | .98565 | 17 |
| 44 | .09990 | .99500 | .11725 | .99310 | .13456 | .99091 | .15184 | .98841 | .16906 | .98561 | 16 |
| 45 | .10019 | .99497 | .11754 | .99307 | .13485 | .99087 | .15212 | .98836 | .16935 | .98556 | 15 |
| 46 | .10048 | .99494 | .11783 | .99303 | .13514 | .99083 | .15241 | .98832 | .16964 | .98551 | 14 |
| 47 | .10077 | .99491 | .11812 | .99300 | .13543 | .99079 | .15270 | .98827 | .16992 | .98546 | 13 |
| 48 | .10106 | .99488 | .11840 | .99297 | .13572 | .99075 | .15299 | .98823 | .17021 | .98541 | 12 |
| 49 | .10135 | .99485 | .11869 | .99293 | .13600 | .99071 | .15327 | .98818 | .17050 | .98536 | 11 |
| 50 | .10164 | .99482 | .11898 | .99290 | .13629 | .99067 | .15356 | .98814 | .17078 | .98531 | 10 |
| 51 | .10192 | .99479 | .11927 | .99286 | .13658 | .99063 | .15385 | .98809 | .17107 | .98526 | 9 |
| 52 | .10221 | .99476 | .11956 | .99283 | .13687 | .99059 | .15414 | .98805 | .17136 | .98521 | 8 |
| 53 | .10250 | .99473 | .11985 | .99279 | .13716 | .99055 | .15442 | .98800 | .17164 | .98516 | 7 |
| 54 | .10279 | .99470 | .12014 | .99276 | .13744 | .99051 | .15471 | .98796 | .17193 | .98511 | 6 |
| 55 | .10308 | .99467 | .12043 | .99272 | .13773 | .99047 | .15500 | .98791 | .17222 | .98506 | 5 |
| 56 | .10337 | .99464 | .12071 | .99269 | .13802 | .99043 | .15529 | .98787 | .17250 | .98501 | 4 |
| 57 | .10366 | .99461 | .12100 | .99265 | .13831 | .99039 | .15557 | .98782 | .17279 | .98496 | 3 |
| 58 | .10395 | .99458 | .12129 | .99262 | .13860 | .99035 | .15586 | .98778 | .17308 | .98491 | 2 |
| 59 | .10424 | .99455 | .12158 | .99258 | .13889 | .99031 | .15615 | .98773 | .17336 | .98486 | 1 |
| 60 | .10453 | .99452 | .12187 | .99255 | .13917 | .99027 | .15643 | .98769 | .17365 | .98481 | 0 |
| ′ | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | ′ |
| | 84° | | 83° | | 82° | | 81° | | 80° | | |

| ′ | 10° Sine | 10° Cosine | 11° Sine | 11° Cosine | 12° Sine | 12° Cosine | 13° Sine | 13° Cosine | 14° Sine | 14° Cosine | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .17365 | .98481 | .19081 | .98163 | .20791 | .97815 | .22495 | .97437 | .24192 | .97030 | 60 |
| 1 | .17393 | .98476 | .19109 | .98157 | .20820 | .97809 | .22523 | .97430 | .24220 | .97023 | 59 |
| 2 | .17422 | .98471 | .19138 | .98152 | .20848 | .97803 | .22552 | .97424 | .24249 | .97015 | 58 |
| 3 | .17451 | .98466 | .19167 | .98146 | .20877 | .97797 | .22580 | .97417 | .24277 | .97008 | 57 |
| 4 | .17479 | .98461 | .19195 | .98140 | .20905 | .97791 | .22608 | .97411 | .24305 | .97001 | 56 |
| 5 | .17508 | .98455 | .19224 | .98135 | .20933 | .97784 | .22637 | .97404 | .24333 | .96994 | 55 |
| 6 | .17537 | .98450 | .19252 | .98129 | .20962 | .97778 | .22665 | .97398 | .24362 | .96987 | 54 |
| 7 | .17565 | .98445 | .19281 | .98124 | .20990 | .97772 | .22693 | .97391 | .24390 | .96980 | 53 |
| 8 | .17594 | .98440 | .19309 | .98118 | .21019 | .97766 | .22722 | .97384 | .24418 | .96973 | 52 |
| 9 | .17623 | .98435 | .19338 | .98112 | .21047 | .97760 | .22750 | .97378 | .24446 | .96966 | 51 |
| 10 | .17651 | .98430 | .19366 | .98107 | .21076 | .97754 | .22778 | .97371 | .24474 | .96959 | 50 |
| 11 | .17680 | .98425 | .19395 | .98101 | .21104 | .97748 | .22807 | .97365 | .24503 | .96952 | 49 |
| 12 | .17708 | .98420 | .19423 | .98096 | .21132 | .97742 | .22835 | .97358 | .24531 | .96945 | 48 |
| 13 | .17737 | .98414 | .19452 | .98090 | .21161 | .97735 | .22863 | .97351 | .24559 | .96937 | 47 |
| 14 | .17766 | .98409 | .19481 | .98084 | .21189 | .97729 | .22892 | .97345 | .24587 | .96930 | 46 |
| 15 | .17794 | .98404 | .19509 | .98079 | .21218 | .97723 | .22920 | .97338 | .24615 | .96923 | 45 |
| 16 | .17823 | .98399 | .19538 | .98073 | .21246 | .97717 | .22948 | .97331 | .24644 | .96916 | 44 |
| 17 | .17852 | .98394 | .19566 | .98067 | .21275 | .97711 | .22977 | .97325 | .24672 | .96909 | 43 |
| 18 | .17880 | .98389 | .19595 | .98061 | .21303 | .97705 | .23005 | .97318 | .24700 | .96902 | 42 |
| 19 | .17909 | .98383 | .19623 | .98056 | .21331 | .97698 | .23033 | .97311 | .24728 | .96894 | 41 |
| 20 | .17937 | .98378 | .19652 | .98050 | .21360 | .97692 | .23062 | .97304 | .24756 | .96887 | 40 |
| 21 | .17966 | .98373 | .19680 | .98044 | .21388 | .97686 | .23090 | .97298 | .24784 | .96880 | 39 |
| 22 | .17995 | .98368 | .19709 | .98039 | .21417 | .97680 | .23118 | .97291 | .24813 | .96873 | 38 |
| 23 | .18023 | .98362 | .19737 | .98033 | .21445 | .97673 | .23146 | .97284 | .24841 | .96866 | 37 |
| 24 | .18052 | .98357 | .19766 | .98027 | .21474 | .97667 | .23175 | .97278 | .24869 | .96858 | 36 |
| 25 | .18081 | .98352 | .19794 | .98021 | .21502 | .97661 | .23203 | .97271 | .24897 | .96851 | 35 |
| 26 | .18109 | .98347 | .19823 | .98016 | .21530 | .97655 | .23231 | .97264 | .24925 | .96844 | 34 |
| 27 | .18138 | .98341 | .19851 | .98010 | .21559 | .97648 | .23260 | .97257 | .24954 | .96837 | 33 |
| 28 | .18166 | .98336 | .19880 | .98004 | .21587 | .97642 | .23288 | .97251 | .24982 | .96829 | 32 |
| 29 | .18195 | .98331 | .19908 | .97998 | .21616 | .97636 | .23316 | .97244 | .25010 | .96822 | 31 |
| 30 | .18224 | .98325 | .19937 | .97992 | .21644 | .97630 | .23345 | .97237 | .25038 | .96815 | 30 |
| 31 | .18252 | .98320 | .19965 | .97987 | .21672 | .97623 | .23373 | .97230 | .25066 | .96807 | 29 |
| 32 | .18281 | .98315 | .19994 | .97981 | .21701 | .97617 | .23401 | .97223 | .25094 | .96800 | 28 |
| 33 | .18309 | .98310 | .20022 | .97975 | .21729 | .97611 | .23429 | .97217 | .25122 | .96793 | 27 |
| 34 | .18338 | .98304 | .20051 | .97969 | .21758 | .97604 | .23458 | .97210 | .25151 | .96786 | 26 |
| 35 | .18367 | .98299 | .20079 | .97963 | .21786 | .97598 | .23486 | .97203 | .25179 | .96778 | 25 |
| 36 | .18395 | .98294 | .20108 | .97958 | .21814 | .97592 | .23514 | .97196 | .25207 | .96771 | 24 |
| 37 | .18424 | .98288 | .20136 | .97952 | .21843 | .97585 | .23542 | .97189 | .25235 | .96764 | 23 |
| 38 | .18452 | .98283 | .20165 | .97946 | .21871 | .97579 | .23571 | .97182 | .25263 | .96756 | 22 |
| 39 | .18481 | .98277 | .20193 | .97940 | .21899 | .97573 | .23599 | .97176 | .25291 | .96749 | 21 |
| 40 | .18509 | .98272 | .20222 | .97934 | .21928 | .97566 | .23627 | .97169 | .25320 | .96742 | 20 |
| 41 | .18538 | .98267 | .20250 | .97928 | .21956 | .97560 | .23656 | .97162 | .25348 | .96734 | 19 |
| 42 | .18567 | .98261 | .20279 | .97922 | .21985 | .97553 | .23684 | .97155 | .25376 | .96727 | 18 |
| 43 | .18595 | .98256 | .20307 | .97916 | .22013 | .97547 | .23712 | .97148 | .25404 | .96719 | 17 |
| 44 | .18624 | .98250 | .20336 | .97910 | .22041 | .97541 | .23740 | .97141 | .25432 | .96712 | 16 |
| 45 | .18652 | .98245 | .20364 | .97905 | .22070 | .97534 | .23769 | .97134 | .25460 | .96705 | 15 |
| 46 | .18681 | .98240 | .20393 | .97899 | .22098 | .97528 | .23797 | .97127 | .25488 | .96697 | 14 |
| 47 | .18710 | .98234 | .20421 | .97893 | .22126 | .97521 | .23825 | .97120 | .25516 | .96690 | 13 |
| 48 | .18738 | .98229 | .20450 | .97887 | .22155 | .97515 | .23853 | .97113 | .25545 | .96682 | 12 |
| 49 | .18767 | .98223 | .20478 | .97881 | .22183 | .97508 | .23882 | .97106 | .25573 | .96675 | 11 |
| 50 | .18795 | .98218 | .20507 | .97875 | .22212 | .97502 | .23910 | .97100 | .25601 | .96667 | 10 |
| 51 | .18824 | .98212 | .20535 | .97869 | .22240 | .97496 | .23938 | .97093 | .25629 | .96660 | 9 |
| 52 | .18852 | .98207 | .20563 | .97863 | .22268 | .97489 | .23966 | .97086 | .25657 | .96653 | 8 |
| 53 | .18881 | .98201 | .20592 | .97857 | .22297 | .97483 | .23995 | .97079 | .25685 | .96645 | 7 |
| 54 | .18910 | .98196 | .20620 | .97851 | .22325 | .97476 | .24023 | .97072 | .25713 | .96638 | 6 |
| 55 | .18938 | .98190 | .20649 | .97845 | .22353 | .97470 | .24051 | .97065 | .25741 | .96630 | 5 |
| 56 | .18967 | .98185 | .20677 | .97839 | .22382 | .97463 | .24079 | .97058 | .25769 | .96623 | 4 |
| 57 | .18995 | .98179 | .20706 | .97833 | .22410 | .97457 | .24108 | .97051 | .25798 | .96615 | 3 |
| 58 | .19024 | .98174 | .20734 | .97827 | .22438 | .97450 | .24136 | .97044 | .25826 | .96608 | 2 |
| 59 | .19052 | .98168 | .20763 | .97821 | .22467 | .97444 | .24164 | .97037 | .25854 | .96600 | 1 |
| 60 | .19081 | .98163 | .20791 | .97815 | .22495 | .97437 | .24192 | .97030 | .25882 | .96593 | 0 |
| ′ | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | ′ |
|  | 79° | | 78° | | 77° | | 76° | | 75° | | |

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| ′ | 15° | | 16° | | 17° | | 18° | | 19° | | ′ |
|---|------|--------|------|--------|------|--------|------|--------|------|--------|---|
| | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | |
| 0 | .25882 | .96593 | .27564 | .96126 | .29237 | .95630 | .30902 | .95106 | .32557 | .94552 | 60 |
| 1 | .25910 | .96585 | .27592 | .96118 | .29265 | .95622 | .30929 | .95097 | .32584 | .94542 | 59 |
| 2 | .25938 | .96578 | .27620 | .96110 | .29293 | .95613 | .30957 | .95088 | .32612 | .94533 | 58 |
| 3 | .25966 | .96570 | .27648 | .96102 | .29321 | .95605 | .30985 | .95079 | .32639 | .94523 | 57 |
| 4 | .25994 | .96562 | .27676 | .96094 | .29348 | .95596 | .31012 | .95070 | .32667 | .94514 | 56 |
| 5 | .26022 | .96555 | .27704 | .96086 | .29376 | .95588 | .31040 | .95061 | .32694 | .94504 | 55 |
| 6 | .26050 | .96547 | .27731 | .96078 | .29404 | .95579 | .31068 | .95052 | .32722 | .94495 | 54 |
| 7 | .26079 | .96540 | .27759 | .96070 | .29432 | .95571 | .31095 | .95043 | .32749 | .94485 | 53 |
| 8 | .26107 | .96532 | .27787 | .96062 | .29460 | .95562 | .31123 | .95033 | .32777 | .94476 | 52 |
| 9 | .26135 | .96524 | .27815 | .96054 | .29487 | .95554 | .31151 | .95024 | .32804 | .94466 | 51 |
| 10 | .26163 | .96517 | .27843 | .96046 | .29515 | .95545 | .31178 | .95015 | .32832 | .94457 | 50 |
| 11 | .26191 | .96509 | .27871 | .96037 | .29543 | .95536 | .31206 | .95006 | .32859 | .94447 | 49 |
| 12 | .26219 | .96502 | .27899 | .96029 | .29571 | .95528 | .31233 | .94997 | .32887 | .94438 | 48 |
| 13 | .26247 | .96494 | .27927 | .96021 | .29599 | .95519 | .31261 | .94988 | .32914 | .94428 | 47 |
| 14 | .26275 | .96486 | .27955 | .96013 | .29626 | .95511 | .31289 | .94979 | .32942 | .94418 | 46 |
| 15 | .26303 | .96479 | .27983 | .96005 | .29654 | .95502 | .31316 | .94970 | .32969 | .94409 | 45 |
| 16 | .26331 | .96471 | .28011 | .95997 | .29682 | .95493 | .31344 | .94961 | .32997 | .94399 | 44 |
| 17 | .26359 | .96463 | .28039 | .95989 | .29710 | .95485 | .31372 | .94952 | .33024 | .94390 | 43 |
| 18 | .26387 | .96456 | .28067 | .95981 | .29737 | .95476 | .31399 | .94943 | .33051 | .94380 | 42 |
| 19 | .26415 | .96448 | .28095 | .95972 | .29765 | .95467 | .31427 | .94933 | .33079 | .94370 | 41 |
| 20 | .26443 | .96440 | .28123 | .95964 | .29793 | .95459 | .31454 | .94924 | .33106 | .94361 | 40 |
| 21 | .26471 | .96433 | .28150 | .95956 | .29821 | .95450 | .31482 | .94915 | .33134 | .94351 | 39 |
| 22 | .26500 | .96425 | .28178 | .95948 | .29849 | .95441 | .31510 | .94906 | .33161 | .94342 | 38 |
| 23 | .26528 | .96417 | .28206 | .95940 | .29876 | .95433 | .31537 | .94897 | .33189 | .94332 | 37 |
| 24 | .26556 | .96410 | .28234 | .95931 | .29904 | .95424 | .31565 | .94888 | .33216 | .94322 | 36 |
| 25 | .26584 | .96402 | .28262 | .95923 | .29932 | .95415 | .31593 | .94878 | .33244 | .94313 | 35 |
| 26 | .26612 | .96394 | .28290 | .95915 | .29960 | .95407 | .31620 | .94869 | .33271 | .94303 | 34 |
| 27 | .26640 | .96386 | .28318 | .95907 | .29987 | .95398 | .31648 | .94860 | .33298 | .94293 | 33 |
| 28 | .26668 | .96379 | .28346 | .95898 | .30015 | .95389 | .31675 | .94851 | .33326 | .94284 | 32 |
| 29 | .26696 | .96371 | .28374 | .95890 | .30043 | .95380 | .31703 | .94842 | .33353 | .94274 | 31 |
| 30 | .26724 | .96363 | .28402 | .95882 | .30071 | .95372 | .31730 | .94832 | .33381 | .94264 | 30 |
| 31 | .26752 | .96355 | .28429 | .95874 | .30098 | .95363 | .31758 | .94823 | .33408 | .94254 | 29 |
| 32 | .26780 | .96347 | .28457 | .95865 | .30126 | .95354 | .31786 | .94814 | .33436 | .94245 | 28 |
| 33 | .26808 | .96340 | .28485 | .95857 | .30154 | .95345 | .31813 | .94805 | .33463 | .94235 | 27 |
| 34 | .26836 | .96332 | .28513 | .95849 | .30182 | .95337 | .31841 | .94795 | .33490 | .94225 | 26 |
| 35 | .26864 | .96324 | .28541 | .95841 | .30209 | .95328 | .31868 | .94786 | .33518 | .94215 | 25 |
| 36 | .26892 | .96316 | .28569 | .95832 | .30237 | .95319 | .31896 | .94777 | .33545 | .94206 | 24 |
| 37 | .26920 | .96308 | .28597 | .95824 | .30265 | .95310 | .31923 | .94768 | .33573 | .94196 | 23 |
| 38 | .26948 | .96301 | .28625 | .95816 | .30292 | .95301 | .31951 | .94758 | .33600 | .94186 | 22 |
| 39 | .26976 | .96293 | .28652 | .95807 | .30320 | .95293 | .31979 | .94749 | .33627 | .94176 | 21 |
| 40 | .27004 | .96285 | .28680 | .95799 | .30348 | .95284 | .32006 | .94740 | .33655 | .94167 | 20 |
| 41 | .27032 | .96277 | .28708 | .95791 | .30376 | .95275 | .32034 | .94730 | .33682 | .94157 | 19 |
| 42 | .27060 | .96269 | .28736 | .95782 | .30403 | .95266 | .32061 | .94721 | .33710 | .94147 | 18 |
| 43 | .27088 | .96261 | .28764 | .95774 | .30431 | .95257 | .32089 | .94712 | .33737 | .94137 | 17 |
| 44 | .27116 | .96253 | .28792 | .95766 | .30459 | .95248 | .32116 | .94702 | .33764 | .94127 | 16 |
| 45 | .27144 | .96246 | .28820 | .95757 | .30486 | .95240 | .32144 | .94693 | .33792 | .94118 | 15 |
| 46 | .27172 | .96238 | .28847 | .95749 | .30514 | .95231 | .32171 | .94684 | .33819 | .94108 | 14 |
| 47 | .27200 | .96230 | .28875 | .95740 | .30542 | .95222 | .32199 | .94674 | .33846 | .94098 | 13 |
| 48 | .27228 | .96222 | .28903 | .95732 | .30570 | .95213 | .32227 | .94665 | .33874 | .94088 | 12 |
| 49 | .27256 | .96214 | .28931 | .95724 | .30597 | .95204 | .32254 | .94656 | .33901 | .94078 | 11 |
| 50 | .27284 | .96206 | .28959 | .95715 | .30625 | .95195 | .32282 | .94646 | .33929 | .94068 | 10 |
| 51 | .27312 | .96198 | .28987 | .95707 | .30653 | .95186 | .32309 | .94637 | .33956 | .94058 | 9 |
| 52 | .27340 | .96190 | .29015 | .95698 | .30680 | .95177 | .32337 | .94627 | .33983 | .94049 | 8 |
| 53 | .27368 | .96182 | .29042 | .95690 | .30708 | .95168 | .32364 | .94618 | .34011 | .94039 | 7 |
| 54 | .27396 | .96174 | .29070 | .95681 | .30736 | .95159 | .32392 | .94609 | .34038 | .94029 | 6 |
| 55 | .27424 | .96166 | .29098 | .95673 | .30763 | .95150 | .32419 | .94599 | .34065 | .94019 | 5 |
| 56 | .27452 | .96158 | .29126 | .95664 | .30791 | .95142 | .32447 | .94590 | .34093 | .94009 | 4 |
| 57 | .27480 | .96150 | .29154 | .95656 | .30819 | .95133 | .32474 | .94580 | .34120 | .93999 | 3 |
| 58 | .27508 | .96142 | .29182 | .95647 | .30846 | .95124 | .32502 | .94571 | .34147 | .93989 | 2 |
| 59 | .27536 | .96134 | .29209 | .95639 | .30874 | .95115 | .32529 | .94561 | .34175 | .93979 | 1 |
| 60 | .27564 | .96126 | .29237 | .95630 | .30902 | .95106 | .32557 | .94552 | .34202 | .93969 | 0 |
| | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | |
| ′ | 74° | | 73° | | 72° | | 71° | | 70° | | ′ |

| ′ | 20° Sine | 20° Cosine | 21° Sine | 21° Cosine | 22° Sine | 22° Cosine | 23° Sine | 23° Cosine | 24° Sine | 24° Cosine | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .34202 | .93969 | .35837 | .93358 | .37461 | .92718 | .39073 | .92050 | .40674 | .91355 | 60 |
| 1 | .34229 | .93959 | .35864 | .93348 | .37488 | .92707 | .39100 | .92039 | .40700 | .91343 | 59 |
| 2 | .34257 | .93949 | .35891 | .93337 | .37515 | .92697 | .39127 | .92028 | .40727 | .91331 | 58 |
| 3 | .34284 | .93939 | .35918 | .93327 | .37542 | .92686 | .39153 | .92016 | .40753 | .91319 | 57 |
| 4 | .34311 | .93929 | .35945 | .93316 | .37569 | .92675 | .39180 | .92005 | .40780 | .91307 | 56 |
| 5 | .34339 | .93919 | .35973 | .93306 | .37595 | .92664 | .39207 | .91994 | .40806 | .91295 | 55 |
| 6 | .34366 | .93909 | .36000 | .93295 | .37622 | .92653 | .39234 | .91982 | .40833 | .91283 | 54 |
| 7 | .34393 | .93899 | .36027 | .93285 | .37649 | .92642 | .39260 | .91971 | .40860 | .91272 | 53 |
| 8 | .34421 | .93889 | .36054 | .93274 | .37676 | .92631 | .39287 | .91959 | .40886 | .91260 | 52 |
| 9 | .34448 | .93879 | .36081 | .93264 | .37703 | .92620 | .39314 | .91948 | .40913 | .91248 | 51 |
| 10 | .34475 | .93869 | .36108 | .93253 | .37730 | .92609 | .39341 | .91936 | .40939 | .91236 | 50 |
| 11 | .34503 | .93859 | .36135 | .93243 | .37757 | .92598 | .39367 | .91925 | .40966 | .91224 | 49 |
| 12 | .34530 | .93849 | .36162 | .93232 | .37784 | .92587 | .39394 | .91914 | .40992 | .91212 | 48 |
| 13 | .34557 | .93839 | .36190 | .93222 | .37811 | .92576 | .39421 | .91902 | .41019 | .91200 | 47 |
| 14 | .34584 | .93829 | .36217 | .93211 | .37838 | .92565 | .39448 | .91891 | .41045 | .91188 | 46 |
| 15 | .34612 | .93819 | .36244 | .93201 | .37865 | .92554 | .39474 | .91879 | .41072 | .91176 | 45 |
| 16 | .34639 | .93809 | .36271 | .93190 | .37892 | .92543 | .39501 | .91868 | .41098 | .91164 | 44 |
| 17 | .34666 | .93799 | .36298 | .93180 | .37919 | .92532 | .39528 | .91856 | .41125 | .91152 | 43 |
| 18 | .34694 | .93789 | .36325 | .93169 | .37946 | .92521 | .39555 | .91845 | .41151 | .91140 | 42 |
| 19 | .34721 | .93779 | .36352 | .93159 | .37973 | .92510 | .39581 | .91833 | .41178 | .91128 | 41 |
| 20 | .34748 | .93769 | .36379 | .93148 | .37999 | .92499 | .39608 | .91822 | .41204 | .91116 | 40 |
| 21 | .34775 | .93759 | .36406 | .93137 | .38026 | .92488 | .39635 | .91810 | .41231 | .91104 | 39 |
| 22 | .34803 | .93748 | .36434 | .93127 | .38053 | .92477 | .39661 | .91799 | .41257 | .91092 | 38 |
| 23 | .34830 | .93738 | .36461 | .93116 | .38080 | .92466 | .39688 | .91787 | .41284 | .91080 | 37 |
| 24 | .34857 | .93728 | .36488 | .93106 | .38107 | .92455 | .39715 | .91775 | .41310 | .91068 | 36 |
| 25 | .34884 | .93718 | .36515 | .93095 | .38134 | .92444 | .39741 | .91764 | .41337 | .91056 | 35 |
| 26 | .34912 | .93708 | .36542 | .93084 | .38161 | .92432 | .39768 | .91752 | .41363 | .91044 | 34 |
| 27 | .34939 | .93698 | .36569 | .93074 | .38188 | .92421 | .39795 | .91741 | .41390 | .91032 | 33 |
| 28 | .34966 | .93688 | .36596 | .93063 | .38215 | .92410 | .39822 | .91729 | .41416 | .91020 | 32 |
| 29 | .34993 | .93677 | .36623 | .93052 | .38241 | .92399 | .39848 | .91718 | .41443 | .91008 | 31 |
| 30 | .35021 | .93667 | .36650 | .93042 | .38268 | .92388 | .39875 | .91706 | .41469 | .90996 | 30 |
| 31 | .35048 | .93657 | .36677 | .93031 | .38295 | .92377 | .39902 | .91694 | .41496 | .90984 | 29 |
| 32 | .35075 | .93647 | .36704 | .93020 | .38322 | .92366 | .39928 | .91683 | .41522 | .90972 | 28 |
| 33 | .35102 | .93637 | .36731 | .93010 | .38349 | .92355 | .39955 | .91671 | .41549 | .90960 | 27 |
| 34 | .35130 | .93626 | .36758 | .92999 | .38376 | .92343 | .39982 | .91660 | .41575 | .90948 | 26 |
| 35 | .35157 | .93616 | .36785 | .92988 | .38403 | .92332 | .40008 | .91648 | .41602 | .90936 | 25 |
| 36 | .35184 | .93606 | .36812 | .92978 | .38430 | .92321 | .40035 | .91636 | .41628 | .90924 | 24 |
| 37 | .35211 | .93596 | .36839 | .92967 | .38456 | .92310 | .40062 | .91625 | .41655 | .90911 | 23 |
| 38 | .35239 | .93585 | .36867 | .92956 | .38483 | .92299 | .40088 | .91613 | .41681 | .90899 | 22 |
| 39 | .35266 | .93575 | .36894 | .92945 | .38510 | .92287 | .40115 | .91601 | .41707 | .90887 | 21 |
| 40 | .35293 | .93565 | .36921 | .92935 | .38537 | .92276 | .40141 | .91590 | .41734 | .90875 | 20 |
| 41 | .35320 | .93555 | .36948 | .92924 | .38564 | .92265 | .40168 | .91578 | .41760 | .90863 | 19 |
| 42 | .35347 | .93544 | .36975 | .92913 | .38591 | .92254 | .40195 | .91566 | .41787 | .90851 | 18 |
| 43 | .35375 | .93534 | .37002 | .92902 | .38617 | .92243 | .40221 | .91555 | .41813 | .90839 | 17 |
| 44 | .35402 | .93524 | .37029 | .92892 | .38644 | .92231 | .40248 | .91543 | .41840 | .90826 | 16 |
| 45 | .35429 | .93514 | .37056 | .92881 | .38671 | .92220 | .40275 | .91531 | .41866 | .90814 | 15 |
| 46 | .35456 | .93502 | .37083 | .92870 | .38698 | .92209 | .40301 | .91519 | .41892 | .90802 | 14 |
| 47 | .35484 | .93493 | .37110 | .92859 | .38725 | .92198 | .40328 | .91508 | .41919 | .90790 | 13 |
| 48 | .35511 | .93483 | .37137 | .92849 | .38752 | .92186 | .40355 | .91496 | .41945 | .90778 | 12 |
| 49 | .35538 | .93472 | .37164 | .92838 | .38778 | .92175 | .40381 | .91484 | .41972 | .90766 | 11 |
| 50 | .35565 | .93462 | .37191 | .92827 | .38805 | .92164 | .40408 | .91472 | .41998 | .90753 | 10 |
| 51 | .35592 | .93452 | .37218 | .92816 | .38832 | .92152 | .40434 | .91461 | .42024 | .90741 | 9 |
| 52 | .35619 | .93441 | .37245 | .92805 | .38859 | .92141 | .40461 | .91449 | .42051 | .90729 | 8 |
| 53 | .35647 | .93431 | .37272 | .92794 | .38886 | .92130 | .40488 | .91437 | .42077 | .90717 | 7 |
| 54 | .35674 | .93420 | .37299 | .92784 | .38912 | .92119 | .40514 | .91425 | .42104 | .90704 | 6 |
| 55 | .35701 | .93410 | .37326 | .92773 | .38939 | .92107 | .40541 | .91414 | .42130 | .90692 | 5 |
| 56 | .35728 | .93400 | .37353 | .92762 | .38966 | .92096 | .40567 | .91402 | .42156 | .90680 | 4 |
| 57 | .35755 | .93389 | .37380 | .92751 | .38993 | .92085 | .40594 | .91390 | .42183 | .90668 | 3 |
| 58 | .35782 | .93379 | .37407 | .92740 | .39020 | .92073 | .40621 | .91378 | .42209 | .90655 | 2 |
| 59 | .35810 | .93368 | .37434 | .92729 | .39046 | .92062 | .40647 | .91366 | .42235 | .90643 | 1 |
| 60 | .35837 | .93358 | .37461 | .92718 | .39073 | .92050 | .40674 | .91355 | .42262 | .90631 | 0 |
| ′ | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | ′ |
| | 69° | | 68° | | 67° | | 66° | | 65° | | |

| ′ | 25° Sine | 25° Cosine | 26° Sine | 26° Cosine | 27° Sine | 27° Cosine | 28° Sine | 28° Cosine | 29° Sine | 29° Cosine | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .42262 | .90631 | .43837 | .89879 | .45399 | .89101 | .46947 | .88295 | .48481 | .87462 | 60 |
| 1 | .42288 | .90618 | .43863 | .89867 | .45425 | .89087 | .46973 | .88281 | .48506 | .87448 | 59 |
| 2 | .42315 | .90606 | .43889 | .89854 | .45451 | .89074 | .46999 | .88267 | .48532 | .87434 | 58 |
| 3 | .42341 | .90594 | .43916 | .89841 | .45477 | .89061 | .47024 | .88254 | .48557 | .87420 | 57 |
| 4 | .42367 | .90582 | .43942 | .89828 | .45503 | .89048 | .47050 | .88240 | .48583 | .87406 | 56 |
| 5 | .42394 | .90569 | .43968 | .89816 | .45529 | .89035 | .47076 | .88226 | .48608 | .87391 | 55 |
| 6 | .42420 | .90557 | .43994 | .89803 | .45554 | .89021 | .47101 | .88213 | .48634 | .87377 | 54 |
| 7 | .42446 | .90545 | .44020 | .89790 | .45580 | .89008 | .47127 | .88199 | .48659 | .87363 | 53 |
| 8 | .42473 | .90532 | .44046 | .89777 | .45606 | .88995 | .47153 | .88185 | .48684 | .87349 | 52 |
| 9 | .42499 | .90520 | .44072 | .89764 | .45632 | .88981 | .47178 | .88172 | .48710 | .87335 | 51 |
| 10 | .42525 | .90507 | .44098 | .89752 | .45658 | .88968 | .47204 | .88158 | .48735 | .87321 | 50 |
| 11 | .42552 | .90495 | .44124 | .89739 | .45684 | .88955 | .47229 | .88144 | .48761 | .87306 | 49 |
| 12 | .42578 | .90483 | .44151 | .89726 | .45710 | .88942 | .47255 | .88130 | .48786 | .87292 | 48 |
| 13 | .42604 | .90470 | .44177 | .89713 | .45736 | .88928 | .47281 | .88117 | .48811 | .87278 | 47 |
| 14 | .42631 | .90458 | .44203 | .89700 | .45762 | .88915 | .47306 | .88103 | .48837 | .87264 | 46 |
| 15 | .42657 | .90446 | .44229 | .89687 | .45787 | .88902 | .47332 | .88089 | .48862 | .87250 | 45 |
| 16 | .42683 | .90433 | .44255 | .89674 | .45813 | .88888 | .47358 | .88075 | .48888 | .87235 | 44 |
| 17 | .42709 | .90421 | .44281 | .89662 | .45839 | .88875 | .47383 | .88062 | .48913 | .87221 | 43 |
| 18 | .42736 | .90408 | .44307 | .89649 | .45865 | .88862 | .47409 | .88048 | .48938 | .87207 | 42 |
| 19 | .42762 | .90396 | .44333 | .89636 | .45891 | .88848 | .47434 | .88034 | .48964 | .87193 | 41 |
| 20 | .42788 | .90383 | .44359 | .89623 | .45917 | .88835 | .47460 | .88020 | .48989 | .87178 | 40 |
| 21 | .42815 | .90371 | .44385 | .89610 | .45942 | .88822 | .47486 | .88006 | .49014 | .87164 | 39 |
| 22 | .42841 | .90358 | .44411 | .89597 | .45968 | .88808 | .47511 | .87993 | .49040 | .87150 | 38 |
| 23 | .42867 | .90346 | .44437 | .89584 | .45994 | .88795 | .47537 | .87979 | .49065 | .87136 | 37 |
| 24 | .42894 | .90334 | .44464 | .89571 | .46020 | .88782 | .47562 | .87965 | .49090 | .87121 | 36 |
| 25 | .42920 | .90321 | .44490 | .89558 | .46046 | .88768 | .47588 | .87951 | .49116 | .87107 | 35 |
| 26 | .42946 | .90309 | .44516 | .89545 | .46072 | .88755 | .47614 | .87937 | .49141 | .87093 | 34 |
| 27 | .42972 | .90296 | .44542 | .89532 | .46097 | .88741 | .47639 | .87923 | .49166 | .87079 | 33 |
| 28 | .42999 | .90284 | .44568 | .89519 | .46123 | .88728 | .47665 | .87909 | .49192 | .87064 | 32 |
| 29 | .43025 | .90271 | .44594 | .89506 | .46149 | .88715 | .47690 | .87896 | .49217 | .87050 | 31 |
| 30 | .43051 | .90259 | .44620 | .89493 | .46175 | .88701 | .47716 | .87882 | .49242 | .87036 | 30 |
| 31 | .43077 | .90246 | .44646 | .89480 | .46201 | .88688 | .47741 | .87868 | .49268 | .87021 | 29 |
| 32 | .43104 | .90233 | .44672 | .89467 | .46226 | .88674 | .47767 | .87854 | .49293 | .87007 | 28 |
| 33 | .43130 | .90221 | .44698 | .89454 | .46252 | .88661 | .47793 | .87840 | .49318 | .86993 | 27 |
| 34 | .43156 | .90208 | .44724 | .89441 | .46278 | .88647 | .47818 | .87826 | .49344 | .86978 | 26 |
| 35 | .43182 | .90196 | .44750 | .89428 | .46304 | .88634 | .47844 | .87812 | .49369 | .86964 | 25 |
| 36 | .43209 | .90183 | .44776 | .89415 | .46330 | .88620 | .47869 | .87798 | .49394 | .86949 | 24 |
| 37 | .43235 | .90171 | .44802 | .89402 | .46355 | .88607 | .47895 | .87784 | .49419 | .86935 | 23 |
| 38 | .43261 | .90158 | .44828 | .89389 | .46381 | .88593 | .47920 | .87770 | .49445 | .86921 | 22 |
| 39 | .43287 | .90146 | .44854 | .89376 | .46407 | .88580 | .47946 | .87756 | .49470 | .86906 | 21 |
| 40 | .43313 | .90133 | .44880 | .89363 | .46433 | .88566 | .47971 | .87743 | .49495 | .86892 | 20 |
| 41 | .43340 | .90120 | .44906 | .89350 | .46458 | .88553 | .47997 | .87729 | .49521 | .86878 | 19 |
| 42 | .43366 | .90108 | .44932 | .89337 | .46484 | .88539 | .48022 | .87715 | .49546 | .86863 | 18 |
| 43 | .43392 | .90095 | .44958 | .89324 | .46510 | .88526 | .48048 | .87701 | .49571 | .86849 | 17 |
| 44 | .43418 | .90082 | .44984 | .89311 | .46536 | .88512 | .48073 | .87687 | .49596 | .86834 | 16 |
| 45 | .43445 | .90070 | .45010 | .89298 | .46561 | .88499 | .48099 | .87673 | .49622 | .86820 | 15 |
| 46 | .43471 | .90057 | .45036 | .89285 | .46587 | .88485 | .48124 | .87659 | .49647 | .86805 | 14 |
| 47 | .43497 | .90045 | .45062 | .89272 | .46613 | .88472 | .48150 | .87645 | .49672 | .86791 | 13 |
| 48 | .43523 | .90032 | .45088 | .89259 | .46639 | .88458 | .48175 | .87631 | .49697 | .86777 | 12 |
| 49 | .43549 | .90019 | .45114 | .89245 | .46664 | .88445 | .48201 | .87617 | .49723 | .86762 | 11 |
| 50 | .43575 | .90007 | .45140 | .89232 | .46690 | .88431 | .48226 | .87603 | .49748 | .86748 | 10 |
| 51 | .43602 | .89994 | .45166 | .89219 | .46716 | .88417 | .48252 | .87589 | .49773 | .86733 | 9 |
| 52 | .43628 | .89981 | .45192 | .89206 | .46742 | .88404 | .48277 | .87575 | .49798 | .86719 | 8 |
| 53 | .43654 | .89968 | .45218 | .89193 | .46767 | .88390 | .48303 | .87561 | .49824 | .86704 | 7 |
| 54 | .43680 | .89956 | .45243 | .89180 | .46793 | .88377 | .48328 | .87546 | .49849 | .86690 | 6 |
| 55 | .43706 | .89943 | .45269 | .89167 | .46819 | .88363 | .48354 | .87532 | .49874 | .86675 | 5 |
| 56 | .43733 | .89930 | .45295 | .89153 | .46844 | .88349 | .48379 | .87518 | .49899 | .86661 | 4 |
| 57 | .43759 | .89918 | .45321 | .89140 | .46870 | .88336 | .48405 | .87504 | .49924 | .86646 | 3 |
| 58 | .43785 | .89905 | .45347 | .89127 | .46896 | .88322 | .48430 | .87490 | .49950 | .86632 | 2 |
| 59 | .43811 | .89892 | .45373 | .89114 | .46921 | .88308 | .48456 | .87476 | .49975 | .86617 | 1 |
| 60 | .43837 | .89879 | .45399 | .89101 | .46947 | .88295 | .48481 | .87462 | .50000 | .86603 | 0 |
| ′ | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | ′ |
| | 64° | | 63° | | 62° | | 61° | | 60° | | |

| ′ | 30° | | 31° | | 32° | | 33° | | 34° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
|  | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine |  |
| 0 | .50000 | .86603 | .51504 | .85717 | .52992 | .84805 | .54464 | .83867 | .55919 | .82904 | 60 |
| 1 | .50025 | .86588 | .51529 | .85702 | .53017 | .84789 | .54488 | .83851 | .55943 | .82887 | 59 |
| 2 | .50050 | .86573 | .51554 | .85687 | .53041 | .84774 | .54513 | .83835 | .55968 | .82871 | 58 |
| 3 | .50076 | .86559 | .51579 | .85672 | .53066 | .84759 | .54537 | .83819 | .55992 | .82855 | 57 |
| 4 | .50101 | .86544 | .51604 | .85657 | .53091 | .84743 | .54561 | .83804 | .56016 | .82839 | 56 |
| 5 | .50126 | .86530 | .51628 | .85642 | .53115 | .84728 | .54586 | .83788 | .56040 | .82822 | 55 |
| 6 | .50151 | .86515 | .51653 | .85627 | .53140 | .84712 | .54610 | .83772 | .56064 | .82806 | 54 |
| 7 | .50176 | .86501 | .51678 | .85612 | .53164 | .84697 | .54635 | .83756 | .56088 | .82790 | 53 |
| 8 | .50201 | .86486 | .51703 | .85597 | .53189 | .84681 | .54659 | .83740 | .56112 | .82773 | 52 |
| 9 | .50227 | .86471 | .51728 | .85582 | .53214 | .84666 | .54683 | .83724 | .56136 | .82757 | 51 |
| 10 | .50252 | .86457 | .51753 | .85567 | .53238 | .84650 | .54708 | .83708 | .56160 | .82741 | 50 |
| 11 | .50277 | .86442 | .51778 | .85551 | .53263 | .84635 | .54732 | .83692 | .56184 | .82724 | 49 |
| 12 | .50302 | .86427 | .51803 | .85536 | .53288 | .84619 | .54756 | .83676 | .56208 | .82708 | 48 |
| 13 | .50327 | .86413 | .51828 | .85521 | .53312 | .84604 | .54781 | .83660 | .56232 | .82692 | 47 |
| 14 | .50352 | .86398 | .51852 | .85506 | .53337 | .84588 | .54805 | .83645 | .56256 | .82675 | 46 |
| 15 | .50377 | .86384 | .51877 | .85491 | .53361 | .84573 | .54829 | .83629 | .56280 | .82659 | 45 |
| 16 | .50403 | .86369 | .51902 | .85476 | .53386 | .84557 | .54854 | .83613 | .56305 | .82643 | 44 |
| 17 | .50428 | .86354 | .51927 | .85461 | .53411 | .84542 | .54878 | .83597 | .56329 | .82626 | 43 |
| 18 | .50453 | .86340 | .51952 | .85446 | .53435 | .84526 | .54902 | .83581 | .56353 | .82610 | 42 |
| 19 | .50478 | .86325 | .51977 | .85431 | .53460 | .84511 | .54927 | .83565 | .56377 | .82593 | 41 |
| 20 | .50503 | .86310 | .52002 | .85416 | .53484 | .84495 | .54951 | .83549 | .56401 | .82577 | 40 |
| 21 | .50528 | .86295 | .52026 | .85401 | .53509 | .84480 | .54975 | .83533 | .56425 | .82561 | 39 |
| 22 | .50553 | .86281 | .52051 | .85385 | .53534 | .84464 | .54999 | .83517 | .56449 | .82544 | 38 |
| 23 | .50578 | .86266 | .52076 | .85370 | .53558 | .84448 | .55024 | .83501 | .56473 | .82528 | 37 |
| 24 | .50603 | .86251 | .52101 | .85355 | .53583 | .84433 | .55048 | .83485 | .56497 | .82511 | 36 |
| 25 | .50628 | .86237 | .52126 | .85340 | .53607 | .84417 | .55072 | .83469 | .56521 | .82495 | 35 |
| 26 | .50654 | .86222 | .52151 | .85325 | .53632 | .84402 | .55097 | .83453 | .56545 | .82478 | 34 |
| 27 | .50679 | .86207 | .52175 | .85310 | .53656 | .84386 | .55121 | .83437 | .56569 | .82462 | 33 |
| 28 | .50704 | .86192 | .52200 | .85294 | .53681 | .84370 | .55145 | .83421 | .56593 | .82446 | 32 |
| 29 | .50729 | .86178 | .52225 | .85279 | .53705 | .84355 | .55169 | .83405 | .56617 | .82429 | 31 |
| 30 | .50754 | .86163 | .52250 | .85264 | .53730 | .84339 | .55194 | .83389 | .56641 | .82413 | 30 |
| 31 | .50779 | .86148 | .52275 | .85249 | .53754 | .84324 | .55218 | .83373 | .56665 | .82396 | 29 |
| 32 | .50804 | .86133 | .52299 | .85234 | .53779 | .84308 | .55242 | .83356 | .56689 | .82380 | 28 |
| 33 | .50829 | .86119 | .52324 | .85218 | .53804 | .84292 | .55266 | .83340 | .56713 | .82363 | 27 |
| 34 | .50854 | .86104 | .52349 | .85203 | .53828 | .84277 | .55291 | .83324 | .56736 | .82347 | 26 |
| 35 | .50879 | .86089 | .52374 | .85188 | .53853 | .84261 | .55315 | .83308 | .56760 | .82330 | 25 |
| 36 | .50904 | .86074 | .52399 | .85173 | .53877 | .84245 | .55339 | .83292 | .56784 | .82314 | 24 |
| 37 | .50929 | .86059 | .52423 | .85157 | .53902 | .84230 | .55363 | .83276 | .56808 | .82297 | 23 |
| 38 | .50954 | .86045 | .52448 | .85142 | .53926 | .84214 | .55388 | .83260 | .56832 | .82281 | 22 |
| 39 | .50979 | .86030 | .52473 | .85127 | .53951 | .84198 | .55412 | .83244 | .56856 | .82264 | 21 |
| 40 | .51004 | .86015 | .52498 | .85112 | .53975 | .84182 | .55436 | .83228 | .56880 | .82248 | 20 |
| 41 | .51029 | .86000 | .52522 | .85096 | .54000 | .84167 | .55460 | .83212 | .56904 | .82231 | 19 |
| 42 | .51054 | .85985 | .52547 | .85081 | .54024 | .84151 | .55484 | .83195 | .56928 | .82214 | 18 |
| 43 | .51079 | .85970 | .52572 | .85066 | .54049 | .84135 | .55509 | .83179 | .56952 | .82198 | 17 |
| 44 | .51104 | .85956 | .52597 | .85051 | .54073 | .84120 | .55533 | .83163 | .56976 | .82181 | 16 |
| 45 | .51129 | .85941 | .52621 | .85035 | .54097 | .84104 | .55557 | .83147 | .57000 | .82165 | 15 |
| 46 | .51154 | .85926 | .52646 | .85020 | .54122 | .84088 | .55581 | .83131 | .57024 | .82148 | 14 |
| 47 | .51179 | .85911 | .52671 | .85005 | .54146 | .84072 | .55605 | .83115 | .57047 | .82132 | 13 |
| 48 | .51204 | .85896 | .52696 | .84989 | .54171 | .84057 | .55630 | .83098 | .57071 | .82115 | 12 |
| 49 | .51229 | .85881 | .52720 | .84974 | .54195 | .84041 | .55654 | .83082 | .57095 | .82098 | 11 |
| 50 | .51254 | .85866 | .52745 | .84959 | .54220 | .84025 | .55678 | .83066 | .57119 | .82082 | 10 |
| 51 | .51279 | .85851 | .52770 | .84943 | .54244 | .84009 | .55702 | .83050 | .57143 | .82065 | 9 |
| 52 | .51304 | .85836 | .52794 | .84928 | .54269 | .83994 | .55726 | .83034 | .57167 | .82048 | 8 |
| 53 | .51329 | .85821 | .52819 | .84913 | .54293 | .83978 | .55750 | .83017 | .57191 | .82032 | 7 |
| 54 | .51354 | .85806 | .52844 | .84897 | .54317 | .83962 | .55775 | .83001 | .57215 | .82015 | 6 |
| 55 | .51379 | .85792 | .52869 | .84882 | .54342 | .83946 | .55799 | .82985 | .57238 | .81999 | 5 |
| 56 | .51404 | .85777 | .52893 | .84866 | .54366 | .83930 | .55823 | .82969 | .57262 | .81982 | 4 |
| 57 | .51429 | .85762 | .52918 | .84851 | .54391 | .83915 | .55847 | .82953 | .57286 | .81965 | 3 |
| 58 | .51454 | .85747 | .52943 | .84836 | .54415 | .83899 | .55871 | .82936 | .57310 | .81949 | 2 |
| 59 | .51479 | .85732 | .52967 | .84820 | .54440 | .83883 | .55895 | .82920 | .57334 | .81932 | 1 |
| 60 | .51504 | .85717 | .52992 | .84805 | .54464 | .83867 | .55919 | .82904 | .57358 | .81915 | 0 |
|  | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine |  |
| ′ | 59° | | 58° | | 57° | | 56° | | 55° | | ′ |

| ′ | 35° Sine | 35° Cosine | 36° Sine | 36° Cosine | 37° Sine | 37° Cosine | 38° Sine | 38° Cosine | 39° Sine | 39° Cosine | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .57358 | .81915 | .58779 | .80902 | .60182 | .79864 | .61566 | .78801 | .62932 | .77715 | 60 |
| 1 | .57381 | .81899 | .58802 | .80885 | .60205 | .79846 | .61589 | .78783 | .62955 | .77696 | 59 |
| 2 | .57405 | .81882 | .58826 | .80867 | .60228 | .79829 | .61612 | .78765 | .62977 | .77678 | 58 |
| 3 | .57429 | .81865 | .58849 | .80850 | .60251 | .79811 | .61635 | .78747 | .63000 | .77660 | 57 |
| 4 | .57453 | .81848 | .58873 | .80833 | .60274 | .79793 | .61658 | .78729 | .63022 | .77641 | 56 |
| 5 | .57477 | .81832 | .58896 | .80816 | .60298 | .79776 | .61681 | .78711 | .63045 | .77623 | 55 |
| 6 | .57501 | .81815 | .58920 | .80799 | .60321 | .79758 | .61704 | .78694 | .63068 | .77605 | 54 |
| 7 | .57524 | .81798 | .58943 | .80782 | .60344 | .79741 | .61726 | .78676 | .63090 | .77586 | 53 |
| 8 | .57548 | .81782 | .58967 | .80765 | .60367 | .79723 | .61749 | .78658 | .63113 | .77568 | 52 |
| 9 | .57572 | .81765 | .58990 | .80748 | .60390 | .79706 | .61772 | .78640 | .63135 | .77550 | 51 |
| 10 | .57596 | .81748 | .59014 | .80730 | .60414 | .79688 | .61795 | .78622 | .63158 | .77531 | 50 |
| 11 | .57619 | .81731 | .59037 | .80713 | .60437 | .79671 | .61818 | .78604 | .63180 | .77513 | 49 |
| 12 | .57643 | .81714 | .59061 | .80696 | .60460 | .79653 | .61841 | .78586 | .63203 | .77494 | 48 |
| 13 | .57667 | .81698 | .59084 | .80679 | .60483 | .79635 | .61864 | .78568 | .63225 | .77476 | 47 |
| 14 | .57691 | .81681 | .59108 | .80662 | .60506 | .79618 | .61887 | .78550 | .63248 | .77458 | 46 |
| 15 | .57715 | .81664 | .59131 | .80644 | .60529 | .79600 | .61909 | .78532 | .63271 | .77439 | 45 |
| 16 | .57738 | .81647 | .59154 | .80627 | .60553 | .79583 | .61932 | .78514 | .63293 | .77421 | 44 |
| 17 | .57762 | .81631 | .59178 | .80610 | .60576 | .79565 | .61955 | .78496 | .63316 | .77402 | 43 |
| 18 | .57786 | .81614 | .59201 | .80593 | .60599 | .79547 | .61978 | .78478 | .63338 | .77384 | 42 |
| 19 | .57810 | .81597 | .59225 | .80576 | .60622 | .79530 | .62001 | .78460 | .63361 | .77366 | 41 |
| 20 | .57833 | .81580 | .59248 | .80558 | .60645 | .79512 | .62024 | .78442 | .63383 | .77347 | 40 |
| 21 | .57857 | .81563 | .59272 | .80541 | .60668 | .79494 | .62046 | .78424 | .63406 | .77329 | 39 |
| 22 | .57881 | .81546 | .59295 | .80524 | .60691 | .79477 | .62069 | .78405 | .63428 | .77310 | 38 |
| 23 | .57904 | .81530 | .59318 | .80507 | .60714 | .79459 | .62092 | .78387 | .63451 | .77292 | 37 |
| 24 | .57928 | .81513 | .59342 | .80489 | .60738 | .79441 | .62115 | .78369 | .63473 | .77273 | 36 |
| 25 | .57952 | .81496 | .59365 | .80472 | .60761 | .79424 | .62138 | .78351 | .63496 | .77255 | 35 |
| 26 | .57976 | .81479 | .59389 | .80455 | .60784 | .79406 | .62160 | .78333 | .63518 | .77236 | 34 |
| 27 | .57999 | .81462 | .59412 | .80438 | .60807 | .79388 | .62183 | .78315 | .63540 | .77218 | 33 |
| 28 | .58023 | .81445 | .59436 | .80420 | .60830 | .79371 | .62206 | .78297 | .63563 | .77199 | 32 |
| 29 | .58047 | .81428 | .59459 | .80403 | .60853 | .79353 | .62229 | .78279 | .63585 | .77181 | 31 |
| 30 | .58070 | .81412 | .59482 | .80386 | .60876 | .79335 | .62251 | .78261 | .63608 | .77162 | 30 |
| 31 | .58094 | .81395 | .59506 | .80368 | .60899 | .79318 | .62274 | .78243 | .63630 | .77144 | 29 |
| 32 | .58118 | .81378 | .59529 | .80351 | .60922 | .79300 | .62297 | .78225 | .63653 | .77125 | 28 |
| 33 | .58141 | .81361 | .59552 | .80334 | .60945 | .79282 | .62320 | .78206 | .63675 | .77107 | 27 |
| 34 | .58165 | .81344 | .59576 | .80316 | .60968 | .79264 | .62342 | .78188 | .63698 | .77088 | 26 |
| 35 | .58189 | .81327 | .59599 | .80299 | .60991 | .79247 | .62365 | .78170 | .63720 | .77070 | 25 |
| 36 | .58212 | .81310 | .59622 | .80282 | .61015 | .79229 | .62388 | .78152 | .63742 | .77051 | 24 |
| 37 | .58236 | .81293 | .59646 | .80264 | .61038 | .79211 | .62411 | .78134 | .63765 | .77033 | 23 |
| 38 | .58260 | .81276 | .59669 | .80247 | .61061 | .79193 | .62433 | .78116 | .63787 | .77014 | 22 |
| 39 | .58283 | .81259 | .59693 | .80230 | .61084 | .79176 | .62456 | .78098 | .63810 | .76996 | 21 |
| 40 | .58307 | .81242 | .59716 | .80212 | .61107 | .79158 | .62479 | .78079 | .63832 | .76977 | 20 |
| 41 | .58330 | .81225 | .59739 | .80195 | .61130 | .79140 | .62502 | .78061 | .63854 | .76959 | 19 |
| 42 | .58354 | .81208 | .59763 | .80178 | .61153 | .79122 | .62524 | .78043 | .63877 | .76940 | 18 |
| 43 | .58378 | .81191 | .59786 | .80160 | .61176 | .79105 | .62547 | .78025 | .63899 | .76921 | 17 |
| 44 | .58401 | .81174 | .59809 | .80143 | .61199 | .79087 | .62570 | .78007 | .63922 | .76903 | 16 |
| 45 | .58425 | .81157 | .59832 | .80125 | .61222 | .79069 | .62592 | .77988 | .63944 | .76884 | 15 |
| 46 | .58449 | .81140 | .59856 | .80108 | .61245 | .79051 | .62615 | .77970 | .63966 | .76866 | 14 |
| 47 | .58472 | .81123 | .59879 | .80091 | .61268 | .79033 | .62638 | .77952 | .63989 | .76847 | 13 |
| 48 | .58496 | .81106 | .59902 | .80073 | .61291 | .79016 | .62660 | .77934 | .64011 | .76828 | 12 |
| 49 | .58519 | .81089 | .59926 | .80056 | .61314 | .78998 | .62683 | .77916 | .64033 | .76810 | 11 |
| 50 | .58543 | .81072 | .59949 | .80038 | .61337 | .78980 | .62706 | .77897 | .64056 | .76791 | 10 |
| 51 | .58567 | .81055 | .59972 | .80021 | .61360 | .78962 | .62728 | .77879 | .64078 | .76772 | 9 |
| 52 | .58590 | .81038 | .59995 | .80003 | .61383 | .78944 | .62751 | .77861 | .64100 | .76754 | 8 |
| 53 | .58614 | .81021 | .60019 | .79986 | .61406 | .78926 | .62774 | .77843 | .64123 | .76735 | 7 |
| 54 | .58637 | .81004 | .60042 | .79968 | .61429 | .78908 | .62796 | .77824 | .64145 | .76717 | 6 |
| 55 | .58661 | .80987 | .60065 | .79951 | .61451 | .78891 | .62819 | .77806 | .64167 | .76698 | 5 |
| 56 | .58684 | .80970 | .60089 | .79934 | .61474 | .78873 | .62842 | .77788 | .64190 | .76679 | 4 |
| 57 | .58708 | .80953 | .60112 | .79916 | .61497 | .78855 | .62864 | .77769 | .64212 | .76661 | 3 |
| 58 | .58731 | .80936 | .60135 | .79899 | .61520 | .78837 | .62887 | .77751 | .64234 | .76642 | 2 |
| 59 | .58755 | .80919 | .60158 | .79881 | .61543 | .78819 | .62909 | .77733 | .64256 | .76623 | 1 |
| 60 | .58779 | .80902 | .60182 | .79864 | .61566 | .78801 | .62932 | .77715 | .64279 | .76604 | 0 |
| ′ | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | ′ |
| | 54° | | 53° | | 52° | | 51° | | 50° | | |

| ′ | 40° Sine | 40° Cosine | 41° Sine | 41° Cosine | 42° Sine | 42° Cosine | 43° Sine | 43° Cosine | 44° Sine | 44° Cosine | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .64279 | .76604 | .65606 | .75471 | .66913 | .74314 | .68200 | .73135 | .69466 | .71934 | 60 |
| 1 | .64301 | .76586 | .65628 | .75452 | .66935 | .74295 | .68221 | .73116 | .69487 | .71914 | 59 |
| 2 | .64323 | .76567 | .65650 | .75433 | .66956 | .74276 | .68242 | .73096 | .69508 | .71894 | 58 |
| 3 | .64346 | .76548 | .65672 | .75414 | .66978 | .74256 | .68264 | .73076 | .69529 | .71873 | 57 |
| 4 | .64368 | .76530 | .65694 | .75395 | .66999 | .74237 | .68285 | .73056 | .69549 | .71853 | 56 |
| 5 | .64390 | .76511 | .65716 | .75375 | .67021 | .74217 | .68306 | .73036 | .69570 | .71833 | 55 |
| 6 | .64412 | .76492 | .65738 | .75356 | .67043 | .74198 | .68327 | .73016 | .69591 | .71813 | 54 |
| 7 | .64435 | .76473 | .65759 | .75337 | .67064 | .74178 | .68349 | .72996 | .69612 | .71792 | 53 |
| 8 | .64457 | .76455 | .65781 | .75318 | .67086 | .74159 | .68370 | .72976 | .69633 | .71772 | 52 |
| 9 | .64479 | .76436 | .65803 | .75299 | .67107 | .74139 | .68391 | .72957 | .69654 | .71752 | 51 |
| 10 | .64501 | .76417 | .65825 | .75280 | .67129 | .74120 | .68412 | .72937 | .69675 | .71732 | 50 |
| 11 | .64524 | .76398 | .65847 | .75261 | .67151 | .74100 | .68434 | .72917 | .69696 | .71711 | 49 |
| 12 | .64546 | .76380 | .65869 | .75241 | .67172 | .74080 | .68455 | .72897 | .69717 | .71691 | 48 |
| 13 | .64568 | .76361 | .65891 | .75222 | .67194 | .74061 | .68476 | .72877 | .69737 | .71671 | 47 |
| 14 | .64590 | .76342 | .65913 | .75203 | .67215 | .74041 | .68497 | .72857 | .69758 | .71650 | 46 |
| 15 | .64612 | .76323 | .65935 | .75184 | .67237 | .74022 | .68518 | .72837 | .69779 | .71630 | 45 |
| 16 | .64635 | .76304 | .65956 | .75165 | .67258 | .74002 | .68539 | .72817 | .69800 | .71610 | 44 |
| 17 | .64657 | .76286 | .65978 | .75146 | .67280 | .73983 | .68561 | .72797 | .69821 | .71590 | 43 |
| 18 | .64679 | .76267 | .66000 | .75126 | .67301 | .73963 | .68582 | .72777 | .69842 | .71569 | 42 |
| 19 | .64701 | .76248 | .66022 | .75107 | .67323 | .73944 | .68603 | .72757 | .69862 | .71549 | 41 |
| 20 | .64723 | .76229 | .66044 | .75088 | .67344 | .73924 | .68624 | .72737 | .69883 | .71529 | 40 |
| 21 | .64746 | .76210 | .66066 | .75069 | .67366 | .73904 | .68645 | .72717 | .69904 | .71508 | 39 |
| 22 | .64768 | .76192 | .66088 | .75050 | .67387 | .73885 | .68666 | .72697 | .69925 | .71488 | 38 |
| 23 | .64790 | .76173 | .66109 | .75030 | .67409 | .73865 | .68688 | .72677 | .69946 | .71468 | 37 |
| 24 | .64812 | .76154 | .66131 | .75011 | .67430 | .73846 | .68709 | .72657 | .69966 | .71447 | 36 |
| 25 | .64834 | .76135 | .66153 | .74992 | .67452 | .73826 | .68730 | .72637 | .69987 | .71427 | 35 |
| 26 | .64856 | .76116 | .66175 | .74973 | .67473 | .73806 | .68751 | .72617 | .70008 | .71407 | 34 |
| 27 | .64878 | .76097 | .66197 | .74953 | .67495 | .73787 | .68772 | .72597 | .70029 | .71386 | 33 |
| 28 | .64901 | .76078 | .66218 | .74934 | .67516 | .73767 | .68793 | .72577 | .70049 | .71366 | 32 |
| 29 | .64923 | .76059 | .66240 | .74915 | .67538 | .73747 | .68814 | .72557 | .70070 | .71345 | 31 |
| 30 | .64945 | .76041 | .66262 | .74896 | .67559 | .73728 | .68835 | .72537 | .70091 | .71325 | 30 |
| 31 | .64967 | .76022 | .66284 | .74876 | .67580 | .73708 | .68857 | .72517 | .70112 | .71305 | 29 |
| 32 | .64989 | .76003 | .66306 | .74857 | .67602 | .73688 | .68878 | .72497 | .70132 | .71284 | 28 |
| 33 | .65011 | .75984 | .66327 | .74838 | .67623 | .73669 | .68899 | .72477 | .70153 | .71264 | 27 |
| 34 | .65033 | .75965 | .66349 | .74818 | .67645 | .73649 | .68920 | .72457 | .70174 | .71243 | 26 |
| 35 | .65055 | .75946 | .66371 | .74799 | .67666 | .73629 | .68941 | .72437 | .70195 | .71223 | 25 |
| 36 | .65077 | .75927 | .66393 | .74780 | .67688 | .73610 | .68962 | .72417 | .70215 | .71203 | 24 |
| 37 | .65100 | .75908 | .66414 | .74760 | .67709 | .73590 | .68983 | .72397 | .70236 | .71182 | 23 |
| 38 | .65122 | .75889 | .66436 | .74741 | .67730 | .73570 | .69004 | .72377 | .70257 | .71162 | 22 |
| 39 | .65144 | .75870 | .66458 | .74722 | .67752 | .73551 | .69025 | .72357 | .70277 | .71141 | 21 |
| 40 | .65166 | .75851 | .66480 | .74702 | .67773 | .73531 | .69046 | .72337 | .70298 | .71121 | 20 |
| 41 | .65188 | .75832 | .66501 | .74683 | .67795 | .73511 | .69067 | .72317 | .70319 | .71100 | 19 |
| 42 | .65210 | .75813 | .66523 | .74664 | .67816 | .73491 | .69088 | .72297 | .70339 | .71080 | 18 |
| 43 | .65232 | .75794 | .66545 | .74644 | .67837 | .73472 | .69109 | .72277 | .70360 | .71059 | 17 |
| 44 | .65254 | .75775 | .66566 | .74625 | .67859 | .73452 | .69130 | .72257 | .70381 | .71039 | 16 |
| 45 | .65276 | .75756 | .66588 | .74606 | .67880 | .73432 | .69151 | .72236 | .70401 | .71019 | 15 |
| 46 | .65298 | .75738 | .66610 | .74586 | .67901 | .73413 | .69172 | .72216 | .70422 | .70998 | 14 |
| 47 | .65320 | .75719 | .66632 | .74567 | .67923 | .73393 | .69193 | .72196 | .70443 | .70978 | 13 |
| 48 | .65342 | .75700 | .66653 | .74548 | .67944 | .73373 | .69214 | .72176 | .70463 | .70957 | 12 |
| 49 | .65364 | .75680 | .66675 | .74528 | .67965 | .73353 | .69235 | .72156 | .70484 | .70937 | 11 |
| 50 | .65386 | .75661 | .66697 | .74509 | .67987 | .73333 | .69256 | .72136 | .70505 | .70916 | 10 |
| 51 | .65408 | .75642 | .66718 | .74489 | .68008 | .73314 | .69277 | .72116 | .70525 | .70896 | 9 |
| 52 | .65430 | .75623 | .66740 | .74470 | .68029 | .73294 | .69298 | .72095 | .70546 | .70875 | 8 |
| 53 | .65452 | .75604 | .66762 | .74451 | .68051 | .73274 | .69319 | .72075 | .70567 | .70855 | 7 |
| 54 | .65474 | .75585 | .66783 | .74431 | .68072 | .73254 | .69340 | .72055 | .70587 | .70834 | 6 |
| 55 | .65496 | .75566 | .66805 | .74412 | .68093 | .73234 | .69361 | .72035 | .70608 | .70813 | 5 |
| 56 | .65518 | .75547 | .66827 | .74392 | .68115 | .73215 | .69382 | .72015 | .70628 | .70793 | 4 |
| 57 | .65540 | .75528 | .66848 | .74373 | .68136 | .73195 | .69403 | .71995 | .70649 | .70772 | 3 |
| 58 | .65562 | .75509 | .66870 | .74353 | .68157 | .73175 | .69424 | .71974 | .70670 | .70752 | 2 |
| 59 | .65584 | .75490 | .66891 | .74334 | .68179 | .73155 | .69445 | .71954 | .70690 | .70731 | 1 |
| 60 | .65606 | .75471 | .66913 | .74314 | .68200 | .73135 | .69466 | .71934 | .70711 | .70711 | 0 |
| ′ | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | Cosine | Sine | ′ |
| | 49° | | 48° | | 47° | | 46° | | 45° | | |

| ′ | 0° Tang | 0° Cotang | 1° Tang | 1° Cotang | 2° Tang | 2° Cotang | 3° Tang | 3° Cotang | 4° Tang | 4° Cotang | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .00000 | Infin. | .01746 | 57.2900 | .03492 | 28.6363 | .05241 | 19.0811 | .06993 | 14.3007 | 60 |
| 1 | .00029 | 3437.75 | .01775 | 56.3506 | .03521 | 28.3994 | .05270 | 18.9755 | .07022 | 14.2411 | 59 |
| 2 | .00058 | 1718.87 | .01804 | 55.4415 | .03550 | 28.1664 | .05299 | 18.8711 | .07051 | 14.1821 | 58 |
| 3 | .00087 | 1145.92 | .01833 | 54.5613 | .03579 | 27.9372 | .05328 | 18.7678 | .07080 | 14.1235 | 57 |
| 4 | .00116 | 859.436 | .01862 | 53.7086 | .03609 | 27.7117 | .05357 | 18.6656 | .07110 | 14.0655 | 56 |
| 5 | .00145 | 687.549 | .01891 | 52.8821 | .03638 | 27.4899 | .05387 | 18.5645 | .07139 | 14.0079 | 55 |
| 6 | .00175 | 572.957 | .01920 | 52.0807 | .03667 | 27.2715 | .05416 | 18.4645 | .07168 | 13.9507 | 54 |
| 7 | .00204 | 491.106 | .01949 | 51.3032 | .03696 | 27.0566 | .05445 | 18.3655 | .07197 | 13.8940 | 53 |
| 8 | .00233 | 429.718 | .01978 | 50.5485 | .03725 | 26.8450 | .05474 | 18.2677 | .07227 | 13.8378 | 52 |
| 9 | .00262 | 381.971 | .02007 | 49.8157 | .03754 | 26.6367 | .05503 | 18.1708 | .07256 | 13.7821 | 51 |
| 10 | .00291 | 343.774 | .02036 | 49.1039 | .03783 | 26.4316 | .05533 | 18.0750 | .07285 | 13.7267 | 50 |
| 11 | .00320 | 312.521 | .02066 | 48.4121 | .03812 | 26.2296 | .05562 | 17.9802 | .07314 | 13.6719 | 49 |
| 12 | .00349 | 286.478 | .02095 | 47.7395 | .03842 | 26.0307 | .05591 | 17.8863 | .07344 | 13.6174 | 48 |
| 13 | .00378 | 264.441 | .02124 | 47.0853 | .03871 | 25.8348 | .05620 | 17.7934 | .07373 | 13.5634 | 47 |
| 14 | .00407 | 245.552 | .02153 | 46.4489 | .03900 | 25.6418 | .05649 | 17.7015 | .07402 | 13.5098 | 46 |
| 15 | .00436 | 229.182 | .02182 | 45.8294 | .03929 | 25.4517 | .05678 | 17.6106 | .07431 | 13.4566 | 45 |
| 16 | .00465 | 214.858 | .02211 | 45.2261 | .03958 | 25.2644 | .05708 | 17.5205 | .07461 | 13.4039 | 44 |
| 17 | .00495 | 202.219 | .02240 | 44.6386 | .03987 | 25.0798 | .05737 | 17.4314 | .07490 | 13.3515 | 43 |
| 18 | .00524 | 190.984 | .02269 | 44.0661 | .04016 | 24.8978 | .05766 | 17.3432 | .07519 | 13.2996 | 42 |
| 19 | .00553 | 180.932 | .02298 | 43.5081 | .04046 | 24.7185 | .05795 | 17.2558 | .07548 | 13.2480 | 41 |
| 20 | .00582 | 171.885 | .02328 | 42.9641 | .04075 | 24.5418 | .05824 | 17.1693 | .07578 | 13.1969 | 40 |
| 21 | .00611 | 163.700 | .02357 | 42.4335 | .04104 | 24.3675 | .05854 | 17.0837 | .07607 | 13.1461 | 39 |
| 22 | .00640 | 156.259 | .02386 | 41.9158 | .04133 | 24.1957 | .05883 | 16.9990 | .07636 | 13.0958 | 38 |
| 23 | .00669 | 149.465 | .02415 | 41.4106 | .04162 | 24.0263 | .05912 | 16.9150 | .07665 | 13.0458 | 37 |
| 24 | .00698 | 143.237 | .02444 | 40.9174 | .04191 | 23.8593 | .05941 | 16.8319 | .07695 | 12.9962 | 36 |
| 25 | .00727 | 137.507 | .02473 | 40.4358 | .04220 | 23.6945 | .05970 | 16.7496 | .07724 | 12.9469 | 35 |
| 26 | .00756 | 132.219 | .02502 | 39.9655 | .04250 | 23.5321 | .05999 | 16.6681 | .07753 | 12.8981 | 34 |
| 27 | .00785 | 127.321 | .02531 | 39.5059 | .04279 | 23.3718 | .06029 | 16.5874 | .07782 | 12.8496 | 33 |
| 28 | .00815 | 122.774 | .02560 | 39.0568 | .04308 | 23.2137 | .06058 | 16.5075 | .07812 | 12.8014 | 32 |
| 29 | .00844 | 118.540 | .02589 | 38.6177 | .04337 | 23.0577 | .06087 | 16.4283 | .07841 | 12.7536 | 31 |
| 30 | .00873 | 114.589 | .02619 | 38.1885 | .04366 | 22.9038 | .06116 | 16.3499 | .07870 | 12.7062 | 30 |
| 31 | .00902 | 110.892 | .02648 | 37.7686 | .04395 | 22.7519 | .06145 | 16.2722 | .07899 | 12.6591 | 29 |
| 32 | .00931 | 107.426 | .02677 | 37.3579 | .04424 | 22.6020 | .06175 | 16.1952 | .07929 | 12.6124 | 28 |
| 33 | .00960 | 104.171 | .02706 | 36.9560 | .04454 | 22.4541 | .06204 | 16.1190 | .07958 | 12.5660 | 27 |
| 34 | .00989 | 101.107 | .02735 | 36.5627 | .04483 | 22.3081 | .06233 | 16.0435 | .07987 | 12.5199 | 26 |
| 35 | .01018 | 98.2179 | .02764 | 36.1776 | .04512 | 22.1640 | .06262 | 15.9687 | .08017 | 12.4742 | 25 |
| 36 | .01047 | 95.4895 | .02793 | 35.8006 | .04541 | 22.0217 | .06291 | 15.8945 | .08046 | 12.4288 | 24 |
| 37 | .01076 | 92.9085 | .02822 | 35.4313 | .04570 | 21.8813 | .06321 | 15.8211 | .08075 | 12.3858 | 23 |
| 38 | .01105 | 90.4633 | .02851 | 35.0695 | .04599 | 21.7426 | .06350 | 15.7483 | .08104 | 12.3390 | 22 |
| 39 | .01135 | 88.1436 | .02881 | 34.7151 | .04628 | 21.6056 | .06379 | 15.6762 | .08134 | 12.2946 | 21 |
| 40 | .01164 | 85.9398 | .02910 | 34.3678 | .04658 | 21.4704 | .06408 | 15.6048 | .08163 | 12.2505 | 20 |
| 41 | .01193 | 83.8435 | .02939 | 34.0273 | .04687 | 21.3369 | .06437 | 15.5340 | .08192 | 12.2067 | 19 |
| 42 | .01222 | 81.8470 | .02968 | 33.6935 | .04716 | 21.2049 | .06467 | 15.4638 | .08221 | 12.1632 | 18 |
| 43 | .01251 | 79.9434 | .02997 | 33.3662 | .04745 | 21.0747 | .06496 | 15.3945 | .08251 | 12.1201 | 17 |
| 44 | .01280 | 78.1263 | .03026 | 33.0452 | .04774 | 20.9460 | .06525 | 15.3254 | .08280 | 12.0772 | 16 |
| 45 | .01309 | 76.3900 | .03055 | 32.7303 | .04803 | 20.8188 | .06554 | 15.2571 | .08309 | 12.0346 | 15 |
| 46 | .01338 | 74.7292 | .03084 | 32.4213 | .04833 | 20.6932 | .06584 | 15.1893 | .08339 | 11.9923 | 14 |
| 47 | .01367 | 73.1390 | .03114 | 32.1181 | .04862 | 20.5691 | .06613 | 15.1222 | .08368 | 11.9504 | 13 |
| 48 | .01396 | 71.6151 | .03143 | 31.8205 | .04891 | 20.4465 | .06642 | 15.0557 | .08397 | 11.9087 | 12 |
| 49 | .01425 | 70.1533 | .03172 | 31.5284 | .04920 | 20.3253 | .06671 | 14.9898 | .08427 | 11.8673 | 11 |
| 50 | .01455 | 68.7501 | .03201 | 31.2416 | .04949 | 20.2056 | .06700 | 14.9244 | .08456 | 11.8262 | 10 |
| 51 | .01484 | 67.4019 | .03230 | 30.9599 | .04978 | 20.0872 | .06730 | 14.8596 | .08485 | 11.7853 | 9 |
| 52 | .01513 | 66.1055 | .03259 | 30.6833 | .05007 | 19.9702 | .06759 | 14.7954 | .08514 | 11.7448 | 8 |
| 53 | .01542 | 64.8580 | .03288 | 30.4116 | .05037 | 19.8546 | .06788 | 14.7317 | .08544 | 11.7045 | 7 |
| 54 | .01571 | 63.6567 | .03317 | 30.1446 | .05066 | 19.7403 | .06817 | 14.6685 | .08573 | 11.6645 | 6 |
| 55 | .01600 | 62.4992 | .03346 | 29.8823 | .05095 | 19.6273 | .06847 | 14.6059 | .08602 | 11.6248 | 5 |
| 56 | .01629 | 61.3829 | .03376 | 29.6245 | .05124 | 19.5156 | .06876 | 14.5438 | .08632 | 11.5853 | 4 |
| 57 | .01658 | 60.3058 | .03405 | 29.3711 | .05153 | 19.4051 | .06905 | 14.4823 | .08661 | 11.5461 | 3 |
| 58 | .01687 | 59.2659 | .03434 | 29.1220 | .05182 | 19.2959 | .06934 | 14.4212 | .08690 | 11.5072 | 2 |
| 59 | .01716 | 58.2612 | .03463 | 28.8771 | .05212 | 19.1879 | .06963 | 14.3607 | .08720 | 11.4685 | 1 |
| 60 | .01746 | 57.2900 | .03492 | 28.6363 | .05241 | 19.0811 | .06993 | 14.3007 | .08749 | 11.4301 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 89° | | 88° | | 87° | | 86° | | 85° | | |

| ′ | 5° | | 6° | | 7° | | 8° | | 9° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .08749 | 11.4301 | .10510 | 9.51436 | .12278 | 8.14435 | .14054 | 7.11537 | .15838 | 6.31375 | 60 |
| 1 | .08778 | 11.3919 | .10540 | 9.48781 | .12308 | 8.12481 | .14084 | 7.10038 | .15868 | 6.30189 | 59 |
| 2 | .08807 | 11.3540 | .10569 | 9.46141 | .12338 | 8.10536 | .14113 | 7.08546 | .15898 | 6.29007 | 58 |
| 3 | .08837 | 11.3163 | .10599 | 9.43515 | .12367 | 8.08600 | .14143 | 7.07059 | .15928 | 6.27829 | 57 |
| 4 | .08866 | 11.2789 | .10628 | 9.40904 | .12397 | 8.06674 | .14173 | 7.05579 | .15958 | 6.26655 | 56 |
| 5 | .08895 | 11.2417 | .10657 | 9.38307 | .12426 | 8.04756 | .14202 | 7.04105 | .15988 | 6.25486 | 55 |
| 6 | .08925 | 11.2048 | .10687 | 9.35724 | .12456 | 8.02848 | .14232 | 7.02637 | .16017 | 6.24321 | 54 |
| 7 | .08954 | 11.1681 | .10716 | 9.33155 | .12485 | 8.00948 | .14262 | 7.01174 | .16047 | 6.23160 | 53 |
| 8 | .08983 | 11.1316 | .10746 | 9.30599 | .12515 | 7.99058 | .14291 | 6.99718 | .16077 | 6.22003 | 52 |
| 9 | .09013 | 11.0954 | .10775 | 9.28058 | .12544 | 7.97176 | .14321 | 6.98268 | .16107 | 6.20851 | 51 |
| 10 | .09042 | 11.0594 | .10805 | 9.25530 | .12574 | 7.95302 | .14351 | 6.96823 | .16137 | 6.19703 | 50 |
| 11 | .09071 | 11.0237 | .10834 | 9.23016 | .12603 | 7.93438 | .14381 | 6.95385 | .16167 | 6.18559 | 49 |
| 12 | .09101 | 10.9882 | .10863 | 9.20516 | .12633 | 7.91582 | .14410 | 6.93952 | .16196 | 6.17419 | 48 |
| 13 | .09130 | 10.9529 | .10893 | 9.18028 | .12662 | 7.89734 | .14440 | 6.92525 | .16226 | 6.16283 | 47 |
| 14 | .09159 | 10.9178 | .10922 | 9.15554 | .12692 | 7.87895 | .14470 | 6.91104 | .16256 | 6.15151 | 46 |
| 15 | .09189 | 10.8829 | .10952 | 9.13093 | .12722 | 7.86064 | .14499 | 6.89688 | .16286 | 6.14023 | 45 |
| 16 | .09218 | 10.8483 | .10981 | 9.10646 | .12751 | 7.84242 | .14529 | 6.88278 | .16316 | 6.12899 | 44 |
| 17 | .09247 | 10.8139 | .11011 | 9.08211 | .12781 | 7.82428 | .14559 | 6.86874 | .16346 | 6.11779 | 43 |
| 18 | .09277 | 10.7797 | .11040 | 9.05789 | .12810 | 7.80622 | .14588 | 6.85475 | .16376 | 6.10664 | 42 |
| 19 | .09306 | 10.7457 | .11070 | 9.03379 | .12840 | 7.78825 | .14618 | 6.84082 | .16405 | 6.09552 | 41 |
| 20 | .09335 | 10.7119 | .11099 | 9.00983 | .12869 | 7.77035 | .14648 | 6.82694 | .16435 | 6.08444 | 40 |
| 21 | .09365 | 10.6783 | .11128 | 8.98598 | .12899 | 7.75254 | .14678 | 6.81312 | .16465 | 6.07340 | 39 |
| 22 | .09394 | 10.6450 | .11158 | 8.96227 | .12929 | 7.73480 | .14707 | 6.79936 | .16495 | 6.06240 | 38 |
| 23 | .09423 | 10.6118 | .11187 | 8.93867 | .12958 | 7.71715 | .14737 | 6.78564 | .16525 | 6.05143 | 37 |
| 24 | .09453 | 10.5789 | .11217 | 8.91520 | .12988 | 7.69957 | .14767 | 6.77199 | .16555 | 6.04051 | 36 |
| 25 | .09482 | 10.5462 | .11246 | 8.89185 | .13017 | 7.68208 | .14796 | 6.75838 | .16585 | 6.02962 | 35 |
| 26 | .09511 | 10.5136 | .11276 | 8.86862 | .13047 | 7.66466 | .14826 | 6.74483 | .16615 | 6.01878 | 34 |
| 27 | .09541 | 10.4813 | .11305 | 8.84551 | .13076 | 7.64732 | .14856 | 6.73133 | .16645 | 6.00797 | 33 |
| 28 | .09570 | 10.4491 | .11335 | 8.82252 | .13106 | 7.63005 | .14886 | 6.71789 | .16674 | 5.99720 | 32 |
| 29 | .09600 | 10.4172 | .11364 | 8.79964 | .13136 | 7.61287 | .14915 | 6.70450 | .16704 | 5.98646 | 31 |
| 30 | .09629 | 10.3854 | .11394 | 8.77689 | .13165 | 7.59575 | .14945 | 6.69116 | .16734 | 5.97576 | 30 |
| 31 | .09658 | 10.3538 | .11423 | 8.75425 | .13195 | 7.57872 | .14975 | 6.67787 | .16764 | 5.96510 | 29 |
| 32 | .09688 | 10.3224 | .11452 | 8.73172 | .13224 | 7.56176 | .15005 | 6.66463 | .16794 | 5.95448 | 28 |
| 33 | .09717 | 10.2913 | .11482 | 8.70931 | .13254 | 7.54487 | .15034 | 6.65144 | .16824 | 5.94390 | 27 |
| 34 | .09746 | 10.2602 | .11511 | 8.68701 | .13284 | 7.52806 | .15064 | 6.63831 | .16854 | 5.93335 | 26 |
| 35 | .09776 | 10.2294 | .11541 | 8.66482 | .13313 | 7.51132 | .15094 | 6.62523 | .16884 | 5.92283 | 25 |
| 36 | .09805 | 10.1988 | .11570 | 8.64275 | .13343 | 7.49465 | .15124 | 6.61219 | .16914 | 5.91236 | 24 |
| 37 | .09834 | 10.1683 | .11600 | 8.62078 | .13372 | 7.47806 | .15153 | 6.59921 | .16944 | 5.90191 | 23 |
| 38 | .09864 | 10.1381 | .11629 | 8.59893 | .13402 | 7.46154 | .15183 | 6.58627 | .16974 | 5.89151 | 22 |
| 39 | .09893 | 10.1080 | .11659 | 8.57718 | .13432 | 7.44509 | .15213 | 6.57339 | .17004 | 5.88114 | 21 |
| 40 | .09923 | 10.0780 | .11688 | 8.55555 | .13461 | 7.42871 | .15243 | 6.56055 | .17033 | 5.87080 | 20 |
| 41 | .09952 | 10.0483 | .11718 | 8.53402 | .13491 | 7.41240 | .15272 | 6.54777 | .17063 | 5.86051 | 19 |
| 42 | .09981 | 10.0187 | .11747 | 8.51259 | .13521 | 7.39616 | .15302 | 6.53503 | .17093 | 5.85024 | 18 |
| 43 | .10011 | 9.98931 | .11777 | 8.49128 | .13550 | 7.37999 | .15332 | 6.52234 | .17123 | 5.84001 | 17 |
| 44 | .10040 | 9.96007 | .11806 | 8.47007 | .13580 | 7.36389 | .15362 | 6.50970 | .17153 | 5.82982 | 16 |
| 45 | .10069 | 9.93101 | .11836 | 8.44896 | .13609 | 7.34786 | .15391 | 6.49710 | .17183 | 5.81966 | 15 |
| 46 | .10099 | 9.90211 | .11865 | 8.42795 | .13639 | 7.33190 | .15421 | 6.48456 | .17213 | 5.80953 | 14 |
| 47 | .10128 | 9.87338 | .11895 | 8.40705 | .13669 | 7.31600 | .15451 | 6.47206 | .17243 | 5.79944 | 13 |
| 48 | .10158 | 9.84482 | .11924 | 8.38625 | .13698 | 7.30018 | .15481 | 6.45961 | .17273 | 5.78938 | 12 |
| 49 | .10187 | 9.81641 | .11954 | 8.36555 | .13728 | 7.28442 | .15511 | 6.44720 | .17303 | 5.77936 | 11 |
| 50 | .10216 | 9.78817 | .11983 | 8.34496 | .13758 | 7.26873 | .15540 | 6.43484 | .17333 | 5.76937 | 10 |
| 51 | .10246 | 9.76009 | .12013 | 8.32446 | .13787 | 7.25310 | .15570 | 6.42253 | .17363 | 5.75941 | 9 |
| 52 | .10275 | 9.73217 | .12042 | 8.30406 | .13817 | 7.23754 | .15600 | 6.41026 | .17393 | 5.74949 | 8 |
| 53 | .10305 | 9.70441 | .12072 | 8.28376 | .13846 | 7.22204 | .15630 | 6.39804 | .17423 | 5.73960 | 7 |
| 54 | .10334 | 9.67680 | .12101 | 8.26355 | .13876 | 7.20661 | .15660 | 6.38587 | .17453 | 5.72974 | 6 |
| 55 | .10363 | 9.64935 | .12131 | 8.24345 | .13906 | 7.19125 | .15689 | 6.37374 | .17483 | 5.71992 | 5 |
| 56 | .10393 | 9.62205 | .12160 | 8.22344 | .13935 | 7.17594 | .15719 | 6.36165 | .17513 | 5.71013 | 4 |
| 57 | .10422 | 9.59490 | .12190 | 8.20352 | .13965 | 7.16071 | .15749 | 6.34961 | .17543 | 5.70037 | 3 |
| 58 | .10452 | 9.56791 | .12219 | 8.18370 | .13995 | 7.14553 | .15779 | 6.33761 | .17573 | 5.69064 | 2 |
| 59 | .10481 | 9.54106 | .12249 | 8.16398 | .14024 | 7.13042 | .15809 | 6.32566 | .17603 | 5.68094 | 1 |
| 60 | .10510 | 9.51436 | .12278 | 8.14435 | .14054 | 7.11537 | .15838 | 6.31375 | .17633 | 5.67128 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 84° | | 83° | | 82° | | 81° | | 80° | | |

| ′ | 10° Tang | Cotang | 11° Tang | Cotang | 12° Tang | Cotang | 13° Tang | Cotang | 14° Tang | Cotang | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .17633 | 5.67128 | .19438 | 5.14455 | .21256 | 4.70463 | .23087 | 4.33148 | .24933 | 4.01078 | 60 |
| 1 | .17663 | 5.66165 | .19468 | 5.13658 | .21286 | 4.69791 | .23117 | 4.32573 | .24964 | 4.00582 | 59 |
| 2 | .17693 | 5.65205 | .19498 | 5.12862 | .21316 | 4.69121 | .23148 | 4.32001 | .24995 | 4.00086 | 58 |
| 3 | .17723 | 5.64248 | .19529 | 5.12069 | .21347 | 4.68452 | .23179 | 4.31430 | .25026 | 3.99592 | 57 |
| 4 | .17753 | 5.63295 | .19559 | 5.11279 | .21377 | 4.67786 | .23209 | 4.30860 | .25056 | 3.99099 | 56 |
| 5 | .17783 | 5.62344 | .19589 | 5.10490 | .21408 | 4.67121 | .23240 | 4.30291 | .25087 | 3.98607 | 55 |
| 6 | .17813 | 5.61397 | .19619 | 5.09704 | .21438 | 4.66458 | .23271 | 4.29724 | .25118 | 3.98117 | 54 |
| 7 | .17843 | 5.60452 | .19649 | 5.08921 | .21469 | 4.65797 | .23301 | 4.29159 | .25149 | 3.97627 | 53 |
| 8 | .17873 | 5.59511 | .19680 | 5.08139 | .21499 | 4.65138 | .23332 | 4.28595 | .25180 | 3.97139 | 52 |
| 9 | .17903 | 5.58573 | .19710 | 5.07360 | .21529 | 4.64480 | .23363 | 4.28032 | .25211 | 3.96651 | 51 |
| 10 | .17933 | 5.57638 | .19740 | 5.06584 | .21560 | 4.63825 | .23393 | 4.27471 | .25242 | 3.96165 | 50 |
| 11 | .17963 | 5.56706 | .19770 | 5.05809 | .21590 | 4.63171 | .23424 | 4.26911 | .25273 | 3.95680 | 49 |
| 12 | .17993 | 5.55777 | .19801 | 5.05037 | .21621 | 4.62518 | .23455 | 4.26352 | .25304 | 3.95196 | 48 |
| 13 | .18023 | 5.54851 | .19831 | 5.04267 | .21651 | 4.61868 | .23485 | 4.25795 | .25335 | 3.94713 | 47 |
| 14 | .18053 | 5.53927 | .19861 | 5.03499 | .21682 | 4.61219 | .23516 | 4.25239 | .25366 | 3.94232 | 46 |
| 15 | .18083 | 5.53007 | .19891 | 5.02734 | .21712 | 4.60572 | .23547 | 4.24685 | .25397 | 3.93751 | 45 |
| 16 | .18113 | 5.52090 | .19921 | 5.01971 | .21743 | 4.59927 | .23578 | 4.24132 | .25428 | 3.93271 | 44 |
| 17 | .18143 | 5.51176 | .19952 | 5.01210 | .21773 | 4.59283 | .23608 | 4.23580 | .25459 | 3.92793 | 43 |
| 18 | .18173 | 5.50264 | .19982 | 5.00451 | .21804 | 4.58641 | .23639 | 4.23030 | .25490 | 3.92316 | 42 |
| 19 | .18203 | 5.49356 | .20012 | 4.99695 | .21834 | 4.58001 | .23670 | 4.22481 | .25521 | 3.91839 | 41 |
| 20 | .18233 | 5.48451 | .20042 | 4.98940 | .21864 | 4.57363 | .23700 | 4.21933 | .25552 | 3.91364 | 40 |
| 21 | .18263 | 5.47548 | .20073 | 4.98188 | .21895 | 4.56726 | .23731 | 4.21387 | .25583 | 3.90890 | 39 |
| 22 | .18293 | 5.46648 | .20103 | 4.97438 | .21925 | 4.56091 | .23762 | 4.20842 | .25614 | 3.90417 | 38 |
| 23 | .18323 | 5.45751 | .20133 | 4.96690 | .21956 | 4.55458 | .23793 | 4.20298 | .25645 | 3.89945 | 37 |
| 24 | .18353 | 5.44857 | .20164 | 4.95945 | .21986 | 4.54826 | .23823 | 4.19756 | .25676 | 3.89474 | 36 |
| 25 | .18384 | 5.43966 | .20194 | 4.95201 | .22017 | 4.54196 | .23854 | 4.19215 | .25707 | 3.89004 | 35 |
| 26 | .18414 | 5.43077 | .20224 | 4.94460 | .22047 | 4.53568 | .23885 | 4.18675 | .25738 | 3.88536 | 34 |
| 27 | .18444 | 5.42192 | .20254 | 4.93721 | .22078 | 4.52941 | .23916 | 4.18137 | .25769 | 3.88068 | 33 |
| 28 | .18474 | 5.41309 | .20285 | 4.92984 | .22108 | 4.52316 | .23946 | 4.17600 | .25800 | 3.87601 | 32 |
| 29 | .18504 | 5.40429 | .20315 | 4.92249 | .22139 | 4.51693 | .23977 | 4.17064 | .25831 | 3.87136 | 31 |
| 30 | .18534 | 5.39552 | .20345 | 4.91516 | .22169 | 4.51071 | .24008 | 4.16530 | .25862 | 3.86671 | 30 |
| 31 | .18564 | 5.38677 | .20376 | 4.90785 | .22200 | 4.50451 | .24039 | 4.15997 | .25893 | 3.86208 | 29 |
| 32 | .18594 | 5.37805 | .20406 | 4.90056 | .22231 | 4.49832 | .24069 | 4.15465 | .25924 | 3.85745 | 28 |
| 33 | .18624 | 5.36936 | .20436 | 4.89330 | .22261 | 4.49215 | .24100 | 4.14934 | .25955 | 3.85284 | 27 |
| 34 | .18654 | 5.36070 | .20466 | 4.88605 | .22292 | 4.48600 | .24131 | 4.14405 | .25986 | 3.84824 | 26 |
| 35 | .18684 | 5.35206 | .20497 | 4.87882 | .22322 | 4.47986 | .24162 | 4.13877 | .26017 | 3.84364 | 25 |
| 36 | .18714 | 5.34345 | .20527 | 4.87162 | .22353 | 4.47374 | .24193 | 4.13350 | .26048 | 3.83906 | 24 |
| 37 | .18745 | 5.33487 | .20557 | 4.86444 | .22383 | 4.46764 | .24223 | 4.12825 | .26079 | 3.83449 | 23 |
| 38 | .18775 | 5.32631 | .20588 | 4.85727 | .22414 | 4.46155 | .24254 | 4.12301 | .26110 | 3.82992 | 22 |
| 39 | .18805 | 5.31778 | .20618 | 4.85013 | .22444 | 4.45548 | .24285 | 4.11778 | .26141 | 3.82537 | 21 |
| 40 | .18835 | 5.30928 | .20648 | 4.84300 | .22475 | 4.44942 | .24316 | 4.11256 | .26172 | 3.82083 | 20 |
| 41 | .18865 | 5.30080 | .20679 | 4.83590 | .22505 | 4.44338 | .24347 | 4.10736 | .26203 | 3.81630 | 19 |
| 42 | .18895 | 5.29235 | .20709 | 4.82882 | .22536 | 4.43735 | .24377 | 4.10216 | .26235 | 3.81177 | 18 |
| 43 | .18925 | 5.28393 | .20739 | 4.82175 | .22567 | 4.43134 | .24408 | 4.09699 | .26266 | 3.80726 | 17 |
| 44 | .18955 | 5.27553 | .20770 | 4.81471 | .22597 | 4.42534 | .24439 | 4.09182 | .26297 | 3.80276 | 16 |
| 45 | .18986 | 5.26715 | .20800 | 4.80769 | .22628 | 4.41936 | .24470 | 4.08666 | .26328 | 3.79827 | 15 |
| 46 | .19016 | 5.25880 | .20830 | 4.80068 | .22658 | 4.41340 | .24501 | 4.08152 | .26359 | 3.79378 | 14 |
| 47 | .19046 | 5.25048 | .20861 | 4.79370 | .22689 | 4.40745 | .24532 | 4.07639 | .26390 | 3.78931 | 13 |
| 48 | .19076 | 5.24218 | .20891 | 4.78673 | .22719 | 4.40152 | .24562 | 4.07127 | .26421 | 3.78485 | 12 |
| 49 | .19106 | 5.23391 | .20921 | 4.77978 | .22750 | 4.39560 | .24593 | 4.06616 | .26452 | 3.78040 | 11 |
| 50 | .19136 | 5.22566 | .20952 | 4.77286 | .22781 | 4.38969 | .24624 | 4.06107 | .26483 | 3.77595 | 10 |
| 51 | .19166 | 5.21744 | .20982 | 4.76595 | .22811 | 4.38381 | .24655 | 4.05599 | .26515 | 3.77152 | 9 |
| 52 | .19197 | 5.20925 | .21013 | 4.75906 | .22842 | 4.37793 | .24686 | 4.05092 | .26546 | 3.76709 | 8 |
| 53 | .19227 | 5.20107 | .21043 | 4.75219 | .22872 | 4.37207 | .24717 | 4.04586 | .26577 | 3.76268 | 7 |
| 54 | .19257 | 5.19293 | .21073 | 4.74534 | .22903 | 4.36623 | .24747 | 4.04081 | .26608 | 3.75828 | 6 |
| 55 | .19287 | 5.18480 | .21104 | 4.73851 | .22934 | 4.36040 | .24778 | 4.03578 | .26639 | 3.75388 | 5 |
| 56 | .19317 | 5.17671 | .21134 | 4.73170 | .22964 | 4.35459 | .24809 | 4.03076 | .26670 | 3.74950 | 4 |
| 57 | .19347 | 5.16863 | .21164 | 4.72490 | .22995 | 4.34879 | .24840 | 4.02574 | .26701 | 3.74512 | 3 |
| 58 | .19378 | 5.16058 | .21195 | 4.71813 | .23026 | 4.34300 | .24871 | 4.02074 | .26733 | 3.74075 | 2 |
| 59 | .19408 | 5.15256 | .21225 | 4.71137 | .23056 | 4.33723 | .24902 | 4.01576 | .26764 | 3.73640 | 1 |
| 60 | .19438 | 5.14455 | .21256 | 4.70463 | .23087 | 4.33148 | .24933 | 4.01078 | .26795 | 3.73205 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 79° | | 78° | | 77° | | 76° | | 75° | | |

| ′ | 15° Tang | 15° Cotang | 16° Tang | 16° Cotang | 17° Tang | 17° Cotang | 18° Tang | 18° Cotang | 19° Tang | 19° Cotang | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .26795 | 3.73205 | .28675 | 3.48741 | .30573 | 3.27085 | .32492 | 3.07768 | .34433 | 2.90421 | 60 |
| 1 | .26826 | 3.72771 | .28706 | 3.48359 | .30605 | 3.26745 | .32524 | 3.07464 | .34465 | 2.90147 | 59 |
| 2 | .26857 | 3.72338 | .28738 | 3.47977 | .30637 | 3.26406 | .32556 | 3.07160 | .34498 | 2.89873 | 58 |
| 3 | .26888 | 3.71907 | .28769 | 3.47596 | .30669 | 3.26067 | .32588 | 3.06857 | .34530 | 2.89600 | 57 |
| 4 | .26920 | 3.71476 | .28800 | 3.47216 | .30700 | 3.25729 | .32621 | 3.06554 | .34563 | 2.89327 | 56 |
| 5 | .26951 | 3.71046 | .28832 | 3.46837 | .30732 | 3.25392 | .32653 | 3.06252 | .34596 | 2.89055 | 55 |
| 6 | .26982 | 3.70616 | .28864 | 3.46458 | .30764 | 3.25055 | .32685 | 3.05950 | .34628 | 2.88783 | 54 |
| 7 | .27013 | 3.70188 | .28895 | 3.46080 | .30796 | 3.24719 | .32717 | 3.05649 | .34661 | 2.88511 | 53 |
| 8 | .27044 | 3.69761 | .28927 | 3.45703 | .30828 | 3.24383 | .32749 | 3.05349 | .34693 | 2.88240 | 52 |
| 9 | .27076 | 3.69335 | .28958 | 3.45327 | .30860 | 3.24049 | .32782 | 3.05049 | .34726 | 2.87970 | 51 |
| 10 | .27107 | 3.68909 | .28990 | 3.44951 | .30891 | 3.23714 | .32814 | 3.04749 | .34758 | 2.87700 | 50 |
| 11 | .27138 | 3.68485 | .29021 | 3.44576 | .30923 | 3.23381 | .32846 | 3.04450 | .34791 | 2.87430 | 49 |
| 12 | .27169 | 3.68061 | .29053 | 3.44202 | .30955 | 3.23048 | .32878 | 3.04152 | .34824 | 2.87161 | 48 |
| 13 | .27201 | 3.67638 | .29084 | 3.43829 | .30987 | 3.22715 | .32911 | 3.03854 | .34856 | 2.86892 | 47 |
| 14 | .27232 | 3.67217 | .29116 | 3.43456 | .31019 | 3.22384 | .32943 | 3.03556 | .34889 | 2.86624 | 46 |
| 15 | .27263 | 3.66796 | .29147 | 3.43084 | .31051 | 3.22053 | .32975 | 3.03260 | .34922 | 2.86356 | 45 |
| 16 | .27294 | 3.66376 | .29179 | 3.42713 | .31083 | 3.21722 | .33007 | 3.02963 | .34954 | 2.86089 | 44 |
| 17 | .27326 | 3.65957 | .29210 | 3.42343 | .31115 | 3.21392 | .33040 | 3.02667 | .34987 | 2.85822 | 43 |
| 18 | .27357 | 3.65538 | .29242 | 3.41973 | .31147 | 3.21063 | .33072 | 3.02372 | .35020 | 2.85555 | 42 |
| 19 | .27388 | 3.65121 | .29274 | 3.41604 | .31178 | 3.20734 | .33104 | 3.02077 | .35052 | 2.85289 | 41 |
| 20 | .27419 | 3.64705 | .29305 | 3.41236 | .31210 | 3.20406 | .33136 | 3.01783 | .35085 | 2.85023 | 40 |
| 21 | .27451 | 3.64289 | .29337 | 3.40869 | .31242 | 3.20079 | .33169 | 3.01489 | .35118 | 2.84758 | 39 |
| 22 | .27482 | 3.63874 | .29368 | 3.40502 | .31274 | 3.19752 | .33201 | 3.01196 | .35150 | 2.84494 | 38 |
| 23 | .27513 | 3.63461 | .29400 | 3.40136 | .31306 | 3.19426 | .33233 | 3.00903 | .35183 | 2.84229 | 37 |
| 24 | .27545 | 3.63048 | .29432 | 3.39771 | .31338 | 3.19100 | .33266 | 3.00611 | .35216 | 2.83965 | 36 |
| 25 | .27576 | 3.62636 | .29463 | 3.39406 | .31370 | 3.18775 | .33298 | 3.00319 | .35248 | 2.83702 | 35 |
| 26 | .27607 | 3.62224 | .29495 | 3.39042 | .31402 | 3.18451 | .33330 | 3.00028 | .35281 | 2.83439 | 34 |
| 27 | .27638 | 3.61814 | .29526 | 3.38679 | .31434 | 3.18127 | .33363 | 2.99738 | .35314 | 2.83176 | 33 |
| 28 | .27670 | 3.61405 | .29558 | 3.38317 | .31466 | 3.17804 | .33395 | 2.99447 | .35346 | 2.82914 | 32 |
| 29 | .27701 | 3.60996 | .29590 | 3.37955 | .31498 | 3.17481 | .33427 | 2.99158 | .35379 | 2.82653 | 31 |
| 30 | .27732 | 3.60588 | .29621 | 3.37594 | .31530 | 3.17159 | .33460 | 2.98868 | .35412 | 2.82391 | 30 |
| 31 | .27764 | 3.60181 | .29653 | 3.37234 | .31562 | 3.16838 | .33492 | 2.98580 | .35445 | 2.82130 | 29 |
| 32 | .27795 | 3.59775 | .29685 | 3.36875 | .31594 | 3.16517 | .33524 | 2.98292 | .35477 | 2.81870 | 28 |
| 33 | .27826 | 3.59370 | .29716 | 3.36516 | .31626 | 3.16197 | .33557 | 2.98004 | .35510 | 2.81610 | 27 |
| 34 | .27858 | 3.58966 | .29748 | 3.36158 | .31658 | 3.15877 | .33589 | 2.97717 | .35543 | 2.81350 | 26 |
| 35 | .27889 | 3.58562 | .29780 | 3.35800 | .31690 | 3.15558 | .33621 | 2.97430 | .35576 | 2.81091 | 25 |
| 36 | .27921 | 3.58160 | .29811 | 3.35443 | .31722 | 3.15240 | .33654 | 2.97144 | .35608 | 2.80833 | 24 |
| 37 | .27952 | 3.57758 | .29843 | 3.35087 | .31754 | 3.14922 | .33686 | 2.96858 | .35641 | 2.80574 | 23 |
| 38 | .27983 | 3.57357 | .29875 | 3.34732 | .31786 | 3.14605 | .33718 | 2.96573 | .35674 | 2.80316 | 22 |
| 39 | .28015 | 3.56957 | .29906 | 3.34377 | .31818 | 3.14288 | .33751 | 2.96288 | .35707 | 2.80059 | 21 |
| 40 | .28046 | 3.56557 | .29938 | 3.34023 | .31850 | 3.13972 | .33783 | 2.96004 | .35740 | 2.79802 | 20 |
| 41 | .28077 | 3.56159 | .29970 | 3.33670 | .31882 | 3.13656 | .33816 | 2.95721 | .35772 | 2.79545 | 19 |
| 42 | .28109 | 3.55761 | .30001 | 3.33317 | .31914 | 3.13341 | .33848 | 2.95437 | .35805 | 2.79289 | 18 |
| 43 | .28140 | 3.55364 | .30033 | 3.32965 | .31946 | 3.13027 | .33881 | 2.95155 | .35838 | 2.79033 | 17 |
| 44 | .28172 | 3.54968 | .30065 | 3.32614 | .31978 | 3.12713 | .33913 | 2.94872 | .35871 | 2.78778 | 16 |
| 45 | .28203 | 3.54573 | .30097 | 3.32264 | .32010 | 3.12400 | .33945 | 2.94591 | .35904 | 2.78523 | 15 |
| 46 | .28234 | 3.54179 | .30128 | 3.31914 | .32042 | 3.12087 | .33978 | 2.94309 | .35937 | 2.78269 | 14 |
| 47 | .28266 | 3.53785 | .30160 | 3.31565 | .32074 | 3.11775 | .34010 | 2.94028 | .35969 | 2.78014 | 13 |
| 48 | .28297 | 3.53393 | .30192 | 3.31216 | .32106 | 3.11464 | .34043 | 2.93748 | .36002 | 2.77761 | 12 |
| 49 | .28329 | 3.53001 | .30224 | 3.30868 | .32139 | 3.11153 | .34075 | 2.93468 | .36035 | 2.77507 | 11 |
| 50 | .28360 | 3.52609 | .30255 | 3.30521 | .32171 | 3.10842 | .34108 | 2.93189 | .36068 | 2.77254 | 10 |
| 51 | .28391 | 3.52219 | .30287 | 3.30174 | .32203 | 3.10532 | .34140 | 2.92910 | .36101 | 2.77002 | 9 |
| 52 | .28423 | 3.51829 | .30319 | 3.29829 | .32235 | 3.10223 | .34173 | 2.92632 | .36134 | 2.76750 | 8 |
| 53 | .28454 | 3.51441 | .30351 | 3.29483 | .32267 | 3.09914 | .34205 | 2.92354 | .36167 | 2.76498 | 7 |
| 54 | .28486 | 3.51053 | .30382 | 3.29139 | .32299 | 3.09606 | .34238 | 2.92076 | .36199 | 2.76247 | 6 |
| 55 | .28517 | 3.50666 | .30414 | 3.28795 | .32331 | 3.09298 | .34270 | 2.91799 | .36232 | 2.75996 | 5 |
| 56 | .28549 | 3.50279 | .30446 | 3.28452 | .32363 | 3.08991 | .34303 | 2.91523 | .36265 | 2.75746 | 4 |
| 57 | .28580 | 3.49894 | .30478 | 3.28109 | .32396 | 3.08685 | .34335 | 2.91246 | .36298 | 2.75496 | 3 |
| 58 | .28612 | 3.49509 | .30509 | 3.27767 | .32428 | 3.08379 | .34368 | 2.90971 | .36331 | 2.75246 | 2 |
| 59 | .28643 | 3.49125 | .30541 | 3.27426 | .32460 | 3.08073 | .34400 | 2.90696 | .36364 | 2.74997 | 1 |
| 60 | .28675 | 3.48741 | .30573 | 3.27085 | .32492 | 3.07768 | .34433 | 2.90421 | .36397 | 2.74748 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 74° | | 73° | | 72° | | 71° | | 70° | | |

| ′ | 20° | | 21° | | 22° | | 23° | | 24° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .36397 | 2.74748 | .38386 | 2.60509 | .40403 | 2.47509 | .42447 | 2.35585 | .44523 | 2.24604 | 60 |
| 1 | .36430 | 2.74499 | .38420 | 2.60283 | .40436 | 2.47302 | .42482 | 2.35395 | .44558 | 2.24428 | 59 |
| 2 | .36463 | 2.74251 | .38453 | 2.60057 | .40470 | 2.47095 | .42516 | 2.35205 | .44593 | 2.24252 | 58 |
| 3 | .36496 | 2.74004 | .38487 | 2.59831 | .40504 | 2.46888 | .42551 | 2.35015 | .44627 | 2.24077 | 57 |
| 4 | .36529 | 2.73756 | .38520 | 2.59606 | .40538 | 2.46682 | .42585 | 2.34825 | .44662 | 2.23902 | 56 |
| 5 | .36562 | 2.73509 | .38553 | 2.59381 | .40572 | 2.46476 | .42619 | 2.34636 | .44697 | 2.23727 | 55 |
| 6 | .36595 | 2.73263 | .38587 | 2.59156 | .40606 | 2.46270 | .42654 | 2.34447 | .44732 | 2.23553 | 54 |
| 7 | .36628 | 2.73017 | .38620 | 2.58932 | .40640 | 2.46065 | .42688 | 2.34258 | .44767 | 2.23378 | 53 |
| 8 | .36661 | 2.72771 | .38654 | 2.58708 | .40674 | 2.45860 | .42722 | 2.34069 | .44802 | 2.23204 | 52 |
| 9 | .36694 | 2.72526 | .38687 | 2.58484 | .40707 | 2.45655 | .42757 | 2.33881 | .44837 | 2.23030 | 51 |
| 10 | .36727 | 2.72281 | .38721 | 2.58261 | .40741 | 2.45451 | .42791 | 2.33693 | .44872 | 2.22857 | 50 |
| 11 | .36760 | 2.72036 | .38754 | 2.58038 | .40775 | 2.45246 | .42826 | 2.33505 | .44907 | 2.22683 | 49 |
| 12 | .36793 | 2.71792 | .38787 | 2.57815 | .40809 | 2.45043 | .42860 | 2.33317 | .44942 | 2.22510 | 48 |
| 13 | .36826 | 2.71548 | .38821 | 2.57593 | .40843 | 2.44839 | .42894 | 2.33130 | .44977 | 2.22337 | 47 |
| 14 | .36859 | 2.71305 | .38854 | 2.57371 | .40877 | 2.44636 | .42929 | 2.32943 | .45012 | 2.22164 | 46 |
| 15 | .36892 | 2.71062 | .38888 | 2.57150 | .40911 | 2.44433 | .42963 | 2.32756 | .45047 | 2.21992 | 45 |
| 16 | .36925 | 2.70819 | .38921 | 2.56928 | .40945 | 2.44230 | .42998 | 2.32570 | .45082 | 2.21819 | 44 |
| 17 | .36958 | 2.70577 | .38955 | 2.56707 | .40979 | 2.44027 | .43032 | 2.32383 | .45117 | 2.21647 | 43 |
| 18 | .36991 | 2.70335 | .38988 | 2.56487 | .41013 | 2.43825 | .43067 | 2.32197 | .45152 | 2.21475 | 42 |
| 19 | .37024 | 2.70094 | .39022 | 2.56266 | .41047 | 2.43623 | .43101 | 2.32012 | .45187 | 2.21304 | 41 |
| 20 | .37057 | 2.69853 | .39055 | 2.56046 | .41081 | 2.43422 | .43136 | 2.31826 | .45222 | 2.21132 | 40 |
| 21 | .37090 | 2.69612 | .39089 | 2.55827 | .41115 | 2.43220 | .43170 | 2.31641 | .45257 | 2.20961 | 39 |
| 22 | .37123 | 2.69371 | .39122 | 2.55608 | .41149 | 2.43019 | .43205 | 2.31456 | .45292 | 2.20790 | 38 |
| 23 | .37157 | 2.69131 | .39156 | 2.55389 | .41183 | 2.42819 | .43239 | 2.31271 | .45327 | 2.20619 | 37 |
| 24 | .37190 | 2.68892 | .39190 | 2.55170 | .41217 | 2.42618 | .43274 | 2.31086 | .45362 | 2.20449 | 36 |
| 25 | .37223 | 2.68653 | .39223 | 2.54952 | .41251 | 2.42418 | .43308 | 2.30902 | .45397 | 2.20278 | 35 |
| 26 | .37256 | 2.68414 | .39257 | 2.54734 | .41285 | 2.42218 | .43343 | 2.30718 | .45432 | 2.20108 | 34 |
| 27 | .37289 | 2.68175 | .39290 | 2.54516 | .41319 | 2.42019 | .43378 | 2.30534 | .45467 | 2.19938 | 33 |
| 28 | .37322 | 2.67937 | .39324 | 2.54299 | .41353 | 2.41819 | .43412 | 2.30351 | .45502 | 2.19769 | 32 |
| 29 | .37355 | 2.67700 | .39357 | 2.54082 | .41387 | 2.41620 | .43447 | 2.30167 | .45538 | 2.19599 | 31 |
| 30 | .37388 | 2.67462 | .39391 | 2.53865 | .41421 | 2.41421 | .43481 | 2.29984 | .45573 | 2.19430 | 30 |
| 31 | .37422 | 2.67225 | .39425 | 2.53648 | .41455 | 2.41223 | .43516 | 2.29801 | .45608 | 2.19261 | 29 |
| 32 | .37455 | 2.66989 | .39458 | 2.53432 | .41490 | 2.41025 | .43550 | 2.29619 | .45643 | 2.19092 | 28 |
| 33 | .37488 | 2.66752 | .39492 | 2.53217 | .41524 | 2.40827 | .43585 | 2.29437 | .45678 | 2.18923 | 27 |
| 34 | .37521 | 2.66516 | .39526 | 2.53001 | .41558 | 2.40629 | .43620 | 2.29254 | .45713 | 2.18755 | 26 |
| 35 | .37554 | 2.66281 | .39559 | 2.52786 | .41592 | 2.40432 | .43654 | 2.29073 | .45748 | 2.18587 | 25 |
| 36 | .37588 | 2.66046 | .39593 | 2.52571 | .41626 | 2.40235 | .43689 | 2.28891 | .45784 | 2.18419 | 24 |
| 37 | .37621 | 2.65811 | .39626 | 2.52357 | .41660 | 2.40038 | .43724 | 2.28710 | .45819 | 2.18251 | 23 |
| 38 | .37654 | 2.65576 | .39660 | 2.52142 | .41694 | 2.39841 | .43758 | 2.28528 | .45854 | 2.18084 | 22 |
| 39 | .37687 | 2.65342 | .39694 | 2.51929 | .41728 | 2.39645 | .43793 | 2.28348 | .45889 | 2.17916 | 21 |
| 40 | .37720 | 2.65109 | .39727 | 2.51715 | .41763 | 2.39449 | .43828 | 2.28167 | .45924 | 2.17749 | 20 |
| 41 | .37754 | 2.64875 | .39761 | 2.51502 | .41797 | 2.39253 | .43862 | 2.27987 | .45960 | 2.17582 | 19 |
| 42 | .37787 | 2.64642 | .39795 | 2.51289 | .41831 | 2.39058 | .43897 | 2.27806 | .45995 | 2.17416 | 18 |
| 43 | .37820 | 2.64410 | .39829 | 2.51076 | .41865 | 2.38863 | .43932 | 2.27626 | .46030 | 2.17249 | 17 |
| 44 | .37853 | 2.64177 | .39862 | 2.50864 | .41899 | 2.38668 | .43966 | 2.27447 | .46065 | 2.17083 | 16 |
| 45 | .37887 | 2.63945 | .39896 | 2.50652 | .41933 | 2.38473 | .44001 | 2.27267 | .46101 | 2.16917 | 15 |
| 46 | .37920 | 2.63714 | .39930 | 2.50440 | .41968 | 2.38279 | .44036 | 2.27088 | .46136 | 2.16751 | 14 |
| 47 | .37953 | 2.63483 | .39963 | 2.50229 | .42002 | 2.38084 | .44071 | 2.26909 | .46171 | 2.16585 | 13 |
| 48 | .37986 | 2.63252 | .39997 | 2.50018 | .42036 | 2.37891 | .44105 | 2.26730 | .46206 | 2.16420 | 12 |
| 49 | .38020 | 2.63021 | .40031 | 2.49807 | .42070 | 2.37697 | .44140 | 2.26552 | .46242 | 2.16255 | 11 |
| 50 | .38053 | 2.62791 | .40065 | 2.49597 | .42105 | 2.37504 | .44175 | 2.26374 | .46277 | 2.16090 | 10 |
| 51 | .38086 | 2.62561 | .40098 | 2.49386 | .42139 | 2.37311 | .44210 | 2.26196 | .46312 | 2.15925 | 9 |
| 52 | .38120 | 2.62332 | .40132 | 2.49177 | .42173 | 2.37118 | .44244 | 2.26018 | .46348 | 2.15760 | 8 |
| 53 | .38153 | 2.62103 | .40166 | 2.48967 | .42207 | 2.36925 | .44279 | 2.25840 | .46383 | 2.15596 | 7 |
| 54 | .38186 | 2.61874 | .40200 | 2.48758 | .42242 | 2.36733 | .44314 | 2.25663 | .46418 | 2.15432 | 6 |
| 55 | .38220 | 2.61646 | .40234 | 2.48549 | .42276 | 2.36541 | .44349 | 2.25486 | .46454 | 2.15268 | 5 |
| 56 | .38253 | 2.61418 | .40267 | 2.48340 | .42310 | 2.36349 | .44384 | 2.25309 | .46489 | 2.15104 | 4 |
| 57 | .38286 | 2.61190 | .40301 | 2.48132 | .42345 | 2.36158 | .44418 | 2.25132 | .46525 | 2.14940 | 3 |
| 58 | .38320 | 2.60963 | .40335 | 2.47924 | .42379 | 2.35967 | .44453 | 2.24956 | .46560 | 2.14777 | 2 |
| 59 | .38353 | 2.60736 | .40369 | 2.47716 | .42413 | 2.35776 | .44488 | 2.24780 | .46595 | 2.14614 | 1 |
| 60 | .38386 | 2.60509 | .40403 | 2.47509 | .42447 | 2.35585 | .44523 | 2.24604 | .46631 | 2.14451 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 69° | | 68° | | 67° | | 66° | | 65° | | |

| ′ | 25° | | 26° | | 27° | | 28° | | 29° | | ′ |
|---|------|--------|------|--------|------|--------|------|--------|------|--------|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .46631 | 2.14451 | .48773 | 2.05030 | .50953 | 1.96261 | .53171 | 1.88073 | .55431 | 1.80405 | 60 |
| 1 | .46666 | 2.14288 | .48809 | 2.04879 | .50989 | 1.96120 | .53208 | 1.87941 | .55469 | 1.80281 | 59 |
| 2 | .46702 | 2.14125 | .48845 | 2.04728 | .51026 | 1.95979 | .53246 | 1.87809 | .55507 | 1.80158 | 58 |
| 3 | .46737 | 2.13963 | .48881 | 2.04577 | .51063 | 1.95838 | .53283 | 1.87677 | .55545 | 1.80034 | 57 |
| 4 | .46772 | 2.13801 | .48917 | 2.04426 | .51099 | 1.95698 | .53320 | 1.87546 | .55583 | 1.79911 | 56 |
| 5 | .46808 | 2.13639 | .48953 | 2.04276 | .51136 | 1.95557 | .53358 | 1.87415 | .55621 | 1.79788 | 55 |
| 6 | .46843 | 2.13477 | .48989 | 2.04125 | .51173 | 1.95417 | .53395 | 1.87283 | .55659 | 1.79665 | 54 |
| 7 | .46879 | 2.13316 | .49026 | 2.03975 | .51209 | 1.95277 | .53432 | 1.87152 | .55697 | 1.79542 | 53 |
| 8 | .46914 | 2.13154 | .49062 | 2.03825 | .51246 | 1.95137 | .53470 | 1.87021 | .55736 | 1.79419 | 52 |
| 9 | .46950 | 2.12993 | .49098 | 2.03675 | .51283 | 1.94997 | .53507 | 1.86891 | .55774 | 1.79296 | 51 |
| 10 | .46985 | 2.12832 | .49134 | 2.03526 | .51319 | 1.94858 | .53545 | 1.86760 | .55812 | 1.79174 | 50 |
| 11 | .47021 | 2.12671 | .49170 | 2.03376 | .51356 | 1.94718 | .53582 | 1.86630 | .55850 | 1.79051 | 49 |
| 12 | .47056 | 2.12511 | .49206 | 2.03227 | .51393 | 1.94579 | .53620 | 1.86499 | .55888 | 1.78929 | 48 |
| 13 | .47092 | 2.12350 | .49242 | 2.03078 | .51430 | 1.94440 | .53657 | 1.86369 | .55926 | 1.78807 | 47 |
| 14 | .47128 | 2.12190 | .49278 | 2.02929 | .51467 | 1.94301 | .53694 | 1.86239 | .55964 | 1.78685 | 46 |
| 15 | .47163 | 2.12030 | .49315 | 2.02780 | .51503 | 1.94162 | .53732 | 1.86109 | .56003 | 1.78563 | 45 |
| 16 | .47199 | 2.11871 | .49351 | 2.02631 | .51540 | 1.94023 | .53769 | 1.85979 | .56041 | 1.78441 | 44 |
| 17 | .47234 | 2.11711 | .49387 | 2.02483 | .51577 | 1.93885 | .53807 | 1.85850 | .56079 | 1.78319 | 43 |
| 18 | .47270 | 2.11552 | .49423 | 2.02335 | .51614 | 1.93746 | .53844 | 1.85720 | .56117 | 1.78198 | 42 |
| 19 | .47305 | 2.11392 | .49459 | 2.02187 | .51651 | 1.93608 | .53882 | 1.85591 | .56156 | 1.78077 | 41 |
| 20 | .47341 | 2.11233 | .49495 | 2.02039 | .51688 | 1.93470 | .53920 | 1.85462 | .56194 | 1.77955 | 40 |
| 21 | .47377 | 2.11075 | .49532 | 2.01891 | .51724 | 1.93332 | .53957 | 1.85333 | .56232 | 1.77834 | 39 |
| 22 | .47412 | 2.10916 | .49568 | 2.01743 | .51761 | 1.93195 | .53995 | 1.85204 | .56270 | 1.77713 | 38 |
| 23 | .47448 | 2.10758 | .49604 | 2.01596 | .51798 | 1.93057 | .54032 | 1.85075 | .56309 | 1.77592 | 37 |
| 24 | .47483 | 2.10600 | .49640 | 2.01449 | .51835 | 1.92920 | .54070 | 1.84946 | .56347 | 1.77471 | 36 |
| 25 | .47519 | 2.10442 | .49677 | 2.01302 | .51872 | 1.92782 | .54107 | 1.84818 | .56385 | 1.77351 | 35 |
| 26 | .47555 | 2.10284 | .49713 | 2.01155 | .51909 | 1.92645 | .54145 | 1.84689 | .56424 | 1.77230 | 34 |
| 27 | .47590 | 2.10126 | .49749 | 2.01008 | .51946 | 1.92508 | .54183 | 1.84561 | .56462 | 1.77110 | 33 |
| 28 | .47626 | 2.09969 | .49786 | 2.00862 | .51983 | 1.92371 | .54220 | 1.84433 | .56501 | 1.76990 | 32 |
| 29 | .47662 | 2.09811 | .49822 | 2.00715 | .52020 | 1.92235 | .54258 | 1.84305 | .56539 | 1.76869 | 31 |
| 30 | .47698 | 2.09654 | .49858 | 2.00569 | .52057 | 1.92098 | .54296 | 1.84177 | .56577 | 1.76749 | 30 |
| 31 | .47733 | 2.09498 | .49894 | 2.00423 | .52094 | 1.91962 | .54333 | 1.84049 | .56616 | 1.76629 | 29 |
| 32 | .47769 | 2.09341 | .49931 | 2.00277 | .52131 | 1.91826 | .54371 | 1.83922 | .56654 | 1.76510 | 28 |
| 33 | .47805 | 2.09184 | .49967 | 2.00131 | .52168 | 1.91690 | .54409 | 1.83794 | .56693 | 1.76390 | 27 |
| 34 | .47840 | 2.09028 | .50004 | 1.99986 | .52205 | 1.91554 | .54446 | 1.83667 | .56731 | 1.76271 | 26 |
| 35 | .47876 | 2.08872 | .50040 | 1.99841 | .52242 | 1.91418 | .54484 | 1.83540 | .56769 | 1.76151 | 25 |
| 36 | .47912 | 2.08716 | .50076 | 1.99695 | .52279 | 1.91282 | .54522 | 1.83413 | .56808 | 1.76032 | 24 |
| 37 | .47948 | 2.08560 | .50113 | 1.99550 | .52316 | 1.91147 | .54560 | 1.83286 | .56846 | 1.75913 | 23 |
| 38 | .47984 | 2.08405 | .50149 | 1.99406 | .52353 | 1.91012 | .54597 | 1.83159 | .56885 | 1.75794 | 22 |
| 39 | .48019 | 2.08250 | .50185 | 1.99261 | .52390 | 1.90876 | .54635 | 1.83033 | .56923 | 1.75675 | 21 |
| 40 | .48055 | 2.08094 | .50222 | 1.99116 | .52427 | 1.90741 | .54673 | 1.82906 | .56962 | 1.75556 | 20 |
| 41 | .48091 | 2.07939 | .50258 | 1.98972 | .52464 | 1.90607 | .54711 | 1.82780 | .57000 | 1.75437 | 19 |
| 42 | .48127 | 2.07785 | .50295 | 1.98828 | .52501 | 1.90472 | .54748 | 1.82654 | .57039 | 1.75319 | 18 |
| 43 | .48163 | 2.07630 | .50331 | 1.98684 | .52538 | 1.90337 | .54786 | 1.82528 | .57078 | 1.75200 | 17 |
| 44 | .48198 | 2.07476 | .50368 | 1.98540 | .52575 | 1.90203 | .54824 | 1.82402 | .57116 | 1.75082 | 16 |
| 45 | .48234 | 2.07321 | .50404 | 1.98396 | .52613 | 1.90069 | .54862 | 1.82276 | .57155 | 1.74964 | 15 |
| 46 | .48270 | 2.07167 | .50441 | 1.98253 | .52650 | 1.89935 | .54900 | 1.82150 | .57193 | 1.74846 | 14 |
| 47 | .48306 | 2.07014 | .50477 | 1.98110 | .52687 | 1.89801 | .54938 | 1.82025 | .57232 | 1.74728 | 13 |
| 48 | .48342 | 2.06860 | .50514 | 1.97966 | .52724 | 1.89667 | .54975 | 1.81899 | .57271 | 1.74610 | 12 |
| 49 | .48378 | 2.06706 | .50550 | 1.97823 | .52761 | 1.89533 | .55013 | 1.81774 | .57309 | 1.74492 | 11 |
| 50 | .48414 | 2.06553 | .50587 | 1.97681 | .52798 | 1.89400 | .55051 | 1.81649 | .57348 | 1.74375 | 10 |
| 51 | .48450 | 2.06400 | .50623 | 1.97538 | .52836 | 1.89266 | .55089 | 1.81524 | .57386 | 1.74257 | 9 |
| 52 | .48486 | 2.06247 | .50660 | 1.97395 | .52873 | 1.89133 | .55127 | 1.81399 | .57425 | 1.74140 | 8 |
| 53 | .48521 | 2.06094 | .50696 | 1.97253 | .52910 | 1.89000 | .55165 | 1.81274 | .57464 | 1.74022 | 7 |
| 54 | .48557 | 2.05942 | .50733 | 1.97111 | .52947 | 1.88867 | .55203 | 1.81150 | .57503 | 1.73905 | 6 |
| 55 | .48593 | 2.05790 | .50769 | 1.96969 | .52985 | 1.88734 | .55241 | 1.81025 | .57541 | 1.73788 | 5 |
| 56 | .48629 | 2.05637 | .50806 | 1.96827 | .53022 | 1.88602 | .55279 | 1.80901 | .57580 | 1.73671 | 4 |
| 57 | .48665 | 2.05485 | .50843 | 1.96685 | .53059 | 1.88469 | .55317 | 1.80777 | .57619 | 1.73555 | 3 |
| 58 | .48701 | 2.05333 | .50879 | 1.96544 | .53096 | 1.88337 | .55355 | 1.80653 | .57657 | 1.73438 | 2 |
| 59 | .48737 | 2.05182 | .50916 | 1.96402 | .53134 | 1.88205 | .55393 | 1.80529 | .57696 | 1.73321 | 1 |
| 60 | .48773 | 2.05030 | .50953 | 1.96261 | .53171 | 1.88073 | .55431 | 1.80405 | .57735 | 1.73205 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 64° | | 63° | | 62° | | 61° | | 60° | | |

| ′ | 30° | | 31° | | 32° | | 33° | | 34° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .57735 | 1.73205 | .60086 | 1.66428 | .62487 | 1.60033 | .64941 | 1.53986 | .67451 | 1.48256 | 60 |
| 1 | .57774 | 1.73089 | .60126 | 1.66318 | .62527 | 1.59930 | .64982 | 1.53888 | .67493 | 1.48163 | 59 |
| 2 | .57813 | 1.72973 | .60165 | 1.66209 | .62568 | 1.59826 | .65024 | 1.53791 | .67536 | 1.48070 | 58 |
| 3 | .57851 | 1.72857 | .60205 | 1.66099 | .62608 | 1.59723 | .65065 | 1.53693 | .67578 | 1.47977 | 57 |
| 4 | .57890 | 1.72741 | .60245 | 1.65990 | .62649 | 1.59620 | .65106 | 1.53595 | .67620 | 1.47885 | 56 |
| 5 | .57929 | 1.72625 | .60284 | 1.65881 | .62689 | 1.59517 | .65148 | 1.53497 | .67663 | 1.47792 | 55 |
| 6 | .57968 | 1.72509 | .60324 | 1.65772 | .62730 | 1.59414 | .65189 | 1.53400 | .67705 | 1.47699 | 54 |
| 7 | .58007 | 1.72393 | .60364 | 1.65663 | .62770 | 1.59311 | .65231 | 1.53302 | .67748 | 1.47607 | 53 |
| 8 | .58046 | 1.72278 | .60403 | 1.65554 | .62811 | 1.59208 | .65272 | 1.53205 | .67790 | 1.47514 | 52 |
| 9 | .58085 | 1.72163 | .60443 | 1.65445 | .62852 | 1.59105 | .65314 | 1.53107 | .67832 | 1.47422 | 51 |
| 10 | .58124 | 1.72047 | .60483 | 1.65337 | .62892 | 1.59002 | .65355 | 1.53010 | .67875 | 1.47330 | 50 |
| 11 | .58162 | 1.71932 | .60522 | 1.65228 | .62933 | 1.58900 | .65397 | 1.52913 | .67917 | 1.47238 | 49 |
| 12 | .58201 | 1.71817 | .60562 | 1.65120 | .62973 | 1.58797 | .65438 | 1.52816 | .67960 | 1.47146 | 48 |
| 13 | ·.58240 | 1.71702 | .60602 | 1.65011 | .63014 | 1.58695 | .65480 | 1.52719 | .68002 | 1.47053 | 47 |
| 14 | .58279 | 1.71588 | .60642 | 1.64903 | .63055 | 1.58593 | .65521 | 1.52622 | .68045 | 1.46962 | 46 |
| 15 | .58318 | 1.71473 | .60681 | 1.64795 | .63095 | 1.58490 | .65563 | 1.52525 | .68088 | 1.46870 | 45 |
| 16 | .58357 | 1.71358 | .60721 | 1.64687 | .63136 | 1.58388 | .65604 | 1.52429 | .68130 | 1.46778 | 44 |
| 17 | .58396 | 1.71244 | .60761 | 1.64579 | .63177 | 1.58286 | .65646 | 1.52332 | .68173 | 1.46686 | 43 |
| 18 | .58435 | 1.71129 | .60801 | 1.64471 | .63217 | 1.58184 | .65688 | 1.52235 | .68215 | 1.46595 | 42 |
| 19 | .58474 | 1.71015 | .60841 | 1.64363 | .63258 | 1.58083 | .65729 | 1.52139 | .68258 | 1.46503 | 41 |
| 20 | .58513 | 1.70901 | .60881 | 1.64256 | .63299 | 1.57981 | .65771 | 1.52043 | .68301 | 1.46411 | 40 |
| 21 | .58552 | 1.70787 | .60921 | 1.64148 | .63340 | 1.57879 | .65813 | 1.51946 | .68343 | 1.46320 | 39 |
| 22 | .58591 | 1.70673 | .60960 | 1.64041 | .63380 | 1.57778 | .65854 | 1.51850 | .68386 | 1.46229 | 38 |
| 23 | .58631 | 1.70560 | .61000 | 1.63934 | .63421 | 1.57676 | .65896 | 1.51754 | .68429 | 1.46137 | 37 |
| 24 | .58670 | 1.70446 | .61040 | 1.63826 | .63462 | 1.57575 | .65938 | 1.51658 | .68471 | 1.46046 | 36 |
| 25 | .58709 | 1.70332 | .61080 | 1.63719 | .63503 | 1.57474 | .65980 | 1.51562 | .68514 | 1.45955 | 35 |
| 26 | .58748 | 1.70219 | .61120 | 1.63612 | .63544 | 1.57372 | .66021 | 1.51466 | .68557 | 1.45864 | 34 |
| 27 | .58787 | 1.70106 | .61160 | 1.63505 | .63584 | 1.57271 | .66063 | 1.51370 | .68600 | 1.45773 | 33 |
| 28 | .58826 | 1.69992 | .61200 | 1.63398 | .63625 | 1.57170 | .66105 | 1.51275 | .68642 | 1.45682 | 32 |
| 29 | .58865 | 1.69879 | .61240 | 1.63292 | .63666 | 1.57069 | .66147 | 1.51179 | .68685 | 1.45592 | 31 |
| 30 | .58905 | 1.69766 | .61280 | 1.63185 | .63707 | 1.56969 | .66189 | 1.51084 | .68728 | 1.45501 | 30 |
| 31 | .58944 | 1.69653 | .61320 | 1.63079 | .63748 | 1.56868 | .66230 | 1.50988 | .68771 | 1.45410 | 29 |
| 32 | .58983 | 1.69541 | .61360 | 1.62972 | .63789 | 1.56767 | .66272 | 1.50893 | .68814 | 1.45320 | 28 |
| 33 | .59022 | 1.69428 | .61400 | 1.62866 | .63830 | 1.56667 | .66314 | 1.50797 | .68857 | 1.45229 | 27 |
| 34 | .59061 | 1.69316 | .61440 | 1.62760 | .63871 | 1.56566 | .66356 | 1.50702 | .68900 | 1.45139 | 26 |
| 35 | .59101 | 1.69203 | .61480 | 1.62654 | .63912 | 1.56466 | .66398 | 1.50607 | .68942 | 1.45049 | 25 |
| 36 | .59140 | 1.69091 | .61520 | 1.62548 | .63953 | 1.56366 | .66440 | 1.50512 | .68985 | 1.44958 | 24 |
| 37 | .59179 | 1.68979 | .61561 | 1.62442 | .63994 | 1.56265 | .66482 | 1.50417 | .69028 | 1.44868 | 23 |
| 38 | .59218 | 1.68866 | .61601 | 1.62336 | .64035 | 1.56165 | .66524 | 1.50322 | .69071 | 1.44778 | 22 |
| 39 | .59258 | 1.68754 | .61641 | 1.62230 | .64076 | 1.56065 | .66566 | 1.50228 | .69114 | 1.44688 | 21 |
| 40 | .59297 | 1.68643 | .61681 | 1.62125 | .64117 | 1.55966 | .66608 | 1.50133 | .69157 | 1.44598 | 20 |
| 41 | .59336 | 1.68531 | .61721 | 1.62019 | .64158 | 1.55866 | .66650 | 1.50038 | .69200 | 1.44508 | 19 |
| 42 | .59376 | 1.68419 | .61761 | 1.61914 | .64199 | 1.55766 | .66692 | 1.49944 | .69243 | 1.44418 | 18 |
| 43 | .59415 | 1.68308 | .61801 | 1.61808 | .64240 | 1.55666 | .66734 | 1.49849 | .69286 | 1.44329 | 17 |
| 44 | .59454 | 1.68196 | .61842 | 1.61703 | .64281 | 1.55567 | .66776 | 1.49755 | .69329 | 1.44239 | 16 |
| 45 | .59493 | 1.68085 | .61882 | 1.61598 | .64322 | 1.55467 | .66818 | 1.49661 | .69372 | 1.44149 | 15 |
| 46 | .59533 | 1.67974 | .61922 | 1.61493 | .64363 | 1.55368 | .66860 | 1.49566 | .69416 | 1.44060 | 14 |
| 47 | .59573 | 1.67863 | .61962 | 1.61388 | .64404 | 1.55269 | .66902 | 1.49472 | .69459 | 1.43970 | 13 |
| 48 | .59612 | 1.67752 | .62003 | 1.61283 | .64446 | 1.55170 | .66944 | 1.49378 | .69502 | 1.43881 | 12 |
| 49 | .59651 | 1.67641 | .62043 | 1.61179 | .64487 | 1.55071 | .66986 | 1.49284 | .69545 | 1.43792 | 11 |
| 50 | .59691 | 1.67530 | .62083 | 1.61074 | .64528 | 1.54972 | .67028 | 1.49190 | .69588 | 1.43703 | 10 |
| 51 | .59730 | 1.67419 | .62124 | 1.60970 | .64569 | 1.54873 | .67071 | 1.49097 | .69631 | 1.43614 | 9 |
| 52 | .59770 | 1.67309 | .62164 | 1.60865 | .64610 | 1.54774 | .67113 | 1.49003 | .69675 | 1.43525 | 8 |
| 53 | .59809 | 1.67198 | .62204 | 1.60761 | .64652 | 1.54675 | .67155 | 1.48909 | .69718 | 1.43436 | 7 |
| 54 | .59849 | 1.67088 | .62245 | 1.60657 | .64693 | 1.54576 | .67197 | 1.48816 | .69761 | 1.43347 | 6 |
| 55 | .59888 | 1.66978 | .62285 | 1.60553 | .64734 | 1.54478 | .67239 | 1.48722 | .69804 | 1.43258 | 5 |
| 56 | .59928 | 1.66867 | .62325 | 1.60449 | .64775 | 1.54379 | .67282 | 1.48629 | .69847 | 1.43169 | 4 |
| 57 | .59967 | 1.66757 | .62366 | 1.60345 | .64817 | 1.54281 | .67324 | 1.48536 | .69891 | 1.43080 | 3 |
| 58 | .60007 | 1.66647 | .62406 | 1.60241 | .64858 | 1.54183 | .67366 | 1.48442 | .69934 | 1.42992 | 2 |
| 59 | .60046 | 1.66538 | .62446 | 1.60137 | .64899 | 1.54085 | .67409 | 1.48349 | .69977 | 1.42903 | 1 |
| 60 | .60086 | 1.66428 | .62487 | 1.60033 | .64941 | 1.53986 | .67451 | 1.48256 | .70021 | 1.42815 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 59° | | 58° | | 57° | | 56° | | 55° | | |

| ′ | 35° | | 36° | | 37° | | 38° | | 39° | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .70021 | 1.42815 | .72654 | 1.37638 | .75355 | 1.32704 | .78129 | 1.27994 | .80978 | 1.23490 | 60 |
| 1 | .70064 | 1.42726 | .72699 | 1.37554 | .75401 | 1.32624 | .78175 | 1.27917 | .81027 | 1.23416 | 59 |
| 2 | .70107 | 1.42638 | .72743 | 1.37470 | .75447 | 1.32544 | .78222 | 1.27841 | .81075 | 1.23343 | 58 |
| 3 | .70151 | 1.42550 | .72788 | 1.37386 | .75492 | 1.32464 | .78269 | 1.27764 | .81123 | 1.23270 | 57 |
| 4 | .70194 | 1.42462 | .72832 | 1.37302 | .75538 | 1.32384 | .78316 | 1.27688 | .81171 | 1.23196 | 56 |
| 5 | .70238 | 1.42374 | .72877 | 1.37218 | .75584 | 1.32304 | .78363 | 1.27611 | .81220 | 1.23123 | 55 |
| 6 | .70281 | 1.42286 | .72921 | 1.37134 | .75629 | 1.32224 | .78410 | 1.27535 | .81268 | 1.23050 | 54 |
| 7 | .70325 | 1.42198 | .72966 | 1.37050 | .75675 | 1.32144 | .78457 | 1.27458 | .81316 | 1.22977 | 53 |
| 8 | .70368 | 1.42110 | .73010 | 1.36967 | .75721 | 1.32064 | .78504 | 1.27382 | .81364 | 1.22904 | 52 |
| 9 | .70412 | 1.42022 | .73055 | 1.36883 | .75767 | 1.31984 | .78551 | 1.27306 | .81413 | 1.22831 | 51 |
| 10 | .70455 | 1.41934 | .73100 | 1.36800 | .75812 | 1.31904 | .78598 | 1.27230 | .81461 | 1.22758 | 50 |
| 11 | .70499 | 1.41847 | .73144 | 1.36716 | .75858 | 1.31825 | .78645 | 1.27153 | .81510 | 1.22685 | 49 |
| 12 | .70542 | 1.41759 | .73189 | 1.36633 | .75904 | 1.31745 | .78692 | 1.27077 | .81558 | 1.22612 | 48 |
| 13 | .70586 | 1.41672 | .73234 | 1.36549 | .75950 | 1.31666 | .78739 | 1.27001 | .81606 | 1.22539 | 47 |
| 14 | .70629 | 1.41584 | .73278 | 1.36466 | .75996 | 1.31586 | .78786 | 1.26925 | .81655 | 1.22467 | 46 |
| 15 | .70673 | 1.41497 | .73323 | 1.36383 | .76042 | 1.31507 | .78834 | 1.26849 | .81703 | 1.22394 | 45 |
| 16 | .70717 | 1.41409 | .73368 | 1.36300 | .76088 | 1.31427 | .78881 | 1.26774 | .81752 | 1.22321 | 44 |
| 17 | .70760 | 1.41322 | .73413 | 1.36217 | .76134 | 1.31348 | .78928 | 1.26698 | .81800 | 1.22249 | 43 |
| 18 | .70804 | 1.41235 | .73457 | 1.36134 | .76180 | 1.31269 | .78975 | 1.26622 | .81849 | 1.22176 | 42 |
| 19 | .70848 | 1.41148 | .73502 | 1.36051 | .76226 | 1.31190 | .79022 | 1.26546 | .81898 | 1.22104 | 41 |
| 20 | .70891 | 1.41061 | .73547 | 1.35968 | .76272 | 1.31110 | .79070 | 1.26471 | .81946 | 1.22031 | 40 |
| 21 | .70935 | 1.40974 | .73592 | 1.35885 | .76318 | 1.31031 | .79117 | 1.26395 | .81995 | 1.21959 | 39 |
| 22 | .70979 | 1.40887 | .73637 | 1.35802 | .76364 | 1.30952 | .79164 | 1.26319 | .82044 | 1.21886 | 38 |
| 23 | .71023 | 1.40800 | .73681 | 1.35719 | .76410 | 1.30873 | .79212 | 1.26244 | .82092 | 1.21814 | 37 |
| 24 | .71066 | 1.40714 | .73726 | 1.35637 | .76456 | 1.30795 | .79259 | 1.26169 | .82141 | 1.21742 | 36 |
| 25 | .71110 | 1.40627 | .73771 | 1.35554 | .76502 | 1.30716 | .79306 | 1.26093 | .82190 | 1.21670 | 35 |
| 26 | .71154 | 1.40540 | .73816 | 1.35472 | .76548 | 1.30637 | .79354 | 1.26018 | .82238 | 1.21598 | 34 |
| 27 | .71198 | 1.40454 | .73861 | 1.35389 | .76594 | 1.30558 | .79401 | 1.25943 | .82287 | 1.21526 | 33 |
| 28 | .71242 | 1.40367 | .73906 | 1.35307 | .76640 | 1.30480 | .79449 | 1.25867 | .82336 | 1.21454 | 32 |
| 29 | .71285 | 1.40281 | .73951 | 1.35224 | .76686 | 1.30401 | .79496 | 1.25792 | .82385 | 1.21382 | 31 |
| 30 | .71329 | 1.40195 | .73996 | 1.35142 | .76733 | 1.30323 | .79544 | 1.25717 | .82434 | 1.21310 | 30 |
| 31 | .71373 | 1.40109 | .74041 | 1.35060 | .76779 | 1.30244 | .79591 | 1.25642 | .82483 | 1.21238 | 29 |
| 32 | .71417 | 1.40022 | .74086 | 1.34978 | .76825 | 1.30166 | .79639 | 1.25567 | .82531 | 1.21166 | 28 |
| 33 | .71461 | 1.39936 | .74131 | 1.34896 | .76871 | 1.30087 | .79686 | 1.25492 | .82580 | 1.21094 | 27 |
| 34 | .71505 | 1.39850 | .74176 | 1.34814 | .76918 | 1.30009 | .79734 | 1.25417 | .82629 | 1.21023 | 26 |
| 35 | .71549 | 1.39764 | .74221 | 1.34732 | .76964 | 1.29931 | .79781 | 1.25343 | .82678 | 1.20951 | 25 |
| 36 | .71593 | 1.39679 | .74267 | 1.34650 | .77010 | 1.29853 | .79829 | 1.25268 | .82727 | 1.20879 | 24 |
| 37 | .71637 | 1.39593 | .74312 | 1.34568 | .77057 | 1.29775 | .79877 | 1.25193 | .82776 | 1.20808 | 23 |
| 38 | .71681 | 1.39507 | .74357 | 1.34487 | .77103 | 1.29696 | .79924 | 1.25118 | .82825 | 1.20736 | 22 |
| 39 | .71725 | 1.39421 | .74402 | 1.34405 | .77149 | 1.29618 | .79972 | 1.25044 | .82874 | 1.20665 | 21 |
| 40 | .71769 | 1.39336 | .74447 | 1.34323 | .77196 | 1.29541 | .80020 | 1.24969 | .82923 | 1.20593 | 20 |
| 41 | .71813 | 1.39250 | .74492 | 1.34242 | .77242 | 1.29463 | .80067 | 1.24895 | .82972 | 1.20522 | 19 |
| 42 | .71857 | 1.39165 | .74538 | 1.34160 | .77289 | 1.29385 | .80115 | 1.24820 | .83022 | 1.20451 | 18 |
| 43 | .71901 | 1.39079 | .74583 | 1.34079 | .77335 | 1.29307 | .80163 | 1.24746 | .83071 | 1.20379 | 17 |
| 44 | .71946 | 1.38994 | .74628 | 1.33998 | .77382 | 1.29229 | .80211 | 1.24672 | .83120 | 1.20308 | 16 |
| 45 | .71990 | 1.38909 | .74674 | 1.33916 | .77428 | 1.29152 | .80258 | 1.24597 | .83169 | 1.20237 | 15 |
| 46 | .72034 | 1.38824 | .74719 | 1.33835 | .77475 | 1.29074 | .80306 | 1.24523 | .83218 | 1.20166 | 14 |
| 47 | .72078 | 1.38738 | .74764 | 1.33754 | .77521 | 1.28997 | .80354 | 1.24449 | .83268 | 1.20095 | 13 |
| 48 | .72122 | 1.38653 | .74810 | 1.33673 | .77568 | 1.28919 | .80402 | 1.24375 | .83317 | 1.20024 | 12 |
| 49 | .72167 | 1.38568 | .74855 | 1.33592 | .77615 | 1.28842 | .80450 | 1.24301 | .83366 | 1.19953 | 11 |
| 50 | .72211 | 1.38484 | .74900 | 1.33511 | .77661 | 1.28764 | .80498 | 1.24227 | .83415 | 1.19882 | 10 |
| 51 | .72255 | 1.38399 | .74946 | 1.33430 | .77708 | 1.28687 | .80546 | 1.24153 | .83465 | 1.19811 | 9 |
| 52 | .72299 | 1.38314 | .74991 | 1.33349 | .77754 | 1.28610 | .80594 | 1.24079 | .83514 | 1.19740 | 8 |
| 53 | .72344 | 1.38229 | .75037 | 1.33268 | .77801 | 1.28533 | .80642 | 1.24005 | .83564 | 1.19669 | 7 |
| 54 | .72388 | 1.38145 | .75082 | 1.33187 | .77848 | 1.28456 | .80690 | 1.23931 | .83613 | 1.19599 | 6 |
| 55 | .72432 | 1.38060 | .75128 | 1.33107 | .77895 | 1.28379 | .80738 | 1.23858 | .83662 | 1.19528 | 5 |
| 56 | .72477 | 1.37976 | .75173 | 1.33026 | .77941 | 1.28302 | .80786 | 1.23784 | .83712 | 1.19457 | 4 |
| 57 | .72521 | 1.37891 | .75219 | 1.32946 | .77988 | 1.28225 | .80834 | 1.23710 | .83761 | 1.19387 | 3 |
| 58 | .72565 | 1.37807 | .75264 | 1.32865 | .78035 | 1.28148 | .80882 | 1.23637 | .83811 | 1.19316 | 2 |
| 59 | .72610 | 1.37722 | .75310 | 1.32785 | .78082 | 1.28071 | .80930 | 1.23563 | .83860 | 1.19246 | 1 |
| 60 | .72654 | 1.37638 | .75355 | 1.32704 | .78129 | 1.27994 | .80978 | 1.23490 | .83910 | 1.19175 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | ′ |
| | 54° | | 53° | | 52° | | 51° | | 50° | | |

| ′ | 40° Tang | 40° Cotang | 41° Tang | 41° Cotang | 42° Tang | 42° Cotang | 43° Tang | 43° Cotang | 44° Tang | 44° Cotang | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .83910 | 1.19175 | .86929 | 1.15037 | .90040 | 1.11061 | .93252 | 1.07237 | .96569 | 1.03553 | 60 |
| 1 | .83960 | 1.19105 | .86980 | 1.14969 | .90093 | 1.10996 | .93306 | 1.07174 | .96625 | 1.03493 | 59 |
| 2 | .84009 | 1.19035 | .87031 | 1.14902 | .90146 | 1.10931 | .93360 | 1.07112 | .96681 | 1.03433 | 58 |
| 3 | .84059 | 1.18964 | .87082 | 1.14834 | .90199 | 1.10867 | .93415 | 1.07049 | .96738 | 1.03372 | 57 |
| 4 | .84108 | 1.18894 | .87133 | 1.14767 | .90251 | 1.10802 | .93469 | 1.06987 | .96794 | 1.03312 | 56 |
| 5 | .84158 | 1.18824 | .87184 | 1.14699 | .90304 | 1.10737 | .93524 | 1.06925 | .96850 | 1.03252 | 55 |
| 6 | .84208 | 1.18754 | .87236 | 1.14632 | .90357 | 1.10672 | .93578 | 1.06862 | .96907 | 1.03192 | 54 |
| 7 | .84258 | 1.18684 | .87287 | 1.14565 | .90410 | 1.10607 | .93633 | 1.06800 | .96963 | 1.03132 | 53 |
| 8 | .84307 | 1.18614 | .87338 | 1.14498 | .90463 | 1.10543 | .93688 | 1.06738 | .97020 | 1.03072 | 52 |
| 9 | .84357 | 1.18544 | .87389 | 1.14430 | .90516 | 1.10478 | .93742 | 1.06676 | .97076 | 1.03012 | 51 |
| 10 | .84407 | 1.18474 | .87441 | 1.14363 | .90569 | 1.10414 | .93797 | 1.06613 | .97133 | 1.02952 | 50 |
| 11 | .84457 | 1.18404 | .87492 | 1.14296 | .90621 | 1.10349 | .93852 | 1.06551 | .97189 | 1.02892 | 49 |
| 12 | .84507 | 1.18334 | .87543 | 1.14229 | .90674 | 1.10285 | .93906 | 1.06489 | .97246 | 1.02832 | 48 |
| 13 | .84556 | 1.18264 | .87595 | 1.14162 | .90727 | 1.10220 | .93961 | 1.06427 | .97302 | 1.02772 | 47 |
| 14 | .84606 | 1.18194 | .87646 | 1.14095 | .90781 | 1.10156 | .94016 | 1.06365 | .97359 | 1.02713 | 46 |
| 15 | .84656 | 1.18125 | .87698 | 1.14028 | .90834 | 1.10091 | .94071 | 1.06303 | .97416 | 1.02653 | 45 |
| 16 | .84706 | 1.18055 | .87749 | 1.13961 | .90887 | 1.10027 | .94125 | 1.06241 | .97472 | 1.02593 | 44 |
| 17 | .84756 | 1.17986 | .87801 | 1.13894 | .90940 | 1.09962 | .94180 | 1.06179 | .97529 | 1.02533 | 43 |
| 18 | .84806 | 1.17916 | .87852 | 1.13828 | .90993 | 1.09899 | .94235 | 1.06117 | .97586 | 1.02474 | 42 |
| 19 | .84856 | 1.17846 | .87904 | 1.13761 | .91046 | 1.09834 | .94290 | 1.06056 | .97643 | 1.02414 | 41 |
| 20 | .84906 | 1.17777 | .87955 | 1.13694 | .91099 | 1.09770 | .94345 | 1.05994 | .97700 | 1.02355 | 40 |
| 21 | .84956 | 1.17708 | .88007 | 1.13627 | .91153 | 1.09706 | .94400 | 1.05932 | .97756 | 1.02295 | 39 |
| 22 | .85006 | 1.17638 | .88059 | 1.13561 | .91206 | 1.09642 | .94455 | 1.05870 | .97813 | 1.02236 | 38 |
| 23 | .85057 | 1.17569 | .88110 | 1.13494 | .91259 | 1.09578 | .94510 | 1.05809 | .97870 | 1.02176 | 37 |
| 24 | .85107 | 1.17500 | .88162 | 1.13428 | .91313 | 1.09514 | .94565 | 1.05747 | .97927 | 1.02117 | 36 |
| 25 | .85157 | 1.17430 | .88214 | 1.13361 | .91366 | 1.09450 | .94620 | 1.05685 | .97984 | 1.02057 | 35 |
| 26 | .85207 | 1.17361 | .88265 | 1.13295 | .91419 | 1.09386 | .94676 | 1.05624 | .98041 | 1.01998 | 34 |
| 27 | .85257 | 1.17292 | .88317 | 1.13228 | .91473 | 1.09322 | .94731 | 1.05562 | .98098 | 1.01939 | 33 |
| 28 | .85308 | 1.17223 | .88369 | 1.13162 | .91526 | 1.09258 | .94786 | 1.05501 | .98155 | 1.01879 | 32 |
| 29 | .85358 | 1.17154 | .88421 | 1.13096 | .91580 | 1.09195 | .94841 | 1.05439 | .98213 | 1.01820 | 31 |
| 30 | .85408 | 1.17085 | .88473 | 1.13029 | .91633 | 1.09131 | .94896 | 1.05378 | .98270 | 1.01761 | 30 |
| 31 | .85458 | 1.17016 | .88524 | 1.12963 | .91687 | 1.09067 | .94952 | 1.05317 | .98327 | 1.01702 | 29 |
| 32 | .85509 | 1.16947 | .88576 | 1.12897 | .91740 | 1.09003 | .95007 | 1.05255 | .98384 | 1.01642 | 28 |
| 33 | .85559 | 1.16878 | .88628 | 1.12831 | .91794 | 1.08940 | .95062 | 1.05194 | .98441 | 1.01583 | 27 |
| 34 | .85609 | 1.16809 | .88680 | 1.12765 | .91847 | 1.08876 | .95118 | 1.05133 | .98499 | 1.01524 | 26 |
| 35 | .85660 | 1.16741 | .88732 | 1.12699 | .91901 | 1.08813 | .95173 | 1.05072 | .98556 | 1.01465 | 25 |
| 36 | .85710 | 1.16672 | .88784 | 1.12633 | .91955 | 1.08749 | .95229 | 1.05010 | .98613 | 1.01406 | 24 |
| 37 | .85761 | 1.16603 | .88836 | 1.12567 | .92008 | 1.08686 | .95284 | 1.04949 | .98671 | 1.01347 | 23 |
| 38 | .85811 | 1.16535 | .88888 | 1.12501 | .92062 | 1.08622 | .95340 | 1.04888 | .98728 | 1.01288 | 22 |
| 39 | .85862 | 1.16466 | .88940 | 1.12435 | .92116 | 1.08559 | .95395 | 1.04827 | .98786 | 1.01229 | 21 |
| 40 | .85912 | 1.16398 | .88992 | 1.12369 | .92170 | 1.08496 | .95451 | 1.04766 | .98843 | 1.01170 | 20 |
| 41 | .85963 | 1.16329 | .89045 | 1.12303 | .92224 | 1.08432 | .95506 | 1.04705 | .98901 | 1.01112 | 19 |
| 42 | .86014 | 1.16261 | .89097 | 1.12238 | .92277 | 1.08369 | .95562 | 1.04644 | .98958 | 1.01053 | 18 |
| 43 | .86064 | 1.16192 | .89149 | 1.12172 | .92331 | 1.08306 | .95618 | 1.04583 | .99016 | 1.00994 | 17 |
| 44 | .86115 | 1.16124 | .89201 | 1.12106 | .92385 | 1.08243 | .95673 | 1.04522 | .99073 | 1.00935 | 16 |
| 45 | .86166 | 1.16056 | .89253 | 1.12041 | .92439 | 1.08179 | .95729 | 1.04461 | .99131 | 1.00876 | 15 |
| 46 | .86216 | 1.15987 | .89306 | 1.11975 | .92493 | 1.08116 | .95785 | 1.04401 | .99189 | 1.00818 | 14 |
| 47 | .86267 | 1.15919 | .89358 | 1.11909 | .92547 | 1.08053 | .95841 | 1.04340 | .99247 | 1.00759 | 13 |
| 48 | .86318 | 1.15851 | .89410 | 1.11844 | .92601 | 1.07990 | .95897 | 1.04279 | .99304 | 1.00701 | 12 |
| 49 | .86368 | 1.15783 | .89463 | 1.11778 | .92655 | 1.07927 | .95952 | 1.04218 | .99362 | 1.00642 | 11 |
| 50 | .86419 | 1.15715 | .89515 | 1.11713 | .92709 | 1.07864 | .96008 | 1.04158 | .99420 | 1.00583 | 10 |
| 51 | .86470 | 1.15647 | .89567 | 1.11648 | .92763 | 1.07801 | .96064 | 1.04097 | .99478 | 1.00525 | 9 |
| 52 | .86521 | 1.15579 | .89620 | 1.11582 | .92817 | 1.07738 | .96120 | 1.04036 | .99536 | 1.00467 | 8 |
| 53 | .86572 | 1.15511 | .89672 | 1.11517 | .92872 | 1.07676 | .96176 | 1.03976 | .99594 | 1.00408 | 7 |
| 54 | .86623 | 1.15443 | .89725 | 1.11452 | .92926 | 1.07613 | .96232 | 1.03915 | .99652 | 1.00350 | 6 |
| 55 | .86674 | 1.15375 | .89777 | 1.11387 | .92980 | 1.07550 | .96288 | 1.03855 | .99710 | 1.00291 | 5 |
| 56 | .86725 | 1.15308 | .89830 | 1.11321 | .93034 | 1.07487 | .96344 | 1.03794 | .99768 | 1.00233 | 4 |
| 57 | .86776 | 1.15240 | .89883 | 1.11256 | .93088 | 1.07425 | .96400 | 1.03734 | .99826 | 1.00175 | 3 |
| 58 | .86827 | 1.15172 | .89935 | 1.11191 | .93143 | 1.07362 | .96457 | 1.03674 | .99884 | 1.00116 | 2 |
| 59 | .86878 | 1.15104 | .89988 | 1.11126 | .93197 | 1.07299 | .96513 | 1.03613 | .99942 | 1.00058 | 1 |
| 60 | .86929 | 1.15037 | .90040 | 1.11061 | .93252 | 1.07237 | .96569 | 1.03553 | 1.00000 | 1.00000 | 0 |
| ′ | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | |
| | 49° | | 48° | | 47° | | 46° | | 45° | | |

# LOGARITHMIC TABLES

**To Find the Logarithmic Sine, Cosine, Tangent, or Cotangent of an Angle From 0° to 45°.**—In the table entitled Logarithms of Trigonometric Functions, find the number of degrees at the top of the page, and the number of minutes in the left-hand column headed (′); opposite the latter, and under the proper head, find the desired logarithmic sine, cosine, tangent, or cotangent.

**To Find the Logarithmic Sine, Cosine, Tangent, or Cotangent of an Angle From 45° to 90°.**—In the table entitled Logarithms of Trigonometric Functions, find the number of degrees at the bottom of the page, and the number of minutes in the right-hand column headed (′); opposite the latter, and above the proper head, find the desired logarithmic sine, cosine, tangent, or cotangent.

**To Find the Logarithmic Functions for an Angle Containing Degrees, Minutes, and Seconds.**—Find the logarithm for the degrees and minutes in the manner just given, then from the column headed d. take the number next below the logarithm thus found; under the heading P.P., find a column headed by this number, and find in this column the number opposite the given number of seconds; add it to the logarithm already found for the degrees and minutes. If the exact number of seconds is not given under P.P., the proper values may be found by interpolating between the values given. As the differences in the column headed d. represent differences corresponding to 60 sec., the amount to be added after the logarithm of the degrees and minutes has been found may be obtained by multiplying the difference by the number of seconds, and dividing the result by 60.

The columns headed Cpl. S. and Cpl. T. on pages 1028 to 1030 can be used to find logarithms of angles including seconds less than 3° and greater than 86°. Reduce the degrees, minutes, and seconds to seconds, and use the following formulas, substituting for Cpl. S and Cpl. T. the values given in the table, and for S. and T., the difference between 10 and Cpl. S. and Cpl. T. as given.

*For angles less than* 4°, log sin $\alpha$ = log $\alpha''$ + S.; log tang $\alpha$ = log $\alpha''$ + T.; log cotg $\alpha$ = Cpl. log $\alpha''$ + Cpl. T. = Cpl. log tang $\alpha$; log $\alpha''$ = log sin $\alpha$ + Cpl. S. = log tang $\alpha$ + Cpl. T. = Cpl. log cotg $\alpha$ + Cpl. T.

*For angles greater than* 86°, log cos $\alpha$ = log $(90° - \alpha'')$ + S.; log cotg $\alpha$ = log $(90° - \alpha'')$ + T.; log tang $\alpha$ = Cpl. log $(90° - \alpha'')$ + Cpl. T. = Cpl. log cotg $\alpha$; log $(90° - \alpha'')$ = log cos $\alpha$ + Cpl. S. = log cotg $\alpha$ + Cpl. T. = Cpl. log tang $\alpha$ + Cpl. T.

## COMMON LOGARITHMS OF NUMBERS

| No. | Log. | No. | Log. | No. | Log. | No. | Log. | No. | Log. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | − ∞ | 20 | 30 103 | 40 | 60 206 | 60 | 77 815 | 80 | 90 309 |
| 1 | 00 000 | 21 | 32 222 | 41 | 61 278 | 61 | 78 533 | 81 | 90 849 |
| 2 | 30 103 | 22 | 34 242 | 42 | 62 325 | 62 | 79 239 | 82 | 91 381 |
| 3 | 47 712 | 23 | 36 173 | 43 | 63 347 | 63 | 79 934 | 83 | 91 908 |
| 4 | 60 206 | 24 | 38 021 | 44 | 64 345 | 64 | 80 618 | 84 | 92 428 |
| 5 | 69 897 | 25 | 39 794 | 45 | 65 321 | 65 | 81 291 | 85 | 92 942 |
| 6 | 77 815 | 26 | 41 497 | 46 | 66 276 | 66 | 81 954 | 86 | 93 450 |
| 7 | 84 510 | 27 | 43 136 | 47 | 67 210 | 67 | 82 607 | 87 | 93 952 |
| 8 | 90 309 | 28 | 44 716 | 48 | 68 124 | 68 | 83 251 | 88 | 94 448 |
| 9 | 95 424 | 29 | 46 240 | 49 | 69 020 | 69 | 83 885 | 89 | 94 939 |
| 10 | 00 000 | 30 | 47 712 | 50 | 69 897 | 70 | 84 510 | 90 | 95 424 |
| 11 | 04 139 | 31 | 49 136 | 51 | 70 757 | 71 | 85 126 | 91 | 95 904 |
| 12 | 07 918 | 32 | 50 515 | 52 | 71 600 | 72 | 85 733 | 92 | 96 379 |
| 13 | 11 394 | 33 | 51 851 | 53 | 72 428 | 73 | 86 332 | 93 | 96 848 |
| 14 | 14 613 | 34 | 53 148 | 54 | 73 239 | 74 | 86 923 | 94 | 97 313 |
| 15 | 17 609 | 35 | 54 407 | 55 | 74 036 | 75 | 87 506 | 95 | 97 772 |
| 16 | 20 412 | 36 | 55 630 | 56 | 74 819 | 76 | 88 081 | 96 | 98 227 |
| 17 | 23 045 | 37 | 56 820 | 57 | 75 587 | 77 | 88 649 | 97 | 98 677 |
| 18 | 25 527 | 38 | 57 978 | 58 | 76 343 | 78 | 89 209 | 98 | 99 123 |
| 19 | 27 875 | 39 | 59 106 | 59 | 77 085 | 79 | 89 763 | 99 | 99 564 |
| 20 | 30 103 | 40 | 60 206 | 60 | 77 815 | 80 | 90 309 | 100 | 00 000 |

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| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **100** | 00 000 | 043 | 087 | 130 | 173 | 217 | 260 | 303 | 346 | 389 |
| 101 | 432 | 475 | 518 | 561 | 604 | 647 | 689 | 732 | 775 | 817 |
| 102 | 860 | 903 | 945 | 988 | *030 | *072 | *115 | *157 | *199 | *242 |
| 103 | 01 284 | 326 | 368 | 410 | 452 | 494 | 536 | 578 | 620 | 662 |
| 104 | 703 | 745 | 787 | 828 | 870 | 912 | 953 | 995 | *036 | *078 |
| 105 | 02 119 | 160 | 202 | 243 | 284 | 325 | 366 | 407 | 449 | 490 |
| 106 | 531 | 572 | 612 | 653 | 694 | 735 | 776 | 816 | 857 | 898 |
| 107 | 938 | 979 | *019 | *060 | *100 | *141 | *181 | *222 | *262 | *302 |
| 108 | 03 342 | 383 | 423 | 463 | 503 | 543 | 583 | 623 | 663 | 703 |
| 109 | 743 | 782 | 822 | 862 | 902 | 941 | 981 | *021 | *060 | *100 |
| **110** | 04 139 | 179 | 218 | 258 | 297 | 336 | 376 | 415 | 454 | 493 |
| 111 | 532 | 571 | 610 | 650 | 689 | 727 | 766 | 805 | 844 | 883 |
| 112 | 922 | 961 | 999 | *038 | *077 | *115 | *154 | *192 | *231 | *269 |
| 113 | 05 308 | 346 | 385 | 423 | 461 | 500 | 538 | 576 | 614 | 652 |
| 114 | 690 | 729 | 767 | 805 | 843 | 881 | 918 | 956 | 994 | *032 |
| 115 | 06 070 | 108 | 145 | 183 | 221 | 258 | 296 | 333 | 371 | 408 |
| 116 | 446 | 483 | 521 | 558 | 595 | 633 | 670 | 707 | 744 | 781 |
| 117 | 819 | 856 | 893 | 930 | 967 | *004 | *041 | *078 | *115 | *151 |
| 118 | 07 188 | 225 | 262 | 298 | 335 | 372 | 408 | 445 | 482 | 518 |
| 119 | 555 | 591 | 628 | 664 | 700 | 737 | 773 | 809 | 846 | 882 |
| **120** | 918 | 954 | 990 | *027 | *063 | *099 | *135 | *171 | *207 | *243 |
| 121 | 08 279 | 314 | 350 | 386 | 422 | 458 | 493 | 529 | 565 | 600 |
| 122 | 636 | 672 | 707 | 743 | 778 | 814 | 849 | 884 | 920 | 955 |
| 123 | 991 | *026 | *061 | *096 | *132 | *167 | *202 | *237 | *272 | *307 |
| 124 | 09 342 | 377 | 412 | 447 | 482 | 517 | 552 | 587 | 621 | 656 |
| 125 | 691 | 726 | 760 | 795 | 830 | 864 | 899 | 934 | 968 | *003 |
| 126 | 10 037 | 072 | 106 | 140 | 175 | 209 | 243 | 278 | 312 | 346 |
| 127 | 380 | 415 | 449 | 483 | 517 | 551 | 585 | 619 | 653 | 687 |
| 128 | 721 | 755 | 789 | 823 | 857 | 890 | 924 | 958 | 992 | *025 |
| 129 | 11 059 | 093 | 126 | 160 | 193 | 227 | 261 | 294 | 327 | 361 |
| **130** | 394 | 428 | 461 | 494 | 528 | 561 | 594 | 628 | 661 | 694 |
| 131 | 727 | 760 | 793 | 826 | 860 | 893 | 926 | 959 | 992 | *024 |
| 132 | 12 057 | 090 | 123 | 156 | 189 | 222 | 254 | 287 | 320 | 352 |
| 133 | 385 | 418 | 450 | 483 | 516 | 548 | 581 | 613 | 646 | 678 |
| 134 | 710 | 743 | 775 | 808 | 840 | 872 | 905 | 937 | 969 | *001 |
| 135 | 13 033 | 066 | 098 | 130 | 162 | 194 | 226 | 258 | 290 | 322 |
| 136 | 354 | 386 | 418 | 450 | 481 | 513 | 545 | 577 | 609 | 640 |
| 137 | 672 | 704 | 735 | 767 | 799 | 830 | 862 | 893 | 925 | 956 |
| 138 | 988 | *019 | *051 | *082 | *114 | *145 | *176 | *208 | *239 | *270 |
| 139 | 14 301 | 333 | 364 | 395 | 426 | 457 | 489 | 520 | 551 | 582 |
| **140** | 613 | 644 | 675 | 706 | 737 | 768 | 799 | 829 | 860 | 891 |
| 141 | 922 | 953 | 983 | *014 | *045 | *076 | *106 | *137 | *168 | *198 |
| 142 | 15 229 | 259 | 290 | 320 | 351 | 381 | 412 | 442 | 473 | 503 |
| 143 | 534 | 564 | 594 | 625 | 655 | 685 | 715 | 746 | 776 | 806 |
| 144 | 836 | 866 | 897 | 927 | 957 | 987 | *017 | *047 | *077 | *107 |
| 145 | 16 137 | 167 | 197 | 227 | 256 | 286 | 316 | 346 | 376 | 406 |
| 146 | 435 | 465 | 495 | 524 | 554 | 584 | 613 | 643 | 673 | 702 |
| 147 | 732 | 761 | 791 | 820 | 850 | 879 | 909 | 938 | 967 | 997 |
| 148 | 17 026 | 056 | 085 | 114 | 143 | 173 | 202 | 231 | 260 | 289 |
| 149 | 319 | 348 | 377 | 406 | 435 | 464 | 493 | 522 | 551 | 580 |
| **150** | 609 | 638 | 667 | 696 | 725 | 754 | 782 | 811 | 840 | 869 |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|

**P. P.**

| | 44 | 43 | 42 |
|---|---|---|---|
| 1 | 4.4 | 4.3 | 4.2 |
| 2 | 8.8 | 8.6 | 8.4 |
| 3 | 13.2 | 12.9 | 12.6 |
| 4 | 17.6 | 17.2 | 16.8 |
| 5 | 22.0 | 21.5 | 21.0 |
| 6 | 26.4 | 25.8 | 25.2 |
| 7 | 30.8 | 30.1 | 29.4 |
| 8 | 35.2 | 34.4 | 33.6 |
| 9 | 39.6 | 38.7 | 37.8 |

| | 41 | 40 | 39 |
|---|---|---|---|
| 1 | 4.1 | 4.0 | 3.9 |
| 2 | 8.2 | 8.0 | 7.8 |
| 3 | 12.3 | 12.0 | 11.7 |
| 4 | 16.4 | 16.0 | 15.6 |
| 5 | 20.5 | 20.0 | 19.5 |
| 6 | 24.6 | 24.0 | 23.4 |
| 7 | 28.7 | 28.0 | 27.3 |
| 8 | 32.8 | 32.0 | 31.2 |
| 9 | 36.9 | 36.0 | 35.1 |

| | 38 | 37 | 36 |
|---|---|---|---|
| 1 | 3.8 | 3.7 | 3.6 |
| 2 | 7.6 | 7.4 | 7.2 |
| 3 | 11.4 | 11.1 | 10.8 |
| 4 | 15.2 | 14.8 | 14.4 |
| 5 | 19.0 | 18.5 | 18.0 |
| 6 | 22.8 | 22.2 | 21.6 |
| 7 | 26.6 | 25.9 | 25.2 |
| 8 | 30.4 | 29.6 | 28.8 |
| 9 | 34.2 | 33.3 | 32.4 |

| | 35 | 34 | 33 |
|---|---|---|---|
| 1 | 3.5 | 3.4 | 3.3 |
| 2 | 7.0 | 6.8 | 6.6 |
| 3 | 10.5 | 10.2 | 9.9 |
| 4 | 14.0 | 13.6 | 13.2 |
| 5 | 17.5 | 17.0 | 16.5 |
| 6 | 21.0 | 20.4 | 19.8 |
| 7 | 24.5 | 23.8 | 23.1 |
| 8 | 28.0 | 27.2 | 26.4 |
| 9 | 31.5 | 30.6 | 29.7 |

| | 32 | 31 | 30 |
|---|---|---|---|
| 1 | 3.2 | 3.1 | 3.0 |
| 2 | 6.4 | 6.2 | 6.0 |
| 3 | 9.6 | 9.3 | 9.0 |
| 4 | 12.8 | 12.4 | 12.0 |
| 5 | 16.0 | 15.5 | 15.0 |
| 6 | 19.2 | 18.6 | 18.0 |
| 7 | 22.4 | 21.7 | 21.0 |
| 8 | 25.6 | 24.8 | 24.0 |
| 9 | 28.8 | 27.9 | 27.0 |

| N. | L.0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **150** | 17 609 | 638 | 667 | 696 | 725 | 754 | 782 | 811 | 840 | 869 |
| 151 | 898 | 926 | 955 | 984 | *013 | *041 | *070 | *099 | *127 | *156 |
| 152 | 18 184 | 213 | 241 | 270 | 298 | 327 | 355 | 384 | 412 | 441 |
| 153 | 469 | 498 | 526 | 554 | 583 | 611 | 639 | 667 | 696 | 724 |
| 154 | 752 | 780 | 808 | 837 | 865 | 893 | 921 | 949 | 977 | *005 |
| 155 | 19 033 | 061 | 089 | 117 | 145 | 173 | 201 | 229 | 257 | 285 |
| 156 | 312 | 340 | 368 | 396 | 424 | 451 | 479 | 507 | 535 | 562 |
| 157 | 590 | 618 | 645 | 673 | 700 | 728 | 756 | 783 | 811 | 838 |
| 158 | 866 | 893 | 921 | 948 | 976 | *003 | *030 | *058 | *085 | *112 |
| 159 | 20 140 | 167 | 194 | 222 | 249 | 276 | 303 | 330 | 358 | 385 |
| **160** | 412 | 439 | 466 | 493 | 520 | 548 | 575 | 602 | 629 | 656 |
| 161 | 683 | 710 | 737 | 763 | 790 | 817 | 844 | 871 | 898 | 925 |
| 162 | 952 | 978 | *005 | *032 | *059 | *085 | *112 | *139 | *165 | *192 |
| 163 | 21 219 | 245 | 272 | 299 | 325 | 352 | 378 | 405 | 431 | 458 |
| 164 | 484 | 511 | 537 | 564 | 590 | 617 | 643 | 669 | 696 | 722 |
| 165 | 748 | 775 | 801 | 827 | 854 | 880 | 906 | 932 | 958 | 985 |
| 166 | 22 011 | 037 | 063 | 089 | 115 | 141 | 167 | 194 | 220 | 246 |
| 167 | 272 | 298 | 324 | 350 | 376 | 401 | 427 | 453 | 479 | 505 |
| 168 | 531 | 557 | 583 | 608 | 634 | 660 | 686 | 712 | 737 | 763 |
| 169 | 789 | 814 | 840 | 866 | 891 | 917 | 943 | 968 | 994 | *019 |
| **170** | 23 045 | 070 | 096 | 121 | 147 | 172 | 198 | 223 | 249 | 274 |
| 171 | 300 | 325 | 350 | 376 | 401 | 426 | 452 | 477 | 502 | 528 |
| 172 | 553 | 578 | 603 | 629 | 654 | 679 | 704 | 729 | 754 | 779 |
| 173 | 805 | 830 | 855 | 880 | 905 | 930 | 955 | 980 | *005 | *030 |
| 174 | 24 055 | 080 | 105 | 130 | 155 | 180 | 204 | 229 | 254 | 279 |
| 175 | 304 | 329 | 353 | 378 | 403 | 428 | 452 | 477 | 502 | 527 |
| 176 | 551 | 576 | 601 | 625 | 650 | 674 | 699 | 724 | 748 | 773 |
| 177 | 797 | 822 | 846 | 871 | 895 | 920 | 944 | 969 | 993 | *018 |
| 178 | 25 042 | 066 | 091 | 115 | 139 | 164 | 188 | 212 | 237 | 261 |
| 179 | 285 | 310 | 334 | 358 | 382 | 406 | 431 | 455 | 479 | 503 |
| **180** | 527 | 551 | 575 | 600 | 624 | 648 | 672 | 696 | 720 | 744 |
| 181 | 768 | 792 | 816 | 840 | 864 | 888 | 912 | 935 | 959 | 983 |
| 182 | 26 007 | 031 | 055 | 079 | 102 | 126 | 150 | 174 | 198 | 221 |
| 183 | 245 | 269 | 293 | 316 | 340 | 364 | 387 | 411 | 435 | 458 |
| 184 | 482 | 505 | 529 | 553 | 576 | 600 | 623 | 647 | 670 | 694 |
| 185 | 717 | 741 | 764 | 788 | 811 | 834 | 858 | 881 | 905 | 928 |
| 186 | 951 | 975 | 998 | *021 | *045 | *068 | *091 | *114 | *138 | *161 |
| 187 | 27 184 | 207 | 231 | 254 | 277 | 300 | 323 | 346 | 370 | 393 |
| 188 | 416 | 439 | 462 | 485 | 508 | 531 | 554 | 577 | 600 | 623 |
| 189 | 646 | 669 | 692 | 715 | 738 | 761 | 784 | 807 | 830 | 852 |
| **190** | 875 | 898 | 921 | 944 | 967 | 989 | *012 | *035 | *058 | *081 |
| 191 | 28 103 | 126 | 149 | 171 | 194 | 217 | 240 | 262 | 285 | 307 |
| 192 | 330 | 353 | 375 | 398 | 421 | 443 | 466 | 488 | 511 | 533 |
| 193 | 556 | 578 | 601 | 623 | 646 | 668 | 691 | 713 | 735 | 758 |
| 194 | 780 | 803 | 825 | 847 | 870 | 892 | 914 | 937 | 959 | 981 |
| 195 | 29 003 | 026 | 048 | 070 | 092 | 115 | 137 | 159 | 181 | 203 |
| 196 | 226 | 248 | 270 | 292 | 314 | 336 | 358 | 380 | 403 | 425 |
| 197 | 447 | 469 | 491 | 513 | 535 | 557 | 579 | 601 | 623 | 645 |
| 198 | 667 | 688 | 710 | 732 | 754 | 776 | 798 | 820 | 842 | 863 |
| 199 | 885 | 907 | 929 | 951 | 973 | 994 | *016 | *038 | *060 | *081 |
| **200** | 30 103 | 125 | 146 | 168 | 190 | 211 | 233 | 255 | 276 | 298 |

| N. | L.0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|

**P.P.**

| | 29 | 28 |
|---|---|---|
| 1 | 2.9 | 2.8 |
| 2 | 5.8 | 5.6 |
| 3 | 8.7 | 8.4 |
| 4 | 11.6 | 11.2 |
| 5 | 14.5 | 14.0 |
| 6 | 17.4 | 16.8 |
| 7 | 20.3 | 19.6 |
| 8 | 23.2 | 22.4 |
| 9 | 26.1 | 25.2 |

| | 27 | 26 |
|---|---|---|
| 1 | 2.7 | 2.6 |
| 2 | 5.4 | 5.2 |
| 3 | 8.1 | 7.8 |
| 4 | 10.8 | 10.4 |
| 5 | 13.5 | 13.0 |
| 6 | 16.2 | 15.6 |
| 7 | 18.9 | 18.2 |
| 8 | 21.6 | 20.8 |
| 9 | 24.3 | 23.4 |

| | 25 |
|---|---|
| 1 | 2.5 |
| 2 | 5.0 |
| 3 | 7.5 |
| 4 | 10.0 |
| 5 | 12.5 |
| 6 | 15.0 |
| 7 | 17.5 |
| 8 | 20.0 |
| 9 | 22.5 |

| | 24 | 23 |
|---|---|---|
| 1 | 2.4 | 2.3 |
| 2 | 4.8 | 4.6 |
| 3 | 7.2 | 6.9 |
| 4 | 9.6 | 9.2 |
| 5 | 12.0 | 11.5 |
| 6 | 14.4 | 13.8 |
| 7 | 16.8 | 16.1 |
| 8 | 19.2 | 18.4 |
| 9 | 21.6 | 20.7 |

| | 22 | 21 |
|---|---|---|
| 1 | 2.2 | 2.1 |
| 2 | 4.4 | 4.2 |
| 3 | 6.6 | 6.3 |
| 4 | 8.8 | 8.4 |
| 5 | 11.0 | 10.5 |
| 6 | 13.2 | 12.6 |
| 7 | 15.4 | 14.7 |
| 8 | 17.6 | 16.8 |
| 9 | 19.8 | 18.9 |

P.P.

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **200** | 30 103 | 125 | 146 | 168 | 190 | 211 | 233 | 255 | 276 | 298 |
| 201 | 320 | 341 | 363 | 384 | 406 | 428 | 449 | 471 | 492 | 514 |
| 202 | 535 | 557 | 578 | 600 | 621 | 643 | 664 | 685 | 707 | 728 |
| 203 | 750 | 771 | 792 | 814 | 835 | 856 | 878 | 899 | 920 | 942 |
| 204 | 963 | 984 | *006 | *027 | *048 | *069 | *091 | *112 | *133 | *154 |
| 205 | 31 175 | 197 | 218 | 239 | 260 | 281 | 302 | 323 | 345 | 366 |
| 206 | 387 | 408 | 429 | 450 | 471 | 492 | 513 | 534 | 555 | 576 |
| 207 | 597 | 618 | 639 | 660 | 681 | 702 | 723 | 744 | 765 | 785 |
| 208 | 806 | 827 | 848 | 869 | 890 | 911 | 931 | 952 | 973 | 994 |
| 209 | 32 015 | 035 | 056 | 077 | 098 | 118 | 139 | 160 | 181 | 201 |
| **210** | 222 | 243 | 263 | 284 | 305 | 325 | 346 | 366 | 387 | 408 |
| 211 | 428 | 449 | 469 | 490 | 510 | 531 | 552 | 572 | 593 | 613 |
| 212 | 634 | 654 | 675 | 695 | 715 | 736 | 756 | 777 | 797 | 818 |
| 213 | 838 | 858 | 879 | 899 | 919 | 940 | 960 | 980 | *001 | *021 |
| 214 | 33 041 | 062 | 082 | 102 | 122 | 143 | 163 | 183 | 203 | 224 |
| 215 | 244 | 264 | 284 | 304 | 325 | 345 | 365 | 385 | 405 | 425 |
| 216 | 445 | 465 | 486 | 506 | 526 | 546 | 566 | 586 | 606 | 626 |
| 217 | 646 | 666 | 686 | 706 | 726 | 746 | 766 | 786 | 806 | 826 |
| 218 | 846 | 866 | 885 | 905 | 925 | 945 | 965 | 985 | *005 | *025 |
| 219 | 34 044 | 064 | 084 | 104 | 124 | 143 | 163 | 183 | 203 | 223 |
| **220** | 242 | 262 | 282 | 301 | 321 | 341 | 361 | 380 | 400 | 420 |
| 221 | 439 | 459 | 479 | 498 | 518 | 537 | 557 | 577 | 596 | 616 |
| 222 | 635 | 655 | 674 | 694 | 713 | 733 | 753 | 772 | 792 | 811 |
| 223 | 830 | 850 | 869 | 889 | 908 | 928 | 947 | 967 | 986 | *005 |
| 224 | 35 025 | 044 | 064 | 083 | 102 | 122 | 141 | 160 | 180 | 199 |
| 225 | 218 | 238 | 257 | 276 | 295 | 315 | 334 | 353 | 372 | 392 |
| 226 | 411 | 430 | 449 | 468 | 488 | 507 | 526 | 545 | 564 | 583 |
| 227 | 603 | 622 | 641 | 660 | 679 | 698 | 717 | 736 | 755 | 774 |
| 228 | 793 | 813 | 832 | 851 | 870 | 889 | 908 | 927 | 946 | 965 |
| 229 | 984 | *003 | *021 | *040 | *059 | *078 | *097 | *116 | *135 | *154 |
| **230** | 36 173 | 192 | 211 | 229 | 248 | 267 | 286 | 305 | 324 | 342 |
| 231 | 361 | 380 | 399 | 418 | 436 | 455 | 474 | 493 | 511 | 530 |
| 232 | 549 | 568 | 586 | 605 | 624 | 642 | 661 | 680 | 698 | 717 |
| 233 | 736 | 754 | 773 | 791 | 810 | 829 | 847 | 866 | 884 | 903 |
| 234 | 922 | 940 | 959 | 977 | 996 | *014 | *033 | *051 | *070 | *088 |
| 235 | 37 107 | 125 | 144 | 162 | 181 | 199 | 218 | 236 | 254 | 273 |
| 236 | 291 | 310 | 328 | 346 | 365 | 383 | 401 | 420 | 438 | 457 |
| 237 | 475 | 493 | 511 | 530 | 548 | 566 | 585 | 603 | 621 | 639 |
| 238 | 658 | 676 | 694 | 712 | 731 | 749 | 767 | 785 | 803 | 822 |
| 239 | 840 | 858 | 876 | 894 | 912 | 931 | 949 | 967 | 985 | *003 |
| **240** | 38 021 | 039 | 057 | 075 | 093 | 112 | 130 | 148 | 166 | 184 |
| 241 | 202 | 220 | 238 | 256 | 274 | 292 | 310 | 328 | 346 | 364 |
| 242 | 382 | 399 | 417 | 435 | 453 | 471 | 489 | 507 | 525 | 543 |
| 243 | 561 | 578 | 596 | 614 | 632 | 650 | 668 | 686 | 703 | 721 |
| 244 | 739 | 757 | 775 | 792 | 810 | 828 | 846 | 863 | 881 | 899 |
| 245 | 917 | 934 | 952 | 970 | 987 | *005 | *023 | *041 | *058 | *076 |
| 246 | 39 094 | 111 | 129 | 146 | 164 | 182 | 199 | 217 | 235 | 252 |
| 247 | 270 | 287 | 305 | 322 | 340 | 358 | 375 | 393 | 410 | 428 |
| 248 | 445 | 463 | 480 | 498 | 515 | 533 | 550 | 568 | 585 | 602 |
| 249 | 620 | 637 | 655 | 672 | 690 | 707 | 724 | 742 | 759 | 777 |
| **250** | 794 | 811 | 829 | 846 | 863 | 881 | 898 | 915 | 933 | 950 |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|

**P. P.**

| | 22 | 21 |
|---|---|---|
| 1 | 2.2 | 2.1 |
| 2 | 4.4 | 4.2 |
| 3 | 6.6 | 6.3 |
| 4 | 8.8 | 8.4 |
| 5 | 11.0 | 10.5 |
| 6 | 13.2 | 12.6 |
| 7 | 15.4 | 14.7 |
| 8 | 17.6 | 16.8 |
| 9 | 19.8 | 18.9 |

| | 20 |
|---|---|
| 1 | 2.0 |
| 2 | 4.0 |
| 3 | 6.0 |
| 4 | 8.0 |
| 5 | 10.0 |
| 6 | 12.0 |
| 7 | 14.0 |
| 8 | 16.0 |
| 9 | 18.0 |

| | 19 |
|---|---|
| 1 | 1.9 |
| 2 | 3.8 |
| 3 | 5.7 |
| 4 | 7.6 |
| 5 | 9.5 |
| 6 | 11.4 |
| 7 | 13.3 |
| 8 | 15.2 |
| 9 | 17.1 |

| | 18 |
|---|---|
| 1 | 1.8 |
| 2 | 3.6 |
| 3 | 5.4 |
| 4 | 7.2 |
| 5 | 9.0 |
| 6 | 10.8 |
| 7 | 12.6 |
| 8 | 14.4 |
| 9 | 16.2 |

| | 17 |
|---|---|
| 1 | 1.7 |
| 2 | 3.4 |
| 3 | 5.1 |
| 4 | 6.8 |
| 5 | 8.5 |
| 6 | 10.2 |
| 7 | 11.9 |
| 8 | 13.6 |
| 9 | 15.3 |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **250** | 39 794 | 811 | 829 | 846 | 863 | 881 | 898 | 915 | 933 | 950 | |
| 251 | 967 | 985 | *002 | *019 | *037 | *054 | *071 | *088 | *106 | *123 | **18** |
| 252 | 40 140 | 157 | 175 | 192 | 209 | 226 | 243 | 261 | 278 | 295 | |
| 253 | 312 | 329 | 346 | 364 | 381 | 398 | 415 | 432 | 449 | 466 | 1  1.8 |
| 254 | 483 | 500 | 518 | 535 | 552 | 569 | 586 | 603 | 620 | 637 | 2  3.6 |
| 255 | 654 | 671 | 688 | 705 | 722 | 739 | 756 | 773 | 790 | 807 | 3  5.4 |
| 256 | 824 | 841 | 858 | 875 | 892 | 909 | 926 | 943 | 960 | 976 | 4  7.2 |
| 257 | 993 | *010 | *027 | *044 | *061 | *078 | *095 | *111 | *128 | *145 | 5  9.0 |
| 258 | 41 162 | 179 | 196 | 212 | 229 | 246 | 263 | 280 | 296 | 313 | 6  10.8 |
| 259 | 330 | 347 | 363 | 380 | 397 | 414 | 430 | 447 | 464 | 481 | 7  12.6 |
|  |  |  |  |  |  |  |  |  |  |  | 8  14.4 |
| **260** | 497 | 514 | 531 | 547 | 564 | 581 | 597 | 614 | 631 | 647 | 9  16.2 |
| 261 | 664 | 681 | 697 | 714 | 731 | 747 | 764 | 780 | 797 | 814 | |
| 262 | 830 | 847 | 863 | 880 | 896 | 913 | 929 | 946 | 963 | 979 | **17** |
| 263 | 996 | *012 | *029 | *045 | *062 | *078 | *095 | *111 | *127 | *144 | |
| 264 | 42 160 | 177 | 193 | 210 | 226 | 243 | 259 | 275 | 292 | 308 | 1  1.7 |
| 265 | 325 | 341 | 357 | 374 | 390 | 406 | 423 | 439 | 455 | 472 | 2  3.4 |
| 266 | 488 | 504 | 521 | 537 | 553 | 570 | 586 | 602 | 619 | 635 | 3  5.1 |
| 267 | 651 | 667 | 684 | 700 | 716 | 732 | 749 | 765 | 781 | 797 | 4  6.8 |
| 268 | 813 | 830 | 846 | 862 | 878 | 894 | 911 | 927 | 943 | 959 | 5  8.5 |
| 269 | 975 | 991 | *008 | *024 | *040 | *056 | *072 | *088 | *104 | *120 | 6  10.2 |
|  |  |  |  |  |  |  |  |  |  |  | 7  11.9 |
| **270** | 43 136 | 152 | 169 | 185 | 201 | 217 | 233 | 249 | 265 | 281 | 8  13.6 |
| 271 | 297 | 313 | 329 | 345 | 361 | 377 | 393 | 409 | 425 | 441 | 9  15.3 |
| 272 | 457 | 473 | 489 | 505 | 521 | 537 | 553 | 569 | 584 | 600 | |
| 273 | 616 | 632 | 648 | 664 | 680 | 696 | 712 | 727 | 743 | 759 | **16** |
| 274 | 775 | 791 | 807 | 823 | 838 | 854 | 870 | 886 | 902 | 917 | |
| 275 | 933 | 949 | 965 | 981 | 996 | *012 | *028 | *044 | *059 | *075 | 1  1.6 |
| 276 | 44 091 | 107 | 122 | 138 | 154 | 170 | 185 | 201 | 217 | 232 | 2  3.2 |
| 277 | 248 | 264 | 279 | 295 | 311 | 326 | 342 | 358 | 373 | 389 | 3  4.8 |
| 278 | 404 | 420 | 436 | 451 | 467 | 483 | 498 | 514 | 529 | 545 | 4  6.4 |
| 279 | 560 | 576 | 592 | 607 | 623 | 638 | 654 | 669 | 685 | 700 | 5  8.0 |
|  |  |  |  |  |  |  |  |  |  |  | 6  9.6 |
| **280** | 716 | 731 | 747 | 762 | 778 | 793 | 809 | 824 | 840 | 855 | 7  11.2 |
| 281 | 871 | 886 | 902 | 917 | 932 | 948 | 963 | 979 | 994 | *010 | 8  12.8 |
| 282 | 45 025 | 040 | 056 | 071 | 086 | 102 | 117 | 133 | 148 | 163 | 9  14.4 |
| 283 | 179 | 194 | 209 | 225 | 240 | 255 | 271 | 286 | 301 | 317 | |
| 284 | 332 | 347 | 362 | 378 | 393 | 408 | 423 | 439 | 454 | 469 | **15** |
| 285 | 484 | 500 | 515 | 530 | 545 | 561 | 576 | 591 | 606 | 621 | |
| 286 | 637 | 652 | 667 | 682 | 697 | 712 | 728 | 743 | 758 | 773 | 1  1.5 |
| 287 | 788 | 803 | 818 | 834 | 849 | 864 | 879 | 894 | 909 | 924 | 2  3.0 |
| 288 | 939 | 954 | 969 | 984 | *000 | *015 | *030 | *045 | *060 | *075 | 3  4.5 |
| 289 | 46 090 | 105 | 120 | 135 | 150 | 165 | 180 | 195 | 210 | 225 | 4  6.0 |
|  |  |  |  |  |  |  |  |  |  |  | 5  7.5 |
| **290** | 240 | 255 | 270 | 285 | 300 | 315 | 330 | 345 | 359 | 374 | 6  9.0 |
| 291 | 389 | 404 | 419 | 434 | 449 | 464 | 479 | 494 | 509 | 523 | 7  10.5 |
| 292 | 538 | 553 | 568 | 583 | 598 | 613 | 627 | 642 | 657 | 672 | 8  12.0 |
| 293 | 687 | 702 | 716 | 731 | 746 | 761 | 776 | 790 | 805 | 820 | 9  13.5 |
| 294 | 835 | 850 | 864 | 879 | 894 | 909 | 923 | 938 | 953 | 967 | |
| 295 | 982 | 997 | *012 | *026 | *041 | *056 | *070 | *085 | *100 | *114 | **14** |
| 296 | 47 129 | 144 | 159 | 173 | 188 | 202 | 217 | 232 | 246 | 261 | |
| 297 | 276 | 290 | 305 | 319 | 334 | 349 | 363 | 378 | 392 | 407 | 1  1.4 |
| 298 | 422 | 436 | 451 | 465 | 480 | 494 | 509 | 524 | 538 | 553 | 2  2.8 |
| 299 | 567 | 582 | 596 | 611 | 625 | 640 | 654 | 669 | 683 | 698 | 3  4.2 |
|  |  |  |  |  |  |  |  |  |  |  | 4  5.6 |
| **300** | 712 | 727 | 741 | 756 | 770 | 784 | 799 | 813 | 828 | 842 | 5  7.0 |
|  |  |  |  |  |  |  |  |  |  |  | 6  8.4 |
|  |  |  |  |  |  |  |  |  |  |  | 7  9.8 |
|  |  |  |  |  |  |  |  |  |  |  | 8  11.2 |
|  |  |  |  |  |  |  |  |  |  |  | 9  12.6 |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |
|---|---|---|---|---|---|---|---|---|---|---|---|

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **300** | 47 712 | 727 | 741 | 756 | 770 | 784 | 799 | 813 | 828 | 842 | |
| 301 | 857 | 871 | 885 | 900 | 914 | 929 | 943 | 958 | 972 | 986 | |
| 302 | 48 001 | 015 | 029 | 044 | 058 | 073 | 087 | 101 | 116 | 130 | |
| 303 | 144 | 159 | 173 | 187 | 202 | 216 | 230 | 244 | 259 | 273 | |
| 304 | 287 | 302 | 316 | 330 | 344 | 359 | 373 | 387 | 401 | 416 | |
| 305 | 430 | 444 | 458 | 473 | 487 | 501 | 515 | 530 | 544 | 558 | |
| 306 | 572 | 586 | 601 | 615 | 629 | 643 | 657 | 671 | 686 | 700 | |
| 307 | 714 | 728 | 742 | 756 | 770 | 785 | 799 | 813 | 827 | 841 | |
| 308 | 855 | 869 | 883 | 897 | 911 | 926 | 940 | 954 | 968 | 982 | |
| 309 | 996 | *010 | *024 | *038 | *052 | *066 | *080 | *094 | *108 | *122 | |
| **310** | 49 136 | 150 | 164 | 178 | 192 | 206 | 220 | 234 | 248 | 262 | |
| 311 | 276 | 290 | 304 | 318 | 332 | 346 | 360 | 374 | 388 | 402 | |
| 312 | 415 | 429 | 443 | 457 | 471 | 485 | 499 | 513 | 527 | 541 | |
| 313 | 554 | 568 | 582 | 596 | 610 | 624 | 638 | 651 | 665 | 679 | |
| 314 | 693 | 707 | 721 | 734 | 748 | 762 | 776 | 790 | 803 | 817 | |
| 315 | 831 | 845 | 859 | 872 | 886 | 900 | 914 | 927 | 941 | 955 | |
| 316 | 969 | 982 | 996 | *010 | *024 | *037 | *051 | *065 | *079 | *092 | |
| 317 | 50 106 | 120 | 133 | 147 | 161 | 174 | 188 | 202 | 215 | 229 | |
| 318 | 243 | 256 | 270 | 284 | 297 | 311 | 325 | 338 | 352 | 365 | |
| 319 | 379 | 393 | 406 | 420 | 433 | 447 | 461 | 474 | 488 | 501 | |
| **320** | 515 | 529 | 542 | 556 | 569 | 583 | 596 | 610 | 623 | 637 | |
| 321 | 651 | 664 | 678 | 691 | 705 | 718 | 732 | 745 | 759 | 772 | |
| 322 | 786 | 799 | 813 | 826 | 840 | 853 | 866 | 880 | 893 | 907 | |
| 323 | 920 | 934 | 947 | 961 | 974 | 987 | *001 | *014 | *028 | *041 | |
| 324 | 51 055 | 068 | 081 | 095 | 108 | 121 | 135 | 148 | 162 | 175 | |
| 325 | 188 | 202 | 215 | 228 | 242 | 255 | 268 | 282 | 295 | 308 | |
| 326 | 322 | 335 | 348 | 362 | 375 | 388 | 402 | 415 | 428 | 441 | |
| 327 | 455 | 468 | 481 | 495 | 508 | 521 | 534 | 548 | 561 | 574 | |
| 328 | 587 | 601 | 614 | 627 | 640 | 654 | 667 | 680 | 693 | 706 | |
| 329 | 720 | 733 | 746 | 759 | 772 | 786 | 799 | 812 | 825 | 838 | |
| **330** | 851 | 865 | 878 | 891 | 904 | 917 | 930 | 943 | 957 | 970 | |
| 331 | 983 | 996 | *009 | *022 | *035 | *048 | *061 | *075 | *088 | *101 | |
| 332 | 52 114 | 127 | 140 | 153 | 166 | 179 | 192 | 205 | 218 | 231 | |
| 333 | 244 | 257 | 270 | 284 | 297 | 310 | 323 | 336 | 349 | 362 | |
| 334 | 375 | 388 | 401 | 414 | 427 | 440 | 453 | 466 | 479 | 492 | |
| 335 | 504 | 517 | 530 | 543 | 556 | 569 | 582 | 595 | 608 | 621 | |
| 336 | 634 | 647 | 660 | 673 | 686 | 699 | 711 | 724 | 737 | 750 | |
| 337 | 763 | 776 | 789 | 802 | 815 | 827 | 840 | 853 | 866 | 879 | |
| 338 | 892 | 905 | 917 | 930 | 943 | 956 | 969 | 982 | 994 | *007 | |
| 339 | 53 020 | 033 | 046 | 058 | 071 | 084 | 097 | 110 | 122 | 135 | |
| **340** | 148 | 161 | 173 | 186 | 199 | 212 | 224 | 237 | 250 | 263 | |
| 341 | 275 | 288 | 301 | 314 | 326 | 339 | 352 | 364 | 377 | 390 | |
| 342 | 403 | 415 | 428 | 441 | 453 | 466 | 479 | 491 | 504 | 517 | |
| 343 | 529 | 542 | 555 | 567 | 580 | 593 | 605 | 618 | 631 | 643 | |
| 344 | 656 | 668 | 681 | 694 | 706 | 719 | 732 | 744 | 757 | 769 | |
| 345 | 782 | 794 | 807 | 820 | 832 | 845 | 857 | 870 | 882 | 895 | |
| 346 | 908 | 920 | 933 | 945 | 958 | 970 | 983 | 995 | *008 | *020 | |
| 347 | 54 033 | 045 | 058 | 070 | 083 | 095 | 108 | 120 | 133 | 145 | |
| 348 | 158 | 170 | 183 | 195 | 208 | 220 | 233 | 245 | 258 | 270 | |
| 349 | 283 | 295 | 307 | 320 | 332 | 345 | 357 | 370 | 382 | 394 | |
| **350** | 407 | 419 | 432 | 444 | 456 | 469 | 481 | 494 | 506 | 518 | |
| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |

**P. P.**

15
1 1.5
2 3.0
3 4.5
4 6.0
5 7.5
6 9.0
7 10.5
8 12.0
9 13.5

14
1 1.4
2 2.8
3 4.2
4 5.6
5 7.0
6 8.4
7 9.8
8 11.2
9 12.6

13
1 1.3
2 2.6
3 3.9
4 5.2
5 6.5
6 7.8
7 9.1
8 10.4
9 11.7

12
1 1.2
2 2.4
3 3.6
4 4.8
5 6.0
6 7.2
7 8.4
8 9.6
9 10.8

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **350** | 54 407 | 419 | 432 | 444 | 456 | 469 | 481 | 494 | 506 | 518 | |
| 351 | 531 | 543 | 555 | 568 | 580 | 593 | 605 | 617 | 630 | 642 | |
| 352 | 654 | 667 | 679 | 691 | 704 | 716 | 728 | 741 | 753 | 765 | |
| 353 | 777 | 790 | 802 | 814 | 827 | 839 | 851 | 864 | 876 | 888 | |
| 354 | 900 | 913 | 925 | 937 | 949 | 962 | 974 | 986 | 998 | *011 | |
| 355 | 55 023 | 035 | 047 | 060 | 072 | 084 | 096 | 108 | 121 | 133 | |
| 356 | 145 | 157 | 169 | 182 | 194 | 206 | 218 | 230 | 242 | 255 | |
| 357 | 267 | 279 | 291 | 303 | 315 | 328 | 340 | 352 | 364 | 376 | |
| 358 | 388 | 400 | 413 | 425 | 437 | 449 | 461 | 473 | 485 | 497 | |
| 359 | 509 | 522 | 534 | 546 | 558 | 570 | 582 | 594 | 606 | 618 | |
| **360** | 630 | 642 | 654 | 666 | 678 | 691 | 703 | 715 | 727 | 739 | |
| 361 | 751 | 763 | 775 | 787 | 799 | 811 | 823 | 835 | 847 | 859 | |
| 362 | 871 | 883 | 895 | 907 | 919 | 931 | 943 | 955 | 967 | 979 | |
| 363 | 991 | *003 | *015 | *027 | *038 | *050 | *062 | *074 | *086 | *098 | |
| 364 | 56 110 | 122 | 134 | 146 | 158 | 170 | 182 | 194 | 205 | 217 | |
| 365 | 229 | 241 | 253 | 265 | 277 | 289 | 301 | 312 | 324 | 336 | |
| 366 | 348 | 360 | 372 | 384 | 396 | 407 | 419 | 431 | 443 | 455 | |
| 367 | 467 | 478 | 490 | 502 | 514 | 526 | 538 | 549 | 561 | 573 | |
| 368 | 585 | 597 | 608 | 620 | 632 | 644 | 656 | 667 | 679 | 691 | |
| 369 | 703 | 714 | 726 | 738 | 750 | 761 | 773 | 785 | 797 | 808 | |
| **370** | 820 | 832 | 844 | 855 | 867 | 879 | 891 | 902 | 914 | 926 | |
| 371 | 937 | 949 | 961 | 972 | 984 | 996 | *008 | *019 | *031 | *043 | |
| 372 | 57 054 | 066 | 078 | 089 | 101 | 113 | 124 | 136 | 148 | 159 | |
| 373 | 171 | 183 | 194 | 206 | 217 | 229 | 241 | 252 | 264 | 276 | |
| 374 | 287 | 299 | 310 | 322 | 334 | 345 | 357 | 368 | 380 | 392 | |
| 375 | 403 | 415 | 426 | 438 | 449 | 461 | 473 | 484 | 496 | 507 | |
| 376 | 519 | 530 | 542 | 553 | 565 | 576 | 588 | 600 | 611 | 623 | |
| 377 | 634 | 646 | 657 | 669 | 680 | 692 | 703 | 715 | 726 | 738 | |
| 378 | 749 | 761 | 772 | 784 | 795 | 807 | 818 | 830 | 841 | 852 | |
| 379 | 864 | 875 | 887 | 898 | 910 | 921 | 933 | 944 | 955 | 967 | |
| **380** | 978 | 990 | *001 | *013 | *024 | *035 | *047 | *058 | *070 | *081 | |
| 381 | 58 092 | 104 | 115 | 127 | 138 | 149 | 161 | 172 | 184 | 195 | |
| 382 | 206 | 218 | 229 | 240 | 252 | 263 | 274 | 286 | 297 | 309 | |
| 383 | 320 | 331 | 343 | 354 | 365 | 377 | 388 | 399 | 410 | 422 | |
| 384 | 433 | 444 | 456 | 467 | 478 | 490 | 501 | 512 | 524 | 535 | |
| 385 | 546 | 557 | 569 | 580 | 591 | 602 | 614 | 625 | 636 | 647 | |
| 386 | 659 | 670 | 681 | 692 | 704 | 715 | 726 | 737 | 749 | 760 | |
| 387 | 771 | 782 | 794 | 805 | 816 | 827 | 838 | 850 | 861 | 872 | |
| 388 | 883 | 894 | 906 | 917 | 928 | 939 | 950 | 961 | 973 | 984 | |
| 389 | 995 | *006 | *017 | *028 | *040 | *051 | *062 | *073 | *084 | *095 | |
| **390** | 59 106 | 118 | 129 | 140 | 151 | 162 | 173 | 184 | 195 | 207 | |
| 391 | 218 | 229 | 240 | 251 | 262 | 273 | 284 | 295 | 306 | 318 | |
| 392 | 329 | 340 | 351 | 362 | 373 | 384 | 395 | 406 | 417 | 428 | |
| 393 | 439 | 450 | 461 | 472 | 483 | 494 | 506 | 517 | 528 | 539 | |
| 394 | 550 | 561 | 572 | 583 | 594 | 605 | 616 | 627 | 638 | 649 | |
| 395 | 660 | 671 | 682 | 693 | 704 | 715 | 726 | 737 | 748 | 759 | |
| 396 | 770 | 780 | 791 | 802 | 813 | 824 | 835 | 846 | 857 | 868 | |
| 397 | 879 | 890 | 901 | 912 | 923 | 934 | 945 | 956 | 966 | 977 | |
| 398 | 988 | 999 | *010 | *021 | *032 | *043 | *054 | *065 | *076 | *086 | |
| 399 | 60 097 | 108 | 119 | 130 | 141 | 152 | 163 | 173 | 184 | 195 | |
| **400** | 206 | 217 | 228 | 239 | 249 | 260 | 271 | 282 | 293 | 304 | |
| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | ·8 | 9 | P. P. |

P. P. side columns:

**13**

| 1 | 1.3 |
| 2 | 2.6 |
| 3 | 3.9 |
| 4 | 5.2 |
| 5 | 6.5 |
| 6 | 7.8 |
| 7 | 9.1 |
| 8 | 10.4 |
| 9 | 11.7 |

**12**

| 1 | 1.2 |
| 2 | 2.4 |
| 3 | 3.6 |
| 4 | 4.8 |
| 5 | 6.0 |
| 6 | 7.2 |
| 7 | 8.4 |
| 8 | 9.6 |
| 9 | 10.8 |

**11**

| 1 | 1.1 |
| 2 | 2.2 |
| 3 | 3.3 |
| 4 | 4.4 |
| 5 | 5.5 |
| 6 | 6.6 |
| 7 | 7.7 |
| 8 | 8.8 |
| 9 | 9.9 |

**10**

| 1 | 1.0 |
| 2 | 2.0 |
| 3 | 3.0 |
| 4 | 4.0 |
| 5 | 5.0 |
| 6 | 6.0 |
| 7 | 7.0 |
| 8 | 8.0 |
| 9 | 9.0 |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **400** | 60 206 | 217 | 228 | 239 | 249 | 260 | 271 | 282 | 293 | 304 |
| 401 | 314 | 325 | 336 | 347 | 358 | 369 | 379 | 390 | 401 | 412 |
| 402 | 423 | 433 | 444 | 455 | 466 | 477 | 487 | 498 | 509 | 520 |
| 403 | 531 | 541 | 552 | 563 | 574 | 584 | 595 | 606 | 617 | 627 |
| 404 | 638 | 649 | 660 | 670 | 681 | 692 | 703 | 713 | 724 | 735 |
| 405 | 746 | 756 | 767 | 778 | 788 | 799 | 810 | 821 | 831 | 842 |
| 406 | 853 | 863 | 874 | 885 | 895 | 906 | 917 | 927 | 938 | 949 |
| 407 | 959 | 970 | 981 | 991 | *002 | *013 | *023 | *034 | *045 | *055 |
| 408 | 61 066 | 077 | 087 | 098 | 109 | 119 | 130 | 140 | 151 | 162 |
| 409 | 172 | 183 | 194 | 204 | 215 | 225 | 236 | 247 | 257 | 268 |
| **410** | 278 | 289 | 300 | 310 | 321 | 331 | 342 | 352 | 363 | 374 |
| 411 | 384 | 395 | 405 | 416 | 426 | 437 | 448 | 458 | 469 | 479 |
| 412 | 490 | 500 | 511 | 521 | 532 | 542 | 553 | 563 | 574 | 584 |
| 413 | 595 | 606 | 616 | 627 | 637 | 648 | 658 | 669 | 679 | 690 |
| 414 | 700 | 711 | 721 | 731 | 742 | 752 | 763 | 773 | 784 | 794 |
| 415 | 805 | 815 | 826 | 836 | 847 | 857 | 868 | 878 | 888 | 899 |
| 416 | 909 | 920 | 930 | 941 | 951 | 962 | 972 | 982 | 993 | *003 |
| 417 | 62 014 | 024 | 034 | 045 | 055 | 066 | 076 | 086 | 097 | 107 |
| 418 | 118 | 128 | 138 | 149 | 159 | 170 | 180 | 190 | 201 | 211 |
| 419 | 221 | 232 | 242 | 252 | 263 | 273 | 284 | 294 | 304 | 315 |
| **420** | 325 | 335 | 346 | 356 | 366 | 377 | 387 | 397 | 408 | 418 |
| 421 | 428 | 439 | 449 | 459 | 469 | 480 | 490 | 500 | 511 | 521 |
| 422 | 531 | 542 | 552 | 562 | 572 | 583 | 593 | 603 | 613 | 624 |
| 423 | 634 | 644 | 655 | 665 | 675 | 685 | 696 | 706 | 716 | 726 |
| 424 | 737 | 747 | 757 | 767 | 778 | 788 | 798 | 808 | 818 | 829 |
| 425 | 839 | 849 | 859 | 870 | 880 | 890 | 900 | 910 | 921 | 931 |
| 426 | 941 | 951 | 961 | 972 | 982 | 992 | *002 | *012 | *022 | *033 |
| 427 | 63 043 | 053 | 063 | 073 | 083 | 094 | 104 | 114 | 124 | 134 |
| 428 | 144 | 155 | 165 | 175 | 185 | 195 | 205 | 215 | 225 | 236 |
| 429 | 246 | 256 | 266 | 276 | 286 | 296 | 306 | 317 | 327 | 337 |
| **430** | 347 | 357 | 367 | 377 | 387 | 397 | 407 | 417 | 428 | 438 |
| 431 | 448 | 458 | 468 | 478 | 488 | 498 | 508 | 518 | 528 | 538 |
| 432 | 548 | 558 | 568 | 579 | 589 | 599 | 609 | 619 | 629 | 639 |
| 433 | 649 | 659 | 669 | 679 | 689 | 699 | 709 | 719 | 729 | 739 |
| 434 | 749 | 759 | 769 | 779 | 789 | 799 | 809 | 819 | 829 | 839 |
| 435 | 849 | 859 | 869 | 879 | 889 | 899 | 909 | 919 | 929 | 939 |
| 436 | 949 | 959 | 969 | 979 | 989 | 998 | *008 | *018 | *028 | *038 |
| 437 | 64 048 | 058 | 068 | 078 | 088 | 098 | 108 | 118 | 128 | 137 |
| 438 | 147 | 157 | 167 | 177 | 187 | 197 | 207 | 217 | 227 | 237 |
| 439 | 246 | 256 | 266 | 276 | 286 | 296 | 306 | 316 | 326 | 335 |
| **440** | 345 | 355 | 365 | 375 | 385 | 395 | 404 | 414 | 424 | 434 |
| 441 | 444 | 454 | 464 | 473 | 483 | 493 | 503 | 513 | 523 | 532 |
| 442 | 542 | 552 | 562 | 572 | 582 | 591 | 601 | 611 | 621 | 631 |
| 443 | 640 | 650 | 660 | 670 | 680 | 689 | 699 | 709 | 719 | 729 |
| 444 | 738 | 748 | 758 | 768 | 777 | 787 | 797 | 807 | 816 | 826 |
| 445 | 836 | 846 | 856 | 865 | 875 | 885 | 895 | 904 | 914 | 924 |
| 446 | 933 | 943 | 953 | 963 | 972 | 982 | 992 | *002 | *011 | *021 |
| 447 | 65 031 | 040 | 050 | 060 | 070 | 079 | 089 | 099 | 108 | 118 |
| 448 | 128 | 137 | 147 | 157 | 167 | 176 | 186 | 196 | 205 | 215 |
| 449 | 225 | 234 | 244 | 254 | 263 | 273 | 283 | 292 | 302 | 312 |
| **450** | 321 | 331 | 341 | 350 | 360 | 369 | 379 | 389 | 398 | 408 |
| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

P. P.

**11**

| 1 | 1.1 |
|---|---|
| 2 | 2.2 |
| 3 | 3.3 |
| 4 | 4.4 |
| 5 | 5.5 |
| 6 | 6.6 |
| 7 | 7.7 |
| 8 | 8.8 |
| 9 | 9.9 |

**10**

| 1 | 1.0 |
|---|---|
| 2 | 2.0 |
| 3 | 3.0 |
| 4 | 4.0 |
| 5 | 5.0 |
| 6 | 6.0 |
| 7 | 7.0 |
| 8 | 8.0 |
| 9 | 9.0 |

**9**

| 1 | 0.9 |
|---|---|
| 2 | 1.8 |
| 3 | 2.7 |
| 4 | 3.6 |
| 5 | 4.5 |
| 6 | 5.4 |
| 7 | 6.3 |
| 8 | 7.2 |
| 9 | 8.1 |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **450** | 65 321 | 331 | 341 | 350 | 360 | 369 | 379 | 389 | 398 | 408 | |
| 451 | 418 | 427 | 437 | 447 | 456 | 466 | 475 | 485 | 495 | 504 | |
| 452 | 514 | 523 | 533 | 543 | 552 | 562 | 571 | 581 | 591 | 600 | |
| 453 | 610 | 619 | 629 | 639 | 648 | 658 | 667 | 677 | 686 | 696 | |
| 454 | 706 | 715 | 725 | 734 | 744 | 753 | 763 | 772 | 782 | 792 | |
| 455 | 801 | 811 | 820 | 830 | 839 | 849 | 858 | 868 | 877 | 887 | |
| 456 | 896 | 906 | 916 | 925 | 935 | 944 | 954 | 963 | 973 | 982 | **10** |
| 457 | 992 | *001 | *011 | *020 | *030 | *039 | *049 | *058 | *068 | *077 | 1  1.0 |
| 458 | 66 087 | 096 | 106 | 115 | 124 | 134 | 143 | 153 | 162 | 172 | 2  2.0 |
| 459 | 181 | 191 | 200 | 210 | 219 | 229 | 238 | 247 | 257 | 266 | 3  3.0 |
| **460** | 276 | 285 | 295 | 304 | 314 | 323 | 332 | 342 | 351 | 361 | 4  4.0 |
| | | | | | | | | | | | 5  5.0 |
| 461 | 370 | 380 | 389 | 398 | 408 | 417 | 427 | 436 | 445 | 455 | 6  6.0 |
| 462 | 464 | 474 | 483 | 492 | 502 | 511 | 521 | 530 | 539 | 549 | 7  7.0 |
| 463 | 558 | 567 | 577 | 586 | 596 | 605 | 614 | 624 | 633 | 642 | 8  8.0 |
| 464 | 652 | 661 | 671 | 680 | 689 | 699 | 708 | 717 | 727 | 736 | 9  9.0 |
| 465 | 745 | 755 | 764 | 773 | 783 | 792 | 801 | 811 | 820 | 829 | |
| 466 | 839 | 848 | 857 | 867 | 876 | 885 | 894 | 904 | 913 | 922 | |
| 467 | 932 | 941 | 950 | 960 | 969 | 978 | 987 | 997 | *006 | *015 | |
| 468 | 67 025 | 034 | 043 | 052 | 062 | 071 | 080 | 089 | 099 | 108 | |
| 469 | 117 | 127 | 136 | 145 | 154 | 164 | 173 | 182 | 191 | 201 | |
| **470** | 210 | 219 | 228 | 237 | 247 | 256 | 265 | 274 | 284 | 293 | |
| | | | | | | | | | | | **9** |
| 471 | 302 | 311 | 321 | 330 | 339 | 348 | 357 | 367 | 376 | 385 | |
| 472 | 394 | 403 | 413 | 422 | 431 | 440 | 449 | 459 | 468 | 477 | 1  0.9 |
| 473 | 486 | 495 | 504 | 514 | 523 | 532 | 541 | 550 | 560 | 569 | 2  1.8 |
| 474 | 578 | 587 | 596 | 605 | 614 | 624 | 633 | 642 | 651 | 660 | 3  2.7 |
| 475 | 669 | 679 | 688 | 697 | 706 | 715 | 724 | 733 | 742 | 752 | 4  3.6 |
| 476 | 761 | 770 | 779 | 788 | 797 | 806 | 815 | 825 | 834 | 843 | 5  4.5 |
| 477 | 852 | 861 | 870 | 879 | 888 | 897 | 906 | 916 | 925 | 934 | 6  5.4 |
| 478 | 943 | 952 | 961 | 970 | 979 | 988 | 997 | *006 | *015 | *024 | 7  6.3 |
| 479 | 68 034 | 043 | 052 | 061 | 070 | 079 | 088 | 097 | 106 | 115 | 8  7.2 |
| **480** | 124 | 133 | 142 | 151 | 160 | 169 | 178 | 187 | 196 | 205 | 9  8.1 |
| 481 | 215 | 224 | 233 | 242 | 251 | 260 | 269 | 278 | 287 | 296 | |
| 482 | 305 | 314 | 323 | 332 | 341 | 350 | 359 | 368 | 377 | 386 | |
| 483 | 395 | 404 | 413 | 422 | 431 | 440 | 449 | 458 | 467 | 476 | |
| 484 | 485 | 494 | 502 | 511 | 520 | 529 | 538 | 547 | 556 | 565 | |
| 485 | 574 | 583 | 592 | 601 | 610 | 619 | 628 | 637 | 646 | 655 | |
| 486 | 664 | 673 | 681 | 690 | 699 | 708 | 717 | 726 | 735 | 744 | |
| 487 | 753 | 762 | 771 | 780 | 789 | 797 | 806 | 815 | 824 | 833 | **8** |
| 488 | 842 | 851 | 860 | 869 | 878 | 886 | 895 | 904 | 913 | 922 | 1  0.8 |
| 489 | 931 | 940 | 949 | 958 | 966 | 975 | 984 | 993 | *002 | *011 | 2  1.6 |
| **490** | 69 020 | 028 | 037 | 046 | 055 | 064 | 073 | 082 | 090 | 099 | 3  2.4 |
| | | | | | | | | | | | 4  3.2 |
| 491 | 108 | 117 | 126 | 135 | 144 | 152 | 161 | 170 | 179 | 188 | 5  4.0 |
| 492 | 197 | 205 | 214 | 223 | 232 | 241 | 249 | 258 | 267 | 276 | 6  4.8 |
| 493 | 285 | 294 | 302 | 311 | 320 | 329 | 338 | 346 | 355 | 364 | 7  5.6 |
| 494 | 373 | 381 | 390 | 399 | 408 | 417 | 425 | 434 | 443 | 452 | 8  6.4 |
| 495 | 461 | 469 | 478 | 487 | 496 | 504 | 513 | 522 | 531 | 539 | 9  7.2 |
| 496 | 548 | 557 | 566 | 574 | 583 | 592 | 601 | 609 | 618 | 627 | |
| 497 | 636 | 644 | 653 | 662 | 671 | 679 | 688 | 697 | 705 | 714 | |
| 498 | 723 | 732 | 740 | 749 | 758 | 767 | 775 | 784 | 793 | 801 | |
| 499 | 810 | 819 | 827 | 836 | 845 | 854 | 862 | 871 | 880 | 888 | |
| **500** | 897 | 906 | 914 | 923 | 932 | 940 | 949 | 958 | 966 | 975 | |
| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **500** | 69 897 | 906 | 914 | 923 | 932 | 940 | 949 | 958 | 966 | 975 | |
| 501 | 984 | 992 | *001 | *010 | *018 | *027 | *036 | *044 | *053 | *062 | |
| 502 | 70 070 | 079 | 088 | 096 | 105 | 114 | 122 | 131 | 140 | 148 | |
| 503 | 157 | 165 | 174 | 183 | 191 | 200 | 209 | 217 | 226 | 234 | |
| 504 | 243 | 252 | 260 | 269 | 278 | 286 | 295 | 303 | 312 | 321 | |
| 505 | 329 | 338 | 346 | 355 | 364 | 372 | 381 | 389 | 398 | 406 | |
| 506 | 415 | 424 | 432 | 441 | 449 | 458 | 467 | 475 | 484 | 492 | |
| 507 | 501 | 509 | 518 | 526 | 535 | 544 | 552 | 561 | 569 | 578 | |
| 508 | 586 | 595 | 603 | 612 | 621 | 629 | 638 | 646 | 655 | 663 | |
| 509 | 672 | 680 | 689 | 697 | 706 | 714 | 723 | 731 | 740 | 749 | |
| **510** | 757 | 766 | 774 | 783 | 791 | 800 | 808 | 817 | 825 | 834 | |
| 511 | 842 | 851 | 859 | 868 | 876 | 885 | 893 | 902 | 910 | 919 | |
| 512 | 927 | 935 | 944 | 952 | 961 | 969 | 978 | 986 | 995 | *003 | |
| 513 | 71 012 | 020 | 029 | 037 | 046 | 054 | 063 | 071 | 079 | 088 | |
| 514 | 096 | 105 | 113 | 122 | 130 | 139 | 147 | 155 | 164 | 172 | |
| 515 | 181 | 189 | 198 | 206 | 214 | 223 | 231 | 240 | 248 | 257 | |
| 516 | 265 | 273 | 282 | 290 | 299 | 307 | 315 | 324 | 332 | 341 | |
| 517 | 349 | 357 | 366 | 374 | 383 | 391 | 399 | 408 | 416 | 425 | |
| 518 | 433 | 441 | 450 | 458 | 466 | 475 | 483 | 492 | 500 | 508 | |
| 519 | 517 | 525 | 533 | 542 | 550 | 559 | 567 | 575 | 584 | 592 | |
| **520** | 600 | 609 | 617 | 625 | 634 | 642 | 650 | 659 | 667 | 675 | |
| 521 | 684 | 692 | 700 | 709 | 717 | 725 | 734 | 742 | 750 | 759 | |
| 522 | 767 | 775 | 784 | 792 | 800 | 809 | 817 | 825 | 834 | 842 | |
| 523 | 850 | 858 | 867 | 875 | 883 | 892 | 900 | 908 | 917 | 925 | |
| 524 | 933 | 941 | 950 | 958 | 966 | 975 | 983 | 991 | 999 | *008 | |
| 525 | 72 016 | 024 | 032 | 041 | 049 | 057 | 066 | 074 | 082 | 090 | |
| 526 | 099 | 107 | 115 | 123 | 132 | 140 | 148 | 156 | 165 | 173 | |
| 527 | 181 | 189 | 198 | 206 | 214 | 222 | 230 | 239 | 247 | 255 | |
| 528 | 263 | 272 | 280 | 288 | 296 | 304 | 313 | 321 | 329 | 337 | |
| 529 | 346 | 354 | 362 | 370 | 378 | 387 | 395 | 403 | 411 | 419 | |
| **530** | 428 | 436 | 444 | 452 | 460 | 469 | 477 | 485 | 493 | 501 | |
| 531 | 509 | 518 | 526 | 534 | 542 | 550 | 558 | 567 | 575 | 583 | |
| 532 | 591 | 599 | 607 | 616 | 624 | 632 | 640 | 648 | 656 | 665 | |
| 533 | 673 | 681 | 689 | 697 | 705 | 713 | 722 | 730 | 738 | 746 | |
| 534 | 754 | 762 | 770 | 779 | 787 | 795 | 803 | 811 | 819 | 827 | |
| 535 | 835 | 843 | 852 | 860 | 868 | 876 | 884 | 892 | 900 | 908 | |
| 536 | 916 | 925 | 933 | 941 | 949 | 957 | 965 | 973 | 981 | 989 | |
| 537 | 997 | *006 | *014 | *022 | *030 | *038 | *046 | *054 | *062 | *070 | |
| 538 | 73 078 | 086 | 094 | 102 | 111 | 119 | 127 | 135 | 143 | 151 | |
| 539 | 159 | 167 | 175 | 183 | 191 | 199 | 207 | 215 | 223 | 231 | |
| **540** | 239 | 247 | 255 | 263 | 272 | 280 | 288 | 296 | 304 | 312 | |
| 541 | 320 | 328 | 336 | 344 | 352 | 360 | 368 | 376 | 384 | 392 | |
| 542 | 400 | 408 | 416 | 424 | 432 | 440 | 448 | 456 | 464 | 472 | |
| 543 | 480 | 488 | 496 | 504 | 512 | 520 | 528 | 536 | 544 | 552 | |
| 544 | 560 | 568 | 576 | 584 | 592 | 600 | 608 | 616 | 624 | 632 | |
| 545 | 640 | 648 | 656 | 664 | 672 | 679 | 687 | 695 | 703 | 711 | |
| 546 | 719 | 727 | 735 | 743 | 751 | 759 | 767 | 775 | 783 | 791 | |
| 547 | 799 | 807 | 815 | 823 | 830 | 838 | 846 | 854 | 862 | 870 | |
| 548 | 878 | 886 | 894 | 902 | 910 | 918 | 926 | 933 | 941 | 949 | |
| 549 | 957 | 965 | 973 | 981 | 989 | 997 | *005 | *013 | *020 | *028 | |
| **550** | 74 036 | 044 | 052 | 060 | 068 | 076 | 084 | 092 | 099 | 107 | |
| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |

P. P. columns:

**9**
1   0.9
2   1.8
3   2.7
4   3.6
5   4.5
6   5.4
7   6.3
8   7.2
9   8.1

**8**
1   0.8
2   1.6
3   2.4
4   3.2
5   4.0
6   4.8
7   5.6
8   6.4
9   7.2

**7**
1   0.7
2   1.4
3   2.1
4   2.8
5   3.5
6   4.2
7   4.9
8   5.6
9   6.3

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **550** | 74 036 | 044 | 052 | 060 | 068 | 076 | 084 | 092 | 099 | 107 | |
| 551 | 115 | 123 | 131 | 139 | 147 | 155 | 162 | 170 | 178 | 186 | |
| 552 | 194 | 202 | 210 | 218 | 225 | 233 | 241 | 249 | 257 | 265 | |
| 553 | 273 | 280 | 288 | 296 | 304 | 312 | 320 | 327 | 335 | 343 | |
| 554 | 351 | 359 | 367 | 374 | 382 | 390 | 398 | 406 | 414 | 421 | |
| 555 | 429 | 437 | 445 | 453 | 461 | 468 | 476 | 484 | 492 | 500 | |
| 556 | 507 | 515 | 523 | 531 | 539 | 547 | 554 | 562 | 570 | 578 | |
| 557 | 586 | 593 | 601 | 609 | 617 | 624 | 632 | 640 | 648 | 656 | |
| 558 | 663 | 671 | 679 | 687 | 695 | 702 | 710 | 718 | 726 | 733 | |
| 559 | 741 | 749 | 757 | 764 | 772 | 780 | 788 | 796 | 803 | 811 | |
| **560** | 819 | 827 | 834 | 842 | 850 | 858 | 865 | 873. | 881 | 889 | |
| 561 | 896 | 904 | 912 | 920 | 927 | 935 | 943 | 950 | 958 | 966 | |
| 562 | 974 | 981 | 989 | 997 | *005 | *012 | *020 | *028 | *035 | *043 | |
| 563 | 75 051 | 059 | 066 | 074 | 082 | 089 | 097 | 105 | 113 | 120 | |
| 564 | 128 | 136 | 143 | 151 | 159 | 166 | 174 | 182 | 189 | 197 | |
| 565 | 205 | 213 | 220 | 228 | 236 | 243 | 251 | 259 | 266 | 274 | |
| 566 | 282 | 289 | 297 | 305 | 312 | 320 | 328 | 335 | 343 | 351 | |
| 567 | 358 | 366 | 374 | 381 | 389 | 397 | 404 | 412 | 420 | 427 | |
| 568 | 435 | 442 | 450 | 458 | 465 | 473 | 481 | 488 | 496 | 504 | |
| 569 | 511 | 519 | 526 | 534 | 542 | 549 | 557 | 565 | 572 | 580 | |
| **570** | 587 | 595 | 603 | 610 | 618 | 626 | 633 | 641 | 648 | 656 | |
| 571 | 664 | 671 | 679 | 686 | 694 | 702 | 709 | 717 | 724 | 732 | |
| 572 | 740 | 747 | 755 | 762 | 770 | 778 | 785 | 793 | 800 | 808 | |
| 573 | 815 | 823 | 831 | 838 | 846 | 853 | 861 | 868 | 876 | 884 | |
| 574 | 891 | 899 | 906 | 914 | 921 | 929 | 937 | 944 | 952 | 959 | |
| 575 | 967 | 974 | 982 | 989 | 997 | *005 | *012 | *020 | *027 | *035 | |
| 576 | 76 042 | 050 | 057 | 065 | 072 | 080 | 087 | 095 | 103 | 110 | |
| 577 | 118 | 125 | 133 | 140 | 148 | 155 | 163 | 170 | 178 | 185 | |
| 578 | 193 | 200 | 208 | 215 | 223 | 230 | 238 | 245 | 253 | 260 | |
| 579 | 268 | 275 | 283 | 290 | 298 | 305 | 313 | 320 | 328 | 335 | |
| **580** | 343 | 350 | 358 | 365 | 373 | 380 | 388 | 395 | 403 | 410 | |
| 581 | 418 | 425 | 433 | 440 | 448 | 455 | 462 | 470 | 477 | 485 | |
| 582 | 492 | 500 | 507 | 515 | 522 | 530 | 537 | 545 | 552 | 559 | |
| 583 | 567 | 574 | 582 | 589 | 597 | 604 | 612 | 619 | 626 | 634 | |
| 584 | 641 | 649 | 656 | 664 | 671 | 678 | 686 | 693 | 701 | 708 | |
| 585 | 716 | 723 | 730 | 738 | 745 | 753 | 760 | 768 | 775 | 782 | |
| 586 | 790 | 797 | 805 | 812 | 819 | 827 | 834 | 842 | 849 | 856 | |
| 587 | 864 | 871 | 879 | 886 | 893 | 901 | 908 | 916 | 923 | 930 | |
| 588 | 938 | 945 | 953 | 960 | 967 | 975 | 982 | 989 | 997 | *004 | |
| 589 | 77 012 | 019 | 026 | 034 | 041 | 048 | 056 | 063 | 070 | 078 | |
| **590** | 085 | 093 | 100 | 107 | 115 | 122 | 129 | 137 | 144 | 151 | |
| 591 | 159 | 166 | 173 | 181 | 188 | 195 | 203 | 210 | 217 | 225 | |
| 592 | 232 | 240 | 247 | 254 | 262 | 269 | 276 | 283 | 291 | 298 | |
| 593 | 305 | 313 | 320 | 327 | 335 | 342 | 349 | 357 | 364 | 371 | |
| 594 | 379 | 386 | 393 | 401 | 408 | 415 | 422 | 430 | 437 | 444 | |
| 595 | 452 | 459 | 466 | 474 | 481 | 488 | 495 | 503 | 510 | 517 | |
| 596 | 525 | 532 | 539 | 546 | 554 | 561 | 568 | 576 | 583 | 590 | |
| 597 | 597 | 605 | 612 | 619 | 627 | 634 | 641 | 648 | 656 | 663 | |
| 598 | 670 | 677 | 685 | 692 | 699 | 706 | 714 | 721 | 728 | 735 | |
| 599 | 743 | 750 | 757 | 764 | 772 | 779 | 786 | 793 | 801 | 808 | |
| **600** | 815 | 822 | 830 | 837 | 844 | 851 | 859 | 866 | 873 | 880 | |
| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |

**P. P.**

**8**

| 1 | 0.8 |
|---|---|
| 2 | 1.6 |
| 3 | 2.4 |
| 4 | 3.2 |
| 5 | 4.0 |
| 6 | 4.8 |
| 7 | 5.6 |
| 8 | 6.4 |
| 9 | 7.2 |

**7**

| 1 | 0.7 |
|---|---|
| 2 | 1.4 |
| 3 | 2.1 |
| 4 | 2.8 |
| 5 | 3.5 |
| 6 | 4.2 |
| 7 | 4.9 |
| 8 | 5.6 |
| 9 | 6.3 |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **600** | 77 815 | 822 | 830 | 837 | 844 | 851 | 859 | 866 | 873 | 880 |
| 601 | 887 | 895 | 902 | 909 | 916 | 924 | 931 | 938 | 945 | 952 |
| 602 | 960 | 967 | 974 | 981 | 988 | 996 | *003 | *010 | *017 | *025 |
| 603 | 78 032 | 039 | 046 | 053 | 061 | 068 | 075 | 082 | 089 | 097 |
| 604 | 104 | 111 | 118 | 125 | 132 | 140 | 147 | 154 | 161 | 168 |
| 605 | 176 | 183 | 190 | 197 | 204 | 211 | 219 | 226 | 233 | 240 |
| 606 | 247 | 254 | 262 | 269 | 276 | 283 | 290 | 297 | 305 | 312 |
| 607 | 319 | 326 | 333 | 340 | 347 | 355 | 362 | 369 | 376 | 383 |
| 608 | 390 | 398 | 405 | 412 | 419 | 426 | 433 | 440 | 447 | 455 |
| 609 | 462 | 469 | 476 | 483 | 490 | 497 | 504 | 512 | 519 | 526 |
| **610** | 533 | 540 | 547 | 554 | 561 | 569 | 576 | 583 | 590 | 597 |
| 611 | 604 | 611 | 618 | 625 | 633 | 640 | 647 | 654 | 661 | 668 |
| 612 | 675 | 682 | 689 | 696 | 704 | 711 | 718 | 725 | 732 | 739 |
| 613 | 746 | 753 | 760 | 767 | 774 | 781 | 789 | 796 | 803 | 810 |
| 614 | 817 | 824 | 831 | 838 | 845 | 852 | 859 | 866 | 873 | 880 |
| 615 | 888 | 895 | 902 | 909 | 916 | 923 | 930 | 937 | 944 | 951 |
| 616 | 958 | 965 | 972 | 979 | 986 | 993 | *000 | *007 | *014 | *021 |
| 617 | 79 029 | 036 | 043 | 050 | 057 | 064 | 071 | 078 | 085 | 092 |
| 618 | 099 | 106 | 113 | 120 | 127 | 134 | 141 | 148 | 155 | 162 |
| 619 | 169 | 176 | 183 | 190 | 197 | 204 | 211 | 218 | 225 | 232 |
| **620** | 239 | 246 | 253 | 260 | 267 | 274 | 281 | 288 | 295 | 302 |
| 621 | 309 | 316 | 323 | 330 | 337 | 344 | 351 | 358 | 365 | 372 |
| 622 | 379 | 386 | 393 | 400 | 407 | 414 | 421 | 428 | 435 | 442 |
| 623 | 449 | 456 | 463 | 470 | 477 | 484 | 491 | 498 | 505 | 511 |
| 624 | 518 | 525 | 532 | 539 | 546 | 553 | 560 | 567 | 574 | 581 |
| 625 | 588 | 595 | 602 | 609 | 616 | 623 | 630 | 637 | 644 | 650 |
| 626 | 657 | 664 | 671 | 678 | 685 | 692 | 699 | 706 | 713 | 720 |
| 627 | 727 | 734 | 741 | 748 | 754 | 761 | 768 | 775 | 782 | 789 |
| 628 | 796 | 803 | 810 | 817 | 824 | 831 | 837 | 844 | 851 | 858 |
| 629 | 865 | 872 | 879 | 886 | 893 | 900 | 906 | 913 | 920 | 927 |
| **630** | 934 | 941 | 948 | 955 | 962 | 969 | 975 | 982 | 989 | 996 |
| 631 | 80 003 | 010 | 017 | 024 | 030 | 037 | 044 | 051 | 058 | 065 |
| 632 | 072 | 079 | 085 | 092 | 099 | 106 | 113 | 120 | 127 | 134 |
| 633 | 140 | 147 | 154 | 161 | 168 | 175 | 182 | 188 | 195 | 202 |
| 634 | 209 | 216 | 223 | 229 | 236 | 243 | 250 | 257 | 264 | 271 |
| 635 | 277 | 284 | 291 | 298 | 305 | 312 | 318 | 325 | 332 | 339 |
| 636 | 346 | 353 | 359 | 366 | 373 | 380 | 387 | 393 | 400 | 407 |
| 637 | 414 | 421 | 428 | 434 | 441 | 448 | 455 | 462 | 468 | 475 |
| 638 | 482 | 489 | 496 | 502 | 509 | 516 | 523 | 530 | 536 | 543 |
| 639 | 550 | 557 | 564 | 570 | 577 | 584 | 591 | 598 | 604 | 611 |
| **640** | 618 | 625 | 632 | 638 | 645 | 652 | 659 | 665 | 672 | 679 |
| 641 | 686 | 693 | 699 | 706 | 713 | 720 | 726 | 733 | 740 | 747 |
| 642 | 754 | 760 | 767 | 774 | 781 | 787 | 794 | 801 | 808 | 814 |
| 643 | 821 | 828 | 835 | 841 | 848 | 855 | 862 | 868 | 875 | 882 |
| 644 | 889 | 895 | 902 | 909 | 916 | 922 | 929 | 936 | 943 | 949 |
| 645 | 956 | 963 | 969 | 976 | 983 | 990 | 996 | *003 | *010 | *017 |
| 646 | 81 023 | 030 | 037 | 043 | 050 | 057 | 064 | 070 | 077 | 084 |
| 647 | 090 | 097 | 104 | 111 | 117 | 124 | 131 | 137 | 144 | 151 |
| 648 | 158 | 164 | 171 | 178 | 184 | 191 | 198 | 204 | 211 | 218 |
| 649 | 224 | 231 | 238 | 245 | 251 | 258 | 265 | 271 | 278 | 285 |
| **650** | 291 | 298 | 305 | 311 | 318 | 325 | 331 | 338 | 345 | 351 |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

**P. P.**

**8**
| 1 | 0.8 |
|---|---|
| 2 | 1.6 |
| 3 | 2.4 |
| 4 | 3.2 |
| 5 | 4.0 |
| 6 | 4.8 |
| 7 | 5.6 |
| 8 | 6.4 |
| 9 | 7.2 |

**7**
| 1 | 0.7 |
|---|---|
| 2 | 1.4 |
| 3 | 2.1 |
| 4 | 2.8 |
| 5 | 3.5 |
| 6 | 4.2 |
| 7 | 4.9 |
| 8 | 5.6 |
| 9 | 6.3 |

**6**
| 1 | 0.6 |
|---|---|
| 2 | 1.2 |
| 3 | 1.8 |
| 4 | 2.4 |
| 5 | 3.0 |
| 6 | 3.6 |
| 7 | 4.2 |
| 8 | 4.8 |
| 9 | 5.4 |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **650** | 81 291 | 298 | 305 | 311 | 318 | 325 | 331 | 338 | 345 | 351 | |
| 651 | 358 | 365 | 371 | 378 | 385 | 391 | 398 | 405 | 411 | 418 | |
| 652 | 425 | 431 | 438 | 445 | 451 | 458 | 465 | 471 | 478 | 485 | |
| 653 | 491 | 498 | 505 | 511 | 518 | 525 | 531 | 538 | 544 | 551 | |
| 654 | 558 | 564 | 571 | 578 | 584 | 591 | 598 | 604 | 611 | 617 | |
| 655 | 624 | 631 | 637 | 644 | 651 | 657 | 664 | 671 | 677 | 684 | |
| 656 | 690 | 697 | 704 | 710 | 717 | 723 | 730 | 737 | 743 | 750 | |
| 657 | 757 | 763 | 770 | 776 | 783 | 790 | 796 | 803 | 809 | 816 | |
| 658 | 823 | 829 | 836 | 842 | 849 | 856 | 862 | 869 | 875 | 882 | |
| 659 | 889 | 895 | 902 | 908 | 915 | 921 | 928 | 935 | 941 | 948 | |
| **660** | 954 | 961 | 968 | 974 | 981 | 987 | 994 | *000 | *007 | *014 | |
| 661 | 82 020 | 027 | 033 | 040 | 046 | 053 | 060 | 066 | 073 | 079 | |
| 662 | 086 | 092 | 099 | 105 | 112 | 119 | 125 | 132 | 138 | 145 | |
| 663 | 151 | 158 | 164 | 171 | 178 | 184 | 191 | 197 | 204 | 210 | |
| 664 | 217 | 223 | 230 | 236 | 243 | 249 | 256 | 263 | 269 | 276 | |
| 665 | 282 | 289 | 295 | 302 | 308 | 315 | 321 | 328 | 334 | 341 | |
| 666 | 347 | 354 | 360 | 367 | 373 | 380 | 387 | 393 | 400 | 406 | |
| 667 | 413 | 419 | 426 | 432 | 439 | 445 | 452 | 458 | 465 | 471 | |
| 668 | 478 | 484 | 491 | 497 | 504 | 510 | 517 | 523 | 530 | 536 | |
| 669 | 543 | 549 | 556 | 562 | 569 | 575 | 582 | 588 | 595 | 601 | |
| **670** | 607 | 614 | 620 | 627 | 633 | 640 | 646 | 653 | 659 | 666 | |
| 671 | 672 | 679 | 685 | 692 | 698 | 705 | 711 | 718 | 724 | 730 | |
| 672 | 737 | 743 | 750 | 756 | 763 | 769 | 776 | 782 | 789 | 795 | |
| 673 | 802 | 808 | 814 | 821 | 827 | 834 | 840 | 847 | 853 | 860 | |
| 674 | 866 | 872 | 879 | 885 | 892 | 898 | 905 | 911 | 918 | 924 | |
| 675 | 930 | 937 | 943 | 950 | 956 | 963 | 969 | 975 | 982 | 988 | |
| 676 | 995 | *001 | *008 | *014 | *020 | *027 | *033 | *040 | *046 | *052 | |
| 677 | 83 059 | 065 | 072 | 078 | 085 | 091 | 097 | 104 | 110 | 117 | |
| 678 | 123 | 129 | 136 | 142 | 149 | 155 | 161 | 168 | 174 | 181 | |
| 679 | 187 | 193 | 200 | 206 | 213 | 219 | 225 | 232 | 238 | 245 | |
| **680** | 251 | 257 | 264 | 270 | 276 | 283 | 289 | 296 | 302 | 308 | |
| 681 | 315 | 321 | 327 | 334 | 340 | 347 | 353 | 359 | 366 | 372 | |
| 682 | 378 | 385 | 391 | 398 | 404 | 410 | 417 | 423 | 429 | 436 | |
| 683 | 442 | 448 | 455 | 461 | 467 | 474 | 480 | 487 | 493 | 499 | |
| 684 | 506 | 512 | 518 | 525 | 531 | 537 | 544 | 550 | 556 | 563 | |
| 685 | 569 | 575 | 582 | 588 | 594 | 601 | 607 | 613 | 620 | 626 | |
| 686 | 632 | 639 | 645 | 651 | 658 | 664 | 670 | 677 | 683 | 689 | |
| 687 | 696 | 702 | 708 | 715 | 721 | 727 | 734 | 740 | 746 | 753 | |
| 688 | 759 | 765 | 771 | 778 | 784 | 790 | 797 | 803 | 809 | 816 | |
| 689 | 822 | 828 | 835 | 841 | 847 | 853 | 860 | 866 | 872 | 879 | |
| **690** | 885 | 891 | 897 | 904 | 910 | 916 | 923 | 929 | 935 | 942 | |
| 691 | 948 | 954 | 960 | 967 | 973 | 979 | 985 | 992 | 998 | *004 | |
| 692 | 84 011 | 017 | 023 | 029 | 036 | 042 | 048 | 055 | 061 | 067 | |
| 693 | 073 | 080 | 086 | 092 | 098 | 105 | 111 | 117 | 123 | 130 | |
| 694 | 136 | 142 | 148 | 155 | 161 | 167 | 173 | 180 | 186 | 192 | |
| 695 | 198 | 205 | 211 | 217 | 223 | 230 | 236 | 242 | 248 | 255 | |
| 696 | 261 | 267 | 273 | 280 | 286 | 292 | 298 | 305 | 311 | 317 | |
| 697 | 323 | 330 | 336 | 342 | 348 | 354 | 361 | 367 | 373 | 379 | |
| 698 | 386 | 392 | 398 | 404 | 410 | 417 | 423 | 429 | 435 | 442 | |
| 699 | 448 | 454 | 460 | 466 | 473 | 479 | 485 | 491 | 497 | 504 | |
| **700** | 510 | 516 | 522 | 528 | 535 | 541 | 547 | 553 | 559 | 566 | |
| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |

P. P.

7
| 1 | 0.7 |
| 2 | 1.4 |
| 3 | 2.1 |
| 4 | 2.8 |
| 5 | 3.5 |
| 6 | 4.2 |
| 7 | 4.9 |
| 8 | 5.6 |
| 9 | 6.3 |

6
| 1 | 0.6 |
| 2 | 1.2 |
| 3 | 1.8 |
| 4 | 2.4 |
| 5 | 3.0 |
| 6 | 3.6 |
| 7 | 4.2 |
| 8 | 4.8 |
| 9 | 5.4 |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **700** | 84 510 | 516 | 522 | 528 | 535 | 541 | 547 | 553 | 559 | 566 |
| 701 | 572 | 578 | 584 | 590 | 597 | 603 | 609 | 615 | 621 | 628 |
| 702 | 634 | 640 | 646 | 652 | 658 | 665 | 671 | 677 | 683 | 689 |
| 703 | 696 | 702 | 708 | 714 | 720 | 726 | 733 | 739 | 745 | 751 |
| 704 | 757 | 763 | 770 | 776 | 782 | 788 | 794 | 800 | 807 | 813 |
| 705 | 819 | 825 | 831 | 837 | 844 | 850 | 856 | 862 | 868 | 874 |
| 706 | 880 | 887 | 893 | 899 | 905 | 911 | 917 | 924 | 930 | 936 |
| 707 | 942 | 948 | 954 | 960 | 967 | 973 | 979 | 985 | 991 | 997 |
| 708 | 85 003 | 009 | 016 | 022 | 028 | 034 | 040 | 046 | 052 | 058 |
| 709 | 065 | 071 | 077 | 083 | 089 | 095 | 101 | 107 | 114 | 120 |
| **710** | 126 | 132 | 138 | 144 | 150 | 156 | 163 | 169 | 175 | 181 |
| 711 | 187 | 193 | 199 | 205 | 211 | 217 | 224 | 230 | 236 | 242 |
| 712 | 248 | 254 | 260 | 266 | 272 | 278 | 285 | 291 | 297 | 303 |
| 713 | 309 | 315 | 321 | 327 | 333 | 339 | 345 | 352 | 358 | 364 |
| 714 | 370 | 376 | 382 | 388 | 394 | 400 | 406 | 412 | 418 | 425 |
| 715 | 431 | 437 | 443 | 449 | 455 | 461 | 467 | 473 | 479 | 485 |
| 716 | 491 | 497 | 503 | 509 | 516 | 522 | 528 | 534 | 540 | 546 |
| 717 | 552 | 558 | 564 | 570 | 576 | 582 | 588 | 594 | 600 | 606 |
| 718 | 612 | 618 | 625 | 631 | 637 | 643 | 649 | 655 | 661 | 667 |
| 719 | 673 | 679 | 685 | 691 | 697 | 703 | 709 | 715 | 721 | 727 |
| **720** | 733 | 739 | 745 | 751 | 757 | 763 | 769 | 775 | 781 | 788 |
| 721 | 794 | 800 | 806 | 812 | 818 | 824 | 830 | 836 | 842 | 848 |
| 722 | 854 | 860 | 866 | 872 | 878 | 884 | 890 | 896 | 902 | 908 |
| 723 | 914 | 920 | 926 | 932 | 938 | 944 | 950 | 956 | 962 | 968 |
| 724 | 974 | 980 | 986 | 992 | 998 | *004 | *010 | *016 | *022 | *028 |
| 725 | 86 034 | 040 | 046 | 052 | 058 | 064 | 070 | 076 | 082 | 088 |
| 726 | 094 | 100 | 106 | 112 | 118 | 124 | 130 | 136 | 141 | 147 |
| 727 | 153 | 159 | 165 | 171 | 177 | 183 | 189 | 195 | 201 | 207 |
| 728 | 213 | 219 | 225 | 231 | 237 | 243 | 249 | 255 | 261 | 267 |
| 729 | 273 | 279 | 285 | 291 | 297 | 303 | 308 | 314 | 320 | 326 |
| **730** | 332 | 338 | 344 | 350 | 356 | 362 | 368 | 374 | 380 | 386 |
| 731 | 392 | 398 | 404 | 410 | 415 | 421 | 427 | 433 | 439 | 445 |
| 732 | 451 | 457 | 463 | 469 | 475 | 481 | 487 | 493 | 499 | 504 |
| 733 | 510 | 516 | 522 | 528 | 534 | 540 | 546 | 552 | 558 | 564 |
| 734 | 570 | 576 | 581 | 587 | 593 | 599 | 605 | 611 | 617 | 623 |
| 735 | 629 | 635 | 641 | 646 | 652 | 658 | 664 | 670 | 676 | 682 |
| 736 | 688 | 694 | 700 | 705 | 711 | 717 | 723 | 729 | 735 | 741 |
| 737 | 747 | 753 | 759 | 764 | 770 | 776 | 782 | 788 | 794 | 800 |
| 738 | 806 | 812 | 817 | 823 | 829 | 835 | 841 | 847 | 853 | 859 |
| 739 | 864 | 870 | 876 | 882 | 888 | 894 | 900 | 906 | 911 | 917 |
| **740** | 923 | 929 | 935 | 941 | 947 | 953 | 958 | 964 | 970 | 976 |
| 741 | 982 | 988 | 994 | 999 | *005 | *011 | *017 | *023 | *029 | *035 |
| 742 | 87 040 | 046 | 052 | 058 | 064 | 070 | 075 | 081 | 087 | 093 |
| 743 | 099 | 105 | 111 | 116 | 122 | 128 | 134 | 140 | 146 | 151 |
| 744 | 157 | 163 | 169 | 175 | 181 | 186 | 192 | 198 | 204 | 210 |
| 745 | 216 | 221 | 227 | 233 | 239 | 245 | 251 | 256 | 262 | 268 |
| 746 | 274 | 280 | 286 | 291 | 297 | 303 | 309 | 315 | 320 | 326 |
| 747 | 332 | 338 | 344 | 349 | 355 | 361 | 367 | 373 | 378 | 384 |
| 748 | 390 | 396 | 402 | 408 | 413 | 419 | 425 | 431 | 437 | 442 |
| 749 | 448 | 454 | 460 | 466 | 471 | 477 | 483 | 489 | 495 | 500 |
| **750** | 506 | 512 | 518 | 523 | 529 | 535 | 541 | 547 | 552 | 558 |
| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

**P. P.**

**7**

| 1 | 0.7 |
|---|---|
| 2 | 1.4 |
| 3 | 2.1 |
| 4 | 2.8 |
| 5 | 3.5 |
| 6 | 4.2 |
| 7 | 4.9 |
| 8 | 5.6 |
| 9 | 6.3 |

**6**

| 1 | 0.6 |
|---|---|
| 2 | 1.2 |
| 3 | 1.8 |
| 4 | 2.4 |
| 5 | 3.0 |
| 6 | 3.6 |
| 7 | 4.2 |
| 8 | 4.8 |
| 9 | 5.4 |

**5**

| 1 | 0.5 |
|---|---|
| 2 | 1.0 |
| 3 | 1.5 |
| 4 | 2.0 |
| 5 | 2.5 |
| 6 | 3.0 |
| 7 | 3.5 |
| 8 | 4.0 |
| 9 | 4.5 |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **750** | 87 506 | 512 | 518 | 523 | 529 | 535 | 541 | 547 | 552 | 558 | |
| 751 | 564 | 570 | 576 | 581 | 587 | 593 | 599 | 604 | 610 | 616 | |
| 752 | 622 | 628 | 633 | 639 | 645 | 651 | 656 | 662 | 668 | 674 | |
| 753 | 679 | 685 | 691 | 697 | 703 | 708 | 714 | 720 | 726 | 731 | |
| 754 | 737 | 743 | 749 | 754 | 760 | 766 | 772 | 777 | 783 | 789 | |
| 755 | 795 | 800 | 806 | 812 | 818 | 823 | 829 | 835 | 841 | 846 | |
| 756 | 852 | 858 | 864 | 869 | 875 | 881 | 887 | 892 | 898 | 904 | |
| 757 | 910 | 915 | 921 | 927 | 933 | 938 | 944 | 950 | 955 | 961 | |
| 758 | 967 | 973 | 978 | 984 | 990 | 996 | *001 | *007 | *013 | *018 | |
| 759 | 88 024 | 030 | 036 | 041 | 047 | 053 | 058 | 064 | 070 | 076 | |
| **760** | 081 | 087 | 093 | 098 | 104 | 110 | 116 | 121 | 127 | 133 | |
| 761 | 138 | 144 | 150 | 156 | 161 | 167 | 173 | 178 | 184 | 190 | **6** |
| 762 | 195 | 201 | 207 | 213 | 218 | 224 | 230 | 235 | 241 | 247 | 1 0.6 |
| 763 | 252 | 258 | 264 | 270 | 275 | 281 | 287 | 292 | 298 | 304 | 2 1.2 |
| 764 | 309 | 315 | 321 | 326 | 332 | 338 | 343 | 349 | 355 | 360 | 3 1.8 |
| 765 | 366 | 372 | 377 | 383 | 389 | 395 | 400 | 406 | 412 | 417 | 4 2.4 |
| 766 | 423 | 429 | 434 | 440 | 446 | 451 | 457 | 463 | 468 | 474 | 5 3.0 |
| 767 | 480 | 485 | 491 | 497 | 502 | 508 | 513 | 519 | 525 | 530 | 6 3.6 |
| 768 | 536 | 542 | 547 | 553 | 559 | 564 | 570 | 576 | 581 | 587 | 7 4.2 |
| 769 | 593 | 598 | 604 | 610 | 615 | 621 | 627 | 632 | 638 | 643 | 8 4.8 |
| **770** | 649 | 655 | 660 | 666 | 672 | 677 | 683 | 689 | 694 | 700 | 9 5.4 |
| 771 | 705 | 711 | 717 | 722 | 728 | 734 | 739 | 745 | 750 | 756 | |
| 772 | 762 | 767 | 773 | 779 | 784 | 790 | 795 | 801 | 807 | 812 | |
| 773 | 818 | 824 | 829 | 835 | 840 | 846 | 852 | 857 | 863 | 868 | |
| 774 | 874 | 880 | 885 | 891 | 897 | 902 | 908 | 913 | 919 | 925 | |
| 775 | 930 | 936 | 941 | 947 | 953 | 958 | 964 | 969 | 975 | 981 | |
| 776 | 986 | 992 | 997 | *003 | *009 | *014 | *020 | *025 | *031 | *037 | |
| 777 | 89 042 | 048 | 053 | 059 | 064 | 070 | 076 | 081 | 087 | 092 | |
| 778 | 098 | 104 | 109 | 115 | 120 | 126 | 131 | 137 | 143 | 148 | |
| 779 | 154 | 159 | 165 | 170 | 176 | 182 | 187 | 193 | 198 | 204 | |
| **780** | 209 | 215 | 221 | 226 | 232 | 237 | 243 | 248 | 254 | 260 | **5** |
| 781 | 265 | 271 | 276 | 282 | 287 | 293 | 298 | 304 | 310 | 315 | 1 0.5 |
| 782 | 321 | 326 | 332 | 337 | 343 | 348 | 354 | 360 | 365 | 371 | 2 1.0 |
| 783 | 376 | 382 | 387 | 393 | 398 | 404 | 409 | 415 | 421 | 426 | 3 1.5 |
| 784 | 432 | 437 | 443 | 448 | 454 | 459 | 465 | 470 | 476 | 481 | 4 2.0 |
| 785 | 487 | 492 | 498 | 504 | 509 | 515 | 520 | 526 | 531 | 537 | 5 2.5 |
| 786 | 542 | 548 | 553 | 559 | 564 | 570 | 575 | 581 | 586 | 592 | 6 3.0 |
| 787 | 597 | 603 | 609 | 614 | 620 | 625 | 631 | 636 | 642 | 647 | 7 3.5 |
| 788 | 653 | 658 | 664 | 669 | 675 | 680 | 686 | 691 | 697 | 702 | 8 4.0 |
| 789 | 708 | 713 | 719 | 724 | 730 | 735 | 741 | 746 | 752 | 757 | 9 4.5 |
| **790** | 763 | 768 | 774 | 779 | 785 | 790 | 796 | 801 | 807 | 812 | |
| 791 | 818 | 823 | 829 | 834 | 840 | 845 | 851 | 856 | 862 | 867 | |
| 792 | 873 | 878 | 883 | 889 | 894 | 900 | 905 | 911 | 916 | 922 | |
| 793 | 927 | 933 | 938 | 944 | 949 | 955 | 960 | 966 | 971 | 977 | |
| 794 | 982 | 988 | 993 | 998 | *004 | *009 | *015 | *020 | *026 | *031 | |
| 795 | 90 037 | 042 | 048 | 053 | 059 | 064 | 069 | 075 | 080 | 086 | |
| 796 | 091 | 097 | 102 | 108 | 113 | 119 | 124 | 129 | 135 | 140 | |
| 797 | 146 | 151 | 157 | 162 | 168 | 173 | 179 | 184 | 189 | 195 | |
| 798 | 200 | 206 | 211 | 217 | 222 | 227 | 233 | 238 | 244 | 249 | |
| 799 | 255 | 260 | 266 | 271 | 276 | 282 | 287 | 293 | 298 | 304 | |
| **800** | 309 | 314 | 320 | 325 | 331 | 336 | 342 | 347 | 352 | 358 | |
| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **800** | 90 309 | 314 | 320 | 325 | 331 | 336 | 342 | 347 | 352 | 358 | |
| 801 | 363 | 369 | 374 | 380 | 385 | 390 | 396 | 401 | 407 | 412 | |
| 802 | 417 | 423 | 428 | 434 | 439 | 445 | 450 | 455 | 461 | 466 | |
| 803 | 472 | 477 | 482 | 488 | 493 | 499 | 504 | 509 | 515 | 520 | |
| 804 | 526 | 531 | 536 | 542 | 547 | 553 | 558 | 563 | 569 | 574 | |
| 805 | 580 | 585 | 590 | 596 | 601 | 607 | 612 | 617 | 623 | 628 | |
| 806 | 634 | 639 | 644 | 650 | 655 | 660 | 666 | 671 | 677 | 682 | |
| 807 | 687 | 693 | 698 | 703 | 709 | 714 | 720 | 725 | 730 | 736 | |
| 808 | 741 | 747 | 752 | 757 | 763 | 768 | 773 | 779 | 784 | 789 | |
| 809 | 795 | 800 | 806 | 811 | 816 | 822 | 827 | 832 | 838 | 843 | |
| **810** | 849 | 854 | 859 | 865 | 870 | 875 | 881 | 886 | 891 | 897 | |
| 811 | 902 | 907 | 913 | 918 | 924 | 929 | 934 | 940 | 945 | 950 | |
| 812 | 956 | 961 | 966 | 972 | 977 | 982 | 988 | 993 | 998 | *004 | |
| 813 | 91 009 | 014 | 020 | 025 | 030 | 036 | 041 | 046 | 052 | 057 | |
| 814 | 062 | 068 | 073 | 078 | 084 | 089 | 094 | 100 | 105 | 110 | |
| 815 | 116 | 121 | 126 | 132 | 137 | 142 | 148 | 153 | 158 | 164 | |
| 816 | 169 | 174 | 180 | 185 | 190 | 196 | 201 | 206 | 212 | 217 | |
| 817 | 222 | 228 | 233 | 238 | 243 | 249 | 254 | 259 | 265 | 270 | |
| 818 | 275 | 281 | 286 | 291 | 297 | 302 | 307 | 312 | 318 | 323 | |
| 819 | 328 | 334 | 339 | 344 | 350 | 355 | 360 | 365 | 371 | 376 | |
| **820** | 381 | 387 | 392 | 397 | 403 | 408 | 413 | 418 | 424 | 429 | |
| 821 | 434 | 440 | 445 | 450 | 455 | 461 | 466 | 471 | 477 | 482 | |
| 822 | 487 | 492 | 498 | 503 | 508 | 514 | 519 | 524 | 529 | 535 | |
| 823 | 540 | 545 | 551 | 556 | 561 | 566 | 572 | 577 | 582 | 587 | |
| 824 | 593 | 598 | 603 | 609 | 614 | 619 | 624 | 630 | 635 | 640 | |
| 825 | 645 | 651 | 656 | 661 | 666 | 672 | 677 | 682 | 687 | 693 | |
| 826 | 698 | 703 | 709 | 714 | 719 | 724 | 730 | 735 | 740 | 745 | |
| 827 | 751 | 756 | 761 | 766 | 772 | 777 | 782 | 787 | 793 | 798 | |
| 828 | 803 | 808 | 814 | 819 | 824 | 829 | 834 | 840 | 845 | 850 | |
| 829 | 855 | 861 | 866 | 871 | 876 | 882 | 887 | 892 | 897 | 903 | |
| **830** | 908 | 913 | 918 | 924 | 929 | 934 | 939 | 944 | 950 | 955 | |
| 831 | 960 | 965 | 971 | 976 | 981 | 986 | 991 | 997 | *002 | *007 | |
| 832 | 92 012 | 018 | 023 | 028 | 033 | 038 | 044 | 049 | 054 | 059 | |
| 833 | 065 | 070 | 075 | 080 | 085 | 091 | 096 | 101 | 106 | 111 | |
| 834 | 117 | 122 | 127 | 132 | 137 | 143 | 148 | 153 | 158 | 163 | |
| 835 | 169 | 174 | 179 | 184 | 189 | 195 | 200 | 205 | 210 | 215 | |
| 836 | 221 | 226 | 231 | 236 | 241 | 247 | 252 | 257 | 262 | 267 | |
| 837 | 273 | 278 | 283 | 288 | 293 | 298 | 304 | 309 | 314 | 319 | |
| 838 | 324 | 330 | 335 | 340 | 345 | 350 | 355 | 361 | 366 | 371 | |
| 839 | 376 | 381 | 387 | 392 | 397 | 402 | 407 | 412 | 418 | 423 | |
| **840** | 428 | 433 | 438 | 443 | 449 | 454 | 459 | 464 | 469 | 474 | |
| 841 | 480 | 485 | 490 | 495 | 500 | 505 | 511 | 516 | 521 | 526 | |
| 842 | 531 | 536 | 542 | 547 | 552 | 557 | 562 | 567 | 572 | 578 | |
| 843 | 583 | 588 | 593 | 598 | 603 | 609 | 614 | 619 | 624 | 629 | |
| 844 | 634 | 639 | 645 | 650 | 655 | 660 | 665 | 670 | 675 | 681 | |
| 845 | 686 | 691 | 696 | 701 | 706 | 711 | 716 | 722 | 727 | 732 | |
| 846 | 737 | 742 | 747 | 752 | 758 | 763 | 768 | 773 | 778 | 783 | |
| 847 | 788 | 793 | 799 | 804 | 809 | 814 | 819 | 824 | 829 | 834 | |
| 848 | 840 | 845 | 850 | 855 | 860 | 865 | 870 | 875 | 881 | 886 | |
| 849 | 891 | 896 | 901 | 906 | 911 | 916 | 921 | 927 | 932 | 937 | |
| **850** | 942 | 947 | 952 | 957 | 962 | 967 | 973 | 978 | 983 | 988 | |
| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |

P. P.

**6**

| 1 | 0.6 |
|---|---|
| 2 | 1.2 |
| 3 | 1.8 |
| 4 | 2.4 |
| 5 | 3.0 |
| 6 | 3.6 |
| 7 | 4.2 |
| 8 | 4.8 |
| 9 | 5.4 |

**5**

| 1 | 0.5 |
|---|---|
| 2 | 1.0 |
| 3 | 1.5 |
| 4 | 2.0 |
| 5 | 2.5 |
| 6 | 3.0 |
| 7 | 3.5 |
| 8 | 4.0 |
| 9 | 4.5 |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P.P. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **850** | 92 942 | 947 | 952 | 957 | 962 | 967 | 973 | 978 | 983 | 988 | |
| 851 | 993 | 998 | *003 | *008 | *013 | *018 | *024 | *029 | *034 | *039 | |
| 852 | 93 044 | 049 | 054 | 059 | 064 | 069 | 075 | 080 | 085 | 090 | |
| 853 | 095 | 100 | 105 | 110 | 115 | 120 | 125 | 131 | 136 | 141 | |
| 854 | 146 | 151 | 156 | 161 | 166 | 171 | 176 | 181 | 186 | 192 | |
| 855 | 197 | 202 | 207 | 212 | 217 | 222 | 227 | 232 | 237 | 242 | |
| 856 | 247 | 252 | 258 | 263 | 268 | 273 | 278 | 283 | 288 | 293 | |
| 857 | 298 | 303 | 308 | 313 | 318 | 323 | 328 | 334 | 339 | 344 | **6** |
| 858 | 349 | 354 | 359 | 364 | 369 | 374 | 379 | 384 | 389 | 394 | 1  0.6 |
| 859 | 399 | 404 | 409 | 414 | 420 | 425 | 430 | 435 | 440 | 445 | 2  1.2 |
| **860** | 450 | 455 | 460 | 465 | 470 | 475 | 480 | 485 | 490 | 495 | 3  1.8 |
| 861 | 500 | 505 | 510 | 515 | 520 | 526 | 531 | 536 | 541 | 546 | 4  2.4 |
| 862 | 551 | 556 | 561 | 566 | 571 | 576 | 581 | 586 | 591 | 596 | 5  3.0 |
| 863 | 601 | 606 | 611 | 616 | 621 | 626 | 631 | 636 | 641 | 646 | 6  3.6 |
| 864 | 651 | 656 | 661 | 666 | 671 | 676 | 682 | 687 | 692 | 697 | 7  4.2 |
| 865 | 702 | 707 | 712 | 717 | 722 | 727 | 732 | 737 | 742 | 747 | 8  4.8 |
| 866 | 752 | 757 | 762 | 767 | 772 | 777 | 782 | 787 | 792 | 797 | 9  5.4 |
| 867 | 802 | 807 | 812 | 817 | 822 | 827 | 832 | 837 | 842 | 847 | |
| 868 | 852 | 857 | 862 | 867 | 872 | 877 | 882 | 887 | 892 | 897 | |
| 869 | 902 | 907 | 912 | 917 | 922 | 927 | 932 | 937 | 942 | 947 | |
| **870** | 952 | 957 | 962 | 967 | 972 | 977 | 982 | 987 | 992 | 997 | |
| 871 | 94 002 | 007 | 012 | 017 | 022 | 027 | 032 | 037 | 042 | 047 | **5** |
| 872 | 052 | 057 | 062 | 067 | 072 | 077 | 082 | 086 | 091 | 096 | 1  0.5 |
| 873 | 101 | 106 | 111 | 116 | 121 | 126 | 131 | 136 | 141 | 146 | 2  1.0 |
| 874 | 151 | 156 | 161 | 166 | 171 | 176 | 181 | 186 | 191 | 196 | 3  1.5 |
| 875 | 201 | 206 | 211 | 216 | 221 | 226 | 231 | 236 | 240 | 245 | 4  2.0 |
| 876 | 250 | 255 | 260 | 265 | 270 | 275 | 280 | 285 | 290 | 295 | 5  2.5 |
| 877 | 300 | 305 | 310 | 315 | 320 | 325 | 330 | 335 | 340 | 345 | 6  3.0 |
| 878 | 349 | 354 | 359 | 364 | 369 | 374 | 379 | 384 | 389 | 394 | 7  3.5 |
| 879 | 399 | 404 | 409 | 414 | 419 | 424 | 429 | 433 | 438 | 443 | 8  4.0 |
| **880** | 448 | 453 | 458 | 463 | 468 | 473 | 478 | 483 | 488 | 493 | 9  4.5 |
| 881 | 498 | 503 | 507 | 512 | 517 | 522 | 527 | 532 | 537 | 542 | |
| 882 | 547 | 552 | 557 | 562 | 567 | 571 | 576 | 581 | 586 | 591 | |
| 883 | 596 | 601 | 606 | 611 | 616 | 621 | 626 | 630 | 635 | 640 | |
| 884 | 645 | 650 | 655 | 660 | 665 | 670 | 675 | 680 | 685 | 689 | |
| 885 | 694 | 699 | 704 | 709 | 714 | 719 | 724 | 729 | 734 | 738 | |
| 886 | 743 | 748 | 753 | 758 | 763 | 768 | 773 | 778 | 783 | 787 | **4** |
| 887 | 792 | 797 | 802 | 807 | 812 | 817 | 822 | 827 | 832 | 836 | 1  0.4 |
| 888 | 841 | 846 | 851 | 856 | 861 | 866 | 871 | 876 | 880 | 885 | 2  0.8 |
| 889 | 890 | 895 | 900 | 905 | 910 | 915 | 919 | 924 | 929 | 934 | 3  1.2 |
| **890** | 939 | 944 | 949 | 954 | 959 | 963 | 968 | 973 | 978 | 983 | 4  1.6 |
| 891 | 988 | 993 | 998 | *002 | *007 | *012 | *017 | *022 | *027 | *032 | 5  2.0 |
| 892 | 95 036 | 041 | 046 | 051 | 056 | 061 | 066 | 071 | 075 | 080 | 6  2.4 |
| 893 | 085 | 090 | 095 | 100 | 105 | 109 | 114 | 119 | 124 | 129 | 7  2.8 |
| 894 | 134 | 139 | 143 | 148 | 153 | 158 | 163 | 168 | 173 | 178 | 8  3.2 |
| 895 | 182 | 187 | 192 | 197 | 202 | 207 | 211 | 216 | 221 | 226 | 9  3.6 |
| 896 | 231 | 236 | 240 | 245 | 250 | 255 | 260 | 265 | 270 | 274 | |
| 897 | 279 | 284 | 289 | 294 | 299 | 303 | 308 | 313 | 318 | 323 | |
| 898 | 328 | 332 | 337 | 342 | 347 | 352 | 357 | 361 | 366 | 371 | |
| 899 | 376 | 381 | 386 | 390 | 395 | 400 | 405 | 410 | 415 | 419 | |
| **900** | 424 | 429 | 434 | 439 | 444 | 448 | 453 | 458 | 463 | 468 | |
| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P.P. |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **900** | 95 424 | 429 | 434 | 439 | 444 | 448 | 453 | 458 | 463 | 468 | |
| 901 | 472 | 477 | 482 | 487 | 492 | 497 | 501 | 506 | 511 | 516 | |
| 902 | 521 | 525 | 530 | 535 | 540 | 545 | 550 | 554 | 559 | 564 | |
| 903 | 569 | 574 | 578 | 583 | 588 | 593 | 598 | 602 | 607 | 612 | |
| 904 | 617 | 622 | 626 | 631 | 636 | 641 | 646 | 650 | 655 | 660 | |
| 905 | 665 | 670 | 674 | 679 | 684 | 689 | 694 | 698 | 703 | 708 | |
| 906 | 713 | 718 | 722 | 727 | 732 | 737 | 742 | 746 | 751 | 756 | |
| 907 | 761 | 766 | 770 | 775 | 780 | 785 | 789 | 794 | 799 | 804 | |
| 908 | 809 | 813 | 818 | 823 | 828 | 832 | 837 | 842 | 847 | 852 | |
| 909 | 856 | 861 | 866 | 871 | 875 | 880 | 885 | 890 | 895 | 899 | |
| **910** | 904 | 909 | 914 | 918 | 923 | 928 | 933 | 938 | 942 | 947 | |
| 911 | 952 | 957 | 961 | 966 | 971 | 976 | 980 | 985 | 990 | 995 | |
| 912 | 999 | *004 | *009 | *014 | *019 | *023 | *028 | *033 | *038 | *042 | |
| 913 | 96 047 | 052 | 057 | 061 | 066 | 071 | 076 | 080 | 085 | 090 | |
| 914 | 095 | 099 | 104 | 109 | 114 | 118 | 123 | 128 | 133 | 137 | |
| 915 | 142 | 147 | 152 | 156 | 161 | 166 | 171 | 175 | 180 | 185 | |
| 916 | 190 | 194 | 199 | 204 | 209 | 213 | 218 | 223 | 227 | 232 | |
| 917 | 237 | 242 | 246 | 251 | 256 | 261 | 265 | 270 | 275 | 280 | |
| 918 | 284 | 289 | 294 | 298 | 303 | 308 | 313 | 317 | 322 | 327 | |
| 919 | 332 | 336 | 341 | 346 | 350 | 355 | 360 | 365 | 369 | 374 | |
| **920** | 379 | 384 | 388 | 393 | 398 | 402 | 407 | 412 | 417 | 421 | |
| 921 | 426 | 431 | 435 | 440 | 445 | 450 | 454 | 459 | 464 | 468 | |
| 922 | 473 | 478 | 483 | 487 | 492 | 497 | 501 | 506 | 511 | 515 | |
| 923 | 520 | 525 | 530 | 534 | 539 | 544 | 548 | 553 | 558 | 562 | |
| 924 | 567 | 572 | 577 | 581 | 586 | 591 | 595 | 600 | 605 | 609 | |
| 925 | 614 | 619 | 624 | 628 | 633 | 638 | 642 | 647 | 652 | 656 | |
| 926 | 661 | 666 | 670 | 675 | 680 | 685 | 689 | 694 | 699 | 703 | |
| 927 | 708 | 713 | 717 | 722 | 727 | 731 | 736 | 741 | 745 | 750 | |
| 928 | 755 | 759 | 764 | 769 | 774 | 778 | 783 | 788 | 792 | 797 | |
| 929 | 802 | 806 | 811 | 816 | 820 | 825 | 830 | 834 | 839 | 844 | |
| **930** | 848 | 853 | 858 | 862 | 867 | 872 | 876 | 881 | 886 | 890 | |
| 931 | 895 | 900 | 904 | 909 | 914 | 918 | 923 | 928 | 932 | 937 | |
| 932 | 942 | 946 | 951 | 956 | 960 | 965 | 970 | 974 | 979 | 984 | |
| 933 | 988 | 993 | 997 | *002 | *007 | *011 | *016 | *021 | *025 | *030 | |
| 934 | 97 035 | 039 | 044 | 049 | 053 | 058 | 063 | 067 | 072 | 077 | |
| 935 | 081 | 086 | 090 | 095 | 100 | 104 | 109 | 114 | 118 | 123 | |
| 936 | 128 | 132 | 137 | 142 | 146 | 151 | 155 | 160 | 165 | 169 | |
| 937 | 174 | 179 | 183 | 188 | 192 | 197 | 202 | 206 | 211 | 216 | |
| 938 | 220 | 225 | 230 | 234 | 239 | 243 | 248 | 253 | 257 | 262 | |
| 939 | 267 | 271 | 276 | 280 | 285 | 290 | 294 | 299 | 304 | 308 | |
| **940** | 313 | 317 | 322 | 327 | 331 | 336 | 340 | 345 | 350 | 354 | |
| 941 | 359 | 364 | 368 | 373 | 377 | 382 | 387 | 391 | 396 | 400 | |
| 942 | 405 | 410 | 414 | 419 | 424 | 428 | 433 | 437 | 442 | 447 | |
| 943 | 451 | 456 | 460 | 465 | 470 | 474 | 479 | 483 | 488 | 493 | |
| 944 | 497 | 502 | 506 | 511 | 516 | 520 | 525 | 529 | 534 | 539 | |
| 945 | 543 | 548 | 552 | 557 | 562 | 566 | 571 | 575 | 580 | 585 | |
| 946 | 589 | 594 | 598 | 603 | 607 | 612 | 617 | 621 | 626 | 630 | |
| 947 | 635 | 640 | 644 | 649 | 653 | 658 | 663 | 667 | 672 | 676 | |
| 948 | 681 | 685 | 690 | 695 | 699 | 704 | 708 | 713 | 717 | 722 | |
| 949 | 727 | 731 | 736 | 740 | 745 | 749 | 754 | 759 | 763 | 768 | |
| **950** | 772 | 777 | 782 | 786 | 791 | 795 | 800 | 804 | 809 | 813 | |

P. P.

**5**

| | |
|---|---|
| 1 | 0.5 |
| 2 | 1.0 |
| 3 | 1.5 |
| 4 | 2.0 |
| 5 | 2.5 |
| 6 | 3.0 |
| 7 | 3.5 |
| 8 | 4.0 |
| 9 | 4.5 |

**4**

| | |
|---|---|
| 1 | 0.4 |
| 2 | 0.8 |
| 3 | 1.2 |
| 4 | 1.6 |
| 5 | 2.0 |
| 6 | 2.4 |
| 7 | 2.8 |
| 8 | 3.2 |
| 9 | 3.6 |

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |
|---|---|---|---|---|---|---|---|---|---|---|---|

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **950** | 97 772 | 777 | 782 | 786 | 791 | 795 | 800 | 804 | 809 | 813 | |
| 951 | 818 | 823 | 827 | 832 | 836 | 841 | 845 | 850 | 855 | 859 | |
| 952 | 864 | 868 | 873 | 877 | 882 | 886 | 891 | 896 | 900 | 905 | |
| 953 | 909 | 914 | 918 | 923 | 928 | 932 | 937 | 941 | 946 | 950 | |
| 954 | 955 | 959 | 964 | 968 | 973 | 978 | 982 | 987 | 991 | 996 | |
| 955 | 98 000 | 005 | 009 | 014 | 019 | 023 | 028 | 032 | 037 | 041 | |
| 956 | 046 | 050 | 055 | 059 | 064 | 068 | 073 | 078 | 082 | 087 | |
| 957 | 091 | 096 | 100 | 105 | 109 | 114 | 118 | 123 | 127 | 132 | |
| 958 | 137 | 141 | 146 | 150 | 155 | 159 | 164 | 168 | 173 | 177 | |
| 959 | 182 | 186 | 191 | 195 | 200 | 204 | 209 | 214 | 218 | 223 | |
| **960** | 227 | 232 | 236 | 241 | 245 | 250 | 254 | 259 | 263 | 268 | **5** |
| 961 | 272 | 277 | 281 | 286 | 290 | 295 | 299 | 304 | 308 | 313 | 1   0.5 |
| 962 | 318 | 322 | 327 | 331 | 336 | 340 | 345 | 349 | 354 | 358 | 2   1.0 |
| 963 | 363 | 367 | 372 | 376 | 381 | 385 | 390 | 394 | 399 | 403 | 3   1.5 |
| 964 | 408 | 412 | 417 | 421 | 426 | 430 | 435 | 439 | 444 | 448 | 4   2.0 |
| 965 | 453 | 457 | 462 | 466 | 471 | 475 | 480 | 484 | 489 | 493 | 5   2.5 |
| 966 | 498 | 502 | 507 | 511 | 516 | 520 | 525 | 529 | 534 | 538 | 6   3.0 |
| 967 | 543 | 547 | 552 | 556 | 561 | 565 | 570 | 574 | 579 | 583 | 7   3.5 |
| 968 | 588 | 592 | 597 | 601 | 605 | 610 | 614 | 619 | 623 | 628 | 8   4.0 |
| 969 | 632 | 637 | 641 | 646 | 650 | 655 | 659 | 664 | 668 | 673 | 9   4.5 |
| **970** | 677 | 682 | 686 | 691 | 695 | 700 | 704 | 709 | 713 | 717 | |
| 971 | 722 | 726 | 731 | 735 | 740 | 744 | 749 | 753 | 758 | 762 | |
| 972 | 767 | 771 | 776 | 780 | 784 | 789 | 793 | 798 | 802 | 807 | |
| 973 | 811 | 816 | 820 | 825 | 829 | 834 | 838 | 843 | 847 | 851 | |
| 974 | 856 | 860 | 865 | 869 | 874 | 878 | 883 | 887 | 892 | 896 | |
| 975 | 900 | 905 | 909 | 914 | 918 | 923 | 927 | 932 | 936 | 941 | |
| 976 | 945 | 949 | 954 | 958 | 963 | 967 | 972 | 976 | 981 | 985 | |
| 977 | 989 | 994 | 998 | *003 | *007 | *012 | *016 | *021 | *025 | *029 | |
| 978 | 99 034 | 038 | 043 | 047 | 052 | 056 | 061 | 065 | 069 | 074 | |
| 979 | 078 | 083 | 087 | 092 | 096 | 100 | 105 | 109 | 114 | 118 | |
| **980** | 123 | 127 | 131 | 136 | 140 | 145 | 149 | 154 | 158 | 162 | **4** |
| 981 | 167 | 171 | 176 | 180 | 185 | 189 | 193 | 198 | 202 | 207 | 1   0.4 |
| 982 | 211 | 216 | 220 | 224 | 229 | 233 | 238 | 242 | 247 | 251 | 2   0.8 |
| 983 | 255 | 260 | 264 | 269 | 273 | 277 | 282 | 286 | 291 | 295 | 3   1.2 |
| 984 | 300 | 304 | 308 | 313 | 317 | 322 | 326 | 330 | 335 | 339 | 4   1.6 |
| 985 | 344 | 348 | 352 | 357 | 361 | 366 | 370 | 374 | 379 | 383 | 5   2.0 |
| 986 | 388 | 392 | 396 | 401 | 405 | 410 | 414 | 419 | 423 | 427 | 6   2.4 |
| 987 | 432 | 436 | 441 | 445 | 449 | 454 | 458 | 463 | 467 | 471 | 7   2.8 |
| 988 | 476 | 480 | 484 | 489 | 493 | 498 | 502 | 506 | 511 | 515 | 8   3.2 |
| 989 | 520 | 524 | 528 | 533 | 537 | 542 | 546 | 550 | 555 | 559 | 9   3.6 |
| **990** | 564 | 568 | 572 | 577 | 581 | 585 | 590 | 594 | 599 | 603 | |
| 991 | 607 | 612 | 616 | 621 | 625 | 629 | 634 | 638 | 642 | 647 | |
| 992 | 651 | 656 | 660 | 664 | 669 | 673 | 677 | 682 | 686 | 691 | |
| 993 | 695 | 699 | 704 | 708 | 712 | 717 | 721 | 726 | 730 | 734 | |
| 994 | 739 | 743 | 747 | 752 | 756 | 760 | 765 | 769 | 774 | 778 | |
| 995 | 782 | 787 | 791 | 795 | 800 | 804 | 808 | 813 | 817 | 822 | |
| 996 | 826 | 830 | 835 | 839 | 843 | 848 | 852 | 856 | 861 | 865 | |
| 997 | 870 | 874 | 878 | 883 | 887 | 891 | 896 | 900 | 904 | 909 | |
| 998 | 913 | 917 | 922 | 926 | 930 | 935 | 939 | 944 | 948 | 952 | |
| 999 | 957 | 961 | 965 | 970 | 974 | 978 | 983 | 987 | 991 | 996 | |
| **1000** | 00 000 | 004 | 009 | 013 | 017 | 022 | 026 | 030 | 035 | 039 | |
| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | P. P. |

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| ″ | ′ | L. Sin. | d. | Cpl. S. | Cpl. T. | L. Tang. | d. c. | L. Cotg. | L. Cos. | |
|---|---|---|---|---|---|---|---|---|---|---|
| o | 0 | — | | 5.31443 | 5.31443 | — | | — | 0.00000 | 60 |
| 60 | 1 | 6.46373 | | 5.31443 | 5.31443 | 6.46373 | | 3.53627 | 0.00000 | 59 |
| 120 | 2 | 6.76476 | 30103 | 5.31443 | 5.31443 | 6.76476 | 30103 | 3.23524 | 0.00000 | 58 |
| 180 | 3 | 6.94085 | 17609 | 5.31443 | 5.31443 | 6.94085 | 17609 | 3.05915 | 0.00000 | 57 |
| 240 | 4 | 7.06579 | 12494 | 5.31443 | 5.31442 | 7.06579 | 12494 | 2.93421 | 0.00000 | 56 |
| 300 | 5 | 7.16270 | 9691 | 5.31443 | 5.31442 | 7.16270 | 9691 | 2.83730 | 0.00000 | 55 |
| 360 | 6 | 7.24188 | 7918 | 5.31443 | 5.31442 | 7.24188 | 7918 | 2.75812 | 0.00000 | 54 |
| 420 | 7 | 7.30882 | 6694 | 5.31443 | 5.31442 | 7.30882 | 6694 | 2.69118 | 0.00000 | 53 |
| 480 | 8 | 7.36682 | 5800 | 5.31443 | 5.31442 | 7.36682 | 5800 | 2.63318 | 0.00000 | 52 |
| 540 | 9 | 7.41797 | 5115 | 5.31443 | 5.31442 | 7.41797 | 5115 | 2.58203 | 0.00000 | 51 |
| 600 | 10 | 7.46373 | 4576 | 5.31443 | 5.31442 | 7.46373 | 4576 | 2.53627 | 0.00000 | 50 |
| 660 | 11 | 7.50512 | 4139 | 5.31443 | 5.31442 | 7.50512 | 4139 | 2.49488 | 0.00000 | 49 |
| 720 | 12 | 7.54291 | 3779 | 5.31443 | 5.31442 | 7.54291 | 3779 | 2.45709 | 0.00000 | 48 |
| 780 | 13 | 7.57767 | 3476 | 5.31443 | 5.31442 | 7.57767 | 3476 | 2.42233 | 0.00000 | 47 |
| 840 | 14 | 7.60985 | 3218 | 5.31443 | 5.31442 | 7.60986 | 3219 | 2.39014 | 0.00000 | 46 |
| 900 | 15 | 7.63982 | 2997 | 5.31443 | 5.31442 | 7.63982 | 2996 | 2.36018 | 0.00000 | 45 |
| 960 | 16 | 7.66784 | 2802 | 5.31443 | 5.31442 | 7.66785 | 2803 | 2.33215 | 0.00000 | 44 |
| 1020 | 17 | 7.69417 | 2633 | 5.31443 | 5.31442 | 7.69418 | 2633 | 2.30582 | 0.00000 | 43 |
| 1080 | 18 | 7.71900 | 2483 | 5.31443 | 5.31442 | 7.71900 | 2482 | 2.28100 | 9.99999 | 42 |
| 1140 | 19 | 7.74248 | 2348 | 5.31443 | 5.31442 | 7.74248 | 2348 | 2.25752 | 9.99999 | 41 |
| 1200 | 20 | 7.76475 | 2227 | 5.31443 | 5.31442 | 7.76476 | 2228 | 2.23524 | 9.99999 | 40 |
| 1260 | 21 | 7.78594 | 2119 | 5.31443 | 5.31442 | 7.78595 | 2119 | 2.21405 | 9.99999 | 39 |
| 1320 | 22 | 7.80615 | 2021 | 5.31443 | 5.31442 | 7.80615 | 2020 | 2.19385 | 9.99999 | 38 |
| 1380 | 23 | 7.82545 | 1930 | 5.31443 | 5.31442 | 7.82546 | 1931 | 2.17454 | 9.99999 | 37 |
| 1440 | 24 | 7.84393 | 1848 | 5.31443 | 5.31442 | 7.84394 | 1848 | 2.15606 | 9.99999 | 36 |
| 1500 | 25 | 7.86166 | 1773 | 5.31443 | 5.31442 | 7.86167 | 1773 | 2.13833 | 9.99999 | 35 |
| 1560 | 26 | 7.87870 | 1704 | 5.31443 | 5.31442 | 7.87871 | 1704 | 2.12129 | 9.99999 | 34 |
| 1620 | 27 | 7.89509 | 1639 | 5.31443 | 5.31442 | 7.89510 | 1639 | 2.10490 | 9.99999 | 33 |
| 1680 | 28 | 7.91088 | 1579 | 5.31443 | 5.31442 | 7.91089 | 1579 | 2.08911 | 9.99999 | 32 |
| 1740 | 29 | 7.92612 | 1524 | 5.31443 | 5.31441 | 7.92613 | 1524 | 2.07387 | 9.99998 | 31 |
| 1800 | 30 | 7.94084 | 1472 | 5.31443 | 5.31441 | 7.94086 | 1473 | 2.05914 | 9.99998 | 30 |
| 1860 | 31 | 7.95508 | 1424 | 5.31443 | 5.31441 | 7.95510 | 1424 | 2.04490 | 9.99998 | 29 |
| 1920 | 32 | 7.96887 | 1379 | 5.31443 | 5.31441 | 7.96889 | 1379 | 2.03111 | 9.99998 | 28 |
| 1980 | 33 | 7.98223 | 1336 | 5.31443 | 5.31441 | 7.98225 | 1336 | 2.01775 | 9.99998 | 27 |
| 2040 | 34 | 7.99520 | 1297 | 5.31443 | 5.31441 | 7.99522 | 1297 | 2.00478 | 9.99998 | 26 |
| 2100 | 35 | 8.00779 | 1259 | 5.31443 | 5.31441 | 8.00781 | 1259 | 1.99219 | 9.99998 | 25 |
| 2160 | 36 | 8.02002 | 1223 | 5.31443 | 5.31441 | 8.02004 | 1223 | 1.97996 | 9.99998 | 24 |
| 2220 | 37 | 8.03192 | 1190 | 5.31443 | 5.31441 | 8.03194 | 1190 | 1.96806 | 9.99997 | 23 |
| 2280 | 38 | 8.04350 | 1158 | 5.31443 | 5.31441 | 8.04353 | 1159 | 1.95647 | 9.99997 | 22 |
| 2340 | 39 | 8.05478 | 1128 | 5.31443 | 5.31441 | 8.05481 | 1128 | 1.94519 | 9.99997 | 21 |
| 2400 | 40 | 8.06578 | 1100 | 5.31443 | 5.31441 | 8.06581 | 1100 | 1.93419 | 9.99997 | 20 |
| 2460 | 41 | 8.07650 | 1072 | 5.31444 | 5.31440 | 8.07653 | 1072 | 1.92347 | 9.99997 | 19 |
| 2520 | 42 | 8.08696 | 1046 | 5.31444 | 5.31440 | 8.08700 | 1047 | 1.91300 | 9.99997 | 18 |
| 2580 | 43 | 8.09718 | 1022 | 5.31444 | 5.31440 | 8.09722 | 1022 | 1.90278 | 9.99997 | 17 |
| 2640 | 44 | 8.10717 | 999 | 5.31444 | 5.31440 | 8.10720 | 998 | 1.89280 | 9.99996 | 16 |
| 2700 | 45 | 8.11693 | 976 | 5.31444 | 5.31440 | 8.11696 | 976 | 1.88304 | 9.99996 | 15 |
| 2760 | 46 | 8.12647 | 954 | 5.31444 | 5.31440 | 8.12651 | 955 | 1.87349 | 9.99996 | 14 |
| 2820 | 47 | 8.13581 | 934 | 5.31444 | 5.31440 | 8.13585 | 934 | 1.86415 | 9.99996 | 13 |
| 2880 | 48 | 8.14495 | 914 | 5.31444 | 5.31440 | 8.14500 | 915 | 1.85500 | 9.99996 | 12 |
| 2940 | 49 | 8.15391 | 896 | 5.31444 | 5.31440 | 8.15395 | 895 | 1.84605 | 9.99996 | 11 |
| 3000 | 50 | 8.16268 | 877 | 5.31444 | 5.31439 | 8.16273 | 878 | 1.83727 | 9.99995 | 10 |
| 3060 | 51 | 8.17128 | 860 | 5.31444 | 5.31439 | 8.17133 | 860 | 1.82867 | 9.99995 | 9 |
| 3120 | 52 | 8.17971 | 843 | 5.31444 | 5.31439 | 8.17976 | 843 | 1.82024 | 9.99995 | 8 |
| 3180 | 53 | 8.18798 | 827 | 5.31444 | 5.31439 | 8.18804 | 828 | 1.81196 | 9.99995 | 7 |
| 3240 | 54 | 8.19610 | 812 | 5.31444 | 5.31439 | 8.19616 | 812 | 1.80384 | 9.99995 | 6 |
| 3300 | 55 | 8.20407 | 797 | 5.31444 | 5.31439 | 8.20413 | 797 | 1.79587 | 9.99994 | 5 |
| 3360 | 56 | 8.21189 | 782 | 5.31444 | 5.31439 | 8.21195 | 782 | 1.78805 | 9.99994 | 4 |
| 3420 | 57 | 8.21958 | 769 | 5.31445 | 5.31439 | 8.21964 | 769 | 1.78036 | 9.99994 | 3 |
| 3480 | 58 | 8.22713 | 755 | 5.31445 | 5.31438 | 8.22720 | 756 | 1.77280 | 9.99994 | 2 |
| 3540 | 59 | 8.23456 | 743 | 5.31445 | 5.31438 | 8.23462 | 742 | 1.76538 | 9.99994 | 1 |
| 3600 | 60 | 8.24186 | 730 | 5.31445 | 5.31438 | 8.24192 | 730 | 1.75808 | 9.99993 | 0 |
| | | L. Cos. | d. | | | L. Cotg. | d. c. | L. Tang. | L. Sin. | ′ |

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| '' | ' | L. Sin. | d. | Cpl. S. | Cpl. T. | L. Tang. | d. c. | L. Cotg. | L. Cos. | |
|----|---|---------|-----|---------|---------|----------|-------|----------|---------|---|
| 3600 | 0 | 8.24186 | 717 | 5.31445 | 5.31438 | 8.24192 | 718 | 1.75808 | 9.99993 | 60 |
| 3660 | 1 | 8.24903 | 706 | 5.31445 | 5.31438 | 8.24910 | 706 | 1.75090 | 9.99993 | 59 |
| 3720 | 2 | 8.25609 | 695 | 5.31445 | 5.31438 | 8.25616 | 696 | 1.74384 | 9.99993 | 58 |
| 3780 | 3 | 8.26304 | 684 | 5.31445 | 5.31438 | 8.26312 | 684 | 1.73688 | 9.99993 | 57 |
| 3840 | 4 | 8.26988 | 673 | 5.31445 | 5.31437 | 8.26996 | 673 | 1.73004 | 9.99992 | 56 |
| 3900 | 5 | 8.27661 | 663 | 5.31445 | 5.31437 | 8.27669 | 663 | 1.72331 | 9.99992 | 55 |
| 3960 | 6 | 8.28324 | 653 | 5.31445 | 5.31437 | 8.28332 | 654 | 1.71668 | 9.99992 | 54 |
| 4020 | 7 | 8.28977 | 644 | 5.31445 | 5.31437 | 8.28986 | 643 | 1.71014 | 9.99992 | 53 |
| 4080 | 8 | 8.29621 | 634 | 5.31445 | 5.31437 | 8.29629 | 634 | 1.70371 | 9.99992 | 52 |
| 4140 | 9 | 8.30255 | 624 | 5.31445 | 5.31437 | 8.30263 | 625 | 1.69737 | 9.99991 | 51 |
| 4200 | 10 | 8.30879 | 616 | 5.31446 | 5.31437 | 8.30888 | 617 | 1.69112 | 9.99991 | 50 |
| 4260 | 11 | 8.31495 | 608 | 5.31446 | 5.31436 | 8.31505 | 607 | 1.68495 | 9.99991 | 49 |
| 4320 | 12 | 8.32103 | 599 | 5.31446 | 5.31436 | 8.32112 | 599 | 1.67888 | 9.99990 | 48 |
| 4380 | 13 | 8.32702 | 590 | 5.31446 | 5.31436 | 8.32711 | 591 | 1.67289 | 9.99990 | 47 |
| 4440 | 14 | 8.33292 | 583 | 5.31446 | 5.31436 | 8.33302 | 584 | 1.66698 | 9.99990 | 46 |
| 4500 | 15 | 8.33875 | 575 | 5.31446 | 5.31436 | 8.33886 | 575 | 1.66114 | 9.99990 | 45 |
| 4560 | 16 | 8.34450 | 568 | 5.31446 | 5.31435 | 8.34461 | 568 | 1.65539 | 9.99989 | 44 |
| 4620 | 17 | 8.35018 | 560 | 5.31446 | 5.31435 | 8.35029 | 561 | 1.64971 | 9.99989 | 43 |
| 4680 | 18 | 8.35578 | 553 | 5.31446 | 5.31435 | 8.35590 | 553 | 1.64410 | 9.99989 | 42 |
| 4740 | 19 | 8.36131 | 547 | 5.31446 | 5.31435 | 8.36143 | 546 | 1.63857 | 9.99989 | 41 |
| 4800 | 20 | 8.36678 | 539 | 5.31446 | 5.31435 | 8.36689 | 540 | 1.63311 | 9.99988 | 40 |
| 4860 | 21 | 8.37217 | 533 | 5.31447 | 5.31434 | 8.37229 | 533 | 1.62771 | 9.99988 | 39 |
| 4920 | 22 | 8.37750 | 526 | 5.31447 | 5.31434 | 8.37762 | 527 | 1.62238 | 9.99988 | 38 |
| 4980 | 23 | 8.38276 | 520 | 5.31447 | 5.31434 | 8.38289 | 520 | 1.61711 | 9.99987 | 37 |
| 5040 | 24 | 8.38796 | 514 | 5.31447 | 5.31434 | 8.38809 | 514 | 1.61191 | 9.99987 | 36 |
| 5100 | 25 | 8.39310 | 508 | 5.31447 | 5.31434 | 8.39323 | 509 | 1.60677 | 9.99987 | 35 |
| 5160 | 26 | 8.39818 | 502 | 5.31447 | 5.31433 | 8.39832 | 502 | 1.60168 | 9.99986 | 34 |
| 5220 | 27 | 8.40320 | 496 | 5.31447 | 5.31433 | 8.40334 | 496 | 1.59666 | 9.99986 | 33 |
| 5280 | 28 | 8.40816 | 491 | 5.31447 | 5.31433 | 8.40830 | 491 | 1.59170 | 9.99986 | 32 |
| 5340 | 29 | 8.41307 | 485 | 5.31447 | 5.31433 | 8.41321 | 486 | 1.58679 | 9.99985 | 31 |
| 5400 | 30 | 8.41792 | 480 | 5.31447 | 5.31433 | 8.41807 | 480 | 1.58193 | 9.99985 | 30 |
| 5460 | 31 | 8.42272 | 474 | 5.31448 | 5.31432 | 8.42287 | 475 | 1.57713 | 9.99985 | 29 |
| 5520 | 32 | 8.42746 | 470 | 5.31448 | 5.31432 | 8.42762 | 470 | 1.57238 | 9.99984 | 28 |
| 5580 | 33 | 8.43216 | 464 | 5.31448 | 5.31432 | 8.43232 | 464 | 1.56768 | 9.99984 | 27 |
| 5640 | 34 | 8.43680 | 459 | 5.31448 | 5.31432 | 8.43696 | 460 | 1.56304 | 9.99984 | 26 |
| 5700 | 35 | 8.44139 | 455 | 5.31448 | 5.31431 | 8.44156 | 455 | 1.55844 | 9.99983 | 25 |
| 5760 | 36 | 8.44594 | 450 | 5.31448 | 5.31431 | 8.44611 | 450 | 1.55389 | 9.99983 | 24 |
| 5820 | 37 | 8.45044 | 445 | 5.31448 | 5.31431 | 8.45061 | 446 | 1.54939 | 9.99983 | 23 |
| 5880 | 38 | 8.45489 | 441 | 5.31448 | 5.31431 | 8.45507 | 441 | 1.54493 | 9.99982 | 22 |
| 5940 | 39 | 8.45930 | 436 | 5.31449 | 5.31431 | 8.45948 | 437 | 1.54052 | 9.99982 | 21 |
| 6000 | 40 | 8.46366 | 433 | 5.31449 | 5.31430 | 8.46385 | 432 | 1.53615 | 9.99982 | 20 |
| 6060 | 41 | 8.46799 | 427 | 5.31449 | 5.31430 | 8.46817 | 428 | 1.53183 | 9.99981 | 19 |
| 6120 | 42 | 8.47226 | 424 | 5.31449 | 5.31430 | 8.47245 | 424 | 1.52755 | 9.99981 | 18 |
| 6180 | 43 | 8.47650 | 419 | 5.31449 | 5.31430 | 8.47669 | 420 | 1.52331 | 9.99981 | 17 |
| 6240 | 44 | 8.48069 | 416 | 5.31449 | 5.31429 | 8.48089 | 416 | 1.51911 | 9.99980 | 16 |
| 6300 | 45 | 8.48485 | 411 | 5.31449 | 5.31429 | 8.48505 | 412 | 1.51495 | 9.99980 | 15 |
| 6360 | 46 | 8.48896 | 408 | 5.31449 | 5.31429 | 8.48917 | 408 | 1.51083 | 9.99979 | 14 |
| 6420 | 47 | 8.49304 | 404 | 5.31450 | 5.31428 | 8.49325 | 404 | 1.50675 | 9.99979 | 13 |
| 6480 | 48 | 8.49708 | 400 | 5.31450 | 5.31428 | 8.49729 | 401 | 1.50271 | 9.99979 | 12 |
| 6540 | 49 | 8.50108 | 396 | 5.31450 | 5.31428 | 8.50130 | 397 | 1.49870 | 9.99978 | 11 |
| 6600 | 50 | 8.50504 | 393 | 5.31450 | 5.31428 | 8.50527 | 393 | 1.49473 | 9.99978 | 10 |
| 6660 | 51 | 8.50897 | 390 | 5.31450 | 5.31427 | 8.50920 | 390 | 1.49080 | 9.99977 | 9 |
| 6720 | 52 | 8.51287 | 386 | 5.31450 | 5.31427 | 8.51310 | 386 | 1.48690 | 9.99977 | 8 |
| 6780 | 53 | 8.51673 | 382 | 5.31450 | 5.31427 | 8.51696 | 383 | 1.48304 | 9.99977 | 7 |
| 6840 | 54 | 8.52055 | 379 | 5.31450 | 5.31427 | 8.52079 | 380 | 1.47921 | 9.99976 | 6 |
| 6900 | 55 | 8.52434 | 376 | 5.31451 | 5.31426 | 8.52459 | 376 | 1.47541 | 9.99976 | 5 |
| 6960 | 56 | 8.52810 | 373 | 5.31451 | 5.31426 | 8.52835 | 373 | 1.47165 | 9.99975 | 4 |
| 7020 | 57 | 8.53183 | 369 | 5.31451 | 5.31426 | 8.53208 | 370 | 1.46792 | 9.99975 | 3 |
| 7080 | 58 | 8.53552 | 367 | 5.31451 | 5.31425 | 8.53578 | 363 | 1.46422 | 9.99974 | 2 |
| 7140 | 59 | 8.53919 | 363 | 5.31451 | 5.31425 | 8.53945 | | 1.46055 | 9.99974 | 1 |
| 7200 | 60 | 8.54282 | | 5.31451 | 5.31425 | 8.54308 | | 1.45692 | 9.99974 | 0 |
| | | L. Cos. | d. | | | L. Cotg. | d. c. | L. Tang. | L. Sin. | ' |

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**2°**

| " | ' | L. Sin. | d. | Cpl. S. | Cpl. T. | L. Tang. | d. c. | L. Cotg. | L. Cos. | |
|---|---|---------|-----|---------|---------|----------|-------|----------|---------|---|
| 7200 | **0** | 8.54282 | 360 | 5.31451 | 5.31425 | 8.54308 | 361 | 1.45692 | 9.99974 | **60** |
| 7260 | 1 | 8.54642 | 360 | 5.31451 | 5.31425 | 8.54669 | 361 | 1.45331 | 9.99973 | 59 |
| 7320 | 2 | 8.54999 | 357 | 5.31452 | 5.31424 | 8.55027 | 358 | 1.44973 | 9.99973 | 58 |
| 7380 | 3 | 8.55354 | 355 | 5.31452 | 5.31424 | 8.55382 | 355 | 1.44618 | 9.99972 | 57 |
| 7440 | 4 | 8.55705 | 351 | 5.31452 | 5.31424 | 8.55734 | 352 | 1.44266 | 9.99972 | 56 |
| 7500 | 5 | 8.56054 | 349 | 5.31452 | 5.31423 | 8.56083 | 349 | 1.43917 | 9.99971 | 55 |
| 7560 | 6 | 8.56400 | 346 | 5.31452 | 5.31423 | 8.56429 | 346 | 1.43571 | 9.99971 | 54 |
| 7620 | 7 | 8.56743 | 343 | 5.31452 | 5.31423 | 8.56773 | 344 | 1.43227 | 9.99970 | 53 |
| 7680 | 8 | 8.57084 | 341 | 5.31453 | 5.31422 | 8.57114 | 341 | 1.42886 | 9.99970 | 52 |
| 7740 | 9 | 8.57421 | 337 | 5.31453 | 5.31422 | 8.57452 | 338 | 1.42548 | 9.99969 | 51 |
| 7800 | **10** | 8.57757 | 336 | 5.31453 | 5.31422 | 8.57788 | 336 | 1.42212 | 9.99969 | **50** |
| 7860 | 11 | 8.58089 | 332 | 5.31453 | 5.31421 | 8.58121 | 333 | 1.41879 | 9.99968 | 49 |
| 7920 | 12 | 8.58419 | 330 | 5.31453 | 5.31421 | 8.58451 | 330 | 1.41549 | 9.99968 | 48 |
| 7980 | 13 | 8.58747 | 328 | 5.31453 | 5.31421 | 8.58779 | 328 | 1.41221 | 9.99967 | 47 |
| 8040 | 14 | 8.59072 | 325 | 5.31454 | 5.31421 | 8.59105 | 326 | 1.40895 | 9.99967 | 46 |
| 8100 | 15 | 8.59395 | 323 | 5.31454 | 5.31420 | 8.59428 | 323 | 1.40572 | 9.99967 | 45 |
| 8160 | 16 | 8.59715 | 320 | 5.31454 | 5.31420 | 8.59749 | 321 | 1.40251 | 9.99966 | 44 |
| 8220 | 17 | 8.60033 | 318 | 5.31454 | 5.31420 | 8.60068 | 319 | 1.39932 | 9.99966 | 43 |
| 8280 | 18 | 8.60349 | 316 | 5.31454 | 5.31419 | 8.60384 | 316 | 1.39616 | 9.99965 | 42 |
| 8340 | 19 | 8.60662 | 313 | 5.31454 | 5.31419 | 8.60698 | 314 | 1.39302 | 9.99964 | 41 |
| 8400 | **20** | 8.60973 | 311 | 5.31455 | 5.31418 | 8.61009 | 311 | 1.38991 | 9.99964 | **40** |
| 8460 | 21 | 8.61282 | 309 | 5.31455 | 5.31418 | 8.61319 | 310 | 1.38681 | 9.99963 | 39 |
| 8520 | 22 | 8.61589 | 307 | 5.31455 | 5.31418 | 8.61626 | 307 | 1.38374 | 9.99963 | 38 |
| 8580 | 23 | 8.61894 | 305 | 5.31455 | 5.31417 | 8.61981 | 305 | 1.38069 | 9.99962 | 37 |
| 8640 | 24 | 8.62196 | 302 | 5.31455 | 5.31417 | 8.62234 | 303 | 1.37766 | 9.99962 | 36 |
| 8700 | 25 | 8.62497 | 301 | 5.31455 | 5.31417 | 8.62535 | 301 | 1.37465 | 9.99961 | 35 |
| 8760 | 26 | 8.62795 | 298 | 5.31456 | 5.31416 | 8.62834 | 299 | 1.37166 | 9.99961 | 34 |
| 8820 | 27 | 8.63091 | 296 | 5.31456 | 5.31416 | 8.63131 | 297 | 1.36869 | 9.99960 | 33 |
| 8880 | 28 | 8.63385 | 294 | 5.31456 | 5.31416 | 8.63426 | 295 | 1.36574 | 9.99960 | 32 |
| 8940 | 29 | 8.63678 | 293 | 5.31456 | 5.31415 | 8.63718 | 292 | 1.36282 | 9.99959 | 31 |
| 9000 | **30** | 8.63968 | 290 | 5.31456 | 5.31415 | 8.64009 | 291 | 1.35991 | 9.99959 | **30** |
| 9060 | 31 | 8.64256 | 288 | 5.31456 | 5.31415 | 8.64298 | 289 | 1.35702 | 9.99958 | 29 |
| 9120 | 32 | 8.64543 | 287 | 5.31457 | 5.31414 | 8.64585 | 287 | 1.35415 | 9.99958 | 28 |
| 9180 | 33 | 8.64827 | 284 | 5.31457 | 5.31414 | 8.64870 | 285 | 1.35130 | 9.99957 | 27 |
| 9240 | 34 | 8.65110 | 283 | 5.31457 | 5.31413 | 8.65154 | 284 | 1.34846 | 9.99956 | 26 |
| 9300 | 35 | 8.65391 | 281 | 5.31457 | 5.31413 | 8.65435 | 281 | 1.34565 | 9.99956 | 25 |
| 9360 | 36 | 8.65670 | 279 | 5.31457 | 5.31413 | 8.65715 | 280 | 1.34285 | 9.99955 | 24 |
| 9420 | 37 | 8.65947 | 277 | 5.31458 | 5.31412 | 8.65993 | 278 | 1.34007 | 9.99955 | 23 |
| 9480 | 38 | 8.66223 | 276 | 5.31458 | 5.31412 | 8.66269 | 276 | 1.33731 | 9.99954 | 22 |
| 9540 | 39 | 8.66497 | 274 | 5.31458 | 5.31412 | 8.66543 | 274 | 1.33457 | 9.99954 | 21 |
| 9600 | **40** | 8.66769 | 272 | 5.31458 | 5.31411 | 8.66816 | 273 | 1.33184 | 9.99953 | **20** |
| 9660 | 41 | 8.67039 | 270 | 5.31458 | 5.31411 | 8.67087 | 271 | 1.32913 | 9.99952 | 19 |
| 9720 | 42 | 8.67308 | 269 | 5.31459 | 5.31410 | 8.67356 | 269 | 1.32644 | 9.99952 | 18 |
| 9780 | 43 | 8.67575 | 267 | 5.31459 | 5.31410 | 8.67624 | 268 | 1.32376 | 9.99951 | 17 |
| 9840 | 44 | 8.67841 | 266 | 5.31459 | 5.31410 | 8.67890 | 266 | 1.32110 | 9.99951 | 16 |
| 9900 | 45 | 8.68104 | 263 | 5.31459 | 5.31409 | 8.68154 | 264 | 1.31846 | 9.99950 | 15 |
| 9960 | 46 | 8.68367 | 263 | 5.31459 | 5.31409 | 8.68417 | 263 | 1.31583 | 9.99949 | 14 |
| 10020 | 47 | 8.68627 | 260 | 5.31460 | 5.31408 | 8.68678 | 261 | 1.31322 | 9.99949 | 13 |
| 10080 | 48 | 8.68886 | 259 | 5.31460 | 5.31408 | 8.68938 | 260 | 1.31062 | 9.99948 | 12 |
| 10140 | 49 | 8.69144 | 258 | 5.31460 | 5.31408 | 8.69196 | 258 | 1.30804 | 9.99948 | 11 |
| 10200 | **50** | 8.69400 | 256 | 5.31460 | 5.31407 | 8.69453 | 257 | 1.30547 | 9.99947 | **10** |
| 10260 | 51 | 8.69654 | 254 | 5.31460 | 5.31407 | 8.69708 | 255 | 1.30292 | 9.99946 | 9 |
| 10320 | 52 | 8.69907 | 253 | 5.31461 | 5.31406 | 8.69962 | 254 | 1.30038 | 9.99946 | 8 |
| 10380 | 53 | 8.70159 | 252 | 5.31461 | 5.31406 | 8.70214 | 252 | 1.29786 | 9.99945 | 7 |
| 10440 | 54 | 8.70409 | 250 | 5.31461 | 5.31405 | 8.70465 | 251 | 1.29535 | 9.99944 | 6 |
| 10500 | 55 | 8.70658 | 249 | 5.31461 | 5.31405 | 8.70714 | 249 | 1.29286 | 9.99944 | 5 |
| 10560 | 56 | 8.70905 | 247 | 5.31461 | 5.31405 | 8.70962 | 248 | 1.29038 | 9.99943 | 4 |
| 10620 | 57 | 8.71151 | 246 | 5.31462 | 5.31404 | 8.71208 | 246 | 1.28792 | 9.99942 | 3 |
| 10680 | 58 | 8.71395 | 244 | 5.31462 | 5.31404 | 8.71453 | 245 | 1.28547 | 9.99942 | 2 |
| 10740 | 59 | 8.71638 | 243 | 5.31462 | 5.31403 | 8.71697 | 244 | 1.28303 | 9.99941 | 1 |
| 10800 | **60** | 8.71880 | 242 | 5.31462 | 5.31403 | 8.71940 | 243 | 1.28060 | 9.99940 | **0** |
| | | L. Cos. | d. | | | L. Cotg. | d. c. | L. Tang. | L. Sin. | ' |

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**3°**

| ' | L. Sin. | d. | L. Tang. | d. c. | L. Cotg. | L. Cos. | | | P. P. | | |
|---|---------|----|----------|-------|----------|---------|---|---|-------|---|---|
| 0 | 8.71880 | 240 | 8.71940 | 241 | 1.28060 | 9.99940 | 60 | | **238** | **234** | **229** |
| 1 | 8.72120 | 239 | 8.72181 | 241 | 1.27819 | 9.99940 | 59 | 6 | 23.8 | 23.4 | 22.9 |
| 2 | 8.72359 | 238 | 8.72420 | 239 | 1.27580 | 9.99939 | 58 | 7 | 27.8 | 27.3 | 26.7 |
| 3 | 8.72597 | 237 | 8.72659 | 239 | 1.27341 | 9.99938 | 57 | 8 | 31.7 | 31.2 | 30.5 |
| 4 | 8.72834 | 235 | 8.72896 | 237 | 1.27104 | 9.99938 | 56 | 9 | 35.7 | 35.1 | 34.4 |
| 5 | 8.73069 | 234 | 8.73132 | 236 | 1.26868 | 9.99937 | 55 | 10 | 39.7 | 39.0 | 38.2 |
| 6 | 8.73303 | 232 | 8.73366 | 234 | 1.26634 | 9.99936 | 54 | 20 | 79.3 | 78.0 | 76.3 |
| 7 | 8.73535 | 232 | 8.73600 | 234 | 1.26400 | 9.99936 | 53 | 30 | 119.0 | 117.0 | 114.5 |
| 8 | 8.73767 | 230 | 8.73832 | 232 | 1.26168 | 9.99935 | 52 | 40 | 158.7 | 156.0 | 152.7 |
| 9 | 8.73997 | 229 | 8.74063 | 231 | 1.25937 | 9.99934 | 51 | 50 | 198.3 | 195.0 | 190.8 |
| 10 | 8.74226 | 229 | 8.74292 | 229 | 1.25708 | 9.99934 | 50 | | **225** | **220** | **216** |
| 11 | 8.74454 | 226 | 8.74521 | 227 | 1.25479 | 9.99933 | 49 | 6 | 22.5 | 22.0 | 21.6 |
| 12 | 8.74680 | 226 | 8.74748 | 226 | 1.25252 | 9.99932 | 48 | 7 | 26.3 | 25.7 | 25.2 |
| 13 | 8.74906 | 224 | 8.74974 | 225 | 1.25026 | 9.99932 | 47 | 8 | 30.0 | 29.3 | 28.8 |
| 14 | 8.75130 | 223 | 8.75199 | 224 | 1.24801 | 9.99931 | 46 | 9 | 33.8 | 33.0 | 32.4 |
| 15 | 8.75353 | 222 | 8.75423 | 222 | 1.24577 | 9.99930 | 45 | 10 | 37.5 | 36.7 | 36.0 |
| 16 | 8.75575 | 220 | 8.75645 | 222 | 1.24355 | 9.99929 | 44 | 20 | 75.0 | 73.3 | 72.0 |
| 17 | 8.75795 | 220 | 8.75867 | 220 | 1.24133 | 9.99929 | 43 | 30 | 112.5 | 110.0 | 108.0 |
| 18 | 8.76015 | 219 | 8.76087 | 219 | 1.23913 | 9.99928 | 42 | 40 | 150.0 | 146.7 | 144.0 |
| 19 | 8.76234 | 217 | 8.76306 | 219 | 1.23694 | 9.99927 | 41 | 50 | 187.5 | 183.3 | 180.0 |
| 20 | 8.76451 | 216 | 8.76525 | 217 | 1.23475 | 9.99926 | 40 | | **212** | **208** | **204** |
| 21 | 8.76667 | 216 | 8.76742 | 216 | 1.23258 | 9.99926 | 39 | 6 | 21.2 | 20.8 | 20.4 |
| 22 | 8.76883 | 214 | 8.76958 | 215 | 1.23042 | 9.99925 | 38 | 7 | 24.7 | 24.3 | 23.8 |
| 23 | 8.77097 | 213 | 8.77173 | 214 | 1.22827 | 9.99924 | 37 | 8 | 28.3 | 27.7 | 27.2 |
| 24 | 8.77310 | 212 | 8.77387 | 213 | 1.22613 | 9.99923 | 36 | 9 | 31.8 | 31.2 | 30.6 |
| 25 | 8.77522 | 211 | 8.77600 | 211 | 1.22400 | 9.99923 | 35 | 10 | 35.3 | 34.7 | 34.0 |
| 26 | 8.77733 | 210 | 8.77811 | 211 | 1.22189 | 9.99922 | 34 | 20 | 70.7 | 69.3 | 68.0 |
| 27 | 8.77943 | 209 | 8.78022 | 210 | 1.21978 | 9.99921 | 33 | 30 | 106.0 | 104.0 | 102.0 |
| 28 | 8.78152 | 208 | 8.78232 | 209 | 1.21768 | 9.99920 | 32 | 40 | 141.3 | 138.7 | 136.0 |
| 29 | 8.78360 | 208 | 8.78441 | 208 | 1.21559 | 9.99920 | 31 | 50 | 176.7 | 173.3 | 170.0 |
| 30 | 8.78568 | 206 | 8.78649 | 206 | 1.21351 | 9.99919 | 30 | | **201** | **197** | **193** |
| 31 | 8.78774 | 205 | 8.78855 | 206 | 1.21145 | 9.99918 | 29 | 6 | 20.1 | 19.7 | 19.3 |
| 32 | 8.78979 | 204 | 8.79061 | 205 | 1.20939 | 9.99917 | 28 | 7 | 23.5 | 23.0 | 22.5 |
| 33 | 8.79183 | 204 | 8.79266 | 204 | 1.20734 | 9.99917 | 27 | 8 | 26.8 | 26.3 | 25.7 |
| 34 | 8.79386 | 202 | 8.79470 | 203 | 1.20530 | 9.99916 | 26 | 9 | 30.2 | 29.6 | 29.0 |
| 35 | 8.79588 | 201 | 8.79673 | 202 | 1.20327 | 9.99915 | 25 | 10 | 33.5 | 32.8 | 32.2 |
| 36 | 8.79789 | 201 | 8.79875 | 201 | 1.20125 | 9.99914 | 24 | 20 | 67.0 | 65.7 | 64.3 |
| 37 | 8.79990 | 199 | 8.80076 | 201 | 1.19924 | 9.99913 | 23 | 30 | 100.5 | 98.5 | 96.5 |
| 38 | 8.80189 | 199 | 8.80277 | 199 | 1.19723 | 9.99913 | 22 | 40 | 134.0 | 131.3 | 128.7 |
| 39 | 8.80388 | 197 | 8.80476 | 198 | 1.19524 | 9.99912 | 21 | 50 | 167.5 | 164.2 | 160.8 |
| 40 | 8.80585 | 197 | 8.80674 | 198 | 1.19326 | 9.99911 | 20 | | **189** | **185** | **181** |
| 41 | 8.80782 | 196 | 8.80872 | 196 | 1.19128 | 9.99910 | 19 | 6 | 18.9 | 18.5 | 18.1 |
| 42 | 8.80978 | 195 | 8.81068 | 196 | 1.18932 | 9.99909 | 18 | 7 | 22.1 | 21.6 | 21.1 |
| 43 | 8.81173 | 195 | 8.81264 | 195 | 1.18736 | 9.99909 | 17 | 8 | 25.2 | 24.7 | 24.1 |
| 44 | 8.81367 | 193 | 8.81459 | 194 | 1.18541 | 9.99908 | 16 | 9 | 28.4 | 27.8 | 27.2 |
| 45 | 8.81560 | 192 | 8.81653 | 193 | 1.18347 | 9.99907 | 15 | 10 | 31.5 | 30.8 | 30.2 |
| 46 | 8.81752 | 192 | 8.81846 | 192 | 1.18154 | 9.99906 | 14 | 20 | 63.0 | 61.7 | 60.3 |
| 47 | 8.81944 | 190 | 8.82038 | 192 | 1.17962 | 9.99905 | 13 | 30 | 94.5 | 92.5 | 90.5 |
| 48 | 8.82134 | 190 | 8.82230 | 192 | 1.17770 | 9.99904 | 12 | 40 | 126.0 | 123.3 | 120.7 |
| 49 | 8.82324 | 189 | 8.82420 | 190 | 1.17580 | 9.99904 | 11 | 50 | 157.5 | 154.2 | 150.8 |
| 50 | 8.82513 | 188 | 8.82610 | 189 | 1.17390 | 9.99903 | 10 | | **4** | **3** | **2** | **1** |
| 51 | 8.82701 | 187 | 8.82799 | 188 | 1.17201 | 9.99902 | 9 | | | | | |
| 52 | 8.82888 | 187 | 8.82987 | 188 | 1.17013 | 9.99901 | 8 | 6 | 0.4 | 0.3 | 0.2 | 0.1 |
| 53 | 8.83075 | 186 | 8.83175 | 186 | 1.16825 | 9.99900 | 7 | 7 | 0.5 | 0.4 | 0.2 | 0.1 |
| 54 | 8.83261 | 185 | 8.83361 | 186 | 1.16639 | 9.99899 | 6 | 8 | 0.5 | 0.4 | 0.3 | 0.1 |
| 55 | 8.83446 | 184 | 8.83547 | 185 | 1.16453 | 9.99898 | 5 | 9 | 0.6 | 0.5 | 0.3 | 0.2 |
| 56 | 8.83630 | 183 | 8.83732 | 184 | 1.16268 | 9.99898 | 4 | 10 | 0.7 | 0.5 | 0.3 | 0.2 |
| 57 | 8.83813 | 183 | 8.83916 | 184 | 1.16084 | 9.99897 | 3 | 20 | 1.3 | 1.0 | 0.7 | 0.3 |
| 58 | 8.83996 | 181 | 8.84100 | 182 | 1.15900 | 9.99896 | 2 | 30 | 2.0 | 1.5 | 1.0 | 0.5 |
| 59 | 8.84177 | 181 | 8.84282 | 182 | 1.15718 | 9.99895 | 1 | 40 | 2.7 | 2.0 | 1.3 | 0.7 |
| 60 | 8.84358 | | 8.84464 | | 1.15536 | 9.99894 | 0 | 50 | 3.3 | 2.5 | 1.7 | 0.8 |
| | L. Cos. | d. | L. Cotg. | d. c. | L. Tang. | L. Sin. | ' | | | P. P. | |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | |
|---|---------|-----|---------|-------|----------|---------|---|
| 0 | 8.84358 | 181 | 8.84464 | 182 | 1.15536 | 9.99894 | 60 |
| 1 | 8.84539 | 179 | 8.84646 | 180 | 1.15354 | 9.99893 | 59 |
| 2 | 8.84718 | 179 | 8.84826 | 180 | 1.15174 | 9.99892 | 58 |
| 3 | 8.84897 | 178 | 8.85006 | 180 | 1.14994 | 9.99891 | 57 |
| 4 | 8.85075 | 177 | 8.85185 | 179 | 1.14815 | 9.99891 | 56 |
| 5 | 8.85252 | 177 | 8.85363 | 178 | 1.14637 | 9.99890 | 55 |
| 6 | 8.85429 | 177 | 8.85540 | 177 | 1.14460 | 9.99889 | 54 |
| 7 | 8.85605 | 176 | 8.85717 | 177 | 1.14283 | 9.99888 | 53 |
| 8 | 8.85780 | 175 | 8.85893 | 176 | 1.14107 | 9.99887 | 52 |
| 9 | 8.85955 | 175 | 8.86069 | 176 | 1.13931 | 9.99886 | 51 |
| 10 | 8.86128 | 173 | 8.86243 | 174 | 1.13757 | 9.99885 | 50 |
| 11 | 8.86301 | 173 | 8.86417 | 174 | 1.13583 | 9.99884 | 49 |
| 12 | 8.86474 | 173 | 8.86591 | 174 | 1.13409 | 9.99883 | 48 |
| 13 | 8.86645 | 171 | 8.86763 | 172 | 1.13237 | 9.99882 | 47 |
| 14 | 8.86816 | 171 | 8.86935 | 172 | 1.13065 | 9.99881 | 46 |
| 15 | 8.86987 | 171 | 8.87106 | 171 | 1.12894 | 9.99880 | 45 |
| 16 | 8.87156 | 169 | 8.87277 | 171 | 1.12723 | 9.99879 | 44 |
| 17 | 8.87325 | 169 | 8.87447 | 170 | 1.12553 | 9.99879 | 43 |
| 18 | 8.87494 | 169 | 8.87616 | 169 | 1.12384 | 9.99878 | 42 |
| 19 | 8.87661 | 167 | 8.87785 | 169 | 1.12215 | 9.99877 | 41 |
| 20 | 8.87829 | 168 | 8.87953 | 168 | 1.12047 | 9.99876 | 40 |
| 21 | 8.87995 | 166 | 8.88120 | 167 | 1.11880 | 9.99875 | 39 |
| 22 | 8.88161 | 166 | 8.88287 | 167 | 1.11713 | 9.99874 | 38 |
| 23 | 8.88326 | 165 | 8.88453 | 166 | 1.11547 | 9.99873 | 37 |
| 24 | 8.88490 | 164 | 8.88618 | 165 | 1.11382 | 9.99872 | 36 |
| 25 | 8.88654 | 164 | 8.88783 | 165 | 1.11217 | 9.99871 | 35 |
| 26 | 8.88817 | 163 | 8.88948 | 163 | 1.11052 | 9.99870 | 34 |
| 27 | 8.88980 | 163 | 8.89111 | 163 | 1.10889 | 9.99869 | 33 |
| 28 | 8.89142 | 162 | 8.89274 | 163 | 1.10726 | 9.99868 | 32 |
| 29 | 8.89304 | 162 | 8.89437 | 161 | 1.10563 | 9.99867 | 31 |
| 30 | 8.89464 | 160 | 8.89598 | 162 | 1.10402 | 9.99866 | 30 |
| 31 | 8.89625 | 161 | 8.89760 | 160 | 1.10240 | 9.99865 | 29 |
| 32 | 8.89784 | 159 | 8.89920 | 160 | 1.10080 | 9.99864 | 28 |
| 33 | 8.89943 | 159 | 8.90080 | 160 | 1.09920 | 9.99863 | 27 |
| 34 | 8.90102 | 159 | 8.90240 | 159 | 1.09760 | 9.99862 | 26 |
| 35 | 8.90260 | 158 | 8.90399 | 158 | 1.09601 | 9.99861 | 25 |
| 36 | 8.90417 | 157 | 8.90557 | 158 | 1.09443 | 9.99860 | 24 |
| 37 | 8.90574 | 157 | 8.90715 | 157 | 1.09285 | 9.99859 | 23 |
| 38 | 8.90730 | 156 | 8.90872 | 157 | 1.09128 | 9.99858 | 22 |
| 39 | 8.90885 | 155 | 8.91029 | 156 | 1.08971 | 9.99857 | 21 |
| 40 | 8.91040 | 155 | 8.91185 | 155 | 1.08815 | 9.99856 | 20 |
| 41 | 8.91195 | 155 | 8.91340 | 155 | 1.08660 | 9.99855 | 19 |
| 42 | 8.91349 | 154 | 8.91495 | 155 | 1.08505 | 9.99854 | 18 |
| 43 | 8.91502 | 153 | 8.91650 | 153 | 1.08350 | 9.99853 | 17 |
| 44 | 8.91655 | 153 | 8.91803 | 154 | 1.08197 | 9.99852 | 16 |
| 45 | 8.91807 | 152 | 8.91957 | 153 | 1.08043 | 9.99851 | 15 |
| 46 | 8.91959 | 152 | 8.92110 | 152 | 1.07890 | 9.99850 | 14 |
| 47 | 8.92110 | 151 | 8.92262 | 152 | 1.07738 | 9.99848 | 13 |
| 48 | 8.92261 | 151 | 8.92414 | 151 | 1.07586 | 9.99847 | 12 |
| 49 | 8.92411 | 150 | 8.92565 | 151 | 1.07435 | 9.99846 | 11 |
| 50 | 8.92561 | 150 | 8.92716 | 150 | 1.07284 | 9.99845 | 10 |
| 51 | 8.92710 | 149 | 8.92866 | 150 | 1.07134 | 9.99844 | 9 |
| 52 | 8.92859 | 149 | 8.93016 | 149 | 1.06984 | 9.99843 | 8 |
| 53 | 8.93007 | 148 | 8.93165 | 148 | 1.06835 | 9.99842 | 7 |
| 54 | 8.93154 | 147 | 8.93313 | 149 | 1.06687 | 9.99841 | 6 |
| 55 | 8.93301 | 147 | 8.93462 | 147 | 1.06538 | 9.99840 | 5 |
| 56 | 8.93448 | 146 | 3.93609 | 147 | 1.06391 | 9.99839 | 4 |
| 57 | 8.93594 | 146 | 8.93756 | 147 | 1.06244 | 9.99838 | 3 |
| 58 | 8.93740 | 145 | 8.93903 | 146 | 1.06097 | 9.99837 | 2 |
| 59 | 8.93885 | 145 | 8.94049 | 146 | 1.05951 | 9.99836 | 1 |
| 60 | 8.94030 |  | 8.94195 |  | 1.05805 | 9.99834 | 0 |
| | L. Cos. | d. | L. Cotg. | d. c. | L. Tang. | L. Sin. | ′ |

**85°**

**P. P.**

| | 181 | 179 | 177 |
|---|------|------|------|
| 6 | 18.1 | 17.9 | 17.7 |
| 7 | 21.1 | 20.9 | 20.7 |
| 8 | 24.1 | 23.9 | 23.6 |
| 9 | 27.2 | 26.9 | 26.6 |
| 10 | 30.2 | 29.8 | 29.5 |
| 20 | 60.3 | 59.7 | 59.0 |
| 30 | 90.5 | 89.5 | 88.5 |
| 40 | 120.7 | 119.3 | 118.0 |
| 50 | 150.8 | 149.2 | 147.5 |

| | 175 | 173 | 171 |
|---|------|------|------|
| 6 | 17.5 | 17.3 | 17.1 |
| 7 | 20.4 | 20.2 | 20.0 |
| 8 | 23.3 | 23.1 | 22.8 |
| 9 | 26.3 | 26.0 | 25.7 |
| 10 | 29.2 | 28.8 | 28.5 |
| 20 | 58.3 | 57.7 | 57.0 |
| 30 | 87.5 | 86.5 | 85.5 |
| 40 | 116.7 | 115.3 | 114.0 |
| 50 | 145.8 | 144.2 | 142.5 |

| | 168 | 166 | 164 |
|---|------|------|------|
| 6 | 16.8 | 16.6 | 16.4 |
| 7 | 19.6 | 19.4 | 19.1 |
| 8 | 22.4 | 22.1 | 21.9 |
| 9 | 25.2 | 24.9 | 24.6 |
| 10 | 28.0 | 27.7 | 27.3 |
| 20 | 56.0 | 55.3 | 54.7 |
| 30 | 84.0 | 83.0 | 82.0 |
| 40 | 112.0 | 110.7 | 109.3 |
| 50 | 140.0 | 138.3 | 136.7 |

| | 162 | 159 | 157 |
|---|------|------|------|
| 6 | 16.2 | 15.9 | 15.7 |
| 7 | 18.9 | 18.6 | 18.3 |
| 8 | 21.6 | 21.2 | 20.9 |
| 9 | 24.3 | 23.9 | 23.6 |
| 10 | 27.0 | 26.5 | 26.2 |
| 20 | 54.0 | 53.0 | 52.3 |
| 30 | 81.0 | 79.5 | 78.5 |
| 40 | 108.0 | 106.0 | 104.7 |
| 50 | 135.0 | 132.5 | 130.8 |

| | 155 | 153 | 151 |
|---|------|------|------|
| 6 | 15.5 | 15.3 | 15.1 |
| 7 | 18.1 | 17.9 | 17.6 |
| 8 | 20.7 | 20.4 | 20.1 |
| 9 | 23.3 | 23.0 | 22.7 |
| 10 | 25.8 | 25.5 | 25.2 |
| 20 | 51.7 | 51.0 | 50.3 |
| 30 | 77.5 | 76.5 | 75.5 |
| 40 | 103.3 | 102.0 | 100.7 |
| 50 | 129.2 | 127.5 | 125.8 |

| | 149 | 147 | 1 |
|---|------|------|------|
| 6 | 14.9 | 14.7 | 0.1 |
| 7 | 17.4 | 17.2 | 0.1 |
| 8 | 19.9 | 19.6 | 0.1 |
| 9 | 22.4 | 22.1 | 0.2 |
| 10 | 24.8 | 24.5 | 0.2 |
| 20 | 49.7 | 49.0 | 0.3 |
| 30 | 74.5 | 73.5 | 0.5 |
| 40 | 99.3 | 98.0 | 0.7 |
| 50 | 124.2 | 122.5 | 0.8 |

### 5°

| ′ | L. Sin. | d. | L. Tang. | d. c. | L. Cotg. | L. Cos. | |
|---|---|---|---|---|---|---|---|
| 0 | 8.94030 | 144 | 8.94195 | 145 | 1.05805 | 9.99834 | 60 |
| 1 | 8.94174 | 143 | 8.94340 | 145 | 1.05660 | 9.99833 | 59 |
| 2 | 8.94317 | 144 | 8.94485 | 145 | 1.05515 | 9.99832 | 58 |
| 3 | 8.94461 | 143 | 8.94630 | 143 | 1.05370 | 9.99831 | 57 |
| 4 | 8.94603 | 143 | 8.94773 | 144 | 1.05227 | 9.99830 | 56 |
| 5 | 8.94746 | 141 | 8.94917 | 143 | 1.05083 | 9.99829 | 55 |
| 6 | 8.94887 | 142 | 8.95060 | 142 | 1.04940 | 9.99828 | 54 |
| 7 | 8.95029 | 141 | 8.95202 | 142 | 1.04798 | 9.99827 | 53 |
| 8 | 8.95170 | 141 | 8.95344 | 142 | 1.04656 | 9.99825 | 52 |
| 9 | 8.95311 | 140 | 8.95486 | 141 | 1.04514 | 9.99824 | 51 |
| 10 | 8.95450 | 139 | 8.95627 | 140 | 1.04373 | 9.99823 | 50 |
| 11 | 8.95589 | 139 | 8.95767 | 141 | 1.04233 | 9.99822 | 49 |
| 12 | 8.95728 | 139 | 8.95908 | 139 | 1.04092 | 9.99821 | 48 |
| 13 | 8.95867 | 138 | 8.96047 | 140 | 1.03953 | 9.99820 | 47 |
| 14 | 8.96005 | 138 | 8.96187 | 138 | 1.03813 | 9.99819 | 46 |
| 15 | 8.96143 | 137 | 8.96325 | 139 | 1.03675 | 9.99817 | 45 |
| 16 | 8.96280 | 137 | 8.96464 | 138 | 1.03536 | 9.99816 | 44 |
| 17 | 8.96417 | 136 | 8.96602 | 137 | 1.03398 | 9.99815 | 43 |
| 18 | 8.96553 | 136 | 8.96739 | 138 | 1.03261 | 9.99814 | 42 |
| 19 | 8.96689 | 136 | 8.96877 | 136 | 1.03123 | 9.99813 | 41 |
| 20 | 8.96825 | 135 | 8.97013 | 137 | 1.02987 | 9.99812 | 40 |
| 21 | 8.96960 | 135 | 8.97150 | 135 | 1.02850 | 9.99810 | 39 |
| 22 | 8.97095 | 134 | 8.97285 | 136 | 1.02715 | 9.99809 | 38 |
| 23 | 8.97229 | 134 | 8.97421 | 135 | 1.02579 | 9.99808 | 37 |
| 24 | 8.97363 | 133 | 8.97556 | 135 | 1.02444 | 9.99807 | 36 |
| 25 | 8.97496 | 133 | 8.97691 | 134 | 1.02309 | 9.99806 | 35 |
| 26 | 8.97629 | 133 | 8.97825 | 134 | 1.02175 | 9.99804 | 34 |
| 27 | 8.97762 | 132 | 8.97959 | 133 | 1.02041 | 9.99803 | 33 |
| 28 | 8.97894 | 132 | 8.98092 | 133 | 1.01908 | 9.99802 | 32 |
| 29 | 8.98026 | 132 | 8.98225 | 133 | 1.01775 | 9.99801 | 31 |
| 30 | 8.98157 | 131 | 8.98358 | 132 | 1.01642 | 9.99800 | 30 |
| 31 | 8.98288 | 131 | 8.98490 | 132 | 1.01510 | 9.99798 | 29 |
| 32 | 8.98419 | 130 | 8.98622 | 131 | 1.01378 | 9.99797 | 28 |
| 33 | 8.98549 | 130 | 8.98753 | 131 | 1.01247 | 9.99796 | 27 |
| 34 | 8.98679 | 130 | 8.98884 | 131 | 1.01116 | 9.99795 | 26 |
| 35 | 8.98808 | 129 | 8.99015 | 130 | 1.00985 | 9.99793 | 25 |
| 36 | 8.98937 | 129 | 8.99145 | 130 | 1.00855 | 9.99792 | 24 |
| 37 | 8.99066 | 129 | 8.99275 | 130 | 1.00725 | 9.99791 | 23 |
| 38 | 8.99194 | 128 | 8.99405 | 129 | 1.00595 | 9.99790 | 22 |
| 39 | 8.99322 | 128 | 8.99534 | 128 | 1.00466 | 9.99788 | 21 |
| 40 | 8.99450 | 127 | 8.99662 | 129 | 1.00338 | 9.99787 | 20 |
| 41 | 8.99577 | 127 | 8.99791 | 128 | 1.00209 | 9.99786 | 19 |
| 42 | 8.99704 | 126 | 8.99919 | 127 | 1.00081 | 9.99785 | 18 |
| 43 | 8.99830 | 126 | 9.00046 | 128 | 0.99954 | 9.99783 | 17 |
| 44 | 8.99956 | 126 | 9.00174 | 127 | 0.99826 | 9.99782 | 16 |
| 45 | 9.00082 | 125 | 9.00301 | 126 | 0.99699 | 9.99781 | 15 |
| 46 | 9.00207 | 125 | 9.00427 | 126 | 0.99573 | 9.99780 | 14 |
| 47 | 9.00332 | 124 | 9.00553 | 126 | 0.99447 | 9.99778 | 13 |
| 48 | 9.00456 | 125 | 9.00679 | 126 | 0.99321 | 9.99777 | 12 |
| 49 | 9.00581 | 123 | 9.00805 | 125 | 0.99195 | 9.99776 | 11 |
| 50 | 9.00704 | 124 | 9.00930 | 125 | 0.99070 | 9.99775 | 10 |
| 51 | 9.00828 | 123 | 9.01055 | 124 | 0.98945 | 9.99773 | 9 |
| 52 | 9.00951 | 123 | 9.01179 | 124 | 0.98821 | 9.99772 | 8 |
| 53 | 9.01074 | 122 | 9.01303 | 124 | 0.98697 | 9.99771 | 7 |
| 54 | 9.01196 | 122 | 9.01427 | 123 | 0.98573 | 9.99769 | 6 |
| 55 | 9.01318 | 122 | 9.01550 | 123 | 0.98450 | 9.99768 | 5 |
| 56 | 9.01440 | 121 | 9.01673 | 123 | 0.98327 | 9.99767 | 4 |
| 57 | 9.01561 | 121 | 9.01796 | 122 | 0.98204 | 9.99765 | 3 |
| 58 | 9.01682 | 121 | 9.01918 | 122 | 0.98082 | 9.99764 | 2 |
| 59 | 9.01803 | 120 | 9.02040 | 122 | 0.97960 | 9.99763 | 1 |
| 60 | 9.01923 | | 9.02162 | | 0.97838 | 9.99761 | 0 |
| | L. Cos. | d. | L. Cotg. | d. c. | L. Tang. | L. Sin. | ′ |

**P. P.**

| | 145 | 143 | 141 |
|---|---|---|---|
| 6 | 14.5 | 14.3 | 14.1 |
| 7 | 16.9 | 16.7 | 16.5 |
| 8 | 19.3 | 19.1 | 18.8 |
| 9 | 21.8 | 21.5 | 21.2 |
| 10 | 24.2 | 23.8 | 23.5 |
| 20 | 48.3 | 47.7 | 47.0 |
| 30 | 72.5 | 71.5 | 70.5 |
| 40 | 96.7 | 95.3 | 94.0 |
| 50 | 120.8 | 119.2 | 117.5 |

| | 139 | 138 | 136 |
|---|---|---|---|
| 6 | 13.9 | 13.8 | 13.6 |
| 7 | 16.2 | 16.1 | 15.9 |
| 8 | 18.5 | 18.4 | 18.1 |
| 9 | 20.9 | 20.7 | 20.4 |
| 10 | 23.2 | 23.0 | 22.7 |
| 20 | 46.3 | 46.0 | 45.3 |
| 30 | 69.5 | 69.0 | 68.0 |
| 40 | 92.7 | 92.0 | 90.7 |
| 50 | 115.8 | 115.0 | 113.3 |

| | 135 | 133 | 131 |
|---|---|---|---|
| 6 | 13.5 | 13.3 | 13.1 |
| 7 | 15.8 | 15.5 | 15.3 |
| 8 | 18.0 | 17.7 | 17.5 |
| 9 | 20.3 | 20.0 | 19.7 |
| 10 | 22.5 | 22.2 | 21.8 |
| 20 | 45.0 | 44.3 | 43.7 |
| 30 | 67.5 | 66.5 | 65.5 |
| 40 | 90.0 | 88.7 | 87.3 |
| 50 | 112.5 | 110.8 | 109.2 |

| | 129 | 128 | 126 |
|---|---|---|---|
| 6 | 12.9 | 12.8 | 12.6 |
| 7 | 15.1 | 14.9 | 14.7 |
| 8 | 17.2 | 17.1 | 16.8 |
| 9 | 19.4 | 19.2 | 18.9 |
| 10 | 21.5 | 21.3 | 21.0 |
| 20 | 43.0 | 42.7 | 42.0 |
| 30 | 64.5 | 64.0 | 63.0 |
| 40 | 86.0 | 85.3 | 84.0 |
| 50 | 107.5 | 106.7 | 105.0 |

| | 125 | 123 | 122 |
|---|---|---|---|
| 6 | 12.5 | 12.3 | 12.2 |
| 7 | 14.6 | 14.4 | 14.2 |
| 8 | 16.7 | 16.4 | 16.3 |
| 9 | 18.8 | 18.5 | 18.3 |
| 10 | 20.8 | 20.5 | 20.3 |
| 20 | 41.7 | 41.0 | 40.7 |
| 30 | 62.5 | 61.5 | 61.0 |
| 40 | 83.3 | 82.0 | 81.3 |
| 50 | 104.2 | 102.5 | 101.7 |

| | 121 | 120 | 1 |
|---|---|---|---|
| 6 | 12.1 | 12.0 | 0.1 |
| 7 | 14.1 | 14.0 | 0.1 |
| 8 | 16.1 | 16.0 | 0.1 |
| 9 | 18.2 | 18.0 | 0.2 |
| 10 | 20.2 | 20.0 | 0.2 |
| 20 | 40.3 | 40.0 | 0.3 |
| 30 | 60.5 | 60.0 | 0.5 |
| 40 | 80.7 | 80.0 | 0.7 |
| 50 | 100.8 | 100.0 | 0.8 |

## 6°

| ' | L. Sin. | d. | L. Tang. | d. c. | L. Cotg. | L. Cos. | ' |
|---|---------|----|----------|-------|----------|---------|---|
| 0 | 9.01923 | 120 | 9.02162 | 121 | 0.97838 | 9.99761 | 60 |
| 1 | 9.02043 | 120 | 9.02283 | 121 | 0.97717 | 9.99760 | 59 |
| 2 | 9.02163 | 120 | 9.02404 | 121 | 0.97596 | 9.99759 | 58 |
| 3 | 9.02283 | 119 | 9.02525 | 120 | 0.97475 | 9.99757 | 57 |
| 4 | 9.02402 | 118 | 9.02645 | 121 | 0.97355 | 9.99756 | 56 |
| 5 | 9.02520 | 119 | 9.02766 | 119 | 0.97234 | 9.99755 | 55 |
| 6 | 9.02639 | 118 | 9.02885 | 120 | 0.97115 | 9.99753 | 54 |
| 7 | 9.02757 | 117 | 9.03005 | 119 | 0.96995 | 9.99752 | 53 |
| 8 | 9.02874 | 118 | 9.03124 | 118 | 0.96876 | 9.99751 | 52 |
| 9 | 9.02992 | 117 | 9.03242 | 119 | 0.96758 | 9.99749 | 51 |
| 10 | 9.03109 | 117 | 9.03361 | 118 | 0.96639 | 9.99748 | 50 |
| 11 | 9.03226 | 116 | 9.03479 | 118 | 0.96521 | 9.99747 | 49 |
| 12 | 9.03342 | 116 | 9.03597 | 117 | 0.96403 | 9.99745 | 48 |
| 13 | 9.03458 | 116 | 9.03714 | 118 | 0.96286 | 9.99744 | 47 |
| 14 | 9.03574 | 116 | 9.03832 | 116 | 0.96168 | 9.99742 | 46 |
| 15 | 9.03690 | 115 | 9.03948 | 117 | 0.96052 | 9.99741 | 45 |
| 16 | 9.03805 | 115 | 9.04065 | 116 | 0.95935 | 9.99740 | 44 |
| 17 | 9.03920 | 114 | 9.04181 | 116 | 0.95819 | 9.99738 | 43 |
| 18 | 9.04034 | 115 | 9.04297 | 116 | 0.95703 | 9.99737 | 42 |
| 19 | 9.04149 | 113 | 9.04413 | 115 | 0.95587 | 9.99736 | 41 |
| 20 | 9.04262 | 114 | 9.04528 | 115 | 0.95472 | 9.99734 | 40 |
| 21 | 9.04376 | 114 | 9.04643 | 115 | 0.95357 | 9.99733 | 39 |
| 22 | 9.04490 | 113 | 9.04758 | 115 | 0.95242 | 9.99731 | 38 |
| 23 | 9.04603 | 112 | 9.04873 | 114 | 0.95127 | 9.99730 | 37 |
| 24 | 9.04715 | 113 | 9.04987 | 114 | 0.95013 | 9.99728 | 36 |
| 25 | 9.04828 | 112 | 9.05101 | 113 | 0.94899 | 9.99727 | 35 |
| 26 | 9.04940 | 112 | 9.05214 | 114 | 0.94786 | 9.99726 | 34 |
| 27 | 9.05052 | 112 | 9.05328 | 113 | 0.94672 | 9.99724 | 33 |
| 28 | 9.05164 | 111 | 9.05441 | 112 | 0.94559 | 9.99723 | 32 |
| 29 | 9.05275 | 111 | 9.05553 | 113 | 0.94447 | 9.99721 | 31 |
| 30 | 9.05386 | 111 | 9.05666 | 112 | 0.94334 | 9.99720 | 30 |
| 31 | 9.05497 | 110 | 9.05778 | 112 | 0.94222 | 9.99718 | 29 |
| 32 | 9.05607 | 110 | 9.05890 | 112 | 0.94110 | 9.99717 | 28 |
| 33 | 9.05717 | 110 | 9.06002 | 111 | 0.93998 | 9.99716 | 27 |
| 34 | 9.05827 | 110 | 9.06113 | 111 | 0.93887 | 9.99714 | 26 |
| 35 | 9.05937 | 109 | 9.06224 | 111 | 0.93776 | 9.99713 | 25 |
| 36 | 9.06046 | 109 | 9.06335 | 110 | 0.93665 | 9.99711 | 24 |
| 37 | 9.06155 | 109 | 9.06445 | 111 | 0.93555 | 9.99710 | 23 |
| 38 | 9.06264 | 108 | 9.06556 | 110 | 0.93444 | 9.99708 | 22 |
| 39 | 9.06372 | 109 | 9.06666 | 109 | 0.93334 | 9.99707 | 21 |
| 40 | 9.06481 | 108 | 9.06775 | 110 | 0.93225 | 9.99705 | 20 |
| 41 | 9.06589 | 107 | 9.06885 | 109 | 0.93115 | 9.99704 | 19 |
| 42 | 9.06696 | 108 | 9.06994 | 109 | 0.93006 | 9.99702 | 18 |
| 43 | 9.06804 | 107 | 9.07103 | 108 | 0.92897 | 9.99701 | 17 |
| 44 | 9.06911 | 107 | 9.07211 | 109 | 0.92789 | 9.99699 | 16 |
| 45 | 9.07018 | 106 | 9.07320 | 108 | 0.92680 | 9.99698 | 15 |
| 46 | 9.07124 | 107 | 9.07428 | 108 | 0.92572 | 9.99696 | 14 |
| 47 | 9.07231 | 106 | 9.07536 | 107 | 0.92464 | 9.99695 | 13 |
| 48 | 9.07337 | 105 | 9.07643 | 108 | 0.92357 | 9.99693 | 12 |
| 49 | 9.07442 | 106 | 9.07751 | 107 | 0.92249 | 9.99692 | 11 |
| 50 | 9.07548 | 105 | 9.07858 | 106 | 0.92142 | 9.99690 | 10 |
| 51 | 9.07653 | 105 | 9.07964 | 107 | 0.92036 | 9.99689 | 9 |
| 52 | 9.07758 | 105 | 9.08071 | 106 | 0.91929 | 9.99687 | 8 |
| 53 | 9.07863 | 105 | 9.08177 | 106 | 0.91823 | 9.99686 | 7 |
| 54 | 9.07968 | 104 | 9.08283 | 106 | 0.91717 | 9.99684 | 6 |
| 55 | 9.08072 | 104 | 9.08389 | 106 | 0.91611 | 9.99683 | 5 |
| 56 | 9.08176 | 104 | 9.08495 | 105 | 0.91505 | 9.99681 | 4 |
| 57 | 9.08280 | 103 | 9.08600 | 105 | 0.91400 | 9.99680 | 3 |
| 58 | 9.08383 | 103 | 9.08705 | 105 | 0.91295 | 9.99678 | 2 |
| 59 | 9.08486 | 103 | 9.08810 | 104 | 0.91190 | 9.99677 | 1 |
| 60 | 9.08589 |  | 9.08914 |  | 0.91086 | 9.99675 | 0 |
| | L. Cos. | d. | L. Cotg. | d. c. | L. Tang. | L. Sin. | ' |

**83°**

### P. P.

| | 121 | 120 | 119 |
|---|-----|-----|-----|
| 6 | 12.1 | 12.0 | 11.9 |
| 7 | 14.1 | 14.0 | 13.9 |
| 8 | 16.1 | 16.0 | 15.9 |
| 9 | 18.2 | 18.0 | 17.9 |
| 10 | 20.2 | 20.0 | 19.8 |
| 20 | 40.3 | 40.0 | 39.7 |
| 30 | 60.5 | 60.0 | 59.5 |
| 40 | 80.7 | 80.0 | 79.3 |
| 50 | 100.8 | 100.0 | 99.2 |

| | 118 | 117 | 116 |
|---|-----|-----|-----|
| 6 | 11.8 | 11.7 | 11.6 |
| 7 | 13.8 | 13.7 | 13.5 |
| 8 | 15.7 | 15.6 | 15.5 |
| 9 | 17.7 | 17.6 | 17.4 |
| 10 | 19.7 | 19.5 | 19.3 |
| 20 | 39.3 | 39.0 | 38.7 |
| 30 | 59.0 | 58.5 | 58.0 |
| 40 | 78.7 | 78.0 | 77.3 |
| 50 | 98.3 | 97.5 | 96.7 |

| | 115 | 114 | 113 |
|---|-----|-----|-----|
| 6 | 11.5 | 11.4 | 11.3 |
| 7 | 13.4 | 13.3 | 13.2 |
| 8 | 15.3 | 15.2 | 15.1 |
| 9 | 17.3 | 17.1 | 17.0 |
| 10 | 19.2 | 19.0 | 18.8 |
| 20 | 38.3 | 38.0 | 37.7 |
| 30 | 57.5 | 57.0 | 56.5 |
| 40 | 76.7 | 76.0 | 75.3 |
| 50 | 95.8 | 95.0 | 94.2 |

| | 112 | 111 | 110 |
|---|-----|-----|-----|
| 6 | 11.2 | 11.1 | 11.0 |
| 7 | 13.1 | 13.0 | 12.8 |
| 8 | 14.9 | 14.8 | 14.7 |
| 9 | 16.8 | 16.7 | 16.5 |
| 10 | 18.7 | 18.5 | 18.3 |
| 20 | 37.3 | 37.0 | 36.7 |
| 30 | 56.0 | 55.5 | 55.0 |
| 40 | 74.7 | 74.0 | 73.3 |
| 50 | 93.3 | 92.5 | 91.7 |

| | 109 | 108 | 107 |
|---|-----|-----|-----|
| 6 | 10.9 | 10.8 | 10.7 |
| 7 | 12.7 | 12.6 | 12.5 |
| 8 | 14.5 | 14.4 | 14.3 |
| 9 | 16.4 | 16.2 | 16.1 |
| 10 | 18.2 | 18.0 | 17.8 |
| 20 | 36.3 | 36.0 | 35.7 |
| 30 | 54.5 | 54.0 | 53.5 |
| 40 | 72.7 | 72.0 | 71.3 |
| 50 | 90.8 | 90.0 | 89.2 |

| | 106 | 105 | 104 |
|---|-----|-----|-----|
| 6 | 10.6 | 10.5 | 10.4 |
| 7 | 12.4 | 12.3 | 12.1 |
| 8 | 14.1 | 14.0 | 13.9 |
| 9 | 15.9 | 15.8 | 15.6 |
| 10 | 17.7 | 17.5 | 17.3 |
| 20 | 35.3 | 35.0 | 34.7 |
| 30 | 53.0 | 52.5 | 52.0 |
| 40 | 70.7 | 70.0 | 69.3 |
| 50 | 88.3 | 87.5 | 86.7 |

## 7°

| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | | P. P. |
|---|---------|-----|---------|-------|----------|---------|---|-------|
| **0** | 9.08589 | 103 | 9.08914 | 105 | 0.91086 | 9.99675 | **60** | |
| 1 | 9.08692 | 103 | 9.09019 | 105 | 0.90981 | 9.99674 | 59 | |
| 2 | 9.08795 | 102 | 9.09123 | 104 | 0.90877 | 9.99672 | 58 | **105 104 103** |
| 3 | 9.08897 | 102 | 9.09227 | 104 | 0.90773 | 9.99670 | 57 | 6 \| 10.5 10.4 10.3 |
| 4 | 9.08999 | 102 | 9.09330 | 103 | 0.90670 | 9.99669 | 56 | 7 \| 12.3 12.1 12.0 |
| 5 | 9.09101 | 102 | 9.09434 | 104 | 0.90566 | 9.99667 | 55 | 8 \| 14.0 13.9 13.7 |
| 6 | 9.09202 | 101 | 9.09537 | 103 | 0.90463 | 9.99666 | 54 | 9 \| 15.8 15.6 15.5 |
| 7 | 9.09304 | 101 | 9.09640 | 103 | 0.90360 | 9.99664 | 53 | 10 \| 17.5 17.3 17.2 |
| 8 | 9.09405 | 101 | 9.09742 | 103 | 0.90258 | 9.99663 | 52 | 20 \| 35.0 34.7 34.3 |
| 9 | 9.09506 | 101 | 9.09845 | 102 | 0.90155 | 9.99661 | 51 | 30 \| 52.5 52.0 51.5 |
| | | 100 | | 102 | | | | 40 \| 70.0 69.3 68.7 |
| **10** | 9.09606 | 101 | 9.09947 | 102 | 0.90053 | 9.99660 | **50** | 50 \| 87.5 86.7 85.8 |
| 11 | 9.09707 | 100 | 9.10049 | 101 | 0.89951 | 9.99658 | 49 | |
| 12 | 9.09807 | 100 | 9.10150 | 102 | 0.89850 | 9.99656 | 48 | **102 101 100** |
| 13 | 9.09907 | 99 | 9.10252 | 102 | 0.89748 | 9.99655 | 47 | 6 \| 10.2 10.1 10.0 |
| 14 | 9.10006 | 100 | 9.10353 | 101 | 0.89647 | 9.99653 | 46 | 7 \| 11.9 11.8 11.7 |
| 15 | 9.10106 | 99 | 9.10454 | 101 | 0.89546 | 9.99651 | 45 | 8 \| 13.6 13.5 13.3 |
| 16 | 9.10205 | 99 | 9.10555 | 101 | 0.89445 | 9.99650 | 44 | 9 \| 15.3 15.2 15.0 |
| 17 | 9.10304 | 98 | 9.10656 | 100 | 0.89344 | 9.99648 | 43 | 10 \| 17.0 16.8 16.7 |
| 18 | 9.10402 | 99 | 9.10756 | 100 | 0.89244 | 9.99647 | 42 | 20 \| 34.0 33.7 33.3 |
| 19 | 9.10501 | 98 | 9.10856 | 100 | 0.89144 | 9.99645 | 41 | 30 \| 51.0 50.5 50.0 |
| | | 98 | | 100 | | | | 40 \| 68.0 67.3 66.7 |
| **20** | 9.10599 | 98 | 9.10956 | 100 | 0.89044 | 9.99643 | **40** | 50 \| 85.0 84.2 83.3 |
| 21 | 9.10697 | 98 | 9.11056 | 99 | 0.88944 | 9.99642 | 39 | |
| 22 | 9.10795 | 98 | 9.11155 | 99 | 0.88845 | 9.99640 | 38 | **99 98** |
| 23 | 9.10893 | 97 | 9.11254 | 99 | 0.88746 | 9.99638 | 37 | 6 \| 9.9 9.8 |
| 24 | 9.10990 | 97 | 9.11353 | 99 | 0.88647 | 9.99637 | 36 | 7 \| 11.6 11.4 |
| 25 | 9.11087 | 97 | 9.11452 | 99 | 0.88548 | 9.99635 | 35 | 8 \| 13.2 13.1 |
| 26 | 9.11184 | 97 | 9.11551 | 98 | 0.88449 | 9.99633 | 34 | 9 \| 14.9 14.7 |
| 27 | 9.11281 | 96 | 9.11649 | 98 | 0.88351 | 9.99632 | 33 | 10 \| 16.5 16.3 |
| 28 | 9.11377 | 97 | 9.11747 | 98 | 0.88253 | 9.99630 | 32 | 20 \| 33.0 32.7 |
| 29 | 9.11474 | 96 | 9.11845 | 98 | 0.88155 | 9.99629 | 31 | 30 \| 49.5 49.0 |
| | | 96 | | 98 | | | | 40 \| 66.0 65.3 |
| **30** | 9.11570 | 96 | 9.11943 | 97 | 0.88057 | 9.99627 | **30** | 50 \| 82.5 81.7 |
| 31 | 9.11666 | 95 | 9.12040 | 98 | 0.87960 | 9.99625 | 29 | |
| 32 | 9.11761 | 96 | 9.12138 | 97 | 0.87862 | 9.99624 | 28 | **97 96 95** |
| 33 | 9.11857 | 95 | 9.12235 | 97 | 0.87765 | 9.99622 | 27 | 6 \| 9.7 9.6 9.5 |
| 34 | 9.11952 | 95 | 9.12332 | 96 | 0.87668 | 9.99620 | 26 | 7 \| 11.3 11.2 11.1 |
| 35 | 9.12047 | 95 | 9.12428 | 97 | 0.87572 | 9.99618 | 25 | 8 \| 12.9 12.8 12.7 |
| 36 | 9.12142 | 94 | 9.12525 | 96 | 0.87475 | 9.99617 | 24 | 9 \| 14.6 14.4 14.3 |
| 37 | 9.12236 | 95 | 9.12621 | 96 | 0.87379 | 9.99615 | 23 | 10 \| 16.2 16.0 15.8 |
| 38 | 9.12331 | 94 | 9.12717 | 96 | 0.87283 | 9.99613 | 22 | 20 \| 32.3 32.0 31.7 |
| 39 | 9.12425 | 94 | 9.12813 | 96 | 0.87187 | 9.99612 | 21 | 30 \| 48.5 48.0 47.5 |
| | | 94 | | 96 | | | | 40 \| 64.7 64.0 63.3 |
| **40** | 9.12519 | 93 | 9.12909 | 95 | 0.87091 | 9.99610 | **20** | 50 \| 80.8 80.0 79.2 |
| 41 | 9.12612 | 94 | 9.13004 | 95 | 0.86996 | 9.99608 | 19 | |
| 42 | 9.12706 | 93 | 9.13099 | 95 | 0.86901 | 9.99607 | 18 | **94 93 92** |
| 43 | 9.12799 | 93 | 9.13194 | 95 | 0.86806 | 9.99605 | 17 | 6 \| 9.4 9.3 9.2 |
| 44 | 9.12892 | 93 | 9.13289 | 95 | 0.86711 | 9.99603 | 16 | 7 \| 11.0 10.9 10.7 |
| 45 | 9.12985 | 93 | 9.13384 | 94 | 0.86616 | 9.99601 | 15 | 8 \| 12.5 12.4 12.3 |
| 46 | 9.13078 | 93 | 9.13478 | 95 | 0.86522 | 9.99600 | 14 | 9 \| 14.1 14.0 13.8 |
| 47 | 9.13171 | 92 | 9.13573 | 94 | 0.86427 | 9.99598 | 13 | 10 \| 15.7 15.5 15.3 |
| 48 | 9.13263 | 92 | 9.13667 | 94 | 0.86333 | 9.99596 | 12 | 20 \| 31.3 31.0 30.7 |
| 49 | 9.13355 | 92 | 9.13761 | 93 | 0.86239 | 9.99595 | 11 | 30 \| 47.0 46.5 46.0 |
| | | 92 | | 93 | | | | 40 \| 62.7 62.0 61.3 |
| **50** | 9.13447 | 92 | 9.13854 | 94 | 0.86146 | 9.99593 | **10** | 50 \| 78.3 77.5 76.7 |
| 51 | 9.13539 | 91 | 9.13948 | 93 | 0.86052 | 9.99591 | 9 | |
| 52 | 9.13630 | 92 | 9.14041 | 93 | 0.85959 | 9.99589 | 8 | **91 90 2** |
| 53 | 9.13722 | 91 | 9.14134 | 93 | 0.85866 | 9.99588 | 7 | 6 \| 9.1 9.0 0.2 |
| 54 | 9.13813 | 91 | 9.14227 | 93 | 0.85773 | 9.99586 | 6 | 7 \| 10.6 10.5 0.2 |
| 55 | 9.13904 | 90 | 9.14320 | 92 | 0.85680 | 9.99584 | 5 | 8 \| 12.1 12.0 0.3 |
| 56 | 9.13994 | 91 | 9.14412 | 92 | 0.85588 | 9.99582 | 4 | 9 \| 13.7 13.5 0.3 |
| 57 | 9.14085 | 90 | 9.14504 | 93 | 0.85496 | 9.99581 | 3 | 10 \| 15.2 15.0 0.3 |
| 58 | 9.14175 | 91 | 9.14597 | 92 | 0.85403 | 9.99579 | 2 | 20 \| 30.3 30.0 0.7 |
| 59 | 9.14266 | 90 | 9.14688 | 92 | 0.85312 | 9.99577 | 1 | 30 \| 45.5 45.0 1.0 |
| | | 90 | | 92 | | | | 40 \| 60.7 60.0 1.3 |
| **60** | 9.14356 | | 9.14780 | | 0.85220 | 9.99575 | **0** | 50 \| 75.8 75.0 1.7 |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | ′ | P. P. |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | | P. P. |
|---|---------|-----|---------|-------|----------|---------|---|-------|
| 0 | 9.14356 | 89 | 9.14780 | 92 | 0.85220 | 9.99575 | 60 | |
| 1 | 9.14445 | 90 | 9.14872 | 91 | 0.85128 | 9.99574 | 59 | **92 │ 91 │ 90** |
| 2 | 9.14535 | 89 | 9.14963 | 91 | 0.85037 | 9.99572 | 58 | 6 │ 9.2 │ 9.1 │ 9.0 |
| 3 | 9.14624 | 90 | 9.15054 | 91 | 0.84946 | 9.99570 | 57 | 7 │ 10.7 │ 10.6 │ 10.5 |
| 4 | 9.14714 | 89 | 9.15145 | 91 | 0.84855 | 9.99568 | 56 | 8 │ 12.3 │ 12.1 │ 12.0 |
| | | | | | | | | 9 │ 13.8 │ 13.7 │ 13.5 |
| 5 | 9.14803 | 88 | 9.15236 | 91 | 0.84764 | 9.99566 | 55 | 10 │ 15.3 │ 15.2 │ 15.0 |
| 6 | 9.14891 | 89 | 9.15327 | 90 | 0.84673 | 9.99565 | 54 | 20 │ 30.7 │ 30.3 │ 30.0 |
| 7 | 9.14980 | 89 | 9.15417 | 91 | 0.84583 | 9.99563 | 53 | 30 │ 46.0 │ 45.5 │ 45.0 |
| 8 | 9.15069 | 88 | 9.15508 | 90 | 0.84492 | 9.99561 | 52 | 40 │ 61.3 │ 60.7 │ 60.0 |
| 9 | 9.15157 | 88 | 9.15598 | 90 | 0.84402 | 9.99559 | 51 | 50 │ 76.7 │ 75.8 │ 75.0 |
| 10 | 9.15245 | 88 | 9.15688 | 89 | 0.84312 | 9.99557 | 50 | |
| 11 | 9.15333 | 88 | 9.15777 | 90 | 0.84223 | 9.99556 | 49 | **89 │ 88** |
| 12 | 9.15421 | 87 | 9.15867 | 89 | 0.84133 | 9.99554 | 48 | 6 │ 8.9 │ 8.8 |
| 13 | 9.15508 | 88 | 9.15956 | 90 | 0.84044 | 9.99552 | 47 | 7 │ 10.4 │ 10.3 |
| 14 | 9.15596 | 87 | 9.16046 | 89 | 0.83954 | 9.99550 | 46 | 8 │ 11.9 │ 11.7 |
| | | | | | | | | 9 │ 13.4 │ 13.2 |
| 15 | 9.15683 | 87 | 9.16135 | 89 | 0.83865 | 9.99548 | 45 | 10 │ 14.8 │ 14.7 |
| 16 | 9.15770 | 87 | 9.16224 | 88 | 0.83776 | 9.99546 | 44 | 20 │ 29.7 │ 29.3 |
| 17 | 9.15857 | 87 | 9.16312 | 89 | 0.83688 | 9.99545 | 43 | 30 │ 44.5 │ 44.0 |
| 18 | 9.15944 | 86 | 9.16401 | 88 | 0.83599 | 9.99543 | 42 | 40 │ 59.3 │ 58.7 |
| 19 | 9.16030 | 86 | 9.16489 | 88 | 0.83511 | 9.99541 | 41 | 50 │ 74.2 │ 73.3 |
| 20 | 9.16116 | 87 | 9.16577 | 88 | 0.83423 | 9.99539 | 40 | |
| 21 | 9.16203 | 86 | 9.16665 | 88 | 0.83335 | 9.99537 | 39 | **87 │ 86** |
| 22 | 9.16289 | 85 | 9.16753 | 88 | 0.83247 | 9.99535 | 38 | 6 │ 8.7 │ 8.6 |
| 23 | 9.16374 | 86 | 9.16841 | 87 | 0.83159 | 9.99533 | 37 | 7 │ 10.2 │ 10.0 |
| 24 | 9.16460 | 85 | 9.16928 | 88 | 0.83072 | 9.99532 | 36 | 8 │ 11.6 │ 11.5 |
| | | | | | | | | 9 │ 13.1 │ 12.9 |
| 25 | 9.16545 | 86 | 9.17016 | 87 | 0.82984 | 9.99530 | 35 | 10 │ 14.5 │ 14.3 |
| 26 | 9.16631 | 85 | 9.17103 | 87 | 0.82897 | 9.99528 | 34 | 20 │ 29.0 │ 28.7 |
| 27 | 9.16716 | 85 | 9.17190 | 87 | 0.82810 | 9.99526 | 33 | 30 │ 43.5 │ 43.0 |
| 28 | 9.16801 | 85 | 9.17277 | 86 | 0.82723 | 9.99524 | 32 | 40 │ 58.0 │ 57.3 |
| 29 | 9.16886 | 84 | 9.17363 | 87 | 0.82637 | 9.99522 | 31 | 50 │ 72.5 │ 71.7 |
| 30 | 9.16970 | 85 | 9.17450 | 86 | 0.82550 | 9.99520 | 30 | |
| 31 | 9.17055 | 84 | 9.17536 | 86 | 0.82464 | 9.99518 | 29 | **85 │ 84** |
| 32 | 9.17139 | 84 | 9.17622 | 86 | 0.82378 | 9.99517 | 28 | 6 │ 8.5 │ 8.4 |
| 33 | 9.17223 | 84 | 9.17708 | 86 | 0.82292 | 9.99515 | 27 | 7 │ 9.9 │ 9.8 |
| 34 | 9.17307 | 84 | 9.17794 | 86 | 0.82206 | 9.99513 | 26 | 8 │ 11.3 │ 11.2 |
| | | | | | | | | 9 │ 12.8 │ 12.6 |
| 35 | 9.17391 | 83 | 9.17880 | 85 | 0.82120 | 9.99511 | 25 | 10 │ 14.2 │ 14.0 |
| 36 | 9.17474 | 84 | 9.17965 | 86 | 0.82035 | 9.99509 | 24 | 20 │ 28.3 │ 28.0 |
| 37 | 9.17558 | 83 | 9.18051 | 85 | 0.81949 | 9.99507 | 23 | 30 │ 42.5 │ 42.0 |
| 38 | 9.17641 | 83 | 9.18136 | 85 | 0.81864 | 9.99505 | 22 | 40 │ 56.7 │ 56.0 |
| 39 | 9.17724 | 83 | 9.18221 | 85 | 0.81779 | 9.99503 | 21 | 50 │ 70.8 │ 70.0 |
| 40 | 9.17807 | 83 | 9.18306 | 85 | 0.81694 | 9.99501 | 20 | |
| 41 | 9.17890 | 83 | 9.18391 | 84 | 0.81609 | 9.99499 | 19 | **83 │ 82** |
| 42 | 9.17973 | 82 | 9.18475 | 85 | 0.81525 | 9.99497 | 18 | 6 │ 8.3 │ 8.2 |
| 43 | 9.18055 | 82 | 9.18560 | 84 | 0.81440 | 9.99495 | 17 | 7 │ 9.7 │ 9.6 |
| 44 | 9.18137 | 83 | 9.18644 | 84 | 0.81356 | 9.99494 | 16 | 8 │ 11.1 │ 10.9 |
| | | | | | | | | 9 │ 12.5 │ 12.3 |
| 45 | 9.18220 | 82 | 9.18728 | 84 | 0.81272 | 9.99492 | 15 | 10 │ 13.8 │ 13.7 |
| 46 | 9.18302 | 81 | 9.18812 | 84 | 0.81188 | 9.99490 | 14 | 20 │ 27.7 │ 27.3 |
| 47 | 9.18383 | 82 | 9.18896 | 84 | 0.81104 | 9.99488 | 13 | 30 │ 41.5 │ 41.0 |
| 48 | 9.18465 | 82 | 9.18979 | 84 | 0.81021 | 9.99486 | 12 | 40 │ 55.3 │ 54.7 |
| 49 | 9.18547 | 81 | 9.19063 | 83 | 0.80937 | 9.99484 | 11 | 50 │ 69.2 │ 68.3 |
| 50 | 9.18628 | 81 | 9.19146 | 83 | 0.80854 | 9.99482 | 10 | |
| 51 | 9.18709 | 81 | 9.19229 | 83 | 0.80771 | 9.99480 | 9 | **81 │ 80 │ 2** |
| 52 | 9.18790 | 81 | 9.19312 | 83 | 0.80688 | 9.99478 | 8 | 6 │ 8.1 │ 8.0 │ 0.2 |
| 53 | 9.18871 | 81 | 9.19395 | 83 | 0.80605 | 9.99476 | 7 | 7 │ 9.5 │ 9.3 │ 0.2 |
| 54 | 9.18952 | 81 | 9.19478 | 83 | 0.80522 | 9.99474 | 6 | 8 │ 10.8 │ 10.7 │ 0.3 |
| | | | | | | | | 9 │ 12.2 │ 12.0 │ 0.3 |
| 55 | 9.19033 | 80 | 9.19561 | 82 | 0.80439 | 9.99472 | 5 | 10 │ 13.5 │ 13.3 │ 0.3 |
| 56 | 9.19113 | 80 | 9.19643 | 82 | 0.80357 | 9.99470 | 4 | 20 │ 27.0 │ 26.7 │ 0.7 |
| 57 | 9.19193 | 80 | 9.19725 | 82 | 0.80275 | 9.99468 | 3 | 30 │ 40.5 │ 40.0 │ 1.0 |
| 58 | 9.19273 | 80 | 9.19807 | 82 | 0.80193 | 9.99466 | 2 | 40 │ 54.0 │ 53.3 │ 1.3 |
| 59 | 9.19353 | 80 | 9.19889 | 82 | 0.80111 | 9.99464 | 1 | 50 │ 67.5 │ 66.7 │ 1.7 |
| 60 | 9.19433 | | 9.19971 | | 0.80029 | 9.99462 | 0 | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | ′ | P. P. |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | | P. P. | | |
|---|---------|----|---------|-------|----------|---------|---|---|---|---|
| 0 | 9.19433 | | 9.19971 | | 0.80029 | 9.99462 | 60 | | | |
| 1 | 9.19513 | 80 | 9.20053 | 82 | 0.79947 | 9.99460 | 59 | | 82 | 81 | 80 |
| 2 | 9.19592 | 79 | 9.20134 | 81 | 0.79866 | 9.99458 | 58 | 6 | 8.2 | 8.1 | 8.0 |
| 3 | 9.19672 | 80 | 9.20216 | 82 | 0.79784 | 9.99456 | 57 | 7 | 9.6 | 9.5 | 9.3 |
| 4 | 9.19751 | 79 | 9.20297 | 81 | 0.79703 | 9.99454 | 56 | 8 | 10.9 | 10.8 | 10.7 |
| | | 79 | | 81 | | | | 9 | 12.3 | 12.2 | 12.0 |
| 5 | 9.19830 | | 9.20378 | | 0.79622 | 9.99452 | 55 | 10 | 13.7 | 13.5 | 13.3 |
| 6 | 9.19909 | 79 | 9.20459 | 81 | 0.79541 | 9.99450 | 54 | 20 | 27.3 | 27.0 | 26.7 |
| 7 | 9.19988 | 79 | 9.20540 | 81 | 0.79460 | 9.99448 | 53 | 30 | 41.0 | 40.5 | 40.0 |
| 8 | 9.20067 | 79 | 9.20621 | 81 | 0.79379 | 9.99446 | 52 | 40 | 54.7 | 54.0 | 53.3 |
| 9 | 9.20145 | 78 | 9.20701 | 80 | 0.79299 | 9.99444 | 51 | 50 | 68.3 | 67.5 | 66.7 |
| | | 78 | | 81 | | | | | | | |
| 10 | 9.20223 | | 9.20782 | | 0.79218 | 9.99442 | 50 | | | | |
| 11 | 9.20302 | 79 | 9.20862 | 80 | 0.79138 | 9.99440 | 49 | | | 79 | 78 |
| 12 | 9.20380 | 78 | 9.20942 | 80 | 0.79058 | 9.99438 | 48 | 6 | | 7.9 | 7.8 |
| 13 | 9.20458 | 78 | 9.21022 | 80 | 0.78978 | 9.99436 | 47 | 7 | | 9.2 | 9.1 |
| 14 | 9.20535 | 77 | 9.21102 | 80 | 0.78898 | 9.99434 | 46 | 8 | | 10.5 | 10.4 |
| | | 78 | | 80 | | | | 9 | | 11.9 | 11.7 |
| 15 | 9.20613 | | 9.21182 | | 0.78818 | 9.99432 | 45 | 10 | | 13.2 | 13.0 |
| 16 | 9.20691 | 78 | 9.21261 | 79 | 0.78739 | 9.99429 | 44 | 20 | | 26.3 | 26.0 |
| 17 | 9.20768 | 77 | 9.21341 | 80 | 0.78659 | 9.99427 | 43 | 30 | | 39.5 | 39.0 |
| 18 | 9.20845 | 77 | 9.21420 | 79 | 0.78580 | 9.99425 | 42 | 40 | | 52.7 | 52.0 |
| 19 | 9.20922 | 77 | 9.21499 | 79 | 0.78501 | 9.99423 | 41 | 50 | | 65.8 | 65.0 |
| | | 77 | | 79 | | | | | | | |
| 20 | 9.20999 | | 9.21578 | | 0.78422 | 9.99421 | 40 | | | | |
| 21 | 9.21076 | 77 | 9.21657 | 79 | 0.78343 | 9.99419 | 39 | | | 77 | 76 |
| 22 | 9.21153 | 77 | 9.21736 | 79 | 0.78264 | 9.99417 | 38 | 6 | | 7.7 | 7.6 |
| 23 | 9.21229 | 76 | 9.21814 | 78 | 0.78186 | 9.99415 | 37 | 7 | | 9.0 | 8.9 |
| 24 | 9.21306 | 77 | 9.21893 | 79 | 0.78107 | 9.99413 | 36 | 8 | | 10.3 | 10.1 |
| | | 76 | | 78 | | | | 9 | | 11.6 | 11.4 |
| 25 | 9.21382 | | 9.21971 | | 0.78029 | 9.99411 | 35 | 10 | | 12.8 | 12.7 |
| 26 | 9.21458 | 76 | 9.22049 | 78 | 0.77951 | 9.99409 | 34 | 20 | | 25.7 | 25.3 |
| 27 | 9.21534 | 76 | 9.22127 | 78 | 0.77873 | 9.99407 | 33 | 30 | | 38.5 | 38.0 |
| 28 | 9.21610 | 76 | 9.22205 | 78 | 0.77795 | 9.99404 | 32 | 40 | | 51.3 | 50.7 |
| 29 | 9.21685 | 75 | 9.22283 | 78 | 0.77717 | 9.99402 | 31 | 50 | | 64.2 | 63.3 |
| | | 76 | | 78 | | | | | | | |
| 30 | 9.21761 | | 9.22361 | | 0.77639 | 9.99400 | 30 | | | | |
| 31 | 9.21836 | 75 | 9.22438 | 77 | 0.77562 | 9.99398 | 29 | | | 75 | 74 |
| 32 | 9.21912 | 76 | 9.22516 | 78 | 0.77484 | 9.99396 | 28 | 6 | | 7.5 | 7.4 |
| 33 | 9.21987 | 75 | 9.22593 | 77 | 0.77407 | 9.99394 | 27 | 7 | | 8.8 | 8.6 |
| 34 | 9.22062 | 75 | 9.22670 | 77 | 0.77330 | 9.99392 | 26 | 8 | | 10.0 | 9.9 |
| | | 75 | | 77 | | | | 9 | | 11.3 | 11.1 |
| 35 | 9.22137 | | 9.22747 | | 0.77253 | 9.99390 | 25 | 10 | | 12.5 | 12.3 |
| 36 | 9.22211 | 74 | 9.22824 | 77 | 0.77176 | 9.99388 | 24 | 20 | | 25.0 | 24.7 |
| 37 | 9.22286 | 75 | 9.22901 | 77 | 0.77099 | 9.99385 | 23 | 30 | | 37.5 | 37.0 |
| 38 | 9.22361 | 75 | 9.22977 | 76 | 0.77023 | 9.99383 | 22 | 40 | | 50.0 | 49.3 |
| 39 | 9.22435 | 74 | 9.23054 | 77 | 0.76946 | 9.99381 | 21 | 50 | | 62.5 | 61.7 |
| | | 74 | | 76 | | | | | | | |
| 40 | 9.22509 | | 9.23130 | | 0.76870 | 9.99379 | 20 | | | | |
| 41 | 9.22583 | 74 | 9.23206 | 76 | 0.76794 | 9.99377 | 19 | | | 73 | 72 |
| 42 | 9.22657 | 74 | 9.23283 | 77 | 0.76717 | 9.99375 | 18 | 6 | | 7.3 | 7.2 |
| 43 | 9.22731 | 74 | 9.23359 | 76 | 0.76641 | 9.99372 | 17 | 7 | | 8.5 | 8.4 |
| 44 | 9.22805 | 73 | 9.23435 | 76 | 0.76565 | 9.99370 | 16 | 8 | | 9.7 | 9.6 |
| | | 73 | | 75 | | | | 9 | | 11.0 | 10.8 |
| 45 | 9.22878 | | 9.23510 | | 0.76490 | 9.99368 | 15 | 10 | | 12.2 | 12.0 |
| 46 | 9.22952 | 74 | 9.23586 | 76 | 0.76414 | 9.99366 | 14 | 20 | | 24.3 | 24.0 |
| 47 | 9.23025 | 73 | 9.23661 | 75 | 0.76339 | 9.99364 | 13 | 30 | | 36.5 | 36.0 |
| 48 | 9.23098 | 73 | 9.23737 | 76 | 0.76263 | 9.99362 | 12 | 40 | | 48.7 | 48.0 |
| 49 | 9.23171 | 73 | 9.23812 | 75 | 0.76188 | 9.99359 | 11 | 50 | | 60.8 | 60.0 |
| | | 73 | | 75 | | | | | | | |
| 50 | 9.23244 | | 9.23887 | | 0.76113 | 9.99357 | 10 | | | | |
| 51 | 9.23317 | 73 | 9.23962 | 75 | 0.76038 | 9.99355 | 9 | | 71 | 3 | 2 |
| 52 | 9.23390 | 73 | 9.24037 | 75 | 0.75963 | 9.99353 | 8 | 6 | 7.1 | 0.3 | 0.2 |
| 53 | 9.23462 | 72 | 9.24112 | 75 | 0.75888 | 9.99351 | 7 | 7 | 8.3 | 0.4 | 0.2 |
| 54 | 9.23535 | 73 | 9.24186 | 74 | 0.75814 | 9.99348 | 6 | 8 | 9.5 | 0.4 | 0.3 |
| | | 72 | | 75 | | | | 9 | 10.7 | 0.5 | 0.3 |
| 55 | 9.23607 | | 9.24261 | | 0.75739 | 9.99346 | 5 | 10 | 11.8 | 0.5 | 0.3 |
| 56 | 9.23679 | 72 | 9.24335 | 74 | 0.75665 | 9.99344 | 4 | 20 | 23.7 | 1.0 | 0.7 |
| 57 | 9.23752 | 73 | 9.24410 | 75 | 0.75590 | 9.99342 | 3 | 30 | 35.5 | 1.5 | 1.0 |
| 58 | 9.23823 | 71 | 9.24484 | 74 | 0.75516 | 9.99340 | 2 | 40 | 47.3 | 2.0 | 1.3 |
| 59 | 9.23895 | 72 | 9.24558 | 74 | 0.75442 | 9.99337 | 1 | 50 | 59.2 | 2.5 | 1.7 |
| 60 | 9.23967 | 72 | 9.24632 | 74 | 0.75368 | 9.99335 | 0 | | | | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | ′ | | P. P. | | |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | | P. P. | |
|---|---------|-----|---------|-------|----------|---------|---|---|---|
| 0 | 9.23967 | 72 | 9.24632 | 74 | 0.75368 | 9.99335 | 60 | | |
| 1 | 9.24039 | 72 | 9.24706 | 73 | 0.75294 | 9.99333 | 59 | | **74** \| **73** |
| 2 | 9.24110 | 71 | 9.24779 | 74 | 0.75221 | 9.99331 | 58 | 6 | 7.4 \| 7.3 |
| 3 | 9.24181 | 71 | 9.24853 | 73 | 0.75147 | 9.99328 | 57 | 7 | 8.6 \| 8.5 |
| 4 | 9.24253 | 72 | 9.24926 | 74 | 0.75074 | 9.99326 | 56 | 8 | 9.9 \| 9.7 |
| 5 | 9.24324 | 71 | 9.25000 | 73 | 0.75000 | 9.99324 | 55 | 9 | 11.1 \| 11.0 |
| 6 | 9.24395 | 71 | 9.25073 | 73 | 0.74927 | 9.99322 | 54 | 10 | 12.3 \| 12.2 |
| 7 | 9.24466 | 71 | 9.25146 | 73 | 0.74854 | 9.99319 | 53 | 20 | 24.7 \| 24.3 |
| 8 | 9.24536 | 70 | 9.25219 | 73 | 0.74781 | 9.99317 | 52 | 30 | 37.0 \| 36.5 |
| 9 | 9.24607 | 71 | 9.25292 | 73 | 0.74708 | 9.99315 | 51 | 40 | 49.3 \| 48.7 |
| 10 | 9.24677 | 70 | 9.25365 | 73 | 0.74635 | 9.99313 | 50 | 50 | 61.7 \| 60.8 |
| 11 | 9.24748 | 71 | 9.25437 | 72 | 0.74563 | 9.99310 | 49 | | |
| 12 | 9.24818 | 70 | 9.25510 | 73 | 0.74490 | 9.99308 | 48 | | **72** \| **71** |
| 13 | 9.24888 | 70 | 9.25582 | 72 | 0.74418 | 9.99306 | 47 | 6 | 7.2 \| 7.1 |
| 14 | 9.24958 | 70 | 9.25655 | 73 | 0.74345 | 9.99304 | 46 | 7 | 8.4 \| 8.3 |
| 15 | 9.25028 | 70 | 9.25727 | 72 | 0.74273 | 9.99301 | 45 | 8 | 9.6 \| 9.5 |
| 16 | 9.25098 | 70 | 9.25799 | 72 | 0.74201 | 9.99299 | 44 | 9 | 10.8 \| 10.7 |
| 17 | 9.25168 | 70 | 9.25871 | 72 | 0.74129 | 9.99297 | 43 | 10 | 12.0 \| 11.8 |
| 18 | 9.25237 | 69 | 9.25943 | 72 | 0.74057 | 9.99294 | 42 | 20 | 24.0 \| 23.7 |
| 19 | 9.25307 | 70 | 9.26015 | 72 | 0.73985 | 9.99292 | 41 | 30 | 36.0 \| 35.5 |
| | | | | | | | | 40 | 48.0 \| 47.3 |
| 20 | 9.25376 | 69 | 9.26086 | 72 | 0.73914 | 9.99290 | 40 | 50 | 60.0 \| 59.2 |
| 21 | 9.25445 | 69 | 9.26158 | 71 | 0.73842 | 9.99288 | 39 | | |
| 22 | 9.25514 | 69 | 9.26229 | 72 | 0.73771 | 9.99285 | 38 | | **70** \| **69** |
| 23 | 9.25583 | 69 | 9.26301 | 71 | 0.73699 | 9.99283 | 37 | 6 | 7.0 \| 6.9 |
| 24 | 9.25652 | 69 | 9.26372 | 71 | 0.73628 | 9.99281 | 36 | 7 | 8.2 \| 8.1 |
| 25 | 9.25721 | 69 | 9.26443 | 71 | 0.73557 | 9.99278 | 35 | 8 | 9.3 \| 9.2 |
| 26 | 9.25790 | 68 | 9.26514 | 71 | 0.73486 | 9.99276 | 34 | 9 | 10.5 \| 10.4 |
| 27 | 9.25858 | 69 | 9.26585 | 70 | 0.73415 | 9.99274 | 33 | 10 | 11.7 \| 11.5 |
| 28 | 9.25927 | 68 | 9.26655 | 71 | 0.73345 | 9.99271 | 32 | 20 | 23.3 \| 23.0 |
| 29 | 9.25995 | 68 | 9.26726 | 71 | 0.73274 | 9.99269 | 31 | 30 | 35.0 \| 34.5 |
| | | | | | | | | 40 | 46.7 \| 46.0 |
| 30 | 9.26063 | 68 | 9.26797 | 70 | 0.73203 | 9.99267 | 30 | 50 | 58.3 \| 57.5 |
| 31 | 9.26131 | 68 | 9.26867 | 70 | 0.73133 | 9.99264 | 29 | | |
| 32 | 9.26199 | 68 | 9.26937 | 71 | 0.73063 | 9.99262 | 28 | | **68** \| **67** |
| 33 | 9.26267 | 68 | 9.27008 | 70 | 0.72992 | 9.99260 | 27 | 6 | 6.8 \| 6.7 |
| 34 | 9.26335 | 68 | 9.27078 | 70 | 0.72922 | 9.99257 | 26 | 7 | 7.9 \| 7.8 |
| 35 | 9.26403 | 67 | 9.27148 | 70 | 0.72852 | 9.99255 | 25 | 8 | 9.1 \| 8.9 |
| 36 | 9.26470 | 68 | 9.27218 | 70 | 0.72782 | 9.99252 | 24 | 9 | 10.2 \| 10.1 |
| 37 | 9.26538 | 67 | 9.27288 | 70 | 0.72712 | 9.99250 | 23 | 10 | 11.3 \| 11.2 |
| 38 | 9.26605 | 67 | 9.27357 | 69 | 0.72643 | 9.99248 | 22 | 20 | 22.7 \| 22.3 |
| 39 | 9.26672 | 67 | 9.27427 | 70 | 0.72573 | 9.99245 | 21 | 30 | 34.0 \| 33.5 |
| | | | | | | | | 40 | 45.3 \| 44.7 |
| 40 | 9.26739 | 67 | 9.27496 | 70 | 0.72504 | 9.99243 | 20 | 50 | 56.7 \| 55.8 |
| 41 | 9.26806 | 67 | 9.27566 | 69 | 0.72434 | 9.99241 | 19 | | |
| 42 | 9.26873 | 67 | 9.27635 | 69 | 0.72365 | 9.99238 | 18 | | **66** \| **65** |
| 43 | 9.26940 | 67 | 9.27704 | 69 | 0.72296 | 9.99236 | 17 | 6 | 6.6 \| 6.5 |
| 44 | 9.27007 | 66 | 9.27773 | 69 | 0.72227 | 9.99233 | 16 | 7 | 7.7 \| 7.6 |
| 45 | 9.27073 | 67 | 9.27842 | 69 | 0.72158 | 9.99231 | 15 | 8 | 8.8 \| 8.7 |
| 46 | 9.27140 | 66 | 9.27911 | 69 | 0.72089 | 9.99229 | 14 | 9 | 9.9 \| 9.8 |
| 47 | 9.27206 | 67 | 9.27980 | 69 | 0.72020 | 9.99226 | 13 | 10 | 11.0 \| 10.8 |
| 48 | 9.27273 | 67 | 9.28049 | 68 | 0.71951 | 9.99224 | 12 | 20 | 22.0 \| 21.7 |
| 49 | 9.27339 | 66 | 9.28117 | 69 | 0.71883 | 9.99221 | 11 | 30 | 33.0 \| 32.5 |
| | | | | | | | | 40 | 44.0 \| 43.3 |
| 50 | 9.27405 | 66 | 9.28186 | 68 | 0.71814 | 9.99219 | 10 | 50 | 55.0 \| 54.2 |
| 51 | 9.27471 | 66 | 9.28254 | 68 | 0.71746 | 9.99217 | 9 | | |
| 52 | 9.27537 | 65 | 9.28323 | 69 | 0.71677 | 9.99214 | 8 | | **3** \| **2** |
| 53 | 9.27602 | 66 | 9.28391 | 68 | 0.71609 | 9.99212 | 7 | 6 | 0.3 \| 0.2 |
| 54 | 9.27668 | 66 | 9.28459 | 68 | 0.71541 | 9.99209 | 6 | 7 | 0.4 \| 0.2 |
| 55 | 9.27734 | 65 | 9.28527 | 68 | 0.71473 | 9.99207 | 5 | 8 | 0.4 \| 0.3 |
| 56 | 9.27799 | 65 | 9.28595 | 67 | 0.71405 | 9.99204 | 4 | 9 | 0.5 \| 0.3 |
| 57 | 9.27864 | 66 | 9.28662 | 68 | 0.71338 | 9.99202 | 3 | 10 | 0.5 \| 0.3 |
| 58 | 9.27930 | 65 | 9.28730 | 68 | 0.71270 | 9.99200 | 2 | 10 | 1.0 \| 0.7 |
| 59 | 9.27995 | 65 | 9.28798 | 67 | 0.71202 | 9.99197 | 1 | 30 | 1.5 \| 1.0 |
| | | | | | | | | 40 | 2.0 \| 1.3 |
| 60 | 9.28060 | | 9.28865 | | 0.71135 | 9.99195 | 0 | 50 | 2.5 \| 1.7 |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | ′ | P. P. | |

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| ′ | L. Sin. | d. | L. Tang. | d. c. | L. Cotg. | L. Cos. | | P. P. | |
|---|---------|-----|----------|-------|----------|---------|----|-------|-----|
| 0 | 9.28060 | | 9.28865 | | 0.71135 | 9.99195 | 60 | | |
| 1 | 9.28125 | 65 | 9.28933 | 68 | 0.71067 | 9.99192 | 59 | **58** | **67** |
| 2 | 9.28190 | 65 | 9.29000 | 67 | 0.71000 | 9.99190 | 58 | 6 6.8 | 6.7 |
| 3 | 9.28254 | 64 | 9.29067 | 67 | 0.70933 | 9.99187 | 57 | 7 7.9 | 7.8 |
| 4 | 9.28319 | 65 | 9.29134 | 67 | 0.70866 | 9.99185 | 56 | 8 9.1 | 8.9 |
| | | 65 | | 67 | | | | 9 10.2 | 10.1 |
| 5 | 9.28384 | 64 | 9.29201 | 67 | 0.70799 | 9.99182 | 55 | 10 11.3 | 11.2 |
| 6 | 9.28448 | 64 | 9.29268 | 67 | 0.70732 | 9.99180 | 54 | 20 22.7 | 22.3 |
| 7 | 9.28512 | 65 | 9.29335 | 67 | 0.70665 | 9.99177 | 53 | 30 34.0 | 33.5 |
| 8 | 9.28577 | 64 | 9.29402 | 66 | 0.70598 | 9.99175 | 52 | 40 45.3 | 44.7 |
| 9 | 9.28641 | 64 | 9.29468 | 67 | 0.70532 | 9.99172 | 51 | 50 56.7 | 55.8 |
| 10 | 9.28705 | 64 | 9.29535 | 66 | 0.70465 | 9.99170 | 50 | | |
| 11 | 9.28769 | 64 | 9.29601 | 67 | 0.70399 | 9.99167 | 49 | **56** | **65** |
| 12 | 9.28833 | 63 | 9.29668 | 66 | 0.70332 | 9.99165 | 48 | 6 6.6 | 6.5 |
| 13 | 9.28896 | 64 | 9.29734 | 66 | 0.70266 | 9.99162 | 47 | 7 7.7 | 7.6 |
| 14 | 9.28960 | 64 | 9.29800 | 66 | 0.70200 | 9.99160 | 46 | 8 8.8 | 8.7 |
| 15 | 9.29024 | 63 | 9.29866 | 66 | 0.70134 | 9.99157 | 45 | 9 9.9 | 9.8 |
| 16 | 9.29087 | 63 | 9.29932 | 66 | 0.70068 | 9.99155 | 44 | 10 11.0 | 10.8 |
| 17 | 9.29150 | 64 | 9.29998 | 66 | 0.70002 | 9.99152 | 43 | 20 22.0 | 21.7 |
| 18 | 9.29214 | 63 | 9.30064 | 66 | 0.69936 | 9.99150 | 42 | 30 33.0 | 32.5 |
| 19 | 9.29277 | 63 | 9.30130 | 65 | 0.69870 | 9.99147 | 41 | 40 44.0 | 43.3 |
| | | 63 | | 65 | | | | 50 55.0 | 54.2 |
| 20 | 9.29340 | 63 | 9.30195 | 66 | 0.69805 | 9.99145 | 40 | | |
| 21 | 9.29403 | 63 | 9.30261 | 65 | 0.69739 | 9.99142 | 39 | **64** | **63** |
| 22 | 9.29466 | 63 | 9.30326 | 65 | 0.69674 | 9.99140 | 38 | 6 6.4 | 6.3 |
| 23 | 9.29529 | 62 | 9.30391 | 66 | 0.69609 | 9.99137 | 37 | 7 7.5 | 7.4 |
| 24 | 9.29591 | 63 | 9.30457 | 65 | 0.69543 | 9.99135 | 36 | 8 8.5 | 8.4 |
| 25 | 9.29654 | 62 | 9.30522 | 65 | 0.69478 | 9.99132 | 35 | 9 9.6 | 9.5 |
| 26 | 9.29716 | 63 | 9.30587 | 65 | 0.69413 | 9.99130 | 34 | 10 10.7 | 10.5 |
| 27 | 9.29779 | 62 | 9.30652 | 65 | 0.69348 | 9.99127 | 33 | 20 21.3 | 21.0 |
| 28 | 9.29841 | 62 | 9.30717 | 65 | 0.69283 | 9.99124 | 32 | 30 32.0 | 31.5 |
| 29 | 9.29903 | 63 | 9.30782 | 64 | 0.69218 | 9.99122 | 31 | 40 42.7 | 42.0 |
| | | | | 64 | | | | 50 53.3 | 52.5 |
| 30 | 9.29966 | 62 | 9.30846 | 65 | 0.69154 | 9.99119 | 30 | | |
| 31 | 9.30028 | 62 | 9.30911 | 64 | 0.69089 | 9.99117 | 29 | **62** | **61** |
| 32 | 9.30090 | 61 | 9.30975 | 65 | 0.69025 | 9.99114 | 28 | 6 6.2 | 6.1 |
| 33 | 9.30151 | 62 | 9.31040 | 64 | 0.68960 | 9.99112 | 27 | 7 7.2 | 7.1 |
| 34 | 9.30213 | 62 | 9.31104 | 64 | 0.68896 | 9.99109 | 26 | 8 8.3 | 8.1 |
| 35 | 9.30275 | 61 | 9.31168 | 65 | 0.68832 | 9.99106 | 25 | 9 9.3 | 9.2 |
| 36 | 9.30336 | 62 | 9.31233 | 64 | 0.68767 | 9.99104 | 24 | 10 10.3 | 10.2 |
| 37 | 9.30398 | 61 | 9.31297 | 64 | 0.68703 | 9.99101 | 23 | 20 20.7 | 20.3 |
| 38 | 9.30459 | 62 | 9.31361 | 64 | 0.68639 | 9.99099 | 22 | 30 31.0 | 30.5 |
| 39 | 9.30521 | 61 | 9.31425 | 64 | 0.68575 | 9.99096 | 21 | 40 41.3 | 40.7 |
| | | 61 | | 64 | | | | 50 51.7 | 50.8 |
| 40 | 9.30582 | 61 | 9.31489 | 63 | 0.68511 | 9.99093 | 20 | | |
| 41 | 9.30643 | 61 | 9.31552 | 64 | 0.68448 | 9.99091 | 19 | **60** | **59** |
| 42 | 9.30704 | 61 | 9.31616 | 63 | 0.68384 | 9.99088 | 18 | 6 6.0 | 5.9 |
| 43 | 9.30765 | 61 | 9.31679 | 64 | 0.68321 | 9.99086 | 17 | 7 7.0 | 6.9 |
| 44 | 9.30826 | 61 | 9.31743 | 63 | 0.68257 | 9.99083 | 16 | 8 8.0 | 7.9 |
| 45 | 9.30887 | 60 | 9.31806 | 64 | 0.68194 | 9.99080 | 15 | 9 9.0 | 8.9 |
| 46 | 9.30947 | 61 | 9.31870 | 63 | 0.68130 | 9.99078 | 14 | 10 10.0 | 9.8 |
| 47 | 9.31008 | 60 | 9.31933 | 63 | 0.68067 | 9.99075 | 13 | 20 20.0 | 19.7 |
| 48 | 9.31068 | 61 | 9.31996 | 63 | 0.68004 | 9.99072 | 12 | 30 30.0 | 29.5 |
| 49 | 9.31129 | 60 | 9.32059 | 63 | 0.67941 | 9.99070 | 11 | 40 40.0 | 39.3 |
| | | 60 | | 63 | | | | 50 50.0 | 49.2 |
| 50 | 9.31189 | 61 | 9.32122 | 63 | 0.67878 | 9.99067 | 10 | | |
| 51 | 9.31250 | 60 | 9.32185 | 63 | 0.67815 | 9.99064 | 9 | **3** | **2** |
| 52 | 9.31310 | 60 | 9.32248 | 63 | 0.67752 | 9.99062 | 8 | 6 0.3 | 0.2 |
| 53 | 9.31370 | 60 | 9.32311 | 62 | 0.67689 | 9.99059 | 7 | 7 0.4 | 0.2 |
| 54 | 9.31430 | 60 | 9.32373 | 63 | 0.67627 | 9.99056 | 6 | 8 0.4 | 0.3 |
| 55 | 9.31490 | 59 | 9.32436 | 62 | 0.67564 | 9.99054 | 5 | 9 0.5 | 0.3 |
| 56 | 9.31549 | 60 | 9.32498 | 63 | 0.67502 | 9.99051 | 4 | 10 0.5 | 0.3 |
| 57 | 9.31609 | 60 | 9.32561 | 62 | 0.67439 | 9.99048 | 3 | 20 1.0 | 0.7 |
| 58 | 9.31669 | 59 | 9.32623 | 62 | 0.67377 | 9.99046 | 2 | 30 1.5 | 1.0 |
| 59 | 9.31728 | 60 | 9.32685 | 62 | 0.67315 | 9.99043 | 1 | 40 2.0 | 1.3 |
| 60 | 9.31788 | | 9.32747 | 62 | 0.67253 | 9.99040 | 0 | 50 2.5 | 1.7 |
| | L. Cos. | d. | L. Cotg. | d. c. | L. Tang. | L. Sin. | ′ | P. P. | |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | | P. P. | |
|---|---------|-----|---------|-------|----------|---------|---|-------|---|
| 0 | 9.31788 | 59 | 9.32747 | 63 | 0.67253 | 9.99040 | 60 | | |
| 1 | 9.31847 | 60 | 9.32810 | 62 | 0.67190 | 9.99038 | 59 | **63** | **62** |
| 2 | 9.31907 | 59 | 9.32872 | 62 | 0.67128 | 9.99035 | 58 | 6 6.3 | 6.2 |
| 3 | 9.31966 | 59 | 9.32933 | 62 | 0.67067 | 9.99032 | 57 | 7 7.4 | 7.2 |
| 4 | 9.32025 | 59 | 9.32995 | 62 | 0.67005 | 9.99030 | 56 | 8 8.4 | 8.3 |
| 5 | 9.32084 | 59 | 9.33057 | 62 | 0.66943 | 9.99027 | 55 | 9 9.5 | 9.3 |
| 6 | 9.32143 | 59 | 9.33119 | 62 | 0.66881 | 9.99024 | 54 | 10 10.5 | 10.3 |
| 7 | 9.32202 | 59 | 9.33180 | 62 | 0.66820 | 9.99022 | 53 | 20 21.0 | 20.7 |
| 8 | 9.32261 | 58 | 9.33242 | 61 | 0.66758 | 9.99019 | 52 | 30 31.5 | 31.0 |
| 9 | 9.32319 | 59 | 9.33303 | 62 | 0.66697 | 9.99016 | 51 | 40 42.0 | 41.3 |
| | | | | | | | | 50 52.5 | 51.7 |
| 10 | 9.32378 | 59 | 9.33365 | 61 | 0.66635 | 9.99013 | 50 | | |
| 11 | 9.32437 | 58 | 9.33426 | 61 | 0.66574 | 9.99011 | 49 | **61** | **60** |
| 12 | 9.32495 | 58 | 9.33487 | 61 | 0.66513 | 9.99008 | 48 | 6 6.1 | 6.0 |
| 13 | 9.32553 | 59 | 9.33548 | 61 | 0.66452 | 9.99005 | 47 | 7 7.1 | 7.0 |
| 14 | 9.32612 | 58 | 9.33609 | 61 | 0.66391 | 9.99002 | 46 | 8 8.1 | 8.0 |
| 15 | 9.32670 | 58 | 9.33670 | 61 | 0.66330 | 9.99000 | 45 | 9 9.2 | 9.0 |
| 16 | 9.32728 | 58 | 9.33731 | 61 | 0.66269 | 9.98997 | 44 | 10 10.2 | 10.0 |
| 17 | 9.32786 | 58 | 9.33792 | 61 | 0.66208 | 9.98994 | 43 | 20 20.3 | 20.0 |
| 18 | 9.32844 | 58 | 9.33853 | 60 | 0.66147 | 9.98991 | 42 | 30 30.5 | 30.0 |
| 19 | 9.32902 | 58 | 9.33913 | 61 | 0.66087 | 9.98989 | 41 | 40 40.7 | 40.0 |
| | | | | | | | | 50 50.8 | 50.0 |
| 20 | 9.32960 | 58 | 9.33974 | 60 | 0.66026 | 9.98986 | 40 | | |
| 21 | 9.33018 | 57 | 9.34034 | 61 | 0.65966 | 9.98983 | 39 | **59** | |
| 22 | 9.33075 | 58 | 9.34095 | 60 | 0.65905 | 9.98980 | 38 | 6 5.9 | |
| 23 | 9.33133 | 57 | 9.34155 | 60 | 0.65845 | 9.98978 | 37 | 7 6.9 | |
| 24 | 9.33190 | 58 | 9.34215 | 61 | 0.65785 | 9.98975 | 36 | 8 7.9 | |
| 25 | 9.33248 | 57 | 9.34276 | 60 | 0.65724 | 9.98972 | 35 | 9 8.9 | |
| 26 | 9.33305 | 57 | 9.34336 | 60 | 0.65664 | 9.98969 | 34 | 10 9.8 | |
| 27 | 9.33362 | 58 | 9.34396 | 60 | 0.65604 | 9.98967 | 33 | 20 19.7 | |
| 28 | 9.33420 | 57 | 9.34456 | 60 | 0.65544 | 9.98964 | 32 | 30 29.5 | |
| 29 | 9.33477 | 57 | 9.34516 | 60 | 0.65484 | 9.98961 | 31 | 40 39.3 | |
| | | | | | | | | 50 49.2 | |
| 30 | 9.33534 | 57 | 9.34576 | 59 | 0.65424 | 9.98958 | 30 | | |
| 31 | 9.33591 | 56 | 9.34635 | 60 | 0.65365 | 9.98955 | 29 | **58** | **57** |
| 32 | 9.33647 | 57 | 9.34695 | 60 | 0.65305 | 9.98953 | 28 | 6 5.8 | 5.7 |
| 33 | 9.33704 | 57 | 9.34755 | 59 | 0.65245 | 9.98950 | 27 | 7 6.8 | 6.7 |
| 34 | 9.33761 | 57 | 9.34814 | 60 | 0.65186 | 9.98947 | 26 | 8 7.7 | 7.6 |
| 35 | 9.33818 | 56 | 9.34874 | 59 | 0.65126 | 9.98944 | 25 | 9 8.7 | 8.6 |
| 36 | 9.33874 | 57 | 9.34933 | 59 | 0.65067 | 9.98941 | 24 | 10 9.7 | 9.5 |
| 37 | 9.33931 | 56 | 9.34992 | 59 | 0.65008 | 9.98938 | 23 | 20 19.3 | 19.0 |
| 38 | 9.33987 | 56 | 9.35051 | 60 | 0.64949 | 9.98936 | 22 | 30 29.0 | 28.5 |
| 39 | 9.34043 | 57 | 9.35111 | 59 | 0.64889 | 9.98933 | 21 | 40 38.7 | 38.0 |
| | | | | | | | | 50 48.3 | 47.5 |
| 40 | 9.34100 | 56 | 9.35170 | 59 | 0.64830 | 9.98930 | 20 | | |
| 41 | 9.34156 | 56 | 9.35229 | 59 | 0.64771 | 9.98927 | 19 | **56** | **55** |
| 42 | 9.34212 | 56 | 9.35288 | 59 | 0.64712 | 9.98924 | 18 | | |
| 43 | 9.34268 | 56 | 9.35347 | 58 | 0.64653 | 9.98921 | 17 | 6 5.6 | 5.5 |
| 44 | 9.34324 | 56 | 9.35405 | 59 | 0.64595 | 9.98919 | 16 | 7 6.5 | 6.4 |
| | | | | | | | | 8 7.5 | 7.3 |
| 45 | 9.34380 | 56 | 9.35464 | 59 | 0.64536 | 9.98916 | 15 | 9 8.4 | 8.3 |
| 46 | 9.34436 | 55 | 9.35523 | 58 | 0.64477 | 9.98913 | 14 | 10 9.3 | 9.2 |
| 47 | 9.34491 | 56 | 9.35581 | 59 | 0.64419 | 9.98910 | 13 | 20 18.7 | 18.3 |
| 48 | 9.34547 | 55 | 9.35640 | 58 | 0.64360 | 9.98907 | 12 | 30 28.0 | 27.5 |
| 49 | 9.34602 | 56 | 9.35698 | 59 | 0.64302 | 9.98904 | 11 | 40 37.3 | 36.7 |
| | | | | | | | | 50 46.7 | 45.8 |
| 50 | 9.34658 | 55 | 9.35757 | 58 | 0.64243 | 9.98901 | 10 | | |
| 51 | 9.34713 | 56 | 9.35815 | 58 | 0.64185 | 9.98898 | 9 | **3** | **2** |
| 52 | 9.34769 | 55 | 9.35873 | 58 | 0.64127 | 9.98896 | 8 | 6 0.3 | 0.2 |
| 53 | 9.34824 | 55 | 9.35931 | 58 | 0.64069 | 9.98893 | 7 | 7 0.4 | 0.2 |
| 54 | 9.34879 | 55 | 9.35989 | 58 | 0.64011 | 9.98890 | 6 | 8 0.4 | 0.3 |
| 55 | 9.34934 | 55 | 9.36047 | 58 | 0.63953 | 9.98887 | 5 | 9 0.5 | 0.3 |
| 56 | 9.34989 | 55 | 9.36105 | 58 | 0.63895 | 9.98884 | 4 | 10 0.5 | 0.3 |
| 57 | 9.35044 | 55 | 9.36163 | 58 | 0.63837 | 9.98881 | 3 | 20 1.0 | 0.7 |
| 58 | 9.35099 | 55 | 9.36221 | 58 | 0.63779 | 9.98878 | 2 | 30 1.5 | 1.0 |
| 59 | 9.35154 | 55 | 9.36279 | 57 | 0.63721 | 9.98875 | 1 | 40 2.0 | 1.3 |
| 60 | 9.35209 | | 9.36336 | | 0.63664 | 9.98872 | 0 | 50 2.5 | 1.7 |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | ′ | P. P. | |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | | P. P. | |
|---|---------|----|---------|-------|----------|---------|----|-------|--|
| 0 | 9.35209 |    | 9.36336 |    | 0.63664 | 9.98872 | 60 | | |
| 1 | 9.35263 | 54 | 9.36394 | 58 | 0.63606 | 9.98869 | 59 | | |
| 2 | 9.35318 | 55 | 9.36452 | 58 | 0.63548 | 9.98867 | 58 | **58** | **57** |
| 3 | 9.35373 | 55 | 9.36509 | 57 | 0.63491 | 9.98864 | 57 | 6 5.8 | 5.7 |
| 4 | 9.35427 | 54 | 9.36566 | 57 | 0.63434 | 9.98861 | 56 | 7 6.8 | 6.7 |
| 5 | 9.35481 | 54 | 9.36624 | 58 | 0.63376 | 9.98858 | 55 | 8 7.7 | 7.6 |
| 6 | 9.35536 | 55 | 9.36681 | 57 | 0.63319 | 9.98855 | 54 | 9 8.7 | 8.6 |
| 7 | 9.35590 | 54 | 9.36738 | 57 | 0.63262 | 9.98852 | 53 | 10 9.7 | 9.5 |
| 8 | 9.35644 | 54 | 9.36795 | 57 | 0.63205 | 9.98849 | 52 | 20 19.3 | 19.0 |
| 9 | 9.35698 | 54 | 9.36852 | 57 | 0.63148 | 9.98846 | 51 | 30 29.0 | 28.5 |
|   |         | 54 |         | 57 |          |         |    | 40 38.7 | 38.0 |
| 10 | 9.35752 | 54 | 9.36909 | 57 | 0.63091 | 9.98843 | 50 | 50 48.3 | 47.5 |
| 11 | 9.35806 | 54 | 9.36966 | 57 | 0.63034 | 9.98840 | 49 | | |
| 12 | 9.35860 | 54 | 9.37023 | 57 | 0.62977 | 9.98837 | 48 | **56** | **55** |
| 13 | 9.35914 | 54 | 9.37080 | 57 | 0.62920 | 9.98834 | 47 | 6 5.6 | 5.5 |
| 14 | 9.35968 | 54 | 9.37137 | 56 | 0.62863 | 9.98831 | 46 | 7 6.5 | 6.4 |
| 15 | 9.36022 | 53 | 9.37193 | 57 | 0.62807 | 9.98828 | 45 | 8 7.5 | 7.3 |
| 16 | 9.36075 | 54 | 9.37250 | 56 | 0.62750 | 9.98825 | 44 | 9 8.4 | 8.3 |
| 17 | 9.36129 | 53 | 9.37306 | 57 | 0.62694 | 9.98822 | 43 | 10 9.3 | 9.2 |
| 18 | 9.36182 | 54 | 9.37363 | 56 | 0.62637 | 9.98819 | 42 | 20 18.7 | 18.3 |
| 19 | 9.36236 | 53 | 9.37419 | 57 | 0.62581 | 9.98816 | 41 | 30 28.0 | 27.5 |
|   |         | 53 |         |    |          |         |    | 40 37.3 | 36.7 |
| 20 | 9.36289 | 53 | 9.37476 | 56 | 0.62524 | 9.98813 | 40 | 50 46.7 | 45.8 |
| 21 | 9.36342 | 53 | 9.37532 | 56 | 0.62468 | 9.98810 | 39 | | |
| 22 | 9.36395 | 54 | 9.37588 | 56 | 0.62412 | 9.98807 | 38 | **54** | |
| 23 | 9.36449 | 53 | 9.37644 | 56 | 0.62356 | 9.98804 | 37 | 6 5.4 | |
| 24 | 9.36502 | 53 | 9.37700 | 56 | 0.62300 | 9.98801 | 36 | 7 6.3 | |
| 25 | 9.36555 | 53 | 9.37756 | 56 | 0.62244 | 9.98798 | 35 | 8 7.2 | |
| 26 | 9.36608 | 52 | 9.37812 | 56 | 0.62188 | 9.98795 | 34 | 9 8.1 | |
| 27 | 9.36660 | 53 | 9.37868 | 56 | 0.62132 | 9.98792 | 33 | 10 9.0 | |
| 28 | 9.36713 | 53 | 9.37924 | 56 | 0.62076 | 9.98789 | 32 | 20 18.0 | |
| 29 | 9.36766 | 53 | 9.37980 | 55 | 0.62020 | 9.98786 | 31 | 30 27.0 | |
|   |         | 53 |         | 56 |          |         |    | 40 36.0 | |
| 30 | 9.36819 | 52 | 9.38035 | 56 | 0.61965 | 9.98783 | 30 | 50 45.0 | |
| 31 | 9.36871 | 53 | 9.38091 | 56 | 0.61909 | 9.98780 | 29 | | |
| 32 | 9.36924 | 52 | 9.38147 | 55 | 0.61853 | 9.98777 | 28 | **53** | **52** |
| 33 | 9.36976 | 52 | 9.38202 | 55 | 0.61798 | 9.98774 | 27 | 6 5.3 | 5.2 |
| 34 | 9.37028 | 53 | 9.38257 | 56 | 0.61743 | 9.98771 | 26 | 7 6.2 | 6.1 |
| 35 | 9.37081 | 52 | 9.38313 | 55 | 0.61687 | 9.98768 | 25 | 8 7.1 | 6.9 |
| 36 | 9.37133 | 52 | 9.38368 | 55 | 0.61632 | 9.98765 | 24 | 9 8.0 | 7.8 |
| 37 | 9.37185 | 52 | 9.38423 | 56 | 0.61577 | 9.98762 | 23 | 10 8.8 | 8.7 |
| 38 | 9.37237 | 52 | 9.38479 | 55 | 0.61521 | 9.98759 | 22 | 20 17.7 | 17.3 |
| 39 | 9.37289 | 52 | 9.38534 | 55 | 0.61466 | 9.98756 | 21 | 30 26.5 | 26.0 |
|   |         | 52 |         | 55 |          |         |    | 40 35.3 | 34.7 |
| 40 | 9.37341 | 52 | 9.38589 | 55 | 0.61411 | 9.98753 | 20 | 50 44.2 | 43.3 |
| 41 | 9.37393 | 52 | 9.38644 | 55 | 0.61356 | 9.98750 | 19 | | |
| 42 | 9.37445 | 52 | 9.38699 | 55 | 0.61301 | 9.98746 | 18 | **51** | **4** |
| 43 | 9.37497 | 52 | 9.38754 | 54 | 0.61246 | 9.98743 | 17 | 6 5.1 | 0.4 |
| 44 | 9.37549 | 51 | 9.38808 | 55 | 0.61192 | 9.98740 | 16 | 7 6.0 | 0.5 |
| 45 | 9.37600 | 52 | 9.38863 | 55 | 0.61137 | 9.98737 | 15 | 8 6.8 | 0.5 |
| 46 | 9.37652 | 51 | 9.38918 | 54 | 0.61082 | 9.98734 | 14 | 9 7.7 | 0.6 |
| 47 | 9.37703 | 52 | 9.38972 | 55 | 0.61028 | 9.98731 | 13 | 10 8.5 | 0.7 |
| 48 | 9.37755 | 51 | 9.39027 | 55 | 0.60973 | 9.98728 | 12 | 20 17.0 | 1.3 |
| 49 | 9.37806 | 52 | 9.39082 | 54 | 0.60918 | 9.98725 | 11 | 30 25.5 | 2.0 |
|   |         | 51 |         | 54 |          |         |    | 40 34.0 | 2.7 |
| 50 | 9.37858 | 51 | 9.39136 | 54 | 0.60864 | 9.98722 | 10 | 50 42.5 | 3.3 |
| 51 | 9.37909 | 51 | 9.39190 | 55 | 0.60810 | 9.98719 | 9 | | |
| 52 | 9.37960 | 51 | 9.39245 | 54 | 0.60755 | 9.98715 | 8 | **3** | **2** |
| 53 | 9.38011 | 51 | 9.39299 | 54 | 0.60701 | 9.98712 | 7 | 6 0.3 | 0.2 |
| 54 | 9.38062 | 51 | 9.39353 | 54 | 0.60647 | 9.98709 | 6 | 7 0.4 | 0.2 |
| 55 | 9.38113 | 51 | 9.39407 | 54 | 0.60593 | 9.98706 | 5 | 8 0.4 | 0.3 |
| 56 | 9.38164 | 51 | 9.39461 | 54 | 0.60539 | 9.98703 | 4 | 9 0.5 | 0.3 |
| 57 | 9.38215 | 51 | 9.39515 | 54 | 0.60485 | 9.98700 | 3 | 10 0.5 | 0.3 |
| 58 | 9.38266 | 51 | 9.39569 | 54 | 0.60431 | 9.98697 | 2 | 20 1.0 | 0.7 |
| 59 | 9.38317 | 51 | 9.39623 | 54 | 0.60377 | 9.98694 | 1 | 30 1.5 | 1.0 |
|   |         | 51 |         | 54 |          |         |    | 40 2.0 | 1.3 |
| 60 | 9.38368 |    | 9.39677 |    | 0.60323 | 9.98690 | 0 | 50 2.5 | 1.7 |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | ′ | P. P. | |

## 14°

| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | ′ | P. P. | |
|---|---------|----|---------|-------|----------|---------|----|---|-------|--|
| 0 | 9.38368 | | 9.39677 | | 0.60323 | 9.98663 | | 60 | | |
| 1 | 9.38418 | 50 | 9.39731 | 54 | 0.60269 | 9.98687 | 3 | 59 | | |
| 2 | 9.38469 | 51 | 9.39785 | 54 | 0.60215 | 9.98684 | 3 | 58 | **54** \| **53** | |
| 3 | 9.38519 | 50 | 9.39838 | 53 | 0.60162 | 9.98681 | 3 | 57 | 6 \| 5.4 \| 5.3 | |
| 4 | 9.38570 | 51 | 9.39892 | 54 | 0.60108 | 9.98678 | 3 | 56 | 7 \| 6.3 \| 6.2 | |
| 5 | 9.38620 | 50 | 9.39945 | 53 | 0.60055 | 9.98675 | | 55 | 8 \| 7.2 \| 7.1 | |
| 6 | 9.38670 | 50 | 9.39999 | 54 | 0.60001 | 9.98671 | 4 | 54 | 9 \| 8.1 \| 8.0 | |
| 7 | 9.38721 | 51 | 9.40052 | 53 | 0.59948 | 9.98668 | 3 | 53 | 10 \| 9.0 \| 8.8 | |
| 8 | 9.38771 | 50 | 9.40106 | 54 | 0.59894 | 9.98665 | 3 | 52 | 20 \| 18.0 \| 17.7 | |
| 9 | 9.38821 | 50 | 9.40159 | 53 | 0.59841 | 9.98662 | 3 | 51 | 30 \| 27.0 \| 26.5 | |
| 10 | 9.38871 | 50 | 9.40212 | 53 | 0.59788 | 9.98659 | 3 | 50 | 40 \| 36.0 \| 35.3 | |
| 11 | 9.38921 | 50 | 9.40266 | 54 | 0.59734 | 9.98656 | 3 | 49 | 50 \| 45.0 \| 44.2 | |
| 12 | 9.38971 | 50 | 9.40319 | 53 | 0.59681 | 9.98652 | 4 | 48 | | |
| 13 | 9.39021 | 50 | 9.40372 | 53 | 0.59628 | 9.98649 | 3 | 47 | | |
| 14 | 9.39071 | 50 | 9.40425 | 53 | 0.59575 | 9.98646 | 3 | 46 | **52** \| **51** | |
| 15 | 9.39121 | 49 | 9.40478 | 53 | 0.59522 | 9.98643 | 3 | 45 | 6 \| 5.2 \| 5.1 | |
| 16 | 9.39170 | 50 | 9.40531 | 53 | 0.59469 | 9.98640 | 4 | 44 | 7 \| 6.1 \| 6.0 | |
| 17 | 9.39220 | 50 | 9.40584 | 52 | 0.59416 | 9.98636 | 3 | 43 | 8 \| 6.9 \| 6.8 | |
| 18 | 9.39270 | 49 | 9.40636 | 53 | 0.59364 | 9.98633 | 3 | 42 | 9 \| 7.8 \| 7.7 | |
| 19 | 9.39319 | 50 | 9.40689 | 53 | 0.59311 | 9.98630 | 3 | 41 | 10 \| 8.7 \| 8.5 | |
| 20 | 9.39369 | 49 | 9.40742 | 53 | 0.59258 | 9.98627 | 4 | 40 | 20 \| 17.3 \| 17.0 | |
| 21 | 9.39418 | 49 | 9.40795 | 52 | 0.59205 | 9.98623 | 3 | 39 | 30 \| 26.0 \| 25.5 | |
| 22 | 9.39467 | 49 | 9.40847 | 53 | 0.59153 | 9.98620 | 3 | 38 | 40 \| 34.7 \| 34.0 | |
| 23 | 9.39517 | 50 | 9.40900 | 52 | 0.59100 | 9.98617 | 3 | 37 | 50 \| 43.3 \| 42.5 | |
| 24 | 9.39566 | 49 | 9.40952 | 53 | 0.59048 | 9.98614 | 4 | 36 | | |
| 25 | 9.39615 | 49 | 9.41005 | 52 | 0.58995 | 9.98610 | | 35 | | |
| 26 | 9.39664 | 49 | 9.41057 | 52 | 0.58943 | 9.98607 | 3 | 34 | **50** \| **49** | |
| 27 | 9.39713 | 49 | 9.41109 | 52 | 0.58891 | 9.98604 | 3 | 33 | 6 \| 5.0 \| 4.9 | |
| 28 | 9.39762 | 49 | 9.41161 | 53 | 0.58839 | 9.98601 | 4 | 32 | 7 \| 5.8 \| 5.7 | |
| 29 | 9.39811 | 49 | 9.41214 | 52 | 0.58786 | 9.98597 | 3 | 31 | 8 \| 6.7 \| 6.5 | |
| 30 | 9.39860 | 49 | 9.41266 | 52 | 0.58734 | 9.98594 | 3 | 30 | 9 \| 7.5 \| 7.4 | |
| 31 | 9.39909 | 49 | 9.41318 | 52 | 0.58682 | 9.98591 | 3 | 29 | 10 \| 8.3 \| 8.2 | |
| 32 | 9.39958 | 48 | 9.41370 | 52 | 0.58630 | 9.98588 | 4 | 28 | 20 \| 16.7 \| 16.3 | |
| 33 | 9.40006 | 49 | 9.41422 | 52 | 0.58578 | 9.98584 | 3 | 27 | 30 \| 25.0 \| 24.5 | |
| 34 | 9.40055 | 48 | 9.41474 | 52 | 0.58526 | 9.98581 | 3 | 26 | 40 \| 33.3 \| 32.7 | |
| 35 | 9.40103 | 49 | 9.41526 | 52 | 0.58474 | 9.98578 | 4 | 25 | 50 \| 41.7 \| 40.8 | |
| 36 | 9.40152 | 48 | 9.41578 | 51 | 0.58422 | 9.98574 | 3 | 24 | | |
| 37 | 9.40200 | 49 | 9.41629 | 52 | 0.58371 | 9.98571 | 3 | 23 | | |
| 38 | 9.40249 | 48 | 9.41681 | 52 | 0.58319 | 9.98568 | 3 | 22 | **48** \| **47** | |
| 39 | 9.40297 | 49 | 9.41733 | 51 | 0.58267 | 9.98565 | 4 | 21 | 6 \| 4.8 \| 4.7 | |
| 40 | 9.40346 | 48 | 9.41784 | 52 | 0.58216 | 9.98561 | 3 | 20 | 7 \| 5.6 \| 5.5 | |
| 41 | 9.40394 | 48 | 9.41836 | 51 | 0.58164 | 9.98558 | 3 | 19 | 8 \| 6.4 \| 6.3 | |
| 42 | 9.40442 | 48 | 9.41887 | 52 | 0.58113 | 9.98555 | 4 | 18 | 9 \| 7.2 \| 7.1 | |
| 43 | 9.40490 | 48 | 9.41939 | 51 | 0.58061 | 9.98551 | 3 | 17 | 10 \| 8.0 \| 7.8 | |
| 44 | 9.40538 | 48 | 9.41990 | 51 | 0.58010 | 9.98548 | 3 | 16 | 20 \| 16.0 \| 15.7 | |
| 45 | 9.40586 | 48 | 9.42041 | 52 | 0.57959 | 9.98545 | 4 | 15 | 30 \| 24.0 \| 23.5 | |
| 46 | 9.40634 | 48 | 9.42093 | 51 | 0.57907 | 9.98541 | 3 | 14 | 40 \| 32.0 \| 31.3 | |
| 47 | 9.40682 | 48 | 9.42144 | 51 | 0.57856 | 9.98538 | 3 | 13 | 50 \| 40.0 \| 39.2 | |
| 48 | 9.40730 | 48 | 9.42195 | 51 | 0.57805 | 9.98535 | 3 | 12 | | |
| 49 | 9.40778 | 47 | 9.42246 | 51 | 0.57754 | 9.98531 | 4 | 11 | | |
| 50 | 9.40825 | 48 | 9.42297 | 51 | 0.57703 | 9.98528 | 3 | 10 | **4** \| **3** | |
| 51 | 9.40873 | 48 | 9.42348 | 51 | 0.57652 | 9.98525 | 3 | 9 | 6 \| 0.4 \| 0.3 | |
| 52 | 9.40921 | 47 | 9.42399 | 51 | 0.57601 | 9.98521 | 4 | 8 | 7 \| 0.5 \| 0.4 | |
| 53 | 9.40968 | 48 | 9.42450 | 51 | 0.57550 | 9.98518 | 3 | 7 | 8 \| 0.5 \| 0.4 | |
| 54 | 9.41016 | 47 | 9.42501 | 51 | 0.57499 | 9.98515 | 3 | 6 | 9 \| 0.6 \| 0.5 | |
| 55 | 9.41063 | 48 | 9.42552 | 51 | 0.57448 | 9.98511 | 4 | 5 | 10 \| 0.7 \| 0.5 | |
| 56 | 9.41111 | 47 | 9.42603 | 51 | 0.57397 | 9.98508 | 3 | 4 | 20 \| 1.3 \| 1.0 | |
| 57 | 9.41158 | 47 | 9.42653 | 50 | 0.57347 | 9.98505 | 3 | 3 | 30 \| 2.0 \| 1.5 | |
| 58 | 9.41205 | 47 | 9.42704 | 51 | 0.57296 | 9.98501 | 4 | 2 | 40 \| 2.7 \| 2.0 | |
| 59 | 9.41252 | 48 | 9.42755 | 51 | 0.57245 | 9.98498 | 3 | 1 | 50 \| 3.3 \| 2.5 | |
| 60 | 9.41300 | | 9.42805 | 50 | 0.57195 | 9.98494 | 4 | 0 | | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. | |

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## 15°

| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | ′ |
|---|---------|----|---------|-------|----------|---------|----|---|---|
| 0 | 9.41300 | 47 | 9.42805 | 51 | 0.57195 | 9.98494 |   | | 60 |
| 1 | 9.41347 | 47 | 9.42856 | 51 | 0.57144 | 9.98491 | 3 | | 59 |
| 2 | 9.41394 | 47 | 9.42906 | 50 | 0.57094 | 9.98488 | 3 | | 58 |
| 3 | 9.41441 | 47 | 9.42957 | 51 | 0.57043 | 9.98484 | 4 | | 57 |
| 4 | 9.41488 | 47 | 9.43007 | 50 | 0.56993 | 9.98481 | 3 | | 56 |
| 5 | 9.41535 | 47 | 9.43057 | 50 | 0.56943 | 9.98477 | 4 | | 55 |
| 6 | 9.41582 | 47 | 9.43108 | 51 | 0.56892 | 9.98474 | 3 | | 54 |
| 7 | 9.41628 | 46 | 9.43158 | 50 | 0.56842 | 9.98471 | 3 | | 53 |
| 8 | 9.41675 | 47 | 9.43208 | 50 | 0.56792 | 9.98467 | 4 | | 52 |
| 9 | 9.41722 | 47 | 9.43258 | 50 | 0.56742 | 9.98464 | 3 | | 51 |
| 10 | 9.41768 | 46 | 9.43308 | 50 | 0.56692 | 9.98460 | 4 | | 50 |
| 11 | 9.41815 | 47 | 9.43358 | 50 | 0.56642 | 9.98457 | 3 | | 49 |
| 12 | 9.41861 | 46 | 9.43408 | 50 | 0.56592 | 9.98453 | 4 | | 48 |
| 13 | 9.41908 | 47 | 9.43458 | 50 | 0.56542 | 9.98450 | 3 | | 47 |
| 14 | 9.41954 | 46 | 9.43508 | 50 | 0.56492 | 9.98447 | 3 | | 46 |
| 15 | 9.42001 | 47 | 9.43558 | 50 | 0.56442 | 9.98443 | 4 | | 45 |
| 16 | 9.42047 | 46 | 9.43607 | 49 | 0.56393 | 9.98440 | 3 | | 44 |
| 17 | 9.42093 | 46 | 9.43657 | 50 | 0.56343 | 9.98436 | 4 | | 43 |
| 18 | 9.42140 | 47 | 9.43707 | 50 | 0.56293 | 9.98433 | 3 | | 42 |
| 19 | 9.42186 | 46 | 9.43756 | 49 | 0.56244 | 9.98429 | 3 | | 41 |
| 20 | 9.42232 | 46 | 9.43806 | 50 | 0.56194 | 9.98426 | 3 | | 40 |
| 21 | 9.42278 | 46 | 9.43855 | 49 | 0.56145 | 9.98422 | 4 | | 39 |
| 22 | 9.42324 | 46 | 9.43905 | 50 | 0.56095 | 9.98419 | 3 | | 38 |
| 23 | 9.42370 | 46 | 9.43954 | 49 | 0.56046 | 9.98415 | 4 | | 37 |
| 24 | 9.42416 | 46 | 9.44004 | 50 | 0.55996 | 9.98412 | 3 | | 36 |
| 25 | 9.42461 | 45 | 9.44053 | 49 | 0.55947 | 9.98409 | 3 | | 35 |
| 26 | 9.42507 | 46 | 9.44102 | 49 | 0.55898 | 9.98405 | 4 | | 34 |
| 27 | 9.42553 | 46 | 9.44151 | 49 | 0.55849 | 9.98402 | 3 | | 33 |
| 28 | 9.42599 | 46 | 9.44201 | 50 | 0.55799 | 9.98398 | 4 | | 32 |
| 29 | 9.42644 | 45 | 9.44250 | 49 | 0.55750 | 9.98395 | 3 | | 31 |
| 30 | 9.42690 | 46 | 9.44299 | 49 | 0.55701 | 9.98391 | 4 | | 30 |
| 31 | 9.42735 | 45 | 9.44348 | 49 | 0.55652 | 9.98388 | 3 | | 29 |
| 32 | 9.42781 | 46 | 9.44397 | 49 | 0.55603 | 9.98384 | 3 | | 28 |
| 33 | 9.42826 | 45 | 9.44446 | 49 | 0.55554 | 9.98381 | 3 | | 27 |
| 34 | 9.42872 | 46 | 9.44495 | 49 | 0.55505 | 9.98377 | 4 | | 26 |
| 35 | 9.42917 | 45 | 9.44544 | 49 | 0.55456 | 9.98373 | 3 | | 25 |
| 36 | 9.42962 | 45 | 9.44592 | 48 | 0.55408 | 9.98370 | 3 | | 24 |
| 37 | 9.43008 | 46 | 9.44641 | 49 | 0.55359 | 9.98366 | 4 | | 23 |
| 38 | 9.43053 | 45 | 9.44690 | 49 | 0.55310 | 9.98363 | 3 | | 22 |
| 39 | 9.43098 | 45 | 9.44738 | 48 | 0.55262 | 9.98359 | 4 | | 21 |
| 40 | 9.43143 | 45 | 9.44787 | 49 | 0.55213 | 9.98356 | 3 | | 20 |
| 41 | 9.43188 | 45 | 9.44836 | 49 | 0.55164 | 9.98352 | 4 | | 19 |
| 42 | 9.43233 | 45 | 9.44884 | 48 | 0.55116 | 9.98349 | 3 | | 18 |
| 43 | 9.43278 | 45 | 9.44933 | 49 | 0.55067 | 9.98345 | 4 | | 17 |
| 44 | 9.43323 | 44 | 9.44981 | 48 | 0.55019 | 9.98342 | 3 | | 16 |
| 45 | 9.43367 | 45 | 9.45029 | 49 | 0.54971 | 9.98338 | 4 | | 15 |
| 46 | 9.43412 | 45 | 9.45078 | 48 | 0.54922 | 9.98334 | 3 | | 14 |
| 47 | 9.43457 | 45 | 9.45126 | 48 | 0.54874 | 9.98331 | 4 | | 13 |
| 48 | 9.43502 | 44 | 9.45174 | 48 | 0.54826 | 9.98327 | 3 | | 12 |
| 49 | 9.43546 | 45 | 9.45222 | 49 | 0.54778 | 9.98324 | 4 | | 11 |
| 50 | 9.43591 | 44 | 9.45271 | 48 | 0.54729 | 9.98320 | 3 | | 10 |
| 51 | 9.43635 | 45 | 9.45319 | 48 | 0.54681 | 9.98317 | 4 | | 9 |
| 52 | 9.43680 | 44 | 9.45367 | 48 | 0.54633 | 9.98313 | 4 | | 8 |
| 53 | 9.43724 | 45 | 9.45415 | 48 | 0.54585 | 9.98309 | 3 | | 7 |
| 54 | 9.43769 | 44 | 9.45463 | 48 | 0.54537 | 9.98305 | 4 | | 6 |
| 55 | 9.43813 | 44 | 9.45511 | 48 | 0.54489 | 9.98302 | 3 | | 5 |
| 56 | 9.43857 | 44 | 9.45559 | 47 | 0.54441 | 9.98299 | 3 | | 4 |
| 57 | 9.43901 | 45 | 9.45606 | 48 | 0.54394 | 9.98295 | 4 | | 3 |
| 58 | 9.43946 | 44 | 9.45654 | 48 | 0.54346 | 9.98291 | 4 | | 2 |
| 59 | 9.43990 | 44 | 9.45702 | 48 | 0.54298 | 9.98288 | 4 | | 1 |
| 60 | 9.44034 | | 9.45750 | | 0.54250 | 9.98284 | | | 0 |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | |

**P. P.**

| | 51 | 50 |
|---|---|---|
| 6 | 5.1 | 5.0 |
| 7 | 6.0 | 5.8 |
| 8 | 6.8 | 6.7 |
| 9 | 7.7 | 7.5 |
| 10 | 8.5 | 8.3 |
| 20 | 17.0 | 16.7 |
| 30 | 25.5 | 25.0 |
| 40 | 34.0 | 33.3 |
| 50 | 42.5 | 41.7 |

| | 49 | 48 |
|---|---|---|
| 6 | 4.9 | 4.8 |
| 7 | 5.7 | 5.6 |
| 8 | 6.5 | 6.4 |
| 9 | 7.4 | 7.2 |
| 10 | 8.2 | 8.0 |
| 20 | 16.3 | 16.0 |
| 30 | 24.5 | 24.0 |
| 40 | 32.7 | 32.0 |
| 50 | 40.8 | 40.0 |

| | 47 | 46 |
|---|---|---|
| 6 | 4.7 | 4.6 |
| 7 | 5.5 | 5.4 |
| 8 | 6.3 | 6.1 |
| 9 | 7.1 | 6.9 |
| 10 | 7.8 | 7.7 |
| 20 | 15.7 | 15.3 |
| 30 | 23.5 | 23.0 |
| 40 | 31.3 | 30.7 |
| 50 | 39.2 | 38.3 |

| | 45 | 44 |
|---|---|---|
| 6 | 4.5 | 4.4 |
| 7 | 5.3 | 5.1 |
| 8 | 6.0 | 5.9 |
| 9 | 6.8 | 6.6 |
| 10 | 7.5 | 7.3 |
| 20 | 15.0 | 14.7 |
| 30 | 22.5 | 22.0 |
| 50 | 37.5 | 36.7 |

| | 4 | 3 |
|---|---|---|
| 6 | 0.4 | 0.3 |
| 7 | 0.5 | 0.4 |
| 8 | 0.5 | 0.4 |
| 9 | 0.6 | 0.5 |
| 10 | 0.7 | 0.5 |
| 20 | 1.3 | 1.0 |
| 30 | 2.0 | 1.5 |
| 40 | 2.7 | 2.0 |
| 50 | 3.3 | 2.5 |

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| ' | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | | P. P. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 9.44034 | 44 | 9.45750 | 47 | 0.54250 | 9.98284 | 3 | 60 | | |
| 1 | 9.44078 | 44 | 9.45797 | 47 | 0.54203 | 9.98281 | 3 | 59 | | |
| 2 | 9.44122 | 44 | 9.45845 | 48 | 0.54155 | 9.98277 | 4 | 58 | | 48  47 |
| 3 | 9.44166 | 44 | 9.45892 | 48 | 0.54108 | 9.98273 | 4 | 57 | 6 | 4.8  4.7 |
| 4 | 9.44210 | 44 | 9.45940 | 47 | 0.54060 | 9.98270 | 3 | 56 | 7 | 5.6  5.5 |
| 5 | 9.44253 | 43 | 9.45987 | 48 | 0.54013 | 9.98266 | 4 | 55 | 8 | 6.4  6.3 |
| 6 | 9.44297 | 44 | 9.46035 | 47 | 0.53965 | 9.98262 | 4 | 54 | 9 | 7.2  7.1 |
| 7 | 9.44341 | 44 | 9.46082 | 48 | 0.53918 | 9.98259 | 3 | 53 | 10 | 8.0  7.8 |
| 8 | 9.44385 | 44 | 9.46130 | 47 | 0.53870 | 9.98255 | 4 | 52 | 20 | 16.0  15.7 |
| 9 | 9.44428 | 43 | 9.46177 | 47 | 0.53823 | 9.98251 | 4 | 51 | 30 | 24.0  23.5 |
| 10 | 9.44472 | 44 | 9.46224 | 47 | 0.53776 | 9.98248 | 3 | 50 | 40 | 32.0  31.3 |
| 11 | 9.44516 | 43 | 9.46271 | 47 | 0.53729 | 9.98244 | 4 | 49 | 50 | 40.0  39.2 |
| 12 | 9.44559 | 43 | 9.46319 | 48 | 0.53681 | 9.98240 | 4 | 48 | | |
| 13 | 9.44602 | 44 | 9.46366 | 47 | 0.53634 | 9.98237 | 3 | 47 | | |
| 14 | 9.44646 | 43 | 9.46413 | 47 | 0.53587 | 9.98233 | 4 | 46 | | 46  45 |
| 15 | 9.44689 | 44 | 9.46460 | 47 | 0.53540 | 9.98229 | 3 | 45 | 6 | 4.6  4.5 |
| 16 | 9.44733 | 43 | 9.46507 | 47 | 0.53493 | 9.98226 | 4 | 44 | 7 | 5.4  5.3 |
| 17 | 9.44776 | 43 | 9.46554 | 47 | 0.53446 | 9.98222 | 4 | 43 | 8 | 6.1  6.0 |
| 18 | 9.44819 | 43 | 9.46601 | 47 | 0.53399 | 9.98218 | 3 | 42 | 9 | 6.9  6.8 |
| 19 | 9.44862 | 43 | 9.46648 | 47 | 0.53352 | 9.98215 | 4 | 41 | 10 | 7.7  7.5 |
| 20 | 9.44905 | 43 | 9.46694 | 46 | 0.53306 | 9.98211 | 4 | 40 | 20 | 15.3  15.0 |
| 21 | 9.44948 | 44 | 9.46741 | 47 | 0.53259 | 9.98207 | 3 | 39 | 30 | 23.0  22.5 |
| 22 | 9.44992 | 43 | 9.46788 | 47 | 0.53212 | 9.98204 | 4 | 38 | 40 | 30.7  30.0 |
| 23 | 9.45035 | 42 | 9.46835 | 46 | 0.53165 | 9.98200 | 4 | 37 | 50 | 38.3  37.5 |
| 24 | 9.45077 | 43 | 9.46881 | 47 | 0.53119 | 9.98196 | 4 | 36 | | |
| 25 | 9.45120 | 43 | 9.46928 | 47 | 0.53072 | 9.98192 | 3 | 35 | | |
| 26 | 9.45163 | 43 | 9.46975 | 46 | 0.53025 | 9.98189 | 4 | 34 | | 44  43 |
| 27 | 9.45206 | 43 | 9.47021 | 47 | 0.52979 | 9.98185 | 4 | 33 | 6 | 4.4  4.3 |
| 28 | 9.45249 | 43 | 9.47068 | 46 | 0.52932 | 9.98181 | 4 | 32 | 7 | 5.1  5.0 |
| 29 | 9.45292 | 42 | 9.47114 | 46 | 0.52886 | 9.98177 | 3 | 31 | 8 | 5.9  5.8 |
| 30 | 9.45334 | 43 | 9.47160 | 47 | 0.52840 | 9.98174 | 4 | 30 | 9 | 6.6  6.5 |
| 31 | 9.45377 | 42 | 9.47207 | 46 | 0.52793 | 9.98170 | 4 | 29 | 10 | 7.3  7.2 |
| 32 | 9.45419 | 43 | 9.47253 | 46 | 0.52747 | 9.98166 | 4 | 28 | 20 | 14.7  14.3 |
| 33 | 9.45462 | 42 | 9.47299 | 47 | 0.52701 | 9.98162 | 4 | 27 | 30 | 22.0  21.5 |
| 34 | 9.45504 | 43 | 9.47346 | 46 | 0.52654 | 9.98159 | 4 | 26 | 40 | 29.3  28.7 |
| 35 | 9.45547 | 42 | 9.47392 | 46 | 0.52608 | 9.98155 | 4 | 25 | 50 | 36.7  35.8 |
| 36 | 9.45589 | 43 | 9.47438 | 46 | 0.52562 | 9.98151 | 4 | 24 | | |
| 37 | 9.45632 | 42 | 9.47484 | 46 | 0.52516 | 9.98147 | 4 | 23 | | |
| 38 | 9.45674 | 42 | 9.47530 | 46 | 0.52470 | 9.98144 | 3 | 22 | | 42  41 |
| 39 | 9.45716 | 42 | 9.47576 | 46 | 0.52424 | 9.98140 | 4 | 21 | 6 | 4.2  4.1 |
| 40 | 9.45758 | 43 | 9.47622 | 46 | 0.52378 | 9.98136 | 4 | 20 | 7 | 4.9  4.8 |
| 41 | 9.45801 | 42 | 9.47668 | 46 | 0.52332 | 9.98132 | 4 | 19 | 8 | 5.6  5.5 |
| 42 | 9.45843 | 42 | 9.47714 | 46 | 0.52286 | 9.98129 | 4 | 18 | 9 | 6.3  6.2 |
| 43 | 9.45885 | 42 | 9.47760 | 46 | 0.52240 | 9.98125 | 4 | 17 | 10 | 7.0  6.8 |
| 44 | 9.45927 | 42 | 9.47806 | 46 | 0.52194 | 9.98121 | 4 | 16 | 20 | 14.0  13.7 |
| 45 | 9.45969 | 42 | 9.47852 | 45 | 0.52148 | 9.98117 | 4 | 15 | 30 | 21.0  20.5 |
| 46 | 9.46011 | 42 | 9.47897 | 46 | 0.52103 | 9.98113 | 3 | 14 | 40 | 28.0  27.3 |
| 47 | 9.46053 | 42 | 9.47943 | 46 | 0.52057 | 9.98110 | 4 | 13 | 50 | 35.0  34.2 |
| 48 | 9.46095 | 42 | 9.47989 | 46 | 0.52011 | 9.98106 | 4 | 12 | | |
| 49 | 9.46136 | 41 | 9.48035 | 45 | 0.51965 | 9.98102 | 4 | 11 | | |
| 50 | 9.46178 | 42 | 9.48080 | 46 | 0.51920 | 9.98098 | 4 | 10 | | 4  3 |
| 51 | 9.46220 | 42 | 9.48126 | 45 | 0.51874 | 9.98094 | 4 | 9 | 6 | 0.4  0.3 |
| 52 | 9.46262 | 41 | 9.48171 | 46 | 0.51829 | 9.98090 | 3 | 8 | 7 | 0.5  0.4 |
| 53 | 9.46303 | 42 | 9.48217 | 45 | 0.51783 | 9.98087 | 4 | 7 | 8 | 0.5  0.4 |
| 54 | 9.46345 | 41 | 9.48262 | 45 | 0.51738 | 9.98083 | 4 | 6 | 9 | 0.6  0.5 |
| 55 | 9.46386 | 42 | 9.48307 | 46 | 0.51693 | 9.98079 | 4 | 5 | 10 | 0.7  0.5 |
| 56 | 9.46428 | 41 | 9.48353 | 45 | 0.51647 | 9.98075 | 4 | 4 | 20 | 1.3  1.0 |
| 57 | 9.46469 | 42 | 9.48398 | 45 | 0.51602 | 9.98071 | 4 | 3 | 30 | 2.0  1.5 |
| 58 | 9.46511 | 41 | 9.48443 | 45 | 0.51557 | 9.98067 | 4 | 2 | 40 | 2.7  2.0 |
| 59 | 9.46552 | 42 | 9.48489 | 45 | 0.51511 | 9.98063 | 3 | 1 | 50 | 3.3  2.5 |
| 60 | 9.46594 | | 9.48534 | | 0.51466 | 9.98060 | | 0 | | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ' | | P. P. |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | P. P. |
|---|---------|-----|---------|-------|----------|---------|-----|---|-------|
| 0 | 9.46594 | 41 | 9.48534 | 45 | 0.51466 | 9.98060 | 4 | 60 | |
| 1 | 9.46635 | 41 | 9.48579 | 45 | 0.51421 | 9.98056 | 4 | 59 | |
| 2 | 9.46676 | 41 | 9.48624 | 45 | 0.51376 | 9.98052 | 4 | 58 | |
| 3 | 9.46717 | 41 | 9.48669 | 45 | 0.51331 | 9.98048 | 4 | 57 | **45 44** |
| 4 | 9.46758 | 42 | 9.48714 | 45 | 0.51286 | 9.98044 | 4 | 56 | 6  4.5  4.4 |
| 5 | 9.46800 | 41 | 9.48759 | 45 | 0.51241 | 9.98040 | 4 | 55 | 7  5.3  5.1 |
| 6 | 9.46841 | 41 | 9.48804 | 45 | 0.51196 | 9.98036 | 4 | 54 | 8  6.0  5.9 |
| 7 | 9.46882 | 41 | 9.48849 | 45 | 0.51151 | 9.98032 | 3 | 53 | 9  6.8  6.6 |
| 8 | 9.46923 | 41 | 9.48894 | 45 | 0.51106 | 9.98029 | 4 | 52 | 10  7.5  7.3 |
| 9 | 9.46964 | 41 | 9.48939 | 45 | 0.51061 | 9.98025 | 4 | 51 | 20  15.0  14.7 |
| 10 | 9.47005 | 40 | 9.48984 | 45 | 0.51016 | 9.98021 | 4 | 50 | 30  22.5  22.0 |
| 11 | 9.47045 | 41 | 9.49029 | 44 | 0.50971 | 9.98017 | 4 | 49 | 40  30.0  29.3 |
| 12 | 9.47086 | 41 | 9.49073 | 45 | 0.50927 | 9.98013 | 4 | 48 | 50  37.5  36.7 |
| 13 | 9.47127 | 41 | 9.49118 | 45 | 0.50882 | 9.98009 | 4 | 47 | |
| 14 | 9.47168 | 41 | 9.49163 | 44 | 0.50837 | 9.98005 | 4 | 46 | **43** |
| 15 | 9.47209 | 40 | 9.49207 | 45 | 0.50793 | 9.98001 | 4 | 45 | 6  4.3 |
| 16 | 9.47249 | 41 | 9.49252 | 44 | 0.50748 | 9.97997 | 4 | 44 | 7  5.0 |
| 17 | 9.47290 | 40 | 9.49296 | 45 | 0.50704 | 9.97993 | 4 | 43 | 8  5.7 |
| 18 | 9.47330 | 41 | 9.49341 | 44 | 0.50659 | 9.97989 | 3 | 42 | 9  6.5 |
| 19 | 9.47371 | 40 | 9.49385 | 45 | 0.50615 | 9.97986 | 4 | 41 | 10  7.2 |
| 20 | 9.47411 | 41 | 9.49430 | 44 | 0.50570 | 9.97982 | 4 | 40 | 20  14.3 |
| 21 | 9.47452 | 40 | 9.49474 | 45 | 0.50526 | 9.97978 | 4 | 39 | 30  21.5 |
| 22 | 9.47492 | 41 | 9.49519 | 44 | 0.50481 | 9.97974 | 4 | 38 | 40  28.7 |
| 23 | 9.47533 | 40 | 9.49563 | 44 | 0.50437 | 9.97970 | 4 | 37 | 50  35.8 |
| 24 | 9.47573 | 40 | 9.49607 | 45 | 0.50393 | 9.97966 | 4 | 36 | |
| 25 | 9.47613 | 41 | 9.49652 | 44 | 0.50348 | 9.97962 | 4 | 35 | |
| 26 | 9.47654 | 40 | 9.49696 | 44 | 0.50304 | 9.97958 | 4 | 34 | **42 41** |
| 27 | 9.47694 | 40 | 9.49740 | 44 | 0.50260 | 9.97954 | 4 | 33 | 6  4.2  4.1 |
| 28 | 9.47734 | 40 | 9.49784 | 44 | 0.50216 | 9.97950 | 4 | 32 | 7  4.9  4.8 |
| 29 | 9.47774 | 40 | 9.49828 | 44 | 0.50172 | 9.97946 | 4 | 31 | 8  5.6  5.5 |
| 30 | 9.47814 | 40 | 9.49872 | 44 | 0.50128 | 9.97942 | 4 | 30 | 9  6.3  6.2 |
| 31 | 9.47854 | 40 | 9.49916 | 44 | 0.50084 | 9.97938 | 4 | 29 | 10  7.0  6.8 |
| 32 | 9.47894 | 40 | 9.19960 | 44 | 0.50040 | 9.97934 | 4 | 28 | 20  14.0  13.7 |
| 33 | 9.47934 | 40 | 9.50004 | 44 | 0.49996 | 9.97930 | 4 | 27 | 30  21.0  20.5 |
| 34 | 9.47974 | 40 | 9.50048 | 44 | 0.49952 | 9.97926 | 4 | 26 | 40  28.0  27.3 |
| 35 | 9.48014 | 40 | 9.50092 | 44 | 0.49908 | 9.97922 | 4 | 25 | 50  35.0  34.2 |
| 36 | 9.48054 | 40 | 9.50136 | 44 | 0.49864 | 9.97918 | 4 | 24 | |
| 37 | 9.48094 | 39 | 9.50180 | 43 | 0.49820 | 9.97914 | 4 | 23 | |
| 38 | 9.48133 | 40 | 9.50223 | 44 | 0.49777 | 9.97910 | 4 | 22 | **40 39** |
| 39 | 9.48173 | 40 | 9.50267 | 44 | 0.49733 | 9.97906 | 4 | 21 | 6  4.0  3.9 |
| 40 | 9.48213 | 39 | 9.50311 | 44 | 0.49689 | 9.97902 | 4 | 20 | 7  4.7  4.6 |
| 41 | 9.48252 | 40 | 9.50355 | 43 | 0.49645 | 9.97898 | 4 | 19 | 8  5.3  5.2 |
| 42 | 9.48292 | 40 | 9.50398 | 44 | 0.49602 | 9.97894 | 4 | 18 | 9  6.0  5.9 |
| 43 | 9.48332 | 39 | 9.50442 | 43 | 0.49558 | 9.97890 | 4 | 17 | 10  6.7  6.5 |
| 44 | 9.48371 | 40 | 9.50485 | 44 | 0.49515 | 9.97886 | 4 | 16 | 20  13.3  13.0 |
| 45 | 9.48411 | 39 | 9.50529 | 43 | 0.49471 | 9.97882 | 4 | 15 | 30  20.0  19.5 |
| 46 | 9.48450 | 40 | 9.50572 | 44 | 0.49428 | 9.97878 | 4 | 14 | 40  26.7  26.0 |
| 47 | 9.48490 | 39 | 9.50616 | 43 | 0.49384 | 9.97874 | 4 | 13 | 50  33.3  32.5 |
| 48 | 9.48529 | 39 | 9.50659 | 44 | 0.49341 | 9.97870 | 4 | 12 | |
| 49 | 9.48568 | 39 | 9.50703 | 43 | 0.49297 | 9.97866 | 5 | 11 | |
| 50 | 9.48607 | 40 | 9.50746 | 43 | 0.49254 | 9.97861 | 4 | 10 | **5 4 3** |
| 51 | 9.48647 | 39 | 9.50789 | 44 | 0.49211 | 9.97857 | 4 | 9 | 6  0.5  0.4  0.3 |
| 52 | 9.48686 | 39 | 9.50833 | 43 | 0.49167 | 9.97853 | 4 | 8 | 7  0.6  0.5  0.4 |
| 53 | 9.48725 | 39 | 9.50876 | 43 | 0.49124 | 9.97849 | 4 | 7 | 8  0.7  0.5  0.4 |
| 54 | 9.48764 | 39 | 9.50919 | 43 | 0.49081 | 9.97845 | 4 | 6 | 9  0.8  0.6  0.5 |
| 55 | 9.48803 | 39 | 9.50962 | 43 | 0.49038 | 9.97841 | 4 | 5 | 10  0.8  0.7  0.5 |
| 56 | 9.48842 | 39 | 9.51005 | 43 | 0.48995 | 9.97837 | 4 | 4 | 20  1.7  1.3  1.0 |
| 57 | 9.48881 | 39 | 9.51048 | 44 | 0.48952 | 9.97833 | 4 | 3 | 30  2.5  2.0  1.5 |
| 58 | 9.48920 | 39 | 9.51092 | 43 | 0.48908 | 9.97829 | 4 | 2 | 40  3.3  2.7  2.0 |
| 59 | 9.48959 | 39 | 9.51135 | 43 | 0.48865 | 9.97825 | 4 | 1 | 50  4.2  3.3  2.5 |
| 60 | 9.48998 | | 9.51178 | | 0.48822 | 9.97821 | | 0 | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. |

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| ' | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | ' | P. P. |
|---|---------|----|---------|-------|----------|---------|----|---|-------|
| 0 | 9.48998 | 39 | 9.51178 | 43 | 0.48822 | 9.97821 | 4 | 60 | |
| 1 | 9.49037 | 39 | 9.51221 | 43 | 0.48779 | 9.97817 | 5 | 59 | |
| 2 | 9.49076 | 39 | 9.51264 | 43 | 0.48736 | 9.97812 | 5 | 58 | **43   42** |
| 3 | 9.49115 | 38 | 9.51306 | 43 | 0.48694 | 9.97808 | 4 | 57 | 6   4.3   4.2 |
| 4 | 9.49153 | 39 | 9.51349 | 43 | 0.48651 | 9.97804 | 4 | 56 | 7   5.0   4.9 |
| 5 | 9.49192 | 39 | 9.51392 | 43 | 0.48608 | 9.97800 | 4 | 55 | 8   5.7   5.6 |
| 6 | 9.49231 | 38 | 9.51435 | 43 | 0.48565 | 9.97796 | 4 | 54 | 9   6.5   6.3 |
| 7 | 9.49269 | 39 | 9.51478 | 42 | 0.48522 | 9.97792 | 4 | 53 | 10   7.2   7.0 |
| 8 | 9.49308 | 39 | 9.51520 | 43 | 0.48480 | 9.97788 | 4 | 52 | 20   14.3   14.0 |
| 9 | 9.49347 | 38 | 9.51563 | 43 | 0.48437 | 9.97784 | 5 | 51 | 30   21.5   21.0 |
| 10 | 9.49385 | 39 | 9.51606 | 42 | 0.48394 | 9.97779 | 4 | 50 | 40   28.7   28.0 |
| 11 | 9.49424 | 38 | 9.51648 | 43 | 0.48352 | 9.97775 | 4 | 49 | 50   35.8   35.0 |
| 12 | 9.49462 | 38 | 9.51691 | 43 | 0.48309 | 9.97771 | 4 | 48 | |
| 13 | 9.49500 | 39 | 9.51734 | 42 | 0.48266 | 9.97767 | 4 | 47 | |
| 14 | 9.49539 | 38 | 9.51776 | 43 | 0.48224 | 9.97763 | 4 | 46 | **41** |
| 15 | 9.49577 | 38 | 9.51819 | 42 | 0.48181 | 9.97759 | 5 | 45 | 6   4.1 |
| 16 | 9.49615 | 39 | 9.51861 | 42 | 0.48139 | 9.97754 | 4 | 44 | 7   4.8 |
| 17 | 9.49654 | 38 | 9.51903 | 43 | 0.48097 | 9.97750 | 4 | 43 | 8   5.5 |
| 18 | 9.49692 | 38 | 9.51946 | 42 | 0.48054 | 9.97746 | 4 | 42 | 9   6.2 |
| 19 | 9.49730 | 38 | 9.51988 | 43 | 0.48012 | 9.97742 | 4 | 41 | 10   6.8 |
| 20 | 9.49768 | 38 | 9.52031 | 42 | 0.47969 | 9.97738 | 4 | 40 | 20   13.7 |
| 21 | 9.49806 | 38 | 9.52073 | 42 | 0.47927 | 9.97734 | 5 | 39 | 30   20.5 |
| 22 | 9.49844 | 38 | 9.52115 | 42 | 0.47885 | 9.97729 | 4 | 38 | 40   27.3 |
| 23 | 9.49882 | 38 | 9.52157 | 43 | 0.47843 | 9.97725 | 4 | 37 | 50   34.2 |
| 24 | 9.49920 | 38 | 9.52200 | 42 | 0.47800 | 9.97721 | 4 | 36 | |
| 25 | 9.49958 | 38 | 9.52242 | 42 | 0.47758 | 9.97717 | 4 | 35 | |
| 26 | 9.49996 | 38 | 9.52284 | 42 | 0.47716 | 9.97713 | 5 | 34 | **39   38** |
| 27 | 9.50034 | 38 | 9.52326 | 42 | 0.47674 | 9.97708 | 4 | 33 | 6   3.9   3.8 |
| 28 | 9.50072 | 38 | 9.52368 | 42 | 0.47632 | 9.97704 | 4 | 32 | 7   4.6   4.4 |
| 29 | 9.50110 | 38 | 9.52410 | 42 | 0.47590 | 9.97700 | 4 | 31 | 8   5.2   5.1 |
| 30 | 9.50148 | 37 | 9.52452 | 42 | 0.47548 | 9.97696 | 5 | 30 | 9   5.9   5.7 |
| 31 | 9.50185 | 38 | 9.52494 | 42 | 0.47506 | 9.97691 | 4 | 29 | 10   6.5   6.3 |
| 32 | 9.50223 | 38 | 9.52536 | 42 | 0.47464 | 9.97687 | 4 | 28 | 20   13.0   12.7 |
| 33 | 9.50261 | 37 | 9.52578 | 42 | 0.47422 | 9.97683 | 4 | 27 | 30   19.5   19.0 |
| 34 | 9.50298 | 38 | 9.52620 | 42 | 0.47380 | 9.97679 | 5 | 26 | 40   26.0   25.3 |
| 35 | 9.50336 | 38 | 9.52661 | 42 | 0.47339 | 9.97674 | 4 | 25 | 50   32.5   31.7 |
| 36 | 9.50374 | 37 | 9.52703 | 42 | 0.47297 | 9.97670 | 4 | 24 | |
| 37 | 9.50411 | 38 | 9.52745 | 42 | 0.47255 | 9.97666 | 4 | 23 | |
| 38 | 9.50449 | 37 | 9.52787 | 42 | 0.47213 | 9.97662 | 5 | 22 | **37   36** |
| 39 | 9.50486 | 37 | 9.52829 | 41 | 0.47171 | 9.97657 | 4 | 21 | 6   3.7   3.6 |
| 40 | 9.50523 | 38 | 9.52870 | 42 | 0.47130 | 9.97653 | 4 | 20 | 7   4.3   4.2 |
| 41 | 9.50561 | 37 | 9.52912 | 41 | 0.47088 | 9.97649 | 4 | 19 | 8   4.9   4.8 |
| 42 | 9.50598 | 37 | 9.52953 | 42 | 0.47047 | 9.97645 | 5 | 18 | 9   5.6   5.4 |
| 43 | 9.50635 | 38 | 9.52995 | 42 | 0.47005 | 9.97640 | 4 | 17 | 10   6.2   6.0 |
| 44 | 9.50673 | 37 | 9.53037 | 41 | 0.46963 | 9.97636 | 4 | 16 | 20   12.3   12.0 |
| 45 | 9.50710 | 37 | 9.53078 | 42 | 0.46922 | 9.97632 | 4 | 15 | 30   18.5   18.0 |
| 46 | 9.50747 | 37 | 9.53120 | 41 | 0.46880 | 9.97628 | 5 | 14 | 40   24.7   24.0 |
| 47 | 9.50784 | 37 | 9.53161 | 41 | 0.46839 | 9.97623 | 4 | 13 | 50   30.8   30.0 |
| 48 | 9.50821 | 37 | 9.53202 | 42 | 0.46798 | 9.97619 | 4 | 12 | |
| 49 | 9.50858 | 38 | 9.53244 | 41 | 0.46756 | 9.97615 | 5 | 11 | |
| 50 | 9.50896 | 37 | 9.53285 | 42 | 0.46715 | 9.97610 | 4 | 10 | **5   4** |
| 51 | 9.50933 | 37 | 9.53327 | 41 | 0.46673 | 9.97606 | 4 | 9 | 6   0.5   0.4 |
| 52 | 9.50970 | 37 | 9.53368 | 41 | 0.46632 | 9.97602 | 5 | 8 | 7   0.6   0.5 |
| 53 | 9.51007 | 37 | 9.53409 | 41 | 0.46591 | 9.97597 | 4 | 7 | 8   0.7   0.5 |
| 54 | 9.51043 | 36 | 9.53450 | 42 | 0.46550 | 9.97593 | 4 | 6 | 9   0.8   0.6 |
| 55 | 9.51080 | 37 | 9.53492 | 41 | 0.46508 | 9.97589 | 4 | 5 | 10   0.8   0.7 |
| 56 | 9.51117 | 37 | 9.53533 | 41 | 0.46467 | 9.97584 | 5 | 4 | 20   1.7   1.3 |
| 57 | 9.51154 | 37 | 9.53574 | 41 | 0.46426 | 9.97580 | 4 | 3 | 30   2.5   2.0 |
| 58 | 9.51191 | 37 | 9.53615 | 41 | 0.46385 | 9.97576 | 4 | 2 | 40   3.3   2.7 |
| 59 | 9.51227 | 36 | 9.53656 | 41 | 0.46344 | 9.97571 | 5 | 1 | 50   4.2   3.3 |
| 60 | 9.51264 | 37 | 9.53697 | 41 | 0.46303 | 9.97567 | 4 | 0 | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ' | P. P. |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | ′ | P. P. |
|---|---------|----|---------|-------|----------|---------|----|----|-------|
| 0 | 9.51264 | 37 | 9.53697 | 41 | 0.46303 | 9.97567 | 4 | 60 | |
| 1 | 9.51301 | 37 | 9.53738 | 41 | 0.46262 | 9.97563 | 5 | 59 | |
| 2 | 9.51338 | 36 | 9.53779 | 41 | 0.46221 | 9.97558 | 4 | 58 | **41**   **40** |
| 3 | 9.51374 | 37 | 9.53820 | 41 | 0.46180 | 9.97554 | 4 | 57 | 6   4.1   4.0 |
| 4 | 9.51411 | 36 | 9.53861 | 41 | 0.46139 | 9.97550 | 5 | 56 | 7   4.8   4.7 |
| 5 | 9.51447 | 37 | 9.53902 | 41 | 0.46098 | 9.97545 | 4 | 55 | 8   5.5   5.3 |
| 6 | 9.51484 | 37 | 9.53943 | 41 | 0.46057 | 9.97541 | 5 | 54 | 9   6.2   6.0 |
| 7 | 9.51520 | 37 | 9.53984 | 41 | 0.46016 | 9.97536 | 4 | 53 | 10   6.8   6.7 |
| 8 | 9.51557 | 36 | 9.54025 | 40 | 0.45975 | 9.97532 | 4 | 52 | 20   13.7   13.3 |
| 9 | 9.51593 | 36 | 9.54065 | 41 | 0.45935 | 9.97528 | 5 | 51 | 30   20.5   20.0 |
| 10 | 9.51629 | 37 | 9.54106 | 41 | 0.45894 | 9.97523 | 4 | 50 | 40   27.3   26.7 |
| 11 | 9.51666 | 36 | 9.54147 | 40 | 0.45853 | 9.97519 | 4 | 49 | 50   34.2   33.3 |
| 12 | 9.51702 | 36 | 9.54187 | 41 | 0.45813 | 9.97515 | 5 | 48 | |
| 13 | 9.51738 | 36 | 9.54228 | 41 | 0.45772 | 9.97510 | 4 | 47 | |
| 14 | 9.51774 | 37 | 9.54269 | 40 | 0.45731 | 9.97506 | 5 | 46 | **39** |
| 15 | 9.51811 | 36 | 9.54309 | 41 | 0.45691 | 9.97501 | 4 | 45 | 6   3.9 |
| 16 | 9.51847 | 36 | 9.54350 | 40 | 0.45650 | 9.97497 | 5 | 44 | 7   4.6 |
| 17 | 9.51883 | 36 | 9.54390 | 41 | 0.45610 | 9.97492 | 4 | 43 | 8   5.2 |
| 18 | 9.51919 | 36 | 9.54431 | 40 | 0.45569 | 9.97488 | 4 | 42 | 9   5.9 |
| 19 | 9.51955 | 36 | 9.54471 | 41 | 0.45529 | 9.97484 | 5 | 41 | 10   6.5 |
| 20 | 9.51991 | 36 | 9.54512 | 40 | 0.45488 | 9.97479 | 4 | 40 | 20   13.0 |
| 21 | 9.52027 | 36 | 9.54552 | 41 | 0.45448 | 9.97475 | 5 | 39 | 30   19.5 |
| 22 | 9.52063 | 36 | 9.54593 | 40 | 0.45407 | 9.97470 | 4 | 38 | 40   26.0 |
| 23 | 9.52099 | 36 | 9.54633 | 40 | 0.45367 | 9.97466 | 5 | 37 | 50   32.5 |
| 24 | 9.52135 | 36 | 9.54673 | 41 | 0.45327 | 9.97461 | 4 | 36 | |
| 25 | 9.52171 | 36 | 9.54714 | 40 | 0.45286 | 9.97457 | 4 | 35 | |
| 26 | 9.52207 | 35 | 9.54754 | 40 | 0.45246 | 9.97453 | 5 | 34 | **37**   **36** |
| 27 | 9.52242 | 36 | 9.54794 | 41 | 0.45206 | 9.97448 | 5 | 33 | 6   3.7   3.6 |
| 28 | 9.52278 | 36 | 9.54835 | 40 | 0.45165 | 9.97444 | 4 | 32 | 7   4.3   4.2 |
| 29 | 9.52314 | 36 | 9.54875 | 40 | 0.45125 | 9.97439 | 5 | 31 | 8   4.9   4.8 |
| 30 | 9.52350 | 35 | 9.54915 | 40 | 0.45085 | 9.97435 | 5 | 30 | 9   5.6   5.4 |
| 31 | 9.52385 | 36 | 9.54955 | 40 | 0.45045 | 9.97430 | 4 | 29 | 10   6.2   6.0 |
| 32 | 9.52421 | 35 | 9.54995 | 40 | 0.45005 | 9.97426 | 5 | 28 | 20   12.3   12.0 |
| 33 | 9.52456 | 36 | 9.55035 | 40 | 0.44965 | 9.97421 | 4 | 27 | 30   18.5   18.0 |
| 34 | 9.52492 | 35 | 9.55075 | 40 | 0.44925 | 9.97417 | 5 | 26 | 40   24.7   24.0 |
| 35 | 9.52527 | 36 | 9.55115 | 40 | 0.44885 | 9.97412 | 4 | 25 | 50   30.8   30.0 |
| 36 | 9.52563 | 35 | 9.55155 | 40 | 0.44845 | 9.97408 | 5 | 24 | |
| 37 | 9.52598 | 36 | 9.55195 | 40 | 0.44805 | 9.97403 | 4 | 23 | |
| 38 | 9.52634 | 35 | 9.55235 | 40 | 0.44765 | 9.97399 | 5 | 22 | **35**   **34** |
| 39 | 9.52669 | 36 | 9.55275 | 40 | 0.44725 | 9.97394 | 4 | 21 | 6   3.5   3.4 |
| 40 | 9.52705 | 35 | 9.55315 | 40 | 0.44685 | 9.97390 | 5 | 20 | 7   4.1   4.0 |
| 41 | 9.52740 | 35 | 9.55355 | 40 | 0.44645 | 9.97385 | 5 | 19 | 8   4.7   4.5 |
| 42 | 9.52775 | 36 | 9.55395 | 39 | 0.44605 | 9.97381 | 5 | 18 | 9   5.3   5.1 |
| 43 | 9.52811 | 35 | 9.55434 | 40 | 0.44566 | 9.97376 | 4 | 17 | 10   5.8   5.7 |
| 44 | 9.52846 | 35 | 9.55474 | 40 | 0.44526 | 9.97372 | 5 | 16 | 20   11.7   11.3 |
| 45 | 9.52881 | 35 | 9.55514 | 40 | 0.44486 | 9.97367 | 4 | 15 | 30   17.5   17.0 |
| 46 | 9.52916 | 35 | 9.55554 | 39 | 0.44446 | 9.97363 | 5 | 14 | 40   23.3   22.7 |
| 47 | 9.52951 | 35 | 9.55593 | 40 | 0.44407 | 9.97358 | 5 | 13 | 50   29.2   28.3 |
| 48 | 9.52986 | 35 | 9.55633 | 40 | 0.44367 | 9.97353 | 4 | 12 | |
| 49 | 9.53021 | 35 | 9.55673 | 39 | 0.44327 | 9.97349 | 5 | 11 | |
| 50 | 9.53056 | 36 | 9.55712 | 40 | 0.44288 | 9.97344 | 4 | 10 | **5**   **4** |
| 51 | 9.53092 | 34 | 9.55752 | 39 | 0.44248 | 9.97340 | 5 | 9 | 6   0.5   0.4 |
| 52 | 9.53126 | 35 | 9.55791 | 40 | 0.44209 | 9.97335 | 4 | 8 | 7   0.6   0.5 |
| 53 | 9.53161 | 35 | 9.55831 | 39 | 0.44169 | 9.97331 | 5 | 7 | 8   0.7   0.5 |
| 54 | 9.53196 | 35 | 9.55870 | 40 | 0.44130 | 9.97326 | 4 | 6 | 9   0.8   0.6 |
| 55 | 9.53231 | 35 | 9.55910 | 39 | 0.44090 | 9.97322 | 5 | 5 | 10   0.8   0.7 |
| 56 | 9.53266 | 35 | 9.55949 | 40 | 0.44051 | 9.97317 | 5 | 4 | 20   1.7   1.3 |
| 57 | 9.53301 | 35 | 9.55989 | 39 | 0.44011 | 9.97312 | 4 | 3 | 30   2.5   2.0 |
| 58 | 9.53336 | 34 | 9.56028 | 39 | 0.43972 | 9.97308 | 5 | 2 | 40   3.3   2.7 |
| 59 | 9.53370 | 35 | 9.56067 | 40 | 0.43933 | 9.97303 | 4 | 1 | 50   4.2   3.3 |
| 60 | 9.53405 | | 9.56107 | | 0.43893 | 9.97299 | | 0 | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. |

## 20°

| ' | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | ' | P. P. |
|---|---------|----|---------|-------|----------|---------|----|---|-------|
| 0 | 9.53405 | 35 | 9.56107 | 39 | 0.43893 | 9.97299 |   | 60 | |
| 1 | 9.53440 | 35 | 9.56146 | 39 | 0.43854 | 9.97294 | 5 | 59 | |
| 2 | 9.53475 | 35 | 9.56185 | 39 | 0.43815 | 9.97289 | 5 | 58 | |
| 3 | 9.53509 | 34 | 9.56224 | 39 | 0.43776 | 9.97285 | 4 | 57 | |
| 4 | 9.53544 | 34 | 9.56264 | 40 | 0.43736 | 9.97280 | 5 | 56 | |
| 5 | 9.53578 | 35 | 9.56303 | 39 | 0.43697 | 9.97276 | 4 | 55 | |
| 6 | 9.53613 | 34 | 9.56342 | 39 | 0.43658 | 9.97271 | 5 | 54 | |
| 7 | 9.53647 | 35 | 9.56381 | 39 | 0.43619 | 9.97266 | 5 | 53 | |
| 8 | 9.53682 | 34 | 9.56420 | 39 | 0.43580 | 9.97262 | 4 | 52 | |
| 9 | 9.53716 | 35 | 9.56459 | 39 | 0.43541 | 9.97257 | 5 | 51 | |
| 10 | 9.53751 | 34 | 9.56498 | 39 | 0.43502 | 9.97252 | 5 | 50 | |
| 11 | 9.53785 | 34 | 9.56537 | 39 | 0.43463 | 9.97248 | 4 | 49 | |
| 12 | 9.53819 | 34 | 9.56576 | 39 | 0.43424 | 9.97243 | 5 | 48 | |
| 13 | 9.53853 | 35 | 9.56615 | 39 | 0.43385 | 9.97238 | 5 | 47 | |
| 14 | 9.53888 | 34 | 9.56654 | 39 | 0.43346 | 9.97234 | 4 | 46 | |
| 15 | 9.53922 | 35 | 9.56693 | 39 | 0.43307 | 9.97229 | 5 | 45 | |
| 16 | 9.53957 | 34 | 9.56732 | 39 | 0.43268 | 9.97224 | 4 | 44 | |
| 17 | 9.53991 | 34 | 9.56771 | 39 | 0.43229 | 9.97220 | 5 | 43 | |
| 18 | 9.54025 | 34 | 9.56810 | 39 | 0.43190 | 9.97215 | 5 | 42 | |
| 19 | 9.54059 | 34 | 9.56849 | 38 | 0.43151 | 9.97210 | 4 | 41 | |
| 20 | 9.54093 | 34 | 9.56887 | 39 | 0.43113 | 9.97206 | 5 | 40 | |
| 21 | 9.54127 | 34 | 9.56926 | 39 | 0.43074 | 9.97201 | 5 | 39 | |
| 22 | 9.54161 | 34 | 9.56965 | 39 | 0.43035 | 9.97196 | 4 | 38 | |
| 23 | 9.54195 | 34 | 9.57004 | 38 | 0.42996 | 9.97192 | 5 | 37 | |
| 24 | 9.54229 | 34 | 9.57042 | 39 | 0.42958 | 9.97187 | 5 | 36 | |
| 25 | 9.54263 | 34 | 9.57081 | 39 | 0.42919 | 9.97182 | 4 | 35 | |
| 26 | 9.54297 | 34 | 9.57120 | 38 | 0.42880 | 9.97178 | 5 | 34 | |
| 27 | 9.54331 | 34 | 9.57158 | 39 | 0.42842 | 9.97173 | 5 | 33 | |
| 28 | 9.54365 | 34 | 9.57197 | 38 | 0.42803 | 9.97168 | 4 | 32 | |
| 29 | 9.54399 | 34 | 9.57235 | 39 | 0.42765 | 9.97163 | 5 | 31 | |
| 30 | 9.54433 | 33 | 9.57274 | 38 | 0.42726 | 9.97159 | 4 | 30 | |
| 31 | 9.54466 | 34 | 9.57312 | 39 | 0.42688 | 9.97154 | 5 | 29 | |
| 32 | 9.54500 | 34 | 9.57351 | 38 | 0.42649 | 9.97149 | 5 | 28 | |
| 33 | 9.54534 | 33 | 9.57389 | 39 | 0.42611 | 9.97145 | 4 | 27 | |
| 34 | 9.54567 | 34 | 9.57428 | 38 | 0.42572 | 9.97140 | 5 | 26 | |
| 35 | 9.54601 | 34 | 9.57466 | 38 | 0.42534 | 9.97135 | 5 | 25 | |
| 36 | 9.54635 | 33 | 9.57504 | 39 | 0.42496 | 9.97130 | 4 | 24 | |
| 37 | 9.54668 | 34 | 9.57543 | 38 | 0.42457 | 9.97126 | 5 | 23 | |
| 38 | 9.54702 | 33 | 9.57581 | 38 | 0.42419 | 9.97121 | 5 | 22 | |
| 39 | 9.54735 | 34 | 9.57619 | 39 | 0.42381 | 9.97116 | 5 | 21 | |
| 40 | 9.54769 | 33 | 9.57658 | 38 | 0.42342 | 9.97111 | 4 | 20 | |
| 41 | 9.54802 | 34 | 9.57696 | 38 | 0.42304 | 9.97107 | 5 | 19 | |
| 42 | 9.54836 | 33 | 9.57734 | 38 | 0.42266 | 9.97102 | 5 | 18 | |
| 43 | 9.54869 | 34 | 9.57772 | 38 | 0.42228 | 9.97097 | 5 | 17 | |
| 44 | 9.54903 | 33 | 9.57810 | 39 | 0.42190 | 9.97092 | 5 | 16 | |
| 45 | 9.54936 | 33 | 9.57849 | 38 | 0.42151 | 9.97087 | 4 | 15 | |
| 46 | 9.54969 | 34 | 9.57887 | 38 | 0.42113 | 9.97083 | 5 | 14 | |
| 47 | 9.55003 | 33 | 9.57925 | 38 | 0.42075 | 9.97078 | 5 | 13 | |
| 48 | 9.55036 | 33 | 9.57963 | 38 | 0.42037 | 9.97073 | 5 | 12 | |
| 49 | 9.55069 | 33 | 9.58001 | 38 | 0.41999 | 9.97068 | 5 | 11 | |
| 50 | 9.55102 | 34 | 9.58039 | 38 | 0.41961 | 9.97063 | 4 | 10 | |
| 51 | 9.55136 | 33 | 9.58077 | 38 | 0.41923 | 9.97059 | 5 | 9 | |
| 52 | 9.55169 | 33 | 9.58115 | 38 | 0.41885 | 9.97054 | 5 | 8 | |
| 53 | 9.55202 | 33 | 9.58153 | 38 | 0.41847 | 9.97049 | 5 | 7 | |
| 54 | 9.55235 | 33 | 9.58191 | 38 | 0.41809 | 9.97044 | 5 | 6 | |
| 55 | 9.55268 | 33 | 9.58229 | 38 | 0.41771 | 9.97039 | 4 | 5 | |
| 56 | 9.55301 | 33 | 9.58267 | 38 | 0.41733 | 9.97035 | 5 | 4 | |
| 57 | 9.55334 | 33 | 9.58304 | 37 | 0.41696 | 9.97030 | 5 | 3 | |
| 58 | 9.55367 | 33 | 9.58342 | 38 | 0.41658 | 9.97025 | 5 | 2 | |
| 59 | 9.55400 | 33 | 9.58380 | 38 | 0.41620 | 9.97020 | 5 | 1 | |
| 60 | 9.55433 |   | 9.58418 |   | 0.41582 | 9.97015 |   | 0 | |
|   | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ' | P. P. |

**69°**

### P. P.

| | 40 | 39 |
|---|-----|-----|
| 6 | 4.0 | 3.9 |
| 7 | 4.7 | 4.6 |
| 8 | 5.3 | 5.2 |
| 9 | 6.0 | 5.9 |
| 10 | 6.7 | 6.5 |
| 20 | 13.3 | 13.0 |
| 30 | 20.0 | 19.5 |
| 40 | 26.7 | 26.0 |
| 50 | 33.3 | 32.5 |

| | 38 | 37 |
|---|-----|-----|
| 6 | 3.8 | 3.7 |
| 7 | 4.4 | 4.3 |
| 8 | 5.1 | 4.9 |
| 9 | 5.7 | 5.6 |
| 10 | 6.3 | 6.2 |
| 20 | 12.7 | 12.3 |
| 30 | 19.0 | 18.5 |
| 40 | 25.3 | 24.7 |
| 50 | 31.7 | 30.8 |

| | 35 |
|---|-----|
| 6 | 3.5 |
| 7 | 4.1 |
| 8 | 4.7 |
| 9 | 5.3 |
| 10 | 5.8 |
| 20 | 11.7 |
| 30 | 17.5 |
| 40 | 23.3 |
| 50 | 29.2 |

| | 34 | 33 |
|---|-----|-----|
| 6 | 3.4 | 3.3 |
| 7 | 4.0 | 3.9 |
| 8 | 4.5 | 4.4 |
| 9 | 5.1 | 5.0 |
| 10 | 5.7 | 5.5 |
| 20 | 11.3 | 11.0 |
| 30 | 17.0 | 16.5 |
| 40 | 22.7 | 22.0 |
| 50 | 28.3 | 27.5 |

| | 5 | 4 |
|---|-----|-----|
| 6 | 0.5 | 0.4 |
| 7 | 0.6 | 0.5 |
| 8 | 0.7 | 0.5 |
| 9 | 0.8 | 0.6 |
| 10 | 0.8 | 0.7 |
| 20 | 1.7 | 1.3 |
| 30 | 2.5 | 2.0 |
| 40 | 3.3 | 2.7 |
| 50 | 4.2 | 3.3 |

### 21°

| ′ | L. Sin. | d. | L. Tang. | d. c. | L. Cotg. | L. Cos. | d. | ′ | P. P. |
|---|---------|----|----------|-------|----------|---------|----|---|-------|
| 0 | 9.55433 | 33 | 9.58418 | 37 | 0.41582 | 9.97015 | 5 | 60 | |
| 1 | 9.55466 | 33 | 9.58455 | 37 | 0.41545 | 9.97010 | 5 | 59 | |
| 2 | 9.55499 | 33 | 9.58493 | 38 | 0.41507 | 9.97005 | 5 | 58 | **38**  **37** |
| 3 | 9.55532 | 33 | 9.58531 | 38 | 0.41469 | 9.97001 | 4 | 57 | 6  3.8  3.7 |
| 4 | 9.55564 | 32 | 9.58569 | 38 | 0.41431 | 9.96996 | 5 | 56 | 7  4.4  4.3 |
| 5 | 9.55597 | 33 | 9.58606 | 37 | 0.41394 | 9.96991 | 5 | 55 | 8  5.1  4.9 |
| 6 | 9.55630 | 33 | 9.58644 | 38 | 0.41356 | 9.96986 | 5 | 54 | 9  5.7  5.6 |
| 7 | 9.55663 | 33 | 9.58681 | 37 | 0.41319 | 9.96981 | 5 | 53 | 10  6.3  6.2 |
| 8 | 9.55695 | 32 | 9.58719 | 38 | 0.41281 | 9.96976 | 5 | 52 | 20  12.7  12.3 |
| 9 | 9.55728 | 33 | 9.58757 | 38 | 0.41243 | 9.96971 | 5 | 51 | 30  19.0  18.5 |
| 10 | 9.55761 | 33 | 9.58794 | 37 | 0.41206 | 9.96966 | 5 | 50 | 40  25.3  24.7 |
| 11 | 9.55793 | 32 | 9.58832 | 38 | 0.41168 | 9.96962 | 4 | 49 | 50  31.7  30.8 |
| 12 | 9.55826 | 33 | 9.58869 | 37 | 0.41131 | 9.96957 | 5 | 48 | |
| 13 | 9.55858 | 32 | 9.58907 | 38 | 0.41093 | 9.96952 | 5 | 47 | |
| 14 | 9.55891 | 33 | 9.58944 | 37 | 0.41056 | 9.96947 | 5 | 46 | |
| 15 | 9.55923 | 32 | 9.58981 | 37 | 0.41019 | 9.96942 | 5 | 45 | **36**  **33** |
| 16 | 9.55956 | 33 | 9.59019 | 38 | 0.40981 | 9.96937 | 5 | 44 | 6  3.6  3.3 |
| 17 | 9.55988 | 32 | 9.59056 | 37 | 0.40944 | 9.96932 | 5 | 43 | 7  4.2  3.9 |
| 18 | 9.56021 | 33 | 9.59094 | 38 | 0.40906 | 9.96927 | 5 | 42 | 8  4.8  4.4 |
| 19 | 9.56053 | 32 | 9.59131 | 37 | 0.40869 | 9.96922 | 5 | 41 | 9  5.4  5.0 |
| 20 | 9.56085 | 32 | 9.59168 | 37 | 0.40832 | 9.96917 | 5 | 40 | 10  6.0  5.5 |
| 21 | 9.56118 | 33 | 9.59205 | 38 | 0.40795 | 9.96912 | 5 | 39 | 20  12.0  11.0 |
| 22 | 9.56150 | 32 | 9.59243 | 37 | 0.40757 | 9.96907 | 5 | 38 | 30  18.0  16.5 |
| 23 | 9.56182 | 33 | 9.59280 | 37 | 0.40720 | 9.96903 | 4 | 37 | 40  24.0  22.0 |
| 24 | 9.56215 | 32 | 9.59317 | 37 | 0.40683 | 9.96898 | 5 | 36 | 50  30.0  27.5 |
| 25 | 9.56247 | 32 | 9.59354 | 37 | 0.40646 | 9.96893 | 5 | 35 | |
| 26 | 9.56279 | 32 | 9.59391 | 38 | 0.40609 | 9.96888 | 5 | 34 | **32** |
| 27 | 9.56311 | 32 | 9.59429 | 37 | 0.40571 | 9.96883 | 5 | 33 | 6  3.2 |
| 28 | 9.56343 | 32 | 9.59466 | 37 | 0.40534 | 9.96878 | 5 | 32 | 7  3.7 |
| 29 | 9.56375 | 33 | 9.59503 | 37 | 0.40497 | 9.96873 | 5 | 31 | 8  4.3 |
| 30 | 9.56408 | 32 | 9.59540 | 37 | 0.40460 | 9.96868 | 5 | 30 | 9  4.8 |
| 31 | 9.56440 | 32 | 9.59577 | 37 | 0.40423 | 9.96863 | 5 | 29 | 10  5.3 |
| 32 | 9.56472 | 32 | 9.59614 | 37 | 0.40386 | 9.96858 | 5 | 28 | 20  10.7 |
| 33 | 9.56504 | 32 | 9.59651 | 37 | 0.40349 | 9.96853 | 5 | 27 | 30  16.0 |
| 34 | 9.56536 | 32 | 9.59688 | 37 | 0.40312 | 9.96848 | 5 | 26 | 40  21.3 |
| 35 | 9.56568 | 31 | 9.59725 | 37 | 0.40275 | 9.96843 | 5 | 25 | 50  26.7 |
| 36 | 9.56599 | 32 | 9.59762 | 37 | 0.40238 | 9.96838 | 5 | 24 | |
| 37 | 9.56631 | 32 | 9.59799 | 36 | 0.40201 | 9.96833 | 5 | 23 | |
| 38 | 9.56663 | 32 | 9.59835 | 37 | 0.40165 | 9.96828 | 5 | 22 | **31**  **6** |
| 39 | 9.56695 | 32 | 9.59872 | 37 | 0.40128 | 9.96823 | 5 | 21 | 6  3.1  0.6 |
| 40 | 9.56727 | 32 | 9.59909 | 37 | 0.40091 | 9.96818 | 5 | 20 | 7  3.6  0.7 |
| 41 | 9.56759 | 31 | 9.59946 | 37 | 0.40054 | 9.96813 | 5 | 19 | 8  4.1  0.8 |
| 42 | 9.56790 | 32 | 9.59983 | 36 | 0.40017 | 9.96808 | 5 | 18 | 9  4.7  0.9 |
| 43 | 9.56822 | 32 | 9.60019 | 37 | 0.39981 | 9.96803 | 5 | 17 | 10  5.2  1.0 |
| 44 | 9.56854 | 32 | 9.60056 | 37 | 0.39944 | 9.96798 | 5 | 16 | 20  10.3  2.0 |
| 45 | 9.56886 | 31 | 9.60093 | 37 | 0.39907 | 9.96793 | 5 | 15 | 30  15.5  3.0 |
| 46 | 9.56917 | 32 | 9.60130 | 36 | 0.39870 | 9.96788 | 5 | 14 | 40  20.7  4.0 |
| 47 | 9.56949 | 31 | 9.60166 | 37 | 0.39834 | 9.96783 | 5 | 13 | 50  25.8  5.0 |
| 48 | 9.56980 | 32 | 9.60203 | 37 | 0.39797 | 9.96778 | 6 | 12 | |
| 49 | 9.57012 | 32 | 9.60240 | 36 | 0.39760 | 9.96772 | 5 | 11 | |
| 50 | 9.57044 | 31 | 9.60276 | 37 | 0.39724 | 9.96767 | 5 | 10 | **5**  **4** |
| 51 | 9.57075 | 32 | 9.60313 | 36 | 0.39687 | 9.96762 | 5 | 9 | 6  0.5  0.4 |
| 52 | 9.57107 | 31 | 9.60349 | 37 | 0.39651 | 9.96757 | 5 | 8 | 7  0.6  0.5 |
| 53 | 9.57138 | 31 | 9.60386 | 36 | 0.39614 | 9.96752 | 5 | 7 | 8  0.7  0.5 |
| 54 | 9.57169 | 32 | 9.60422 | 37 | 0.39578 | 9.96747 | 5 | 6 | 9  0.8  0.6 |
| 55 | 9.57201 | 31 | 9.60459 | 36 | 0.39541 | 9.96742 | 5 | 5 | 10  0.8  0.7 |
| 56 | 9.57232 | 32 | 9.60495 | 37 | 0.39505 | 9.96737 | 5 | 4 | 20  1.7  1.3 |
| 57 | 9.57264 | 31 | 9.60532 | 36 | 0.39468 | 9.96732 | 5 | 3 | 30  2.5  2.0 |
| 58 | 9.57295 | 31 | 9.60568 | 37 | 0.39432 | 9.96727 | 5 | 2 | 40  3.3  2.7 |
| 59 | 9.57326 | 32 | 9.60605 | 36 | 0.39395 | 9.96722 | 5 | 1 | 50  4.2  3.3 |
| 60 | 9.57358 | | 9.60641 | 36 | 0.39359 | 9.96717 | | 0 | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. |

## 22°

| ' | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | ' | P. P. | |
|---|---------|----|---------|-------|----------|---------|----|---|---|-------|---|
| 0 | 9.57358 | 31 | 9.60641 | 36 | 0.39359 | 9.96717 | 6 | | 60 | | |
| 1 | 9.57389 | 31 | 9.60677 | 36 | 0.39323 | 9.96711 | 6 | | 59 | | |
| 2 | 9.57420 | 31 | 9.60714 | 37 | 0.39286 | 9.96706 | 5 | | 58 | | **37** / **36** |
| 3 | 9.57451 | 31 | 9.60750 | 36 | 0.39250 | 9.96701 | 5 | | 57 | 6 | 3.7 / 3.6 |
| 4 | 9.57482 | 32 | 9.60786 | 37 | 0.39214 | 9.96696 | 5 | | 56 | 7 | 4.3 / 4.2 |
| 5 | 9.57514 | 31 | 9.60823 | 36 | 0.39177 | 9.96691 | 5 | | 55 | 8 | 4.9 / 4.8 |
| 6 | 9.57545 | 31 | 9.60859 | 36 | 0.39141 | 9.96686 | 5 | | 54 | 9 | 5.6 / 5.4 |
| 7 | 9.57576 | 31 | 9.60895 | 36 | 0.39105 | 9.96681 | 5 | | 53 | 10 | 6.2 / 6.0 |
| 8 | 9.57607 | 31 | 9.60931 | 36 | 0.39069 | 9.96676 | 6 | | 52 | 20 | 12.3 / 12.0 |
| 9 | 9.57638 | 31 | 9.60967 | 37 | 0.39033 | 9.96670 | 5 | | 51 | 30 | 18.5 / 18.0 |
| 10 | 9.57669 | 31 | 9.61004 | 36 | 0.38996 | 9.96665 | 5 | | 50 | 40 | 24.7 / 24.0 |
| 11 | 9.57700 | 31 | 9.61040 | 36 | 0.38960 | 9.96660 | 5 | | 49 | 50 | 30.8 / 30.0 |
| 12 | 9.57731 | 31 | 9.61076 | 36 | 0.38924 | 9.96655 | 5 | | 48 | | |
| 13 | 9.57762 | 31 | 9.61112 | 36 | 0.38888 | 9.96650 | 5 | | 47 | | |
| 14 | 9.57793 | 31 | 9.61148 | 36 | 0.38852 | 9.96645 | 5 | | 46 | | **35** |
| 15 | 9.57824 | 31 | 9.61184 | 36 | 0.38816 | 9.96640 | 6 | | 45 | 6 | 3.5 |
| 16 | 9.57855 | 30 | 9.61220 | 36 | 0.38780 | 9.96634 | 5 | | 44 | 7 | 4.1 |
| 17 | 9.57885 | 30 | 9.61256 | 36 | 0.38744 | 9.96629 | 5 | | 43 | 8 | 4.7 |
| 18 | 9.57916 | 31 | 9.61292 | 36 | 0.38708 | 9.96624 | 5 | | 42 | 9 | 5.3 |
| 19 | 9.57947 | 31 | 9.61328 | 36 | 0.38672 | 9.96619 | 5 | | 41 | 10 | 5.8 |
| 20 | 9.57978 | 30 | 9.61364 | 36 | 0.38636 | 9.96614 | 6 | | 40 | 20 | 11.7 |
| 21 | 9.58008 | 31 | 9.61400 | 36 | 0.38600 | 9.96608 | 5 | | 39 | 30 | 17.5 |
| 22 | 9.58039 | 31 | 9.61436 | 36 | 0.38564 | 9.96603 | 5 | | 38 | 40 | 23.3 |
| 23 | 9.58070 | 31 | 9.61472 | 36 | 0.38528 | 9.96598 | 5 | | 37 | 50 | 29.2 |
| 24 | 9.58101 | 30 | 9.61508 | 36 | 0.38492 | 9.96593 | 5 | | 36 | | |
| 25 | 9.58131 | 31 | 9.61544 | 35 | 0.38456 | 9.96588 | 6 | | 35 | | |
| 26 | 9.58162 | 30 | 9.61579 | 36 | 0.38421 | 9.96582 | 5 | | 34 | | **32** / **31** |
| 27 | 9.58192 | 31 | 9.61615 | 36 | 0.38385 | 9.96577 | 5 | | 33 | 6 | 3.2 / 3.1 |
| 28 | 9.58223 | 30 | 9.61651 | 36 | 0.38349 | 9.96572 | 5 | | 32 | 7 | 3.7 / 3.6 |
| 29 | 9.58253 | 31 | 9.61687 | 35 | 0.38313 | 9.96567 | 5 | | 31 | 8 | 4.3 / 4.1 |
| 30 | 9.58284 | 30 | 9.61722 | 36 | 0.38278 | 9.96562 | 6 | | 30 | 9 | 4.8 / 4.7 |
| 31 | 9.58314 | 31 | 9.61758 | 36 | 0.38242 | 9.96556 | 5 | | 29 | 10 | 5.3 / 5.2 |
| 32 | 9.58345 | 30 | 9.61794 | 36 | 0.38206 | 9.96551 | 5 | | 28 | 20 | 10.7 / 10.3 |
| 33 | 9.58375 | 31 | 9.61830 | 35 | 0.38170 | 9.96546 | 5 | | 27 | 30 | 16.0 / 15.5 |
| 34 | 9.58406 | 30 | 9.61865 | 36 | 0.38135 | 9.96541 | 6 | | 26 | 40 | 21.3 / 20.7 |
| 35 | 9.58436 | 31 | 9.61901 | 35 | 0.38099 | 9.96535 | 5 | | 25 | 50 | 26.7 / 25.8 |
| 36 | 9.58467 | 30 | 9.61936 | 36 | 0.38064 | 9.96530 | 5 | | 24 | | |
| 37 | 9.58497 | 30 | 9.61972 | 36 | 0.38028 | 9.96525 | 5 | | 23 | | |
| 38 | 9.58527 | 30 | 9.62008 | 35 | 0.37992 | 9.96520 | 6 | | 22 | | **30** / **29** |
| 39 | 9.58557 | 31 | 9.62043 | 36 | 0.37957 | 9.96514 | 5 | | 21 | 6 | 3.0 / 2.9 |
| 40 | 9.58588 | 30 | 9.62079 | 35 | 0.37921 | 9.96509 | 5 | | 20 | 7 | 3.5 / 3.4 |
| 41 | 9.58618 | 30 | 9.62114 | 36 | 0.37886 | 9.96504 | 6 | | 19 | 8 | 4.0 / 3.9 |
| 42 | 9.58648 | 30 | 9.62150 | 35 | 0.37850 | 9.96498 | 5 | | 18 | 9 | 4.5 / 4.4 |
| 43 | 9.58678 | 30 | 9.62185 | 36 | 0.37815 | 9.96493 | 5 | | 17 | 10 | 5.0 / 4.8 |
| 44 | 9.58709 | 30 | 9.62221 | 35 | 0.37779 | 9.96488 | 5 | | 16 | 20 | 10.0 / 9.7 |
| 45 | 9.58739 | 30 | 9.62256 | 36 | 0.37744 | 9.96483 | 6 | | 15 | 30 | 15.0 / 14.5 |
| 46 | 9.58769 | 30 | 9.62292 | 35 | 0.37708 | 9.96477 | 5 | | 14 | 40 | 20.0 / 19.3 |
| 47 | 9.58799 | 30 | 9.62327 | 35 | 0.37673 | 9.96472 | 5 | | 13 | 50 | 25.0 / 24.2 |
| 48 | 9.58829 | 30 | 9.62362 | 36 | 0.37638 | 9.96467 | 6 | | 12 | | |
| 49 | 9.58859 | 30 | 9.62398 | 35 | 0.37602 | 9.96461 | 5 | | 11 | | |
| 50 | 9.58889 | 30 | 9.62433 | 35 | 0.37567 | 9.96456 | 5 | | 10 | | **6** / **5** |
| 51 | 9.58919 | 30 | 9.62468 | 36 | 0.37532 | 9.96451 | 6 | | 9 | 6 | 0.6 / 0.5 |
| 52 | 9.58949 | 30 | 9.62504 | 35 | 0.37496 | 9.96445 | 5 | | 8 | 7 | 0.7 / 0.6 |
| 53 | 9.58979 | 30 | 9.62539 | 35 | 0.37461 | 9.96440 | 5 | | 7 | 8 | 0.8 / 0.7 |
| 54 | 9.59009 | 30 | 9.62574 | 35 | 0.37426 | 9.96435 | 6 | | 6 | 9 | 0.9 / 0.8 |
| 55 | 9.59039 | 30 | 9.62609 | 36 | 0.37391 | 9.96429 | 5 | | 5 | 10 | 1.0 / 0.8 |
| 56 | 9.59069 | 30 | 9.62645 | 35 | 0.37355 | 9.96424 | 5 | | 4 | 20 | 2.0 / 1.7 |
| 57 | 9.59098 | 29 | 9.62680 | 35 | 0.37320 | 9.96419 | 6 | | 3 | 30 | 3.0 / 2.5 |
| 58 | 9.59128 | 30 | 9.62715 | 35 | 0.37285 | 9.96413 | 5 | | 2 | 40 | 4.0 / 3.3 |
| 59 | 9.59158 | 30 | 9.62750 | 35 | 0.37250 | 9.96408 | 5 | | 1 | 50 | 5.0 / 4.2 |
| 60 | 9.59188 | | 9.62785 | | 0.37215 | 9.96403 | | | 0 | | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | | ' | P. P. | |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | ′ |
|---|---------|----|---------|-------|----------|---------|----|---|
| 0 | 9.59188 | 30 | 9.62785 | 35 | 0.37215 | 9.96403 | 6 | 60 |
| 1 | 9.59218 | 29 | 9.62820 | 35 | 0.37180 | 9.96397 | 5 | 59 |
| 2 | 9.59247 | 30 | 9.62855 | 35 | 0.37145 | 9.96392 | 5 | 58 |
| 3 | 9.59277 | 30 | 9.62890 | 35 | 0.37110 | 9.96387 | 6 | 57 |
| 4 | 9.59307 | 29 | 9.62926 | 36 35 | 0.37074 | 9.96381 | 5 | 56 |
| 5 | 9.59336 | 30 | 9.62961 | 35 | 0.37039 | 9.96376 | 6 | 55 |
| 6 | 9.59366 | 30 | 9.62996 | 35 | 0.37004 | 9.96370 | 5 | 54 |
| 7 | 9.59396 | 29 | 9.63031 | 35 | 0.36969 | 9.96365 | 5 | 53 |
| 8 | 9.59425 | 30 | 9.63066 | 35 | 0.36934 | 9.96360 | 6 | 52 |
| 9 | 9.59455 | 29 | 9.63101 | 34 | 0.36899 | 9.96354 | 5 | 51 |
| 10 | 9.59484 | 30 | 9.63135 | 35 | 0.36865 | 9.96349 | 6 | 50 |
| 11 | 9.59514 | 29 | 9.63170 | 35 | 0.36830 | 9.96343 | 5 | 49 |
| 12 | 9.59543 | 30 | 9.63205 | 35 | 0.36795 | 9.96338 | 5 | 48 |
| 13 | 9.59573 | 29 | 9.63240 | 35 | 0.36760 | 9.96333 | 6 | 47 |
| 14 | 9.59602 | 30 | 9.63275 | 35 | 0.36725 | 9.96327 | 5 | 46 |
| 15 | 9.59632 | 29 | 9.63310 | 35 | 0.36690 | 9.96322 | 6 | 45 |
| 16 | 9.59661 | 29 | 9.63345 | 34 | 0.36655 | 9.96316 | 5 | 44 |
| 17 | 9.59690 | 30 | 9.63379 | 35 | 0.36621 | 9.96311 | 6 | 43 |
| 18 | 9.59720 | 29 | 9.63414 | 35 | 0.36586 | 9.96305 | 5 | 42 |
| 19 | 9.59749 | 29 | 9.63449 | 35 | 0.36551 | 9.96300 | 6 | 41 |
| 20 | 9.59778 | 30 | 9.63484 | 35 | 0.36516 | 9.96294 | 5 | 40 |
| 21 | 9.59808 | 29 | 9.63519 | 34 | 0.36481 | 9.96289 | 5 | 39 |
| 22 | 9.59837 | 29 | 9.63553 | 35 | 0.36447 | 9.96284 | 6 | 38 |
| 23 | 9.59866 | 29 | 9.63588 | 35 | 0.36412 | 9.96278 | 5 | 37 |
| 24 | 9.59895 | 29 | 9.63623 | 34 | 0.36377 | 9.96273 | 6 | 36 |
| 25 | 9.59924 | 30 | 9.63657 | 35 | 0.36343 | 9.96267 | 5 | 35 |
| 26 | 9.59954 | 29 | 9.63692 | 34 | 0.36308 | 9.96262 | 6 | 34 |
| 27 | 9.59983 | 29 | 9.63726 | 35 | 0.36274 | 9.96256 | 5 | 33 |
| 28 | 9.60012 | 29 | 9.63761 | 35 | 0.36239 | 9.96251 | 6 | 32 |
| 29 | 9.60041 | 29 | 9.63796 | 34 | 0.36204 | 9.96245 | 5 | 31 |
| 30 | 9.60070 | 29 | 9.63830 | 35 | 0.36170 | 9.96240 | 6 | 30 |
| 31 | 9.60099 | 29 | 9.63865 | 34 | 0.36135 | 9.96234 | 5 | 29 |
| 32 | 9.60128 | 29 | 9.63899 | 35 | 0.36101 | 9.96229 | 6 | 28 |
| 33 | 9.60157 | 29 | 9.63934 | 34 | 0.36066 | 9.96223 | 5 | 27 |
| 34 | 9.60186 | 29 | 9.63968 | 35 | 0.36032 | 9.96218 | 6 | 26 |
| 35 | 9.60215 | 29 | 9.64003 | 34 | 0.35997 | 9.96212 | 5 | 25 |
| 36 | 9.60244 | 29 | 9.64037 | 35 | 0.35963 | 9.96207 | 6 | 24 |
| 37 | 9.60273 | 29 | 9.64072 | 34 | 0.35928 | 9.96201 | 5 | 23 |
| 38 | 9.60302 | 29 | 9.64106 | 34 | 0.35894 | 9.96196 | 6 | 22 |
| 39 | 9.60331 | 28 | 9.64140 | 35 | 0.35860 | 9.96190 | 5 | 21 |
| 40 | 9.60359 | 29 | 9.64175 | 34 | 0.35825 | 9.96185 | 6 | 20 |
| 41 | 9.60388 | 29 | 9.64209 | 34 | 0.35791 | 9.96179 | 5 | 19 |
| 42 | 9.60417 | 29 | 9.64243 | 35 | 0.35757 | 9.96174 | 6 | 18 |
| 43 | 9.60446 | 28 | 9.64278 | 34 | 0.35722 | 9.96168 | 6 | 17 |
| 44 | 9.60474 | 29 | 9.64312 | 34 | 0.35688 | 9.96162 | 5 | 16 |
| 45 | 9.60503 | 29 | 9.64346 | 35 | 0.35654 | 9.96157 | 6 | 15 |
| 46 | 9.60532 | 29 | 9.64381 | 34 | 0.35619 | 9.96151 | 5 | 14 |
| 47 | 9.60561 | 28 | 9.64415 | 34 | 0.35585 | 9.96146 | 6 | 13 |
| 48 | 9.60589 | 29 | 9.64449 | 34 | 0.35551 | 9.96140 | 5 | 12 |
| 49 | 9.60618 | 28 | 9.64483 | 34 | 0.35517 | 9.96135 | 6 | 11 |
| 50 | 9.60646 | 29 | 9.64517 | 35 | 0.35483 | 9.96129 | 6 | 10 |
| 51 | 9.60675 | 29 | 9.64552 | 34 | 0.35448 | 9.96123 | 5 | 9 |
| 52 | 9.60704 | 28 | 9.64586 | 34 | 0.35414 | 9.96118 | 6 | 8 |
| 53 | 9.60732 | 28 | 9.64620 | 34 | 0.35380 | 9.96112 | 6 | 7 |
| 54 | 9.60761 | 28 | 9.64654 | 34 | 0.35346 | 9.96107 | 6 | 6 |
| 55 | 9.60789 | 29 | 9.64688 | 34 | 0.35312 | 9.96101 | 6 | 5 |
| 56 | 9.60818 | 29 | 9.64722 | 34 | 0.35278 | 9.96095 | 5 | 4 |
| 57 | 9.60846 | 29 | 9.64756 | 34 | 0.35244 | 9.96090 | 6 | 3 |
| 58 | 9.60875 | 28 | 9.64790 | 34 | 0.35210 | 9.96084 | 5 | 2 |
| 59 | 9.60903 | 28 | 9.64824 | 34 | 0.35176 | 9.96079 | 6 | 1 |
| 60 | 9.60931 | 28 | 9.64858 | | 0.35142 | 9.96073 | | 0 |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ |

P. P.

| | 36 | 35 |
|---|------|------|
| 6 | 3.6 | 3.5 |
| 7 | 4.2 | 4.1 |
| 8 | 4.8 | 4.7 |
| 9 | 5.4 | 5.3 |
| 10 | 6.0 | 5.8 |
| 20 | 12.0 | 11.7 |
| 30 | 18.0 | 17.5 |
| 40 | 24.0 | 23.3 |
| 50 | 30.0 | 29.2 |

| | 34 |
|---|------|
| 6 | 3.4 |
| 7 | 4.0 |
| 8 | 4.5 |
| 9 | 5.1 |
| 10 | 5.7 |
| 20 | 11.3 |
| 30 | 17.0 |
| 40 | 22.7 |
| 50 | 28.3 |

| | 30 | 29 |
|---|------|------|
| 6 | 3.0 | 2.9 |
| 7 | 3.5 | 3.4 |
| 8 | 4.0 | 3.9 |
| 9 | 4.5 | 4.4 |
| 10 | 5.0 | 4.8 |
| 20 | 10.0 | 9.7 |
| 30 | 15.0 | 14.5 |
| 40 | 20.0 | 19.3 |
| 50 | 25.0 | 24.2 |

| | 28 |
|---|------|
| 6 | 2.8 |
| 7 | 3.3 |
| 8 | 3.7 |
| 9 | 4.2 |
| 10 | 4.7 |
| 20 | 9.3 |
| 30 | 14.0 |
| 40 | 18.7 |
| 50 | 23.3 |

| | 5 | 5 |
|---|------|------|
| 6 | 0.6 | 0.5 |
| 7 | 0.7 | 0.6 |
| 8 | 0.8 | 0.7 |
| 9 | 0.9 | 0.8 |
| 10 | 1.0 | 0.8 |
| 20 | 2.0 | 1.7 |
| 30 | 3.0 | 2.5 |
| 40 | 4.0 | 3.3 |
| 50 | 5.0 | 4.2 |

## 24°

| ' | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | P. P. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 9.60931 | 29 | 9.64858 | 34 | 0.35142 | 9.96073 | 6 | 60 | |
| 1 | 9.60960 | 28 | 9.64892 | 34 | 0.35108 | 9.96067 | 6 | 59 | |
| 2 | 9.60988 | 28 | 9.64926 | 34 | 0.35074 | 9.96062 | 5 | 58 | |
| 3 | 9.61016 | 29 | 9.64960 | 34 | 0.35040 | 9.96056 | 6 | 57 | **34**  **33** |
| 4 | 9.61045 | 28 | 9.64994 | 34 | 0.35006 | 9.96050 | 6 | 56 | 6 \| 3.4 \| 3.3 |
| | | | | 34 | | | 5 | | 7 \| 4.0 \| 3.9 |
| 5 | 9.61073 | 28 | 9.65028 | 34 | 0.34972 | 9.96045 | | 55 | 8 \| 4.5 \| 4.4 |
| 6 | 9.61101 | 28 | 9.65062 | 34 | 0.34938 | 9.96039 | 6 | 54 | 9 \| 5.1 \| 5.0 |
| 7 | 9.61129 | 28 | 9.65096 | 34 | 0.34904 | 9.96034 | 5 | 53 | 10 \| 5.7 \| 5.5 |
| 8 | 9.61158 | 29 | 9.65130 | 34 | 0.34870 | 9.96028 | 6 | 52 | 20 \| 11.3 \| 11.0 |
| 9 | 9.61186 | 28 | 9.65164 | 33 | 0.34836 | 9.96022 | 6 | 51 | 30 \| 17.0 \| 16.5 |
| 10 | 9.61214 | 28 | 9.65197 | 34 | 0.34803 | 9.96017 | 5 | 50 | 40 \| 22.7 \| 22.0 |
| 11 | 9.61242 | 28 | 9.65231 | 34 | 0.34769 | 9.96011 | 6 | 49 | 50 \| 28.3 \| 27.5 |
| 12 | 9.61270 | 28 | 9.65265 | 34 | 0.34735 | 9.96005 | 6 | 48 | |
| 13 | 9.61298 | 28 | 9.65299 | 34 | 0.34701 | 9.96000 | 6 | 47 | |
| 14 | 9.61326 | 28 | 9.65333 | 33 | 0.34667 | 9.95994 | 6 | 46 | **29** |
| 15 | 9.61354 | 28 | 9.65366 | 34 | 0.34634 | 9.95988 | 6 | 45 | 6 \| 2.9 |
| 16 | 9.61382 | 29 | 9.65400 | 34 | 0.34600 | 9.95982 | 5 | 44 | 7 \| 3.4 |
| 17 | 9.61411 | 27 | 9.65434 | 33 | 0.34566 | 9.95977 | 6 | 43 | 8 \| 3.9 |
| 18 | 9.61438 | 28 | 9.65467 | 34 | 0.34533 | 9.95971 | 6 | 42 | 9 \| 4.4 |
| 19 | 9.61466 | 28 | 9.65501 | 34 | 0.34499 | 9.95965 | 5 | 41 | 10 \| 4.8 |
| 20 | 9.61494 | 28 | 9.65535 | 33 | 0.34465 | 9.95960 | 6 | 40 | 20 \| 9.7 |
| 21 | 9.61522 | 28 | 9.65568 | 34 | 0.34432 | 9.95954 | 6 | 39 | 30 \| 14.5 |
| 22 | 9.61550 | 28 | 9.65602 | 34 | 0.34398 | 9.95948 | 6 | 38 | 40 \| 19.3 |
| 23 | 9.61578 | 28 | 9.65636 | 33 | 0.34364 | 9.95942 | 5 | 37 | 50 \| 24.2 |
| 24 | 9.61606 | 28 | 9.65669 | 34 | 0.34331 | 9.95937 | 6 | 36 | |
| 25 | 9.61634 | 28 | 9.65703 | 33 | 0.34297 | 9.95931 | 6 | 35 | |
| 26 | 9.61662 | 27 | 9.65736 | 34 | 0.34264 | 9.95925 | 5 | 34 | **28** |
| 27 | 9.61689 | 28 | 9.65770 | 33 | 0.34230 | 9.95920 | 6 | 33 | 6 \| 2.8 |
| 28 | 9.61717 | 28 | 9.65803 | 34 | 0.34197 | 9.95914 | 6 | 32 | 7 \| 3.3 |
| 29 | 9.61745 | 28 | 9.65837 | 33 | 0.34163 | 9.95908 | 6 | 31 | 8 \| 3.7 |
| 30 | 9.61773 | 27 | 9.65870 | 34 | 0.34130 | 9.95902 | 5 | 30 | 9 \| 4.2 |
| 31 | 9.61800 | 28 | 9.65904 | 33 | 0.34096 | 9.95897 | 6 | 29 | 10 \| 4.7 |
| 32 | 9.61828 | 28 | 9.65937 | 34 | 0.34063 | 9.95891 | 6 | 28 | 20 \| 9.3 |
| 33 | 9.61856 | 27 | 9.65971 | 33 | 0.34029 | 9.95885 | 6 | 27 | 30 \| 14.0 |
| 34 | 9.61883 | 28 | 9.66004 | 34 | 0.33996 | 9.95879 | 6 | 26 | 40 \| 18.7 |
| 35 | 9.61911 | 28 | 9.66038 | 33 | 0.33962 | 9.95873 | 5 | 25 | 50 \| 23.3 |
| 36 | 9.61939 | 27 | 9.66071 | 33 | 0.33929 | 9.95868 | 6 | 24 | |
| 37 | 9.61966 | 28 | 9.66104 | 34 | 0.33896 | 9.95862 | 6 | 23 | |
| 38 | 9.61994 | 27 | 9.66138 | 33 | 0.33862 | 9.95856 | 6 | 22 | **27** |
| 39 | 9.62021 | 28 | 9.66171 | 33 | 0.33829 | 9.95850 | 6 | 21 | 6 \| 2.7 |
| 40 | 9.62049 | 27 | 9.66204 | 34 | 0.33796 | 9.95844 | 5 | 20 | 7 \| 3.2 |
| 41 | 9.62076 | 28 | 9.66238 | 33 | 0.33762 | 9.95839 | 6 | 19 | 8 \| 3.6 |
| 42 | 9.62104 | 27 | 9.66271 | 33 | 0.33729 | 9.95833 | 6 | 18 | 9 \| 4.1 |
| 43 | 9.62131 | 28 | 9.66304 | 33 | 0.33696 | 9.95827 | 6 | 17 | 10 \| 4.5 |
| 44 | 9.62159 | 27 | 9.66337 | 34 | 0.33663 | 9.95821 | 6 | 16 | 20 \| 9.0 |
| 45 | 9.62186 | 28 | 9.66371 | 33 | 0.33629 | 9.95815 | 5 | 15 | 30 \| 13.5 |
| 46 | 9.62214 | 27 | 9.66404 | 33 | 0.33596 | 9.95810 | 6 | 14 | 40 \| 18.0 |
| 47 | 9.62241 | 27 | 9.66437 | 33 | 0.33563 | 9.95804 | 6 | 13 | 50 \| 22.5 |
| 48 | 9.62268 | 28 | 9.66470 | 33 | 0.33530 | 9.95798 | 6 | 12 | |
| 49 | 9.62296 | 27 | 9.66503 | 34 | 0.33497 | 9.95792 | 6 | 11 | |
| 50 | 9.62323 | 27 | 9.66537 | 33 | 0.33463 | 9.95786 | 6 | 10 | **6**  **5** |
| 51 | 9.62350 | 27 | 9.66570 | 33 | 0.33430 | 9.95780 | 6 | 9 | 6 \| 0.6 \| 0.5 |
| 52 | 9.62377 | 28 | 9.66603 | 33 | 0.33397 | 9.95775 | 5 | 8 | 7 \| 0.7 \| 0.6 |
| 53 | 9.62405 | 27 | 9.66636 | 33 | 0.33364 | 9.95769 | 6 | 7 | 8 \| 0.8 \| 0.7 |
| 54 | 9.62432 | 27 | 9.66669 | 33 | 0.33331 | 9.95763 | 6 | 6 | 9 \| 0.9 \| 0.8 |
| 55 | 9.62459 | 27 | 9.66702 | 33 | 0.33298 | 9.95757 | 6 | 5 | 10 \| 1.0 \| 0.8 |
| 56 | 9.62486 | 27 | 9.66735 | 33 | 0.33265 | 9.95751 | 6 | 4 | 20 \| 2.0 \| 1.7 |
| 57 | 9.62513 | 28 | 9.66768 | 33 | 0.33232 | 9.95745 | 6 | 3 | 30 \| 3.0 \| 2.5 |
| 58 | 9.62541 | 27 | 9.66801 | 33 | 0.33199 | 9.95739 | 6 | 2 | 40 \| 4.0 \| 3.3 |
| 59 | 9.62568 | 27 | 9.66834 | 33 | 0.33166 | 9.95733 | 5 | 1 | 50 \| 5.0 \| 4.2 |
| 60 | 9.62595 | | 9.66867 | | 0.33133 | 9.95728 | | 0 | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ' | P. P. |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | | P. P. | |
|---|---------|----|---------|-------|----------|---------|----|---|---|-------|---|
| 0 | 9.62595 | 27 | 9.66867 | 33 | 0.33133 | 9.95728 | 6 | 60 | | | |
| 1 | 9.62622 | 27 | 9.66900 | 33 | 0.33100 | 9.95722 | 6 | 59 | | | |
| 2 | 9.62649 | 27 | 9.66933 | 33 | 0.33067 | 9.95716 | 6 | 58 | | | |
| 3 | 9.62676 | 27 | 9.66966 | 33 | 0.33034 | 9.95710 | 6 | 57 | | 33 | 32 |
| 4 | 9.62703 | 27 | 9.66999 | 33 | 0.33001 | 9.95704 | 6 | 56 | 6 | 3.3 | 3.2 |
| 5 | 9.62730 | 27 | 9.67032 | 33 | 0.32968 | 9.95698 | 6 | 55 | 7 | 3.9 | 3.7 |
| 6 | 9.62757 | 27 | 9.67065 | 33 | 0.32935 | 9.95692 | 6 | 54 | 8 | 4.4 | 4.3 |
| 7 | 9.62784 | 27 | 9.67098 | 33 | 0.32902 | 9.95686 | 6 | 53 | 9 | 5.0 | 4.8 |
| 8 | 9.62811 | 27 | 9.67131 | 32 | 0.32869 | 9.95680 | 6 | 52 | 10 | 5.5 | 5.3 |
| 9 | 9.62838 | 27 | 9.67163 | 33 | 0.32837 | 9.95674 | 6 | 51 | 20 | 11.0 | 10.7 |
| 10 | 9.62865 | 27 | 9.67196 | 33 | 0.32804 | 9.95668 | 5 | 50 | 30 | 16.5 | 16.0 |
| 11 | 9.62892 | 27 | 9.67229 | 33 | 0.32771 | 9.95663 | 6 | 49 | 40 | 22.0 | 21.3 |
| 12 | 9.62918 | 26 | 9.67262 | 33 | 0.32738 | 9.95657 | 6 | 48 | 50 | 27.5 | 26.7 |
| 13 | 9.62945 | 27 | 9.67295 | 32 | 0.32705 | 9.95651 | 6 | 47 | | | |
| 14 | 9.62972 | 27 | 9.67327 | 33 | 0.32673 | 9.95645 | 6 | 46 | | | |
| 15 | 9.62999 | 27 | 9.67360 | 33 | 0.32640 | 9.95639 | 6 | 45 | | 27 | |
| 16 | 9.63026 | 26 | 9.67393 | 33 | 0.32607 | 9.95633 | 6 | 44 | 6 | 2.7 | |
| 17 | 9.63052 | 27 | 9.67426 | 32 | 0.32574 | 9.95627 | 6 | 43 | 7 | 3.2 | |
| 18 | 9.63079 | 27 | 9.67458 | 33 | 0.32542 | 9.95621 | 6 | 42 | 8 | 3.6 | |
| 19 | 9.63106 | 27 | 9.67491 | 33 | 0.32509 | 9.95615 | 6 | 41 | 9 | 4.1 | |
| 20 | 9.63133 | 26 | 9.67524 | 32 | 0.32476 | 9.95609 | 6 | 40 | 10 | 4.5 | |
| 21 | 9.63159 | 27 | 9.67556 | 33 | 0.32444 | 9.95603 | 6 | 39 | 20 | 9.0 | |
| 22 | 9.63186 | 27 | 9.67589 | 33 | 0.32411 | 9.95597 | 6 | 38 | 30 | 13.5 | |
| 23 | 9.63213 | 26 | 9.67622 | 32 | 0.32378 | 9.95591 | 6 | 37 | 40 | 18.0 | |
| 24 | 9.63239 | 27 | 9.67654 | 33 | 0.32346 | 9.95585 | 6 | 36 | 50 | 22.5 | |
| 25 | 9.63266 | 26 | 9.67687 | 32 | 0.32313 | 9.95579 | 6 | 35 | | | |
| 26 | 9.63292 | 27 | 9.67719 | 33 | 0.32281 | 9.95573 | 6 | 34 | | 26 | |
| 27 | 9.63319 | 27 | 9.67752 | 33 | 0.32248 | 9.95567 | 6 | 33 | 6 | 2.6 | |
| 28 | 9.63345 | 26 | 9.67785 | 32 | 0.32215 | 9.95561 | 6 | 32 | 7 | 3.0 | |
| 29 | 9.63372 | 26 | 9.67817 | 33 | 0.32183 | 9.95555 | 6 | 31 | 8 | 3.5 | |
| 30 | 9.63398 | 27 | 9.67850 | 32 | 0.32150 | 9.95549 | 6 | 30 | 9 | 3.9 | |
| 31 | 9.63425 | 26 | 9.67882 | 33 | 0.32118 | 9.95543 | 6 | 29 | 10 | 4.3 | |
| 32 | 9.63451 | 27 | 9.67915 | 32 | 0.32085 | 9.95537 | 6 | 28 | 20 | 8.7 | |
| 33 | 9.63478 | 26 | 9.67947 | 33 | 0.32053 | 9.95531 | 6 | 27 | 30 | 13.0 | |
| 34 | 9.63504 | 27 | 9.67980 | 32 | 0.32020 | 9.95525 | 6 | 26 | 40 | 17.3 | |
| 35 | 9.63531 | 26 | 9.68012 | 32 | 0.31988 | 9.95519 | 6 | 25 | 50 | 21.7 | |
| 36 | 9.63557 | 26 | 9.68044 | 33 | 0.31956 | 9.95513 | 6 | 24 | | | |
| 37 | 9.63583 | 27 | 9.68077 | 32 | 0.31923 | 9.95507 | 6 | 23 | | | |
| 38 | 9.63610 | 26 | 9.68109 | 33 | 0.31891 | 9.95500 | 7 | 22 | | 7 | |
| 39 | 9.63636 | 26 | 9.68142 | 32 | 0.31858 | 9.95494 | 6 | 21 | 6 | 0.7 | |
| 40 | 9.63662 | 27 | 9.68174 | 32 | 0.31826 | 9.95488 | 6 | 20 | 7 | 0.8 | |
| 41 | 9.63689 | 26 | 9.68206 | 33 | 0.31794 | 9.95482 | 6 | 19 | 8 | 0.9 | |
| 42 | 9.63715 | 26 | 9.68239 | 32 | 0.31761 | 9.95476 | 6 | 18 | 9 | 1.1 | |
| 43 | 9.63741 | 26 | 9.68271 | 32 | 0.31729 | 9.95470 | 6 | 17 | 10 | 1.2 | |
| 44 | 9.63767 | 27 | 9.68303 | 33 | 0.31697 | 9.95464 | 6 | 16 | 20 | 2.3 | |
| 45 | 9.63794 | 26 | 9.68336 | 32 | 0.31664 | 9.95458 | 6 | 15 | 30 | 3.5 | |
| 46 | 9.63820 | 26 | 9.68368 | 32 | 0.31632 | 9.95452 | 6 | 14 | 40 | 4.7 | |
| 47 | 9.63846 | 26 | 9.68400 | 32 | 0.31600 | 9.95446 | 6 | 13 | 50 | 5.8 | |
| 48 | 9.63872 | 26 | 9.68432 | 33 | 0.31568 | 9.95440 | 6 | 12 | | | |
| 49 | 9.63898 | 26 | 9.68465 | 32 | 0.31535 | 9.95434 | 7 | 11 | | | |
| 50 | 9.63924 | 26 | 9.68497 | 32 | 0.31503 | 9.95427 | 6 | 10 | | 6 | 5 |
| 51 | 9.63950 | 26 | 9.68529 | 32 | 0.31471 | 9.95421 | 6 | 9 | 6 | 0.6 | 0.5 |
| 52 | 9.63976 | 26 | 9.68561 | 32 | 0.31439 | 9.95415 | 6 | 8 | 7 | 0.7 | 0.6 |
| 53 | 9.64002 | 26 | 9.68593 | 33 | 0.31407 | 9.95409 | 6 | 7 | 8 | 0.8 | 0.7 |
| 54 | 9.64028 | 26 | 9.68626 | 32 | 0.31374 | 9.95403 | 6 | 6 | 9 | 0.9 | 0.8 |
| 55 | 9.64054 | 26 | 9.68658 | 32 | 0.31342 | 9.95397 | 6 | 5 | 10 | 1.0 | 0.8 |
| 56 | 9.64080 | 26 | 9.68690 | 32 | 0.31310 | 9.95391 | 7 | 4 | 20 | 2.0 | 1.7 |
| 57 | 9.64106 | 26 | 9.68722 | 32 | 0.31278 | 9.95384 | 6 | 3 | 30 | 3.0 | 2.5 |
| 58 | 9.64132 | 26 | 9.68754 | 32 | 0.31246 | 9.95378 | 6 | 2 | 40 | 4.0 | 3.3 |
| 59 | 9.64158 | 26 | 9.68786 | 32 | 0.31214 | 9.95372 | 6 | 1 | 50 | 5.0 | 4.2 |
| 60 | 9.64184 | | 9.68818 | | 0.31182 | 9.95366 | | 0 | | | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | | P. P. | |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | | P. P. | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 9.64184 | 26 | 9.68818 | 32 | 0.31182 | 9.95366 | 6 | 60 | | | |
| 1 | 9.64210 | 26 | 9.68850 | 32 | 0.31150 | 9.95360 | 6 | 59 | | | |
| 2 | 9.64236 | 26 | 9.68882 | 32 | 0.31118 | 9.95354 | 6 | 58 | | | |
| 3 | 9.64262 | 26 | 9.68914 | 32 | 0.31086 | 9.95348 | 7 | 57 | | **32** | **31** |
| 4 | 9.64288 | 25 | 9.68946 | 32 | 0.31054 | 9.95341 | 6 | 56 | 6 | 3.2 | 3.1 |
| 5 | 9.64313 | 26 | 9.68978 | 32 | 0.31022 | 9.95335 | 6 | 55 | 7 | 3.7 | 3.6 |
| 6 | 9.64339 | 26 | 9.69010 | 32 | 0.30990 | 9.95329 | 6 | 54 | 8 | 4.3 | 4.1 |
| 7 | 9.64365 | 26 | 9.69042 | 32 | 0.30958 | 9.95323 | 6 | 53 | 9 | 4.8 | 4.7 |
| 8 | 9.64391 | 26 | 9.69074 | 32 | 0.30926 | 9.95317 | 7 | 52 | 10 | 5.3 | 5.2 |
| 9 | 9.64417 | 25 | 9.69106 | 32 | 0.30894 | 9.95310 | 6 | 51 | 20 | 10.7 | 10.3 |
| 10 | 9.64442 | 26 | 9.69138 | 32 | 0.30862 | 9.95304 | 6 | 50 | 30 | 16.0 | 15.5 |
| 11 | 9.64468 | 26 | 9.69170 | 32 | 0.30830 | 9.95298 | 6 | 49 | 40 | 21.3 | 20.7 |
| 12 | 9.64494 | 25 | 9.69202 | 32 | 0.30798 | 9.95292 | 6 | 48 | 50 | 26.7 | 25.8 |
| 13 | 9.64519 | 26 | 9.69234 | 32 | 0.30766 | 9.95286 | 7 | 47 | | | |
| 14 | 9.64545 | 26 | 9.69266 | 32 | 0.30734 | 9.95279 | 6 | 46 | | | |
| 15 | 9.64571 | 25 | 9.69298 | 31 | 0.30702 | 9.95273 | 6 | 45 | | **26** | |
| 16 | 9.64596 | 26 | 9.69329 | 32 | 0.30671 | 9.95267 | 6 | 44 | 6 | 2.6 | |
| 17 | 9.64622 | 25 | 9.69361 | 32 | 0.30639 | 9.95261 | 7 | 43 | 7 | 3.0 | |
| 18 | 9.64647 | 26 | 9.69393 | 32 | 0.30607 | 9.95254 | 6 | 42 | 8 | 3.5 | |
| 19 | 9.64673 | 25 | 9.69425 | 32 | 0.30575 | 9.95248 | 6 | 41 | 9 | 3.9 | |
| 20 | 9.64698 | 26 | 9.69457 | 31 | 0.30543 | 9.95242 | 6 | 40 | 10 | 4.3 | |
| 21 | 9.64724 | 25 | 9.69488 | 32 | 0.30512 | 9.95236 | 7 | 39 | 20 | 8.7 | |
| 22 | 9.64749 | 26 | 9.69520 | 32 | 0.30480 | 9.95229 | 6 | 38 | 30 | 13.0 | |
| 23 | 9.64775 | 25 | 9.69552 | 32 | 0.30448 | 9.95223 | 6 | 37 | 40 | 17.3 | |
| 24 | 9.64800 | 26 | 9.69584 | 31 | 0.30416 | 9.95217 | 6 | 36 | 50 | 21.7 | |
| 25 | 9.64826 | 25 | 9.69615 | 32 | 0.30385 | 9.95211 | 7 | 35 | | | |
| 26 | 9.64851 | 26 | 9.69647 | 32 | 0.30353 | 9.95204 | 6 | 34 | | **25** | |
| 27 | 9.64877 | 25 | 9.69679 | 31 | 0.30321 | 9.95198 | 6 | 33 | 6 | 2.5 | |
| 28 | 9.64902 | 25 | 9.69710 | 32 | 0.30290 | 9.95192 | 7 | 32 | 7 | 2.9 | |
| 29 | 9.64927 | 26 | 9.69742 | 32 | 0.30258 | 9.95185 | 6 | 31 | 8 | 3.3 | |
| 30 | 9.64953 | 25 | 9.69774 | 31 | 0.30226 | 9.95179 | 6 | 30 | 9 | 3.8 | |
| 31 | 9.64978 | 25 | 9.69805 | 32 | 0.30195 | 9.95173 | 6 | 29 | 10 | 4.2 | |
| 32 | 9.65003 | 26 | 9.69837 | 31 | 0.30163 | 9.95167 | 7 | 28 | 20 | 8.3 | |
| 33 | 9.65029 | 25 | 9.69868 | 32 | 0.30132 | 9.95160 | 6 | 27 | 30 | 12.5 | |
| 34 | 9.65054 | 25 | 9.69900 | 32 | 0.30100 | 9.95154 | 6 | 26 | 40 | 16.7 | |
| 35 | 9.65079 | 25 | 9.69932 | 31 | 0.30068 | 9.95148 | 7 | 25 | 50 | 20.8 | |
| 36 | 9.65104 | 26 | 9.69963 | 32 | 0.30037 | 9.95141 | 6 | 24 | | | |
| 37 | 9.65130 | 25 | 9.69995 | 31 | 0.30005 | 9.95135 | 6 | 23 | | | |
| 38 | 9.65155 | 25 | 9.70026 | 32 | 0.29974 | 9.95129 | 7 | 22 | | **24** | |
| 39 | 9.65180 | 25 | 9.70058 | 31 | 0.29942 | 9.95122 | 6 | 21 | 6 | 2.4 | |
| 40 | 9.65205 | 25 | 9.70089 | 32 | 0.29911 | 9.95116 | 6 | 20 | 7 | 2.8 | |
| 41 | 9.65230 | 25 | 9.70121 | 31 | 0.29879 | 9.95110 | 7 | 19 | 8 | 3.2 | |
| 42 | 9.65255 | 26 | 9.70152 | 32 | 0.29848 | 9.95103 | 6 | 18 | 9 | 3.6 | |
| 43 | 9.65281 | 25 | 9.70184 | 31 | 0.29816 | 9.95097 | 7 | 17 | 10 | 4.0 | |
| 44 | 9.65306 | 25 | 9.70215 | 32 | 0.29785 | 9.95090 | 6 | 16 | 20 | 8.0 | |
| 45 | 9.65331 | 25 | 9.70247 | 31 | 0.29753 | 9.95084 | 6 | 15 | 30 | 12.0 | |
| 46 | 9.65356 | 25 | 9.70278 | 31 | 0.29722 | 9.95078 | 7 | 14 | 40 | 16.0 | |
| 47 | 9.65381 | 25 | 9.70309 | 32 | 0.29691 | 9.95071 | 6 | 13 | 50 | 20.0 | |
| 48 | 9.65406 | 25 | 9.70341 | 31 | 0.29659 | 9.95065 | 6 | 12 | | | |
| 49 | 9.65431 | 25 | 9.70372 | 32 | 0.29628 | 9.95059 | 7 | 11 | | | |
| 50 | 9.65456 | 25 | 9.70404 | 31 | 0.29596 | 9.95052 | 6 | 10 | | **7** | **6** |
| 51 | 9.65481 | 25 | 9.70435 | 31 | 0.29565 | 9.95046 | 6 | 9 | 6 | 0.7 | 0.6 |
| 52 | 9.65506 | 25 | 9.70466 | 32 | 0.29534 | 9.95039 | 7 | 8 | 7 | 0.8 | 0.7 |
| 53 | 9.65531 | 25 | 9.70498 | 31 | 0.29502 | 9.95033 | 6 | 7 | 8 | 0.9 | 0.8 |
| 54 | 9.65556 | 24 | 9.70529 | 31 | 0.29471 | 9.95027 | 7 | 6 | 9 | 1.1 | 0.9 |
| 55 | 9.65580 | 25 | 9.70560 | 32 | 0.29440 | 9.95020 | 6 | 5 | 10 | 1.2 | 1.0 |
| 56 | 9.65605 | 25 | 9.70592 | 31 | 0.29408 | 9.95014 | 6 | 4 | 20 | 2.3 | 2.0 |
| 57 | 9.65630 | 25 | 9.70623 | 31 | 0.29377 | 9.95007 | 7 | 3 | 30 | 3.5 | 3.0 |
| 58 | 9.65655 | 25 | 9.70654 | 31 | 0.29346 | 9.95001 | 6 | 2 | 40 | 4.7 | 4.0 |
| 59 | 9.65680 | 25 | 9.70685 | 32 | 0.29315 | 9.94995 | 7 | 1 | 50 | 5.8 | 5.0 |
| 60 | 9.65705 | | 9.70717 | | 0.29283 | 9.94988 | | 0 | | | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | | P. P. | |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | ′ |
|---|---------|----|---------|-------|----------|---------|----|---|
| 0 | 9.65705 |    | 9.70717 |    | 0.29283 | 9.94988 |   | 60 |
| 1 | 9.65729 | 24 | 9.70748 | 31 | 0.29252 | 9.94982 | 6 | 59 |
| 2 | 9.65754 | 25 | 9.70779 | 31 | 0.29221 | 9.94975 | 7 | 58 |
| 3 | 9.65779 | 25 | 9.70810 | 31 | 0.29190 | 9.94969 | 6 | 57 |
| 4 | 9.65804 | 25 | 9.70841 | 31 | 0.29159 | 9.94962 | 7 | 56 |
| 5 | 9.65828 | 24 | 9.70873 | 32 | 0.29127 | 9.94956 | 6 | 55 |
| 6 | 9.65853 | 25 | 9.70904 | 31 | 0.29096 | 9.94949 | 7 | 54 |
| 7 | 9.65878 | 25 | 9.70935 | 31 | 0.29065 | 9.94943 | 6 | 53 |
| 8 | 9.65902 | 24 | 9.70966 | 31 | 0.29034 | 9.94936 | 7 | 52 |
| 9 | 9.65927 | 25 | 9.70997 | 31 | 0.29003 | 9.94930 | 6 | 51 |
| 10 | 9.65952 | 24 | 9.71028 | 31 | 0.28972 | 9.94923 | 7 | 50 |
| 11 | 9.65976 | 25 | 9.71059 | 31 | 0.28941 | 9.94917 | 6 | 49 |
| 12 | 9.66001 | 24 | 9.71090 | 31 | 0.28910 | 9.94911 | 6 | 48 |
| 13 | 9.66025 | 25 | 9.71121 | 32 | 0.28879 | 9.94904 | 7 | 47 |
| 14 | 9.66050 | 25 | 9.71153 | 31 | 0.28847 | 9.94898 | 6 | 46 |
| 15 | 9.66075 | 24 | 9.71184 | 31 | 0.28816 | 9.94891 | 7 | 45 |
| 16 | 9.66099 | 25 | 9.71215 | 31 | 0.28785 | 9.94885 | 6 | 44 |
| 17 | 9.66124 | 24 | 9.71246 | 31 | 0.28754 | 9.94878 | 7 | 43 |
| 18 | 9.66148 | 25 | 9.71277 | 31 | 0.28723 | 9.94871 | 7 | 42 |
| 19 | 9.66173 | 24 | 9.71308 | 31 | 0.28692 | 9.94865 | 6 | 41 |
| 20 | 9.66197 | 24 | 9.71339 | 31 | 0.28661 | 9.94858 | 6 | 40 |
| 21 | 9.66221 | 25 | 9.71370 | 31 | 0.28630 | 9.94852 | 7 | 39 |
| 22 | 9.66246 | 24 | 9.71401 | 30 | 0.28599 | 9.94845 | 6 | 38 |
| 23 | 9.66270 | 25 | 9.71431 | 31 | 0.28569 | 9.94839 | 7 | 37 |
| 24 | 9.66295 | 24 | 9.71462 | 31 | 0.28538 | 9.94832 | 6 | 36 |
| 25 | 9.66319 | 24 | 9.71493 | 31 | 0.28507 | 9.94826 | 7 | 35 |
| 26 | 9.66343 | 25 | 9.71524 | 31 | 0.28476 | 9.94819 | 6 | 34 |
| 27 | 9.66368 | 24 | 9.71555 | 31 | 0.28445 | 9.94813 | 7 | 33 |
| 28 | 9.66392 | 24 | 9.71586 | 31 | 0.28414 | 9.94806 | 7 | 32 |
| 29 | 9.66416 | 25 | 9.71617 | 31 | 0.28383 | 9.94799 | 6 | 31 |
| 30 | 9.66441 | 24 | 9.71648 | 31 | 0.28352 | 9.94793 | 7 | 30 |
| 31 | 9.66465 | 24 | 9.71679 | 30 | 0.28321 | 9.94786 | 6 | 29 |
| 32 | 9.66489 | 24 | 9.71709 | 31 | 0.28291 | 9.94780 | 7 | 28 |
| 33 | 9.66513 | 24 | 9.71740 | 31 | 0.28260 | 9.94773 | 6 | 27 |
| 34 | 9.66537 | 25 | 9.71771 | 31 | 0.28229 | 9.94767 | 7 | 26 |
| 35 | 9.66562 | 24 | 9.71802 | 31 | 0.28198 | 9.94760 | 7 | 25 |
| 36 | 9.66586 | 24 | 9.71833 | 30 | 0.28167 | 9.94753 | 6 | 24 |
| 37 | 9.66610 | 24 | 9.71863 | 31 | 0.28137 | 9.94747 | 7 | 23 |
| 38 | 9.66634 | 24 | 9.71894 | 31 | 0.28106 | 9.94740 | 6 | 22 |
| 39 | 9.66658 | 24 | 9.71925 | 30 | 0.28075 | 9.94734 | 7 | 21 |
| 40 | 9.66682 | 24 | 9.71955 | 31 | 0.28045 | 9.94727 | 7 | 20 |
| 41 | 9.66706 | 24 | 9.71986 | 31 | 0.28014 | 9.94720 | 6 | 19 |
| 42 | 9.66731 | 25 | 9.72017 | 31 | 0.27983 | 9.94714 | 7 | 18 |
| 43 | 9.66755 | 24 | 9.72048 | 31 | 0.27952 | 9.94707 | 7 | 17 |
| 44 | 9.66779 | 24 | 9.72078 | 31 | 0.27922 | 9.94700 | 6 | 16 |
| 45 | 9.66803 | 24 | 9.72109 | 31 | 0.27891 | 9.94694 | 7 | 15 |
| 46 | 9.66827 | 24 | 9.72140 | 30 | 0.27860 | 9.94687 | 7 | 14 |
| 47 | 9.66851 | 24 | 9.72170 | 31 | 0.27830 | 9.94680 | 6 | 13 |
| 48 | 9.66875 | 24 | 9.72201 | 30 | 0.27799 | 9.94674 | 7 | 12 |
| 49 | 9.66899 | 23 | 9.72231 | 31 | 0.27769 | 9.94667 | 7 | 11 |
| 50 | 9.66922 | 24 | 9.72262 | 31 | 0.27738 | 9.94660 | 6 | 10 |
| 51 | 9.66943 | 24 | 9.72293 | 30 | 0.27707 | 9.94654 | 7 | 9 |
| 52 | 9.66970 | 24 | 9.72323 | 31 | 0.27677 | 9.94647 | 7 | 8 |
| 53 | 9.66994 | 24 | 9.72354 | 30 | 0.27646 | 9.94640 | 6 | 7 |
| 54 | 9.67018 | 24 | 9.72384 | 31 | 0.27616 | 9.94634 | 7 | 6 |
| 55 | 9.67042 | 24 | 9.72415 | 30 | 0.27585 | 9.94627 | 7 | 5 |
| 56 | 9.67066 | 24 | 9.72445 | 31 | 0.27555 | 9.94620 | 6 | 4 |
| 57 | 9.67090 | 23 | 9.72476 | 30 | 0.27524 | 9.94614 | 7 | 3 |
| 58 | 9.67113 | 24 | 9.72506 | 31 | 0.27494 | 9.94607 | 7 | 2 |
| 59 | 9.67137 | 24 | 9.72537 | 30 | 0.27463 | 9.94600 | 7 | 1 |
| 60 | 9.67161 |    | 9.72567 |    | 0.27433 | 9.94593 |   | 0 |
|   | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ |

**P. P.**

| | 32 | 31 |
|---|-----|-----|
| 6 | 3.2 | 3.1 |
| 7 | 3.7 | 3.6 |
| 8 | 4.3 | 4.1 |
| 9 | 4.8 | 4.7 |
| 10 | 5.3 | 5.2 |
| 20 | 10.7 | 10.3 |
| 30 | 16.0 | 15.5 |
| 40 | 21.3 | 20.7 |
| 50 | 26.7 | 25.8 |

| | 30 |
|---|-----|
| 6 | 3.0 |
| 7 | 3.5 |
| 8 | 4.0 |
| 9 | 4.5 |
| 10 | 5.0 |
| 20 | 10.0 |
| 30 | 15.0 |
| 40 | 20.0 |
| 50 | 25.0 |

| | 25 | 24 |
|---|-----|-----|
| 6 | 2.5 | 2.4 |
| 7 | 2.9 | 2.8 |
| 8 | 3.3 | 3.2 |
| 9 | 3.8 | 3.6 |
| 10 | 4.2 | 4.0 |
| 20 | 8.3 | 8.0 |
| 30 | 12.5 | 12.0 |
| 40 | 16.7 | 16.0 |
| 50 | 20.8 | 20.0 |

| | 23 |
|---|-----|
| 6 | 2.3 |
| 7 | 2.7 |
| 8 | 3.1 |
| 9 | 3.5 |
| 10 | 3.8 |
| 20 | 7.7 |
| 30 | 11.5 |
| 40 | 15.3 |
| 50 | 19.2 |

| | 7 | 6 |
|---|-----|-----|
| 6 | 0.7 | 0.6 |
| 7 | 0.8 | 0.7 |
| 8 | 0.9 | 0.8 |
| 9 | 1.1 | 0.9 |
| 10 | 1.2 | 1.0 |
| 20 | 2.3 | 2.0 |
| 30 | 3.5 | 3.0 |
| 40 | 4.7 | 4.0 |
| 50 | 5.8 | 5.0 |

## 28°

| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | P. P. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 9.67161 | 24 | 9.72567 | 31 | 0.27433 | 9.94593 | 6 | 60 | |
| 1 | 9.67185 | 23 | 9.72598 | 30 | 0.27402 | 9.94587 | 7 | 59 | |
| 2 | 9.67208 | 24 | 9.72628 | 31 | 0.27372 | 9.94580 | 7 | 58 | **31** **30** |
| 3 | 9.67232 | 24 | 9.72659 | 30 | 0.27341 | 9.94573 | 6 | 57 | 6 3.1 3.0 |
| 4 | 9.67256 | 24 | 9.72689 | 31 | 0.27311 | 9.94567 | 7 | 56 | 7 3.6 3.5 |
| 5 | 9.67280 | 23 | 9.72720 | 30 | 0.27280 | 9.94560 | 7 | 55 | 8 4.1 4.0 |
| 6 | 9.67303 | 24 | 9.72750 | 30 | 0.27250 | 9.94553 | 7 | 54 | 9 4.7 4.5 |
| 7 | 9.67327 | 23 | 9.72780 | 31 | 0.27220 | 9.94546 | 6 | 53 | 10 5.2 5.0 |
| 8 | 9.67350 | 24 | 9.72811 | 30 | 0.27189 | 9.94540 | 7 | 52 | 20 10.3 10.0 |
| 9 | 9.67374 | 24 | 9.72841 | 31 | 0.27159 | 9.94533 | 7 | 51 | 30 15.5 15.0 |
| 10 | 9.67398 | 23 | 9.72872 | 30 | 0.27128 | 9.94526 | 7 | 50 | 40 20.7 20.0 |
| 11 | 9.67421 | 24 | 9.72902 | 30 | 0.27098 | 9.94519 | 6 | 49 | 50 25.8 25.0 |
| 12 | 9.67445 | 23 | 9.72932 | 31 | 0.27068 | 9.94513 | 7 | 48 | |
| 13 | 9.67468 | 24 | 9.72963 | 30 | 0.27037 | 9.94506 | 7 | 47 | |
| 14 | 9.67492 | 23 | 9.72993 | 30 | 0.27007 | 9.94499 | 7 | 46 | |
| 15 | 9.67515 | 24 | 9.73023 | 31 | 0.26977 | 9.94492 | 7 | 45 | **29** |
| 16 | 9.67539 | 23 | 9.73054 | 30 | 0.26946 | 9.94485 | 6 | 44 | 6 2.9 |
| 17 | 9.67562 | 24 | 9.73084 | 30 | 0.26916 | 9.94479 | 7 | 43 | 7 3.4 |
| 18 | 9.67586 | 23 | 9.73114 | 30 | 0.26886 | 9.94472 | 7 | 42 | 8 3.9 |
| 19 | 9.67609 | 24 | 9.73144 | 31 | 0.26856 | 9.94465 | 7 | 41 | 9 4.4 |
| 20 | 9.67633 | 23 | 9.73175 | 30 | 0.26825 | 9.94458 | 7 | 40 | 10 4.8 |
| 21 | 9.67656 | 24 | 9.73205 | 30 | 0.26795 | 9.94451 | 6 | 39 | 20 9.7 |
| 22 | 9.67680 | 23 | 9.73235 | 30 | 0.26765 | 9.94445 | 7 | 38 | 30 14.5 |
| 23 | 9.67703 | 23 | 9.73265 | 30 | 0.26735 | 9.94438 | 7 | 37 | 40 19.3 |
| 24 | 9.67726 | 24 | 9.73295 | 31 | 0.26705 | 9.94431 | 7 | 36 | 50 24.2 |
| 25 | 9.67750 | 23 | 9.73326 | 30 | 0.26674 | 9.94424 | 7 | 35 | |
| 26 | 9.67773 | 23 | 9.73356 | 30 | 0.26644 | 9.94417 | 7 | 34 | **24** **23** |
| 27 | 9.67796 | 24 | 9.73386 | 30 | 0.26614 | 9.94410 | 6 | 33 | 6 2.4 2.3 |
| 28 | 9.67820 | 23 | 9.73416 | 30 | 0.26584 | 9.94404 | 7 | 32 | 7 2.8 2.7 |
| 29 | 9.67843 | 23 | 9.73446 | 30 | 0.26554 | 9.94397 | 7 | 31 | 8 3.2 3.1 |
| 30 | 9.67866 | 24 | 9.73476 | 31 | 0.26524 | 9.94390 | 7 | 30 | 9 3.6 3.5 |
| 31 | 9.67890 | 23 | 9.73507 | 30 | 0.26493 | 9.94383 | 7 | 29 | 10 4.0 3.8 |
| 32 | 9.67913 | 23 | 9.73537 | 30 | 0.26463 | 9.94376 | 7 | 28 | 20 8.0 7.7 |
| 33 | 9.67936 | 23 | 9.73567 | 30 | 0.26433 | 9.94369 | 7 | 27 | 30 12.0 11.5 |
| 34 | 9.67959 | 23 | 9.73597 | 30 | 0.26403 | 9.94362 | 7 | 26 | 40 16.0 15.3 |
| 35 | 9.67982 | 24 | 9.73627 | 30 | 0.26373 | 9.94355 | 6 | 25 | 50 20.0 19.2 |
| 36 | 9.68006 | 23 | 9.73657 | 30 | 0.26343 | 9.94349 | 7 | 24 | |
| 37 | 9.68029 | 23 | 9.73687 | 30 | 0.26313 | 9.94342 | 7 | 23 | |
| 38 | 9.68052 | 23 | 9.73717 | 30 | 0.26283 | 9.94335 | 7 | 22 | **22** |
| 39 | 9.68075 | 23 | 9.73747 | 30 | 0.26253 | 9.94328 | 7 | 21 | 6 2.2 |
| 40 | 9.68098 | 23 | 9.73777 | 30 | 0.26223 | 9.94321 | 7 | 20 | 7 2.6 |
| 41 | 9.68121 | 23 | 9.73807 | 30 | 0.26193 | 9.94314 | 7 | 19 | 8 2.9 |
| 42 | 9.68144 | 23 | 9.73837 | 30 | 0.26163 | 9.94307 | 7 | 18 | 9 3.3 |
| 43 | 9.68167 | 23 | 9.73867 | 30 | 0.26133 | 9.94300 | 7 | 17 | 10 3.7 |
| 44 | 9.68190 | 23 | 9.73897 | 30 | 0.26103 | 9.94293 | 7 | 16 | 20 7.3 |
| 45 | 9.68213 | 24 | 9.73927 | 30 | 0.26073 | 9.94286 | 7 | 15 | 30 11.0 |
| 46 | 9.68237 | 23 | 9.73957 | 30 | 0.26043 | 9.94279 | 6 | 14 | 40 14.7 |
| 47 | 9.68260 | 23 | 9.73987 | 30 | 0.26013 | 9.94273 | 7 | 13 | 50 18.3 |
| 48 | 9.68283 | 22 | 9.74017 | 30 | 0.25983 | 9.94266 | 7 | 12 | |
| 49 | 9.68305 | 23 | 9.74047 | 30 | 0.25953 | 9.94259 | 7 | 11 | |
| 50 | 9.68328 | 23 | 9.74077 | 30 | 0.25923 | 9.94252 | 7 | 10 | **7** **8** |
| 51 | 9.68351 | 23 | 9.74107 | 30 | 0.25893 | 9.94245 | 7 | 9 | 6 0.7 0.6 |
| 52 | 9.68374 | 23 | 9.74137 | 29 | 0.25863 | 9.94238 | 7 | 8 | 7 0.8 0.7 |
| 53 | 9.68397 | 23 | 9.74166 | 30 | 0.25834 | 9.94231 | 7 | 7 | 8 0.9 0.8 |
| 54 | 9.68420 | 23 | 9.74196 | 30 | 0.25804 | 9.94224 | 7 | 6 | 9 1.1 0.9 |
| 55 | 9.68443 | 23 | 9.74226 | 30 | 0.25774 | 9.94217 | 7 | 5 | 10 1.2 1.0 |
| 56 | 9.68466 | 23 | 9.74256 | 30 | 0.25744 | 9.94210 | 7 | 4 | 20 2.3 2.0 |
| 57 | 9.68489 | 23 | 9.74286 | 30 | 0.25714 | 9.94203 | 7 | 3 | 30 3.5 3.0 |
| 58 | 9.68512 | 22 | 9.74316 | 29 | 0.25684 | 9.94196 | 7 | 2 | 40 4.7 4.0 |
| 59 | 9.68534 | 23 | 9.74345 | 30 | 0.25655 | 9.94189 | 7 | 1 | 50 5.8 5.0 |
| 60 | 9.68557 | | 9.74375 | | 0.25625 | 9.94182 | | 0 | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. |

## 29°

| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | P. P. |
|---|---------|----|---------|-------|----------|---------|----|--|-------|
| 0 | 9.68557 | 23 | 9.74375 | 30 | 0.25625 | 9.94182 | 7 | 60 | |
| 1 | 9.68580 | 23 | 9.74405 | 30 | 0.25595 | 9.94175 | 7 | 59 | |
| 2 | 9.68603 | 22 | 9.74435 | 30 | 0.25565 | 9.94168 | 7 | 58 | |
| 3 | 9.68625 | 23 | 9.74465 | 29 | 0.25535 | 9.94161 | 7 | 57 | **30** |
| 4 | 9.68648 | 23 | 9.74494 | 30 | 0.25506 | 9.94154 | 7 | 56 | 6  3.0 |
| 5 | 9.68671 | 23 | 9.74524 | 30 | 0.25476 | 9.94147 | 7 | 55 | 7  3.5 |
| 6 | 9.68694 | 22 | 9.74554 | 29 | 0.25446 | 9.94140 | 7 | 54 | 8  4.0 |
| 7 | 9.68716 | 23 | 9.74583 | 30 | 0.25417 | 9.94133 | 7 | 53 | 9  4.5 |
| 8 | 9.68739 | 23 | 9.74613 | 30 | 0.25387 | 9.94126 | 7 | 52 | 10  5.0 |
| 9 | 9.68762 | 22 | 9.74643 | 30 | 0.25357 | 9.94119 | 7 | 51 | 20  10.0 |
| 10 | 9.68784 | 23 | 9.74673 | 29 | 0.25327 | 9.94112 | 7 | 50 | 30  15.0 |
| 11 | 9.68807 | 22 | 9.74702 | 30 | 0.25298 | 9.94105 | 7 | 49 | 40  20.0 |
| 12 | 9.68829 | 23 | 9.74732 | 30 | 0.25268 | 9.94098 | 7 | 48 | 50  25.0 |
| 13 | 9.68852 | 23 | 9.74762 | 29 | 0.25238 | 9.94090 | 8 | 47 | |
| 14 | 9.68875 | 22 | 9.74791 | 30 | 0.25209 | 9.94083 | 7 | 46 | |
| 15 | 9.68897 | 23 | 9.74821 | 30 | 0.25179 | 9.94076 | 7 | 45 | **29** |
| 16 | 9.68920 | 22 | 9.74851 | 29 | 0.25149 | 9.94069 | 7 | 44 | 6  2.9 |
| 17 | 9.68942 | 23 | 9.74880 | 30 | 0.25120 | 9.94062 | 7 | 43 | 7  3.4 |
| 18 | 9.68965 | 22 | 9.74910 | 29 | 0.25090 | 9.94055 | 7 | 42 | 8  3.9 |
| 19 | 9.68987 | 23 | 9.74939 | 30 | 0.25061 | 9.94048 | 7 | 41 | 9  4.4 |
| 20 | 9.69010 | 22 | 9.74969 | 29 | 0.25031 | 9.94041 | 7 | 40 | 10  4.8 |
| 21 | 9.69032 | 23 | 9.74998 | 30 | 0.25002 | 9.94034 | 7 | 39 | 20  9.7 |
| 22 | 9.69055 | 22 | 9.75028 | 30 | 0.24972 | 9.94027 | 7 | 38 | 30  14.5 |
| 23 | 9.69077 | 23 | 9.75058 | 29 | 0.24942 | 9.94020 | 8 | 37 | 50  24.2 |
| 24 | 9.69100 | 22 | 9.75087 | 30 | 0.24913 | 9.94012 | 7 | 36 | |
| 25 | 9.69122 | 22 | 9.75117 | 29 | 0.24883 | 9.94005 | 7 | 35 | |
| 26 | 9.69144 | 23 | 9.75146 | 30 | 0.24854 | 9.93998 | 7 | 34 | **23** |
| 27 | 9.69167 | 22 | 9.75176 | 29 | 0.24824 | 9.93991 | 7 | 33 | 6  2.3 |
| 28 | 9.69189 | 23 | 9.75205 | 30 | 0.24795 | 9.93984 | 7 | 32 | 7  2.7 |
| 29 | 9.69212 | 22 | 9.75235 | 29 | 0.24765 | 9.93977 | 7 | 31 | 8  3.1 |
| 30 | 9.69234 | 22 | 9.75264 | 30 | 0.24736 | 9.93970 | 7 | 30 | 9  3.5 |
| 31 | 9.69256 | 23 | 9.75294 | 29 | 0.24706 | 9.93963 | 8 | 29 | 10  3.8 |
| 32 | 9.69279 | 22 | 9.75323 | 30 | 0.24677 | 9.93955 | 7 | 28 | 20  7.7 |
| 33 | 9.69301 | 22 | 9.75353 | 29 | 0.24647 | 9.93948 | 7 | 27 | 30  11.5 |
| 34 | 9.69323 | 22 | 9.75382 | 29 | 0.24618 | 9.93941 | 7 | 26 | 40  15.3 |
| 35 | 9.69345 | 23 | 9.75411 | 30 | 0.24589 | 9.93934 | 7 | 25 | 50  19.2 |
| 36 | 9.69368 | 22 | 9.75441 | 29 | 0.24559 | 9.93927 | 7 | 24 | |
| 37 | 9.69390 | 22 | 9.75470 | 30 | 0.24530 | 9.93920 | 8 | 23 | |
| 38 | 9.69412 | 22 | 9.75500 | 29 | 0.24500 | 9.93912 | 7 | 22 | **22** |
| 39 | 9.69434 | 22 | 9.75529 | 29 | 0.24471 | 9.93905 | 7 | 21 | 6  2.2 |
| 40 | 9.69456 | 23 | 9.75558 | 30 | 0.24442 | 9.93898 | 7 | 20 | 7  2.6 |
| 41 | 9.69479 | 22 | 9.75588 | 29 | 0.24412 | 9.93891 | 7 | 19 | 8  2.9 |
| 42 | 9.69501 | 22 | 9.75617 | 30 | 0.24383 | 9.93884 | 8 | 18 | 9  3.3 |
| 43 | 9.69523 | 22 | 9.75647 | 29 | 0.24353 | 9.93876 | 7 | 17 | 10  3.7 |
| 44 | 9.69545 | 22 | 9.75676 | 29 | 0.24324 | 9.93869 | 7 | 16 | 20  7.3 |
| 45 | 9.69567 | 22 | 9.75705 | 30 | 0.24295 | 9.93862 | 7 | 15 | 30  11.0 |
| 46 | 9.69589 | 22 | 9.75735 | 29 | 0.24265 | 9.93855 | 8 | 14 | 40  14.7 |
| 47 | 9.69611 | 22 | 9.75764 | 29 | 0.24236 | 9.93847 | 7 | 13 | 50  18.3 |
| 48 | 9.69633 | 22 | 9.75793 | 29 | 0.24207 | 9.93840 | 7 | 12 | |
| 49 | 9.69655 | 22 | 9.75822 | 30 | 0.24178 | 9.93833 | 7 | 11 | |
| 50 | 9.69677 | 22 | 9.75852 | 29 | 0.24148 | 9.93826 | 7 | 10 | **8**  **7** |
| 51 | 9.69699 | 22 | 9.75881 | 29 | 0.24119 | 9.93819 | 8 | 9 | 6  0.8  0.7 |
| 52 | 9.69721 | 22 | 9.75910 | 29 | 0.24090 | 9.93811 | 7 | 8 | 7  0.9  0.8 |
| 53 | 9.69743 | 22 | 9.75939 | 30 | 0.24061 | 9.93804 | 7 | 7 | 8  1.1  0.9 |
| 54 | 9.69765 | 22 | 9.75969 | 29 | 0.24031 | 9.93797 | 8 | 6 | 9  1.2  1.1 |
| 55 | 9.69787 | 22 | 9.75998 | 29 | 0.24002 | 9.93789 | 7 | 5 | 10  1.3  1.2 |
| 56 | 9.69809 | 22 | 9.76027 | 29 | 0.23973 | 9.93782 | 7 | 4 | 20  2.7  2.3 |
| 57 | 9.69831 | 22 | 9.76056 | 30 | 0.23944 | 9.93775 | 7 | 3 | 30  4.0  3.5 |
| 58 | 9.69853 | 22 | 9.76086 | 29 | 0.23914 | 9.93768 | 8 | 2 | 40  5.3  4.7 |
| 59 | 9.69875 | 22 | 9.76115 | 29 | 0.23885 | 9.93760 | 7 | 1 | 50  6.7  5.8 |
| 60 | 9.69897 | | 9.76144 | | 0.23856 | 9.93753 | | 0 | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | | P. P. | |
|---|---------|----|---------|-------|----------|---------|----|---|---|-------|---|
| 0 | 9.69897 |    | 9.76144 |    | 0.23856 | 9.93753 |   | 60 | | | |
| 1 | 9.69919 | 22 | 9.76173 | 29 | 0.23827 | 9.93746 | 7 | 59 | | | |
| 2 | 9.69941 | 22 | 9.76202 | 29 | 0.23798 | 9.93738 | 8 | 58 | | **30** | **29** |
| 3 | 9.69963 | 22 | 9.76231 | 29 | 0.23769 | 9.93731 | 7 | 57 | 6 | 3.0 | 2.9 |
| 4 | 9.69984 | 21 | 9.76261 | 30 | 0.23739 | 9.93724 | 7 | 56 | 7 | 3.5 | 3.4 |
| 5 | 9.70006 | 22 | 9.76290 | 29 | 0.23710 | 9.93717 | 7 | 55 | 8 | 4.0 | 3.9 |
| 6 | 9.70028 | 22 | 9.76319 | 29 | 0.23681 | 9.93709 | 8 | 54 | 9 | 4.5 | 4.4 |
| 7 | 9.70050 | 22 | 9.76348 | 29 | 0.23652 | 9.93702 | 7 | 53 | 10 | 5.0 | 4.8 |
| 8 | 9.70072 | 22 | 9.76377 | 29 | 0.23623 | 9.93695 | 7 | 52 | 20 | 10.0 | 9.7 |
| 9 | 9.70093 | 21 | 9.76406 | 29 | 0.23594 | 9.93687 | 8 | 51 | 30 | 15.0 | 14.5 |
| 10 | 9.70115 | 22 | 9.76435 | 29 | 0.23565 | 9.93680 | 7 | 50 | 40 | 20.0 | 19.3 |
| 11 | 9.70137 | 22 | 9.76464 | 29 | 0.23536 | 9.93673 | 7 | 49 | 50 | 25.0 | 24.2 |
| 12 | 9.70159 | 22 | 9.76493 | 29 | 0.23507 | 9.93665 | 8 | 48 | | | |
| 13 | 9.70180 | 21 | 9.76522 | 29 | 0.23478 | 9.93658 | 7 | 47 | | | |
| 14 | 9.70202 | 22 | 9.76551 | 29 | 0.23449 | 9.93650 | 8 | 46 | | | |
| 15 | 9.70224 | 22 | 9.76580 | 29 | 0.23420 | 9.93643 | 7 | 45 | | **28** | |
| 16 | 9.70245 | 21 | 9.76609 | 29 | 0.23391 | 9.93636 | 7 | 44 | 6 | 2.8 | |
| 17 | 9.70267 | 22 | 9.76639 | 30 | 0.23361 | 9.93628 | 8 | 43 | 7 | 3.3 | |
| 18 | 9.70288 | 21 | 9.76668 | 29 | 0.23332 | 9.93621 | 7 | 42 | 8 | 3.7 | |
| 19 | 9.70310 | 22 | 9.76697 | 29 | 0.23303 | 9.93614 | 7 | 41 | 9 | 4.2 | |
| 20 | 9.70332 | 22 | 9.76725 | 28 | 0.23275 | 9.93606 | 8 | 40 | 10 | 4.7 | |
| 21 | 9.70353 | 21 | 9.76754 | 29 | 0.23246 | 9.93599 | 7 | 39 | 20 | 9.3 | |
| 22 | 9.70375 | 22 | 9.76783 | 29 | 0.23217 | 9.93591 | 8 | 38 | 30 | 14.0 | |
| 23 | 9.70396 | 21 | 9.76812 | 29 | 0.23188 | 9.93584 | 7 | 37 | 40 | 18.7 | |
| 24 | 9.70418 | 22 | 9.76841 | 29 | 0.23159 | 9.93577 | 7 | 36 | 50 | 23.3 | |
| 25 | 9.70439 | 21 | 9.76870 | 29 | 0.23130 | 9.93569 | 8 | 35 | | | |
| 26 | 9.70461 | 22 | 9.76899 | 29 | 0.23101 | 9.93562 | 7 | 34 | | **22** | |
| 27 | 9.70482 | 21 | 9.76928 | 29 | 0.23072 | 9.93554 | 8 | 33 | 6 | 2.2 | |
| 28 | 9.70504 | 22 | 9.76957 | 29 | 0.23043 | 9.93547 | 7 | 32 | 7 | 2.6 | |
| 29 | 9.70525 | 21 | 9.76986 | 29 | 0.23014 | 9.93539 | 8 | 31 | 8 | 2.9 | |
| 30 | 9.70547 | 22 | 9.77015 | 29 | 0.22985 | 9.93532 | 7 | 30 | 9 | 3.3 | |
| 31 | 9.70568 | 21 | 9.77044 | 29 | 0.22956 | 9.93525 | 8 | 29 | 10 | 3.7 | |
| 32 | 9.70590 | 22 | 9.77073 | 28 | 0.22927 | 9.93517 | 7 | 28 | 20 | 7.3 | |
| 33 | 9.70611 | 21 | 9.77101 | 29 | 0.22899 | 9.93510 | 8 | 27 | 30 | 11.0 | |
| 34 | 9.70633 | 22 | 9.77130 | 29 | 0.22870 | 9.93502 | 8 | 26 | 40 | 14.7 | |
| 35 | 9.70654 | 21 | 9.77159 | 29 | 0.22841 | 9.93495 | 7 | 25 | 50 | 18.3 | |
| 36 | 9.70675 | 21 | 9.77188 | 29 | 0.22812 | 9.93487 | 8 | 24 | | | |
| 37 | 9.70697 | 22 | 9.77217 | 29 | 0.22783 | 9.93480 | 7 | 23 | | | |
| 38 | 9.70718 | 21 | 9.77246 | 28 | 0.22754 | 9.93472 | 8 | 22 | | **21** | |
| 39 | 9.70739 | 21 | 9.77274 | 29 | 0.22726 | 9.93465 | 7 | 21 | 6 | 2.1 | |
| 40 | 9.70761 | 22 | 9.77303 | 29 | 0.22697 | 9.93457 | 7 | 20 | 7 | 2.5 | |
| 41 | 9.70782 | 21 | 9.77332 | 29 | 0.22668 | 9.93450 | 8 | 19 | 8 | 2.8 | |
| 42 | 9.70803 | 21 | 9.77361 | 29 | 0.22639 | 9.93442 | 7 | 18 | 9 | 3.2 | |
| 43 | 9.70824 | 22 | 9.77390 | 28 | 0.22610 | 9.93435 | 8 | 17 | 10 | 3.5 | |
| 44 | 9.70846 | 21 | 9.77418 | 29 | 0.22582 | 9.93427 | 8 | 16 | 20 | 7.0 | |
| 45 | 9.70867 | 21 | 9.77447 | 29 | 0.22553 | 9.93420 | 7 | 15 | 30 | 10.5 | |
| 46 | 9.70888 | 21 | 9.77476 | 29 | 0.22524 | 9.93412 | 8 | 14 | 40 | 14.0 | |
| 47 | 9.70909 | 22 | 9.77505 | 29 | 0.22495 | 9.93405 | 7 | 13 | 50 | 17.5 | |
| 48 | 9.70931 | 21 | 9.77533 | 28 | 0.22467 | 9.93397 | 8 | 12 | | | |
| 49 | 9.70952 | 21 | 9.77562 | 29 | 0.22438 | 9.93390 | 7 | 11 | | **8** | **7** |
| 50 | 9.70973 | 21 | 9.77591 | 29 | 0.22409 | 9.93382 | 7 | 10 | 6 | 0.8 | 0.7 |
| 51 | 9.70994 | 21 | 9.77619 | 28 | 0.22381 | 9.93375 | 8 | 9 | 7 | 0.9 | 0.8 |
| 52 | 9.71015 | 21 | 9.77648 | 29 | 0.22352 | 9.93367 | 8 | 8 | 8 | 1.1 | 0.9 |
| 53 | 9.71036 | 22 | 9.77677 | 29 | 0.22323 | 9.93360 | 7 | 7 | 9 | 1.2 | 1.1 |
| 54 | 9.71058 | 21 | 9.77706 | 28 | 0.22294 | 9.93352 | 8 | 6 | 10 | 1.3 | 1.2 |
| 55 | 9.71079 | 21 | 9.77734 | 29 | 0.22266 | 9.93344 | 7 | 5 | 20 | 2.7 | 2.3 |
| 56 | 9.71100 | 21 | 9.77763 | 28 | 0.22237 | 9.93337 | 8 | 4 | 30 | 4.0 | 3.5 |
| 57 | 9.71121 | 21 | 9.77791 | 29 | 0.22209 | 9.93329 | 7 | 3 | 40 | 5.3 | 4.7 |
| 58 | 9.71142 | 21 | 9.77820 | 29 | 0.22180 | 9.93322 | 8 | 2 | 50 | 6.7 | 5.8 |
| 59 | 9.71163 | 21 | 9.77849 | 28 | 0.22151 | 9.93314 | 7 | 1 | | | |
| 60 | 9.71184 |    | 9.77877 |    | 0.22123 | 9.93307 |   | 0 | | | |
|   | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | | P. P. | |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | P. P. |
|---|---------|-----|---------|-------|----------|---------|-----|-----|-------|
| 0 | 9.71184 | 21 | 9.77877 | 29 | 0.22123 | 9.93307 | 8 | 60 | |
| 1 | 9.71205 | 21 | 9.77906 | 29 | 0.22094 | 9.93299 | 8 | 59 | |
| 2 | 9.71226 | 21 | 9.77935 | 29 | 0.22065 | 9.93291 | 8 | 58 | |
| 3 | 9.71247 | 21 | 9.77963 | 28 | 0.22037 | 9.93284 | 7 | 57 | **29** |
| 4 | 9.71268 | 21 | 9.77992 | 29 | 0.22008 | 9.93276 | 8 | 56 | 6 | 2.9 |
| 5 | 9.71289 | 21 | 9.78020 | 28 | 0.21980 | 9.93269 | 7 | 55 | 7 | 3.4 |
| 6 | 9.71310 | 21 | 9.78049 | 29 | 0.21951 | 9.93261 | 8 | 54 | 8 | 3.9 |
| 7 | 9.71331 | 21 | 9.78077 | 28 | 0.21923 | 9.93253 | 8 | 53 | 9 | 4.4 |
| 8 | 9.71352 | 21 | 9.78106 | 29 | 0.21894 | 9.93246 | 7 | 52 | 10 | 4.8 |
| 9 | 9.71373 | 21 | 9.78135 | 28 | 0.21865 | 9.93238 | 8 | 51 | 20 | 9.7 |
| 10 | 9.71393 | 20 | 9.78163 | 28 | 0.21837 | 9.93230 | 8 | 50 | 30 | 14.5 |
| 11 | 9.71414 | 21 | 9.78192 | 29 | 0.21808 | 9.93223 | 7 | 49 | 40 | 19.3 |
| 12 | 9.71435 | 21 | 9.78220 | 28 | 0.21780 | 9.93215 | 8 | 48 | 50 | 24.2 |
| 13 | 9.71456 | 21 | 9.78249 | 29 | 0.21751 | 9.93207 | 8 | 47 | |
| 14 | 9.71477 | 21 | 9.78277 | 28 | 0.21723 | 9.93200 | 7 | 46 | |
| 15 | 9.71498 | 21 | 9.78306 | 29 | 0.21694 | 9.93192 | 8 | 45 | **28** |
| 16 | 9.71519 | 21 | 9.78334 | 28 | 0.21666 | 9.93184 | 8 | 44 | 6 | 2.8 |
| 17 | 9.71539 | 20 | 9.78363 | 29 | 0.21637 | 9.93177 | 7 | 43 | 7 | 3.3 |
| 18 | 9.71560 | 21 | 9.78391 | 28 | 0.21609 | 9.93169 | 8 | 42 | 8 | 3.7 |
| 19 | 9.71581 | 21 | 9.78419 | 28 | 0.21581 | 9.93161 | 8 | 41 | 9 | 4.2 |
| 20 | 9.71602 | 21 | 9.78448 | 29 | 0.21552 | 9.93154 | 7 | 40 | 10 | 4.7 |
| 21 | 9.71622 | 20 | 9.78476 | 28 | 0.21524 | 9.93146 | 8 | 39 | 20 | 9.3 |
| 22 | 9.71643 | 21 | 9.78505 | 29 | 0.21495 | 9.93138 | 8 | 38 | 30 | 14.0 |
| 23 | 9.71664 | 21 | 9.78533 | 28 | 0.21467 | 9.93131 | 7 | 37 | 40 | 18.7 |
| 24 | 9.71685 | 21 | 9.78562 | 29 | 0.21438 | 9.93123 | 8 | 36 | 50 | 23.3 |
| 25 | 9.71705 | 20 | 9.78590 | 28 | 0.21410 | 9.93115 | 8 | 35 | |
| 26 | 9.71726 | 21 | 9.78618 | 28 | 0.21382 | 9.93108 | 7 | 34 | **21** |
| 27 | 9.71747 | 21 | 9.78647 | 29 | 0.21353 | 9.93100 | 8 | 33 | 6 | 2.1 |
| 28 | 9.71767 | 20 | 9.78675 | 28 | 0.21325 | 9.93092 | 8 | 32 | 7 | 2.5 |
| 29 | 9.71788 | 21 | 9.78704 | 29 | 0.21296 | 9.93084 | 7 | 31 | 8 | 2.8 |
| 30 | 9.71809 | 21 | 9.78732 | 28 | 0.21268 | 9.93077 | 8 | 30 | 9 | 3.2 |
| 31 | 9.71829 | 20 | 9.78760 | 28 | 0.21240 | 9.93069 | 8 | 29 | 10 | 3.5 |
| 32 | 9.71850 | 21 | 9.78789 | 29 | 0.21211 | 9.93061 | 8 | 28 | 20 | 7.0 |
| 33 | 9.71870 | 20 | 9.78817 | 28 | 0.21183 | 9.93053 | 8 | 27 | 30 | 10.5 |
| 34 | 9.71891 | 21 | 9.78845 | 28 | 0.21155 | 9.93046 | 7 | 26 | 40 | 14.0 |
| 35 | 9.71911 | 20 | 9.78874 | 29 | 0.21126 | 9.93038 | 8 | 25 | 50 | 17.5 |
| 36 | 9.71932 | 21 | 9.78902 | 28 | 0.21098 | 9.93030 | 8 | 24 | |
| 37 | 9.71952 | 20 | 9.78930 | 28 | 0.21070 | 9.93022 | 8 | 23 | |
| 38 | 9.71973 | 21 | 9.78959 | 29 | 0.21041 | 9.93014 | 8 | 22 | **20** |
| 39 | 9.71994 | 21 | 9.78987 | 28 | 0.21013 | 9.93007 | 7 | 21 | 6 | 2.0 |
| 40 | 9.72014 | 20 | 9.79015 | 28 | 0.20985 | 9.92999 | 8 | 20 | 7 | 2.3 |
| 41 | 9.72034 | 20 | 9.79043 | 28 | 0.20957 | 9.92991 | 8 | 19 | 8 | 2.7 |
| 42 | 9.72055 | 21 | 9.79072 | 29 | 0.20928 | 9.92983 | 8 | 18 | 9 | 3.0 |
| 43 | 9.72075 | 20 | 9.79100 | 28 | 0.20900 | 9.92976 | 7 | 17 | 10 | 3.3 |
| 44 | 9.72096 | 20 | 9.79128 | 28 | 0.20872 | 9.92968 | 8 | 16 | 20 | 6.7 |
| 45 | 9.72116 | 21 | 9.79156 | 29 | 0.20844 | 9.92960 | 8 | 15 | 30 | 10.0 |
| 46 | 9.72137 | 20 | 9.79185 | 28 | 0.20815 | 9.92952 | 8 | 14 | 40 | 13.3 |
| 47 | 9.72157 | 20 | 9.79213 | 28 | 0.20787 | 9.92944 | 8 | 13 | 50 | 16.7 |
| 48 | 9.72177 | 21 | 9.79241 | 28 | 0.20759 | 9.92936 | 7 | 12 | |
| 49 | 9.72198 | 20 | 9.79269 | 28 | 0.20731 | 9.92929 | 8 | 11 | |
| 50 | 9.72218 | 20 | 9.79297 | 29 | 0.20703 | 9.92921 | 8 | 10 | **8** | **7** |
| 51 | 9.72238 | 21 | 9.79326 | 28 | 0.20674 | 9.92913 | 8 | 9 | 6 | 0.8 | 0.7 |
| 52 | 9.72259 | 20 | 9.79354 | 28 | 0.20646 | 9.92905 | 8 | 8 | 7 | 0.9 | 0.8 |
| 53 | 9.72279 | 20 | 9.79382 | 28 | 0.20618 | 9.92897 | 8 | 7 | 8 | 1.1 | 0.9 |
| 54 | 9.72299 | 21 | 9.79410 | 28 | 0.20590 | 9.92889 | 8 | 6 | 9 | 1.2 | 1.1 |
| 55 | 9.72320 | 20 | 9.79438 | 28 | 0.20562 | 9.92881 | 7 | 5 | 10 | 1.3 | 1.2 |
| 56 | 9.72340 | 20 | 9.79466 | 29 | 0.20534 | 9.92874 | 8 | 4 | 20 | 2.7 | 2.3 |
| 57 | 9.72360 | 21 | 9.79495 | 28 | 0.20505 | 9.92866 | 8 | 3 | 30 | 4.0 | 3.5 |
| 58 | 9.72381 | 20 | 9.79523 | 28 | 0.20477 | 9.92858 | 8 | 2 | 40 | 5.3 | 4.7 |
| 59 | 9.72401 | 20 | 9.79551 | 28 | 0.20449 | 9.92850 | 8 | 1 | 50 | 6.7 | 5.8 |
| 60 | 9.72421 | | 9.79579 | | 0.20421 | 9.92842 | | 0 | |
| ′ | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. |

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### 32°

| ' | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | ' |
|---|---------|-----|---------|-------|----------|---------|-----|----|
| 0 | 9.72421 | 20 | 9.79579 | 28 | 0.20421 | 9.92842 | 8 | 60 |
| 1 | 9.72441 | 20 | 9.79607 | 28 | 0.20393 | 9.92834 | 8 | 59 |
| 2 | 9.72461 | 21 | 9.79635 | 28 | 0.20365 | 9.92826 | 8 | 58 |
| 3 | 9.72482 | 20 | 9.79663 | 28 | 0.20337 | 9.92818 | 8 | 57 |
| 4 | 9.72502 | 20 | 9.79691 | 28 | 0.20309 | 9.92810 | 7 | 56 |
| 5 | 9.72522 | 20 | 9.79719 | 28 | 0.20281 | 9.92803 | 8 | 55 |
| 6 | 9.72542 | 20 | 9.79747 | 29 | 0.20253 | 9.92795 | 8 | 54 |
| 7 | 9.72562 | 20 | 9.79776 | 28 | 0.20224 | 9.92787 | 8 | 53 |
| 8 | 9.72582 | 20 | 9.79804 | 28 | 0.20196 | 9.92779 | 8 | 52 |
| 9 | 9.72602 | 20 | 9.79832 | 28 | 0.20168 | 9.92771 | 8 | 51 |
| 10 | 9.72622 | 21 | 9.79860 | 28 | 0.20140 | 9.92763 | 8 | 50 |
| 11 | 9.72643 | 20 | 9.79888 | 28 | 0.20112 | 9.92755 | 8 | 49 |
| 12 | 9.72663 | 20 | 9.79916 | 28 | 0.20084 | 9.92747 | 8 | 48 |
| 13 | 9.72683 | 20 | 9.79944 | 28 | 0.20056 | 9.92739 | 8 | 47 |
| 14 | 9.72703 | 20 | 9.79972 | 28 | 0.20028 | 9.92731 | 8 | 46 |
| 15 | 9.72723 | 20 | 9.80000 | 28 | 0.20000 | 9.92723 | 8 | 45 |
| 16 | 9.72743 | 20 | 9.80028 | 28 | 0.19972 | 9.92715 | 8 | 44 |
| 17 | 9.72763 | 20 | 9.80056 | 28 | 0.19944 | 9.92707 | 8 | 43 |
| 18 | 9.72783 | 20 | 9.80084 | 28 | 0.19916 | 9.92699 | 8 | 42 |
| 19 | 9.72803 | 20 | 9.80112 | 28 | 0.19888 | 9.92691 | 8 | 41 |
| 20 | 9.72823 | 20 | 9.80140 | 28 | 0.19860 | 9.92683 | 8 | 40 |
| 21 | 9.72843 | 20 | 9.80168 | 27 | 0.19832 | 9.92675 | 8 | 39 |
| 22 | 9.72863 | 20 | 9.80195 | 28 | 0.19805 | 9.92667 | 8 | 38 |
| 23 | 9.72883 | 19 | 9.80223 | 28 | 0.19777 | 9.92659 | 8 | 37 |
| 24 | 9.72902 | 20 | 9.80251 | 28 | 0.19749 | 9.92651 | 8 | 36 |
| 25 | 9.72922 | 20 | 9.80279 | 28 | 0.19721 | 9.92643 | 8 | 35 |
| 26 | 9.72942 | 20 | 9.80307 | 28 | 0.19693 | 9.92635 | 8 | 34 |
| 27 | 9.72962 | 20 | 9.80335 | 28 | 0.19665 | 9.92627 | 8 | 33 |
| 28 | 9.72982 | 20 | 9.80363 | 28 | 0.19637 | 9.92619 | 8 | 32 |
| 29 | 9.73002 | 20 | 9.80391 | 28 | 0.19609 | 9.92611 | 8 | 31 |
| 30 | 9.73022 | 19 | 9.80419 | 28 | 0.19581 | 9.92603 | 8 | 30 |
| 31 | 9.73041 | 20 | 9.80447 | 27 | 0.19553 | 9.92595 | 8 | 29 |
| 32 | 9.73061 | 20 | 9.80474 | 28 | 0.19526 | 9.92587 | 8 | 28 |
| 33 | 9.73081 | 20 | 9.80502 | 28 | 0.19498 | 9.92579 | 8 | 27 |
| 34 | 9.73101 | 20 | 9.80530 | 28 | 0.19470 | 9.92571 | 8 | 26 |
| 35 | 9.73121 | 19 | 9.80558 | 28 | 0.19442 | 9.92563 | 8 | 25 |
| 36 | 9.73140 | 20 | 9.80586 | 28 | 0.19414 | 9.92555 | 9 | 24 |
| 37 | 9.73160 | 20 | 9.80614 | 28 | 0.19386 | 9.92546 | 8 | 23 |
| 38 | 9.73180 | 20 | 9.80642 | 27 | 0.19358 | 9.92538 | 8 | 22 |
| 39 | 9.73200 | 19 | 9.80669 | 28 | 0.19331 | 9.92530 | 8 | 21 |
| 40 | 9.73219 | 20 | 9.80697 | 28 | 0.19303 | 9.92522 | 8 | 20 |
| 41 | 9.73239 | 20 | 9.80725 | 28 | 0.19275 | 9.92514 | 8 | 19 |
| 42 | 9.73259 | 19 | 9.80753 | 28 | 0.19247 | 9.92506 | 8 | 18 |
| 43 | 9.73278 | 20 | 9.80781 | 27 | 0.19219 | 9.92498 | 8 | 17 |
| 44 | 9.73298 | 20 | 9.80808 | 28 | 0.19192 | 9.92490 | 8 | 16 |
| 45 | 9.73318 | 19 | 9.80836 | 28 | 0.19164 | 9.92482 | 9 | 15 |
| 46 | 9.73337 | 20 | 9.80864 | 28 | 0.19136 | 9.92473 | 8 | 14 |
| 47 | 9.73357 | 20 | 9.80892 | 27 | 0.19108 | 9.92465 | 8 | 13 |
| 48 | 9.73377 | 19 | 9.80919 | 28 | 0.19081 | 9.92457 | 8 | 12 |
| 49 | 9.73396 | 20 | 9.80947 | 28 | 0.19053 | 9.92449 | 8 | 11 |
| 50 | 9.73416 | 19 | 9.80975 | 28 | 0.19025 | 9.92441 | 8 | 10 |
| 51 | 9.73435 | 20 | 9.81003 | 28 | 0.18997 | 9.92433 | 8 | 9 |
| 52 | 9.73455 | 19 | 9.81030 | 27 | 0.18970 | 9.92425 | 9 | 8 |
| 53 | 9.73474 | 20 | 9.81058 | 28 | 0.18942 | 9.92416 | 8 | 7 |
| 54 | 9.73494 | 19 | 9.81086 | 27 | 0.18914 | 9.92408 | 8 | 6 |
| 55 | 9.73513 | 20 | 9.81113 | 28 | 0.18887 | 9.92400 | 8 | 5 |
| 56 | 9.73533 | 19 | 9.81141 | 28 | 0.18859 | 9.92392 | 8 | 4 |
| 57 | 9.73552 | 20 | 9.81169 | 27 | 0.18831 | 9.92384 | 8 | 3 |
| 58 | 9.73572 | 19 | 9.81196 | 28 | 0.18804 | 9.92376 | 9 | 2 |
| 59 | 9.73591 | 20 | 9.81224 | 28 | 0.18776 | 9.92367 | 8 | 1 |
| 60 | 9.73611 | | 9.81252 | | 0.18748 | 9.92359 | | 0 |
| | L.Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ' |

**P. P.**

| | 29 | 28 |
|---|-----|-----|
| 6 | 2.9 | 2.8 |
| 7 | 3.4 | 3.3 |
| 8 | 3.9 | 3.7 |
| 9 | 4.4 | 4.2 |
| 10 | 4.8 | 4.7 |
| 20 | 9.7 | 9.3 |
| 30 | 14.5 | 14.0 |
| 40 | 19.3 | 18.7 |
| 50 | 24.2 | 23.3 |

| | 27 |
|---|-----|
| 6 | 2.7 |
| 7 | 3.2 |
| 8 | 3.6 |
| 9 | 4.1 |
| 10 | 4.5 |
| 20 | 9.0 |
| 30 | 13.5 |
| 40 | 18.0 |
| 50 | 22.5 |

| | 21 | 20 |
|---|-----|-----|
| 6 | 2.1 | 2.0 |
| 7 | 2.5 | 2.3 |
| 8 | 2.8 | 2.7 |
| 9 | 3.2 | 3.0 |
| 10 | 3.5 | 3.3 |
| 20 | 7.0 | 6.7 |
| 30 | 10.5 | 10.0 |
| 40 | 14.0 | 13.3 |
| 50 | 17.5 | 16.7 |

| | 19 |
|---|-----|
| 6 | 1.9 |
| 7 | 2.2 |
| 8 | 2.5 |
| 9 | 2.9 |
| 10 | 3.2 |
| 20 | 6.3 |
| 30 | 9.5 |
| 40 | 12.7 |
| 50 | 15.8 |

| | 9 | 8 | 7 |
|---|-----|-----|-----|
| 6 | 0.9 | 0.8 | 0.7 |
| 7 | 1.1 | 0.9 | 0.8 |
| 8 | 1.2 | 1.1 | 0.9 |
| 9 | 1.4 | 1.2 | 1.1 |
| 10 | 1.5 | 1.3 | 1.2 |
| 20 | 3.0 | 2.7 | 2.3 |
| 30 | 4.5 | 4.0 | 3.5 |
| 40 | 6.0 | 5.3 | 4.7 |
| 50 | 7.5 | 6.7 | 5.8 |

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## 33°

| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | P. P. |
|---|---------|----|---------|-------|----------|---------|----|----|-------|
| 0 | 9.73611 | 19 | 9.81252 | 27 | 0.18748 | 9.92359 | 8 | 60 | |
| 1 | 9.73630 | 20 | 9.81279 | 28 | 0.18721 | 9.92351 | 8 | 59 | |
| 2 | 9.73650 | 19 | 9.81307 | 28 | 0.18693 | 9.92343 | 8 | 58 | |
| 3 | 9.73669 | 20 | 9.81335 | 28 | 0.18665 | 9.92335 | 8 | 57 | **28 27** |
| 4 | 9.73689 | 19 | 9.81362 | 27 | 0.18638 | 9.92326 | 9 | 56 | 6 2.8 2.7 |
| 5 | 9.73708 | 19 | 9.81390 | 28 | 0.18610 | 9.92318 | 8 | 55 | 7 3.3 3.2 |
| 6 | 9.73727 | 20 | 9.81418 | 27 | 0.18582 | 9.92310 | 8 | 54 | 8 3.7 3.6 |
| 7 | 9.73747 | 19 | 9.81445 | 28 | 0.18555 | 9.92302 | 9 | 53 | 9 4.2 4.1 |
| 8 | 9.73766 | 19 | 9.81473 | 27 | 0.18527 | 9.92293 | 8 | 52 | 10 4.7 4.5 |
| 9 | 9.73785 | 20 | 9.81500 | 28 | 0.18500 | 9.92285 | 8 | 51 | 20 9.3 9.0 |
| 10 | 9.73805 | 19 | 9.81528 | 28 | 0.18472 | 9.92277 | 8 | 50 | 30 14.0 13.5 |
| 11 | 9.73824 | 19 | 9.81556 | 27 | 0.18444 | 9.92269 | 9 | 49 | 40 18.7 18.0 |
| 12 | 9.73843 | 20 | 9.81583 | 28 | 0.18417 | 9.92260 | 8 | 48 | 50 23.3 22.5 |
| 13 | 9.73863 | 19 | 9.81611 | 27 | 0.18389 | 9.92252 | 8 | 47 | |
| 14 | 9.73882 | 19 | 9.81638 | 28 | 0.18362 | 9.92244 | 9 | 46 | |
| 15 | 9.73901 | 20 | 9.81666 | 27 | 0.18334 | 9.92235 | 8 | 45 | **20** |
| 16 | 9.73921 | 19 | 9.81693 | 28 | 0.18307 | 9.92227 | 8 | 44 | 6 2.0 |
| 17 | 9.73940 | 19 | 9.81721 | 27 | 0.18279 | 9.92219 | 8 | 43 | 7 2.3 |
| 18 | 9.73959 | 19 | 9.81748 | 28 | 0.18252 | 9.92211 | 9 | 42 | 8 2.7 |
| 19 | 9.73978 | 19 | 9.81776 | 27 | 0.18224 | 9.92202 | 8 | 41 | 9 3.0 |
| 20 | 9.73997 | 20 | 9.81803 | 28 | 0.18197 | 9.92194 | 8 | 40 | 10 3.3 |
| 21 | 9.74017 | 19 | 9.81831 | 27 | 0.18169 | 9.92186 | 9 | 39 | 20 6.7 |
| 22 | 9.74036 | 19 | 9.81858 | 28 | 0.18142 | 9.92177 | 8 | 38 | 30 10.0 |
| 23 | 9.74055 | 19 | 9.81886 | 27 | 0.18114 | 9.92169 | 8 | 37 | 40 13.3 |
| 24 | 9.74074 | 19 | 9.81913 | 28 | 0.18087 | 9.92161 | 9 | 36 | 50 16.7 |
| 25 | 9.74093 | 20 | 9.81941 | 27 | 0.18059 | 9.92152 | 8 | 35 | |
| 26 | 9.74113 | 19 | 9.81968 | 28 | 0.18032 | 9.92144 | 8 | 34 | **19** |
| 27 | 9.74132 | 19 | 9.81996 | 27 | 0.18004 | 9.92136 | 9 | 33 | 6 1.9 |
| 28 | 9.74151 | 19 | 9.82023 | 28 | 0.17977 | 9.92127 | 8 | 32 | 7 2.2 |
| 29 | 9.74170 | 19 | 9.82051 | 27 | 0.17949 | 9.92119 | 8 | 31 | 8 2.5 |
| 30 | 9.74189 | 19 | 9.82078 | 28 | 0.17922 | 9.92111 | 9 | 30 | 9 2.9 |
| 31 | 9.74208 | 19 | 9.82106 | 27 | 0.17894 | 9.92102 | 8 | 29 | 10 3.2 |
| 32 | 9.74227 | 19 | 9.82133 | 28 | 0.17867 | 9.92094 | 8 | 28 | 20 6.3 |
| 33 | 9.74246 | 19 | 9.82161 | 27 | 0.17839 | 9.92086 | 9 | 27 | 30 9.5 |
| 34 | 9.74265 | 19 | 9.82188 | 27 | 0.17812 | 9.92077 | 8 | 26 | 40 12.7 |
| 35 | 9.74284 | 19 | 9.82215 | 28 | 0.17785 | 9.92069 | 9 | 25 | 50 15.8 |
| 36 | 9.74303 | 19 | 9.82243 | 27 | 0.17757 | 9.92060 | 8 | 24 | |
| 37 | 9.74322 | 19 | 9.82270 | 28 | 0.17730 | 9.92052 | 8 | 23 | |
| 38 | 9.74341 | 19 | 9.82298 | 27 | 0.17702 | 9.92044 | 9 | 22 | **18** |
| 39 | 9.74360 | 19 | 9.82325 | 27 | 0.17675 | 9.92035 | 8 | 21 | 6 1.8 |
| 40 | 9.74379 | 19 | 9.82352 | 28 | 0.17648 | 9.92027 | 9 | 20 | 7 2.1 |
| 41 | 9.74398 | 19 | 9.82380 | 27 | 0.17620 | 9.92018 | 8 | 19 | 8 2.4 |
| 42 | 9.74417 | 19 | 9.82407 | 28 | 0.17593 | 9.92010 | 8 | 18 | 9 2.7 |
| 43 | 9.74436 | 19 | 9.82435 | 27 | 0.17565 | 9.92002 | 9 | 17 | 10 3.0 |
| 44 | 9.74455 | 19 | 9.82462 | 27 | 0.17538 | 9.91993 | 8 | 16 | 20 6.0 |
| 45 | 9.74474 | 19 | 9.82489 | 28 | 0.17511 | 9.91985 | 9 | 15 | 30 9.0 |
| 46 | 9.74493 | 19 | 9.82517 | 27 | 0.17483 | 9.91976 | 8 | 14 | 40 12.0 |
| 47 | 9.74512 | 19 | 9.82544 | 27 | 0.17456 | 9.91968 | 9 | 13 | 50 15.0 |
| 48 | 9.74531 | 18 | 9.82571 | 28 | 0.17429 | 9.91959 | 8 | 12 | |
| 49 | 9.74549 | 19 | 9.82599 | 27 | 0.17401 | 9.91951 | 9 | 11 | |
| 50 | 9.74568 | 19 | 9.82626 | 27 | 0.17374 | 9.91942 | 8 | 10 | **9 8** |
| 51 | 9.74587 | 19 | 9.82653 | 28 | 0.17347 | 9.91934 | 9 | 9 | 6 0.9 0.8 |
| 52 | 9.74606 | 19 | 9.82681 | 27 | 0.17319 | 9.91925 | 8 | 8 | 7 1.1 0.9 |
| 53 | 9.74625 | 19 | 9.82708 | 27 | 0.17292 | 9.91917 | 9 | 7 | 8 1.2 1.1 |
| 54 | 9.74644 | 18 | 9.82735 | 27 | 0.17265 | 9.91908 | 8 | 6 | 9 1.4 1.2 |
| 55 | 9.74662 | 19 | 9.82762 | 28 | 0.17238 | 9.91900 | 9 | 5 | 10 1.5 1.3 |
| 56 | 9.74681 | 19 | 9.82790 | 27 | 0.17210 | 9.91891 | 8 | 4 | 20 3.0 2.7 |
| 57 | 9.74700 | 19 | 9.82817 | 27 | 0.17183 | 9.91883 | 9 | 3 | 30 4.5 4.0 |
| 58 | 9.74719 | 18 | 9.82844 | 27 | 0.17156 | 9.91874 | 8 | 2 | 40 6.0 5.3 |
| 59 | 9.74737 | 19 | 9.82871 | 28 | 0.17129 | 9.91866 | 9 | 1 | 50 7.5 6.7 |
| 60 | 9.74756 | | 9.82899 | | 0.17101 | 9.91857 | | 0 | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | P. P. | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 9.74756 | 19 | 9.82899 | 27 | 0.17101 | 9.91857 | 8 | 60 | | |
| 1 | 9.74775 | 19 | 9.82926 | 27 | 0.17074 | 9.91849 | 9 | 59 | | |
| 2 | 9.74794 | 18 | 9.82953 | 27 | 0.17047 | 9.91840 | 8 | 58 | **28** | **27** |
| 3 | 9.74812 | 19 | 9.82980 | 28 | 0.17020 | 9.91832 | 9 | 57 | 6 \| 2.8 | 2.7 |
| 4 | 9.74831 | 19 | 9.83008 | 27 | 0.16992 | 9.91823 | 8 | 56 | 7 \| 3.3 | 3.2 |
| 5 | 9.74850 | 18 | 9.83035 | 27 | 0.16965 | 9.91815 | 9 | 55 | 8 \| 3.7 | 3.6 |
| 6 | 9.74868 | 19 | 9.83062 | 27 | 0.16938 | 9.91806 | 8 | 54 | 9 \| 4.2 | 4.1 |
| 7 | 9.74887 | 19 | 9.83089 | 28 | 0.16911 | 9.91798 | 9 | 53 | 10 \| 4.7 | 4.5 |
| 8 | 9.74906 | 18 | 9.83117 | 27 | 0.16883 | 9.91789 | 8 | 52 | 20 \| 9.3 | 9.0 |
| 9 | 9.74924 | 19 | 9.83144 | 27 | 0.16856 | 9.91781 | 9 | 51 | 30 \| 14.0 | 13.5 |
| 10 | 9.74943 | 18 | 9.83171 | 27 | 0.16829 | 9.91772 | 9 | 50 | 40 \| 18.7 | 18.0 |
| 11 | 9.74961 | 19 | 9.83198 | 27 | 0.16802 | 9.91763 | 8 | 49 | 50 \| 23.3 | 22.5 |
| 12 | 9.74980 | 19 | 9.83225 | 27 | 0.16775 | 9.91755 | 9 | 48 | | |
| 13 | 9.74999 | 18 | 9.83252 | 28 | 0.16748 | 9.91746 | 8 | 47 | | |
| 14 | 9.75017 | 19 | 9.83280 | 27 | 0.16720 | 9.91738 | 9 | 46 | | |
| 15 | 9.75036 | 18 | 9.83307 | 27 | 0.16693 | 9.91729 | 9 | 45 | **26** | |
| 16 | 9.75054 | 19 | 9.83334 | 27 | 0.16666 | 9.91720 | 8 | 44 | 6 \| 2.6 | |
| 17 | 9.75073 | 18 | 9.83361 | 27 | 0.16639 | 9.91712 | 9 | 43 | 7 \| 3.0 | |
| 18 | 9.75091 | 19 | 9.83388 | 27 | 0.16612 | 9.91703 | 8 | 42 | 8 \| 3.5 | |
| 19 | 9.75110 | 18 | 9.83415 | 27 | 0.16585 | 9.91695 | 9 | 41 | 9 \| 3.9 | |
| 20 | 9.75128 | 19 | 9.83442 | 28 | 0.16558 | 9.91686 | 9 | 40 | 10 \| 4.3 | |
| 21 | 9.75147 | 18 | 9.83470 | 27 | 0.16530 | 9.91677 | 8 | 39 | 20 \| 8.7 | |
| 22 | 9.75165 | 19 | 9.83497 | 27 | 0.16503 | 9.91669 | 9 | 38 | 30 \| 13.0 | |
| 23 | 9.75184 | 18 | 9.83524 | 27 | 0.16476 | 9.91660 | 9 | 37 | 40 \| 17.3 | |
| 24 | 9.75202 | 19 | 9.83551 | 27 | 0.16449 | 9.91651 | 8 | 36 | 50 \| 21.7 | |
| 25 | 9.75221 | 18 | 9.83578 | 27 | 0.16422 | 9.91643 | 9 | 35 | | |
| 26 | 9.75239 | 19 | 9.83605 | 27 | 0.16395 | 9.91634 | 9 | 34 | **19** | |
| 27 | 9.75258 | 18 | 9.83632 | 27 | 0.16368 | 9.91625 | 8 | 33 | 6 \| 1.9 | |
| 28 | 9.75276 | 18 | 9.83659 | 27 | 0.16341 | 9.91617 | 9 | 32 | 7 \| 2.2 | |
| 29 | 9.75294 | 19 | 9.83686 | 27 | 0.16314 | 9.91608 | 9 | 31 | 8 \| 2.5 | |
| 30 | 9.75313 | 18 | 9.83713 | 27 | 0.16287 | 9.91599 | 8 | 30 | 9 \| 2.9 | |
| 31 | 9.75331 | 19 | 9.83740 | 28 | 0.16260 | 9.91591 | 9 | 29 | 10 \| 3.2 | |
| 32 | 9.75350 | 18 | 9.83768 | 27 | 0.16232 | 9.91582 | 9 | 28 | 20 \| 6.3 | |
| 33 | 9.75368 | 18 | 9.83795 | 27 | 0.16205 | 9.91573 | 8 | 27 | 30 \| 9.5 | |
| 34 | 9.75386 | 19 | 9.83822 | 27 | 0.16178 | 9.91565 | 9 | 26 | 40 \| 12.7 | |
| 35 | 9.75405 | 18 | 9.83849 | 27 | 0.16151 | 9.91556 | 9 | 25 | 50 \| 15.8 | |
| 36 | 9.75423 | 18 | 9.83876 | 27 | 0.16124 | 9.91547 | 9 | 24 | | |
| 37 | 9.75441 | 18 | 9.83903 | 27 | 0.16097 | 9.91538 | 8 | 23 | | |
| 38 | 9.75459 | 19 | 9.83930 | 27 | 0.16070 | 9.91530 | 9 | 22 | **18** | |
| 39 | 9.75478 | 18 | 9.83957 | 27 | 0.16043 | 9.91521 | 9 | 21 | 6 \| 1.8 | |
| 40 | 9.75496 | 18 | 9.83984 | 27 | 0.16016 | 9.91512 | 8 | 20 | 7 \| 2.1 | |
| 41 | 9.75514 | 19 | 9.84011 | 27 | 0.15989 | 9.91504 | 9 | 19 | 8 \| 2.4 | |
| 42 | 9.75533 | 18 | 9.84038 | 27 | 0.15962 | 9.91495 | 9 | 18 | 9 \| 2.7 | |
| 43 | 9.75551 | 18 | 9.84065 | 27 | 0.15935 | 9.91486 | 9 | 17 | 10 \| 3.0 | |
| 44 | 9.75569 | 18 | 9.84092 | 27 | 0.15908 | 9.91477 | 8 | 16 | 20 \| 6.0 | |
| 45 | 9.75587 | 18 | 9.84119 | 27 | 0.15881 | 9.91469 | 9 | 15 | 30 \| 9.0 | |
| 46 | 9.75605 | 19 | 9.84146 | 27 | 0.15854 | 9.91460 | 9 | 14 | 40 \| 12.0 | |
| 47 | 9.75624 | 18 | 9.84173 | 27 | 0.15827 | 9.91451 | 9 | 13 | 50 \| 15.0 | |
| 48 | 9.75642 | 18 | 9.84200 | 27 | 0.15800 | 9.91442 | 9 | 12 | | |
| 49 | 9.75660 | 18 | 9.84227 | 27 | 0.15773 | 9.91433 | 8 | 11 | | |
| 50 | 9.75678 | 18 | 9.84254 | 26 | 0.15746 | 9.91425 | 9 | 10 | **9** | **8** |
| 51 | 9.75696 | 18 | 9.84280 | 27 | 0.15720 | 9.91416 | 9 | 9 | 6 \| 0.9 | 0.8 |
| 52 | 9.75714 | 19 | 9.84307 | 27 | 0.15693 | 9.91407 | 9 | 8 | 7 \| 1.1 | 0.9 |
| 53 | 9.75733 | 18 | 9.84334 | 27 | 0.15666 | 9.91398 | 9 | 7 | 8 \| 1.2 | 1.1 |
| 54 | 9.75751 | 18 | 9.84361 | 27 | 0.15639 | 9.91389 | 8 | 6 | 9 \| 1.4 | 1.2 |
| 55 | 9.75769 | 18 | 9.84388 | 27 | 0.15612 | 9.91381 | 9 | 5 | 10 \| 1.5 | 1.3 |
| 56 | 9.75787 | 18 | 9.84415 | 27 | 0.15585 | 9.91372 | 9 | 4 | 20 \| 3.0 | 2.7 |
| 57 | 9.75805 | 18 | 9.84442 | 27 | 0.15558 | 9.91363 | 9 | 3 | 30 \| 4.5 | 4.0 |
| 58 | 9.75823 | 18 | 9.84469 | 27 | 0.15531 | 9.91354 | 9 | 2 | 40 \| 6.0 | 5.3 |
| 59 | 9.75841 | 18 | 9.84496 | 27 | 0.15504 | 9.91345 | 9 | 1 | 50 \| 7.5 | 6.7 |
| 60 | 9.75859 | | 9.84523 | | 0.15477 | 9.91336 | | 0 | | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. | |

**55°**

## 35°

| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | P. P. | |
|---|---------|----|---------|-------|----------|---------|----|---|-------|---|
| 0 | 9.75859 | 18 | 9.84523 | 27 | 0.15477 | 9.91336 | 8 | 60 | | |
| 1 | 9.75877 | 18 | 9.84550 | 26 | 0.15450 | 9.91328 | 9 | 59 | | |
| 2 | 9.75895 | 18 | 9.84576 | 27 | 0.15424 | 9.91319 | 9 | 58 | | 27 26 |
| 3 | 9.75913 | 18 | 9.84603 | 27 | 0.15397 | 9.91310 | 9 | 57 | 6 | 2.7 2.6 |
| 4 | 9.75931 | 18 | 9.84630 | 27 | 0.15370 | 9.91301 | 9 | 56 | 7 | 3.2 3.0 |
| 5 | 9.75949 | 18 | 9.84657 | 27 | 0.15343 | 9.91292 | 9 | 55 | 8 | 3.6 3.5 |
| 6 | 9.75967 | 18 | 9.84684 | 27 | 0.15316 | 9.91283 | 9 | 54 | 9 | 4.1 3.9 |
| 7 | 9.75985 | 18 | 9.84711 | 27 | 0.15289 | 9.91274 | 9 | 53 | 10 | 4.5 4.3 |
| 8 | 9.76003 | 18 | 9.84738 | 26 | 0.15262 | 9.91266 | 8 | 52 | 20 | 9.0 8.7 |
| 9 | 9.76021 | 18 | 9.84764 | 27 | 0.15236 | 9.91257 | 9 | 51 | 30 | 13.5 13.0 |
| 10 | 9.76039 | 18 | 9.84791 | 27 | 0.15209 | 9.91248 | 9 | 50 | 40 | 18.0 17.3 |
| 11 | 9.76057 | 18 | 9.84818 | 27 | 0.15182 | 9.91239 | 9 | 49 | 50 | 22.5 21.7 |
| 12 | 9.76075 | 18 | 9.84845 | 27 | 0.15155 | 9.91230 | 9 | 48 | | |
| 13 | 9.76093 | 18 | 9.84872 | 27 | 0.15128 | 9.91221 | 9 | 47 | | |
| 14 | 9.76111 | 18 | 9.84899 | 26 | 0.15101 | 9.91212 | 9 | 46 | | 18 |
| 15 | 9.76129 | 17 | 9.84925 | 27 | 0.15075 | 9.91203 | 9 | 45 | 6 | 1.8 |
| 16 | 9.76146 | 18 | 9.84952 | 27 | 0.15048 | 9.91194 | 9 | 44 | 7 | 2.1 |
| 17 | 9.76164 | 18 | 9.84979 | 27 | 0.15021 | 9.91185 | 9 | 43 | 8 | 2.4 |
| 18 | 9.76182 | 18 | 9.85006 | 27 | 0.14994 | 9.91176 | 9 | 42 | 9 | 2.7 |
| 19 | 9.76200 | 18 | 9.85033 | 26 | 0.14967 | 9.91167 | 9 | 41 | 10 | 3.0 |
| 20 | 9.76218 | 18 | 9.85059 | 27 | 0.14941 | 9.91158 | 9 | 40 | 20 | 6.0 |
| 21 | 9.76236 | 17 | 9.85086 | 27 | 0.14914 | 9.91149 | 8 | 39 | 30 | 9.0 |
| 22 | 9.76253 | 18 | 9.85113 | 27 | 0.14887 | 9.91141 | 9 | 38 | 40 | 12.0 |
| 23 | 9.76271 | 18 | 9.85140 | 26 | 0.14860 | 9.91132 | 9 | 37 | 50 | 15.0 |
| 24 | 9.76289 | 18 | 9.85166 | 27 | 0.14834 | 9.91123 | 9 | 36 | | |
| 25 | 9.76307 | 17 | 9.85193 | 27 | 0.14807 | 9.91114 | 9 | 35 | | |
| 26 | 9.76324 | 18 | 9.85220 | 27 | 0.14780 | 9.91105 | 9 | 34 | | 17 |
| 27 | 9.76342 | 18 | 9.85247 | 26 | 0.14753 | 9.91096 | 9 | 33 | 6 | 1.7 |
| 28 | 9.76360 | 18 | 9.85273 | 27 | 0.14727 | 9.91087 | 9 | 32 | 7 | 2.0 |
| 29 | 9.76378 | 17 | 9.85300 | 27 | 0.14700 | 9.91078 | 9 | 31 | 8 | 2.3 |
| 30 | 9.76395 | 18 | 9.85327 | 27 | 0.14673 | 9.91069 | 9 | 30 | 9 | 2.6 |
| 31 | 9.76413 | 18 | 9.85354 | 26 | 0.14646 | 9.91060 | 9 | 29 | 10 | 2.8 |
| 32 | 9.76431 | 17 | 9.85380 | 27 | 0.14620 | 9.91051 | 9 | 28 | 20 | 5.7 |
| 33 | 9.76448 | 18 | 9.85407 | 27 | 0.14593 | 9.91042 | 9 | 27 | 30 | 8.5 |
| 34 | 9.76466 | 18 | 9.85434 | 26 | 0.14566 | 9.91033 | 10 | 26 | 40 | 11.3 |
| 35 | 9.76484 | 17 | 9.85460 | 27 | 0.14540 | 9.91023 | 9 | 25 | 50 | 14.2 |
| 36 | 9.76501 | 18 | 9.85487 | 27 | 0.14513 | 9.91014 | 9 | 24 | | |
| 37 | 9.76519 | 18 | 9.85514 | 26 | 0.14486 | 9.91005 | 9 | 23 | | |
| 38 | 9.76537 | 17 | 9.85540 | 27 | 0.14460 | 9.90996 | 9 | 22 | | 10 |
| 39 | 9.76554 | 18 | 9.85567 | 27 | 0.14433 | 9.90987 | 9 | 21 | 6 | 1.0 |
| 40 | 9.76572 | 18 | 9.85594 | 26 | 0.14406 | 9.90978 | 9 | 20 | 7 | 1.2 |
| 41 | 9.76590 | 17 | 9.85620 | 27 | 0.14380 | 9.90969 | 9 | 19 | 8 | 1.3 |
| 42 | 9.76607 | 18 | 9.85647 | 27 | 0.14353 | 9.90960 | 9 | 18 | 9 | 1.5 |
| 43 | 9.76625 | 17 | 9.85674 | 26 | 0.14326 | 9.90951 | 9 | 17 | 10 | 1.7 |
| 44 | 9.76642 | 18 | 9.85700 | 27 | 0.14300 | 9.90942 | 9 | 16 | 20 | 3.3 |
| 45 | 9.76660 | 17 | 9.85727 | 27 | 0.14273 | 9.90933 | 9 | 15 | 30 | 5.0 |
| 46 | 9.76677 | 18 | 9.85754 | 26 | 0.14246 | 9.90924 | 9 | 14 | 40 | 6.7 |
| 47 | 9.76695 | 17 | 9.85780 | 27 | 0.14220 | 9.90915 | 9 | 13 | 50 | 8.3 |
| 48 | 9.76712 | 18 | 9.85807 | 27 | 0.14193 | 9.90906 | 10 | 12 | | |
| 49 | 9.76730 | 17 | 9.85834 | 26 | 0.14166 | 9.90896 | 9 | 11 | | |
| 50 | 9.76747 | 18 | 9.85860 | 27 | 0.14140 | 9.90887 | 9 | 10 | | 9 8 |
| 51 | 9.76765 | 17 | 9.85887 | 26 | 0.14113 | 9.90878 | 9 | 9 | 6 | 0.9 0.8 |
| 52 | 9.76782 | 18 | 9.85913 | 27 | 0.14087 | 9.90869 | 9 | 8 | 7 | 1.1 0.9 |
| 53 | 9.76800 | 17 | 9.85940 | 27 | 0.14060 | 9.90860 | 9 | 7 | 8 | 1.2 1.1 |
| 54 | 9.76817 | 18 | 9.85967 | 26 | 0.14033 | 9.90851 | 9 | 6 | 9 | 1.4 1.2 |
| 55 | 9.76835 | 17 | 9.85993 | 27 | 0.14007 | 9.90842 | 10 | 5 | 10 | 1.5 1.3 |
| 56 | 9.76852 | 18 | 9.86020 | 26 | 0.13980 | 9.90832 | 9 | 4 | 20 | 3.0 2.7 |
| 57 | 9.76870 | 17 | 9.86046 | 27 | 0.13954 | 9.90823 | 9 | 3 | 30 | 4.5 4.0 |
| 58 | 9.76887 | 17 | 9.86073 | 27 | 0.13927 | 9.90814 | 9 | 2 | 40 | 6.0 5.3 |
| 59 | 9.76904 | 18 | 9.86100 | 26 | 0.13900 | 9.90805 | 9 | 1 | 50 | 7.5 6.7 |
| 60 | 9.76922 | | 9.86126 | | 0.13874 | 9.90796 | | 0 | | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. | |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | P. P. |
|---|---------|----|---------|-------|----------|---------|----|-------|
| 0 | 9.76922 | 17 | 9.86126 | 27 | 0.13874 | 9.90796 | 9 | 60 |
| 1 | 9.76939 | 18 | 9.86153 | 27 | 0.13847 | 9.90787 | 10 | 59 |
| 2 | 9.76957 | 17 | 9.86179 | 27 | 0.13821 | 9.90777 | 9 | 58 |
| 3 | 9.76974 | 17 | 9.86206 | 26 | 0.13794 | 9.90768 | 9 | 57 |
| 4 | 9.76991 | 18 | 9.86232 | 27 | 0.13768 | 9.90759 | 9 | 56 |
| 5 | 9.77009 | 17 | 9.86259 | 26 | 0.13741 | 9.90750 | 9 | 55 |
| 6 | 9.77026 | 17 | 9.86285 | 27 | 0.13715 | 9.90741 | 10 | 54 |
| 7 | 9.77043 | 18 | 9.86312 | 26 | 0.13688 | 9.90731 | 9 | 53 |
| 8 | 9.77061 | 17 | 9.86338 | 27 | 0.13662 | 9.90722 | 9 | 52 |
| 9 | 9.77078 | 17 | 9.86365 | 27 | 0.13635 | 9.90713 | 9 | 51 |
| 10 | 9.77095 | 17 | 9.86392 | 26 | 0.13608 | 9.90704 | 10 | 50 |
| 11 | 9.77112 | 18 | 9.86418 | 27 | 0.13582 | 9.90694 | 9 | 49 |
| 12 | 9.77130 | 17 | 9.86445 | 26 | 0.13555 | 9.90685 | 9 | 48 |
| 13 | 9.77147 | 17 | 9.86471 | 27 | 0.13529 | 9.90676 | 9 | 47 |
| 14 | 9.77164 | 17 | 9.86498 | 26 | 0.13502 | 9.90667 | 10 | 46 |
| 15 | 9.77181 | 18 | 9.86524 | 27 | 0.13476 | 9.90657 | 9 | 45 |
| 16 | 9.77199 | 17 | 9.86551 | 26 | 0.13449 | 9.90648 | 9 | 44 |
| 17 | 9.77216 | 17 | 9.86577 | 26 | 0.13423 | 9.90639 | 9 | 43 |
| 18 | 9.77233 | 17 | 9.86603 | 27 | 0.13397 | 9.90630 | 10 | 42 |
| 19 | 9.77250 | 18 | 9.86630 | 26 | 0.13370 | 9.90620 | 9 | 41 |
| 20 | 9.77268 | 17 | 9.86656 | 27 | 0.13344 | 9.90611 | 9 | 40 |
| 21 | 9.77285 | 17 | 9.86683 | 26 | 0.13317 | 9.90602 | 10 | 39 |
| 22 | 9.77302 | 17 | 9.86709 | 27 | 0.13291 | 9.90592 | 9 | 38 |
| 23 | 9.77319 | 17 | 9.86736 | 26 | 0.13264 | 9.90583 | 9 | 37 |
| 24 | 9.77336 | 17 | 9.86762 | 27 | 0.13238 | 9.90574 | 9 | 36 |
| 25 | 9.77353 | 17 | 9.86789 | 26 | 0.13211 | 9.90565 | 10 | 35 |
| 26 | 9.77370 | 17 | 9.86815 | 27 | 0.13185 | 9.90555 | 9 | 34 |
| 27 | 9.77387 | 18 | 9.86842 | 26 | 0.13158 | 9.90546 | 9 | 33 |
| 28 | 9.77405 | 17 | 9.86868 | 26 | 0.13132 | 9.90537 | 10 | 32 |
| 29 | 9.77422 | 17 | 9.86894 | 27 | 0.13106 | 9.90527 | 9 | 31 |
| 30 | 9.77439 | 17 | 9.86921 | 26 | 0.13079 | 9.90518 | 9 | 30 |
| 31 | 9.77456 | 17 | 9.86947 | 27 | 0.13053 | 9.90509 | 10 | 29 |
| 32 | 9.77473 | 17 | 9.86974 | 26 | 0.13026 | 9.90499 | 9 | 28 |
| 33 | 9.77490 | 17 | 9.87000 | 27 | 0.13000 | 9.90490 | 10 | 27 |
| 34 | 9.77507 | 17 | 9.87027 | 26 | 0.12973 | 9.90480 | 9 | 26 |
| 35 | 9.77524 | 17 | 9.87053 | 26 | 0.12947 | 9.90471 | 9 | 25 |
| 36 | 9.77541 | 17 | 9.87079 | 27 | 0.12921 | 9.90462 | 10 | 24 |
| 37 | 9.77558 | 17 | 9.87106 | 26 | 0.12894 | 9.90452 | 9 | 23 |
| 38 | 9.77575 | 17 | 9.87132 | 26 | 0.12868 | 9.90443 | 9 | 22 |
| 39 | 9.77592 | 17 | 9.87158 | 27 | 0.12842 | 9.90434 | 10 | 21 |
| 40 | 9.77609 | 17 | 9.87185 | 26 | 0.12815 | 9.90424 | 9 | 20 |
| 41 | 9.77626 | 17 | 9.87211 | 27 | 0.12789 | 9.90415 | 10 | 19 |
| 42 | 9.77643 | 17 | 9.87238 | 26 | 0.12762 | 9.90405 | 9 | 18 |
| 43 | 9.77660 | 17 | 9.87264 | 26 | 0.12736 | 9.90396 | 10 | 17 |
| 44 | 9.77677 | 17 | 9.87290 | 27 | 0.12710 | 9.90386 | 9 | 16 |
| 45 | 9.77694 | 17 | 9.87317 | 26 | 0.12683 | 9.90377 | 9 | 15 |
| 46 | 9.77711 | 17 | 9.87343 | 26 | 0.12657 | 9.90368 | 10 | 14 |
| 47 | 9.77728 | 16 | 9.87369 | 27 | 0.12631 | 9.90358 | 9 | 13 |
| 48 | 9.77744 | 17 | 9.87396 | 26 | 0.12604 | 9.90349 | 10 | 12 |
| 49 | 9.77761 | 17 | 9.87422 | 26 | 0.12578 | 9.90339 | 9 | 11 |
| 50 | 9.77778 | 17 | 9.87448 | 27 | 0.12552 | 9.90330 | 10 | 10 |
| 51 | 9.77795 | 17 | 9.87475 | 26 | 0.12525 | 9.90320 | 9 | 9 |
| 52 | 9.77812 | 17 | 9.87501 | 26 | 0.12499 | 9.90311 | 10 | 8 |
| 53 | 9.77829 | 17 | 9.87527 | 27 | 0.12473 | 9.90301 | 9 | 7 |
| 54 | 9.77846 | 16 | 9.87554 | 26 | 0.12446 | 9.90292 | 10 | 6 |
| 55 | 9.77862 | 17 | 9.87580 | 26 | 0.12420 | 9.90282 | 9 | 5 |
| 56 | 9.77879 | 17 | 9.87606 | 27 | 0.12394 | 9.90273 | 10 | 4 |
| 57 | 9.77896 | 17 | 9.87633 | 26 | 0.12367 | 9.90263 | 9 | 3 |
| 58 | 9.77913 | 17 | 9.87659 | 26 | 0.12341 | 9.90254 | 10 | 2 |
| 59 | 9.77930 | 16 | 9.87685 | 26 | 0.12315 | 9.90244 | 9 | 1 |
| 60 | 9.77946 | | 9.87711 | | 0.12289 | 9.90235 | | 0 |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ |

P. P.

|  | 27 | 26 |
|---|----|----|
| 6 | 2.7 | 2.6 |
| 7 | 3.2 | 3.0 |
| 8 | 3.6 | 3.5 |
| 9 | 4.1 | 3.9 |
| 10 | 4.5 | 4.3 |
| 20 | 9.0 | 8.7 |
| 30 | 13.5 | 13.0 |
| 40 | 18.0 | 17.3 |
| 50 | 22.5 | 21.7 |

|  | 18 |
|---|----|
| 6 | 1.8 |
| 7 | 2.1 |
| 8 | 2.4 |
| 9 | 2.7 |
| 10 | 3.0 |
| 20 | 6.0 |
| 30 | 9.0 |
| 40 | 12.0 |
| 50 | 15.0 |

|  | 17 |
|---|----|
| 6 | 1.7 |
| 7 | 2.0 |
| 8 | 2.3 |
| 9 | 2.5 |
| 10 | 2.8 |
| 20 | 5.7 |
| 30 | 8.5 |
| 40 | 11.3 |
| 50 | 14.2 |

|  | 16 |
|---|----|
| 6 | 1.6 |
| 7 | 1.9 |
| 8 | 2.1 |
| 9 | 2.4 |
| 10 | 2.7 |
| 20 | 5.3 |
| 30 | 8.0 |
| 40 | 10.7 |
| 50 | 13.3 |

|  | 10 | 9 |
|---|----|----|
| 6 | 1.0 | 0.9 |
| 7 | 1.2 | 1.1 |
| 8 | 1.3 | 1.2 |
| 9 | 1.5 | 1.4 |
| 10 | 1.7 | 1.5 |
| 20 | 3.3 | 3.0 |
| 30 | 5.0 | 4.5 |
| 40 | 6.7 | 6.0 |
| 50 | 8.3 | 7.5 |

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**37°**

| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | P. P. |
|---|---------|-----|---------|-------|----------|---------|-----|----|-------|
| 0 | 9.77946 | 17 | 9.87711 | 27 | 0.12289 | 9.90235 | 10 | **60** | |
| 1 | 9.77963 | 17 | 9.87738 | 26 | 0.12262 | 9.90225 | 9 | 59 | |
| 2 | 9.77980 | 17 | 9.87764 | 26 | 0.12236 | 9.90216 | 10 | 58 | |
| 3 | 9.77997 | 16 | 9.87790 | 27 | 0.12210 | 9.90206 | 9 | 57 | **27** |
| 4 | 9.78013 | 17 | 9.87817 | 26 | 0.12183 | 9.90197 | 10 | 56 | 6 2.7 |
| 5 | 9.78030 | 17 | 9.87843 | 26 | 0.12157 | 9.90187 | 9 | **55** | 7 3.2 |
| 6 | 9.78047 | 16 | 9.87869 | 26 | 0.12131 | 9.90178 | 10 | 54 | 8 3.6 |
| 7 | 9.78063 | 17 | 9.87895 | 27 | 0.12105 | 9.90168 | 9 | 53 | 9 4.1 |
| 8 | 9.78080 | 17 | 9.87922 | 26 | 0.12078 | 9.90159 | 10 | 52 | 10 4.5 |
| 9 | 9.78097 | 16 | 9.87948 | 26 | 0.12052 | 9.90149 | 10 | 51 | 20 9.0 |
| **10** | 9.78113 | 17 | 9.87974 | 26 | 0.12026 | 9.90139 | 9 | **50** | ·30 13.5 |
| 11 | 9.78130 | 17 | 9.88000 | 27 | 0.12000 | 9.90130 | 10 | 49 | 40 18.0 |
| 12 | 9.78147 | 16 | 9.88027 | 26 | 0.11973 | 9.90120 | 9 | 48 | 50 22.5 |
| 13 | 9.78163 | 17 | 9.88053 | 26 | 0.11947 | 9.90111 | 10 | 47 | |
| 14 | 9.78180 | 17 | 9.88079 | 26 | 0.11921 | 9.90101 | 10 | 46 | |
| 15 | 9.78197 | 16 | 9.88105 | 26 | 0.11895 | 9.90091 | 9 | **45** | **26** |
| 16 | 9.78213 | 17 | 9.88131 | 27 | 0.11869 | 9.90082 | 10 | 44 | 6 2.6 |
| 17 | 9.78230 | 16 | 9.88158 | 26 | 0.11842 | 9.90072 | 9 | 43 | 7 3.0 |
| 18 | 9.78246 | 17 | 9.88184 | 26 | 0.11816 | 9.90063 | 10 | 42 | 8 3.5 |
| 19 | 9.78263 | 17 | 9.88210 | 26 | 0.11790 | 9.90053 | 10 | 41 | 9 3.9 |
| **20** | 9.78280 | 16 | 9.88236 | 26 | 0.11764 | 9.90043 | 9 | **40** | 10 4.3 |
| 21 | 9.78296 | 17 | 9.88262 | 27 | 0.11738 | 9.90034 | 10 | 39 | 20 8.7 |
| 22 | 9.78313 | 16 | 9.88289 | 26 | 0.11711 | 9.90024 | 10 | 38 | 30 13.0 |
| 23 | 9.78329 | 17 | 9.88315 | 26 | 0.11685 | 9.90014 | 9 | 37 | 40 17.3 |
| 24 | 9.78346 | 16 | 9.88341 | 26 | 0.11659 | 9.90005 | 10 | 36 | 50 21.7 |
| 25 | 9.78362 | 17 | 9.88367 | 26 | 0.11633 | 9.89995 | 10 | **35** | |
| 26 | 9.78379 | 16 | 9.88393 | 27 | 0.11607 | 9.89985 | 9 | 34 | **17** |
| 27 | 9.78395 | 17 | 9.88420 | 26 | 0.11580 | 9.89976 | 10 | 33 | 6 1.7 |
| 28 | 9.78412 | 16 | 9.88446 | 26 | 0.11554 | 9.89966 | 10 | 32 | 7 2.0 |
| 29 | 9.78428 | 17 | 9.88472 | 26 | 0.11528 | 9.89956 | 9 | 31 | 8 2.3 |
| **30** | 9.78445 | 16 | 9.88498 | 26 | 0.11502 | 9.89947 | 10 | **30** | 9 2.6 |
| 31 | 9.78461 | 17 | 9.88524 | 26 | 0.11476 | 9.89937 | 10 | 29 | 10 2.8 |
| 32 | 9.78478 | 16 | 9.88550 | 27 | 0.11450 | 9.89927 | 9 | 28 | 20 5.7 |
| 33 | 9.78494 | 16 | 9.88577 | 26 | 0.11423 | 9.89918 | 10 | 27 | 30 8.5 |
| 34 | 9.78510 | 17 | 9.88603 | 26 | 0.11397 | 9.89908 | 10 | 26 | 40 11.3 |
| 35 | 9.78527 | 16 | 9.88629 | 26 | 0.11371 | 9.89898 | 10 | **25** | 50 14.2 |
| 36 | 9.78543 | 17 | 9.88655 | 26 | 0.11345 | 9.89888 | 9 | 24 | |
| 37 | 9.78560 | 16 | 9.88681 | 26 | 0.11319 | 9.89879 | 10 | 23 | **16** |
| 38 | 9.78576 | 16 | 9.88707 | 26 | 0.11293 | 9.89869 | 10 | 22 | 6 1.6 |
| 39 | 9.78592 | 17 | 9.88733 | 26 | 0.11267 | 9.89859 | 10 | 21 | 7 1.9 |
| **40** | 9.78609 | 16 | 9.88759 | 27 | 0.11241 | 9.89849 | 9 | **20** | 8 2.1 |
| 41 | 9.78625 | 17 | 9.88786 | 26 | 0.11214 | 9.89840 | 10 | 19 | 9 2.4 |
| 42 | 9.78642 | 16 | 9.88812 | 26 | 0.11188 | 9.89830 | 10 | 18 | 10 2.7 |
| 43 | 9.78658 | 16 | 9.88838 | 26 | 0.11162 | 9.89820 | 10 | 17 | 20 5.3 |
| 44 | 9.78674 | 17 | 9.88864 | 26 | 0.11136 | 9.89810 | 9 | 16 | 30 8.0 |
| 45 | 9.78691 | 16 | 9.88890 | 26 | 0.11110 | 9.89801 | 10 | **15** | 40 10.7 |
| 46 | 9.78707 | 16 | 9.88916 | 26 | 0.11084 | 9.89791 | 10 | 14 | 50 13.3 |
| 47 | 9.78723 | 16 | 9.88942 | 26 | 0.11058 | 9.89781 | 10 | 13 | |
| 48 | 9.78739 | 17 | 9.88968 | 26 | 0.11032 | 9.89771 | 10 | 12 | |
| 49 | 9.78756 | 16 | 9.88994 | 26 | 0.11006 | 9.89761 | 9 | 11 | **10** **9** |
| **50** | 9.78772 | 16 | 9.89020 | 26 | 0.10980 | 9.89752 | 10 | **10** | 6 1.0 0.9 |
| 51 | 9.78788 | 17 | 9.89046 | 27 | 0.10954 | 9.89742 | 10 | 9 | 7 1.2 1.1 |
| 52 | 9.78805 | 16 | 9.89073 | 26 | 0.10927 | 9.89732 | 10 | 8 | 8 1.3 1.2 |
| 53 | 9.78821 | 16 | 9.89099 | 26 | 0.10901 | 9.89722 | 10 | 7 | 9 1.5 1.4 |
| 54 | 9.78837 | 16 | 9.89125 | 26 | 0.10875 | 9.89712 | 10 | 6 | 10 1.7 1.5 |
| 55 | 9.78853 | 16 | 9.89151 | 26 | 0.10849 | 9.89702 | 9 | **5** | 20 3.3 3.0 |
| 56 | 9.78869 | 17 | 9.89177 | 26 | 0.10823 | 9.89693 | 10 | 4 | 30 5.0 4.5 |
| 57 | 9.78886 | 16 | 9.89203 | 26 | 0.10797 | 9.89683 | 10 | 3 | 40 6.7 6.0 |
| 58 | 9.78902 | 16 | 9.89229 | 26 | 0.10771 | 9.89673 | 10 | 2 | 50 8.3 7.5 |
| 59 | 9.78918 | 16 | 9.89255 | 26 | 0.10745 | 9.89663 | 10 | 1 | |
| **60** | 9.78934 | | 9.89281 | | 0.10719 | 9.89653 | | **0** | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. |

**52°**

## 38°

| ′ | L. Sin. | d. | L. Tang. | d. c. | L. Cotg. | L. Cos. | d. | | P. P. | |
|---|---------|-----|----------|-------|----------|---------|-----|-----|-------|-------|
| 0 | 9.78934 | 16 | 9.89281 | 26 | 0.10719 | 9.89653 | 10 | 60 | | |
| 1 | 9.78950 | 16 | 9.89307 | 26 | 0.10693 | 9.89643 | 10 | 59 | | |
| 2 | 9.78967 | 17 | 9.89333 | 26 | 0.10667 | 9.89633 | 10 | 58 | **26** | **25** |
| 3 | 9.78983 | 16 | 9.89359 | 26 | 0.10641 | 9.89624 | 9 | 57 | 6 | 2.6 | 2.5 |
| 4 | 9.78999 | 16 | 9.89385 | 26 | 0.10615 | 9.89614 | 10 | 56 | 7 | 3.0 | 2.9 |
| 5 | 9.79015 | 16 | 9.89411 | 26 | 0.10589 | 9.89604 | 10 | 55 | 8 | 3.5 | 3.3 |
| 6 | 9.79031 | 16 | 9.89437 | 26 | 0.10563 | 9.89594 | 10 | 54 | 9 | 3.9 | 3.8 |
| 7 | 9.79047 | 16 | 9.89463 | 26 | 0.10537 | 9.89584 | 10 | 53 | 10 | 4.3 | 4.2 |
| 8 | 9.79063 | 16 | 9.89489 | 26 | 0.10511 | 9.89574 | 10 | 52 | 20 | 8.7 | 8.3 |
| 9 | 9.79079 | 16 | 9.89515 | 26 | 0.10485 | 9.89564 | 10 | 51 | 30 | 13.0 | 12.5 |
| 10 | 9.79095 | 16 | 9.89541 | 26 | 0.10459 | 9.89554 | 10 | 50 | 40 | 17.3 | 16.7 |
| 11 | 9.79111 | 16 | 9.89567 | 26 | 0.10433 | 9.89544 | 10 | 49 | 50 | 21.7 | 20.8 |
| 12 | 9.79128 | 17 | 9.89593 | 26 | 0.10407 | 9.89534 | 10 | 48 | | |
| 13 | 9.79144 | 16 | 9.89619 | 26 | 0.10381 | 9.89524 | 10 | 47 | | |
| 14 | 9.79160 | 16 | 9.89645 | 26 | 0.10355 | 9.89514 | 10 | 46 | | **17** |
| 15 | 9.79176 | 16 | 9.89671 | 26 | 0.10329 | 9.89504 | 10 | 45 | 6 | 1.7 |
| 16 | 9.79192 | 16 | 9.89697 | 26 | 0.10303 | 9.89495 | 9 | 44 | 7 | 2.0 |
| 17 | 9.79208 | 16 | 9.89723 | 26 | 0.10277 | 9.89485 | 10 | 43 | 8 | 2.3 |
| 18 | 9.79224 | 16 | 9.89749 | 26 | 0.10251 | 9.89475 | 10 | 42 | 9 | 2.6 |
| 19 | 9.79240 | 16 | 9.89775 | 26 | 0.10225 | 9.89465 | 10 | 41 | 10 | 2.8 |
| 20 | 9.79256 | 16 | 9.89801 | 26 | 0.10199 | 9.89455 | 10 | 40 | 20 | 5.7 |
| 21 | 9.79272 | 16 | 9.89827 | 26 | 0.10173 | 9.89445 | 10 | 39 | 30 | 8.5 |
| 22 | 9.79288 | 16 | 9.89853 | 26 | 0.10147 | 9.89435 | 10 | 38 | 40 | 11.3 |
| 23 | 9.79304 | 16 | 9.89879 | 26 | 0.10121 | 9.89425 | 10 | 37 | 50 | 14.2 |
| 24 | 9.79319 | 16 | 9.89905 | 26 | 0.10095 | 9.89415 | 10 | 36 | | |
| 25 | 9.79335 | 16 | 9.89931 | 26 | 0.10069 | 9.89405 | 10 | 35 | | |
| 26 | 9.79351 | 16 | 9.89957 | 26 | 0.10043 | 9.89395 | 10 | 34 | | **16** | **15** |
| 27 | 9.79367 | 16 | 9.89983 | 26 | 0.10017 | 9.89385 | 10 | 33 | 6 | 1.6 | 1.5 |
| 28 | 9.79383 | 16 | 9.90009 | 26 | 0.09991 | 9.89375 | 10 | 32 | 7 | 1.9 | 1.8 |
| 29 | 9.79399 | 16 | 9.90035 | 26 | 0.09965 | 9.89364 | 11 | 31 | 8 | 2.1 | 2.0 |
| 30 | 9.79415 | 16 | 9.90061 | 25 | 0.09939 | 9.89354 | 10 | 30 | 9 | 2.4 | 2.3 |
| 31 | 9.79431 | 16 | 9.90086 | 26 | 0.09914 | 9.89344 | 10 | 29 | 10 | 2.7 | 2.5 |
| 32 | 9.79447 | 16 | 9.90112 | 26 | 0.09888 | 9.89334 | 10 | 28 | 20 | 5.3 | 5.0 |
| 33 | 9.79463 | 15 | 9.90138 | 26 | 0.09862 | 9.89324 | 10 | 27 | 30 | 8.0 | 7.5 |
| 34 | 9.79478 | 16 | 9.90164 | 26 | 0.09836 | 9.89314 | 10 | 26 | 40 | 10.7 | 10.0 |
| 35 | 9.79494 | 16 | 9.90190 | 26 | 0.09810 | 9.89304 | 10 | 25 | 50 | 13.3 | 12.5 |
| 36 | 9.79510 | 16 | 9.90216 | 26 | 0.09784 | 9.89294 | 10 | 24 | | |
| 37 | 9.79526 | 16 | 9.90242 | 26 | 0.09758 | 9.89284 | 10 | 23 | | |
| 38 | 9.79542 | 16 | 9.90268 | 26 | 0.09732 | 9.89274 | 10 | 22 | | **11** |
| 39 | 9.79558 | 15 | 9.90294 | 26 | 0.09706 | 9.89264 | 10 | 21 | 6 | 1.1 |
| 40 | 9.79573 | 16 | 9.90320 | 26 | 0.09680 | 9.89254 | 10 | 20 | 7 | 1.3 |
| 41 | 9.79589 | 16 | 9.90346 | 25 | 0.09654 | 9.89244 | 10 | 19 | 8 | 1.5 |
| 42 | 9.79605 | 16 | 9.90371 | 26 | 0.09629 | 9.89233 | 11 | 18 | 9 | 1.7 |
| 43 | 9.79621 | 15 | 9.90397 | 26 | 0.09603 | 9.89223 | 10 | 17 | 10 | 1.8 |
| 44 | 9.79636 | 16 | 9.90423 | 26 | 0.09577 | 9.89213 | 10 | 16 | 20 | 3.7 |
| 45 | 9.79652 | 16 | 9.90449 | 26 | 0.09551 | 9.89203 | 10 | 15 | 30 | 5.5 |
| 46 | 9.79668 | 16 | 9.90475 | 26 | 0.09525 | 9.89193 | 10 | 14 | 40 | 7.3 |
| 47 | 9.79684 | 15 | 9.90501 | 26 | 0.09499 | 9.89183 | 10 | 13 | 50 | 9.2 |
| 48 | 9.79699 | 16 | 9.90527 | 26 | 0.09473 | 9.89173 | 11 | 12 | | |
| 49 | 9.79715 | 16 | 9.90553 | 25 | 0.09447 | 9.89162 | 10 | 11 | | |
| 50 | 9.79731 | 15 | 9.90578 | 26 | 0.09422 | 9.89152 | 10 | 10 | | **10** | **9** |
| 51 | 9.79746 | 16 | 9.90604 | 26 | 0.09396 | 9.89142 | 10 | 9 | 6 | 1.0 | 0.9 |
| 52 | 9.79762 | 16 | 9.90630 | 26 | 0.09370 | 9.89132 | 10 | 8 | 7 | 1.2 | 1.1 |
| 53 | 9.79778 | 15 | 9.90656 | 26 | 0.09344 | 9.89122 | 10 | 7 | 8 | 1.3 | 1.2 |
| 54 | 9.79793 | 16 | 9.90682 | 26 | 0.09318 | 9.89112 | 11 | 6 | 9 | 1.5 | 1.4 |
| 55 | 9.79809 | 16 | 9.90708 | 26 | 0.09292 | 9.89101 | 10 | 5 | 10 | 1.7 | 1.5 |
| 56 | 9.79825 | 15 | 9.90734 | 25 | 0.09266 | 9.89091 | 10 | 4 | 20 | 3.3 | 3.0 |
| 57 | 9.79840 | 16 | 9.90759 | 26 | 0.09241 | 9.89081 | 10 | 3 | 30 | 5.0 | 4.5 |
| 58 | 9.79856 | 16 | 9.90785 | 26 | 0.09215 | 9.89071 | 11 | 2 | 40 | 6.7 | 6.0 |
| 59 | 9.79872 | 15 | 9.90811 | 26 | 0.09189 | 9.89060 | 10 | 1 | 50 | 8.3 | 7.5 |
| 60 | 9.79887 | | 9.90837 | | 0.09163 | 9.89050 | | 0 | | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. | |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | P. P. |
|---|---------|----|---------|-------|----------|---------|----|---|-------|
| 0 | 9.79887 | 16 | 9.90837 | 26 | 0.09163 | 9.89050 | 10 | 60 | |
| 1 | 9.79903 | 15 | 9.90863 | 26 | 0.09137 | 9.89040 | 10 | 59 | |
| 2 | 9.79918 | 16 | 9.90889 | 26 | 0.09111 | 9.89030 | 10 | 58 | |
| 3 | 9.79934 | 16 | 9.90914 | 25 | 0.09086 | 9.89020 | 11 | 57 | **26** |
| 4 | 9.79950 | 15 | 9.90940 | 26 | 0.09060 | 9.89009 | 10 | 56 | 6 \| 2.6 |
| 5 | 9.79965 | 16 | 9.90966 | 26 | 0.09034 | 9.88999 | 10 | 55 | 7 \| 3.0 |
| 6 | 9.79981 | 15 | 9.90992 | 26 | 0.09008 | 9.88989 | 11 | 54 | 8 \| 3.5 |
| 7 | 9.79996 | 16 | 9.91018 | 25 | 0.08982 | 9.88978 | 10 | 53 | 9 \| 3.9 |
| 8 | 9.80012 | 15 | 9.91043 | 26 | 0.08957 | 9.88968 | 10 | 52 | 10 \| 4.3 |
| 9 | 9.80027 | 16 | 9.91069 | 26 | 0.08931 | 9.88958 | 10 | 51 | 20 \| 8.7 |
| 10 | 9.80043 | 15 | 9.91095 | 26 | 0.08905 | 9.88948 | 11 | 50 | 30 \| 13.0 |
| 11 | 9.80058 | 16 | 9.91121 | 26 | 0.08879 | 9.88937 | 10 | 49 | 40 \| 17.3 |
| 12 | 9.80074 | 15 | 9.91147 | 25 | 0.08853 | 9.88927 | 10 | 48 | 50 \| 21.7 |
| 13 | 9.80089 | 16 | 9.91172 | 26 | 0.08828 | 9.88917 | 11 | 47 | |
| 14 | 9.80105 | 15 | 9.91198 | 26 | 0.08802 | 9.88906 | 10 | 46 | **25** |
| 15 | 9.80120 | 16 | 9.91224 | 26 | 0.08776 | 9.88896 | 10 | 45 | 6 \| 2.5 |
| 16 | 9.80136 | 15 | 9.91250 | 26 | 0.08750 | 9.88886 | 11 | 44 | 7 \| 2.9 |
| 17 | 9.80151 | 15 | 9.91276 | 25 | 0.08724 | 9.88875 | 10 | 43 | 8 \| 3.3 |
| 18 | 9.80166 | 16 | 9.91301 | 26 | 0.08699 | 9.88865 | 10 | 42 | 9 \| 3.8 |
| 19 | 9.80182 | 15 | 9.91327 | 26 | 0.08673 | 9.88855 | 11 | 41 | 10 \| 4.2 |
| 20 | 9.80197 | 16 | 9.91353 | 26 | 0.08647 | 9.88844 | 10 | 40 | 20 \| 8.3 |
| 21 | 9.80213 | 15 | 9.91379 | 25 | 0.08621 | 9.88834 | 10 | 39 | 30 \| 12.5 |
| 22 | 9.80228 | 16 | 9.91404 | 26 | 0.08596 | 9.88824 | 11 | 38 | 40 \| 16.7 |
| 23 | 9.80244 | 15 | 9.91430 | 26 | 0.08570 | 9.88813 | 10 | 37 | 50 \| 20.8 |
| 24 | 9.80259 | 15 | 9.91456 | 26 | 0.08544 | 9.88803 | 10 | 36 | |
| 25 | 9.80274 | 16 | 9.91482 | 25 | 0.08518 | 9.88793 | 11 | 35 | |
| 26 | 9.80290 | 15 | 9.91507 | 26 | 0.08493 | 9.88782 | 10 | 34 | |
| 27 | 9.80305 | 15 | 9.91533 | 26 | 0.08467 | 9.88772 | 11 | 33 | **16** |
| 28 | 9.80320 | 16 | 9.91559 | 26 | 0.08441 | 9.88761 | 10 | 32 | 6 \| 1.6 |
| 29 | 9.80336 | 15 | 9.91585 | 25 | 0.08415 | 9.88751 | 10 | 31 | 7 \| 1.9 |
| 30 | 9.80351 | 15 | 9.91610 | 26 | 0.08390 | 9.88741 | 11 | 30 | 8 \| 2.1 |
| 31 | 9.80366 | 16 | 9.91636 | 26 | 0.08364 | 9.88730 | 10 | 29 | 9 \| 2.4 |
| 32 | 9.80382 | 15 | 9.91662 | 26 | 0.08338 | 9.88720 | 10 | 28 | 10 \| 2.7 |
| 33 | 9.80397 | 15 | 9.91688 | 25 | 0.08312 | 9.88709 | 10 | 27 | 20 \| 5.3 |
| 34 | 9.80412 | 16 | 9.91713 | 26 | 0.08287 | 9.88699 | 11 | 26 | 30 \| 8.0 |
| 35 | 9.80428 | 15 | 9.91739 | 26 | 0.08261 | 9.88688 | 10 | 25 | 40 \| 10.7 |
| 36 | 9.80443 | 15 | 9.91765 | 26 | 0.08235 | 9.88678 | 10 | 24 | 50 \| 13.3 |
| 37 | 9.80458 | 15 | 9.91791 | 25 | 0.08209 | 9.88668 | 11 | 23 | |
| 38 | 9.80473 | 16 | 9.91816 | 26 | 0.08184 | 9.88657 | 10 | 22 | **15** |
| 39 | 9.80489 | 15 | 9.91842 | 26 | 0.08158 | 9.88647 | 11 | 21 | 6 \| 1.5 |
| 40 | 9.80504 | 15 | 9.91868 | 25 | 0.08132 | 9.88636 | 10 | 20 | 7 \| 1.8 |
| 41 | 9.80519 | 15 | 9.91893 | 26 | 0.08107 | 9.88626 | 11 | 19 | 8 \| 2.0 |
| 42 | 9.80534 | 16 | 9.91919 | 26 | 0.08081 | 9.88615 | 10 | 18 | 9 \| 2.3 |
| 43 | 9.80550 | 15 | 9.91945 | 26 | 0.08055 | 9.88605 | 11 | 17 | 10 \| 2.5 |
| 44 | 9.80565 | 15 | 9.91971 | 25 | 0.08029 | 9.88594 | 10 | 16 | 20 \| 5.0 |
| 45 | 9.80580 | 15 | 9.91996 | 26 | 0.08004 | 9.88584 | 11 | 15 | 30 \| 7.5 |
| 46 | 9.80595 | 15 | 9.92022 | 26 | 0.07978 | 9.88573 | 10 | 14 | 40 \| 10.0 |
| 47 | 9.80610 | 15 | 9.92048 | 25 | 0.07952 | 9.88563 | 11 | 13 | 50 \| 12.5 |
| 48 | 9.80625 | 16 | 9.92073 | 26 | 0.07927 | 9.88552 | 10 | 12 | |
| 49 | 9.80641 | 15 | 9.92099 | 26 | 0.07901 | 9.88542 | 11 | 11 | |
| 50 | 9.80656 | 15 | 9.92125 | 25 | 0.07875 | 9.88531 | 10 | 10 | **11  10** |
| 51 | 9.80671 | 15 | 9.92150 | 26 | 0.07850 | 9.88521 | 11 | 9 | 6 \| 1.1 \| 1.0 |
| 52 | 9.80686 | 15 | 9.92176 | 26 | 0.07824 | 9.88510 | 11 | 8 | 7 \| 1.3 \| 1.2 |
| 53 | 9.80701 | 15 | 9.92202 | 25 | 0.07798 | 9.88499 | 10 | 7 | 8 \| 1.5 \| 1.3 |
| 54 | 9.80716 | 15 | 9.92227 | 26 | 0.07773 | 9.88489 | 11 | 6 | 9 \| 1.7 \| 1.5 |
| 55 | 9.80731 | 15 | 9.92253 | 26 | 0.07747 | 9.88478 | 10 | 5 | 10 \| 1.8 \| 1.7 |
| 56 | 9.80746 | 16 | 9.92279 | 25 | 0.07721 | 9.88468 | 11 | 4 | 20 \| 3.7 \| 3.3 |
| 57 | 9.80762 | 15 | 9.92304 | 26 | 0.07696 | 9.88457 | 10 | 3 | 30 \| 5.5 \| 5.0 |
| 58 | 9.80777 | 15 | 9.92330 | 26 | 0.07670 | 9.88447 | 11 | 2 | 40 \| 7.3 \| 6.7 |
| 59 | 9.80792 | 15 | 9.92356 | 25 | 0.07644 | 9.88436 | 11 | 1 | 50 \| 9.2 \| 8.3 |
| 60 | 9.80807 | | 9.92381 | | 0.07619 | 9.88425 | | 0 | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | P. P. |
|---|---------|-----|---------|-------|----------|---------|-----|-----|-------|
| 0 | 9.80807 | 15 | 9.92381 | 26 | 0.07619 | 9.88425 | 10 | 60 | |
| 1 | 9.80822 | 15 | 9.92407 | 26 | 0.07593 | 9.88415 | 10 | 59 | |
| 2 | 9.80837 | 15 | 9.92433 | 26 | 0.07567 | 9.88404 | 11 | 58 | |
| 3 | 9.80852 | 15 | 9.92458 | 25 | 0.07542 | 9.88394 | 10 | 57 | **26** |
| 4 | 9.80867 | 15 | 9.92484 | 26 | 0.07516 | 9.88383 | 11 | 56 | 6 2.6 |
| | | 15 | | 26 | | | 11 | | 7 3.0 |
| 5 | 9.80882 | 15 | 9.92510 | 25 | 0.07490 | 9.88372 | 10 | 55 | 8 3.5 |
| 6 | 9.80897 | 15 | 9.92535 | 26 | 0.07465 | 9.88362 | 11 | 54 | 9 3.9 |
| 7 | 9.80912 | 15 | 9.92561 | 26 | 0.07439 | 9.88351 | 11 | 53 | 10 4.3 |
| 8 | 9.80927 | 15 | 9.92587 | 25 | 0.07413 | 9.88340 | 10 | 52 | 20 8.7 |
| 9 | 9.80942 | 15 | 9.92612 | 26 | 0.07388 | 9.88330 | 11 | 51 | 30 13.0 |
| 10 | 9.80957 | 15 | 9.92638 | 25 | 0.07362 | 9.88319 | 11 | 50 | 40 17.3 |
| 11 | 9.80972 | 15 | 9.92663 | 26 | 0.07337 | 9.88308 | 10 | 49 | 50 21.7 |
| 12 | 9.80987 | 15 | 9.92689 | 26 | 0.07311 | 9.88298 | 11 | 48 | |
| 13 | 9.81002 | 15 | 9.92715 | 25 | 0.07285 | 9.88287 | 11 | 47 | |
| 14 | 9.81017 | 15 | 9.92740 | 26 | 0.07260 | 9.88276 | 10 | 46 | |
| | | 15 | | 26 | | | 11 | | **25** |
| 15 | 9.81032 | 15 | 9.92766 | 26 | 0.07234 | 9.88266 | 11 | 45 | 6 2.5 |
| 16 | 9.81047 | 14 | 9.92792 | 25 | 0.07208 | 9.88255 | 11 | 44 | 7 2.9 |
| 17 | 9.81061 | 15 | 9.92817 | 26 | 0.07183 | 9.88244 | 10 | 43 | 8 3.3 |
| 18 | 9.81076 | 15 | 9.92843 | 25 | 0.07157 | 9.88234 | 11 | 42 | 9 3.8 |
| 19 | 9.81091 | 15 | 9.92868 | 26 | 0.07132 | 9.88223 | 11 | 41 | 10 4.2 |
| 20 | 9.81106 | 15 | 9.92894 | 26 | 0.07106 | 9.88212 | 11 | 40 | 20 8.3 |
| 21 | 9.81121 | 15 | 9.92920 | 25 | 0.07080 | 9.88201 | 10 | 39 | 30 12.5 |
| 22 | 9.81136 | 15 | 9.92945 | 26 | 0.07055 | 9.88191 | 11 | 38 | 40 16.7 |
| 23 | 9.81151 | 15 | 9.92971 | 25 | 0.07029 | 9.88180 | 11 | 37 | 50 20.8 |
| 24 | 9.81166 | 14 | 9.92996 | 26 | 0.07004 | 9.88169 | 11 | 36 | |
| 25 | 9.81180 | 15 | 9.93022 | 26 | 0.06978 | 9.88158 | 10 | 35 | |
| 26 | 9.81195 | 15 | 9.93048 | 25 | 0.06952 | 9.88148 | 11 | 34 | **15** |
| 27 | 9.81210 | 15 | 9.93073 | 26 | 0.06927 | 9.88137 | 11 | 33 | 6 1.5 |
| 28 | 9.81225 | 15 | 9.93099 | 25 | 0.06901 | 9.88126 | 11 | 32 | 7 1.8 |
| 29 | 9.81240 | 14 | 9.93124 | 26 | 0.06876 | 9.88115 | 10 | 31 | 8 2.0 |
| 30 | 9.81254 | 15 | 9.93150 | 25 | 0.06850 | 9.88105 | 11 | 30 | 9 2.3 |
| 31 | 9.81269 | 15 | 9.93175 | 26 | 0.06825 | 9.88094 | 11 | 29 | 10 2.5 |
| 32 | 9.81284 | 15 | 9.93201 | 26 | 0.06799 | 9.88083 | 11 | 28 | 20 5.0 |
| 33 | 9.81299 | 15 | 9.93227 | 25 | 0.06773 | 9.88072 | 11 | 27 | 30 7.5 |
| 34 | 9.81314 | 14 | 9.93252 | 26 | 0.06748 | 9.88061 | 10 | 26 | 40 10.0 |
| 35 | 9.81328 | 15 | 9.93278 | 25 | 0.06722 | 9.88051 | 11 | 25 | 50 12.5 |
| 36 | 9.81343 | 15 | 9.93303 | 26 | 0.06697 | 9.88040 | 11 | 24 | |
| 37 | 9.81358 | 14 | 9.93329 | 26 | 0.06671 | 9.88029 | 11 | 23 | |
| 38 | 9.81372 | 15 | 9.93354 | 26 | 0.06646 | 9.88018 | 11 | 22 | **14** |
| 39 | 9.81387 | 15 | 9.93380 | 26 | 0.06620 | 9.88007 | 11 | 21 | 6 1.4 |
| 40 | 9.81402 | 15 | 9.93406 | 25 | 0.06594 | 9.87996 | 11 | 20 | 7 1.6 |
| 41 | 9.81417 | 14 | 9.93431 | 26 | 0.06569 | 9.87985 | 10 | 19 | 8 1.9 |
| 42 | 9.81431 | 15 | 9.93457 | 25 | 0.06543 | 9.87975 | 11 | 18 | 9 2.1 |
| 43 | 9.81446 | 15 | 9.93482 | 26 | 0.06518 | 9.87964 | 11 | 17 | 10 2.3 |
| 44 | 9.81461 | 14 | 9.93508 | 25 | 0.06492 | 9.87953 | 11 | 16 | 20 4.7 |
| 45 | 9.81475 | 15 | 9.93533 | 26 | 0.06467 | 9.87942 | 11 | 15 | 30 7.0 |
| 46 | 9.81490 | 15 | 9.93559 | 25 | 0.06441 | 9.87931 | 11 | 14 | 40 9.3 |
| 47 | 9.81505 | 14 | 9.93584 | 26 | 0.06416 | 9.87920 | 11 | 13 | 50 11.7 |
| 48 | 9.81519 | 15 | 9.93610 | 26 | 0.06390 | 9.87909 | 11 | 12 | |
| 49 | 9.81534 | 15 | 9.93636 | 25 | 0.06364 | 9.87898 | 11 | 11 | |
| 50 | 9.81549 | 14 | 9.93661 | 26 | 0.06339 | 9.87887 | 10 | 10 | **11** **10** |
| 51 | 9.81563 | 15 | 9.93687 | 25 | 0.06313 | 9.87877 | 11 | 9 | 6 1.1 1.0 |
| 52 | 9.81578 | 14 | 9.93712 | 26 | 0.06288 | 9.87866 | 11 | 8 | 7 1.3 1.2 |
| 53 | 9.81592 | 15 | 9.93738 | 25 | 0.06262 | 9.87855 | 11 | 7 | 8 1.5 1.3 |
| 54 | 9.81607 | 15 | 9.93763 | 26 | 0.06237 | 9.87844 | 11 | 6 | 9 1.7 1.5 |
| 55 | 9.81622 | 14 | 9.93789 | 25 | 0.06211 | 9.87833 | 11 | 5 | 10 1.8 1.7 |
| 56 | 9.81636 | 15 | 9.93814 | 26 | 0.06186 | 9.87922 | 11 | 4 | 20 3.7 3.3 |
| 57 | 9.81651 | 14 | 9.93840 | 25 | 0.06160 | 9.87811 | 11 | 3 | 30 5.5 5.0 |
| 58 | 9.81665 | 15 | 9.93865 | 26 | 0.06135 | 9.87800 | 11 | 2 | 40 7.3 6.7 |
| 59 | 9.81680 | 14 | 9.93891 | 25 | 0.06109 | 9.87789 | 11 | 1 | 50 9.2 8.3 |
| 60 | 9.81694 | | 9.93916 | | 0.06084 | 9.87778 | | 0 | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | P. P. |
|---|---------|----|---------|-------|----------|---------|-----|----|-------|
| **0** | 9.81694 | 15 | 9.93916 | 26 | 0.06084 | 9.87778 | 11 | **60** | |
| 1 | 9.81709 | 14 | 9.93942 | 25 | 0.06058 | 9.87767 | 11 | 59 | |
| 2 | 9.81723 | 15 | 9.93967 | 26 | 0.06033 | 9.87756 | 11 | 58 | |
| 3 | 9.81738 | 14 | 9.93993 | 25 | 0.06007 | 9.87745 | 11 | 57 | **26** |
| 4 | 9.81752 | 15 | 9.94018 | 26 | 0.05982 | 9.87734 | 11 | 56 | 6 \| 2.6 |
| 5 | 9.81767 | 14 | 9.94044 | 25 | 0.05956 | 9.87723 | 11 | 55 | 7 \| 3.0 |
| 6 | 9.81781 | 15 | 9.94069 | 26 | 0.05931 | 9.87712 | 11 | 54 | 8 \| 3.5 |
| 7 | 9.81796 | 14 | 9.94095 | 25 | 0.05905 | 9.87701 | 11 | 53 | 9 \| 3.9 |
| 8 | 9.81810 | 15 | 9.94120 | 26 | 0.05880 | 9.87690 | 11 | 52 | 10 \| 4.3 |
| 9 | 9.81825 | 14 | 9.94146 | 25 | 0.05854 | 9.87679 | 11 | 51 | 20 \| 8.7 |
| **10** | 9.81838 | 15 | 9.94171 | 26 | 0.05829 | 9.87668 | 11 | **50** | 30 \| 13.0 |
| 11 | 9.81854 | 14 | 9.94197 | 25 | 0.05803 | 9.87657 | 11 | 49 | 40 \| 17.3 |
| 12 | 9.81868 | 14 | 9.94222 | 26 | 0.05778 | 9.87646 | 11 | 48 | 50 \| 21.7 |
| 13 | 9.81882 | 15 | 9.94248 | 25 | 0.05752 | 9.87635 | 11 | 47 | |
| 14 | 9.81897 | 14 | 9.94273 | 26 | 0.05727 | 9.87624 | 11 | 46 | |
| 15 | 9.81911 | 15 | 9.94299 | 25 | 0.05701 | 9.87613 | 12 | 45 | **25** |
| 16 | 9.81926 | 14 | 9.94324 | 26 | 0.05676 | 9.87601 | 11 | 44 | 6 \| 2.5 |
| 17 | 9.81940 | 15 | 9.94350 | 25 | 0.05650 | 9.87590 | 11 | 43 | 7 \| 2.9 |
| 18 | 9.81955 | 14 | 9.94375 | 26 | 0.05625 | 9.87579 | 11 | 42 | 8 \| 3.3 |
| 19 | 9.81969 | 14 | 9.94401 | 25 | 0.65599 | 9.87568 | 11 | 41 | 9 \| 3.8 |
| **20** | 9.81983 | 15 | 9.94426 | 26 | 0.05574 | 9.87557 | 11 | **40** | 10 \| 4.2 |
| 21 | 9.81998 | 14 | 9.94452 | 25 | 0.05548 | 9.87546 | 11 | 39 | 20 \| 8.3 |
| 22 | 9.82012 | 14 | 9.94477 | 26 | 0.05523 | 9.87535 | 11 | 38 | 30 \| 12.5 |
| 23 | 9.82026 | 15 | 9.94503 | 25 | 0.05497 | 9.87524 | 11 | 37 | 40 \| 16.7 |
| 24 | 9.82041 | 14 | 9.94528 | 26 | 0.05472 | 9.87513 | 12 | 36 | 50 \| 20.8 |
| **25** | 9.82055 | 14 | 9.94554 | 25 | 0.05446 | 9.87501 | 11 | **35** | |
| 26 | 9.82069 | 15 | 9.94579 | 25 | 0.05421 | 9.87490 | 11 | 34 | |
| 27 | 9.82084 | 14 | 9.94604 | 26 | 0.05396 | 9.87479 | 11 | 33 | **15** |
| 28 | 9.82098 | 14 | 9.94630 | 25 | 0.05370 | 9.87468 | 11 | 32 | 6 \| 1.5 |
| 29 | 9.82112 | 14 | 9.94655 | 26 | 0.05345 | 9.87457 | 11 | 31 | 7 \| 1.8 |
| **30** | 9.82126 | 15 | 9.94681 | 25 | 0.05319 | 9.87446 | 12 | **30** | 8 \| 2.0 |
| 31 | 9.82141 | 14 | 9.94706 | 26 | 0.05294 | 9.87434 | 11 | 29 | 9 \| 2.3 |
| 32 | 9.82155 | 14 | 9.94732 | 25 | 0.05268 | 9.87423 | 11 | 28 | 10 \| 2.5 |
| 33 | 9.82169 | 15 | 9.94757 | 26 | 0.05243 | 9.87412 | 11 | 27 | 20 \| 5.0 |
| 34 | 9.82184 | 14 | 5.94783 | 25 | 0.05217 | 9.87401 | 11 | 26 | 30 \| 7.5 |
| **35** | 9.82198 | 14 | 9.94808 | 26 | 0.05192 | 9.87390 | 12 | **25** | 40 \| 10.0 |
| 36 | 9.82212 | 14 | 9.94834 | 25 | 0.05166 | 9.87378 | 11 | 24 | 50 \| 12.5 |
| 37 | 9.82226 | 14 | 9.94859 | 25 | 0.05141 | 9.87367 | 11 | 23 | |
| 38 | 9.82240 | 15 | 9.94884 | 26 | 0.05116 | 9.87356 | 11 | 22 | **14** |
| 39 | 9.82255 | 14 | 9.94910 | 25 | 0.05090 | 9.87345 | 11 | 21 | 6 \| 1.4 |
| **40** | 9.82269 | 14 | 9.94935 | 26 | 0.05065 | 9.87334 | 12 | **20** | 7 \| 1.6 |
| 41 | 9.82283 | 14 | 9.94961 | 25 | 0.05039 | 9.87322 | 11 | 19 | 8 \| 1.9 |
| 42 | 9.82297 | 14 | 9.94986 | 26 | 0.05014 | 9.87311 | 11 | 18 | 9 \| 2.1 |
| 43 | 9.82311 | 15 | 9.95012 | 25 | 0.04988 | 9.87300 | 12 | 17 | 10 \| 2.3 |
| 44 | 9.82326 | 14 | 9.95037 | 25 | 0.04963 | 9.87288 | 11 | 16 | 20 \| 4.7 |
| **45** | 9.82340 | 14 | 9.95062 | 26 | 0.04938 | 9.87277 | 11 | **15** | 30 \| 7.0 |
| 46 | 9.82354 | 14 | 9.95088 | 25 | 0.04912 | 9.87266 | 11 | 14 | 40 \| 9.3 |
| 47 | 9.82368 | 14 | 9.95113 | 26 | 0.04887 | 9.87255 | 12 | 13 | 50 \| 11.7 |
| 48 | 9.82382 | 14 | 9.95139 | 25 | 0.04861 | 9.87243 | 11 | 12 | |
| 49 | 9.82396 | 14 | 9.95164 | 26 | 0.04836 | 9.87232 | 11 | 11 | |
| **50** | 9.82410 | 14 | 9.95190 | 25 | 0.04810 | 9.87221 | 12 | **10** | **12** \| **11** |
| 51 | 9.82424 | 15 | 9.95215 | 25 | 0.04785 | 9.87209 | 11 | 9 | 6 \| 1.2 \| 1.1 |
| 52 | 9.82439 | 14 | 9.95240 | 26 | 0.04760 | 9.87198 | 11 | 8 | 7 \| 1.4 \| 1.3 |
| 53 | 9.82453 | 14 | 9.95266 | 25 | 0.04734 | 9.87187 | 12 | 7 | 8 \| 1.6 \| 1.5 |
| 54 | 9.82467 | 14 | 9.95291 | 26 | 0.04709 | 9.87175 | 11 | 6 | 9 \| 1.8 \| 1.7 |
| **55** | 9.82481 | 14 | 9.95317 | 25 | 0.04683 | 9.87164 | 11 | **5** | 10 \| 2.0 \| 1.8 |
| 56 | 9.82495 | 14 | 9.95342 | 26 | 0.04658 | 9.87153 | 12 | 4 | 20 \| 4.0 \| 3.7 |
| 57 | 9.82509 | 14 | 9.95368 | 25 | 0.04632 | 9.87141 | 11 | 3 | 30 \| 6.0 \| 5.5 |
| 58 | 9.82523 | 14 | 9.95393 | 25 | 0.04607 | 9.87130 | 11 | 2 | 40 \| 8.0 \| 7.3 |
| 59 | 9.82537 | 14 | 9.95418 | 26 | 0.04582 | 9.87119 | 12 | 1 | 50 \| 10.0 \| 9.2 |
| **60** | 9.82551 | | 9.95444 | | 0.04556 | 9.87107 | | **0** | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | | P. P. |
|---|---------|-----|---------|-------|----------|---------|-----|-----|---|-------|
| 0 | 9.82551 | 14 | 9.95444 | 25 | 0.04556 | 9.87107 | 11 | 60 | | |
| 1 | 9.82565 | 14 | 9.95469 | 26 | 0.04531 | 9.87096 | 11 | 59 | | |
| 2 | 9.82579 | 14 | 9.95495 | 25 | 0.04505 | 9.87085 | 11 | 58 | | |
| 3 | 9.82593 | 14 | 9.95520 | 25 | 0.04480 | 9.87073 | 12 | 57 | | **26** |
| 4 | 9.82607 | 14 | 9.95545 | 26 | 0.04455 | 9.87062 | 11 | 56 | | 6 \| 2.6 |
| 5 | 9.82621 | 14 | 9.95571 | 25 | 0.04429 | 9.87050 | 12 | 55 | | 7 \| 3.0 |
| 6 | 9.82635 | 14 | 9.95596 | 26 | 0.04404 | 9.87039 | 11 | 54 | | 8 \| 3.5 |
| 7 | 9.82649 | 14 | 9.95622 | 25 | 0.04378 | 9.87028 | 11 | 53 | | 9 \| 3.9 |
| 8 | 9.82663 | 14 | 9.95647 | 25 | 0.04353 | 9.87016 | 12 | 52 | | 10 \| 4.3 |
| 9 | 9.82677 | 14 | 9.95672 | 26 | 0.04328 | 9.87005 | 11 | 51 | | 20 \| 8.7 |
| 10 | 9.82691 | 14 | 9.95698 | 25 | 0.04302 | 9.86993 | 11 | 50 | | 30 \| 13.0 |
| 11 | 9.82705 | 14 | 9.95723 | 25 | 0.04277 | 9.86982 | 12 | 49 | | 40 \| 17.3 |
| 12 | 9.82719 | 14 | 9.95748 | 26 | 0.04252 | 9.86970 | 11 | 48 | | 50 \| 21.7 |
| 13 | 9.82733 | 14 | 9.95774 | 25 | 0.04226 | 9.86959 | 12 | 47 | | |
| 14 | 9.82747 | 14 | 9.95799 | 26 | 0.04201 | 9.86947 | 11 | 46 | | |
| 15 | 9.82761 | 14 | 9.95825 | 25 | 0.04175 | 9.86936 | 12 | 45 | | **25** |
| 16 | 9.82775 | 13 | 9.95850 | 25 | 0.04150 | 9.86924 | 11 | 44 | | 6 \| 2.5 |
| 17 | 9.82788 | 14 | 9.95875 | 26 | 0.04125 | 9.86913 | 11 | 43 | | 7 \| 2.9 |
| 18 | 9.82802 | 14 | 9.95901 | 25 | 0.04099 | 9.86902 | 12 | 42 | | 8 \| 3.3 |
| 19 | 9.82816 | 14 | 9.95926 | 26 | 0.04074 | 9.86890 | 11 | 41 | | 9 \| 3.8 |
| 20 | 9.82830 | 14 | 9.95952 | 25 | 0.04048 | 9.86879 | 12 | 40 | | 10 \| 4.2 |
| 21 | 9.82844 | 14 | 9.95977 | 25 | 0.04023 | 9.86867 | 12 | 39 | | 20 \| 8.3 |
| 22 | 9.82858 | 14 | 9.96002 | 26 | 0.03998 | 9.86855 | 11 | 38 | | 30 \| 12.5 |
| 23 | 9.82872 | 13 | 9.96028 | 25 | 0.03972 | 9.86844 | 12 | 37 | | 40 \| 16.7 |
| 24 | 9.82885 | 14 | 9.96053 | 25 | 0.03947 | 9.86832 | 11 | 36 | | 50 \| 20.8 |
| 25 | 9.82899 | 14 | 9.96078 | 26 | 0.03922 | 9.86821 | 12 | 35 | | |
| 26 | 9.82913 | 14 | 9.96104 | 25 | 0.03896 | 9.86809 | 11 | 34 | | |
| 27 | 9.82927 | 14 | 9.96129 | 26 | 0.03871 | 9.86798 | 12 | 33 | | **14** |
| 28 | 9.82941 | 14 | 9.96155 | 25 | 0.03845 | 9.86786 | 11 | 32 | | 6 \| 1.4 |
| 29 | 9.82955 | 13 | 9.96180 | 25 | 0.03820 | 9.86775 | 12 | 31 | | 7 \| 1.6 |
| 30 | 9.82968 | 14 | 9.96205 | 26 | 0.03795 | 9.86763 | 11 | 30 | | 8 \| 1.9 |
| 31 | 9.82982 | 14 | 9.96231 | 25 | 0.03769 | 9.86752 | 12 | 29 | | 9 \| 2.1 |
| 32 | 9.82996 | 14 | 9.96256 | 25 | 0.03744 | 9.86740 | 12 | 28 | | 10 \| 2.3 |
| 33 | 9.83010 | 13 | 9.96281 | 26 | 0.03719 | 9.86728 | 11 | 27 | | 20 \| 4.7 |
| 34 | 9.83023 | 14 | 9.96307 | 25 | 0.03693 | 9.86717 | 12 | 26 | | 30 \| 7.0 |
| 35 | 9.83037 | 14 | 9.96332 | 25 | 0.03668 | 9.86705 | 11 | 25 | | 40 \| 9.3 |
| 36 | 9.83051 | 14 | 9.96357 | 26 | 0.03643 | 9.86694 | 12 | 24 | | 50 \| 11.7 |
| 37 | 9.83065 | 13 | 9.96383 | 25 | 0.03617 | 9.86682 | 12 | 23 | | |
| 38 | 9.83078 | 14 | 9.96408 | 25 | 0.03592 | 9.86670 | 11 | 22 | | **13** |
| 39 | 9.83092 | 14 | 9.96433 | 26 | 0.03567 | 9.86659 | 12 | 21 | | 6 \| 1.3 |
| 40 | 9.83106 | 14 | 9.96459 | 25 | 0.03541 | 9.86647 | 12 | 20 | | 7 \| 1.5 |
| 41 | 9.83120 | 13 | 9.96484 | 26 | 0.03516 | 9.86635 | 11 | 19 | | 8 \| 1.7 |
| 42 | 9.83133 | 14 | 9.96510 | 25 | 0.03490 | 9.86624 | 12 | 18 | | 9 \| 2.0 |
| 43 | 9.83147 | 14 | 9.96535 | 25 | 0.03465 | 9.86612 | 12 | 17 | | 10 \| 2.2 |
| 44 | 9.83161 | 13 | 9.96560 | 26 | 0.03440 | 9.86600 | 11 | 16 | | 20 \| 4.3 |
| 45 | 9.83174 | 14 | 9.96586 | 25 | 0.03414 | 9.86589 | 12 | 15 | | 30 \| 6.5 |
| 46 | 9.83188 | 14 | 9.96611 | 25 | 0.03389 | 9.86577 | 12 | 14 | | 40 \| 8.7 |
| 47 | 9.83202 | 13 | 9.96636 | 26 | 0.03364 | 9.86565 | 11 | 13 | | 50 \| 10.8 |
| 48 | 9.83215 | 14 | 9.96662 | 25 | 0.03338 | 9.86554 | 12 | 12 | | |
| 49 | 9.83229 | 13 | 9.96687 | 25 | 0.03313 | 9.86542 | 12 | 11 | | |
| 50 | 9.83242 | 14 | 9.96712 | 26 | 0.03288 | 9.86530 | 12 | 10 | | **12** \| **11** |
| 51 | 9.83256 | 14 | 9.96738 | 25 | 0.03262 | 9.86518 | 11 | 9 | | 6 \| 1.2 \| 1.1 |
| 52 | 9.83270 | 13 | 9.96763 | 25 | 0.03237 | 9.86507 | 12 | 8 | | 7 \| 1.4 \| 1.3 |
| 53 | 9.83283 | 14 | 9.96788 | 26 | 0.03212 | 9.86495 | 12 | 7 | | 8 \| 1.6 \| 1.5 |
| 54 | 9.83297 | 13 | 9.96814 | 25 | 0.03186 | 9.86483 | 11 | 6 | | 9 \| 1.8 \| 1.7 |
| 55 | 9.83310 | 14 | 9.96839 | 25 | 0.03161 | 9.86472 | 12 | 5 | | 10 \| 2.0 \| 1.8 |
| 56 | 9.83324 | 14 | 9.96864 | 26 | 0.03136 | 9.86460 | 12 | 4 | | 20 \| 4.0 \| 3.7 |
| 57 | 9.83338 | 13 | 9.96890 | 25 | 0.03110 | 9.86448 | 12 | 3 | | 30 \| 6.0 \| 5.5 |
| 58 | 9.83351 | 14 | 9.96915 | 25 | 0.03085 | 9.86436 | 11 | 2 | | 40 \| 8.0 \| 7.3 |
| 59 | 9.83365 | 13 | 9.96940 | 26 | 0.03060 | 9.86425 | 12 | 1 | | 50 \| 10.0 \| 9.2 |
| 60 | 9.83378 | | 9.96966 | | 0.03034 | 9.86413 | | 0 | | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | | P. P. |

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| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | | P. P. | |
|---|---------|-----|---------|-------|----------|---------|-----|-----|---|-------|---|
| **0** | 9.83378 | 14 | 9.96966 | 25 | 0.03034 | 9.86413 | 12 | **60** | | | |
| 1 | 9.83392 | 13 | 9.96991 | 25 | 0.03009 | 9.86401 | 12 | 59 | | | |
| 2 | 9.83405 | 14 | 9.97016 | 25 | 0.02984 | 9.86389 | 12 | 58 | | **26** | |
| 3 | 9.83419 | 13 | 9.97042 | 25 | 0.02958 | 9.86377 | 12 | 57 | | 6 \| 2.6 | |
| 4 | 9.83432 | 14 | 9.97067 | 25 | 0.02933 | 9.86366 | 11 | 56 | | 7 \| 3.0 | |
| **5** | 9.83446 | 13 | 9.97092 | 26 | 0.02908 | 9.86354 | 12 | **55** | | 8 \| 3.5 | |
| 6 | 9.83459 | 14 | 9.97118 | 25 | 0.02882 | 9.86342 | 12 | 54 | | 9 \| 3.9 | |
| 7 | 9.83473 | 13 | 9.97143 | 25 | 0.02857 | 9.86330 | 12 | 53 | | 10 \| 4.3 | |
| 8 | 9.83486 | 14 | 9.97168 | 25 | 0.02832 | 9.86318 | 12 | 52 | | 20 \| 8.7 | |
| 9 | 9.83500 | 13 | 9.97193 | 26 | 0.02807 | 9.86306 | 11 | 51 | | 30 \| 13.0 | |
| **10** | 9.83513 | 14 | 9.97219 | 25 | 0.02781 | 9.86295 | 12 | **50** | | 40 \| 17.3 | |
| 11 | 9.83527 | 13 | 9.97244 | 25 | 0.02756 | 9.86283 | 12 | 49 | | 50 \| 21.7 | |
| 12 | 9.83540 | 14 | 9.97269 | 26 | 0.02731 | 9.86271 | 12 | 48 | | | |
| 13 | 9.83554 | 13 | 9.97295 | 25 | 0.02705 | 9.86259 | 12 | 47 | | | |
| 14 | 9.83567 | 14 | 9.97320 | 25 | 0.02680 | 9.86247 | 12 | 46 | | **25** | |
| **15** | 9.83581 | 13 | 9.97345 | 26 | 0.02655 | 9.86235 | 12 | **45** | | 6 \| 2.5 | |
| 16 | 9.83594 | 14 | 9.97371 | 25 | 0.02629 | 9.86223 | 12 | 44 | | 7 \| 2.9 | |
| 17 | 9.83608 | 13 | 9.97396 | 25 | 0.02604 | 9.86211 | 11 | 43 | | 8 \| 3.3 | |
| 18 | 9.83621 | 13 | 9.97421 | 26 | 0.02579 | 9.86200 | 12 | 42 | | 9 \| 3.8 | |
| 19 | 9.83634 | 14 | 9.97447 | 25 | 0.02553 | 9.86188 | 12 | 41 | | 10 \| 4.2 | |
| **20** | 9.83648 | 13 | 9.97472 | 25 | 0.02528 | 9.86176 | 12 | **40** | | 20 \| 8.3 | |
| 21 | 9.83661 | 13 | 9.97497 | 26 | 0.02503 | 9.86164 | 12 | 39 | | 30 \| 12.5 | |
| 22 | 9.83674 | 13 | 9.97523 | 25 | 0.02477 | 9.86152 | 12 | 38 | | 40 \| 16.7 | |
| 23 | 9.83688 | 14 | 9.97548 | 25 | 0.02452 | 9.86140 | 12 | 37 | | 50 \| 20.8 | |
| 24 | 9.83701 | 13 | 9.97573 | 25 | 0.02427 | 9.86128 | 12 | 36 | | | |
| **25** | 9.83715 | 14 | 9.97598 | 26 | 0.02402 | 9.86116 | 12 | **35** | | | |
| 26 | 9.83728 | 13 | 9.97624 | 25 | 0.02376 | 9.86104 | 12 | 34 | | **14** | |
| 27 | 9.83741 | 13 | 9.97649 | 25 | 0.02351 | 9.86092 | 12 | 33 | | 6 \| 1.4 | |
| 28 | 9.83755 | 14 | 9.97674 | 26 | 0.02326 | 9.86080 | 12 | 32 | | 7 \| 1.6 | |
| 29 | 9.83768 | 13 | 9.97700 | 25 | 0.02300 | 9.86068 | 12 | 31 | | 8 \| 1.9 | |
| **30** | 9.83781 | 13 | 9.97725 | 25 | 0.02275 | 9.86056 | 12 | **30** | | 9 \| 2.1 | |
| 31 | 9.83795 | 14 | 9.97750 | 26 | 0.02250 | 9.86044 | 12 | 29 | | 10 \| 2.3 | |
| 32 | 9.83808 | 13 | 9.97776 | 25 | 0.02224 | 9.86032 | 12 | 28 | | 20 \| 4.7 | |
| 33 | 9.83821 | 13 | 9.97801 | 25 | 0.02199 | 9.86020 | 12 | 27 | | 30 \| 7.0 | |
| 34 | 9.83834 | 14 | 9.97826 | 25 | 0.02174 | 9.86008 | 12 | 26 | | 40 \| 9.3 | |
| **35** | 9.83848 | 13 | 9.97851 | 26 | 0.02149 | 9.85996 | 12 | **25** | | 50 \| 11.7 | |
| 36 | 9.83861 | 13 | 9.97877 | 25 | 0.02123 | 9.85984 | 12 | 24 | | | |
| 37 | 9.83874 | 13 | 9.97902 | 25 | 0.02098 | 9.85972 | 12 | 23 | | | |
| 38 | 9.83887 | 14 | 9.97927 | 26 | 0.02073 | 9.85960 | 12 | 22 | | **13** | |
| 39 | 9.83901 | 13 | 9.97953 | 25 | 0.02047 | 9.85948 | 12 | 21 | | 6 \| 1.3 | |
| **40** | 9.83914 | 13 | 9.97978 | 25 | 0.02022 | 9.85936 | 12 | **20** | | 7 \| 1.5 | |
| 41 | 9.83927 | 13 | 9.98003 | 26 | 0.01997 | 9.85924 | 12 | 19 | | 8 \| 1.7 | |
| 42 | 9.83940 | 14 | 9.98029 | 25 | 0.01971 | 9.85912 | 12 | 18 | | 9 \| 2.0 | |
| 43 | 9.83954 | 13 | 9.98054 | 25 | 0.01946 | 9.85900 | 12 | 17 | | 10 \| 2.2 | |
| 44 | 9.83967 | 13 | 9.98079 | 25 | 0.01921 | 9.85888 | 12 | 16 | | 20 \| 4.3 | |
| **45** | 9.83980 | 13 | 9.98104 | 26 | 0.01896 | 9.85876 | 12 | **15** | | 30 \| 6.5 | |
| 46 | 9.83993 | 13 | 9.98130 | 25 | 0.01870 | 9.85864 | 13 | 14 | | 40 \| 8.7 | |
| 47 | 9.84006 | 14 | 9.98155 | 25 | 0.01845 | 9.85851 | 12 | 13 | | 50 \| 10.8 | |
| 48 | 9.84020 | 13 | 9.98180 | 26 | 0.01820 | 9.85839 | 12 | 12 | | | |
| 49 | 9.84033 | 13 | 9.98206 | 25 | 0.01794 | 9.85827 | 12 | 11 | | **12** \| **11** | |
| **50** | 9.84046 | 13 | 9.98231 | 25 | 0.01769 | 9.85815 | 12 | **10** | | 6 \| 1.2 \| 1.1 | |
| 51 | 9.84059 | 13 | 9.98256 | 25 | 0.01744 | 9.85803 | 12 | 9 | | 7 \| 1.4 \| 1.3 | |
| 52 | 9.84072 | 13 | 9.98281 | 26 | 0.01719 | 9.85791 | 12 | 8 | | 8 \| 1.6 \| 1.5 | |
| 53 | 9.84085 | 13 | 9.98307 | 25 | 0.01693 | 9.85779 | 13 | 7 | | 9 \| 1.8 \| 1.7 | |
| 54 | 9.84098 | 14 | 9.98332 | 25 | 0.01668 | 9.85766 | 12 | 6 | | 10 \| 2.0 \| 1.8 | |
| **55** | 9.84112 | 13 | 9.98357 | 26 | 0.01643 | 9.85754 | 12 | **5** | | 20 \| 4.0 \| 3.7 | |
| 56 | 9.84125 | 13 | 9.98383 | 25 | 0.01617 | 9.85742 | 12 | 4 | | 30 \| 6.0 \| 5.5 | |
| 57 | 9.84138 | 13 | 9.98408 | 25 | 0.01592 | 9.85730 | 12 | 3 | | 40 \| 8.0 \| 7.3 | |
| 58 | 9.84151 | 13 | 9.98433 | 25 | 0.01567 | 9.85718 | 12 | 2 | | 50 \| 10.0 \| 9.2 | |
| 59 | 9.84164 | 13 | 9.98458 | 26 | 0.01542 | 9.85706 | 13 | 1 | | | |
| **60** | 9.84177 | | 9.98484 | | 0.01516 | 9.85693 | | **0** | | | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | | P. P. | |

**46°**

**44°**

| ′ | L. Sin. | d. | L.Tang. | d. c. | L. Cotg. | L. Cos. | d. | | P. P. | |
|---|---------|----|---------|-------|----------|---------|----|----|----|----|
| 0 | 9.84177 | 13 | 9.98484 | 25 | 0.01516 | 9.85693 | 12 | 60 | | |
| 1 | 9.84190 | 13 | 9.98509 | 25 | 0.01491 | 9.85681 | 12 | 59 | | |
| 2 | 9.84203 | 13 | 9.98534 | 26 | 0.01466 | 9.85669 | 12 | 58 | | **26** |
| 3 | 9.84216 | 13 | 9.98560 | 25 | 0.01440 | 9.85657 | 12 | 57 | 6 | 2.6 |
| 4 | 9.84229 | 13 | 9.98585 | 25 | 0.01415 | 9.85645 | 13 | 56 | 7 | 3.0 |
| 5 | 9.84242 | 13 | 9.98610 | 25 | 0.01390 | 9.85632 | 12 | 55 | 8 | 3.5 |
| 6 | 9.84255 | 14 | 9.98635 | 26 | 0.01365 | 9.85620 | 12 | 54 | 9 | 3.9 |
| 7 | 9.84269 | 13 | 9.98661 | 25 | 0.01339 | 9.85608 | 12 | 53 | 10 | 4.3 |
| 8 | 9.84282 | 13 | 9.98686 | 25 | 0.01314 | 9.85596 | 13 | 52 | 20 | 8.7 |
| 9 | 9.84295 | 13 | 9.98711 | 26 | 0.01289 | 9.85583 | 12 | 51 | 30 | 13.0 |
| 10 | 9.84308 | 13 | 9.98737 | 25 | 0.01263 | 9.85571 | 12 | 50 | 40 | 17.3 |
| 11 | 9.84321 | 13 | 9.98762 | 25 | 0.01238 | 9.85559 | 12 | 49 | 50 | 21.7 |
| 12 | 9.84334 | 13 | 9.98787 | 25 | 0.01213 | 9.85547 | 13 | 48 | | |
| 13 | 9.84347 | 13 | 9.98812 | 26 | 0.01188 | 9.85534 | 12 | 47 | | |
| 14 | 9.84360 | 13 | 9.98838 | 25 | 0.01162 | 9.85522 | 12 | 46 | | |
| 15 | 9.84373 | 12 | 9.98863 | 25 | 0.01137 | 9.85510 | 13 | 45 | | **25** |
| 16 | 9.84385 | 13 | 9.98888 | 25 | 0.01112 | 9.85497 | 12 | 44 | 6 | 2.5 |
| 17 | 9.84398 | 13 | 9.98913 | 26 | 0.01087 | 9.85485 | 12 | 43 | 7 | 2.9 |
| 18 | 9.84411 | 13 | 9.98939 | 25 | 0.01061 | 9.85473 | 13 | 42 | 8 | 3.3 |
| 19 | 9.84424 | 13 | 9.98964 | 25 | 0.01036 | 9.85460 | 12 | 41 | 9 | 3.8 |
| 20 | 9.84437 | 13 | 9.98989 | 26 | 0.01011 | 9.85448 | 12 | 40 | 10 | 4.2 |
| 21 | 9.84450 | 13 | 9.99015 | 25 | 0.00985 | 9.85436 | 13 | 39 | 20 | 8.3 |
| 22 | 9.84463 | 13 | 9.99040 | 25 | 0.00960 | 9.85423 | 12 | 38 | 30 | 12.5 |
| 23 | 9.84476 | 13 | 9.99065 | 25 | 0.00935 | 9.85411 | 12 | 37 | 40 | 16.7 |
| 24 | 9.84489 | 13 | 9.99090 | 26 | 0.00910 | 9.85399 | 13 | 36 | 50 | 20.8 |
| 25 | 9.84502 | 13 | 9.99116 | 25 | 0.00884 | 9.85386 | 12 | 35 | | |
| 26 | 9.84515 | 13 | 9.99141 | 25 | 0.00859 | 9.85374 | 13 | 34 | | **14** |
| 27 | 9.84528 | 12 | 9.99166 | 25 | 0.00834 | 9.85361 | 12 | 33 | 6 | 1.4 |
| 28 | 9.84540 | 13 | 9.99191 | 26 | 0.00809 | 9.85349 | 12 | 32 | 7 | 1.6 |
| 29 | 9.84553 | 13 | 9.99217 | 25 | 0.00783 | 9.85337 | 13 | 31 | 8 | 1.9 |
| 30 | 9.84566 | 13 | 9.99242 | 25 | 0.00758 | 9.85324 | 12 | 30 | 9 | 2.1 |
| 31 | 9.84579 | 13 | 9.99267 | 26 | 0.00733 | 9.85312 | 13 | 29 | 10 | 2.3 |
| 32 | 9.84592 | 13 | 9.99293 | 25 | 0.00707 | 9.85299 | 12 | 28 | 20 | 4.7 |
| 33 | 9.84605 | 13 | 9.99318 | 25 | 0.00682 | 9.85287 | 13 | 27 | 30 | 7.0 |
| 34 | 9.84618 | 12 | 9.99343 | 25 | 0.00657 | 9.85274 | 12 | 26 | 40 | 9.3 |
| 35 | 9.84630 | 13 | 9.99368 | 26 | 0.00632 | 9.85262 | 12 | 25 | 50 | 11.7 |
| 36 | 9.84643 | 13 | 9.99394 | 25 | 0.00606 | 9.85250 | 13 | 24 | | |
| 37 | 9.84656 | 13 | 9.99419 | 25 | 0.00581 | 9.85237 | 12 | 23 | | |
| 38 | 9.84669 | 13 | 9.99444 | 25 | 0.00556 | 9.85225 | 13 | 22 | | **13** |
| 39 | 9.84682 | 12 | 9.99469 | 26 | 0.00531 | 9.85212 | 12 | 21 | 6 | 1.3 |
| 40 | 9.84694 | 13 | 9.99495 | 25 | 0.00505 | 9.85200 | 13 | 20 | 7 | 1.5 |
| 41 | 9.84707 | 13 | 9.99520 | 25 | 0.00480 | 9.85187 | 12 | 19 | 8 | 1.7 |
| 42 | 9.84720 | 13 | 9.99545 | 25 | 0.00455 | 9.85175 | 13 | 18 | 9 | 2.0 |
| 43 | 9.84733 | 12 | 9.99570 | 26 | 0.00430 | 9.85162 | 12 | 17 | 10 | 2.2 |
| 44 | 9.84745 | 13 | 9.99596 | 25 | 0.00404 | 9.85150 | 13 | 16 | 20 | 4.3 |
| 45 | 9.84758 | 13 | 9.99621 | 25 | 0.00379 | 9.85137 | 12 | 15 | 30 | 6.5 |
| 46 | 9.84771 | 13 | 9.99646 | 26 | 0.00354 | 9.85125 | 13 | 14 | 40 | 8.7 |
| 47 | 9.84784 | 12 | 9.99672 | 25 | 0.00328 | 9.85112 | 12 | 13 | 50 | 10.8 |
| 48 | 9.84796 | 13 | 9.99697 | 25 | 0.00303 | 9.85100 | 13 | 12 | | |
| 49 | 9.84809 | 13 | 9.99722 | 25 | 0.00278 | 9.85087 | 13 | 11 | | **12** |
| 50 | 9.84822 | 13 | 9.99747 | 26 | 0.00253 | 9.85074 | 12 | 10 | 6 | 1.2 |
| 51 | 9.84835 | 12 | 9.99773 | 25 | 0.00227 | 9.85062 | 13 | 9 | 7 | 1.4 |
| 52 | 9.84847 | 13 | 9.99798 | 25 | 0.00202 | 9.85049 | 12 | 8 | 8 | 1.6 |
| 53 | 9.84860 | 13 | 9.99823 | 25 | 0.00177 | 9.85037 | 13 | 7 | 9 | 1.8 |
| 54 | 9.84873 | 12 | 9.99848 | 26 | 0.00152 | 9.85024 | 12 | 6 | 10 | 2.0 |
| 55 | 9.84885 | 13 | 9.99874 | 25 | 0.00126 | 9.85012 | 13 | 5 | 20 | 4.0 |
| 56 | 9.84898 | 13 | 9.99899 | 25 | 0.00101 | 9.84999 | 13 | 4 | 30 | 6.0 |
| 57 | 9.84911 | 12 | 9.99924 | 25 | 0.00076 | 9.84986 | 13 | 3 | 40 | 8.0 |
| 58 | 9.84923 | 13 | 9.99949 | 26 | 0.00051 | 9.84974 | 13 | 2 | 50 | 10.0 |
| 59 | 9.84936 | 13 | 9.99975 | 25 | 0.00025 | 9.84961 | 12 | 1 | | |
| 60 | 9.84949 | | 0.00000 | | 0.00000 | 9.84949 | | 0 | | |
| | L. Cos. | d. | L. Cotg. | d. c. | L.Tang. | L. Sin. | d. | ′ | P. P. | |

**45°**

# TRAVERSE TABLES

To use the tables, find the number of degrees in the left-hand column if the angle is less than 45°, and in the right-hand column if greater than 45°. The numbers on the same line running across the page are the latitudes and departures for that angle for the respective distances, 1, 2, 3, 4, 5, 6, 7, 8, 9, which appear at the top and bottom of the pages. Thus, if the bearing of a line is 10° and the distance 4, the latitude will be 3.939 and the departure .695; with the same bearing, and the distance 8, the latitude will be 7.878 and the departure 1.389. The latitude and departure for 80 is 10 times the latitude and departure for 8, and is found by moving the decimal point one place to the right; that for 500 is 100 times the latitude and departure for 5, and is found by moving the decimal point two places to the right and so on. By moving the decimal point one, two, or more places to the right, the latitude and departure may be found for any multiple of any number given in the table. In finding the latitude and departure for any number such as 453, the number is resolved into three numbers viz.: 400, 50, 3, and the latitude and departure for each taken from the table and then added together.

**Rule.**—*Write down the latitude and departure, neglecting the decimal points, for the first figure of the given distance; write under them the latitude and departure for the second figure, setting them one place farther to the right; under these, place the latitude and departure for the third figure, setting them one place still farther to the right, and so continue until all the figures of the given distance have been used; add these latitudes and departures, and point off on the right of their sums a number of decimal places equal to the number of decimal places to which the tables being used are carried; the resulting numbers will be the latitude and departure of the given distance in feet, links, chains, or whatever unit of measurement is adopted.*

**Example.**—A bearing is 16° and the distance 725 ft.; what is the latitude and departure?

**Solution.**—Applying the rule just given:

| Distances | Latitudes | Departures |
|---|---|---|
| 700 | 6729 | 1929 |
| 20 | 1923 | 0551 |
| 5 | 4806 | 1378 |
| 725 | 696.936 | 199.788 |

Taking the nearest whole numbers and rejecting the decimals the latitude and departure are found to be 697 and 200 respectively.

When a 0 occurs in the given number, the next figure must be set two places to the right as in the following example:

**Examples.**—The bearing is 22° and the distance 907 ft.; required, the latitude and departure.

**Solution.**—Applying the rule just given.

| Distances | Latitudes | Departures |
|---|---|---|
| 900 | 8345 | 3371 |
| 7 | 6490 | 2622 |
| 907 | 840.990 | 339.722 |

Here the place of 0 both in the distance column and in the latitude and departure columns is occupied by a dash —. Rejecting the decimals, the latitude is 841 ft. and the departure 340 ft.

When the bearing is more than 45°, the names of the columns must be read from the bottom of the page. The latitude of any bearing, as 60°, is the departure of its complement, 30°; and the departure of any bearing, as 30°, is the latitude of its complement, 60°. Where the bearings are given in smaller fractions of degrees than is found in the table, the latitudes and departures can be found by interpolation.

| Bearing | 1 | | 2 | | 3 | | 4 | | 5 | Bearing |
|---|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | |
| 0° | 1.000 | 0.000 | 2.000 | 0.000 | 3.000 | 0.000 | 4.000 | 0.000 | 5.000 | 90° |
| 0¼ | 1.000 | 0.004 | 2.000 | 0.009 | 3.000 | 0.013 | 4.000 | 0.017 | 5.000 | 89¾ |
| 0½ | 1.000 | 0.009 | 2.000 | 0.017 | 3.000 | 0.026 | 4.000 | 0.035 | 5.000 | 89½ |
| 0¾ | 1.000 | 0.013 | 2.000 | 0.026 | 3.000 | 0.039 | 4.000 | 0.052 | 5.000 | 89¼ |
| 1° | 1.000 | 0.017 | 2.000 | 0.035 | 3.000 | 0.052 | 3.999 | 0.070 | 4.999 | 89° |
| 1¼ | 1.000 | 0.022 | 2.000 | 0.044 | 2.999 | 0.065 | 3.999 | 0.087 | 4.999 | 88¾ |
| 1½ | 1.000 | 0.026 | 1.999 | 0.052 | 2.999 | 0.079 | 3.999 | 0.105 | 4.998 | 88½ |
| 1¾ | 1.000 | 0.031 | 1.999 | 0.061 | 2.999 | 0.092 | 3.998 | 0.122 | 4.998 | 88¼ |
| 2° | 0.999 | 0.035 | 1.999 | 0.070 | 2.998 | 0.105 | 3.998 | 0.140 | 4.997 | 88° |
| 2¼ | 0.999 | 0.039 | 1.998 | 0.079 | 2.998 | 0.118 | 3.997 | 0.157 | 4.996 | 87¾ |
| 2½ | 0.999 | 0.044 | 1.998 | 0.087 | 2.997 | 0.131 | 3.996 | 0.174 | 4.995 | 87½ |
| 2¾ | 0.999 | 0.048 | 1.998 | 0.096 | 2.997 | 0.144 | 3.995 | 0.192 | 4.994 | 87¼ |
| 3° | 0.999 | 0.052 | 1.997 | 0.105 | 2.996 | 0.157 | 3.995 | 0.209 | 4.993 | 87° |
| 3¼ | 0.998 | 0.057 | 1.997 | 0.113 | 2.995 | 0.170 | 3.994 | 0.227 | 4.992 | 86¾ |
| 3½ | 0.998 | 0.061 | 1.996 | 0.122 | 2.994 | 0.183 | 3.993 | 0.244 | 4.991 | 86½ |
| 3¾ | 0.998 | 0.065 | 1.996 | 0.131 | 2.994 | 0.196 | 3.991 | 0.262 | 4.989 | 86¼ |
| 4° | 0.998 | 0.070 | 1.995 | 0.140 | 2.993 | 0.209 | 3.990 | 0.279 | 4.988 | 86° |
| 4¼ | 0.997 | 0.074 | 1.995 | 0.148 | 2.992 | 0.222 | 3.989 | 0.296 | 4.986 | 85¾ |
| 4½ | 0.997 | 0.078 | 1.994 | 0.157 | 2.991 | 0.235 | 3.988 | 0.314 | 4.985 | 85½ |
| 4¾ | 0.997 | 0.083 | 1.993 | 0.166 | 2.990 | 0.248 | 3.986 | 0.331 | 4.983 | 85¼ |
| 5° | 0.996 | 0.087 | 1.992 | 0.174 | 2.989 | 0.261 | 3.985 | 0.349 | 4.981 | 85° |
| 5¼ | 0.996 | 0.092 | 1.992 | 0.183 | 2.987 | 0.275 | 3.983 | 0.366 | 4.979 | 84¾ |
| 5½ | 0.995 | 0.096 | 1.991 | 0.192 | 2.986 | 0.288 | 3.982 | 0.383 | 4.977 | 84½ |
| 5¾ | 0.995 | 0.100 | 1.990 | 0.200 | 2.985 | 0.301 | 3.980 | 0.401 | 4.975 | 84¼ |
| 6° | 0.995 | 0.105 | 1.989 | 0.209 | 2.984 | 0.314 | 3.978 | 0.418 | 4.973 | 84° |
| 6¼ | 0.994 | 0.109 | 1.988 | 0.218 | 2.982 | 0.327 | 3.976 | 0.435 | 4.970 | 83¾ |
| 6½ | 0.994 | 0.113 | 1.987 | 0.226 | 2.981 | 0.340 | 3.974 | 0.453 | 4.968 | 83½ |
| 6¾ | 0.993 | 0.118 | 1.986 | 0.235 | 2.979 | 0.353 | 3.972 | 0.470 | 4.965 | 83¼ |
| 7° | 0.993 | 0.122 | 1.985 | 0.244 | 2.978 | 0.366 | 3.970 | 0.487 | 4.963 | 83° |
| 7¼ | 0.992 | 0.126 | 1.984 | 0.252 | 2.976 | 0.379 | 3.968 | 0.505 | 4.960 | 82¾ |
| 7½ | 0.991 | 0.131 | 1.983 | 0.261 | 2.974 | 0.392 | 3.966 | 0.522 | 4.957 | 82½ |
| 7¾ | 0.991 | 0.135 | 1.982 | 0.270 | 2.973 | 0.405 | 3.963 | 0.539 | 4.954 | 82¼ |
| 8° | 0.990 | 0.139 | 1.981 | 0.278 | 2.971 | 0.418 | 3.961 | 0.557 | 4.951 | 82° |
| 8¼ | 0.990 | 0.143 | 1.979 | 0.287 | 2.969 | 0.430 | 3.959 | 0.574 | 4.948 | 81¾ |
| 8½ | 0.989 | 0.148 | 1.978 | 0.296 | 2.967 | 0.443 | 3.956 | 0.591 | 4.945 | 81½ |
| 8¾ | 0.988 | 0.152 | 1.977 | 0.304 | 2.965 | 0.456 | 3.953 | 0.608 | 4.942 | 81¼ |
| 9° | 0.988 | 0.156 | 1.975 | 0.313 | 2.963 | 0.469 | 3.951 | 0.626 | 4.938 | 81° |
| 9¼ | 0.987 | 0.161 | 1.974 | 0.321 | 2.961 | 0.482 | 3.948 | 0.643 | 4.935 | 80¾ |
| 9½ | 0.986 | 0.165 | 1.973 | 0.330 | 2.959 | 0.495 | 3.945 | 0.660 | 4.931 | 80½ |
| 9¾ | 0.986 | 0.169 | 1.971 | 0.339 | 2.957 | 0.508 | 3.942 | 0.677 | 4.928 | 80¼ |
| 10° | 0.985 | 0.174 | 1.970 | 0.347 | 2.954 | 0.521 | 3.939 | 0.695 | 4.924 | 80° |
| 10¼ | 0.984 | 0.178 | 1.968 | 0.356 | 2.952 | 0.534 | 3.936 | 0.712 | 4.920 | 79¾ |
| 10½ | 0.983 | 0.182 | 1.967 | 0.364 | 2.950 | 0.547 | 3.933 | 0.729 | 4.916 | 79½ |
| 10¾ | 0.982 | 0.187 | 1.965 | 0.373 | 2.947 | 0.560 | 3.930 | 0.746 | 4.912 | 79¼ |
| 11° | 0.982 | 0.191 | 1.963 | 0.382 | 2.945 | 0.572 | 3.927 | 0.763 | 4.908 | 79° |
| 11¼ | 0.981 | 0.195 | 1.962 | 0.390 | 2.942 | 0.585 | 3.923 | 0.780 | 4.904 | 78¾ |
| 11½ | 0.980 | 0.199 | 1.960 | 0.399 | 2.940 | 0.598 | 3.920 | 0.797 | 4.900 | 78½ |
| 11¾ | 0.979 | 0.204 | 1.958 | 0.407 | 2.937 | 0.611 | 3.916 | 0.815 | 4.895 | 78¼ |
| 12° | 0.978 | 0.208 | 1.956 | 0.416 | 2.934 | 0.624 | 3.913 | 0.832 | 4.891 | 78° |
| 12¼ | 0.977 | 0.212 | 1.954 | 0.424 | 2.932 | 0.637 | 3.909 | 0.849 | 4.886 | 77¾ |
| 12½ | 0.976 | 0.216 | 1.953 | 0.433 | 2.929 | 0.649 | 3.905 | 0.866 | 4.881 | 77½ |
| 12¾ | 0.975 | 0.221 | 1.951 | 0.441 | 2.926 | 0.662 | 3.901 | 0.883 | 4.877 | 77¼ |
| 13° | 0.974 | 0.225 | 1.949 | 0.450 | 2.923 | 0.675 | 3.897 | 0.900 | 4.872 | 77° |
| Bearing | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Bearing |
| | 1 | | 2 | | 3 | | 4 | | 5 | |

| Bearing | 5 | | 6 | | 7 | | 8 | | 9 | | Bearing |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | |
| 0° | 0.000 | 6.000 | 0.000 | 7.000 | 0.000 | 8.000 | 0.000 | 9.000 | 0.000 | | 90° |
| 0¼ | 0.022 | 6.000 | 0.026 | 7.000 | 0.031 | 8.000 | 0.035 | 9.000 | 0.039 | | 89¾ |
| 0½ | 0.044 | 6.000 | 0.052 | 7.000 | 0.061 | 8.000 | 0.070 | 9.000 | 0.079 | | 89½ |
| 0¾ | 0.065 | 5.999 | 0.079 | 6.999 | 0.092 | 7.999 | 0.105 | 8.999 | 0.118 | | 89¼ |
| 1° | 0.087 | 5.999 | 0.105 | 6.999 | 0.122 | 7.999 | 0.140 | 8.999 | 0.157 | | 89° |
| 1¼ | 0.109 | 5.999 | 0.131 | 6.998 | 0.153 | 7.998 | 0.175 | 8.998 | 0.196 | | 88¾ |
| 1½ | 0.131 | 5.998 | 0.157 | 6.998 | 0.183 | 7.997 | 0.209 | 8.997 | 0.236 | | 88½ |
| 1¾ | 0.153 | 5.997 | 0.183 | 6.997 | 0.214 | 7.996 | 0.244 | 8.996 | 0.275 | | 88¼ |
| 2° | 0.174 | 5.996 | 0.209 | 6.996 | 0.244 | 7.995 | 0.279 | 8.995 | 0.314 | | 88° |
| 2¼ | 0.196 | 5.995 | 0.236 | 6.995 | 0.275 | 7.994 | 0.314 | 8.993 | 0.353 | | 87¾ |
| 2½ | 0.218 | 5.994 | 0.262 | 6.993 | 0.305 | 7.992 | 0.349 | 8.991 | 0.393 | | 87½ |
| 2¾ | 0.240 | 5.993 | 0.288 | 6.992 | 0.336 | 7.991 | 0.384 | 8.990 | 0.432 | | 87¼ |
| 3° | 0.262 | 5.992 | 0.314 | 6.990 | 0.366 | 7.989 | 0.419 | 8.988 | 0.471 | | 87° |
| 3¼ | 0.283 | 5.990 | 0.340 | 6.989 | 0.397 | 7.987 | 0.454 | 8.986 | 0.510 | | 86¾ |
| 3½ | 0.305 | 5.989 | 0.366 | 6.987 | 0.427 | 7.985 | 0.488 | 8.983 | 0.549 | | 86½ |
| 3¾ | 0.327 | 5.987 | 0.392 | 6.985 | 0.458 | 7.983 | 0.523 | 8.981 | 0.589 | | 86¼ |
| 4° | 0.349 | 5.985 | 0.419 | 6.983 | 0.488 | 7.981 | 0.558 | 8.978 | 0.628 | | 86° |
| 4¼ | 0.371 | 5.984 | 0.445 | 6.981 | 0.519 | 7.978 | 0.593 | 8.975 | 0.667 | | 85¾ |
| 4½ | 0.392 | 5.982 | 0.471 | 6.978 | 0.549 | 7.975 | 0.628 | 8.972 | 0.706 | | 85½ |
| 4¾ | 0.414 | 5.979 | 0.497 | 6.976 | 0.580 | 7.973 | 0.662 | 8.969 | 0.745 | | 85¼ |
| 5° | 0.436 | 5.977 | 0.523 | 6.973 | 0.610 | 7.970 | 0.697 | 8.966 | 0.784 | | 85° |
| 5¼ | 0.458 | 5.975 | 0.549 | 6.971 | 0.641 | 7.966 | 0.732 | 8.962 | 0.824 | | 84¾ |
| 5½ | 0.479 | 5.972 | 0.575 | 6.968 | 0.671 | 7.963 | 0.767 | 8.959 | 0.863 | | 84½ |
| 5¾ | 0.501 | 5.970 | 0.601 | 6.965 | 0.701 | 7.960 | 0.802 | 8.955 | 0.902 | | 84¼ |
| 6° | 0.523 | 5.967 | 0.627 | 6.962 | 0.732 | 7.956 | 0.836 | 8.951 | 0.941 | | 84° |
| 6¼ | 0.544 | 5.964 | 0.653 | 6.958 | 0.762 | 7.952 | 0.871 | 8.947 | 0.980 | | 83¾ |
| 6½ | 0.566 | 5.961 | 0.679 | 6.955 | 0.792 | 7.949 | 0.906 | 8.942 | 1.019 | | 83½ |
| 6¾ | 0.588 | 5.958 | 0.705 | 6.951 | 0.823 | 7.945 | 0.940 | 8.938 | 1.058 | | 83¼ |
| 7° | 0.609 | 5.955 | 0.731 | 6.948 | 0.853 | 7.940 | 0.975 | 8.933 | 1.097 | | 83° |
| 7¼ | 0.631 | 5.952 | 0.757 | 6.944 | 0.888 | 7.936 | 1.010 | 8.928 | 1.136 | | 82¾ |
| 7½ | 0.653 | 5.949 | 0.783 | 6.940 | 0.914 | 7.932 | 1.044 | 8.923 | 1.175 | | 82½ |
| 7¾ | 0.674 | 5.945 | 0.809 | 6.936 | 0.944 | 7.927 | 1.079 | 8.918 | 1.214 | | 82¼ |
| 8° | 0.696 | 5.942 | 0.835 | 6.932 | 0.974 | 7.922 | 1.113 | 8.912 | 1.253 | | 82° |
| 8¼ | 0.717 | 5.938 | 0.861 | 6.928 | 1.004 | 7.917 | 1.148 | 8.907 | 1.291 | | 81¾ |
| 8½ | 0.739 | 5.934 | 0.887 | 6.923 | 1.035 | 7.912 | 1.182 | 8.901 | 1.330 | | 81½ |
| 8¾ | 0.761 | 5.930 | 0.913 | 6.919 | 1.065 | 7.907 | 1.217 | 8.895 | 1.369 | | 81¼ |
| 9° | 0.782 | 5.926 | 0.939 | 6.914 | 1.095 | 7.902 | 1.251 | 8.889 | 1.408 | | 81° |
| 9¼ | 0.804 | 5.922 | 0.964 | 6.909 | 1.125 | 7.896 | 1.286 | 8.883 | 1.447 | | 80¾ |
| 9½ | 0.825 | 5.918 | 0.990 | 6.904 | 1.155 | 7.890 | 1.320 | 8.877 | 1.485 | | 80½ |
| 9¾ | 0.847 | 5.913 | 1.016 | 6.899 | 1.185 | 7.884 | 1.355 | 8.870 | 1.524 | | 80¼ |
| 10° | 0.868 | 5.909 | 1.042 | 6.894 | 1.216 | 7.878 | 1.389 | 8.863 | 1.563 | | 80° |
| 10¼ | 0.890 | 5.904 | 1.068 | 6.888 | 1.246 | 7.872 | 1.424 | 8.856 | 1.601 | | 79¾ |
| 10½ | 0.911 | 5.900 | 1.093 | 6.883 | 1.276 | 7.866 | 1.458 | 8.849 | 1.640 | | 79½ |
| 10¾ | 0.933 | 5.895 | 1.119 | 6.877 | 1.306 | 7.860 | 1.492 | 8.842 | 1.679 | | 79¼ |
| 11° | 0.954 | 5.890 | 1.145 | 6.871 | 1.336 | 7.853 | 1.526 | 8.835 | 1.717 | | 79° |
| 11¼ | 0.975 | 5.885 | 1.171 | 6.866 | 1.366 | 7.846 | 1.561 | 8.827 | 1.756 | | 78¾ |
| 11½ | 0.997 | 5.880 | 1.196 | 6.859 | 1.396 | 7.839 | 1.595 | 8.819 | 1.794 | | 78½ |
| 11¾ | 1.018 | 5.874 | 1.222 | 6.853 | 1.425 | 7.832 | 1.629 | 8.811 | 1.833 | | 78¼ |
| 12° | 1.040 | 5.869 | 1.247 | 6.847 | 1.455 | 7.825 | 1.663 | 8.803 | 1.871 | | 78° |
| 12¼ | 1.061 | 5.863 | 1.273 | 6.841 | 1.485 | 7.818 | 1.697 | 8.795 | 1.910 | | 77¾ |
| 12½ | 1.082 | 5.858 | 1.299 | 6.834 | 1.515 | 7.810 | 1.732 | 8.787 | 1.948 | | 77½ |
| 12¾ | 1.103 | 5.852 | 1.324 | 6.827 | 1.545 | 7.803 | 1.766 | 8.778 | 1.986 | | 77¼ |
| 13° | 1.125 | 5.846 | 1.350 | 6.821 | 1.575 | 7.795 | 1.800 | 8.769 | 2.025 | | 77° |

| Bearing | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Bearing |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 5 | | 6 | | 7 | | 8 | | 9 | |

| Bearing | 1 | | 2 | | 3 | | 4 | | 5 | Bearing |
|---|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | |
| 13° | 0.974 | 0.225 | 1.949 | 0.450 | 2.923 | 0.675 | 3.897 | 0.900 | 4.872 | 77° |
| 13¼ | 0.973 | 0.229 | 1.947 | 0.458 | 2.920 | 0.688 | 3.894 | 0.917 | 4.867 | 76¾ |
| 13½ | 0.972 | 0.233 | 1.945 | 0.467 | 2.917 | 0.700 | 3.889 | 0.934 | 4.862 | 76½ |
| 13¾ | 0.971 | 0.238 | 1.943 | 0.475 | 2.914 | 0.713 | 3.885 | 0.951 | 4.857 | 76¼ |
| 14° | 0.970 | 0.242 | 1.941 | 0.484 | 2.911 | 0.726 | 3.881 | 6.968 | 4.851 | 76° |
| 14¼ | 0.969 | 0.246 | 1.938 | 0.492 | 2.908 | 0.738 | 3.877 | 0.985 | 4.846 | 75¾ |
| 14½ | 0.968 | 0.250 | 1.936 | 0.501 | 2.904 | 0.751 | 3.873 | 1.002 | 4.841 | 75½ |
| 14¾ | 0.967 | 0.255 | 1.934 | 0.509 | 2.901 | 0.764 | 3.868 | 1.018 | 4.835 | 75¼ |
| 15° | 0.966 | 0.259 | 1.932 | 0.518 | 2.898 | 0.776 | 3.864 | 1.035 | 4.830 | 75° |
| 15¼ | 0.965 | 0.263 | 1.930 | 0.526 | 2.894 | 0.789 | 3.859 | 1.052 | 4.824 | 74¾ |
| 15½ | 0.964 | 0.267 | 1.927 | 0.534 | 2.891 | 0.802 | 3.855 | 1.069 | 4.818 | 74½ |
| 15¾ | 0.962 | 0.271 | 1.925 | 0.543 | 2.887 | 0.814 | 3.850 | 1.086 | 4.812 | 74¼ |
| 16° | 0.961 | 0.276 | 1.923 | 0.551 | 2.884 | 0.827 | 3.845 | 1.103 | 4.806 | 74° |
| 16¼ | 0.960 | 0.280 | 1.920 | 0.560 | 2.880 | 0.839 | 3.840 | 1.119 | 4.800 | 73¾ |
| 16½ | 0.959 | 0.284 | 1.918 | 0.568 | 2.876 | 0.852 | 3.835 | 1.136 | 4.794 | 73½ |
| 16¾ | 0.958 | 0.288 | 1.915 | 0.576 | 2.873 | 0.865 | 3.830 | 1.153 | 4.788 | 73¼ |
| 17° | 0.956 | 0.292 | 1.913 | 0.585 | 2.869 | 0.877 | 3.825 | 1.169 | 4.782 | 73° |
| 17¼ | 0.955 | 0.297 | 1.910 | 0.593 | 2.865 | 0.890 | 3.820 | 1.186 | 4.775 | 72¾ |
| 17½ | 0.954 | 0.301 | 1.907 | 0.601 | 2.861 | 0.902 | 3.815 | 1.203 | 4.769 | 72½ |
| 17¾ | 0.952 | 0.305 | 1.905 | 0.610 | 2.857 | 0.915 | 3.810 | 1.220 | 4.762 | 72¼ |
| 18° | 0.951 | 0.309 | 1.902 | 0.618 | 2.853 | 0.927 | 3.804 | 1.236 | 4.755 | 72° |
| 18¼ | 0.950 | 0.313 | 1.899 | 0.626 | 2.849 | 0.939 | 3.799 | 1.253 | 4.748 | 71¾ |
| 18½ | 0.948 | 0.317 | 1.897 | 0.635 | 2.845 | 0.952 | 3.793 | 1.269 | 4.742 | 71½ |
| 18¾ | 0.947 | 0.321 | 1.894 | 0.643 | 2.841 | 0.964 | 3.788 | 1.286 | 4.735 | 71¼ |
| 19° | 0.946 | 0.326 | 1.891 | 0.651 | 2.837 | 0.977 | 3.782 | 1.302 | 4.728 | 71° |
| 19¼ | 0.944 | 0.330 | 1.888 | 0.659 | 2.832 | 0.989 | 3.776 | 1.319 | 4.720 | 70¾ |
| 19½ | 0.943 | 0.334 | 1.885 | 0.668 | 2.828 | 1.001 | 3.771 | 1.335 | 4.713 | 70½ |
| 19¾ | 0.941 | 0.338 | 1.882 | 0.676 | 2.824 | 1.014 | 3.765 | 1.352 | 4.706 | 70¼ |
| 20° | 0.940 | 0.342 | 1.879 | 0.684 | 2.819 | 1.026 | 3.759 | 1.368 | 4.698 | 70° |
| 20¼ | 0.938 | 0.346 | 1.876 | 0.692 | 2.815 | 1.038 | 3.753 | 1.384 | 4.691 | 69¾ |
| 20½ | 0.937 | 0.350 | 1.873 | 0.700 | 2.810 | 1.051 | 3.747 | 1.401 | 4.683 | 69½ |
| 20¾ | 0.935 | 0.354 | 1.870 | 0.709 | 2.805 | 1.063 | 3.741 | 1.417 | 4.676 | 69¼ |
| 21° | 0.934 | 0.358 | 1.867 | 0.717 | 2.801 | 1.075 | 3.734 | 1.433 | 4.668 | 69° |
| 21¼ | 0.932 | 0.362 | 1.864 | 0.725 | 2.796 | 1.087 | 3.728 | 1.450 | 4.660 | 68¾ |
| 21½ | 0.930 | 0.367 | 1.861 | 0.733 | 2.791 | 1.100 | 3.722 | 1.466 | 4.652 | 68½ |
| 21¾ | 0.929 | 0.371 | 1.858 | 0.741 | 2.786 | 1.112 | 3.715 | 1.482 | 4.644 | 68¼ |
| 22° | 0.927 | 0.375 | 1.854 | 0.749 | 2.782 | 1.124 | 3.709 | 1.498 | 4.636 | 68° |
| 22¼ | 0.926 | 0.379 | 1.851 | 0.757 | 2.777 | 1.136 | 3.702 | 1.515 | 4.628 | 67¾ |
| 22½ | 0.924 | 0.382 | 1.848 | 0.765 | 2.772 | 1.148 | 3.696 | 1.531 | 4.619 | 67½ |
| 22¾ | 0.922 | 0.387 | 1.844 | 0.773 | 2.767 | 1.160 | 3.689 | 1.547 | 4.611 | 67¼ |
| 23° | 0.921 | 0.391 | 1.841 | 0.781 | 2.762 | 1.172 | 3.682 | 1.563 | 4.603 | 67° |
| 23¼ | 0.919 | 0.395 | 1.838 | 0.789 | 2.756 | 1.184 | 3.675 | 1.579 | 4.594 | 66¾ |
| 23½ | 0.917 | 0.399 | 1.834 | 0.797 | 2.751 | 1.196 | 3.668 | 1.595 | 4.585 | 66½ |
| 23¾ | 0.915 | 0.403 | 1.831 | 0.805 | 2.746 | 1.208 | 3.661 | 1.611 | 4.577 | 66¼ |
| 24° | 0.914 | 0.407 | 1.827 | 0.813 | 2.741 | 1.220 | 3.654 | 1.627 | 4.568 | 66° |
| 24¼ | 0.912 | 0.411 | 1.824 | 0.821 | 2.735 | 1.232 | 3.647 | 1.643 | 4.559 | 65¾ |
| 24½ | 0.910 | 0.415 | 1.820 | 0.829 | 2.730 | 1.244 | 3.640 | 1.659 | 4.550 | 65½ |
| 24¾ | 0.908 | 0.419 | 1.816 | 0.837 | 2.724 | 1.256 | 3.633 | 1.675 | 4.541 | 65¼ |
| 25° | 0.906 | 0.423 | 1.813 | 0.845 | 2.719 | 1.268 | 3.625 | 1.690 | 4.532 | 65° |
| 25¼ | 0.904 | 0.427 | 1.809 | 0.853 | 2.713 | 1.280 | 3.618 | 1.706 | 4.522 | 64¾ |
| 25½ | 0.903 | 0.431 | 1.805 | 0.861 | 2.708 | 1.292 | 3.610 | 1.722 | 4.513 | 64½ |
| 25¾ | 0.901 | 0.434 | 1.801 | 0.869 | 2.702 | 1.303 | 3.603 | 1.738 | 4.503 | 64¼ |
| 26° | 0.899 | 0.438 | 1.798 | 0.877 | 2.696 | 1.315 | 3.595 | 1.753 | 4.494 | 64° |
| Bearing | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Bearing |
| | 1 | | 2 | | 3 | | 4 | | 5 | |

| Bearing | 5 | 6 | | 7 | | 8 | | 9 | | Bearing |
|---|---|---|---|---|---|---|---|---|---|---|
| | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | |
| 13° | 1.125 | 5.846 | 1.350 | 6.821 | 1.575 | 7.795 | 1.800 | 8.769 | 2.025 | 77° |
| 13¼ | 1.146 | 5.840 | 1.375 | 6.814 | 1.604 | 7.787 | 1.834 | 8.760 | 2.063 | 76¾ |
| 13½ | 1.167 | 5.834 | 1.401 | 6.807 | 1.634 | 7.779 | 1.868 | 8.751 | 2.101 | 76½ |
| 13¾ | 1.188 | 5.828 | 1.426 | 6.799 | 1.664 | 7.771 | 1.902 | 8.742 | 2.139 | 76¼ |
| 14° | 1.210 | 5.822 | 1.452 | 6.792 | 1.693 | 7.762 | 1.935 | 8.733 | 2.177 | 76° |
| 14¼ | 1.231 | 5.815 | 1.477 | 6.785 | 1.723 | 7.754 | 1.969 | 8.723 | 2.215 | 75¾ |
| 14½ | 1.252 | 5.809 | 1.502 | 6.777 | 1.753 | 7.745 | 2.003 | 8.713 | 2.253 | 75½ |
| 14¾ | 1.273 | 5.802 | 1.528 | 6.769 | 1.782 | 7.736 | 2.037 | 8.703 | 2.291 | 75¼ |
| 15° | 1.294 | 5.796 | 1.553 | 6.761 | 1.812 | 7.727 | 2.071 | 8.693 | 2.329 | 75° |
| 15¼ | 1.315 | 5.789 | 1.578 | 6.754 | 1.841 | 7.718 | 2.104 | 8.683 | 2.367 | 74¾ |
| 15½ | 1.336 | 5.782 | 1.603 | 6.745 | 1.871 | 7.709 | 2.138 | 8.673 | 2.405 | 74½ |
| 15¾ | 1.357 | 5.775 | 1.629 | 6.737 | 1.900 | 7.700 | 2.172 | 8.662 | 2.443 | 74¼ |
| 16° | 1.378 | 5.768 | 1.654 | 6.729 | 1.929 | 7.690 | 2.205 | 8.651 | 2.481 | 74° |
| 16¼ | 1.399 | 5.760 | 1.679 | 6.720 | 1.959 | 7.680 | 2.239 | 8.640 | 2.518 | 73¾ |
| 16½ | 1.420 | 5.753 | 1.704 | 6.712 | 1.988 | 7.671 | 2.272 | 8.629 | 2.556 | 73½ |
| 16¾ | 1.441 | 5.745 | 1.729 | 6.703 | 2.017 | 7.661 | 2.306 | 8.618 | 2.594 | 73¼ |
| 17° | 1.462 | 5.738 | 1.754 | 6.694 | 2.047 | 7.650 | 2.339 | 8.607 | 2.631 | 73° |
| 17¼ | 1.483 | 5.730 | 1.779 | 6.685 | 2.076 | 7.640 | 2.372 | 8.595 | 2.669 | 72¾ |
| 17½ | 1.504 | 5.722 | 1.804 | 6.676 | 2.105 | 7.630 | 2.406 | 8.583 | 2.706 | 72½ |
| 17¾ | 1.524 | 5.714 | 1.829 | 6.667 | 2.134 | 7.619 | 2.439 | 8.572 | 2.744 | 72¼ |
| 18° | 1.545 | 5.706 | 1.854 | 6.657 | 2.163 | 7.608 | 2.472 | 8.560 | 2.781 | 72° |
| 18¼ | 1.566 | 5.698 | 1.879 | 6.648 | 2.192 | 7.598 | 2.505 | 8.547 | 2.818 | 71¾ |
| 18½ | 1.587 | 5.690 | 1.904 | 6.638 | 2.221 | 7.587 | 2.538 | 8.535 | 2.856 | 71½ |
| 18¾ | 1.607 | 5.682 | 1.929 | 6.629 | 2.250 | 7.575 | 2.572 | 8.522 | 2.893 | 71¼ |
| 19° | 1.628 | 5.673 | 1.953 | 6.619 | 2.279 | 7.564 | 2.605 | 8.510 | 2.930 | 71° |
| 19¼ | 1.648 | 5.665 | 1.978 | 6.609 | 2.308 | 7.553 | 2.638 | 8.497 | 2.967 | 70¾ |
| 19½ | 1.669 | 5.656 | 2.003 | 6.598 | 2.337 | 7.541 | 2.670 | 8.484 | 3.004 | 70½ |
| 19¾ | 1.690 | 5.647 | 2.028 | 6.588 | 2.365 | 7.529 | 2.703 | 8.471 | 3.041 | 70¼ |
| 20° | 1.710 | 5.638 | 2.052 | 6.578 | 2.394 | 7.518 | 2.736 | 8.457 | 3.078 | 70° |
| 20¼ | 1.731 | 5.629 | 2.077 | 6.567 | 2.423 | 7.506 | 2.769 | 8.444 | 3.115 | 69¾ |
| 20½ | 1.751 | 5.620 | 2.101 | 6.557 | 2.451 | 7.493 | 2.802 | 8.430 | 3.152 | 69½ |
| 20¾ | 1.771 | 5.611 | 2.126 | 6.546 | 2.480 | 7.481 | 2.834 | 8.416 | 3.189 | 69¼ |
| 21° | 1.792 | 5.601 | 2.150 | 6.535 | 2.509 | 7.469 | 2.867 | 8.402 | 3.225 | 69° |
| 21¼ | 1.812 | 5.592 | 2.175 | 6.524 | 2.537 | 7.456 | 2.900 | 8.388 | 3.262 | 68¾ |
| 21½ | 1.833 | 5.582 | 2.199 | 6.513 | 2.566 | 7.443 | 2.932 | 8.374 | 3.299 | 68½ |
| 21¾ | 1.853 | 5.573 | 2.223 | 6.502 | 2.594 | 7.430 | 2.964 | 8.359 | 3.335 | 68¼ |
| 22° | 1.873 | 5.563 | 2.248 | 6.490 | 2.622 | 7.417 | 2.997 | 8.345 | 3.371 | 68° |
| 22¼ | 1.893 | 5.553 | 2.272 | 6.479 | 2.651 | 7.404 | 3.029 | 8.330 | 3.408 | 67¾ |
| 22½ | 1.913 | 5.543 | 2.296 | 6.467 | 2.679 | 7.391 | 3.061 | 8.315 | 3.444 | 67½ |
| 22¾ | 1.934 | 5.533 | 2.320 | 6.455 | 2.707 | 7.378 | 3.094 | 8.300 | 3.480 | 67¼ |
| 23° | 1.954 | 5.523 | 2.344 | 6.444 | 2.735 | 7.364 | 3.126 | 8.285 | 3.517 | 67° |
| 23¼ | 1.974 | 5.513 | 2.368 | 6.432 | 2.763 | 7.350 | 3.158 | 8.269 | 3.553 | 66¾ |
| 23½ | 1.994 | 5.502 | 2.392 | 6.419 | 2.791 | 7.336 | 3.190 | 8.254 | 3.589 | 66½ |
| 23¾ | 2.014 | 5.492 | 2.416 | 6.407 | 2.819 | 7.322 | 3.222 | 8.238 | 3.625 | 66¼ |
| 24° | 2.034 | 5.481 | 2.440 | 6.395 | 2.847 | 7.308 | 3.254 | 8.222 | 3.661 | 66° |
| 24¼ | 2.054 | 5.471 | 2.464 | 6.382 | 2.875 | 7.294 | 3.286 | 8.206 | 3.696 | 65¾ |
| 24½ | 2.073 | 5.460 | 2.488 | 6.370 | 2.903 | 7.280 | 3.318 | 8.190 | 3.732 | 65½ |
| 24¾ | 2.093 | 5.449 | 2.512 | 6.357 | 2.931 | 7.265 | 3.349 | 8.173 | 3.768 | 65¼ |
| 25° | 2.113 | 5.438 | 2.536 | 6.344 | 2.958 | 7.250 | 3.381 | 8.157 | 3.804 | 65° |
| 25¼ | 2.133 | 5.427 | 2.559 | 6.331 | 2.986 | 7.236 | 3.413 | 8.140 | 3.839 | 64¾ |
| 25½ | 2.153 | 5.416 | 2.583 | 6.318 | 3.014 | 7.221 | 3.444 | 8.123 | 3.875 | 64½ |
| 25¾ | 2.172 | 5.404 | 2.607 | 6.305 | 3.041 | 7.206 | 3.476 | 8.106 | 3.910 | 64¼ |
| 26° | 2.192 | 5.393 | 2.630 | 6.292 | 3.069 | 7.190 | 3.507 | 8.089 | 3.945 | 64° |
| Bearing | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Bearing |
| | 5 | 6 | | 7 | | 8 | | 9 | | |

| Bearing | 1 Lat. | 1 Dep. | 2 Lat. | 2 Dep. | 3 Lat. | 3 Dep. | 4 Lat. | 4 Dep. | 5 Lat. | Bearing |
|---|---|---|---|---|---|---|---|---|---|---|
| 26° | 0.899 | 0.438 | 1.798 | 0.877 | 2.696 | 1.315 | 3.595 | 1.753 | 4.494 | 64° |
| 26¼ | 0.897 | 0.442 | 1.794 | 0.885 | 2.691 | 1.327 | 3.587 | 1.769 | 4.484 | 63¾ |
| 26½ | 0.895 | 0.446 | 1.790 | 0.892 | 2.685 | 1.339 | 3.580 | 1.785 | 4.475 | 63½ |
| 26¾ | 0.893 | 0.450 | 1.786 | 0.900 | 2.679 | 1.350 | 3.572 | 1.800 | 4.465 | 63¼ |
| 27° | 0.891 | 0.454 | 1.782 | 0.908 | 2.673 | 1.362 | 3.564 | 1.816 | 4.455 | 63° |
| 27¼ | 0.889 | 0.458 | 1.778 | 0.916 | 2.667 | 1.374 | 3.556 | 1.831 | 4.445 | 62¾ |
| 27½ | 0.887 | 0.462 | 1.774 | 0.923 | 2.661 | 1.385 | 3.548 | 1.847 | 4.435 | 62½ |
| 27¾ | 0.885 | 0.466 | 1.770 | 0.931 | 2.655 | 1.397 | 3.540 | 1.862 | 4.425 | 62¼ |
| 28° | 0.883 | 0.469 | 1.766 | 0.939 | 2.649 | 1.408 | 3.532 | 1.878 | 4.415 | 62° |
| 28¼ | 0.881 | 0.473 | 1.762 | 0.947 | 2.643 | 1.420 | 3.524 | 1.893 | 4.404 | 61¾ |
| 28½ | 0.879 | 0.477 | 1.758 | 0.954 | 2.636 | 1.431 | 3.515 | 1.909 | 4.394 | 61½ |
| 28¾ | 0.877 | 0.481 | 1.753 | 0.962 | 2.630 | 1.443 | 3.507 | 1.924 | 4.384 | 61¼ |
| 29° | 0.875 | 0.485 | 1.749 | 0.970 | 2.624 | 1.454 | 3.498 | 1.939 | 4.373 | 61° |
| 29¼ | 0.872 | 0.489 | 1.745 | 0.977 | 2.617 | 1.466 | 3.490 | 1.954 | 4.362 | 60¾ |
| 29½ | 0.870 | 0.492 | 1.741 | 0.985 | 2.611 | 1.477 | 3.481 | 1.970 | 4.352 | 60½ |
| 29¾ | 0.868 | 0.496 | 1.736 | 0.992 | 2.605 | 1.489 | 3.473 | 1.985 | 4.341 | 60¼ |
| 30° | 0.866 | 0.500 | 1.732 | 1.000 | 2.598 | 1.500 | 3.464 | 2.000 | 4.330 | 60° |
| 30¼ | 0.864 | 0.504 | 1.728 | 1.008 | 2.592 | 1.511 | 3.455 | 2.015 | 4.319 | 59¾ |
| 30½ | 0.862 | 0.508 | 1.723 | 1.015 | 2.585 | 1.523 | 3.447 | 2.030 | 4.308 | 59½ |
| 30¾ | 0.859 | 0.511 | 1.719 | 1.023 | 2.578 | 1.534 | 3.438 | 2.045 | 4.297 | 59¼ |
| 31° | 0.857 | 0.515 | 1.714 | 1.030 | 2.572 | 1.545 | 3.429 | 2.060 | 4.286 | 59° |
| 31¼ | 0.855 | 0.519 | 1.710 | 1.038 | 2.565 | 1.556 | 3.420 | 2.075 | 4.275 | 58¾ |
| 31½ | 0.853 | 0.522 | 1.705 | 1.045 | 2.558 | 1.567 | 3.411 | 2.090 | 4.263 | 58½ |
| 31¾ | 0.850 | 0.526 | 1.701 | 1.052 | 2.551 | 1.579 | 3.401 | 2.105 | 4.252 | 58¼ |
| 32° | 0.848 | 0.530 | 1.696 | 1.060 | 2.544 | 1.590 | 3.392 | 2.120 | 4.240 | 58° |
| 32¼ | 0.846 | 0.534 | 1.691 | 1.067 | 2.537 | 1.601 | 3.383 | 2.134 | 4.229 | 57¾ |
| 32½ | 0.843 | 0.537 | 1.687 | 1.075 | 2.530 | 1.612 | 3.374 | 2.149 | 4.217 | 57½ |
| 32¾ | 0.841 | 0.541 | 1.682 | 1.082 | 2.523 | 1.623 | 3.364 | 2.164 | 4.205 | 57¼ |
| 33° | 0.839 | 0.545 | 1.677 | 1.089 | 2.516 | 1.634 | 3.355 | 2.179 | 4.193 | 57° |
| 33¼ | 0.836 | 0.548 | 1.673 | 1.097 | 2.509 | 1.645 | 3.345 | 2.193 | 4.181 | 56¾ |
| 33½ | 0.834 | 0.552 | 1.668 | 1.104 | 2.502 | 1.656 | 3.336 | 2.208 | 4.169 | 56½ |
| 33¾ | 0.831 | 0.556 | 1.663 | 1.111 | 2.494 | 1.667 | 3.326 | 2.222 | 4.157 | 56¼ |
| 34° | 0.829 | 0.559 | 1.658 | 1.118 | 2.487 | 1.678 | 3.316 | 2.237 | 4.145 | 56° |
| 34¼ | 0.827 | 0.563 | 1.653 | 1.126 | 2.480 | 1.688 | 3.306 | 2.251 | 4.133 | 55¾ |
| 34½ | 0.824 | 0.566 | 1.648 | 1.133 | 2.472 | 1.699 | 3.297 | 2.266 | 4.121 | 55½ |
| 34¾ | 0.822 | 0.570 | 1.643 | 1.140 | 2.465 | 1.710 | 3.287 | 2.280 | 4.108 | 55¼ |
| 35° | 0.819 | 0.574 | 1.638 | 1.147 | 2.457 | 1.721 | 3.277 | 2.294 | 4.096 | 55° |
| 35¼ | 0.817 | 0.577 | 1.633 | 1.154 | 2.450 | 1.731 | 3.267 | 2.309 | 4.083 | 54¾ |
| 35½ | 0.814 | 0.581 | 1.628 | 1.161 | 2.442 | 1.742 | 3.257 | 2.323 | 4.071 | 54½ |
| 35¾ | 0.812 | 0.584 | 1.623 | 1.168 | 2.435 | 1.753 | 3.246 | 2.337 | 4.058 | 54¼ |
| 36° | 0.809 | 0.588 | 1.618 | 1.176 | 2.427 | 1.763 | 3.236 | 2.351 | 4.045 | 54° |
| 36¼ | 0.806 | 0.591 | 1.613 | 1.183 | 2.419 | 1.774 | 3.226 | 2.365 | 4.032 | 53¾ |
| 36½ | 0.804 | 0.595 | 1.608 | 1.190 | 2.412 | 1.784 | 3.215 | 2.379 | 4.019 | 53½ |
| 36¾ | 0.801 | 0.598 | 1.603 | 1.197 | 2.404 | 1.795 | 3.205 | 2.393 | 4.006 | 53¼ |
| 37° | 0.799 | 0.602 | 1.597 | 1.204 | 2.396 | 1.805 | 3.195 | 2.407 | 3.993 | 53° |
| 37¼ | 0.796 | 0.605 | 1.592 | 1.211 | 2.388 | 1.816 | 3.184 | 2.421 | 3.980 | 52¾ |
| 37½ | 0.793 | 0.609 | 1.587 | 1.218 | 2.380 | 1.826 | 3.173 | 2.435 | 3.967 | 52½ |
| 37¾ | 0.791 | 0.612 | 1.581 | 1.224 | 2.372 | 1.837 | 3.163 | 2.449 | 3.953 | 52¼ |
| 38° | 0.788 | 0.616 | 1.576 | 1.231 | 2.364 | 1.847 | 3.152 | 2.463 | 3.940 | 52° |
| 38¼ | 0.785 | 0.619 | 1.571 | 1.238 | 2.356 | 1.857 | 3.141 | 2.476 | 3.927 | 51¾ |
| 38½ | 0.783 | 0.623 | 1.565 | 1.245 | 2.348 | 1.868 | 3.130 | 2.490 | 3.913 | 51½ |
| 38¾ | 0.780 | 0.626 | 1.560 | 1.252 | 2.340 | 1.878 | 3.120 | 2.504 | 3.899 | 51¼ |
| 39° | 0.777 | 0.629 | 1.554 | 1.259 | 2.331 | 1.888 | 3.109 | 2.517 | 3.886 | 51° |

| Bearing | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Bearing |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | | 2 | | 3 | | 4 | 5 | |

| Bearing | 5 | 8 | | 7 | | 8 | | 9 | | Bearing |
|---|---|---|---|---|---|---|---|---|---|---|
| | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | |
| 26° | 2.192 | 5.393 | 2.630 | 6.292 | 3.069 | 7.190 | 3.507 | 8.089 | 3.945 | 64° |
| 26¼ | 2.211 | 5.381 | 2.654 | 6.278 | 3.096 | 7.175 | 3.538 | 8.072 | 3.981 | 63¾ |
| 26½ | 2.231 | 5.370 | 2.677 | 6.265 | 3.123 | 7.160 | 3.570 | 8.054 | 4.016 | 63½ |
| 26¾ | 2.250 | 5.358 | 2.701 | 6.251 | 3.151 | 7.144 | 3.601 | 8.037 | 4.051 | 63¼ |
| 27° | 2.270 | 5.346 | 2.724 | 6.237 | 3.178 | 7.128 | 3.632 | 8.019 | 4.086 | 63° |
| 27¼ | 2.289 | 5.334 | 2.747 | 6.223 | 3.205 | 7.112 | 3.663 | 8.001 | 4.121 | 62¾ |
| 27½ | 2.309 | 5.322 | 2.770 | 6.209 | 3.232 | 7.096 | 3.694 | 7.983 | 4.156 | 62½ |
| 27¾ | 2.328 | 5.310 | 2.794 | 6.195 | 3.259 | 7.080 | 3.725 | 7.965 | 4.190 | 62¼ |
| 28° | 2.347 | 5.298 | 2.817 | 6.181 | 3.286 | 7.064 | 3.756 | 7.947 | 4.225 | 62° |
| 28¼ | 2.367 | 5.285 | 2.840 | 6.166 | 3.313 | 7.047 | 3.787 | 7.928 | 4.260 | 61¾ |
| 28½ | 2.386 | 5.273 | 2.863 | 6.152 | 3.340 | 7.031 | 3.817 | 7.909 | 4.294 | 61½ |
| 28¾ | 2.405 | 5.260 | 2.886 | 6.137 | 3.367 | 7.014 | 3.848 | 7.891 | 4.329 | 61¼ |
| 29° | 2.424 | 5.248 | 2.909 | 6.122 | 3.394 | 6.997 | 3.878 | 7.872 | 4.363 | 61° |
| 29¼ | 2.443 | 5.235 | 2.932 | 6.107 | 3.420 | 6.980 | 3.909 | 7.852 | 4.398 | 60¾ |
| 29½ | 2.462 | 5.222 | 2.955 | 6.093 | 3.447 | 6.963 | 3.939 | 7.833 | 4.432 | 60½ |
| 29¾ | 2.481 | 5.209 | 2.977 | 6.077 | 3.474 | 6.946 | 3.970 | 7.814 | 4.466 | 60¼ |
| 30° | 2.500 | 5.196 | 3.000 | 6.062 | 3.500 | 6.928 | 4.000 | 7.794 | 4.500 | 60° |
| 30¼ | 2.519 | 5.183 | 3.023 | 6.047 | 3.526 | 6.911 | 4.030 | 7.775 | 4.534 | 59¾ |
| 30½ | 2.538 | 5.170 | 3.045 | 6.031 | 3.553 | 6.893 | 4.060 | 7.755 | 4.568 | 59½ |
| 30¾ | 2.556 | 5.156 | 3.068 | 6.016 | 3.579 | 6.875 | 4.090 | 7.735 | 4.602 | 59¼ |
| 31° | 2.575 | 5.143 | 3.090 | 6.000 | 3.605 | 6.857 | 4.120 | 7.715 | 4.635 | 59° |
| 31¼ | 2.594 | 5.129 | 3.113 | 5.984 | 3.631 | 6.839 | 4.150 | 7.694 | 4.669 | 58¾ |
| 31½ | 2.612 | 5.116 | 3.135 | 5.968 | 3.657 | 6.821 | 4.180 | 7.674 | 4.702 | 58½ |
| 31¾ | 2.631 | 5.102 | 3.157 | 5.952 | 3.683 | 6.803 | 4.210 | 7.653 | 4.736 | 58¼ |
| 32° | 2.650 | 5.088 | 3.180 | 5.936 | 3.709 | 6.784 | 4.239 | 7.632 | 4.769 | 58° |
| 32¼ | 2.668 | 5.074 | 3.202 | 5.920 | 3.735 | 6.766 | 4.269 | 7.612 | 4.802 | 57¾ |
| 32½ | 2.686 | 5.060 | 3.224 | 5.904 | 3.761 | 6.747 | 4.298 | 7.591 | 4.836 | 57½ |
| 32¾ | 2.705 | 5.046 | 3.246 | 5.887 | 3.787 | 6.728 | 4.328 | 7.569 | 4.869 | 57¼ |
| 33° | 2.723 | 5.032 | 3.268 | 5.871 | 3.812 | 6.709 | 4.357 | 7.548 | 4.902 | 57° |
| 33¼ | 2.741 | 5.018 | 3.290 | 5.854 | 3.838 | 6.690 | 4.386 | 7.527 | 4.935 | 56¾ |
| 33½ | 2.760 | 5.003 | 3.312 | 5.837 | 3.864 | 6.671 | 4.416 | 7.505 | 4.967 | 56½ |
| 33¾ | 2.778 | 4.989 | 3.333 | 5.820 | 3.889 | 6.652 | 4.445 | 7.483 | 5.000 | 56¼ |
| 34° | 2.796 | 4.974 | 3.355 | 5.803 | 3.914 | 6.632 | 4.474 | 7.461 | 5.033 | 56° |
| 34¼ | 2.814 | 4.960 | 3.377 | 5.786 | 3.940 | 6.613 | 4.502 | 7.439 | 5.065 | 55¾ |
| 34½ | 2.832 | 4.945 | 3.398 | 5.769 | 3.965 | 6.593 | 4.531 | 7.417 | 5.098 | 55½ |
| 34¾ | 2.850 | 4.930 | 3.420 | 5.752 | 3.990 | 6.573 | 4.560 | 7.395 | 5.130 | 55¼ |
| 35° | 2.868 | 4.915 | 3.441 | 5.734 | 4.015 | 6.553 | 4.589 | 7.372 | 5.162 | 55° |
| 35¼ | 2.886 | 4.900 | 3.463 | 5.716 | 4.040 | 6.533 | 4.617 | 7.350 | 5.194 | 54¾ |
| 35½ | 2.904 | 4.885 | 3.484 | 5.699 | 4.065 | 6.513 | 4.646 | 7.327 | 5.226 | 54½ |
| 35¾ | 2.921 | 4.869 | 3.505 | 5.681 | 4.090 | 6.493 | 4.674 | 7.304 | 5.258 | 54¼ |
| 36° | 2.939 | 4.854 | 3.527 | 5.663 | 4.115 | 6.472 | 4.702 | 7.281 | 5.290 | 54° |
| 36¼ | 2.957 | 4.839 | 3.548 | 5.645 | 4.139 | 6.452 | 4.730 | 7.258 | 5.322 | 53¾ |
| 36½ | 2.974 | 4.823 | 3.569 | 5.627 | 4.164 | 6.431 | 4.759 | 7.235 | 5.353 | 53½ |
| 36¾ | 2.992 | 4.808 | 3.590 | 5.609 | 4.188 | 6.410 | 4.787 | 7.211 | 5.385 | 53¼ |
| 37° | 3.009 | 4.792 | 3.611 | 5.590 | 4.213 | 6.389 | 4.815 | 7.188 | 5.416 | 53° |
| 37¼ | 3.026 | 4.776 | 3.632 | 5.572 | 4.237 | 6.368 | 4.842 | 7.164 | 5.448 | 52¾ |
| 37½ | 3.044 | 4.760 | 3.653 | 5.554 | 4.261 | 6.347 | 4.870 | 7.140 | 5.479 | 52½ |
| 37¾ | 3.061 | 4.744 | 3.673 | 5.535 | 4.286 | 6.326 | 4.898 | 7.116 | 5.510 | 52¼ |
| 38° | 3.078 | 4.728 | 3.694 | 5.516 | 4.310 | 6.304 | 4.925 | 7.092 | 5.541 | 52° |
| 38¼ | 3.095 | 4.712 | 3.715 | 5.497 | 4.334 | 6.283 | 4.953 | 7.068 | 5.572 | 51¾ |
| 38½ | 3.113 | 4.696 | 3.735 | 5.478 | 4.358 | 6.261 | 4.980 | 7.043 | 5.603 | 51½ |
| 38¾ | 3.130 | 4.679 | 3.756 | 5.459 | 4.381 | 6.239 | 5.007 | 7.019 | 5.633 | 51¼ |
| 39° | 3.147 | 4.663 | 3.776 | 5.440 | 4.405 | 6.217 | 5.035 | 6.994 | 5.664 | 51° |
| Bearing | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Bearing |
| | 5 | 6 | | 7 | | 8 | | 9 | | |

| Bearing | 1 | | 2 | | 3 | | 4 | | 5 | Bearing |
|---|---|---|---|---|---|---|---|---|---|---|
| | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | |
| **39°** | 0.777 | 0.629 | 1.554 | 1.259 | 2.331 | 1.888 | 3.109 | 2.517 | 3.886 | **51°** |
| 39¼ | 0.774 | 0.633 | 1.549 | 1.265 | 2.323 | 1.898 | 3.098 | 2.531 | 3.872 | 50¾ |
| 39½ | 0.772 | 0.636 | 1.543 | 1.272 | 2.315 | 1.908 | 3.086 | 2.544 | 3.858 | 50½ |
| 39¾ | 0.769 | 0.639 | 1.538 | 1.279 | 2.307 | 1.918 | 3.075 | 2.558 | 3.844 | 50¼ |
| **40°** | 0.766 | 0.643 | 1.532 | 1.286 | 2.298 | 1.928 | 3.064 | 2.571 | 3.830 | **50°** |
| 40¼ | 0.763 | 0.646 | 1.526 | 1.292 | 2.290 | 1.938 | 3.053 | 2.584 | 3.816 | 49¾ |
| 40½ | 0.760 | 0.649 | 1.521 | 1.299 | 2.281 | 1.948 | 3.042 | 2.598 | 3.802 | 49½ |
| 40¾ | 0.758 | 0.653 | 1.515 | 1.306 | 2.273 | 1.958 | 3.030 | 2.611 | 3.788 | 49¼ |
| **41°** | 0.755 | 0.656 | 1.509 | 1.312 | 2.264 | 1.968 | 3.019 | 2.624 | 3.774 | **49°** |
| 41¼ | 0.752 | 0.659 | 1.504 | 1.319 | 2.256 | 1.978 | 3.007 | 2.637 | 3.759 | 48¾ |
| 41½ | 0.749 | 0.663 | 1.498 | 1.325 | 2.247 | 1.988 | 2.996 | 2.650 | 3.745 | 48½ |
| 41¾ | 0.746 | 0.666 | 1.492 | 1.332 | 2.238 | 1.998 | 2.984 | 2.664 | 3.730 | 48¼ |
| **42°** | 0.743 | 0.669 | 1.486 | 1.338 | 2.229 | 2.007 | 2.973 | 2.677 | 3.716 | **48°** |
| 42¼ | 0.740 | 0.672 | 1.480 | 1.345 | 2.221 | 2.017 | 2.961 | 2.689 | 3.701 | 47¾ |
| 42½ | 0.737 | 0.676 | 1.475 | 1.351 | 2.212 | 2.027 | 2.949 | 2.702 | 3.686 | 47½ |
| 42¾ | 0.734 | 0.679 | 1.469 | 1.358 | 2.203 | 2.036 | 2.937 | 2.715 | 3.672 | 47¼ |
| **43°** | 0.731 | 0.682 | 1.463 | 1.364 | 2.194 | 2.046 | 2.925 | 2.728 | 3.657 | **47°** |
| 43¼ | 0.728 | 0.685 | 1.457 | 1.370 | 2.185 | 2.056 | 2.913 | 2.741 | 3.642 | 46¾ |
| 43½ | 0.725 | 0.688 | 1.451 | 1.377 | 2.176 | 2.065 | 2.901 | 2.753 | 3.627 | 46½ |
| 43¾ | 0.722 | 0.692 | 1.445 | 1.383 | 2.167 | 2.075 | 2.889 | 2.766 | 3.612 | 46¼ |
| **44°** | 0.719 | 0.695 | 1.439 | 1.389 | 2.158 | 2.084 | 2.877 | 2.779 | 3.597 | **46°** |
| 44¼ | 0.716 | 0.698 | 1.433 | 1.396 | 2.149 | 2.093 | 2.865 | 2.791 | 3.582 | 45¾ |
| 44½ | 0.713 | 0.701 | 1.427 | 1.402 | 2.140 | 2.103 | 2.853 | 2.804 | 3.566 | 45½ |
| 44¾ | 0.710 | 0.704 | 1.420 | 1.408 | 2.131 | 2.112 | 2.841 | 2.816 | 3.551 | 45¼ |
| **45°** | 0.707 | 0.707 | 1.414 | 1.414 | 2.121 | 2.121 | 2.828 | 2.828 | 3.536 | **45°** |
| Bearing. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Bearing. |

| Bearing | 5 | | 6 | | 7 | | 8 | | 9 | | Bearing |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | |
| **39°** | 3.147 | 4.663 | 3.776 | 5.440 | 4.405 | 6.217 | 5.035 | 6.994 | 5.664 | | **51°** |
| 39¼ | 3.164 | 4.646 | 3.796 | 5.421 | 4.429 | 6.195 | 5.062 | 6.970 | 5.694 | | 50¾ |
| 39½ | 3.180 | 4.630 | 3.816 | 5.401 | 4.453 | 6.173 | 5.089 | 6.945 | 5.725 | | 50½ |
| 39¾ | 3.197 | 4.613 | 3.837 | 5.382 | 4.476 | 6.151 | 5.116 | 6.920 | 5.755 | | 50¼ |
| **40°** | 3.214 | 4.596 | 3.857 | 5.362 | 4.500 | 6.128 | 5.142 | 6.894 | 5.785 | | **50°** |
| 40¼ | 3.231 | 4.579 | 3.877 | 5.343 | 4.523 | 6.106 | 5.169 | 6.869 | 5.815 | | 49¾ |
| 40½ | 3.247 | 4.562 | 3.897 | 5.323 | 4.546 | 6.083 | 5.196 | 6.844 | 5.845 | | 49½ |
| 40¾ | 3.264 | 4.545 | 3.917 | 5.303 | 4.569 | 6.061 | 5.222 | 6.818 | 5.875 | | 49¼ |
| **41°** | 3.280 | 4.528 | 3.936 | 5.283 | 4.592 | 6.038 | 5.248 | 6.792 | 5.905 | | **49°** |
| 41¼ | 3.297 | 4.511 | 3.956 | 5.263 | 4.615 | 6.015 | 5.275 | 6.767 | 5.934 | | 48¾ |
| 41½ | 3.313 | 4.494 | 3.976 | 5.243 | 4.638 | 5.992 | 5.301 | 6.741 | 5.964 | | 48½ |
| 41¾ | 3.329 | 4.476 | 3.995 | 5.222 | 4.661 | 5.968 | 5.327 | 6.715 | 5.993 | | 48¼ |
| **42°** | 3.346 | 4.459 | 4.015 | 5.202 | 4.684 | 5.945 | 5.353 | 6.688 | 6.022 | | **48°** |
| 42¼ | 3.362 | 4.441 | 4.034 | 5.182 | 4.707 | 5.922 | 5.379 | 6.662 | 6.051 | | 47¾ |
| 42½ | 3.378 | 4.424 | 4.054 | 5.161 | 4.729 | 5.898 | 5.405 | 6.635 | 6.080 | | 47½ |
| 42¾ | 3.394 | 4.406 | 4.073 | 5.140 | 4.752 | 5.875 | 5.430 | 6.609 | 6.109 | | 47¼ |
| **43°** | 3.410 | 4.388 | 4.092 | 5.119 | 4.774 | 5.851 | 5.456 | 6.582 | 6.138 | | **47°** |
| 43¼ | 3.426 | 4.370 | 4.111 | 5.099 | 4.796 | 5.827 | 5.481 | 6.555 | 6.167 | | 46¾ |
| 43½ | 3.442 | 4.352 | 4.130 | 5.078 | 4.818 | 5.803 | 5.507 | 6.528 | 6.195 | | 46½ |
| 43¾ | 3.458 | 4.334 | 4.149 | 5.057 | 4.841 | 5.779 | 5.532 | 6.501 | 6.224 | | 46¼ |
| **44°** | 3.473 | 4.316 | 4.168 | 5.035 | 4.863 | 5.755 | 5.557 | 6.474 | 6.252 | | **46°** |
| 44¼ | 3.489 | 4.298 | 4.187 | 5.014 | 4.885 | 5.730 | 5.582 | 6.447 | 6.280 | | 45¾ |
| 44½ | 3.505 | 4.280 | 4.206 | 4.993 | 4.906 | 5.706 | 5.607 | 6.419 | 6.308 | | 45½ |
| 44¾ | 3.520 | 4.261 | 4.224 | 4.971 | 4.928 | 5.681 | 5.632 | 6.392 | 6.336 | | 45¼ |
| **45°** | 3.536 | 4.243 | 4.243 | 4.950 | 4.950 | 5.657 | 5.657 | 6.364 | 6.364 | | **45°** |
| Bearing | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. | Bearing |

## SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1.0000 | 1.0000 | 1.000000000 | 3.1416 | 0.7854 |
| 2 | 4 | 8 | 1.4142 | 1.2599 | .500000000 | 6.2832 | 3.1416 |
| 3 | 9 | 27 | 1.7321 | 1.4422 | .333333333 | 9.4248 | 7.0686 |
| 4 | 16 | 64 | 2.0000 | 1.5874 | .250000000 | 12.5664 | 12.5664 |
| 5 | 25 | 125 | 2.2361 | 1.7100 | .200000000 | 15.7080 | 19.635 |
| 6 | 36 | 216 | 2.4495 | 1.8171 | .166666666 | 18.850 | 28.274 |
| 7 | 49 | 343 | 2.6458 | 1.9129 | .142857143 | 21.991 | 38.485 |
| 8 | 64 | 512 | 2.8284 | 2.0000 | .125000000 | 25.133 | 50.266 |
| 9 | 81 | 729 | 3.0000 | 2.0801 | .111111111 | 28.274 | 63.617 |
| 10 | 100 | 1,000 | 3.1623 | 2.1544 | .100000000 | 31.416 | 78.540 |
| 11 | 121 | 1,331 | 3.3166 | 2.2240 | .090909091 | 34.558 | 95.033 |
| 12 | 144 | 1,728 | 3.4641 | 2.2894 | .083333333 | 37.699 | 113.10 |
| 13 | 169 | 2,197 | 3.6056 | 2.3513 | .076923077 | 40.841 | 132.73 |
| 14 | 196 | 2,744 | 3.7417 | 2.4101 | .071428571 | 43.982 | 153.94 |
| 15 | 225 | 3,375 | 3.8730 | 2.4662 | .066666666 | 47.124 | 176.71 |
| 16 | 256 | 4,096 | 4.0000 | 2.5198 | .062500000 | 50.265 | 201.06 |
| 17 | 289 | 4,913 | 4.1231 | 2.5713 | .058823529 | 53.407 | 226.98 |
| 18 | 324 | 5,832 | 4.2426 | 2.6207 | .055555556 | 56.549 | 254.47 |
| 19 | 361 | 6,859 | 4.3589 | 2.6684 | .052631579 | 59.690 | 283.53 |
| 20 | 400 | 8,000 | 4.4721 | 2.7144 | .050000000 | 62.832 | 314.16 |
| 21 | 441 | 9,261 | 4.5826 | 2.7589 | .047619048 | 65.973 | 346.36 |
| 22 | 484 | 10,648 | 4.6904 | 2.8020 | .045454545 | 69.115 | 380.13 |
| 23 | 529 | 12,167 | 4.7958 | 2.8439 | .043478261 | 72.257 | 415.48 |
| 24 | 576 | 13,824 | 4.8990 | 2.8845 | .041666667 | 75.398 | 452.39 |
| 25 | 625 | 15,625 | 5.0000 | 2.9240 | .040000000 | 78.540 | 490.87 |
| 26 | 676 | 17,576 | 5.0990 | 2.9625 | .038461538 | 81.681 | 530.93 |
| 27 | 729 | 19,683 | 5.1962 | 3.0000 | .037037037 | 84.823 | 572.56 |
| 28 | 784 | 21,952 | 5.2915 | 3.0366 | .035714286 | 87.965 | 615.75 |
| 29 | 841 | 24,389 | 5.3852 | 3.0723 | .034482759 | 91.106 | 660.52 |
| 30 | 900 | 27,000 | 5.4772 | 3.1072 | .033333333 | 94.248 | 706.86 |
| 31 | 961 | 29,791 | 5.5678 | 3.1414 | .032258065 | 97.389 | 754.77 |
| 32 | 1,024 | 32,768 | 5.6569 | 3.1748 | .031250000 | 100.53 | 804.25 |
| 33 | 1,089 | 35,937 | 5.7446 | 3.2075 | .030303030 | 103.67 | 855.30 |
| 34 | 1,156 | 39,304 | 5.8310 | 3.2396 | .029411765 | 106.81 | 907.92 |
| 35 | 1,225 | 42,875 | 5.9161 | 3.2711 | .028571429 | 109.96 | 962.11 |
| 36 | 1,296 | 46,656 | 6.0000 | 3.3019 | .027777778 | 113.10 | 1,017.88 |
| 37 | 1,369 | 50,653 | 6.0828 | 3.3322 | .027027027 | 116.24 | 1,075.21 |
| 38 | 1,444 | 54,872 | 6.1644 | 3.3620 | .026315789 | 119.38 | 1,134.11 |
| 39 | 1,521 | 59,319 | 6.2450 | 3.3912 | .025641026 | 122.52 | 1,194.59 |
| 40 | 1,600 | 64,000 | 6.3246 | 3.4200 | .025000000 | 125.66 | 1,256.64 |
| 41 | 1,681 | 68,921 | 6.4031 | 3.4482 | .024390244 | 128.81 | 1,320.25 |
| 42 | 1,764 | 74,088 | 6.4807 | 3.4760 | .023809524 | 131.95 | 1,385.44 |
| 43 | 1,849 | 79,507 | 6.5574 | 3.5034 | .023255814 | 135.09 | 1,452.20 |
| 44 | 1,936 | 85,184 | 6.6332 | 3.5303 | .022727273 | 138.23 | 1,520.53 |
| 45 | 2,025 | 91,125 | 6.7082 | 3.5569 | .022222222 | 141.37 | 1,590.43 |
| 46 | 2,116 | 97,336 | 6.7823 | 3.5830 | .021739130 | 144.51 | 1,661.90 |
| 47 | 2,209 | 103,823 | 6.8557 | 3.6088 | .021276600 | 147.65 | 1,734.94 |
| 48 | 2,304 | 110,592 | 6.9282 | 3.6342 | .020833333 | 150.80 | 1,809.56 |
| 49 | 2,401 | 117,649 | 7.0000 | 3.6593 | .020408163 | 153.94 | 1,885.74 |
| 50 | 2,500 | 125,000 | 7.0711 | 3.6840 | .020000000 | 157.08 | 1,963.50 |
| 51 | 2,601 | 132,651 | 7.1414 | 3.7084 | .019607843 | 160.22 | 2,042.82 |
| 52 | 2,704 | 140,608 | 7.2111 | 3.7325 | .019230769 | 163.36 | 2,123.72 |
| 53 | 2,809 | 148,877 | 7.2801 | 3.7563 | .018867925 | 166.50 | 2,206.18 |
| 54 | 2,916 | 157,464 | 7.3485 | 3.7798 | .018518519 | 169.65 | 2,290.22 |
| 55 | 3,025 | 166,375 | 7.4162 | 3.8030 | .018181818 | 172.79 | 2,375.83 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum | Area |
|---|---|---|---|---|---|---|---|
| 56 | 3,136 | 175,616 | 7.4833 | 3.8259 | .017857143 | 175.93 | 2,463.01 |
| 57 | 3,249 | 185,193 | 7.5498 | 3.8485 | .017543860 | 179.07 | 2,551.76 |
| 58 | 3,364 | 195,112 | 7.6158 | 3.8709 | .017241379 | 182.21 | 2,642.08 |
| 59 | 3,481 | 205,379 | 7.6811 | 3.8930 | .016949153 | 185.35 | 2,733.97 |
| 60 | 3,600 | 216,000 | 7.7460 | 3.9149 | .016666667 | 188.50 | 2,827.43 |
| 61 | 3,721 | 226,981 | 7.8102 | 3.9365 | .016393443 | 191.64 | 2,922.47 |
| 62 | 3,844 | 238,328 | 7.8740 | 3.9579 | .016129032 | 194.78 | 3,019.07 |
| 63 | 3,969 | 250,047 | 7.9373 | 3.9791 | .015873016 | 197.92 | 3,117.25 |
| 64 | 4,096 | 262,144 | 8.0000 | 4.0000 | .015625000 | 201.06 | 3,216.99 |
| 65 | 4,225 | 274,625 | 8.0623 | 4.0207 | .015384615 | 204.20 | 3,318.31 |
| 66 | 4,356 | 287,496 | 8.1240 | 4.0412 | .015151515 | 207.34 | 3,421.19 |
| 67 | 4,489 | 300,763 | 8.1854 | 4.0615 | .014925373 | 210.49 | 3,525.65 |
| 68 | 4,624 | 314,432 | 8.2462 | 4.0817 | .014705882 | 213.63 | 3,631.68 |
| 69 | 4,761 | 328,509 | 8.3066 | 4.1016 | .014492754 | 216.77 | 3,739.28 |
| 70 | 4,900 | 343,000 | 8.3666 | 4.1213 | .014285714 | 219.91 | 3,848.45 |
| 71 | 5,041 | 357,911 | 8.4261 | 4.1408 | .014084507 | 223.05 | 3,959.19 |
| 72 | 5,184 | 373,248 | 8.4853 | 4.1602 | .013888889 | 226.19 | 4,071.50 |
| 73 | 5,329 | 389,017 | 8.5440 | 4.1793 | .013698630 | 229.34 | 4,185.39 |
| 74 | 5,476 | 405,224 | 8.6023 | 4.1983 | .013513514 | 232.48 | 4,300.84 |
| 75 | 5,625 | 421,875 | 8.6603 | 4.2172 | .013333333 | 235.62 | 4,417.86 |
| 76 | 5,776 | 438,976 | 8.7178 | 4.2358 | .013157895 | 238.76 | 4,536.46 |
| 77 | 5,929 | 456,533 | 8.7750 | 4.2543 | .012987013 | 241.90 | 4,656.63 |
| 78 | 6,084 | 474,552 | 8.8318 | 4.2727 | .012820513 | 245.04 | 4,778.36 |
| 79 | 6,241 | 493,039 | 8.8882 | 4.2908 | .012658228 | 248.19 | 4,901.67 |
| 80 | 6,400 | 512,000 | 8.9443 | 4.3089 | .012500000 | 251.33 | 5,026.55 |
| 81 | 6,561 | 531,441 | 9.0000 | 4.3267 | .012345679 | 254.47 | 5,153.00 |
| 82 | 6,724 | 551,368 | 9.0554 | 4.3445 | .012195122 | 257.61 | 5,281.02 |
| 83 | 6,889 | 571,787 | 9.1104 | 4.3621 | .012048193 | 260.75 | 5,410.61 |
| 84 | 7,056 | 592,704 | 9.1652 | 4.3795 | .011904762 | 263.89 | 5,541.77 |
| 85 | 7,225 | 614,125 | 9.2195 | 4.3968 | .011764706 | 267.04 | 5,674.50 |
| 86 | 7,396 | 636,056 | 9.2736 | 4.4140 | .011627907 | 270.18 | 5,808.80 |
| 87 | 7,569 | 658,503 | 9.3274 | 4.4310 | .011494253 | 273.32 | 5,944.68 |
| 88 | 7,744 | 681,472 | 9.3808 | 4.4480 | .011363636 | 276.46 | 6,082.12 |
| 89 | 7,921 | 704,969 | 9.4340 | 4.4647 | .011235955 | 279.60 | 6,221.14 |
| 90 | 8,100 | 729,000 | 9.4868 | 4.4814 | .011111111 | 282.74 | 6,361.73 |
| 91 | 8,281 | 753,571 | 9.5394 | 4.4979 | .010989011 | 285.88 | 6,503.88 |
| 92 | 8,464 | 778,688 | 9.5917 | 4.5144 | .010869565 | 289.03 | 6,647.61 |
| 93 | 8,649 | 804,357 | 9.6437 | 4.5307 | .010752688 | 292.17 | 6,792.91 |
| 94 | 8,836 | 830,584 | 9.6954 | 4.5468 | .010638298 | 295.31 | 6,939.78 |
| 95 | 9,025 | 857,375 | 9.7468 | 4.5629 | .010526316 | 298.45 | 7,088.22 |
| 96 | 9,216 | 884,736 | 9.7980 | 4.5789 | .010416667 | 301.59 | 7,238.23 |
| 97 | 9,409 | 912,673 | 9.8489 | 4.5947 | .010309278 | 304.73 | 7,389.81 |
| 98 | 9,604 | 941,192 | 9.8995 | 4.6104 | .010204082 | 307.88 | 7,542.96 |
| 99 | 9,801 | 970,299 | 9.9499 | 4.6261 | .010101010 | 311.02 | 7,697.69 |
| 100 | 10,000 | 1,000,000 | 10.0000 | 4.6416 | .010000000 | 314.16 | 7,853.98 |
| 101 | 10,201 | 1,030,301 | 10.0499 | 4.6570 | .009900990 | 317.30 | 8,011.85 |
| 102 | 10,404 | 1,061,208 | 10.0995 | 4.6723 | .009803922 | 320.44 | 8,171.28 |
| 103 | 10,609 | 1,092,727 | 10.1489 | 4.6875 | .009708738 | 323.58 | 8,332.29 |
| 104 | 10,816 | 1,124,864 | 10.1980 | 4.7027 | .009615385 | 326.73 | 8,494.87 |
| 105 | 11,025 | 1,157,625 | 10.2470 | 4.7177 | .009523810 | 329.87 | 8,659.01 |
| 106 | 11,236 | 1,191,016 | 10.2956 | 4.7326 | .009433962 | 333.01 | 8,824.73 |
| 107 | 11,449 | 1,225,043 | 10.3441 | 4.7475 | .009345794 | 336.15 | 8,992.02 |
| 108 | 11,664 | 1,259,712 | 10.3923 | 4.7622 | .009259259 | 339.29 | 9,160.88 |
| 109 | 11,881 | 1,295,029 | 10.4403 | 4.7769 | .009174312 | 342.43 | 9,331.32 |
| 110 | 12,100 | 1,331,000 | 10.4881 | 4.7914 | .009090909 | 345.58 | 9,503.32 |
| 111 | 12,321 | 1,367,631 | 10.5357 | 4.8059 | .009009009 | 348.72 | 9,676.89 |
| 112 | 12,544 | 1,404,928 | 10.5830 | 4.8203 | .008928571 | 351.86 | 9,852.03 |
| 113 | 12,769 | 1,442,897 | 10.6301 | 4.8346 | .008849558 | 355.00 | 10,028.75 |
| 114 | 12,996 | 1,481,544 | 10.6771 | 4.8488 | .008771930 | 358.14 | 10,207.03 |
| 115 | 13,225 | 1,520,875 | 10.7238 | 4.8629 | .008695652 | 361.28 | 10,386.89 |
| 116 | 13,456 | 1,560,896 | 10.7703 | 4.8770 | .008620690 | 364.42 | 10,568.32 |
| 117 | 13,689 | 1,601,613 | 10.8167 | 4.8910 | .008547009 | 367.57 | 10,751.32 |
| 118 | 13,924 | 1,643,032 | 10.8628 | 4.9049 | .008474576 | 370.71 | 10,935.88 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|---|---|---|---|---|---|---|---|
| 119 | 14,161 | 1,685,159 | 10.9087 | 4.9187 | .008403361 | 373.85 | 11,122.02 |
| 120 | 14,400 | 1,728,000 | 10.9545 | 4.9324 | .008333333 | 376.99 | 11,309.73 |
| 121 | 14,641 | 1,771,561 | 11.0000 | 4.9461 | .008264463 | 380.13 | 11,499.01 |
| 122 | 14,834 | 1,815,848 | 11.0454 | 4.9597 | .008196721 | 383.27 | 11,689.87 |
| 123 | 15,129 | 1,860,867 | 11.0905 | 4.9732 | .008130081 | 386.42 | 11,882.29 |
| 124 | 15,376 | 1,906,624 | 11.1355 | 4.9866 | .008064516 | 389.56 | 12,076.28 |
| 125 | 15,625 | 1,953,125 | 11.1803 | 5.0000 | .008000000 | 392.70 | 12,271.85 |
| 126 | 15,876 | 2,000,376 | 11.2250 | 5.0133 | .007936508 | 395.84 | 12,468.98 |
| 127 | 16,129 | 2,048,383 | 11.2694 | 5.0265 | .007874016 | 398.98 | 12,667.69 |
| 128 | 16,384 | 2,097,152 | 11.3137 | 5.0397 | .007812500 | 402.12 | 12,867.96 |
| 129 | 16,641 | 2,146,689 | 11.3578 | 5.0528 | .007751938 | 405.27 | 13,069.81 |
| 130 | 16,900 | 2,197,000 | 11.4018 | 5.0658 | .007692308 | 408.41 | 13,273.23 |
| 131 | 17,161 | 2,248,091 | 11.4455 | 5.0788 | .007633588 | 411.55 | 13,478.22 |
| 132 | 17,424 | 2,299,968 | 11.4891 | 5.0916 | .007575758 | 414.69 | 13,684.78 |
| 133 | 17,689 | 2,352,637 | 11.5326 | 5.1045 | .007518797 | 417.83 | 13,892.91 |
| 134 | 17,956 | 2,406,104 | 11.5758 | 5.1172 | .007462687 | 420.97 | 14,102.61 |
| 135 | 18,225 | 2,460,375 | 11.6190 | 5.1299 | .007407407 | 424.12 | 14,313.88 |
| 136 | 18,496 | 2,515,456 | 11.6619 | 5.1426 | .007352941 | 427.26 | 14,526.72 |
| 137 | 18,769 | 2,571,353 | 11.7047 | 5.1551 | .007299270 | 430.40 | 14,741.14 |
| 138 | 19,044 | 2,628,072 | 11.7473 | 5.1676 | .007246377 | 433.54 | 14,957.12 |
| 139 | 19,321 | 2,685,619 | 11.7898 | 5.1801 | .007194245 | 436.68 | 15,174.68 |
| 140 | 19,600 | 2,744,000 | 11.8322 | 5.1925 | .007142857 | 439.82 | 15,393.80 |
| 141 | 19,881 | 2,803,221 | 11.8743 | 5.2048 | .007092199 | 442.96 | 15,614.50 |
| 142 | 20,164 | 2,863,288 | 11.9164 | 5.2171 | .007042254 | 446.11 | 15,836.77 |
| 143 | 20,449 | 2,924,207 | 11.9583 | 5.2293 | .006993007 | 449.25 | 16,060.61 |
| 144 | 20,736 | 2,985,984 | 12.0000 | 5.2415 | .006944444 | 452.39 | 16,286.02 |
| 145 | 21,025 | 3,048,625 | 12.0416 | 5.2536 | .006896552 | 455.53 | 16,513.00 |
| 146 | 21,316 | 3,112,136 | 12.0830 | 5.2656 | .006849315 | 458.67 | 16,741.55 |
| 147 | 21,609 | 3,176,523 | 12.1244 | 5.2776 | .006802721 | 461.81 | 16,971.67 |
| 148 | 21,904 | 3,241,792 | 12.1655 | 5.2896 | .006756757 | 464.96 | 17,203.36 |
| 149 | 22,201 | 3,307,949 | 12.2066 | 5.3015 | .006711409 | 468.10 | 17,436.62 |
| 150 | 22,500 | 3,375,000 | 12.2474 | 5.3133 | .006666667 | 471.24 | 17,671.46 |
| 151 | 22,801 | 3,442,951 | 12.2882 | 5.3251 | .006622517 | 474.38 | 17,907.86 |
| 152 | 23,104 | 3,511,808 | 12.3288 | 5.3368 | .006578947 | 477.52 | 18,145.84 |
| 153 | 23,409 | 3,581,577 | 12.3693 | 5.3485 | .006535948 | 480.66 | 18,385.39 |
| 154 | 23,716 | 3,652,264 | 12.4097 | 5.3601 | .006493506 | 483.81 | 18,626.50 |
| 155 | 24,025 | 3,723,875 | 12.4499 | 5.3717 | .006451613 | 486.95 | 18,869.19 |
| 156 | 24,336 | 3,796,416 | 12.4900 | 5.3832 | .006410256 | 490.09 | 19,113.45 |
| 157 | 24,649 | 3,869,893 | 12.5300 | 5.3947 | .006369427 | 493.23 | 19,359.28 |
| 158 | 24,964 | 3,944,312 | 12.5698 | 5.4061 | .006329114 | 496.37 | 19,606.68 |
| 159 | 25,281 | 4,019,679 | 12.6095 | 5.4175 | .006289308 | 499.51 | 19,855.65 |
| 160 | 25,600 | 4,096,000 | 12.6491 | 5.4288 | .006250000 | 502.65 | 20,106.19 |
| 161 | 25,921 | 4,173,281 | 12.6886 | 5.4401 | .006211180 | 505.80 | 20,358.31 |
| 162 | 26,244 | 4,251,528 | 12.7279 | 5.4514 | .006172840 | 508.94 | 20,611.99 |
| 163 | 26,569 | 4,330,747 | 12.7671 | 5.4626 | .006134969 | 512.08 | 20,867.24 |
| 164 | 26,896 | 4,410,944 | 12.8062 | 5.4737 | .006097561 | 515.22 | 21,124.07 |
| 165 | 27,225 | 4,492,125 | 12.8452 | 5.4848 | .006060606 | 518.36 | 21,382.46 |
| 166 | 27,556 | 4,574,296 | 12.8841 | 5.4959 | .006024096 | 521.50 | 21,642.43 |
| 167 | 27,889 | 4,657,463 | 12.9228 | 5.5069 | .005988024 | 524.65 | 21,903.97 |
| 168 | 28,224 | 4,741,632 | 12.9615 | 5.5178 | .005952381 | 527.79 | 22,167.08 |
| 169 | 28,561 | 4,826,809 | 13.0000 | 5.5288 | .005917160 | 530.93 | 22,431.76 |
| 170 | 28,900 | 4,913,000 | 13.0384 | 5.5397 | .005882353 | 534.07 | 22,698.01 |
| 171 | 29,241 | 5,000,211 | 13.0767 | 5.5505 | .005847953 | 537.21 | 22,965.83 |
| 172 | 29,584 | 5,088,448 | 13.1149 | 5.5613 | .005813953 | 540.35 | 23,235.22 |
| 173 | 29,929 | 5,177,717 | 13.1529 | 5.5721 | .005780347 | 543.50 | 23,506.18 |
| 174 | 30,276 | 5,268,024 | 13.1909 | 5.5828 | .005747126 | 546.64 | 23,778.71 |
| 175 | 30,625 | 5,359,375 | 13.2288 | 5.5934 | .005714286 | 549.78 | 24,052.82 |
| 176 | 30,976 | 5,451,776 | 13.2665 | 5.6041 | .005681818 | 552.92 | 24,328.49 |
| 177 | 31,329 | 5,545,233 | 13.3041 | 5.6147 | .005649718 | 556.06 | 24,605.74 |
| 178 | 31,684 | 5,639,752 | 13.3417 | 5.6252 | .005617978 | 559.20 | 24,884.56 |
| 179 | 32,041 | 5,735,339 | 13.3791 | 5.6357 | .005586592 | 562.35 | 25,164.94 |
| 180 | 32,400 | 5,832,000 | 13.4164 | 5.6462 | .005555556 | 565.49 | 25,446.90 |
| 181 | 32,761 | 5,929,741 | 13.4536 | 5.6567 | .005524862 | 568.63 | 25,730.48 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|-----|--------|------|----------|----------|------------|---------|------|
| 182 | 33,124 | 6,028,568 | 13.4907 | 5.6671 | .005494505 | 571.77 | 26,015.53 |
| 183 | 33,489 | 6,128,487 | 13.5277 | 5.6774 | .005464481 | 574.91 | 26,302.20 |
| 184 | 33,856 | 6,229,504 | 13.5647 | 5.6877 | .005434783 | 578.05 | 26,590.44 |
| 185 | 34,225 | 6,331,625 | 13.6015 | 5.6980 | .005405405 | 581.19 | 26,880.25 |
| 186 | 34,596 | 6,434,856 | 13.6382 | 5.7083 | .005376344 | 584.34 | 27,171.63 |
| 187 | 34,969 | 6,539,203 | 13.6748 | 5.7185 | .005347594 | 587.48 | 27,464.59 |
| 188 | 35,344 | 6,644,672 | 13.7113 | 5.7287 | .005319149 | 590.62 | 27,759.11 |
| 189 | 35,721 | 6,751,269 | 13.7477 | 5.7388 | .005291005 | 593.76 | 28,055.21 |
| 190 | 36,100 | 6,859,000 | 13.7840 | 5.7489 | .005263158 | 596.90 | 28,352.87 |
| 191 | 36,481 | 6,967,871 | 13.8203 | 5.7590 | .005235602 | 600.04 | 28,652.11 |
| 192 | 36,864 | 7,077,888 | 13.8564 | 5.7690 | .005208333 | 603.19 | 28,952.92 |
| 193 | 37,249 | 7,189,017 | 13.8924 | 5.7790 | .005181347 | 606.33 | 29,255.30 |
| 194 | 37,636 | 7,301,384 | 13.9284 | 5.7890 | .005154639 | 609.47 | 29,559.25 |
| 195 | 38,025 | 7,414,875 | 13.9642 | 5.7989 | .005128205 | 612.61 | 29,864.77 |
| 196 | 38,416 | 7,529,536 | 14.0000 | 5.8088 | .005102041 | 615.75 | 30,171.86 |
| 197 | 38,809 | 7,645,373 | 14.0357 | 5.8186 | .005076142 | 618.89 | 30,480.52 |
| 198 | 39,204 | 7,762,392 | 14.0712 | 5.8285 | .005050505 | 622.04 | 30,790.75 |
| 199 | 39,601 | 7,880,599 | 14.1067 | 5.8383 | .005025126 | 625.18 | 31,102.55 |
| 200 | 40,000 | 8,000,000 | 14.1421 | 5.8480 | .005000000 | 628.32 | 31,415.93 |
| 201 | 40,401 | 8,120,601 | 14.1774 | 5.8578 | .004975124 | 631.46 | 31,730.87 |
| 202 | 40,804 | 8,242,408 | 14.2127 | 5.8675 | .004950495 | 634.60 | 32,047.39 |
| 203 | 41,209 | 8,365,427 | 14.2478 | 5.8771 | .004926108 | 637.74 | 32,365.47 |
| 204 | 41,616 | 8,489,664 | 14.2829 | 5.8868 | .004901961 | 640.88 | 32,685.13 |
| 205 | 42,025 | 8,615,125 | 14.3178 | 5.8964 | .004878049 | 644.03 | 33,006.36 |
| 206 | 42,436 | 8,741,816 | 14.3527 | 5.9059 | .004854369 | 647.17 | 33,329.16 |
| 207 | 42,849 | 8,869,743 | 14.3875 | 5.9155 | .004830918 | 650.31 | 33,653.53 |
| 208 | 43,264 | 8,998,912 | 14.4222 | 5.9250 | .004807692 | 653.45 | 33,979.47 |
| 209 | 43,681 | 9,129,329 | 14.4568 | 5.9345 | .004784689 | 656.59 | 34,306.98 |
| 210 | 44,100 | 9,261,000 | 14.4914 | 5.9439 | .004761905 | 659.73 | 34,636.06 |
| 211 | 44,521 | 9,393,931 | 14.5258 | 5.9533 | .004739336 | 662.88 | 34,966.71 |
| 212 | 44,944 | 9,528,128 | 14.5602 | 5.9627 | .004716981 | 666.02 | 35,298.94 |
| 213 | 45,369 | 9,663,597 | 14.5945 | 5.9721 | .004694836 | 669.16 | 35,632.73 |
| 214 | 45,796 | 9,800,344 | 14.6287 | 5.9814 | .004672897 | 672.30 | 35,968.09 |
| 215 | 46,225 | 9,938,375 | 14.6629 | 5.9907 | .004651163 | 675.44 | 36,305.03 |
| 216 | 46,656 | 10,077,696 | 14.6969 | 6.0000 | .004629630 | 678.58 | 36,643.54 |
| 217 | 47,089 | 10,218,313 | 14.7309 | 6.0092 | .004608295 | 681.73 | 36,983.61 |
| 218 | 47,524 | 10,360,232 | 14.7648 | 6.0185 | .004587156 | 684.87 | 37,325.26 |
| 219 | 47,961 | 10,503,459 | 14.7986 | 6.0277 | .004566210 | 688.01 | 37,668.48 |
| 220 | 48,400 | 10,648,000 | 14.8324 | 6.0368 | .004545455 | 691.15 | 38,013.27 |
| 221 | 48,841 | 10,793,861 | 14.8661 | 6.0459 | .004524887 | 694.29 | 38,359.63 |
| 222 | 49,284 | 10,941,048 | 14.8997 | 6.0550 | .004504505 | 697.43 | 38,707.56 |
| 223 | 49,729 | 11,089,567 | 14.9332 | 6.0641 | .004484305 | 700.58 | 39,057.07 |
| 224 | 50,176 | 11,239,424 | 14.9666 | 6.0732 | .004464286 | 703.72 | 39,408.14 |
| 225 | 50,625 | 11,390,625 | 15.0000 | 6.0822 | .004444444 | 706.86 | 39,760.78 |
| 226 | 51,076 | 11,543,176 | 15.0333 | 6.0912 | .004424779 | 710.00 | 40,115.00 |
| 227 | 51,529 | 11,697,083 | 15.0665 | 6.1002 | .004405286 | 713.14 | 40,470.78 |
| 228 | 51,984 | 11,852,352 | 15.0997 | 6.1091 | .004385965 | 716.28 | 40,828.14 |
| 229 | 52,441 | 12,008,989 | 15.1327 | 6.1180 | .004366812 | 719.42 | 41,187.07 |
| 230 | 52,900 | 12,167,000 | 15.1658 | 6.1269 | .004347826 | 722.57 | 41,547.56 |
| 231 | 53,361 | 12,326,391 | 15.1987 | 6.1358 | .004329004 | 725.71 | 41,909.63 |
| 232 | 53,824 | 12,487,168 | 15.2315 | 6.1446 | .004310345 | 728.85 | 42,273.27 |
| 233 | 54,289 | 12,649,337 | 15.2643 | 6.1534 | .004291845 | 731.99 | 42,638.48 |
| 234 | 54,756 | 12,812,904 | 15.2971 | 6.1622 | .004273504 | 735.13 | 43,005.26 |
| 235 | 55,225 | 12,977,875 | 15.3297 | 6.1710 | .004255319 | 738.27 | 43,373.61 |
| 236 | 55,696 | 13,144,256 | 15.3623 | 6.1797 | .004237288 | 741.42 | 43,743.54 |
| 237 | 56,169 | 13,312,053 | 15.3948 | 6.1885 | .004219409 | 744.56 | 44,115.08 |
| 238 | 56,644 | 13,481,272 | 15.4272 | 6.1672 | .004201681 | 747.70 | 44,488.09 |
| 239 | 57,121 | 13,651,919 | 15.4596 | 6.2058 | .004184100 | 750.84 | 44,862.73 |
| 240 | 57,600 | 13,824,000 | 15.4919 | 6.2145 | .004166667 | 753.98 | 45,238.93 |
| 241 | 58,081 | 13,997,521 | 15.5242 | 6.2231 | .004149378 | 757.12 | 45,616.71 |
| 242 | 58,564 | 14,172,488 | 15.5563 | 6.2317 | .004132231 | 760.27 | 45,996.06 |
| 243 | 59,049 | 14,348,907 | 15.5885 | 6.2403 | .004115226 | 763.41 | 46,376.98 |
| 244 | 59,536 | 14,526,784 | 15.6205 | 6.2488 | .004098361 | 766.55 | 46,759.47 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|-----|--------|------|----------|----------|------------|---------|------|
| 245 | 60,025 | 14,706,125 | 15.6525 | 6.2573 | .004081633 | 769.69 | 47,143.52 |
| 246 | 60,516 | 14,886,936 | 15.6844 | 6.2658 | .004065041 | 772.83 | 47,529.16 |
| 247 | 61,009 | 15,069,223 | 15.7162 | 6.2743 | .004048583 | 775.97 | 47,916.36 |
| 248 | 61,504 | 15,252,992 | 15.7480 | 6.2828 | .004032258 | 779.11 | 48,305.13 |
| 249 | 62,001 | 15,438,249 | 15.7797 | 6.2912 | .004016064 | 782.26 | 48,695.47 |
| 250 | 62,500 | 15,625,000 | 15.8114 | 6.2996 | .004000000 | 785.40 | 49,087.39 |
| 251 | 63,001 | 15,813,251 | 15.8430 | 6.3080 | .003984064 | 788.54 | 49,480.87 |
| 252 | 63,504 | 16,003,008 | 15.8745 | 6.3164 | .003968254 | 791.68 | 49,875.92 |
| 253 | 64,009 | 16,194,277 | 15.9060 | 6.3247 | .003952569 | 794.82 | 50,272.55 |
| 254 | 64,516 | 16,387,064 | 15.9374 | 6.3330 | .003937008 | 797.96 | 50,670.75 |
| 255 | 65,025 | 16,581,375 | 15.9687 | 6.3413 | .003921569 | 801.11 | 51,070.52 |
| 256 | 65,536 | 16,777,216 | 16.0000 | 6.3496 | .003906250 | 804.25 | 51,471.85 |
| 257 | 66,049 | 16,974,593 | 16.0312 | 6.3579 | .003891051 | 807.39 | 51,874.76 |
| 258 | 66,564 | 17,173,512 | 16.0624 | 6.3661 | .003875969 | 810.53 | 52,279.24 |
| 259 | 67,081 | 17,373,979 | 16.0935 | 6.3743 | .003861004 | 813.67 | 52,685.29 |
| 260 | 67,600 | 17,576,000 | 16.1245 | 6.3825 | .003846154 | 816.81 | 53,092.92 |
| 261 | 68,121 | 17,779,581 | 16.1555 | 6.3907 | .003831418 | 819.96 | 53,502.11 |
| 262 | 68,644 | 17,984,728 | 16.1864 | 6.3988 | .003816794 | 823.10 | 53,912.87 |
| 263 | 69,169 | 18,191,447 | 16.2173 | 6.4070 | .003802281 | 826.24 | 54,325.21 |
| 264 | 69,696 | 18,399,744 | 16.2481 | 6.4151 | .003787879 | 829.38 | 54,739.11 |
| 265 | 70,225 | 18,609,625 | 16.2788 | 6.4232 | .003773585 | 832.52 | 55,154.59 |
| 266 | 70,756 | 18,821,096 | 16.3095 | 6.4312 | .003759398 | 835.66 | 55,571.63 |
| 267 | 71,289 | 19,034,163 | 16.3401 | 6.4393 | .003745318 | 838.81 | 55,990.25 |
| 268 | 71,824 | 19,248,832 | 16.3707 | 6.4473 | .003731343 | 841.95 | 56,410.44 |
| 269 | 72,361 | 19,465,109 | 16.4012 | 6.4553 | .003717472 | 845.09 | 56,832.20 |
| 270 | 72,900 | 19,683,000 | 16.4317 | 6.4633 | .003703704 | 848.23 | 57,255.53 |
| 271 | 73,441 | 19,902,511 | 16.4621 | 6.4713 | .003690037 | 851.87 | 57,680.43 |
| 272 | 73,984 | 20,123,648 | 16.4924 | 6.4792 | .003676471 | 854.51 | 58,106.90 |
| 273 | 74,529 | 20,346,417 | 16.5227 | 6.4872 | .003663004 | 857.65 | 58,534.94 |
| 274 | 75,076 | 20,570,824 | 16.5529 | 6.4951 | .003649635 | 860.80 | 58,964.55 |
| 275 | 75,625 | 20,796,875 | 16.5831 | 6.5030 | .003636364 | 863.94 | 59,395.74 |
| 276 | 76,176 | 21,024,576 | 16.6132 | 6.5108 | .003623188 | 867.08 | 59,828.49 |
| 277 | 76,729 | 21,253,933 | 16.6433 | 6.5187 | .003610108 | 870.22 | 60,262.82 |
| 278 | 77,284 | 21,484,952 | 16.6733 | 6.5265 | .003597122 | 873.36 | 60,698.71 |
| 279 | 77,841 | 21,717,639 | 16.7033 | 6.5343 | .003584229 | 876.50 | 61,136.18 |
| 280 | 78,400 | 21,952,000 | 16.7332 | 6.5421 | .003571429 | 879.65 | 61,575.22 |
| 281 | 78,961 | 22,188,041 | 16.7631 | 6.5499 | .003558719 | 882.79 | 62,015.82 |
| 282 | 79,524 | 22,425,768 | 16.7929 | 6.5577 | .003546099 | 885.93 | 62,458.00 |
| 283 | 80,089 | 22,665,187 | 16.8226 | 6.5654 | .003533569 | 889.07 | 62,901.75 |
| 284 | 80,656 | 22,906,304 | 16.8523 | 6.5731 | .003521127 | 892.21 | 63,347.07 |
| 285 | 81,225 | 23,149,125 | 16.8819 | 6.5808 | .003508772 | 895.35 | 63,793.97 |
| 286 | 81,796 | 23,393,656 | 16.9115 | 6.5885 | .003496503 | 898.50 | 64,242.43 |
| 287 | 82,369 | 23,639,903 | 16.9411 | 6.5962 | .003484321 | 901.64 | 64,692.46 |
| 288 | 82,944 | 23,887,872 | 16.9706 | 6.6039 | .003472222 | 904.78 | 65,144.07 |
| 289 | 83,521 | 24,137,569 | 17.0000 | 6.6115 | .003460208 | 907.92 | 65,597.24 |
| 290 | 84,100 | 24,389,000 | 17.0294 | 6.6191 | .003448276 | 911.06 | 66,051.99 |
| 291 | 84,681 | 24,642,171 | 17.0587 | 6.6267 | .003436426 | 914.20 | 66,508.30 |
| 292 | 85,264 | 24,897,088 | 17.0880 | 6.6343 | .003424658 | 917.35 | 66,966.19 |
| 293 | 85,849 | 25,153,757 | 17.1172 | 6.6419 | .003412969 | 920.49 | 67,425.65 |
| 294 | 86,436 | 25,412,184 | 17.1464 | 6.6494 | .003401361 | 923.63 | 67,886.68 |
| 295 | 87,025 | 25,672,375 | 17.1756 | 6.6569 | .003389831 | 926.77 | 68,349.28 |
| 296 | 87,616 | 25,934,336 | 17.2047 | 6.6644 | .003378378 | 929.91 | 68,813.45 |
| 297 | 88,209 | 26,198,073 | 17.2337 | 6.6719 | .003367003 | 933.05 | 69,279.19 |
| 298 | 88,804 | 26,463,592 | 17.2627 | 6.6794 | .003355705 | 936.19 | 69,746.50 |
| 299 | 89,401 | 26,730,899 | 17.2916 | 6.6869 | .003344482 | 939.34 | 70,215.38 |
| 300 | 90,000 | 27,000,000 | 17.3205 | 6.6943 | .003333333 | 942.48 | 70,685.83 |
| 301 | 90,601 | 27,270,901 | 17.3494 | 6.7018 | .003322259 | 945.62 | 71,157.86 |
| 302 | 91,204 | 27,543,608 | 17.3781 | 6.7092 | .003311258 | 948.76 | 71,631.45 |
| 303 | 91,809 | 27,818,127 | 17.4069 | 6.7166 | .003300330 | 951.90 | 72,106.62 |
| 304 | 92,416 | 28,094,464 | 17.4356 | 6.7240 | .003289474 | 955.04 | 72,583.36 |
| 305 | 93,025 | 28,372,625 | 17.4642 | 6.7313 | .003278689 | 958.19 | 73,061.66 |
| 306 | 93,636 | 28,652,616 | 17.4929 | 6.7387 | .003267974 | 961.33 | 73,541.54 |
| 307 | 94,249 | 28,934,443 | 17.5214 | 6.7460 | .003257329 | 964.47 | 74,022.99 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|-----|--------|------|----------|----------|------------|---------|------|
| 308 | 94,864 | 29,218,112 | 17.5499 | 6.7533 | .003246753 | 967.61 | 74,506.01 |
| 309 | 95,481 | 29,503,629 | 17.5784 | 6.7606 | .003236063 | 970.75 | 74,990.60 |
| 310 | 96,100 | 29,791,000 | 17.6068 | 6.7679 | .003225806 | 973.89 | 75,476.76 |
| 311 | 96,721 | 30,080,231 | 17.6352 | 6.7752 | .003215434 | 977.04 | 75,964.50 |
| 312 | 97,344 | 30,371,328 | 17.6635 | 6.7824 | .003205128 | 980.18 | 76,453.80 |
| 313 | 97,969 | 30,664,297 | 17.6918 | 6.7897 | .003194888 | 983.32 | 76,944.67 |
| 314 | 98,596 | 30,959,144 | 17.7200 | 6.7969 | .003184713 | 986.46 | 77,437.12 |
| 315 | 99,225 | 31,255,875 | 17.7482 | 6.8041 | .003174603 | 989.60 | 77,931.13 |
| 316 | 99,856 | 31,554,496 | 17.7764 | 6.8113 | .003164557 | 992.74 | 78,426.72 |
| 317 | 100,489 | 31,855,013 | 17.8045 | 6.8185 | .003154574 | 995.88 | 78,923.88 |
| 318 | 101,124 | 32,157,432 | 17.8326 | 6.8256 | .003144654 | 999.03 | 79,422.60 |
| 319 | 101,761 | 32,461,759 | 17.8606 | 6.8328 | .003134796 | 1,002.17 | 79,922.90 |
| 320 | 102,400 | 32,768,000 | 17.8885 | 6.8399 | .003125000 | 1,005.31 | 80,424.77 |
| 321 | 103,041 | 33,076,161 | 17.9165 | 6.8470 | .003115265 | 1,008.45 | 80,928.21 |
| 322 | 103,684 | 33,386,248 | 17.9444 | 6.8541 | .003105590 | 1,011.59 | 81,433.22 |
| 323 | 104,329 | 33,698,267 | 17.9722 | 6.8612 | .003095975 | 1,014.73 | 81,939.80 |
| 324 | 104,976 | 34,012,224 | 18.0000 | 6.8683 | .003086420 | 1,017.88 | 82,447.96 |
| 325 | 105,625 | 34,328,125 | 18.0278 | 6.8753 | .003076923 | 1,021.02 | 82,957.68 |
| 326 | 106,276 | 34,645,976 | 18.0555 | 6.8824 | .003067485 | 1,024.16 | 83,468.98 |
| 327 | 106,929 | 34,965,783 | 18.0831 | 6.8894 | .003058104 | 1,027.30 | 83,981.84 |
| 328 | 107,584 | 35,287,552 | 18.1108 | 6.8964 | .003048780 | 1,030.44 | 84,496.28 |
| 329 | 108,241 | 35,611,289 | 18.1384 | 6.9034 | .003039514 | 1,033.58 | 85,012.28 |
| 330 | 108,900 | 35,937,000 | 18.1659 | 6.9104 | .003030303 | 1,036.73 | 85,529.86 |
| 331 | 109,561 | 36,264,691 | 18.1934 | 6.9174 | .003021148 | 1,039.87 | 86,049.01 |
| 332 | 110,224 | 36,594,368 | 18.2209 | 6.9244 | .003012048 | 1,043.01 | 86,569.73 |
| 333 | 110,889 | 36,926,037 | 18.2483 | 6.9313 | .003003003 | 1,046.15 | 87,092.02 |
| 334 | 111,556 | 37,259,704 | 18.2757 | 6.9382 | .002994012 | 1,049.29 | 87,615.88 |
| 335 | 112,225 | 37,595,375 | 18.3030 | 6.9451 | .002985075 | 1,052.43 | 88,141.31 |
| 336 | 112,896 | 37,933,056 | 18.3303 | 6.9521 | .002976190 | 1,055.58 | 88,668.31 |
| 337 | 113,569 | 38,272,753 | 18.3576 | 6.9589 | .002967359 | 1,058.72 | 89,196.88 |
| 338 | 114,244 | 38,614,472 | 18.3848 | 6.9658 | .002958580 | 1,061.86 | 89,727.03 |
| 339 | 114,921 | 38,958,219 | 18.4120 | 6.9727 | .002949853 | 1,065.00 | 90,258.74 |
| 340 | 115,600 | 39,304,000 | 18.4391 | 6.9795 | .002941176 | 1,068.14 | 90,792.03 |
| 341 | 116,281 | 39,651,821 | 18.4662 | 6.9864 | .002932551 | 1,071.28 | 91,326.88 |
| 342 | 116,964 | 40,001,688 | 18.4932 | 6.9932 | .002923977 | 1,074.42 | 91,863.31 |
| 343 | 117,649 | 40,353,607 | 18.5203 | 7.0000 | .002915452 | 1,077.57 | 92,401.31 |
| 344 | 118,336 | 40,707,584 | 18.5472 | 7.0068 | .002906977 | 1,080.71 | 92,940.88 |
| 345 | 119,025 | 41,063,625 | 18.5742 | 7.0136 | .002898551 | 1,083.85 | 93,482.02 |
| 346 | 119,716 | 41,421,736 | 18.6011 | 7.0203 | .002890173 | 1,086.99 | 94,024.73 |
| 347 | 120,409 | 41,781,923 | 18.6279 | 7.0271 | .002881844 | 1,090.13 | 94,569.01 |
| 348 | 121,104 | 42,144,192 | 18.6548 | 7.0338 | .002873563 | 1,093.27 | 95,114.86 |
| 349 | 121,801 | 42,508,549 | 18.6815 | 7.0406 | .002865330 | 1,096.42 | 95,662.28 |
| 350 | 122,500 | 42,875,000 | 18.7083 | 7.0473 | .002857143 | 1,099.56 | 96,211.28 |
| 351 | 123,201 | 43,243,551 | 18.7350 | 7.0540 | .002849003 | 1,102.70 | 96,761.84 |
| 352 | 123,904 | 43,614,208 | 18.7617 | 7.0607 | .002840909 | 1,105.84 | 97,313.97 |
| 353 | 124,609 | 43,986,977 | 18.7883 | 7.0674 | .002832861 | 1,108.98 | 97,867.68 |
| 354 | 125,316 | 44,361,864 | 18.8149 | 7.0740 | .002824859 | 1,112.12 | 98,422.96 |
| 355 | 126,025 | 44,738,875 | 18.8414 | 7.0807 | .002816901 | 1,115.27 | 98,979.80 |
| 356 | 126,736 | 45,118,016 | 18.8680 | 7.0873 | .002808989 | 1,118.41 | 99,538.22 |
| 357 | 127,449 | 45,499,293 | 18.8944 | 7.0940 | .002801120 | 1,121.55 | 100,098.21 |
| 358 | 128,164 | 45,882,712 | 18.9209 | 7.1006 | .002793296 | 1,124.69 | 100,659.77 |
| 359 | 128,881 | 46,268,279 | 18.9473 | 7.1072 | .002785515 | 1,127.83 | 101,222.90 |
| 360 | 129,600 | 46,656,000 | 18.9737 | 7.1138 | .002777778 | 1,130.97 | 101,787.60 |
| 361 | 130,321 | 47,045,881 | 19.0000 | 7.1204 | .002770083 | 1,134.11 | 102,353.87 |
| 362 | 131,044 | 47,437,928 | 19.0263 | 7.1269 | .002762431 | 1,137.26 | 102,921.72 |
| 363 | 131,769 | 47,832,147 | 19.0526 | 7.1335 | .002754821 | 1,140.40 | 103,491.13 |
| 364 | 132,496 | 48,228,544 | 19.0788 | 7.1400 | .002747253 | 1,143.54 | 104,062.12 |
| 365 | 133,225 | 48,627,125 | 19.1050 | 7.1466 | .002739726 | 1,146.68 | 104,634.67 |
| 366 | 133,956 | 49,027,896 | 19.1311 | 7.1531 | .002732240 | 1,149.82 | 105,208.80 |
| 367 | 134,689 | 49,430,863 | 19.1572 | 7.1596 | .002724796 | 1,152.96 | 105,784.49 |
| 368 | 135,424 | 49,836,032 | 19.1833 | 7.1661 | .002717391 | 1,156.11 | 106,361.76 |
| 369 | 136,161 | 50,243,409 | 19.2094 | 7.1726 | .002710027 | 1,159.25 | 106,940.60 |
| 370 | 136,900 | 50,653,000 | 19.2354 | 7.1791 | .002702703 | 1,162.39 | 107,521.01 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|---|---|---|---|---|---|---|---|
| 371 | 137,641 | 51,064,811 | 19.2614 | 7.1855 | .002695418 | 1,165.53 | 108,102.99 |
| 372 | 138,384 | 51,478,848 | 19.2873 | 7.1920 | .002688172 | 1,168.67 | 108,686.54 |
| 373 | 139,129 | 51,895,117 | 19.3132 | 7.1984 | .002680965 | 1,171.81 | 109,271.66 |
| 374 | 139,876 | 52,313,624 | 19.3391 | 7.2048 | .002673797 | 1,174.96 | 109,858.35 |
| 375 | 140,625 | 52,734,375 | 19.3649 | 7.2112 | .002666667 | 1,178.10 | 110,446.62 |
| 376 | 141,376 | 53,157,376 | 19.3907 | 7.2177 | .002659574 | 1,181.24 | 111,036.45 |
| 377 | 142,129 | 53,582,633 | 19.4165 | 7.2240 | .002652520 | 1,184.38 | 111,627.86 |
| 378 | 142,884 | 54,010,152 | 19.4422 | 7.2304 | .002645503 | 1,187.52 | 112,220.83 |
| 379 | 143,641 | 54,439,939 | 19.4679 | 7.2368 | .002638521 | 1,190.66 | 112,815.38 |
| 380 | 144,400 | 54,872,000 | 19.4936 | 7.2432 | .002631579 | 1,193.81 | 113,411.49 |
| 381 | 145,161 | 55,306,341 | 19.5192 | 7.2495 | .002624672 | 1,196.95 | 114,009.18 |
| 382 | 145,924 | 55,742,968 | 19.5448 | 7.2558 | .002617801 | 1,200.09 | 114,608.44 |
| 383 | 146,689 | 56,181,887 | 19.5704 | 7.2622 | .002610966 | 1,203.23 | 115,209.27 |
| 384 | 147,456 | 56,623,104 | 19.5959 | 7.2685 | .002604167 | 1,206.37 | 115,811.67 |
| 385 | 148,225 | 57,066,625 | 19.6214 | 7.2748 | .002597403 | 1,209.51 | 116,415.64 |
| 386 | 148,996 | 57,512,456 | 19.6469 | 7.2811 | .002590674 | 1,212.65 | 117,021.18 |
| 387 | 149,769 | 57,960,603 | 19.6723 | 7.2874 | .002583979 | 1,215.80 | 117,628.30 |
| 388 | 150,544 | 58,411,072 | 19.6977 | 7.2936 | .002577320 | 1,218.94 | 118,236.98 |
| 389 | 151,321 | 58,863,869 | 19.7231 | 7.2999 | .002570694 | 1,222.08 | 118,847.24 |
| 390 | 152,100 | 59,319,000 | 19.7484 | 7.3061 | .002564103 | 1,225.22 | 119,459.06 |
| 391 | 152,881 | 59,776,471 | 19.7737 | 7.3124 | .002557545 | 1,228.36 | 120,072.46 |
| 392 | 153,664 | 60,236,288 | 19.7990 | 7.3186 | .002551020 | 1,231.50 | 120,687.42 |
| 393 | 154,449 | 60,698,457 | 19.8242 | 7.3248 | .002544529 | 1,234.65 | 121,303.96 |
| 394 | 155,236 | 61,162,984 | 19.8494 | 7.3310 | .002538071 | 1,237.79 | 121,922.07 |
| 395 | 156,025 | 61,629,875 | 19.8746 | 7.3372 | .002531646 | 1,240.93 | 122,541.75 |
| 396 | 156,816 | 62,099,136 | 19.8997 | 7.3434 | .002525253 | 1,244.07 | 123,163.00 |
| 397 | 157,609 | 62,570,773 | 19.9249 | 7.3496 | .002518892 | 1,247.21 | 123,785.82 |
| 398 | 158,404 | 63,044,792 | 19.9499 | 7.3558 | .002512563 | 1,250.35 | 124,410.21 |
| 399 | 159,201 | 63,521,199 | 19.9750 | 7.3619 | .002506266 | 1,253.50 | 125,036.17 |
| 400 | 160,000 | 64,000,000 | 20.0000 | 7.3681 | .002500000 | 1,256.64 | 125,663.71 |
| 401 | 160,801 | 64,481,201 | 20.0250 | 7.3742 | .002493766 | 1,259.78 | 126,292.81 |
| 402 | 161,604 | 64,964,808 | 20.0499 | 7.3803 | .002487562 | 1,262.92 | 126,923.48 |
| 403 | 162,409 | 65,450,827 | 20.0749 | 7.3864 | .002481390 | 1,266.06 | 127,555.73 |
| 404 | 163,216 | 65,939,264 | 20.0998 | 7.3925 | .002475248 | 1,269.20 | 128,189.55 |
| 405 | 164,025 | 66,430,125 | 20.1246 | 7.3986 | .002469136 | 1,272.35 | 128,824.93 |
| 406 | 164,836 | 66,923,416 | 20.1494 | 7.4047 | .002463054 | 1,275.49 | 129,461.89 |
| 407 | 165,649 | 67,419,143 | 20.1742 | 7.4108 | .002457002 | 1,278.68 | 130,100.42 |
| 408 | 166,464 | 67,917,312 | 20.1990 | 7.4169 | .002450980 | 1,281.77 | 130,740.52 |
| 409 | 167,281 | 68,417,929 | 20.2237 | 7.4229 | .002444988 | 1,284.91 | 131,382.19 |
| 410 | 168,100 | 68,921,000 | 20.2485 | 7.4290 | .002439024 | 1,288.05 | 132,025.43 |
| 411 | 168,921 | 69,426,531 | 20.2731 | 7.4350 | .002433090 | 1,291.19 | 132,670.24 |
| 412 | 169,744 | 69,934,528 | 20.2978 | 7.4410 | .002427184 | 1,294.34 | 133,316.63 |
| 413 | 170,569 | 70,444,997 | 20.3224 | 7.4470 | .002421308 | 1,297.48 | 133,964.58 |
| 414 | 171,396 | 70,957,944 | 20.3470 | 7.4530 | .002415459 | 1,300.62 | 134,614.10 |
| 415 | 172,225 | 71,473,375 | 20.3715 | 7.4590 | .002409639 | 1,303.76 | 135,265.20 |
| 416 | 173,056 | 71,991,296 | 20.3961 | 7.4650 | .002403846 | 1,306.90 | 135,917.86 |
| 417 | 173,889 | 72,511,713 | 20.4206 | 7.4710 | .002398082 | 1,310.04 | 136,572.10 |
| 418 | 174,724 | 73,034,632 | 20.4450 | 7.4770 | .002392344 | 1,313.19 | 137,227.91 |
| 419 | 175,561 | 73,560,059 | 20.4695 | 7.4829 | .002386635 | 1,316.33 | 137,885.29 |
| 420 | 176,400 | 74,088,000 | 20.4939 | 7.4889 | .002380952 | 1,319.47 | 138,544.24 |
| 421 | 177,241 | 74,618,461 | 20.5183 | 7.4948 | .002375297 | 1,322.61 | 139,204.76 |
| 422 | 178,084 | 75,151,448 | 20.5426 | 7.5007 | .002369668 | 1,325.75 | 139,866.85 |
| 423 | 178,929 | 75,686,967 | 20.5670 | 7.5067 | .002364066 | 1,328.89 | 140,530.51 |
| 424 | 179,776 | 76,225,024 | 20.5913 | 7.5126 | .002358491 | 1,332.04 | 141,195.74 |
| 425 | 180,625 | 76,765,625 | 20.6155 | 7.5185 | .002352941 | 1,335.18 | 141,862.54 |
| 426 | 181,476 | 77,308,776 | 20.6398 | 7.5244 | .002347418 | 1,338.32 | 142,530.92 |
| 427 | 182,329 | 77,854,483 | 20.6640 | 7.5302 | .002341920 | 1,341.46 | 143,200.86 |
| 428 | 183,184 | 78,402,752 | 20.6882 | 7.5361 | .002336449 | 1,344.60 | 143,872.38 |
| 429 | 184,041 | 78,953,589 | 20.7123 | 7.5420 | .002331002 | 1,347.74 | 144,545.46 |
| 430 | 184,900 | 79,507,000 | 20.7364 | 7.5478 | .002325581 | 1,350.88 | 145,220.12 |
| 431 | 185,761 | 80,062,991 | 20.7605 | 7.5537 | .002320186 | 1,354.03 | 145,896.35 |
| 432 | 186,624 | 80,621,568 | 20.7846 | 7.5595 | .002314815 | 1,357.17 | 146,574.15 |
| 433 | 187,489 | 81,182,737 | 20.8087 | 7.5654 | .002309469 | 1,360.31 | 147,253.52 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|---|---|---|---|---|---|---|---|
| 434 | 188,356 | 81,746,504 | 20.8327 | 7.5712 | .002304147 | 1,363.45 | 147,934.46 |
| 435 | 189,225 | 82,312,875 | 20.8567 | 7.5770 | .002298851 | 1,366.59 | 148,616.97 |
| 436 | 190,096 | 82,881,856 | 20.8806 | 7.5828 | .002293578 | 1,369.73 | 149,301.05 |
| 437 | 190,969 | 83,453,453 | 20.9045 | 7.5886 | .002288330 | 1,372.88 | 149,986.70 |
| 438 | 191,844 | 84,027,672 | 20.9284 | 7.5944 | .002283105 | 1,376.02 | 150,673.93 |
| 439 | 192,721 | 84,604,519 | 20.9523 | 7.6001 | .002277904 | 1,379.16 | 151,362.72 |
| 440 | 193,600 | 85,184,000 | 20.9762 | 7.6059 | .002272727 | 1,382.30 | 152,053.08 |
| 441 | 194,481 | 85,766,121 | 21.0000 | 7.6117 | .002267574 | 1,385.44 | 152,745.02 |
| 442 | 195,364 | 86,350,888 | 21.0238 | 7.6174 | .002262443 | 1,388.58 | 153,438.53 |
| 443 | 196,249 | 86,938,307 | 21.0476 | 7.6232 | .002257336 | 1,391.73 | 154,133.60 |
| 444 | 197,136 | 87,528,384 | 21.0713 | 7.6289 | .002252252 | 1,394.87 | 154,830.25 |
| 445 | 198,025 | 88,121,125 | 21.0950 | 7.6346 | .002247191 | 1,398.01 | 155,528.47 |
| 446 | 198,916 | 88,716,536 | 21.1187 | 7.6403 | .002242152 | 1,401.15 | 156,228.26 |
| 447 | 199,809 | 89,314,623 | 21.1424 | 7.6460 | .002237136 | 1,404.29 | 156,929.62 |
| 448 | 200,704 | 89,915,392 | 21.1660 | 7.6517 | .002232143 | 1,407.43 | 157,632.55 |
| 449 | 201,601 | 90,518,849 | 21.1896 | 7.6574 | .002227171 | 1,410.58 | 158,337.06 |
| 450 | 202,500 | 91,125,000 | 21.2132 | 7.6631 | .002222222 | 1,413.72 | 159,043.13 |
| 451 | 203,401 | 91,733,851 | 21.2368 | 7.6688 | .002217295 | 1,416.86 | 159,750.77 |
| 452 | 204,304 | 92,345,408 | 21.2603 | 7.6744 | .002212389 | 1,420.00 | 160,459.99 |
| 453 | 205,209 | 92,959,677 | 21.2838 | 7.6801 | .002207506 | 1,423.14 | 161,170.77 |
| 454 | 206,116 | 93,576,664 | 21.3073 | 7.6857 | .002202643 | 1,426.28 | 161,883.13 |
| 455 | 207,025 | 94,196,375 | 21.3307 | 7.6914 | .002197802 | 1,429.42 | 162,597.05 |
| 456 | 207,936 | 94,818,816 | 21.3542 | 7.6970 | .002192982 | 1,432.57 | 163,312.55 |
| 457 | 208,849 | 95,443,993 | 21.3776 | 7.7026 | .002188184 | 1,435.71 | 164,029.62 |
| 458 | 209,764 | 96,071,912 | 21.4009 | 7.7082 | .002183406 | 1,438.85 | 164,748.26 |
| 459 | 210,681 | 96,702,579 | 21.4243 | 7.7138 | .002178649 | 1,441.99 | 165,468.47 |
| 460 | 211,600 | 97,336,000 | 21.4476 | 7.7194 | .002173913 | 1,445.13 | 166,190.25 |
| 461 | 212,521 | 97,972,181 | 21.4709 | 7.7250 | .002169197 | 1,448.27 | 166,913.60 |
| 462 | 213,444 | 98,611,128 | 21.4942 | 7.7306 | .002164502 | 1,451.42 | 167,638.53 |
| 463 | 214,369 | 99,252,847 | 21.5174 | 7.7362 | .002159827 | 1,454.56 | 168,365.02 |
| 464 | 215,296 | 99,897,344 | 21.5407 | 7.7418 | .002155172 | 1,457.70 | 169,093.08 |
| 465 | 216,225 | 100,544,625 | 21.5639 | 7.7473 | .002150538 | 1,460.84 | 169,822.72 |
| 466 | 217,156 | 101,194,696 | 21.5870 | 7.7529 | .002145923 | 1,463.98 | 170,553.92 |
| 467 | 218,089 | 101,847,563 | 21.6102 | 7.7584 | .002141328 | 1,467.12 | 171,286.70 |
| 468 | 219,024 | 102,503,232 | 21.6333 | 7.7639 | .002136752 | 1,470.27 | 172,021.05 |
| 469 | 219,961 | 103,161,709 | 21.6564 | 7.7695 | .002132196 | 1,473.41 | 172,756.97 |
| 470 | 220,900 | 103,823,000 | 21.6795 | 7.7750 | .002127660 | 1,476.55 | 173,494.45 |
| 471 | 221,841 | 104,487,111 | 21.7025 | 7.7805 | .002123142 | 1,479.69 | 174,233.51 |
| 472 | 222,784 | 105,154,048 | 21.7256 | 7.7860 | .002118644 | 1,482.83 | 174,974.14 |
| 473 | 223,729 | 105,823,817 | 21.7486 | 7.7915 | .002114165 | 1,485.97 | 175,716.35 |
| 474 | 224,676 | 106,496,424 | 21.7715 | 7.7970 | .002109705 | 1,489.11 | 176,460.12 |
| 475 | 225,625 | 107,171,875 | 21.7945 | 7.8025 | .002105263 | 1,492.26 | 177,205.46 |
| 476 | 226,576 | 107,850,176 | 21.8174 | 7.8079 | .002100840 | 1,495.40 | 177,952.37 |
| 477 | 227,529 | 108,531,333 | 21.8403 | 7.8134 | .002096484 | 1,498.54 | 178,700.86 |
| 478 | 228,484 | 109,215,352 | 21.8632 | 7.8188 | .002092050 | 1,501.68 | 179,450.91 |
| 479 | 229,441 | 109,902,239 | 21.8861 | 7.8243 | .002087683 | 1,504.82 | 180,202.54 |
| 480 | 230,400 | 110,592,000 | 21.9089 | 7.8297 | .002083333 | 1,507.96 | 180,955.74 |
| 481 | 231,361 | 111,284,641 | 21.9317 | 7.8352 | .002079002 | 1,511.11 | 181,710.50 |
| 482 | 232,324 | 111,980,168 | 21.9545 | 7.8406 | .002074689 | 1,514.25 | 182,466.84 |
| 483 | 233,289 | 112,678,587 | 21.9775 | 7.8460 | .002070393 | 1,517.39 | 183,224.75 |
| 484 | 234,256 | 113,379,904 | 22.0000 | 7.8514 | .002066116 | 1,520.53 | 183,984.23 |
| 485 | 235,225 | 114,084,125 | 22.0227 | 7.8568 | .002061856 | 1,523.67 | 184,745.28 |
| 486 | 236,196 | 114,791,256 | 22.0454 | 7.8622 | .002057613 | 1,526.81 | 185,507.90 |
| 487 | 237,169 | 115,501,303 | 22.0681 | 7.8676 | .002053388 | 1,529.96 | 186,272.10 |
| 488 | 238,144 | 116,214,272 | 22.0907 | 7.8730 | .002049180 | 1,533.10 | 187,037.86 |
| 489 | 239,121 | 116,930,169 | 22.1133 | 7.8784 | .002044990 | 1,536.24 | 187,805.19 |
| 490 | 240,100 | 117,649,000 | 22.1359 | 7.8837 | .002040816 | 1,539.38 | 188,574.10 |
| 491 | 241,081 | 118,370,771 | 22.1585 | 7.8891 | .002036660 | 1,542.52 | 189,344.57 |
| 492 | 242,064 | 119,095,488 | 22.1811 | 7.8944 | .002032520 | 1,545.66 | 190,116.62 |
| 493 | 243,049 | 119,823,157 | 22.2036 | 7.8998 | .002028398 | 1,548.81 | 190,890.24 |
| 494 | 244,036 | 120,553,784 | 22.2261 | 7.9051 | .002024291 | 1,551.95 | 191,665.43 |
| 495 | 245,025 | 121,287,375 | 22.2486 | 7.9105 | .002020202 | 1,555.09 | 192,442.18 |
| 496 | 246,016 | 122,023,936 | 22.2711 | 7.9158 | .002016129 | 1,558.23 | 193,220.51 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|-----|--------|------|----------|----------|------------|---------|------|
| 497 | 247,009 | 122,763,473 | 22.2935 | 7.9211 | .002012072 | 1,561.37 | 194,000.41 |
| 498 | 248,004 | 123,505,992 | 22.3159 | 7.9264 | .002008032 | 1,564.51 | 194,781.89 |
| 499 | 249,001 | 124,251,499 | 22.3383 | 7.9317 | .002004008 | 1,567.65 | 195,564.93 |
| 500 | 250,000 | 125,000,000 | 22.3607 | 7.9370 | .002000000 | 1,570.80 | 196,349.54 |
| 501 | 251,001 | 125,751,501 | 22.3830 | 7.9423 | .001996008 | 1,573.94 | 197,135.72 |
| 502 | 252,004 | 126,506,008 | 22.4054 | 7.9476 | .001992032 | 1,577.08 | 197,923.48 |
| 503 | 253,009 | 127,263,527 | 22.4277 | 7.9528 | .001988072 | 1,580.22 | 198,712.80 |
| 504 | 254,016 | 128,024,064 | 22.4499 | 7.9581 | .001984127 | 1,583.36 | 199,503.70 |
| 505 | 255,025 | 128,787,625 | 22.4722 | 7.9634 | .001980198 | 1,586.50 | 200,296.17 |
| 506 | 256,036 | 129,554,216 | 22.4944 | 7.9686 | .001976285 | 1,589.65 | 201,090.20 |
| 507 | 257,049 | 130,323,843 | 22.5167 | 7.9739 | .001972387 | 1,592.79 | 201,885.81 |
| 508 | 258,064 | 131,096,512 | 22.5389 | 7.9791 | .001968504 | 1,595.93 | 202,682.99 |
| 509 | 259,081 | 131,872,229 | 22.5610 | 7.9843 | .001964637 | 1,599.07 | 203,481.74 |
| 510 | 260,100 | 132,651,000 | 22.5832 | 7.9895 | .001960785 | 1,602.21 | 204,282.06 |
| 511 | 261,121 | 133,432,831 | 22.6053 | 7.9948 | .001956947 | 1,605.35 | 205,083.95 |
| 512 | 262,144 | 134,217,728 | 22.6274 | 8.0000 | .001953125 | 1,608.50 | 205,887.42 |
| 513 | 263,169 | 135,005,697 | 22.6495 | 8.0052 | .001949318 | 1,611.64 | 206,692.45 |
| 514 | 264,196 | 135,796,744 | 22.6716 | 8.0104 | .001945525 | 1,614.78 | 207,499.05 |
| 515 | 265,225 | 136,590,875 | 22.6936 | 8.0156 | .001941748 | 1,617.92 | 208,307.23 |
| 516 | 266,256 | 137,388,096 | 22.7156 | 8.0208 | .001937984 | 1,621.06 | 209,116.97 |
| 517 | 267,289 | 138,188,413 | 22.7376 | 8.0260 | .001934236 | 1,624.20 | 209,928.29 |
| 518 | 268,324 | 138,991,832 | 22.7596 | 8.0311 | .001930502 | 1,627.34 | 210,741.18 |
| 519 | 269,361 | 139,798,359 | 22.7816 | 8.0363 | .001926782 | 1,630.49 | 211,555.63 |
| 520 | 270,400 | 140,608,000 | 22.8035 | 8.0415 | .001923077 | 1,633.63 | 212,371.66 |
| 521 | 271,411 | 141,420,761 | 22.8254 | 8.0466 | .001919386 | 1,636.77 | 213,189.26 |
| 522 | 272,484 | 142,236,648 | 22.8473 | 8.0517 | .001915709 | 1,639.91 | 214,008.43 |
| 523 | 273,529 | 143,055,667 | 22.8692 | 8.0569 | .001912046 | 1,643.05 | 214,829.17 |
| 524 | 274,576 | 143,877,824 | 22.8910 | 8.0620 | .001908397 | 1,646.19 | 215,651.49 |
| 525 | 275,625 | 144,703,125 | 22.9129 | 8.0671 | .001904762 | 1,649.34 | 216,475.37 |
| 526 | 276,676 | 145,531,576 | 22.9347 | 8.0723 | .001901141 | 1,652.48 | 217,300.82 |
| 527 | 277,729 | 146,363,183 | 22.9565 | 8.0774 | .001897533 | 1,655.62 | 218,127.85 |
| 528 | 278,784 | 147,197,952 | 22.9783 | 8.0825 | .001893939 | 1,658.76 | 218,956.44 |
| 529 | 279,841 | 148,035,889 | 23.0000 | 8.0876 | .001890359 | 1,661.90 | 219,786.61 |
| 530 | 280,900 | 148,877,001 | 23.0217 | 8.0927 | .001886792 | 1,665.04 | 220,618.34 |
| 531 | 281,961 | 149,721,291 | 23.0434 | 8.0978 | .001883239 | 1,668.19 | 221,451.65 |
| 532 | 283,024 | 150,568,768 | 23.0651 | 8.1028 | .001879699 | 1,671.33 | 222,286.53 |
| 533 | 284,089 | 151,419,437 | 23.0868 | 8.1079 | .001876173 | 1,674.47 | 223,122.98 |
| 534 | 285,156 | 152,273,304 | 23.1084 | 8.1130 | .001872659 | 1,677.61 | 223,961.00 |
| 535 | 286,225 | 153,130,375 | 23.1301 | 8.1180 | .001869159 | 1,680.75 | 224,800.59 |
| 536 | 287,296 | 153,990,656 | 23.1517 | 8.1231 | .001865672 | 1,683.89 | 225,641.75 |
| 537 | 288,369 | 154,854,153 | 23.1733 | 8.1281 | .001862197 | 1,687.04 | 226,484.48 |
| 538 | 289,444 | 155,720,872 | 23.1948 | 8.1332 | .001858736 | 1,690.18 | 227,328.79 |
| 539 | 290,521 | 156,590,819 | 23.2164 | 8.1382 | .001855288 | 1,693.32 | 228,174.66 |
| 540 | 291,600 | 157,464,000 | 23.2379 | 8.1433 | .001851852 | 1,696.46 | 229,022.10 |
| 541 | 292,681 | 158,340,421 | 23.2594 | 8.1483 | .001848429 | 1,699.60 | 229,871.12 |
| 542 | 293,764 | 159,220,088 | 23.2809 | 8.1533 | .001845018 | 1,702.74 | 230,721.71 |
| 543 | 294,849 | 160,103,007 | 23.3024 | 8.1583 | .001841621 | 1,705.88 | 231,573.86 |
| 544 | 295,936 | 160,989,184 | 23.3238 | 8.1633 | .001838235 | 1,709.03 | 232,427.59 |
| 545 | 297,025 | 161,878,625 | 23.3452 | 8.1683 | .001834862 | 1,712.17 | 233,282.89 |
| 546 | 298,116 | 162,771,336 | 23.3666 | 8.1733 | .001831502 | 1,715.31 | 234,139.76 |
| 547 | 299,209 | 163,667,323 | 23.3880 | 8.1783 | .001828154 | 1,718.45 | 234,998.20 |
| 548 | 300,304 | 164,566,592 | 23.4094 | 8.1833 | .001824818 | 1,721.59 | 235,858.21 |
| 549 | 301,401 | 165,469,149 | 23.4307 | 8.1882 | .001821494 | 1,724.73 | 236,719.79 |
| 550 | 302,500 | 166,375,000 | 23.4521 | 8.1932 | .001818182 | 1,727.88 | 237,582.94 |
| 551 | 303,601 | 167,284,151 | 23.4734 | 8.1982 | .001814882 | 1,731.02 | 238,447.67 |
| 552 | 304,704 | 168,196,608 | 23.4947 | 8.2031 | .001811594 | 1,734.16 | 239,313.96 |
| 553 | 305,809 | 169,112,377 | 23.5160 | 8.2081 | .001808318 | 1,737.30 | 240,181.83 |
| 554 | 306,916 | 170,031,464 | 23.5372 | 8.2130 | .001805054 | 1,740.44 | 241,051.26 |
| 555 | 308,025 | 170,953,875 | 23.5584 | 8.2180 | .001801802 | 1,743.58 | 241,922.27 |
| 556 | 309,136 | 171,879,616 | 23.5797 | 8.2229 | .001798561 | 1,746.73 | 242,794.85 |
| 557 | 310,249 | 172,808,693 | 23.6008 | 8.2278 | .001795332 | 1,749.87 | 243,668.99 |
| 558 | 311,364 | 173,741,112 | 23.6220 | 8.2327 | .001792115 | 1,753.01 | 244,544.71 |
| 559 | 312,481 | 174,676,879 | 23.6432 | 8.2377 | .001788909 | 1,756.15 | 245,422.00 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|---|---|---|---|---|---|---|---|
| 560 | 313,600 | 175,616,000 | 23.6643 | 8.2426 | .001785714 | 1,759.29 | 246,300.86 |
| 561 | 314,721 | 176,558,481 | 23.6854 | 8.2475 | .001782531 | 1,762.43 | 247,181.30 |
| 562 | 315,844 | 177,504,328 | 23.7065 | 8.2524 | .001779359 | 1,765.58 | 248,063.30 |
| 563 | 316,969 | 178,453,547 | 23.7276 | 8.2573 | .001776199 | 1,768.72 | 248,946.87 |
| 564 | 318,096 | 179,406,144 | 23.7487 | 8.2621 | .001773050 | 1,771.86 | 249,832.01 |
| 565 | 319,225 | 180,362,125 | 23.7697 | 8.2670 | .001769912 | 1,775.00 | 250,718.73 |
| 566 | 320,356 | 181,321,496 | 23.7908 | 8.2719 | .001766784 | 1,778.14 | 251,607.01 |
| 567 | 321,489 | 182,284,263 | 23.8118 | 8.2768 | .001763668 | 1,781.28 | 252,496.87 |
| 568 | 322,624 | 183,250,432 | 23.8328 | 8.2816 | .001760563 | 1,784.42 | 253,388.30 |
| 569 | 323,761 | 184,220,009 | 23.8537 | 8.2865 | .001757469 | 1,787.57 | 254,281.29 |
| 570 | 324,900 | 185,193,000 | 23.8747 | 8.2913 | .001754386 | 1,790.71 | 255,175.86 |
| 571 | 326,041 | 186,169,411 | 23.8956 | 8.2962 | .001751313 | 1,793.85 | 256,072.00 |
| 572 | 327,184 | 187,149,248 | 23.9165 | 8.3010 | .001748252 | 1,796.99 | 256,969.71 |
| 573 | 328,329 | 188,132,517 | 23.9374 | 8.3059 | .001745201 | 1,800.13 | 257,868.99 |
| 574 | 329,476 | 189,119,224 | 23.9583 | 8.3107 | .001742164 | 1,803.27 | 258,769.85 |
| 575 | 330,625 | 190,109,375 | 23.9792 | 8.3155 | .001739130 | 1,806.42 | 259,672.27 |
| 576 | 331,776 | 191,102,976 | 24.0000 | 8.3203 | .001736111 | 1,809.56 | 260,576.26 |
| 577 | 332,929 | 192,100,033 | 24.0208 | 8.3251 | .001733102 | 1,812.70 | 261,481.83 |
| 578 | 334,084 | 193,100,552 | 24.0416 | 8.3300 | .001730104 | 1,815.84 | 262,388.96 |
| 579 | 335,241 | 194,104,539 | 24.0624 | 8.3348 | .001727116 | 1,818.98 | 263,297.67 |
| 580 | 336,400 | 195,112,000 | 24.0832 | 8.3396 | .001724138 | 1,822.12 | 264,207.94 |
| 581 | 337,561 | 196,122,941 | 24.1039 | 8.3443 | .001721170 | 1,825.27 | 265,119.79 |
| 582 | 338,724 | 197,137,368 | 24.1247 | 8.3491 | .001718213 | 1,828.41 | 266,033.21 |
| 583 | 339,889 | 198,155,287 | 24.1454 | 8.3539 | .001715266 | 1,831.55 | 266,948.20 |
| 584 | 341,056 | 199,176,704 | 24.1661 | 8.3587 | .001712329 | 1,834.69 | 267,864.76 |
| 585 | 342,225 | 200,201,625 | 24.1868 | 8.3634 | .001709402 | 1,837.83 | 268,782.89 |
| 586 | 343,396 | 201,230,056 | 24.2074 | 8.3682 | .001706485 | 1,840.97 | 269,702.59 |
| 587 | 344,569 | 202,262,003 | 24.2281 | 8.3730 | .001703578 | 1,844.11 | 270,623.86 |
| 588 | 345,744 | 203,297,472 | 24.2487 | 8.3777 | .001700680 | 1,847.26 | 271,546.70 |
| 589 | 346,921 | 204,336,469 | 24.2693 | 8.3825 | .001697793 | 1,850.40 | 272,471.12 |
| 590 | 348,100 | 205,379,000 | 24.2899 | 8.3872 | .001694915 | 1,853.54 | 273,397.10 |
| 591 | 349,281 | 206,425,071 | 24.3105 | 8.3919 | .001692047 | 1,856.68 | 274,324.66 |
| 592 | 350,464 | 207,474,688 | 24.3311 | 8.3967 | .001689189 | 1,859.82 | 275,253.78 |
| 593 | 351,649 | 208,527,857 | 24.3516 | 8.4014 | .001686341 | 1,862.96 | 276,184.48 |
| 594 | 352,836 | 209,584,584 | 24.3721 | 8.4061 | .001683502 | 1,866.11 | 277,116.75 |
| 595 | 354,025 | 210,644,875 | 24.3926 | 8.4108 | .001680672 | 1,869.25 | 278,050.58 |
| 596 | 355,216 | 211,708,736 | 24.4131 | 8.4155 | .001677852 | 1,872.39 | 278,985.99 |
| 597 | 356,409 | 212,776,173 | 24.4336 | 8.4202 | .001675042 | 1,875.53 | 279,922.97 |
| 598 | 357,604 | 213,847,192 | 24.4540 | 8.4249 | .001672241 | 1,878.67 | 280,861.52 |
| 599 | 358,801 | 214,921,799 | 24.4745 | 8.4296 | .001669449 | 1,881.81 | 281,801.65 |
| 600 | 360,000 | 216,000,000 | 24.4949 | 8.4343 | .001666667 | 1,884.96 | 282,743.34 |
| 601 | 361,201 | 217,081,801 | 24.5153 | 8.4390 | .001663894 | 1,888.10 | 283,686.60 |
| 602 | 362,404 | 218,167,208 | 24.5357 | 8.4437 | .001661130 | 1,891.24 | 284,631.44 |
| 603 | 363,609 | 219,256,227 | 24.5561 | 8.4484 | .001658375 | 1,894.38 | 285,577.84 |
| 604 | 364,816 | 220,348,864 | 24.5764 | 8.4530 | .001655629 | 1,897.52 | 286,525.82 |
| 605 | 366,025 | 221,445,125 | 24.5968 | 8.4577 | .001652893 | 1,900.66 | 287,475.36 |
| 606 | 367,236 | 222,545,016 | 24.6171 | 8.4623 | .001650165 | 1,903.81 | 288,426.48 |
| 607 | 368,449 | 223,648,543 | 24.6374 | 8.4670 | .001647446 | 1,906.95 | 289,379.17 |
| 608 | 369,664 | 224,755,712 | 24.6577 | 8.4716 | .001644737 | 1,910.09 | 290,333.43 |
| 609 | 370,881 | 225,866,529 | 24.6779 | 8.4763 | .001642036 | 1,913.23 | 291,289.26 |
| 610 | 372,100 | 226,981,000 | 24.6982 | 8.4809 | .001639344 | 1,916.37 | 292,246.66 |
| 611 | 373,321 | 228,099,131 | 24.7184 | 8.4856 | .001636661 | 1,919.51 | 293,205.63 |
| 612 | 374,544 | 229,220,928 | 24.7386 | 8.4902 | .001633987 | 1,922.65 | 294,166.17 |
| 613 | 375,769 | 230,346,397 | 24.7588 | 8.4948 | .001631321 | 1,925.80 | 295,128.28 |
| 614 | 376,996 | 231,475,544 | 24.7790 | 8.4994 | .001628664 | 1,928.94 | 296,091.97 |
| 615 | 378,225 | 232,608,375 | 24.7992 | 8.5040 | .001626016 | 1,932.08 | 297,057.22 |
| 616 | 379,456 | 233,744,896 | 24.8193 | 8.5086 | .001623377 | 1,935.22 | 298,024.05 |
| 617 | 380,689 | 234,885,113 | 24.8395 | 8.5132 | .001620746 | 1,938.36 | 298,992.44 |
| 618 | 381,924 | 236,029,032 | 24.8596 | 8.5178 | .001618123 | 1,941.50 | 299,962.41 |
| 619 | 383,161 | 237,176,659 | 24.8797 | 8.5224 | .001615509 | 1,944.65 | 300,933.95 |
| 620 | 384,400 | 238,328,000 | 24.8998 | 8.5270 | .001612903 | 1,947.79 | 301,907.05 |
| 621 | 385,641 | 239,483,061 | 24.9199 | 8.5316 | .001610306 | 1,950.93 | 302,881.73 |
| 622 | 386,884 | 240,641,848 | 24.9399 | 8.5362 | .001607717 | 1,954.07 | 303,857.98 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|---|---|---|---|---|---|---|---|
| 623 | 388,129 | 241,804,367 | 24.9600 | 8.5408 | .001605136 | 1,957.21 | 304,835.80 |
| 624 | 389,376 | 242,970,624 | 24.9800 | 8.5453 | .001602564 | 1,960.35 | 305,815.20 |
| 625 | 390,625 | 244,140,625 | 25.0000 | 8.5499 | .001600000 | 1,963.50 | 306,796.16 |
| 626 | 391,876 | 245,314,376 | 25.0200 | 8.5544 | .001597444 | 1,966.64 | 307,778.69 |
| 627 | 393,129 | 246,491,883 | 25.0400 | 8.5589 | .001594896 | 1,969.78 | 308,762.79 |
| 628 | 394,384 | 247,673,152 | 25.0599 | 8.5635 | .001592357 | 1,972.92 | 309,748.47 |
| 629 | 395,641 | 248,858,189 | 25.0799 | 8.5681 | .001589825 | 1,976.05 | 310,735.71 |
| 630 | 396,900 | 250,047,000 | 25.0998 | 8.5726 | .001587302 | 1,979.20 | 311,724.53 |
| 631 | 398,161 | 251,239,591 | 25.1197 | 8.5772 | .001584786 | 1,982.35 | 312,714.92 |
| 632 | 399,424 | 252,435,968 | 25.1396 | 8.5817 | .001582278 | 1,985.49 | 313,706.88 |
| 633 | 400,689 | 253,636,137 | 25.1595 | 8.5862 | .001579779 | 1,988.63 | 314,700.40 |
| 634 | 401,956 | 254,840,104 | 25.1794 | 8.5907 | .001577287 | 1,991.77 | 315,695.50 |
| 635 | 403,225 | 256,047,875 | 25.1992 | 8.5952 | .001574803 | 1,994.91 | 316,692.17 |
| 636 | 404,496 | 257,259,456 | 25.2190 | 8.5997 | .001572327 | 1,998.05 | 317,690.42 |
| 637 | 405,769 | 258,474,853 | 25.2389 | 8.6043 | .001569859 | 2,001.19 | 318,690.23 |
| 638 | 407,044 | 259,694,072 | 25.2587 | 8.6088 | .001567398 | 2,004.34 | 319,691.61 |
| 639 | 408,321 | 260,917,119 | 25.2784 | 8.6132 | .001564945 | 2,007.48 | 320,694.56 |
| 640 | 409,600 | 262,144,000 | 25.2982 | 8.6177 | .001562500 | 2,010.62 | 321,699.09 |
| 641 | 410,881 | 263,374,721 | 25.3180 | 8.6222 | .001560062 | 2,013.76 | 322,705.18 |
| 642 | 412,164 | 264,609,288 | 25.3377 | 8.6267 | .001557632 | 2,016.90 | 323,712.85 |
| 643 | 413,449 | 265,847,707 | 25.3574 | 8.6312 | .001555210 | 2,020.04 | 324,722.09 |
| 644 | 414,736 | 267,089,984 | 25.3772 | 8.6357 | .001552795 | 2,023.19 | 325,732.89 |
| 645 | 416,125 | 268,336,125 | 25.3969 | 8.6401 | .001550388 | 2,026.33 | 326,745.27 |
| 646 | 417,316 | 269,585,136 | 25.4165 | 8.6446 | .001547988 | 2,029.47 | 327,759.22 |
| 647 | 418,609 | 270,840,023 | 25.4362 | 8.6490 | .001545595 | 2,032.61 | 328,774.74 |
| 648 | 419,904 | 272,097,792 | 25.4558 | 8.6535 | .001543210 | 2,035.75 | 329,791.83 |
| 649 | 421,201 | 273,359,449 | 25.4755 | 8.6579 | .001540832 | 2,038.89 | 330,810.49 |
| 650 | 422,500 | 274,625,000 | 25.4951 | 8.6624 | .001538462 | 2,042.04 | 331,830.72 |
| 651 | 423,801 | 275,894,451 | 25.5147 | 8.6668 | .001536098 | 2,045.18 | 332,852.53 |
| 652 | 425,104 | 277,167,808 | 25.5343 | 8.6713 | .001533742 | 2,048.32 | 333,875.90 |
| 653 | 426,409 | 278,445,077 | 25.5539 | 8.6757 | .001531394 | 2,051.46 | 334,900.85 |
| 654 | 427,716 | 279,726,264 | 25.5734 | 8.6801 | .001529052 | 2,054.60 | 335,927.36 |
| 655 | 429,025 | 281,011,375 | 25.5930 | 8.6845 | .001526718 | 2,057.74 | 336,955.45 |
| 656 | 430,336 | 282,300,416 | 25.6125 | 8.6890 | .001524390 | 2,060.88 | 337,985.10 |
| 657 | 431,639 | 283,593,393 | 25.6320 | 8.6934 | .001522070 | 2,064.03 | 339,016.33 |
| 658 | 432,964 | 284,890,312 | 25.6515 | 8.6978 | .001519751 | 2,067.17 | 340,049.13 |
| 659 | 434,281 | 286,191,179 | 25.6710 | 8.7022 | .001517451 | 2,070.31 | 341,083.50 |
| 660 | 435,600 | 287,496,000 | 25.6905 | 8.7066 | .001515152 | 2,073.45 | 342,119.44 |
| 661 | 436,921 | 288,804,781 | 25.7099 | 8.7110 | .001512859 | 2,076.59 | 343,156.95 |
| 662 | 438,244 | 290,117,528 | 25.7294 | 8.7154 | .001510574 | 2,079.73 | 344,196.03 |
| 663 | 439,569 | 291,434,247 | 25.7488 | 8.7198 | .001508296 | 2,082.88 | 345,236.69 |
| 664 | 440,896 | 292,754,944 | 25.7682 | 8.7241 | .001506024 | 2,086.02 | 346,278.91 |
| 665 | 442,225 | 294,079,625 | 25.7876 | 8.7285 | .001503759 | 2,089.16 | 347,322.70 |
| 666 | 443,556 | 295,408,296 | 25.8070 | 8.7329 | .001501502 | 2,092.30 | 348,368.07 |
| 667 | 444,899 | 296,740,963 | 25.8263 | 8.7373 | .001499250 | 2,095.44 | 349,415.00 |
| 668 | 446,224 | 298,077,632 | 25.8457 | 8.7416 | .001497006 | 2,098.58 | 350,463.51 |
| 669 | 447,561 | 299,418,309 | 25.8650 | 8.7460 | .001494768 | 2,101.73 | 351,513.59 |
| 670 | 448,900 | 300,763,000 | 25.8844 | 8.7503 | .001492537 | 2,104.87 | 352,565.24 |
| 671 | 450,241 | 302,111,711 | 25.9037 | 8.7547 | .001490313 | 2,108.01 | 353,618.45 |
| 672 | 451,584 | 303,464,448 | 25.9230 | 8.7590 | .001488095 | 2,111.15 | 354,673.24 |
| 673 | 452,929 | 304,821,217 | 25.9422 | 8.7634 | .001485884 | 2,114.29 | 355,729.60 |
| 674 | 454,276 | 306,182,024 | 25.9615 | 8.7677 | .001483680 | 2,117.43 | 356,787.54 |
| 675 | 455,625 | 307,546,875 | 25.9808 | 8.7721 | .001481481 | 2,120.58 | 357,847.04 |
| 676 | 456,976 | 308,915,776 | 26.0000 | 8.7764 | .001479290 | 2,123.72 | 358,908.11 |
| 677 | 458,329 | 310,288,733 | 26.0192 | 8.7807 | .001477105 | 2,126.86 | 359,970.75 |
| 678 | 459,684 | 311,665,752 | 26.0384 | 8.7850 | .001474926 | 2,130.00 | 361,034.97 |
| 679 | 461,041 | 313,046,839 | 26.0576 | 8.7893 | .001472754 | 2,133.14 | 362,100.75 |
| 680 | 462,400 | 314,432,000 | 26.0768 | 8.7937 | .001470588 | 2,136.28 | 363,168.11 |
| 681 | 463,761 | 315,821,241 | 26.0960 | 8.7980 | .001468429 | 2,139.42 | 364,237.04 |
| 682 | 465,124 | 317,214,568 | 26.1151 | 8.8023 | .001466276 | 2,142.57 | 365,307.54 |
| 683 | 466,489 | 318,611,987 | 26.1343 | 8.8066 | .001464129 | 2,145.71 | 366,379.60 |
| 684 | 467,856 | 320,013,504 | 26.1534 | 8.8109 | .001461988 | 2,148.85 | 367,453.24 |
| 685 | 469,225 | 321,419,125 | 26.1725 | 8.8152 | .001459854 | 2,151.99 | 368,528.45 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|-----|--------|------|----------|----------|------------|---------|------|
| 686 | 470,596 | 322,828,856 | 26.1916 | 8.8194 | .001457726 | 2,155.13 | 369,605.23 |
| 687 | 471,969 | 324,242,703 | 26.2107 | 8.8237 | .001455604 | 2,158.27 | 370,683.59 |
| 688 | 473,344 | 325,660,672 | 26.2298 | 8.8280 | .001453488 | 2,161.42 | 371,763.51 |
| 689 | 474,721 | 327,082,769 | 26.2488 | 8.8323 | .001451379 | 2,164.56 | 372,845.00 |
| 690 | 476,100 | 328,509,000 | 26.2679 | 8.8366 | .001449275 | 2,167.70 | 373,928.07 |
| 691 | 477,481 | 329,939,371 | 26.2869 | 8.8408 | .001447178 | 2,170.84 | 375,012.70 |
| 692 | 478,864 | 331,373,888 | 26.3059 | 8.8451 | .001445087 | 2,173.98 | 376,098.91 |
| 693 | 480,249 | 332,812,557 | 26.3249 | 8.8493 | .001443001 | 2,177.12 | 377,186.68 |
| 694 | 481,636 | 334,255,384 | 26.3439 | 8.8536 | .001440922 | 2,180.27 | 378,276.03 |
| 695 | 483,025 | 335,702,375 | 26.3629 | 8.8578 | .001438849 | 2,183.41 | 379,366.95 |
| 696 | 484,416 | 337,153,536 | 26.3818 | 8.8621 | .001436782 | 2,186.55 | 380,459.44 |
| 697 | 485,809 | 338,608,873 | 26.4008 | 8.8663 | .001434720 | 2,189.69 | 381,553.50 |
| 698 | 487,204 | 340,068,392 | 26.4197 | 8.8706 | .001432665 | 2,192.83 | 382,649.13 |
| 699 | 488,601 | 341,532,099 | 26.4386 | 8.8748 | .001430615 | 2,195.97 | 383,746.33 |
| 700 | 490,000 | 343,000,000 | 26.4575 | 8.8790 | .001428571 | 2,199.11 | 384,845.10 |
| 701 | 491,401 | 344,472,101 | 26.4764 | 8.8833 | .001426534 | 2,202.26 | 385,945.44 |
| 702 | 492,804 | 345,948,408 | 26.4953 | 8.8875 | .001424501 | 2,205.40 | 387,047.36 |
| 703 | 494,209 | 347,428,927 | 26.5141 | 8.8917 | .001422475 | 2,208.54 | 388,150.84 |
| 704 | 495,616 | 348,913,664 | 26.5330 | 8.8959 | .001420454 | 2,211.68 | 389,255.90 |
| 705 | 497,025 | 350,402,625 | 26.5518 | 8.9001 | .001418440 | 2,214.82 | 390,362.52 |
| 706 | 498,436 | 351,895,816 | 26.5707 | 8.9043 | .001416431 | 2,217.96 | 391,470.72 |
| 707 | 499,849 | 353,393,243 | 26.5895 | 8.9085 | .001414427 | 2,221.11 | 392,580.49 |
| 708 | 501,264 | 354,894,912 | 26.6083 | 8.9127 | .001412429 | 2,224.25 | 393,691.82 |
| 709 | 502,681 | 356,400,829 | 26.6271 | 8.9169 | .001410437 | 2,227.39 | 394,804.73 |
| 710 | 504,100 | 357,911,000 | 26.6458 | 8.9211 | .001408451 | 2,230.53 | 395,919.21 |
| 711 | 505,521 | 359,425,431 | 26.6646 | 8.9253 | .001406470 | 2,233.67 | 397,035.26 |
| 712 | 506,944 | 360,944,128 | 26.6833 | 8.9295 | .001404494 | 2,236.81 | 398,152.89 |
| 713 | 508,369 | 362,467,097 | 26.7021 | 8.9337 | .001402525 | 2,239.96 | 399,272.08 |
| 714 | 509,796 | 363,994,344 | 26.7208 | 8.9378 | .001400560 | 2,243.10 | 400,392.84 |
| 715 | 511,225 | 365,525,875 | 26.7395 | 8.9420 | .001398601 | 2,246.24 | 401,515.18 |
| 716 | 512,656 | 367,061,696 | 26.7582 | 8.9462 | .001396648 | 2,249.38 | 402,639.08 |
| 717 | 514,089 | 368,601,813 | 26.7769 | 8.9503 | .001394700 | 2,252.52 | 403,764.56 |
| 718 | 515,524 | 370,146,232 | 26.7955 | 8.9545 | .001392758 | 2,255.66 | 404,891.60 |
| 719 | 516,961 | 371,694,959 | 26.8142 | 8.9587 | .001390821 | 2,258.81 | 406,020.22 |
| 720 | 518,400 | 373,248,000 | 26.8328 | 8.9628 | .001388889 | 2,261.95 | 407,150.41 |
| 721 | 519,841 | 374,805,361 | 26.8514 | 8.9670 | .001386963 | 2,265.09 | 408,282.17 |
| 722 | 521,284 | 376,367,048 | 26.8701 | 8.9711 | .001385042 | 2,268.23 | 409,415.50 |
| 723 | 522,729 | 377,933,067 | 26.8887 | 8.9752 | .001383126 | 2,271.37 | 410,550.40 |
| 724 | 524,176 | 379,503,424 | 26.9072 | 8.9794 | .001381215 | 2,274.51 | 411,686.87 |
| 725 | 525,625 | 381,078,125 | 26.9258 | 8.9835 | .001379310 | 2,277.65 | 412,824.91 |
| 726 | 527,076 | 382,657,176 | 26.9444 | 8.9876 | .001377410 | 2,280.80 | 413,964.52 |
| 727 | 528,529 | 384,240,583 | 26.9629 | 8.9918 | .001375516 | 2,283.94 | 415,105.71 |
| 728 | 529,984 | 385,828,352 | 26.9815 | 8.9959 | .001373626 | 2,287.08 | 416,248.46 |
| 729 | 531,441 | 387,420,489 | 27.0000 | 9.0000 | .001371742 | 2,290.22 | 417,392.79 |
| 730 | 532,900 | 389,017,000 | 27.0185 | 9.0041 | .001369863 | 2,293.36 | 418,538.68 |
| 731 | 534,361 | 390,617,891 | 27.0370 | 9.0082 | .001367989 | 2,296.50 | 419,686.15 |
| 732 | 535,824 | 392,223,168 | 27.0555 | 9.0123 | .001366120 | 2,299.65 | 420,835.19 |
| 733 | 537,289 | 393,832,837 | 27.0740 | 9.0164 | .001364256 | 2,302.79 | 421,985.79 |
| 734 | 538,756 | 395,446,904 | 27.0924 | 9.0205 | .001362398 | 2,305.93 | 423,137.97 |
| 735 | 540,225 | 397,065,375 | 27.1109 | 9.0246 | .001360544 | 2,309.07 | 424,291.72 |
| 736 | 541,696 | 398,688,256 | 27.1293 | 9.0287 | .001358696 | 2,312.21 | 425,447.04 |
| 737 | 543,169 | 400,315,553 | 27.1477 | 9.0328 | .001356852 | 2,315.35 | 426,603.94 |
| 738 | 544,644 | 401,947,272 | 27.1662 | 9.0369 | .001355014 | 2,318.50 | 427,762.40 |
| 739 | 546,121 | 403,583,419 | 27.1846 | 9.0410 | .001353180 | 2,321.64 | 428,922.43 |
| 740 | 547,600 | 405,224,000 | 27.2029 | 9.0450 | .001351351 | 2,324.78 | 430,084.03 |
| 741 | 549,081 | 406,869,021 | 27.2213 | 9.0491 | .001349528 | 2,327.92 | 431,247.21 |
| 742 | 550,564 | 408,518,488 | 27.2397 | 9.0532 | .001347709 | 2,331.06 | 432,411.95 |
| 743 | 552,049 | 410,172,407 | 27.2580 | 9.0572 | .001345895 | 2,334.20 | 433,578.27 |
| 744 | 553,536 | 411,830,784 | 27.2764 | 9.0613 | .001344086 | 2,337.34 | 434,746.16 |
| 745 | 555,025 | 413,493,625 | 27.2947 | 9.0654 | .001342282 | 2,340.49 | 435,915.62 |
| 746 | 556,516 | 415,160,936 | 27.3130 | 9.0694 | .001340483 | 2,343.63 | 437,086.64 |
| 747 | 558,009 | 416,832,723 | 27.3313 | 9.0735 | .001338688 | 2,346.77 | 438,259.24 |
| 748 | 559,504 | 418,508,992 | 27.3496 | 9.0775 | .001336898 | 2,349.91 | 439,433.41 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|-----|--------|------|----------|----------|------------|---------|------|
| 749 | 561,001 | 420,189,749 | 27.3679 | 9.0816 | .001335113 | 2,353.05 | 440,609.16 |
| 750 | 562,500 | 421,875,000 | 27.3861 | 9.0856 | .001333333 | 2,356.19 | 441,786.47 |
| 751 | 564,001 | 423,564,751 | 27.4044 | 9.0896 | .001331558 | 2,359.34 | 442,965.35 |
| 752 | 565,504 | 425,259,008 | 27.4226 | 9.0937 | .001329787 | 2,362.48 | 444,145.80 |
| 753 | 567,009 | 426,957,777 | 27.4408 | 9.0977 | .001328021 | 2,365.62 | 445,327.83 |
| 754 | 568,516 | 428,661,064 | 27.4591 | 9.1017 | .001326260 | 2,368.76 | 446,511.42 |
| 755 | 570,025 | 430,368,875 | 27.4773 | 9.1057 | .001324503 | 2,371.90 | 447,696.59 |
| 756 | 571,536 | 432,081,216 | 27.4955 | 9.1098 | .001322751 | 2,375.04 | 448,883.32 |
| 757 | 573,049 | 433,798,093 | 27.5136 | 9.1138 | .001321004 | 2,378.19 | 450,071.63 |
| 758 | 574,564 | 435,519,512 | 27.5318 | 9.1178 | .001319261 | 2,381.33 | 451,261.51 |
| 759 | 576,081 | 437,245,479 | 27.5500 | 9.1218 | .001317523 | 2,384.47 | 452,452.96 |
| 760 | 577,600 | 438,976,000 | 27.5681 | 9.1258 | .001315789 | 2,387.61 | 453,645.98 |
| 761 | 579,121 | 440,711,081 | 27.5862 | 9.1298 | .001314060 | 2,390.75 | 454,840.57 |
| 762 | 580,644 | 442,450,728 | 27.6043 | 9.1338 | .001312336 | 2,393.89 | 456,036.73 |
| 763 | 582,169 | 444,194,947 | 27.6225 | 9.1378 | .001310616 | 2,397.04 | 457,234.46 |
| 764 | 583,696 | 445,943,744 | 27.6405 | 9.1418 | .001308901 | 2,400.18 | 458,433.77 |
| 765 | 585,225 | 447,697,125 | 27.6586 | 9.1458 | .001307190 | 2,403.32 | 459,634.64 |
| 766 | 586,756 | 449,455,096 | 27.6767 | 9.1498 | .001305483 | 2,406.46 | 460,837.08 |
| 767 | 588,289 | 451,217,663 | 27.6948 | 9.1537 | .001303781 | 2,409.60 | 462,041.10 |
| 768 | 589,824 | 452,984,832 | 27.7128 | 9.1577 | .001302083 | 2,412.74 | 463,246.69 |
| 769 | 591,361 | 454,756,609 | 27.7308 | 9.1617 | .001300390 | 2,415.88 | 464,453.84 |
| 770 | 592,900 | 456,533,000 | 27.7489 | 9.1657 | .001298701 | 2,419.03 | 465,662.57 |
| 771 | 594,441 | 458,314,011 | 27.7669 | 9.1696 | .001297017 | 2,422.17 | 466,872.87 |
| 772 | 595,984 | 460,099,648 | 27.7849 | 9.1736 | .001295337 | 2,425.31 | 468,084.74 |
| 773 | 597,529 | 461,889,917 | 27.8029 | 9.1775 | .001293661 | 2,428.45 | 469,298.18 |
| 774 | 599,076 | 463,684,824 | 27.8209 | 9.1815 | .001291990 | 2,431.59 | 470,513.19 |
| 775 | 600,625 | 465,484,375 | 27.8388 | 9.1855 | .001290323 | 2,434.73 | 471,729.77 |
| 776 | 602,176 | 467,288,576 | 27.8568 | 9.1894 | .001288660 | 2,437.88 | 472,947.92 |
| 777 | 603,729 | 469,097,433 | 27.8747 | 9.1933 | .001287001 | 2,441.02 | 474,167.65 |
| 778 | 605,284 | 470,910,952 | 27.8927 | 9.1973 | .001285347 | 2,444.16 | 475,388.94 |
| 779 | 606,841 | 472,729,139 | 27.9106 | 9.2012 | .001283697 | 2,447.30 | 476,611.81 |
| 780 | 608,400 | 474,552,000 | 27.9285 | 9.2052 | .001282051 | 2,450.44 | 477,836.24 |
| 781 | 609,961 | 476,379,541 | 27.9464 | 9.2091 | .001280410 | 2,453.58 | 479,062.25 |
| 782 | 611,524 | 478,211,768 | 27.9643 | 9.2130 | .001278772 | 2,456.73 | 480,289.83 |
| 783 | 613,089 | 480,048,687 | 27.9821 | 9.2170 | .001277139 | 2,459.87 | 481,518.97 |
| 784 | 614,656 | 481,890,304 | 28.0000 | 9.2209 | .001275510 | 2,463.01 | 482,749.69 |
| 785 | 616,225 | 483,736,625 | 28.0179 | 9.2248 | .001273885 | 2,466.15 | 483,981.98 |
| 786 | 617,796 | 485,587,656 | 28.0357 | 9.2287 | .001272265 | 2,469.29 | 485,215.84 |
| 787 | 619,369 | 487,443,403 | 28.0535 | 9.2326 | .001270648 | 2,472.43 | 486,451.28 |
| 788 | 620,944 | 489,303,872 | 28.0713 | 9.2365 | .001269036 | 2,475.58 | 487,688.28 |
| 789 | 622,521 | 491,169,069 | 28.0891 | 9.2404 | .001267427 | 2,478.72 | 488,926.85 |
| 790 | 624,100 | 493,039,000 | 28.1069 | 9.2443 | .001265823 | 2,481.86 | 490,166.99 |
| 791 | 625,681 | 494,913,671 | 28.1247 | 9.2482 | .001264223 | 2,485.00 | 491,408.71 |
| 792 | 627,264 | 496,793,088 | 28.1425 | 9.2521 | .001262626 | 2,488.14 | 492,651.99 |
| 793 | 628,849 | 498,677,257 | 28.1603 | 9.2560 | .001261034 | 2,491.28 | 493,896.85 |
| 794 | 630,436 | 500,566,184 | 28.1780 | 9.2599 | .001259446 | 2,494.42 | 495,143.28 |
| 795 | 632,025 | 502,459,875 | 28.1957 | 9.2638 | .001257862 | 2,497.57 | 496,391.27 |
| 796 | 633,616 | 504,358,336 | 28.2135 | 9.2677 | .001256281 | 2,500.71 | 497,640.84 |
| 797 | 635,209 | 506,261,573 | 28.2312 | 9.2716 | .001254705 | 2,503.85 | 498,891.98 |
| 798 | 636,804 | 508,169,592 | 28.2489 | 9.2754 | .001253133 | 2,506.99 | 500,144.69 |
| 799 | 638,401 | 510,082,399 | 28.2666 | 9.2793 | .001251564 | 2,510.13 | 501,398.97 |
| 800 | 640,000 | 512,000,000 | 28.2843 | 9.2832 | .001250000 | 2,513.27 | 502,654.82 |
| 801 | 641,601 | 513,922,401 | 28.3019 | 9.2870 | .001248439 | 2,516.42 | 503,912.25 |
| 802 | 643,204 | 515,849,608 | 28.3196 | 9.2909 | .001246883 | 2,519.56 | 505,171.24 |
| 803 | 644,809 | 517,781,627 | 28.3373 | 9.2948 | .001245330 | 2,522.70 | 506,431.80 |
| 804 | 646,416 | 519,718,464 | 28.3549 | 9.2986 | .001243781 | 2,525.84 | 507,693.94 |
| 805 | 648,025 | 521,660,125 | 28.3725 | 9.3025 | .001242236 | 2,528.98 | 508,957.64 |
| 806 | 649,636 | 523,606,616 | 28.3901 | 9.3063 | .001240695 | 2,532.12 | 510,222.92 |
| 807 | 651,249 | 525,557,943 | 28.4077 | 9.3102 | .001239157 | 2,535.27 | 511,489.77 |
| 808 | 652,864 | 527,514,112 | 28.4253 | 9.3140 | .001237624 | 2,538.41 | 512,758.19 |
| 809 | 654,481 | 529,475,129 | 28.4429 | 9.3179 | .001236094 | 2,541.55 | 514,028.18 |
| 810 | 656,100 | 531,441,000 | 28.4605 | 9.3217 | .001234568 | 2,544.69 | 515,299.74 |
| 811 | 657,721 | 533,411,731 | 28.4781 | 9.3255 | .001233046 | 2,547.83 | 516,572.87 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|---|---|---|---|---|---|---|---|
| 812 | 659,344 | 535,387,328 | 28.4956 | 9.3294 | .001231527 | 2,550.97 | 517,847.57 |
| 813 | 660,969 | 537,367,797 | 28.5132 | 9.3332 | .001230012 | 2,554.11 | 519,123.84 |
| 814 | 662,596 | 539,353,144 | 28.5307 | 9.3370 | .001228501 | 2,557.26 | 520,401.68 |
| 815 | 664,225 | 541,343,375 | 28.5482 | 9.3408 | .001226994 | 2,560.40 | 521,681.10 |
| 816 | 665,856 | 543,338,496 | 28.5657 | 9.3447 | .001225490 | 2,563.54 | 522,962.08 |
| 817 | 667,489 | 545,338,513 | 28.5832 | 9.3485 | .001223990 | 2,566.68 | 524,244.63 |
| 818 | 669,124 | 547,343,432 | 28.6007 | 9.3523 | .001222494 | 2,569.82 | 525,528.76 |
| 819 | 670,761 | 549,353,259 | 28.6182 | 9.3561 | .001221001 | 2,572.96 | 526,814.46 |
| 820 | 672,400 | 551,368,000 | 28.6356 | 9.3599 | .001219512 | 2,576.11 | 528,101.73 |
| 821 | 674,041 | 553,387,661 | 28.6531 | 9.3637 | .001218027 | 2,579.25 | 529,390.56 |
| 822 | 675,584 | 555,412,248 | 28.6705 | 9.3675 | .001216545 | 2,582.39 | 530,680.97 |
| 823 | 677,329 | 557,441,767 | 28.6880 | 9.3713 | .001215067 | 2,585.53 | 531,972.95 |
| 824 | 678,976 | 559,476,224 | 28.7054 | 9.3751 | .001213592 | 2,588.67 | 533,266.50 |
| 825 | 680,625 | 561,515,625 | 28.7228 | 9.3789 | .001212121 | 2,591.81 | 534,561.62 |
| 826 | 682,276 | 563,559,976 | 28.7402 | 9.3827 | .001210654 | 2,594.96 | 535,858.32 |
| 827 | 683,929 | 565,609,283 | 28.7576 | 9.3865 | .001209190 | 2,598.10 | 537,156.58 |
| 828 | 685,584 | 567,663,552 | 28.7750 | 9.3902 | .001207729 | 2,601.24 | 538,456.41 |
| 829 | 687,241 | 569,722,789 | 28.7924 | 9.3940 | .001206273 | 2,604.38 | 539,757.82 |
| 830 | 688,900 | 571,787,000 | 28.8097 | 9.3978 | .001204819 | 2,607.52 | 541,060.79 |
| 831 | 690,561 | 573,856,191 | 28.8271 | 9.4016 | .001203369 | 2,610.66 | 542,365.34 |
| 832 | 692,224 | 575,930,368 | 28.8444 | 9.4053 | .001201923 | 2,613.81 | 543,671.46 |
| 833 | 693,889 | 578,009,537 | 28.8617 | 9.4091 | .001200480 | 2,616.95 | 544,979.15 |
| 834 | 695,556 | 580,093,704 | 28.8791 | 9.4129 | .001199041 | 2,620.09 | 546,288.40 |
| 835 | 697,225 | 582,182,875 | 28.8964 | 9.4166 | .001197605 | 2,623.23 | 547,599.23 |
| 836 | 698,896 | 584,277,056 | 28.9137 | 9.4204 | .001196172 | 2,626.37 | 548,911.63 |
| 837 | 700,569 | 586,376,253 | 28.9310 | 9.4241 | .001194743 | 2,629.51 | 550,225.61 |
| 838 | 702,244 | 588,480,472 | 28.9482 | 9.4279 | .001193317 | 2,632.65 | 551,541.15 |
| 839 | 703,921 | 590,589,719 | 28.9655 | 9.4316 | .001191895 | 2,635.80 | 552,858.26 |
| 840 | 705,600 | 592,704,000 | 28.9828 | 9.4354 | .001190476 | 2,638.94 | 554,176.94 |
| 841 | 707,281 | 594,823,321 | 29.0000 | 9.4391 | .001189061 | 2,642.08 | 555,497.20 |
| 842 | 708,964 | 596,947,688 | 29.0172 | 9.4429 | .001187648 | 2,645.22 | 556,819.02 |
| 843 | 710,649 | 599,077,107 | 29.0345 | 9.4466 | .001186240 | 2,648.36 | 558,142.42 |
| 344 | 712,336 | 601,211,584 | 29.0517 | 9.4503 | .001184834 | 2,651.50 | 559,467.39 |
| 845 | 714,025 | 603,351,125 | 29.0689 | 9.4541 | .001183432 | 2,654.65 | 560,793.92 |
| 846 | 715,716 | 605,495,736 | 29.0861 | 9.4578 | .001182033 | 2,657.79 | 562,122.03 |
| 847 | 717,409 | 607,645,423 | 29.1033 | 9.4615 | .001180638 | 2,660.93 | 563,451.71 |
| 848 | 719,104 | 609,800,192 | 29.1204 | 9.4652 | .001179245 | 2,664.07 | 564,782.96 |
| 849 | 720,801 | 611,960,049 | 29.1376 | 9.4690 | .001177856 | 2,667.21 | 566,115.78 |
| 850 | 722,500 | 614,125,000 | 29.1548 | 9.4727 | .001176471 | 2,670.35 | 567,450.17 |
| 851 | 724,201 | 616,295,051 | 29.1719 | 9.4764 | .001175088 | 2,673.50 | 568,786.14 |
| 852 | 725,904 | 618,470,208 | 29.1890 | 9.4801 | .001173709 | 2,676.64 | 570,123.67 |
| 853 | 727,609 | 620,650,477 | 29.2062 | 9.4838 | .001172333 | 2,679.78 | 571,462.77 |
| 854 | 729,316 | 622,835,864 | 29.2233 | 9.4875 | .001170960 | 2,682.92 | 572,803.45 |
| 855 | 731,025 | 625,026,375 | 29.2404 | 9.4912 | .001169591 | 2,686.06 | 574,145.69 |
| 856 | 732,736 | 627,222,016 | 29.2575 | 9.4949 | .001168224 | 2,689.20 | 575,489.51 |
| 857 | 734,449 | 629,422,793 | 29.2746 | 9.4986 | .001166861 | 2,692.34 | 576,834.90 |
| 858 | 736,164 | 631,628,712 | 29.2916 | 9.5023 | .001165501 | 2,695.49 | 578,181.85 |
| 859 | 737,881 | 633,839,779 | 29.3087 | 9.5060 | .001164144 | 2,698.63 | 579,530.38 |
| 860 | 739,600 | 636,056,000 | 29.3258 | 9.5097 | .001162791 | 2,701.77 | 580,880.48 |
| 861 | 741,321 | 638,277,381 | 29.3428 | 9.5135 | .001161440 | 2,704.91 | 582,232.15 |
| 862 | 743,044 | 640,503,928 | 29.3598 | 9.5171 | .001160093 | 2,708.05 | 583,585.39 |
| 863 | 744,769 | 642,735,647 | 29.3769 | 9.5207 | .001158749 | 2,711.19 | 584,940.20 |
| 864 | 746,496 | 644,972,544 | 29.3939 | 9.5244 | .001157407 | 2,714.34 | 586,296.59 |
| 865 | 748,225 | 647,214,625 | 29.4109 | 9.5281 | .001156069 | 2,717.48 | 587,654.54 |
| 866 | 749,956 | 649,461,896 | 29.4279 | 9.5317 | .001154734 | 2,720.62 | 589,014.07 |
| 867 | 751,689 | 651,714,363 | 29.4449 | 9.5354 | .001153403 | 2,723.76 | 590,375.16 |
| 868 | 753,424 | 653,972,032 | 29.4618 | 9.5391 | .001152074 | 2,726.90 | 591,737.83 |
| 869 | 755,161 | 656,234,909 | 29.4788 | 9.5427 | .001150748 | 2,730.04 | 593,102.06 |
| 870 | 756,900 | 658,503,000 | 29.4958 | 9.5464 | .001149425 | 2,733.19 | 594,467.87 |
| 871 | 758,641 | 660,776,311 | 29.5127 | 9.5501 | .001148106 | 2,736.33 | 595,835.25 |
| 872 | 760,384 | 663,054,848 | 29.5296 | 9.5537 | .001146789 | 2,739.47 | 597,204.20 |
| 873 | 762,129 | 665,338,617 | 29.5466 | 9.5574 | .001145475 | 2,742.61 | 598,574.72 |
| 874 | 763,876 | 667,627,624 | 29.5635 | 9.5610 | .001144165 | 2,745.75 | 599,946.81 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|---|---|---|---|---|---|---|---|
| 875 | 765,625 | 669,921,875 | 29.5804 | 9.5647 | .001142857 | 2,748.89 | 601,320.47 |
| 876 | 767,376 | 672,221,376 | 29.5973 | 9.5683 | .001141553 | 2,752.04 | 602,695.70 |
| 877 | 769,129 | 674,526,133 | 29.6142 | 9.5719 | .001140251 | 2,755.18 | 604,072.50 |
| 878 | 770,884 | 676,836,152 | 29.6311 | 9.5756 | .001138952 | 2,758.32 | 605,450.88 |
| 879 | 772,641 | 679,151,439 | 29.6479 | 9.5792 | .001137656 | 2,761.46 | 606,830.82 |
| 880 | 774,400 | 681,472,000 | 29.6648 | 9.5828 | .001136364 | 2,764.60 | 608,212.34 |
| 881 | 776,161 | 683,797,841 | 29.6816 | 9.5865 | .001135074 | 2,767.74 | 609,595.42 |
| 882 | 777,924 | 686,128,968 | 29.6985 | 9.5901 | .001133787 | 2,770.88 | 610,980.08 |
| 883 | 779,689 | 688,465,387 | 29.7153 | 9.5937 | .001132503 | 2,774.03 | 612,366.31 |
| 884 | 781,456 | 690,807,104 | 29.7321 | 9.5973 | .001131222 | 2,777.17 | 613,754.11 |
| 885 | 783,225 | 693,154,125 | 29.7489 | 9.6010 | .001129944 | 2,780.31 | 615,143.48 |
| 886 | 784,996 | 695,506,456 | 29.7658 | 9.6046 | .001128668 | 2,783.45 | 616,534.42 |
| 887 | 786,769 | 697,864,103 | 29.7825 | 9.6082 | .001127396 | 2,786.59 | 617,926.93 |
| 888 | 788,544 | 700,227,072 | 29.7993 | 9.6118 | .001126126 | 2,789.73 | 619,321.01 |
| 889 | 790,321 | 702,595,369 | 29.8161 | 9.6154 | .001124859 | 2,792.88 | 620,716.66 |
| 890 | 792,100 | 704,969,000 | 29.8329 | 9.6190 | .001123596 | 2,796.02 | 622,113.89 |
| 891 | 793,881 | 707,347,971 | 29.8496 | 9.6226 | .001122334 | 2,799.16 | 623,512.68 |
| 892 | 795,664 | 707,982,288 | 29.8664 | 9.6262 | .001121076 | 2,802.30 | 624,913.04 |
| 893 | 797,449 | 712,121,957 | 29.8831 | 9.6298 | .001119821 | 2,805.44 | 626,314.98 |
| 894 | 799,236 | 714,516,984 | 29.8998 | 9.6334 | .001118568 | 2,808.58 | 627,718.49 |
| 895 | 801,025 | 716,917,375 | 29.9166 | 9.6370 | .001117818 | 2,811.73 | 629,123.56 |
| 896 | 802,816 | 719,323,136 | 29.9333 | 9.6406 | .001116071 | 2,814.87 | 630,530.21 |
| 897 | 804,609 | 721,734,273 | 29.9500 | 9.6442 | .001114827 | 2,818.01 | 631,938.43 |
| 898 | 806,404 | 724,150,792 | 29.9666 | 9.6477 | .001113586 | 2,821.15 | 633,348.22 |
| 899 | 808,201 | 726,572,699 | 29.9833 | 9.6513 | .001112347 | 2,824.29 | 634,759.58 |
| 900 | 810,000 | 729,000,000 | 30.0000 | 9.6549 | .001111111 | 2,827.43 | 636,172.51 |
| 901 | 811,801 | 731,432,701 | 30.0167 | 9.6585 | .001109878 | 2,830.58 | 637,587.01 |
| 902 | 813,604 | 733,870,808 | 30.0333 | 9.6620 | .001108647 | 2,833.72 | 639,003.09 |
| 903 | 815,409 | 736,314,327 | 30.0500 | 9.6656 | .001107420 | 2,836.86 | 640,420.73 |
| 904 | 817,216 | 738,763,264 | 30.0666 | 9.6692 | .001106195 | 2,840.00 | 641,839.95 |
| 905 | 819,025 | 741,217,625 | 30.0832 | 9.6727 | .001104972 | 2,843.14 | 643,260.73 |
| 906 | 820,836 | 743,677,416 | 30.0998 | 9.6763 | .001103753 | 2,846.28 | 644,683.09 |
| 907 | 822,649 | 746,142,643 | 30.1164 | 9.6799 | .001102536 | 2,849.42 | 646,107.01 |
| 908 | 824,464 | 748,613,312 | 30.1330 | 9.6834 | .001101322 | 2,852.57 | 647,532.51 |
| 909 | 826,281 | 751,089,429 | 30.1496 | 9.6870 | .001100110 | 2,855.71 | 648,959.58 |
| 910 | 828,100 | 753,571,000 | 30.1662 | 9.6905 | .001098901 | 2,858.85 | 650,388.22 |
| 911 | 829,921 | 756,058,031 | 30.1828 | 9.6941 | .001091695 | 2,861.99 | 651,818.43 |
| 912 | 831,744 | 758,550,825 | 30.1993 | 9.6976 | .001096491 | 2,865.13 | 653,250.21 |
| 913 | 833,569 | 761,048,497 | 30.2159 | 9.7012 | .001095290 | 2,868.27 | 654,683.56 |
| 914 | 835,396 | 763,551,944 | 30.2324 | 9.7047 | .001094092 | 2,871.42 | 656,118.48 |
| 915 | 837,225 | 766,060,875 | 30.2490 | 9.7082 | .001092896 | 2,874.56 | 657,554.98 |
| 916 | 839,056 | 768,575,296 | 30.2655 | 9.7118 | .001091703 | 2,877.70 | 658,993.04 |
| 917 | 840,889 | 771,095,213 | 30.2820 | 9.7153 | .001090513 | 2,880.84 | 660,432.68 |
| 918 | 842,724 | 773,620,632 | 30.2985 | 9.7188 | .001089325 | 2,883.98 | 661,873.88 |
| 919 | 844,561 | 776,151,559 | 30.3150 | 9.7224 | .001088139 | 2,887.12 | 663,316.66 |
| 920 | 846,400 | 778,688,000 | 30.3315 | 9.7259 | .001086957 | 2,890.27 | 664,761.01 |
| 921 | 848,241 | 781,229,961 | 30.3480 | 9.7294 | .001085776 | 2,893.41 | 666,206.92 |
| 922 | 850,084 | 783,777,448 | 30.3645 | 9.7329 | .001084599 | 2,896.55 | 667,654.41 |
| 923 | 851,929 | 786,330,467 | 30.3809 | 9.7364 | .001083423 | 2,899.69 | 669,103.47 |
| 924 | 853,776 | 788,889,024 | 30.3974 | 9.7400 | .001082251 | 2,902.83 | 670,554.10 |
| 925 | 855,625 | 791,453,125 | 30.4138 | 9.7435 | .001081081 | 2,905.97 | 672,006.30 |
| 926 | 857,476 | 794,022,776 | 30.4302 | 9.7470 | .001079914 | 2,909.11 | 673,460.08 |
| 927 | 859,329 | 796,597,983 | 30.4467 | 9.7505 | .001078749 | 2,912.26 | 674,915.42 |
| 928 | 861,184 | 799,178,752 | 30.4631 | 9.7540 | .001077586 | 2,915.40 | 676,372.33 |
| 929 | 863,041 | 801,765,089 | 30.4795 | 9.7575 | .001076426 | 2,918.54 | 677,830.82 |
| 930 | 864,900 | 804,357,000 | 30.4959 | 9.7610 | .001075269 | 2,921.68 | 679,290.87 |
| 931 | 866,761 | 806,954,491 | 30.5123 | 9.7645 | .001074114 | 2,924.82 | 680,752.50 |
| 932 | 868,624 | 809,557,568 | 30.5287 | 9.7680 | .001072961 | 2,927.96 | 682,215.69 |
| 933 | 870,489 | 812,166,237 | 30.5450 | 9.7715 | .001071811 | 2,931.11 | 683,680.46 |
| 934 | 872,356 | 814,780,504 | 30.5614 | 9.7750 | .001070664 | 2,934.25 | 685,146.80 |
| 935 | 874,225 | 817,400,375 | 30.5778 | 9.7785 | .001069519 | 2,937.39 | 686,614.71 |
| 936 | 876,096 | 820,025,856 | 30.5941 | 9.7829 | .001068376 | 2,940.53 | 688,084.19 |
| 937 | 877,969 | 822,656,953 | 30.6105 | 9.7854 | .001067236 | 2,943.67 | 689,555.24 |

| No. | Square | Cube | Sq. Root | Cu. Root | Reciprocal | Circum. | Area |
|---|---|---|---|---|---|---|---|
| 938 | 879,844 | 825,293,672 | 30.6268 | 9.7889 | .001066098 | 2,946.81 | 691,027.86 |
| 939 | 881,721 | 827,936,019 | 30.6431 | 9.7924 | .001064963 | 2,949.96 | 692,502.05 |
| 940 | 883,600 | 830,584,000 | 30.6594 | 9.7959 | .001063830 | 2,953.10 | 693,977.82 |
| 941 | 885,481 | 833,237,621 | 30.6757 | 9.7993 | .001062699 | 2,956.24 | 695,455.15 |
| 942 | 887,364 | 835,896,888 | 30.6920 | 9.8028 | .001061571 | 2,959.38 | 696,934.06 |
| 943 | 889,249 | 838,561,807 | 30.7083 | 9.8063 | .001060445 | 2,962.52 | 698,414.53 |
| 944 | 891,136 | 841,232,384 | 30.7246 | 9.8097 | .001059322 | 2,965.66 | 699,896.58 |
| 945 | 893,025 | 843,908,625 | 30.7409 | 9.8132 | .001058201 | 2,968.81 | 701,380.19 |
| 946 | 894,916 | 846,590,536 | 30.7571 | 9.8167 | .001057082 | 2,971.95 | 702,865.38 |
| 947 | 896,808 | 849,278,123 | 30.7734 | 9.8201 | .001055966 | 2,975.09 | 704,352.14 |
| 948 | 898,704 | 851,971,392 | 30.7896 | 9.8236 | .001054852 | 2,978.23 | 705,840.47 |
| 949 | 900,601 | 854,670,349 | 30.8058 | 9.8270 | .001053741 | 2,981.37 | 707,330.37 |
| 950 | 902,500 | 857,375,000 | 30.8221 | 9.8305 | .001052632 | 2,984.51 | 708,821.84 |
| 951 | 904,401 | 860,085,351 | 30.8383 | 9.8339 | .001051525 | 2,987.65 | 710,314.88 |
| 952 | 906,304 | 862,801,408 | 30.8545 | 9.8374 | .001050420 | 2,990.80 | 711,809.50 |
| 953 | 908,209 | 865,523,177 | 30.8707 | 9.8408 | .001049318 | 2,993.94 | 713,305.68 |
| 954 | 910,116 | 868,250,664 | 30.8869 | 9.8443 | .001048218 | 2,997.08 | 714,803.43 |
| 955 | 912,025 | 870,983,875 | 30.9031 | 9.8477 | .001047120 | 3,000.22 | 716,302.76 |
| 956 | 913,936 | 873,722,816 | 30.9192 | 9.8511 | .001046025 | 3,003.36 | 717,803.66 |
| 957 | 915,849 | 876,467,493 | 30.9354 | 9.8546 | .001044932 | 3,006.50 | 719,306.12 |
| 958 | 917,764 | 879,217,912 | 30.9516 | 9.8580 | .001043841 | 3,009.65 | 720,810.16 |
| 959 | 919,681 | 881,974,079 | 30.9677 | 9.8614 | .001042753 | 3,012.79 | 722,315.77 |
| 960 | 921,600 | 884,736,000 | 30.9839 | 9.8648 | .001041667 | 3,015.93 | 723,822.95 |
| 961 | 923,521 | 887,503,681 | 31.0000 | 9.8683 | .001040583 | 3,019.07 | 725,331.70 |
| 962 | 925,444 | 890,277,128 | 31.0161 | 9.8717 | .001039501 | 3,022.21 | 726,842.02 |
| 963 | 927,369 | 893,056,347 | 31.0322 | 9.8751 | .001038422 | 3,025.35 | 728,353.91 |
| 964 | 929,296 | 895,841,344 | 31.0483 | 9.8785 | .001037344 | 3,028.50 | 729,867.37 |
| 965 | 931,225 | 898,632,125 | 31.0644 | 9.8819 | .001036269 | 3,031.64 | 731,382.40 |
| 966 | 933,156 | 901,428,696 | 31.0805 | 9.8854 | .001035197 | 3,034.78 | 732,899.01 |
| 967 | 935,089 | 904,231,063 | 31.0966 | 9.8888 | .001034126 | 3,037.92 | 734,417.18 |
| 968 | 937,024 | 907,039,232 | 31.1127 | 9.8922 | .001033058 | 3,041.06 | 735,936.93 |
| 969 | 938,961 | 909,853,209 | 31.1288 | 9.8956 | .001031992 | 3,044.20 | 737,458.24 |
| 970 | 940,900 | 912,673,000 | 31.1448 | 9.8990 | .001030928 | 3,047.34 | 738,981.13 |
| 971 | 942,841 | 915,498,611 | 31.1609 | 9.9024 | .001029866 | 3,050.49 | 740,505.59 |
| 972 | 944,784 | 918,330,048 | 31.1769 | 9.9058 | .001028807 | 3,053.63 | 742,031.62 |
| 973 | 946,729 | 921,167,317 | 31.1929 | 9.9092 | .001027749 | 3,056.77 | 743,559.22 |
| 974 | 948,676 | 924,010,424 | 31.2090 | 9.9126 | .001026694 | 3,059.91 | 745,088.39 |
| 975 | 950,625 | 926,859,375 | 31.2250 | 9.9160 | .001025641 | 3,063.05 | 746,619.13 |
| 976 | 952,576 | 929,714,176 | 31.2410 | 9.9194 | .001024590 | 3,066.19 | 748,151.44 |
| 977 | 954,529 | 932,574,833 | 31.2570 | 9.9228 | .001023541 | 3,069.34 | 749,685.32 |
| 978 | 956,484 | 935,441,352 | 31.2730 | 9.9261 | .001022495 | 3,072.48 | 751,220.78 |
| 979 | 958,441 | 938,313,739 | 31.2890 | 9.9295 | .001021450 | 3,075.62 | 752,757.80 |
| 980 | 960,400 | 941,192,000 | 31.3050 | 9.9329 | .001020408 | 3,078.76 | 754,296.40 |
| 981 | 962,361 | 944,076,141 | 31.3209 | 9.9363 | .001019168 | 3,081.90 | 755,836.56 |
| 982 | 964,324 | 946,966,168 | 31.3369 | 9.9396 | .001018330 | 3,085.04 | 757,378.30 |
| 983 | 966,289 | 949,862,087 | 31.3528 | 9.9430 | .001017294 | 3,088.19 | 758,921.61 |
| 984 | 968,256 | 952,763,904 | 31.3688 | 9.9464 | .001016260 | 3,091.33 | 760,466.48 |
| 985 | 970,225 | 955,671,625 | 31.3847 | 9.9497 | .001015228 | 3,094.47 | 762,012.93 |
| 986 | 972,196 | 958,585,256 | 31.4006 | 9.9531 | .001014199 | 3,097.61 | 763,560.95 |
| 987 | 974,169 | 961,504,803 | 31.4166 | 9.9565 | .001013171 | 3,100.75 | 765,110.54 |
| 988 | 976,144 | 964,430,272 | 31.4325 | 9.9598 | .001012146 | 3,103.89 | 766,661.70 |
| 989 | 978,121 | 967,361,669 | 31.4484 | 9.9632 | .001011122 | 3,107.04 | 768,214.44 |
| 990 | 980,100 | 970,299,000 | 31.4643 | 9.9666 | .001010101 | 3,110.18 | 769,768.74 |
| 991 | 982,081 | 973,242,271 | 31.4802 | 9.9699 | .001009082 | 3,113.32 | 771,324.61 |
| 992 | 984,064 | 976,191,488 | 31.4960 | 9.9733 | .001008065 | 3,116.46 | 772,882.06 |
| 993 | 986,049 | 979,146,657 | 31.5119 | 9.9766 | .001007049 | 3,119.60 | 774,441.07 |
| 994 | 988,036 | 982,107,784 | 31.5278 | 9.9800 | .001006036 | 3,122.74 | 776,001.66 |
| 995 | 990,025 | 985,074,875 | 31.5436 | 9.9833 | .001005025 | 3,125.88 | 777,563.82 |
| 996 | 992,016 | 988,047,936 | 31.5595 | 9.9866 | .001004016 | 3,129.03 | 779,127.54 |
| 997 | 994,009 | 991,026,973 | 31.5753 | 9.9900 | .001003009 | 3,132.17 | 780,692.84 |
| 998 | 996,004 | 994,011,992 | 31.5911 | 9.9933 | .001002004 | 3,135.31 | 782,259.71 |
| 999 | 998,001 | 997,002,999 | 31.6070 | 9.9967 | .001001001 | 3,138.45 | 783,828.15 |
| 1000 | 1,000,000 | 1,000,000,000 | 31.6228 | 10.0000 | .001000000 | 3,141.59 | 785,398.16 |

## CIRCUMFERENCES AND AREAS OF CIRCLES FROM 1-64 to 100

| Diam. | Circum. | Area | Diam. | Circum. | Area | Diam. | Circum. | Area |
|---|---|---|---|---|---|---|---|---|
| 1/64 | .0491 | .0002 | 6 | 18.8496 | 28.2744 | 13⅛ | 41.2335 | 135.297 |
| 1/32 | .0982 | .0008 | 6⅛ | 19.2423 | 29.4648 | 13¼ | 41.6262 | 137.887 |
| 1/16 | .1963 | .0031 | 6¼ | 19.6350 | 30.6797 | 13⅜ | 42.0189 | 140.501 |
| 1/8 | .3927 | .0123 | 6⅜ | 20.0277 | 31.9191 | 13½ | 42.4116 | 143.139 |
| 3/16 | .5890 | .0276 | 6½ | 20.4204 | 33.1831 | 13⅝ | 42.8043 | 145.802 |
| 1/4 | .7854 | .0491 | 6⅝ | 20.8131 | 34.4717 | 13¾ | 43.1970 | 148.490 |
| 5/16 | .9817 | .0767 | 6¾ | 21.2058 | 35.7848 | 13⅞ | 43.5897 | 151.202 |
| 3/8 | 1.1781 | .1104 | 6⅞ | 21.5985 | 37.1224 | 14 | 43.9824 | 153.938 |
| 7/16 | 1.3744 | .1503 | 7 | 21.9912 | 38.4846 | 14⅛ | 44.3751 | 156.700 |
| 1/2 | 1.5708 | .1963 | 7⅛ | 22.3839 | 39.8713 | 14¼ | 44.7678 | 159.485 |
| 9/16 | 1.7671 | .2485 | 7¼ | 22.7766 | 41.2826 | 14⅜ | 45.1605 | 162.296 |
| 5/8 | 1.9635 | .3068 | 7⅜ | 23.1693 | 42.7184 | 14½ | 45.5532 | 165.130 |
| 11/16 | 2.1598 | .3712 | 7½ | 23.5620 | 44.1787 | 14⅝ | 45.9459 | 167.990 |
| 3/4 | 2.3562 | .4418 | 7⅝ | 23.9547 | 45.6636 | 14¾ | 46.3386 | 170.874 |
| 13/16 | 2.5525 | .5185 | 7¾ | 24.3474 | 47.1731 | 14⅞ | 46.7313 | 173.782 |
| 7/8 | 2.7489 | .6013 | 7⅞ | 24.7401 | 48.7071 | 15 | 47.1240 | 176.715 |
| 15/16 | 2.9452 | .6903 | 8 | 25.1328 | 50.2656 | 15⅛ | 47.5167 | 179.673 |
| 1 | 3.1416 | .7854 | 8⅛ | 25.5255 | 51.8487 | 15¼ | 47.9094 | 182.655 |
| 1⅛ | 3.5343 | .9940 | 8¼ | 25.9182 | 53.4563 | 15⅜ | 48.3021 | 185.661 |
| 1¼ | 3.9270 | 1.2272 | 8⅜ | 26.3109 | 55.0884 | 15½ | 48.6948 | 188.692 |
| 1⅜ | 4.3197 | 1.4849 | 8½ | 26.7036 | 56.7451 | 15⅝ | 49.0875 | 191.748 |
| 1½ | 4.7124 | 1.7671 | 8⅝ | 27.0963 | 58.4264 | 15¾ | 49.4802 | 194.828 |
| 1⅝ | 5.1051 | 2.0739 | 8¾ | 27.4890 | 60.1322 | 15⅞ | 49.8729 | 197.933 |
| 1¾ | 5.4978 | 2.4053 | 8⅞ | 27.8817 | 61.8625 | 16 | 50.2656 | 201.062 |
| 1⅞ | 5.8905 | 2.7612 | 9 | 28.2744 | 63.6174 | 16⅛ | 50.6583 | 204.216 |
| 2 | 6.2832 | 3.1416 | 9⅛ | 28.6671 | 65.3968 | 16¼ | 51.0510 | 207.395 |
| 2⅛ | 6.6759 | 3.5466 | 9¼ | 29.0598 | 67.2008 | 16⅜ | 51.4437 | 210.598 |
| 2¼ | 7.0686 | 3.9761 | 9⅜ | 29.4525 | 69.0293 | 16½ | 51.8364 | 213.825 |
| 2⅜ | 7.4613 | 4.4301 | 9½ | 29.8452 | 70.8823 | 16⅝ | 52.2291 | 217.077 |
| 2½ | 7.8540 | 4.9087 | 9⅝ | 30.2379 | 72.7599 | 16¾ | 52.6218 | 220.354 |
| 2⅝ | 8.2467 | 5.4119 | 9¾ | 30.6306 | 74.6621 | 16⅞ | 53.0145 | 223.655 |
| 2¾ | 8.6394 | 5.9396 | 9⅞ | 31.0233 | 76.589 | 17 | 53.4072 | 226.981 |
| 2⅞ | 9.0321 | 6.4918 | 10 | 31.4160 | 78.540 | 17⅛ | 53.7999 | 230.331 |
| 3 | 9.4248 | 7.0686 | 10⅛ | 31.8087 | 80.516 | 17¼ | 54.1926 | 233.706 |
| 3⅛ | 9.8175 | 7.6699 | 10¼ | 32.2014 | 82.516 | 17⅜ | 54.5853 | 237.105 |
| 3¼ | 10.2102 | 8.2258 | 10⅜ | 32.5941 | 84.541 | 17½ | 54.9780 | 240.529 |
| 3⅜ | 10.6029 | 8.9462 | 10½ | 32.9868 | 86.590 | 17⅝ | 55.3707 | 243.977 |
| 3½ | 10.9956 | 9.6211 | 10⅝ | 33.3795 | 88.664 | 17¾ | 55.7634 | 247.450 |
| 3⅝ | 11.3883 | 10.3206 | 10¾ | 33.7722 | 90.763 | 17⅞ | 56.1561 | 250.948 |
| 3¾ | 11.7810 | 11.0447 | 10⅞ | 34.1649 | 92.886 | 18 | 56.5488 | 254.470 |
| 3⅞ | 12.1737 | 11.7933 | 11 | 34.5576 | 95.033 | 18⅛ | 56.9415 | 258.016 |
| 4 | 12.5664 | 12.5664 | 11⅛ | 34.9503 | 97.205 | 18¼ | 57.3342 | 261.587 |
| 4⅛ | 12.9591 | 13.3641 | 11¼ | 35.3430 | 99.402 | 18⅜ | 57.7269 | 265.183 |
| 4¼ | 13.3518 | 14.1863 | 11⅜ | 35.7357 | 101.623 | 18½ | 58.1196 | 268.803 |
| 4⅜ | 13.7445 | 15.0330 | 11½ | 36.1284 | 103.869 | 18⅝ | 58.5123 | 272.448 |
| 4½ | 14.1372 | 15.9043 | 11⅝ | 36.5211 | 106.139 | 18¾ | 58.9050 | 276.117 |
| 4⅝ | 14.5299 | 16.8002 | 11¾ | 36.9138 | 108.434 | 18⅞ | 59.2977 | 279.811 |
| 4¾ | 14.9226 | 17.7206 | 11⅞ | 37.3065 | 110.754 | 19 | 59.6904 | 283.529 |
| 4⅞ | 15.3153 | 18.6655 | 12 | 37.6992 | 113.098 | 19⅛ | 60.0831 | 287.272 |
| 5 | 15.7080 | 19.6350 | 12⅛ | 38.0919 | 115.466 | 19¼ | 60.4758 | 291.040 |
| 5⅛ | 16.1007 | 20.6290 | 12¼ | 38.4846 | 117.859 | 19⅜ | 60.8685 | 294.832 |
| 5¼ | 16.4934 | 21.6476 | 12⅜ | 38.8773 | 120.277 | 19½ | 61.2612 | 298.648 |
| 5⅜ | 16.8861 | 22.6907 | 12½ | 39.2700 | 122.719 | 19⅝ | 61.6539 | 302.489 |
| 5½ | 17.2788 | 23.7583 | 12⅝ | 39.6627 | 125.185 | 19¾ | 62.0466 | 306.355 |
| 5⅝ | 17.6715 | 24.8505 | 12¾ | 40.0554 | 127.677 | 19⅞ | 62.4393 | 310.245 |
| 5¾ | 18.0642 | 25.9673 | 12⅞ | 40.4481 | 130.192 | 20 | 62.8320 | 314.160 |
| 5⅞ | 18.4569 | 27.1086 | 13 | 40.8408 | 132.733 | 20⅛ | 63.2247 | 318.099 |

| Diam. | Circum. | Area | Diam. | Circum. | Area | Diam. | Circum. | Area |
|---|---|---|---|---|---|---|---|---|
| 20¼ | 63.6174 | 322.063 | 28⅛ | 88.3575 | 621.264 | 36 | 113.098 | 1,017.878 |
| 20⅜ | 64.0101 | 326.051 | 28¼ | 88.7502 | 626.798 | 36⅛ | 113.490 | 1,024.960 |
| 20½ | 64.4028 | 330.064 | 28⅜ | 89.1429 | 632.357 | 36¼ | 113.883 | 1,032.065 |
| 20⅝ | 64.7955 | 334.102 | 28½ | 89.5356 | 637.941 | 36⅜ | 114.276 | 1,039.195 |
| 20¾ | 65.1882 | 338.164 | 28⅝ | 89.9283 | 643.549 | 36½ | 114.668 | 1,046.349 |
| 20⅞ | 65.5809 | 342.250 | 28¾ | 90.3210 | 649.182 | 36⅝ | 115.061 | 1,053.528 |
| 21 | 65.9736 | 346.361 | 28⅞ | 90.7137 | 654.840 | 36¾ | 115.454 | 1,060.732 |
| 21⅛ | 66.3663 | 350.497 | 29 | 91.1064 | 660.521 | 36⅞ | 115.846 | 1,067.960 |
| 21¼ | 66.7590 | 354.657 | 29⅛ | 91.4991 | 666.228 | 37 | 116.239 | 1,075.213 |
| 21⅜ | 67.1517 | 358.842 | 29¼ | 91.8918 | 671.959 | 37⅛ | 116.632 | 1,082.490 |
| 21½ | 67.5444 | 363.051 | 29⅜ | 92.2845 | 677.714 | 37¼ | 117.025 | 1,089.792 |
| 21⅝ | 67.9371 | 367.285 | 29½ | 92.6772 | 683.494 | 37⅜ | 117.417 | 1,097.118 |
| 21¾ | 68.3298 | 371.543 | 29⅝ | 93.0699 | 689.299 | 37½ | 117.810 | 1,104.469 |
| 21⅞ | 68.7225 | 375.826 | 29¾ | 93.4626 | 695.128 | 37⅝ | 118.203 | 1,111.844 |
| 22 | 69.1152 | 380.134 | 29⅞ | 93.8553 | 700.982 | 37¾ | 118.595 | 1,119.244 |
| 22⅛ | 69.5079 | 384.465 | 30 | 94.2480 | 706.860 | 37⅞ | 118.988 | 1,126.669 |
| 22¼ | 69.9006 | 388.822 | 30⅛ | 94.6407 | 712.763 | 38 | 119.381 | 1,134.118 |
| 22⅜ | 70.2933 | 393.203 | 30¼ | 95.0334 | 718.690 | 38⅛ | 119.773 | 1,141.591 |
| 22½ | 70.6860 | 397.609 | 30⅜ | 95.4261 | 724.642 | 38¼ | 120.166 | 1,149.089 |
| 22⅝ | 71.0787 | 402.038 | 30½ | 95.8188 | 730.618 | 38⅜ | 120.559 | 1,156.612 |
| 22¾ | 71.4714 | 406.494 | 30⅝ | 96.2115 | 736.619 | 38½ | 120.952 | 1,164.159 |
| 22⅞ | 71.8641 | 410.973 | 30¾ | 96.6042 | 742.645 | 38⅝ | 121.344 | 1,171.731 |
| 23 | 72.2568 | 415.477 | 30⅞ | 96.9969 | 748.695 | 38¾ | 121.737 | 1,179.327 |
| 23⅛ | 72.6495 | 420.004 | 31 | 97.3896 | 754.769 | 38⅞ | 122.130 | 1,186.948 |
| 23¼ | 73.0422 | 424.558 | 31⅛ | 97.7823 | 760.869 | 39 | 122.522 | 1,194.593 |
| 23⅜ | 73.4349 | 429.135 | 31¼ | 98.1750 | 766.992 | 39⅛ | 122.915 | 1,202.263 |
| 23½ | 73.8276 | 433.737 | 31⅜ | 98.5677 | 773.140 | 39¼ | 123.308 | 1,209.958 |
| 23⅝ | 74.2203 | 438.364 | 31½ | 98.9604 | 779.313 | 39⅜ | 123.700 | 1,217.677 |
| 23¾ | 74.6130 | 443.015 | 31⅝ | 99.3531 | 785.510 | 39½ | 124.093 | 1,225.420 |
| 23⅞ | 75.0057 | 447.697 | 31¾ | 99.7458 | 791.732 | 39⅝ | 124.486 | 1,233.188 |
| 24 | 75.3984 | 452.390 | 31⅞ | 100.1385 | 797.979 | 39¾ | 124.879 | 1,240.981 |
| 24⅛ | 75.7911 | 457.115 | 32 | 100.5312 | 804.250 | 39⅞ | 125.271 | 1,248.798 |
| 24¼ | 76.1838 | 461.864 | 32⅛ | 100.9239 | 810.545 | 40 | 125.664 | 1,256.640 |
| 24⅜ | 76.5765 | 466.638 | 32¼ | 101.3166 | 816.865 | 40⅛ | 126.057 | 1,264.510 |
| 24½ | 76.9692 | 471.436 | 32⅜ | 101.7093 | 823.210 | 40¼ | 126.449 | 1,272.400 |
| 24⅝ | 77.3619 | 476.259 | 32½ | 102.1020 | 829.579 | 40⅜ | 126.842 | 1,280.310 |
| 24¾ | 77.7546 | 481.107 | 32⅝ | 102.4947 | 835.972 | 40½ | 127.235 | 1,288.250 |
| 24⅞ | 78.1473 | 485.979 | 32¾ | 102.8874 | 842.391 | 40⅝ | 127.627 | 1,296.220 |
| 25 | 78.5400 | 490.875 | 32⅞ | 103.2801 | 848.833 | 40¾ | 128.020 | 1,304.210 |
| 25⅛ | 78.9327 | 495.796 | 33 | 103.6728 | 855.301 | 40⅞ | 128.413 | 1,312.220 |
| 25¼ | 79.3254 | 500.742 | 33⅛ | 104.0655 | 861.792 | 41 | 128.806 | 1,320.260 |
| 25⅜ | 79.7181 | 505.711 | 33¼ | 104.4582 | 868.309 | 41⅛ | 129.198 | 1,328.320 |
| 25½ | 80.1108 | 510.706 | 33⅜ | 104.8509 | 874.850 | 41¼ | 129.591 | 1,336.410 |
| 25⅝ | 80.5035 | 515.726 | 33½ | 105.2436 | 881.415 | 41⅜ | 129.984 | 1,344.520 |
| 25¾ | 80.8962 | 520.769 | 33⅝ | 105.6363 | 888.005 | 41½ | 130.376 | 1,352.660 |
| 25⅞ | 81.2889 | 525.838 | 33¾ | 106.0290 | 894.620 | 41⅝ | 130.769 | 1,360.820 |
| 26 | 81.6816 | 530.930 | 33⅞ | 106.4217 | 901.259 | 41¾ | 131.162 | 1,369.000 |
| 26⅛ | 82.0743 | 536.048 | 34 | 106.8144 | 907.922 | 41⅞ | 131.554 | 1,377.210 |
| 26¼ | 82.4670 | 541.190 | 34⅛ | 107.2071 | 914.611 | 42 | 131.947 | 1,385.450 |
| 26⅜ | 82.8597 | 546.356 | 34¼ | 107.5998 | 921.323 | 42⅛ | 132.340 | 1,393.700 |
| 26½ | 83.2524 | 551.547 | 34⅜ | 107.9925 | 928.061 | 42¼ | 132.732 | 1,401.990 |
| 26⅝ | 83.6451 | 556.763 | 34½ | 108.3852 | 934.822 | 42⅜ | 133.125 | 1,410.300 |
| 26¾ | 84.0378 | 562.002 | 34⅝ | 108.7779 | 941.609 | 42½ | 133.518 | 1,418.630 |
| 26⅞ | 84.4305 | 567.267 | 34¾ | 109.1706 | 948.420 | 42⅝ | 133.911 | 1,426.990 |
| 27 | 84.8232 | 572.557 | 34⅞ | 109.5633 | 955.255 | 42¾ | 134.303 | 1,435.370 |
| 27⅛ | 85.2159 | 577.870 | 35 | 109.9560 | 962.115 | 42⅞ | 134.696 | 1,443.770 |
| 27¼ | 85.6086 | 583.209 | 35⅛ | 110.3487 | 969.000 | 43 | 135.089 | 1,452.200 |
| 27⅜ | 86.0013 | 588.571 | 35¼ | 110.7414 | 975.909 | 43⅛ | 135.481 | 1,460.660 |
| 27½ | 86.3940 | 593.959 | 35⅜ | 111.1341 | 982.842 | 43¼ | 135.874 | 1,469.140 |
| 27⅝ | 86.7867 | 599.371 | 35½ | 111.5268 | 989.800 | 43⅜ | 136.267 | 1,477.640 |
| 27¾ | 87.1794 | 604.807 | 35⅝ | 111.9195 | 996.783 | 43½ | 136.660 | 1,486.170 |
| 27⅞ | 87.5721 | 610.268 | 35¾ | 112.3122 | 1,003.790 | 43⅝ | 137.052 | 1,494.730 |
| 28 | 87.9648 | 615.754 | 35⅞ | 112.7049 | 1,010.822 | 43¾ | 137.445 | 1,503.300 |

| Diam. | Circum. | Area | Diam. | Circum. | Area | Diam. | Circum. | Area |
|---|---|---|---|---|---|---|---|---|
| 43⅞ | 137.838 | 1,511.910 | 51¾ | 162.578 | 2,103.35 | 59⅝ | 187.318 | 2,792.21 |
| 44 | 138.230 | 1,520.530 | 51⅞ | 162.970 | 2,113.52 | 59¾ | 187.711 | 2,803.93 |
| 44⅛ | 138.623 | 1,529.190 | 52 | 163.363 | 2,123.72 | 59⅞ | 188.103 | 2,815.67 |
| 44¼ | 139.016 | 1,537.860 | 52⅛ | 163.756 | 2,133.94 | 60 | 188.496 | 2,827.44 |
| 44⅜ | 139.408 | 1,546.56 | 52¼ | 164.149 | 2,144.19 | 60⅛ | 188.889 | 2,839.23 |
| 44½ | 139.801 | 1,555.29 | 52⅜ | 164.541 | 2,154.46 | 60¼ | 189.281 | 2,851.05 |
| 44⅝ | 140.194 | 1,564.04 | 52½ | 164.934 | 2,164.76 | 60⅜ | 189.674 | 2,862.89 |
| 44¾ | 140.587 | 1,572.81 | 52⅝ | 165.327 | 2,175.08 | 60½ | 190.067 | 2,874.76 |
| 44⅞ | 140.979 | 1,581.61 | 52¾ | 165.719 | 2,185.42 | 60⅝ | 190.459 | 2,886.65 |
| 45 | 141.372 | 1,590.43 | 52⅞ | 166.112 | 2,195.79 | 60¾ | 190.852 | 2,898.57 |
| 45⅛ | 141.765 | 1,599.28 | 53 | 166.505 | 2,206.19 | 60⅞ | 191.245 | 2,910.51 |
| 45¼ | 142.157 | 1,608.16 | 53⅛ | 166.897 | 2,216.61 | 61 | 191.638 | 2,922.47 |
| 45⅜ | 142.550 | 1,617.05 | 53¼ | 167.290 | 2,227.05 | 61⅛ | 192.030 | 2,934.46 |
| 45½ | 142.943 | 1,625.97 | 53⅜ | 167.683 | 2,237.52 | 61¼ | 192.423 | 2,946.48 |
| 45⅝ | 143.335 | 1,634.92 | 53½ | 168.076 | 2,248.01 | 61⅜ | 192.816 | 2,958.52 |
| 45¾ | 143.728 | 1,643.89 | 53⅝ | 168.468 | 2,258.53 | 61½ | 193.208 | 2,970.58 |
| 45⅞ | 144.121 | 1,652.89 | 53¾ | 168.861 | 2,269.07 | 61⅝ | 193.601 | 2,982.67 |
| 46 | 144.514 | 1,661.91 | 53⅞ | 169.254 | 2,279.64 | 61¾ | 193.994 | 2,994.78 |
| 46⅛ | 144.906 | 1,670.95 | 54 | 169.646 | 2,290.23 | 61⅞ | 194.386 | 3,006.92 |
| 46¼ | 145.299 | 1,680.02 | 54⅛ | 170.039 | 2,300.84 | 62 | 194.779 | 3,019.08 |
| 46⅜ | 145.692 | 1,689.11 | 54¼ | 170.432 | 2,311.48 | 62⅛ | 195.172 | 3,031.26 |
| 46½ | 146.084 | 1,698.23 | 54⅜ | 170.824 | 2,322.15 | 62¼ | 195.565 | 3,043.47 |
| 46⅝ | 146.477 | 1,707.37 | 54½ | 171.217 | 2,332.83 | 62⅜ | 195.957 | 3,055.71 |
| 46¾ | 146.870 | 1,716.54 | 54⅝ | 171.610 | 2,343.55 | 62½ | 196.350 | 3,067.97 |
| 46⅞ | 147.262 | 1,725.73 | 54¾ | 172.003 | 2,354.29 | 62⅝ | 196.743 | 3,080.25 |
| 47 | 147.655 | 1,734.95 | 54⅞ | 172.395 | 2,365.05 | 62¾ | 197.135 | 3,092.56 |
| 47⅛ | 148.048 | 1,744.19 | 55 | 172.788 | 2,375.83 | 62⅞ | 197.528 | 3,104.89 |
| 47¼ | 148.441 | 1,753.46 | 55⅛ | 173.181 | 2,386.65 | 63 | 197.921 | 3,117.25 |
| 47⅜ | 148.833 | 1,762.74 | 55¼ | 173.573 | 2,397.48 | 63⅛ | 198.313 | 3,129.64 |
| 47½ | 149.226 | 1,772.06 | 55⅜ | 173.966 | 2,408.34 | 63¼ | 198.706 | 3,142.04 |
| 47⅝ | 149.619 | 1,781.40 | 55½ | 174.359 | 2,419.23 | 63⅜ | 199.099 | 3,154.47 |
| 47¾ | 150.011 | 1,790.76 | 55⅝ | 174.751 | 2,430.14 | 63½ | 199.492 | 3,166.93 |
| 47⅞ | 150.404 | 1,800.15 | 55¾ | 175.144 | 2,441.07 | 63⅝ | 199.884 | 3,179.41 |
| 48 | 150.797 | 1,809.56 | 55⅞ | 175.537 | 2,452.03 | 63¾ | 200.277 | 3,191.91 |
| 48⅛ | 151.189 | 1,819.00 | 56 | 175.930 | 2,463.01 | 63⅞ | 200.670 | 3,204.44 |
| 48¼ | 151.582 | 1,828.46 | 56⅛ | 176.322 | 2,474.02 | 64 | 201.062 | 3,217.00 |
| 48⅜ | 151.975 | 1,837.95 | 56¼ | 176.715 | 2,485.05 | 64⅛ | 201.455 | 3,229.58 |
| 48½ | 152.368 | 1,847.46 | 56⅜ | 177.108 | 2,496.11 | 64¼ | 201.848 | 3,242.18 |
| 48⅝ | 152.760 | 1,856.99 | 56½ | 177.500 | 2,507.19 | 64⅜ | 202.240 | 3,254.81 |
| 48¾ | 153.153 | 1,866.55 | 56⅝ | 177.893 | 2,518.30 | 64½ | 202.633 | 3,267.46 |
| 48⅞ | 153.546 | 1,876.14 | 56¾ | 178.286 | 2,529.43 | 64⅝ | 203.026 | 3,280.14 |
| 49 | 153.938 | 1,885.75 | 56⅞ | 178.678 | 2,540.58 | 64¾ | 203.419 | 3,292.84 |
| 49⅛ | 154.331 | 1,895.38 | 57 | 179.071 | 2,551.76 | 64⅞ | 203.811 | 3,305.56 |
| 49¼ | 154.724 | 1,905.04 | 57⅛ | 179.464 | 2,562.97 | 65 | 204.204 | 3,318.31 |
| 49⅜ | 155.116 | 1,914.72 | 57¼ | 179.857 | 2,574.20 | 65⅛ | 204.597 | 3,331.09 |
| 49½ | 155.509 | 1,924.43 | 57⅜ | 180.249 | 2,585.45 | 65¼ | 204.989 | 3,343.89 |
| 49⅝ | 155.902 | 1,934.16 | 57½ | 180.642 | 2,596.73 | 65⅜ | 205.382 | 3,356.71 |
| 49¾ | 156.295 | 1,943.91 | 57⅝ | 181.035 | 2,608.03 | 65½ | 205.775 | 3,369.56 |
| 49⅞ | 156.687 | 1,953.69 | 57¾ | 181.427 | 2,619.36 | 65⅝ | 206.167 | 3,382.44 |
| 50 | 157.080 | 1,963.50 | 57⅞ | 181.820 | 2,630.71 | 65¾ | 206.560 | 3,395.33 |
| 50⅛ | 157.473 | 1,973.33 | 58 | 182.213 | 2,642.09 | 65⅞ | 206.953 | 3,408.26 |
| 50¼ | 157.865 | 1,983.18 | 58⅛ | 182.605 | 2,653.49 | 66 | 207.346 | 3,421.20 |
| 50⅜ | 158.258 | 1,993.06 | 58¼ | 182.998 | 2,664.91 | 66⅛ | 207.738 | 3,434.17 |
| 50½ | 158.651 | 2,002.97 | 58⅜ | 183.391 | 2,676.36 | 66¼ | 208.131 | 3,447.17 |
| 50⅝ | 159.043 | 2,012.89 | 58½ | 183.784 | 2,687.84 | 66⅜ | 208.524 | 3,460.19 |
| 50¾ | 159.436 | 2,022.85 | 58⅝ | 184.176 | 2,699.33 | 66½ | 208.916 | 3,473.24 |
| 50⅞ | 159.829 | 2,032.82 | 58¾ | 184.569 | 2,710.86 | 66⅝ | 209.309 | 3,486.30 |
| 51 | 160.222 | 2,042.83 | 58⅞ | 184.962 | 2,722.41 | 66¾ | 209.702 | 3,499.40 |
| 51⅛ | 160.614 | 2,052.85 | 59 | 185.354 | 2,733.98 | 66⅞ | 210.094 | 3,512.52 |
| 51¼ | 161.007 | 2,062.90 | 59⅛ | 185.747 | 2,745.57 | 67 | 210.487 | 3,525.66 |
| 51⅜ | 161.400 | 2,072.98 | 59¼ | 186.140 | 2,757.20 | 67⅛ | 210.880 | 3,538.83 |
| 51½ | 161.792 | 2,083.08 | 59⅜ | 186.532 | 2,768.84 | 67¼ | 211.273 | 3,552.02 |
| 51⅝ | 162.185 | 2,093.20 | 59½ | 186.925 | 2,780.51 | 67⅜ | 211.665 | 3,565.24 |

| Diam. | Circum. | Area | Diam. | Circum. | Area | Diam. | Circum. | Area |
|---|---|---|---|---|---|---|---|---|
| 67⅛ | 212.058 | 3,578.48 | 75⅜ | 236.798 | 4,462.16 | 83¼ | 261.538 | 5,443.26 |
| 67⅜ | 212.451 | 3,591.74 | 75½ | 237.191 | 4,476.98 | 83⅜ | 261.931 | 5,459.62 |
| 67⅝ | 212.843 | 3,605.04 | 75⅝ | 237.583 | 4,491.81 | 83½ | 262.324 | 5,476.01 |
| 67⅞ | 213.236 | 3,618.35 | 75¾ | 237.976 | 4,506.67 | 83⅝ | 262.716 | 5,492.41 |
| 68 | 213.629 | 3,631.69 | 75⅞ | 238.369 | 4,521.56 | 83¾ | 263.109 | 5,508.84 |
| 68⅛ | 214.021 | 3,645.05 | 76 | 238.762 | 4,536.47 | 83⅞ | 263.502 | 5,525.30 |
| 68¼ | 214.414 | 3,658.44 | 76⅛ | 239.154 | 4,551.41 | 84 | 263.894 | 5,541.78 |
| 68⅜ | 214.807 | 3,671.86 | 76¼ | 239.547 | 4,566.36 | 84⅛ | 264.287 | 5,558.29 |
| 68½ | 215.200 | 3,685.29 | 76⅜ | 239.940 | 4,581.35 | 84¼ | 264.680 | 5,574.82 |
| 68⅝ | 215.592 | 3,698.76 | 76½ | 240.332 | 4,596.36 | 84⅜ | 265.072 | 5,591.37 |
| 68¾ | 215.985 | 3,712.24 | 76⅝ | 240.725 | 4,611.39 | 84½ | 265.465 | 5,607.95 |
| 68⅞ | 216.378 | 3,725.75 | 76¾ | 241.118 | 4,626.45 | 84⅝ | 265.858 | 5,624.56 |
| 69 | 216.770 | 3,739.29 | 76⅞ | 241.510 | 4,641.53 | 84¾ | 266.251 | 5,641.18 |
| 69⅛ | 217.163 | 3,752.85 | 77 | 241.903 | 4,656.64 | 84⅞ | 266.643 | 5,657.84 |
| 69¼ | 217.556 | 3,766.43 | 77⅛ | 242.296 | 4,671.77 | 85 | 267.036 | 5,674.51 |
| 69⅜ | 217.948 | 3,780.04 | 77¼ | 242.689 | 4,686.92 | 85⅛ | 267.429 | 5,691.22 |
| 69½ | 218.341 | 3,793.68 | 77⅜ | 243.081 | 4,702.10 | 85¼ | 267.821 | 5,707.94 |
| 69⅝ | 218.734 | 3,807.34 | 77½ | 243.474 | 4,717.31 | 85⅜ | 268.214 | 5,724.69 |
| 69¾ | 219.127 | 3,821.02 | 77⅝ | 243.867 | 4,732.54 | 85½ | 268.607 | 5,741.47 |
| 69⅞ | 219.519 | 3,834.73 | 77¾ | 244.259 | 4,747.79 | 85⅝ | 268.999 | 5,758.27 |
| 70 | 219.912 | 3,848.46 | 77⅞ | 244.652 | 4,763.07 | 85¾ | 269.392 | 5,775.10 |
| 70⅛ | 220.305 | 3,862.22 | 78 | 245.045 | 4,778.37 | 85⅞ | 269.785 | 5,791.94 |
| 70¼ | 220.697 | 3,876.00 | 78⅛ | 245.437 | 4,793.70 | 86 | 270.178 | 5,808.82 |
| 70⅜ | 221.090 | 3,889.80 | 78¼ | 245.830 | 4,809.05 | 86⅛ | 270.570 | 5,825.72 |
| 70½ | 221.483 | 3,903.63 | 78⅜ | 246.223 | 4,824.43 | 86¼ | 270.963 | 5,842.64 |
| 70⅝ | 221.875 | 3,917.49 | 78½ | 246.616 | 4,839.83 | 86⅜ | 271.356 | 5,859.59 |
| 70¾ | 222.268 | 3,931.37 | 78⅝ | 247.008 | 4,855.26 | 86½ | 271.748 | 5,876.56 |
| 70⅞ | 222.661 | 3,945.27 | 78¾ | 247.401 | 4,870.71 | 86⅝ | 272.141 | 5,893.55 |
| 71 | 223.054 | 3,959.20 | 78⅞ | 247.794 | 4,886.18 | 86¾ | 272.534 | 5,910.58 |
| 71⅛ | 223.446 | 3,973.15 | 79 | 248.186 | 4,901.68 | 86⅞ | 272.926 | 5,927.62 |
| 71¼ | 223.839 | 3,987.13 | 79⅛ | 248.579 | 4,917.21 | 87 | 273.319 | 5,944.69 |
| 71⅜ | 224.232 | 4,001.13 | 79¼ | 248.972 | 4,932.75 | 87⅛ | 273.712 | 5,961.79 |
| 71½ | 224.624 | 4,015.16 | 79⅜ | 249.364 | 4,948.33 | 87¼ | 274.105 | 5,978.91 |
| 71⅝ | 225.017 | 4,029.21 | 79½ | 249.757 | 4,963.92 | 87⅜ | 274.497 | 5,996.05 |
| 71¾ | 225.410 | 4,043.29 | 79⅝ | 250.150 | 4,979.55 | 87½ | 274.890 | 6,013.22 |
| 71⅞ | 225.802 | 4,057.39 | 79¾ | 250.543 | 4,995.19 | 87⅝ | 275.283 | 6,030.41 |
| 72 | 226.195 | 4,071.51 | 79⅞ | 250.935 | 5,010.86 | 87¾ | 275.675 | 6,047.63 |
| 72⅛ | 226.588 | 4,085.66 | 80 | 251.328 | 5,026.56 | 87⅞ | 276.068 | 6,064.87 |
| 72¼ | 226.981 | 4,099.84 | 80⅛ | 251.721 | 5,042.28 | 88 | 276.461 | 6,082.14 |
| 72⅜ | 227.373 | 4,114.04 | 80¼ | 252.113 | 5,058.03 | 88⅛ | 276.853 | 6,099.43 |
| 72½ | 227.766 | 4,128.26 | 80⅜ | 252.506 | 5,073.79 | 88¼ | 277.246 | 6,116.74 |
| 72⅝ | 228.159 | 4,142.51 | 80½ | 252.899 | 5,089.59 | 88⅜ | 277.629 | 6,134.08 |
| 72¾ | 228.551 | 4,156.78 | 80⅝ | 253.291 | 5,105.41 | 88½ | 278.032 | 6,151.45 |
| 72⅞ | 228.944 | 4,171.08 | 80¾ | 253.684 | 5,121.25 | 88⅝ | 278.424 | 6,168.84 |
| 73 | 229.337 | 4,185.40 | 80⅞ | 254.077 | 5,137.12 | 88¾ | 278.817 | 6,186.25 |
| 73⅛ | 229.729 | 4,199.74 | 81 | 254.470 | 5,153.01 | 88⅞ | 279.210 | 6,203.69 |
| 73¼ | 230.122 | 4,214.11 | 81⅛ | 254.862 | 5,168.93 | 89 | 279.602 | 6,221.15 |
| 73⅜ | 230.515 | 4,228.51 | 81¼ | 255.255 | 5,184.87 | 89⅛ | 279.995 | 6,238.64 |
| 73½ | 230.908 | 4,242.93 | 81⅜ | 255.648 | 5,200.83 | 89¼ | 280.388 | 6,256.15 |
| 73⅝ | 231.300 | 4,257.37 | 81½ | 256.040 | 5,216.82 | 89⅜ | 280.780 | 6,273.69 |
| 73¾ | 231.693 | 4,271.84 | 81⅝ | 256.433 | 5,232.84 | 89½ | 281.173 | 6,291.25 |
| 73⅞ | 232.086 | 4,286.33 | 81¾ | 256.826 | 5,248.88 | 89⅝ | 281.566 | 6,308.84 |
| 74 | 232.478 | 4,300.85 | 81⅞ | 257.218 | 5,264.94 | 89¾ | 281.959 | 6,326.45 |
| 74⅛ | 232.871 | 4,315.39 | 82 | 257.611 | 5,281.03 | 89⅞ | 282.351 | 6,344.08 |
| 74¼ | 233.264 | 4,329.96 | 82⅛ | 258.004 | 5,297.14 | 90 | 282.744 | 6,361.74 |
| 74⅜ | 233.656 | 4,344.55 | 82¼ | 258.397 | 5,313.28 | 90⅛ | 283.137 | 6,379.42 |
| 74½ | 234.049 | 4,359.17 | 82⅜ | 258.789 | 5,329.44 | 90¼ | 283.529 | 6,397.13 |
| 74⅝ | 234.442 | 4,373.81 | 82½ | 259.182 | 5,345.63 | 90⅜ | 283.922 | 6,414.86 |
| 74¾ | 234.835 | 4,388.47 | 82⅝ | 259.575 | 5,361.84 | 90½ | 284.315 | 6,432.62 |
| 74⅞ | 235.227 | 4,403.16 | 82¾ | 259.967 | 5,378.08 | 90⅝ | 284.707 | 6,450.40 |
| 75 | 235.620 | 4,417.87 | 82⅞ | 260.360 | 5,394.34 | 90¾ | 285.100 | 6,468.21 |
| 75⅛ | 236.013 | 4,432.61 | 83 | 260.753 | 5,410.62 | 90⅞ | 285.493 | 6,486.04 |
| 75¼ | 236.405 | 4,447.38 | 83⅛ | 261.145 | 5,426.93 | 91 | 285.886 | 6,503.90 |

| Diam. | Circum. | Area | Diam. | Circum. | Area | Diam. | Circum. | Area |
|---|---|---|---|---|---|---|---|---|
| 91⅛ | 286.278 | 6,521.78 | 94⅛ | 295.703 | 6,958.26 | 97⅛ | 305.128 | 7,408.89 |
| 91¼ | 286.671 | 6,539.68 | 94¼ | 296.096 | 6,976.76 | 97¼ | 305.521 | 7,427.97 |
| 91⅜ | 287.064 | 6,557.61 | 94⅜ | 296.488 | 6,995.28 | 97⅜ | 305.913 | 7,447.08 |
| 91½ | 287.456 | 6,575.56 | 94½ | 296.881 | 7,013.82 | 97½ | 306.306 | 7,466.21 |
| 91⅝ | 287.849 | 6,593.54 | 94⅝ | 297.274 | 7,032.39 | 97⅝ | 306.699 | 7,485.37 |
| 91¾ | 288.242 | 6,611.55 | 94¾ | 297.667 | 7,050.98 | 97¾ | 307.091 | 7,504.55 |
| 91⅞ | 288.634 | 6,629.57 | 94⅞ | 298.059 | 7,069.59 | 97⅞ | 307.484 | 7,523.75 |
| 92 | 289.027 | 6,647.63 | 95 | 298.452 | 7,088.24 | 98 | 307.877 | 7,542.98 |
| 92⅛ | 289.420 | 6,665.70 | 95⅛ | 298.845 | 7,106.90 | 98⅛ | 308.270 | 7,562.24 |
| 92¼ | 289.813 | 6,683.80 | 95¼ | 299.237 | 7,125.59 | 98¼ | 308.662 | 7,581.52 |
| 92⅜ | 290.205 | 6,701.93 | 95⅜ | 299.630 | 7,144.31 | 98⅜ | 309.055 | 7,600.82 |
| 92½ | 290.598 | 6,720.08 | 95½ | 300.023 | 7,163.04 | 98½ | 309.448 | 7,620.15 |
| 92⅝ | 290.991 | 6,738.25 | 95⅝ | 300.415 | 7,181.81 | 98⅝ | 309.840 | 7,639.50 |
| 92¾ | 291.383 | 6,756.45 | 95¾ | 300.808 | 7,200.60 | 98¾ | 310.233 | 7,658.88 |
| 92⅞ | 291.776 | 6,774.68 | 95⅞ | 301.201 | 7,219.41 | 98⅞ | 310.626 | 7,678.28 |
| 93 | 292.169 | 6,792.92 | 96 | 301.594 | 7,238.25 | 99 | 311.018 | 7,697.71 |
| 93⅛ | 292.562 | 6,811.20 | 96⅛ | 301.986 | 7,257.11 | 99⅛ | 311.411 | 7,717.16 |
| 93¼ | 292.954 | 6,829.49 | 96¼ | 302.379 | 7,275.99 | 99¼ | 311.804 | 7,736.63 |
| 93⅜ | 293.347 | 6,847.82 | 96⅜ | 302.772 | 7,294.91 | 99⅜ | 312.196 | 7,756.13 |
| 93½ | 293.740 | 6,866.16 | 96½ | 303.164 | 7,313.84 | 99½ | 312.589 | 7,775.66 |
| 93⅝ | 294.132 | 6,884.53 | 96⅝ | 303.557 | 7,332.80 | 99⅝ | 312.982 | 7,795.21 |
| 93¾ | 294.525 | 6,902.93 | 96¾ | 303.950 | 7,351.79 | 99¾ | 313.375 | 7,814.78 |
| 93⅞ | 294.918 | 6,921.35 | 96⅞ | 304.342 | 7,370.79 | 99⅞ | 313.767 | 7,834.38 |
| 94 | 295.310 | 6,939.79 | 97 | 304.735 | 7,389.83 | 100 | 314.160 | 7,854.00 |

The preceding table may be used to determine the diameter when the circumference or area is known. Thus, the diameter of a circle having an area of 7,200 sq. in. is approximately 95¾ in.

# A GLOSSARY OF MINING TERMS

The glossary of mining terms here presented is taken largely from that given in the 10th Edition of the Coal and Metal Miners' Pocketbook. This was a combination of glossaries including: Raymond's Glossary of Mining and Metallurgical Terms, Powers' Pocketbook for Miners and Metallurgists, Locke's Miners' Pocketbook, Vol. AC, Second Pennsylvania Geological Survey, Ihlseng's Manual of Mining, Chism's Encyclopedia of Mexican Mining Law, a Glossary of Terms as Used in Coal Mining, by W. S. Gresley, 11th Annual Report of the State Mine Inspector of Missouri, Bullman's Colliery Working and Management, Reynolds's Handbook of Mining Laws, Report of the Mine Inspector of Tennessee for 1897, Smithsonian Report for 1886, and words from other stray sources. In the present glossary words which may be applied only to metal mining have been omitted. Many words also not found in the original list have been added. Other terms also have been included which are by no means peculiar to the coal-mining industry. It has been the aim, however, to include such terms as are in common use by those engaged in coal production regardless of whether they apply only to that industry or to others as well. Various foreign words have been selected as being those which an American is most likely to encounter. This list is, however, by no means exhaustive. For a large number of purely local terms used in the various coal fields of Great Britain the reader is referred to Mr. Gresley's glossary.

# GLOSSARY

**Abattis** (Leicester).—Cross-packing of branches or rough wood, used to keep roads open for ventilation.

*Absolute Pressure.*—The pressure reckoned from a vacuum.

*Absolute Temperature.*—The temperature reckoned from the absolute zero, −459.2° F. or −273° C.

*Accompt* (Cornish).—Settling day or place.

*Achicar* (Mexican).—To diminish the quantity of water in any gallery or working, generally by carrying it out in buckets or in leather bags. *Achicadores.*—Laborers employed for said purpose. *Achichinques.*—Same as *Achicadores.* Also applied to hangers-on about police courts, etc. Such people as are generally called strikers in the United States.

*Acreage Rent* (English).—Royalty or rent for working minerals.

*Addlings* (North of England).—Earnings.

*Ademador* (Spanish).—Mine carpenter, or timberman.

*Ademar* (Spanish).—To timber.

*Adit.*—A nearly horizontal passage from the surface, by which a mine is entered and unwatered with just sufficient slope to insure drainage. In the United States, an adit driven across the measures is usually called a tunnel, though the latter, strictly speaking, passes entirely through a hill, and is open at both ends.

*Aodobe.*—Sun-dried brick.

*Adventurers.*—Original prospectors.

*Adverse.*—To oppose the granting of a patent to a mining claim.

*Adze.*—A curved cutting instrument for dressing timber.

*Aerage* (French).—Ventilation.

*Aerometers.*—The air pistons of a Struve ventilator.

*Aerophore.*—The name given to an apparatus that will enable a man to enter places in mines filled with explosive or other deadly gases, with safety.

*Afterdamp.*—The gaseous mixture resulting from an explosion of firedamp.

*Agent.*—The manager of a mining property.

*Ahondar* (Spanish).—To sink.

*Air.*—The current of atmospheric air circulating through and ventilating the workings of a mine.

*Air Box.*—Wooden tubes used to convey air for ventilating headings or sinkings or other local ventilation.

*Air Compartment.*—An air-tight portion of any shaft, winze, rise, or level, used for improving ventilation.

*Air-Course.*—See *Airway.*

*Air Crossing.*—A bridge that carries one air-course over another, an overcast

*Air Cushion.*—A cushion or spring caused by confined air.

*Air Door.*—A door for the regulation of currents of air through the workings of a mine.

*Air-End Way* (Locke).—Ventilation levels run parallel with main level.

*Air Furnace.*—A ventilation furnace.

*Air Gates* (Locke).—(1) Underground roadways, used principally for ventilating purposes. (2) An air regulator.

*Air Head* (Staff).—Ventilation ways.

*Air Heading.*—An airway, or air course.

*Air Hole* (Powers).—A hole drilled in advance to improve ventilation by communication with other workings or the surface.

*Airless End.*—The extremity of a stall in longwall workings in which there is no current of air, or circulation of ventilation, but which is kept pure by diffusion and by the ingress and egress of cars, men, etc.

*Air Level.*—A level or airway of former workings made use of in subsequent deeper mining operations for ventilating purposes.

*Air Oven.*—A heated chamber for drying samples of ore, coal, etc.

*Air Pipe.*—A pipe made of canvas or metal, or a wooden box used in conveying air to the workmen, or for rock drills or air locomotives.

*Air-Shaft.*—A shaft or pit used expressly for ventilation.

*Air Slit* (Yorks).—A short head between other air heads.

*Air Sollar.*—A brattice carried beneath the tram rails or road bed in a heading or gangway.

*Air Stack.*—A stack or chimney built over a shaft for ventilation.

*Airway.*—Any passage through which air is carried.

*Aitch Piece.*—Parts of a pump in which the valves are fixed.

*Albanil* (Spanish).—Mason.

*Alive* (Cornish).—Productive.

*Alluvium.*—Gravel, sand, and mud deposited by streams.

*Almagre* (Spanish).—Red ocher.

*Alternating Motion.*—Up and down, or backward and forward motion.

*Alto* (Mexican).—The hanging wall of a vein. See *Respaldos.*

*Amygdaloidal.*—Almond-shaped.

*Analysis.*—The determination of the original elements and the proportions of each in a substance.

*Anemometer.*—An instrument used for measuring the velocity of a ventilating current by means of a revolving vane wheel.

*Angle Beam.*—A two-limbed beam used for turning angles in shafts, etc.

*Anhydrous.*—Without water in its composition.

*Anneal.*—To toughen or soften metals, glass, etc., by first heating and then cooling very slowly or quickly depending on the metal.

*Anthracite.*—Coal containing a small percentage of volatile matter.

*Anticline.*—A flexure or fold in which the rocks on the opposite sides of the fold dip away from each other, like the two legs of the letter A. The inclination on one side may be much greater than on the opposite side. An anticline is said to be overturned when the rocks on both sides dip in the same direction.

*Anticlinal Axis.*—The ridge of a saddle in a mineral vein, or the line along the summit of a vein, from which the vein dips in opposite directions.

*Anticlinal Flexure; Anticlinal Fold.*—An anticline.

*Antiguos, Los* (Mexican).—The Spanish or Indian miners of colonial times.

*Aparejo* (Mexican).—A rigid pair of large stuffed pads connected over the back of a pack mule by an unpadded portion to protect body of mule when heavy or irregularly shaped loads are carried.

*Aperos* (Mexican).—All kinds of mining supplies in general. *Aperador.*—A storekeeper.

*Apex.*—The landing point at the top of a slope or inclined plane, the knuckle; also, the top of an anticlinal. In the U. S. Revised Statutes, the end or edge of a vein nearest the surface.

*A pique* (Mexican).—Perpendicular.

*Apron* (English).—(1) A covering of timber, stone, or metal, to protect a surface against the action of water flowing over it. (2) A hinged extension to a loading chute.

*Aqua Fortis.*—Nitric acid.

*Aquo Regia.*—A mixture of hydrochloric acid and nitric acid.

*Aqueduct.*—An artificial channel for carrying water.

*Arajo* (Mexican).—See *Hatajo.*

*Arch* (Cornish).—Portion of lode left standing to support hanging wall, or because too poor.

*Archean.*—An early period of geological time.

*Arching.*—Brickwork or stonework forming the roof of any underground roadway.

*Arenaceous.*—Sandy; rocks are arenaceous when they contain a considerable percentage of sand.

*Arenillas* (Spanish).—Refuse earth.

*Argillaceous.*—Clayey; rocks are argillaceous when they contain a considerable percentage of clay, or have some of the characteristics of clay.

*Argol.*—Crude tartar deposited from wine.

*Arm.*—The inclined leg of a set of timber.

*Arrastra* (Mexican).—To drag along the ground. *Arrastrar el Agua.*—To almost completely exhaust the water in a sump or working.

*Arroba* (Mexican).—25 lb.

*Artesian Well.*—An artificial channel of escape, made by a bore hole, for a subterranean stream, subject to hydrostatic pressure.

*Ascensional Ventilation.*—The arrangement of the ventilating currents in such a manner that the air shall continuously rise until reaching the bottom of the upcast shaft. Particularly applicable to steep seams.

*Ashlar.*—A facing of cut stone applied to a backing of rubble or rough masonry or brickwork.

*Aspirail* (French).—Opening for ventilation.

*Assay.*—The determination of the quality and quantity of any particular substance in a mineral. *Assayer.*—One who performs assays.

*Assessment Work.*—The annual work necessary to hold a mining claim.

*Astel.*—Overhead boarding in a gallery.

*Astyllen* (Cornish).—Small dam in an adit; partition between ore and deads on grass.

*Atacador* (Mexican).—A tamping bar or tamping stick.

*Atecas* (Mexican).—Same as *Achicadores*, etc.

*Atierres* (Spanish).—Refuse rock or dirt inside a mine, gob.

*Atle* (Cornish).—Refuse rock.

*Atle* (*Addle*).—The waste of a mine.

*Attrition.*—The act of wearing away by friction.

*Auger Stem.*—The iron rod or bar to which the bit is attached.

*Auget.*—Priming tube.

*Ausscharen* (German).—Junction of lodes.

*Auszimmern* (German).—Timbering.

*Aviador* (Spanish).—One who provides the capital to work a mine. *Avio.*—Money furnished to the proprietors of a mine to work the mine by another person, the *Aviador*. *Avio Contract.*—A contract between two parties for working a mine by which one of the parties, the *aviador*, furnishes the money to the proprietors for working the mine.

*Axis.*—An imaginary line passing through a body that may be supposed to revolve around it.

*Azimuth.*—The azimuth of a body is that arc of the horizon that is included between the meridian circle at the given place and a vertical plane passing through the body. It is always measured from due north around to the right.

*Azoic.*—The age of rocks that were formed before animal life existed.

**Back.**—(1) A plane or cleavage in coal, etc., having frequently a smooth parting and some sooty coal included in it. (2) The inner end of a heading or gangway. (3) To throw back into the gob or waste the small slack, dirt, etc. (4) To roll large coals out of a waste for loading into cars.

*Back Balance.*—(1) A self-acting incline in the mine, where a balance car and a carriage in which the mine car is placed are used. The loaded car upon the carriage will hoist the balance car, and the balance car will hoist the carriage and empty car. (2) A weight moving vertically or on an incline which places tension upon a tension carriage.

*Backbye Work.*—Work done between the shaft and the working face, in contradistinction to face work, or work done at the face.

*Back Casing.*—A wall or lining of dry bricks used in sinking through drift deposits, the permanent walling being built up within it. The use of timber cribs and planking serves the same purpose.

*Back End* (England).—The last portion of a jud.

*Backing.*—(1) The rough masonry of a wall faced with finer work. (2) Earth deposited behind a retaining wall, etc. (3) Timbers let into notches in the rock across the top of a level.

*Backing Deals.*—Deal boards or planking placed at the back of curbs for supporting the sides of a shaft that is liable to run.

*Back Joint.*—Joint plane more or less parallel to the strike of the cleavage, and frequently vertical.

*Backlash.*—(1) Backward suction of air-currents produced after an explosion of firedamp. (2) Reëntry of air into a fan. (3) Lost motion or play between the teeth of gears.

*Back Pressure.*—The loss, expressed in pounds per square inch, due to getting the steam out of the cylinder of an engine after it has done its work.

*Back Shift.*—Afternoon shift.

*Back Skin* (North of England).—A leather jacket for wet workings.

*Backstay.*—A wrought-iron forked bar attached to the back of cars when ascending an inclined plane, which throws them off the rails if the rope or coupling breaks.

*Baff Ends.*—Long wooden edges for adjusting linings in sinking shafts during the operation of fixing the lining.

*Baffle.*—(1) To brush out firedamp. (2) A firebrick partition to guide the flue gases through a boiler.

*Bait.*—Provisions.

*Bajo* (Mexican)—The footwall of a vein. See *Respaldo.*

*Bal* (Cornish).—A mine.

*Balance.*—(1) The counterpoise or weights attached to the drum of a winding engine, to assist the engine in lifting the load out of a shaft bottom and in helping it to slacken speed when the cage reaches the surface. It consists often of a bunch of heavy chains suspended in a shallow shaft,

the chains resting on the shaft bottom as unwound off the balance drum attached to the main shaft of the engine. (2) Scales used in chemical analysis and assaying.

*Balance Bob.*—A large beam or lever attached to the main rods of a Cornish pumping engine, carrying on its outer end a counterpoise.

*Balance Box.*—A large box placed on one end of a balance bob and filled with old iron, rock, etc., to counterbalance the weight of the pump rods.

*Balance Brow.*—An inclined plane in steep seams on which a platform on wheels travels and carries the cars of coal.

*Balance Car.*—A small weighted truck mounted upon a short inclined track, and carrying a sheave around which the rope of an endless haulage system passes as it winds off the drum.

*Balance Pit.*—A pit or shaft in which a balance rises or falls.

*Balanzon* (Mexican).—The balance bob of a Cornish pump.

*Balk.*—(1) A more or less sudden thinning out of a seam of coal. (2) Irregular-shaped masses of stone intruding into a coal seam, or bulgings out of the stone roof into the seam. (3) A bar of timber supporting the roof of a mine, or for carrying any heavy load.

*Ballast.*—Broken stone, gravel, sand, etc., used for keeping railroad ties steady.

*Bancos* (Spanish).—Horses in a vein or cross-courses.

*Band.*—A seam or thin stratum of stone or other refuse in a seam of coal, a parting.

*Bank.*—(1) The top of the shaft, or out of the shaft. (2) The surface around the mouth of a shaft. (3) To manipulate coals, etc., on the bank. (4) The whole or sometimes only one side or one end of a working place underground. (5) A large heap of mineral on the surface.

*Bank Chain.*—A chain that includes the bank of a river or creek.

*Bank Claim* (Australian).—Mining right on bank of stream.

*Bank Head.*—The upper end of an inclined plane, next to the engine or drum, made nearly level.

*Bank Right* (Australian).—Right to divert water to bank claim.

*Banksman.*—The man in attendance at the top of the shaft, superintending the work of banking.

*Bankwork.*—A system of working coal in South Yorkshire.

*Bank to Bank.*—A shift.

*Bannocking.*—See *Kirving.*

*Bar.*—A length of timber placed horizontally for supporting the roof. In some cases, bars of wrought iron, about 3 in. × 1 in. × 5 ft. are used.

*Bargain.*—Portion of mine worked by a gang on contract.

*Baring.*—See *Stripping.*

*Barmaster* (Derbyshire).—Mine manager, agent, and engineer.

*Barney.*—A small car, used on inclined planes and slopes to push the mine car up the slope. *Barney Pit.*—A pit at the bottom of a slope or plane into which the barney runs to allow the mine car to pass over it.

*Barra* (Mexican).—(1) A bar, as of gold, silver, iron, steel, etc. (2) A certain share in a mine. The ancient Spanish laws, from time immemorial, considered a mine as divided into 24 parts, and each part was called a "barra."

*Barra Viuda,* or *Aviada* (Mexican).—These are "barras" or shares that participate in the profits, but not in the expenses, of mining concerns. Their share of the expenses is paid by the other shares. Non-assessable shares.

*Barranca* (Mexican).—A ravine, a gulch. What is improperly called in the United States a canyon or cañon.

*Barrena* (Mexican).—A hand drill for opening holes in rocks for blasting purposes.

*Barrenarse* (Mexican).—When two mines or two workings (as a shaft or winze, or a gallery) communicate with each other.

*Barren Ground.*—Strata unproductive of seams of coal, etc., of a workable thickness.

*Barreno* (Mexican).—(1) A drill hole for blasting purposes. In mechanics, any bored hole. (2) A communication between two mines or two workings.

*Barretero* (Mexican).—A miner of the first class; one that knows how to point his holes, drill, and blast, or work with a gad.

*Barrier Pillar.*—A solid block or rib of coal, etc., left unworked between two collieries or mines for security against accidents arising from influx of water.

*Barrier System.*—The method of working a colliery by pillar and stall, where solid ribs or barriers of coal are left in between a set or series of working places.

*Barrow.*—(1) A box with two handles at one end and a wheel at the other. (2) Heap of waste stuff raised from a mine: a dump.

*Bar Timbering.*—A system of supporting a tunnel roof by long top bars, while the whole lower tunnel core is taken out, leaving an open space for the masons to run up the arching. Under certain conditions, the bars are withdrawn after the masonry is completed, otherwise they are bricked in and not drawn.

*Basin.*—(1) A coal field having some resemblance in form to a basin. (2) The synclinal axis of a seam of coal or stratum of rock.

*Basket.*—A measure of weight = 2 cwt.

*Basque.*—Crucible or furnace lining.

*Bass* (Derbyshire).—Indurated clay.

*Basset.*—Outcrop of a lode or stratum.

*Bastard.*—A particularly hard massive rock or boulder.

*Batch.*—An amount of concrete material.

*Batt* (English).—(1) A highly bituminous shale found in the coal measures. (2) Hardened clay, but not fireclay. Same as *Bend* and *Bind*.

*Batten.*—A piece of thin board less than 12 in. in width.

*Batter.*—The inclination of a face of masonry or of any inclined portion of a frame or metal structure.

*Battery.*—(1) A structure built to keep coal from sliding down a chute or breast. (2) An embankment or platform on which miners work. (3) A set of stamps. (4) Two or more boilers with a common setting.

*Bay.*—An open space for waste between two packs in a longwall working. See *Board.*

*Bay of Biscay Country.*—(Geological).—See *Crab Holes.*

*Beans* (North of England).—All coal that will pass through about ½-in. screen.

*Bear.*—A deposit of iron at the bottom of a furnace.

*Bear; to Bear In.*—Underholing or undermining; driving in at the top or at the side of a working.

*Bearers.*—Pieces of timber 3 or 4 ft. longer than the breadth of a shaft, which are fixed into the solid rock at the sides at certain intervals apart; used as foundations for sets of timber.

*Bearing.*—(1) The course by a compass. (2) The span or length in the clear between the points of support of a beam, etc. (3) The points of support of a beam, shaft, axle, etc.

*Bearing Door.*—A door placed for the purpose of directing and regulating the amount of ventilation passing through an entire district of a mine.

*Bearing In.*—The depth or distance under of the undercut or holing.

*Bearing-up Pulley.*—A pulley wheel fixed in a frame and arranged to tighten up or take up the slack rope in endless-rope haulage.

*Bearing-up Stop.*—A partition of brattice or plank that serves to conduct air to a face.

*Beat* (Cornish).—To cut away a lode.

*Beataway.*—Working hard ground by means of wedges and sledge hammers.

*Bed.*—(1) The level surface of a rock upon which a curb or crib is laid. (2) A stratum of coal, ironstone, clay, etc.

*Bed Claim* (Australian).—A claim that includes the bed of a river or creek.

*Bede.*—Miners' pickax.

*Bedplate.*—A large plate of iron used as a foundation for an engine or other machine.

*Bed Rock.*—The solid rock underlying the soil, drift, or alluvial deposits.

*Beehive Oven.*—The ordinary circular or rectangular arched oven in which coke is made without the recovery of any byproducts other than in some instances the heat.

*Before-Breast.*—Rock or vein, which still lies ahead.

*Bell.*—Overhanging rock or slate, of a bell-like form, disconnected from the main roof.

*Belly.*—A swelling mass of mineral in a lode.

*Bench.*—(1) A natural terrace marking the outcrop of any stratum. (2) A stratum of coal forming a portion of the vein.

*Benching.*—To break up with wedges the bottom coals when the holing is done in the middle of the seam.

*Benching Up* (North of England).—Working on top of coal.

*Bench Mark.*—A mark cut in a tree, rock or on some solid structure whose

elevation is known. Used by surveyors for reference in determining elevations.

*Bench Working.*—The system of working one or more seams or beds of mineral by open working or stripping, in stages or steps.

*Bend* (Derbyshire).—Indurated clay.

*Bessemer Steel.*—Steel made by the Bessemer process.

*Beton* (English).—Concrete of hydraulic cement with broken stone, bricks, gravel, etc.

*Bevel.*—The slope formed by trimming away on edge.

*Bevel Gear.*—A gear-wheel whose teeth are inclined to the axis of the wheel.

*Biche.*—A hollow-ended tool for recovering boring rods.

*Billy Boy.*—A boy who attends a Billy Playfair.

*Billy Playfair.*—A mechanical contrivance for weighing coal, consisting of an iron trough with a sort of hopper bottom, into which all the small coal passing through the screen is conducted and weighed off and emptied from time to time.

*Bin.*—A box or receptacle used for tools, stones, ore, coal, etc.

*Bind*, or *Binder.*—Indurated argillaceous shales or clay, very commonly forming the roof of a coal seam and frequently containing clay ironstone. See *Batt.*

*Binding.*—Hiring men.

*Bit.*—(1) A piece of steel placed in the cutting edge of a drill or point of a pick. (2) The cutting tool of a mining machine.

*Blackband.*—Carbonaceous ironstone in beds, mingled with coaly matter sufficient for its own calcination.

*Black Batt*, or *Black Stone.*—Black carbonaceous shale.

*Black Butts.*—See *Black Ends.*

*Blackdamp.*—Carbonic-acid gas.

*Black Dimaonds.*—Coal.

*Black Ends.*—Beehive coke of inferior quality due to mismanipulation or discoloration.

*Black Jack.*—(1) Properly speaking, dark varieties of zinc blend, but many miners apply it to any black mineral. (2) Crude black oil used to oil mine cars. Often called Black Strap.

*Black Lead.*—Graphite.

*Black Stone.*—A carbonaceous shale.

*Blast.*—(1) The sudden rush of fire, gas, and dust of an explosion through the workings and roadways of a mine. (2) To cut or bring down coal, rocks, etc., by the explosion of gunpowder, dynamite, etc.

*Blasting Barrel.*—A small pipe used for blasting in wet or gaseous places.

*Blast Pipe.*—A pipe for supplying air to furnaces.

*Blind Coal.*—Coal altered by the heat of a trap dike.

*Blind Creek.*—(1) A creek in which water flows only in very wet weather. (2) (Australasian) Dry watercourse.

*Blind Drift.*—(1) A horizontal passage in the mine not yet connected with the other workings. (2) A drift not opening to daylight.

*Blind Joint.*—Obscure bedding plane.

*Blind Lead*, or *Blind Lode.*—A vein having no visible outcrop.

*Blind Level.*—(1) An incomplete level. (2) A drainage level.

*Blind Shaft*, or *Blind Pit.*—A shaft not coming to the surface.

*Bloat.*—A hammer swelled at the eye.

*Block Claim* (Australian).—A square mining claim.

*Block Coal.*—Coal that breaks in large rectangular lumps.

*Block Reefs.*—Reefs showing frequent contractions longitudinally.

*Block Tin.*—Cast tin.

*Bloomary.*—A forge for making wrought iron.

*Blossom.*—The decomposed outcrop, float, surface stain, or any indicating traces of a coal bed or mineral deposit. *Blossom Rock.*—(1) Colored veinstone detached from an outcrop. (2) The rock detached from a vein, but which has not been transported.

*Blow.*—(1) To blast with gunpowder, etc. (2) A dam or stopping is said to blow when gas escapes through it.

*Blower.*—(1) A sudden emission or outburst of gas in a mine. (2) Any emission of gas from a coal seam similar to that from an ordinary gas burner. (3) A type of centrifugal fan used largely to force air into furnaces. (4) A blowdown ventilating fan.

*Blow Fan.*—A small centrifugal fan used to force air through canvas pipes or wooden boxes to the workmen.

*Blowdown Fan.*—A force fan.

*Blown-out Shot.*—A shot that has blown out the tamping, but not broken the coal or rock.

*Blow Off.*—To let off excess of steam from a boiler.

*Blow Out.*—(1) To finish a smelting campaign. (2) A blown-out shot. (3) The decomposed mineral exposure of a vein.

*Blowpipe.*—An instrument for creating a blast whereby the heat of a flame or lamp can be better utilized.

*Blue Cap.*—The blue halo of ignited gas (firedamp and air) on the top of the flame in a safety lamp.

*Blue Elvan* (Cornish).—Greenstone.

*Blue John.*—Fluorspar.

*Blue Metal.*—A local term for shale possessing a bluish color.

*Blue Peach* (Cornish).—A slate-blue fine-grained schorl.

*Bluestone.*—(1) Sulphate of copper. (2) Lapis lazuli. (3) Basalt. (4) Maryland, a gray gneiss; in Ohio, a gray sandstone; in the District of Columbia, a mica schist; in New York, a blue-gray sandstone; in Penusylvania, a blue-gray sandstone. (5) A popular term among stone men not sufficiently definite to be of value.

*Bluff.*—Blunt.

*Board.*—A wide heading usually from 3 to 5 yd. wide.

*Board-and-Pillar.*—A system of working coal where the first stage of excavation is accomplished with the roof sustained by pillars of coal left between the breasts; often called *Breast-and-Pillar*.

*Bob.*—An oscillating bell-crank, or lever, through which the motion of an engine is transmitted to the pump rods in an engine or pumping pit. There are ⊥ bobs, L bobs, and V bobs.

*Boca* or *Boca Mina* (Mexican).—Mouth or mine mouth. This is the name applied to the principal or first opening of a mine, or to the one where the miners are accustomed to descend.

*Bochorno* (Mexican).—Excessive heat, with want of ventilation, so that the lights go out. See *Vapores*.

*Body.*—(1) An ore body, or pocket of mineral deposit. (2) The thickness of a lubricating oil or other liquid; also the measure of that thickness expressed in the number of seconds in which a given quantity of the oil at a given temperature flows through a given aperture.

*Boleo* (Mexican).—A dump pile for waste rock.

*Bond.*—(1) The arrangement of blocks of stone or brickwork to form a firm structure by a judicious overlapping of each other so as to break joint. (2) An agreement for hiring men. (3) Apparatus for electrically joining the ends of adjacent rails. A cross bond joins both rails of a track.

*Bone.*—Slaty coal or carbonaceous shale found in coal seams.

*Bone Ash.*—Burnt bones pulverized and sifted.

*Bonnet.*—(1) The overhead cover of a cage. (2) A cover for the gauze of a safety lamp. (3) A cap piece for an upright timber. (4) The upper part of a valve containing the stuffing box.

*Booming.*—Ground sluicing on a large scale by emptying the contents of a reservoir at once on material collected below, thus removing boulders.

*Bord* (English).—A narrow breast.

*Bord-and-Pillar* (English).—See *Pillar-and-Breast*.

*Bord Room.*—The space excavated in driving a bord. The term is used in connection with the "ridding" of the fallen stone in old bords when driving roads across them in pillar working; thus, "ridding across the old bord room."

*Bord Ways Course.*—The direction at right angles to the main cleavage planes. In some mining districts it is termed "on face."

*Bore.*—To drill.

*Bore Hole.*—A hole made with a drill, auger, or other tools, in coal, rock, or other material.

*Bort.*—Amorphous dark diamond.

*Bosh.*—The plane in a blast furnace where the greatest diameter is reached.

*Boss* (English).—(1) An increase of the diameter at any part of the shaft. (2) A person in charge of a piece of work.

*Botas* (Mexican).—Buckets made of an entire ox skin, to take out water.

*Botryoidal.*—Grape-like in appearance.

*Bottle Chock.*—A pulley with a wide grooved face for guiding a cable around a turn in the track, an angle sheave.

*Bottle Jack* (English).—An appliance for lifting heavy weights.

*Bottom.*—(1) The landing at the bottom of the shaft or slope. (2) The lowest point of mining operations. (3) The floor, bottom rock, or stratum underlying a coal bed. (4) In alluvial, the bed rock or reef.

*Bottomer, Bottomman.*—The person that loads the cages at the pit bottom and gives the signal to bank. The onsetter or bottom cager.

*Bottom Joint.*—Joint or bedding plane, horizontal or nearly so.

*Bottom Lift.*—(1) The deepest column of a pump. (2) The lowest or deepest lift or level of a mine.

*Bottom Pillars.*—Large pillars left around the bottom of a shaft.

*Boulders.*—Loose rounded masses of stone detached from the parent rock.

*Bounce.*—A sudden spalling off of the sides of ribs and pillars due to excessive pressure; a bump.

*Bow.*—The handle of a kibble.

*Bowk.*—An iron barrel or tub used for hoisting rock and other débris when sinking a shaft.

*Bowke* (Staffordshire).—A small wooden box for hauling ironstone underground.

*Box.*—(1) A 12′ to 14′ section of a sluice. (2) A mine car.

*Box Bill.*—Tool for recovering boring rods.

*Boxing.*—A method of securing shafts solely by slabs and wooden pegs.

*Brace.*—(1) An inclined beam, bar, or strut for sustaining compression or tension. See *Tie-Brace, Sway-Brace.* (2) A platform at the top of a shaft on which miners stand to work the tackle. (3) (Cornish) Building at pit mouth.

*Brace Heads.*—Wooden handles or bars for raising and rotating the rods when boring a deep hole.

*Braize.*—(1) Charcoal dust. (2) Fine coke refuse or breeze.

*Brake Sieve.*—Hand jigger.

*Brances.*—Iron pyrites in coal.

*Branch.*—Small vein shooting off from main lode.

*Brashy.*—Short and tender.

*Brasque.*—A mixture of clay and coke or charcoal used for furnace bottoms.

*Brass.*—(1) Iron pyrites in coal. (2) An alloy of copper and zinc.

*Brasses* (English).—Fitting of brass in plummer blocks, etc., for diminishing the friction of revolving journals that rest upon them.

*Brat.*—A thin bed of coal mixed with pyrites or limestone.

*Brattice.*—A lining or partition.

*Brattice Cloth.*—Ducking or canvas used for making a brattice.

*Brazzil* (North of England).—Iron pyrites in coal.

*Breaker.*—In anthracite mining, the structure in which the coal is broken, sized, and cleaned for market. Known also as *Coal Breaker.*

*Breaker Boy.*—A boy who works in a coal breaker.

*Breakstaff.*—The lever for blowing a blacksmiths' bellows, or for working bore rods up and down.

*Breakthrough.*—A narrow passage cut through a pillar connecting rooms.

*Breast.*—(1) A stall, board, or room in which coal is mined. (2) The face or wall of a quarry is sometimes called by this name.

*Breast-and-Pillar.*—A system of working coal by boards or rooms with pillars of coal between them.

*Breast Wall* (English).—A wall built to prevent the falling of a vertical face cut into the natural soil.

*Breccia.*—A rock composed of angular fragments cemented together.

*Breeding Fire.*—See *Gob Fire.*

*Breese.*—Fine slack.

*Breeze.*—Small coke, probably same as braize or braise.

*Brettis* (Derbyshire).—A timber crib filled with slack.

*Bridge.*—(1) A platform on wheels running on rails for covering the mouth of a shaft or slope. (2) A track or platform passing over an inclined haulageway and which can be raised out of the way of ascending and descending cars. (3) An air crossing.

*Bridle Bar.*—The transverse bar connecting the points of a switch.

*Bridle Chains.*—Short chains by which a cage, car, or gunboat is attached to a winding rope; of use in case the rope pulls out of its socket.

*Briquets.*—Fuel made of slack or culm and pressed into brick form.

*Broaching Bit.*—A tool for reopening a bore hole that has been partially closed by swelling of the walls.

*Brob.*—A spike to prevent timber slipping.

*Broil* (Cornish).—Traces of a vein in loose matter.

*Broken.*—A district of coal pillars in process of removal, so called in contra-distinction to the first working of a seam by bord-and-wall, or working in the "whole." See *Whole Working*.

*Broken Coal.*—Anthracite coal that will pass through a mesh or bars about 3¼ to 4⅛ in., and over a mesh 2⅜ in. square.

*Bronce* (Mexican).—In mining, copper or iron pyrites.

*Brooching.*—Smoothing.

*Brow.*—An underground roadway leading to a working place driven either to the rise or to the dip.

*Brown Coal.*—Lignite. A fuel classed between peat and bituminous coal.

*Brown Spar.*—Dolomite containing carbonate of iron.

*Brownstone.*—(1) Decomposed iron pyrites. (2) Brown sandstone.

*Brujula* (Mexican).—A surveyors' (or marine) magnetic compass.

*Brush.*—(1) To mix air with the gas in a mine working by swinging a jacket, etc., which creates a current. (2) To "*brush*" the roof is to take down some of the roof slate to increase the height or headroom.

*Bryle* (Cornish).—Traces of a vein in loose matter.

*Bucket.*—(1) An iron or wooden receptacle for hoisting ore, or for raising rock in shaft sinking. (2) The top valve or clack of a pump.

*Bucket Pump.*—A lifting pump, consisting of buckets fastened to an endless belt or chain.

*Bucket Sword.*—A wrought-iron rod to which the pump bucket is attached.

*Bucket Tree.*—The pipe between the working barrel and the wind bore.

*Bucking Hammer.*—An iron disk, provided with a handle, used for breaking up minerals by hand.

*Buckstay.*—An iron or steel brace resting upon or built into a boiler setting or furnace wall to support the brickwork.

*Buckwheat.*—Anthracite coal that will pass through a mesh of about ½ in. and over a mesh ¼ in.

*Buddling.*—Washing.

*Bug Dust.*—Fine coal; the cutting produced by a chain machine or puncher.

*Buggy.*—A small mine car.

*Bug Hole.*—A small cavity usually lined with crystals.

*Building.*—A built-up block or pillar of stone or coal to support the roof.

*Bulkhead.*—(1) A tight partition or stopping. (2) The end of a flume carrying water for hydraulicking.

*Bull.*—An iron rod used in ramming clay to line a shot hole.

*Bulldog.*—A barney.

*Bull Engine.*—A single, direct-acting pumping engine, the pump rods forming a continuation of the piston rod.

*Buller Shot.*—A second shot put in close to, and to do the work not done by, a blown-out shot, loose powder being used.

*Bulling.*—Lining a shot hole with clay.

*Bull Pump.*—A single-acting pumping engine in which the steam cylinder is placed over the shaft or slope and the pump rods are attached directly to the piston rod. The steam enters below the piston and raises the pump rods; the water is pumped on the down stroke by the weight of the rods.

*Bull Pup.*—A worthless claim.

*Bull Wheel.*—(1) A wheel on which the rope carrying the boring rod is coiled when boring by steam machinery. (2) The principal wheel of any machine, usually a driving wheel.

*Bully.*—A miners' hammer.

*Bump.*—See *Bounce*.

*Bunding.*—A staging in a level for carrying débris.

*Bunkers.*—(1) Steam coal consumed on board ship. (2) Receptacles placed near a boiler for holding a supply of fuel.

*Buntons.*—Timbers placed horizontally across a shaft or slope to carry the cage guides, pump rods, column pipe, etc.; also, to strengthen the shaft timbering.

*Burden.*—(1) Earth overlying a bed of useful mineral. (2) The proportion of ore and flux to fuel in the charge of a blast furnace.

*Burr.*—Solid rock.

*Burrow.*—Refuse heap.

*Buscones* (Spanish).—Prospectors, fossickers, tribute workers.

*Bush.*—To line a circular hole with a ring of metal, to prevent the hole from wearing out.

*Butt.*—(1) Coal surface exposed at right angles to the face; the "*ends*" of

the coal. (2) The butt of a slate quarry is where the overlying rock comes in contact with an inclined stratum of slate rock.

*Butt Entry.*—A gallery driven at right angles with the butt joint.

*Butterfly Valve.*—A circular valve that revolves on an axis passing through its center.

*Butt Heading.*—See *Butt Entry.*

*Butty.*—A partner in a contract for driving or mining; a comrade, crony. Sometimes called "*Buddy.*"

*By Level.*—A side level driven for some unusual but necessary purpose.

*Byproduct Oven.*—A coke oven arranged to conserve and recover the various byproducts of the coking process.

*Byproducts.*—Products of coking other than coke. The more common byproducts are gas, tar, benzol and ammonium sulphate.

*Caballo* (Mexican).—A "horse" or mass of barren rock in a vein.

*Cabin.*—(1) A miner's house. (2) A small room in the mine for the use of the officials.

*Cable Drilling.*—Rope drilling.

*Cage.*—A platform on which mine cars are raised to the surface.

*Cage Guides.*—Vertical rods of pine, iron, or steel, or wire rope, fixed in a shaft, between which cages run, and whereby they are prevented from striking one another or against any portion of the shaft.

*Cager.*—The person that puts the cars on the cage at the bottom or top of the shaft.

*Cage Seat.*—Scaffolding, sometimes fitted with strong springs, to receive the shock, and on which the cage drops when reaching the pit bottom.

*Cage Sheets.*—Short props or catches on which cages stand during caging or changing cars.

*Caking Coal.*—Coal that agglomerates on the grate.

*Cala* (Spanish).—Prospecting pit.

*Calcareous.*—Containing lime.

*Calcine.*—To heat a substance, not sufficiently to melt it, but enough to drive off the volatile contents.

*California Pump.*—A rude pump made of a wooden box through which an endless belt with floats circulates; used for pumping water from shallow ground.

*Callys* (Cornish).—Stratified rocks traversed by lodes.

*Cam.*—(1) A curved arm attached to a revolving shaft for raising stamps. (2) Carbonate of lime and fluorspar, found on the joints of lodes.

*Camino* (Mexican).—Any gallery, winze, or shaft inside of a mine used for general transit.

*Campaign.*—The length of time a furnace remains in blast.

*Cañada* (Mexican).—See *Barranca.*

*Canch,* or *Caunche.*—(1) A thickness of stone required to be removed to make height or to improve the gradient of a road at a fault. If above a seam, it is termed a "top canch;" if below, a "bottom canch." (2) A trench with sloping sides and very narrow bottom.

*Cand* (Cornish).—Fluorspar.

*Cank* (Derbyshire).—Whinstone.

*Canker.*—The ocherous sediment in coal-pit waters.

*Cannel Coal.*—See Classification of Coals (page 378).

*Cañon* (Mexican).—(1) A level, drift, or gallery within a mine. (2) A steep-sided ravine. *Cañon de Guia.*—A drift along the vein.

*Cants* (English).—The pieces forming the ends of buckets of a waterwheel.

*Cap.*—(1) A piece of plank placed on top of a prop. See, also, *Collar.* (2) The pale bluish elongation of the flame of a lamp caused by the presence of gas.

*Cap Rock.*—The upper rock that covers the bed rock.

*Capstan.*—A vertical axle used for heavy hoisting, and worked by horizontal arms or bars.

*Captain.*—Cornish name for manager or boss of a mine.

*Car.*—Any car used for the conveyance of coal along the gangways or haulage roads of a mine.

*Carat.*—A weight nearly equal to 4 grains.

*Carbon.*—A combustible elementary substance forming the largest component part of coal.

*Carbonaceous.*—Coaly, containing carbon or coal.

*Carbonate.*—Carbonic acid combined with a base.

*Carboniferous.*—Containing or carrying coal.

*Carga* (Mexican).—A charge.  A mule load, generally of 300 lb., but variable in different parts of Mexico.

*Carriage.*—See *Cage* and *Slope Cage.*

*Cartridge.*—Paper or waterproof cylindrical case filled with explosive forming the charge for blasting.

*Cascajo* (Mexican).—Gravel.

*Case.*—A fissure admitting water into a mine.

*Case-harden.*—To convert the outer surface of wrought iron into hard steel by heating it while in contact with charcoal, cyanide, etc., and quenching.

*Casing.*—Tubing inserted in a bore hole to keep out water or to protect the sides from collapsing.

*Cast Iron.*—Pig iron that contains carbon (up to 5%), silicon, sulphur, phosphorus, etc.

*Cata* (Spanish).—A mine denounced but not worked.

*Catches.*—(1) Iron levers or props at the top and bottom of a shaft.  (2) Stops fitted on a cage to prevent cars from running off.

*Cauf* (North of England).—A coal bucket or basket.

*Cauldron Bottoms.*—The fossil remains or the "casts" of the trunks of sigillaria that have remained vertical above or below the seam.

*Caulk.*—To fill seams or joints with something to prevent leaking.

*Caunter*, or *Caunter Lode* (Cornish).—A vein running obliquely across the regular veins of the district.

*Cave*, or *Cave In.*—A caving-in of the roof strata of a mine, sometimes extending to the surface.

*Cavils.*—Lots drawn by the hewers each quarter year to determine their working places.

*Cement.*—A binding material.

*Center.*—A temporary support, serving at the same time as a guide to the masons, placed under an arch during the progress of its construction.

*Centrifugal Force.*—A force drawing away from the center.

*Centripetal Force.*—A force drawing toward the center.

*CH₄.*—Marsh gas (see page 859).

*Chain.*—A measure 66 or 100 ft. long, divided into 100 links.

*Chain-Brow Way.*—An underground inclined plane worked on the endless-chain system of haulage.

*Chain Pillar.*—A pillar left to protect the gangway and air-course, and running parallel to these passages.

*Chain Road.*—An underground wagonway worked on the endless-chain system of haulage.

*Chair.*—Sometimes applied to *keeps.*

*Chamber.*—See *Breast.*

*Charco* (Mexican).—A pool of water.

*Charge.*—(1) The amount of powder or other explosive used in one blast or shot.  (2) The material fed into a furnace at one time.

*Charquear*(Mexican).—To dip out water from pools within the mine, throwing it into gutters or pipes that will conduct it to the shaft.

*Check.*—A metal token used to identify the cars loaded by each particular miner.

*Check-Battery.*—A battery to close the lower part of a chute, acting as a check to the flow of coal and as an air stopping.

*Checker Coal.*—Anthracite coal that seems to be made up of rectangular grains.

*Check-Weighman.*—A man appointed and paid by the miners to check the weighing of the coal at the surface.

*Cheek.*—Wall.

*Chestnut Coal.*—Anthracite coal that will pass through a mesh 1⅝ in. square and over a mesh ¾ in. square (see page 952).

*Chiflon* (Mexican).—A narrow drift directed obliquely downwards, any pipe from which issues water or air under pressure, or at high velocity.

*Chilian Mill.*—A roller mill for crushing ore or other material.

*Chill Hardening.*—Giving a greater hardness to the outside of cast iron by pouring it into iron molds, which causes the skin of the casting to cool rapidly.

*Chimney.*—A furnace or air stack.

*Chinese Pump.*—Like a California pump, but made entirely of wood.

*Chock.*—(1) A square pillar for supporting the roof, constructed of prop timber laid up in alternate cross-layers, in log-cabin style, the center

being filled with waste. (2) A wooden or other block used to prevent the movement of a car or other body. (3) To secure with chocks.

*Chokedamp.*—See *Blackdamp.*

*Churn Drill.*—A long iron bar with a cutting end of steel, worked by raising and letting it fall. When worked by blows of a hammer or sledge, it is called a "jumper."

*Chute* (also spelled *Shute*).—(1) A narrow inclined passage in a mine, down which coal or ore is either pushed or slides by gravity. (2) The loading chute of a tipple.

*Cielo.*— (Mexican).—A ceiling. *Trabajar de Cielo.*—Overhead stoping.

*Clack.*—A valve that is opened and closed by the force of the water; a check valve.

*Clack Door.*—The opening into the valve chamber to facilitate repairs and renewals without unseating the pump or breaking the connections.

*Clack Piece.*—The casting forming the valve chamber.

*Clack Seal.*—The receptacle for the valve to rest on.

*Claggy* (North of England).—When coal is tightly joined to the roof.

*Claim.*—A portion of ground staked out and held by virtue of a miner's right.

*Clanny.*—A type of safety lamp invented by Dr. Clanny.

*Clastic.*—Constituted of rocks or minerals that are fragments derived from other rocks.

*Clay Band.*—Argillaceous iron ore; common in many coal measures.

*Clay Course.*—A clay seam or gouge found at the sides of some veins.

*Claying Bar.*—A bar for molding clay in a wet bore hole.

*Clearance.*—(1) The distance between the piston at the end of its stroke and the end of the cylinder. (2) The volume or entire space filled with steam at end of a stroke, including the space between piston and cylinder head, and the steam ducts to the valve seat.

*Cleat.*—(1) Vertical cleavage of coal seams, irrespective of dip or strike. (2) A small piece of wood nailed to two planks to keep them together, or nailed to any structure to make a support for something else.

*Cleavage.*—The property of splitting more readily in some directions than in others.

*Clinometer.*—An instrument used to measure the angle of dip.

*Clod.*—Soft and tough shale or slate forming the roof or floor of a coal seam.

*Clunch* (English).—Under clay, fireclay.

*Clutch.*—A device for transmitting motion at will from one shaft to another or to some other machine part such as a pulley, or *vice versa*.

*Coal Breaker.*—See *Breaker.*

*Coal Cutter.*—A machine for holing or undercutting coal.

*Coal Dust.*—Very finely powdered coal suspended in the airways or deposited along the passages of a mine.

*Coal Measures.*—Strata of coal with the attendant rocks.

*Coal Pipes* (North of England).—Very thin irregular coal beds.

*Coal Road.*—An underground roadway or heading in coal.

*Coal Smut.*—See *Blossom.*

*Coaly Rashings.*—Soft dark shale, in small pieces, containing much carbonaceous matter.

*Cobbing Hammer.*—A short two-faced hammer for breaking minerals to sizes.

*Cockermeg,* or *Cockers.*—Timber used to hold coal face while it is being undercut.

*Cod* (North of England).—The bearing of an axle.

*Cofer* (Derbyshire).—To calk a shaft by ramming clay behind the lining.

*Coffer.*—Mortar box of a battery.

*Coffer Dam.*—An enclosure built in the water, and then pumped dry so as to permit masonry or other work to be carried on inside of it.

*Coffin* (Cornish).—An old pit.

*Cog.*—(1) A chock. (2) A wooden gear tooth. (3) Loosely, any gear tooth or even gear wheel

*Cohete* (Mexican).—A rocket; applied to a blast within a mine or outside.

*Coil Drag.*—A tool for picking pebbles, etc., from drill holes.

*Coke.*—The fixed carbon and ash of coal sintered together.

*Collar.*—(1) A flat ring surrounding anything closely. (2) Collar of a mine shaft is the first wood frame of the shaft. (3) The bar or crosspiece of a framing in entry timbering. (4) The mouth or portal of a slope or the first set of timber therein.

*Colliery.*—The whole coal mine plant, including the mine and all adjuncts.

*Colliery Warnings* (English).—Telegraphic messages sent from signal-service

stations to the principal colliery centers to warn managers of mines when sudden falls of the barometer occur.

*Column, or Column Pipe.*—The pipe conveying the drainage water from the mine to the surface.

*Comer* (Mexican).—To eat. *Comerse los Pilares.*—To take out the last vestiges of mineral from the sides and rock pillars of a mine.

*Conchoidal.*—Shell-like, such as the curved fracture of flint.

*Concrete.*—Artificial stone, formed by mixing broken stone, gravel, etc., with lime, cement, tar, or other binder. When hydraulic cement is used instead of lime, the mixture is called *beton* (English).

*Concretion.*—A cemented aggregation of one or more kinds of minerals around a nucleus.

*Conduit.*—(1) A covered waterway. (2) An airway. (3) A pipe or box for enclosing and protecting an electrical conductor.

*Conduit Hole.*—A flat hole drilled for blasting up a thin piece in the bottom of a level.

*Conductors* (English).—See *Guides.*

*Conformable.*—Strata are conformable when they lie one over the other with the same dip.

*Conglomerate.*—The rock formation underlying the *Coal Measures*; a rock containing or consisting of pebbles, or of fragments of other rocks cemented together; English *Pudding Rock* or millstone grit.

*Conical Drum.*—The rope roll or drum of a winding engine, constructed in the form of two truncated cones placed back to back, the outer ends being usually the smaller in diameter.

*Contact.*—Union of different formations.

*Contact Lode or Vein.*—A vein lying between two differently constituted rocks.

*Contour.*—(1) The line that bounds the figure of an object. (2) In surveying, a contour line is a line every point of which is at an equal elevation.

*Contramina* (Mexican).—Countermine. Any communication between two or more mines. Also, a tunnel communicating with a shaft.

*Cope,* or *Coup.*—An exchange of working places between hewers.

*Corbond.*—An irregular mass from a lode.

*Cord.*—(1) A cord weighs about 8 tons. (2) ·128 cu. ft. of firewood.

*Core Drill.*—A diamond or other hollow drill for securing cores.

*Cores.*—Cylinder-shaped pieces of rock produced by the diamond-drill system of boring.

*Corf.*—A mine wagon or tub.

*Cornish Pumps.*—A single-acting pump, in which the motion is transmitted through a walking beam; in other respects similar to a *Bull Pump.*

*Cortar Pillar* (Mexican).—To form a rock support or pillar within a mine, at the opening of a cross-cut or elsewhere.

*Cortar Sogas* (Mexican).—Literally, to cut the ropes. To abandon the mine, taking away everything useful or movable.

*Corve.*—A mining wagon or tub.

*Costean* (Cornish).—To prospect a lode by sinking pits on its supposed course.

*Costeaning.*—Trenching for a lode.

*Cost Book* (Cornish).—Mining accounts.

*Cotton Rock.*—(1) Decomposed chert. (2) A variety of earthy limestone.

*Coulee.*—(1) A solidified stream or sheet of lava extending down a volcano, often forming a ridge or spur. (2) A deep gulch or water channel, usually dry.

*Counter.*—(1) A cross-vein. (2) (English) An apparatus for recording the number of strokes made by a Cornish pumping engine or other machine, or the revolutions of a shaft or pully. (3) A secondary haulageway in a coal mine.

*Counterchute.*—A chute down which coal is dumped to a lower level or gangway.

*Countergangway.*—A level or gangway driven at a higher level than the main one.

*Country.*—The formation traversed by a lode.

*Country Rock.*—The main rock of the region through which the veins cut, or that surrounding the veins.

*Course.*—The direction of a line in regard to the points of compass.

*Coursing* or *Coursing the Air.*—Conducting it through the different portions of a mine by means of doors, stoppings, and brattices.

*Cow.*—A self-acting brake.

*Coyoting.*—Irregular mining by small pits.

*Crab.*—A variety of windlass or capstan consisting of a short shaft or axle, either horizontal or vertical, which serves as a rope drum for raising weights; it may be worked by a winch or handspikes.

*Crab Holes.*—Holes often met with in the bed rock of alluvial. Also depressions on the surface owing to unequal disintegration of the underlying rock; a sink hole or pot hole.

*Cradle Dump.*—A rocking tipple for dumping cars. See *Dump.*

*Cramp* (English).—(1) A short bar of metal having its two ends bent downwards at right angles for insertion into two adjoining pieces of stone, wood, etc., to hold them together. (2) A pillar left for support in a mine.

*Cranch.*—Part of a vein left by previous workers.

*Crane* (English).—A hoisting machine consisting of a revolving vertical post or stalk, a projecting jib, and a stay for sustaining the outer end of the jib; these do not change their relative positions as they do in a derrick. There is also a rope drum with winding rope, etc. (2) Any movable or traveling lifting device.

*Creep.*—The gradual upheaval of the floor or sagging of the roof of mine workings due to the weighting action of the roof and a tender floor.

*Creston* (Mexican).—The outcrop or apex of a vein or mineral deposit.

*Crevice.*—A fissure.

*Criadero* (Mexican).—(1) A mineral deposit of irregular form, not vein-like. (2) Any mineral deposit. This latter is the more modern sense, and the word is so used in the mining laws at present in force in Mexico.

*Crib.*—(1) A structure composed of horizontal timbers laid on one another, or a framework built like a log cabin. See *Chock.* (2) A miner's luncheon. (3) See *Curb.*

*Crib Kettle.*—A dinner pail.

*Cribbing.*—Close timbering, as the lining of a shaft, or the construction of cribs of timber, or timber and earth or rock to support a roof.

*Cribble.*—A sieve.

*Crisol* (Mexican).—A crucible of any kind.

*Crop.*—See *Outcrop.*

*Crop Fall.*—A caving in of the surface at or near the outcrop of a bed of coal.

*Cropping Coal.*—The leaving of a small thickness of coal at the bottom of the seam in a working place, usually in order to keep back water. The coal so left is termed "*Cropper Coal.*"

*Cropping Out.*—Appearing at the surface; outcropping.

*Croppings.*—Portions of a vein as seen exposed at the surface.

*Cross-Course.*—A vein lying more or less at right angles to the regular vein of the district.

*Crosscut.*—(1) A tunnel driven through or across the measures from one seam to another. (2) A small passageway driven at right angles to the main gangway to connect it with a parallel gangway or air-course.

*Crosses and Holes* (Derbyshire).—Made in the ground by the discoverer of a lode to temporarily secure possession.

*Cross-Heading.*—A passage driven for ventilation from the airway to the gangway, or from one breast through the pillar to the adjoining working.

*Cross-Heading,* or *Cross-Gateway.*—A road kept through goaf and cutting off the gateways at right angles or diagonally.

*Cross-Hole.*—See *Crosscut* (2).

*Cross-Latches.*—See *Latches.*

*Cross-Vein.*—An intersecting vein.

*Crouan* (Cornish).—Granite.

*Crowbar.*—A strong iron bar with a slightly curved or flattened end.

*Crowfoot.*—A tool for drawing boring rods.

*Crown Tree.*—A piece of timber set on props to support the roof.

*Crucero* (Mexican).—A crosscut for ventilation to get around a horse, or to prospect for the vein.

*Crucible.*—(1) The bottom of a cupola furnace in which the molten materials collect. (2) Pots for smelting assays in or used in making coal analyses.

*Crush.*—See *Squeeze, Thrust.*

*Crusher.*—A machine used for crushing ores, rock or coal.

*Crushing.*—Reduction of mineral in size by machinery.

*Crystal.*—A solid of definite geometrical form which mineral (or sometimes organic) matter has assumed.

*Culm.*—Anthracite-coal dirt.

*Culm Bank,* or *Culm Dump.*—Heaps of culm now generally kept separate from the rock and slate dumps.

*Cuña (Mexican).*—Literally, a wedge. A short drill or picker generally known in the United States as a "gad."

*Cundy.*—The open space in the gob of long-wall work.

*Cupel.*—A cup made of bone ash for absorbing litharge.

*Curb.*—(1) A timber frame intended as a support or foundation for the lining of a shaft. (2) The heavy frame or sill at the top of a shaft.

*Curbing.*—The wooden lining of a shaft.

*Curtain.*—A sheet of canvas or other material used to control or deflect an air current.

*Cut.*—(1) To strike or reach a vein. (2) To excavate in the side of a hill.

*Cutter.*—A term employed in speaking of any coal-cutting or rock-cutting machines, the men operating them, or the men engaged in underholing by pick or drill.

*Cutting Down.*—To cut down a shaft is to increase its sectional area.

*Dam.*—A timber bulkhead, or a masonry or brick stopping built to prevent the water in old workings from flooding other workings, or to confine the water in a mine flooded to drown out a mine fire.

*Damp.*—Mine gases and gaseous mixtures are called damps. See also *Afterdamp, Blackdamp, Firedamp, Stinkdamp.*

*Dan (North of England).*—A truck without wheels.

*Danger Board.*—See *Fireboard.*

*Dant (North of England).*—Soft inferior coal.

*Dap.*—A notch cut in a timber to receive another timber.

*Datum Water Level.*—The level at which water is first struck in a shaft sunk on a reef or gutter.

*Davy.*—A safety lamp invented by Sir Humphrey Davy.

*Day.*—Light seen at the top of a shaft.

*Day Fall.*—See *Crop Fall.*

*Day Shift.*—The relay of men working in the daytime.

*Dead.*—The air of a mine is said to be dead or heavy when it contains carbonic-acid gas, or when the ventilation is sluggish.

*Dead.*—(1) Unproductive. (2) Unventilated.

*Dead Roast.*—To completely drive off all volatile substances.

*Deads.*—Waste or rubbish from a mine.

*Dead Work.*—Exploratory or prospecting work that is not directly productive; brushing roof, lifting bottom, cleaning up falls, blowing rock, etc.

*Dean (Cornish).*—The end of a level.

*Débris.*—Fragments from any kind of disintegration.

*Deep (English).*—(1) "To the deep," toward the lower portion of a mine; hence, the lower workings. (2) A passage driven downward in the measure being worked. The main deep is the principal or hoisting slope.

*Delta.*—A triangularly shaped piece of alluvial land at the mouth of a river.

*Demasia (Mexican).*—A piece of unoccupied ground between two mining concessions.

*Denudation.*—The laying bare by water or other agency.

*Denuncio (Mexican).*—Denouncement. The act of applying for a mining concession under the old mining laws.

*Deposit.*—(1) Irregular mineral bodies not veins. (2) A bed or any sedimentary formation.

*Deputy (English).*—(1) A man who fixes and withdraws the timber supporting the roof of a mine, and attends to the safety of the roof and sides, builds stoppings, puts up bratticing, and looks after the safety of the hewers, etc. (2) An underground official who sees to the general safety of a certain number of stalls or of a district, but does not set the timber himself, although he has to see that it is properly and sufficiently done. (3) (American) A deputy sheriff.

*Derrick.*—(1) A crane in which the rope or chain forming the stay can be let out or hauled in at pleasure, thus altering the inclination of a jib. (2) The structure erected to sink a drill hole and the framework above shafts are sometimes called by this name.

*Derrumbe,* or *Derrumbamineto (Mexican).*—The caving in of the whole or a portion of a mine.

*Desaguador (Spanish).*—A water pipe or drain.

*Desague (Mexican).*—Drainage of a mine by any means.

*Descargar* (Mexican).—Literally, "to unload." *Descargar un Horno.*—To tear down a furnace.

*Descubridora* (Mexican).—The first mine opened in a new district or on a new mineral deposit.

*Desmontar* (Mexican).—Literally, to clear away underbrush. In mining, to take away useless and barren rocks; to remove rubbish.

*Despensa* (Mexican).—(1) A pantry or storeroom. (2) A secure room to lock up rich ore.

*Despoblar* (Mexican).—To suspend work in a mine.

*Dessue* (Cornish).—To cut away the ground beside a thin vein so as to remove the latter entire.

*Destajo* (Mexican).—(1) A contract to do any kind of work in or about a mine or elsewhere for a fixed price. (2) Piece work, as distinguished from time work. *Destajero.*—A contractor for piece work.

*Detaching Hook.*—A self-acting mechanical contrivance for setting free a winding rope from a cage when the latter is raised beyond a certain point in the head-gear; the rope being released, the cage remains suspended in the frame.

*Diagonal Joints.*—Joints diagonal to the strike of the cleavage.

*Dial* (English).—An instrument similar to a surveyor's compass, with vernier attached.

*Dialing.*—Surveying.

*Die.*—The bottom iron block of a battery, or grinding pan on which the shoe acts.

*Digger or Dredge.*—A machine for removing coal from the bed of streams, the coal having washed down from collieries or culm banks above.

*Digging.*—Mining operations in coal or other minerals.

*Dike.*—See also *Dyke.*

*Dillies*, or *Ginneys.*—Short self-acting inclines where one or two tubs at a time are run.

*Dip.*—(1) To slope downwards. (2) The inclination of strata with a horizontal plane. (3) The lower workings of a mine.

*Dip Joint.*—Vertical joints about parallel to the direction of the cleavage dip.

*Dippa* (Cornish).—A small catch-water pit.

*Dirt Fault.*—A confusion in a seam of coal, the top and bottom of the seam being well defined, but the body of the vein being soft and dirty.

*Disintegration.*—Separation by mechanical means, not by decomposition.

*Ditch.*—(1) The drainage gutter in a mine. (2) A drainage gutter on the surface. (3) An open conveyor of water for hydraulic or irrigation purposes.

*Divide.*—The top of a ridge, hill, or mountain.

*Dividing Slate.*—A stratum of slate separating two benches of coal. See *Parting.*

*Divining*, or *Dowsing Rod.*—A small forked hazel twig that, when held loosely in the hands, is supposed to dip downwards when passing over water or metallic minerals.

*Dizzue* (Cornish).—See *Dessue.*

*Dog.*—(1) An iron bar, spiked at the ends, with which timbers are held together or steadied. (2) A short heavy iron bar, used as a drag behind a car or trip of cars when ascending a slope to prevent their running back back down the slope in case of accident. See *Drag.* (3) A pawl.

*Dog Hole.*—A little opening from one place in a mine to another, smaller than a breakthrough.

*Dog Iron.*—A short bar of iron with both ends pointed and bent down so as to hold together two pieces of wood into which the points are driven. Or one end may be bent down and pointed, while the other is formed into an eye, so that if the point be driven into a log, the other end may be used to haul on.

*Dolly.*—(1) A machine for breaking up minerals, being a rough pestle and mortar, the former being attached to a spring pole by a rope. (2) A tool used to sharpen drills. (3) A bar against which rivets are driven.

*Donk* (North of England).—Soft mineral found in cross-veins.

*Donkey Engine* (English).—(1) A small steam engine attached to a large one, and fed from the same boiler; used for pumping water into the boiler. (2) A small steam engine.

*Door Piece* (English).—The portion of a lift of pumps in which the clack or valve is situated.

*Doors.*—Wooden doors in underground roads or airways to deflect the air current.

*Door Tender.*—A boy whose duty it is to open and close a mine door before and after the passage of a train of mine cars; a trapper.

*Dope.*—(1) An absorbent for holding a thick liquid. The material that absorbs the nitroglycerine in explosives. (2) Powder cartridges.

*Double Shift.*—When there are two sets of men at work, one set relieving the other.

*Double Tape Fuse.*—Fuse of superior quality, or having a heavier and stronger covering.

*Double Timber.*—Two props with a bar placed across the tops of them to support the roof and sides.

*Downcast.*—The opening through which the fresh air is drawn or forced into the mine; the intake.

*Draftage.*—A deduction made from the gross weight of mineral when transported, to allow for loss.

*Drag.*—(1) The frictional resistance offered to a current of air in a mine. (2) See *Dog.*

*Draw.*—(1) To "draw" the pillars; robbing the pillars after the breasts are exhausted. (2) An effect of creep upon the pillars of a mine.

*Draw a Charge.*—(1) To take a charge from a furnace. (2) Remove explosives (3) Removing the coke from an oven.

*Drawlift.*—A pump that receives its water by suction and will not force it above its head.

*Draw-Hole.*—An aperture in a battery through which the coal is drawn.

*Draw Slate.*—Fragile slate above a coal measure, which must be removed to prevent caving.

*Drawing an Entry.*—Removing the last of the coal from an entry.

*Drawn.*—The condition in which an entry or room is left after all the coal has been removed. See *Robbed.*

*Dresser* (Staffordshire).—A large coal pick.

*Drift.*—(1) A horizontal passage underground. A drift follows the vein, as distinguished from a crosscut, which intersects it, or a level or gallery, which may do either. (2) In coal mining, a gangway above water, level, driven from the surface in the seam. (3) Unstratified diluvium.

*Drifting.*—(1) Driving a drift. (2) Cars, locomotives, etc., "drift" when they will run by gravity but not attain a dangerous speed.

*Drill.*—An instrument used in boring holes.

*Drive* (*Drift*).—A horizontal passage in a lode.

*Drive.*—To cut an opening through strata.

*Driving.*—Excavating horizontal passages, in contradistinction to sinking or raising.

*Driving on Line.*—Keeping a heading or breast accurately on a given course by means of a compass or transit.

*Dropper.*—(1) A spur dropping into the lode. (2) A feeder. (3) A branch leaving the vein on the footwall side. (4) Water dropping from the roof.

*Drop Shaft.*—A monkey shaft down which earth and other matter are lowered by means of a drop (i.e., a kind of pulley with break attached); the empty bucket is brought up as the full one is lowered.

*Druggon* (Staffordshire).—A vessel for carrying fresh water into a mine.

*Drum.*—The cylinder or pulley on which the winding ropes are coiled or wound.

*Drum Rings.*—Cast-iron rings with projections to which are bolted the lagging forming the surface for the ropes to lap upon.

*Drummy.*—Sounding loose, open, shaky, or dangerous when tested.

*Druse.*—A cavity lined with small crystals.

*Duck Machine.*—An arrangement of two boxes, one working within the other, for forcing air into mines.

*Duelas* (Mexican).—Staves of a barrel or cask, etc.

*Dumb'd.*—Choked, said of a sieve or grating.

*Dumb Drift.*—A short tunnel or passage connecting the main return airways of a mine with the upcast shaft some distance above the furnace, in order to prevent the return air laden with mine gases from passing through or over the ventilating furnace.

*Dump.*—(1) A pile or heap of ore, coal, culm, slate, or rock. (2) The tipple by which the cars are dumped. (3) To unload a car by tipping it up. (4) The pile of mullock as discharged from a mine.

*Dumper.*—A car so constructed that the body may be revolved to dump the material in front or on either side of the track.

*Durn* (Cornish).—A timber frame.

*Durr* (German).—Barren ground.

*Dust.*—See *Coal Dust.*

*Duty.*—The unit of measure of the work of a pumping engine expressed in foot-pounds of work obtained from a bushel, or 100 lb., or other unit of fuel.

*Dyke,* or *Dike.*—(1) A wall of igneous rock passing through strata, with or without accompanying dislocation of the strata. (2) A fissure filled with igneous matter. (3) Barren rock.

*Dzhu* (Cornish).—See *dessue.*

**Ear.**—The inlet or intake of a fan.

*Echado* (Mexican).—The dip of the vein.

*Edge Coals* (English).—Highly inclined seams of coal, or those having a dip greater than 30°.

*Efflorescence.*—An incrustation by a secondary mineral, due to loss of water of crystallization.

*Egg Coal.*—Anthracite coal that will pass through a 2⅜-in. square mesh and over a 2-in. square mesh (see page 952).

*Elbow.*—A sharp bend, as in a lode or pipe, a pipe-bend fitting.

*Electric Blast.*—Instantaneous blasting of material by means of electricity.

*Elevator Pump.*—An endless band with buckets attached, running over two drums for draining shallow ground.

*Elvan.*—A Cornish name applied to most dike rocks of that county, irrespective of the mineral constitution, but in the present day restricted to quartz porphyries.

*Emborrascarse* (Mexican).—To go barren by the vein terminating or pinching out, etc.

*Empties.*—Empty mine or railroad cars.

*Encino* (Mexican).—Live oak.

*End Joint* (End Cleat).—A joint or cleat in a seam about at right angles to the principal or face cleats.

*Endless Chain.*—A system of haulage or pumping by the moving of an endless chain.

*Endless Rope.*—A system of haulage same as endless chain, except that a wire rope is used instead of chain.

*End,* or *End-On.*—Working a seam of coal at right angles to the principal or face cleats.

*Engine Plane.*—An incline up which loaded cars are drawn by a rope operated by an engine located at the top or bottom of the incline. The empty cars descend by gravity, pulling the rope after them.

*Engineer.*—(1) One who has charge of the surveying or machinery about a mine. (2) One who runs an engine.

*Entibar* (Mexican).—To timber a mine or any part thereof.

*Entry.*—A main haulage road, gangway, or airway. An underground passage used for haulage or ventilation, or as a manway.

*Entry Stumps.*—Pillars of coal left in the mouths of abandoned rooms to support the road, entry, or gangway till the entry pillars are drawn.

*Erosion.*—The wearing away of rocks by the elements.

*Escaleras* (Mexican).—Ladders, generally made of notched sticks.

*Escarpment.*—A nearly vertical natural face of rock or soil.

*Escoria* (Mexican).—Slag of cinders.

*Escorial.*—Slag pile.

*Exploder.*—A chemical employed for the instantaneous explosion of powder.

*Exploitation.*—The working of a mine, and similar undertakings; the examination instituted for that purpose.

*Exploration.*—Development.

*Explosion.*—Sudden ignition of a body of firedamp, dust, etc.

*Eye* (English).—(1) A circular hole in a bar for receiving a pin and for other purposes. (2) The eye of a shaft is the very beginning of a pit. (3) The eye of a fan is the central or intake opening.

**Face.**—(1) The place at which the material is actually being worked, either in a breast or heading or in longwall. (2) The end of a drift or tunnel.

*Face-On.*—When the face of the breast or entry is parallel to the face cleats of the seam (see page 614).

*Face Wall.*—A wall built to sustain a face cut into the natural earth, in distinction to a retaining wall, which supports earth deposited behind it.

*Faenas* (Mexican).—Dead work, in the way of development.

*Fahlband* (German).—A course impregnated with metallic sulphides.

*Fall.*—(1) A masss of roof or side which has fallen in any part of a mine. (2) To blast or wedge down coal.

*False Bedding.*—Irregular lamination, wherein the laminæ, though for short distances parallel to each other, are oblique to the general stratification of the mass at varying angles and directions.

*False Bottom.*—A movable bottom in some apparatus.

*False Cleavage.*—A secondary slip cleavage superinduced on slaty cleavage.

*False Set.*—A temporary set of timber used until work is far enough advanced to put in a permanent set.

*Famp* (North of England).—Thin beds of soft tough shale.

*Fan.*—A machine for creating a circulation of air in a mine.

*Fan Drift.*—A short tunnel or conduit leading from the top of the air-shaft to the fan.

*Fanega* (Mexican).—A spanish measure of about 2½ bushels.

*Fang* (Derbyshire).—An air-course.

*Fascines* (English).—Bunches of twigs and small branches for forming foundations or retaining walls in soft ground.

*Fast.*—(1) A road driven in a seam with the solid coal at each side. "Fast at an end," or "fast at one side," implies that one side is solid coal and the other open to the goaf or some previous excavation. (2) Bed rock.

*Fast End.*—An end of a breast of coal that requires cutting.

*Fat Coals.*—Those containing volatile oily matters.

*Fathom* (English).—6 ft.

*Fault.*—A fracture or disturbance of the strata breaking the continuity of the formation.

*Feather.*—A slightly projecting narrow rib lengthwise into a shaft, arranged to catch into a corresponding groove in anything that surrounds and slides along the shaft.

*Feather Edge.*—(1) A passage from false to true bottom. (2) The thin end of a wedge-shaped piece of rock or coal.

*Feed.*—Forward motion imparted to the cutters or drills of rock-drilling or coal-cutting machinery, either hand or automatic.

*Feeder.*—(1) A runner of water. (2) A small blower of gas. (3) A device for feeding at a uniform rate to any machine process.

*Fend-Off* (English).—A sort of bell-crank for turning a pump rod past the angle of a crooked shaft.

*Fiery.*—Containing explosive gas.

*Fines.*—Very small material produced in breaking up large lumps.

*Fire.*—(1) A miners' term for firedamp. (2) To blast with gunpowder or other explosive. (3) A word shouted by miners to warn one another when a shot is to be fired.

*Fire-Bars* (English).—The iron bars of a grate on which the fuel rests.

*Fireboard.*—A piece of board with the word *fire* painted upon it and suspended to a prop, etc., in the workings, to caution men not to take a naked light beyond it, or to pass it without the consent of the foreman or his assistants.

*Fire Boss.*—An underground official who examines the mine for gas and inspects safety lamps taken into the mine.

*Fireclay.*—Any clay that will withstand a great heat without vitrifying.

*Firedamp.*—(1) A mixture of light carburetted hydrogen ($CH_4$) and air in explosive proportions; often applied to $CH_4$ alone or to any explosive mixture of mine gases.

*Fireman.*—See *Fire Boss.*

*Fire-Setting.*—The process of exposing very hard rock to intense heat, rendering it thereby easier for breaking down.

*First Aid.*—The assistance or treatment which should be given an injured person immediately upon injury or as soon thereafter as possible.

*First Working.*—See *Whole Working.*

*Firsts.*—The best mineral picked from a mine.

*Fish.*—To join two beams, rails, etc., together by long pieces at their sides.

*Fish Plates.*—The bars used to join the ends of adjacent rails in a track

*Fissure.*—An extensive crack.

*Fissure Vein.*—Any mineralized crevice in the rock of very great depth.

*Flag.*—A track signal or target.

*Flags.*—Broad flat stones for paving.

*Flagstone.*—Any kind of a stone that separates naturally into thin tabular plates suitable for pavements and curbing. Especially applicable to sandstone and schists.

*Flang* (Cornish).—A double-pointed pick.

*Flange* (English).—A projecting ledge or rim.

*Flat.*—(1) A district or set of workings separated by faults, old workings, or barriers of solid coal. (2) The siding or station laid with two or more lines of railway, to which the putters bring the full cars from the working face, and where they get the empty cars to take back. (3) The area of working places, from which coal is brought to the same station, is also called "flat."

*Flat Rod.*—A horizontal rod for conveying power to a distance.

*Flats.*—(1) Narrow decomposed parts of limestones that are mineralized. (2) Flatcars.

*Flat Sheet.*—Sheet-iron flooring at landings and in the plats, chambers, and junctions of drives, to facilitate the turning and management of trucks.

*Flat Wall* (Cornish).—Foot-wall.

*Float.*—Broken and transported particles or boulders of vein matter.

*Float Stones.*—Loose boulders from lodes lying on or near the surface.

*Flood Gate* (English).—A gate to let off excess of water in flood or other times.

*Floor.*—(1) The stratum of rock upon which a seam of coal immediately lies. (2) That part of a mine upon which you walk or upon which the road bed is laid.

*Flucan.*—A soft, greasy, clayey substance found in the joints of veins.

*Fluke.*—A rod for cleaning out drill holes.

*Flume.*—An artificial watercourse.

*Flush.*—(1) To clean out a line of pipes, gutters, etc., by letting in a sudden rush of water. (2) The splitting of the edges of stone under pressure. (3) Forming an even continuous line or surface. (4) To fill a mine with fine material. Sometimes called slush.

*Following Stone.*—Roof stone that falls on the removal of the seam.

*Foot-Hole.*—Holes cut in the sides of shafts or winzes to enable miners to ascend and descend.

*Foot-Piece.*—(1) A wedge of wood or part of a slab placed on the foot-wall against which a stull piece is jammed. (2) A piece of wood placed on the floor of a drive to support a leg or prop of timber.

*Foot-Wall.*—The lower boundary of a lode.

*Footway.*—Ladders in mines.

*Force Fan.*—See *Blowdown Fan.*

*Force Piece.*—Diagonal timbering to secure the ground.

*Force Pump.*—A pump that forces water above its valves.

*Forebay.*—Penstock. The reservoir from which water passes directly to a waterwheel.

*Forepoling.*—Driving the poles over the timbers so that their ends project beyond the last set of timber, so as to protect the miner from roof falls; used also in quicksand or other loose material.

*Forewinning.*—The first working of a seam in distinction from pillar drawing.

*Fork.*—(1) A deep receptacle in the rock, to enable a pump to extract the bottom water. A pump is said to be "going in fork" when the water is so low that air is sucked through the windbore. (2) (Cornish) Bottom of sump. (3) (Derbyshire) Prop for soft ground.

*Formation.*—A series of strata that belong to a single geological age.

*Fossil.*—Organic remains or impressions of them found in mineral matter.

*Fother* (North of England).—$\frac{1}{3}$ chaldron.

*Frame Set.*—The legs and cap or collar arranged so as to support a passage mined out of the rock or lode; also called *Framing.*

*Free.*—Coal is said to be "free" when it is loose and easily mined, or when it will "run" without mining.

*Free Miner.*—Licensed miner.

*Fresno* (Mexican).—An ash tree.

*Fronton* (Mexican).—Any working face.

*Fuelle* (Mexican).—A bellows.

*Furnace.*—A large coal fire at or near the bottom of an upcast shaft, for producing a current of air for ventilating the mine.

*Furnace Shaft.*—The upcast shaft in furnace ventilation.

*Fuse.*—(1) A hollow tube filled with an explosive mixture for igniting cartridges. (2) To melt.

**Gad.**—(1) A small steel wedge used for loosening jointy ground.   (2) A pointed chisel.

*Gale.*—A grant of mining ground.

*Galera* (Mexican).—A shed; any long or large room; a storehouse.

*Galiage.*—See *Firedamp.*

*Gallery.*—A horizontal passage.

*Gallows Frame.*—The frame supporting a pulley over which the hoisting rope passes to the engine.

*Gang.*—A set of miners, a "shift."

*Gangway.*—The main haulage road or level.

*Ganister.*—A hard, compact, extremely silicious fireclay.

*Gas.*—See *Firedamp.*   Any firedamp mixture in a mine is called *gas.*

*Gas Coal.*—Bituminous coal containing a large percentage of volatile matter.

*Gash Vein.*—A wedge-shaped vein.

*Gasket.*—A band or ring of any material put between the flanges of pipes, etc., before bolting, to make them water-tight or steam-tight.

*Gate.*—An underground road connecting a stall or breast with a main road.

*Gateway.*—(1) A road kept through goaf in longwall working.   (2) A gangway having ventilating doors.

*Gauge Door.*—A wooden door fixed in an airway for regulating the supply of ventilation necessary for a certain district or number of men.

*Gauge Pressure.*—The pressure shown by an ordinary steam gauge.   It is the pressure above that of the atmosphere.

*Gears,* or *Pair of Gears.*—(1) Two props and a plank, the plank being supported by the props at either end.   (2) Toothed wheels for transmitting motion.

*Geodes.*—Large nodules of stone with a hollow in the center.

*Geordie.*—A safety lamp invented by George Stephenson.

*Geyser.*—Natural fountain of hot water and steam.

*Gib.*—(1) A short prop of timber by which coal is supported while being holed or undercut.   (2) A piece of metal often used in the same hole with a wedge-shaped key for holding pieces together.

*Ginneys.*—See *Dillies.*

*Gin,* or *Horse Gin.*—A vertical drum and framework by which the minerals and dirt are raised from a shallow pit.

*Giraffe.*—A mechanical appliance for receiving and tipping a car full of mineral or waste rock when it arrives at the surface.

*Girdle.*—A thin bed or band of stone.   A roof is described as a post roof with metal girdles, or a metal roof with post girdles, according as the post or the metal predominates.

*Goaf,* or *Goave.*—That part of a mine from which the coal has been worked away, and the space more or less filled up with waste.

*Gob.*—(1) Another word for *Goaf.*   (2) To leave coal and other minerals that are not marketable in the mine.   (3) To stow or pack any useless underground roadway with rubbish.

*Gob Fire.*—Spontaneous combustion underground of fine coal and slack in the gob.

*Gobbing Up.*—Filling with waste.

*Gob Road.*—A roadway in a mine carried through the goaf.

*Going Headways,* or *Going Bord.*—A headway or bord laid with rails, and used for conveying the coal tubs to and from the face.

*Golpeador* (Mexican).—A striker, in hand drilling.

*Goths* (Staffordshire).—Sudden burstings of coal from the face, owing to tension caused by unequal pressure.

*Gouge.*—The layer of clay, or decomposed rock, that lies along the wall or walls of a vein.   It is not always valueless.

*Grade.*—The amount of fall or inclination in ditches, flumes, roads, etc.

*Grain.*—An obscure vertical cleavage usually more or less parallel to the end or dip joints.

*Granza* (Mexican).—Metallic minerals from the size of rice to that of hens' eggs.

*Grass.*—The surface of the ground.

*Grate Coal.*—See *Broken Coal.*

*Grating.*—A perforated iron sheet or wire gauze placed in front of reducing machinery.

*Gravel.*—Water-worn stones about the size of marbles.

*Gray Metal.*—Shale of a grayish color.

*Graywacke.*—A compact gray sandstone frequently found in Paleozoic formations.

*Greenstone.*—A general term employed to designate green-colored igneous rocks, as diorite, dolerite, diabase, gabbro, etc.

*Grid.*—(1) A grated opening. (2) A section of electrical resistance, usually made of cast iron.

*Griddle.*—A coarse sieve used for sifting ores, clay, etc.

*Grizzly.*—A bar screen.

*Ground Rent.*—Rent paid for surface occupied by the plant, etc., of a colliery.

*Groundsill.*—A log laid on the floor of a drive on which the legs of a set of timber rest.

*Grout* (English).—Thin mortar poured into the interstices between stones and bricks.

*Grove* (Derbyshire).—A mine.

*Grub Stake.*—The mining outfit or supplies furnished to a prospector on condition of sharing in his finds.

*Guag* (Cornish).—Worked-out ground.

*Gualdria* (Mexican).—A long and stout beam, generally sustaining other beams or some heavy weight.

*Guarda Raya* (Mexican).—A landmark; a monument.

*Guides.*—See *Cage Guides.*

*Guijo* (Mexican).—A pointed pivot, upon which turns the upright center piece of an arrastre, of a door, etc.

*Gunboat.*—A self-dumping car, holding from 5 to 8 tons of coal, used upon inclined planes or slopes. They are filled by emptying the mine cars into them at the foot of the slope.

*Gunnies* (Cornish).—3 ft.

*Gutter.*—(1) A small water-draining channel.

*Hade.*—The inclination of a vein or fault, taking the vertical as zero.

*Half Course.*—(1) At an angle of 45° from general or previous course. (2) Half on the level and half on the dip.

*Half Set.*—One leg piece and a cap.

*Hammer-and-Plate.*—A signaling apparatus.

*Hand Barrow.*—A long box or platform with handles at each end.

*Hand Dog.*—A kind of spanner or wrench for screwing up and disconnecting the joints of boring rods at the surface.

*Handspike.*—A wooden lever for working a capstan or windlass.

*Hanger-On.*—The man that runs the loaded cars on to the cages and gives the signal to hoist. See *Cager.*

*Hanging Spear Rod.*—Wooden pump rods adjustable by screws, etc., by which a sinking set of pumps is suspended in a shaft.

*Hanging Wall.*—In metalliferous mining, the stratum lying geologically directly above a bed or vein.

*Hatajo* (Mexican).—A drove of pack mules.

*Hat Rollers.*—Cast-iron or steel rollers shaped like a hat, revolving on a vertical pin, for guiding inclined haulage ropes around curves.

*Hatter.*—A miner working by himself on his own account.

*Haulage Clip.*—Levers, jaws, wedges, etc., by which cars, singly or in trains, are connected to the hauling ropes.

*Hauling.*—The drawing of conveying of the product of the mine from the working places to the bottom of the hoisting shaft, or slope.

*Haunches.*—The parts of an arch from the keystone to the skew back.

*Hazle* (North of England).—Sandstone mixed with shale.

*Head.*—(1) Pressure of water in pounds per square inch. (2) Any subterranean passage driven in solid coal. (3) That part of a face nearest the roof.

*Head,* or *Sluice Head* (Australia and New Zealand).—A supply of 1 cu. ft. of water per second, regardless of the head, pressure, or size of orifice.

*Head-Block.*—(1) A stop at the head of a slope or shaft to stop cars from going down the shaft or slope. (2) A cap piece.

*Headboard.*—A wedge of wood placed against the hanging wall, and against which one end of the stull piece is jammed.

*Header.*—(1) A rock that heads off or delays progress. (2) A blast hole at or above the head. (3) A stone or brick laid lengthwise at right angles to the face of the masonry. (4) The Stanley Header is an entry boring machine that bores the entire section of the entry in one operation.

*Head-Gear.*—The pulley frame erected over a shaft.

*Head-House.*—When the head-frame is housed in, the structure is known by this name.

*Heading.*—(1) A continuous passage for air or for use as a manway; a gangway or entry. (2) A connecting passage between two rooms, breasts, or other working places.

*Head-Piece.*—A cap; a collar.

*Headrace.*—An aqueduct for bringing a supply of water on to the ground.

*Headstocks.*—Gallows frame; head-frame.

*Headways.*—(1) A road; usually 9 ft. wide, in a direction parallel to the main-cleavage planes of the coal seams, which direction is called "headways course," and is generally about north and south in the Newcastle coal field. It is termed "on end" in other districts. (2) Cross-headings.

*Heave.*—The shifting of rocks, seams, or lodes on the face of a cross-course, etc.

*Heaving.*—The rising of the thill (or floor) of a seam where the coal has been removed.

*Hechado* (Spanish).—Dip.

*Heel of Coal.*—A small body of coal left under a larger body as a support.

*Heel of a Shot.*—In blasting, the front of a shot, or the face of the shot farthest from the charge.

*Heep Stead* (English).—The entire surface plant of a colliery.

*Helper.*—A miner's assistant, who works under the direction of the miner.

*Helve.*—A handle.

*Hewer.*—A collier that cuts coal; a digger.

*High Reef.*—The bed rock or reef is frequently found to rise more abruptly on one side of a gutter than on the other, and this abrupt reef is termed a high reef.

*Hitch.*—(1) A fault or dislocation of less throw than the thickness of the seam in which it occurs. (2) Step cut in the rock or lode for holding stay-beams, or timber, etc., for various purposes.

*Hoarding.*—A temporary close fence of boards placed around a work in progress.

*Hogback.*—A roll occurring in the floor and not in the roof, the coal being cut out or nearly so, for a distance.

*Hoister.*—A machine used in hoisting the product. It may be operated by steampower or horsepower.

*Hole.*—(1) To undercut a seam of coal by hand or machine. (2) A bore hole. (3) To make a communication from one part of a mine to another.

*Holing.*—(1) The portion of the seam or underclay removed from beneath the coal before it is broken down. (2) A short passage connecting two roads. (3) See *Kirving*.

*Holing Through.*—Driving a passage through to make connection with another part of the same workings, or with those in an adjacent mine.

*Hood.*—See *Bonnet*.

*Hopper.*—A coal pocket; a funnel-shaped feeding trough.

*Horn Coal.*—Coal worked partly end-on and partly face-on.

*Horse Gin.*—A gearing for winding by horsepower.

*Horsepower.*—The power that will raise 33,000 lb. 1 ft. high per minute.

*Horse,* or *Horsebacks.*—(1) Natural channels cut or washed away by water in a coal seam and filled up with shale and sandstone. Sometimes a bank or ridge of foreign matter in a coal seam. (2) A mass of country rock lying within a vein or bed. (3) Any irregularity cutting out a portion of the vein. See *Dirt Fault* and *Rock Fault*.

*Horse Whim.*—A vertical drum worked by a horse, for hauling or hoisting. Called also *Horse Gin*.

*Hose.*—A strong flexible pipe made of leather, canvas, rubber, etc., and used for the conveyance of water, steam, or air under pressure to any particular point.

*H Piece.*—The portion of a column pipe containing the valves of the pump.

*Hueco* (Mexican).—See *Demasia*.

*Hulk* (Cornish).—To pick out the soft portions of a lode.

*Hundido* (Mexican).—See *Derrumbe*.

*Hungry.*—Worthless looking.

*Hurdy Gurdy.*—A waterwheel that receives motion from the force of traveling water.

*Hung Shot.*—A shot which does not explode immediately upon detonation or ignition.

*Hutch.*—(English) A mine car.

*Hydraulic Cement.*—A mixture of lime, magnesia, alumina, and silica that solidifies beneath water.

*Hydraulicking.*—Working or removing auriferous or other gravel beds by hydraulic power.

*Hydrocarbons.*—Compounds of hydrogen and carbon.

**Igneous Rocks.**—Those that have been in a more or less fused state.

*Inbye.*—In a direction inward toward the face of the workings, or away from the entrance.

*Incline.*—Short for inclined plane. Any inclined heading or slope road or track having a general inclination or grade in one direction.

*Indicator.*—(1) A mechanical contrivance attached to winding, hauling, or other machinery, which shows the position of the cages in the shaft or the cars on an incline during their journey or run. (2) An apparatus for showing the presence of firedamp in mines, the temperature of goaves, the speed of a ventilator, pressure of steam, air, or water, etc.

*Indicator Card,* or *Diagram.*—A diagram showing the variation of steam pressure in the cylinder of an engine during an entire stroke or revolution.

*Indoor Catches.*—Strong beams in Cornish pumping-engine houses to catch the beam in case of a smash, thus preventing damage to the engine itself.

*In-Fork.*—When a pump continues working after water has receded below the holes of the wind bore.

*Ingot.*—A lump of cast metal.

*In Place.*—A vein or deposit in its original position.

*Inset.*—The entrance to a mine at the bottom, or part way down a shaft where the cages are loaded.

*Inside Slope.*—A slope on which coal is raised from a lower to a higher gangway.

*Inspector.*—A government official whose duties are to enforce the laws regulating the working of mines.

*Instroke.*—The right to take coal from a royalty to the surface by a shaft in an adjoining royalty. A rent is usually charged for this privilege.

*Intake.*—(1) The passage through which the fresh air is drawn or forced in a mine, commencing at the bottom of a downcast shaft, or the mouth of a slope. (2) The fresh air passing into a colliery.

*Inversion.*—Such a change in the dip of a vein or seam as makes the footwall or floor the upper and the hanging wall or roof the lower of the two.

*Irestone* (Cornish).—Any hard tough stone.

*Iron Man.*—A coal-cutting machine.

**Jacal** (Mexican).—See *Xacal.*

*Jack.*—(1) A lantern-shaped case made of tin, in which safety lamps are carried in strong currents of air. (2) A device for lifting heavy weights.

*Jacket.*—(1) An extra surface covering, as a steam jacket. (2) A water-jacket is a furnace having double iron walls, between which water circulates.

*Jack-Lamp.*—A Davy lamp, with the addition of a glass cylinder outside the gauze.

*Jars.*—In rope drilling, two long links which take up the shock of impact when the falling tools strike the bottom of the hole.

*Jenkin.*—A road cut in a pillar of coal in a bordways direction, that is, at right angles to the main cleavage planes.

*Jig.*—(1) A self-acting incline. (2) A machine for separating ores or minerals from worthless rock by means of their difference in specific gravity; also called *Jigger* or *Washer.*

*Jigger.*—(1) A kind of coupling hook for connecting cars on an incline. (2) An allowance of liquor sometimes issued to workmen (almost obsolete). (3) See *Jig.*

*Jigging.*—Separating heavy from light particles by agitation in water.

*Jockey.*—A self-acting apparatus carried on the front truck of a set for releasing it from the hauling rope.

*Joggle.*—A joint of trusses or sets of timber for receiving pressure at right angles, or nearly so.

*Joints.*—(1) Divisional planes that divide the rock in a quarry into natural blocks. There are usually two or three nearly parallel series, called by quarrymen end joints, back joints, and bottom joints, according to their position. (2) In coal seams, the less pronounced cleats or vertical cleavages in the coal. The shorter cleats, about at right angles to the face cleats and the bedding plane of the coal.

*Jud.*—(1) A portion of the working face loosened by "kirving" underneath, and "nicking" up one side. The operation of kirving and nicking is spoken of as "making a jud." (2) The term jud is also applied to a working place, usually 6 to 8 yd. wide, driven in a pillar of coal. When a jud has been driven the distance required, the timber and rails are removed, and this is termed "drawing a jud."

*Judge* (Derbyshire and North of England).—A measuring staff.

*Jugglers,* or *Jugulars.*—Timbers set obliquely against the rib in a breast, to form a triangular passage to be used as a manway, airway, or chute.

*Jump.*—An upthrow or a downthrow fault.

*Jumper.*—A hand drill used in boring holes in rock for blasting.

**Kann** (Cornish).—Fluorspar.

*Kazen* (Cornish).—A sieve.

*Keeker.*—An official that superintends the screening and cleaning of the coal.

*Keel Wedge.*—A long iron wedge for driving over the top of a pick hilt.

*Keeps,* or *Keps.*—Wings, catches, or rests to hold the cage at rest when it reaches any landing.

*Kenner.*—Time for quitting work.

*Kerf.*—The undercut made to assist the breaking of the coal.

*Kerve* (North of England).—In coal mining, to cut under.

*Kettle* or *Kettle Bottom.*—The petrified stump of a tree or other fossil in the roof of a mine.

*Key.*—(1) An iron bar of suitable size and taper for filling the keyways of shaft and pulley so as to keep both together. (2) A kind of spanner used in deep boring by hand.

*Kibble.*—See *Bowk.* Often made with a bow or handle, and carrying over a ton of débris.

*Kick Back.*—A track arrangement for reversing the direction of travel of cars moving by gravity.

*Kickup.*—An apparatus for emptying trucks.

*Kieve.*—Tossing tub.

*Killas* (Cornish).—Clay slate.

*Kiln.*—A chamber built of stone or brick, or sunk in the ground, for burning minerals in.

*Kind.*—(1) Tender, soft, easy. (2) Likely looking stone.

*Kind-Chaudron.*—A system of sinking shafts through water-bearing strata.

*Kirving* (North of England).—The cutting made beneath the coal seam.

*Kist.*—The wooden box or chest in which the deputy keeps his tools. The chest is always placed at the flat or lamp station, and this spot is often referred to by the expression "at the kist."

*Kit.*—Any workman's necessary outfit, as tools, etc.

*Kitty.*—A squib made of a straw tube filled with powder.

*Knee Piece.*—A bent piece of piping.

*Knocker.*—A lever that strikes on a plate of iron at the mouth of a shaft, by means of which miners below can signal to those on the top.

*Knocker Line.*—The signal line extending down the shaft from the knocker.

*Koepe System.*—A system of hoisting without using drums, the rope being endless and passing over pulleys instead of around a drum.

**Labor** (Mexican).—Mine workings in general. Specifically, a stope or any other place where mineral is being taken out.

*Ladderway, Ladder Road.*—The particular shaft, or compartment of a shaft, used for ladders.

*Lagging.*—(1) Small round timbers, slabs, or plank, driven in behind the legs and over the collar, to prevent pieces of the sides or roof from falling through. (2) Long pieces of timber closely fitted together and fastened to the drum rings to form a surface for the rope to wind onto.

*Laminæ.*—Sheets not naturally separated, but which may be forced apart.

*Lamp Men.*—Cleaners, repairers, and those having charge of the safety lamps at a colliery.

*Lamp Stations.*—Certain fixed stations in a mine at which safety lamps

are allowed to be opened and relighted by men appointed for that purpose, or beyond which, on no pretense, is a naked light allowed to be taken.

*Lander.*—The man that receives a load of mineral at the mouth of a shaft.

*Lander's Crook.*—A hook or tongs for upsetting the bucket of hoisted rock.

*Landing.*—(1) A level stage for loading or unloading a cage or skip. (2) The top or bottom of a slope, shaft, or inclined plane.

*Land Sale.*—The sale of coal loaded into carts or wagons for local consumption.

*Land-Sale Collieries.*—Those selling the entire product for local consumption, and shipping none by rail or water.

*Lap.*—One coil of rope on a drum or pulley.

*Large.*—The largest lumps of coal sent to the surface, or all coal that is hand picked or does not pass over screens; also, the large coal that passes over screens.

*Larry.*—(1) A car to which an endless rope is attached, fixed at the inside end of the road, forming part of the appliance for taking up slack rope. See *Balance Car.* (2) See *Barney.* (3) A car with a hopper bottom and adjustable chutes for feeding coke ovens.

*Latches.*—(1) A synonym of switch. Applied to the split rail and hinged switches. (2) Hinged switch points, or short pieces of rail that form rail crossings.

*Lateral.*—From the side.

*Lath.*—A plank laid over a framed center or used in poling.

*Launder.*—Water trough.

*Laundry Box.*—The box at the surface receiving the water pumped up from below.

*Lava.*—A common term for all rock matter that has flowed from a volcano or fissure.

*Lazadores* (Mexican).—Men formerly employed in recruiting Indians for work in the mines by the gentle persuasion of a lasso.

*Lazy Back* (Staffordshire).—A coal stack, or pile of coal.

*Leader.*—A seam of coal too small to be worked profitably, but often being a guide to larger seams lying in known proximity to it.

*Leat.*—A small water ditch.

*Leg.*—A wooden prop supporting one end of a collar.

*Leg Piece.*—An upright log placed against the side of a drive to support the cap piece.

*Leñador* (Mexican).—One that cuts, carries, or furnishes wood for combustible.

*Level.*—A road or gangway running parallel or nearly so with the strike of the seam.

*Lid.*—A cap piece used in timbering.

*Lift.*—(1) The vertical height traveled by a cage in a shaft. (2) The *lift of a pump* is the theoretical height from the level of the water in the sump to the point of discharge. (3) The distance between the first level and the surface, or between two levels. (4) The levels of a shaft or slope.

*Lifting Guards.*—Fencing placed around the mouth of a shaft, which is lifted out of the way by the ascending cage.

*Lignite.*—A coal of a woody character containing about 66% carbon and having a brown streak.

*Lime Cartridge.*—A charge or measured quantity of compressed dry caustic lime made up into a cartridge and used instead of gunpowder for breaking down coal. Water is applied to the cartridge, and the expansion breaks down the coal without producing a flame.

*Lime Coal.*—Small coal suitable for lime burning.

*Lines.*—Plumb-lines, not less than two in number, hung from hooks or spads driven in wooden plugs. A line drawn through the centers of the two strings or wires, as the case may be, represents the bearing or course to be driven on.

*Lining.*—The planks arranged against frame sets.

*Linternilla* (Mexican).—The drum of a *Horse Whim.*

*Lip Screen.*—(1) A small screen or screen bars, placed at the draw hole of a coal pocket to take out the fine coal. (2) A stepped coal screen.

*Little Giant.*—The name given to a special sort of hydraulic nozzle used for sluicing purposes.

*Llaves* (Mexican).—Horizontal cross-beams in a shaft, or the upright pieces that sustain the roof beams in a drift or tunnel.

*Loaded Track.*—Track used for loaded cars.

*Loader.*—One that fills the mine cars at the working places.

*Loam.*—Any natural mixture of sand and clay that is neither distinctly sandy nor clayey.

*Location.*—The first approximate staking out or survey of a mining claim, in distinction from a *Patent Survey*, or a *Patented Claim*.

*Location Survey.*—See *Location*.

*Lode* (Cornish).—Strictly a fissure in the country rock filled with mineral; usually applied to metalliferous lodes.   In general miners' usage, a lode, vein, or ledge is a tabular deposit of valuable minerals between definite boundaries.    Whether it be a fissure formation or not is not always known, and does not affect the legal title under the United States federal and local statutes and customs relative to lodes.   But it must not be a placer, i.e., it must consist of quartz or other rock in place, and bearing valuable mineral.

*Logs.*—Portions of trunks of trees cut to length and built up so as to raise the mouth or collar of a shaft from the surface, in order to give the requisite space for the dumping of mullock and mineral.

*Long-Pillar Work.*—A system of working coal seams in three separate operations: (*a*) Large pillars are left; (*b*) a number of parallel headings are driven through the block; and (*c*) the ribs or narrow pillars are worked away in both directions.

*Long Ton.*—2,240 lb.

*Longwall.*—A system of working a seam of coal in which the whole seam is taken out and no pillars left, excepting the shaft pillars, and sometimes the main-road pillars.

*Loose End.*—(1) A portion of a seam worked on two sides.   (2) A portion that projects in the shape of a wedge between previous workings.

*Low Grade.*—Not rich in mineral.

*Lumber.*—Timber cut to the various sizes and shapes for carpenters' purposes.

*Lumbreras* (Mexican).—Ventilating shafts in a mine or other underground work.

*Lump Coal.*—(1) All coal (anthracite only) larger than broken coal, or, when steamboat coal is made, lumps larger than this size.   (2) In soft coal, all coal passing over the screen.

*Lute.*—An adhesive clay used either to protect any iron vessel from too strong a heat or for securing air- and gas-tight joints.

*Lye* (English).—A siding or turnout.

*Machote* (Mexican).—A stake or permanent bench mark fixed in an underground working, from which the length and progress thereof is measured.

*Magnetic Needle.*—Needle used in surveying.

*Magnetic North.*—The direction indicated by the north end of the magnetic needle.

*Magnetic Meridian.*—The line or great circle in which the magnetic needle sets at any given place.

*Main Road.*—The principal haulage road of a mine from which the several crossroads lead to the working face.

*Main Rod* (English).—See *Pump Rod*.

*Main Rope.*—In tail-rope haulage, the rope that draws the loaded cars out.

*Makings* (North of England).—Small coal produced in kirving; fines.

*Malacate* (Mexican).—A *Horse Whim;* now extended to any hoisting machine used in mines.

*Mamposteria* (Mexican).—Mason work.

*Manager.*—An official who has the control and supervision of a mine, both under and above ground.

*Man Engine.*—An apparatus consisting of one or two reciprocating rods, to which suitable stages are attached, used for lowering and raising men in shafts.

*Manhole.*—(1) A refuge hole constructed in the side of a gangway, tunnel, or slope.   (2) A hole in boilers through which a man can get into the boiler to examine and repair it.

*Manway.*—A small passage used as a traveling way for the miner, and also often used as an airway or chute, or both.

*Marco* (Mexican).—A weight of 8 oz.

*Marcus.*—A patented shaker screen with a non-harmonic or quick-return motion.

*Marl.*—Clay containing calcareous matter.

*Marlinespike.*—A sharp-pointed and gradually tapered round iron, used in splicing ropes.

*Marrow.*—A partner.

*Marsaut Lamp.*—A type of safety lamp whose chief characteristic is the multiple-gauze chimneys.

*Marsh Gas.*—$CH_4$, often used synonymously with *Firedamp* (see page 859).

*Match.*—(1) A charge of gunpowder put into a paper several inches long, and used for igniting explosives. (2) The touch end of a squib.

*Mattock.*—A kind of pick with broad ends for digging.

*Maul.*—A driver's hammer.

*Maundril.*—A pick with two shanks and points, used for getting coal, etc.

*Mear* (Derbyshire).—32 yd. along the vein.

*Measures.*—Strata.

*Mecha* (Mexican).—A wick for a lamp or candle; a torch.

*Merced* (Mexican).—A gift, grant, or concession.

*Meridian.*—A north and south line, either true or approximate.

*Metal.*—(1) In coal mining, indurated clay or slate. (2) An element that forms a base by combining with oxygen and which is solid at ordinary temperature (with exception of quicksilver), opaque (except in the thinnest possible films), has a metallic luster, and is a good conductor of heat and electricity, and, as a rule, of a higher specific gravity than the non-metals. (3) (Mexican) All kinds of metalliferous minerals are called "metal" in Mexico.

*Mill Cinder.*—The slag from the puddling furnace of a rolling mill.

*Mill Hole.*—An auxiliary shaft connecting a stope or other excavation with the level below.

*Mine.*—Any excavation made for the extraction of minerals.

*Miner.*—One who mines.

*Mineral.*—Any constituent of the earth's crust that has a definite composition.

*Mineral Oil.*—Petroleum obtained from the earth, and its distillates.

*Minero* (Mexican).—A mine owner; a mining captain; an underground boss.

*Mine Road.*—Any mine track used for general haulage.

*Mine Run.*—The entire unscreened output of a mine.

*Minero Mayor* (Mexican).—The head mining captain. A mining workman is called *Operario.*

*Miners' Dial.*—An instrument used in surveying underground workings.

*Miners' Inch.*—A measure of water varying in different districts, being the quantity of water that passes through a slit 1 in. high, of a certain width under a given head (see page 309).

*Miner's Right.*—An annual permit from the Government to occupy and work mineral land.

*Mining.*—In its broad sense, it embraces all that is concerned with the extraction of minerals and their complete utilization.

*Mining Engineer.*—A man having knowledge and experience in the many departments of mining.

*Mining Retreating.*—A process of mining by which the vein is untouched until after all the gangways, etc., are driven, when the mineral extraction begins at the boundary and progresses toward the shaft.

*Mistress* (North of England).—A miner's lamp.

*Moil.*—(1) A short length of steel rod tapered to a point, used for cutting hitches, etc. (2) To cut with a moil.

*Monitor.*—See *Gunboat.*

*Monkey.*—The hammer or ram of a pile driver.

*Monkey Drift.*—A small drift driven in for prospecting purposes, or a crosscut driven to an airway above the gangway.

*Monkey Gangway.*—A small gangway parallel with the main gangway.

*Monkey Rolls.*—The smaller rolls in an anthracite breaker.

*Monkey Shaft.*—A shaft rising from a lower to a higher level.

*Monoclinal.*—Applied to an area in which the rocks all dip in the same direction.

*Mop.*—Some material surrounding a drill in the form of a disk, to prevent water from splashing up.

*Morgan.*—(Cape of Good Hope). A surface measure = 2.11 acres.

*Mortise.*—A hole cut in one piece of timber, etc., to receive the tenon that projects from another piece.

*Mote* (*Moat*).—A straw filled with gunpowder, for igniting a shot.

*Mother Gate.*—The main road of a district in longwall working.

*Mother Lode (Main Lode).*—The principal vein of any district.

*Motive Column.*—The length of a column of air whose weight is equal to the difference in weight of like columns of air in downcast and upcast shafts. The ventilating pressure in furnace ventilation is measured by the difference of the weights of the air columns in the two shafts.

*Mouth.*—The top of a shaft or slope, or the entrance to a drift or tunnel.

*Moyle.*—An iron with a sharp steel point, for driving into clefts when levering off rock.

*Muck.*—(1) Any material, particularly refuse, removed from a mine, shaft or slope. (2) To remove refuse.

*Mucker.*—One who mucks or removes refuse; a shoveler.

*Muckle.*—Soft clay overlying or underlying coal.

*Mucks* (Staffordshire).—Bad earthy coal.

*Muescas* (Mexican).—Notches in a stick; mortises; notches cut in a round or square beam, for the purpose of using it as a ladder.

*Mueseler Lamp.*—A type of safety lamp invented and used in the collieries of Belgium. Its chief characteristic is the inner sheet-iron chimney for increasing the draft of the lamp.

*Mufle.*—A thin clay oven heated from the outside.

*Mullock.*—Country rock and worthless minerals taken from a mine.

*Mundic.*—Iron pyrites.

**Naked Light.**—A candle or any form of lamp that is not a safety lamp.

*Narrow Work.*—(1) All work for which a price per yard of length driven is paid, and which, therefore, must be measured. (2) Headings, chutes, crosscuts, gangways, etc.

*Natas* (Mexican).—Same as *Escoria* or *Grasa.*

*Natural Ventilation.*—Ventilation of a mine without either furnace or other artificial means; the heat imparted to the air by the strata, men, animals, and lights in the mine, causing it to flow in one direction, or to ascend.

*Neck.*—A cylindrical body of rock differing from the country around it.

*Needle.*—(1) A sharp-pointed metal rod with which a small hole is made through the stemming to the cartridge in blasting operations. (2) A hitch cut in the side rock to receive the end of a timber.

*Nick.*—To cut or shear coal after holing.

*Nicking.*—(1) A vertical cutting or shearing up one side of a face of coal. (2) The chipping of the coal along the rib of an entry or room which is usually the first indications of a squeeze.

*Night Shift.*—The set of men that work during the night.

*Nip.*—When the roof and floor of a coal seam come close together, pinching the coal between them.

*Nipper.*—An errand boy, particularly one who carries steel, bits, etc., to be sharpened.

*Nip Out.*—The disappearance of a coal seam by the thickening of the adjoining strata, which takes its place.

*Nitro.*—A corrupted abbreviation for nitroglycerine or dynamite.

*Nodules.*—Concretions that are frequently found to enclose organic remains.

*Nogs.*—Logs of wood piled one on another to support the roof. See *Chock.*

*Nook.*—The corner of a working place made by the face with one side.

*Noria* (Spanish).—An endless chain of buckets.

*Nozzle.*—The front nose piece of bellows or blast pipe for a furnace, or of a water pipe.

*Nut Coal.*—A contraction of the term chestnut coal.

*Nuts.*—Small lumps of coal that will pass through a screen or bars, the spaces between which vary in width from ½ to 2½ in.

**Ocote** (Mexican).—Pitch pine.

*Odd Work.*—Work other than that done by contract, such as repairing roads, constructing stoppings, dams, etc.

*Offtake.*—The raised portion of an upcast shaft above the surface, for carrying off smoke and steam, etc., produced by the furnaces and engines underground.

*Oil Shale.*—Shale containing such a proportion of hydrocarbons as to be capable of yielding mineral oil on slow distillation.

*Oil Smellers.*—Men that profess to be able to indicate where petroleum oil is to be found.

*Old Man.*—(1) Old workings in a mine. (2) An appliance for holding a drill ratchet.

*Oolitic.*—A structure peculiar to certain rocks, resembling the roe of a fish.

*Open Cast.*—Workings having no roof.

*Open Cutting.*—(1) An excavation made on the surface for the purpose of getting a face wherein a tunnel can be driven. (2) Any surface excavation.

*Openings, an Opening.*—Any excavation on a coal or ore bed, or to reach the same; a mine.

*Openwork.*—An open cut.

*Operario* (Mexican).—A working miner.

*Operator.*—The individual or company actually working a colliery.

*Ores.*—Minerals or mineral masses from which metals or metallic combinations can be extracted on a large scale in an economic manner.

*Outburst.*—A blower. A sudden emission of large quantities of occluded gas.

*Oulbye.*—In the direction of the shaft or slope bottom, or toward the outside.

*Outcrop.*—The portion of a vein or bed, or any stratum appearing at the surface, or occurring immediately below the soil or diluvial drift.

*Outcropping.*—See *Cropping Out.*

*Outlet.*—A passage furnishing an outlet for air, for the miners, for water, or for the mineral mined.

*Output.*—The product of a mine sent to market, or the total product of a mine.

*Outset.*—The walling of shafts built up above the original level of the ground.

*Outstroke Rent.*—The rent that the owner of a royalty receives on coal brought into his royalty from adjacent properties.

*Outtake.*—The passage by which the ventilating current is taken out of the mine; the upcast.

*Overburden.*—The covering of rock, earth, etc., overlying a mineral deposit that must be removed before effective work can be performed.

*Overcast.*—A passage through which the ventilating current is conveyed over a gangway or airway.

*Overhand Stoping.*—The ordinary method of stoping upwards.

*Overlap Fault.*—A fault in which the shifted strata double back over themselves.

*Overman.*—One who has charge of the workings while the men are in the mine. He takes his orders from the *Underviewer.*

*Overwind.*—To hoist the cage into or over the top of the head-frame.

*Oyamel* (Mexican).—White pine.

*Pack.*—A rough wall or block of coal or stone built up to support the roof.

*Packing.*—The material placed in stuffingboxes, etc., to prevent leaks.

*Pack Wall.*—A wall of stone or rubbish built on either side of a mine road, to carry the roof and keep the sides up.

*Paleozoic.*—The oldest series of rocks in which fossils of animals occur.

*Palero* (Mexican).—A mine carpenter.

*Palm.*—A piece of stout leather fitting the palm of the hand, and secured by a loop to the thumb; this has a flat indented plate for forcing the needle.

*Palm Needle.*—A straight triangular-sectioned needle, used for sewing canvas.

*Palo* (Mexican).—A stick; a piece of timber.

*Panel.*—(1) A large rectangular block or pillar of coal measuring, say, 130 by 100 yd. (2) A group of breasts or rooms separated from the other workings by large pillars.

*Panel Working.*—A system of working coal seams in which the colliery is divided up into large squares or panels, isolated or surrounded by solid ribs of coal, in each of which a separate set of breasts and pillars is worked, and the ventilation is kept distinct, that is, every panel has its own circulation, the air of one not passing into the adjoining one, but being carried direct to the main return airway.

*Parcionero* (Mexican).—A partner in a mining contract.

*Parrot Coal.*—A kind of coal that splits or cracks with a chattering noise when on the fire.

*Parting.*—(1) Any thin interstratified bed of earthy material. (2) A side track or turnout in a haulage road.

*Pass.*—(1) A convenient hole for throwing down ore to a lower level. (2) A passage left in old workings for men to travel in from one level to another.

*Pass-By.*—A siding in which cars pass one another underground; a turnout.

*Pass-Into.*—When one mineral gradually passes into another without any sudden change.

*Patch* or *Patcher.*—A driver's assistant or helper; a brakeman or triprider.

*Patented Claim.*—A claim to which a patent right has been secured from the government, by compliance with the laws relating to such claims.

*Patent Fuel.*—Small coal mixed with small amounts of pitch, tar or other binder and compressed by machinery into bricks.

*Patent Survey.*—An accurate survey of a claim by a deputized surveyor as required by law in order to secure a patent right to the claim.

*Pavement.*—The floor.

*Pay Out.*—To slacken or let out rope.

*Pay Rock.*—Mineralized rock.

*Pay Streak.*—Mineralized part of rock.

*Peach Stone* (Cornish).—Chlorite schist.

*Pea Coal.*—A small size of anthracite coal (see page 952).

*Peas.*—Small coal about ½ to ¾ in. cube.

*Peat.*—The decomposed partly carbonized organic matter of bogs, swamps, etc.

*Penstock.*—See *Forebay.*

*Pent House.*—A wooden covering for the protection of sinkers working in a pit bottom.

*Pentice.*—A few pieces of timber laid as a roof over men's heads, to screen them when working in dangerous places, e.g., at the bottom of shafts.

*Pestle.*—A hard rod for pounding minerals, etc.

*Peter Out.*—To "peter out" is to thin out, or gradually decrease in thickness.

*Petrifaction.*—Organic remains converted into stone.

*Petrol.*—Variant for petroleum or its derivatives, particularly gasoline or motor spirit.

*Pick.*—(1) A tool for cutting and holing coal. (2) To dress the sides or face of an excavation with a pick.

*Picker.*—(1) A small tool used to pull up the wick of a miner's lamp. (2) A person who picks the slate from the coal in a coal breaker or tipple.

*Picking Chute.*—A chute in an anthracite breaker along which boys are stationed to pick the slate from coal.

*Picking Table.*—(1) A flat or slightly inclined platform on which anthracite coal is run to be picked free from slate. (2) A sorting table. (3) A moving belt or steel apron on which coal is picked.

*Pico* (Mexican).—A striking or sledge hammer.

*Picture.*—A screen to keep off falling water from men at work.

*Pig.*—A mass of lead or iron cast into a rough mold.

*Pigsty Timbering.*—Hollow pillars built up of logs of wood laid crosswise for supporting heavy weights.

*Pike.*—A pick.

*Pileta* (Mexican).—A sump.

*Piling.*—Long pieces of timber driven into soft ground for the purpose of securing a solid base on which to build any superstructure. Sheet piling consists of planks or steel shapes driven into the ground to prevent an influx of water, quicksand and the like.

*Pillar.*—(1) A solid block of coal, etc., varying in area from a few square yards to several acres. (2) Sometimes applied to a timber support.

*Pillar-and-Room.*—A system of working coal by which solid blocks of coal are left on either side of the rooms, entries, etc., to support the roof until the rooms are driven up, after which they are drawn out.

*Pillar-and-Stall.*—See *Breast-and-Pillar.*

*Pillar Roads.*—Working roads or inclines in pillars having a range of longwall faces on either side.

*Pinch.*—A contraction in the vein.

*Pinch Out.*—When a lode or stratum runs out to nothing.

*Pipe.*—An elongated body of mineral. Also the name given to the fossil trunks of trees found in coal veins.

*Pipe Clay.*—A soft white clay.

*Piped Air.*—Air carried into the working place by pipes or brattices.

*Pit.*—(1) A shaft. (2) The underground portion of a colliery, including all workings. (3) A gravel pit.

*Pit Bank.*—The raised ground or platform where the coal is sorted and screened at the surface.

*Pit Bottom.*—The portion of a mine immediately around the bottom of a shaft or slope. See *Shaft Bottom.*

*Pitch.*—(1) Rise of a seam. (2) Grade of an incline. (3) Inclination. (4) (Cornish) A part of a lode let out to be worked on shares or by the piece.

*Pit Coal.*—Generally signifies the bituminous varieties of coal.

*Pit Frame.*—See *Head-Frame.*

*Pit Headman.*—The man who has charge at the top of the shaft or slope.

*Pitman.*—A miner; also, one who looks after the pumps, etc.

*Pit Prop.*—A piece of timber used as a temporary support for the roof.

*Pit Rails.*—Mine rails for underground roads.

*Pit Room.*—The extent of underground workings in use or available for use.

*Pit's Eye.*—Pit bottom or entrance into a shaft.

*Pit Top.*—The mouth of a shaft or slope.

*Place.*—The portion of coal face allotted to a hewer is spoken of as his "working place," or simply "place."

*Plan.*—(1) The system on which a colliery is worked as *Longwall, Pillar-and-Breast,* etc. (2) A map or plan of the colliery showing outside improvements and underground workings. (3) (Mexican) The very lowest working in a mine. *Trabajar de Plan.*—To work to gain depth.

*Plane.*—A main road, either level or inclined, along which coal is conveyed by engine power or gravity.

*Plane Table.*—A simple surveying instrument by means of which one can plot in the field.

*Plank Dam.*—A water-tight stopping fixed in a heading constructed of timber placed across the passage, one upon another, sidewise, and tightly wedged.

*Plank Tubbing.*—Shaft lining of planks driven down vertically behind wooden cribs all around the shaft, all joints being tightly wedged, to keep back the water.

*Plant.*—The shafts or slope, tunnels, engine houses, railways, machinery, workshops, etc., of a colliery or other mine.

*Plat,* or *Map.*—A map of the surface and underground workings, or of either; to draw such a map from survey.

*Plate* (North of England).—Scaly shale in limestone beds.

*Plates.*—Metal rails 4 ft. long.

*Plenum.*—A mode of ventilating a mine or a heading by forcing fresh air into it.

*Plomada* (Mexican).—A plumb-line or plumb-bob.

*Plugging.*—When drift water forces its way through the puddle clay into the shaft, holes are bored through the slabs near the leakage point, and plugs of clay forced into them until the leakage is stopped.

*Plumb.*—Vertical.

*Plummet.*—(1) A heavy weight attached to a string or fine copper wire used for determining the verticality of shaft timbering. (2) A plumb-bob setting a surveying instrument over a point.

*Plunger.*—The solid ram of a force pump working in the plunger case.

*Plunger Case.*—The pump cylinder or barrel in which the plunger works.

*Poblar* (Mexican).—To set men at work in a mine.

*Pocket.*—(1) A thickening out of a seam of coal or other mineral over a small area. (2) A hopper-shaped receptacle from which coal or ore is loaded into cars or boats.

*Pole Tools.*—Drilling tools used in drilling in the old fashion, with rods, now superseded by the rope-drilling method.

*Poll Pick.*—A pick having the longer end pointed and the shorter end hammer-shaped.

*Polrox* (Cornish).—Waterwheel pit.

*Poppet Heads.*—The pulley frame or hoisting gear over a shaft.

*Poppet (Puppet).*—(1) A pulley frame or the head-gear over a shaft. (2) A valve that lifts bodily from its seat instead of being hinged.

*Post.*—(1) Any upright timber; applied particularly to the timbers used for propping. See *Prop.* (2) Local term for sandstone. Post stone may be "strong," "framey," "short," or "broken."

*Post-and-Stall.*—A system of working coal much the same as *Pillar-and-Stall.*

*Post-Tertiary.*—Strata younger than the Tertiary formation.

*Pot Bottom.*—A large boulder in the roof slate, having the appearance of the rounded bottom of a pot, and which easily becomes detached.

*Pot Growan* (Cornish).—Decomposed granite.

*Pot Hole.*—A circular hole in the rock caused by the action of stones whirled

around by the water when the strata was covered by water. They are generally filled with sand and drift.

*Power Drill.*—A rock drill employing steam, air, or electricity as a motor.

*Prian* (Cornish).—Soft white clay.

*Pricker.*—(1) A thin brass rod for making a hole in the stemming when blasting, for the insertion of a fuse. (2) A piece of bent wire by which the size of the flame in a safety lamp is regulated without removing the top of the lamp.

*Prong* (English).—The forked end of the bucket-pump rods for attachment to the traveling valve and seat.

*Prop.*—A wooden or metal temporary support for the roof.

*Propping.*—The timbering of a mine.

*Prospect.*—The name given to underground workings whose value has not yet been made manifest. A prospect is to a mine what mineral is to ore.

*Prospect Hole.*—Any shaft or drift hole put down for the purpose of prospecting the ground.

*Prospecting.*—Examining a tract of country in search of minerals.

*Prospector.*—One engaged in searching for minerals.

*Prospect Tunnel or Entry.*—A tunnel or entry driven through barren measures or a fault to ascertain the character of strata beyond.

*Protector Lamp.*—A safety lamp whose flame cannot be exposed to the outside atmosphere, as the action of opening the lamp extinguishes the light.

*Prove.*—(1) To ascertain, by boring, driving, etc., the position and character of a coal seam, a fault, etc. (2) To examine a mine in search of firedamp, etc., known as "proving the pit."

*Proving Hole.*—(1) A bore hole driven for prospecting purposes. (2) A small heading driven in to find a bed or vein lost by a dislocation of the strata, or to prove the quality of the mineral in advance of the other workings.

*Pudding Rock.*—Conglomerate.

*Puddle.*—(1) Earth well rammed into a trench, etc., to prevent leaking. (2) A process for converting cast iron into wrought iron.

*Pueble* (Mexican).—The actual working of a mine; the aggregation of persons employed therein.

*Puertas* (Mexican).—Massive barren rocks, or "horses," occurring in a vein.

*Pug Mill.*—A mill for preparing clay for making bricks, pottery, etc.

*Pulley.*—(1) The wheel over which a winding rope passes at the top of the head-gear. (2) Small wooden cylinders over which a winding rope is carried on the floor or sides of a plane.

*Pulleying.*—Overwinding or drawing up a cage into the pulley frame.

*Pump.*—Any mechanism for raising water.

*Pump Bob.*—See *Bob.*

*Pump Ring.*—A flat iron ring that, when lapped with tarred baize or engine shag, secures the joints of water columns.

*Pump Rods.*—Heavy timbers by which the motion of the engine is transmitted to the pump. In Cornish and bull pumps, the weight of the rods makes the effective (pumping) stroke, the engine merely lifting the rods on the up stroke.

*Pump Slope.*—A slope used for pumping machinery.

*Pump Station.*—An enlargement made in the shaft, slope, or gangway, to receive the pump.

*Pump Tree.*—Cast-iron pipes, generally 9 ft. long, of which the column or set is formed.

*Punch-and-Thirl.*—A kind of pillar-and-stall system of working.

*Punch Prop.*—A short timber prop set on the top of a crown tree, or used in holding, as a sprag.

*Pyran* (Cornish).—See *Prian.*

*Pyrites.*—Sulphide of iron, copper, etc.

*Pyrometer.*—An instrument for measuring high degrees of heat.

Quarry.—(1) An open surface excavation for working valuable rocks or minerals. (2) An underground excavation for obtaining stone for stowage or pack walls.

*Quarternary.*—Post-tertiary period.

*Quemados* (Mexican).—Burnt rock. Any dark cinder-like mineral encountered in a vein or mineral deposit, generally manganiferous.

*Quick* (Adjective).—Soft, running ground. *Quick* (Noun).—Productive.

*Quicksand.*—Soft watery strata easily moved, or readily yielding to pressure.
*Quicksilver.*—Mercury.
*Quitapepena* (Mexican).—A watchman that searches the miners as they come out at the mouth of a mine.

*Race.*—A channel for conducting water to or from the place where it performs work. The former is termed the headrace, and the latter the tailrace.
*Rack* (Cornish).—A toothed gear of infinite radius, *i.e.* a straight gear or one whose pitch line has no curvature.
*Rafter Timbering.*—That in which the timbers appear like roof rafters.
*Rag Wheel.*—Sprocket wheel. A wheel with teeth or pins that catch into the links of chains.
*Rails.*—The iron or steel portion of the tramway or railroad or their wooden counterparts.
*Rake* (Cornish).—(1) A vein. (2) (Derbyshire) Fissure vein crossing strata.
*Ram.*—(1) The plunger of a pump. (2) A device for raising water. (3) A machine for drawing a coke charge from an oven.
*Ramal* (Mexican).—A branch vein.
*Ramalear* (Mexican).—To branch off into various divisions.
*Ramble.*—Stone of little coherence above a seam that falls readily on the removal of the coal. See *Following Stone.*
*Rance.*—A pillar of coal.
*Rapper.*—A lever with a hammer attached at one end, which signals by striking a plate of metal, when the signaling wire to which it is attached is pulled.
*Rash.*—A term used to designate the bottom of a mine when soft and slaty; also the top.
*Reacher.*—A slim prop reaching from one wall to the other.
*Reamer.*—An enlarging tool.
*Reaming.*—Enlarging the diameter of a bore hole.
*Receiving Pit.*—A shallow pit for containing material run into it.
*Red-Ash Coal.*—Coal that produces a reddish ash when burnt.
*Red Rab* (Cornish).—Red slaty rock.
*Refuge Chamber.*—A chamber shut off from the rest of the mine, stored with food, etc., and to be used by the survivors in case of a mine disaster.
*Refuge Hole.*—A place formed in the side of an underground passage in which a man can take refuge during the passing of a train, or when shots are fired.
*Regulator.*—A door in a mine, the opening or shutting of which regulates the supply of ventilation to a district of the mine.
*Reliz* (Spanish).—Wall of lode.
*Rendrock.*—A variety of dynamite.
*Repairman.*—A workman whose duty it is to repair tracks, doors, brattices, or to reset timbers, etc., under the direction of the foreman.
*Rescue and Recovery.*—The work of removing live men or dead bodies after a mine disaster; also putting the mine in shape for operation again.
*Reserve.*—Mineral already opened up by shafts, winzes, levels, etc., which may be secured at short notice for any emergency.
*Reservoir.*—An artificially built, dammed, or excavated place for holding a reserve of water.
*Respaldos* (Mexican).—The walls enclosing a vein. *Respaldo Alto.*—The hanging wall. *Respaldo Bajo.*—The foot-wall.
*Rests, Keeps, Wings.*—Supports on which a cage rests when the loaded car is being taken off and the empty one put on.
*Resue.*—See *Stripping.*
*Retort Oven.*—A coke oven which conserves the gas evolved.
*Return.*—The air-course along which the vitiated air of a mine is returned or conducted back to the upcast shaft.
*Return Air.*—The air that has been passed through the workings.
*Reverberatory.*—A class of furnaces in which the flame from the fire-grate is made to beat down on the charge in the body of the furnace.
*Reversed Fault.*—See *Overlap Fault.*
*Rib.*—The side of a pillar.
*Rib-and-Pillar.*—A system of working similar to *Pillar-and-Stall.*
*Ribbon.*—A line of bedding or a thin bed appearing on the cleavage surface and sometimes of a different color.
*Rick.*—Open heap in which coal is coked.

*Ridding.*—Clearing away fallen stone and débris.

*Riddle.*—(1) An oblong frame holding iron bars parallel to each other, used for sifting material that is thrown against it. (2) A hand operated sieve.

*Ride, Riding.*—To be conveyed on a cage or mine car.

*Rider.*—(1) A guide frame for steadying a sinking bucket. (2) Boys that ride on trips on mechanical haulage roads. (3) A thin seam of coal overlying a thicker one.

*Right Shore.*—The right shore of a river is on the right hand when descending the river.

*Rim Rock.*—Bed rock forming a boundary to gravel deposit.

*Ring.*—(1) A complete circle of tubbing plates placed round a circular shaft. (2) Troughs placed in shafts to catch the falling water, and so arranged as to convey it to a certain point.

*Ripping.*—Removing stone from its natural position above the seam.

*Rise.*—The inclination of the strata, when looking up the pitch.

*Rise Workings.*—Underground workings carried on to the rise or high side of the shaft.

*Road.*—(1) Any underground passageway or gallery. (2) The **iron** rails, etc., of underground roads.

*Rob.*—To cut away or reduce the size of pillars of coal.

*Robbing.*—The taking of mineral from pillars.

*Robbing an Entry.*—See *Drawing an Entry.*

*Rock.*—A mixture of different minerals in varying proportions.

*Rock Chute.*—See *Slate Chute.*

*Rock Drill.*—A rock-boring machine worked by hand, compressed air, steam, or electrical power.

*Rock Fault.*—A replacement of a coal seam over greater or less area, by some other rock, usually sandstone.

*Rodding.*—The operation of fixing or repairing wooden eye guides in shafts.

*Roll.*—An inequality in the roof or floor of a mine.

*Roller.*—A small steel, iron, or wooden wheel or cylinder upon which the hauling rope is carried just above the floor.

*Rolleyway.*—A main haulage road.

*Rolling Ground.*—When the surface is much varied by many small hills and valleys.

*Rolls.*—Cast-iron cylinders, either plain or fitted with steel teeth, used to break coal and other materials into various sizes.

*Roof.*—The top of any subterranean passage.

*Room.*—Synonymous with *Breast.*

*Room-and-Rance.*—A system of working coal similar to *Pillar-and-Stall.*

*Rope Roll.*—The drum of a winding engine.

*Round Coal.*—Coal in large lumps, either hand-picked, or, after passing over screens, to take out the small.

*Royalty.*—The price paid per ton to the owner of mineral land by the lessee.

*Rubbing Surface.*—The total area of a given length of airway; that is, the area of top, bottom, and sides added together, or the perimeter multiplied by the length.

*Rubble.*—Coarse pieces of rock.

*Rumbo* (Mexican).—The course or direction of a vein.

*Run.*—(1) The sliding and crushing of pillars of coal. (2) The length of a lease or tract on the strike of the seam.

*Run Coal.*—Soft bituminous coal.

*Rung, Rundle,* or *Round.*—A step or cross-bar of a ladder.

*Runner.*—A man or boy whose duty it is to run mine cars by gravity from working places to the gangway.

*Running Lift.*—A sinking set o pumps constructed to lengthen or shorten at will, by means of a sliding or telescoping wind bore.

*Rush.*—An old-fashioned way of exploding blasts by filling a hollow stalk with slow powder and then igniting it.

*Rush Together.*—See *Caved In.*

*Rusty.*—Stained by iron oxide.

**Saddle.**—An anticlinal, a hogback.

*Saddleback.*—A depression in the strata. See *Roll.*

*Safety Cage.*—A cage fitted with an apparatus for arresting its motion in the shaft in case the rope breaks.

*Safety Car.*—See *Barney.*

*Safety Catches.*—Appliances fitted to cages, to make them safety cages.

*Safety Door.*—A strongly constructed door, hinged to the roof, and always kept open and hung near to the main door, for immediate use when main door is damaged by an explosion or otherwise.

*Safety First.*—A term often applied to accident prevention and first aid and to rescue and recovery training in general.

*Safety Fuse.*—A cord with slow-burning powder in the center for exploding charged blast holes.

*Safety Lamp.*—(1) A miner's lamp in which the flame is protected in such a manner that an explosive mixture of air and firedamp can be detected by the mixture burning inside the gauze. (2) An electric cap or hand lamp which will not ignite gas even when broken.

*Sag.*—A depression, e.g., in ropes, ranges of mountains, etc., also in mine floors.

*Sagre,* or *Seggar.*—A local term for fireclay, often forming the floor (or thill) of coal seams.

*Salting.*—(1) Changing the value of the ore in a mine or of ore samples before they have been assayed, so that the assay will show much higher values than it should. (2) Sprinkling salt on the floors of underground passages in very dry mines, in order to lay the dust.

*Sample.*—A representative specimen of coal from a much larger amount as a carload, shipload, or from the face of a room.

*Sampler.*—(1) An instrument or apparatus for taking samples. (2) One whose duty it is to select the samples for an assay or analysis, or to prepare the mineral to be tested, by grinding and sampling.

*Samson Post.*—An upright supporting the working beam that communicates oscillatory motion to pump or drill rod.

*Sand Bag.*—A bag filled with sand for preventing a washout by obstructing the flow.

*Sand Pump.*—A sludger; a cylinder provided with a stem (or other) valve, lowered into a drill hole to remove the pulverized rock.

*Scaffolding.*—(1) Incrustations on the inside of a blast furnace. (2) False-work employed in building.

*Scale.*—(1) A small portion of the ventilating current in a mine passing through a certain size of aperture. (2) The rate of wages to be paid, which varies under certain contingencies. (3) A weighing apparatus (4) Incrustation on the inside of a boiler.

*Scale Door.*—See *Regulator.*

*Scallop.*—To hew coal without kirving or nicking or shot firing.

*Schist.*—Crystalline or metamorphic rocks having a slaty structure.

*Schute.*—See *Chute.*

*Scissors Fault.*—A fault of dislocation, in which two beds are thrown so as to cross each other.

*Scoop.*—A large-sized shovel with a scoop-shaped blade.

*Scoria.*—Ashes.

*Scrap.*—(1) Worthless or obsolete iron, copper, machinery, etc. (2) To discard.

*Scraper.*—(1) A tool for cleaning the dust out of the bore hole. (2) A mechanical contrivance used at collieries to scrape the culm or slack along a trough to the place of deposit.

*Screen.*—(1) A mechanical apparatus for sizing materials. (2) A cloth brattice or curtain hung across a road in a mine, to direct the ventilation.

*Scrin* (Derbyshire).—A small vein.

*Sculping.*—Fracturing the state along the grain, i.e., across the cleavage.

*Scupper Nails.*—Nails with broad heads, for nailing down canvas, etc.

*Sea Coal.*—That which is transported by sea.

*Sealing.*—Shutting off all air from a mine or a part of a mine by stoppings.

*Seam.*—Synonymous with *Bed, Vein,* etc.

*Seam-Out.*—A term applied to a shot or blast that has simply blown out a softer stratum of the deposit in which it was placed, without dislodging the other strata or layers of the seam.

*Second Outlet (Second Opening).*—A passageway out of a mine, for use in case of accident to the main outlet.

*Seconds.*—Second-class coal, not best.

*Second Working.*—The operation of getting or working out the pillars formed by the first working.

*Section.*—(1) A vertical or horizontal exposure of strata. (2) A drawing or sketch representing the rock strata as cut by a vertical or a horizontal plane.

*Sedimentary Rocks.*—Rocks formed from deposits of sediment by wind or water.

*Seedbag.*—A water-tight packing of flaxseed around the tube in a drill hole, to prevent the influx into the hole of water from above.

*Segregations.*—Detached portions of veins in place.

*Self-Acting Plane.*—An inclined plane upon which the weight or force of gravity acting on the full cars is sufficient to overcome the resistance of the empties; in other words, the full car, running down, pulls the other car up.

*Self-Detaching Hook.*—A self-acting hook for setting free a hoisting rope in case of overwinding.

*Selvage.*—The clay seam on the walls of veins; gouge.

*Separation Doors.*—The main doors at or near the shaft or slope bottom, which separate the intake from the return airways.

*Separation Valve.*—A massive cast-iron plate suspended from the roof of a return airway through which all the return air of a separate district flows, allowing the air to always flow past or underneath it; but in the event of an explosion of gas, the force of the blast closes it against its frame or seating, and prevents a communication with other districts. The blast being over, the weight of the valve allows it to return to its normal position.

*Set.*—To fix in place a prop or sprag.

*Set Hammer.*—The flat-faced hammer held on hot iron by a blacksmith when shaping or smoothing a surface by aid of his striker's sledge.

*Set of Timber.*—The timbers which compose any framing, whether used in a shaft, slope, level, or gangway. Thus, the four pieces forming a single course in the curbing of a shaft, or the three or four pieces forming the legs and collar, and sometimes the sill of an entry framing are together called a set of timber, or timber set.

*Shackle.*—A **U**-shaped link in a chain closed by a pin; when the latter is withdrawn the chain is severed at that point.

*Shaft.*—A vertical or highly inclined pit or hole made through strata, through which the product of the mine is hoisted, and through which the ventilation is passed either into or out of the mine. A shaft sunk from one seam to another is called a "blind shaft."

*Shaft Pillar.*—Solid material left unworked beneath buildings and around the shaft, to support them against subsidence.

*Shaking Screen or Shaker.*—A flat screen, often inclined, which is given an oscillatory motion and is used for sizing coal.

*Shale.*—(1) Slightly clay, all argillaceous strata that split up or peel off in thin laminæ. (2) A laminated and stratified sedimentary deposit of clay, often impregnated with bituminous matter.

*Shank.*—The body portion of any tool, up from its cutting edge or bit.

*Shearing.*—Cutting a vertical groove in a coal face or breast. The cutting of a "fast end" of coal.

*Shear Legs.*—A high wooden frame placed over an engine or pumping shaft fitted with small pulleys and rope for lifting heavy weights.

*Shears,* or *Sheers* (English).—Two tall poles, with their feet some distance apart and their tops fastened together, for supporting hoisting tackle.

*Shear Zone.*—Hogback.

*Sheave.*—A wheel with a grooved circumference over which a rope is passed either for the transmission of power or for winding or hauling.

*Sheet Pump.*—See *Sludger.*

*Sheets.*—Coarse cloth curtains or screens for directing the ventilating current underground.

*Shelly.*—A name applied to coal that has been so crushed and fractured that it easily breaks up into small pieces. The term is also applied to a laminated roof that sounds hollow and breaks into thin layers of slate or shale.

*Shet* (Staffordshire).—Fallen roof of coal mine.

*Sheth.*—An old term denoting a district of about eight or nine adjacent bords. Thus, a "sheth of bords," or a "sheth of pillars."

*Shift.*—(1) The number of hours worked without change. (2) A gang or force of workmen employed at one time upon any work, as the day shift, or the night shift.

*Shoading* (Cornish).—Prospecting.

*Shoe.*—(1) A steel or iron guide piece fixed to the ends or sides of cages, to fit or run on the conductors. (2) The lower capping of any post or pile,

to protect its end while driving. (3) A wooden or sheet-iron frame or muff arranged at the bottom of a shaft while sinking through quicksand, to prevent the inflow of sand while inserting the shaft lining.

*Shoot, Chute, Shute.*—An inclined or vertical trough or pipe for conveying materials from a higher to a lower level.

*Shoot.*—To break rock or coal by means of explosives.

*Shooting.*—Blasting in a mine.

*Shore* (English).—A studdle or thrusting stay.

*Shore Up.*—To stay, prop up, or support by braces.

*Shot.*—(1) A charge or blast. (2) The firing of a blast. (3) Injured by a blast.

*Shot Firer.*—See *Shot Lighter.*

*Shot Hole.*—The bore hole in which an explosive substance is placed for blasting.

*Shot Lighter*, or *Shot Firer.*—A man specially appointed by the manager of the mine to fire off every shot in a certain district, if, after he has examined the immediate neighborhood of the shot, he finds it free from gas, and otherwise safe.

*Show.*—When the flame of a safety lamp becomes elongated or unsteady, owing to the presence of firedamp in the air, it is said to show.

*Showing.*—The first appearance of float, indicating the approach to an out-cropping vein or seam. Blossom.

*Shroud.*—A housing or jacket.

*Shute.*—See *Chute, Shoot*, and *Schute.*

*Shutter.*—(1) A movable sliding door, fitted within the outer casing of a Guibal or other closed fan, for regulating the size of the opening from the fan, to suit the ventilation and secure economical working of the machine. (2) A slide covering the opening in a door or brattice, and forming a regulator for the proportionate division of the air current between two or more districts of a mine.

*Siddle.*—Inclination.

*Side.*—(1) The more or less vertical face or wall of coal or goaf forming one side of an underground working place. (2) Rib. (3) A district.

*Side Chain.*—A chain hooked on to the sides of cars running on an incline or along a gangway, to keep the cars together in case the coupling breaks.

*Siding.*—A short piece of track parallel to the main track, to serve as a passing place.

*Siding Over.*—A short road driven in a pillar in a headwise direction.

*Sight.*—(1) A bearing or angle taken with a compass or transit when making a survey. (2) Any established point of a survey.

*Sights.*—Bobs or weighted strings hung from two or more established points in the roof of a room or entry, to give direction to the men driving the entry or room.

*Sill.*—(1) The floor piece of a timber set, or that on which the track rests; the base of any framing or structure. (2) The floor of a seam.

*Sing.*—The noise made by a feeder of gas issuing from the coal.

*Singing Coal.*—Coal from which gas is issuing with a hissing sound.

*Singing Lamp.*—A safety lamp, which, when placed in an atmosphere of explosive gas, gives out a peculiar sound or note, the strength of the note varying in proportion to the percentage of firedamp present.

*Single-Entry System.*—A system of opening a mine by driving a single entry only, in place of a pair of entries. The air current returns along the face of the rooms, which must be kept open.

*Single-Intake Fan.*—A ventilating fan that takes or receives its air upon one side only.

*Single-Rope Haulage.*—A system of underground haulage in which a single rope is used, the empty trip running in by gravity. This is engine-plane haulage.

*Sink.*—To excavate a shaft or slope; to bore or put down a bore hole.

*Sinker.*—A man who works at the bottom of a shaft or face of a slope during the course of sinking.

*Sinker Bar.*—In rope drilling, a heavy bar attached above the jars, to give force to the up stroke, so as to dislodge the bit in the hole.

*Sinking.*—The process of excavating a shaft or slope or boring a hole.

*Siphon.*—A simple, effective, and economical mode of conveying water over a hill whose height is not greater than what the atmospheric pressure will raise the water. Its form is that of an iron pipe, bent like an in-

verted **U**; the vertical height between the surface of the water in the upper basin and the top of the hill is called the lift of the siphon: while the vertical height between the surfaces of the water in the upper and lower basins is called the fall of the siphon.

*Sirdar.*—A foreman.

*Sizing.*—To sort minerals into sizes.

*Skew Back.*—The beveled member from which an arch springs, and upon which it rests.

*Skids.*—Slides upon which heavy bodies are slid from place to place.

*Skip.*—(1) A mine car. (2) A car for hoisting out of a slope. (3) A thin slice taken off from a breast or pillar or rib along its entire length or part of its length.

*Skit* (Cornish).—A pump.

*Slab.*—Split pieces of timber from 2 in. to 3 in. thick, 4 ft. to 6 ft. long, and 7 in. to 14 in. wide, placed behind sets or frames of timber in shafts or levels.

*Slack.*—(1) Fine coal that will pass through the smallest sized screen. The fine coal and dust resulting from the handling of coal, and the disintegration of soft coal. (2) The process by which lignite disintegrates when exposed to the air and weather.

*Slant.*—(1) An underground roadway driven at an angle between the full rise or dip of the seam and the strike or level. (2) Any inclined road in a seam.

*Slant Chutes.*—Chutes driven diagonally across a pillar, to connect a breast manway with a manway chute.

*Slate.*—(1) A hardened clay having a peculiar cleavage. (2) About coal mines, slate is any shale accompanying the coal, also sometimes applied to bony coal.

*Slate Chute.*—(1) A chute for conveying slate or bony coal to a pocket from which it is loaded into "dumpers." (2) A chute driven through slate.

*Slate Picker.*—(1) A man or boy that picks the slate or bone from coal. (2) A mechanical contrivance for separating slate and coal.

*Sleck* (Derbyshire).—Mud in a mine.

*Sled.*—A drag used to convey coal along the face to the road head where it is loaded, or to the chute.

*Sledge.*—A heavy double-handed hammer.

*Sleeper* (English).—The foundation pieces or cross-ties on which rails rest.

*Sleeve.*—A hollow cylinder usually fitting over two pieces, to hold them together.

*Slickensides.*—Polished surfaces of vein walls.

*Slide.*—Loose deposit covering the outcrop of a seam.

*Slides.*—See *Guides.*

*Sliding Scale.*—A mode of regulating the wages paid workingmen by taking as a basis for calculation the market price of coal, the wages rising and falling with the state of trade.

*Sliding Wind Bore* (English).—The bottom pipe or suction piece of a sinking set of pumps having a lining made to slide like a telescope within it, to give length without altering the adjustment of the whole column of pipes.

*Slime, Sludge.*—The pulp or fine mud from a drill hole.

*Slings.*—Pieces of ropes or chains to be put around stones, etc., for raising them.

*Slip.*—(1) A fault. (2) A smooth joint or crack where the strata have moved upon each other.

*Slip Cleavage.*—Microscopic folding and fracture accompanied by slippage; quarrymen's "false cleavage."

*Slit.*—A short heading put through to connect two other headings.

*Slitter.*—See *Pick.*

*Slope.*—A plane or inclined roadway, usually driven in the seam from the surface. A rock slope is a slope driven across the strata, to connect two seams; or a slope opening driven from the surface, to reach a seam below that does not outcrop at an accessible point.

*Sludge.*—See *Slime.*

*Sludger, Sludge Pump.*—A cylinder having an upward opening valve at the bottom, which is lowered into a bore hole, to pump out the sludge or fine rock resulting from drillings.

*Sluice.*—Any overflow channel.

*Sluice Head,* or *Head* (Australia and New Zealand).—A supply of 1 cu. ft. of water per second, regardless of the head, pressure, or size of orifice.

*Small.*—See *Slack*.

*Smift, Snift.*—A bit of touch paper, touch wood, etc., attached by a bit of clay or grease to the outside end of the train of gunpowder when blasting.

*Smut* (Staffordshire).—Soft, bad coal.

*Snore, Snore Piece.*—The hole in the lower part of a sinking or Cornish pump, through which water enters.

*Snub or Snubbing.*—(1) To undercut by means of explosives or otherwise. (2) To lower, as a car, by a turn of a rope around a post.

*Soapstone.*—A term incorrectly applied by the miner to any soft, unctuous rock.

*Socavon* (Mexican).—A mining tunnel; an adit. *Socavon á hilo de veta.*—A drift tunnel. *Socavon crucero.*—A crosscut tunnel or adit.

*Socket.*—(1) The innermost end of a shot hole, not blown away after firing. (2) A wrought-iron contrivance by means of which a wire rope is securely attached to a chain or block.

*Sole, Sole Plate.*—A piece of timber set underneath a prop.

*Sollar.*—(1) A wooden platform fixed in a shaft, for the ladders to rest on. (2) A division of the air compartment in a drift or slope.

*Sondear* (Mexican).—To bore for prospecting purposes.

*Sondeo* (Mexican).—A boring for prospecting purposes.

*Soplete* (Mexican).—A blowpipe.

*Sorting.*—Separating valuable from worthless material.

*Sounding.*—(1) Knocking on a roof to see whether it is sound or safe to work under. (2) Rapping on a pillar so that a person on the other side of it may be signaled to, or to enable him to estimate its width.

*Sow.*—(1) A tool used for sharpening drills. (2) Iron deposits at the bottom of furnaces.

*Spad.*—A horseshoe nail with a hole in the head, or a similar device for driving into the mine timbers, or into a wooden plug fitted into the roof, to mark a survey station.

*Spall.*—To break up rocks with a large hammer, for hand sorting.

*Spalls.*—The chips and other waste material cut from a block of stone in process of dressing.

*Spar.*—A name given to certain white quartz-like minerals, e.g., calcspar, feldspar, fluorspar.

*Spears.*—Pump-rods.

*Specimen.*—A picked piece of mineral.

*Spelter.*—The commercial name for zinc.

*Spent Shot.*—A blast hole that has been fired, but has not done its work.

*Spiders.*—See *Drum Rings*.

*Spiegeleisen.*—Manganiferous white cast iron.

*Spiking Curbs.*—A light ring of wood to which planks are spiked when plank tubbing is used.

*Spiles* (Cornish).—A temporary lagging driven ahead on levels in loose ground. Short pieces of planking sharpened flatways, and used for driving into watery strata as sheath piling. to assist in checking the flow; used much in sinking through quicksands.

*Spiling.*—A process of timbering through soft ground.

*Spiral.*—A spiral coal chute which mechanically separates the slate from the coal.

*Spiral Drum.*—See *Conical Drum*.

*Splint, or Splent.*—A hard, high volatile coal, producing a white ash, intermediate between cannel and bituminous coal.

*Split.*—(1) To divide an air current into two or more separate currents. (2) Any division or branch of the ventilating current. (3) The workings ventilated by that branch. (4) Any member of a coal bed split by thick partings into two or more seams. (5) A bench separated by a considerable interval from the other benches of a coal bed.

*Spoil.*—Débris from a coal mine.

*Spoon.*—A slender iron rod with a cup-shaped projection at right angles to the rod, used for scraping drillings out of a bore hole.

*Spout.*—A short underground passage connecting a main road with an aircourse.

*Sprag.*—(1) A short wooden prop set in a slanting position for keeping up the coal during the operation of holing. (2) A short round piece of hard wood, pointed at both ends, to act as a brake when placed between the spokes of mine-car wheels. (3) The horizontal member of a square set of timber running longitudinally with the deposit.

*Spragger.*—One who attends to the spragging of cars.

*Sprag Road.*—A mine road having such a sharp grade that sprags are needed to control the speed of the cars.

*Spreader.*—A timber stretched across a shaft or stope.

*Spring Beams.*—Two short parallel timber beams, built with a Cornish pumping engine house, nearly on a level with the engine beam, for catching the beam, etc., and preventing a smash in case of a breakdown.

*Spring Latch.*—The latch or tongue of an automatic switch, operated by a spring pole at the side of the track.

*Spring Pole.*—An elastic wooden pole from which boring rods are suspended. Used also to operate a spring latch.

*Sprocket Wheel* (English).—Rag wheel. A wheel with teeth or pins which engage the links of a chain.

*Spur.*—(1) A short ridge or offsetting pointed branch from a main ridge or mountain. (2) A short branch or feeder from the main lode of a vein. (3) A branch road.

*Square Set.*—A variety of timbering for large excavations.

*Squealer.*—A shot which breaks the coal only enough to allow the gases of detonition to escape with a whistling or squealing sound; also called a whistler.

*Squeeze.*—See *Creep.*

*Squib.*—A straw, rush, paper, or quill tube filled with a priming of gunpowder, with a slow match on one end.

*Stage.*—A platform on which mine cars stand.

*Stage Pumping.*—Draining a mine by means of two or more pumps placed at different levels, each of which raises the water to the next pump above, or to the surface.

*Stage Working.*—A system of working minerals by removing the strata above the beds, after which the various beds are removed in steps or stages.

*Staging.*—A temporary flooring or scaffold, or platform.

*Stalactites.*—Icicle-shaped formations of mineral matter depending from roof strata.

*Stalagmites.*—Accumulations of mineral matter that form on the floor, caused by the continual dripping of water impregnated with mineral matter.

*Stall.*—A narrow breast, or chamber.

*Stall Gate.*—A road along which the mineral worked in a stall is conveyed to the main road.

*Stanchion.*—A vertical prop or strut.

*Standage.*—Pump reservoir.

*Standing.*—Not at work, not going forwards, idle.

*Standing Gas.*—A body of firedamp known to exist in a mine, but not in circulation; sometimes fenced off.

*Standing Sett* (English).—A fixed lift of pumps in a sinking set.

*Staple.*—(1) A shallow pit within a mine. (2) An underground shaft.

*Starter.*—A man who ascends a chute to the battery and starts the coal to running.

*Starved* (English).—When a pump is choked at the brass holes.

*Station.*—A flat or convenient resting place in a shaft or level.

*Stave.*—A ladder step.

*Stay* (English).—Props, struts, or ties for keeping anything in its place.

*Steamboat Coal.*—In anthracite only, coal small enough to pass through bars set 6 to 8 in. apart, but too large to pass through bars from 3½ to 5 in. Comparatively few collieries make steamboat coal except to fill special contracts or orders.

*Steam Coal.*—A hard, free-burning, non-caking coal.

*Steam Jet.*—A system of ventilating a mine by means of a number of jets of steam, at high pressure, kept constantly blowing off from a series of pipes in the bottom of the upcast shaft.

*Steel Mill.*—An apparatus for obtaining light in a fiery mine. It consisted of a revolving steel wheel, to which a piece of flint was held, to produce sparking.

*Steel Needle.*—An instrument used in preparing blasting holes, before the safety fuse was invented.

*Steening,* or *Steining.*—The brick or stone lining of a shaft.

*Stemmer.*—A copper or wooden bar used for stemming.

*Stemming.*—(1) Fine shale or dirt put into a shot hole after the powder, and rammed hard. (2) Tamping a shot.

*Step* (English).—(1) The cavity in a piece for receiving the pivot of an

upright shaft, **or** the end of an upright piece. (2) The shearing in a coal face.

*Stint.*—The amount of work to be done by a man in a specified time.

*Stitch.*—To fasten a timber by toe nailing.

*Stobb.*—A long steel wedge used in bringing down coal after it has been holed.

*Stomp.*—A short wooden plug fixed in the roof of a level, to serve as a bench mark for timbering.

*Stone Coal.*—Anthracite; also other hard varieties of coal.

*Stone Head.*—A heading or gangway driven in stone. A tunnel.

*Stone Tubbing.*—Water-tight stone walling of a shaft cemented at the back.

*Stook.*—A pillar of coal about 4 yd. square, being the last portion of a full-sized pillar to be worked away in bord-and-pillar workings.

*Stook-and-Feather.*—A wedge for breaking down coal, worked by hydraulic power, the pressure being applied at the extreme inner end of the drilled hole.

*Stoop.*—A pillar of coal.

*Stoop-and-Room.*—A system of working coal very similar to pillar-and-stall.

*Stop.*—Any cleat or beam to check the descent of a cage, car, pump rods, etc.

*Stope.*—(1) To excavate mineral in a series of steps. (2) A place in a mine that is worked by stoping.

*Stoping.*—Working out mineral between two levels or on the surface, by stopes or steps. *Stoping Overhand.*—Mining a stope upwards, the flight of steps being inverted. *Stoping Underhand.*—Mining a stope downwards in such a series that it presents the appearance of a flight of steps.

*Stopping.*—An air-tight wall built across any passageway in a mine.

*Stove Coal.*—In anthracite only; two sizes of stove coal are made, large and small: large stove, known as No. 3, passes through a $2\frac{1}{2}$-in. to 2-in. mesh and over a $1\frac{1}{2}$-in. to $1\frac{1}{2}$-in. mesh; small stove, known as No. 4, passes through a $1\frac{1}{2}$-in. to $1\frac{1}{2}$-in. mesh and over a $1\frac{1}{8}$-in. to 1-in. mesh. Only one size of stove coal is now usually made. It passes through a 2-in. square mesh and over $1\frac{1}{8}$-in. square mesh.

*Stove Up,* or *Stoved.*—Upset. When a rod of iron heated at one end is hammered endwise the diameter of that end is enlarged, and it is said to be upset or stove up.

*Stow.*—To pack away rubbish into goaves or old workings.

*Stowce.*—(1) Windlass. (2) Landmarks.

*Stowing.*—The débris of a vein thrown back of a miner and which supports the roof or hanging wall of the excavation.

*Straight Ends and Walls.*—A system of working coal somewhat similar to bord-and-pillar. Straight ends are headings from 4 ft. 6 in. to 6 ft. in width. Walls are pillars 30 ft. wide.

*Straight Work.*—A system of getting coal by headings or narrow work.

*Strake.*—A slightly inclined table for separating heavier minerals from lighter ones.

*Stratification.*—Arrangement in layers.

*Stratum* (plural, *strata*).—A layer or bed of rocks, or other deposit.

*Streak.*—The color of the mark made when a mineral is scratched against a white surface.

*Strett.*—The system of getting coal by headings or narrow work. See *Bord-and-Pillar.*

*Strike* (of a seam or vein).—The intersection of an inclined seam or a vein with a horizontal plane. A level course in the seam. The direction of strike is always at right angles to the direction of the dip of the seam.

*Strike Joints.*—Joints or cleavages that are parallel to the strike of the seam.

*Striking Deal.*—Planks fixed in a sloping direction just within the mouth of a shaft, to guide the tub to the surface.

*Stringer* (English).—Any longitudinal timber or beam.

*Stringpump.*—A system of pumping whereby the motion of the engine is transmitted to the pump by timbers or stringers bolted together.

*String Rods.*—A line of surface rods connected rigidly for the transmission of power; used for operating small pumps in adjoining shafts from a central station.

*Strip.*—(1) To remove the overlying strata of a bed or vein. (2) Mining a deposit by first taking off the overlying material.

*Strut* (English).—A prop to sustain compression, whether vertical or inclined.

*Struve Ventilator.*—A pneumatic ventilating apparatus consisting of two vessel-like gas holders, which are moved up and down in a tank of water. By this means, the air is sucked out of the mine as required.

*Studdle.*—A piece of squared timber placed vertically between two sets of timber in a shaft.

*Stull.*—A post for supporting the wall or roof in a mine; a prop timber.

*Stump.*—The pillar between the gangway and each room turned off the gangway. Sometimes the entry pillars are called stumps.

*Stumping.*—A kind of pillar-and-stall plan of getting coal.

*Stup.*—Powdered coke or coal mixed with clay.

*Sturt.*—A tribute bargain profitable to the miner.

*Stuttle*, or *Sprag.*—The horizontal member of a square set of timber running longitudinally with the deposit.

*Stythe.*—Carbonic-acid gas (blackdamp).

*Sucker Rod.*—The pump rod of an oil or artesian well.

*Suction Pump* (English).—A pump wherein, by the movement of the piston, water is drawn up into the vacuum caused.

*Sulphur.*—(1) One of the elements. (2) Iron pyrites.

*Sulphuret.*—See *Sulphide.*

*Sulphide.*—A combination of sulphur and a base.

*Sump*, or *Sumpi.*—A catch basin into which the drainage of a mine flows and from which it is pumped to the surface.

*Sumping*, or *Sumping Cut.*—Forcing the cutter bar of a coal cutter into or under the coal.

*Surface Deposits.*—Those that are exposed and can be mined from the surface.

*Swab Stick.*—A short wooden rod, bruised into a kind of stumpy brush at one end, for cleaning out a drill hole.

*Swally*, or *Swelly.*—A trough, or syncline, in a coal seam.

*Swamp.*—A depression or natural hollow in a seam, a basin.

*Sweeping Table.*—A stationary buddle.

*Sweet.*—Free from deleterious gases.

*Swing.*—The arc or curve described by the point of an instrument, such as a pick or hammer, when being used.

*Switch.*—(1) The movable tongue or rail by which a train is diverted from one track to another. (2) The junction of two tracks. (3) An apparatus for changing the course of or interrupting an electrical current.

*Switchboard.*—A board where several electrical wires terminate, and where, by means of switches, connection may be established between any of these wires and the main wire.

*Synclinal Axis.*—The line or course of a syncline.

*Syncline.*—The point or axis of a basin toward which the strata upon either side dip. An inverted anticline. A basin.

*Systematic Timbering.*—Placing mine timbers according to a predetermined plan, regardless of roof conditions.

**Tackle** (English).—(1) Ropes, chain, detaching hooks, cages, and all other apparatus for raising coal or ore in shafts. (2) Any rope for hoisting, as a tackle rope, block and tackle, etc.

*Tail-Back.*—When the firedamp ignites and the flame is elongated or creeps backwards against the current of air, it is said to tail-back.

*Tailing.*—The blossom; the outcrop or smut.

*Tailings.*—The refuse from a jig.

*Tail-Pipe.*—The suction pipe of a pump.

*Tailrace.*—The channel along which water flows after it has done its work.

*Tail-Rope.*—(1) In a tail-rope system of haulage, the rope that is used to draw the empties back into the mine. (2) A wire rope attached beneath cages, as a balance.

*Tail-Rope System of Haulage.*—A haulage system in which the full trip is drawn out by the main rope and the empty trip is drawn in by the tail-rope, these ropes being attached to the opposite ends of the trip.

*Tail-Sheave.*—The sheave at the inbye end of any haulage system. See *Turn Pulley.*

*Take the Air.*—(1) To measure the ventilating current. (2) Applied to a ventilating fan as working well, or working poorly.

*Taladro* (Mexican).—A drill for mechanical or mining purposes. *Taladrar.*—To bore or drill.

*Tally.*—(1) A mark or number placed by the miner on every car of coal sent out of his place, usually a tin ticket. By counting these, a tally is made of all the cars of coal he sends out. (2) Any numbering, or counting, or memorandum, as a tally sheet.

*Tamp.*—To fill a bore hole, after inserting the charge, with some substance

which is rammed hard as it is put into the hole. Vertical holes are often tamped with water, when blasting with dynamite.

*Tamping.*—The process of stemming or filling a bore hole.

*Tamping Bar.*—A copper-tipped bar, for ramming the tamping or stemming.

*Tanates* (Mexican).—Leather, hide, or jute bags, to carry ore or waste rock within or out of a mine. *Tanatero.*—A laborer or bag carrier.

*Tap.*—(1) To cut or bore into old workings, for the purpose of liberating accumulations of gas or water. (2) To pierce or open any gas or water feeder. (3) To win coal in a new district.

*Tapextle* (Mexican).—A working platform or stage built up in a stope or anywhere in a mine; a landing place between two flights of ladders.

*Teem.*—To pour or tip.

*Teeming Trough.*—A trough into which the water from a mine is pumped.

*Telegraph.*—A sheet-iron trough-shaped chute, for conveying coal or slate from the screens to the pockets, or boilers.

*Temper.*—(1) To change the hardness of metals by first heating and then plunging them into water, oil, etc. (2) To mix mortar, or to prepare clay for bricks, etc.

*Tempering.*—The act of reheating and properly cooling a bar of metal to any desired degree of hardness.

*Temper Screw.*—In rope drilling, a screw for gradually lowering the clamped (upper) end of the rope as the hole is deepened.

*Tenon.*—A projecting tongue fitting into a corresponding cavity called a mortise.

*Tequio* (Mexican).—A task set for a drillman or for any laborer in a mine, to be regarded as a day's work.

*Terrace.*—A raised level bank, such as river terraces, lake terraces, etc.

*Terrero* (Mexican).—The dump of a mine.

*Test.*—A trial of an engine, fan, or other appliance or substance.

*Theodolite.*—An instrument used in surveying, for taking both vertical and horizontal angular measurements. An engineer's large transit, with attachments.

*Thill.*—See *Floor.*

*Thimble.*—(1) A short piece of tube slid over another piece, to strengthen a joint, etc. (2) An iron ring with a groove around it on the outside, used as an eye when a rope is doubled about it.

*Thirl.*—See *Crosscut.*

*Through-and-Through.*—A system of getting bituminous coal, without regard to the size of the lump.

*Throw.*—(1) A fault of dislocation. (2) The vertical distance between the two ends of a faulted bed of coal.

*Thrown.*—Faulted; broken by a fault.

*Thrust.*—Creep or squeeze due to excessive weight, hard floor, and too small pillars.

*Thurl* (Staffordshire).—To cut through from one working into another.

*Tie-Back.*—(1) A beam serving a purpose similar to a fend-off beam, but fixed at the opposite side of the shaft or inclined road. (2) The wire ropes or stayrods which are sometimes used on the side of the tower opposite the hoisting engine, in place of or to reenforce the engine braces.

*Tiff.*—Calcite or carbonate of lime.

*Timber.*—(1) Props, bars, collars, legs, laggings, etc. (2) To set or place timber in a mine or shaft.

*Timberer, Timberman.*—A man who sets timber.

*Time.*—(1) Hours of work performed by workmen. (2) To count the strokes of a pump or revolutions of an engine or fan.

*Tin-Can Safety Lamp.*—A Davy lamp placed inside a tin can or cylinder having a glass in front, air holes near the bottom, and open-topped, making the lamp safer in a rapid current of air.

*Tip.*—A dump. See *Tipper*, or *Tipple.*

*Tipper, Tipple*, or *Tippler.*—An apparatus for emptying cars of coal or ore, by tipping or turning them upside down, and then bringing them back to original position, with a minimum of manual labor.

*Tipple.*—The dump trestle and tracks at the mouth of a shaft or slope, where the output of a mine is dumped, screened, and loaded.

*Tiro* (Mexican).—A mining shaft. *Tiro Vertical.*—A vertical shaft.

*Token.*—(1) A piece of leather or metal stamped with the hewer's or putter's number or distinctive mark, and fastened to the tub he is filling or putting.

**Ton.**—A measure of weight. Long ton is 2,240 lb.; short ton is 2,000 lb.; metric ton is 1,000 kilograms = 2,204.6 lb.

**Top.**—(1) See *Roof.* (2) Top of a shaft; surface over a mine.

**Topit.**—A kind of brace head screwed to the top of boring rods, when withdrawing them from the hole.

**Topping.**—The coal on a car above the top of the car box.

**Track.**—Railways or tramways.

**Tracking.**—Wooden rails.

**Train Boy.**—A boy that rides on a trip, to attend to rope attachments, signal in case of derailment of cars, etc. Trip rider.

**Train,** or **Trip.**—The cars taken at one time by mules, or by any motor, or run at one time on a slope, plane, or sprag road, always together.

**Tram.**—A mine car, or the track on which it runs.

**Trammer.**—One who pushes cars along the track.

**Tramroad.**—A mine track or railroad.

**Tram Rope.**—A hauling rope, to which the cars are attached by a clip or chain, either singly or in trips.

**Tramway.**—A small, roughly constructed track for running wagons or trucks on.

**Transfer Carriage.**—Movable platform or truck used to transfer mine cars from one track to another.

**Transome** (English).—A heavy wooden bed or supporting piece.

**Trap.**—(1) A steep heading along which men travel. (2) A fault of dislocation. (3) An eruptive rock. (4) A dangerous place. (5) To tend door.

**Trap Dike.**—A fault (not necessarily accompanied by displacement of strata) in which the spaces between the fractured edges of the beds are filled up by a thick wall of igneous rock.

**Trap Door.**—A small door, kept locked, fixed in a stoping, for giving access to firemen and certain others to the return airways, dams, or other unused portions of the mine.

**Trapper.**—A boy employed underground to tend doors.

**Traveling Road.**—An underground passage or way used expressly, though not always exclusively, for men to travel along to and from their working places.

**Treenail.**—A long wooden pin for securing planks or beams together.

**Trend.**—The course of a vein, fault, or other feature.

**Tribute.**—A method of working mines by contract, whereby the miners receive a certain share of the products won. *Tributers.*—Miners paid by results.

**Trig.**—A sprag used to block or stop a wheel or any machinery.

**Trip.**—The mine cars in one train or set. See *Train.*

**Triple-Entry System.**—A system of opening a mine by driving three parallel entries for the main entries.

**Triturate.**—To grind or pulverize.

**Trolley.**—(1) A small four-wheeled truck, used for carrying the ore bucket underground. (2) An electric locomotive. (3) The arm of a locomotive or other machine that conducts the electric current from the wire above the track to the machine.

**Trompe.**—An apparatus for producing ventilation by the fall of water down a shaft.

**Trouble.**—A dislocation or fault; any irregularity in the bed.

**Trough Fault.**—A wedge-shaped fault, or, more correctly, a mass of rock, coal, etc., let down in between two faults, which faults, however, are not necessarily of equal throw.

**Troughs,** or **Thirling.**—A passage cut through a pillar to connect two rooms.

**Truck.**—Used synonymously with *Barney.*

**Truck System.**—Paying miners in food instead of money.

**Trunnions.**—Cylindrical projections or journals, attached to the sides of a vessel, so that it can rotate in a vertical plane.

**Trying the Lamp.**—The examination of the flame of a safety lamp for the purpose of forming a judgment as to the quantity of firedamp mixed with the air.

**Tub.**—(1) A mine car. (2) An iron or wooden barrel used in a shaft, for hoisting material.

**Tubbing.**—Cast iron, and sometimes timber, lining or walling of a circular shaft.

**Tubbing Wedges.**—Small wooden wedges hammered between the joints of tubbing plates.

*Tubing.*—Iron pipes or tubes used for lining bore holes, to prevent caving.

*Tunnel.*—A horizontal passage driven across the measures and open to day at both ends; applied also to such passages open to day at only one end, or not open to day at either end.

*Turbary.*—A peat bog.

*Turbine.*—(1) A rapidly revolving waterwheel, impelled by the pressure of water upon blades. (2) A similar type of power generator propelled by steam or air.

*Turn.*—(1) The hours during which coal, etc., is being raised from the mine. (2) See *Shift*. (3) To open rooms, headings, or chutes off from an entry or gangway. (4) The number of cars allowed each miner.

*Turnout.*—A siding or passing on any tram or haulage road.

*Turn Pulley.*—A sheave fixed at the inside end of an endless- or tail-rope haulage plane, around which the rope returns. See *Tail-Sheave*.

*Turntable.*—A revolving platform on which cars or locomotives are turned around.

*Tut Work.*—Breaking ground at so much per foot or fathom.

*Tuyère.*—The tubes through which air is forced into a furnace.

*Two-Throw.*—When, in sinking, a depth of about 12 ft. has been reached, and the débris has to be raised to the surface by two lifts or throws with the shovel, one man working on staging above another.

*Unconformability.*—When one layer of rock, resting on another layer, does not correspond in its angle of bedding.

*Undercast.*—An air-course carried under another air-course or roadway.

*Underclay.*—A bed of fireclay or other less clayey stratum, lying immediately beneath a seam of coal.

*Undercut.*—To remove a small portion of the bottom of the bed or the underclay, so that the mass of coal or mineral can be wedged or blasted down.

*Underhand Stoping.*—See *Stoping Underhand*.

*Underhand Work.*—Picking or drilling downwards.

*Underholing, Undermining.*—To mine out a portion of the bottom of a seam or the underclay, by pick or powder, thus leaving the top unsupported and ready to be blown down by shots, broken down by wedges, or mined with a pick or bar.

*Underlie,* or *Underlay.*—The inclination of a stratum at right angles to its course or strike; the true dip.

*Underviewer,* or *Underlooker.*—An inside foreman.

*Unwater.*—To drain or pump the water from a mine or shaft.

*Upcast.*—The shaft through which the return air ascends.

*Upraise.*—An auxiliary shaft, a mill hole, or heading carried from one level up toward another.

*Upthrow.*—A fault in which the displacement has been upward.

*Vapor* (Mexican).—Steam; heated and stinking gas sometimes found in mines, which causes candles to burn dimly and go out.

*Vein.*—See *Lode*. Often applied incorrectly to a seam or bed of coal or other mineral.

*Vena* (Mexican).—A thin vein, not over 3 in. thick—a knife-blade vein.

*Vend* (North of England).—Total sales of coal from a mine.

*Vent,* or *Vent Hole.*—(1) A small passage made with a needle through the tamping, which is used for admitting a squib, to enable the charge to be lighted. (2) Any opening made into a confined space.

*Ventilating Column.*—See *Motive Column*.

*Ventilating Pressure.*—The total pressure or force required to overcome the friction of the air in mines; the unit of ventilating pressure or pressure per sq. ft. of area multiplied by the area of the airway.

*Ventilation.*—Circulation. The atmospheric air circulating in a mine.

*Ventilator.*—Any means or apparatus for producing a current of air in a mine or other airway.

*Vestry* (North of England).—A refuse.

*Viewer.*—The general manager or mining engineer of one or more collieries, who has control of the whole of the underground works, and also generally of those on the surface.

*Vug,* or *Vugh* (Cornish).—A cavity in the rock.

*Wagon.*—A mine car.

*Wagon Breast.*—A breast in which the mine cars are taken up to the working face.

*Wailing.*—Picking stones and dirt from among coals.

*Wale* (North of England).—Hand-dressing coal.

*Walking Beam.*—See *Working Beam*.

*Wall.*—(1) The face of a longwall working or breast.   (2) A rib of solid coal between two breasts.   (3) A crosscut driven between bords.

*Walling.*—See *Steening*.

*Walling Cribs.*—Oak cribs or curbs upon which walling is built.

*Walling Stage.*—A movable wooden scaffold suspended from a crab on the surface, upon which the workmen stand when walling or lining a shaft.

*Wall Plates.*—The two longest pieces of timber in a set used in a rectangular shaft.

*Warners.*—Apparatus consisting of a variety of delicately constructed machines, actuated by chemical, physical, electrical, and mechanical properties, for indicating the presence of small quantities of firedamp in the mines.   At present, most of these ingenious contrivances are more suited to the laboratory than for practical application underground.

*Warning Lamp.*—A safety lamp fitted with certain delicate apparatus, for indicating very small proportions of firedamp in the atmosphere of a mine.   As small a quantity as 3 % can be determined by this means.

*Wash.*—Drift, clay, stones, etc., overlying the strata.

*Washer.*—A jig.

*Wash Fault.*—A portion of a seam of coal replaced by shale or sandstone.

*Washing Apparatus*, or *Washery.*—(1) Machinery and appliances erected on the surface at a colliery, often in connection with coke ovens, for extracting, by washing with water, the impurities mixed with the coal dust or small slack.   (2) Machinery for removing impurities from small sizes of anthracite coal.

*Washout.*—The erosion of an appreciable extent of a coal seam by aqueous agency.

*Waste.*—(1) See *Goaf*.   (2) Very small coal or slack.   (3) The portion of a mine occupied by the return airways.   (4) Also used to denote the spaces between the pack walls in the gob of longwall working.   (5) Refuse material.

*Waste Gate* (English).—A door for regulating discharge of surplus water.

*Water Blast.*—The sudden escape of air pent up in rise workings, under considerable pressure from a head of water that has accumulated in a connecting shaft.

*Water Cartridge.*—A waterproof cartridge surrounded by an outer case.   The space between being filled with water, which is employed to destroy the flame produced when the shot is fired, thereby lessening the chance of an explosion should gas be present in the place.

*Water Gauge.*—An instrument for measuring the pressure per square foot producing ventilation in a mine.

*Water Hammer.*—The hammering noise caused by the intermittent escape of gas through water in pipes.

*Water-Jacket.*—A jacket filled with water, to keep cool a cylinder or furnace.

*Water Level.*—An underground passage or heading driven very nearly dead level or with sufficient grade only to drain off the water.

*Water Right.*—The privilege of taking a certain quantity of water from a watercourse.

*Watershed.*—The elevated land or ridge that divides drainage areas.

*Waterwheel* (English).—Overshot, undershot, breast wheels, etc.   A wheel provided with buckets, which is set in motion by the weight or impact of a stream of water.

*Weather.*—To crumble by exposure to the atmosphere.

*Weather Door.*—See *Trap Door*.

*Web.*—The face of a longwall stall in course of being holed and broken down for removal.   The length of breast or face brought down by one mining.

*Wedging.*—The material, moss or wood, used to render the shaft lining tight.

*Wedging Crib.*—A curb or crib of wood or cast iron wedged tightly in place and packed, in order to form a water-tight joint and upon which tubbing is built.

*Wedging Down.*—Breaking down the coal at the face with hammers and wedges instead of by blasting.

*Weigh Bridge* (English).—A platform large enough to carry a wagon, resting on a series of levers, by means of which heavy bodies are weighed.

*Weize.*—A band or ring of spun yarn, rope, rubber, lead, etc., put in between the flanges of pipes before bolting them together, in order to make a water-tight joint; a gasket.

*Well.*—A sump, or a branch from the sump.

*Whim.*—A winding drum worked by a horse.

*Whim Shaft.*—A shaft through which coal, ore, water, etc., are raised from a mine by means of a whim.

*Whin.*—A hard, compact rock.

*Whin Dike.*—A fault or fissure filled with whin and the débris of other rocks, sometimes accompanied by a dislocation of the strata.

*Whip.*—A hoisting appliance consisting of a pulley supporting the hoisting rope to which the horse is directly attached.

*Whitedamp.*—Carbonic oxide ($CO$). A gas found in coal mines, generally where ventilation is slack. A product of slow combustion in a limited supply of air. It burns and will support combustion. It is extremely poisonous.

*Whole Working.*—The first working of a seam, which divides it into pillars.

*Wild Rock.*—Any rock not fit for commercial slate.

*Win.*—To sink a shaft or slope, or drive a drift to a workable seam of mineral in such a manner as to permit its being successfully worked.

*Winch*, or *Windlass.*—A hoisting machine consisting of a horizontal drum operated by crank-arm and manual labor.

*Wind Bore* (England).—The bottom or suction pipe of a lift of pumps, which has suitable brass holes or perforations for suction of water or air.

*Wind Gauge.*—An anemometer for testing the velocity of air in mines.

*Winding.*—The operation of raising or hauling the product of a mine by means of an engine and ropes.

*Winding Engines.*—Hoisting or haulage engines.

*Wind Method.*—A system of separating coal into various sizes, and extracting the dirt from it, which in principle depends on the specific gravity or size of the coal and the strength of the current of air.

*Wind Sail.*—The top part of canvas piping, which is used for conveying air down shallow shafts.

*Wing Bore.*—A side or flank bore hole.

*Wings.*—See *Rests* and *Keeps.*

*Winning.*—A sinking shaft, a new coal, ironstone, clay, shale, or other mine of stratified material. A working place in a mine.

*Winze.*—Interior shaft connecting levels, sometimes used as an ore chute.

*Won.*—Proved, sunk to, and tested.

*Work.*—(1) To mine. (2) Applied to mine working when affected by squeeze or creep.

*Workable.*—Any seam that can be profitably mined.

*Worked Out.*—When all available mineral has been extracted from a mine it is worked out.

*Working.*—Applied to mine workings when squeezing.

*Working Barrel.*—The water cylinder of a pump.

*Working Beam* (English).—A beam having a vertical motion on a rock shaft at its center, one end being connected with the piston rod and the other with a crank or pump rod, etc.

*Working Cost.*—The total cost of producing the mineral.

*Working Face.*—See *Face.*

*Working Home.*—Getting or working out a seam of coal, etc., from the boundary or far end of the mine toward the shaft bottom.

*Working on Air.*—A pump works on air when air is sucked up with the water.

*Working Out.*—Working outwards or in the direction of the boundaries of the colliery.

*Working Place.*—The actual place in a mine at which the coal is being mined.

*Workings.*—The openings of a colliery, including all roads, ways, levels, dips, airways, etc.

*Wrought Iron.*—Iron in its minimum state of carburization.

*Wythern* (Wales).—Lode.

**Xacal** (Mexican).—A miner's cabin; a storehouse for mining goods; a shaft house.

**Yardage, Yard Work.**—Price paid per yard for cutting coal.

*Yard Price.*—Various prices, per yard driven (in addition to the tonnage prices), paid for roads of certain widths and driven in certain directions.

*Yield.*—The proportion of a seam sent to market.

**Zone.**—In coal-mining phraseology this word means a certain series of coal seams with their accompanying shales, etc., which contain, for example, much firedamp, called a fiery zone, or, if much water, a watery zone.

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